Willard Evan Bleick

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UNITED STATES NAVAL POSTGRADUATE SCHOOL
Monterey, California

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ABSTRACT:

This paper presents an elementary treatment of the first order differential effects of the earth's oblateness on a close satellite using the simple notion of a varied orbit. The usual result of this approach is that the radial variation is unbounded for infinite time. The standard method of celestial mechanics for removing this difficulty is to analyze the perturbed orbit as an ellipse whose shape and space orientation are functions of time. It is shown here that the secular term may be avoided more simply by relating points on the varied and unvaried orbits by a type of radial velocity correspondence instead of the usual time correspondence, and by making the varied orbit osculate at a latus rectum chord end point of the unvaried orbit.

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Oblateness-Perturbed Orbits by Velocity-Correspondence Variations

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(Received 1966)

ABSTRACT

This paper presents an elementary treatment of the first order differential effects of the earth's oblateness on a close satellite using the simple notion of a varied orbit. The usual result of this approach is that the radial variation contains a secular term which is unbounded for infinite time. The standard method of celestial mechanics for removing this difficulty is to analyse the perturbed orbit as an ellipse whose shape and space orientation are functions of time. It is shown here that the secular term may be avoided more simply by relating points on the varied and unvaried orbits by a type of radial velocity correspondence instead of the usual time correspondence, and by making the varied orbit osculate at a latus rectum chord end point of the unvaried orbit.

INTRODUCTION

Let the origin O of a non-rotating rectangular coordinate frame Oxyz be at the center of the earth with the Oz axis pointing toward the North pole. Let the unit mass earth satellite at S in Fig. 1 have the coordinates x,y,z. Adopt the equatorial radius of the earth as a unit of length. If it be assumed that the earth's mass distribution has axial symmetry about the Oz polar axis, the gravitational potential V of the satellite may be expanded[1] in a series of zonal harmonics. Retention of the two lowest order terms in this series gives

$$V=-\mu[\rho^{-1}+\epsilon\rho^{-3}(1-3z^2\rho^{-2})] \tag{1}$$

where μ is the gravitational constant and $\rho^2=x^2+y^2+z^2$. A spherically symmetric distribution of mass corresponds to $\epsilon=0$ in (1). Lecar, Sorenson and Eckels[2] have determined that the earth's oblate-spheroidal shape gives rise to the value $\epsilon=0.541\times10^{-3}$ squared units of length. The problem is to find the variations of an unperturbed Keplerian elliptical orbit, corresponding to $\epsilon=0$, caused by the small ϵ perturbation term in (1).

Let z cosI-y sinI=0 be the equation of the unperturbed orbit plane Oxv of Fig. 1, passing through the Ox axis with inclination angle I to the earth's equatorial plane. Introduce the cylindrical coordinates r,ξ,w shown in Fig. 1 with the Ow cylinder axis perpendicular to the Oxv plane. The equations

$$\begin{aligned} x&=r\cos\xi, \\ y&=r\cos I\sin\xi-w\sin I, \\ z&=r\sin I\sin\xi+w\cos I, \end{aligned} \tag{2}$$

