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COMPUTATIONAL AND EXPERIMENTAL INVESTIGATION OF FLAPPING-FOIL PROPULSION

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Abstract-It is the objective of this paper to summarize the authors' recent work on flapping foils, including the application of flapping foils for boundary layer and flow separation control. Water tunnel experiments on sinusoidally plunging foils are described which elucidate the change in vortical wake pattern shed from the foil's trailing edge. These experiments were carried out using dye flow visualization and laser-Doppler velocimetry. It is found that the wake pattern is a strong function of the non-dimensional plunge velocity, with the wake topology changing from a typical Kármán vortex street to an inverse Kármán vortex street and to an asymmetric wake structure as the non-dimensional plunge velocity increases. The transition points between the various structures is dependent on scaling effects and wing and flow quality. These results are partly reproducible with inviscid panel code and Navier-Stokes code predictions. Additional interesting features are obtained if two degrees of freedom are permitted (pitch-plunge motions) or two airfoils in close proximity to each other are studied. Finally, experimental and computational results are presented which demonstrate the use of flapping foils for boundary layer and flow separation control.

I. INTRODUCTION

Bird flight and aquatic animal propulsion have fascinated many people for a long time. However, the lack of immediately apparent commercial or military applications has kept the funding level of studies devoted to these problems relatively low. It appears that there is an increasing interest in the development of micro air and water vehicles which, in turn, raises the question whether *unconventional* propulsion systems, such as those used by birds and aquatic animals for millions of years, deserve a *second look*.

The first explanation of the bird's ability to generate a thrust force by means of flapping its wings seems to have been published by Knoller (1909) in Vienna and Betz (1912) in Göttingen. Prandtl's student Birnbaum (1924) first presented a solution for incompressible flow past flapping airfoils, while Katzmayr (1922) in Vienna produced the first wind tunnel measurements which conclusively showed that an airfoil mounted in an oscillating wind stream experiences a thrust force. Garrick (1936) applied Theodorsen's theory to the problem of sinusoidally plunging and/or pitching airfoils and presented results valid for the whole reduced frequency range. In the 1940's and 50's Schmidt (1965) in East Germany started to conduct systematic experiments on flapping foil propellers which led him to the development of the *wave propeller* and its demonstration on a catamaran boat.

Theodorsen's oscillatory thin airfoil theory, as applied by Garrick, shows that the propulsive efficiency of a single harmonically plunging airfoil is only about 50 percent unless the airfoil oscillates rather slowly (which in turn requires a large airfoil in order to obtain significant thrust values). Schmidt sought to overcome this deficiency by arranging two airfoils in tandem, where the forward foil is oscillating and the rear-ward foil is stationary. This makes it possible to convert the vortical energy generated by the forward foil into additional thrust rather than being wasted. Schmidt claimed that his wave propeller achieved efficiencies comparable to those of conventional propellers and had the additional advantage of enabling operation in shallow waters.

The only predictive tool available to Schmidt was Theodorsen's inviscid oscillatory thin airfoil theory - a tool of only marginal utility. Therefore, he arrived at his wave propeller by pure experimentation. Future progress in the design and development of *unconventional* propellers will depend on a better understanding of the flow physics of such propellers and on the acquisition of reliable aerodynamic design data. This requires the systematic measurement and computation of the flow over flapping single foils and foil combinations.

It is the purpose of this paper to review our work on this problem over the past few years, to assess the current state-of-the-art, to point out discrepancies and deficiencies in our present understanding of flapping foil aerodynamics, and to indicate needs for future investigations.

II. SINGLE FLAPPING FOILS

Inviscid oscillatory thin-airfoil theory leaves many important questions unanswered. Among them are the effect of flapping amplitude, airfoil geometry, and most importantly, viscous flow effects, especially at the small Reynolds numbers encountered by micro air and water vehicles. Any improved prediction method must be validated by well controlled experiments. In this paper we limit ourselves to the discussion of two-dimensional flows.

Garrick (1936), using thin-airfoil theory, showed that both pitching and plunging airfoils develop a thrust force. The dependence of thrust and propulsive efficiency on reduced frequency is shown in Figs. 3,4,5,6 of Platzer *et al.* (1993). It is seen that the switch from drag to thrust occurs at a rather high frequency for pitching airfoils, whereas plunging foils develop thrust over the whole frequency range. However, note the decreasing thrust but increasing propulsive efficiency as the frequency decreases toward zero. It is therefore of interest to assess the validity of this prediction against available measurements. To this end it is important to recognize the difference between drag-indicative and thrust-indicative vortical wake patterns shed from flapping foils. Figures 1 and 2 (taken from Jones *et al.*, 1998) show the two wake patterns.

