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## GEOPHYSICS OF DEEP CONVECTION AND DEEP WATER FORMATION IN OCEANS

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### ABSTRACT

An important task for geophysicists studying deep convection and deep water formation in the ocean is to investigate various physical mechanisms generating vertical circulation. Two major types of such mechanisms are available so far, namely, thermodynamical instability, which generates convection and mixing due largely to surface buoyancy flux brought by net brine release or surface cooling, and dynamical instabilities, which also produce various vertical cells exchanging water masses at different depths. After reviewing various mechanisms, in this short note, symmetric instability is found to be very important for both near-boundary and open ocean deep convections. The symmetric instability is a combined inertial (or centrifugal) and baroclinic instability. Vertical cells driven by the symmetric instability are in planes perpendicular to the shear flow. However, vertical cells generated by the baroclinic instability are in the same plane as the shear flow. For the near-boundary deep convection, especially in off-shore wind prevailing regions (e.g., the Weddell Sea, the Ross Sea), where onshore Ekman flux is not available, the symmetric instability plays an important role in driving dense water off the shelf. For the open-ocean deep convection, the symmetric instability of a background cyclonic circulation generates vertical cells, exchanging water masses and reducing the stratification of the water column within the gyre. Therefore, the symmetric instability is a strong candidate for being a dominant mechanism for 'preconditioning' for open ocean deep convection.

### INTRODUCTION

Deep convection in oceans is closely related to climate change on long time scales. Without deep convection in oceans, there is only a traditional positive radiation/climate feedback mechanism (Budyko, 1969; Sellers, 1969). A colder surface of the earth yields more ice, which increases the albedo, which leads to a decrease in net heat absorption and so a colder surface. With deep convection in oceans, a negative feedback mechanism becomes available (Killworth, 1983). A colder ocean surface does produce more ice, which by brine ejection gives a cold, saline surface layer. This leads to convection, bringing warmer water from deep layer; the warm water melts some of the ice and decreases the surface albedo, allowing more heat to be absorbed, warming the surface.

Since deep convection in oceans is a primary ingredient in the earth climate system, it becomes important to understand the basic mechanisms driving deep convection in

global warming and climate change before coupling this component (deep ocean convection) into climate models. From the geophysical point of view, the mechanisms driving vertical cells are treated as the main forcing for deep ocean convection and are classified into two categories: thermodynamical and dynamical.

Thermodynamical mechanisms driving deep convection invoke directly or indirectly to a convective instability. Any mechanisms that cause a denser water mass to lie over a lighter water mass belong to this category, e.g., convective instability, double diffusion, thermobaric instability, cabbeling instability, and potential instability.

Dynamical mechanisms contributing to deep convection are referred to as dynamical instabilities. Any dynamic instability generating vertical cells belongs to this category, e.g., Kelvin-Helmholtz instability, baroclinic instability, centrifugal instability, and symmetric instability. The vertical cells driven by dynamical instabilities not only exchange water mass in the vertical direction, which reduces the stratification and contributes to the preconditioning of open ocean deep convection, but also play important role in driving dense water mass off the continental shelves in near-boundary convection.

## 2. VERTICAL CELLS DRIVEN BY THERMODYNAMIC INSTABILITIES

This convection takes place inside the fluid, where the vertical gradient of the density  $\rho$  becomes positive, i.e.,

$$\frac{\partial \rho}{\partial z} > 0 \quad (1)$$

The equation of state of sea water is given by

$$\rho = \rho_0 [1 - \alpha(\theta - \theta_0) + \beta(S - S_0)] \quad (2)$$

where  $\theta$ ,  $S$  are potential temperature and salinity, and  $\theta_0$ ,  $S_0$ , and  $\rho_0$  are reference values. The thermal expansion coefficient ( $\alpha$ ) and saline contraction coefficient ( $\beta$ ) are defined by (McDougall, 1987)

$$\begin{aligned} \alpha &= -\frac{1}{\rho_0} \frac{\partial \rho}{\partial \theta} \Big|_{S,p} = -\frac{1}{\rho_0} \frac{\partial \rho}{\partial T} \Big|_{S,p} \left[ \frac{\partial \theta}{\partial T} \Big|_{S,p} \right]^{-1} \\ \beta &= \frac{1}{\rho_0} \frac{\partial \rho}{\partial S} \Big|_{\theta,p} = \frac{1}{\rho_0} \frac{\partial \rho}{\partial S} \Big|_{T,p} + \alpha \frac{\partial \theta}{\partial S} \Big|_{T,p} \end{aligned} \quad (3)$$

