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Performance analysis of FFH/BPSK receivers with convolutional coding and soft decision Viterbi decoding over channels with partial-band noise interference

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THESIS

PERFORMANCE ANALYSIS OF FFH/BPSK RECEIVERS WITH CONVOLUTIONAL CODING AND SOFT DECISION VITERBI DECODING OVER CHANNELS WITH PARTIAL-BAND NOISE INTERFERENCE

by

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March 1996

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ABSTRACT

An analysis of the performance of a binary phase shift keying (BPSK) communication system employing fast frequency-hopped (FFH) spread spectrum modulation, under conditions of hostile partial-band noise interference, is performed in this thesis. The data are assumed to be encoded using convolutional coding and the receivers are assumed to use soft decision Viterbi decoding.

The receiver structures to be examined are the conventional FFH/BPSK receiver with diversity, the conventional FFH/BPSK receiver with diversity and the assumption of perfect side information, and the noise-normalized FFH/BPSK combining receiver with diversity. The FFH/BPSK noise-normalized receiver with diversity minimizes the effects of hostile partial-band noise interference and alleviates the effects of fading. The effect of inaccurate measurement of the noise power present in each hop is also examined, and it is found that noise measurement error does not significantly degrade receiver performance. For the conventional FFH/BPSK receiver with perfect side information, the effect of a Ricean fading channel is also examined.

TABLE OF CONTENTS

I. INTRODUCTION

A. BACKGROUND

The conventional modulation/demodulation techniques like binary phase-shift keying (BPSK), quadrature phase-shift keying (QPSK), M-ary frequency-shift keying (MFSK) have good performance for digital communications in an additive white Gaussian noise (AWGN) environment. In reality, there are many occasions when the channel cannot be modeled as AWGN. For example, a military communication system is more likely to encounter narrowband interference such as a continuous wave (CW) signal near the carrier frequency in addition to AWGN. Also, the transmitted signal usually follows many different propagation paths in order to reach the receiver, so it suffers from another type of interference called multipath interference.

The necessity for a more robust modulation/demodulation technique in the presence of narrowband interference for military applications drove the development of spread spectrum. Spread spectrum is so called because the transmission bandwidth employed is much greater than the minimum bandwidth required to transmit the information. Some types of spread spectrum systems decrease the ability of a hostile observer to detect communication. This type of system is referred to as a low probability of detection (LPD) communication system. Other types of spread spectrum systems decrease the ability of a hostile observer to intercept communications. This type of system is referred to as a low probability of intercept (LPI) communication system. Spread spectrum systems also decrease the ability of a hostile jammer to efficiently jam the communication band. In the past, spread spectrum systems were primarily used for military applications. Nowadays, spread spectrum systems are coming into widespread use for commercial applications such as cellular communications and wireless communications. There are three primary types of spread spectrum systems: direct-sequence, frequency-hopping, and time-hopping systems. Hybrids of such systems can also be obtained.

As was mentioned above, one spread-spectrum modulation technique is frequency-hopping spread spectrum. Typically, in frequency-hopped spread spectrum the frequency of the carrier changes periodically according to some pseudorandom pattern. If more than one symbol is transmitted before the carrier frequency hops, then the system is referred to as a slow frequency-hopping (SFH) spread spectrum system. If the carrier frequency changes one or more times during the transmission of one symbol, then the system is referred to as a fast frequency-hopped (FFH) spread spectrum system. Fast frequency-hopping is a form of frequency/time diversity when multiple hops per symbol are used. Diversity is a form of repetition coding for the transmission of one symbol. A signal configured with multiple replicate copies, each transmitted over a different frequency, has a greater possibility of surviving than a signal with no diversity.

One of the most severe forms of narrowband interference is partial-band noise interference. In this case the jammer selects a fraction of the signal transmission bandwidth to jam with the result that significant performance degradation can occur. Fast frequency-hopping with diversity can be an effective countermeasure against partial-band noise jamming.

B. OBJECTIVE

In this thesis, the performance of a fast frequency-hopped BPSK receiver using noise-normalization is investigated. Noise-normalization combining is a technique that can be useful in reducing the effect of hostile interference. The noise-normalized receiver will be discussed in Chapter III. Partial-band noise interference is assumed in addition to AWGN. The communication system also employs diversity, convolutional coding, and soft decision Viterbi decoding. For purposes of comparison, a fast frequency-hopped BPSK receiver with perfect side information is used. The perfect side information receiver will be discussed in Chapter IV. The assumption of perfect side information is unrealistic but provides an ideal against which other systems can be compared.

The effect of partial-band interference on communication systems for noncoherent channels was investigated in [Ref. 1]. The performance of a noncoherent FFH/BFSK system with diversity and partial-band noise interference, was investigated [Ref. 2]. In

[Ref. 3] the effect of partial-band noise interference and channel fading on a non-coherent FFH/MFSK noise-normalized receiver was investigated, but the noise measurement was assumed to be ideal. In [Ref. 4] the performance of a FH/DPSK receiver with partial-band jamming and in [Ref. 5] the performance of a FH/QPSK receiver in the presence of jamming was investigated.

Previous investigations of FFH systems have concentrated on noncoherent systems such as noncoherent FFH/MFSK since, given current technology, implementation of coherent systems is not practical. This will not remain true in the future given sufficient technological advances, and FFH coherent systems with diversity have the advantage of zero noncoherent combining losses. Noncoherent combining losses result in a severe limitation of the effectiveness of noncoherent FFH systems with diversity. Hence, in this thesis the performance of a noise-normalized FFH/BPSK receiver with non-ideal noise power measurement and partial band noise jamming is investigated. Non-ideal noise power measurement is an important problem for the noise-normalized receiver since accurate, real time noise power estimation is difficult to implement for a FFH system.

II. DESCRIPTION OF SYSTEMS

The fast frequency-hopped BPSK transmitter is assumed to perform L hops per data bit where L is an integer greater than or equal to one. A block diagram of the FFH/BPSK receiver with noise normalization combining is shown in Fig. 1. At the receiver, the FFH signal is assumed perfectly dehopped. After the multiplier used for frequency translation, or heterodyning, an integrator circuit integrates the signal over the duration of one hop. The integrator acts as a low pass filter and provides optimum detection in AWGN. The integrator output is modeled as a Gaussian random variable X_k where k is an integer number between 1 and L corresponding to the specific hop.

For signals transmitted over fading channels, simple linear combining of the hops of a bit is sufficient to improve performance dramatically. For narrowband noise such as the worst case partial-band noise jamming considered in this thesis, simple linear combining is ineffective. One technique for reducing performance degradation due to partial-band noise jamming is noise-normalization combining. In noise-normalized receiver, the integrator output is normalized by the measured noise power $\hat{\sigma}_k$ of hop k. We consider both ideal and non-ideal noise power measurement. The decision variable Z of the receiver is formed by the summation of the noise-normalized integrator outputs Z_k k=1...L. The decision variable Z is also a Gaussian random variable since it is the sum of L independent Gaussian random variables. The signal Z is routed to a comparator where the final decision for an one or zero takes place. The effect of the noise-normalization procedure is to de-emphasize jammed hops with respect to unjammed hops; hence, the influence of jammed hops on the overall decision statistic is minimized.

The other receiver to be considered uses a knowledge of which hops are jammed and which are not. This information is called side information. When we know with certainty which hops are jammed and which are not, then we have perfect side information. In a receiver with perfect side information, only unjammed hops contribute to the final decision statistic unless all hops are jammed, in which case all hops are used with equal weight. If the system operates at a very high ratio of bit energy-to-thermal noise power spectral density (E_b/N_o), then the effect of thermal noise may be neglected;

and if any of the L bits are received without jamming, the decoder makes a perfect decision based only on the unjammed hops. Perfect side information is not realistic, but it gives us a standard against which to measure receivers which have imperfect side information.

The type of interference that is considered in this thesis is partial-band noise interference caused either by a partial-band jammer or by some unintended narrowband interference. The partial-band noise interference is modeled as a Gaussian process. Let ρ denote the fraction of the spread spectrum bandwidth jammed. Thus, ρ is the probability that narrowband interference is present, and (1-ρ) is the probability that narrowband interference is not present. If $\frac{N_j}{2N}$ is the average power spectral density of partial-band noise interference over the entire spread bandwidth, where N is the total number of frequency bins and N_j is the noise power spectral density of the jammer without partial-band jamming, then $\frac{N_j}{2N\rho}$ is the power spectral density of partial-band interference when it is present. In addition to narrowband interference, the signal suffers from the existence of thermal noise. The power spectral density of this wideband noise is defined as $\frac{N_o}{2}$. Thus, the total power spectral density of wideband noise and partial-band noise interference combined is given by

$$N_T = \frac{N_o}{2} + \frac{N_j}{2N\rho} \tag{1}$$

If we assume that the equivalent noise bandwidth of the receiver is B Hz, then for each hop the noise power N_oB is received with probability (1-ρ) when interference is not present and noise power $(N_o + N_j/N\rho)B$ with probability ρ when interference is present. In this thesis the bit rate is assume fixed. So for an FFH/BPSK system with L^{th} order diversity the hop rate is given by

$$R_h = LR_b \quad \text{or} \quad T_h = \frac{T_b}{L} \tag{2}$$

where R_h and T_h are the hop rate and the hop duration, respectively, and R_b and T_b are the bit rate and the bit duration, respectively. It is obvious that when $L = 1$ there is no diversity and noise-normalization has no effect. Also, the average energy per bit is related to the average energy per hop by

$$E_b = LE_h \Rightarrow E_h = \frac{E_b}{L} \qquad (3)$$

where E_b is the average energy per bit and E_h is the average energy per hop. So for a fixed bit rate and a fixed average energy per bit, both hop duration and average energy per hop decreases as diversity increases.

The equivalent noise bandwidth in the noise-normalized BPSK demodulator must be at least as wide as the hop rate, and for the correlation demodulator assumed in this thesis $B = R_h = \frac{1}{T_h}$. So the received noise power is $(N_o + \frac{N_j}{\rho N})/T_h$ when partial-band noise interference is present and N_o/T_h when is not present.

Note that there are two ways to investigate the performance of the FFH/BPSK noise-normalized receiver with L^{th} order diversity. The first is by assuming that the spread spectrum transmission bandwidth W is fixed. The bit rate is already assumed fixed for this thesis. Thus, as diversity increases, the number of frequency bins must decrease when bandwidth is fixed since $W = KNLR_b$ where L is the diversity, N the number of frequency bins, and K is an integer.

Alternatively, we can assume that the number of bins N is fixed. Now increasing diversity L also increases bandwidth by a factor of L. This means that the power of jammer P_j must be spread over a bandwidth larger by a factor of L, and the jamming power spectral density decreases by a factor of L. For this case $N_{jnew} = \frac{N_j}{L}$ where N_{jnew} is the jamming power spectral density after the spreading over the larger bandwidth.

When a system utilizes forward error correction coding (FEC), then for every k information data bits, n coded bits are transmitted where n>k. The FEC can help improve overall system performance. It is obvious that the n coded bits must be transmitted in the same time that the k data bits are transmitted in order to maintain a fixed data bit rate. So

$$nT_{b_c} = kT_b \qquad (4)$$

and

$$T_{b_c} = \frac{k}{n} T_b = rT_b \tag{5}$$

where $r = \frac{k}{n}$ is the code rate, T_{b_c} is the coded bit duration, and R_{b_c} is the coded bit rate which is given by

$$R_{b_c} = \frac{n}{k} R_b = \frac{R_b}{r} \tag{6}$$

Since r<1, the coded bit rate is higher than the uncoded bit rate. On the other hand, the average transmitted power is the same regardless of whether coded or uncoded bits are transmitted and is given by

$$P_c = E_{b_c} R_{b_c} = E_b R_b \tag{7}$$

which implies

$$E_{b_c} = \frac{R_b}{R_{b_c}} E_b = rE_b \tag{8}$$

Since r<1, the average energy per coded bit is less than the energy per data bit, and for fixed average energy per data bit, increasing the level of coding increases the probability of coded bit error. In this thesis the FEC used is convolutional coding with constraint length v=3 and code rate r=1/2. A convolutional code produces n coded bits that are determined by the k data bits and the k(v-1) preceding data bits.

If we assume that the number of bins N is fixed, then the bandwidth with coding is increased by a factor 1/r and the power of jammer has to be spread over a bandwidth larger by a factor 1/r. Hence, $N_{jnew} = \frac{N_j}{\frac{1}{r}} = rN_j$. Taking diversity into account, we see that the jamming power spectral density decreases in total by a factor L/r. Hence, the jamming power spectral density is $N_{jnew} = \frac{rN_j}{L}$.

We modeled the channel as a Ricean fading channel in which the signal consists of two components: a direct signal component and a diffuse signal component [Ref . 6]. In this case the channel for each hop is modeled as a frequency-nonselective, slowly fading

Ricean process. So the bandwidth of a hop is assumed to be smaller than the coherence bandwidth of the channel, and the hop duration is assumed smaller than the coherence time of the channel [Ref. 6]-[Ref. 7]. As a result of the above assumptions, the dehopped signal amplitude is modeled as a Ricean random variable. In this thesis, only the receiver with perfect side information is assumed to receive a signal over a fading channel.

III. PERFORMANCE ANALYSIS OF FAST FREQUENCY-HOPPED NON-IDEAL NOISE-NORMALIZED COMBINING BPSK RECEIVERS WITH SOFT DECISION VITERBI DECODING

The performance of the noise-normalized receiver is evaluated in this thesis by obtaining the bit error probability versus the bit energy-to-interference power spectral density for the receiver in Fig 1. The analysis requires the statistics of the random variables that model the integrator output X_k k=1,2,...,L, for each hop k , as well as the statistics of the noise-normalized random variables Z_k k=1,2,..., L, where

$$Z_k = \frac{X_k}{\hat{\sigma}_k} \tag{9}$$

where $\hat{\sigma}_k^2$ is the inaccurate noise power measurement for hop k. The summation of Z_k's yields the final decision random variable Z

$$Z = \sum_{k=1}^{L} Z_k \tag{10}$$

From Fig 1. the mean of X_k is simply the integrator output when there is no noise and is given by

$$\overline{X}_k = \sqrt{2}A_c \tag{11}$$

For this thesis we assumed perfect dehopping .

A. NOISE POWER ESTIMATOR

We have already discussed in the previous chapter that the noise power for each hop is

$$\sigma_k^2 = \sigma_{k_o}^2 = N_o B = \frac{N_o}{T_h} \tag{12}$$

with probability $1-\rho$ when interference is not present and is

$$\sigma_k^2 = \sigma_{k_j}^2 = \frac{N_o + \frac{N_j}{\rho N}}{T_h} \tag{13}$$

with probability ρ when interference is present. When the estimation of the noise power is ideal then

$$\hat{\sigma}_{k_{ideal}}^2 = \sigma_k^2 \tag{14}$$

When the noise power estimation is not ideal, then the total estimated noise power when interference is present is given by

$$\hat{\sigma}_{k_j}^2 = \frac{N_o + \frac{\hat{N}_j}{\rho N}}{T_h} \tag{15}$$

where $\hat{N}_j$ is the estimated noise power spectral density of the jammer and thermal noise power is assumed to be estimated without error. In this thesis $\hat{N}_j$ is modeled as a parameter so

$$\hat{N}_j = kN_j \tag{16}$$

where k is a real number and takes different values. For values of k such that $k<1$, $\hat{N}_j$ is underestimated . For values of k such that $k>1$, $\hat{N}_j$ is overestimated. When $k=1$, we have ideal noise power estimation and $\hat{N}_j = N_j$.

When only thermal noise is present the estimated noise power is given by

$$\hat{\sigma}_{k_o}^2 = \sigma_{k_o}^2 = \frac{N_o}{T_h} \tag{17}$$

B. PROBABILITY DENSITY FUNCTION OF THE DECISION VARIABLES

The random variable X_k is a Gaussian random variable, and its probability density function is

$$f_{X_k}(X_k) = \frac{1}{\sqrt{2\pi}\,\sigma_k} \exp\left(-\frac{(X_k - \overline{X_k})^2}{2\sigma_k^2}\right) \tag{18}$$

Making the linear transformation of equation (9) and using equation (11), we get the probability density function

$$f_{Z_k}(Z_k) = \frac{1}{\sqrt{2\pi}\,\frac{\sigma_k}{\hat{\sigma}_k}} \exp\left(-\frac{(Z_k - \frac{\sqrt{2}\,A_c}{\hat{\sigma}_k})^2}{2(\frac{\sigma_k}{\hat{\sigma}_k})^2}\right) \tag{19}$$

We note that equation (19) is a Gaussian random variable with mean

$$\overline{Z}_k = \frac{\sqrt{2}\,A_c}{\hat{\sigma}_k} \tag{20}$$

and variance

$$\sigma_{Z_k}^2 = \frac{\sigma_k^2}{\hat{\sigma}_k^2} \tag{21}$$

Since Z_K is a Gaussian random variable, then from equation (10) Z is also a Gaussian random variable, hence the mean of Z is

$$\overline{Z} = \sum_{k=1}^{L} \overline{Z_k} \tag{22}$$

and the variance is

$$\sigma_Z^2 = \sum_{k=1}^{L} \sigma_{Z_k}^2 \tag{23}$$

Substituting equations (20) and (21) into equations (22) and (23), respectively, we obtain the mean

$$\overline{Z} = \sum_{k=1}^{L} \frac{\sqrt{2}\,A_c}{\hat{\sigma}_k} \tag{24}$$

and the variance

$$\sigma_Z^2 = \sum_{k=1}^{L} \frac{\sigma_k^2}{\hat{\sigma}_k^2} \tag{25}$$

of the decision variable Z.

If i of the L hops are jammed, then from equation (24) the mean of the random variable Z, is given by

$$\overline{Z} = \frac{\sqrt{2}\,A_c}{\hat{\sigma}_{k_j}} i + \frac{\sqrt{2}\,A_c}{\sigma_{k_o}} (L - i) \tag{26}$$

The variance, or the noise power, of the random variable Z is obtained from equation (25) as

$$\sigma_Z^2 = \frac{\sigma_{k_j}^2}{\hat{\sigma}_{k_j}^2} i + \frac{\sigma_{k_o}^2}{\hat{\sigma}_{k_o}^2} (L - i) = \frac{\sigma_{k_j}^2}{\hat{\sigma}_{k_j}^2} i + (L - i) \tag{27}$$

where $\hat{\sigma}_{k_j}^2$ is the estimated noise power of the jammer and $\sigma_{k_o}^2$ is the thermal noise power. We have already assumed in equation (17) that $\hat{\sigma}_{k_o}^2 = \sigma_{k_o}^2$

C. PROBABILITY OF BIT ERROR

1. Without Coding

Since the decision statistic Z is a Gaussian random variable, the conditional bit error probability is

$$P_b(i) = Q\left(\sqrt{\frac{\bar{Z}^2}{\sigma_Z^2}}\right) \tag{28}$$

where the Q-function is defined as

$$Q(z) = \frac{1}{\sqrt{2\pi}} \int_z^{\infty} \exp(-\frac{\lambda^2}{2}) d\lambda \tag{29}$$

Substituting equations (26) and (27) into equation (28), we get the conditional bit error probability given i jammed hops as

$$P_b(i) = Q\left(\sqrt{\frac{2A_c^2(\frac{i}{\hat{\sigma}_{k_j}} + \frac{L-i}{\sigma_{ko}})^2}{i\frac{\sigma_{k_j}^2}{\hat{\sigma}_{k_j}^2} + L - i}}\right) \tag{30}$$

The argument of the Q-function can be rewritten

$$\frac{2A_c^2(\frac{i}{\hat{\sigma}_{k_j}} + \frac{L-i}{\sigma_{ko}})^2}{i\frac{\sigma_{k_j}^2}{\hat{\sigma}_{k_j}^2} + L - i} = \frac{\frac{2A_c^2}{\sigma_{ko}^2}(\frac{i}{\hat{\sigma}_{k_j}/\sigma_{ko}} + L - i)^2}{i\frac{\sigma_{k_j}^2}{\hat{\sigma}_{k_j}^2} + L - i} \tag{31}$$

and we already know from equations (2) and (17) that

$$\frac{2A_c^2}{\sigma_{k_o}^2} = \frac{2A_c^2 T_h}{N_o} = \frac{2A_c^2 L T_h}{L N_o} = \frac{2A_c^2 T_b}{L N_o} = \frac{2E_b}{L N_o} \tag{32}$$

Also from equations (15) and (17)

$$\frac{\hat{\sigma}_{k_j}^2}{\sigma_{k_o}^2} = \frac{N_o + \frac{\hat{N}_j}{\rho N}}{N_o} = 1 + \frac{\hat{N}_j}{\rho N N_o} \tag{33}$$

which can be rewritten

$$\frac{\hat{\sigma}_{k_j}}{\sigma_{k_o}} = \left(1 + \frac{\hat{N}_j}{\rho N N_o}\right)^{\frac{1}{2}} \tag{34}$$

Similarly, from equations (13) and (15)

$$\frac{\sigma_{k_j}^2}{\hat{\sigma}_{k_j}^2} = \frac{1+\frac{N_j}{\rho N N_o}}{1+\frac{\hat{N}_j}{\rho N N_o}} \tag{35}$$

Substituting equations (31), (32), (34), and (35) into equation (30), we get the conditional probability of bit error for the noize-normalized receiver without coding and non-ideal noise power estimation as

$$P_b(i,\hat{N}_j) = Q\left(\sqrt{\frac{\frac{2E_b}{LN_o}\left(\frac{i}{\sqrt{1+\frac{\hat{N}_j}{\rho N N_o}}}+L-i\right)^2}{i\left(\frac{1+\frac{N_j}{\rho N N_o}}{1+\frac{\hat{N}_j}{\rho N N_o}}\right)+L-i}}\right) \tag{36}$$

We note that for i=0 (no jammed hops)

$$P_b(0) = P_b = Q\left(\sqrt{\frac{2E_b}{N_o}}\right) \tag{37}$$

which is the bit error probability for the conventional BPSK receiver.

Since whether or not a hop is jammed is independent of whether other hops are jammed or not the probability that i of L hops are jammed is given by

$$\rho^i(1-\rho)^{L-i} \tag{38}$$

and the number of different ways that i of L hops can be jammed is given by the binomial coefficient

$$\begin{pmatrix} L \\ i \end{pmatrix} \tag{39}$$

Therefore, the probability of the event that i of L hops are jammed is

$$P_r(\text{i hops jammed}) = \begin{pmatrix} L \\ i \end{pmatrix} \rho^i (1-\rho)^{L-i} \tag{40}$$

Finally, the probability of bit error for any type of FFH system with diversity and partial-band noise jamming is given by

$$P_b = \sum_{i=0}^{L} \begin{pmatrix} L \\ i \end{pmatrix} \rho^i (1-\rho)^{L-i} P_b(i) \tag{41}$$

In our case $P_b(i)$ is given by equation (36). From equation (41) we can calculate the bit error probability of a FFH/BPSK noise-normalized receiver with diversity.

In order to estimate the worst case ρ, the ρ which maximizes the probability of bit error, we assume that P_b is dominated by the all hops jammed case. Hence

$$P_b \approx \rho^L Q\left(\sqrt{\frac{2\rho N E_b}{N_j}}\right) \tag{42}$$

According to Leibnitz's rule, we get from equation (29)

$$\frac{dQ(z)}{dz} = -\frac{1}{\sqrt{2\pi}} \exp\left(-\frac{z^2}{2}\right) \tag{43}$$

Now

$$\frac{dP_b}{d\rho} = L\rho^{L-i} Q\left(\sqrt{\frac{2\rho NE_b}{N_j}}\right) + \rho^L \frac{dQ(z)}{dz}\frac{dz}{d\rho} \tag{44}$$

and applying equation (43) to equation (44), we get

$$\frac{dP_b}{d\rho} = L\rho^{L-i} Q\left(\sqrt{\frac{2\rho NE_b}{N_j}}\right) - \frac{\rho^L}{\sqrt{2\pi}} \exp(-\frac{\rho NE_b}{N_j})\frac{1}{2}\left(\frac{2\rho NE_b}{N_j}\right)^{-\frac{1}{2}}\left(\frac{2NE_b}{N_j}\right) \tag{45}$$

By using an approximate expression of Q(z), [Ref. 8]

$$Q(z) \approx \frac{\exp(-\frac{z^2}{2})}{\sqrt{2\pi}\, z}\left(\frac{z^2}{z^2+1}\right) \qquad z > 2 \tag{46}$$

we can simplify equation (45) as follows

$$\frac{dP_b}{d\rho} \approx L\rho^{L-i} \frac{\exp(-\frac{\rho NE_b}{N_j})}{\sqrt{\frac{4\pi\rho NE_b}{N_j}}}\left(\frac{\frac{2\rho NE_b}{N_j}}{\frac{2\rho NE_b}{N_j}+1}\right) - \frac{\rho^L \exp(-\frac{\rho NE_b}{N_j})}{\sqrt{\frac{4\pi\rho NE_b}{N_j}}}\left(\frac{NE_b}{N_j}\right) \tag{47}$$

which reduces to

$$\frac{dP_b}{d\rho} \approx \frac{\rho^{L-\frac{1}{2}} \exp(-\frac{\rho NE_b}{N_j})}{\sqrt{\frac{4\pi NE_b}{N_j}}}\left(\frac{NE_b}{N_j}\right)\left(\frac{2L}{\frac{2\rho NE_b}{N_j}+1} - 1\right) \tag{48}$$

We require $\frac{dP_b}{d\rho} = 0$. From equation (48), only the factor $\frac{2L}{\frac{2\rho NE_b}{N_j}+1} - 1$ can be equal to zero for $1 \geq \rho > 0$. Hence,

$$\frac{2L}{\frac{2\rho_{wc} NE_b}{N_j}+1} - 1 = 0 \tag{49}$$

which can be solved to obtain

$$\rho_{wc} \approx \frac{L-\frac{1}{2}}{NE_b/N_j} \qquad (50)$$

Equation (50) provides an estimate of ρ_{wc}.

