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Abstract

This paper derives and applies preference models for decision analysis when consequences of the decision can vary across a geographic region. These models address both decisions where decision consequences are constant within specified subregions of the overall geographic region, and where consequences can vary in a continuous fashion across the region. We define preference conditions for such decisions, and derive specific value and utility function forms that are implied by these conditions. The functions are applied to two planning decisions involving water use and temperature reduction in regional urban development, and fire coverage across a city. These examples illustrate the applicability of the approach and the insights that can be gained from using it. With the increasing use of computer-based geographic information systems, it is now practical to use sophisticated decision making procedures of this type in situations where decision alternatives have geographically varying consequences, and these approaches can yield additional insights into the key drivers of a decision.

Subject classifications: Decision analysis; geographic information systems; multiattribute utility; multiattribute value; spatial additive independence; spatial preferential independence; spatial homogeneity; utility independence.

Area of review: Decision Analysis.

1. Introduction

This paper derives and applies preference models for decision analysis when consequences of the decision occur across a geographic region. As an example, consider a situation where regional planners are addressing alternative regional development plans that could have varying environmental or socioeconomic impacts across a city or other geographic entity. The regional planners and other stakeholders in the decision making process must consider multiple geographic maps showing current and potential future levels of environmental pollutants, urban development, water availability, air temperature, etc., that could vary across the region in different ways depending on which alternative is implemented.

In such decision making situations, the use of maps generated by computer-based geographic information systems (GIS) has become widespread due to increasing computer capabilities and decreasing costs (Obermeyer and Pinto 2007), but there has been limited application of preference models to address these decisions. Instead, most GIS research has focused on statistical approaches for analyzing the data or improved methods to display mapped data to decision stakeholders, and decision stakeholders are left to make geographic tradeoffs among the varying consequences using informal methods. The availability of powerful geographic information systems with associated geographic databases now makes it feasible to support such decisions with preference models based on established decision analysis principles.

Using decision analysis terminology, the consequences of selecting different alternatives in decisions supported by geographic information can be described in terms of one or more attributes (variables) whose levels (scores) are known, or can be estimated with some uncertainty, over a region. The levels of these attributes may be a function of both the alternative that is selected and geographic location. Thus, to judge the relative desirability of the decision alternatives, the decision maker(s) must combine the geographically-varying attribute levels of each alternative. The question we address is: When each decision alternative can be represented by one or more maps showing attribute levels over a geographic region that would result from selecting each decision alternative, how can a decision maker determine the relative desirability of each alternative?

This paper develops preference models for a decision analysis approach to such decisions. We use the term *preference model* to designate conditions on decision maker preferences and the possible forms for a value or utility function that will obey those conditions. The models are applied to two illustrative real-world problem domains: Water use and temperature reduction tradeoffs in regional urban development and fire coverage across a city.

2. Previous Related Work

The most relevant previous work includes concepts from multiattribute preference theory and specific applications of geographic information systems, including some previous limited applications of decision analysis to spatial models.

2.1. Applications of Geographic Information Systems

Geographic information systems are now widely used for formulating and analyzing spatial problems (Obermeyer and Pinto 2007) and are the basis for a large stream of literature. For example, Knox and Weatherfield (1999) consider their use in irrigation and water resource management, Pendleton et al. (1998) analyze them in studying wildlife habitat selection in Alaska, and Kohlin and Parks (2001) look at a GIS model to analyze deforestation. An overview is provided by Bond and Devine (1991), who illustrate GIS as effectively incorporating techniques from classical statistics. Arbia (1993) provides a more detailed overview of GIS, including consideration of sampling and modeling errors but not analysis of uncertainty over outcomes or consideration of preferences. The lack of formal decision analysis methods is a limitation in most previous GIS work that reduces its usefulness for decision making. Most previous work demonstrates how GIS can illuminate the geographic characteristics of a system but does not directly address the decision process that uses the outputs from the GIS analysis. Worrall and Bond (1997) explore some of the reasons GIS tools have yielded fewer benefits than expected in the public sector; one of these reasons is a lack of effective spatial decision support systems. As we will demonstrate, adding a decision analysis component to the analysis of geographic information can increase the power of GIS tools to support policy decision making.

While most GIS literature does not directly address decision making, a small subset of the literature does consider using GIS in making decisions. De Silva and Eglese (2000) discuss the development of a spatial decision support system which connects GIS data to a simulation model for evacuations. Malczewski (1999) and Jankowski (1995, 2006) provide more detailed analysis of multi-criteria decisions using GIS data. Chan (2005) examines the use of multi-criteria decision making in a broad context of spatial applications. However, the decisions considered by these authors do not directly involve preferences over a spatial distribution of consequences. Rather, they are "siting" decisions, in which individual locations must be chosen for facilities or infrastructure to optimize one or more measures of performance. Keisler and Sundell (1997) use a somewhat different approach for a park planning problem. They examine multiattribute utility over aggregated attribute levels within a geographic region. These aggregated levels are affected by the decision maker's choice of where the boundary of the region is drawn. In contrast, we consider preference functions that directly address spatially varying evaluation attributes.

2.2. Multiattribute Preference Theory

We use Keeney and Raiffa's (1976) terminology and notation for preference models with multiple evaluation attributes. The potential consequences of a decision are characterized by multiple evaluation attributes, $X_1, X_2, \ldots, X_n$, where x_i designates a specific level of X_i, and we assume conditions on decision maker preferences such that:

1. For decisions with no uncertainty, the overall value of a consequence is specified by a *multiattribute value function* $V(x_1, x_2, \ldots x_n)$, and

2. For decisions under uncertainty, the overall preferability of an alternative is specified by the expected value of a *multiattribute utility function* $U(x_1, x_2 \ldots x_n)$, where probabilities are used to characterize the likelihoods that an alternative will yield various consequences.

See Fishburn (1970) and Krantz et al. (1971) for expositions of the relevant preference theory. Keeney and Raiffa (1976), Keeney (1992), and Kirkwood (1997) have more elementary presentations.

In decision analysis, specifying a multiattribute value or utility function requires assessments from a decision maker, and this can be difficult because it requires the determination of an *n*-dimensional function. To simplify this, researchers have established conditions on preferences under which the form of the value or utility function is simplified, and we use two of these conditions in our work, *preferential independence* and *additive independence*. A subset of $X_1, X_2, \ldots X_n$, when $n \geq 3$, is defined to be preferentially independent of its complement if the rank ordering of alternatives with no uncertainty that have common levels for the complementary attributes do not depend on those common levels. If this property holds for all subsets of $X_1, X_2, \ldots, X_n$, then *mutual preferential independence* is said to hold, and in this case

$$V(x_1, x_2, \ldots, x_n) = \sum_{i=1}^{n} a_i v_i(x_i), \tag{1}$$

where $v_i(x_i)$ is called the single attribute value function over x_i, and $a_i \geq 0$ is called the weighting constant for attribute *i* (Debreu 1960).

For decisions under uncertainty, *additive independence* is defined to hold if the rank-ordering of alternatives using expected utility depends only on the marginal probability distributions over the attributes for each alternative. When additive independence holds, the multiattribute utility function can be written in the same weighted sum form as (1), but in this case the single attribute functions are utility functions over each attribute (Fishburn 1965). See Keeney and Raiffa (1976, Sections 3.6.2, pp. 111-112, and 6.5, pp. 295-297) and Kirkwood (1997, pp. 238-241 and 249-50) for further background, including assessment procedures for value and utility functions.