Generally,  $\alpha$  and  $\beta$  are functions of potential temperature, salinity, and pressure:

$$\alpha = \alpha(\theta, S, p), \quad \beta = \beta(S, p) \quad (4)$$

which implies that the dependence of  $\beta$  on  $\theta$  is very small. The Brunt-Vaisala frequency is

$$N^2 \equiv -\frac{g}{\rho_0} \frac{\partial \rho}{\partial z} = g \frac{\partial}{\partial z} [\alpha(\theta - \theta_0) - \beta(S - S_0)] \quad (5)$$

The convective instability takes place where the square of the Brunt-Vaisala frequency becomes negative, i.e.,

$$N^2 < 0, \quad (6)$$

which causes rapid overturning in the fluid.

Since the stratification of sea water is generally stable, convective instability does not occur very often in the deep ocean. Therefore, an important task for physical oceanographers is to investigate physical processes that cause the reduction of stratification, i.e.,

$$\frac{\partial}{\partial t} (g^{-1} N^2) < 0 \quad (7)$$

Substitution of (5) into (7) leads to

$$\begin{aligned} \frac{\partial}{\partial t} (g^{-1} N^2) = & \frac{\partial}{\partial z} \left\{ (\alpha \frac{\partial \theta}{\partial t} - \beta \frac{\partial S}{\partial t}) + (\theta - \theta_0) \left[ \frac{\partial \alpha}{\partial \theta} \frac{\partial \theta}{\partial t} + \frac{\partial \alpha}{\partial S} \frac{\partial S}{\partial t} + \frac{\partial \alpha}{\partial p} \frac{\partial p}{\partial t} \right] \right. \\ & \left. - (S - S_0) \left[ \frac{\partial \beta}{\partial S} \frac{\partial S}{\partial t} + \frac{\partial \beta}{\partial p} \frac{\partial p}{\partial t} \right] \right\} < 0 \end{aligned} \quad (8)$$

From Gregg (1984) it can be shown that, for all practical purposes, the first law of thermodynamics can be regarded as a conservation equation for potential temperature  $\theta$ . The conservation equations for heat and salt are

$$\frac{\partial \theta}{\partial t} = A_\theta + M_\theta - \frac{\partial D_T}{\partial z} \quad (9)$$

$$\frac{\partial S}{\partial t} = A_S + M_S - \frac{\partial D_S}{\partial z} \quad (10)$$

Here

$$A_\theta = -\vec{V} \cdot \nabla \theta, \quad A_S = -\vec{V} \cdot \nabla S$$

are advection of potential temperature and salinity, and

$$M_\theta = \nabla_2(K\nabla_2\theta) - \frac{\partial F_\theta}{\partial z}, \quad M_S = \nabla_2(K\nabla_2S) - \frac{\partial F_S}{\partial z}$$

are the divergence of the fluxes of  $\theta$  and  $S$  due to lateral mixing and small-scale turbulent mixing in the vertical direction, and  $F_\theta$  and  $F_S$  are upward heat and salt turbulent fluxes. Horizontal eddy diffusivity is  $K$ . Heat and salt fluxes due to double diffusion are  $D_T$  and  $D_S$ . Substitution of (9) and (10) into (8) leads to

$$\frac{\partial}{\partial t}(g^{-1}N) = G_1 + G_2 + G_3 + G_4 + G_5 < 0 \quad (11)$$

where  $G_1, G_2, G_3, G_4$ , and  $G_5$  are five terms representing different physical processes of thermodynamical destabilization: buoyancy flux induced instability, potential instability, double diffusion, thermobaric instability, and cabbeling instability.

The buoyancy induced instability arises from the condition

$$G_1 = \frac{\partial}{\partial z}(\alpha M_\theta - \beta M_S) < 0 \quad (12)$$

The effect of horizontal mixing on this instability has not been studied much, largely due to the uncertainty in the value of  $K$ . An approximate form of  $G_1$ , widely used by oceanographers, is

$$G_1 \approx -\frac{(\alpha F_\theta - \beta F_S)|_{z=0}}{H^2} < 0 \quad (12a)$$

which indicates the convection generated by surface buoyancy loss (Fig.1).