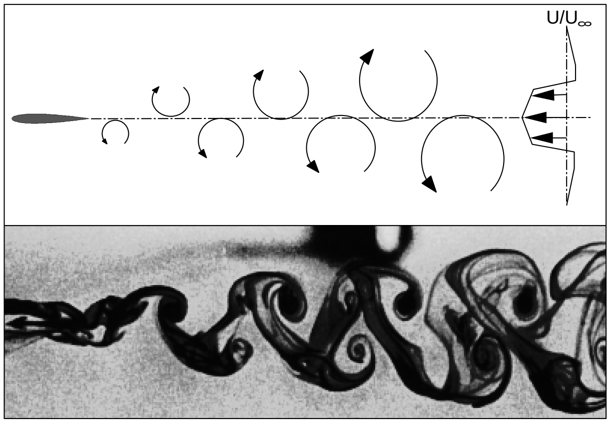

Fig. 1: Drag-indicative vortex street.

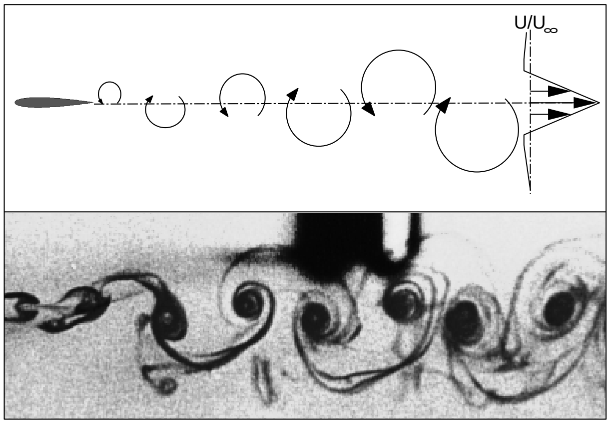

Fig. 2: Thrust-indicative vortex street.

Kármán vortex streets, shown in Fig. 1, produce a time-averaged velocity distribution with a distinct velocity deficit, indicative of drag. Reverse Kármán vortex streets, shown in Fig. 2, with counterclockwise upper vortices and clockwise lower vortices, produce a time-averaged jet-type velocity distribution, indicative of thrust.

Koochesfahani (1989) visualized the vortices shed from a NACA 0012 airfoil pitching sinusoidally about its quarter chord point. He also obtained LDV measurements of the time-averaged velocity profiles downstream of the trailing edge. He found that the vortex pattern indeed switches from drag-indicative Kármán vortex streets to thrust-indicative streets. A direct assessment of the drag/thrust from the measured wake deficit or surplus, however, is quite difficult and we refer to Fig. 13 and our discussion of this problem in Jones *et al.* (1998).

Lai and Platzer (1999) visualized and measured the vortical wakes shed from harmonically plunging NACA 0012 foils, using dye flow visualization and LDV for the measurement of the time-averaged velocity profiles. Similar to Koochesfahani, the tests were conducted in water at freestream speeds ranging from 0.05 to 0.21 m/sec, corresponding to Reynolds numbers from 500 to 20100. The amplitude/chord ratio and the frequency of the plunge oscillation were varied between 0 and 0.1 and 0 and 10 Hz, respectively. It was found that a good parameter to categorize the resulting phenomena is the product of the amplitude/chord ratio and the reduced frequency, representing the maximum plunge velocity-to-freestream speed ratio, or the maximum non-dimensional plunge velocity, which for pure plunge motions is the Strouhal number times π. The changes in observed vortex patterns as this speed ratio is increased is shown schematically in Fig. 7 of Lai and Platzer (1999). For zero and very low ratios the Kármán vortex street is observed which changes to the reverse Kármán street at values of approximately 0.3 and to a deflected street at values of approximately 1. For actual pictures of the vortex patterns we refer to Figs. 3, 4 and 5 of Lai and Platzer (1999). The LDV-measured time-averaged velocity profiles show that the maximum streamwise velocity measured downstream of the trailing edge starts to exceed the freestream speed as soon as the non-dimensional plunge velocity exceeds 0.25.