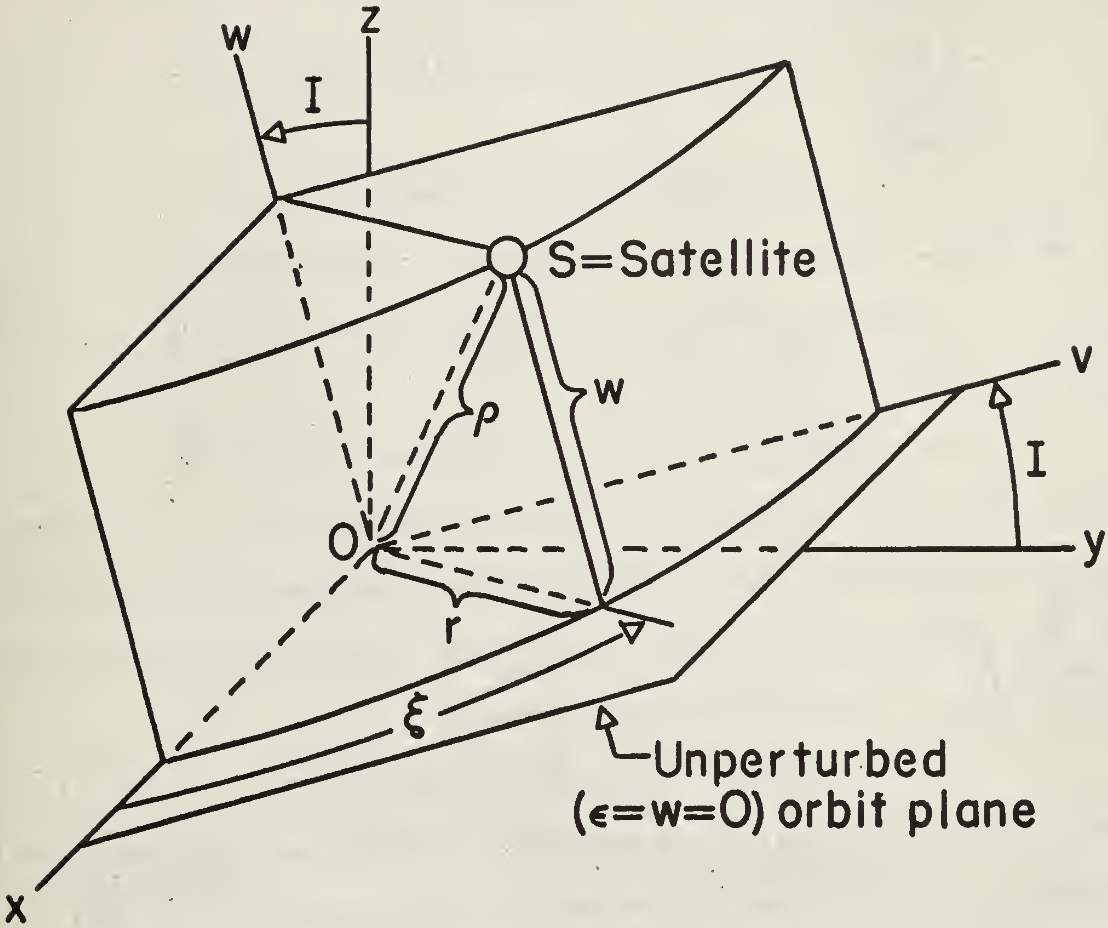

FIG. 1. Cylindrical coordinates based on the Oxv plane of the unperturbed orbit.

relate the x,y,z and r,ξ,w coordinates. The kinetic energy T and potential energy V of the unit mass perturbed satellite at S are then

$$T=\tfrac{1}{2}(\dot{r}^2+r^2\dot{\xi}^2+\dot{w}^2) \tag{3}$$

and

$$V=-\mu[\rho^{-1}+\epsilon\rho^{-3}-3\epsilon(r\ \sin I\sin\xi+w\ \cos I)^2\rho^{-5}] \tag{4}$$

where $\rho^2=r^2+w^2$.

VARIATION OUT OF ORBIT PLANE

The Lagrangian function T-V may be constructed from (3) and (4), and the equation of satellite motion in the w coordinate found to be

$$\ddot{w}+\mu\rho^{-3}w=-\mu\epsilon\frac{\partial}{\partial w}[\rho^{-3}-3(r\ \sin I\sin\xi+w\ \cos I)^2\rho^{-5}]. \tag{5}$$

Using the fact that w=0 in the unperturbed orbit, one finds from (5) that the first order time-correspondence variation δw of the perturbed satellite out of the unperturbed orbit plane Oxv of Fig. 1 satisfies the equation

$$\delta\ddot{w}+\mu r^{-3}\delta w=-3\epsilon\mu r^{-4}\sin 2I\sin\xi. \tag{6}$$

Note that the operators δ and d/dt commute in (6) since δw is defined to be a time-correspondence variation; i.e., δw, w, r and ξ are all defined for the same value of time. Let $\xi=\omega$ be the argument of the perigee P of the unperturbed elliptical orbit of Fig. 2 measured from the ascending node N on the Ox axis. Let $\xi=\lambda=\omega-\frac{1}{2}\pi$ be the argument of the latus rectum chord end point L which precedes perigee. If the satellite position argument α is measured from L the equation of the unperturbed Keplerian orbit may be written as