2. With Coding

The bit error probability of the FFH/BPSK noise-normalized receiver with L^{th} order diversity and convolutional coding is bounded by [Ref. 6]

$$P_b < \frac{1}{k} \sum_{d=d_{free}}^{\infty} w_d P_2(d) \qquad (51)$$

where w_d is the information weight of all paths of weight d and $P_2(d)$ is the probability of selecting a code word that is a distance d from the all zero code word. For soft decision decoding, $P_2(d)$ is equivalent to the probability of coded bit error for binary signaling with d^{th} order diversity [Ref. 6]. Since we have already assumed that our system utilizes L^{th} order diversity, the total effective diversity is Ld and $P_2(d)$ is obtained by modifying equation (41) to get

$$P_2(d) = \sum_{i=0}^{Ld} \binom{Ld}{i} \rho^i (1-\rho)^{Ld-i} P_2(d/i) \qquad (52)$$

where $P_2(d/i)$ is given by modifying equation (36) to get

$$P_2(d/i) = Q\left(\sqrt{ \frac{ \frac{2rE_b}{LN_o} \left(\frac{i}{\sqrt{1+\frac{\hat{N}_j}{\rho N N_o}}} + Ld - i \right)^2 }{ i\left(\frac{1+\frac{N_j}{\rho N N_o}}{1+\frac{\hat{N}_j}{\rho N N_o}} \right) + Ld - i } } \right) \qquad (53)$$

where r is the ratio of the number of information bits to the number of coded bits and is called the code rate. In order to find the worst case value of ρ, we follow the same procedure as before, beginning with

$$P_b \approx \rho^{Ld} Q\left(\sqrt{\frac{2rdE_b\rho N}{N_j}}\right) \tag{54}$$

and we get

$$\rho_{wc} \approx \frac{\left(dL-\frac{1}{2}\right)N_j}{NE_b rd} \tag{55}$$

This concludes the analysis of the FFH noise-normalized combining BPSK receiver. The discussion of numerical results is deferred until Chapter V where results for both the noise-normalized and the receiver with perfect side information, discussed in the next Chapter, are presented.

IV. PERFORMANCE ANALYSIS OF FFH/BPSK RECEIVERS WITH PERFECT SIDE INFORMATION, RICEAN FADING AND SOFT DECISION VITERBI DECODING

A. PROBABILITY OF BIT ERROR

1. Without Coding

As mentioned in the previous chapter, the total bit error probability of any type of FFH spread spectrum system with diversity and partial-band noise interference is given by

$$P_b = \sum_{i=0}^{L} \binom{L}{i} \rho^i (1-\rho)^{L-i} P_b(i) \tag{56}$$

For a receiver with perfect side information, when the number of jammed hops is less than L, then the computation of the probability of bit error is based on the L-i unjammed hops. Hence, the only noise source is AWGN, and the system has an effective diversity of L-i, where the effective energy per bit is reduced by the factor $\frac{L-i}{L}$. In this case, the conditional probability of bit error $P_b(i)$ is given by

$$P_b(i) = Q\left(\sqrt{\frac{2(L-i)E_b}{N_o L}}\right) \qquad i \neq L \tag{57}$$

When all hops are jammed, all hops are used, and the conditional probability of bit error is given by

$$P_b(L) = Q\left(\sqrt{\frac{2E_b}{N_o + \frac{N_j}{\rho N}}}\right) \qquad i = L \tag{58}$$

Assuming that $\frac{2(L-i)E_b}{LN_o} >> 1$, we come to the conclusion that $P_b(i) \approx 0$ when $i \neq L$. Assuming also that N_o is very small, we obtain the total probability of bit error

$$P_b \approx \rho^L \left(\sqrt{\frac{2E_b \rho N}{N_j}} \right) \tag{59}$$

The approximate worst case value of ρ, as previously, is given by equation (50).

2. With Coding

We have already shown that the probability of bit error for convolutional coding with soft decision Viterbi decoding is given by equation (51), while $P_2(d)$ is given by equation (52). For the receiver with perfect side information, when $i \neq Ld$, we adapt equation (53) to obtain the conditional probability of bit error.

$$P_2(d/i) = Q\left(\sqrt{\frac{2rE_b}{LN_o}(Ld - i)} \right) \tag{60}$$

and when $i = Ld$

$$P_2(d/i) = Q\left(\sqrt{\frac{2rdE_b}{N_o + \frac{N_j}{\rho N}}} \right) \tag{61}$$

The approximate worst case value of ρ is again given by equation (55).

3. With Fading

By modeling the channel as a Ricean fading channel we assume that the signal amplitude is no longer fixed but varies. In this thesis, only slowly fading frequency-nonselective Ricean fading channels are examined [Ref. 6]. The problem of BPSK with diversity transmitted over a Ricean fading channel has been examined [Ref.9]. The result obtained is cumbersome in the extreme. In this subsection, the probability of bit error is obtained in terms of an integral that must be evaluated numerically. The resulting numerical integration is significantly easier to implement than is applying the analytic solution obtained in [Ref. 9].

The probability density function of the Ricean random variable a_{c_k} is [Ref. 10]

$$f_{A_k}(a_{c_k}) = \frac{a_{c_k}}{\sigma^2}\exp(-\frac{a_{c_k}^2+\alpha^2}{2\sigma^2})I_o(\frac{a_{c_k}\alpha}{\sigma^2})u(a_{c_k}) \tag{62}$$

where $I(\bullet)$ represents the modified Bessel function of order zero and $u(\bullet)$ is the unit step function. The total average received signal power of hop k is

$$\overline{a_{c_k}^2} = \alpha^2 + 2\sigma^2 \tag{63}$$

where α^2 is the power of the received non-faded (direct) signal component and $2\sigma^2$ is the power of the received Rayleigh faded (diffuse) signal component. The noise-normalized signal power of hop k is given by

$$\frac{\overline{a_{c_k}^2}}{\sigma_k^2} = \frac{\alpha^2}{\sigma_k^2} + \frac{2\sigma^2}{\sigma_k^2} = \rho_k + \xi_k \tag{64}$$

where $\rho_k = \alpha^2/\sigma_k^2$ is the direct signal-to-noise power ratio and $\xi_k = 2\sigma^2/\sigma_k^2$ is the diffuse signal-to-noise ratio. We specify the ratio of the direct component of the signal power to the power of the Rayleigh- faded component of the signal as

$$\eta = \frac{\alpha^2}{2\sigma^2} = \frac{\alpha^2/\sigma_k^2}{2\sigma^2/\sigma_k^2} = \frac{\rho_k}{\xi_k} \tag{65}$$

from equation (65),

$$\rho_k = \xi_k\eta \tag{66}$$

Substituting equation (65) into equation (64), we get

$$\frac{\overline{a_{c_k}^2}}{\sigma_k^2} = \xi_k\eta + \xi_k = \xi_k(\eta + 1) \tag{67}$$

and the final expression for the diffuse signal-to-noise ratio is

$$\xi_k = \left(\frac{1}{\eta+1}\right)\left(\frac{\overline{a_{c_k}^2}}{\sigma_k^2}\right) \tag{68}$$

We notice that if there is no fading then $\frac{\overline{a_{c_k}^2}}{\sigma_k^2} = \frac{A_c^2}{\sigma_k^2}$, which implies that $\eta \to \infty$.
Also, when $\eta \to 0$ we have a Rayleigh fading channel, and $\rho_k \to 0$; i.e., the direct signal component goes to zero.
In equation (62), for the change of variables $a_{c_k}^2 = b_k$, we get

$$\frac{da_{c_k}}{db_k} = \frac{1}{2a_{c_k}} \tag{69}$$

and the probability density function becomes

$$f_{B_k}(b_k) = \frac{1}{2\sigma^2}\exp(-\frac{b_k+\alpha^2}{2\sigma^2})I_o(\frac{\alpha}{\sigma^2}\sqrt{b_k})u(b_k) \tag{70}$$

The Laplace transform of equation (70) is

$$F_{B_k}(s) = \int_0^\infty f_{B_k}(b_k)\exp(-sb_k)db_k \tag{71}$$

which can be evaluated to obtain

$$F_{B_k} = \frac{1}{2\sigma^2}\exp[-\frac{\alpha^2}{2\sigma^2}(\frac{s}{s+\frac{\sigma^2}{2}})](\frac{1}{s+\frac{\sigma^2}{2}}) \tag{72}$$

We define $b = \sum_{k=1}^{L} a_{c_k}^2$; and since $F_B(s) = [F_{B_k}(s)]^L$, we get after inverse transforming

$$f_B(b) = \frac{b^{(L-1)/2}}{2\sigma^2(L\alpha^2)^{(L-1)/2}}\exp\left(-\frac{b+L\alpha^2}{2\sigma^2}\right)I_{L-1}\left(\frac{\alpha}{\sigma^2}\sqrt{Lb}\right)u(b) \tag{73}$$

We know that the probability density function of a Gaussian random variable is [Ref. 6]

$$f_X(x) = \frac{1}{\sqrt{2\pi}\,\sigma_x} \exp\left(\frac{-(x-\bar{x})^2}{2\sigma_x^2}\right) \tag{74}$$

where σ_x and $\bar{x}$ are the variance and the mean of the Gaussian random variable x, respectively. Thus, the bit error probability is [Ref. 6]

$$P_b = \frac{1}{\sqrt{2\pi}} \int_0^{\infty} \frac{1}{\sigma_x} \exp\left(-\frac{(x-\bar{x})^2}{2\sigma_x^2}\right) dx \tag{75}$$

Substituting in equation (75) $\lambda = \frac{x-\bar{x}}{\sigma_x}$ and $d\lambda = \frac{dx}{\sigma_x}$, we get

$$P_b = \frac{1}{\sqrt{2\pi}} \int_{\frac{\bar{x}}{\sigma_x}}^{\infty} \exp(-\tfrac{\lambda^2}{2}) d\lambda \tag{76}$$

Modifying equation (76) by using

$$\frac{\bar{x}}{\sigma_x} = \frac{\sqrt{2} \sum_{k=1}^{L} a_{c_k}^2}{\sqrt{\frac{N_o}{T_h} \sum_{k=1}^{L} a_{c_k}^2}} \tag{77}$$

which leads to

$$\frac{\bar{x}^2}{\sigma_x^2} = \frac{2T_h \sum_{k=1}^{L} a_{c_k}^2}{N_o} = \frac{2T_h b}{N_o} \tag{78}$$

we obtain bit error probability conditioned on b as

$$P_b(b) = \frac{1}{\sqrt{2\pi}} \int_{\sqrt{\frac{2T_h b}{N_o}}}^{\infty} \exp(-\tfrac{\lambda^2}{2}) d\lambda \tag{79}$$

which can be expressed in terms of the Q-function as

$$P_b = Q\left(\sqrt{\frac{2T_h b}{N_o}}\right) \tag{80}$$

Now the total P_b is

$$P_b = \int_0^\infty f_B(b) P_b(b) db \tag{81}$$

An approximate expression for equation (80) that is valid for b>>1 is

$$P_b(b) \approx P_{b_{app}}(b) = \frac{1}{\sqrt{4\pi T_h b/N_o}} \exp\left(-\frac{T_h b}{N_o}\right) \tag{82}$$

Now we add and subtract $P_{b_{app}}$ from the integral of equation (81) to get

$$P_b = \int_0^\infty f_B(b)[P_b(b) + P_{b_{app}} - P_{b_{app}}] db \tag{83}$$

which is equal to

$$P_b = \int_0^\infty f_B(b)\left[P_b(b) - P_{b_{app}}(b)\right] db + \int_0^\infty f_B(b) P_{b_{app}} db \tag{84}$$

We define

$$I_B = \int_0^\infty f_B(b) P_{b_{app}} db \tag{85}$$

and if in equation (82) we let

$$\sqrt{\frac{2T_h b}{N_o}} = \sqrt{c} \tag{86}$$

where c is some constant, then equation (82) becomes

$$P_{b_{app}}(b) = \frac{1}{\sqrt{2\pi c}} \exp(\frac{-T_h b}{N_o}) \tag{87}$$

Now equation (85) becomes

$$I_B = \int_0^{\infty} \frac{b^{(L-1)/2} \exp(-L\alpha^2/2\sigma^2)}{2\sigma^2 (L\alpha^2)^{(L-1)/2} \sqrt{2\pi c}} \exp\left(-b(\frac{1}{2\sigma^2} + \frac{1}{N_o/T_h}\right) I_{L-1}\left(\frac{\alpha}{\sigma^2}\sqrt{Lb}\right) db \tag{88}$$

and if we define $\xi = \frac{2\sigma^2}{\sigma_k^2}$ and $\rho = \frac{\alpha^2}{\sigma_k^2}$, equation (88) can be evaluated to obtain

$$I_B = \frac{1}{(1+\xi_k)^L \sqrt{2\pi c}} \exp(-\frac{L\rho_k}{1+\xi_k}) \tag{89}$$

Rewritting equation (73), we get

$$f_B(b) = \frac{(b/\sigma_k^2)^{(L-1)/2}}{2\sigma^2 (L\alpha^2/\sigma_k^2)^{(L-1)/2}} \exp\left(-\frac{b/\sigma_k^2 + L\alpha^2/\sigma_k^2}{2\sigma^2/\sigma_k^2}\right) I_{L-1}\left(\frac{2\sqrt{L\alpha^2 b/\sigma_k^4}}{2\sigma^2/\sigma_k^2}\right) \tag{90}$$

If we define $b' = b/\sigma_k^2$ and $db' = db/\sigma_k^2$, then equation (90) becomes

$$f_B(b') = \frac{(b')^{(L-1)/2}}{\xi_k (L\rho_k)^{(L-1)/2}} \exp\left(-\frac{b' + L\rho_k}{\xi_k}\right) I_{L-1}\left(\frac{2}{\xi_k}\sqrt{L\rho_k b'}\right) \tag{91}$$

and equation (84) can be expressed as

$$P_b = \int_0^{\infty} f_B(b')\left[P_b(b') - P_{b_{app}}(b')\right] db' + \frac{1}{\sqrt{2\pi c}\,(1+\xi_k)^L} \exp(-\frac{L\rho_k}{1+\xi_k}) \tag{92}$$

In equation (92),

$$P_b(b') = Q(\sqrt{2b'}) \tag{93}$$

$$P_{b_{app}}(b') = \frac{1}{\sqrt{2\pi c}} \exp(-b') \tag{94}$$

and

$$f_B(b') = \frac{(b')^{(L-i-1)/2}}{\xi_k[(L-i)\rho_k]^{(L-i-1)/2}} \exp\left(\frac{-b'-(L-i)\rho_k}{\xi_k}\right) I_{L-i-1}\left(\frac{2}{\xi_k}\sqrt{(L-i)\rho_k b'}\right) \tag{95}$$

and $I_{L-i-1}(\frac{2}{\xi_k}\sqrt{(L-i)\rho_k b'})$ is the modified Bessel function of order L-i-1. Equation (92) is now in a form that can be easily evaluated by numerical integration since the integral rapidly approaches zero to increasing b'.

The conditional probability of bit error, assuming that i out of L hops are jammed, is obtained from equation (92) as

$$P_b(i) = \int_0^\infty f_B(b')[P_b(b') - P_{b_{app}}(b')]db' + \frac{1}{\sqrt{2\pi c}\,(1+\xi_k)^{L-i}} \exp\left(-\frac{(L-i)\rho_k}{1+\xi_k}\right) \tag{96}$$

When all hops are jammed, $i = L$, the conditional bit error probability is given by

$$P_b(L) = \int_0^\infty f_B(b')[P_b(b') - P_{bapp}(b')]db' + \frac{1}{\sqrt{2\pi c}\,(1+\xi_k)^{L}} \exp\left(-\frac{L\rho_k}{1+\xi_k}\right) \tag{97}$$

Finally, substituting equations (96) and (97) into equation (56), we obtain the total bit error probability for a perfect side information receiver and a Ricean fading channel. Numerical results are presented in the next Chapter.

V. NUMERICAL RESULTS AND DISCUSSION

A. NOISE-NORMALIZED RECEIVER

1. Without Coding

By using equations (36) and (41), we obtained the probability of bit error of the noise-normalized receiver without coding. Figs. 2-13, are an illustration of an examination of the performance of the receiver for non-ideal estimation of noise power. Specifically, in Figs. 2-4, a 50% underestimation of noise power is assumed, in Figs.5-7 a 75% underestimation of noise power is assumed, in Figs.8-10 a 50% overestimation of noise power is assumed, and in Figs.11-13 a 100% overestimation of noise power is assumed. The order of diversity is L=2, 4, and 6, and the fraction of the jammed band ρ is taken as ρ=0.001,0.02,0.1,0.5,1and ρ_{wc}. We assumed that the ratio of bit energy-to-jamming noise power spectral density varied from -30 dB to 10 dB. In all cases, by increasing diversity we achieved better performance, contrary to what is obtained with noncoherent systems where noncoherent combining losses are a factor [Ref. 11].

The performance of the receiver is better when the noise power is underestimated 50% rather than 75%. The performance of the receiver is better for 100% overestimation than for 50% overestimation. In general, we noticed that overestimation gave better performance than underestimation. The higher the overestimation error the better the performance of the receiver. This result is not unexpected and is consistent with results obtained for noncoherent systems. For all cases of noise-normalization error considered and, for L=2, ρ=1 is not worst case for all values of E_b/N_j. For some specific values of E_b/N_j, when the whole band is jammed receiver performance is best. For 75% underestimation of noise power, L=2, and values of E_b/N_j between -1 dB and -17 dB, ρ=1 is best case for the receiver; while for values of E_b/N_j between -9 dB and 10 dB, ρ=0.001 is worst case. For 50% overestimation of noise power, L=2, and E_b/N_j between -1 dB and -12 dB, ρ=1 is best case for the receiver; while for E_b/N_j between -5 dB and 10 dB, ρ=0.001 is the worst case. Clearly, a diversity of L=2 is insufficient to completely eliminate the effect of partial-band noise jamming.

For 50% and 100% overestimation of noise power and for L=4 and 6, $\rho=1$ is worst case for the receiver. For 75% and 50% underestimation of noise power, as L is increased, the range of values of E_b/N_j over which $\rho=1$ is best case becomes smaller. For a specific fraction of the band jammed, the receiver has the best performance for 100% overestimation of noise power and diversity of order L=6.

Ideal noise normalization, illustrated in Figs.14-16, is seen to have better performance than that obtained when noise power is underestimated. On the other hand, ideal noise-normalization does not perform as well as when noise power is overestimated. As can be seen from an examination of the figures, the effect of partial-band noise jamming can be completely negated, given sufficient diversity, by intentionally overestimating noise power.

2. With Coding

By using a combination of equations (51), (52), and (53), we obtained the probability of bit error of the noise-normalized receiver with coding. In Figs. 17-26 we illustrate the performance of the receiver for the same cases as above, but this time with the addition of convolutional coding. The order of diversity examined is L=1 and 2. As before, overestimation of noise power gives better results than underestimation. For 75% and 50% underestimation of noise power and for both L=1 and 2, $\rho=1$ is not worst case for the receiver for all values of E_b/N_j. For L=1, different values of ρ yield worst case depending on E_b/N_j. For 75% underestimation of noise power and E_b/N_j between -4 dB and -11dB, worst case is $\rho=0.001$ and between -11 dB and -16 dB, worst case is $\rho=0.02$. For 50% and 100% overestimation of noise power and L=2, $\rho=1$ is the worst case for all values of E_b/N_j. In all cases equation (50) provided a satisfactory approximation to ρ_{wc}. Again, ideal noise-normalization gave better results than noise power underestimation and worse than overestimation. As can be seen, much lower levels of diversity are required to eliminate the effects of partial-band noise jamming when convolutional coding is used.

3. Comparisons

In Figs. 27-29 the improvement in performance of the noise-normalized receiver without coding for worst case partial-band jamming and increasing the estimation error is shown. For L=2 and E_b/N_j <-25 dB and for L=4, 6 and E_b/N_j <-23 dB the performance remains the same, regardless of the estimation error. Especially noteworthy is that the same performance is obtained for a wide range of overestimation. It is clear that with overestimation of noise power the receiver performance is better than with underestimation. The higher the overestimation error, the better the performance of the receiver. We also noticed that for bit energy-to-jamming noise ratios less than -23 dB and worst case partial-band jamming that the performance is unaffected by estimation error.

In Figs. 30-31 the improvement in performance of the noise-normalized receiver with coding for worst case partial-band jamming and increasing the estimation error is shown. For L=1 and E_b/N_j <-25 dB and for L=2 and E_b/N_j <-23 dB, the performance remains the same, regardless of the estimation error.

In Figs. 32-37, we illustrate the probability of bit error for the noise-normalized receiver both with and without coding for worst case partial-band noise jamming and different values of diversity for each case of noise power estimation. In Figs. 32-34 we assumed a system with constant bandwidth, which is the classical way of evaluating a system. No matter what the noise estimation is, the utilization of coding allows the receiver to have better performance than without coding even with high order diversity (L=6). In Figs. 35-37, we assume a system with a constant number of bins. This kind of system in general has better performance than a system using constant bandwidth. The reason is that in this system the bandwidth is increased by a factor of L without coding and by a factor of L/r with coding. Consequently, the power of the jammer is spread over a larger bandwidth, decreasing its effectiveness and enhancing the performance of the receiver. So for the case of 50% noise power underestimation without coding and L=2, there is a gain of 3 dB and for L=6 there is a gain of 8 dB at probability of 10^{-3}. This is in agreement with theory as we expected a gain of $10\log_{10}L$ dB. For the noise-normalization with coding the gain is expected to be $10\log_{10}\left(\frac{L}{r}\right)$ dB. For example, regardless of estimation error, at a probability of bit error 10^{-9} and L_c=2 the gain is 6 dB. Again, the utilization of coding yields better performance than no coding and diversity L=6.

In Figs. 38-40 we compare the performance of the noise-normalized receiver for each case of noise power estimation, with and without coding, with diversity L=2, to the performance of the comparable conventional FFH/BPSK receiver assuming a system with constant bandwidth. It is apparent that the noise-normalized receiver with coding gives the best performance for almost the entire range of E_b/N_j. The FFH/BPSK with noise-normalization receiver with coding has a significantly better performance, about 23 dB, than the conventional FFH/BPSK with coding at a probability of bit error 10^{-7}. We also notice that for all cases of noise power estimation error and for E_b/N_j between -17 dB and -11 dB that the conventional FFH/BPSK receiver with coding has the worst performance; while for values of E_b/N_j >-9dB the conventional FFH/BPSK receiver with coding has better performance than the FFH/BPSK noise-normalized receiver without coding .

In Figs. 41-43, we illustrate the same comparisons as above but for a system with a constant number of bins. The same general trends previously observed are repeated, but the performance of the receivers with coding is improved about 7 dB, while without coding the performance is improved about 3 dB as compared to a system with constant bandwidth.

B. PERFECT SIDE INFORMATION RECEIVER

1. Without Fading

a. Without Coding

In order to evaluate the performance of the perfect side information receiver, we computed the probability of bit error using equations (56), (57), and (58). We assume that the ratio of the bit energy-to-thermal noise power spectral density E_b/N_o is 20 dB rather 10.52 dB. In Figs. 44-46 the performance of the perfect side information receiver without coding and for L=2, 4, and 6 is illustrated. The fraction of the band that must be jammed for worst case performance is calculated from equation (50). Broadband jamming, ρ=1, is not worst case for all values of E_b/N_j. In general, as diversity increases the range of values of E_b/N_j for which ρ=1 is worst case increases. We also note that as the bit energy, E_b, decreases smaller values of ρ result in worst case performance. As diversity increases, worst case partial-band noise jamming approaches broadband noise jamming (ρ_{wc} =1).

b. With Coding

In Figs.47-48 we evaluated the performance of the perfect side information receiver by computing the bit error probability using equations (60) and (61). As is expected, the utilization of coding gives better results than without coding. For ρ=1 and L=2, there is a coding gain of 4 dB for a probability of bit error of 10^{-6}.

When using convolutional coding, there is no need to use high orders of diversity. Even with a diversity of L=2, the performance of the receiver is better than without coding and a diversity of L=6. Performance degradation due to partial-band noise jamming is essentially eliminated with very small level of diversity when coding is used.

2. With Fading

Without Coding

In Figs 49-57 we evaluate the performance of the perfect side information receiver for Ricean fading channels using equations (96), (97), and (56) .

In Figs. 49-51, we examine the performance of the receiver for L=2, 4, and 6 with a ratio of the direct-to-diffuse component of the signal of 10, η=10. In all cases, ρ=1 yields worst case. The smaller the fraction of the band that is jammed, the better the performance of the receiver. As we increase the direct component of the signal so that η=100 (Figs.52-54), the overall performance of the receiver improves and approaches what is expected in the absence of fading.

For the case of Rayleigh fading, the direct component of the signal is zero. In Figs. 55-57, we examine the performance of the receiver for this case. We note that for L=2 even small values of ρ are effective in degrading receiver performance. For values of

E_b/N_j between -30 dB and -13 dB, ρ=1 is the worst case for the receiver and between -7 dB and 1 dB is the best. Receiver performance improves when diversity is increased to L=6, and only ρ=0.1 results in a high probability of bit error. For values of E_b/N_j greater than -8 dB, the effectiveness of the jammer is negligible when the whole band is jammed.

VI. CONCLUSIONS

There is a noticeable difference in the performance of a FFH/BPSK noise-normalized receiver in partial-band noise jamming conditions when the noise power estimation is inaccurate. The poorest receiver performance is obtained when the noise jamming power is underestimated, and the best performance is obtained when the noise jamming power is overestimated. This is the result we intuitively expect and is consistent with the results obtained for noncoherent noise-normalized MFSK [Ref 11]. The effect of diversity in minimizing receiver degradation due to partial-band noise jamming is significant. The performance of the receiver improves as diversity is increased.