Potential instability is primarily a result of differential flows at synoptic scale, causing

$$G_2 = \frac{\partial}{\partial z}(\alpha A_\theta - \beta A_S) < 0 \quad (13)$$

This mechanism is recognized by the meteorologists in explaining the meso-scale vertical circulations within extra-tropical cyclones due to a cool, dry flow moving over a warm, moist flow (Browning, 1974). However, the validity of this mechanism as it pertains to the deep convection in oceans is still an open question. This is mostly due to the lack of de-

tailed current measurements in the deep ocean. The generation of convection in deep currents by vertical-differential advection and mixing will be verified when good data sets become available.

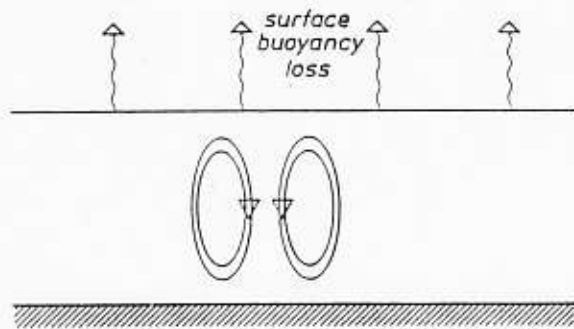


Fig. 1. Vertical circulation driven by the upward surface buoyancy flux.

Double diffusive convection takes place when

$$G_3 = \frac{\partial}{\partial z} \left( \beta \frac{\partial F_S}{\partial z} - \alpha \frac{\partial F_T}{\partial z} \right) < 0 \quad (14)$$

This type of convection is driven by the difference between heat diffusion (fast) and salt diffusion (slow). The idea of double diffusion of heat and salt was developed by Stern (1960) and Stommel (1962), and this gave rise to a series of theoretical and laboratory investigations of thermohaline convection. Laboratory experiments by Turner and Stommel (1964) and Turner (1965) showed that the occurrence of cold, and relatively fresh, water over dense, warm, and salty water results in a successive convective layers in which convection is induced by fast upward heat diffusion. The penetrative convection through the upper boundary of each layer is delayed by a density microjump created at this boundary by mixing in a stable gradient. There are two kinds of double diffusive convection, namely, diffusive staircases (in which relatively fresh, cold water overlies warmer, saltier water) and salt finger staircases (in which warm, salty water overlies cooler, fresher water). From the northwestern Weddell Sea during March 1986 (Muench et al., 1990), two typical vertical length-scales were found (1-5 m, and 100 m) in the layers in the staircases of the upper, steeper portion of the thermocline. This mechanism cannot explain the formation of extensive convective layers such as "chimneys".

Thermobaric instability appears when

$$G_4 = \frac{\partial}{\partial z} \left\{ [(\theta - \theta_0) \frac{\partial \alpha}{\partial p} - (S - S_0) \frac{\partial \beta}{\partial p}] \frac{\partial p}{\partial t} \right\} < 0 \quad (15)$$

and cabbeling instability occurs when

$$G_5 = \frac{\partial}{\partial z} \left\{ (\theta - \theta_0) \frac{\partial \alpha}{\partial \theta} \frac{\partial \theta}{\partial t} + [(\theta - \theta_0) \frac{\partial \alpha}{\partial S} - (S - S_0) \frac{\partial \beta}{\partial S}] \frac{\partial S}{\partial t} \right\} < 0 \quad (16)$$

Both instabilities are caused by nonlinearities in the equation of the state of seawater (McDougall, 1987).

### 3. VERTICAL CELLS DRIVEN BY DYNAMIC INSTABILITIES

The thermodynamic instabilities (buoyancy driven, potential, double diffusive, thermobaric, cabbeling) generally generate sporadic convection events. Only by being associated with some dynamical mechanism or some "preconditioning" may these thermodynamical instabilities take an active role in generating two main types of convection, namely, open-ocean deep convection and near ocean boundary convection (Killworth, 1983). It becomes necessary to review the different features of various dynamical instabilities, e.g., Kelvin-Helmholtz, baroclinic, symmetric, and centrifugal instabilities.