We say that preferences modeled by a value or utility function are i) *continuous* if the value or utility function is defined over a continuous domain and is continuous, ii) *twice-differentiable* if the second partial derivatives of the value or utility function exist over the entire domain, and iii) *monotonic* if the value or utility function is either always increasing or always decreasing with respect to any X_i for any specified set of levels for the other attributes. When mutual preferential independence holds so that (1) is valid, and preferences are continuous and monotonic, we define the *midvalue* x_i^{mid} of an interval $[x_i', x_i'']$ on X_i as the attribute level such that $v_i(x_i^{mid}) = [v_i(x_i') + v_i(x_i'')]/2$. See Keeney and Raiffa (1976, pp. 94, 120) and Kirkwood (1997, Definition 9.13 and Theorem 9.14 on pp. 233-234) for further discussion of the midvalue, and see Harvey (1995, p. 384, 393) for discussion of a midvalue independence condition in a time-preference context and assessment of the midvalue.

3. Spatially-Oriented Preference Models Under Certainty

This section develops new preference models for geographically-oriented decision making where potential consequences of decision alternatives are known with no uncertainty. Each preference model designates conditions on decision maker preferences and the possible forms for a value function that will obey those conditions. We assume that preferences among consequences obey conditions that imply that an ordinal value function exists over these consequences. (See Debreu 1954, 1964, Fishburn 1970, and Krantz et al. 1971 for specification of such conditions. Harvey and Østerdal 2011 show the development of this theory in the context of decisions with consequences that occur over time.) Initially, we assume there is a single evaluation attribute Z, where z designates a specific level of Z. We also assume the region of interest is partitioned into m subregions, labeled 1, ..., m, such that z does not vary within any specified subregion, and we designate the level of Z in subregion i by z_i. The value function for decisions under certainty taking into account the variation between the subregions is designated by $V(z_1, z_2, \ldots, z_m)$. After developing the theory for this situation, we then consider decisions that yield consequences that can vary more continuously over the geographic region of interest or have multiple evaluation attributes. We consider situations with uncertainty in Section 6.

3.1 Value Models for Spatial Decisions with a Single Evaluation Attribute

This section considers a single attribute Z, and develops a value model to evaluate alternatives that can have varying consequences over a region of interest in terms of the attribute. The derivations for the value models are in the Appendix.

DEFINITION 1. Preferences for consequences over the region of interest exhibit *mutual spatial preferential independence* with respect to Z if the rank-ordering of alternatives that have common levels of Z for any specified subset of subregions does not depend on those common levels. (Note that the common levels do not have to be the same for different subregions.)

It follows directly from Debreu's (1960) theorem presented in Section 2.2 that for a region with three or more subregions, mutual spatial preferential independence holds if and only if the value function can be written as

$$V(z_1, z_2, \ldots, z_m) = \sum_{i=1}^{m} a_i v_i(z_i), \tag{2}$$

where z_i is the level of Z in subregion i, v_i is called the single attribute value function over z_i, and $a_i \geq 0$ is called the weighting constant associated with subregion i. While (2) has the same form as (1), the z_i in (2) represent the levels of the same attribute in different subregions, rather than the levels of different attributes $X_1, X_2, \ldots, X_n$ as in (1).

DEFINITION 2. Continuous monotonic preferences for the attribute Z over a region of interest that exhibit mutual spatial preferential independence are *spatially homogeneous* if the midvalue z_i^{mid} of any interval $[z_i', z_i'']$ does not depend on the subregion *i*.

MODEL 1. *For a region with three or more subregions, assuming continuous twice-differentiable, monotonic preferences over Z that exhibit mutual spatial preferential independence, spatial homogeneity holds if and only if the value function can be written as*

$$V(z_1, z_2, \ldots, z_m) = \sum_{i=1}^{m} a_i v(z_i), \tag{3}$$

where $v(z_i)$ *is the (common) single attribute value function over* z_i*, and* $a_i \geq 0$.

Thus, when spatial homogeneity holds it is only necessary to assess one single attribute value function and a weighting constant for each subregion. This decomposition has intuitive appeal because it separates the returns to scale characteristics for preferences over the attribute, which are addressed in $v(z_i)$, from the priority or weighting assigned to each subregion, which is encoded in a_i. In some cases subregions will be weighted equally, and then all a_i can be set equal to one so the weights can be eliminated from the equation. For the analogous context of decisions with a stream of outcomes over time, the form of equation (3) has been previously applied with the a_i's being interpreted as time discounting weights. In that context, Krantz et al. (1971, pp. 303-305) and Harvey (1986, 1995) show conditions for the existence of a value function of the form of (3), following up on a question raised about this by Fishburn (1970, p. 93).

The concepts of spatial preferential independence and spatial homogeneity generalize in a natural way to situations where *Z* can vary continuously over a region. In this case, there are no designated subregions, but rather the level of *Z* is a function of the geographic coordinates *x* and *y* that designate a location within the region of interest.

DEFINITION 3. Preferences for consequences for a spatially varying attribute Z over a region exhibit *mutual spatial preferential independence* with respect to Z if the rank ordering of alternatives that have common levels of Z for any specified subregion does not depend on those common values.

DEFINITION 4. Continuous monotonic preferences for consequences for a spatially varying attribute Z that are mutually spatially preferentially independent with respect to Z are *spatially homogeneous* if the midvalue $z^{mid}(x, y)$ for any interval $[z_i'(x, y), z_i''(x, y)]$ depends on z_i' and z_i'' but not on x and y.

The following model extends Model 1 to situations where Z and the geographic weight can vary continuously across the region. To derive this model, it is necessary to make two assumptions that will hold in realistic decision situations: 1) The region of interest A has a finite area with a reasonably well-behaved boundary, and 2) The value function $v[z(x,y)]$ and weighting function $a(x,y)$, which are defined in the next paragraph, are continuous, except on a set of zero area, and bounded. Assumption 2 ensures that there are no locations in the region which have infinite weight or value.

MODEL 2. *Given the conditions in the preceding paragraph, spatial homogeneity holds for continuous, twice-differentiable, monotonic preferences over an attribute Z that can vary over a region of interest A if and only if the overall value over A can be written as*

$$V(z) = \iint_A a(x, y) v[z(x, y)]\, dxdy\,, \tag{4}$$

where x and y are coordinates within the region, v is the (common) single attribute value function over z, and a(x,y) *is the non-negative weight for the* (x,y) *location.*

Equation (4) has a satisfying interpretation in geographic terms in that it separates two different aspects of the decision evaluation, the *importance* of a specific location (x, y) with respect to the evaluation attribute, which is encoded by $a(x, y)$, and the impact of *returns to scale* on the attribute as a function of z, which is encoded by $v[z(x, y)]$.

Spatial homogeneity is likely to be necessary for tractability in most realistic decision situations since it would be difficult to assess a mathematical expression for a single attribute value function whose shape can vary continuously throughout the region. Without this condition, there could be an infinite number of different single attribute value functions, one for each location. We interpret $a(x,y)$ as a weighting function, but it could also be viewed more broadly as a generating function for a transform of $v[z(x,y)]$.

3.2 Multiple Evaluation Attributes

Thus far, we have considered only a single evaluation attribute defined across a region. Some decisions will address multiple attributes, one or more of which can vary geographically. Incorporating multiple attributes is a conceptually straightforward extension provided that the appropriate preference independence conditions hold over the multiple attributes. We first consider the situation where there are m subregions, and within each subregion the levels for the attributes do not vary. Let n designate the number of attributes, and let Z_{ij} designate the j^{th} attribute in the i^{th} subregion, where z_{ij} stands for a specific level of that attribute. Modify the notation presented earlier so that Z_i now designates the vector of n attributes $Z_{i1}, Z_{i2}, \ldots, Z_{in}$ in subregion i, and Z^j designates the vector $Z_{1j}, Z_{2j}, \ldots, Z_{mj}$ of the j^{th} attribute across the m subregions. Let Z designate all $m \times n$ attribute-subregion combinations. Further, let z_i designate a vector of specified levels for Z_i, z^j designate a vector of specified levels for Z^j, and z designate specified levels for all the attribute-subregion combinations Z.

