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Improving the Army's joint platform allocation tool (JPAT)

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THESIS

IMPROVING THE ARMY'S JOINT PLATFORM ALLOCATION TOOL (JPAT)

by

John P. Harrop

September 2013

Thesis Advisor:	Emily Craparo
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The U.S. Army's joint platform allocation tool (JPAT) is an integer linear program that was developed by the Army's Training and Doctrine Command Analysis Center and the Naval Postgraduate School to help inform acquisition decisions involving aerial reconnaissance and surveillance (R&S) resources. JPAT evaluates inputs such as mission requirements, locations of available equipment, and budgetary constraints to determine an effective assignment of unmanned aerial R&S assets to missions.

As of September 2013, JPAT is solved using a rolling horizon approach, which produces a sub-optimal solution, and requires substantial computational resources to solve a problem of realistic size. Because JPAT is an integer linear program, it is a suitable candidate for using decomposition techniques to improve its computational efficiency.

This thesis conducts an analysis of multiple approaches for increasing JPAT's computational efficiency. First, we reformulate JPAT using Benders decomposition. Then, we solve both the original and decomposed formulations using the simplex and barrier algorithms with multiple size datasets. In addition, we experiment with an initial heuristic solution and other techniques in our attempts to improve JPAT's runtime. We find that while Benders decomposition does not result in significant improvements in computation time for the instances considered in this thesis, initial solution heuristics and other modifications to the model improve JPAT's performance.

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IMPROVING THE ARMY'S JOINT PLATFORM ALLOCATION TOOL (JPAT)

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Submitted in partial fulfillment of the
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ABSTRACT

The U.S. Army's joint platform allocation tool (JPAT) is an integer linear program that was developed by the Army's Training and Doctrine Command Analysis Center and the Naval Postgraduate School to help inform acquisition decisions involving aerial reconnaissance and surveillance (R&S) resources. JPAT evaluates inputs such as mission requirements, locations of available equipment, and budgetary constraints to determine an effective assignment of unmanned aerial R&S assets to missions.

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LIST OF ACRONYMS AND ABBREVIATIONS

BCA	Budget Control Act
DoD	Department of Defense
EODSKED	explosive ordnance disposal scheduling tool
EODTEU TWO	Explosive Ordnance Disposal Training and Evaluation Unit Two
FY	fiscal year
GAMS	General Algebraic Modeling System
JPAT	joint platform allocation tool
LP	linear program
MIP	mixed integer program
MUVAM	maritime UV assignment model
NAVICP	Naval Inventory Control Point
NPS	Naval Postgraduate School
R&S	reconnaissance and surveillance
SIP model	strategic inventory positioning model
TRAC	Training and Doctrine Command Analysis Center
TRADOC	Training and Doctrine Command
UAV	unmanned aerial vehicle
UV	unmanned vehicle
VCSA	Vice Chief of Staff, Army

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EXECUTIVE SUMMARY

The U.S. Army's joint platform allocation Tool (JPAT) is an integer linear program that was developed by the Army's Training and Doctrine Command Analysis Center and the Naval Postgraduate School to help inform acquisition decisions involving aerial reconnaissance and surveillance (R&S) resources. JPAT evaluates inputs such as mission requirements, locations of available equipment, and budgetary constraints to determine an effective assignment of unmanned aerial R&S assets to missions. It is currently solved iteratively using a rolling horizon approach, which produces a sub-optimal solution, and requires substantial computational resources to solve a problem of realistic size. Because JPAT is an integer linear program, it is a suitable candidate for using decomposition techniques to improve its computational efficiency.

This thesis conducts an analysis of the outcome of solving a problem of realistic size with multiple approaches. First, we reformulate JPAT using Benders decomposition. Then, we solve both the original and decomposed formulations using the simplex and barrier algorithms, and we solve each version with and without a heuristically generated initial solution, or "warm start." We evaluate the impact of each of the modifications on a number of datasets. In addition, we experiment with different transfer frequencies. The original formulation allows equipment to transfer from one geographic location to another every month, while this thesis also considers the impacts of quarterly and semi-annual equipment transfer frequencies.

We find that the Benders decomposition formulation has longer runtimes than the original formulation in every problem instance tested in this thesis. We also find that when transfers are done every quarter instead of every month, there is a significant decrease in runtime, a 99% decrease in some instances, with a 20% average decrease in the objective value. There is a further decrease in runtime when transfers are only allowed semi-annually (every 6 months), with an additional 20% average decrease in the objective value. We find that while Benders decomposition does not result in significant improvements in computation time for the instances considered in this thesis, initial solution heuristics and other modifications to the model improve JPAT's performance.

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Last, and most importantly, words cannot express my profound gratitude for the love and support of Jennifer, my amazing wife of 17 years. This thesis would not have been possible without her sacrifices and devotion to our three children during the long hours spent on completing this research.

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I. INTRODUCTION

A. BACKGROUND

Over the past decade, the Department of Defense (DoD) has spent an estimated $2 trillion in direct support of the wars in Iraq and Afghanistan (Blimes, 2013, p. 2). Much of this was financed through borrowing and has added significantly to the national debt (Blimes, 2013, p. 3). In an effort to control spending, Congress enacted the Budget Control Act (BCA) of 2011 which is aimed at reducing the national debt by $1.2 trillion over the next decade. Mandated by the BCA is a series of automatic spending cuts known as the budget sequestration which went into effect on March 1, 2013 and reduced the defense budget by 10 percent or approximately $55 billion per year (Harrison, 2012, p. 2). Budget cuts notwithstanding, the Department of Defense still needs to repair and replace aging equipment that has been battered from more than a decade of heavy use. According to Blimes (2013),

> the Pentagon also faces the task of replacing years of worn-out equipment, which will cost more than the amounts appropriated for this purpose. Equipment, materiel, vehicles and other fixed assets have depreciated at an estimated 6 times the peace-time rate, due to heavy utilization, poor repair and upkeep in the field, and the harsh conditions in the region. (p. 3)

In light of these recent events, the Army is seeking ways to more efficiently use its limited budget resources to meet mission requirements. This goal is outlined in a directive issued by the Vice Chief of Staff, Army (VCSA) in September 2011 (Craparo, Smead, & Tabacca, 2013, p. 1). One way to accomplish this is through the use of mathematical optimization, which provides a method for informing future acquisition decisions based on data available today. In response to the VCSA directive, analysts from the Army's Training and Doctrine Command Analysis Center (TRAC) and the Naval Postgraduate School (NPS) developed the joint platform allocation tool (JPAT). JPAT is a mixed-integer linear program (MIP) that evaluates inputs such as mission requirements, equipment and sensor locations, costs, and budget constraints to help determine an effective assignment, procurement, and retirement schedule of aerial R&S assets (Craparo et al., 2013).

JPAT is currently solved iteratively using a rolling horizon approach, which produces a sub-optimal solution, and requires substantial computational resources to solve a problem of realistic size. Because JPAT is an integer linear program, it is a suitable candidate for decomposition techniques such as Benders decomposition, as well as other heuristic approaches designed to improve computational efficiency.

This thesis evaluates a variety of modifications to the JPAT formulation and implementation, as described in Chapter II. Our analysis of the impact of these modifications appears in Chapter III. In Chapter IV, we discuss our findings and recommendations for further research.

B. OBJECTIVES AND APPROACH

The goal of this thesis is to identify techniques to improve JPAT's runtime. To accomplish this, we study two types of modifications. First, we modify the JPAT formulation. We reformulate it using Benders decomposition as described in Chapter II, and experiment with restrictions to the problem that may improve its tractability. The second area in which we seek improvement is through manipulating how the solver software operates on the problem. In this approach, we experiment with two different algorithms for solving linear programs: the simplex algorithm and a barrier method. The simplex and barrier algorithms are explained further in Chapter II. In addition, we solve each formulation both with and without an initial heuristic solution.

C. SCOPE, LIMITATIONS AND ASSUMPTIONS

This thesis is based on the existing JPAT model as described in Craparo et al. (2013). While this thesis is unclassified, the actual input data used for JPAT is classified. Thus, for this thesis we generate random data that does not necessarily reflect the characteristics of the actual data. In addition, the data sets that we use for this research may not be indicative of future data and requirements. Therefore, the main assumption of this thesis is that algorithm performance on randomly generated datasets is reflective of performance on the real dataset. This research also inherits all the assumptions of the original JPAT model including the following: JPAT does not consider threats faced by equipment from enemy action while performing missions; the only method for removing a system from inventory is through retirement. The time required to perform a mission accounts for only the time an asset is performing the mission, not transit time to and from the base of operations. Transfer time accounts for preparation, travel, and system set up time. Setup time accounts for all pre-flight requirements including fueling, and pre-flight checks. Production rates account for manufacture, assembly, and shipment to the field (Craparo et al. 2013).

D. LITERATURE REVIEW

As outlined by Craparo et al. (2013, p. 1), mathematical optimization is used extensively in military applications to help decision makers determine the most economical allocation of scarce financial, personnel, and material resources. Benders decomposition is used to increase computational efficiency in some of these instances with varying degrees of success. This literature review focuses on research in the area of capital planning and asset allocation including the use of Benders decomposition and other techniques to improve algorithm speed and memory usage.

Alvarez (2004) uses Benders decomposition to reformulate a DC power-flow model to test the reliability of power grids against terrorist attacks. He also attempts to increase the speed of the formulation through the use of several different techniques. One technique he tries is "solving the mixed-integer master problem exactly every k^{th} iteration only, and solving its easier, linear program relaxation otherwise" (Alvarez, 2004, p. xviii). Additional techniques explored are limiting the number of cuts in the master problem, using sub-optimal integer solutions of the master problem to solve the subproblem, and experimenting with the optimality gap between upper and lower bounds.

Bessman (2010) uses global Benders decomposition in a defender-attacker model to efficiently allocate available resources to interdict potential drug smugglers. Using the attacker model as the subproblem and the linearization of the defender-attacker model as the master problem, the author is able to determine the optimal allocation of limited interdiction resources over a wide geographic area.

Jackson (1995) uses a combination of Benders decomposition and Lagrangean relaxation referred to as cross decomposition in a facility location problem that ascertains the optimal placement of military units globally. The author compares the results obtained by cross decomposition with the branch and bound method with varying results. Cross decomposition is faster for some problems, while the branch and bound method is faster for others.