As previously noted in connection with Koochesfahani's measurements, it is difficult to convert the measured velocity defect or surplus into drag or thrust. However, it is clear that drag is generated at speed ratios less than 0.25. It is therefore interesting to compare the measurements with Navier-Stokes computed force values. Figure 6, from Tuncer and Platzer (1996), shows Navier-Stokes computed thrust and propulsive efficiency values for the sinusoidally plunging NACA 0012 airfoil at a chord-Reynolds number of 3 million and the comparison with oscillatory thin-airfoil theory. It is seen that thrust is predicted for all values of amplitude and frequency down to vanishingly small values. This prediction disagrees with the experimental findings. However, it must be noted that these Navier-Stokes calculations account only for the pressure drag and not for the viscous drag. Clearly, at low amplitudes and frequencies the viscous effects are dominant. Linear theory and Navier-Stokes predictions are in good agreement for small amplitude values. As one would expect, the thrust increases with increasing values of plunge amplitude and frequency. As already mentioned, linear theory predicts very high values of propulsive efficiency at low frequencies, decreasing to 50 percent with increasing frequency. It is apparent from the measurements and from the Navier-Stokes computations that the viscous effects nullify this optimistic prediction for the low frequency range.

Another interesting comparison can be made by using the inviscid panel code described by Platzer *et al.* (1993). This code accounts for the effect of airfoil geometry and vortex roll-up. Plotting the measured wake wave lengths as a function of plunge amplitude and frequency one obtains the results shown in Figs. 10 and 11 of Jones *et al.* (1998). It is seen that the wave length increases with increasing amplitude of oscillation, an effect which is reasonably well predicted by the panel code. On the other hand, the wave length decreases with frequency for a given amplitude. Again, the panel code predicts the experimental observations quite well. The panel code can also be used for a direct comparison with the measured time-averaged velocity profiles, as shown in Figs. 14, 15 and 16 of Jones *et al.* (1998). For the two lower non-dimensional plunge velocities the agreement is quite good. For a plunge velocity of 2.3 there is a deflected vortex wake and the panel code prediction is marginal. However, it is remarkable that the panel code correctly predicts the phenomenon of the asymmetric vortex pattern, as shown in Fig. 3 (taken from Jones *et al.*, 1998). Further work is necessary to clarify whether the measured profiles in this case are exclusively caused by the shedding of trailing edge vortices or by a combination of leading and trailing edge vortices.

Since thrust increases with increasing amplitude and frequency, the question arises whether it is more efficient to achieve a required thrust value by increasing amplitude and minimizing frequency or vice versa. This question can be answered to some extent by looking again at Fig. 6 of Tuncer and Platzer (1996). A given thrust coefficient, say 0.1, can be obtained with an amplitude of 0.3 and a reduced frequency of 0.5. If the amplitude is increased to 0.4 the reduced frequency is lowered to approximately 0.4 and the propulsive efficiency is generally increased (unless too small values of amplitude and frequency are chosen). This conclusion is further substantiated by Navier-Stokes computations of Tuncer *et al.* (1998) who investigated the dynamic stall characteristics of a sinusoidally plunging NACA 0012 airfoil at a Reynolds number of 1 million. As seen from their Fig. 3, dynamic stall is predicted as soon as the non-dimensional plunge velocity hk exceeds a value of approximately 0.35. Hence one can choose either a large amplitude and a small frequency or vice versa. However, if one wants to optimize the propulsive efficiency it is advantageous to operate in the large amplitude/low frequency range, as shown in Figs. 11a and 11b of Jones and Platzer (1997) and Fig. 4 of Tuncer *et al.* (1998).

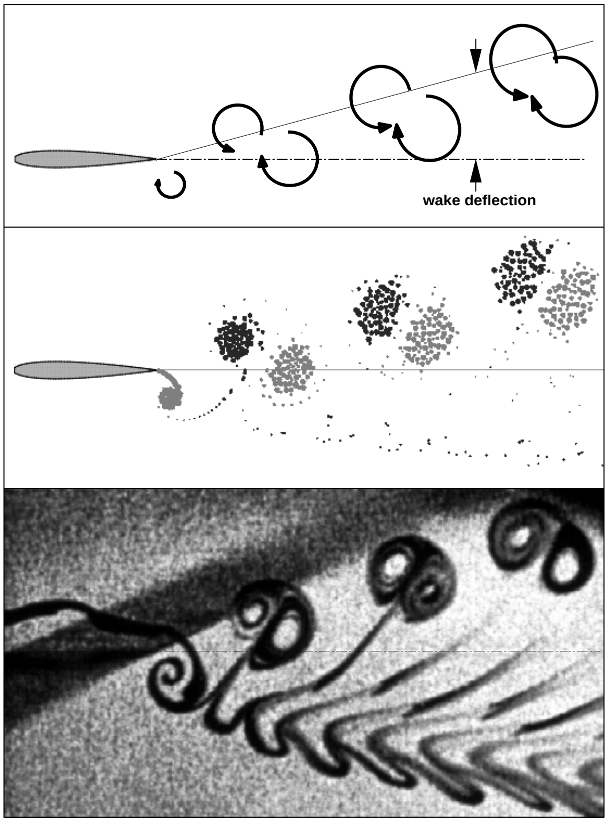

Fig. 3: Deflected wake comparison (kh=1.5).

