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# Radial-Inflow Turbines

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DEPARTMENT OF AERONAUTICS



TECHNICAL NOTE

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RADIAL-INFLOW TURBINES

by

*Michael*  
M. H. VAVRA  
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UNITED STATES NAVAL POSTGRADUATE SCHOOL  
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PROPULSION LABORATORIES  
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## INTRODUCTION

During the past year the writer has been associated with three projects where radial turbines were used instead of axial turbines. In a 1 MW Brayton-cycle space power plant, operating with a mixture of Helium and Xenon to obtain reasonably high molecular weights of the operating fluid, a radial-inflow turbine of 7 in. diameter was found to be the best choice to generate abt. 6000 KW at 170 psia and 1500 °F inlet conditions. Its speed would be 38,000 rpm and an efficiency of about 89% should be obtained. This nuclear gas turbine would have a centrifugal compressor, and with a radial turbine it is possible to completely balance the axial thrust of the rotor. This condition is necessary since it is desirable to use gas bearings with a limited thrust capability, to avoid a lubrication system which could contaminate the reactor and the heat transfer surfaces. Besides that, the simplicity of the turbine itself, its inlet and discharge manifolds, and the ease with which they can be arranged in the loop were additional considerations which lead to the choice of a radial turbine.

In a project for a nuclear gas turbine of 1.5MW for land use, that operates with air in a closed system at high pressures, a radial turbine was preferred to an axial one for similar reasons. A wheel of about 8 in. diameter at 40,000 rpm can generate 5300KW at inlet conditions of 1100 psia and 1400 °F at a pressure ratio of about 2. Efficiencies of 88% are expected. With a radial turbine it is possible to obtain a favorable arrangement of the inlet casing which is then stressed only by a small pressure difference. Thrust balancing was of great importance at the high operating pressure although oil-lubricated outboard bearings will be used. As planned, the unit could have a simple tapered - land thrust bearing to take up the resulting axial force of about 2000 pounds. Ruggedness, simplicity, and



cost were other factors that made the radial turbine more advantageous than an axial type.

A very special application of a radial turbine was found for a turbo-pump unit of a rocket motor with extreme pressures. Because of the small volume flow rate in the turbine the leakage losses in an axial turbine would have become excessive without special sealing arrangements at the blade tips, which were thought undesirable because of the distortions of the casing at the high pressures. Complete thrust balancing is necessary and the radial turbine in question is symmetrical with dual discharge because of this requirement.

Radial turbines have been in use for many years, especially as expanders for air-conditioning systems in aircraft, for cryogenic purposes, and small gas turbines. At present the tendency toward smaller and smaller turbomachines has given a great boost to radial machines since they can cover a field where the axial turbine cannot operate at good efficiency because of its small blade heights. Interesting applications and designs are shown in Reference [1]\*. While, until recently, it was not possible to fabricate small radial-type rotors from forgings, such designs have now become a reality with modern production methods, especially by means of electric discharge machining. Excellent tolerances can be maintained for very thin blades since no force is applied to the rotor blades during machining.

The life support system of the Mercury capsule has two compressor wheels of 2 in. diameter running at 40,000 rpm. Efforts are now made to build a self-contained life support system for extravehicular space explor-

\*References - refer to bibliography at end of paper.



ations which would use a turbo-compressor set with a radial turbine of 4.4 in. diameter, operating at 80,000 rpm, and driving a centrifugal compressor with a wheel of 1.32 in. diameter [2]. It is of interest to note that the waste heat of the astronaut and his  $\text{CO}_2$  production will be used as the sole heat source of the little turbopump and refrigeration unit.

Recently, the literature dealing with radial turbines has increased greatly. Concepts that are somewhat foreign to the designer of axial turbines are being applied, such as specific speed and specific diameter, which have been used in hydraulic turbine designs for many years. Actually, radial hydraulic turbines, so-called Francis turbines, have been built since 1900, and the designers of radial turbines for compressible fluids now try to benefit from the experiences made in the hydraulic field.

However, some of the recommendations and design criteria one finds in the literature are confusing and contradictory. From Fig.1, for instance, it seems that the only requirement for high efficiency is a high specific speed  $N_s$ . (This value and others will be defined later in the paper). However, Ref.5 indicates also that the value of  $U_1/C_o$  for best efficiency should be between 0.69 and 0.725. The corresponding head coefficients  $k_{is} = (C_o/U_1)^2$ , which the writer prefers to use, are therefore between 2.1 and 1.9. Fig.2, which has been adapted from Ref.6, seems to convey the same recommendations, but it contains one curve from another source (Hydro-Aire, Inc.) that seems to indicate that there exists an optimum value of the specific speed, so that the efficiency tends to decrease if this value of  $N_s$  is exceeded. In Ref.6 it is shown that the value of  $U_1/C_o$  at which the best efficiencies were measured is 0.65, corresponding to  $k_{is} = 2.37$ .



Ref.4 introduces the so-called specific diameter  $D_s$ , and in Fig.3 it is shown that a relation exists between  $D_s$  and  $N_s$  for optimum efficiencies. Further, the highest efficiency occurs at values of  $U_1/C_o$  between 0.6 and 0.7, or at values of  $k_{is}$  between 2.78 and 2.04.

The results of Figs.1 and 2 differ from those of Fig.3. The first-mentioned figures show the efficiency  $\eta_T$  of a turbine if it is referred to the isentropic enthalpy drop from the total inlet pressure to the total discharge pressure, whereas in Fig.3 are given the efficiencies  $\eta_i$  referred to the enthalpy drop from the total inlet to the static discharge pressure. If  $\eta_T$  is used as a criterion, two turbines with the same efficiency  $\eta_T$  can generate different powers for the same inlet conditions and the same static discharge pressure at the wheel exit, depending on the velocity of the flow at the turbine discharge. It is possible of course, to recuperate part of the kinetic discharge energy by means of a diffuser, similar to the use of a draft tube in hydraulic turbines, but this effect should not be considered if different turbines are compared which must produce the same power. It is necessary, at least, to compare not only the "total-to-total" efficiencies  $\eta_T$  but also the "total-to static" efficiencies  $\eta_i$ .

In order to clarify the situation the writer has undertaken theoretical studies to define more clearly the parameters that bear on the operating performance of radial turbines. The purpose of the study was to establish the range of applicability and the efficiency of such turbines. In particular, the writer does not like to operate with parameters that are semi-dimensionless, which require changes and conversion factors if different systems of units are used. Not only is the use of such para-



meters awkward, but in most cases they do not show up the actual physical nature of the phenomenon they describe.

The study is limited to radial turbines where the blades extend from the outer diameter of the rotor to the discharge annulus, and whose meridional channel is such that nearly uniform discharge velocity can be expected. In particular, the rotor blades are supposed to be in meridional planes at the inlet to avoid bending stresses in the blades. Rotors with such "radial" blades can operate at higher peripheral speeds than others for given physical properties of the disk material. Furthermore, since the use of multi-stage radial turbines is associated with considerable design difficulties, the turbines treated in this paper will be single-stage units with whirl-free flows at the discharge to minimize the so-called leaving loss, which equals the kinetic energy of the absolute flow at the turbine discharge.

The investigation has been carried out because of the research work in radial turbines that is now under way, or is planned, at the new Propulsion Laboratories of the United States Naval Postgraduate School. It is hoped that these efforts will create a better understanding of the flow phenomena in such machines, and establish more clearly when and where a radial turbine should be used. The study is preliminary in nature. Three-dimensional flow conditions are not treated, and it does not touch on the effects of Mach and Reynolds numbers, although the latter have a pronounced influence on the performance of turbines, both at very low and very high values.



## DIMENSIONLESS PERFORMANCE PARAMETERS

The analysis of the radial turbine of Fig.4 will be carried out with the simplified one-dimensional approach described in chapter 15 of Ref.7. The conditions along a mean stream surface from station 1 to station 2 are considered to be representative of the stage, and  $R_2$  is the mean square radius at the discharge. For the assumed uniform velocity distribution at the rotor discharge, equal flow rates will pass between the radii  $R_{20}$  and  $R_2$ , as do between  $R_2$  and  $R_{21}$ . The expansion process in the turbine is shown in Fig.5.