Explosive ordnance disposal scheduling tool (EODSKED) is a scheduling optimization tool that supports Explosive Ordnance Disposal Training and Evaluation Unit (EODTEU) Two in developing an optimal personnel training schedule (DeWinter, 2012). EODSKED attempts to fulfill all training requirements while constrained by units' deployment schedules and availability of instructors, equipment, and facilities. EODSKED then devises a training schedule based on these constraints.

The Strategic inventory positioning (SIP) model is a mixed integer linear program that helps to determine the most cost effective inventory storage locations for Naval Inventory Control Point (NAVICP) managed repairable items (Burton, 2005). With global transportation expenses exceeding $400 million annually, SIP is projected to result in significant cost savings to NAVICP by strategically locating inventory items in areas that minimize transportation costs and distance to end users.

Duhan (2005) developed the Maritime UV assignment model (MUVAM), an integer linear program that assists in the scheduling and allocation of unmanned vehicles (UVs) in the maritime environment. Similar to JPAT, MUVAM takes user inputs such as mission types, available equipment, location and priority to find the most efficient UV mission assignment schedule.

II. METHODOLOGY

This chapter discusses the original JPAT formulation and the techniques applied in the decomposition of the original model. In addition, we discuss the use of different solution algorithms and starting points for the solver.

A. JPAT MODEL OVERVIEW

The original JPAT Model is designed by the Army Training and Doctrine Command Analysis Center (TRAC) Monterey and the Naval Postgraduate School (NPS) to determine the optimal assignment and acquisition schedule for aerial R&S equipment.

JPAT assigns *configurations* consisting of one *platform* and one or more *sensors* to fulfill prioritized *mission demands* in required geographic *locations*. JPAT places greater utility on fulfilling higher priority missions and maximizes that utility when assigning configurations to fulfill mission demands. A platform is an unmanned aerial vehicle (UAV) that carries a limited payload of sensors. Each sensor is capable of satisfying one or more *intelligence* (INT) requirements or required sensing tasks such as infrared, optical, radar, signals intelligence, etc. Assets are procured, retired and transferred between geographic locations in specific combinations of platforms and sensors. JPAT refers to these combinations as *systems.* Each platform has exactly one combination in which it can be procured, retired, or transferred and thus makes up exactly one system. More succinctly, only complete systems are distributed to and from locations, while configurations are used to fulfill mission demands; see Figures 1 and 2. Systems can be disassembled after arrival at a new geographic location and its sensors can be installed on other platforms to make different configurations (Craparo et al., 2013, p. 4).

JPAT is a discrete time model in which each equipment type has a limited number of hours available for missions and transfers between locations in each time step. The available hours reflect the total quantity of each equipment type at each location and not each individual piece of equipment. The time required for routine maintenance is accounted for and deducted from the number of available hours, and JPAT is constrained to not exceed the number of available hours when assigning equipment to fulfill mission demands. JPAT also accounts for budgetary constraints. The total cost of operating, maintaining, transferring, procuring, and retiring systems cannot exceed the budget for each month. Unused funds from one month will be available in subsequent months until used or until the end of the fiscal year (FY). In accordance with federal government fiscal policies, JPAT does not allow transfer of funds from one FY to another (Craparo et al. 2013).

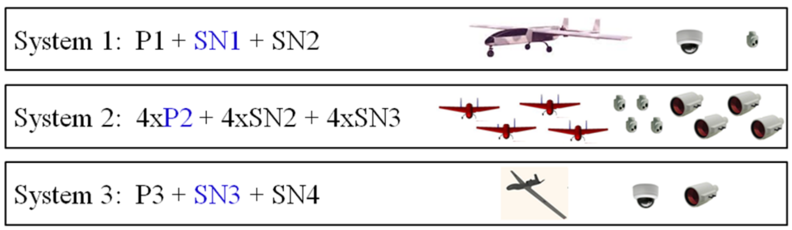

Figure 1. Three example systems composed of platforms P1, P2, and P3, and sensors SN1, SN2, SN3, and SN4 (from Craparo et al., 2013)

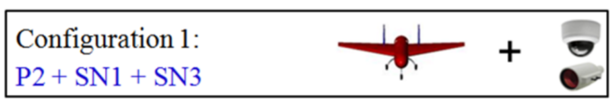

Figure 2. A configuration consisting of equipment derived from multiple systems (from Craparo et al., 2013)

We now present JPAT's mathematical formulation.

B. DATA, VARIABLES AND SETS

The following formulation is adapted from Craparo et al. (2013, pp. 5-7).

1. Indices and Sets

$l, l' \in L$ Locations

$m \in M$ Mission demands

$i \in I$ INT (intelligence) types

$r \in R$ Iterations in the rolling horizon model

$t, t' \in TIME$ Time steps

$t \in T(r) \subseteq TIME$	Time steps considered in iteration r
$t \in N \subseteq TIME$	Time steps occurring at the beginning of a fiscal year
$c \in C$	Configurations
$e \in E$	Equipment
$y, y' \in Y$	Systems
$M(l)$	Set of mission demands residing in location l
$l(m)$	Location of mission demand m (each mission demand resides in exactly one location)
$(y, y') \in REP$	Identifies the system y' replacing a retiring system y
$(t, y, l, l') \in GP$	Identifies systems y eligible to transfer from location l to location l' at time t

2. Input Data [Units]

$iq_{e,l}$	Initial quantity of equipment e in location l at time 0 [number of items]
$d_{t,m}$	Number of times mission demand m is present at time t [number of occurrences]
$ok_{m,i,c}$	Number between 0 and 1 indicating the ability of configuration c to fulfill INT type i in mission demand m [unitless]
omc_e	Operation and maintenance (O&M) cost per month for equipment e [$M]
pc_y	Procurement cost for system y [$M]
rc_y	Retirement cost for system y [$M]
$b_{t,y}$	Maximum budget for system y at time t [$M]
$mpr_{t,y}$	Maximum production rate of system y at time t [number of items]
p_m	Number between 0 and 1 indicating the importance of mission demand m [unitless]
$ec_{c,e}$	Number of equipment e in configuration c [number of items]
$es_{y,e}$	Number of equipment e in system y [number of items]

he_e	Hours available for transport and missions per time period for equipment e. Accounts for regular maintenance hours, etc. [hours]
hm_m	Hours required to perform mission demand m, not including equipment-specific setup and takedown time [hours]
$hi_{m,i}$	Hours required for INT type i in mission demand m [hours]
su_e	Time to set up, take down, and maintain equipment e per assignment [hours]
$transdays_{y,l,l'}$	Time required to transfer system y from location l to location l'. Includes actual transit time as well as packing, unpacking, etc. [days]
$sr_{m,c}$	Sorties required in order for configuration c to fully complete mission demand m [number of sorties]
$newyear_t$	1 if time t is the start of a new year; 0 otherwise [binary]
$upperbounds_y$	Maximum number of system y that can ever be distributed, total across all time [number of items]
$mr_{t,y}$	Total number of system y that must be retired by time t [number of items]
$initial_y$	Number of system y initially in theater [number of items]

3. Calculated Data

$\max_{t,y}$ — Maximum total number of system y that can have been distributed as of time t [number of items]

$$\max_{t,y} = \min(upperbounds_y \text{ - } initial_{y'} \sum_{t'=1}^{t} mpr_{t',y}$$

$ht_{e,y,l,l'}$ — Hours to decrement from equipment type e when transporting system y from location l to location l'. Assumes that any transfers that require more than 1 month (26 operational days) are completed in 1 month. [hours]

$$ht_{e,y,l,l'} = \min\left(1, \frac{transdays_{y,l,l'}}{26}\right) he_e es_{y,e}$$

4. Positive Integer Variables

$G_{t,y,l,l'}$	Number of system y transferring from location l to location l' at time t
$Z_{t,y,l}$	Number of system y retiring from location l at time t
$D_{t,y,l}$	Number of system y distributed to location l at time t

5. Binary Variables

$P_{t,c,l}$	1 if sufficient equipment is present to create configuration c at time t in location l; 0 otherwise

6. Positive Variables

$X_{t,m,c,i}$	Number of hours configuration c is assigned to INT type i for mission demand m at time t
$S_{t,m,c}$	Number of sorties flown by configuration c against mission demand m at time t
$Q_{t,e,l}$	Quantity of equipment e present in location l at time t (equation (3) in Figure 3 ensures the integrality of this variable)
B_t	Budget rolled over from previous time period at time t

C. ORIGINAL FORMULATION

The original JPAT formulation is shown in Figure 3.

The following explanation is reproduced from Craparo et al. (2013, p. 7).

> The objective function (1) maximizes the weighted mission demand coverage, weighted by mission demand priority and configuration performance. Constraint set (2) ensures that intelligence requirements are not over satisfied by the assigned configurations. Constraint sets (3-4) maintain a record of the quantity of each equipment type available in each location, beginning with the initial quantity (4) and updating the quantity based on system procurements, retirements, and transfers in subsequent time steps (3).
>
> Constraint sets (5-8) ensure that configurations are employed appropriately based on equipment availability. Constraint set (5) forces $P_{t,c,l}$ to take on a value of zero if any piece of equipment require to

construct configuration *c* is not present in a sufficient quantity in location *l* at time *t*; otherwise, $P_{t,c,l}$ is allowed to take on a value of one. Constraint set (6) uses the variables $P_{t,c,l}$ to control the number of sorties flown by configuration *c*: if $P_{t,c,l} = 0$, then configuration *c* cannot fly any sorties against any mission demands in location *l* at time *t*. Otherwise, configuration *c* can fly any number of sorties so long as it does not exceed the number of sorties required to completely satisfy the mission demand. Constraint set (7) ensures that the time spent covering intelligence requirements is appropriate given the number of sorties flown. Finally, constraint set (8) ensures that the hours spent fulfilling mission demands and transferring from one location to another do not exceed the "pool" of hours available for each equipment type.

Constraint sets (9-11) ensure that budgetary limitations are observed. Constraint set (9) calculates the monthly budget rollover B_t while accounting for equipment maintenance, system procurement, and system retirement costs. Because B_t is a nonnegative variable, constraint set (9) ensures that the available budget is not exceeded on months that do not mark the beginning of a fiscal year. Likewise, constraint set (10) performs this function for months that do mark the beginning of a fiscal year, while constraint set (11) sets B_t to zero for months at the beginning of a fiscal year.