Birds generally use a combination of pitching and plunging motion rather than a single degree of freedom pitch or plunge motion as discussed above. This expands the parameter space considerably. In addition to the pitch and plunge amplitudes one now has to consider the phase angle between the pitch and plunge motions. It is important to realize that the key parameter for determining whether an airfoil generates thrust or extracts power from a flow is the effective angle of attack, as illustrated in Fig. 4. Cases (a) and (b) represent the pure plunge and pitch modes. Case (c) is the neutral case (pure feathering) between thrust generation and power extraction. A negative effective angle of attack (relative to the flight path) leads to thrust generation, case (d), a positive angle to power extraction, case (e). Using the panel code, thrust, power, and propulsive efficiency can be studied as a function of phasing between pitch and plunge. This is shown in Figs. 5 and 6 (taken from Jones and Platzer, 1997). It is seen that for the parameter combination chosen in Fig. 5 thrust is generated for any phase angle between pitch and plunge, but the optimum efficiency occurs at around 90 degrees. Unfortunately, this corresponds to a minimum in the thrust coefficient. On the other hand, for the parameter combination of Fig. 6, power is extracted for phase angles near 90 degrees at both the highest efficiency and maximum power extraction. These findings are consistent with earlier results obtained by Lan (1979) for a rectangular wing of aspect ratio 8 using an unsteady quasi-vortex-lattice method.

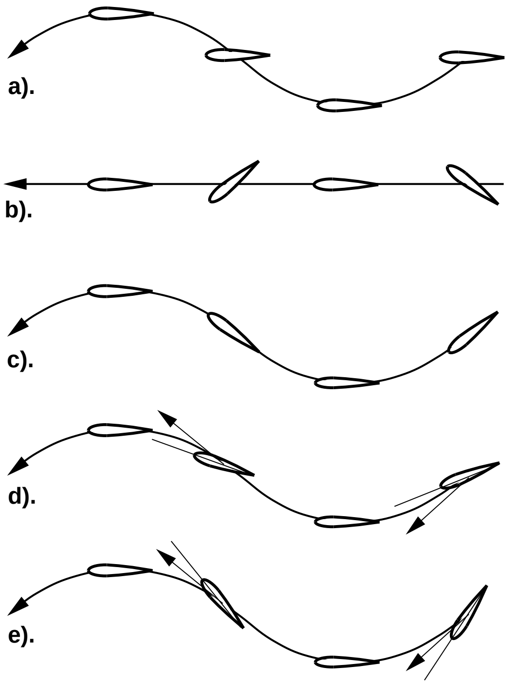

Fig. 4: Effective versus geometric α.

The predictive power of any inviscid flow theory must be evaluated by comparison with direct measurements of the thrust generated by flapping wings. Anderson *et al.* (1998) presented thrust and power measurements for a NACA 0012 airfoil executing combined plunge and pitch motions at a Reynolds number of 40000 and found good agreement with inviscid flow models over a certain parametric range. However, they also identified other test cases which revealed large discrepancies between inviscid theory and experiment. Consistent with the findings of Lai and Platzer (1999) mentioned earlier poor agreement was obtained with low frequency tests (where viscous effects are expected to be quite strong). Another phenomenon which cannot be easily modeled with inviscid flow theories is the shedding of leading edge vortices. In a series of flow visualization experiments Anderson *et al.* (1998) demonstrated the formation of leading edge vortices and, most significantly, showed that high thrust values can be achieved with high propulsive efficiency if the shedding of the leading edge vortex is properly timed. This seemed to occur for phase angles of about 75 degrees between the pitch and plunge motion.

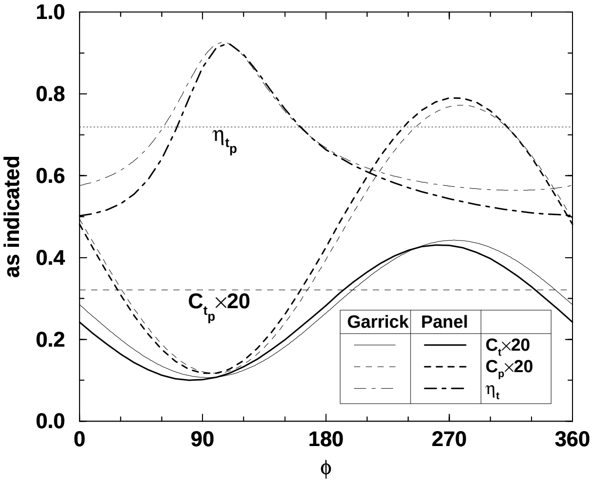

Fig. 5: Performance for k=0.5, h=0.2 and $\Delta\alpha$=4°.