The utilization of convolutional coding with soft decision Viterbi decoding dramatically improves the performance of the receiver and increases significantly the robustness of the receiver in partial-band noise jamming. The combination of coding and a diversity of L=2 gives performance superior to that obtained when only a high order diversity (L=6) with no coding is used.

We investigated the performance of the FFH/BPSK noise-normalized system for both a constant bandwidth and constant number of frequency bins. In general systems with a constant number of bins have better performance against a jammer than systems with constant bandwidth. In both cases, the FFH/BPSK noise-normalized receiver with coding and L^{th} order diversity performs better in a partial-band noise jamming environment than the traditional FFH/BPSK receiver either with or without coding.

The performance of a FFH/BPSK receiver with the assumption of perfect side information with fading and partial-band noise jamming is also examined. As mentioned previously, the assumption of perfect side information is unrealistic but provides an ideal against which other systems can be compared. We note that the performance of the receiver improves in fading channels with partial-band jamming for increasing diversity. The opposite occurs in channels without fading and with partial-band noise jamming. The worst performance degradation of the receiver occurs for Rayleigh fading channels with partial-band noise interference. Even with a diversity of L=6, partial-band noise jamming with ρ=0.1 resulted in a high probability of bit error.

We conclude that the noise-normalized receiver is relatively insensitive to noise power measurement error. The larger the overestimation error, the better the performance of the receiver. Hence, the noise-normalized receiver is very robust in worst case partial-band noise jamming even when noise power measurement error is large.

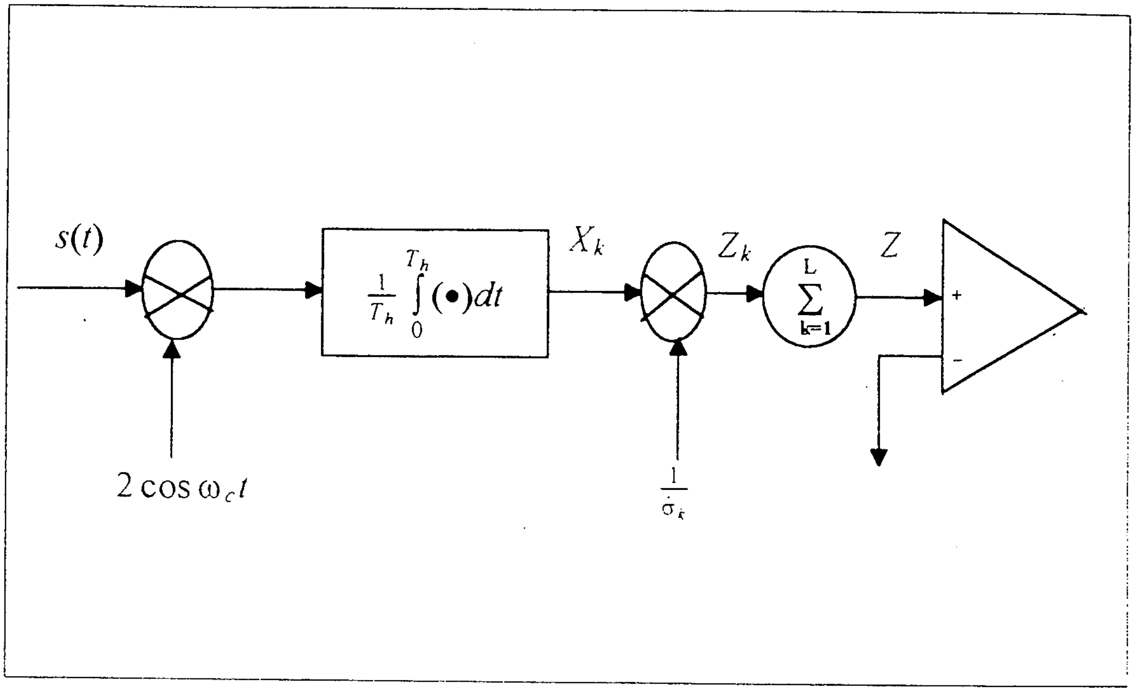

Figure 1 FFH/BPSK noise - normalized receiver assuming perfect dehopping.

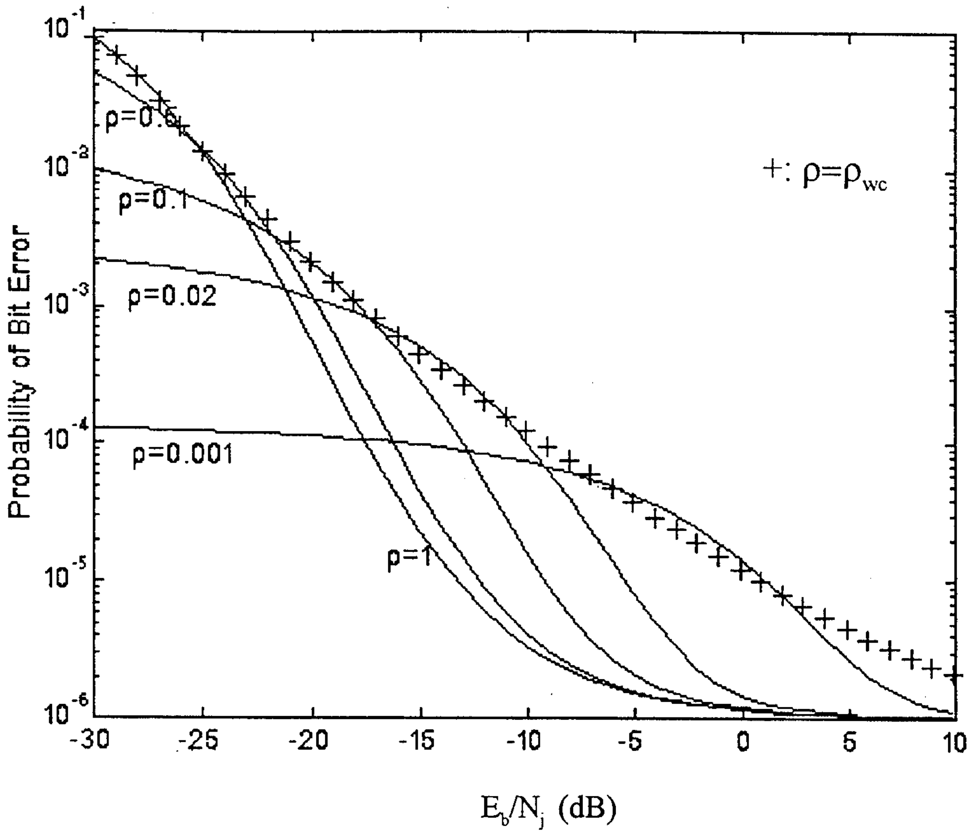

Figure 2. Non-ideal noise-normalization with 75% underestimation of noise power. Performance of the receiver without coding, L=2 and E_b/N_o=10.52 dB.

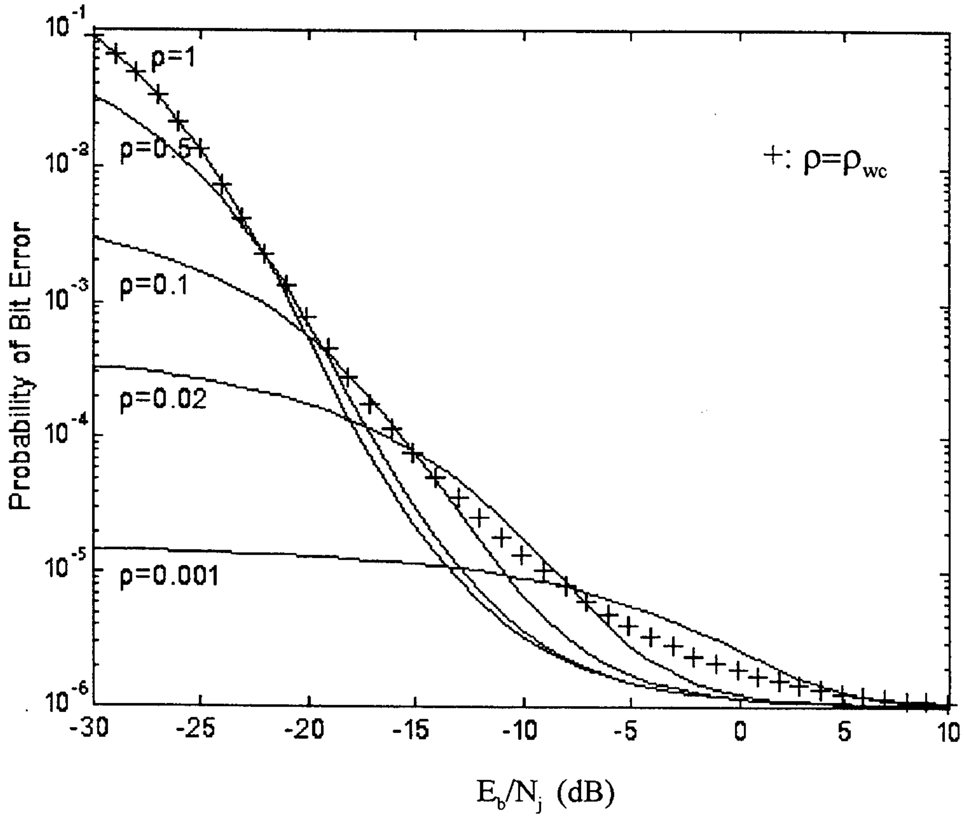

Figure 3. Non-ideal noise-normalization with 75% underestimation of noise power. Performance of the receiver without coding, L=4 and E_b/N_o=10.52 dB.

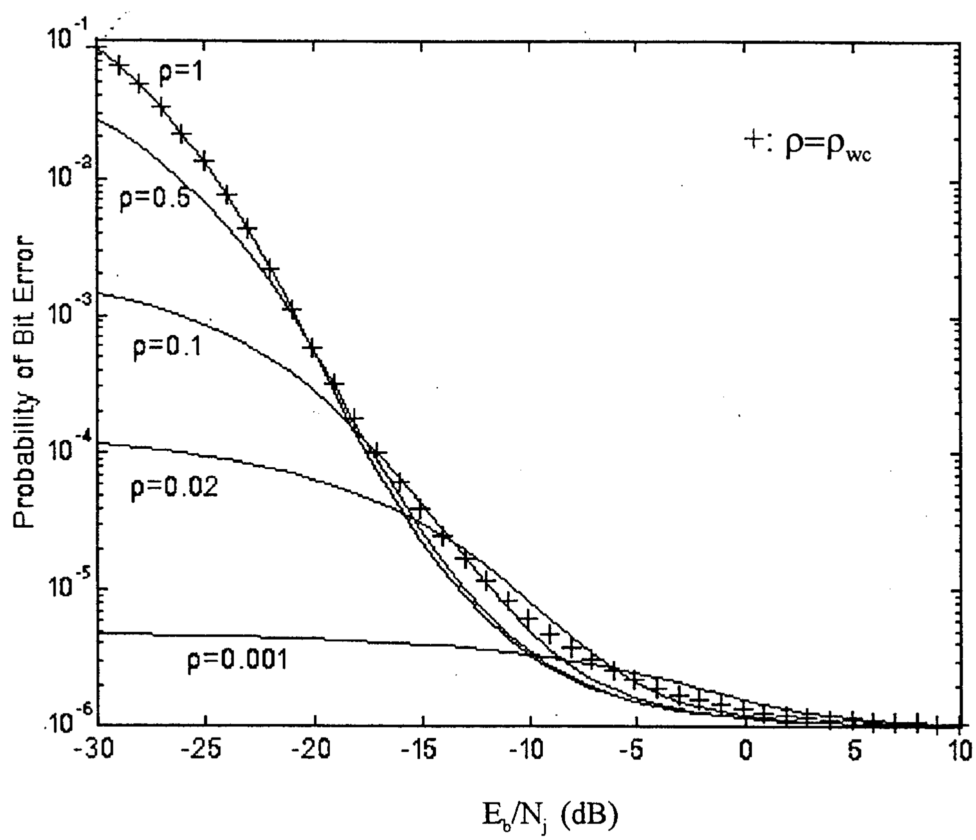

Figure 4. Non-ideal noise-normalization with 75% underestimation of noise power. Performance of the receiver without coding, L=6 and E_b/N_o=10.52 dB.

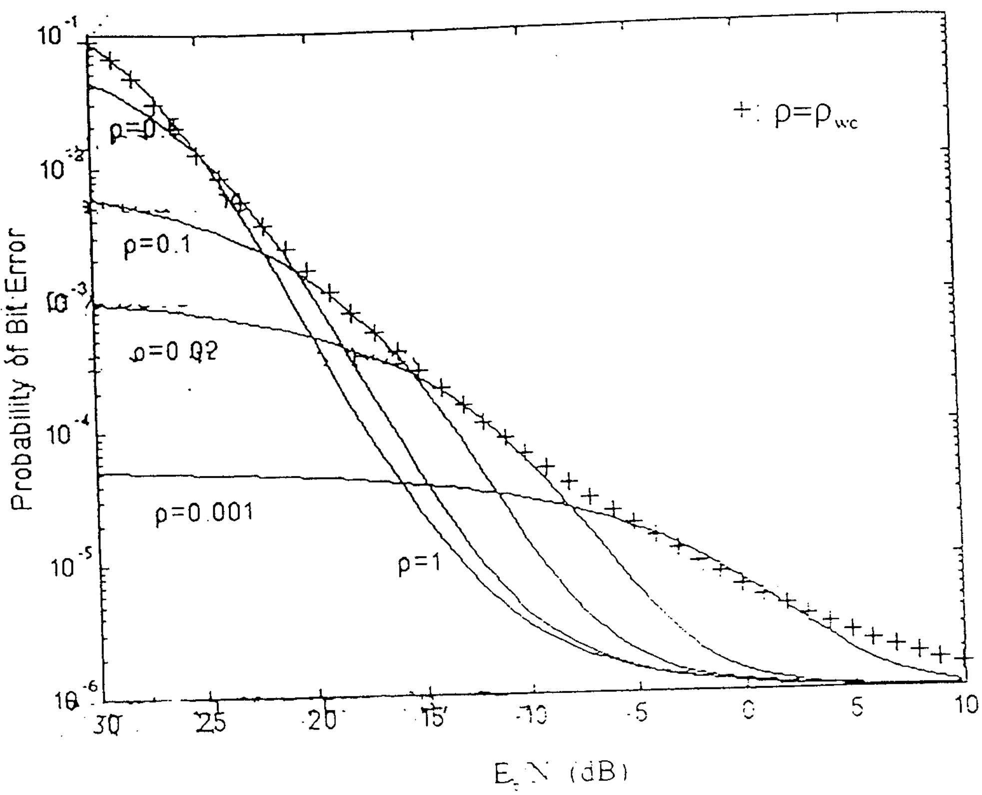

Figure 5. Non-ideal noise-normalization with 50% underestimation of noise power. Performance of the receiver without coding, L=2 and E_b/N_o=10.52 dB.

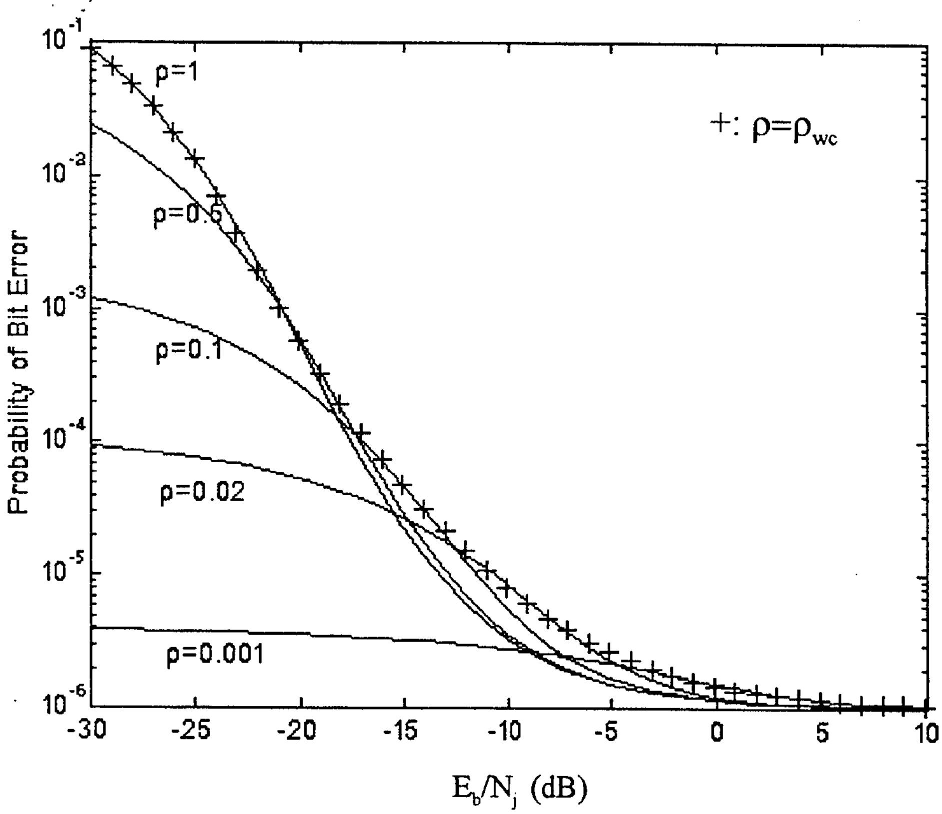

Figure 6. Non-ideal noise-normalization with 50% underestimation of noise power. Performance of the receiver without coding, L=4 and E_b/N_o=10.52 dB.

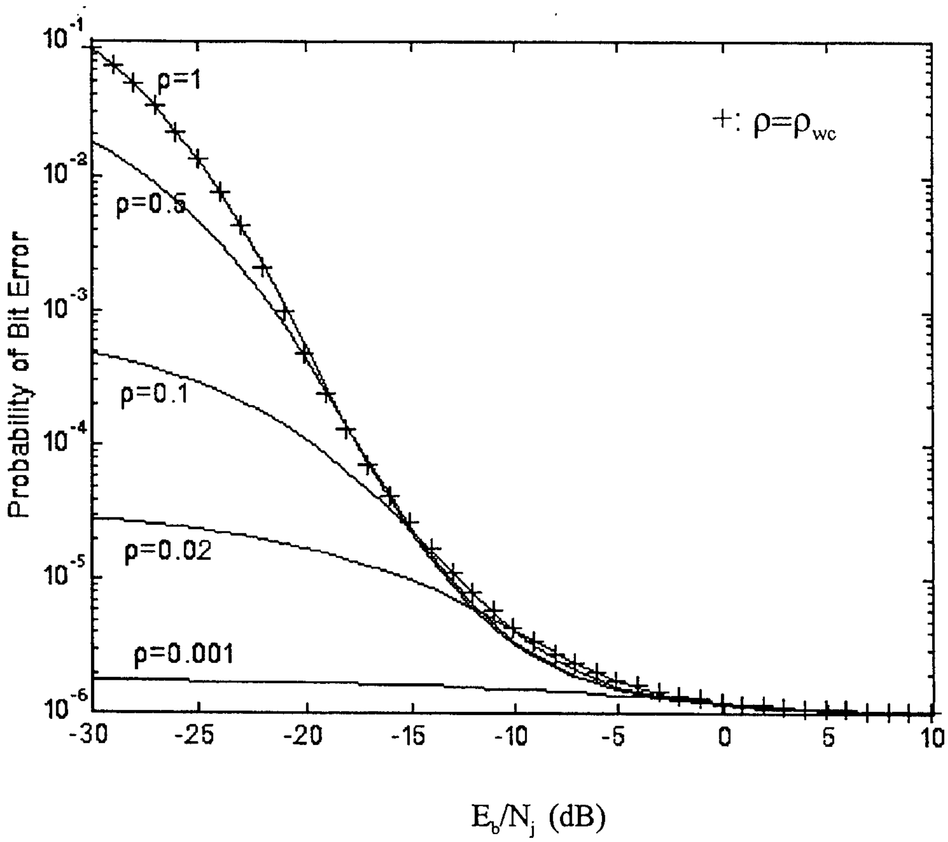

Figure 7. Non-ideal noise-normalization with 50% underestimation of noise power. Performance of the receiver without coding, L=6 and E_b/N_o=10.52 dB.

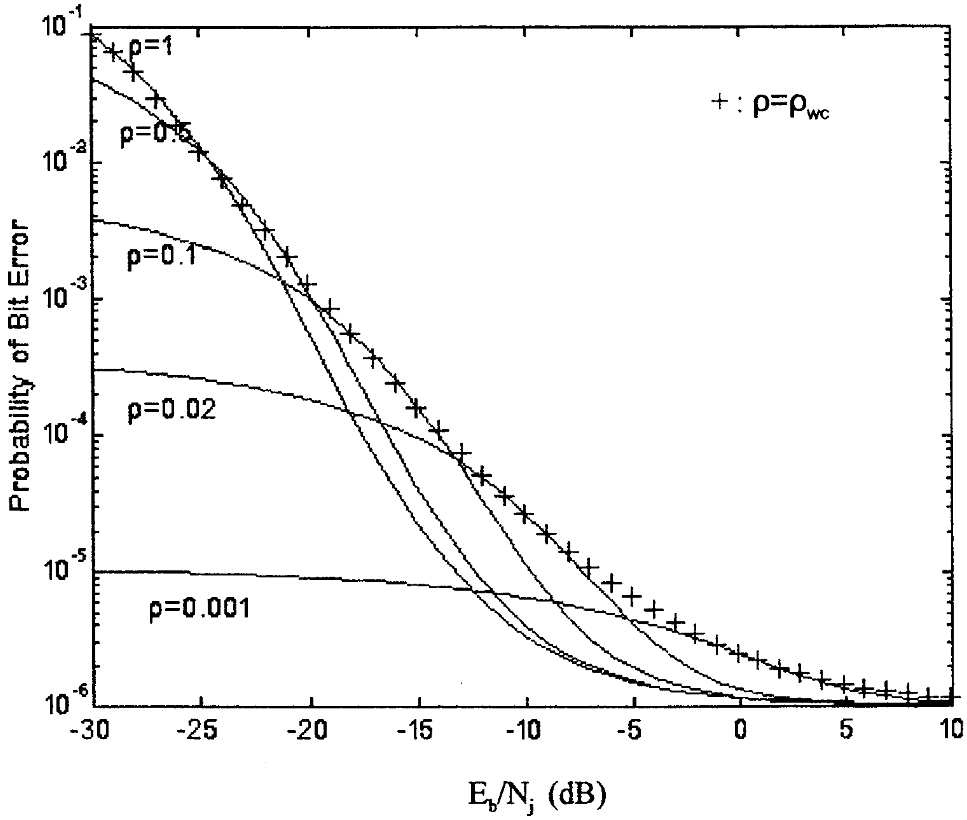

Figure 8. Non-ideal noise-normalization with 50% overestimation of noise power. Performance of the receiver without coding, L=2 and E_b/N_o=10.52 dB.

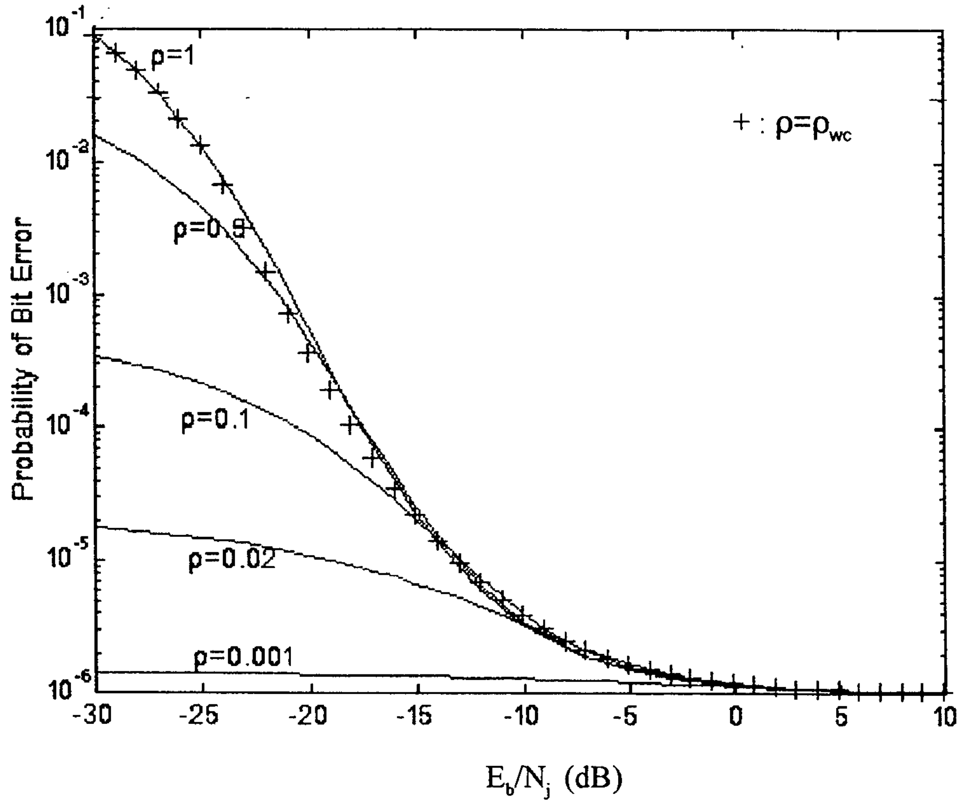

Figure 9. Non-ideal noise-normalization with 50% overestimation of noise power. Performance of the receiver without coding, L=4 and E_b/N_o=10.52 dB.

