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Some Interoperability Issues for Computer-Generated Forces: The Theoretical Chasm between Entity-Level and Aggregated-Force Combat Simulations

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ABSTRACT: *This paper presents two important interoperability issues for computer-generated forces. The first is fundamental and concerns the inconsistent representation of interfiring times of direct-fire weapons in ground combat in entity-level (i.e. discrete-event) and aggregated-force (with attrition modeled by Lanchester-type equations) combat simulations. The second is behavioral and again concerns inconsistent representation of platform-level command and control in such combat in entity-level and aggregated-force simulations. This second inconsistency concerns combat-system behavior in acquiring and attacking (i.e. firing at) enemy systems, i.e. whether or not new targets can be acquired while an enemy target is being engaged. Computational evidence of the seriousness of the consequences of such inconsistencies on simulation output is presented. This paper shows how they can be avoided by appropriate mathematical modeling of Lanchester attrition-rate coefficients in aggregated-force combat simulations. Without such modeling, however, fundamental inconsistencies (with significant consequences) currently exist between all entity-level and aggregated-force combat simulations.*

1. Introduction

Modeling and simulation (M&S) is widely used in DoD for a variety of purposes. Such models basically come in two different varieties: entity-level simulations (e.g. Janus (see references [1], [2], [3]), CASTFOREM (see references [4], [5], [6]), JCATS) and aggregated-force simulations (VIC, EAGLE, JWARS). For a variety of reasons one may want to play computer-generated forces in an entity-level simulation with an aggregated-force model, or vice versa. This paper points out that there are inherent inconsistencies between the play of an entity-level simulation and an aggregated-force simulation for direct-fire weapons in ground combat. The two areas of such inconsistencies, pointed out for the first time here, are:

(1) representation of interfiring times of direct-fire weapons in ground combat,
(2) acquisition of new targets while a target is being engaged by a particular firer.

Preliminary computational experience shows that such inconsistencies can lead to an order of magnitude difference in kill rates achieved in such simulations. Hence, major interoperability problems can exist between an entity-level simulation and an aggregated-force one for direct-fire combat under these circumstances.

2. Background

Essentially all aggregated-force ground-combat models used by the U.S. Army and ground-component models used in DoD campaign models are based on some variety of Lanchester-type attrition paradigm (see reference [7]). The practical use of such differential-equation-based models depends critically on one's ability to obtain realistic values for the Lanchester attrition-rate coefficients. Such a coefficient denotes the rate at which an individual weapon-system type kills enemy targets of a particular type. Two approaches for determining numerical values for them are (Section 5.1 of reference [7])

(1) the freestanding-analytical-model approach (which generates these values from an analytical submodel, not dependent on the running of any entity-level simulation),
(2) the hierarchy-of-models approach (which estimates parameter values from the output of an entity-level Monte-Carlo combat simulation).

The paper at hand considers only the first of these.

The freestanding-analytical-model approach conceptually consists of considering a single typical firer and then computing the rate at which this firer type kills a particular enemy target type according to a micro-combat model. Very few people are aware of this conceptual basis. If interoperability is desired for computer-generated forces, this model must be in consonance with any entity-level simulation with which the aggregated-force model must interface. Significant mathematical modeling is involved in determining an analytical expression for this single-weapon-system-type kill rate (Lanchester attrition-rate coefficient). These attrition-rate coefficients are then used in a deterministic differential-equation model (implemented as a system of difference equations) to assess outcomes of engagements between military units at the force-on-force level of detail.

On the other hand, entity-level simulations are basically discrete-event simulations that employ Monte-Carlo methods for determining the outcomes of all random events (e.g. see references [8] and [9]). Although each of these two approaches (i.e. entity-level and aggregated-force approaches) assess battle outcomes in fundamentally different ways, from the standpoint of simulation interoperability it is essential that their basic conceptualizations of the combat process are basically the same.

3. Playing Attrition in Combat Simulations

Entity-level and aggregated-force (i.e. Lanchester-type) simulations play attrition in fundamentally different ways. A thumbnail sketch of each of these will now be given. However, underlying both approaches is a similar conceptualization of the attrition process for a firer engaging an enemy target. They each must consider the same factors, namely

(1) terrain effects,
(2) target acquisition,
(3) target-engagement policy (rules of engagement),
(4) target-weapon-pair firing process,
(5) target-projectile-pair vulnerability/lethality effects.

Some type of target-priority list is always involved in playing rules of engagement. Both types of simulations also have to consider how a firer behaves over time. Figure 3.1 depicts such a conceptualization for aggregated forces (see reference [10]). Because line of sight (LOS) is played statistically in such an aggregated-force model, its determination does not appear explicitly in the target-engagement cycle.

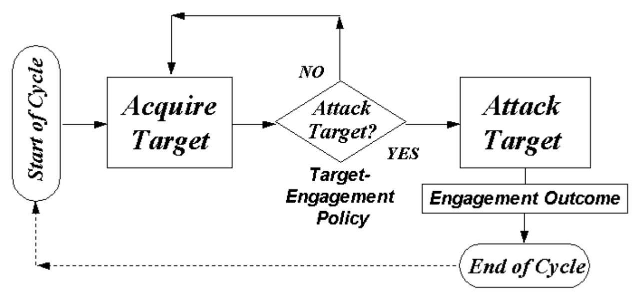

Figure 3.1: Target-engagement cycle (serial acquisition of targets).