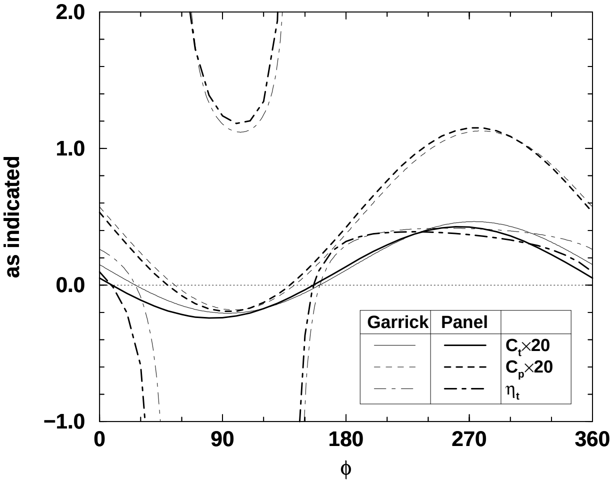

Fig. 6: Performance for k=0.5, h=0.2 and $\Delta\alpha$=8°.

These results show the importance of determining the dynamic stall boundaries of flapping airfoils, both experimentally and computationally. Tuncer *et al.* (1998) and Tuncer and Platzer (2000) therefore attempted to determine the dynamic stall boundary of the NACA 0012 airfoil at high Reynolds numbers using a Navier-Stokes analysis. In Fig. 1 of Tuncer *et al.* (1998) it is shown that a drastic loss of thrust is predicted for the sinusoidally plunging airfoil as soon as the dynamic stall limit is reached. Also, it is seen that the comparison with the inviscid panel code prediction is quite good below the stall limit.

Further Navier-Stokes calculations were made by Tuncer *et al.* (1998) and by Isogai *et al.* (1999) to determine the dynamic stall boundaries of NACA 0012 airfoils in combined pitch and plunge motion. Their calculations are generally in good agreement and show that the highest efficiency is obtained when the pitch oscillation leads the plunge motion by 90 degrees and no appreciable flow separation occurs. Using a finite element incompressible Navier-Stokes solver, Ramamurti *et al.* (1999) extended these calculations to the experiments of Anderson *et al.* (1998) involving the shedding of strong leading-edge vortices at Reynolds numbers of 1100. Their computations confirmed Anderson's observations that the phase angle between the pitch and plunge oscillation is the critical parameter in maximizing thrust or propulsive efficiency. Further computations and experiments are clearly needed to draw more definitive conclusions.

The special case of airfoil oscillation in a still medium provides additional interesting insights into the flow physics of flapping airfoils and confronts the computational aerodynamicist with a considerable challenge. Lai and Platzer (1998) studied the flow generated by a sinusoidally plunging NACA 0012 airfoil in still water using dye visualization and LDV techniques. They found that vortices are shed from the *trailing edge* and a jet is produced with a time-averaged jet velocity which is greater than the peak plunge velocity for several chord lengths downstream of the trailing edge. Consistent with the earlier findings the jet axis is deflected relative to the chord line because the non-dimensional plunge velocity now tends toward infinitely large values. Airfoils having a sharp trailing edge and a rounded leading edge therefore generate static thrust. However, the precise flow details around the leading edge could not be ascertained in these tests. The effective angle of attack induced by the plunge oscillation is certainly very large and it is likely that dynamic stall vortices are shed from the leading edge. Further experimental and computational studies are required to identify the precise flow features.

III. INTERFERENCE EFFECTS

It is apparent from the above review of the flow physics of single flapping foils that our understanding and prediction of flapping foil aero/hydrodynamics is largely limited to cases where no or little flow separation occurs. Yet, it is also clear that dynamic stall is likely to occur in many practical applications. Unfortunately, it is still unclear whether the dynamic stall phenomenon limits the use of flapping foil propellers or whether it is possible to take advantage of it for the generation of significant thrust values with high propulsive efficiency.

As already mentioned, Schmidt (1965) sought to develop an efficient flapping foil propeller by exploiting the interference effect between a flapping fore-wing and a non-flapping hind-wing. The stationary hind-wing is exposed to an oscillatory flow and therefore can exploit the Katzmayr effect, i.e., convert the vortical energy generated by the flapping fore-wing into additional thrust. Bosch (1978) presented the first computational analysis of this tandem foil arrangement using oscillatory thin-airfoil theory. Platzer *et al.* (1993) generalized the previously mentioned unsteady panel code to the computation of incompressible flow past two oscillating airfoils whereby the position of the two airfoils relative to each other can be quite arbitrary. Hence it is possible to study the interference effects between two oscillating airfoils. Two arrangements are of greatest interest, namely the tandem and the biplane arrangement, shown as Fig. 7b and 7c, respectively.