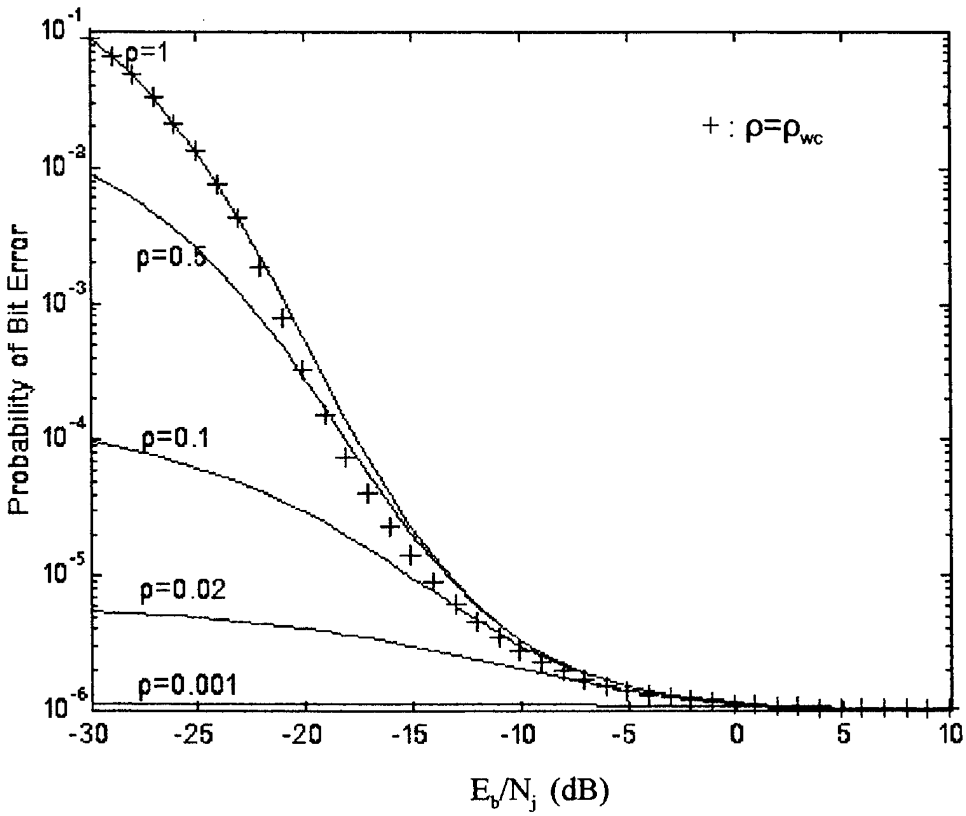

Figure 10. Non-ideal noise-normalization with 50% overestimation of noise power. Performance of the receiver without coding, L=6 and E_b/N_o=10.52 dB.

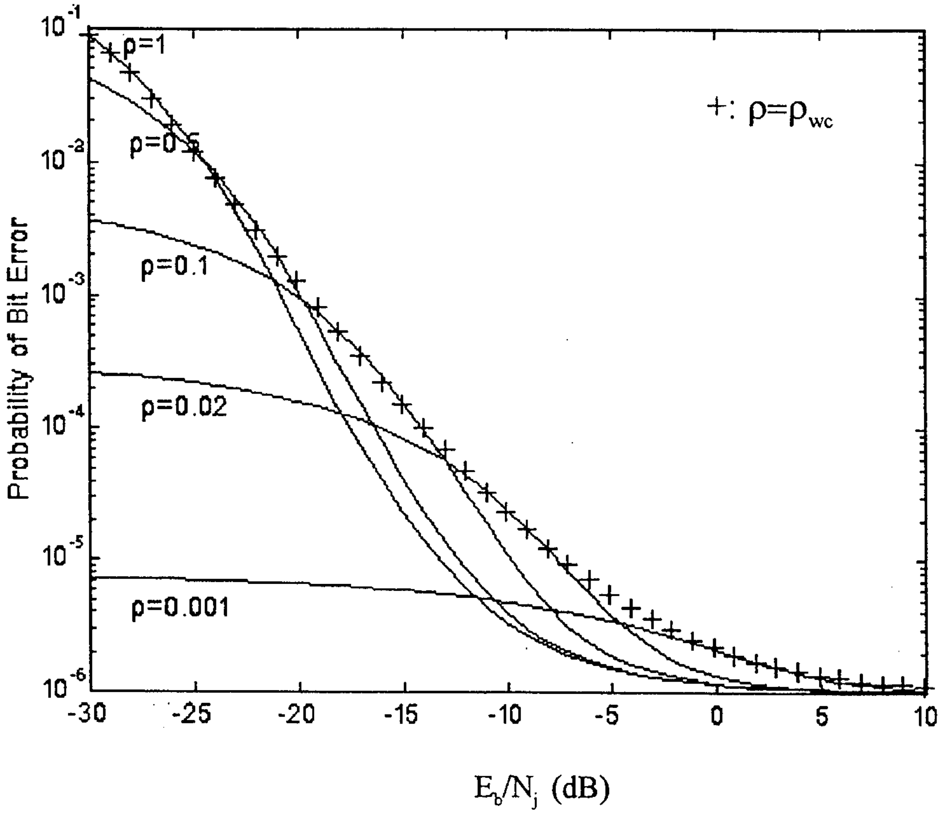

Figure 11. Non-ideal noise-normalization with 100% overestimation of noise power. Performance of the receiver without coding, L=2 and E_b/N_o=10.52 dB.

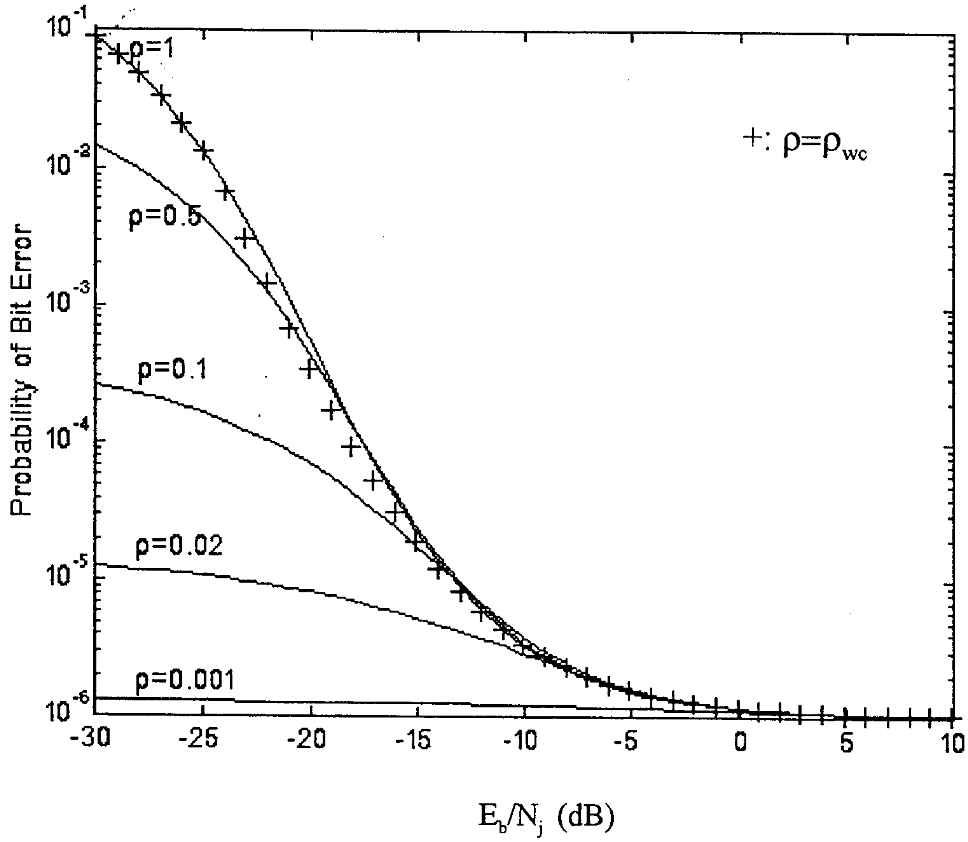

Figure 12. Non-ideal noise-normalization with 100% overestimation of noise power. Performance of the receiver without coding, L=4 and E_b/N_o=10.52 dB.

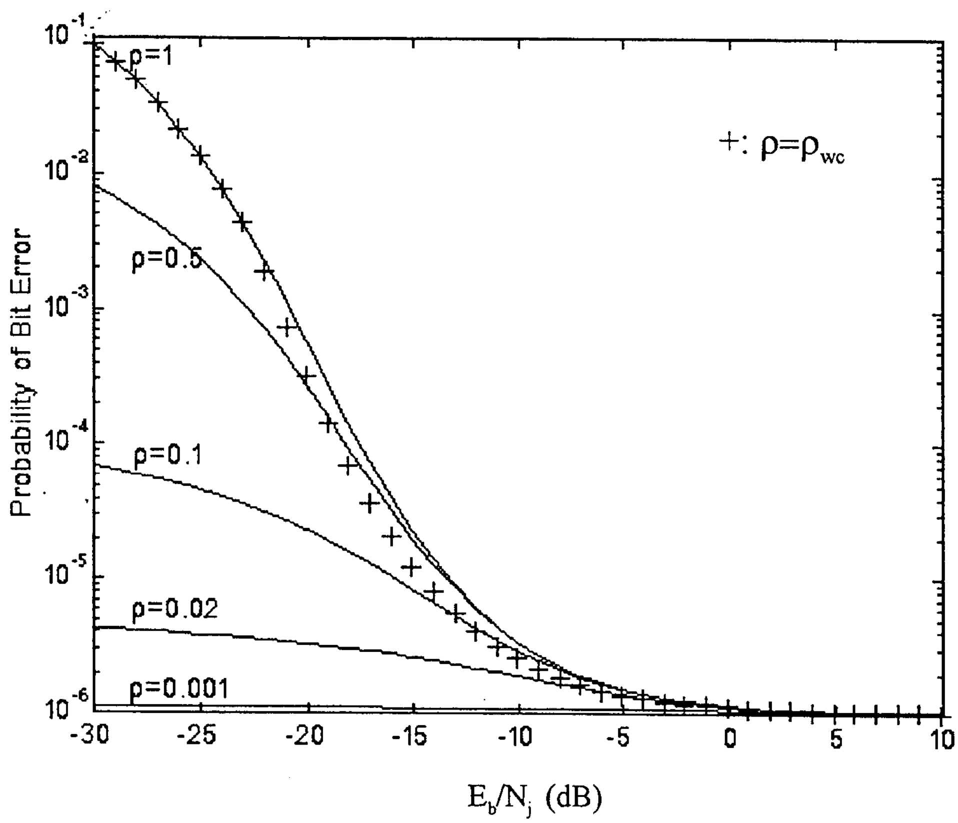

Figure 13. Non-ideal noise-normalization with 100% overestimation of noise power. Performance of the receiver without coding, L=6 and E_b/N_o=10.52 dB.

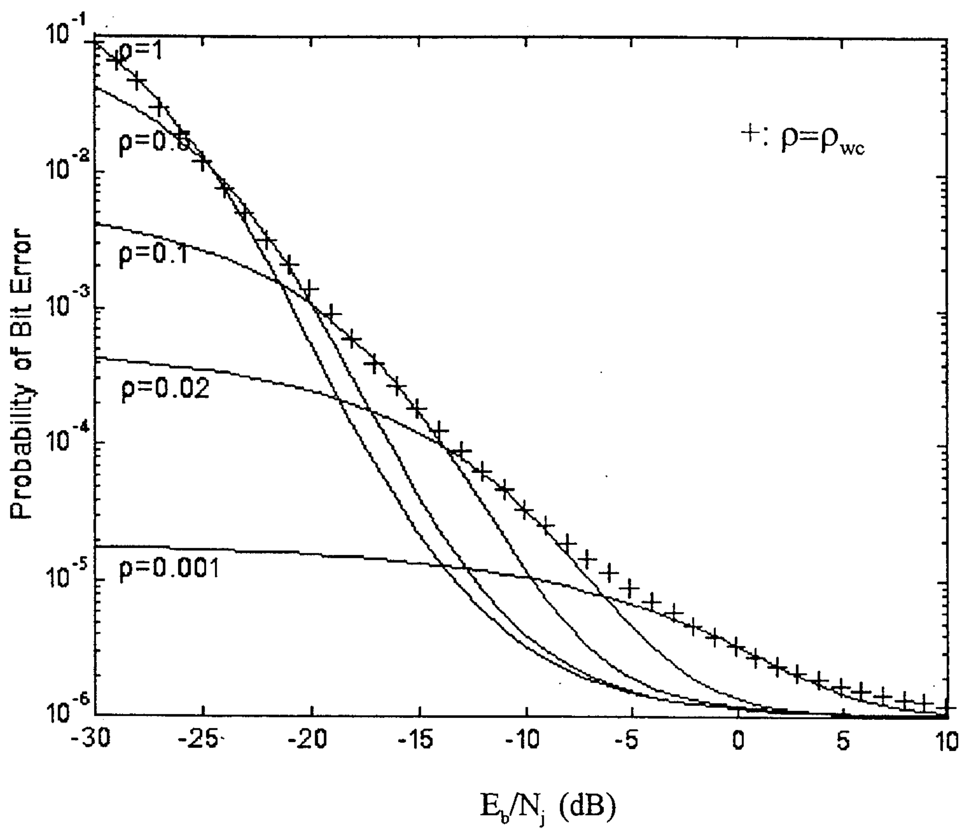

Figure 14. Ideal noise-normalization . Performance of the receiver without coding, L=2 and E_b/N_o=10.52 dB.

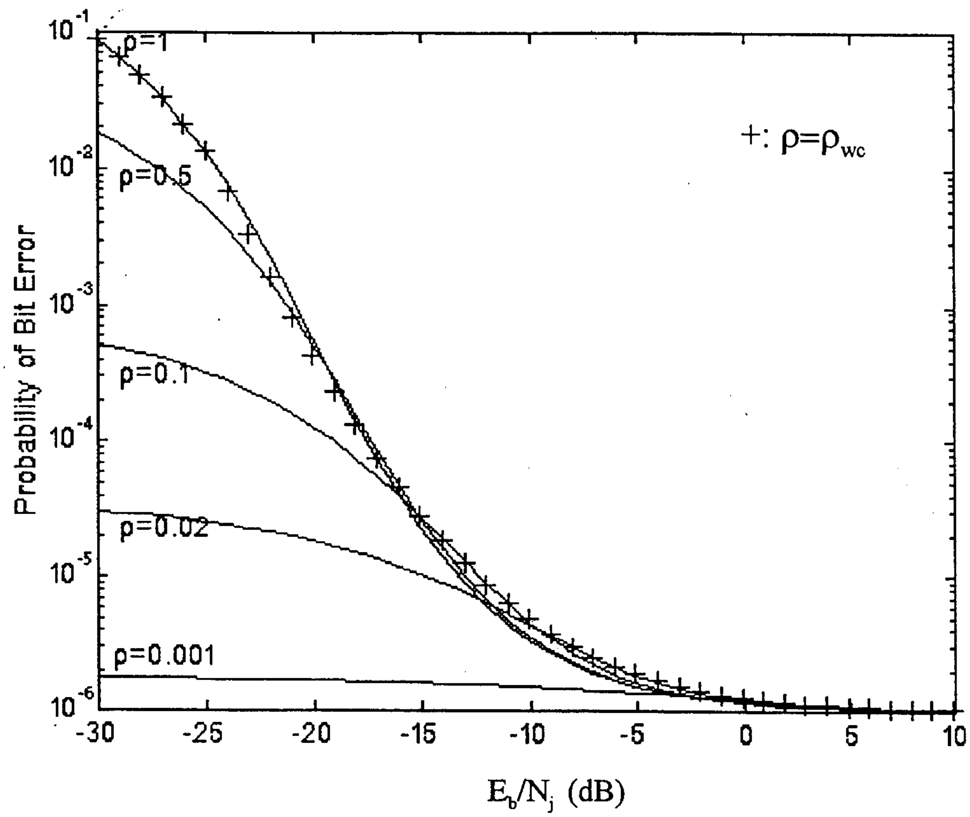

Figure 15. Ideal noise-normalization . Performance of the receiver without coding, L=4 and E_b/N_o=10.52 dB.

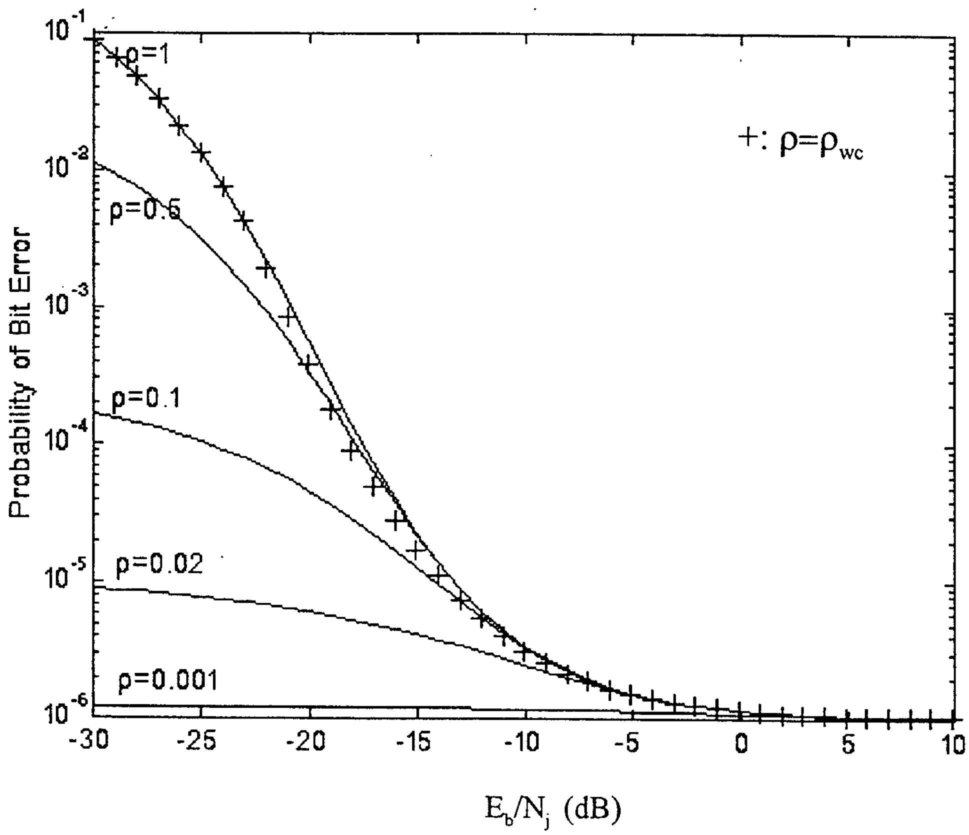

Figure 16. Ideal noise-normalization . Performance of the receiver without coding, L=6 and E_b/N_o=10.52 dB.

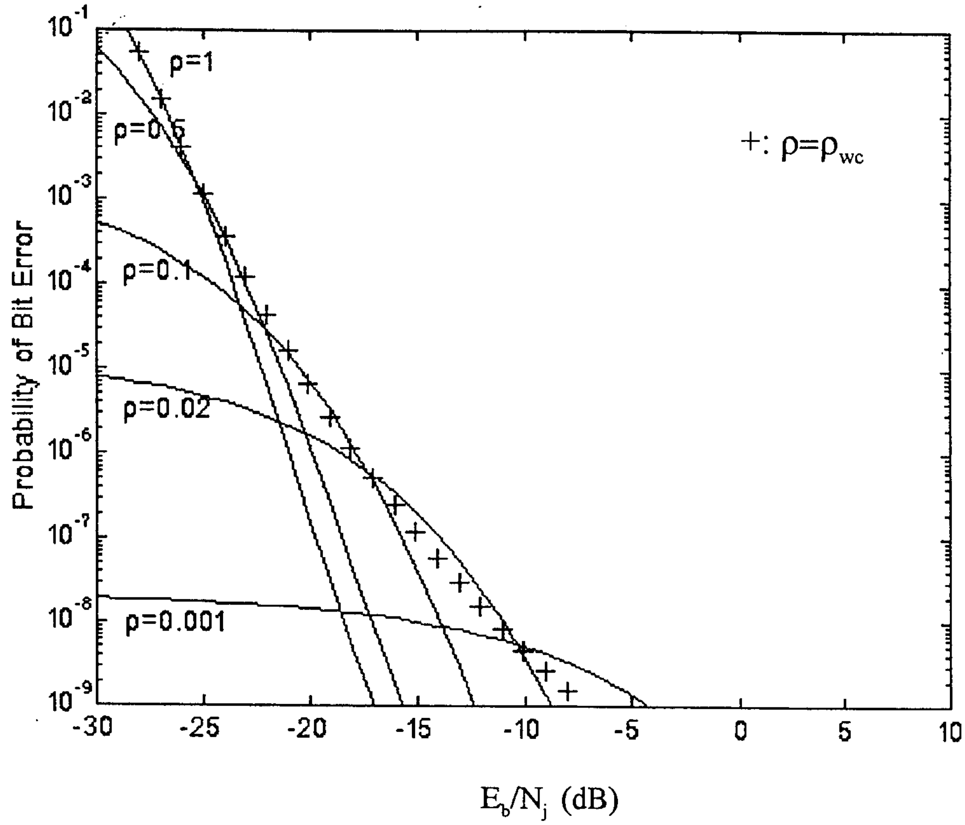

Figure 17. Non-ideal noise-normalization with 75% underestimation of noise power. Performance of the receiver with convolutional coding, constraint length $v=3$, code rate $r=1/2$, $L=1$ and $E_b/N_o=10.52$ dB.

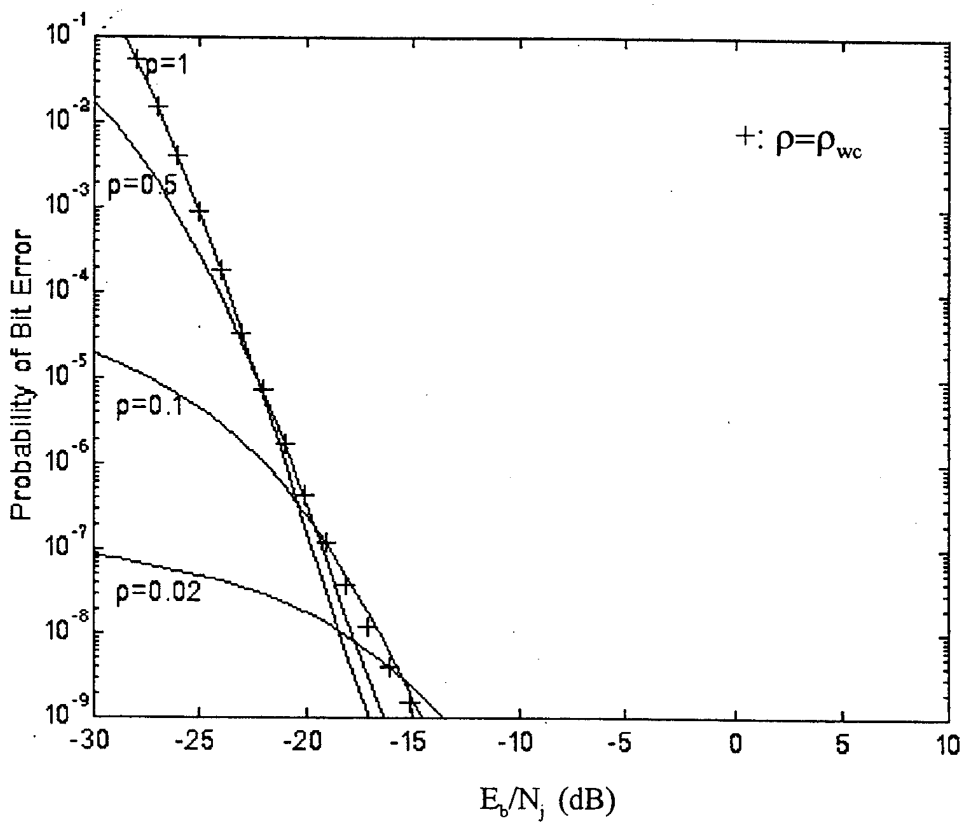

Figure 18. Non-ideal noise-normalization with 75% underestimation of noise power. Performance of the receiver with convolutional coding, constraint length $v=3$, code rate r =1/2, L=2 and E_b/N_o=10.52 dB.

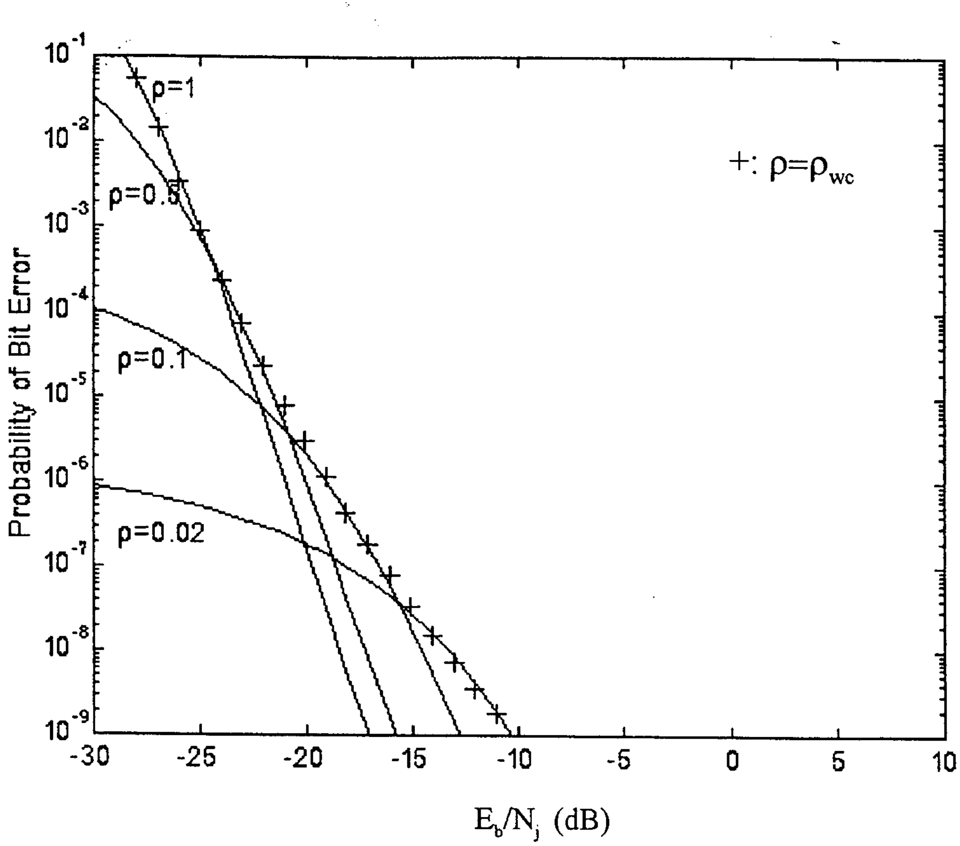

Figure 19. Non-ideal noise-normalization with 50% underestimation of noise power. Performance of the receiver with convolutional coding, constraint length $\nu=3$, code rate r =1/2, L=1 and E_b/N_o=10.52 dB.