For assumed radial inlet of the relative velocity  $W_1$ , and axial direction of the absolute discharge velocity  $V_2$ , there are from Eqs.15 (30) and 15 (32) of Ref.7

$$v = \frac{U_1}{C_o} = \varphi \sin \alpha_1 \sqrt{1 - r^*} = \frac{\Psi X}{R_2/R_1} \quad (1)$$

where, in addition to the symbols of Fig.4,

$\varphi$  = velocity coefficient of nozzles

$\Psi$  = velocity coefficient of rotor blades

$r^*$  = theoretical degree of reaction of the stage (see Fig.5)

$X$  = function, given by Eq.15 (27) of Ref.7

$C_o$  = theoretical velocity of isentropic expansion from  $P_o$  to  $P_2$ .

$v$  = velocity ratio of stage

Neglecting the reheat factor  $f$  in Eq.15 (27) of Ref.1, and assuming complete recovery of the relative velocity  $W_1$  in the rotor ( $\Phi_R = 1$ ), there is with Eq.1, and the symbols of Fig.4,



$$\frac{r^*}{1-r^*} = C = \varphi^2 \left\{ \sin^2 \alpha_1 \left[ 2 + (R_2/R_1)^2 \left( \frac{1}{\psi^2} - 1 \right) \right] + \cos^2 \alpha_1 \frac{(V_{m2}/V_{m1})^2}{\psi^2} - 1 \right\} \quad (2)$$

This relation shows that the degree of reaction  $r^*$  of a radial turbine with radial blades at the outer diameter, and axial absolute velocity at the discharge, cannot be chosen arbitrarily for chosen radius ratios, and ratios  $V_{m2}/V_{m1}$  of the through-flow velocities.

From Eq.2

$$r^* = \frac{C}{1+C}$$

The following performance parameters of the stage are introduced:

$$k_{is} = \Delta h_{is} / (U_1^2 / 2 \text{ g J}) \quad (\text{Head Coefficient}) \quad (3)$$

$$k = \Delta h_w / (U_1^2 / 2 \text{ g J}) \quad (\text{Work Coefficient}) \quad (4)$$

$$\eta_i = \frac{\Delta h_w}{\Delta h_{is}} \quad (\text{Total-to Static Efficiency}) \quad (5)$$

$$\eta_T = \frac{\Delta h_w}{\Delta H_{is}} \quad (\text{Total-to-Total Efficiency}) \quad (6)$$

The head coefficient  $k_{is}$  represents the isentropic enthalpy difference  $\Delta h_{is}$  of Fig.5, from the total inlet pressure  $P_0$  to the static discharge pressure  $p_2$  that can be handled by the stage, as a multiple of the "kinetic energy" of the peripheral speed  $U_1$  at the outer radius  $R_1$  of the rotor. This head coefficient is related to the velocity ratio  $\psi$  of the stage by

$$k_{is} = \frac{1}{\psi^2} = \left( \frac{C_0}{U_1} \right)^2 = \frac{1+C}{\varphi^2 \sin^2 \alpha_1} \quad (7)$$

The work coefficient  $k$  expresses the useful mechanical energy  $\Delta h_w$  of Fig.5, generated by the rotor, in terms of  $U_1^2 / 2 \text{ g J}$ . From Eq.15 (28) of Ref. 7



$$\eta_{i1} = \frac{2}{k_{is}} = \frac{2 \varphi^2 \sin^2 \alpha_1}{1 + C} \quad (8)$$

and, from Eq.15 (19), if  $\eta_s$  is replaced by  $\eta_T$ ,

$$\begin{aligned} \eta_T &= \frac{\eta_{i1}}{1 - (V_{m2}/V_{m1})^2 \frac{\cot^2 \alpha_1}{k_{is}}} = \frac{\eta_{i1}}{1 - (V_{m2}/U_1)^2 \frac{1}{k_{is}}} \\ &= \frac{2}{k_{is} - (V_{m2}/U_1)^2} \end{aligned} \quad (9)$$

Other useful relations are:

$$\frac{V_{m2}}{U_1} = \frac{V_{m2}}{V_{m1}} \cot \alpha_1 \quad (10)$$

and

$$\beta_2 = \tan^{-1} \left( \frac{R_2/R_1}{V_{m2}/U_1} \right) \quad (11)$$

The nozzle discharge angle  $\alpha_1$  cannot exceed about  $80^\circ$  to  $84^\circ$ . The writer has carried out tests on a turbine inlet scroll with guide vanes, where average flow angles  $\alpha_1$  of about  $83^\circ$  were measured. The losses from the scroll inlet to the rotor inlet were about 6% of the theoretical velocity head from inlet to nozzle discharge, corresponding to a value of  $\varphi$  of about 0.97. For the calculations presented later a velocity coefficient  $\varphi = 0.96$  will be used. The velocity coefficients  $\Psi$  of the rotor blades are not known with great accuracy, in fact, this investigation was undertaken to establish more precisely the values of  $\Psi$  for rotors with different geometry.

The radius ratios  $R_2/R_1$  cannot vary greatly since the outer radius  $R_{20}$  at the discharge should not be larger than about 75% to 80% of  $R_1$  to



obtain reasonable wheel shapes. Fig.6 shows that for values  $R_2/R_1$  smaller than 0.4 the available annulus area at the discharge becomes too small, and for  $R_2/R_1$  larger than 0.6 the radius  $R_{2o}$  becomes excessive. Hence  $R_2/R_1$  can vary from about 0.4 to 0.6.

One major difficulty with the design of radial turbine wheels is produced by the fact that the volume flow rate at the discharge is larger than that at the inlet, and that this flow rate must pass through the outlet annulus which is a fraction only of the projected area  $\pi R_1^2$  of the rotor. Hence in many cases it is necessary to use through-flow velocities  $V_{m2}$  that are larger than  $V_{m1}$  at the rotor inlet. In general, therefore, the three parameters  $\alpha_1$ ,  $R_2/R_1$ , and  $V_{m2}/V_{m1} = V_2/W_1$  can be varied to satisfy particular operating conditions. However, certain combinations of these quantities may produce angles  $\beta_2$  that are larger than those which can be realized in actual rotors. It must be borne in mind that the velocity triangles of Fig.4(b) represent the actual average velocities and flow angles at the different stations. Hence the actual blade angle  $\beta_{2B}$  at the rotor discharge, that produces a flow angle  $\beta_2$ , must be larger than  $\beta_2$  because of the thickness of the blades, and because of an effect similar to that which is known as "slip" in centrifugal compressors. For these reasons the flow angle  $\beta_2$  cannot exceed about  $65^\circ$  to  $70^\circ$ .

#### SPECIFIC SPEED AND SPECIFIC DIAMETER

These concepts are obtained from similarity considerations, in particular from Buckingham's  $\pi$  - Theorem [8] if changes in mass density are ignored. Specific speed  $N_s$  and specific diameter  $D_s$  are regularly used



for hydraulic pumps and turbines. For compressible fluids it is generally accepted to apply these quantities to the station where the lowest mass density of the fluid occurs. For radial turbines then

$$N_s \equiv \frac{N Q_2^{\frac{1}{2}}}{H^{\frac{3}{4}}} \quad (12)$$

where:  $N$  = rotative speed of impeller  
 $Q_2$  = volume flow rate at rotor discharge  
 $H$  = isentropic head

Further

$$D_s \equiv \frac{D_1 H^{\frac{1}{4}}}{Q_2^{\frac{1}{2}}} \quad (13)$$

where  $D_1$  is the outer diameter of the rotor. Evidently the values obtained from Eqs. 12 and 13 for particular designs depend on the units that are used to express the different quantities. Even in the English system of units a number of conversion factors must be used to correlate different values.