3.1. Attrition in Entity-Level Simulations

In an entity-level simulation various algorithms (e.g. see [11]) related to the factors given above are woven together by some type of event-based, time-preserving, sequencing scheme. Moreover, the reader should realize that in entity-level simulations additional events and processes not reflected in Figure 3.1 are also emulated (e.g. misidentification, false targets, battle-damage assessment). However, these are not important for the purposes of the paper at hand.

In such simulations, the outcomes of all random events are determined by Monte-Carlo methods, although not every event need be random. For example, the LOS process is played by explicitly determining whether or not intervisibility exists between each pair of opposing entities on the battlefield. Such a determination involves a mathematical representation of the terrain (possibly including micro-terrain features) (e.g. see [8]). Other deterministic) factors influencing when an observer will acquire a target include the distance between observer and target, the constituents of the intervening atmosphere, the exposure of the target, the desired level of target acquisition, etc. However (with the exception of automatic-target-recognition systems), the length of time required for target acquisition is determined by the ability of a human observer and is therefore a random variable. The “ACQUIRE” model (see reference [12]), which takes this time to be distributed according to an exponential distribution, is the basis of the algorithms used by Janus and CASTFOREM to play target acquisition. This model was developed by what is now the Center for Night Vision and Electro-Optics (CNVEO), which continues to maintain and improve it. Although it had been in existence for some time, attention was first called to ACQUIRE in 1977 (see reference [13]), and it has been the basis for playing target acquisition in both entity and aggregated combat simulations ever since.

Rules of engagement can be directly implemented as decision rules in the computer code in systemic simulations such as CASTFOREM. They are developed from concepts given in U.S. Army Field Manuals with the assistance of military experts. In war games such as Janus, these decision rules are implemented by human players who are part of the simulation (i.e. man “in the loop”) and thus are able to react to unforeseen events. The firing process consists of firing rounds/missiles at the target and determining the outcomes of these firings. The times at which rounds are fired is played by sampling from an interfiring-time distribution that the U.S. Army takes to have both fixed and variable components. To the authors’ best knowledge, every entity-level simulation used by the Army employs a lognormal distribution for the variable component. The oldest source for this procedure that the authors have at hand is reference [14], which cites a 1977 JMEM document for support. We suspect that previous documents (e.g. reference [15]) would yield similar results, but we have been unable to discover the empirical basis for use of the lognormal distribution. The outcomes of all rounds are usually assumed to be independent for direct-fire weapons. However, substantially different models are used for artillery and other indirect-fire weapons, especially for target-projectile vulnerability/lethality effects (e.g. see [12]). These effects are modeled more simply for most direct-fire weapons that fire a non-fragmenting projectile through a conditional-kill probability, i.e. the probability that a round that hits the target will kill (i.e. incapacitate) it (further details are to be found in Chapter 15 of [16]). Very recently, though, more comprehensive mission-oriented methodology has been developed (e.g. see reference [17]).

3.2. Attrition in Aggregated-Force Simulations

As discussed above in Section 2, losses from direct-fire combat between engaged opposing forces are assessed in an aggregated-force simulation by means of deterministic differential equations (implemented as difference equations and solved recursively) with additional equations to determine numerical values for the Lanchester attrition-rate coefficients (i.e. single-weapon-system-type kill rates) (e.g. see references [7] and [18]). Traditionally, linear scaling has been used in the differential-equation model (see reference [7], however, for other alternatives). Although considering exactly the same attrition factors as an entity-level simulation, an aggregated-force simulation uses mathematical modeling to develop an analytical expression for such a kill rate and therefore frequently must make simplifying assumptions for reasons of mathematical tractability. These simplifying assumptions can result in inconsistencies between the two types of simulations when they differ significantly from how attrition is played in the entity-level simulation.

The basic attrition paradigm played in such simulations is for heterogeneous-force Lanchester-type combat given by (for $i = 1,2,\ldots ,m$ and $j = 1,2,\ldots ,n$ (see Fig. 3.2.1)

$$\begin{cases} \dfrac{dx_i}{dt} = -\sum_{j=1}^{n} a_{ij}\, y_j \quad \text{with } x_i(0) = x_{0i}\,, \\ \dfrac{dy_j}{dt} = -\sum_{i=1}^{m} b_{ji}\, x_i \quad \text{with } y_j(0) = y_{0j}\,, \end{cases} \qquad (3.2.1)$$

where a_{ij} denotes the rate at which a single Y_j firer (with force level denoted as y_j) kills X_i targets (with force level denoted as x_i) and similarly for b_{ji}.

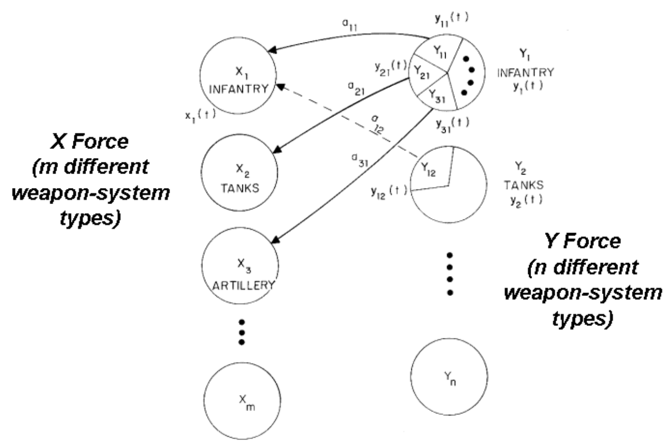

Figure 3.2.1: Combat between two heterogeneous forces.