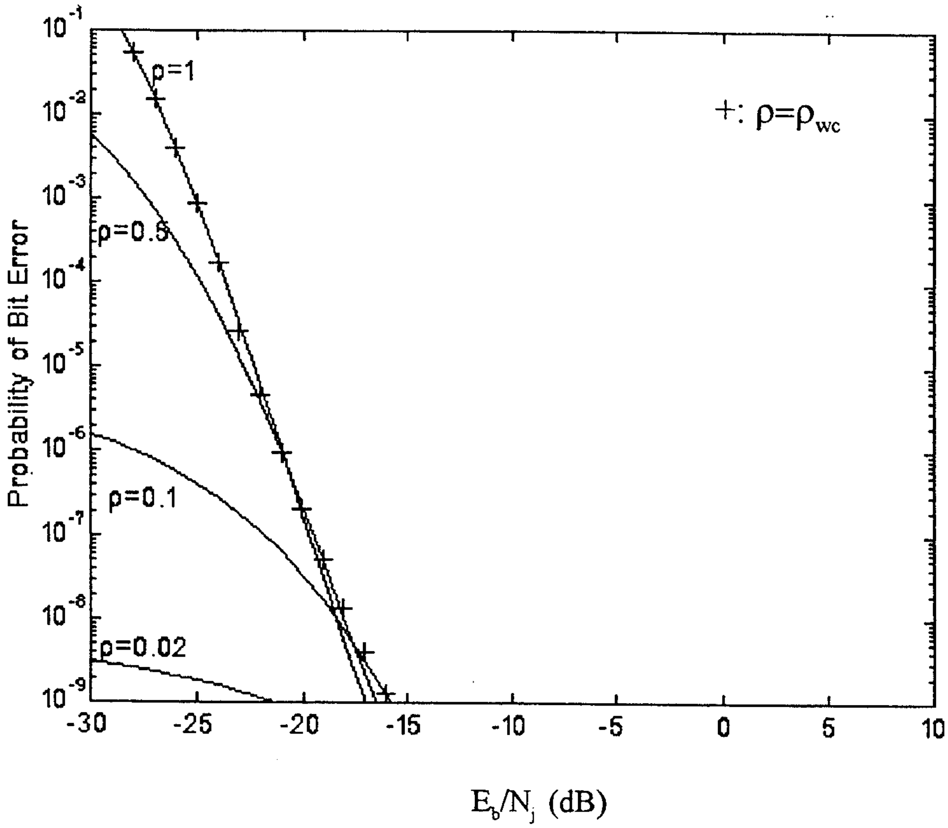

Figure 20. Non-ideal noise-normalization with 50% underestimation of noise power. Performance of the receiver with convolutional coding, constraint length $v=3$, code rate $r=1/2$, $L=2$ and $E_b/N_o=10.52$ dB.

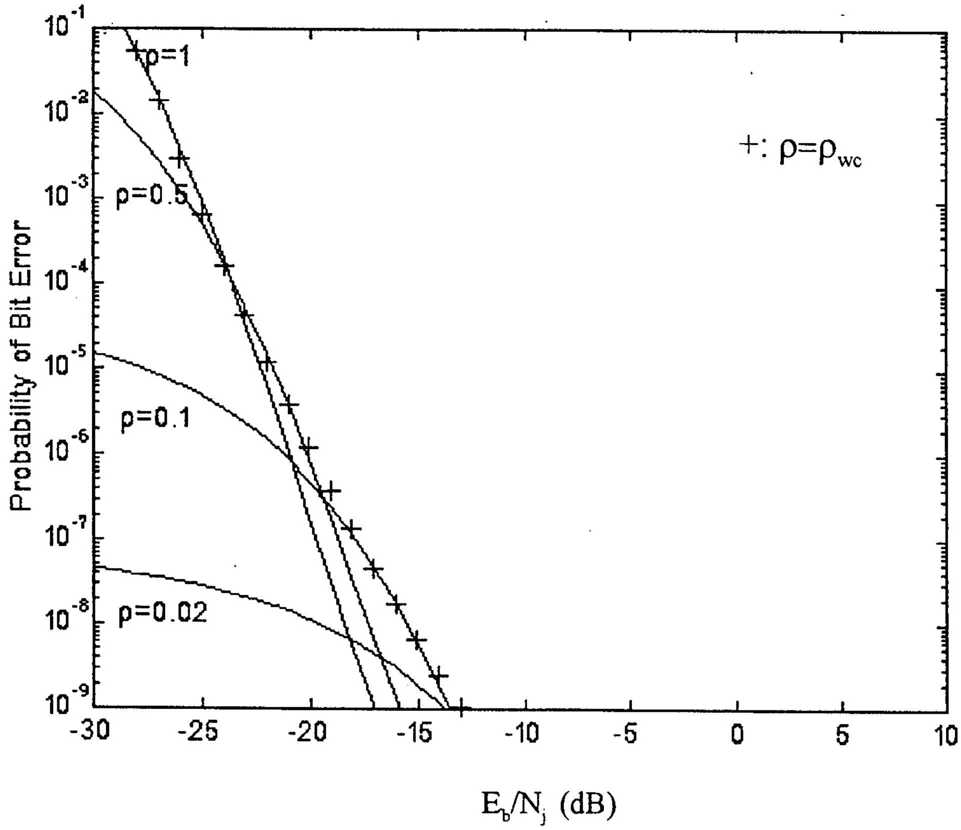

Figure 21. Non-ideal noise-normalization with 50% overestimation of noise power. Performance of the receiver with convolutional coding, constraint length $v=3$, code rate $r = 1/2$, $L=1$ and $E_b/N_o = 10.52$ dB.

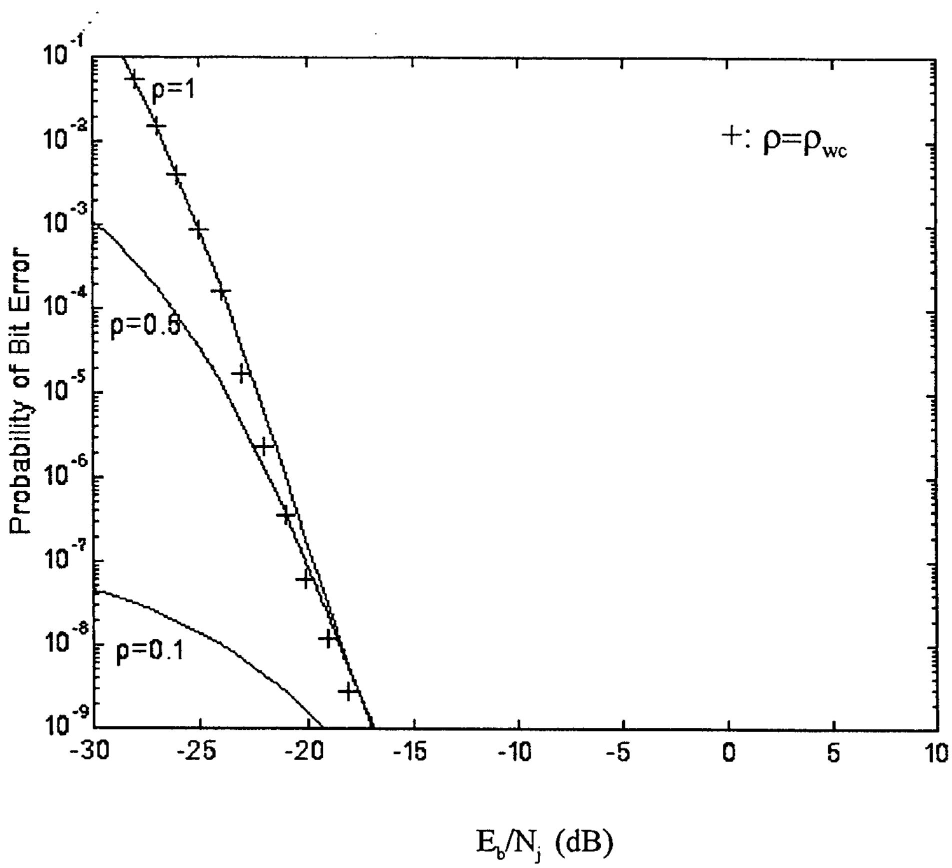

Figure 22. Non-ideal noise-normalization with 50% overestimation of noise power. Performance of the receiver with convolutional coding, constraint length $v=3$, code rate $r=1/2$, $L=2$ and $E_b/N_o=10.52$ dB.

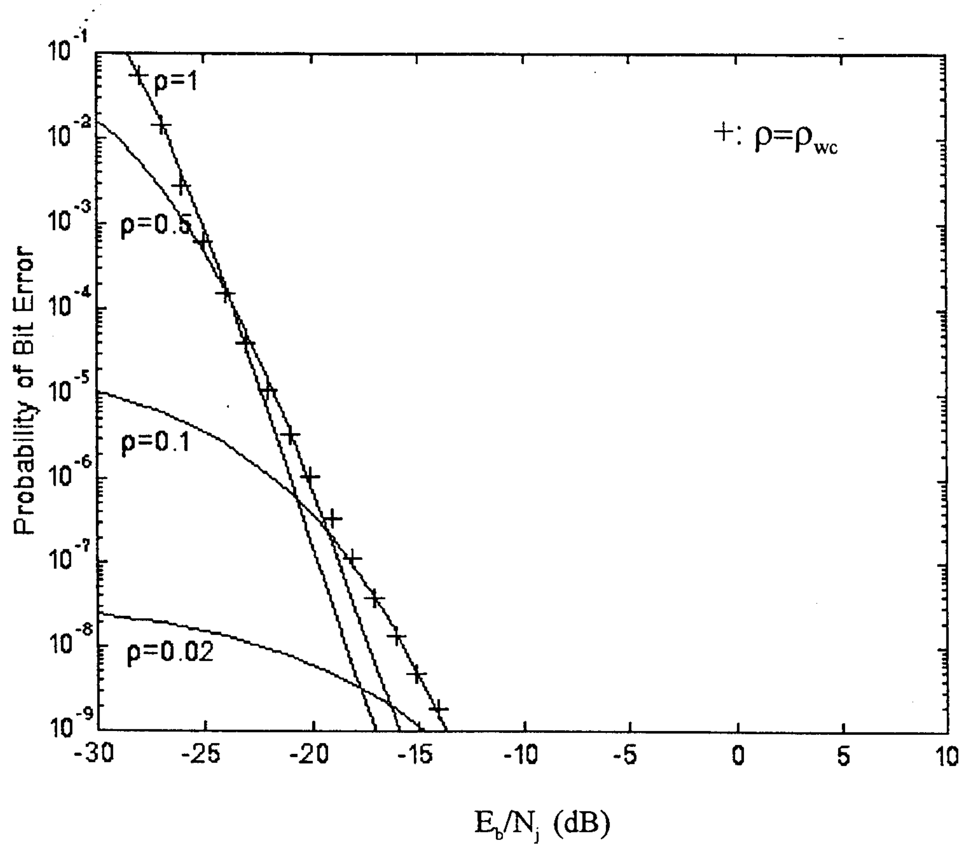

Figure 23. Non-ideal noise-normalization with 100% overestimation of noise power. Performance of the receiver with convolutional coding, constraint length $v=3$, code rate $r=1/2$, $L=1$ and $E_b/N_o=10.52$ dB.

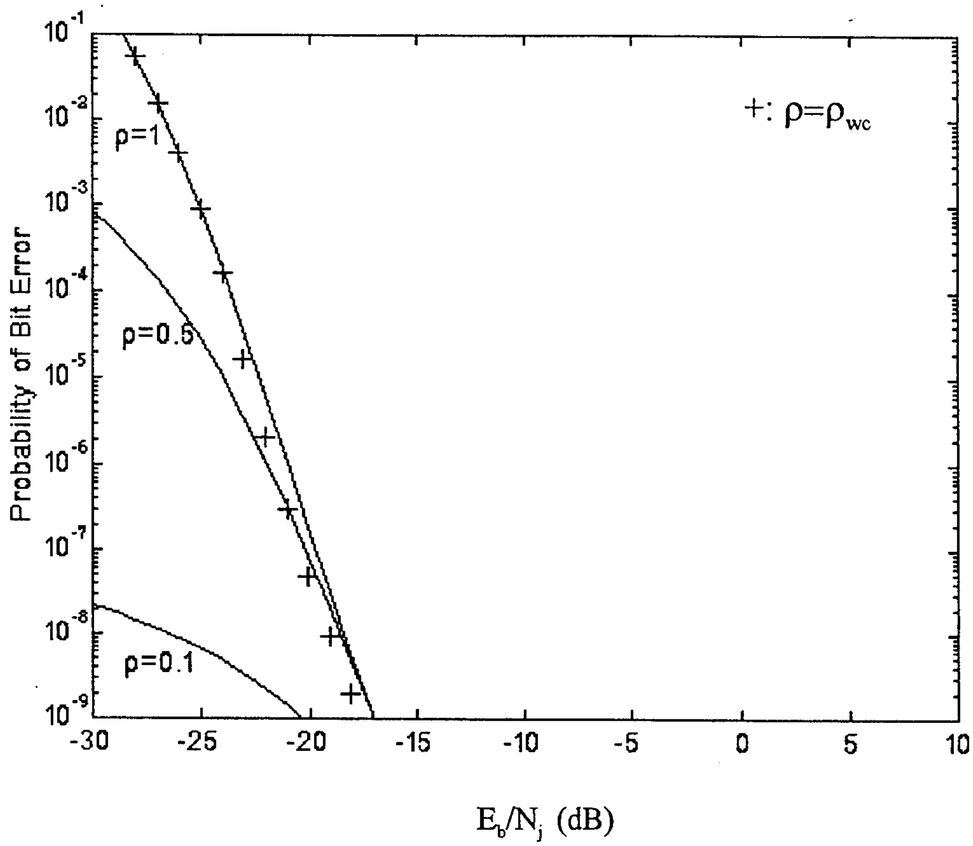

Figure 24. Non-ideal noise-normalization with 100% overestimation of noise power. Performance of the receiver with convolutional coding, constraint length $v=3$, code rate $r=1/2$, $L=2$ and $E_b/N_o=10.52$ dB.

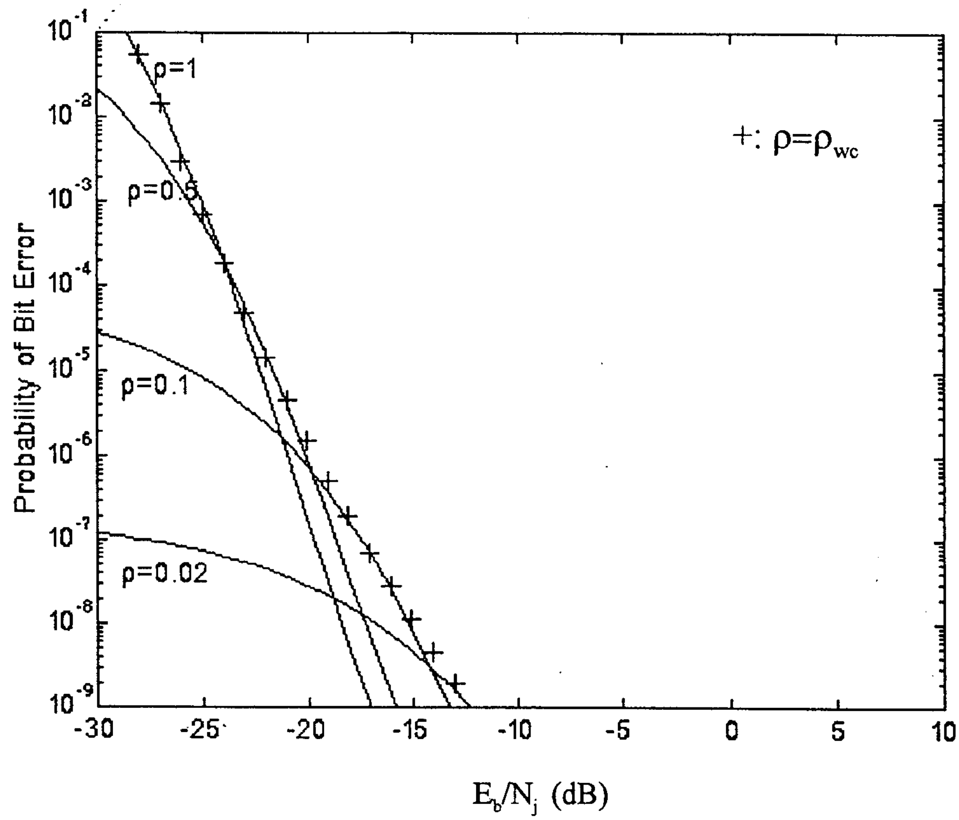

Figure 25. Ideal noise-normalization. Performance of the receiver with convolutional coding, constraint length $v=3$, code rate $r=1/2$, $L=1$ and $E_b/N_o=10.52$ dB.

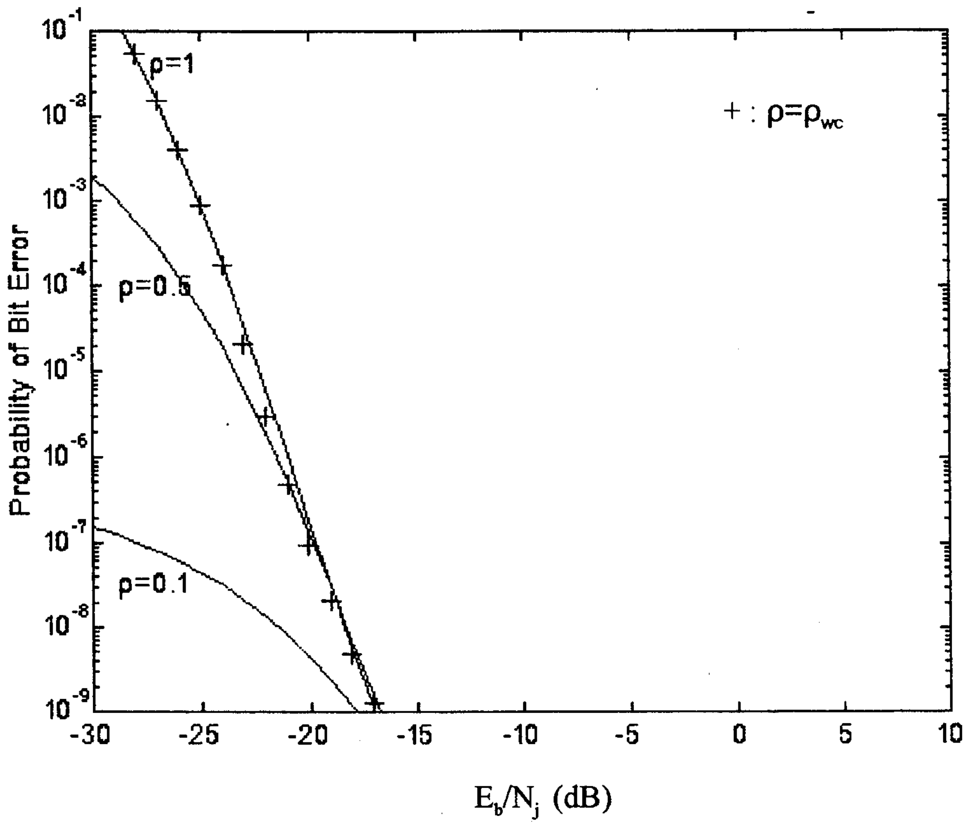

Figure 26. Ideal noise-normalization. Performance of the receiver with convolutional coding, constraint length v=3, code rate r =1/2, L=2 and E_b/N_o=10.52 dB.

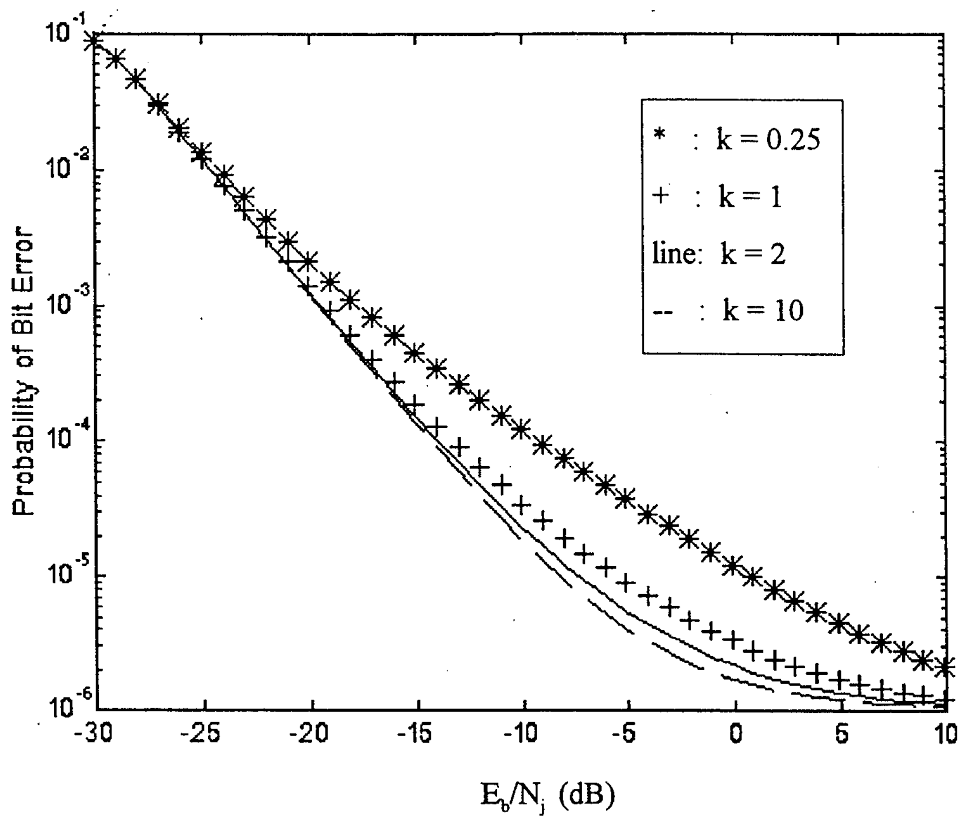

Figure 27. Non-ideal noise-normalization without coding and L=2. Comparison of the different values of noise estimation for worst case ($\rho=\rho_{wc}$) partial band jamming.

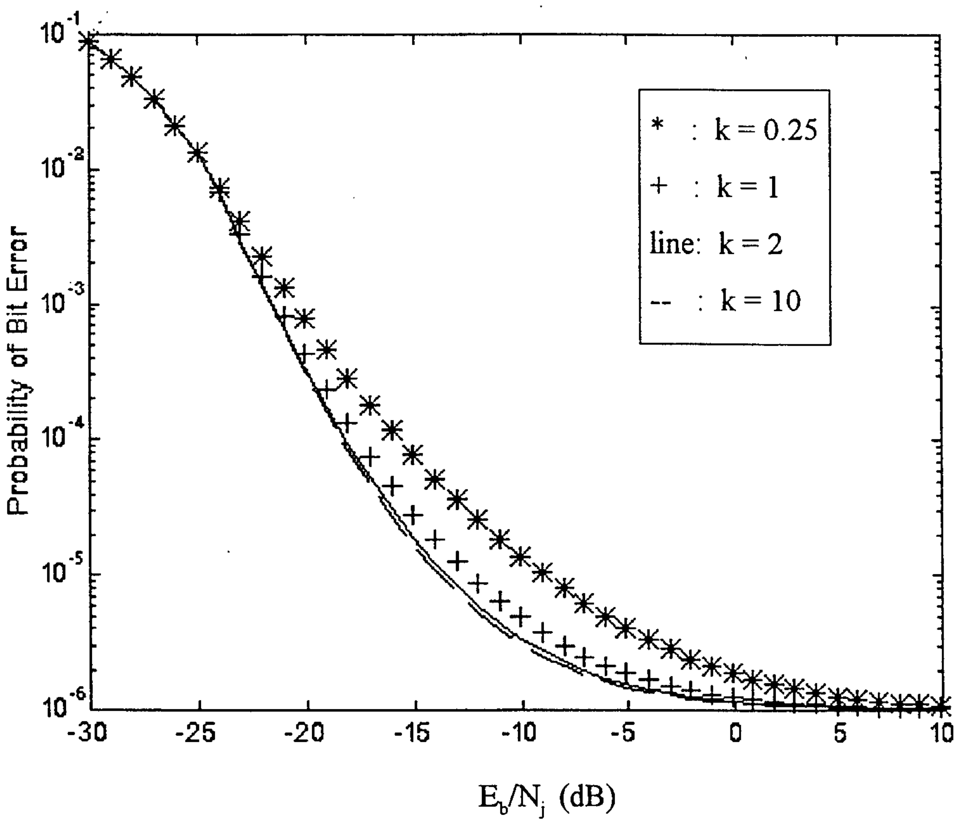

Figure 28. Non-ideal noise-normalization without coding and L=4. Comparison of the different values of noise estimation for worst case ($\rho=\rho_{wc}$) partial band jamming.