$$r=p/(1+e\ \sin\alpha) \tag{7}$$

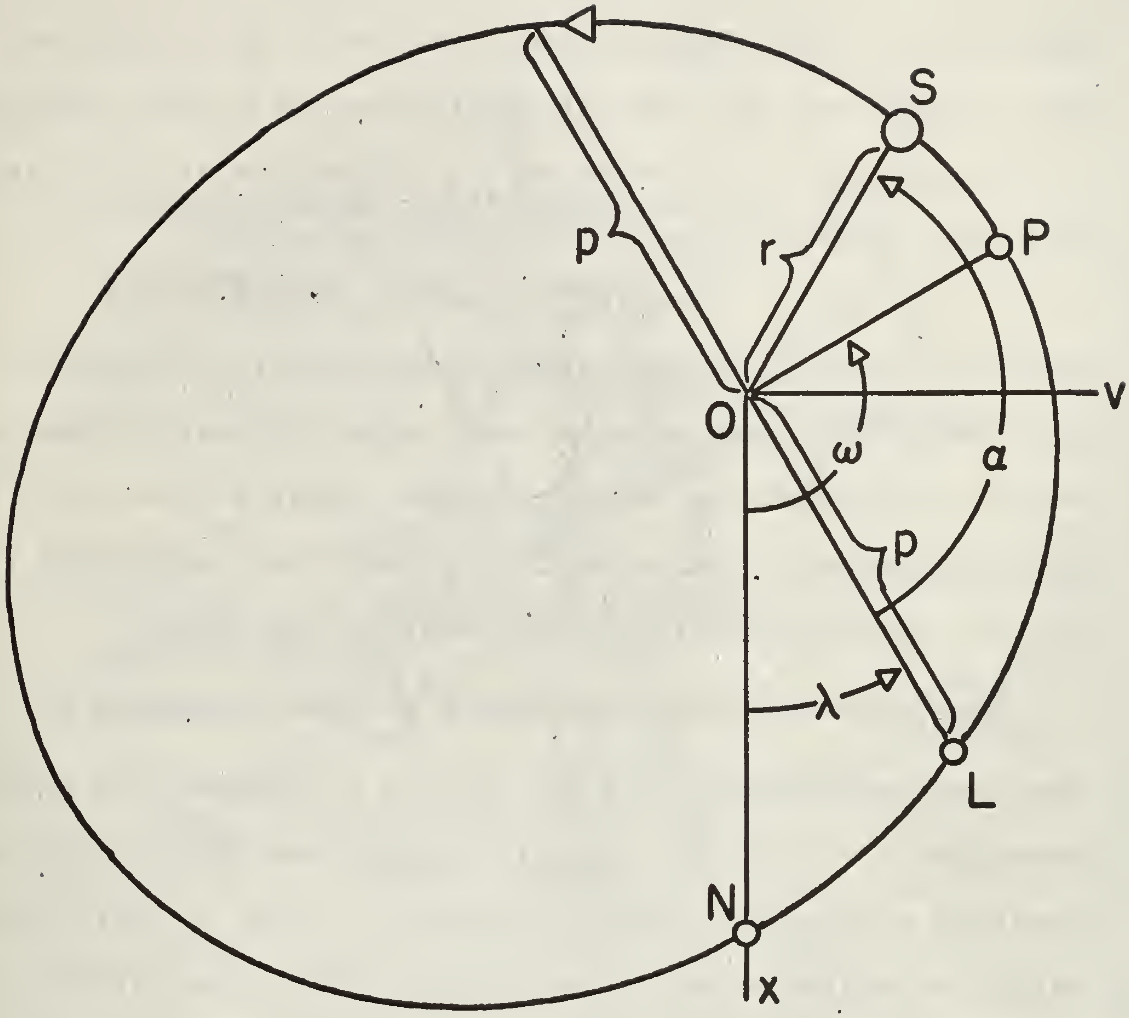

FIG. 2. Unperturbed Keplerian orbit.

where e is the orbit eccentricity and p is the semi latus rectum. The constant angular momentum of the unit mass satellite in this orbit is

$$h=r^2\dot{\alpha}=(p\mu)^{\frac{1}{2}}=2\pi ab/T \tag{8}$$

where a,b are the semiprincipal axes and T is the period of the orbit. Equations (7) and (8) may be used to replace time by α as the independent variable and to replace δw by $r^{-1}\delta w$ as the dependent variable in (6) so that it reduces to

$$(r^{-1}\delta w)''+r^{-1}\delta w=-3\epsilon p^{-2}\sin 2I(1+e\ \sin\alpha)\sin(\lambda+\alpha) \tag{9}$$

where the primes indicate differentiation with respect to α. The simplicity of presentation of this paper depends on making the varied orbit osculate the unperturbed orbit at the latus rectum chord end point L. Equation (9) is therefore integrated with the initial conditions $\delta w(0)=\delta w'(0)=0$ at $\alpha=0$ to obtain

$$\frac{2p^2\delta w}{\epsilon r\sin 2I}=3\alpha\cos\xi-\cos\lambda(3\sin\alpha+8e\sin^4\tfrac{1}{2}\alpha)-4e\sin\lambda\sin\alpha\sin^2\tfrac{1}{2}\alpha. \tag{10}$$

The term proportional to α in (10) is a secular term which is unbounded for infinite time. To remove this undesirable term consider a plane through the origin O in Fig. 3, which is initially coincident with the Oxv plane of the unperturbed orbit at $\alpha=0$, and which precesses around the Oz polar axis to the west at the differential angular rate $\delta\dot{\varphi}$ always maintaining a constant inclination angle I with the Oxy equatorial plane. The equation of this precessing plane is

$$-x\sin I\sin\delta\varphi-y\sin I\cos\delta\varphi+z\cos I=0 \tag{11}$$

where $\delta\varphi=0$ when $\alpha=0$. The distance δw^* of the perturbed satellite from the precessing plane may be found by substituting its coordinates $x+\delta x, y+\delta y, z+\delta z$ from (2) into the left member of (11),