In a stably stratified ocean, Kelvin-Helmholtz instability takes place when the destabilizing influence of the shear flow overcomes the stabilizing effect of the buoyancy force. This can happen when the Richardson number is locally below its critical value of 0.25, i.e.,

$$Ri \equiv N^2/U_z^2 < 0.25 \quad (17)$$

where  $U_z = \partial U/\partial z$ . If the mean flow,  $U$  is geostrophically balanced, vertical shear (i.e.,  $U_z^2 > 0$ ) may cause baroclinic instability. The vertical cells generated by both Kelvin-Helmholtz and baroclinic instabilities are in the same plane of the shear flow (Fig.2), however, the horizontal scales are quite different. The vertical cells generated by the Kelvin-Helmholtz instability have a much smaller horizontal scale (comparable with the thermal convection scale) than that associated with baroclinic instability (Table 1).

Symmetric instability is known to occur as a series of two-dimensional rolls aligned with vertical shear flow (i.e., vertical cells are generally perpendicular to the flow, see



Fig.3) in a stably stratified ocean when the product of the Coriolis parameter  $f$  and the Ertel potential vorticity  $q$  becomes negative.

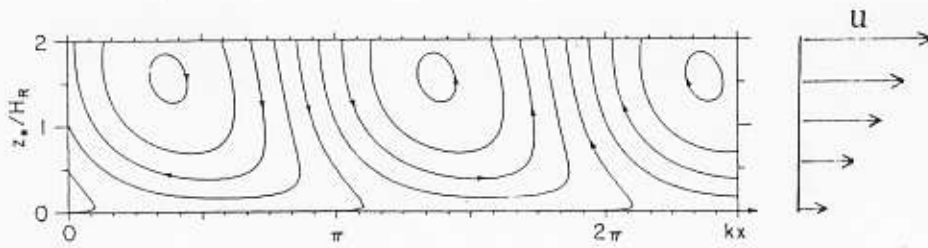


Fig. 2. Vertical circulation driven by the baroclinic instability and predicted by the Charney model, show that the vertical cells are in the same plane of the shear flow (Gill, 1982).

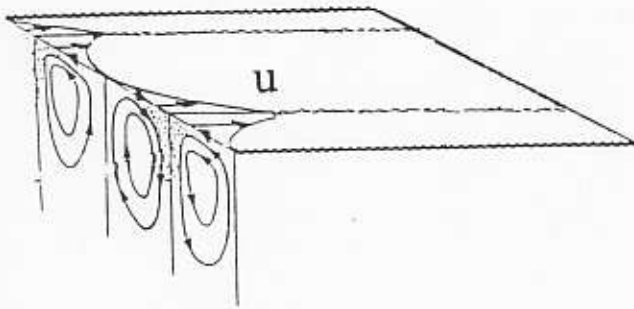


Fig. 3. The vertical circulation driven by the symmetric instability, show that the vertical cells are in planes perpendicular to the shear flow.

For a basic flow with thermal wind balance,  $q$  may be expressed in terms of the Richardson number, stratification, and vertical and horizontal shears (Emanuel, 1983):

$$q = \frac{fN^2}{g} \left( \frac{f + \bar{\zeta}}{f} - \frac{1}{Ri} \right), \quad \bar{\zeta} = -U_y \quad (18)$$

which leads to a necessary condition for the symmetric instability to occur (Table 1):

$$Ri + \frac{\bar{\zeta}}{f} Ri < 1 \quad (19)$$



Centrifugal instability, which is the symmetric instability in a vortex, was discovered by Rayleigh (1880). The vortex becomes unstable when the square of the angular momentum decreases outward (Rayleigh, 1916). Later, Taylor (1923) confirmed Rayleigh's discovery by his experimental and theoretical research on flow between coaxial rotating cylinders. When the outer cylinder rotates (with angular velocity  $\Omega_2$ ) in the same direction as the inner cylinder (with angular velocity  $\Omega_1$ ), but with a larger angular velocity that the square of the angular momentum increases outward, the flow is stable. When the outer cylinder rotates in the opposite direction ( $\Omega_1 < 0, \Omega_2 > 0$ ), or rotates in the same direction with a smaller angular velocity ( $\Omega_1 > \Omega_2 > 0$ ), the square of the angular momentum decreases with radius somewhere between the two cylinders, where the flow becomes unstable, and the vertical cells are intensified (Fig.4). Charney (1973) derived a general form for this instability. Chu (1991) discussed the formation of vertical cells inside the vortex. Analogous to the convection, the vertical cells generated inside the vortex can also transport heat and salt vertically and reduce the stratification. Therefore, this mechanism can be considered as another possible contributor to preconditioning.