For such attrition-rate-coefficient calculations (see [7], [10], and [18]), LOS is played as a stochastic process (i.e. continuous-time Markov chain) for computational feasibility (see [10] for details), with two parameters (prob-ability of LOS, denoted as P_{LOS}, and mean time that a target is exposed to enemy fire, denoted as $1/\mu$). Existing simulations have assumed that all events are independent and all times between events have exponential distributions [10]. Very few people are aware of this fact.

Unlike entity-level simulations, the mathematical modeling of kill rates is not nearly as well developed, with much room for improvement. Taylor [10] has recently developed new methodology that extends existing methodology and explicitly relates four key intermediate quantities (see below). However, Taylor has obtained significantly different results than those given in reference [18]. Even fewer people are aware of the above facts.

Target acquisition is simplified by assuming target independence (in practice exactly like an entity-level simulation does) and assuming there is only one level of target acquisition, with single-target acquisition rate (for an exponential distribution of time to acquire), denoted as (for example) $\lambda_{X_i Y_j}$ for a Y_j observer acquiring X_i targets. Only Taylor's [10] work has emphasized that different target-engagement policies lead to different kill rates and contains kill-rate results for a number of simple policies (and one not so simple).

The last two factors influencing attrition given in Section 3 above are mathematically combined to produce a distribution of time to kill an acquired target under conditions of continuous LOS. The reciprocal of the time is called the conditional kill rate and is usually taken to be equal to the product of a firing rate times a single-shot kill probability (see Chapter 5 of reference [7], however, for other useful expressions for a conditional kill rate). All previous work has assumed that the time to kill an acquired target under conditions of continuous LOS has an exponential distribution. This assumption had to be made for reasons of mathematical tractability, since the development of an expression for a Lanchester attrition-rate coefficient involves determination of the probability that an acquired target is killed before LOS is lost. Taylor's [10] new methodology explicitly focuses on such probabilities and associated expected values. New results by Taylor for the probability that one nonnegative random variable is less than another have led to one being able to develop kill-rate expressions under more general conditions than heretofore possible.

4. Current Representation of Interfiring Times in Lanchester-Type Models

As noted above, all current aggregated-force simulations that assess casualties according to some Lanchester-type paradigm (e.g. VIC, EAGLE, AWARS), assume that there is an exponential distribution for the time to kill an acquired target under conditions of continuous LOS. However, Taylor [19] has shown that this situation arises only when the interfiring times are exponentially distributed. Consequently, all current aggregated-force simulations assume that there is an exponential distribution for the interfiring times.

5. Interfiring-Time Inconsistency between Entity-Level and Aggregated-Force Simulations

To summarize, all entity-level simulations assume that all interfiring times have a lognormal distribution, while all aggregated-force simulations assume that they have an exponential distribution. Furthermore, these distributions are quite different (see Figure 5.1). From the shapes of these distributions (shown for the same mean value) it should be clear that one could obtain significantly different values for a Lanchester attrition-rate coefficient. This conjecture has been confirmed by experimental computing on Excel spreadsheets, with significantly different kill rates arising in "choppy" terrain (terrain for which the mean time that a target is exposed to enemy fire is less than the time to kill an acquired target). In such terrain, a Lanchester attrition-rate coefficient computed under the assumption of an exponential distribution for the interfiring times may be significantly higher than that for an Erlang approximation to the lognormal interfiring times, since an exponential distribution has as larger proportion of interfiring times close to zero (see Figure 5.1). Hence, the exponential interfiring times biases outcomes towards killing the target before LOS is lost, i.e. the battle runs "too hot."

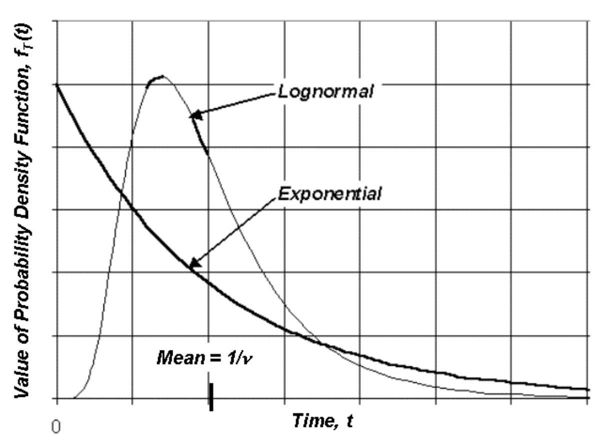

Figure 5.1: Exponential and lognormal distributions with the same mean value. The shape-parameter value of the lognormal distribution is that used in all U.S. Army entity-level simulations (see reference [12]).

6. Current Representation of Sequencing of Target Acquisition and Engagement in Lanchester-Type Models

A key question concerning a firer's behavior to be asked for developing a conceptual model of the attrition process is the following, "Can new targets be acquired while an acquired target is being attacked by this firer?" The simplest conceptual model for answering this key question consists of the following two basic cases:

(1) no new target can be acquired (serial acquisition of targets),

(2) new targets can be acquired at the same rate as when no target is being attacked (parallel acquisition of targets).

Thus, the target-engagement cycle shown in Figure 3.1 is for serial acquisition of targets. Moreover, current aggregated-force simulations apparently only consider serial acquisition of targets.

Recently, Taylor [10] has developed new theoretically correct analytical expressions for a Lanchester attrition-rate coefficient for parallel acquisition (that differs significantly from the results given in Section 3.3 of reference [18] for the VIC model). Taylor was told that these results were not that useful in practice, since the parallel-acquisition option in VIC has never really been exercised. Thus, apparently only serial acquisition of targets is being currently played in VIC and related aggregated-force models.