The values of  $N_s$  and  $D_s$  in Figs. 1, 2, and 3 are calculated with the usually applied units of

$N$  in rpm  
 $Q_2$  in  $\text{ft}^3/\text{sec}$   
 $H$  in  $(\text{ft-lb})/\text{lbm}$   
 $D_1$  in ft

Then  $N_s$  and  $D_s$  have the dimensions  $(\text{ft}/\text{sec}^2)^{3/4} \text{ min}^{-1}$ , and  $(\text{ft}/\text{sec}^2)^{-1/4}$ , respectively. Sometimes a so-called dimensionless specific speed  $N_{s1}$  and a specific diameter  $D_{s1}$  are used, where  $N$  is introduced in revolutions per second,  $Q_2$  in  $\text{ft}^3/\text{sec}$ ,  $D_1$  in feet, and  $H$  in  $\text{ft-lb}/\text{slug}$



or  $\text{ft}^2/\text{sec}^2$ . Then

$$N_s = N_{s1} \quad (810.55)$$

and

$$D_s = D_{s1} \quad (0.420)$$

Adding to the confusion is the situation that some sources introduce the speed  $N$  in radians per second. The resulting specific speed  $N_{s2}$  is then related to  $N_s$  by

$$N_s = N_{s2} \quad (129.01)$$

It can be shown that turbines with similar geometry and equal specific speed and specific diameter must have the same efficiency if Mach number and Reynolds number effects are ignored. Furthermore it is evident that  $N_s$  and  $D_s$  represent the speed and the diameter, respectively, of a turbomachine that discharges a unit of volume flow rate at a unit of head.

It is possible however to relate these concepts directly to the performance parameters that were established earlier. For  $N_s$  and  $D_s$  of Figs.1, 2, and 3, with

$$\begin{aligned} N &= \frac{30}{\pi} \frac{U_1}{R_1} \\ Q_2 &= A_2 V_{m2} = \zeta_A \pi R_1^2 V_{m2} \\ H &= \Delta h_{is} J = k_{is} \frac{U_1^2}{2g} \end{aligned} \quad (14)$$

there are

$$N_s = 384.55 \frac{(\zeta_A V_{m2}/U_1)^{\frac{1}{2}}}{k_{is}^{3/4}} \quad (15)$$

$$\text{and } D_s = 0.3984 \frac{k_{is}^{\frac{1}{4}}}{(\zeta_A V_{m2}/U_1)^{\frac{1}{2}}} \quad (16)$$



Further

$$k_{is} = 2.3472 (10^4) (N_s D_s)^{-2} \quad (17)$$

and

$$\zeta_A V_{m2}/U_1 = 6.7624 (10^{-6}) k_{is}^{3/2} N_s^2 \quad (18)$$

The factor  $\zeta_A$  of Eq.14 is shown in Fig.6. It represents the annulus area at the discharge as a fraction of the area of a circle with the wheel radius  $R_1$ . Hence, the quantity  $\zeta_A$  establishes the ratios  $R_{20}/R_1$  and  $R_{2i}/R_1$  of a turbine rotor with a given radius ratio  $R_2/R_1$ , operating at a certain value of  $V_{m2}/U_1$ . The value  $\zeta_A$  has not appeared in the preceding calculations, nor is there a way to establish it by means of other, more detailed flow calculations without knowing more about the interdependence of wheel geometry and flow losses. The principal advantage of introducing the specific speed concept lies exactly in the fact that  $\zeta_A$  can be obtained from it. However instead of making a detour via the specific speed it would be just as easy and useful to establish the dimensionless quantity  $\zeta_A V_{m2}/U_1$  as a function of, say, the head coefficient  $k_{is}$ , since Eq.17 shows that giving a relation between  $N_s$  and  $D_s$ , as in Fig.3, is tantamount to specifying the magnitude of  $k_{is}$ .

These investigations also show that it is not sufficient to state simply that a turbomachine must have a certain specific speed for best efficiency, without implying or specifying that it must also operate at a definite head coefficient.

Although Fig.3 presents calculated data only, a curve was drawn through the points that give the optimum efficiency for chosen values of  $N_s$ . For the corresponding values of  $D_s$  and  $\eta_i$  it is then possible to



draw the curves of Fig.7, which in turn can be used to establish the relation between  $\zeta_A V_{m2}/U_1$  and  $k_{is}$  with the efficiency  $\eta_i$  as parameter. Fig.7 indicates that a value of  $\zeta_A V_{m2}/U_1 = 0.11$  produces the highest efficiency with a  $k_{is} = 2.35$ , corresponding to  $N_s = 68$  and  $D_s = 1.47$ . If the curves of Figs.3 and 7 were correct they would definitely be at variance with the presently used criterion that a radial turbine must have an  $N_s$  of at least 80 to reach optimum efficiencies.

#### CALCULATED PERFORMANCE VALUES

Eqs.2 to 11 were used to establish the performance of radial-inflow turbines for different values of  $\alpha_1$ ,  $R_2/R_1$ , and  $V_{m2}/V_{m1}$ . For the data presented in Figs.8 and 9, a constant value  $\Psi = 0.9$  was used for the losses in the rotor, and  $\varphi$  was taken to be 0.96. Because the ratio  $V_{m2}/U_1$  appears in Eqs.15 and 16 the results of the calculations were plotted as functions of this quantity. Extremely interesting is the fact that  $V_{m2}/U_1$  has the strongest influence on the performance of the turbine. Although not clearly seen in Fig.8, the curves for  $\alpha_1 = 75^\circ$  and  $70^\circ$  overlap almost completely with the curves for  $\alpha_1 = 80^\circ$ . The influence of the radius ratio is small also and the curves indicate that, the smaller the value of the ratio  $V_{m2}/U_1$  is, the higher the internal efficiency  $\eta_i$  will be. However, because of the limitations imposed by too large flow angles  $\beta_2$ , the minimum value of  $V_{m2}/U_1$  cannot be smaller than about 0.25, depending on  $R_2/R_1$ .

Fig.8 further shows that the total efficiency  $\eta_T$  is not affected greatly by  $V_{m2}/U_1$ . Hence, the total efficiencies of 90% to 93% quoted in Ref.5 are not surprising, all the more as the quantity  $(V_{m2}/U_1)^2$  is equal to the leaving loss coefficient  $k_E$  which is defined by Eq.15 (17) of Ref.7.



Experience with axial-flow turbines shows that the rotor losses, or the velocity coefficients  $\Psi$ , are primarily depending on the deflection of the flow in the rotor blades (see Fig.15(5), p.434, Ref.7). A similar inter-dependence between  $\Psi$  and  $\beta_2$  seems likely in radial turbines. Since the flow is turned by  $90^\circ$  even though the angle  $\beta_2$  is zero, it can be argued that the flow deflection in a radial turbine is  $90^\circ + \beta_2$ . Based on this premise the curve for  $\Psi$  of Fig.10 has been established, and the corresponding results are plotted in Fig.11. It is seen that the curve  $\eta_i$  vs.  $V_{m2}/U_1$  now shows a decrease in efficiency for values of  $V_{m2}/U_1$  below about 0.15. The writer believes however that  $\Psi$  will decrease more with increasing angles  $\beta_2$  than given by Fig.10, and there is reason to assume that the best efficiency will occur at values of  $V_{m2}/U_1$  between 0.3 and 0.35. The optimum value of  $V_{m2}/U_1$  is affected also by the disk friction losses of the impeller. The frictional moment  $M_f$  is obtained from

$$M_f = C_f \rho_1 U_1^2 R_1^3$$

where, according to von Kármán,

$$C_f = \text{constant } Re^{-1/5}$$

The mass density  $\rho_1$  is that at the rotor inlet. Ignoring Reynolds number effects, or taking  $C_f$  as a constant, the power  $P_f$  absorbed by disk friction is

$$P_f = M_f \omega = C \rho_1 U_1^2 R_1^2$$

The power  $P_w$  generated by the rotor is

$$P_w = w \Delta h_{is} \eta_i$$

Hence the efficiency  $\eta_o$  of the turbine, by taking account of disk



friction, is

$$\eta_o = \eta_i - \frac{C \rho_1 U_1^2 R_1^3}{w \Delta h_{is}}$$

Now, with the mass density  $\rho_2$  at the wheel discharge,

$$w = \rho_2 A_2 V_{m2} = \rho_2 \zeta_A \pi R_1^2 V_{m2}$$

and, with  $\Delta h_{is} = k_{is} U_1^2 / 2 g J$ ,

$$\eta_o = \eta_i - K \frac{\rho_1 / \rho_2}{\zeta_A \frac{V_{m2}}{U_1}} k_{is} \quad (19)$$

where  $K$  is a constant if Reynolds number influences are ignored. It is evident from Eq. 19 that the optimum value of the efficiency  $\eta_o$  will be shifted toward higher values of  $V_{m2}/U_1$  because of disk friction.