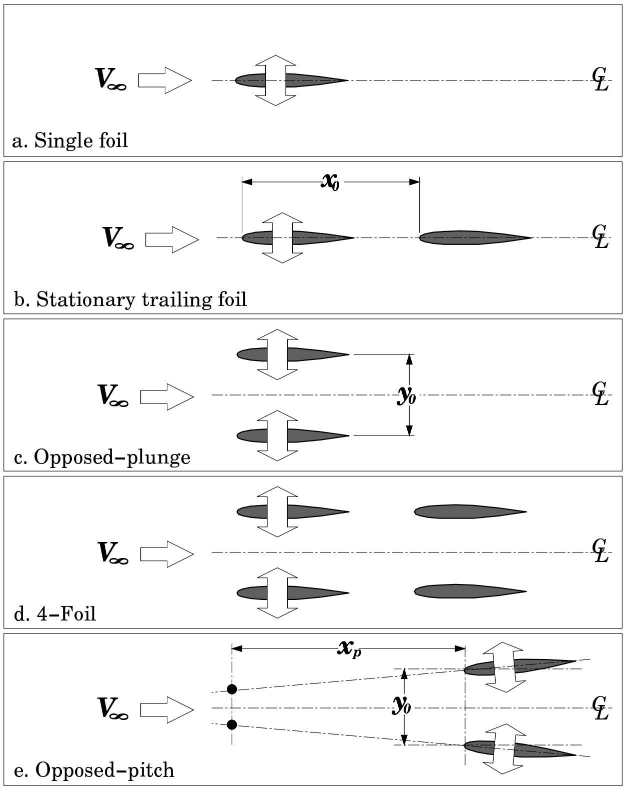

Fig. 7: Numerical and experimental configurations.

The latter arrangement is equivalent to a single airfoil oscillating in ground effect provided the oscillation of the two airfoils in the biplane arrangement occurs in counter-phase. Jones and Platzer (1999) performed a more detailed computational and experimental investigation of both arrangements. To this end the model shown in Fig. 8 was built and tested which allowed the flapping of the two fore-wings in either a pure plunge mode or a combined pitch/plunge mode with the optional mounting of two stationary hind-wings as shown in Fig. 7d. For a detailed description of the laser technique used to measure the thrust we refer to this paper. The panel code analysis reveals that the biplane arrangement (or flight close to a ground plane) has a quite favorable thrust enhancement effect. This prediction could be confirmed with the thrust measurements shown in Fig. 18 of Jones and Platzer

(1999). On the other hand, the Schmidt effect could not be confirmed in the experiments conducted to date. While the inviscid panel code analysis shows significant thrust enhancement generated by the hind-wing, the experiment indicates that the hind-wing's viscous drag nullifies this additional thrust. However, it remains to be seen whether this conclusion still holds if the hind-wing also executes a flapping motion. Lan (1979) performed an inviscid vortex lattice analysis of two rectangular flapping wings in tandem arrangement and he showed that this dragonfly arrangement can produce high thrust with high efficiency if the pitching is in advance of the flapping and the hind-wing leads the fore-wing with some optimum phase angle.

Recently, Jones and Platzer (2000) presented additional thrust measurement on the 6 gram micro-air vehicle configuration shown in Figs. 9 and 10. The measured static thrust values are in good agreement with the panel code prediction, but there is a significant decrease in thrust for increasing free-stream speeds, as seen in their Figs. 24 and 25. This discrepancy is likely to be caused by the shedding of leading edge vortices (dynamic stall). However, it is unclear why good thrust values are achieved at zero free-stream (in the presence of dynamic stall) and a rapid thrust loss occurs at small wind speeds. Flow visualizations and measurements are presently in progress to clarify the flow features which cause this loss of thrust.

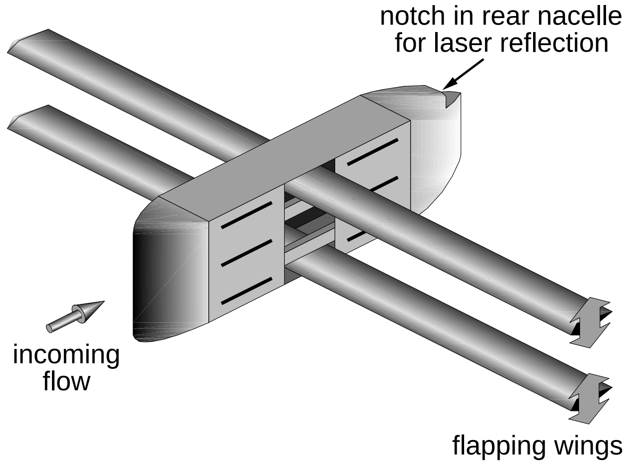

Fig. 8: Isometric view of the large model.