TABLE 1

Criteria and horizontal scales for different types of instability.

Instability type	Criterion for instability	Horizontal scale
Baroclinic	$ \bar{U}_z  > 0$	$NH f$
Kelvin-Helmholtz	$Ri < 1/4$	$\bar{U}_z \bar{U}_{zz}$
Symmetric	$Ri + \bar{\zeta} Ri f < 1$	$\bar{U}_z H f$
Convective	$Ri < 0$	$H$

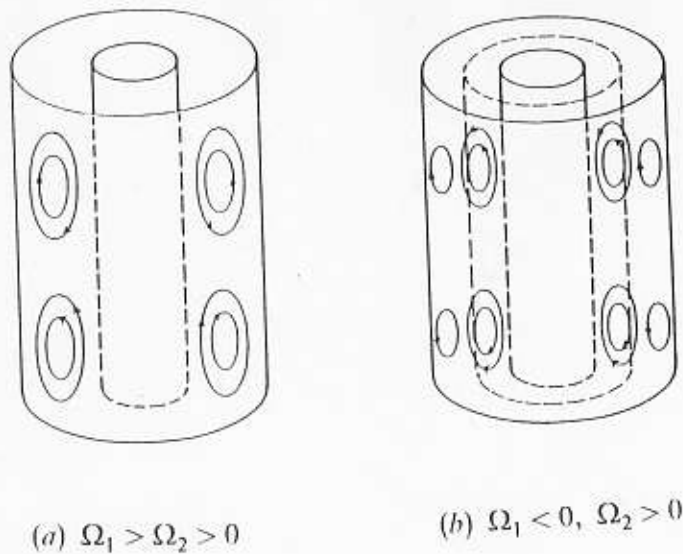


Fig. 4. Vertical cells generated between coaxial rotating cylinders (after Charney, 1973).

#### 4. IMPORTANCE OF SYMMETRIC INSTABILITY IN DEEP CONVECTION

##### 4.1 Near-Boundary Convection

An important dynamical component for the near boundary convection, as mentioned by Killworth (1983), is the onshore surface Ekman transport by the prevailing winds. However, observation (Fig.5) shows that the prevailing coast-parallel winds near Weddell Sea and Ross Sea ice edges are virtually across the ice edge. The climatological data collected by the Antarctic Automatic Weather Station also shows the cross ice edge winds prevailing in the Ross Sea (Stearns and Weidner, 1987). The Ekman transport generated by such a wind field is parallel to the coast, not cross-shelf. Therefore, we have to find other dynamical mechanisms for driving dense water off the shelf.

Vertical cells generated by the symmetric instability are in the plane perpendicular to the flow (Fig.3). The importance of this instability has been recognized by meteorologists for a long time (e.g., Solberg, 1933; Kuo, 1954). However, it has been given less attention by the oceanographic community. In order to investigate the structures of these cells, Emanuel (1979) formulated an equation for the streamfunction  $\psi$  for the disturbances generated by a mean flow  $U$  (in x-direction) with constant vertical and horizontal shears ( $U_z, U_y$ ), and constant stratification. The mean flow, confined between two horizontal rigid boundaries, is in hydrostatic and geostrophic equilibrium. The disturbances are stationary overturning cells in the (y,z)-plane.