DEFINITION 5. Preferences over the region of interest that are spatially preferentially independent with respect to a set of attribute vectors $Z_1, Z_2, \ldots, Z_m$ are *multiattribute spatially homogeneous* if when two alternatives that differ only in the attribute levels for a specified subregion are indifferent, then the same indifference relation holds for those same attribute levels in any subregion. (In this definition, the scalar attribute for each subregion considered in Definition 2 is replaced with a vector Z_i that measures the decision consequences in each subregion *i* with respect to the *n* evaluation attributes $Z_{i1}, Z_{i2}, \ldots, Z_{in}$.)

MODEL 3. *Multiattribute spatial homogeneity holds, and the* Z^j *for j=1, ..., n are mutually preferentially independent, if and only if the overall value can be written as*

$$V(z) = \sum_{i=1}^{m} a_i \sum_{j=1}^{n} b_j v_j(z_{ij}), \tag{5}$$

where $a_i \geq 0$ *is called the weighting constant for subregion i,* $b_j \geq 0$ *is called the weighting constant for the* j^{th} *attribute, and* $v_j(z_{ij})$ *is called the single attribute value function over attribute level* z_{ij}. (Note that *v* only depends on the attribute index *j*.)

The following Model 4 extends Model 3 to the situation where the multiple attribute levels and weights can vary continuously across the region. Thus, this model extends Model 3 analogously to the way that Model 2 extends Model 1. Let $Z^j(x,y)$ designate the j^{th} attribute at location (x,y) and $z^j(x,y)$ designate the level of $Z^j(x,y)$. We assume analogous conditions to those assumed for Model 2: 1) The region of interest *A* has a finite area with a reasonably well-behaved boundary, and 2) The value functions $v_j[z^j(x,y)]$ and weighting function $a(x,y)$ are continuous, except on a set of zero area, and bounded.

DEFINITION 6. Preferences over the region of interest that are spatially preferentially independent with respect to the n attributes are *continuously multiattribute spatially homogeneous* if the preference relation between any two specified vectors of n attribute levels at a location (x, y) does not depend on (x, y).

MODEL 4. *Given the conditions in the paragraph preceding Definition 6, continuous multiattribute spatial homogeneity holds and the* $Z^j(x, y)$ *are mutually preferentially independent for all* (x, y) *for a set of multiple attributes* Z^j *that can vary over a region of interest A if and only if the overall value can be written as*

$$V(z) = \iint_A a(x, y) \sum_{j=1}^{n} b_j v_j [z^j(x, y)] dxdy, \quad (6)$$

where x and y are coordinates within the region, v_j *is the j^{th} single attribute value function,* $a(x, y) \geq 0$ *is the weight for location* (x, y)*, and* $b_j \geq 0$ *is the weight for the j^{th} attribute.*

Some assumptions needed for models in this section may seem restrictive for some decision situations with geographically varying consequences. However, the resulting formulas are more general than most summary metrics typically used in GIS analysis. For example, those summary metrics are often simple averages, which are special cases of the more general formulas derived in this section. The functions and conditions on preference structures presented in this section serve as the foundation for Section 5, in which we apply these spatial decision tools to two illustrative applications.

4. Value Model Assessment Procedures

This section summarizes approaches for assessing the single-attribute value functions and weights required for the preference models presented in Section 3, and presents references to sources with more details on value function assessment.

Standard assessment procedures can be used to assess the single-attribute value function $v(z)$. (See, for example, Keeney and Raiffa 1976, Section 3.7.2 or Kirkwood, 1997, Section 4.3.) Often value functions will increase or decrease monotonically over levels of the attribute, such as value functions for median family incomes or levels of pollution, as assumed in our theoretical development. (If a value function is not monotonic, it can be possible to redefine the attribute as the distance from an "ideal point" level, in which case the redefined attribute will be monotonic.) With monotonic preferences, single attribute value functions can be assessed using a midvalue splitting approach. For example, suppose a value function is being assessed for profit, ranging from \$0 to \$100,000, with higher profits being preferred. The value function can first be scaled by setting $v(\$0) = 0$, and $v(\$100,000) = 1$. If the midvalue of [\$0, \$100,000], as defined in Section 2.2, is determined to be \$40,000, then $v(\$40,000) = 0.5$. The midvalue of [\$0, \$40,000] or [\$40,000, \$100,000] could then be assessed, yielding attribute levels with values of 0.25 and 0.75, respectively. This procedure could be continued as long as desired to approximate the decision maker's value function over profit to any level of accuracy. Alternatively, a functional form for the value function, such as the exponential forms for evaporation rate and night cooling in the example in Section 5.1, can be fitted to a set of directly assessed points on the value function.

Spatial homogeneity could be verified by asking the decision maker whether the midvalue is the same for different subregions. Provided that spatial homogeneity is satisfied, the procedure for assessing a single-attribute value function is the same whether the attribute is defined over discrete subregions or continuously over the entire region.

The value tradeoff method (Keeney and Raiffa 1976, Section 3.7.3, Eisenführ et al 2010, Section 6.4.2) or the swing weighting approach (Eisenführ et al. 2010, Section 6.4.3, Kirkwood, 1997, Section 4.4) can be used to determine the weights a_i, where the weights are on the subregions as shown in equation (3). By convention weights are assumed to sum to 1. If a decision involves spatially varying levels of night cooling, for example, then to begin the assessment of weights using the swing weighting method, the decision maker can be asked to imagine that two subregions are both at the worst level of night cooling. The decision maker is then asked to consider a situation where only one subregion could be improved to the best night cooling level and asked to select which subregion to improve, by "swinging" its amount of night cooling to the best level. The subregion chosen for improvement would thus have a higher weight. Continuing with the standard swing weighting approach would quantify specific numbers for the weights, though this approach requires stronger preference assumptions than are used to derive the models in Section 3. These stronger conditions, difference consistency and difference independence of one attribute from the others, were developed by Dyer and Sarin (1979). See Kirkwood (1997, pp. 241-244) for further discussion of the conditions. With a continuous weighting function, similar assessments might be done to fit an analytical form, as we illustrate in Section 5.2, or weight assessments might be done for a subset of points in the region with interpolation used to calculate weights for other points in the region. Weights for the continuous case could be normalized to integrate to 1.

A swing weighting approach may also be used to determine the attribute weights b_j in Equations (5) and (6). Since these weights refer to the single-attribute value functions themselves rather than the value achieved in different locations, the traditional swing weighting or value tradeoff approaches can be used with no need to incorporate further geographic considerations.

5. Illustrative Applications

This section presents two examples which apply the preference models developed above and shows insights that can be gained from using these models. The analysis for these applications was conducted using Excel, with some use of Visual Basic for Applications and the Excel Solver. As discussed in Section 2.1, the use of these preference models differs from the approaches in previous applications of GIS data in that decision maker preferences are explicitly specified over spatially-varying attributes, rather than assessed at an aggregate attribute level, such as the average attribute level over the region.