Now, if the results from Fig.7 were considered to be correct, hence if the optimum ratio  $\zeta_A V_{m2}/U_1$  would have to be equal to about 0.11, a value of  $\zeta_A = (0.11)/(0.3) = 0.37$  should give the best turbine for  $V_{m2}/U_1 = 0.3$ . This value of  $\zeta_A$  then establishes the radii  $R_{2o}$  and  $R_{2i}$  in relation to  $R_1$  for given ratios  $R_2/R_1$ . For  $R_2/R_1 = 0.5$ , for instance, there are from Fig.6:

$$R_{2o}/R_1 = 0.25 ; R_{2i}/R_1 = 0.66$$

For given operating conditions and flow rates of the turbine the value of  $\zeta_A$  not only gives these radius ratios but also determines the actual diameter of the wheel and, because of  $k_{is}$ , the rpm of the turbine.

To establish the blade width  $b_1$  at the rotor inlet it is necessary to know the relation between the mass densities  $\rho_1$  and  $\rho_2$ . Modifying Eq.15(40) of Ref.7 for the present application

$$\frac{\rho_1}{\rho_2} = \frac{1 - \left[ \frac{2 + (V_{m2}/U_1)^2}{k_{is}} \right] Y}{1 - \varphi^2 (1 - r^*) Y} \left[ \frac{1 - (1 - r^*) Y}{1 - Y} \right]^{\gamma/(\gamma-1)} \quad (20)$$



where

$$Y = \frac{\gamma - 1}{2} k_{is} (U_1/a_o)^2 \quad (21)$$

The quantity  $a_o$  in Eq.21 is the velocity of sound of the fluid at the total inlet temperature  $T_o$ . With this information the geometry of the rotor is completely determined.

### CONCLUSIONS

The study shows that the efficiency of an inward-flow radial turbine with radial blades at the inlet of the rotor, and whirl-free discharge, is primarily depending of the ratios  $V_{m2}/U_1$  and  $\zeta_A V_{m2}/U_1$ . If the interdependence between  $\zeta_A V_{m2}/U_1$ ,  $k_{is}$ , and  $\eta_i$  were known it would be possible to establish the dimensions of designs with optimum performance. Whereas the quantity  $V_{m2}/U_1$  seems to be a measure for the effect of the flow deflection on the losses, the ratio  $\zeta_A V_{m2}/U_1$  seems to include also the influence of the geometry of the flow path.

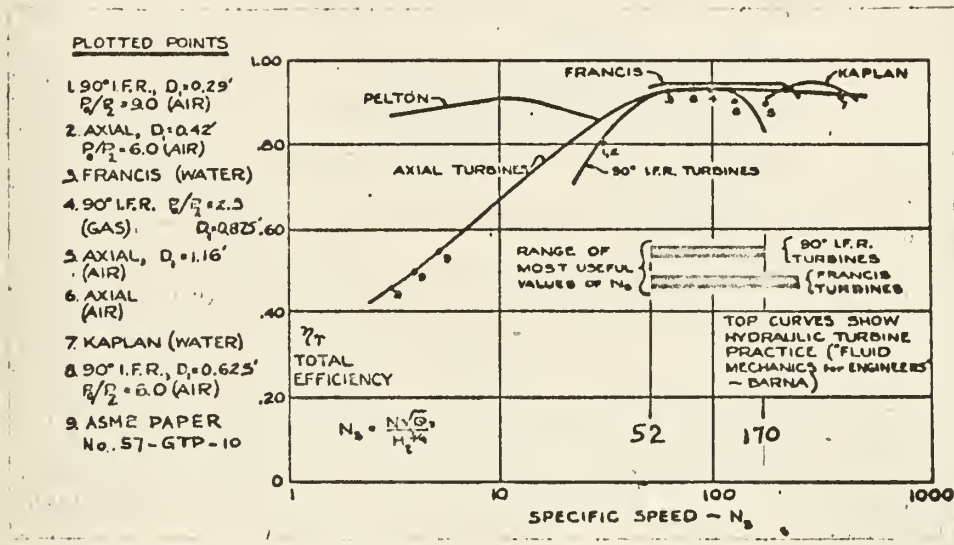
Various approximate and tentative limits of the design parameters are indicated in the paper, but it is hoped that research at the Propulsion Laboratories of the U.S. Naval Postgraduate School will eventually produce more accurate design criteria which are necessary to enhance the state of the art in this promising field of turbomachines.



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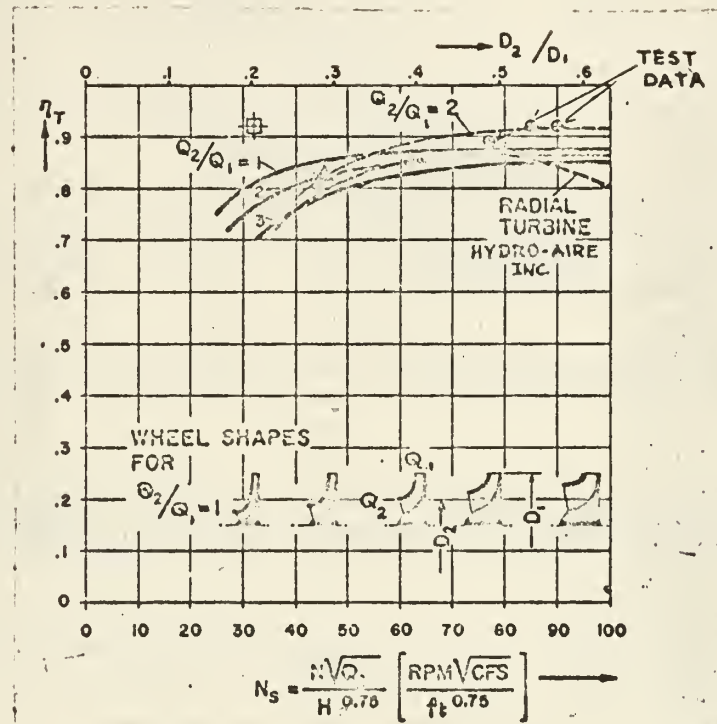


Fig. 2 Influence of Specific Speed on Efficiency of Radial Turbines

( Adapted from Fig. 9 of Reference 6 )