Constraint sets (12-13) control distribution and retirement of systems. Constraint set (12) ensures that the total number of system *y* distributed as of time *t* does not exceed the limits posed by system production rates and fielding restrictions. Constraint set (13) ensures that any system y' that "upgrades" a system *y* is not distributed until its predecessor *y* is retired.

Finally, constraint sets (14-21) declare variable types.

$$\max_{\substack{P,\, G,\, Z\\ D,\, S,\, X,\\ B,\, Q,}} \quad z = \sum_{(t,m,c,i):t\in T(r),d_{t,m}>0,hi_{m,i}>0} p_m ok_{m,i,c} \frac{X_{t,m,c,i}}{\sum_{i'} hi_{m,i'}} \tag{1}$$

$$\text{s.t.} \quad \sum_{c:ok_{m,i,c}>0} X_{t,m,c,i} \le hi_{m,i} d_{t,m} \qquad \forall\ t \in T(r), m, i : d_{t,m} > 0, hi_{m,i} > 0 \tag{2}$$

$$Q_{t,e,l} = Q_{t-1,e,l} + es_{y,e} \sum_{y} (D_{t,y,l} - Z_{t,y,l} + \sum_{l'} (G_{t,y,l',l} - G_{t,y,l,l'})) \qquad \forall\ t \in T(r), e, l : t > 1 \tag{3}$$

$$Q_{t=1,e,l} = iq_{e,l} \qquad \forall\ e, l \tag{4}$$

$$P_{t,c,l} \le \frac{Q_{t,e,l}}{ec_{c,e}} \qquad \forall\ t \in T(r), l, c, e : ec_{c,e} > 0, \exists m \in M(l) : d_{t,m} > 0 \tag{5}$$

$$S_{t,m,c} \le sr_{m,c} d_{t,m} P_{t,c,l(m)} \qquad \forall\ t \in T(r), m, c \tag{6}$$

$$X_{t,m,c,i} \le \frac{hm_m S_{t,m,c}}{sr_{m,c}} \qquad \forall t \in T(r), m, c, i : ok_{m,i,c} > 0, hm_{m,i} > 0, d_{t,m} > 0 \tag{7}$$

$$\sum_{y,l'} ht_{e,y,l,l'} G_{t,y,l,l'} + \sum_{c,m\in M(l)} ec_{c,e} \left(\frac{hm_m}{sr_{m,c}} + su_e \right) S_{t,m,c} \le he_e Q_{t,e,l} \qquad \forall\ t, e, l \tag{8}$$

$$B_t = B_{t-1} + \sum_{y} b_{t,y} - \sum_{y,l} (pc_y D_{t,y,l} + rc_y Z_{t,y,l}) - \sum_{e,l} omc_e Q_{t,e,l} \qquad \forall\ t \in T(r) \setminus N : t > 1 \tag{9}$$

$$\sum_{y,l} pc_y D_{t,y,l} + \sum_{y,l} rc_y Z_{t,y,l} + \sum_{e,l} omc_e Q_{t,e,l} \le \sum_{y} b_{t,y} \qquad \forall\ t \in T(r) \cap N \tag{10}$$

$$B_t = 0 \qquad \forall\ t \in T(r) \cap N \tag{11}$$

$$\sum_{l,t'\le t} D_{t',y,l} \le max_{t,y} \qquad \forall\ t \in T(r), y \tag{12}$$

$$\sum_{t'\le t, y:(y,y')\in REP} Z_{t',y,l} \ge \sum_{t'\le t} D_{t',y',l} \qquad \forall\ t \in T(r), l, y' : \exists y : (y, y') \in REP \tag{13}$$

$$P_{t,c,l} \in \{0, 1\} \qquad \forall\ t \in T(r), c, l \tag{14}$$

$$G_{t,y,l,l'} \in \mathbb{Z}^+ \qquad \forall\ (t, y, l, l') \in GP : t \in T(r) \tag{15}$$

$$Z_{t,y,l} \in \mathbb{Z}^+ \qquad \forall\ t \in T(r), y, l \tag{16}$$

$$D_{t,y,l} \in \mathbb{Z}^+ \qquad \forall\ t \in T(r), y, l \tag{17}$$

$$X_{t,m,c,i} \ge 0 \qquad \forall\ t \in T(r), m, c, i \tag{18}$$

$$S_{t,m,c} \ge 0 \qquad \forall\ t \in T(r), m, c \tag{19}$$

$$Q_{t,e,l} \ge 0 \qquad \forall\ t \in T(r), e, l \tag{20}$$

$$B_t \ge 0 \qquad \forall\ t \in T(r) \tag{21}$$

Figure 3. JPAT's mathematical formulation (from Craparo et al., 2013)

D. A BRIEF OVERVIEW OF BRANCH AND BOUND AND BENDERS DECOMPOSITION

In this section, we provide a brief overview of two approaches for solving integer linear programs: branch and bound algorithm and Benders decomposition. The branch and bound algorithm first solves the relaxed MIP in order to determine an upper bound for a maximization problem. The next step is to estimate which branches in the solution tree may contain a potential optimal solution and which ones do not need to be explored. It then explores the branches that were not eliminated to obtain an optimal solution. While branch and bound may be able to solve a small problem relatively quickly, it can require significantly longer when solving a medium sized or larger problem (Chinneck, 2010, Chapter 12). One method used to potentially decrease the runtime of large problem instances is Benders decomposition. In Benders decomposition, the problem is reformulated into two separate problems: a relaxed master problem and one or more subproblems. The master problem is often a MIP that is formulated over the discrete ("hard") variables from the original MIP, while the subproblem is often a liner program (LP) that solves for the continuous ("easy") variables (Conejo, Castillo, Minguez, & Garcia-Bertrand 2010, pp. 110-119). For a maximization master problem and a minimization dual subproblem, the subproblem is solved initially with a trivial solution or guess to obtain a lower bound for the optimal objective value of the original formulation. Then, for the upper bound, the master problem is solved for the integer variables given the fixed values of the continuous variables from the subproblem (Conejo, et al., pp. 110-119). After a number of iterations between the master and subproblems, the lower and upper bounds converge on the optimal solution to the original formulation (Fischetti, Salvagnin, & Zanette, 2008, p. 1). Termination criteria can be set for the maximum number of iterations or the proportional difference between upper and lower bounds. In this thesis, we terminate the problem when the lower bound is within 10% of the upper bound.

E. JPAT MODEL DECOMPOSITION FORMULATION

We now describe the Benders decomposition formulation of the JPAT model. JPAT is broken up into a subproblem (described in section III.E.1), and a master problem (described in section III.E.3). The dual of the subproblem is derived in section III.E.2. The master problem primarily models procurement and movement of assets (strategic level decisions), while the subproblem assigns assets to fulfill mission demands (tactical level decisions) given their fixed locations obtained in the master problem.

1. Primal Subproblem

The objective function (22) in the primal subproblem is the same as the objective function (1) in the original formulation. It "maximizes the weighted mission demand coverage, weighted by mission demand priority and configuration performance" (Craparo et al., 2013). Constraint sets (23) through (26) in the primal subproblem are largely the same as constraint sets (2), and (6) through (8) respectively in the original formulation and model low level functionality of equipment when placed in geographic locations. Constraint set (27) declares variable types. The values for $\tilde{P}_{t,c,l(m),it}$, in constraint set (24) and $\tilde{Q}_{t,e,l,it}$, and $\tilde{G}_{t,y,l,l',it}$ in constraint set (26) are held fixed in the subproblem and correspond to the optimal values of the master problem variables $P_{t,c,l(m)}$, $Q_{t,e,l}$, and $G_{t,y,l,l'}$ from the previous problem iteration.

Equation [Dual Variable]

$$\text{maximize} \quad z_S = \sum_{(t\in T,m,c,i)|d_{t,m}>0,hi_{m,i}>0} p_m ok_{m,i,c} \frac{X_{t,m,c,i}}{\sum_{i'} hi_{m,i'}} \tag{22}$$

s.t.

$$\sum_{c|ok_{m,i,c\geq 0}} X_{t,m,c,i} \leq hi_{m,i} d_{t,m} \qquad \forall (t\in T(r),m,i)\,|\, d_{t,m}>0, hi_{m,i}>0 \qquad \left[\pi^1_{t,m,i}\right] \tag{23}$$

$$S_{t,m,c} \leq \sum_{M(l),l(m)} sr_{m,c} d_{t,m} \tilde{P}_{t,c,l(m),it} \qquad \forall (t\in T(r),m,c)\,|\, \sum_i ok_{m,i,c}>0, d_{t,m}>0 \qquad \left[\pi^5_{t,m,c}\right] \tag{24}$$

$$X_{t,m,c,i} \leq \frac{S_{t,m,c} hm_m}{sr_{m,c}} \qquad \forall (t\in T(r),m,c,i)\,|\, ok_{m,i,c}>0, hi_{m,i}>0, d_{t,m}>0 \qquad \left[\pi^6_{t,m,c,i}\right] \tag{25}$$

$$\sum_{\substack{(c,m)|ec_{c,e}>0,ok_{m,i,c}>0,\\ m\in l(m),l\in M(l),sr_{m,c}>0,d_{t,m}>0}} ec_{c,e}\left(\frac{hm_m}{sr_{m,c}}+su_e\right)S_{t,m,c} \le he_e\tilde{Q}_{t,e,l,it} - \sum_{y,l'|es_{y,e}>0} ht_{e,y,l,l'}\tilde{G}_{t,y,l,l',it}$$

$$\forall t\in T(r),e,l \qquad \left[\pi^7_{t,e,l}\right] \qquad (26)$$

$$S_{t,m,c},\ \ X_{t,m,c,i}\ge 0 \qquad \forall t\in T(r),m,c,i \qquad (27)$$

2. Dual Subproblem

We take the dual of the primal subproblem to obtain the dual subproblem. Constraint sets (31) through (34) set continuous positive variables and are the dual variables $\pi^1_{t,m,i}$, $\pi^5_{t,m,c}$, $\pi^6_{t,m,c,i}$, and $\pi^7_{t,e,l}$ that correspond to the primal subproblem constraint sets (23) through (26) respectively. Again, the values for $\tilde{P}_{t,c,l(m),it}$, $\tilde{Q}_{t,e,l,it}$, and $\tilde{G}_{t,y,l,l',it}$ in the objective function (28) are held fixed in the subproblem and correspond to the optimal values of the master problem variables $P_{t,c,l(m)}$, $Q_{t,e,l}$, and $G_{t,y,l,l'}$ from the previous problem iteration.