7. Sequencing Inconsistency between Entity-Level and Aggregated-Force Simulations.

Many platforms (e.g. M1A2 SEP Abrams tank) today have a second set of eyes and frequently additional sensors that allow a new target to be acquired while a previously acquired target is being attacked (i.e. fired at). Furthermore, the importance of such parallel acquisition has been just recently greatly increased by the U.S. Army's attempt at an internetted environment for its future combat system that would have assets that acquire targets and firers that shoot at them operate in parallel. Such an occurrence is easily incorporated into (and, indeed, has been) an entity-level simulation. Thus, parallel acquisition of targets is played for a number of weapon systems in current entity-level simulations. For an aggregated-force simulation, which sees the battlefield attrition process through the construct of the target-engagement cycle, this means that a different target-engagement cycle must be considered. Consideration of a different structure for the target-engagement cycle (see Figure 7.1) leads to a significantly different expression for the Lanchester attrition-rate coefficient in this case.

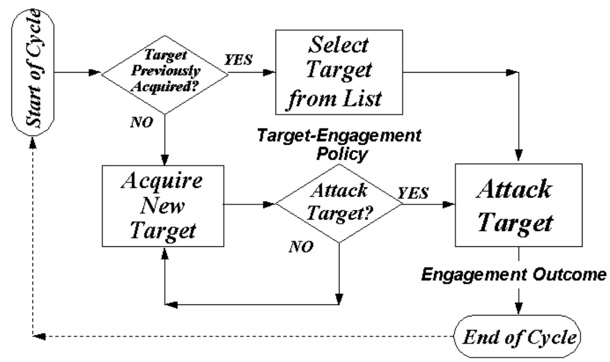

Figure 7.1: Target-engagement cycle for parallel acquisition of targets (no preemption by higher-priority target).

Moreover, Figure 7.1 is not the end of the story, since rules of engagement in an entity-level simulation may say that in the case of parallel acquisition when a higher priority target than the one currently being attacked is acquired, fire should be shifted to the higher-priority target (i.e. acquisition of a higher-priority target can preempt the engagement of a lower-priority one). Thus, Figure 7.2 depicts the target-engagement cycle for such a case in which preemption by a higher-priority target can occur. In this figure X_1 denotes the higher-priority target.

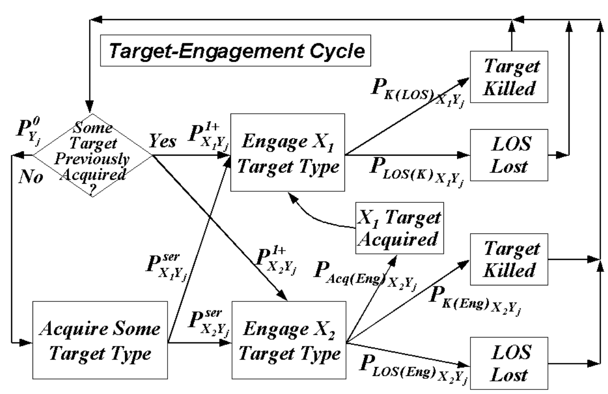

Figure 7.2: Diagram of target-engagement cycle for parallel acquisition of targets with preemption by higher-priority target type (case of two target types).

8. Current Target-Engagement Policies Played in Lanchester-Type Models

Currently, VIC and related models play Bonder and Farrell's m-period target-engagement policy for serial acquisition of targets. It is a very complicated policy (see reference [10] for further details) that has no analogue in any current entity-level simulation. Furthermore, we have been unable to find documentation concerning the source or any justification of this policy, particularly from any U.S. Army Field Manual. It is depicted for the case of three target types in Figure 8.1 below. In this case, different rules of engagement are played in each of three periods of time, the same number as the number of different target types. During the first period of time, only the highest-priority target type will be engaged when acquired. During the second period, whichever of the two highest-priority target types is acquired first will be engaged when acquired. The low-priority target type is ignored when acquired, but kept under surveillance. Furthermore, it should be clear from Figures 7.1 and 7.2 above that a slightly different type of target-engagement policy is required for parallel acquisition of targets. Moreover, such policies can be either open loop or closed loop (see reference [10] for further details). So far, however, only open-loop policies have been considered.

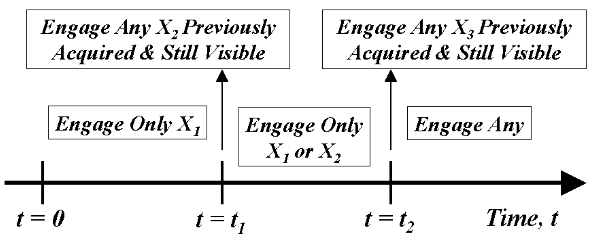

Figure 8.1: Bonder and Farrell's m-period target-engagement policy (three periods).

Recently Taylor [10] has proposed the following relatively simple target-engagement policy for serial acquisition. When a target is acquired, a decision is made whether or not to attack it immediately. This decision is based on consideration of a specific probability of immediate attack for each given target type. This probability for a given target type is constant over time (and hence the name Taylor's constant-probability-of-engagement-for-a-given-target-type policy). It will be convenient, however, to refer to this policy simply as the constant-probability target-engagement policy. Moreover, it is assumed that if the target is not immediately attacked, knowledge about its location will be lost and it must consequently be acquired again to be engaged at some later time. Use of this policy allows one to skip low-priority targets that are given a low probability for being immediately engaged.