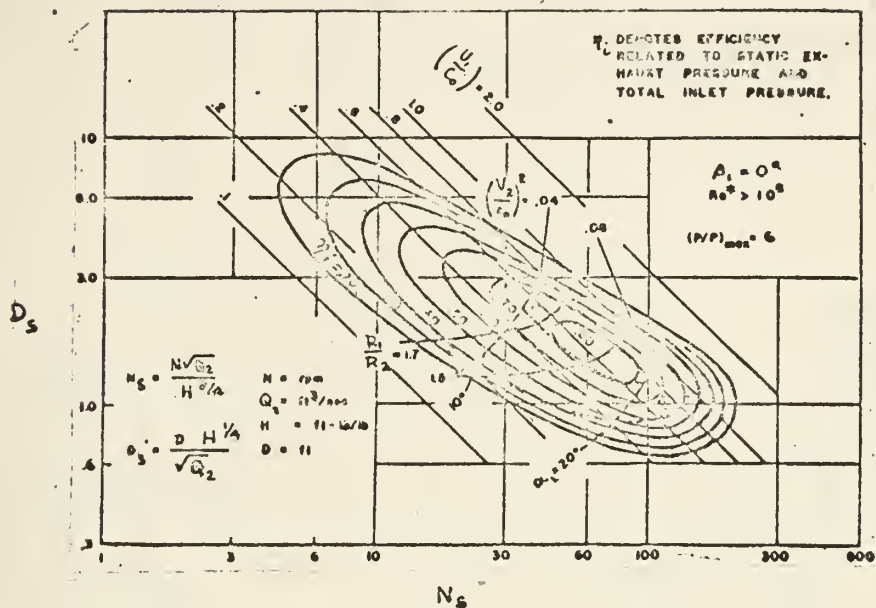


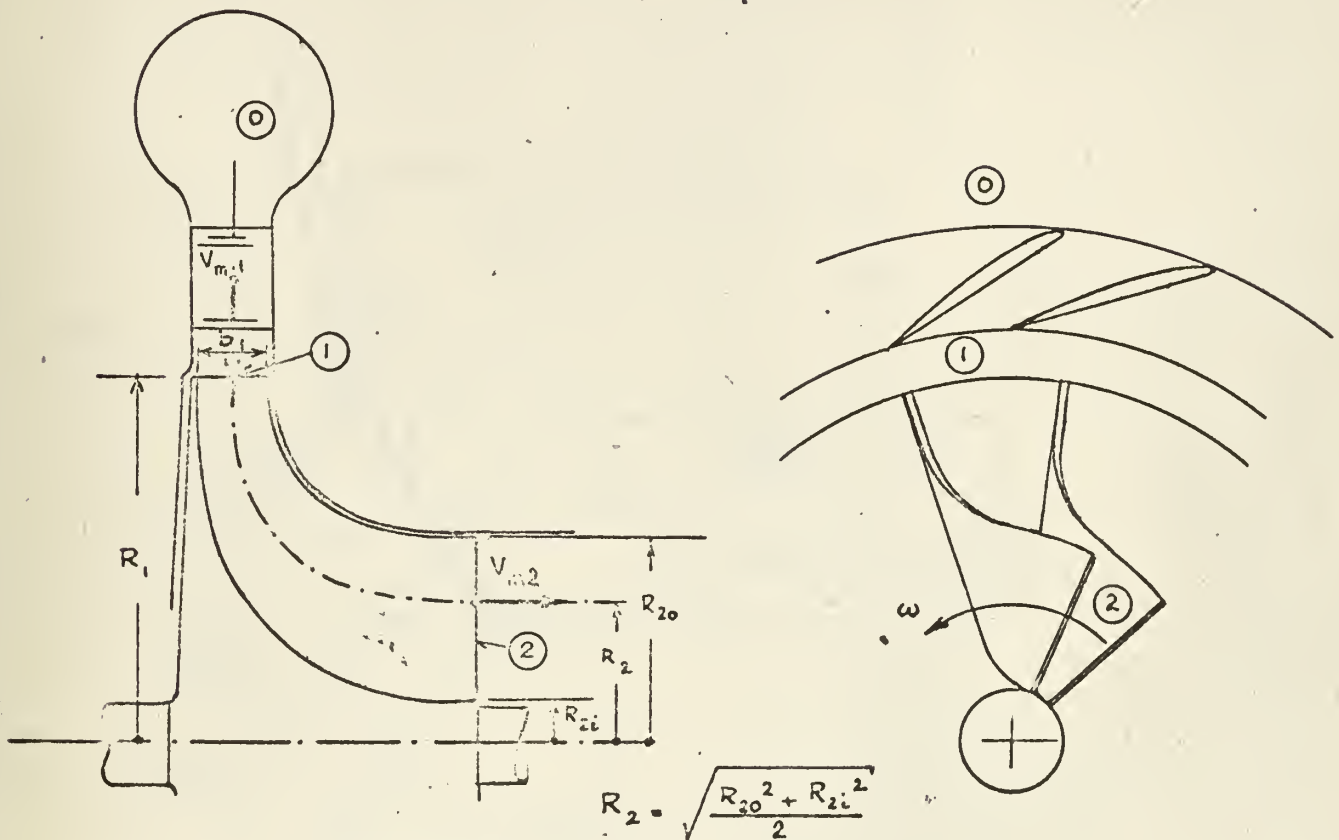
Fig. 3 Calculated  $N_s - D_s$  Diagram for Single-Stage Radial  
Turbines

( Adapted from Fig. 16 of Reference 4 )

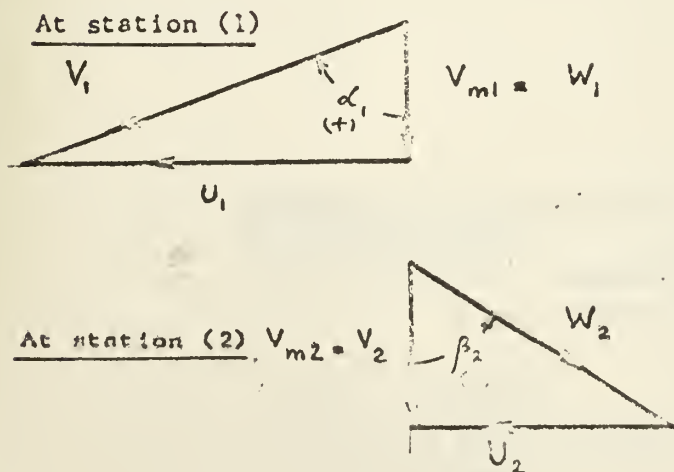


**Fig. 4 Radial - Inflow Turbines**

- (a) Dimensions
- (b) Velocity Triangles at Inlet and Discharge of Rotor

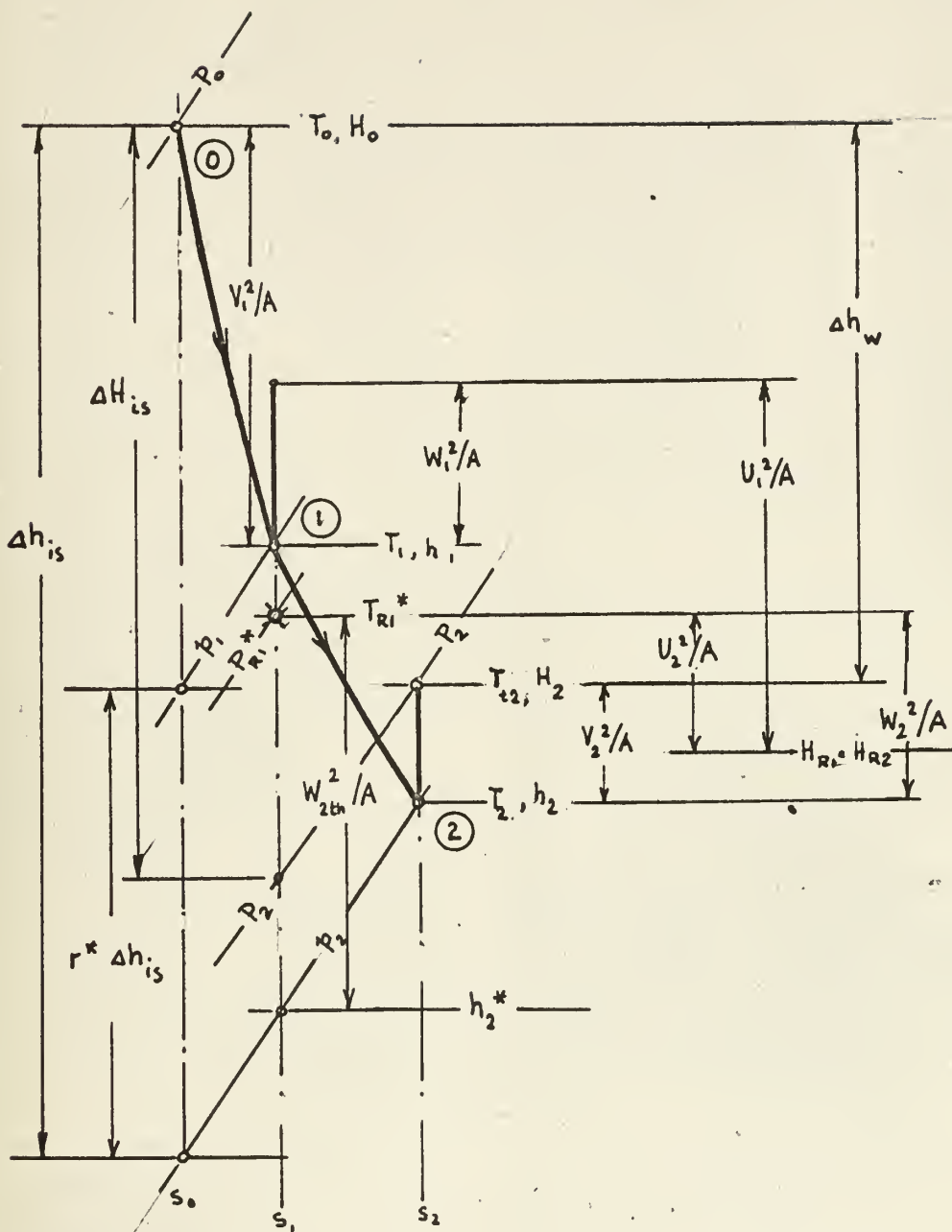


**Fig. 4 (a)**



**Fig. 4(b)**





**Fig. 5 Expansion Process in Radial Turbine represented in Entropy Diagram**

$A = 2 \text{ g J } c_p$  for temperature differences  
 $A = 2 \text{ g J}$  for enthalpy differences