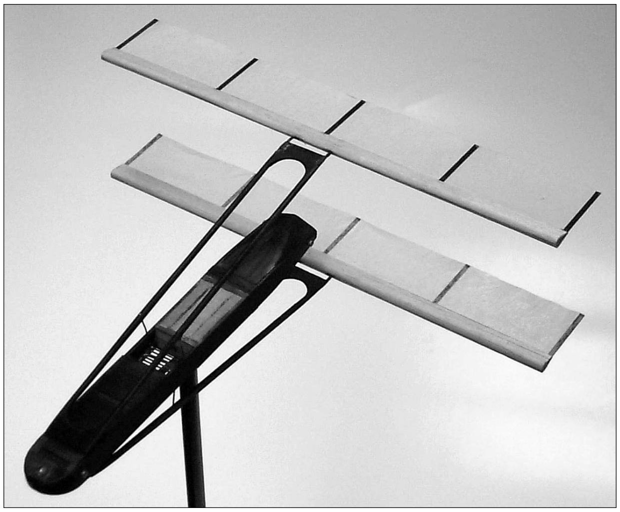

Fig. 9: Isometric view of the MAV.

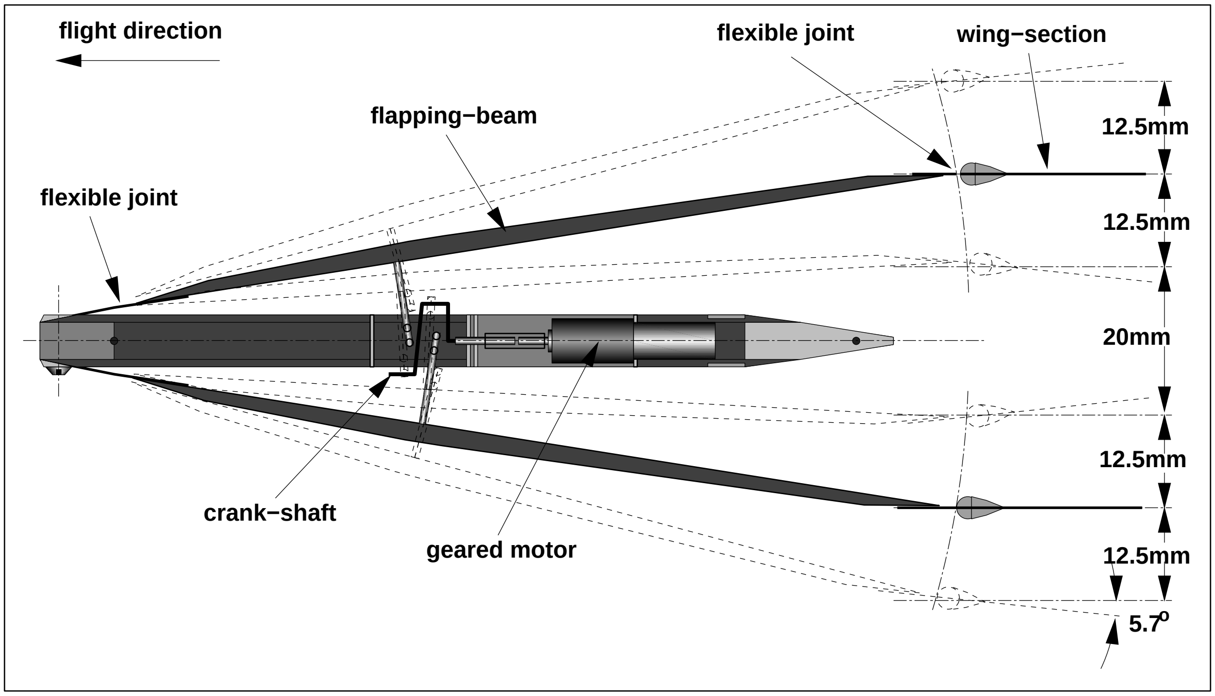

Fig. 10: Schematic side view of the 15cm MAV model (actual size).