Let $p_{X_i Y_j}$ denote the probability that an acquired target type X_i will be immediately engaged by a Y_j firer type using serial acquisition. This probability allows one to reflect the priority of a particular X_i target type to a specific Y_j firer type. A small value for such a probability can be used to model a low target priority. Use of such a constant-probability policy leads to a relatively simple expression of a Lanchester attrition-rate coefficient.

9. Rules-of-Engagement Inconsistency

Thus, entity-level simulations use rules of engagement to determine whether or not an acquired target will be engaged. A wide spectrum of such rules are used in contemporary entity-level simulations. On the other hand, only Bonder and Farrell's m-period target-engagement policy has ever been used in aggregated-force simulations like VIC. It is unlikely that all rules of engagement will yield the same results as those obtained for the single target-engagement policy apparently presumed by VIC. Moreover, all target-engagement policies in entity-level simulations have been closed-loop, while the single policy used in VIC and related models has been open loop. The consequences of this difference could be profound and should be investigated (see reference [10] for further information).

Additionally, although Taylor [10] has obtained explicit analytical results for a Lanchester attrition-rate coefficient for serial acquisition with Bonder and Farrell's m-period policy, no such explicit formulas are to be found in the VIC documentation (e.g. see Section 3.3 of reference [18]). Thus, the effects of using a single target-engagement policy in VIC may be compounded by lack of the availability of an explicit analytical expression for such a kill rate. Furthermore, only Taylor [10] has emphasized the importance of investigating the dependence of such kill rates on the target-engagement policy assumed. In fact, he is the only person to consider other target-engagement policies.

10. Removing Such Inconsistencies

It is our hypothesis that such inconsistencies can only be removed by mathematically modeling the same processes and events in a Lanchester attrition-rate coefficient as are played in entity-level simulations. We also believe that the latter should be viewed as a source of ideas about what and how to play combat effects in a Lanchester attrition-rate coefficient. To be able to do this, however, one must have some sound methodology for reflecting such micro-combat detail in such kill rates. It further appears to us that the place to start is to consider a single typical firer and then develop a mathematical expression for the rate at which he kills a particular enemy target type under the influence of the factors considered above in Section 3.

10.1. New Methodology for Computing Lanchester Attrition-Rate Coefficients

Taylor [10] explicitly considers the target-engagement cycle and a general principle on which to base the computation of a Lanchester attrition-rate coefficient with LOS modeled as a stochastic process in a heterogeneous-target environment as described above in order to develop a mathematical expression for such a kill rate. Taylor's principle states that a single-weapon-system-type kill rate should be computed as the expected number of kills in the target-engagement cycle divided by the expected length of this target-engagement cycle. This same principle is taken to hold for both serial and also parallel acquisition of targets.

If one assumes that all targets act independently of each other and that there is statistical independence between the processes of acquiring and attacking a target of a given type, then consideration of Taylor's principle and the target-engagement cycle for serial acquisition depicted in Figure 3.1 leads immediately to the following general expression for a Lanchester attrition-rate coefficient for the case of serial acquisition of targets

$$a_{ij}^{ser} = \frac{P_{X_iY_j}^{eng}\, P_{K(LOS)_{X_iY_j}}}{\left\{ E\left[T_{acq_{Y_j}}\right] + \sum_{k=1}^{m} P_{X_kY_j}^{eng}\, E\left[T_{atk|acq_{X_kY_j}}\right] \right\}} \quad (10.1.1)$$

where a_{ij}^{ser} denotes the rate at which an individual Y_j firer type (using serial acquisition) kills X_i target types, $P_{X_iY_j}^{eng}$ denotes the probability that the next target type to be engaged by a Y_j firer type will be an X_i target type, $P_{K(LOS)_{X_iY_j}}$ denotes the probability that a Y_j firer type will kill an X_i target type before line of sight (LOS) is lost, $E\left[T_{acq_{Y_j}}\right]$ denotes the expected time for a Y_j firer type to acquire the next target that will be engaged, $E\left[T_{atk|acq_{X_kY_j}}\right]$ denotes that expected time that a Y_j firer type will spend attacking (i.e. firing at) an acquired X_i target type until the target is either killed or LOS is lost, and m denotes the number of different X target types (see Figure 3.2.1 above).

Equation (10.1.1) is a very general result from which all other expressions for a Lanchester attrition-rate coefficient for serial acquisition of targets can be derived simply by evaluating its various component parts (e.g. $P_{X_iY_j}^{eng}$) under different conditions. Except for the existence of the probabilities and expected values that it contains, it only assumes that all targets act independently of each other and that there is statistical independence of the processes of acquiring and attacking any such target. It can be derived by straightforward probability arguments. In some sense it is the analogue of Little's formula, which has proven to be so useful in queueing theory (see pp. 10-11 of reference [20]). Finally, one should note that equation (10.1.1) holds for all probability distributions such that the quantities contained in the equation (e.g. probability that the target is killed before LOS lost) exist.

10.2. Functional Dependence of Kill Rates

Because of the overall complexity of the attrition process considered, it is convenient to consider a factorization of the target-engagement cycle into two phases and to examine which of the intermediate quantities appearing in (10.1.1) are effected by a parameter from one of these phases. One can gain an overall understanding of what model inputs effect the various quantities in (10.1.1) (referred to as key intermediate quantities) by examining the target-engagement cycle (see Figure 3.1). It factors naturally into two phases

(1) acquire-and-choose-target phase,
(2) attack-of-chosen-target phase.

Parameters of the acquire-and-choose-target phase (referred to as Phase I) effect the following two key intermediate quantities

(1) probability of next target type to be engaged,
(2) expected time to acquire next target to be engaged.