$h$  = static enthalpy  
 $H$  = total enthalpy  
 $H_R$  = relative total enthalpy  
 $p$  = static pressure  
 $P$  = total pressure  
 $P_R$  = total relative pressure

$T$  = static temperature  
 $T_t$  = total temperature  
 $s$  = entropy  
 $V$  = absolute velocity  
 $W$  = relative velocity  
 $U$  = peripheral velocity

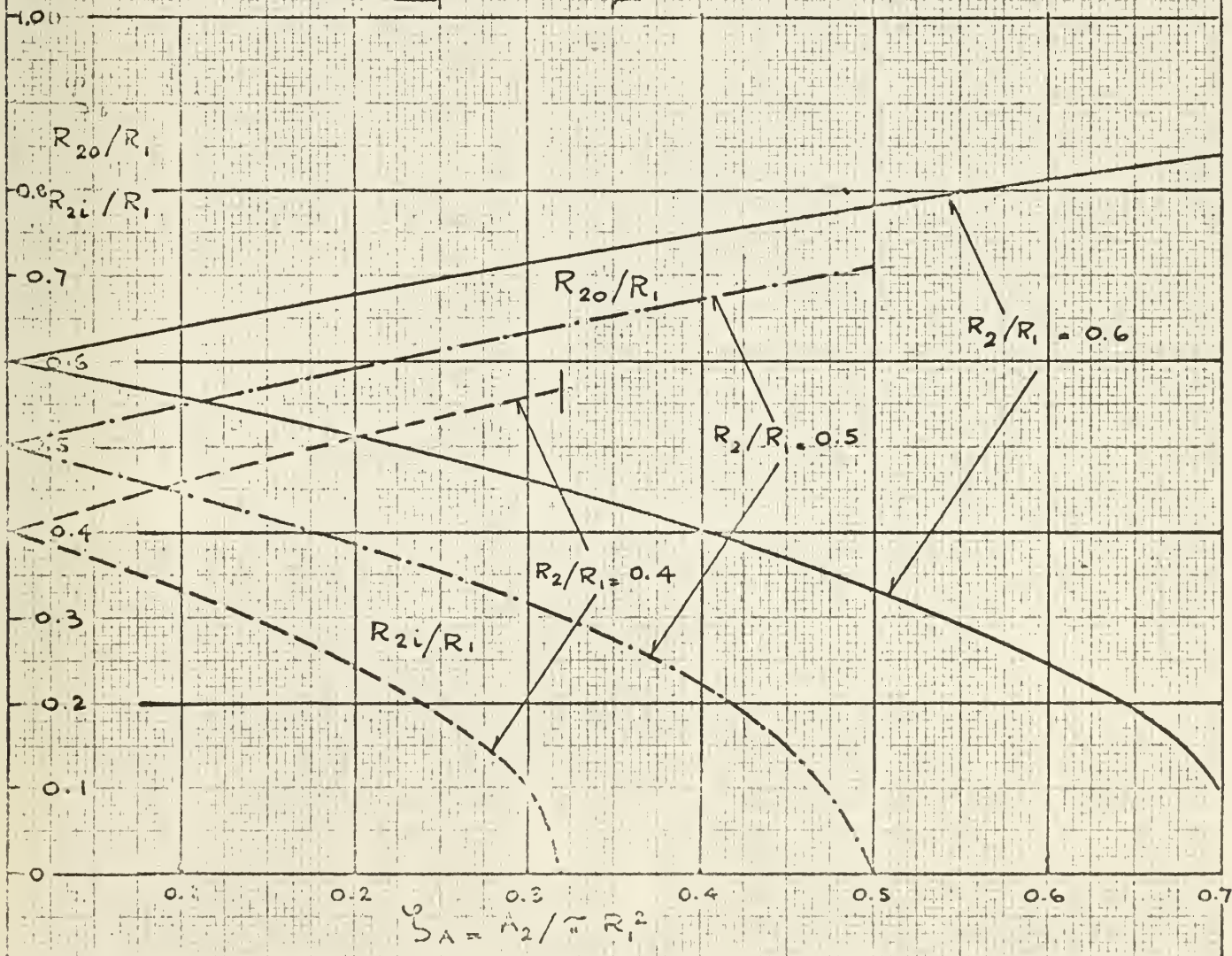


OUTER AND INNER RADII OF DISCHARGE ANNULUS  