5.1. Water Use and Temperature Reduction in Regional Urban Development

Many decisions involving spatial data address multiple attributes. (For example, Keller et al. 2010 found multiple attributes used by stakeholders in water resource planning in Arizona.) The application in this section illustrates the use of Model 3 from section 3.2 in such decisions. Gober et al. (2010) applied a heat flux model known as the "Local-Scale Urban Meteorological Parameterization Scheme" (LUMPS) to investigate urban heat island effects in Phoenix, Arizona. Urban development has led to increased temperatures in Phoenix, mostly by reducing the amount of night cooling that occurs. As a result, there is motivation to increase the quantity of vegetation, as "green" areas acquire and retain less heat. However, this would also require more water, as green areas lose more to evaporation than developed urban areas. Thus, night cooling and evaporation rate are both important considerations when choosing development strategies.

Using the LUMPS model, evaporation rate and night cooling results were estimated by Gober et al. for ten different tracts of land with three different land use classifications (industrial, xeric, and mesic) in the greater Phoenix area using each of three potential development strategies for each tract (compact city, oasis city, and desert city). The current levels of evaporation rate and night cooling for the ten tracts are shown in Figure 1. As shown in the figure, the ten tracts included in the study are not contiguous. Figure 2 shows the changes that would result from applying each strategy to each tract, as projected by the LUMPS model. The different shades of the tracts represent the current classifications: the darkest shade represents industrial tracts, the lightest shade xeric (desert vegetation) tracts, and the middle shade mesic (non-desert vegetation) tracts.

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>>INSERT FIGURE 1 HERE<<
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>>INSERT FIGURE 2 HERE<<
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Given only the data shown in Figures 1 and 2, it is not clear which development strategy should be implemented in each tract, since reductions in evaporation rate (which are desirable) are accompanied by increases in night temperature (which are undesirable), and the magnitudes of these effects vary from tract to tract. Thus, to defensibly choose the optimal development strategy, we should specify a value function to determine an overall value for different combinations of evaporation rate and night cooling across the ten tracts. If the conditions for Model 3 hold, then to determine a value function we need only specify single attribute value functions for evaporation rate and night cooling, as well as weights for the two attributes and each tract. To illustrate the analysis, assume an equal weight of 0.10 on each tract and use the following normalized exponential single-attribute value functions for evaporation rate and night cooling, where higher levels of evaporation are less desirable, while higher levels of night cooling are more desirable, and the functions are normalized between zero and one over the ranges of possible levels for the two attributes, as graphed in Figure 3:

$$v_E\left(z_{iE}\right)=\frac{1-e^{-0.905\left(1-\left(\frac{z_{iE}-0.047}{0.113}\right)\right)}}{1-e^{-0.905}} \tag{7a}$$

$$v_N\left(z_{iN}\right)=\frac{1-e^{-3.35\left(\frac{z_{iN}-0.031}{2.374}\right)}}{1-e^{-3.35}}, \tag{7b}$$

where 0.905 and 3.35 are the exponential constants, z_{iE} represents the evaporation rate in tract *i*, and z_{iN} represents the amount of night cooling in tract *i*. [Analysis of the information in Figures 1 and 2 shows that the smallest achievable evaporation rate is 0.047, and the largest is 0.160 (equal to 0.047 + 0.113). Subtracting 0.047 from the evaporation rate and dividing by 0.113 normalizes it to range from 0 to 1. The smallest achievable level of night cooling is 0.031, and the largest is 2.405 (equal to 0.031 + 2.374).]

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>>INSERT FIGURE 3 HERE<<
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To illustrate the analysis approach, assume the decision maker places a 40% weight on the evaporation rate value and a 60% weight on the night cooling value. Then the overall value function is obtained using Model 3:

$$V(z) = \sum_{i=1}^{10} 0.1\left(0.4v_E\left(z_{iE}\right) + 0.6v_N\left(z_{iN}\right)\right), \tag{8}$$

and the resulting optimal development plan is shown in Figure 4, assuming there are no constraints on which development strategy can be applied to each tract.

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>>INSERT FIGURE 4 HERE<<
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Once we framed this decision using a multiattribute value function, analyzing the decision problem became more straightforward. We specified the single attribute value functions over evaporation rate and night cooling, as well as weights on the two evaluation attributes and the ten tracts. These clarified the geographic value structure, and with this structure it was a straightforward calculation to determine the preferred decision for each tract.

With this formulation, the decision problem is effectively made up of ten smaller decision problems, one for each tract, which can be solved independently. This is because using equation (8) the total value is the sum of the values for each tract, and there is no constraint across tracts. We can extend this model to incorporate cross-tract constraints on the development plans. For example, assume there are constraints on the average decrease in evaporation rate and the average increase in night cooling allowed across the tracts. Restrictions such as these can be included without altering the preference model, and will allow decision makers to consider "what if?" questions about the impact of different constraints. For example, Figure 5 shows the optimal plan still using (8), but now requiring a minimum average decrease of 7% in evaporation rate and a minimum average increase of 12% in night cooling across the tracts. In this case, it is optimal to neglect the oasis development strategy entirely. This is because the oasis strategy leads to increased evaporation rates in tracts where it is imposed, leaving little flexibility in other tracts to satisfy the overall evaporation constraint. This example illustrates the type of constrained optimization analysis that can be done once a spatial value function is determined. This type of analysis is not realistic by simply examining mapped projections of the impacts of various policies, such as those shown in Figure 2.

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>>INSERT FIGURE 5 HERE<<
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5.2. Fire Coverage Across a City

The second example is a stylized fire coverage problem motivated by the discussion in Church and Roberts (1983) that illustrates the application of Model 2 from Section 3.1. In this example, the decision is where to locate three fire stations within a city, and we initially consider a solution to minimize average response times, where average response time is calculated as a continuous function over the city. An optimization model for this is

$$\min_{K} \iint_{A} z(x, y, K)\,dxdy,$$
$$K = \left(\left(K_x^1, K_y^1\right), \left(K_x^2, K_y^2\right), \left(K_x^3, K_y^3\right)\right), \qquad (9)$$

where K is a vector representing the x and y coordinates of the three stations, and z is the average response time for a point (x, y) in the city region A given the locations of the three fire stations. For this illustrative example, region A is assumed to be square with dimensions normalized from 0 to 1 in both x and y.

To develop a specific functional form for (9), we assume that for some fraction of incidents α, the fire station assigned to respond is not the closest one. Of those incidents, the same fraction α are not assigned to the next closest station. Finally, of those incidents not assigned to the two closest stations, the same fraction α will end up unassigned to any of the three stations. With these assumptions, the expression in equation (9) becomes

$$z\left(x,y,K\right)=\left(\sum_{i=1}^{3}\alpha^{i-1}\left(1-\alpha\right)f\left(d\left(\left(x,y\right),K^{(i)}\right)\right)\right)+\alpha^{3}\overline{f}\ , \tag{10}$$

where $K^{(i)}$ is the location of the ith closest station, $d\left(\left(x,y\right),K^{(i)}\right)$ is the distance between (x,y) and $K^{(i)}$, $f(d)$ is the average response time from a station at distance d, and $\overline{f}$ is the average "unassigned" response time (occurring when none of the three stations is properly equipped to respond). For $d(\cdot)$, we use a "metropolitan" distance measure, which is the sum of the x and y distances to account for travel along gridlines in a metropolitan area. The range of $f(d)$ is assumed to be [0,1]. We assume that $f(d)$ is linear in d below an upper bound d'. We can think of d' as a large enough distance between the station and the incident such that there is no benefit to responding, and the station therefore will not respond if $d \geq d'$. Provided α is not large, $\alpha^{3}\overline{f}$ will be close to zero. Since this term is constant and close to zero, the exact choice of $\overline{f}$ is immaterial, and we assume $\alpha^{3}\overline{f}$ can be ignored. In this illustrative example, we set α = 0.15. While (9) was developed directly from the definition of average response time, it is a special case of (4) where all locations are given equal weight so that $a(x, y) = 1$ for all x and y, and $v(z) = z$, and we will now generalize (9) based on Model 2.