Likewise, parameters of the attack-of-chosen-target phase (referred to as Phase II) effect the other two key intermediate quantities, namely

(1) probability that target is killed before LOS lost,
(2) expected time spent attacking until engagement ends.

Thus, changing the target-engagement policy effects only the two key intermediate quantities for Phase I.

10.3. Kill Rates for Exponential Interfiring Times

This is the baseline situation from which excursions will be made by considering different cases that then change parameters in one of the two phases of the target-engagement cycle discussed above. The assumption of exponential interfiring times implies that the time to kill an acquired target will be exponentially distributed. Making other standard assumptions (i.e. times between all events are exponentially distributed) (see reference [10] for details), one finds that specification of the parameters in the attack-of-chosen-target phase (i.e. Phase II) leads to

$$P_{K(LOS)_{X_iY_j}} = \frac{\alpha_{ij}}{\alpha_{ij} + \mu}, \qquad (10.3.1)$$

and

$$E\left[T_{atk|acq_{X_iY_j}}\right] = \frac{1}{\alpha_{ij} + \mu}, \qquad (10.3.2)$$

where α_{ij} denotes the rate at which a Y_j firer type kills an acquired X_i target type under conditions of continuous LOS (conditional kill rate) and μ denotes the rate of losing LOS. Substituting (10.3.1) and (10.3.2) into (10.1.1), one finds that

$$a_{ij}^{ser} = \frac{\left\{\frac{P^{eng}_{X_iY_j}}{\alpha_{ij} + \mu}\right\}\alpha_{ij}}{\left[E\left[T_{a_{Y_j}}\right] + \sum_{k=1}^{m}\left\{\frac{P^{eng}_{X_kY_j}}{\alpha_{kj} + \mu}\right\}\right]} \qquad (10.3.3)$$

Specifying parameters of the acquire-and-choose-target phase (i.e. Phase II) and assuming Taylor's constant-probability target-engagement policy (see Section 8 above) , one finds that

$$P^{eng}_{X_iY_j} = \frac{p_{X_iY_j} P_{LOS} \lambda_{X_iY_j} x_i}{\sum_{k=1}^{m} p_{X_kY_j} P_{LOS} \lambda_{X_kY_j} x_k}, \qquad (10.3.4)$$

and

$$E\left[T_{a_{Y_j}}\right] = \frac{1}{\sum_{k=1}^{m} p_{X_kY_j} P_{LOS} \lambda_{X_kY_j} x_k}, \qquad (10.3.5)$$

where $\lambda_{X_iY_j}$ denotes the rate at which a Y_j firer type acquires a particular X_i target type. Substituting (10.3.4) and (10.3.5) into (10.3.3), one obtains the desired final result that

$$a_{ij}^{ser} = \frac{\left\{\frac{p_{X_iY_j} P_{LOS} \lambda_{X_iY_j} x_i}{\alpha_{ij} + \mu}\right\}\alpha_{ij}}{\left[1 + \sum_{k=1}^{m}\left\{\frac{p_{X_kY_j} P_{LOS} \lambda_{X_kY_j} x_k}{\alpha_{kj} + \mu}\right\}\right]} \qquad (10.3.6)$$

10.4. Kill Rates for Erlang Interfiring Times

This case arises when lognormal interfiring times (see reference [12]) are approximated by an Erlang distribution (see Figure 10.4.1 that is drawn for shape-parameter value of the U.S. Army data). Thus, interfiring times were modeled by

$$f_T(t) = r\nu \frac{(r\nu t)^{r-1}}{\Gamma(r)} e^{-r\nu t}, \qquad (10.4.1)$$

where $f_T(t)$ denotes the probability density function (p.d.f.) of the interfiring times, ν denotes the firing rate given by $\nu = 1/E[T]$, and r denotes a shape parameter. Values for these two parameters were estimated from the lognormal distribution for interfiring times by the method of moments.

Taylor (see reference [21]) has shown that for an arbitrary distribution of interfiring times the probability that a Y_j firer type will kill an X_i target type before losing LOS is given by

$$P_{K(LOS)_{X_iY_j}} = \frac{P_{SSK_{X_iY_j}} \mathcal{F}(\mu)}{1 - \left[1 - P_{SSK_{X_iY_j}}\right]\mathcal{F}(\mu)}, \qquad (10.4.2)$$

where $P_{SSK_{X_iY_j}}$ denotes the single-shot kill probability of a Y_j firer type firing at an X_i target type (assumed constant over time for a fixed set of circumstances) and $\mathcal{F}(z)$ denotes the Laplace transform of the p.d.f. of the common interfiring times. For an Erlang distribution with p.d.f. given by (10.4.1) this becomes

$$P_{K(LOS)_{X_iY_j}} = \frac{P_{SSK_{X_iY_j}}\left(\nu_{X_iY_j}\right)^r}{\left(\nu_{X_iY_j} + \mu\right)^r - \left[1 - P_{SSK_{X_iY_j}}\right]\left(\nu_{X_iY_j}\right)^r}. \qquad (10.4.3)$$

Taylor [21] has also shown that in general

$$E\left[T_{atk|acq_{X_iY_j}}\right] = \frac{1}{\mu}\left\{1 - P_{K(LOS)_{X_iY_j}}\right\} \qquad (10.4.4)$$

Again assuming Taylor's constant-probability target-engagement policy, one can then substitute (10.3.4), (10.3.5), (10.4.3), and (10.4.4) into(10.1.1) to obtain an expression for a Lanchester attrition-rate coefficient for serial acquisition.