OF RADIAL TURBINE FOR DIFFERENT RATIOS  $\phi_A$   
AND  $R_2/R_1$



$$R_2 = \sqrt{\frac{R_{2i}^2 + R_{20}^2}{2}}$$

$$A_2 = \pi(R_{20}^2 - R_{2i}^2)$$

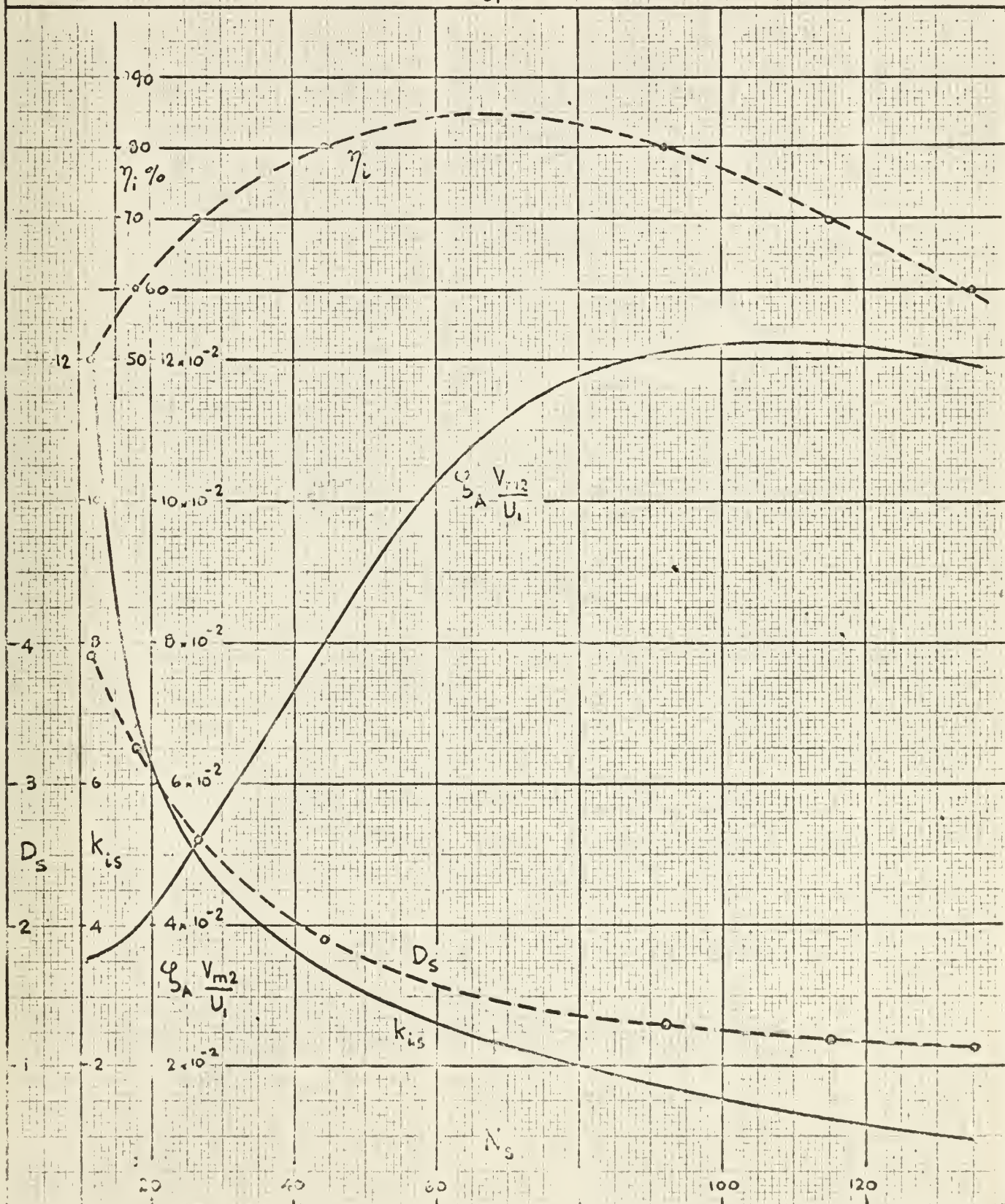




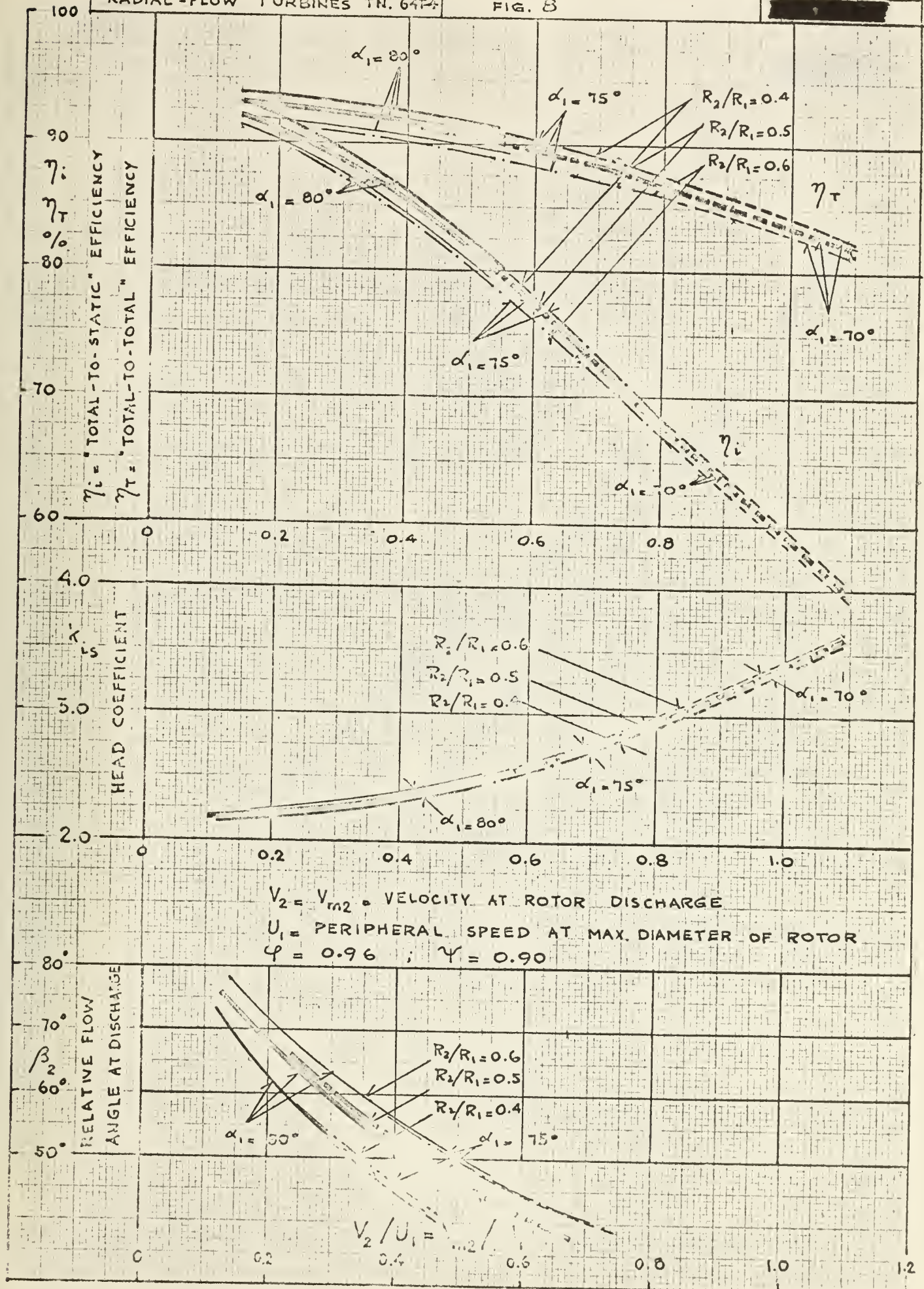
$N_s$  = SPECIFIC SPEED $D_s$  = SPECIFIC DIAMETER $\eta_i$  = "TOTAL-TO-STATIC" EFFICIENCY $K_{is}$  = HEAD COEFFICIENT  $\propto \frac{Q}{A} \frac{V_{m2}}{U_1}$  CALCULATED FROM ABOVE DATA