Using the notation of Model 2, we can say that the approach shown in (9) assumes that all areas of the city are equally important, and that the value function over the average response time at a given location is linear with respect to the response time. Given the relationship between response time and the size of the fire that the responder will have to fight, it is reasonable to assume there are diminishing returns for decreases in response time from the perspective of a policy maker. That is, high values will be placed on the range of response times which will likely assure the survival of the building(s), while changes in response times slightly above this range will be associated with steeper decreases in value. For example, consider the following nonlinear value function that incorporates this consideration:

$$v\left(z\left(x,y,K\right)\right)=\frac{1-e^{-3.86\left(1-z\left(x,y,K\right)\right)}}{1-e^{-3.86}}\ . \tag{11}$$

An exponential constant of 3.86 corresponds to a midvalue of 0.826 for the range from zero to one, where the midvalue would be obtained using the assessment procedures discussed in Section 4. Equation (11) is shown in Figure 6. It is normalized so that $v(0) = 1$ and $v(1) = 0$.

****
>>INSERT FIGURE 6 HERE<<
****

It is reasonable that a rapid average response time could be more critical in some areas than others, due to differences in, for example, population or economic importance. To illustrate the impact of this on the optimal fire station locations, assume that development in this city is concentrated along a river extending upstream from the center of the eastern boundary of the city, which is on a bay, to the southwestern area, and therefore more emphasis is placed on protecting areas which are closer to the bay and the river. A weighting function that represents this geographic weighting is specified by:

$$a(x,y) \sim x^{1.1}(1-x)^{0.1}\, y^{1.5}(1-y)^{\frac{1.425-0.6x}{0.05+0.4x}}, \tag{12}$$

normalized to integrate to 1 over the region, as shown in Figure 7. (In this illustrative analysis, we ignore the impact of the river on response times.) This mathematical representation of a is illustrative, and the weighting function for any decision problem would be constructed to capture the region-specific pattern of variation in the characteristics which are relevant to the decision.

****
>>INSERT FIGURE 7 HERE<<
****

The resulting optimization problem is now

$$\begin{gathered}\max_K \iint_A a(x,y)\frac{1-e^{-3.86(1-z(x,y,K))}}{1-e^{-3.86}}\,dxdy, \\ K = \left(\left(K_x^1,K_y^1\right),\left(K_x^2,K_y^2\right),\left(K_x^3,K_y^3\right)\right),\end{gathered} \tag{13}$$

where $a(x,y)$ is given by (12) normalized to integrate to one over the region of interest.

The optimization problems given in both (9) and (13) were solved numerically using a grid search with a distance of 0.025 between adjacent points, and a numerical integration that divides the region into 400 cells, computing the average response time in the center of each cell. Changes in the search and integration parameters did not noticeably affect the results. Figure 8 shows the optimal fire station locations for (13) designated with diamonds, along with the locations determined with an unweighted linear value function as given in (9) designated with circles. Conforming with the preference to protect areas closer to the bay and river, the locations of the fire stations have been "pulled" toward the higher weighted part of the city relative to their locations when only average response time is considered.

****
>>INSERT FIGURE 8 HERE<<
****

6. Extensions of the Preference Models to Address Uncertainty

In this section, we consider preference models for decisions where the consequences of decision alternatives are uncertain. We assume that decision maker preferences obey a set of conditions (such as those provided in the sources referenced in Section 2) so that probabilities can be assigned to the possible consequences of each alternative and expected utility can be used to rank alternatives. The preference conditions in Section 3 can be extended to decisions under uncertainty to determine the requirements for an additive utility function. Instead of preferential independence, however, the use of an additive multiattribute utility function requires the considerably stronger condition of additive independence.

We first consider the case in which the level for the single evaluation attribute Z does not vary within each subregion, and then consider the continuous case.

DEFINITION 6. *Spatial additive independence* with respect to Z over the region of interest holds if the rank ordering for any set of alternatives depends only on the marginal probability distributions for each alternative over the levels z_1, z_2, …, z_m of Z in each of the subregions 1, ..., m.

In spatial decision problems with uncertainty, instead of maximizing a spatial *value* function, we maximize expected utility using a spatial *utility* function. With this assumption, it is straightforward to show that spatial additive independence is satisfied if and only if the utility is given by

$$U(z_1, z_2, \dots, z_m) = \sum_{i=1}^{m} a_i u_i(z_i) . \qquad (14)$$

where $u_i(z_i)$ is the single attribute utility function over z_i, and a_i is the weighting constant associated with subregion i. Equation (14) follows directly from Definition 6 using the same approach used to prove that additive independence implies an additive utility function. (See Section 2.2 for references to previous related work that proved this result.) When spatial homogeneity also holds, the $u_i(z_i)$ are all equal to a common function $u(z_i)$ by analogous reasoning to Model 1.

Analogously to Model 2, we can extend (14) to situations where the single evaluation attribute Z can vary continuously over the region of interest, resulting in utility given by

$$U(z) = \iint_A a(x, y) u[z(x, y)] \, dxdy . \qquad (15)$$

Establishing equation (15) requires analogous assumptions about A, $a(x,y)$, and u as were required for A, $a(x,y)$, and v in Model 2. With these assumptions, equation (15) is obtained under the assumption of spatial homogeneity using reasoning analogous to the development in the Appendix for equation (4), but to develop (15) the spatial preferential independence condition in Definition 4 is replaced with spatial additive independence. Results analogous to Models 3 and 4, but with a utility function and assuming additive independence can also be derived. As with the case of no uncertainty, spatial homogeneity is an important assumption to make the analysis tractable. However, the required preference assumptions for an additive utility function are strong and may not be appropriate in some decision situations.

An approach to developing more general spatial utility function forms with less restrictive requirements would be to construct the utility function over the value functions that were developed in Section 3 using methods such as those presented by Dyer and Sarin (1982) and Matheson and Abbas (2005). If the conditions needed for a spatially homogeneous additive value function hold, then in the case with discrete subregions, a utility function *U* could be constructed over the value function in (3) with the form:

$$U\left(V(z_1, z_2, \ldots, z_m)\right) = U\left(\sum_{i=1}^{m} a_i v(z_i)\right). \tag{16}$$

Similarly, in the continuous case the utility function could be constructed over the value function in (4) with the form:

$$U\left(V(z)\right) = U\left(\iint_A a(x,y) v[z(x,y)]\, dxdy\right). \tag{17}$$

These are less restrictive than (14) and (15) in that they have unspecified utility functions *U*, and hence can represent more general preference conditions than (14) or (15). Standard utility function assessment procedures can be modified to determine *U*. For example, a possible approach for assessing this utility function is to identify the potential decision consequences with the highest and lowest possible values, and visualize a hypothetical binary gamble between them with probability *p* of the highest-value outcome occurring and probability 1-*p* of the lowest-value outcome occurring. The utility of the value placed on a specified consequence could then be determined by finding the value of *p* for which the decision maker is indifferent between the specified consequence and the gamble. By equating expected utilities, the assessed *p* would be the utility associated with the specified consequence.

7. Concluding Comments

This paper develops preference theory for decisions with geographically varying consequences. As shown by the illustrative applications in this paper, these types of decisions are important in a variety of decision contexts, and with the widespread use of geographic information systems, it is now practical to apply more rigorous decision analysis methods to these decisions. When faced with a decision that has spatially-varying consequences, formulating specific structures and conditions for the decision stakeholders' preferences will allow an analyst to elicit an appropriate value or utility function using the results in this paper. This can help to provide a more defensible gauge of the desirability of the proposed decision alternatives. We believe the methods in this paper can be applied to a wide range of real-world policy decisions with geographically varying consequences, such as regional development planning, pollution abatement, facility location, and utility service provision.