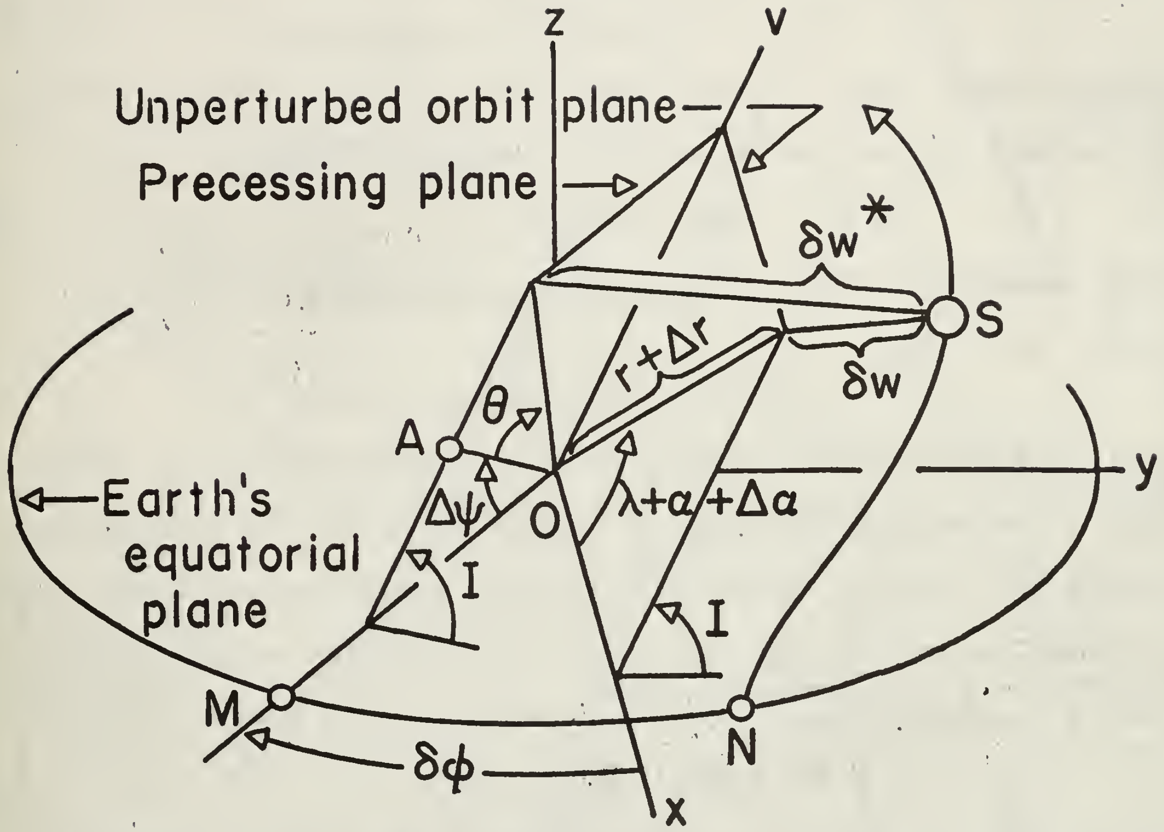

FIG. 3. Satellite variations δw out of the unperturbed orbit plane and δw* out of the precessing perturbed orbit plane.

noting that $w=0$. Using the approximations $\sin\delta\varphi \approx \delta\varphi$ and $\cos\delta\varphi \approx 1$, and neglecting other second order differentials, one obtains

$$\delta w^* = \delta w - r\sin I\cos\xi\,\delta\varphi. \tag{12}$$

On comparing (12) with (10) it is seen that the distance δw^* is periodic and contains no secular term if

$$\delta\varphi = 3\varepsilon\alpha\cos I/p^2. \tag{13}$$

We conclude that the mean plane (11) of the perturbed orbit precesses around the Oz axis at the angular rate

$$\delta\dot{\varphi} = 3\varepsilon\dot{\alpha}\cos I/p^2, \tag{14}$$

and that the satellite has periodic excursions δw^* from this plane given by

$$\delta w^* = \delta w - [3\varepsilon\alpha r\sin 2I\cos(\lambda+\alpha)/2p^2]. \tag{15}$$

The relation between $\delta w^*/r$ and α is shown graphically in Fig. 4 for a few values of λ and e. The trace OM of the precessing plane (11) in the equatorial Oxy plane has the equation $y=-x\delta\varphi$, so that the ascending node N of the perturbed orbit in Fig. 3 moves westward along the equator through the angle

$$\delta\varphi(2\pi) = 6\pi\varepsilon\cos I/p^2 \tag{16}$$

on successive transits of the equator. This nodal regression from N_1 to N_2 is shown in Fig. 5. If the node N_1 corresponds to α on the unperturbed orbit one must not conclude that the next ascending node N_2 corresponds to $\alpha+2\pi$ since the variations Δr and $\Delta\alpha$ in the unperturbed orbit plane of Fig. 3 have not yet been considered. It is shown later in (46) that the point Q of Fig. 5, corresponding to $\alpha+2\pi$, is at a positive or negative differential angle $\Delta\psi(2\pi)$ of order ε from N_2 thus justifying the assignment of $\delta\varphi(2\pi)$ of (16) to the angle N_1ON_2.