IV. BOUNDARY LAYER PROPULSION AND FLOW SEPARATION CONTROL

As shown in the previously described experiments and computations, a flapping foil is a device which generates a jet flow downstream of the trailing edge and imparts additional momentum to the flow. Hence the question arises whether small flapping foils can be used to energize a boundary layer flow or to prevent/delay flow separation. Lai, Yue, Platzer (1997), Dohring (1998), and Dohring *et al.* (1998) performed several experiments to explore the flow physics of foils flapping in a boundary layer or in a separated flow region.

Flow visualization studies, LDV measurements and Navier-Stokes computations were performed to determine the flow characteristics generated by a small foil which executes a sinusoidal plunge motion in a flat-plate boundary layer. It was found that the jet velocity generated downstream of the plunging foil increases with decreasing distance from the flat plate. The plunge amplitude, frequency and distance from the wall were varied quite systematically and it was found that there is an optimum spacing between plate and foil. The jet velocity could be almost doubled compared to the values measured in a free-stream. This finding is quite consistent with the previously mentioned favorable biplane interference effect where the two foils oscillate in counter-phase, simulating the ground effect. The measured and the Navier-Stokes computed velocity profiles downstream of the oscillating foil agree quite well, as shown for example in Fig. 8 of Dohring *et al.* (1998). Similarly, the dye visualizations of the vortical flow patterns could be reproduced quite well with the Navier-Stokes computations, as documented by Dohring (1998).

In a second experiment, a small plunging airfoil was mounted in the separated wake flow region of an airfoil which had a semi-circular or a cusped trailing edge. Complete flow re-attachment could be achieved by selecting the proper plunge amplitude and frequency. As shown by Dohring (1998) and Dohring *et al.* (1998), Fig. 14, the controlling parameter was again found to be the non-dimensional plunge velocity.

In a third experiment, the sinusoidally plunging foil was mounted in the recirculatory flow region caused by the flow over a backward-facing step. The experimental setup is shown in Fig. 1 of Lai *et al.* (1997). The extent of the separated flow region could be reduced by more than 60 percent by proper placement, frequency, and amplitude of the plunging foil, as shown in Figs. 9-12 *(Lai et al., 1997).*

Finally, in a fourth series of experiments a small plunging foil was placed in the wake of a conventional airfoil. Lai and Platzer (1998) could show that the velocity defect in the wake could be substantially reduced, making the wake almost indistinguishable from the free-stream flow.

V. SUMMARY

In this paper we attempted to summarize the current knowledge about the unsteady aerodynamic and propulsive characteristics of flapping single foils and foil combinations. It is apparent that the vortical wakes shed from flapping foils are a strong function of the non-dimensional plunge velocity (Strouhal number $\times \pi$), changing from drag-indicative wakes at low plunge velocity to thrust-indicative symmetrical wakes and to asymmetric thrust/lift-indicative wakes with increasing plunge velocity. The flow visualizations, LDV measured time-averaged wake velocity profiles and the direct thrust measurements are predicted well with the panel and Navier-Stokes codes for thrust-indicative symmetric wakes shed from foils in pure plunge oscillation. At low plunge velocities the prediction of the unsteady aerodynamic characteristics is quite difficult because the viscous effects are dominant. The change from a symmetric wake to an asymmetric one is predicted both by the panel and the Navier-Stokes code. However, further experimental and computational work is required to clarify the question of how much vorticity shed from the leading edge contributes to this.

Considerable insight could be gained about the merits of using two foils. The tandem arrangement, used in Schmidt's wave propellers, where the upstream foil is oscillating and the downstream foil is stationary, appears to be relatively unattractive. The inviscid panel code analysis shows improved efficiency, but the inclusion of the viscous effects nullifies this advantage. On the other hand, the biplane arrangement offers considerable benefits in optimizing achievable thrust and efficiency compared to the single foil. However, it remains to be studied whether the dragonfly arrangement, where both the upstream and downstream foils are oscillating, is superior.

Also, it could be shown that flapping foils can be used successfully to influence the boundary layer and flow separation characteristics. Small flapping foils mounted in the boundary layer of a flat-plate and oscillating in the pure plunge mode generate a time-averaged jet velocity which is significantly larger than obtainable in the free-stream, thus exhibiting a very favorable ground effect. The flapping foil can be regarded as a two-dimensional propeller which entrains the upstream flow and imparts a significant momentum to the downstream flow. This effect could be demonstrated by suppressing or delaying the flow separation from the rounded trailing edge of a stationary foil by means of a small flapping foil mounted in the wake of the stationary foil. Also, the wake velocity defect could be eliminated or reduced by imparting flow momentum to the wake with the small flapping foil. In another experiment, the recirculatory flow region generated by the flow over a backward-facing step could be reduced quite substantially by positioning a small flapping foil inside the recirculatory flow region.

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