Appendix

DERIVATION OF MODEL 1. Since our definition of spatial homogeneity requires preferential independence to hold, (2) must hold, so the only additional step required is to establish that all of the v_i are identical. If z_i^{mid} is the midvalue of [z_i', z_i''], then $v_i(z_i^{mid}) = [v_i(z_i') + v_i(z_i'')]/2$. For notational convenience, define $z \equiv (z_i' + z_i'')/2$, $h \equiv z_i'' - z = z - z_i'$, and $\pi = z - z_i^{mid}$. Then the midvalue equation can be rewritten as $v_i(z - \pi) = [v_i(z - h) + v_i(z + h)]/2$. Performing a Taylor series expansion around z and keeping only the first terms that do not cancel out results in $\pi = -(1/2)[v_i''(z)/v_i'(z)]h^2$, where v_i' and v_i'' represent the first and second derivatives, respectively. This is a second order linear differential equation, although since π may depend on z, it is not a constant coefficient linear differential equation. However, since it is a second order linear differential equation, any solutions must be the same to within two constants of integration. Therefore, since the v_i can always be scaled to lie between zero and one, the solutions for all the different v_i can be set equal to the same function, which is called v in (3). The converse of the model result follows by direct substitution into (3). Harvey (1995, Theorem 2.1) proves a result with similar mathematical structure to Model 1 in the context of decisions with consequences that vary over time with an infinite planning horizon.

DERIVATION OF MODEL 2. Start with Model 1, as stated in (2), and define $\lambda_i \equiv a_i / A_i$ in (2), where A_i is the area (for example, in square miles) of subregion *i*. Then (2) can be rewritten as

$$V(z_1, z_2, \ldots, z_m) = \sum_{i=1}^{m} \lambda_i A_i v(z_i) . \qquad \text{(A-1)}$$

We extend (A-1) to an attribute that varies over the region as follows: Partition the region into a uniform grid, where the two dimensions of the grid are designed by *x* and *y*, and where the *x* and *y* dimensions of each cell in the grid are designated by Δx and Δy, respectively, so that the area A_i of any cell is $\Delta x \times \Delta y$. While it is easiest to visualize this partition if *A* is rectangular, the result holds for more general regions, as established in the references given below. If λ_i and $v_i(z_i)$ did not vary within a grid cell, then from the assumptions for Model 2, the conditions of Model 1 would hold and therefore (A-1) could be written as

$$V(z_1, z_2, \ldots, z_m) = \sum_{i=1}^{m} \lambda(x_i, y_i) v[z(x_i, y_i)] \Delta x \Delta y , \qquad \text{(A-2)}$$

where x_i and y_i designate some specified but arbitrary point within grid cell *i*, $\lambda(x_i, y_i) \equiv \lambda_i$, and $v[z(x_i, y_i)] \equiv v(z_i)$.

However, in Model 2 λ_i and $v_i(z_i)$ *can* vary within a grid cell so that (A-2) is only an approximation to the value of $V(z)$. However, equation (A-2) is a Riemann sum of $\lambda(x, y)v[z(x, y)]$ over *A*. From the assumptions for Model 2, λ and v are both bounded, and therefore their product is also bounded. Also by the assumptions of Model 2, these functions are continuous almost everywhere (that is, except on a subset of *A* with measure zero), and therefore $\lambda(x, y)v[z(x, y)]$ is Riemann integrable over *A* if the boundary of *A* obeys conditions that will be met in any practical situation. (For proofs of this, see Apostol 1962, Section 2.12, or Trench 2003, Theorem 7.1.19.) Since $\lambda(x, y)v[z(x, y)]$ is integrable, the Riemann sum in (A-2) will converge to a unique value (which by definition is the integral) as the partition of A is made finer so that m approaches infinity and both Δx and Δy approach zero. Thus, in the limit, (A-2) becomes

$$V(z) = \iint_A \lambda(x, y) v[z(x, y)] dx dy, \tag{A-3}$$

where $V(z)$ is the value from a decision-making perspective associated with the distribution of the attribute over the region of interest. The converse of the model result follows by direct substitution into (4). Note that in (4), $\lambda(x, y)$ in (A-3) has been replaced with $a(x, y)$ to make the notation more parallel to (2). However, the units for a in (2) and (4) are different.

Harvey and Østerdal (2011) more completely specify the steps necessary to rigorously establish a result analogous to Model 2 in the context of continuous time decisions.

DERIVATION OF MODEL 3. The notation used in this proof is defined in Section 3.2. Since the Z_i are multiattribute spatially homogenous by assumption, it follows from the definition of multiattribute spatial homogeneity that the Z_i are mutually preferentially independent. From the assumptions of the model statement, the Z^j are also mutually preferentially independent. From the definition of mutual preferential independence, any subset of the (vector) attributes in either the set of $Z_i, i = 1,2,\ldots n$, or the set of $Z^j, j = 1,2,\ldots m$, is preferentially independent of its complement. Consider the specific case of two pairs of vector attributes $\{Z_a, Z_b\}$ and $\{Z^c, Z^d\}$, where a, b, c, and d are specified but arbitrary indices in the feasible ranges for the number of subregions or the number of attributes, as appropriate. Then $\{Z_a, Z_b\}$ and $\{Z^c, Z^d\}$ are each mutually independent of their complements. Also, the intersection of these two subsets of attributes is the two attribute-subregion combination Z_{ac} and Z_{bd}, and it follows from Gorrman (1968), Theorem 2, (also stated as condition ii of Keeney and Raiffa 1976, Theorem 3.7), that $\{Z_{ac}, Z_{bd}\}$ is preferentially independent of its complement. However, a, b, c, and d are arbitrary, and therefore this establishes that every pair of attribute-subregion combinations is preferentially independent of its complement. Hence, from inductive application of Gorman (1968), Theorem 2 (also stated as a Corollary on page 114 of Keeney and Raiffa 1976), the Z_{ij} are mutually preferentially independent, and therefore

$$V(z)=\sum_{i=1}^{m}\sum_{j=1}^{n}k_{ij}v_{ij}(z_{ij}) \qquad \text{(A-4)}$$

(Debreu 1960). Since the attributes are multiattribute spatially homogeneous by assumption, it follows that if z_{ij}^{mid} is the midvalue of an interval $[z_{ij}', z_{ij}'']$ for some specified i and j, it must be the midvalue for any i. Therefore an analogous argument to the one given in the derivation of Model 1 establishes that the single-attribute value functions v_{ij} cannot be a function of the subregion i, and hence

$$V(z)=\sum_{i=1}^{m}\sum_{j=1}^{n}k_{ij}v_{j}(z_{ij}) \qquad \text{(A-5)}$$

holds in this case.