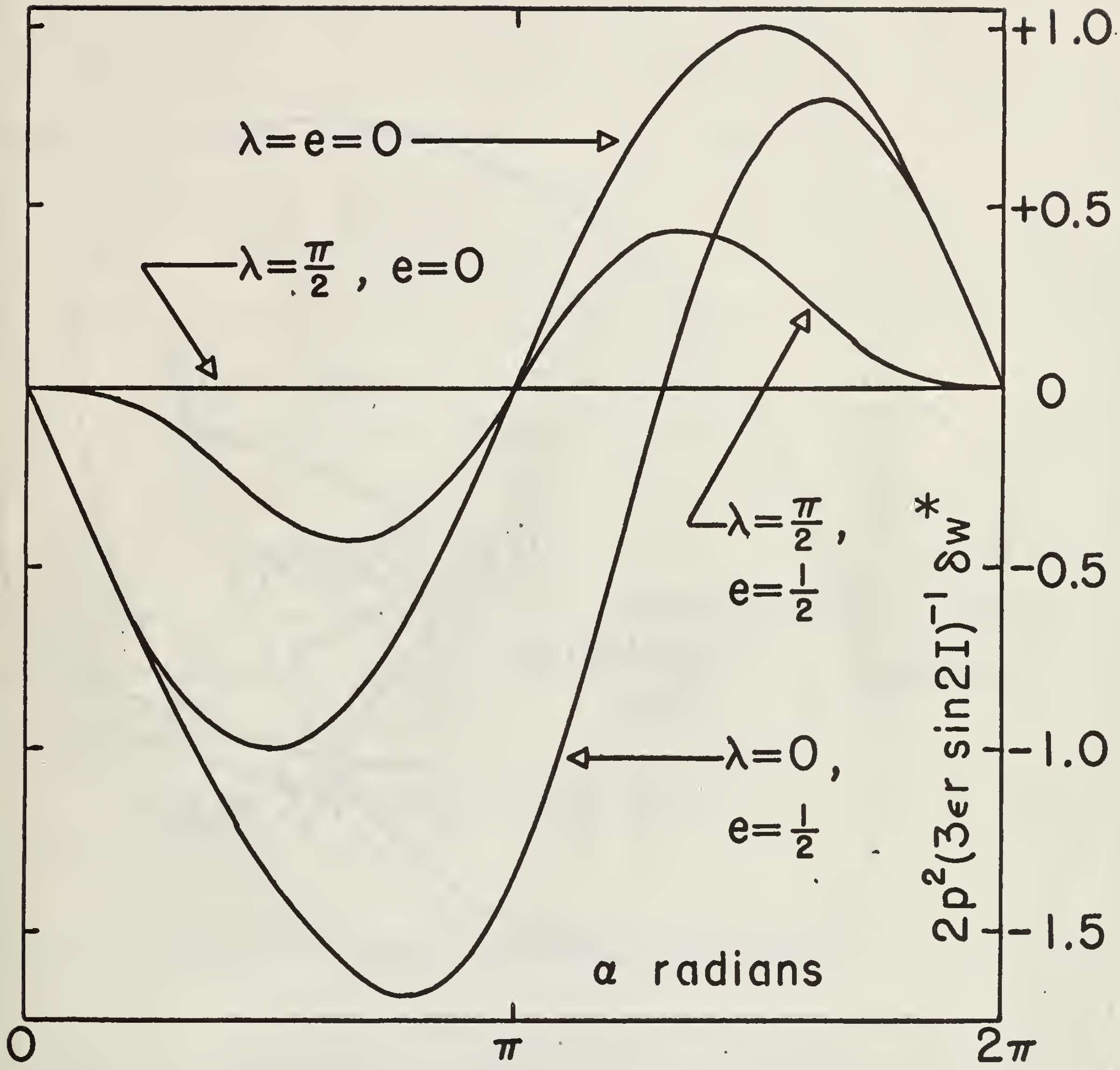

FIG. 4. Relation between $r^{-1}\delta w^*$ and α.

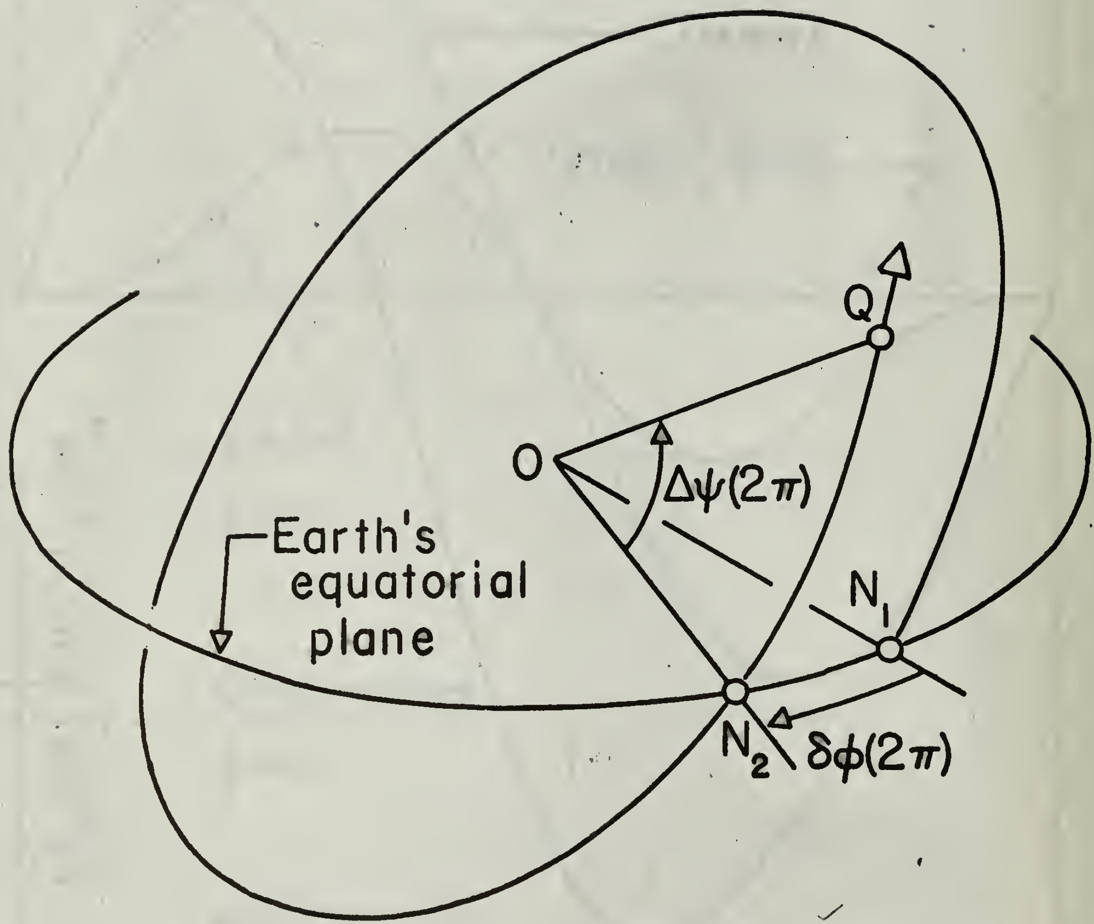

FIG. 5. Regression $\delta\varphi(2\pi)$ of the ascending node in the equatorial plane and precession $\Delta\psi(2\pi)$ of the perturbed orbit.