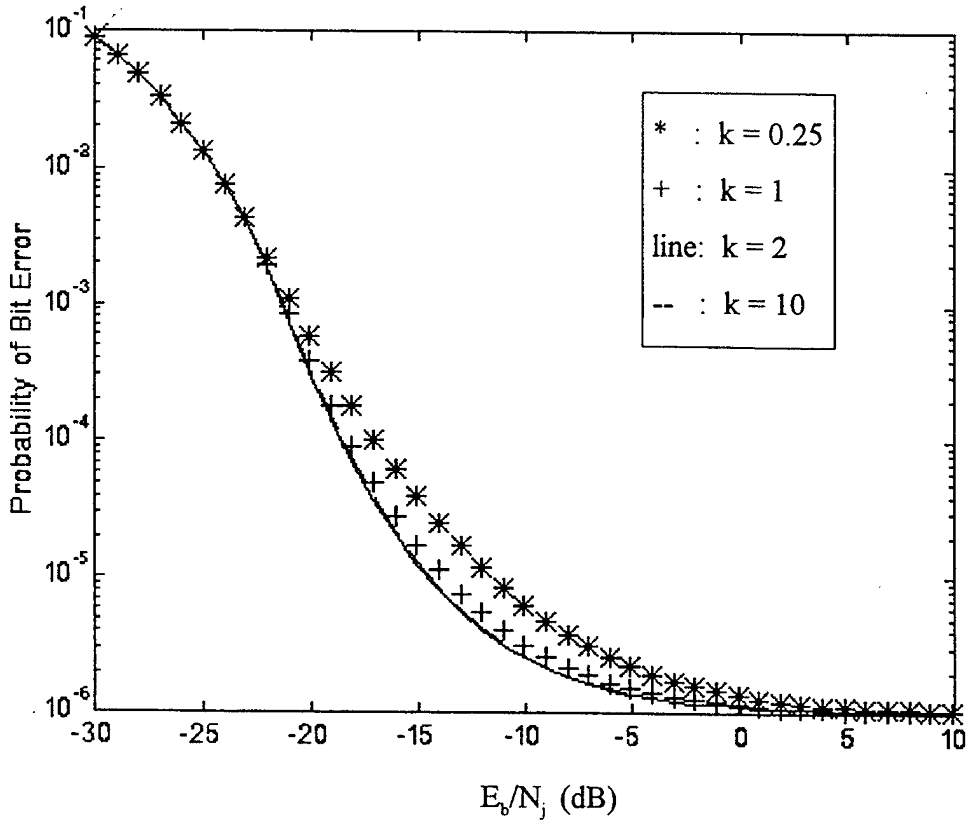

Figure 29. Non-ideal noise-normalization without coding and L=6. Comparison of the different values of noise estimation for worst case ($\rho=\rho_{wc}$) partial band jamming.

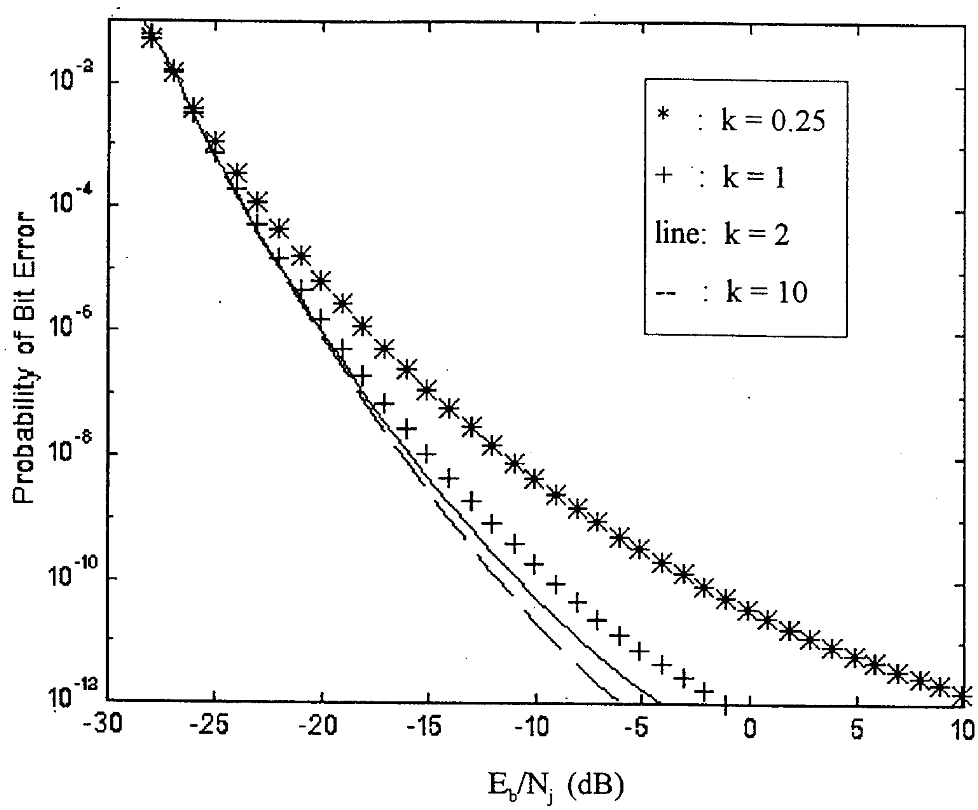

Figure 30. Non-ideal noise-normalization with coding and L=1. Comparison of the different values of noise estimation for worst case ($\rho=\rho_{wc}$) partial band jamming.

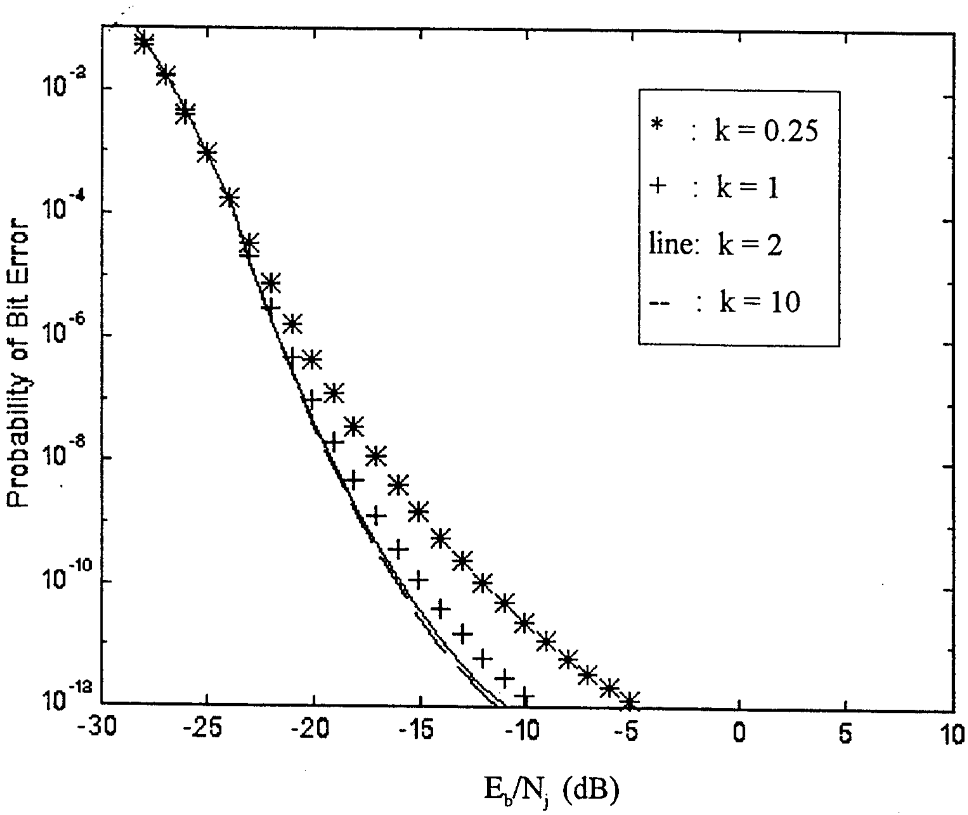

Figure 31. Non-ideal noise-normalization with coding and L=2. Comparison of the different values of noise estimation for worst case ($\rho = \rho_{wc}$) partial band jamming.

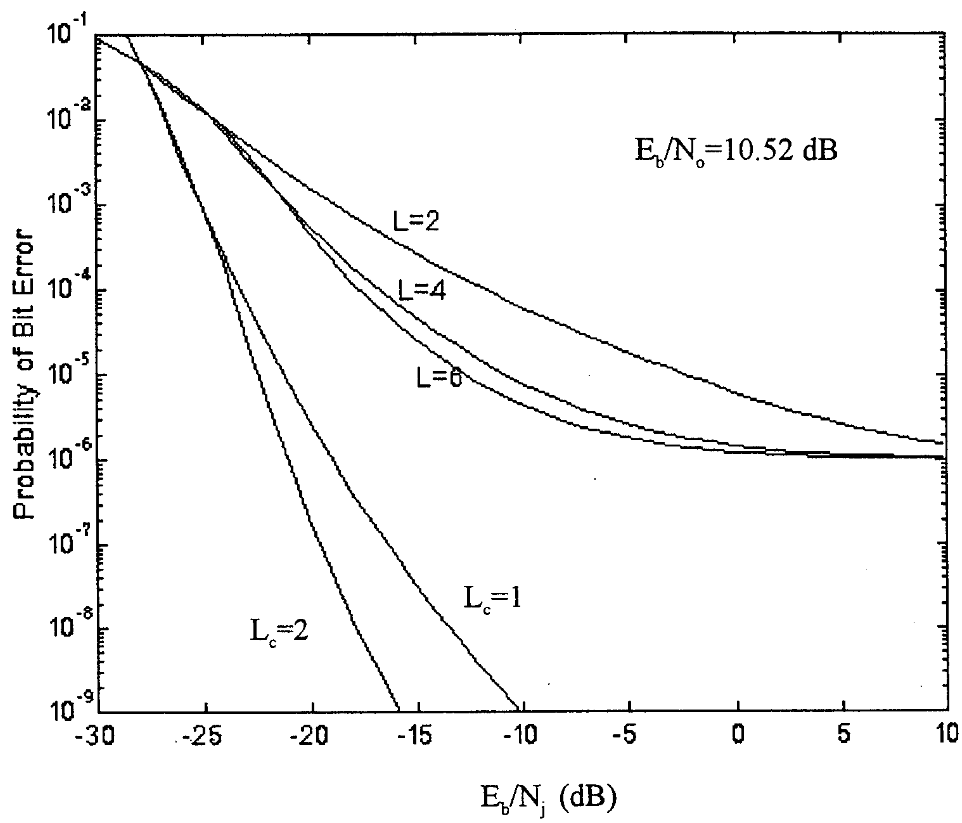

Figure 32. Non-ideal noise-normalization with 50% underestimation of noise power. Performance of the receiver for worst case partial-band jamming for different values of diversity with and without coding and assuming constant bandwidth.

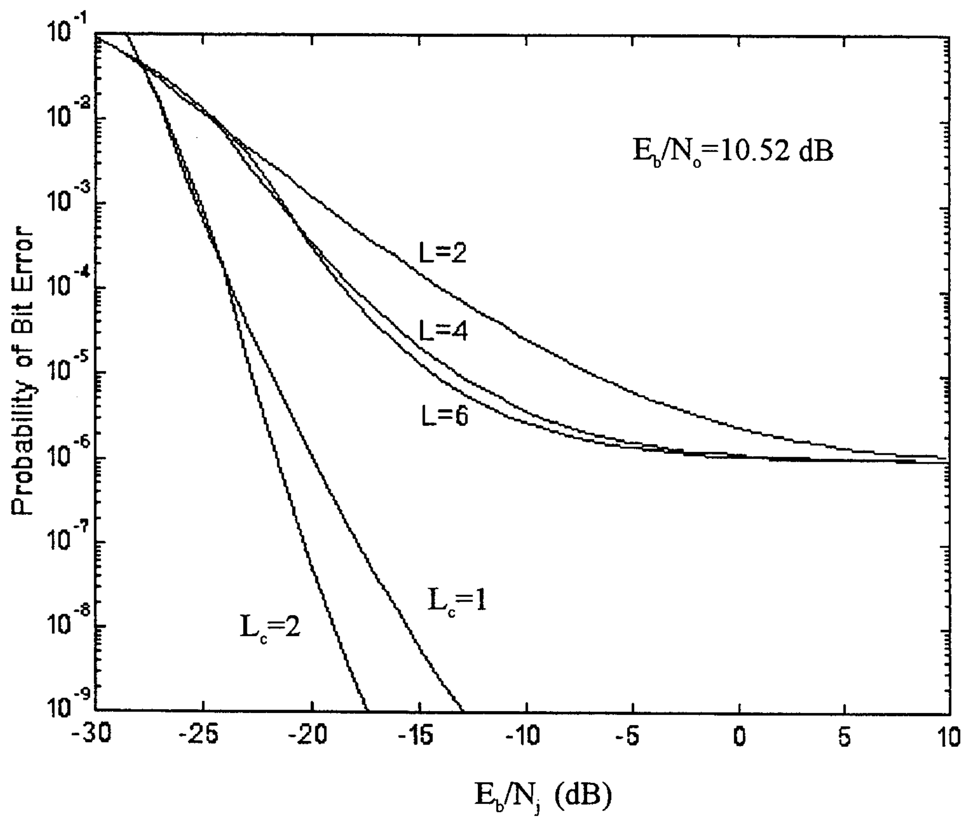

Figure 33. Non-ideal noise-normalization with 50% overestimation of noise power. Performance of the receiver for worst case partial-band jamming for different values of diversity with and without coding and assuming constant bandwidth.

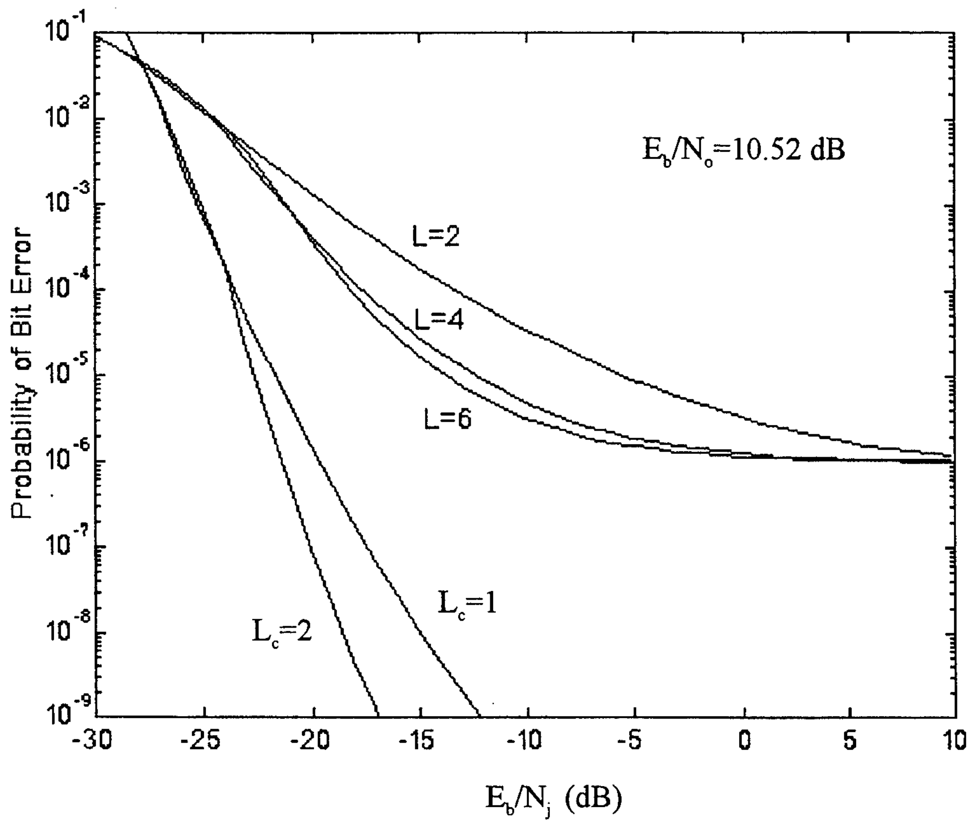

Figure 34. Ideal noise-normalization. Performance of the receiver for worst case partial-band jamming for different values of diversity with and without coding and assuming constant bandwidth.

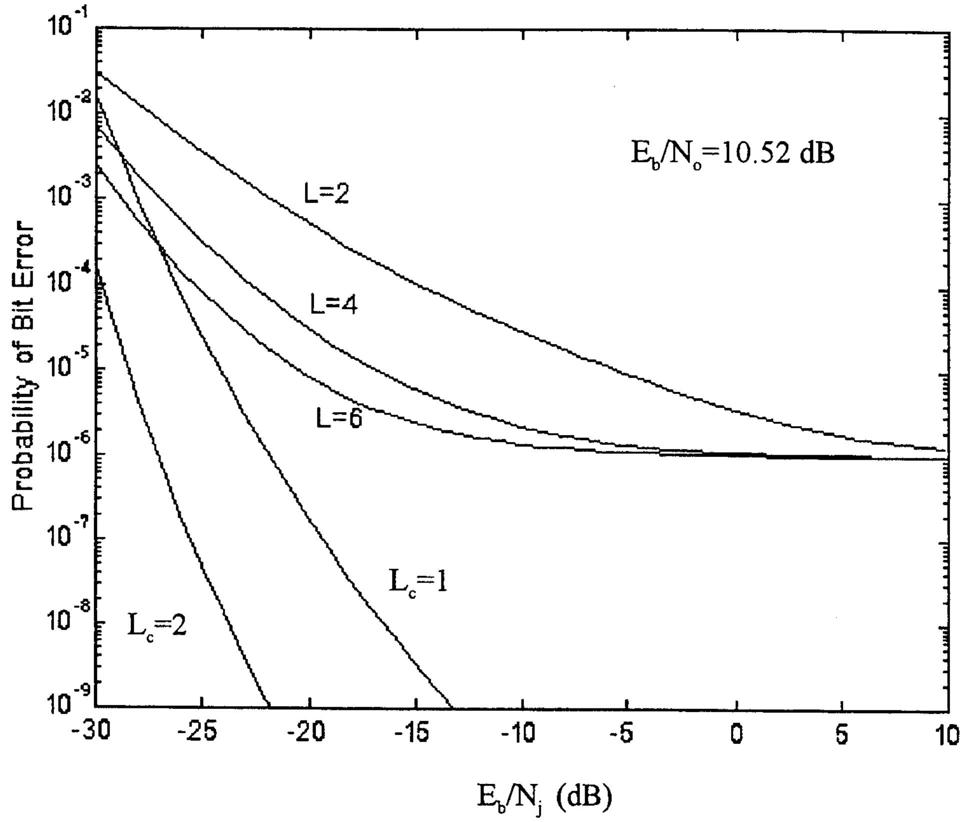

Figure 35. Non-ideal noise-normalization with 50% underestimation of noise power. Performance of the receiver for worst case partial-band jamming, for different values of diversity with and without coding and assuming constant number of bins.

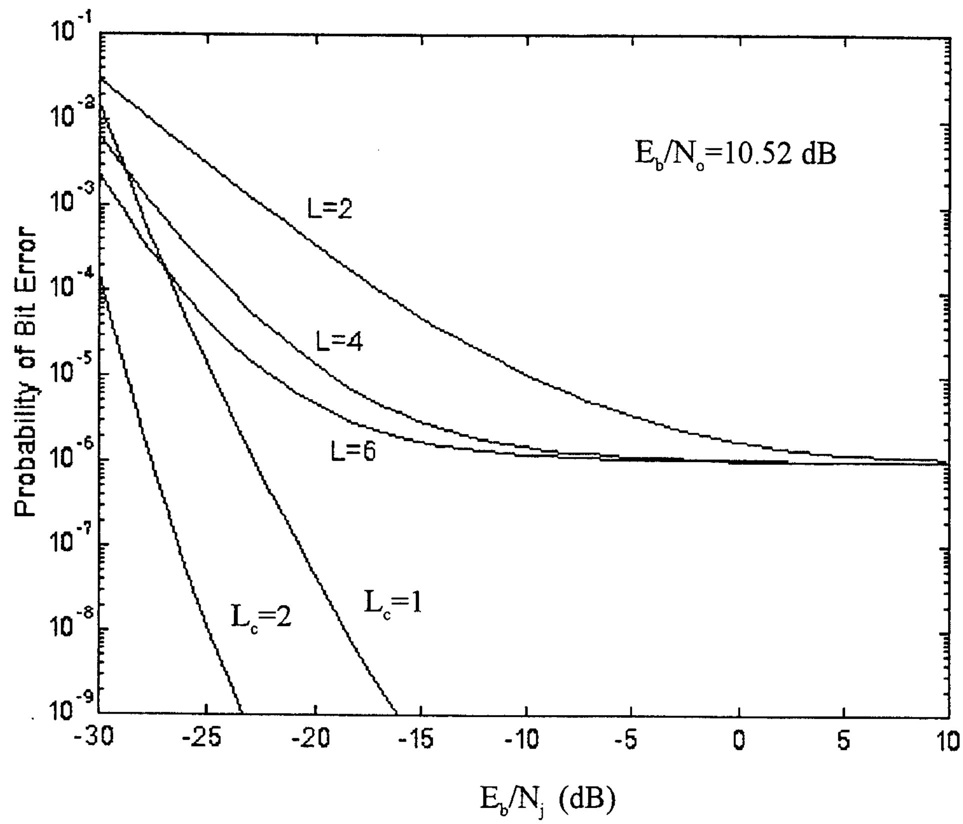

Figure 36. Non-ideal noise-normalization with 50% overestimation of noise power. Performance of the receiver for worst case partial-band jamming, for different values of diversity with and without coding and assuming constant number of bins.

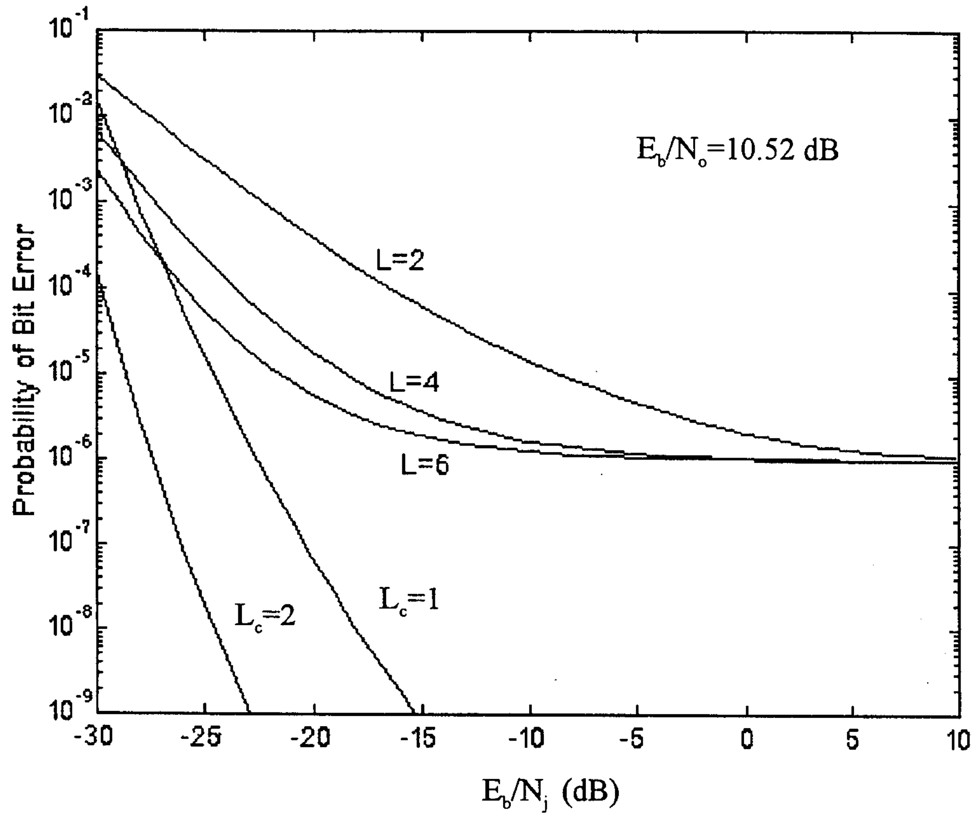

Figure 37. Ideal noise-normalization. Performance of the receiver for worst case partial-band jamming, for different values of diversity with and without coding and assuming constant number of bins.

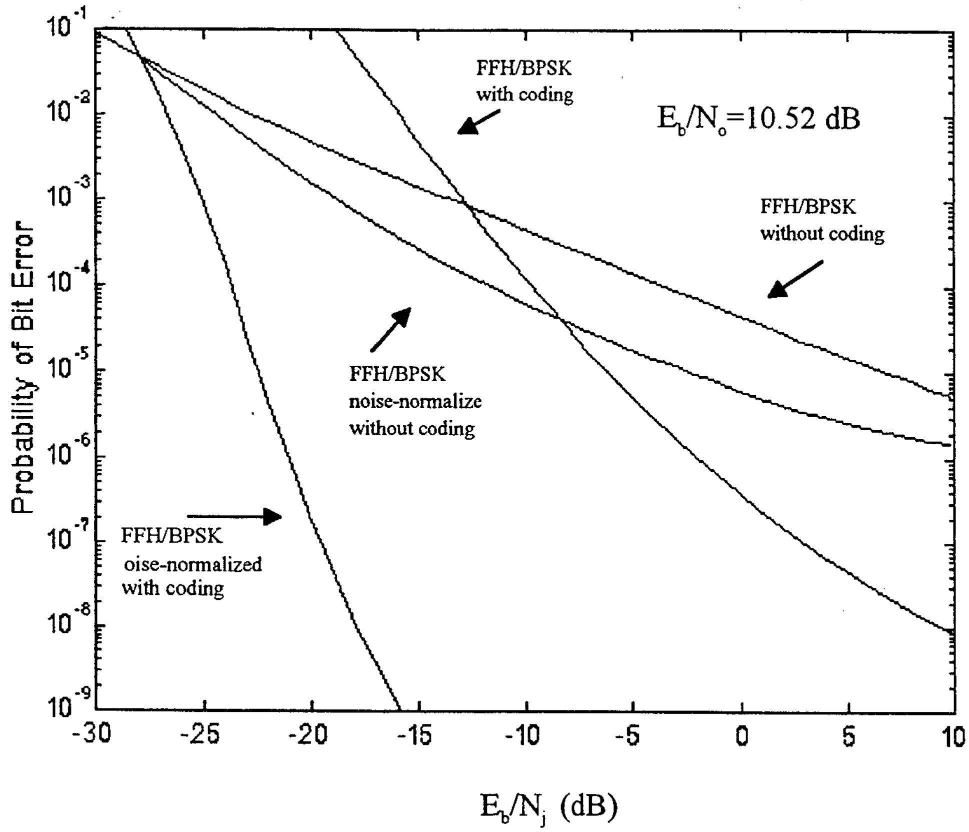

Figure 38. Comparison of different types of receivers for L=2 and 50% underestimation of noise power for the noise normalized receiver, assuming constant bandwidth.