To show that multiattribute spatial homogeneity also implies $k_{ij}=a_i b_j$, and hence (5) holds, first assume without loss of generality that the subregions and attributes are labeled so that the largest scaling constant is k_{11}. Consider two hypothetical alternatives: 1) all the attribute-subregion combinations except Z_{11} and Z_{1j} are set to arbitrary levels, Z_{11} is set to its least preferred level so that $v_j(z_{11})=0$ in (A-5), and Z_{1j} is set to its most preferable level so that $v_j(z_{1j})=1$, and 2) another hypothetical alternative with all the attribute-subregion combinations except Z_{11} and Z_{1j} set to the same arbitrary levels as the first alternative, Z_{1j} set to its least preferred level so $v_j(z_{1j})=0$, and Z_{11} set to the level z_{11}^{j} such that the two alternatives are equally preferred. Then equating the values for each of these two alternatives calculated using (A-5) and cancelling common terms results in $k_{11}v_1(z_{11}^{j})=k_{1j}$ for any $j\neq 1$. However, by the multiattribute spatial homogeneity condition, if this equation holds for subregion 1, then the same level z_{11}^{j} must make the analogous equation true for any subregion i, and hence

$k_{i1}v_1(z_{11}^j) = k_{ij}$ for any i. Define $b_j \equiv v_1(z_{11}^j)$ and $a_i \equiv k_{i1}$. Substituting these definitions into $k_{i1}v_1(z_{11}^j) = k_{ij}$ gives $k_{ij} = a_i b_j$. Substitute this into (A-5), and (5) follows. The converse of the model result follows by direct substitution into (5).

DERIVATION OF MODEL 4. The derivation of Model 4 from Model 3 is analogous to the derivation of Model 2 from Model 1. By the assumptions of Model 4, the conditions for (5) hold, and therefore this formula can be converted to a Riemann sum analogous to (A-2) using an analogous procedure to that used in the Model 2 derivation. A further derivation analogous to the Model 2 derivation leads to (6), and direct substitution into (6) yields the converse of the result.

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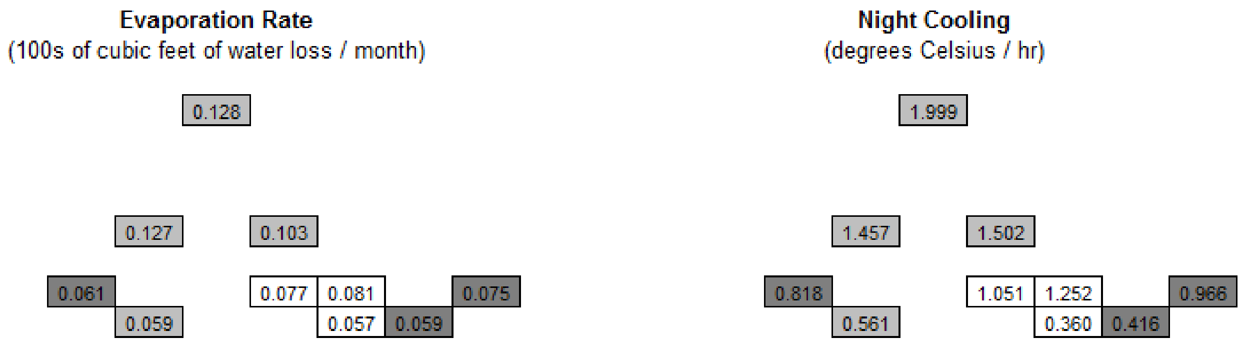

Figure 1. Current evaporation rate and night cooling for each of the ten tracts. The darkest shade represents industrial tracts, the lightest shade xeric, and the middle shade mesic.

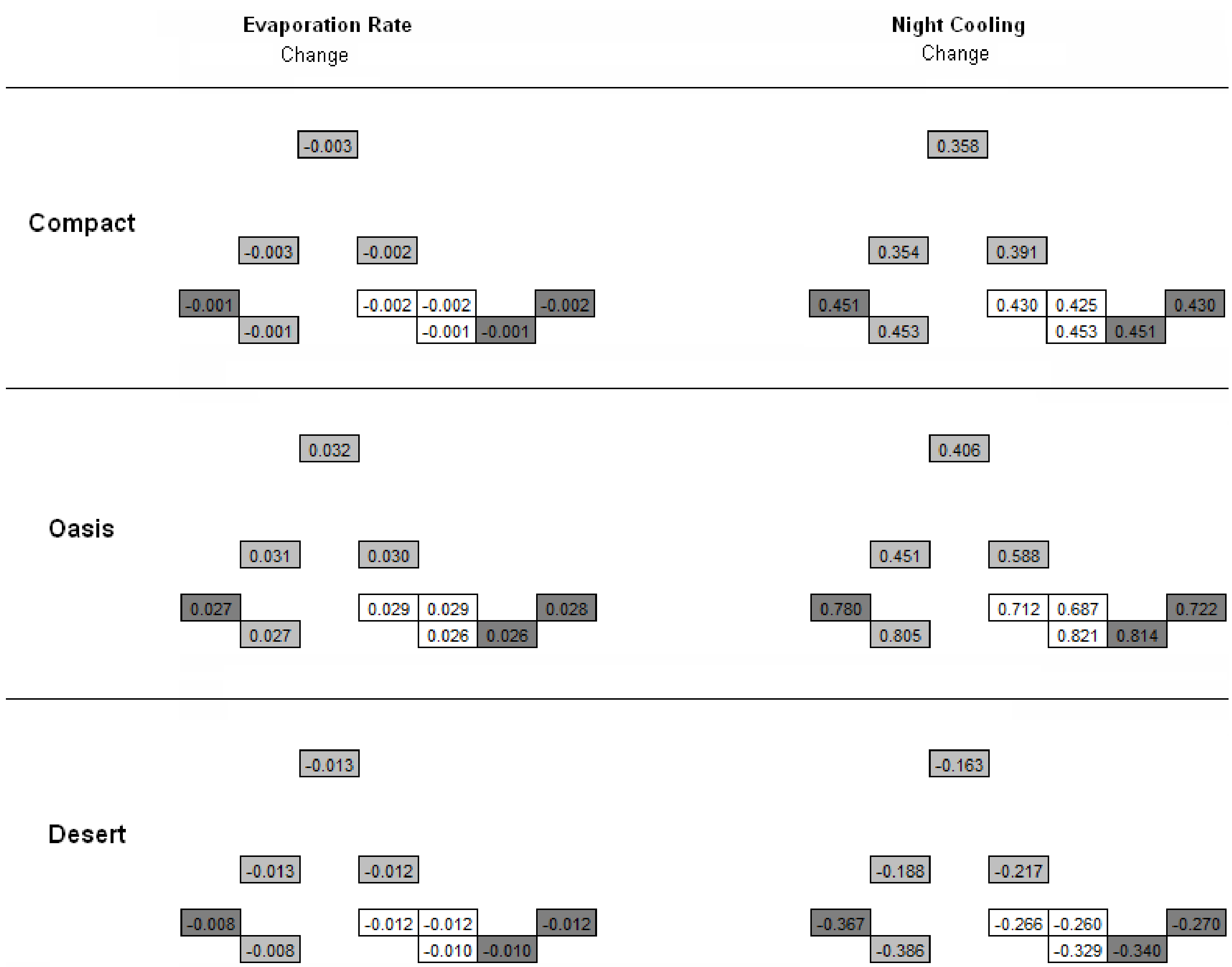

Figure 2. Changes in evaporation rate and night cooling that would result from implementing each of the three strategies in the ten tracts. The different shades represent the current classification.