VARIATIONS IN THE ORBIT PLANE

The gravitational force on the satellite has no moment around the earth's polar Oz axis because of the axial symmetry of the earth's mass distribution assumed in (1). Hence the moment of momentum of the perturbed satellite around the Oz axis is conserved. The moment of momentum vector is found most easily by first computing its components in the Oxvw frame of Fig. 1, and then transforming them to the Oxyz frame. It follows from (8) that conservation of the Oz component of moment of momentum of the unit mass perturbed satellite requires

$$[(w\dot{r}-r\dot{w})\cos\xi-wr\dot{\xi}\sin\xi]\sin I+r^2\dot{\xi}\cos I=h\cos I \qquad (17)$$

where it is assumed that the orbit osculates the unperturbed orbit (7) at $\xi=\lambda$, i.e. $\alpha=0$. On expressing (17) in terms of first order time-correspondence variations from the $w=\dot{w}=0$ unperturbed orbit, and using (8) to replace time by α as the independent variable, one obtains

$$2r^{-1}\delta r+\delta\alpha'=\tan I\cos^2\xi\frac{d}{d\alpha}\left(\frac{\delta w}{r\cos\xi}\right)=F_1 \qquad (18)$$

where, using (10),

$$\frac{-p^2F_1}{\epsilon\sin^2 I}=2e[\sin^2\tfrac{1}{2}\alpha(2\cos\alpha+\cos 2\alpha)\sin 2\lambda+\sin^3\alpha\cos 2\lambda]+3\sin\alpha\sin(2\lambda+\alpha). \qquad (19)$$

From (3) and (4) it is seen that conservation of the total energy T+V of the unit mass perturbed satellite requires

$$\tfrac{1}{2}(\dot{r}^2+r^2\dot{\alpha}^2+\dot{w}^2)-\mu[\rho^{-1}+\epsilon\rho^{-3}-3\epsilon(r\sin I\sin\xi+w\cos I)^2\rho^{-5}] \qquad (20)$$
$$=E-\mu\epsilon p^{-3}(1-3\sin^2 I\sin^2\lambda)$$

where it is assumed that the orbit osculates the unperturbed orbit (7) of total energy E at $\xi=\lambda$, i.e. $\alpha=0$. On expressing (20) in terms of first order time-correspondence variations from the

$w=\dot{w}=0$ unperturbed orbit one obtains

$$\dot{r}\delta\dot{r}+(r\dot{\alpha}^2+\mu r^{-2})\delta r+r^2\dot{\alpha}\delta\dot{\alpha}=\mu\epsilon[r^{-3}-p^{-3}+3\sin^2 I(p^{-3}\sin^2\lambda-r^{-3}\sin^2\xi)]. \quad (21)$$

On using (8) to replace time by α as the independent variable (21) becomes

$$r^{-2}r'\delta r'+(r^{-1}+p^{-1})\delta r+\delta\alpha'=F_2 \quad (22)$$

where

$$F_2=\epsilon[1-r^3p^{-3}-3\sin^2 I(\sin^2\xi-r^3p^{-3}\sin^2\lambda)]/pr. \quad (23)$$

If the rather complicated Eqs. (18) and (22) are integrated as they stand it will be found that δr contains a secular term, exhibited later in (53), which is unbounded for infinite values of α. The secular nature of δr raises doubt about its ability to give a sufficiently accurate description of the radial perturbation when α is more than a few multiples of 2π, and makes for unwieldy mathematics in the determination of $\delta\alpha$. Sterne[3] has removed the radial secular term in the present problem by a standard method of celestial mechanics which analyzes the perturbed orbit as an ellipse whose shape and space orientation are set up from the beginning to be functions of time. The secular term difficulty is avoided here in a simpler fashion by relating points on the varied and unvaried orbits by a type of radial velocity correspondence instead of the usual time correspondence implied by the use of the symbols δr and $\delta\alpha$. We assume that the differential change in radial velocity $\Delta\dot{r}$ from a point on the unperturbed orbit to a corresponding point on the varied orbit is

$$\Delta\dot{r}=\delta\dot{r}+\ddot{r}\Delta t=\ddot{r}F(\alpha)/\dot{r} \quad (24)$$

where Δt is the corresponding time difference. The function $F(\alpha)$, of order ϵ, is chosen later to avoid a singularity. The function F will also be chosen so that exact radial velocity correspondence, $\Delta\dot{r}=0$, is achieved at perigee and apogee, thus facilitating a discussion of the precession of the apsidal line of the perturbed orbit. The symbol Δ, used in this section to denote the velocity-correspondence variation operator, does not commute with the operator d/dt since the correspondence of points defined in (24) is not based on equal values of time. The velocity-correspondence variations in r, α and $\dot{\alpha}$, consistent with (24), are

$$\Delta r=\delta r+\dot{r}\Delta t, \tag{25}$$

$$\Delta\alpha=\delta\alpha+\dot{\alpha}\Delta t, \tag{26}$$

and

$$\Delta\dot{\alpha}=\delta\dot{\alpha}+\ddot{\alpha}\Delta t. \tag{27}$$

Note that Δt may be found from (25) after $\Delta r-\delta r$ has been determined in (53). Note also that $\Delta w=\delta w+\dot{w}\Delta t=\delta w$ and $\Delta w^*=\delta w^*$ since $\dot{w}=0$ on the unperturbed orbit.