Equation [Primal Variable]

$$\text{minimize}\ \ z_D = \sum_{t\in T(r),m,i|d_{t,m}>0,hi_{m,i}>0} hi_{m,i}d_{t,m}\pi^1_{t,m,i} + \sum_{t\in T(r),m,c|\sum_i ok_{m,i,c}>0,d_{t,m}>0}\left(sr_{m,c}d_{t,m}\tilde{P}_{t,c,l(m),it}\right)\pi^5_{t,m,c}$$

$$+\sum_{t\in T(r),e,l}\left(he_e\tilde{Q}_{t,e,l,it} - \sum_{(y,l')|es_{y,e}>0} ht_{e,y,l,l'}\tilde{G}_{t,y,l,l',it}\right)\pi^7_{t,e,l} \qquad (28)$$

s.t.

$$\pi^1_{t,m,i}+\pi^6_{t,m,c,i}\ge \frac{p_m ok_{m,i,c}}{\sum_{i'} hi_{m,i'}} \qquad \forall t\in T(r),m,c,i:\sum_i hi_{m,i'}\ge 0 \qquad \left[X_{t,m,c,i}\right] \quad (29)$$

$$\pi^5_{t,m,c} - \sum_{i|ok_{m,i,c}\ge 0}\left(\frac{hm_m}{sr_{m,c}}\right)\pi^6_{t,m,c,i} + \sum_{e|ec_{c,e}\ge 0} ec_{c,e}\left(\frac{hm_m}{sr_{m,c}}+su_e\right)\pi^7_{t,e,l(m)}\ge 0$$

$$\forall t\in T(r),m,c: sr_{m,c}\ge 0 \qquad \left[S_{t,m,c}\right] \quad (30)$$

$$\pi^1_{t,m,i} \geq 0 \qquad \forall t \in T(r), m, i \tag{31}$$

$$\pi^5_{t,m,c} \geq 0 \qquad \forall t \in T(r), m, c \tag{32}$$

$$\pi^6_{t,m,c,i} \geq 0 \qquad \forall t \in T(r), m, c, i \tag{33}$$

$$\pi^7_{t,e,l} \geq 0 \qquad \forall t \in T(r), e, l \tag{34}$$

3. Master Problem

The master problem contains all integer variables ($G_{t,y,l,l'}, Z_{t,y,l}, D_{t,y,l}, P_{t,c,l}$) along with the continuous variables $Q_{t,e,l}$ and B_t as the "hard" variables. The objective function (35) maximizes across all the "hard" variables. Constraint set (36) ensures that the total number of hours used for mission demands and transfers does not exceed the total numbers available for each mission demand in each time period. The values for $\tilde{\pi}^1_{t,m,i,it}$, $\tilde{\pi}^5_{t,m,c,i,it}$, and $\tilde{\pi}^7_{t,e,l,it}$ in constraint set (36) are held fixed in the master problem and correspond to the values of the subproblem variables $\pi^1_{t,m,i}$, $\pi^5_{t,m,c,i}$, and $\pi^7_{t,e,l}$ from iteration it. Constraint sets (37) through (42) are largely the same as constraint sets (3), (5), (9), (10), (12), and (13) in the original formulation and model high-level decisions of where to place assets. Constraint set (43) ensures that the number of systems retiring from each location equals the number required for mandatory retirement. Constraint set (44) ensures that the number of hours used for mission fulfillment and transfers does not exceed the total hours available. Constraint sets (45) through (50) set variable types.

$$\underset{P,G,Z,D,B,Q}{\text{maximize}} \; Z_m \tag{35}$$

s.t.

$$z_m \leq \sum_{t,m,i \mid hi_{m,i}>0, d_{t,m}>0, \tilde{\pi}^1_{t,m,i,it}>0} hi_{m,i} d_{t,m} \tilde{\pi}^1_{t,m,i,it} + \sum_{t,m,c \mid d_{t,m}>0, \sum_i ok_{m,i,c}>0, \tilde{\pi}^5_{t,m,c,i,it}>0} sr_{m,c} d_{t,m} \sum_{l(m)} P_{t,c,l} \tilde{\pi}^5_{t,m,c,i,it}$$

$$+ \sum_{t,e,l \mid \tilde{\pi}^7_{t,e,l,it}>0} he_e Q_{t,e,l} - \sum_{transable_{t,y,l',l} \mid y>0, l'>0} ht_{e,y,l',l} G_{t,y,l',l} \tilde{\pi}^7_{t,e,l,it} \quad \forall it \tag{36}$$

$$Q_{t,e,l} = Q_{t-1,e,l} + \sum_{y|\max dist_{t,y}>0} es_{y,e} D_{t,y,l} - \sum_{y|mr_{t,y}>0} es_{y,e} Z_{t,y,l} + \sum_{transable_{t,y,l',l}|es_{y,e}>0,y>0,l'>0} es_{y,e} G_{t,y,l',l}$$

$$- \sum_{transable_{t,y,l,l'}|es_{y,e}>0,y>0,l'>0} es_{y,e} G_{t,y,l,l'} \qquad \forall t \in T(r), e, l : t > 1 \quad (37)$$

$$P_{t,c,l} \le \frac{Q_{t,e,l}}{ec_{c,e}} \qquad \forall t, l, c, e : ec_{c,e} > 0, l \in M(l), \sum_i ok_{m,i,c} > 0, d_{t,m} > 0 \quad (38)$$

$$B_t = B_{t-1} + \sum_Y b_{t,y} - \sum_{y,l|\max_{t,y}>0} pc_y D_{t,y,l} - \sum_{y,l|mr_{t,y}>0} Z_{t,y,l} pc_y - \sum_{e,l} Q_{t,e,l} omc_e$$

$$\forall t : t > 1, newyear_t = 0 \quad (39)$$

$$\sum_{e,l} Q_{t,e,l} omc_e + \sum_{y,l,t|t>1,\max_{t,y}>0} pc_y D_{t,y,l} + \sum_{y,l|mr_{t,y}>0} Z_{t,y,l} pc_y \le \sum_y b_{t,y} \quad \forall t : newyear_t > 0 \quad (40)$$

$$\sum_{l,t'|t'>t,t'>1,\max_{t',y}>0} D_{t,y,l} \le \max_{t,y} \qquad \forall t, y : \max_{t,y} > 0 \quad (41)$$

$$\sum_{t',replace_{y,y'}|t'\le t,t'>1,mr_{t,y}>0} Z_{t',y,l} \ge \sum_{t'|t'>t,t'>1,\max_{t,y}>0} D_{t',y',l} \qquad \forall t, y', l : \sum_y replace_{y,y'} > 0 \quad (42)$$

$$\sum_l Z_{t,y,l} = mr_{t,y} \qquad \forall t, y : mr_{t,y} > 0 \quad (43)$$

$$\sum_{y,l'} ht_{e,y,l,l'} G_{t,y,l,l'} \le he_e Q_{t,e,l} \qquad \forall t, e, l \quad (44)$$

$$P_{t,c,l} \in \{0,1\} \qquad \forall t \in T(r), c, l \quad (45)$$

$$Q_{t,e,l} \ge 0 \qquad \forall t \in T(r), e, l \quad (46)$$

$$G_{t,y,l,l'} \in \mathbb{Z}^+ \qquad \forall (t, y, l, l') \in GP : t \in T(r) \quad (47)$$

$$D_{t,y,l} \in \mathbb{Z}^+ \qquad \forall t \in T(r), y, l \quad (48)$$

$$Z_{t,y,l} \in \mathbb{Z}^+ \qquad \forall t \in T(r), y, l \quad (49)$$

$$B_t \ge 0 \qquad \forall t \in T(r) \quad (50)$$

III. ANALYSIS

In this chapter, we discuss the results of our computational experiments. For our analysis, we experiment with datasets of several different sizes. Because the number of missions, time steps, and configurations were identified as parameters that will be increased in future runs of JPAT, we perform experiments to determine the impact of the cardinality of each of these sets on JPAT's runtime. We begin with a small dataset containing 10 missions, 5 time steps, and 5 configurations. We then conduct 5 experiments in which we increase the cardinality of each of these sets, as well as the percentage of nonzero elements in the $ok_{m,i,c}$ parameter, which we refer to as the *ok* density. We solve each experiment with the techniques described in III.B using IBM's CPLEX solver with General Algebraic Modeling System (GAMS) software version 24.0.2.

A. PROBLEM INSTANCES

To eliminate redundancy in the data and reduce the computational resources needed, TRAC performs preprocessing techniques on JPAT's input data prior to solving the model in GAMS. Specific information on how the preprocessing is performed is outlined in Craparo, Smead, and Tabacca (2013, pp. 10-11). Table 1 displays the cardinalities of the parameters that we vary in our experiments and their cardinalities in the original and preprocessed datasets. For each experiment, we use ten different random seeds within GAMS to generate data. The appendix contains a psuedocode representation of our data generation procedure.

	Original dataset	Preprocessed dataset	Experiment 1a	Experiment 1b	Experiment 2	Experiment 3	Experiment 4	Experiment 5
Missions	2200	250	10	10	10	10	50	250
Time steps	144	60	5	5	5	20	24	60
Configurations	20	20	5	5	10	10	10	20
Ok density*	Baseline	Baseline	Baseline	2X Baseline	Baseline	Baseline	Baseline	Baseline
Random Seed	NA	NA	1 – 10	1 – 10	1 – 10	1 – 10	1 – 10	1 – 10

Table 1. Relative sizes of problem instances considered by TRAC analysts and in this thesis. This thesis runs Experiments 1a, 1b, and 2-5. Because the density of the actual ok_{mic} parameter is classified, we refer only to a baseline density and its variations.