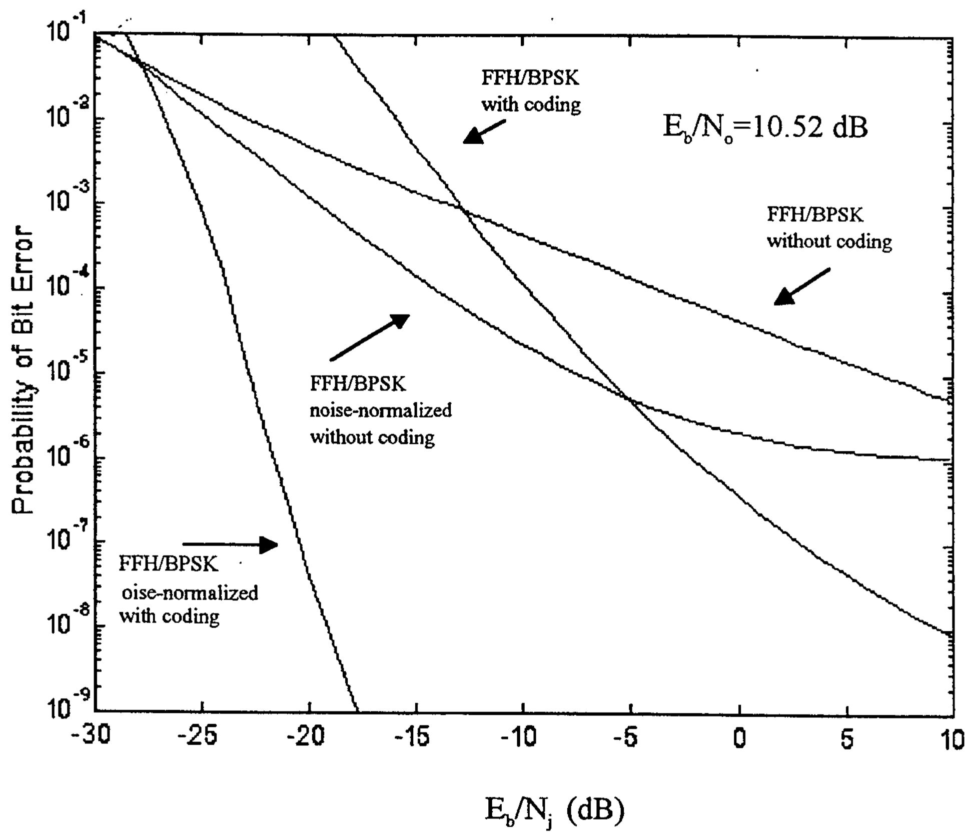

Figure 39. Comparison of different types of receivers for L=2 and 100% overestimation of noise power for the noise-normalized receiver, assuming constant bandwidth.

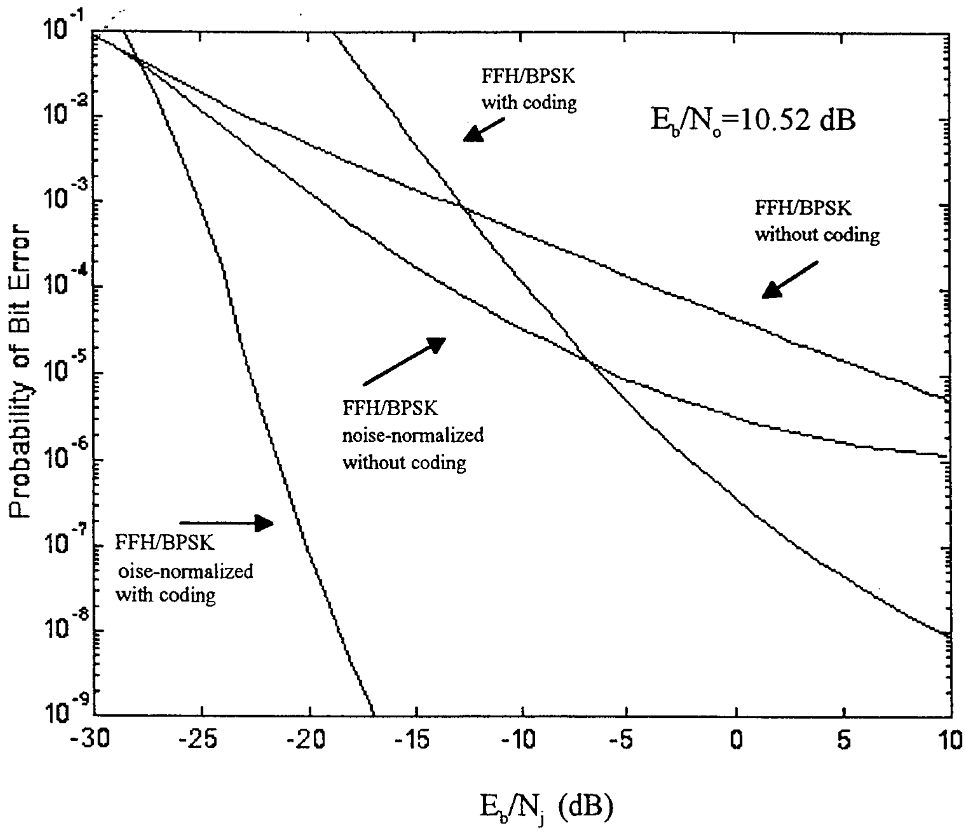

Figure 40. Comparison of different types of receivers for L=2 with ideal noise normalization for the noise-normalized receiver, assuming constant bandwidth.

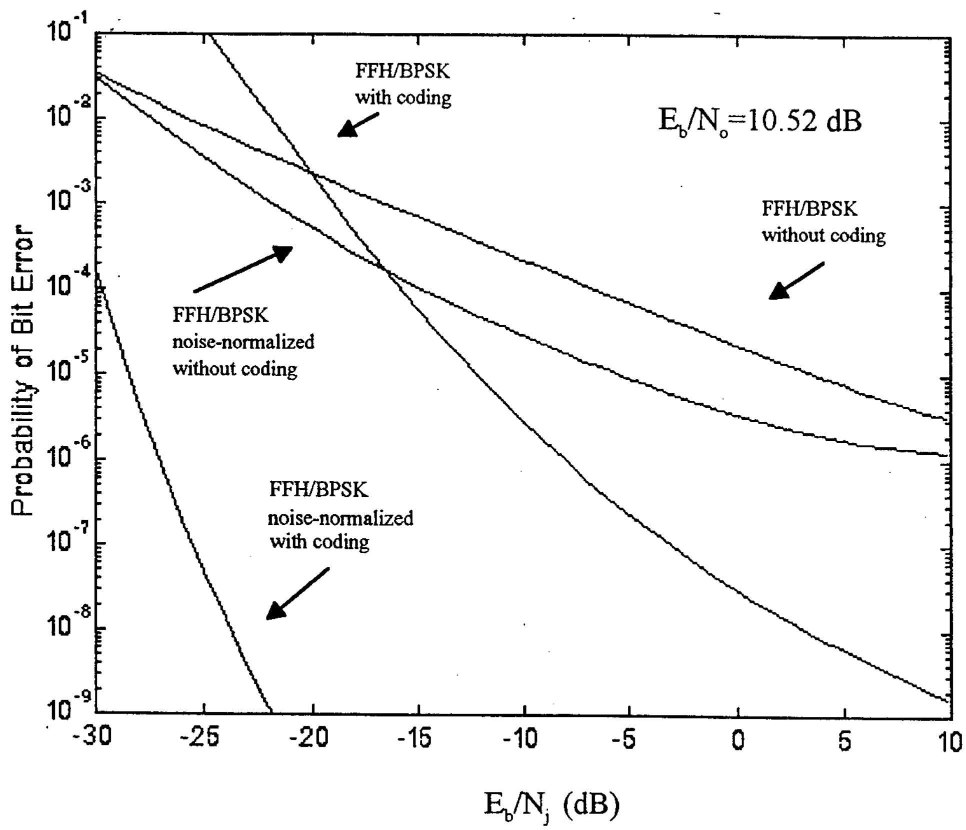

Figure 41. Comparison of different types of receivers for L=2 and 50% underestimation of noise power for the noise normalized receiver, assuming constant number of bins.

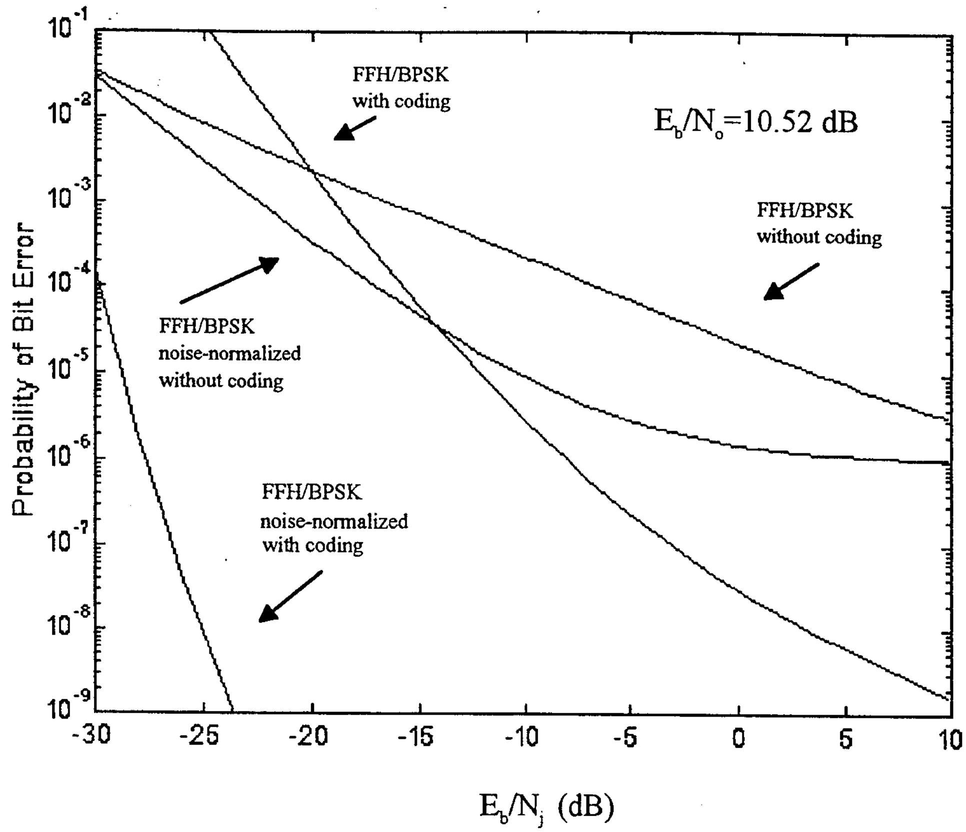

Figure 42. Comparison of different types of receivers for L=2 and 100% overestimation of noise power for the noise normalized receiver, assuming constant number of bins.

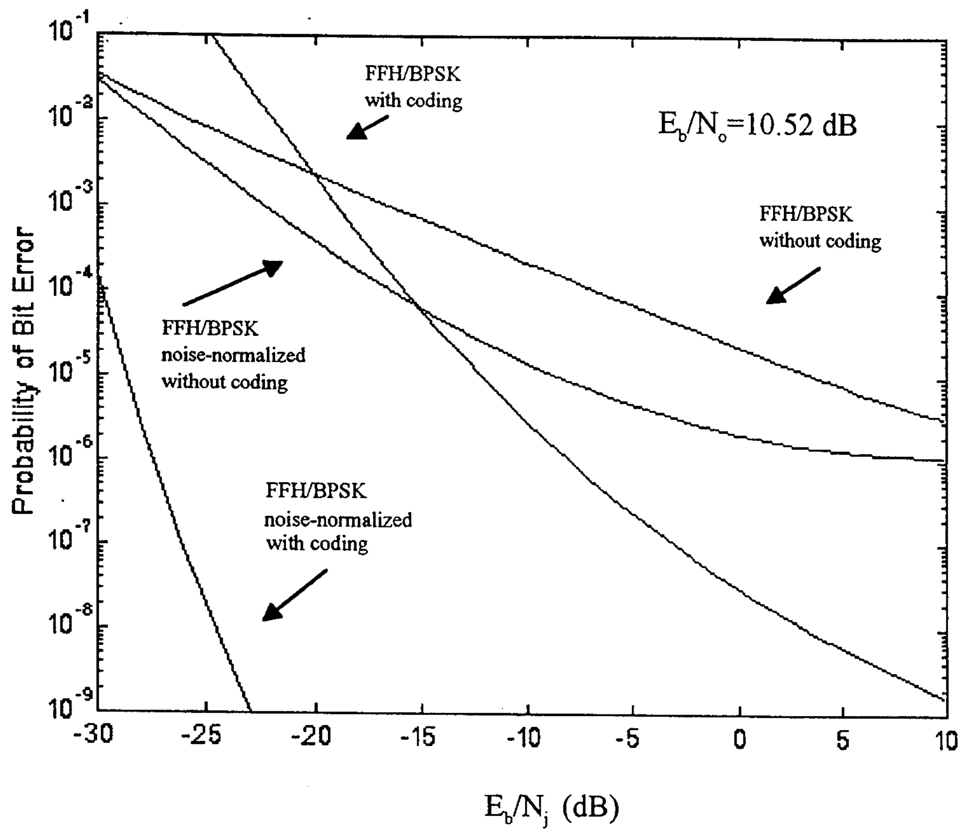

Figure 43. Comparison of different types of receivers for L=2 and ideal noise normalization for the noise-normalized receiver, assuming constant number of bins.

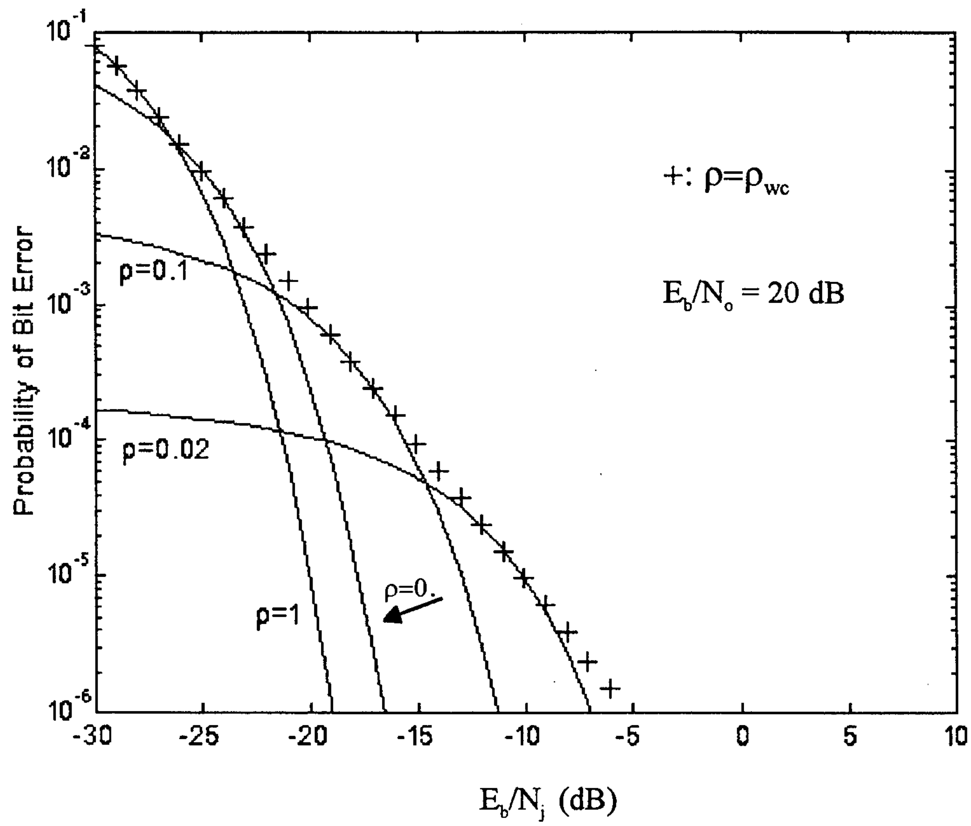

Figure 44. Performance of the perfect side information receiver without coding and with diversity L=2.

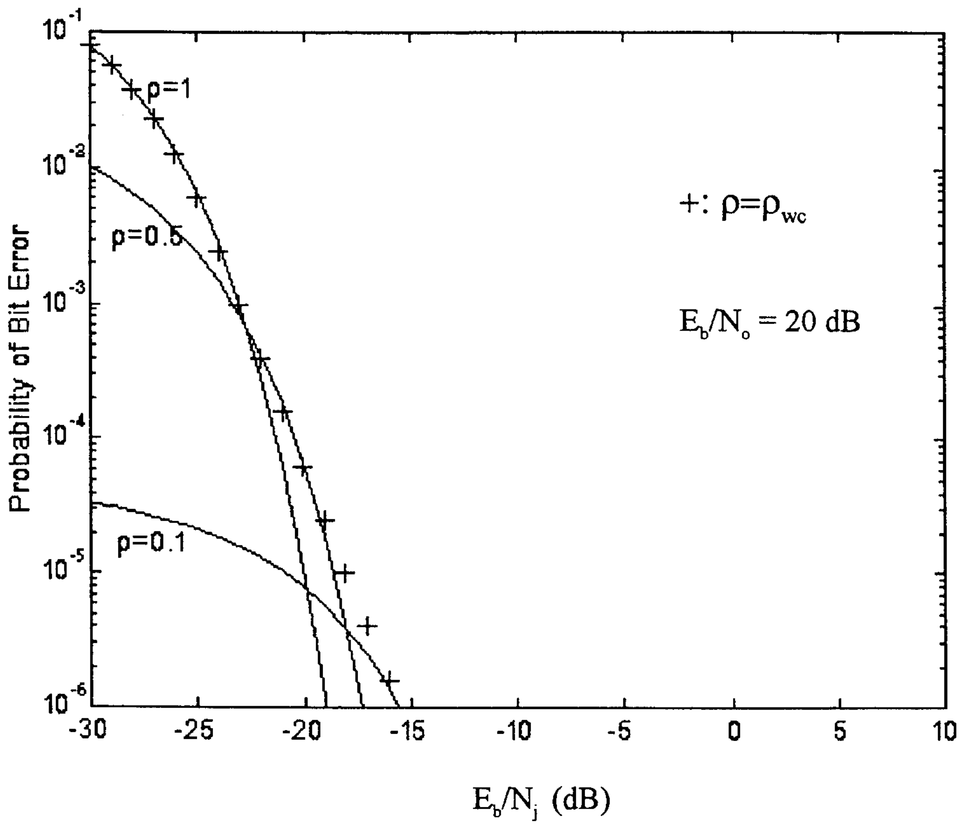

Figure 45. Performance of the perfect side information receiver without coding and with diversity L=4.

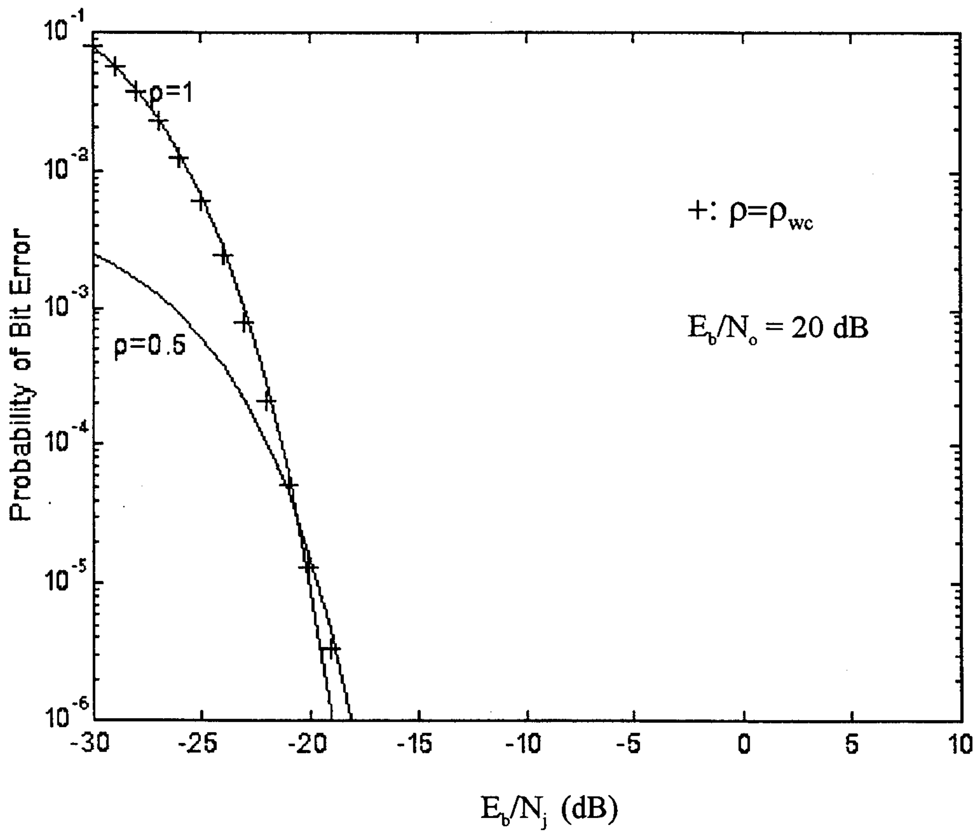

Figure 46. Performance of the perfect side information receiver without coding and with diversity L=6.

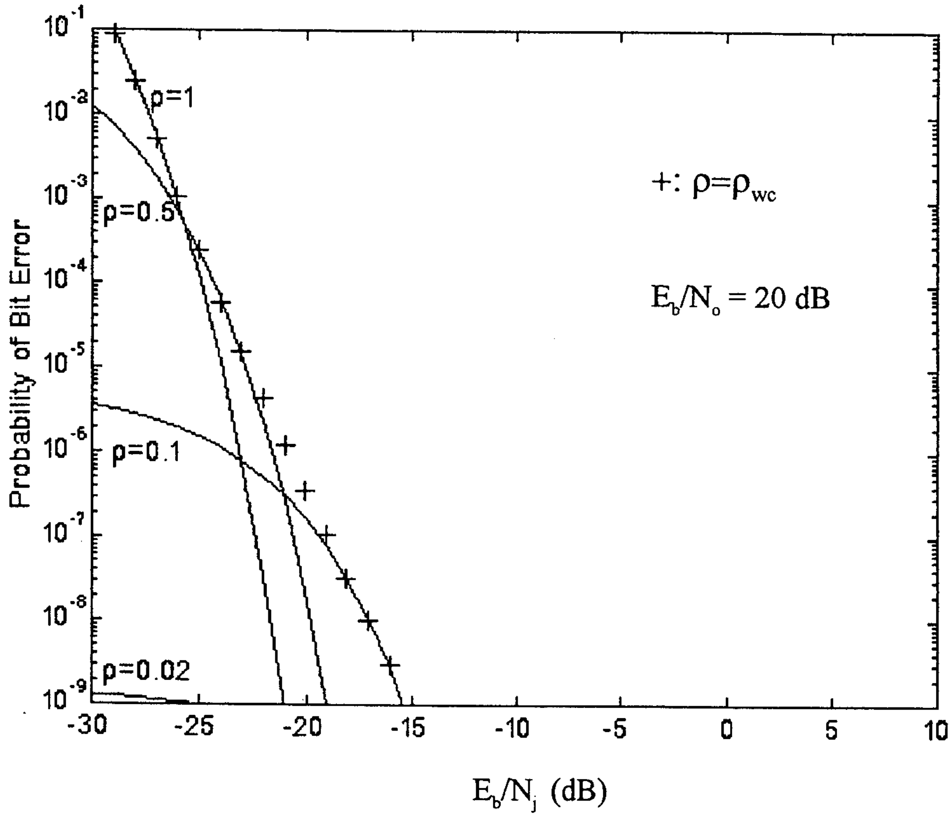

Figure 47. Performance of the perfect side information receiver with coding, constraint length v=3, code rate r=1/2 and with diversity L=1.