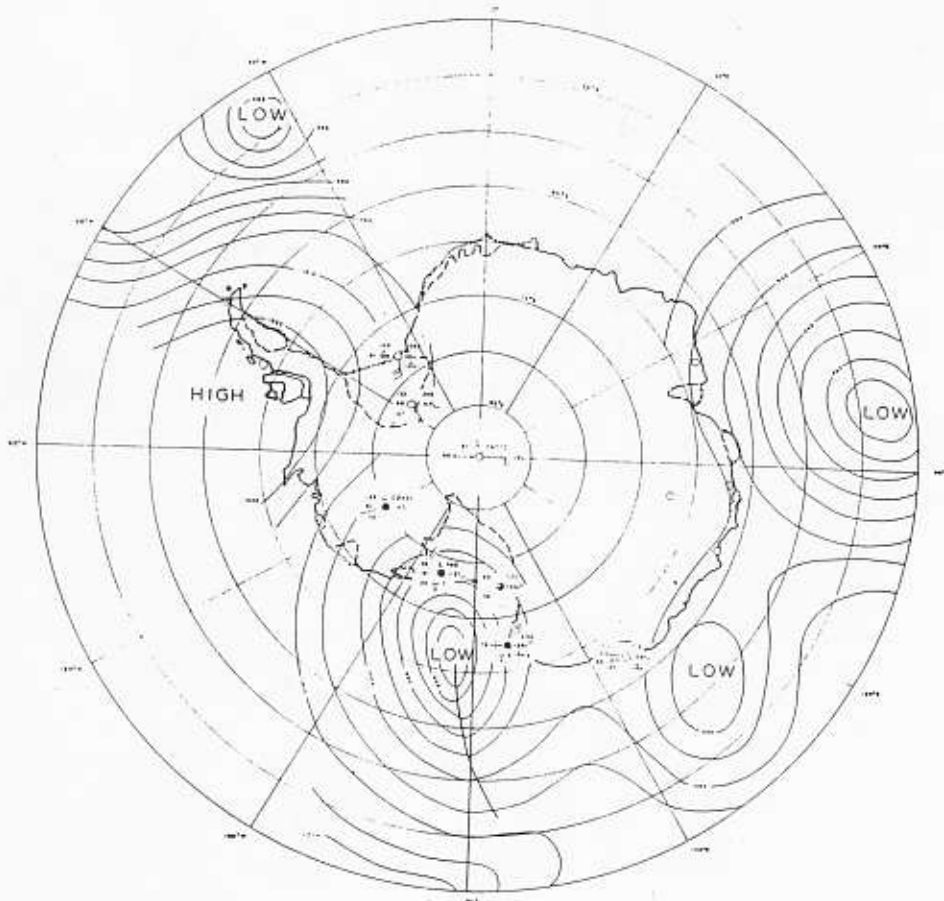


Fig. 5. Surface weather map at 1200, 11 August 1958 (after Vickers, 1966).

The streamfunction of the disturbances  $\psi$  satisfies the following equation as the growth rate is just zero (Emanuel, 1979)

$$P \frac{\partial^6 \psi}{\partial z^6} + \chi \left( \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial y \partial z} \right) + \frac{\partial^2 \psi}{\partial z^2} = 0 \quad (20)$$

where the viscosity and shear parameters  $P$  and  $\chi$  are defined by

$$P = \frac{f}{f - U_y} \frac{v^2}{f^2 H^4}, \quad \chi = \frac{f}{f - U_y} \frac{(1 + Pr)^2}{RiPr} \quad (21)$$

Here  $\nu$  is the eddy viscosity,  $H$  is the depth of the fluid, and  $Pr$  is the Prandtl number,  $\nu/\kappa$ , where  $\kappa$  is the eddy diffusivity. The vertical and lateral coordinates in (20) have been normalized according to

$$z_* = H z, \quad y_* = H \frac{N^2 Pr}{fU_2(1 + Pr)} y \quad (22)$$

where the asterisks denote dimensional quantities. An example of the structure of symmetric instability is shown in Fig.6. The streamlines take the form of sloped rolls. In the absence of onshore Ekman transport due to the cross-coastal winds (Fig.5), the current is virtually parallel to the ice edge. The overturning cells generated by the symmetric instability is a possible dynamical mechanism to drive the dense water off the shelf. The importance of this mechanism for the near boundary convection needs to be verified by a more sophisticated model with a complex topography.