B. SOLUTION APPROACHES

Figure 4 shows the 24 approaches that we use to solve Experiments 3-5 from Table 1. For Experiments 1a, 1b, and 2, we do not vary the transfer frequencies and use only 8 approaches with the transfer frequency held fixed at monthly. As illustrated, we solve the original and Benders formulations using both the barrier and simplex algorithms. We also experiment with an initial solution heuristic that disables transfers from one geographic location to another, i.e., $G_{t,y,l,l'} = 0$. This provides an initial integer feasible solution of varying quality. Initially, transfers between geographic locations occur monthly in the original JPAT model. For Experiment 3, we analyze the effects of restricting the transfer frequencies to only occur quarterly or semi-annually.

By default, and in our experiments, GAMS generates the MIP model before solving each iteration of Benders decomposition. GAMS can be manipulated to only generate the MIP model prior to the first iteration. Thus, we do not include model generation times in our reported runtimes for Benders iterations greater than 1. Overall, the generation times do not have a significant effect on the runtimes and do not change the outcome of the experiments. For both the original formulation and each iteration of the Benders master and subproblems, we set a 10% gap between the relaxed MIP and the current integer solution as the termination criterion. We terminate the Benders run when the lower bound is within 10% of the upper bound. In addition, for this analysis, we prevent CPLEX from automatically generating cuts.

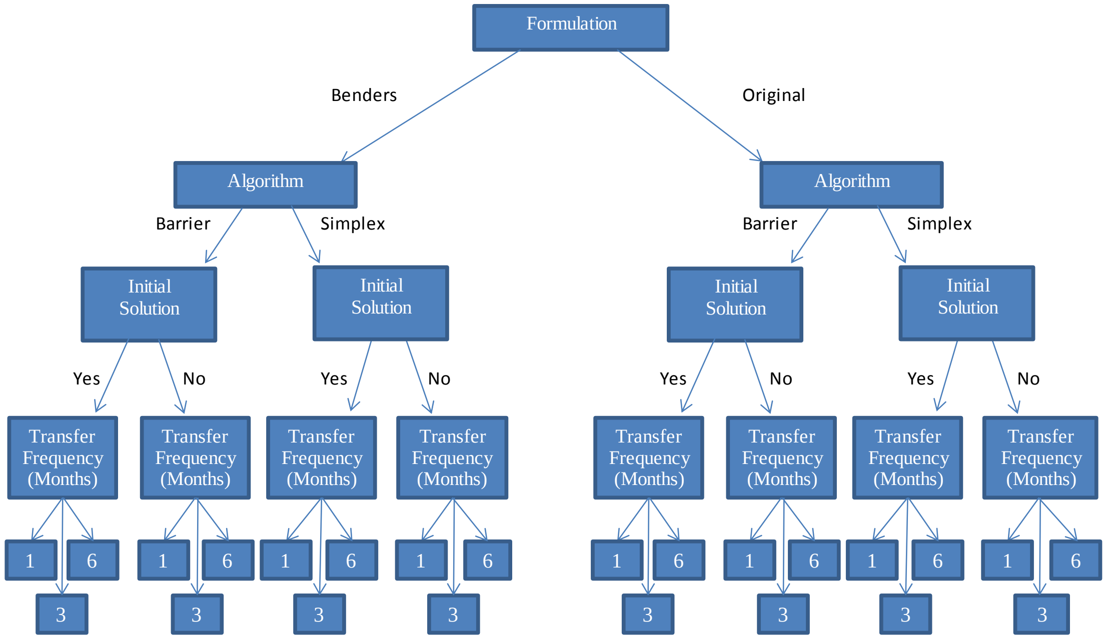

Figure 4. Approaches used to solve each problem instance from Table 1. Transfer frequencies are varied only for Experiments 3-5 in Table 1.

C. OUTPUT ANALYSIS

1. Experiment 1a

The experiment is run on a relatively small dataset. The runtimes for each of the 10 seeds for the indicated formulation, algorithm, and initial solution status are shown in Figure 5. Note that although we consider ten discrete random seed values, we connect the data points corresponding to these values for clarity. This also occurs in Figures 12, 14, and 15. In addition, the x-axis, labeled "Scenario" for these figures, represents the ten random seeds used for each experiment. In each figure these random seeds are sorted in increasing order according to their runtimes in the original formulation using the simplex method and no initial solution. Consequently, because some seeds had faster runtimes on some experiments than others, the ordering of the ten random seeds on the x-axis varies among the indicated figures. For Experiment 1a, transfers are allowed to occur monthly. The average runtimes of the 10 seeds are shown in Figure 6. As shown, every instance of the original formulation is very efficient and reaches a solution in less than 1/3 seconds for this experiment. The Benders formulations exhibit significantly more variability for the different seeds than the original formulations do, as displayed in Figure 5. The

Benders formulation requires five times longer to solve the problem using the simplex method with no initial solution. When we solve both formulations with the initial solution and the simplex method, the Benders formulation takes six times longer to reach an optimal solution than the original formulation. The original formulation using the simplex method with an initial solution is slightly faster than the other versions of the original formulation and the fastest overall for this run. In contrast, the fastest of the Benders formulations (the barrier method with an initial solution) requires nearly five times as long as the original formulation. Thus, for small instances, the original formulation using the simplex method and an initial solution appears to be the most efficient. For the Benders decomposition formulation, the barrier method with an initial solution is fastest at 12% faster than the barrier method with no initial solution.

2. Experiment 1b

Figure 7 shows the average solution times for Experiment 1b, which is the same as Experiment 1a with a two-fold increase in the *ok* density. As shown, there is a significant increase in runtime for the Benders formulations using the simplex method and an initial solution; this version requires over two hours to reach a near-optimal solution. These times are skewed by one random seed that takes significantly longer to solve when compared to the other nine seeds. This particular random seed requires between 35 and 40 Benders iterations to reach optimality, whereas the other nine seeds require on average less than ten Benders iterations. The average solution times are much lower when this seed is removed, as shown in Figure 8. The original formulation again performs better than the Benders formulation when the outlier is removed, terminating in less than 0.5 seconds. In contrast, the Benders formulation using simplex and no initial solution requires nine times longer to reach a near-optimal solution. The Benders formulation performs best with the barrier method and an initial solution. However, this version requires six times longer than its original counterpart. The barrier method and initial solution do not provide any significant improvements in the original formulation. However, for the Benders formulation, the initial solution with the barrier method solves the same problem in 1/3 the time required by the barrier method without the initial solution.

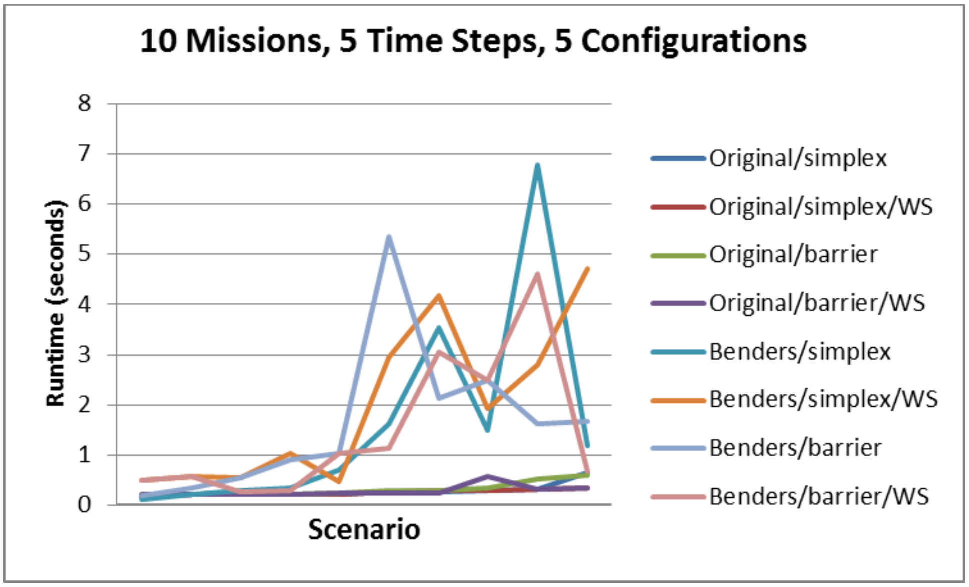

Figure 5. Runtimes (seconds) for Experiment 1a across different seeds; "WS" indicates that an initial solution ("warm start") is used