Equations (7), (24), (25) and (26) yield

$$\Delta r=\delta r+[r'\delta r'/(r^2p^{-1}-r)]+F \tag{28}$$

and

$$\Delta\alpha=\delta\alpha+[\delta r'/(r^2p^{-1}-r)]+F/r'. \tag{29}$$

Subtraction of (18) from (22) yields

$$r'r^{-2}\delta r'+(p^{-1}-r^{-1})\delta r=F_2-F_1. \tag{30}$$

Substitution of (30) into (28) yields

$$\Delta r=F+[(F_2-F_1)/(p^{-1}-r^{-1})]. \tag{31}$$

Elimination of $\delta r'$ from (28) and (29) yields

$$\Delta\alpha-\delta\alpha=(\Delta r-\delta r)/r'. \tag{32}$$

Differentiation of (32), together with the use of (7), (18) and (28), yields

$$\frac{d}{d\alpha}\Delta\alpha=F_1+(r^2/r')[Fr^{-2}\tan\alpha+\frac{d}{d\alpha}(\Delta r/r^2)]. \tag{33}$$

Use of (7), (19) and (23) gives

$$\frac{p(F_2-F_1)}{\epsilon(p^{-1}-r^{-1})}=\sin^2 I[\sin^2\alpha\cos 2\lambda+(\sin\alpha+2\sin^2\tfrac{1}{2}\alpha\cos^2\alpha\csc\alpha)\sin 2\lambda] \tag{34}$$
$$+(3\sin^2 I\sin^2\lambda-1)[1+(r/p)+(r/p)^2]$$

which is singular at $\alpha=\pi$. To avoid the same singularity in Δr choose F of (31) to be

$$F=\epsilon\sin^2 I\sin 2\lambda(\cos\alpha-1)\cos^2\alpha/p\sin\alpha. \tag{35}$$

The differential change in radial velocity $\Delta\dot{r}$ is then found from (24), with the aid of (7), (8) and (35), to be

$$\Delta\dot{r}=\epsilon\dot{\alpha}p^{-1}\sin^2 I\sin 2\lambda(1-\cos\alpha)\cos\alpha, \tag{36}$$

giving exact radial velocity correspondence at osculation, perigee and apogee. The differential change Δr is found from (31), (34) and (35) to be

$$\Delta r=\epsilon p^{-1}\{\sin^2 I(\sin^2\alpha\cos 2\lambda+\sin\alpha\sin 2\lambda) \tag{37}$$
$$-(1-3\sin^2 I\sin^2\lambda)[1+(r/p)+(r/p)^2]\},$$

giving

$$\Delta r(0)=3\epsilon p^{-1}(3\sin^2 I\sin^2\lambda-1). \tag{38}$$

The variation $\Delta\alpha(0)$ at osculation, where $\delta r(0)=\delta\alpha(0)=0$, is found from (7), (32) and (38) to be

$$\Delta\alpha(0)=3\epsilon(1-3\sin^2 I\sin^2\lambda)/p^2 e. \tag{39}$$

Equation (39) shows that the notion of a velocity-correspondence variation cannot be used for e=0 as one would expect. An integration of (33) with the aid of (7), (19), (35) and (37) yields

$$\Delta\alpha-\Delta\alpha(0)= \tag{40}$$
$$\tfrac{1}{2}\epsilon p^{-2}\{[(2-3\sin^2 I)(3\alpha+4e\sin^2\tfrac{1}{2}\alpha)+\sin^2 I\sin\alpha(9\cos\alpha\cos 2\lambda-2e^{-1}\sin 2\lambda)]+$$
$$2\sin^2 I\sin^2\tfrac{1}{2}\alpha[2e(\cos 2\alpha+2\cos\alpha-2e^{-2})\cos 2\lambda-(9+5\cos\alpha+4e\sin\alpha+e\sin 2\alpha)\sin 2\lambda]\},$$

periodic in α except for the secular term

$$3\epsilon\alpha(2-3\sin^2 I)/2p^2. \tag{41}$$

The change in $\alpha+\Delta\alpha$ from $\alpha=\frac{1}{2}\pi$ to $\alpha=3\pi/2$ may be calculated from (40) to be

$$\pi+\tfrac{1}{2}\epsilon p^{-2}[3\pi(2-3\sin^2 I)-4(2e+e^{-1})\sin^2 I\sin 2\lambda] \tag{42}$$

so that the apsidal line of the perturbed orbit does not pass through the earth's center unless the last term of (42) happens to be zero. The complex variable describing the projection of the perturbed satellite orbit in the Oxv plane of Fig. 3 is

$$(r+\Delta r)\exp[i(\lambda+\alpha+\Delta\alpha)] \tag{43}$$

if the Ox axis is taken as the axis of the real part. Let OM of Fig. 3 be the trace of the precessing perturbed orbit plane (11) in the earth's equatorial plane. Let $\Delta\psi(\alpha)$ of Fig. 3 be a differential angle measured from the trace OM to a moving reference radial line OA in the precessing plane. Then the complex variable describing the projection of the perturbed satellite orbit in the precessing plane relative to the reference radial line OA as the axis of real numbers is

$$(r+\Delta r)\exp i\theta \tag{44}$$

where the angle θ of Fig. 3 is

$$\theta=\lambda+\alpha+\Delta\alpha+\delta\varphi\cos I-\Delta\psi. \tag{45}$$

On substituting $\Delta\alpha$ from (40) and $\delta\varphi$ from (13) into (45) it is found that the perturbed orbit projection (44) is periodic relative to the moving line OA and contains no secular term if