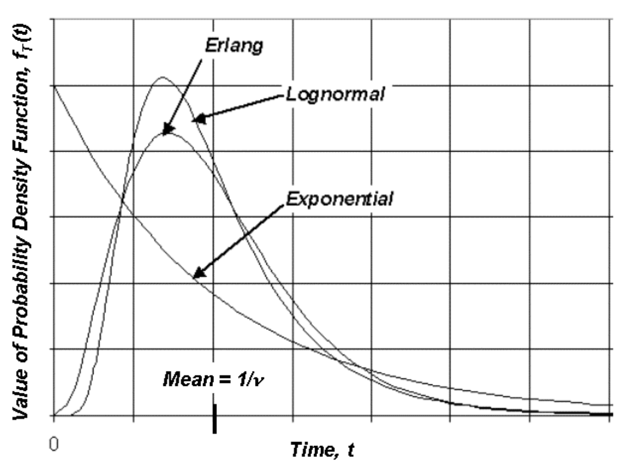

Figure 10.4.1: Density functions with the same mean. The shape parameter of the Erlang fits the lognormal.

The Erlang distribution is used to approximate the lognormal because it is mathematically tractable (i.e. has a simple Laplace transform) in the calculation of the probability of killing a target before LOS is lost. Eyeballing Figure 10.4.1, one sees that it also is a much better approximation to the lognormal distribution with shape-parameter value of U.S. Army data (see reference [12]) than is the exponential. In this case, not only is the Erlang distribution mathematically tractable, but also it looks like the lognormal. We leave it to others to see if one can do better.

Some experimental computing has shown that the effect of changing from exponential interfiring times to Erlang is to reduce the kill rate (i.e. "cool off" the battle). The extent of this cooling off depends on the value of the mean time that an enemy target is exposed to fire and the effectiveness of this fire. As seen in Figure 10.4.1 for the same mean value, an exponential distribution has more probability density near zero than does the Erlang distribution. Consequently, the probability that the target is killed before LOS is lost is higher for the exponential distribution (all other things being equal). This effect becomes particularly pronounced when the mean time to kill the target is less than the mean time a target is exposed to enemy fire.

10.5. Kill Rates for Parallel Acquisition

In the case of parallel acquisition of targets (see Figure 7.1 for the case of no preemption by higher-priority target) one must consider whether or not a previously acquired target is available for immediate engagement at the beginning of a new target-engagement cycle. For understanding the kill-rate expression given below, it is convenient to show certain key probabilities on the diagram of the target-engagement cycle (see Figure 10.5.1 below). Taylor [10] has shown that for parallel acquisition of targets (no preemption by higher-priority target) and Taylor's target-engagement policy for acquiring a new target to be engaged, a Lanchester attrition-rate coefficient is given by

$$a_{ij}^{par} = \frac{\left\{ S_{Y_j} P^{1+}_{X_iY_j} + P^{0}_{Y_j} \right\} \theta_{X_iY_j} P_{LOS}\, \lambda_{X_iY_j}\, x_i \left\{ \frac{\alpha_{ij}}{\alpha_{ij}+\mu} \right\}}{\left\{ P^{0}_{Y_j} + \sum_{k=1}^{m} \left[\frac{\left(S_{Y_j} P^{1+}_{X_kY_j} + P^{0}_{Y_j} \right) \theta_{X_kY_j} P_{LOS}\, \lambda_{X_kY_j}\, x_k}{\alpha_{ij}+\mu} \right] \right\}} \qquad (10.5.1)$$

where the probability that a Y_j firer has one or more X_i targets available for immediate attack at the beginning of a new target-engagement cycle is given by

$$P^{1+}_{X_iY_j} = 1 - (1 - A_{ij})^{x_i} \prod_{k \in I_{ij}} (1 - A_{kj})^{x_k}, \qquad (10.5.2)$$

the probability that a Y_j firer has no target available for immediate attack at the beginning of a new target-engagement cycle is given by

$$P^{0}_{Y_j} = \prod_{i=1}^{m} (1 - A_{ij})^{x_i}, \qquad (10.5.3)$$

and $A_{ij} = A_{ij}(t)$ denotes the probability that a particular X_i target is available to a Y_j firer for immediate attack at the beginning of a new target-engagement cycle. The latter quantity is called target availability and is determined from a three-state Markov-chain model for a particular target (see reference [10] for further details).

Experimental computing (e.g. see reference [10]) has shown that just changing from serial to parallel acquisition of targets has the effect of being a significant force multiplier. Through more efficient acquisition of targets, a force is able to inflict more casualties on the enemy (say 60% or more sometimes) and also suffer fewer casualties (say approximately 20% or less). Frequently, such a change from serial to parallel acquisition of targets can literally change defeat into victory.

10.6. Other Extensions

Other cases of interest for which explicit kill-rate results have been obtained (and hence for which interoperability can be computationally investigated) include the following

(1) fixed component of interfiring times,
(2) Bonder and Farrell's m-period target-engagement policy,
(3) some other target-engagement policies of interest.

It turns out that Army data indicates that for direct-fire weapons an interfering time can have both a fixed and also a random (or variable) component. The case of a purely random interfiring time has been considered in Section 10.4 above. The same approach also works for a purely fixed (i.e. deterministic) interfiring time. Complete details and verification of formulas are in the process of being worked out right now.

Although Taylor [10] has emphasized the importance of determining and understanding the influence of the target-engagement policy on kill rates, relatively little work has been done on investigating this dependence. Can the target-engagement policy have a significant impact on a kill rate? The simple fact is that we just do not know.