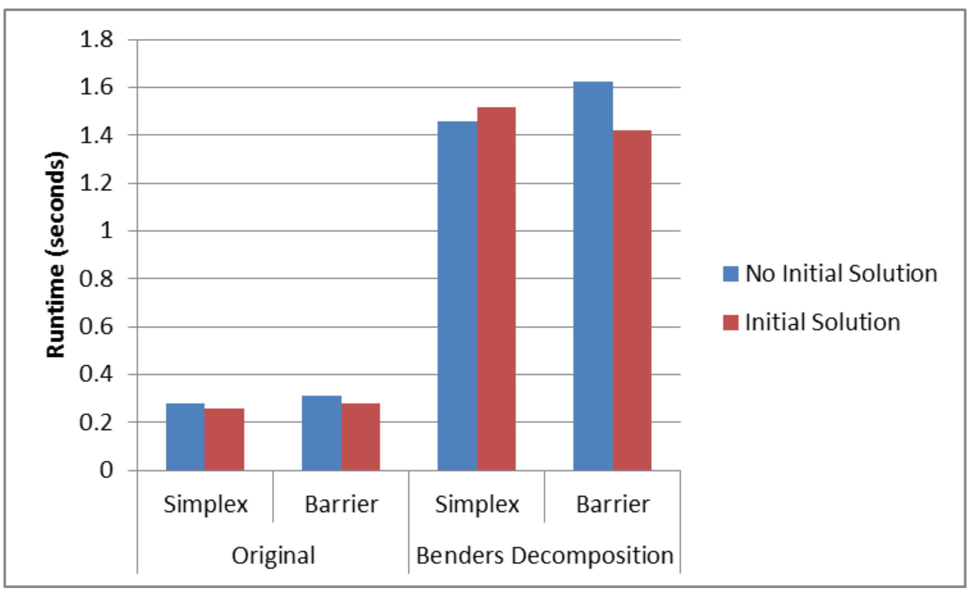

Figure 6. Average runtimes (seconds) for Experiment 1a

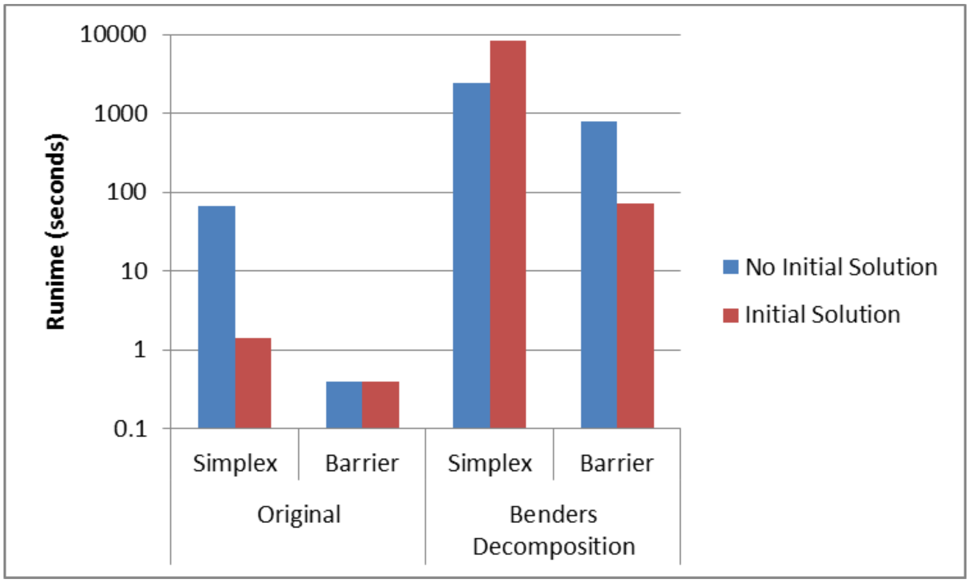

Figure 7. Average runtimes (seconds) for Experiment 1b (logarithmic scale)

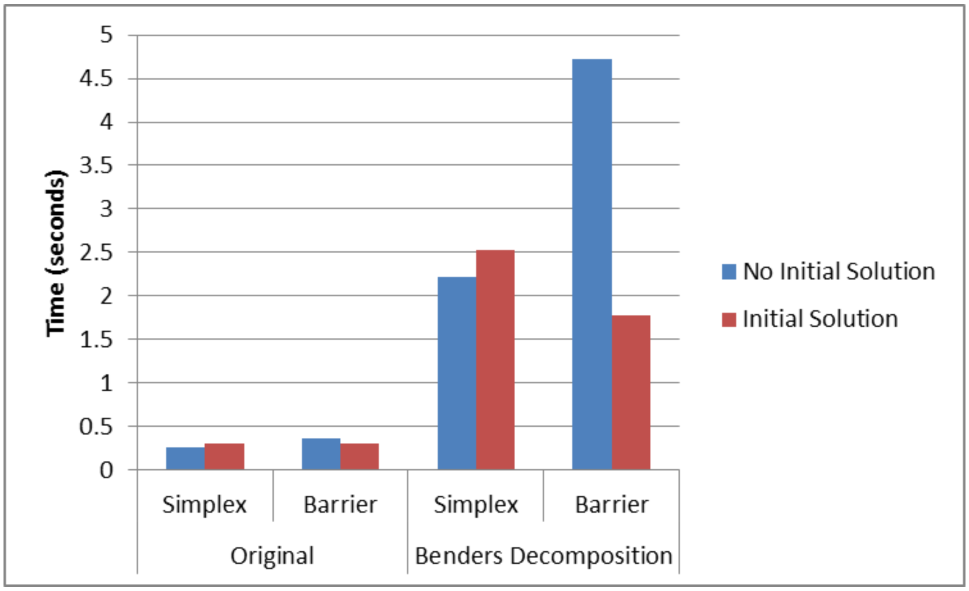

Figure 8. Average runtimes (seconds) for Experiment 1b (outlier removed)

3. Experiment 2

For Experiment 2, average runtimes are shown in Figure 9. As with Experiments 1a and 1b, the original formulation performs more efficiently than the Benders formulation. However, the initial solution and barrier algorithms significantly reduce the runtimes of the Benders formulation. When Benders decomposition is used with the simplex method, the average runtime is 39 seconds without the initial solution and around 15 seconds with the initial solution. This is a 61% improvement in average runtime. When the barrier method and the initial solution are combined, the efficiency of the Benders decomposition formulation is increased slightly more, to an average runtime of 10 seconds. However, the original formulation terminates in much less time, requiring less than a second for each version, regardless of the algorithm and initial solution used.

Comparing the results of Experiments 1b and 2, we see that a two-fold increase in the number of configurations results in a larger runtime increase than a two-fold increase in *ok* density does. Figures 10 and 11 illustrate the differences in solution times for the original and Benders decomposition formulations respectively. These figures illustrate that in every instance, the original formulation outperforms the Benders decomposition formulation. To further illustrate this point, Figure 12 shows the variation across all seeds for this experiment. This figure confirms that the original formulation outperforms the Benders formulation in all cases. The blue bar represents the solution times for Experiment 1a, while the red and green bars represent solution times for the same data set with a two-fold increase in the *ok* density (Experiment 1b, outlier removed) and the number of configurations (Experiment 2), respectively. As indicated, the number of configurations has a greater impact on runtimes than the *ok* density does. Increasing the number of configurations has a much greater impact on the Benders formulations than it does on the original formulations.

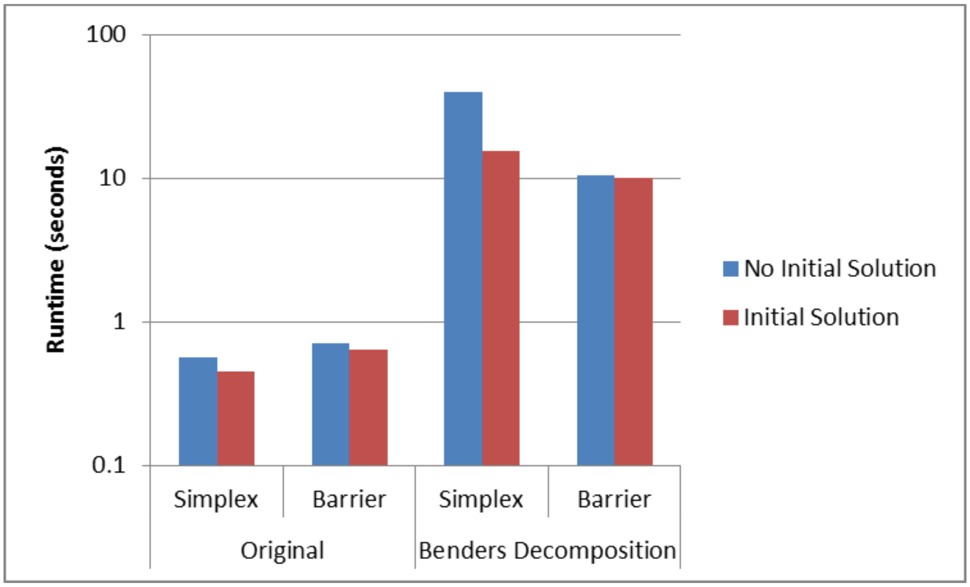

Figure 9. Average runtimes (seconds) for Experiment 2 (logarithmic scale)

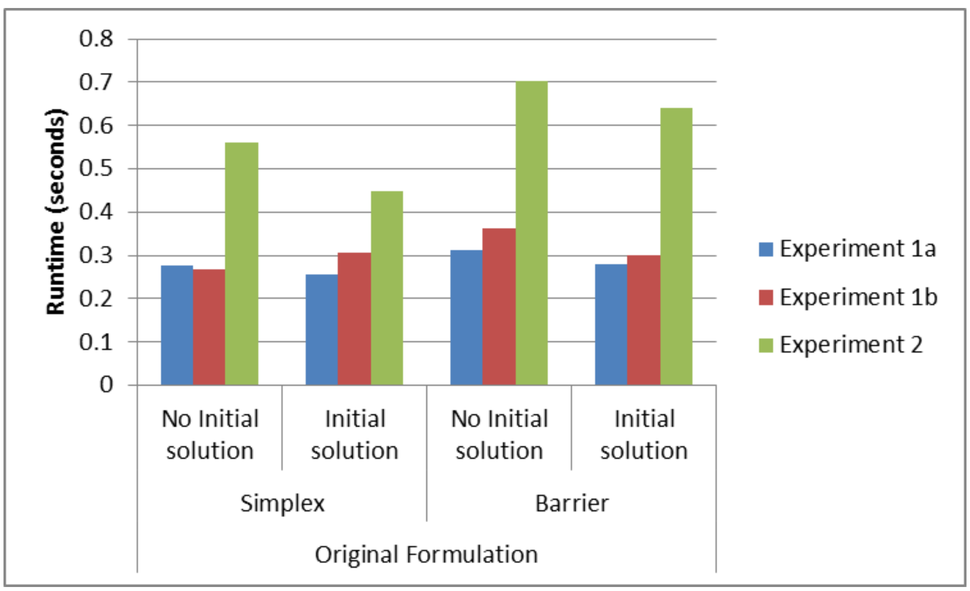

Figure 10. Average runtimes for Experiments 1a, 1b, and 2 (seconds), using the original formulation

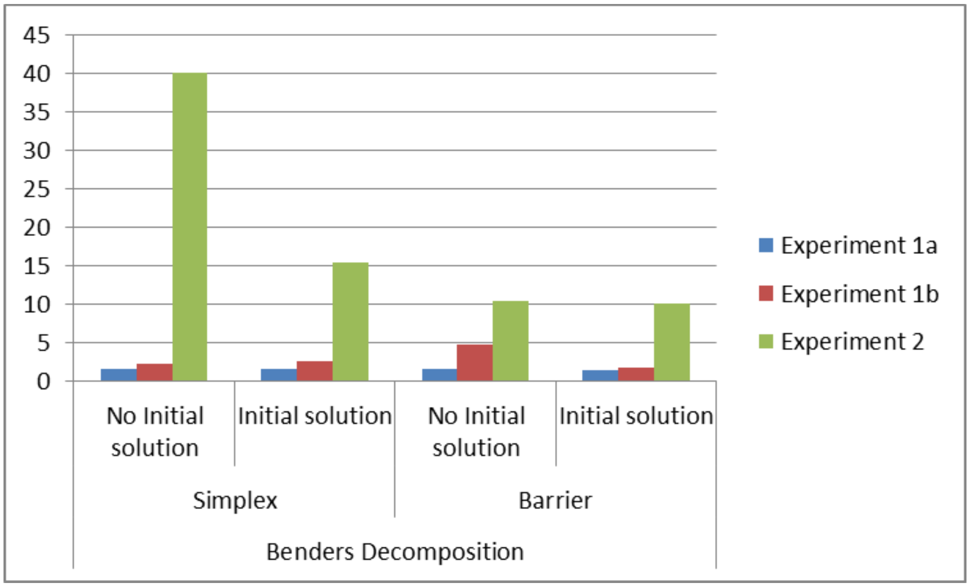

Figure 11. Average runtimes for Experiments 1a, 1b, and 2 (seconds), using the Benders formulation