DATA POINTS O

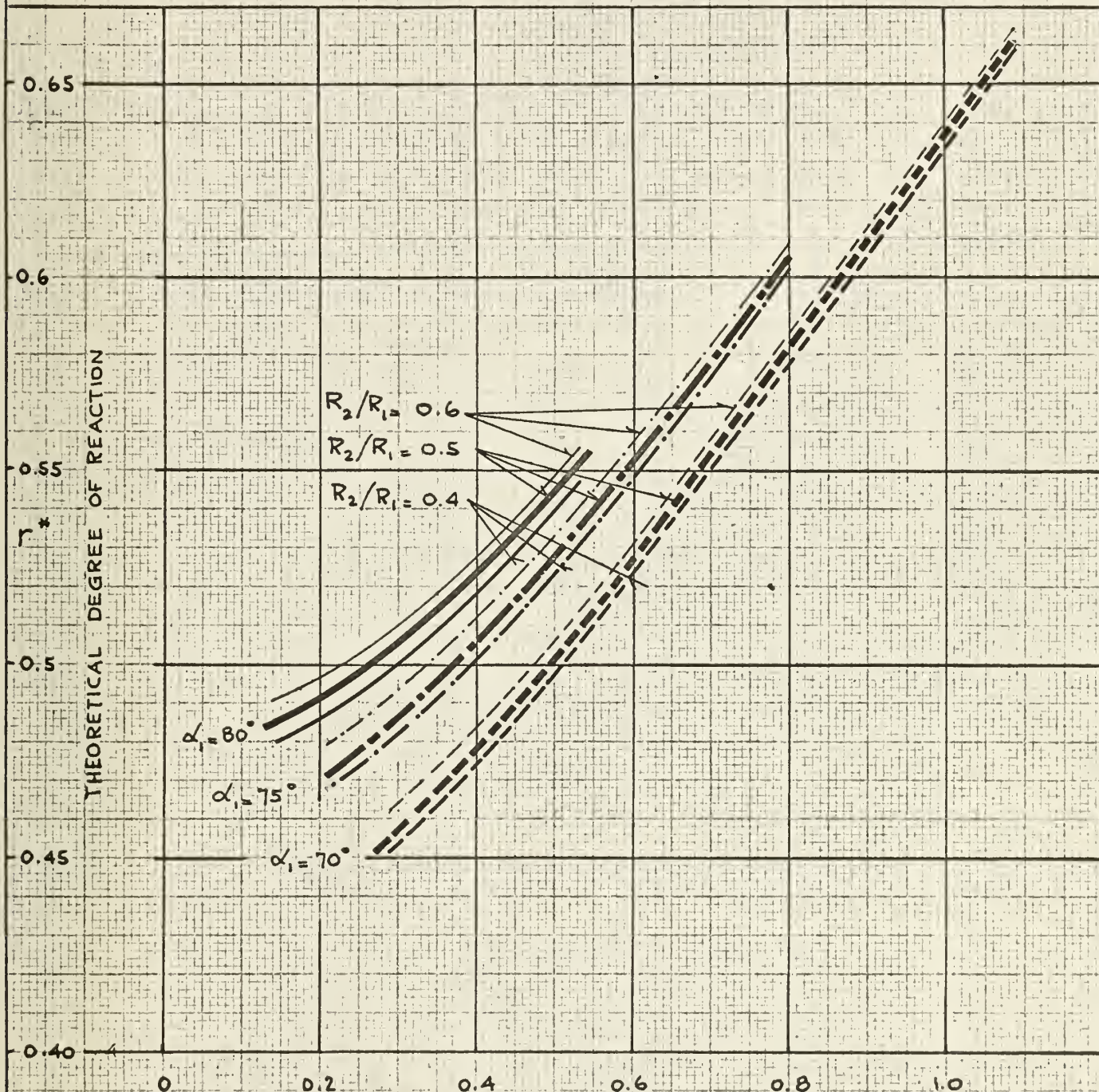
FROM REF. 4 FOR OPTIMUM CONDITIONS











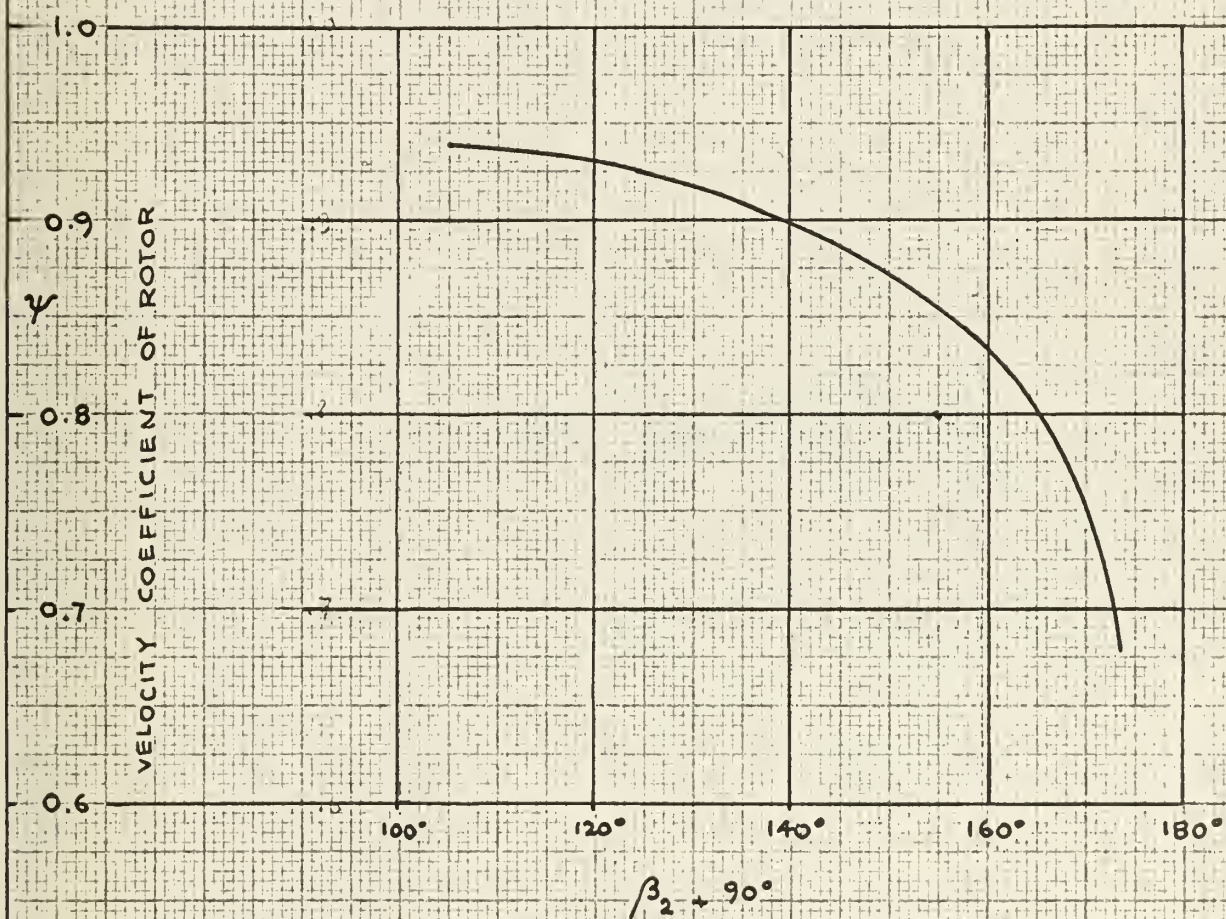
$$V_2/U_1 = V_{m2}/U_1$$

$V_2 = V_{m2}$  = VELOCITY AT ROTOR DISCHARGE

$U_1$  = PERIPHERAL SPEED AT MAX. DIAMETER OF ROTOR

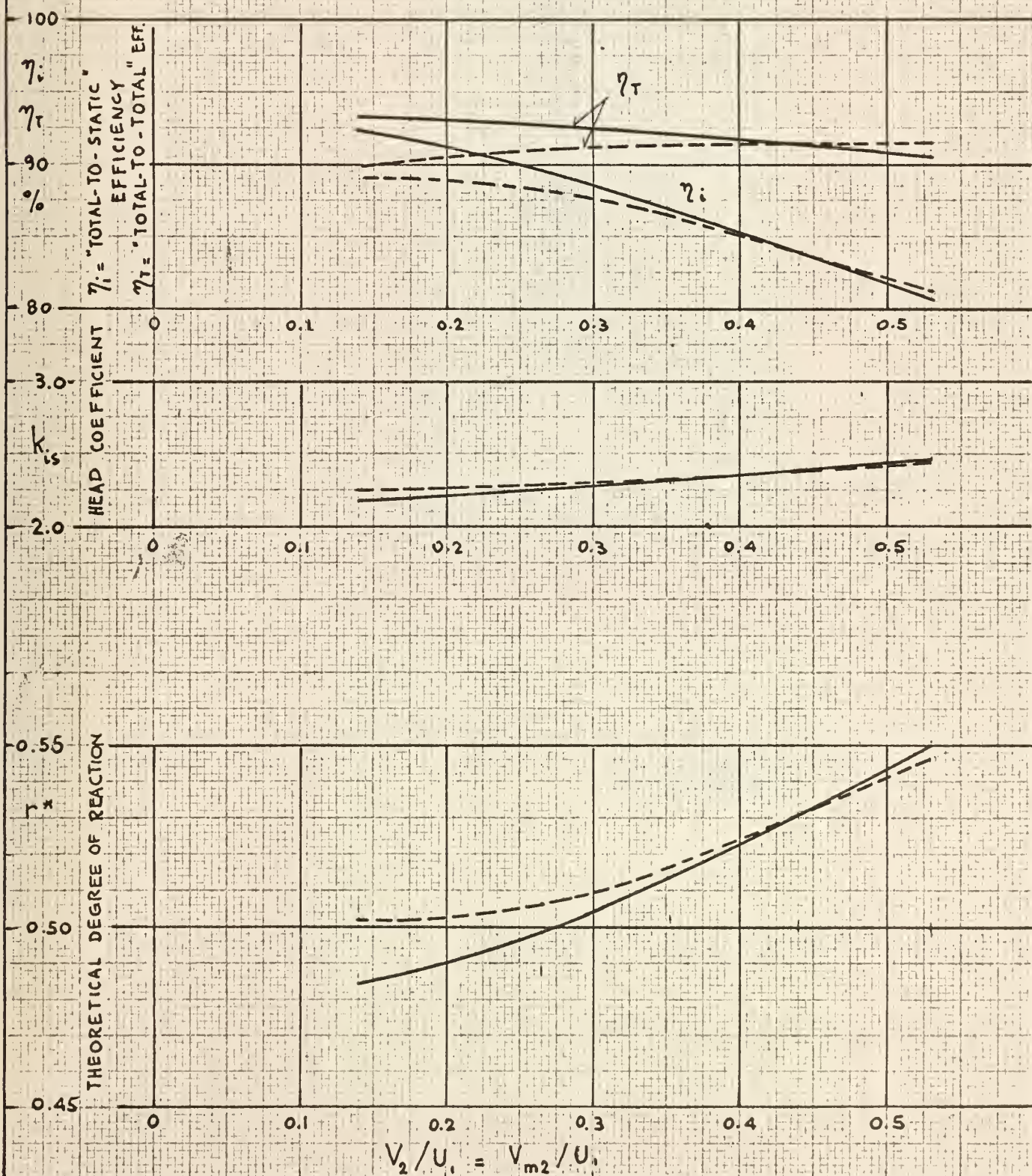


ASSUMED CHANGE OF VELOCITY COEFFICIENT  $\psi$  OF ROTOR  
WITH RELATIVE DISCHARGE ANGLE  $\beta_2$





————— CONDITIONS FOR CONSTANT LOSS IN ROTOR  $\psi = 0.9$   
 - - - - - CONDITIONS FOR LOSSES ACCORDING TO FIG. 10  
 $R_2/R_1 = 0.5$  ;  $\alpha_1 = 80^\circ$  ;  $\varphi = 0.96$





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Nuclear  
Gas  
Turbine  
Radial  
Thrust  
Balance  
Applicability  
Efficiency  
Rotor  
Blade  
Flow  
Phenomena

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