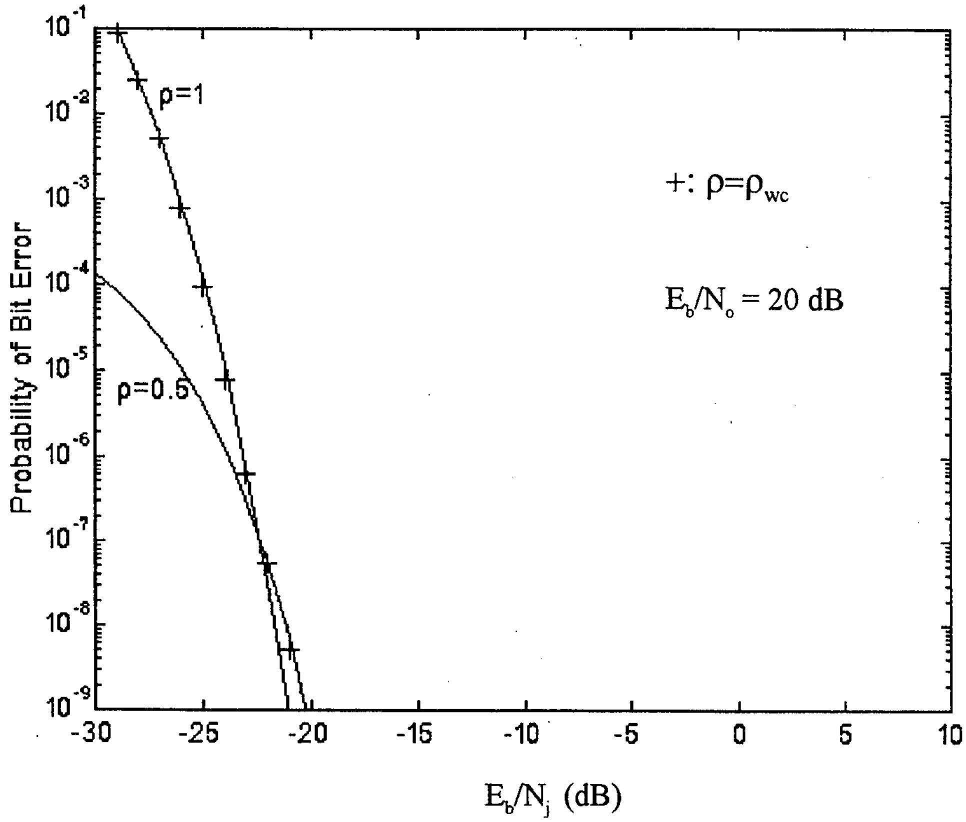

Figure 48. Performance of the perfect side information receiver with coding,constraint length v =3, code rate r =1/2 and with diversity L=2.

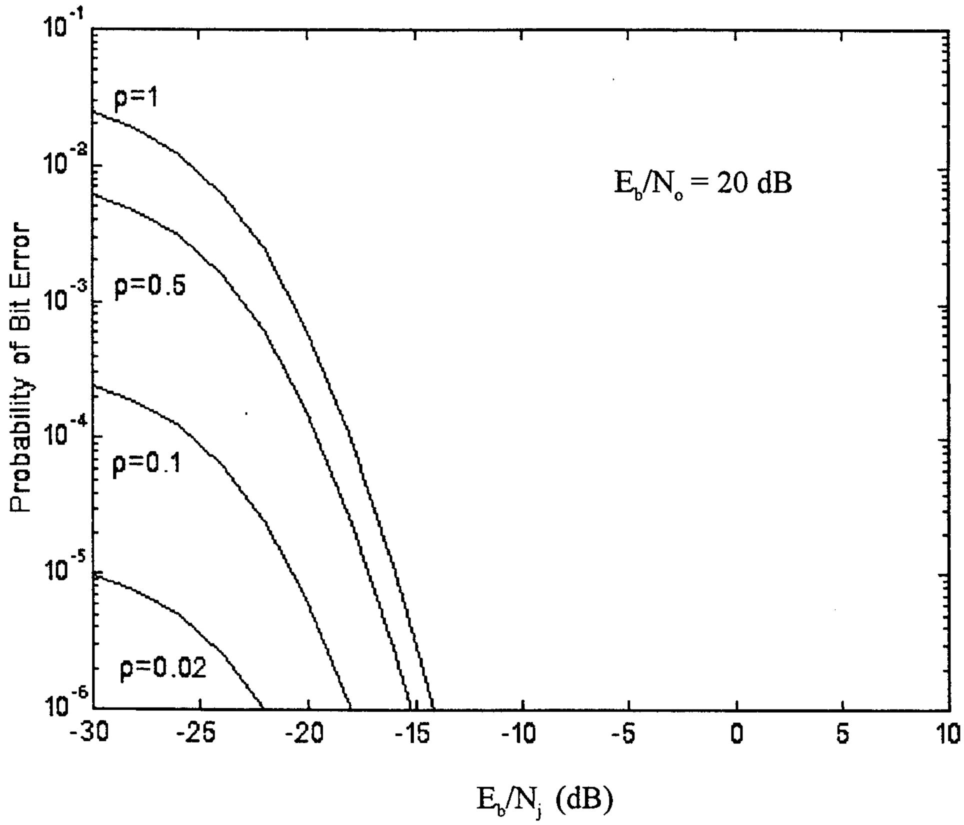

Figure 49. Performance of the perfect side information receiver with fading, $\eta = 10$, without coding and with diversity L=2.

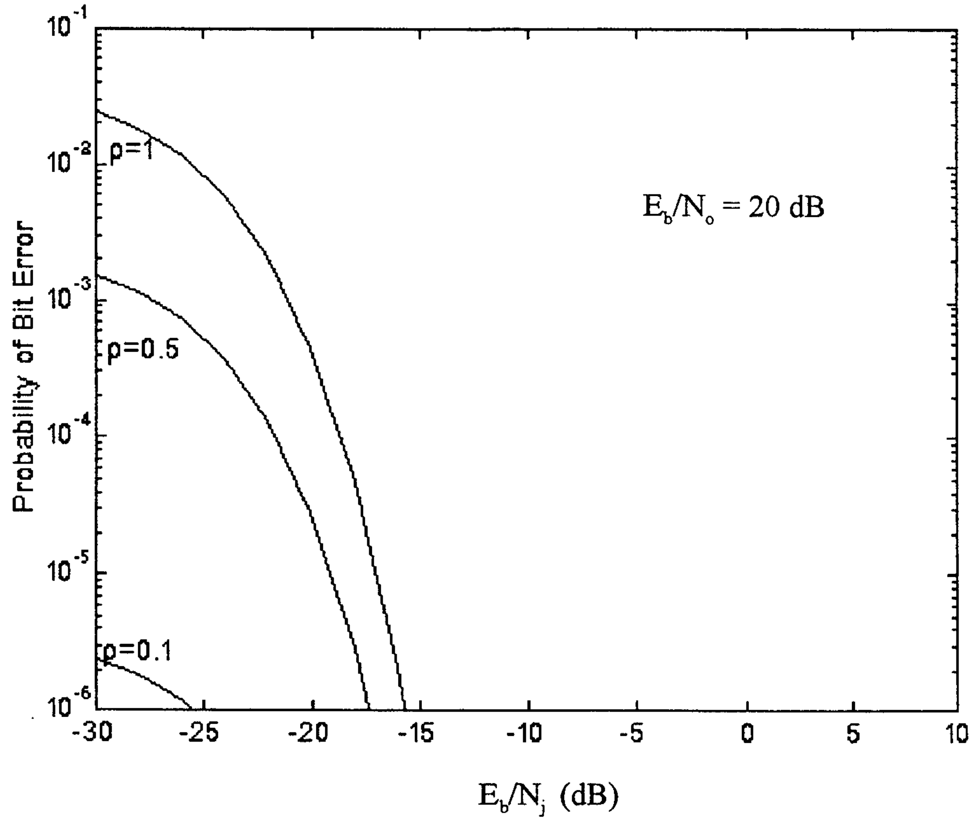

Figure 50. Performance of the perfect side information receiver with fading, $\eta = 10$, without coding and with diversity L=4.

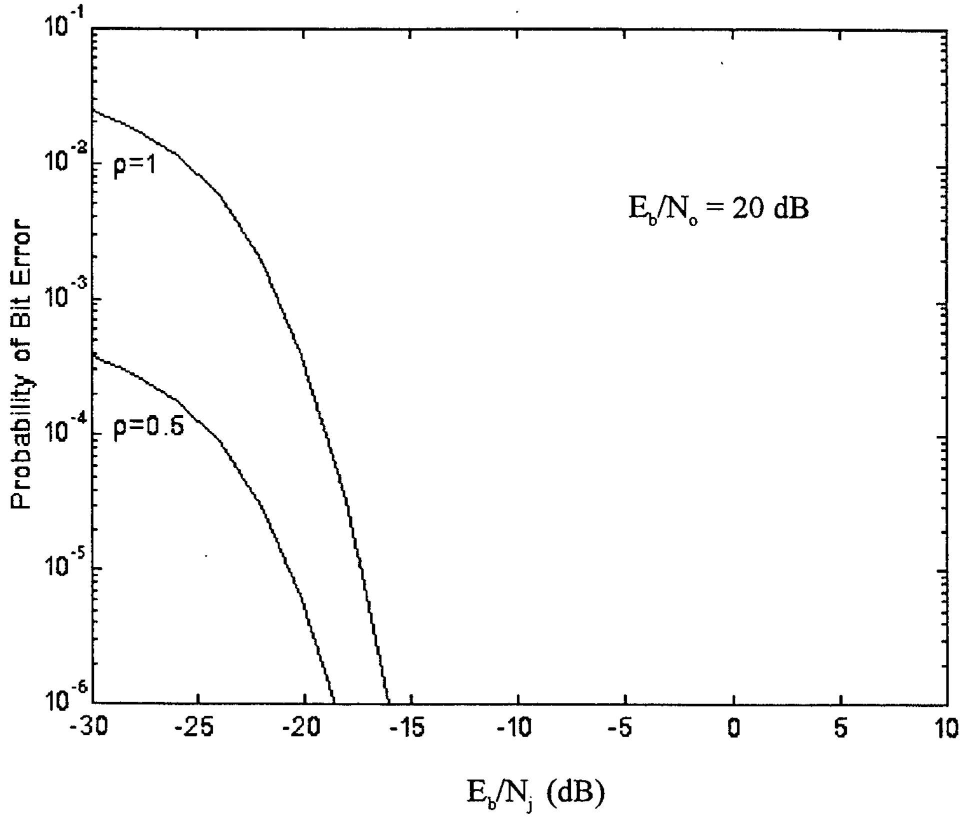

Figure 51. Performance of the perfect side information receiver with fading, $\eta = 10$, without coding and with diversity L=6.

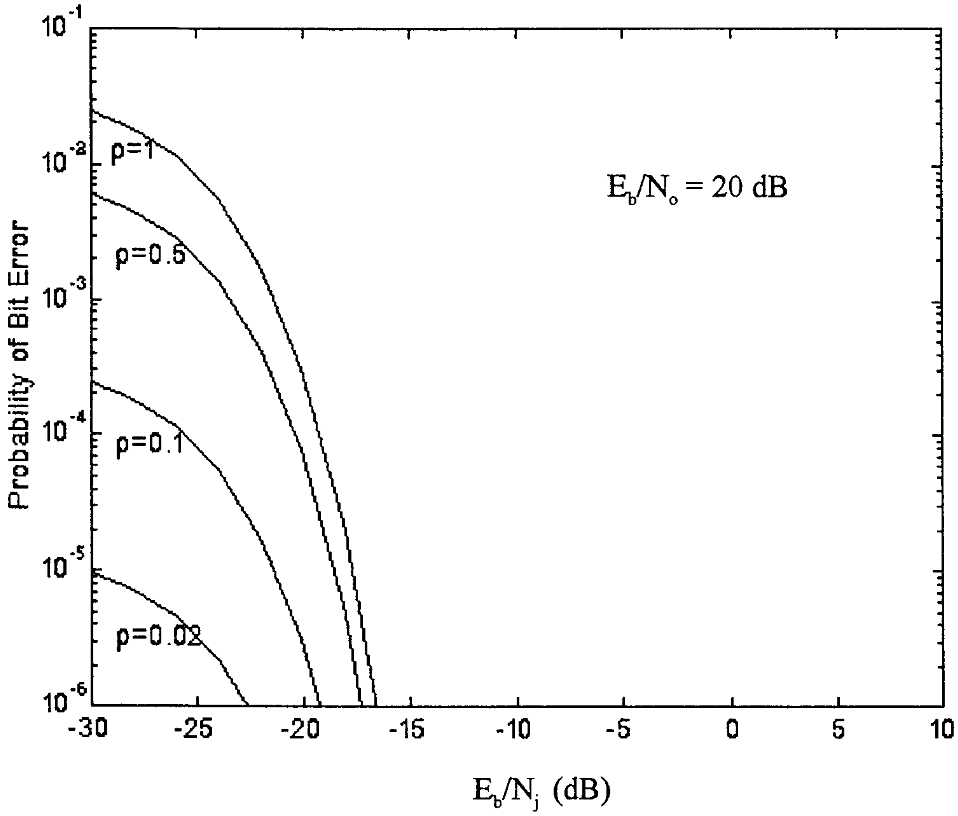

Figure 52. Performance of the perfect side information receiver with fading, $\eta = 100$, without coding and with diversity L=2.

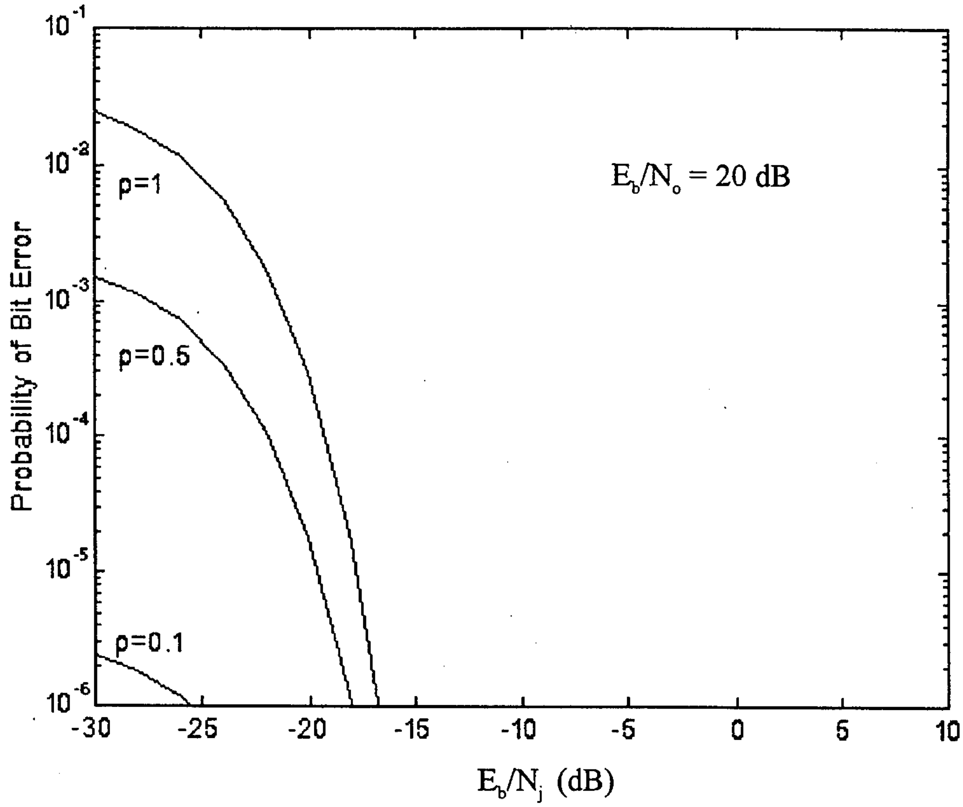

Figure 53. Performance of the perfect side information receiver with fading, $\eta=100$, without coding and with diversity L=4.

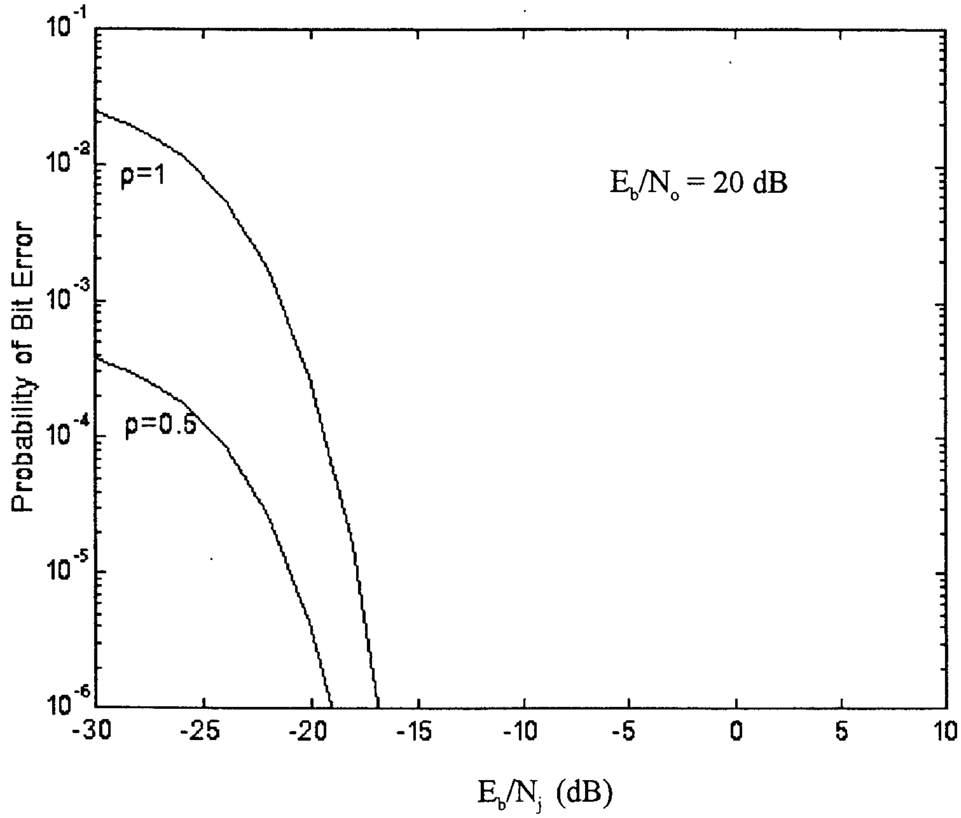

Figure 54. Performance of the perfect side information receiver with fading, $\eta = 100$, without coding and with diversity L=6.

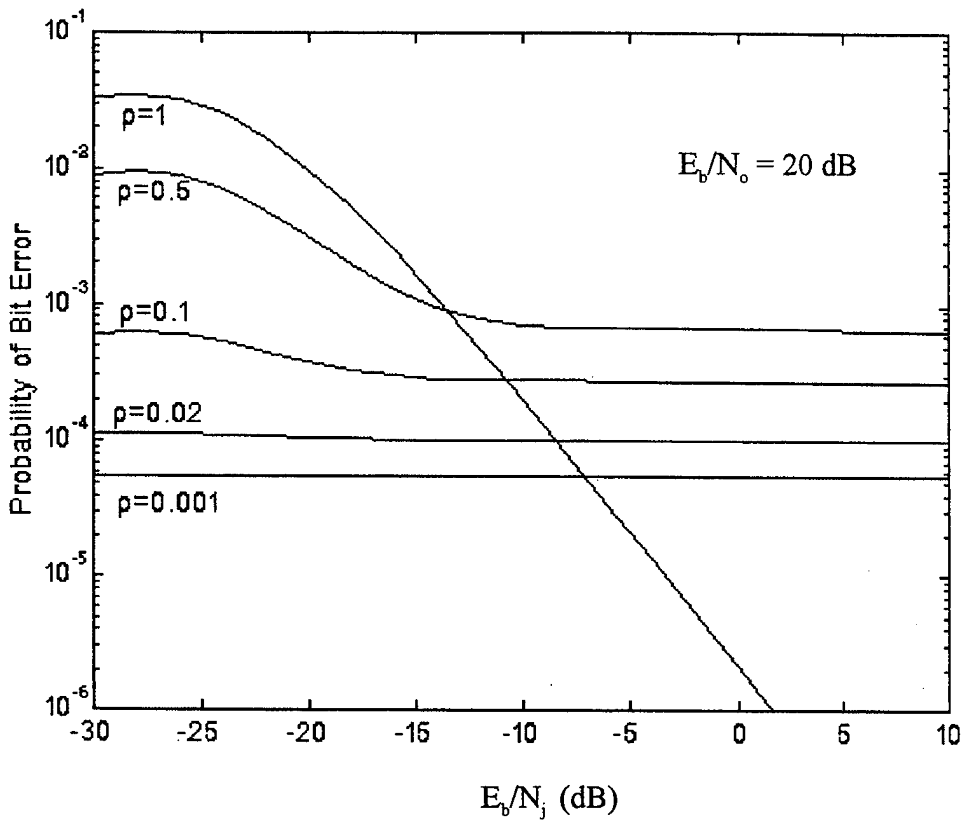

Figure 55. Performance of the perfect side information receiver with Rayleigh fading, $\eta=0$, without coding and with diversity L=2.

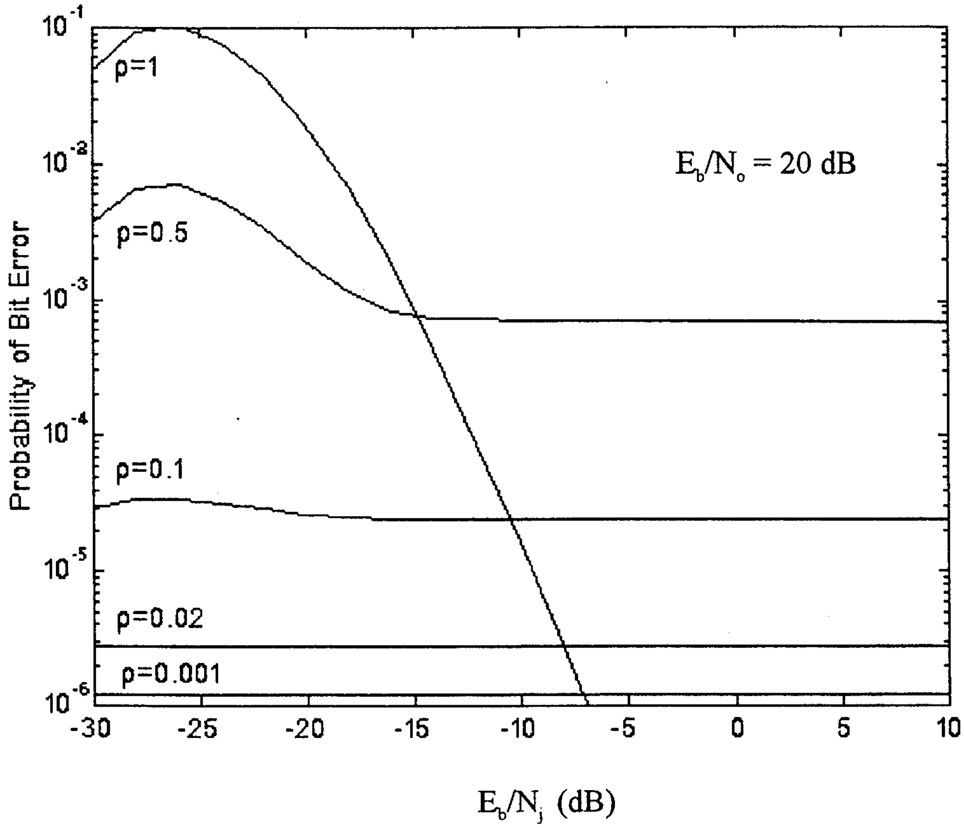

Figure 56. Performance of the perfect side information receiver with Rayleigh fading, $\eta=0$, without coding and with diversity L=4.

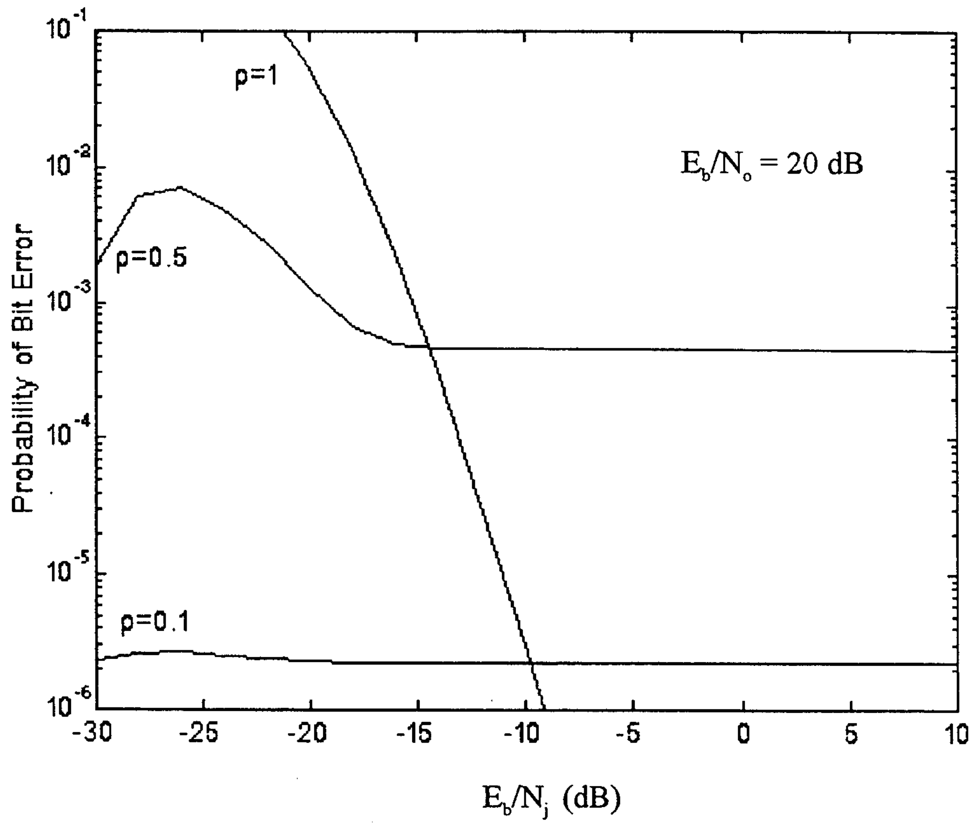

Figure 57. Performance of the perfect side information receiver with Rayleigh fading, $\eta=0$, without coding and with diversity L=6.

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