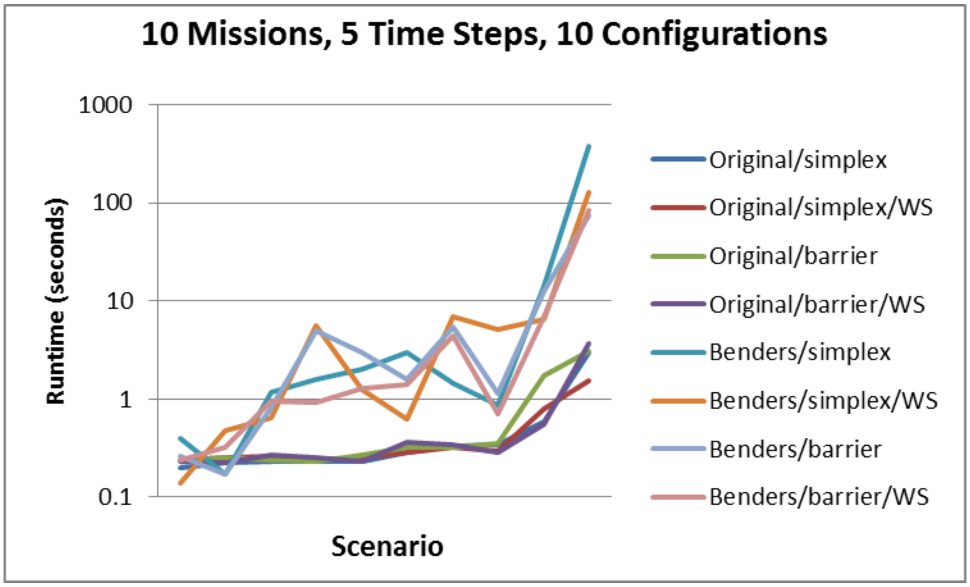

Figure 12. Runtimes (seconds) for Experiment 2 across different seeds (logarithmic scale); "WS" indicates that an initial solution is used

4. Experiment 3

For Experiment 3, we increase the number of time steps from 5 to 20. Additionally, we also vary the allowed transfer frequencies from monthly to quarterly (every 3 months) and semi-annually (every 6 months) in order to examine the impact of this restriction on problem instances involving relatively long time horizons.

We have a limited amount of time and computational resources in which to conduct our experiments. Thus, it is sometimes necessary to terminate experiments when it appears that no solution is forthcoming in a reasonable amount of time. This allows us to use the available resources to conduct additional experiments. For this reason, we terminate Experiment 3 after a runtime 100 hours with no solution for all Benders instances with monthly transfers and for seed ten only for all Benders instances with quarterly transfers.

Figure 13 shows the average runtime (in minutes) for Experiment 3 when transfers are allowed to occur monthly, quarterly, and semi-annually. This figure illustrates that there is a significant decrease in runtimes when transfers occur quarterly instead of monthly. There is a further decrease in runtime when transfers are allowed semi-annually for all but 1 of the 8 approaches (original formulation using the simplex method and an initial solution). As in Experiment 1b, this can be attributed to one random seed that took significantly longer to solve than the other 9. When this seed is removed, the average runtime for quarterly transfers is slightly more than the average runtime for semi-annual transfers. Figure 13 shows the biggest improvements with the original formulation. For instance, the original formulation with the barrier method and an initial solution goes from a runtime of 113 minutes for the monthly transfers to a runtime of about 2 seconds for the quarterly transfers. This is a 99.9% improvement in runtime. The original formulation using the barrier method without an initial solution goes from a runtime of 271 minutes to a runtime of 2.6 minutes or a runtime improvement of over 99%. Changing the transfer frequency from monthly to quarterly also results in reduced runtime for the Benders formulations, which time out at 100 hours for the monthly transfers.

With this increase in computational efficiency comes a decrease in the optimal objective value. Figures 14 and 15 show the proportion of initial (monthly transfer frequency) objective value obtained when using the indicated transfer frequency instead of monthly. As the figures indicate, there is significant variation in the proportion of objective values obtained across the different seeds when different transfer frequencies are used. Restricting the transfer frequency could result in mission demands not being met while a location is awaiting transfer of equipment, but this is not the case for every problem instance.

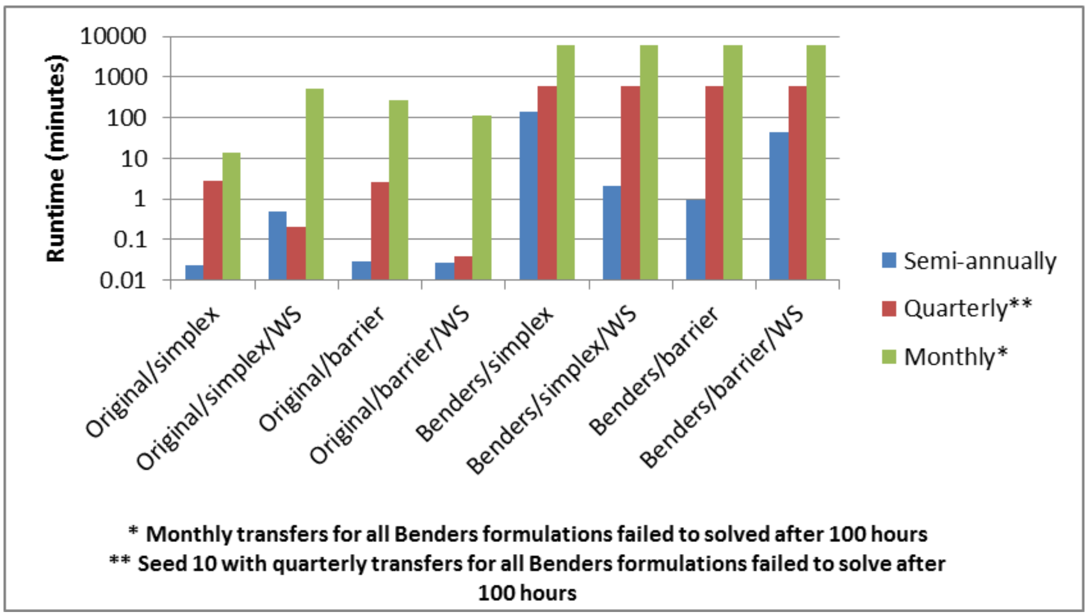

Figure 13. Runtimes (minutes) for Experiment 3 for indicated transfer periodicity (logarithmic scale; "WS" indicates that an initial solution is used.

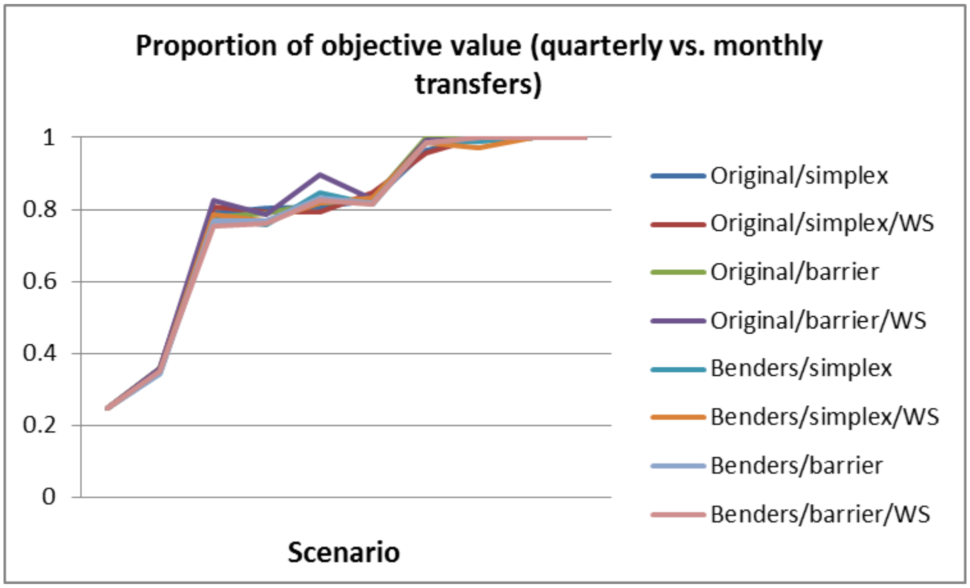

Figure 14. Proportion of monthly transfer frequency objective value attained with quarterly transfers; “WS” indicates that an initial solution is used

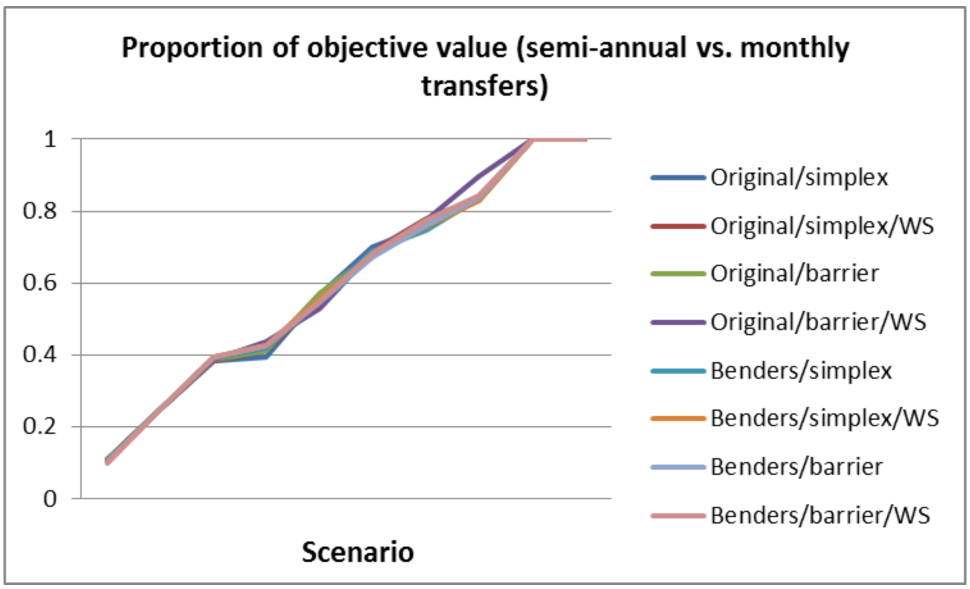

Figure 15. Proportion of monthly transfer frequency objective value obtained with semi-annual transfers; “WS” indicates that an initial solution is used