Essentially no computational investigations have been conducted on this important topic. However, explicit analytical results for a Lanchester attrition-rate coefficient are available for a few cases of interest, particularly Bonder and Farrell's m-period target-engagement policy (see reference [10] for further details).

For the reasons stated above, Taylor [10] reports that he has not been able to check any of his explicit analytical results for a Lanchester attrition-rate coefficient for serial acquisition with Bonder and Farrell's m-period policy. No such explicit analytical result is given in reference [18] (see Section 3.3). Moreover, one should note that the results obtained by Taylor and Neta (see reference [22]) for the two key intermediate quantities for Phase I are extremely complicated in the general case. Consequently, they will not be given here (see reference [22] for further details). One should also investigate the consequences of using a closed loop policy versus of using a closed loop policy (see reference [10] for further details).

11. Discussion

This paper suggests that on purely theoretical grounds significant interoperability problems may lurk beneath the coordinated use of entity-level simulations and aggregated-force simulations for computer generated forces. We base this somewhat speculative conclusion on the fact that certain key components of attrition are played in significantly different ways in the two different types of combat simulations. Experimental computing with the basic Lanchester-type paradigm that underlies essentially all aggregated-force simulations suggests that such differences in the conceptual basis of a model can have a significant impact on the model's output. Ultimately, however, one should compare the results of interest from both types of simulations for comparable circumstances to verify that such interoperability problems really exist.

Thus, the conclusions of this paper have been based on some (limited) experimental computing with a basic Lanchester-type paradigms that plays attrition in two fundamentally different ways

(1) how it is currently played in aggregated-force simulations,
(2) how it should be played to be consistent with current entity-level simulations.

Moreover, in order to be able to make such theoretical comparisons, one has to be able to compute a Lanchester attrition-rate coefficient under the two different sets of conditions. Until recently, however, this has just not been possible. Taylor's new methodology based on consideration of the target-engagement cycle provides a theoretical basis for computing such attrition-rate coefficients (see reference [10]). As the results presented here should make perfectly clear, however, considerable mathematical modeling is involved in such calculations. Unfortunately, most (if not all) of the community that uses such mathematical models is not aware that a significant capital investment in mathematical-modeling research is necessary to do the calculations that it assumed that it has been doing. The results of such research should be explicit analytical formulas for computing kill rates under well-delineated conditions. Interestingly enough, the authors could not find such formulas in the VIC documentation (e.g. see reference [18]) (nor any evidence of the necessary prerequisites for obtaining them).

Moreover, having explicit analytical results for kill rates greatly facilitates conveniently obtaining combat outcomes from an aggregated-force combat model. We were

able to do virtually all the computational work reported here on Excel spreadsheets. It is not difficult to play up to three weapon-system types on each side and still be able to compute force-level trajectories on an Excel spreadsheet. One should remember that to do such calculations (which greatly facilitates parametric excursions) one first needs an explicit analytical formula for a Lanchester attrition-rate coefficient.

The immediate motivation for the direction of the work reported here was our investigation of the playing of the time to kill an acquired target (under conditions of continuous LOS) in a Lanchester attrition-rate coefficient. All previous work (see references [7] and [10]) had assumed that this time was exponentially distributed, but AMSAA took interfiring times to have both a fixed and a variable component, with the variable component having a lognormal distribution. Therefore, in order to be consistent, one must calculate a Lanchester attrition-rate coefficient under the same conditions. Taylor's general formula (10.1.1) for a Lanchester attrition-rate coefficient was very useful for providing insight into specific mathematical expressions that had to be developed. For example, one would have to be able to calculate the probability that one nonnegative random variable would be less than another (e.g. the probability that the target would be killed before LOS lost). Fortunately, we had already been investigating such problems that frequently arise in military operations research (see Appendix B of reference [7]). In particular, the theory of stochastic duels (see reference [23]) investigates such questions and contains many ideas and results that have been essential for the work reported here. Later it became apparent that one did not have to be able to compute actually compute such a kill rate under the conditions played in entity-level simulations in order to identify circumstances for a possible interoperability problem.

Finally, this work has made us aware that if one takes a bottom-up approach to developing an aggregated-force model based on Lanchester-type attrition paradigms, then micro-combat models are required for calculating Lanchester attrition-rate coefficients. In particular, Taylor's methodology requires analysis and modeling of the target-engagement cycle at the micro-combat level of detail. Although the dynamics of combat may be very complex and poorly understood, entity-level simulations play combat in great detail (i.e. at the micro-combat level of detail) and are widely accepted. Why not mine them for ideas on what and how to play the target-engagement cycle? Although one might not develop a valid aggregated-force model, at least it might be interoperable with entity-level simulations.

12. Final Comments

Significantly more mathematical modeling is required for initially obtaining an expression for a Lanchester attrition-rate coefficient (one of the basic building blocks for any aggregated-force simulation) than for developing an entity-level simulation. Moreover, an aggregated-force simulation should play the attrition process in a way consistent with how it is done in entity-level simulations. The latter play combat at the micro-combat level of detail. Furthermore, Taylor [10] has developed methodology for computing Lanchester attrition-rate coefficients that reflect such a level of detail by focusing on the analysis and modeling of the target-engagement cycle. When such detail is played in consistent manner as that portrayed in entity-level simulations, one should obtain some consistency in results, i.e. achieve some degree of interoperability between entity-level and aggregated-force simulations. However, communications between two distinct communities of workers is required to achieve this latter goal.

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