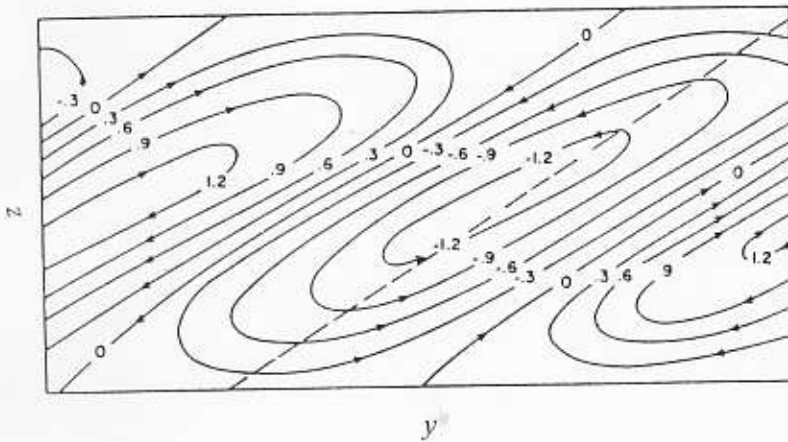


Fig. 6. Stream function in  $(y,z)$  plane (after Emanuel, 1982).

#### 4.2 Open Ocean Deep Convection

There are two important requirements for open-ocean deep convection: background cyclonic circulation and preconditioning (Killworth, 1983). The main ideas are that the cyclonic circulation is necessary to form an upward "doming" of isopycnals in the center of the cyclonic gyre and so reduce the stratification of the water column within the gyre, and that the preconditioning (operating over a period of weeks) creates a region of very weak stratification within the cyclonic dome which then become suitable for convection

if the surface forcing is sufficiently intense. In the absence of baroclinic instability, Chu (1991) showed that vertical cells can also be generated by the vortex (Fig.7). The vertical circulation generated by the vortex exchanges the heat and salt at different levels and reduces the stratification. Therefore, this effect of the vortex on the secondary circulation, i.e., symmetric instability, should play an important role in the "preconditioning" for open ocean deep convection.

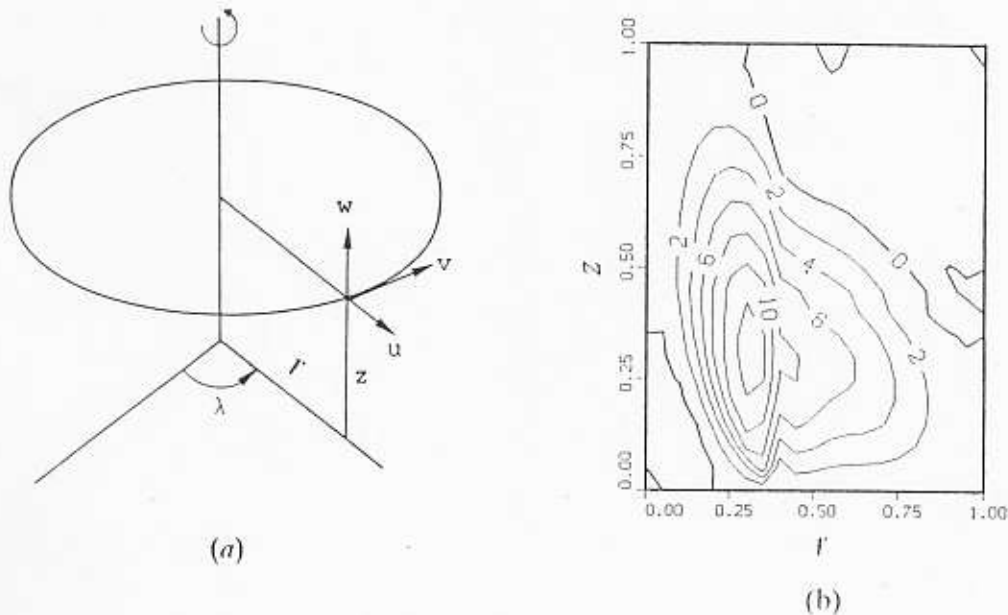


Fig. 7. (a) The cylindrical coordinates  $(r, \lambda, z)$  with the vertical axis at the center of the vortex. (b) The radial-vertical sectional streamfunction indicating the vertical cells generated by a barotropic Gaussian-type cyclonic vortex.

## 5. CONCLUSIONS

This short article is intended to bring the importance of the symmetric instability of currents or of vortices in the deep convection to the attention of the oceanographic community. In the near-boundary convection, the overturning cells generated by the symmetric instability of alongshore currents is a possible dynamical mechanism to drive dense water off the shelf. In the open ocean deep convection, the vertical cells generated inside a vortex by symmetric instability reduce stratification by the vertical exchange of heat and salt. It is reasonable to consider this mechanism as part of the "preconditioning".

## 6. ACKNOWLEDGEMENTS

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