5. Experiments 4 and 5

Experiments 4 and 5 are beyond the scope of the tools and time available for this research. Using a monthly transfer frequency, both experiments are run for 400 hours for both the original and Benders formulations with no solution reached for any version of the problem for either experiment. Experiment 4 has a significant increase in mission demands relative to Experiment 3, as well as a slight increase in time steps. For Experiment 4, the original formulation with a transfer frequency of monthly reaches a 35% gap between the current MIP solution and the relaxed MIP after 400 hours or nearly 17 days of runtime. For Experiment 4 using the Benders formulation and a monthly transfer frequency, the problem reaches a 74% gap between the upper and lower bounds on the first Benders iteration after 400 hours. When the transfer frequency is changed to quarterly for Experiment 4, no solution is reached using either formulation after 95 hours. In this instance, the Benders formulation reaches a 26% gap between upper and lower bounds for the first iteration of the master problem, while the original formulation reaches a 40% gap between the current MIP solution and the relaxed MIP solution. When the transfer frequency is changed to semi-annually, the first five seeds of the original formulation reach a solution that is within 10% of the optimal solution, while seeds 5–10 time out after 90 hours. With semi-annual transfers, the Benders formulation reaches a near-optimal solution for the first seed only, while the remaining seeds time out after 90 hours. The average runtimes of the solutions we obtained for Experiment 4 with semi-annual transfers are shown in Table 2.

Experiment 5 has a significant increase in missions, time steps and configurations to equal that of the preprocessed dataset used for the original JPAT model. The gaps for Experiment 5 are significantly larger than those of Experiment 4. After 400 hours, the original formulation with monthly transfers reaches an optimality gap of 514% for both the barrier and simplex algorithms. The first iteration of the Benders formulations with monthly transfers reaches an optimality gap of 520% after 400 hours for both algorithms. These problems need to run for several more weeks or perhaps months to reach an optimal solution and require further research to accomplish. In addition, Experiment 5 is run for both the original and Benders formulations with transfer frequencies of quarterly and semi-annually. For the quarterly transfer frequency, the original formulation reaches a 175% optimality gap after 330 hours and the Benders formulation reaches a 409% gap after 233 hours. For the semi-annual transfers, the original formulation reaches a 136% gap after 260 hours, while the Benders formulation reaches a 160% gap after 100 hours.

6. Average runtimes for all experiments

Table 2 summarizes the average runtimes for each of the experiments conducted in this research.

	Original formulation				Benders Decomposition			
	Simplex Method		Barrier Method		Simplex Method		Barrier Method	
	No Initial Solution	Initial Solution	No Initial Solution	Initial Solution	No Initial Solution	Initial Solution	No Initial Solution	Initial Solution
Experiment 1a (Monthly transfers)	0.277	0.257	0.311	0.279	1.458	1.519	1.625	1.418
Experiment 1b (Monthly transfers)	65.9	1.41	0.39	0.40	2,390	8,239	794	72.0
Experiment 1b (Monthly transfers) (outlier removed)	0.27	0.31	0.36	0.30	2.21	2.53	4.73	1.78
Experiment 2 (Monthly transfers)	0.56	0.45	0.70	0.64	40.0	15.4	10.4	10.0
Experiment 3 (Monthly transfers)	811	31,042	16,289	6,813	100 hrs*	100 hrs*	100 hrs*	100 hrs*
Experiment 3 (Quarterly transfers)	161.3	12.6	156.9	2.2	36018**	36030**	36003**	36083**
Experiment 3 (Semi-annual transfers)	1.38	28.86	1.67	1.57	8,455	127.2	55.8	2,578
Experiment 4 (Monthly transfers)	400 hrs*	400 hrs*	400 hrs*	400 hrs*	400 hrs*	400 hrs*	400 hrs*	400 hrs*
Experiment 4 (Quarterly transfers)	95 hrs*	95 hrs*	95 hrs*	95 hrs*	95 hrs*	95 hrs*	95 hrs*	95 hrs*
Experiment 4 (Semi-annual transfers)	300.066***	100.102***	2371.956***	73.054***	5117****	3362****	52474****	2645****
Experiment 5 (Monthly transfers)	400 hrs*	400 hrs*	400 hrs*	400 hrs*	400 hrs*	400 hrs*	400 hrs*	400 hrs*
Experiment 5 (Quarterly transfers)	330 hrs*	330 hrs*	330 hrs*	330 hrs*	233 hrs*	233 hrs*	233 hrs*	233 hrs*
Experiment 5 (Semi-annual transfers)	260 hrs*	260 hrs*	260 hrs*	260 hrs*	92 hrs*	92 hrs*	92 hrs*	92 hrs*

Table 2. Average runtimes (in seconds) for each approach.
*all 10 seeds timed out after indicated amount of time
**seed 10 timed out after 100 hours
***seeds 5 – 10 timed out after 90 hours
****seeds 2 – 10 timed out after 90 hours

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IV. CONCLUSIONS AND RECOMMENDATIONS

The original JPAT formulation is already very efficient, and the only modification identified by this research to significantly increase its efficiency is to reduce the frequency of transfers in the model. Overall, the barrier algorithm performs better than the simplex algorithm, and the initial solution improves the performance of both the original formulation and the Benders formulation. This improvement is more significant in the Benders formulation than in the original formulation. Part A of this chapter presents a summary of findings. There are many other techniques that may benefit JPAT that we are unable to research further due to time constraints, some of which are described in part B of this chapter.

A. SUMMARY OF FINDINGS

Overall, the original formulation performs more efficiently than the Benders formulation. However, on larger problem instances than those considered in this thesis, the Benders formulations may prove to outperform the original formulation. We find that the barrier method and the initial solution speed up the Benders formulation more than the original formulation, and also that the number of configurations has more of an impact on solution times than the ok density does. The number of time steps causes the largest increase in solution times. Preliminary experiments indicate that an increase in the number of systems has a similar increasing effect on runtime as the number of time steps does. This is an area that needs further study as this thesis only conducts a small trial experiment varying the number of systems. This is most likely due to the additional model complexity brought about by increasing the number of variables $G_{t,y,l,l'}$.

We noted a significant decrease in runtimes when transfers are only allowed every three time steps (quarterly) instead of every time step (monthly). However, we find that this restriction may result in a significant decrease in the optimal objective value, although the magnitude of this decrease varies with the problem instance. Such a restriction could result in unfulfilled mission demands due to lack of equipment available at the location needed and the inability to transfer equipment in the current time step.

Allowing transfers only every 6 time steps results in a further decrease in runtimes, however, at the cost of a greater decrease in objective value obtained. Further study on the effects of placing restrictions on transfers may help increase model efficiency. One promising area for future research involves using the solution to an instance with restricted transfer frequency as an initial solution for the original model.

B. RECOMMENDATIONS FOR USAGE AND IMPLEMENTATION OF MODEL

1. Recommendations

JPAT has thus far been solved by TRAC without an initial solution and with the simplex algorithm using a rolling horizon approach. The initial solution and barrier method are beneficial for some instances, but not for others. Consequently, this research does not uncover a more efficient JPAT formulation that differs from the original formulation. Furthermore, as demonstrated by the variation in runtimes between different seeds within the same experiment, JPAT is sensitive to specific input data and highly variable in nature. Thus, it is difficult to make sweeping recommendations based on this research. However, testing these modifications on the original dataset rather than on randomly-generated data may result in differing recommendations.

2. Future Work

Some potential areas for future research are with different forms of decomposition techniques other than Benders decomposition. Further experimentation with different heuristics for initial solutions, including solutions obtained by restricting transfer frequencies, may also be beneficial. Additional areas for further research include testing on the original or other non-random datasets, experimenting further with a rolling horizon approach, manipulating solver settings, setting solver priorities, and letting a large problem run to completion.

APPENDIX. RANDOMLY GENERATED DATA FORMULATION

Following is the psuedocode for generating the random data used in this research.

$iqy_{y,l}$	$\max\left(0, floor\left[uniform\left(-15,5\right)\right]\right)$
$iq_{e,l}$	$\sum_y iqy_{y,l} es_{y,e}$
$d_{t,m}$	$\max\left(0, floor\left[uniform\left(-15,5\right)\right]\right) floor\left[uniform\left(1,20\right)\right]$
$ok_{m,i,c}$	$rand = uniform\left(0,1\right); if\left[rand < okdensity(okrun), ok_{m,i,c} = uniform\left(0,1\right)\right]$
omc_e	0
pc_y	$floor\left[uniform\left(5,50\right)\right]$
rc_y	0
$b_{t,y}$	$10,000,000$
p_m	$\min\left[uniform\left(0,1.2\right)\right]$
$ec_{c,e}$	$\max\left(0, floor\left[uniform\left(-15,5\right)\right]\right)$
$es_{y,e}$	$\max\left(0, floor\left[uniform\left(-15,5\right)\right]\right)$
he_e	$100\, floor\left[uniform\left(2,5\right)\right]$
hm_m	$s\max\left(i, hi_{m,i}\right)$
$hi_{m,i}$	$uniform\left(1,10\right)$
su_e	$floor\left[uniform\left(1,4\right)\right]$
$sr_{m,c}$	$floor\left[uniform\left(0,4\right)\right]$

$newyear_t$	$if\left\{ord\left(\frac{t}{12}\right)= floor\left[ord\left(\frac{t}{12}\right)\right]\right\}, newyear_t = 1, else \;\; newyear_t = 0$
$mr_{t,y}$	0
$ht_{e,y,l,l'}$	$uniform(24,48)$

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