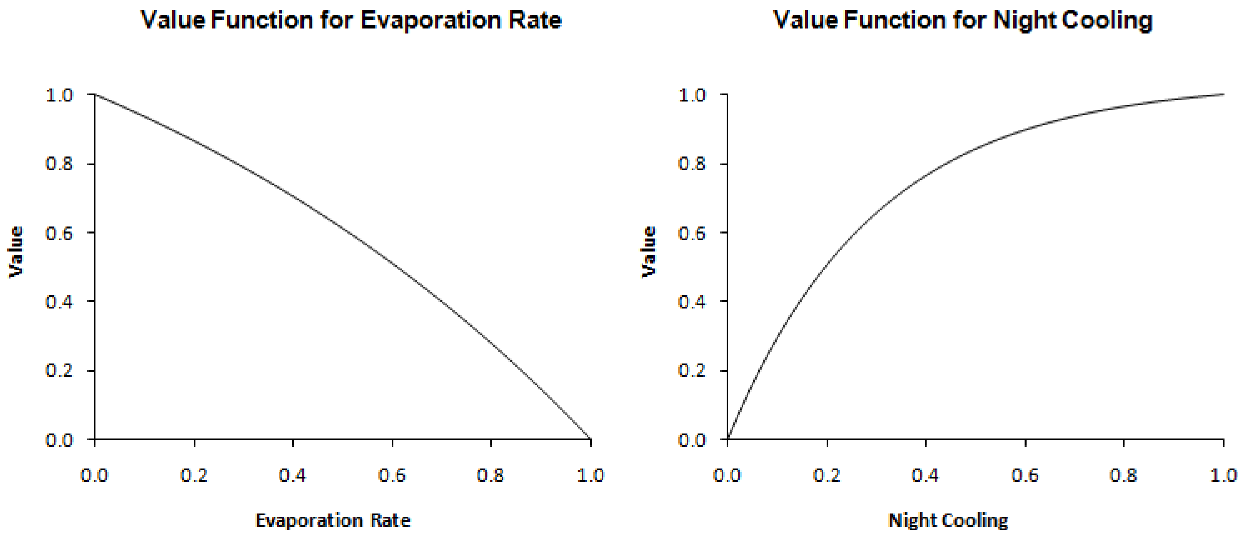

Figure 3. The exponential value functions over (normalized) evaporation rate and night cooling, with exponential constants c = 0.905 and c = 3.35, respectively.

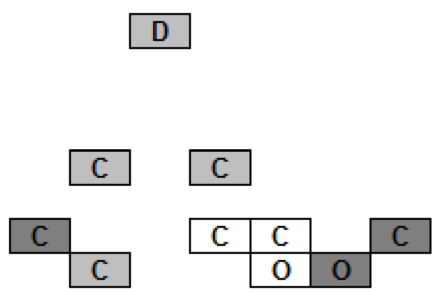

Figure 4. The optimal development plan using (7) and no cross-tract constraints. C=Compact, O=Oasis, D=Desert

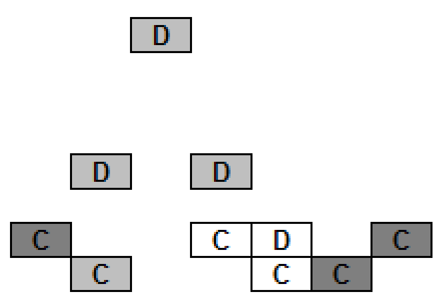

Figure 5. The optimal development plan with constraints on overall levels of evaporation rate and night cooling. C=Compact, O=Oasis, D=Desert

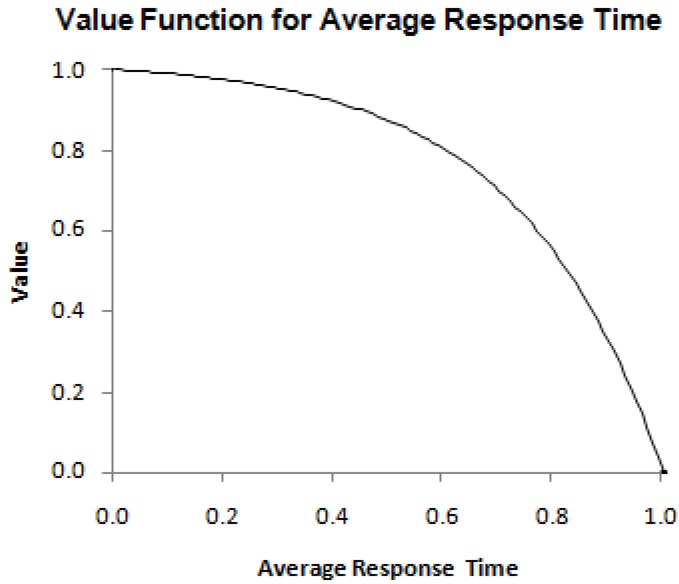

Figure 6. The exponential value function over average response time, with exponential constant $c = 3.86$.

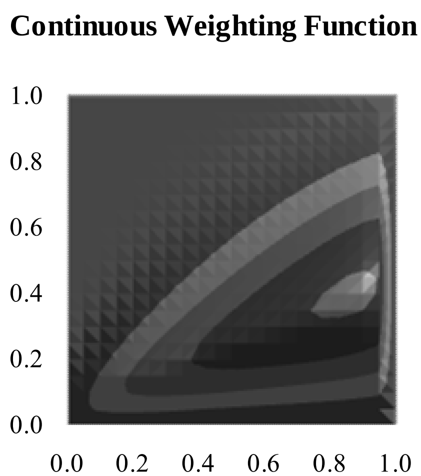

Figure 7. A contour map of the illustrative weighting function expressing the weight assigned to any point in the city. The maximum weight is assigned to (0.85, 0.4).

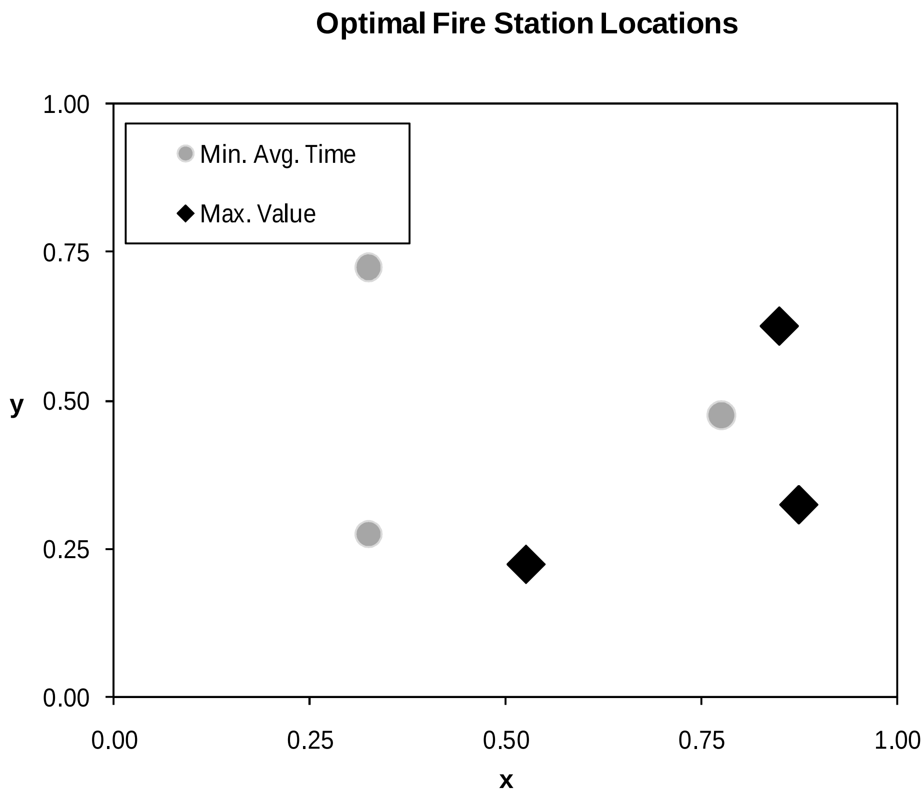

Figure 8. Optimal fire station locations when minimizing unweighted average response time, and when maximizing a geographically-weighted nonlinear value function.