$$\Delta\psi=3\epsilon\alpha(4-5\sin^2 I)/2p^2. \tag{46}$$

Note that the rate of precession

$$(d/dt)\Delta\psi=3\epsilon\dot{\alpha}(4-5\sin^2 I)/2p^2 \tag{47}$$

of the periodic perturbed orbit (44 to 46) relative to the trace OM is direct or retrograde according as $\sin I$ is less than or greater than $(4/5)^{\frac{1}{2}}$. The $\arcsin(4/5)^{\frac{1}{2}}$ is 63°26'. The precession $\Delta\psi(2\pi)=3\epsilon\pi(4-5\sin^2 I)/p^2$ of the periodic perturbed orbit in the precessing plane between the times of two successive ascending nodes N_1 and N_2 has been pictured earlier in Fig. 5.

VARIATION IN NODAL PERIOD

The change from the period T of the unperturbed orbit to the nodal period $T+\Delta T$ between the two successive nodes N_1 and N_2 of Fig. 5 can be calculated only in terms of time-correspondence variations. Let Δt represent the differential time changes from the points $z=\delta z$ at $\alpha=-\lambda$ and $\alpha=-\lambda+2\pi$ on the varied orbit to the corresponding nodes N_1 and N_2 at $z=0$ on the varied orbit, so that

$$\delta z+\dot{z}\Delta t=0. \tag{48}$$

The differential change in the nodal period is then

$$\begin{aligned}\Delta T&=\Delta t(-\lambda+2\pi)-\Delta t(-\lambda) \\ &=[\delta z(-\lambda)-\delta z(-\lambda+2\pi)]/\dot{z}(-\lambda).\end{aligned} \tag{49}$$

Substitution of (2) into (49) gives

$$\dot{\alpha}(-\lambda)\Delta T=[\delta\alpha+r^{-1}\cot I\delta w]_{-\lambda+2\pi}^{-\lambda}. \tag{50}$$

Equation (10) yields

$$r^{-1}\cot I\delta w\Big|_{-\lambda+2\pi}^{-\lambda}=-6\epsilon\pi p^{-2}\cos^2 I. \tag{51}$$

The calculation of the time-correspondence variation $\delta\alpha$, evaded thus far, is now required. Eliminate F_2-F_1 from (30) and (31) to obtain

$$\frac{d}{d\alpha}[(\delta r-\Delta r)\sec\alpha]=-(F\tan\alpha+\frac{d\Delta r}{d\alpha})\sec\alpha. \tag{52}$$

Integrate (52) with the aid of (37), using (38) and $\delta r(0)=0$

as initial conditions at osculation, to obtain

$$(\delta r-\Delta r)\sec\alpha = \tag{53}$$

$$\frac{\epsilon e(3\sin^2 I\sin^2\lambda-1)}{p(1-e^2)^2}\{e\cos\alpha[(4-e^2)\frac{r}{p}+(1-e^2)\frac{r^2}{p^2}]-e(5-2e^2)-\frac{3}{e}(1-e^2)^2$$

$$+3(1-e^2)^{-\frac{1}{2}}[\alpha+\sum_{n=1}^{\infty}\frac{a_n\sin n\alpha+b_n(1-\cos n\alpha)}{n}]\}$$

$$-\epsilon p^{-1}\sin^2 I[4\sin^2\tfrac{1}{2}\alpha\cos 2\lambda+\sin\alpha\sin 2\lambda]$$

where a_n and b_n are coefficients in the Fourier series

$$(1-e^2)^{\frac{1}{2}}(1+e\sin\alpha)^{-1}=\tfrac{1}{2}a_0+\sum_{n=1}^{\infty}(a_n\cos n\alpha+b_n\sin n\alpha) \tag{54}$$

and are defined by the complex number

$$a_n+ib_n=2(ie)^{-n}[1-(1-e^2)^{\frac{1}{2}}]^n. \tag{55}$$

Comparison of (37) with (53) shows that the time-correspondence variation δr contains a secular term proportional to $\alpha\cos\alpha$, and also shows the simplicity of the velocity-correspondence variation Δr versus the complicated δr. Use of (32), (40) and (53) gives

$$\delta\alpha\Big|_{-\lambda+2\pi}^{-\lambda}=6\pi\epsilon p^{-2}[(3\sin^2 I\sin^2\lambda-1)(1-e^2)^{-5/2}+\tfrac{3}{2}\sin^2 I-1]. \tag{56}$$

Use of (8), (50), (51) and (56) then shows that the fractional change in the nodal period is

$$\frac{\Delta T}{T}=\frac{3\epsilon}{ab(1-e\sin\lambda)^2}[\frac{3\sin^2 I\sin^2\lambda-1}{(1-e^2)^{5/2}}+\tfrac{5}{2}\sin^2 I-2]. \tag{57}$$

CONCLUDING REMARKS

The notion of varying an orbit by velocity-correspondence variations instead of time-correspondence variations has been shown to lead to a simple discussion of first order oblateness effects. The variations obtained are less complicated than those derived by an analysis[3] in terms of an ellipse whose shape and space orientation are functions of time, and have a more direct physical interpretation. The analysis can be carried through for a general point of osculation at the price of an increase in mathematical complexity.

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[3] Reference 1, pp. 100-103, 116-127.

FIGURE CAPTIONS

FIG. 1. Cylindrical coordinates based on the Oxv plane of the unperturbed orbit.

FIG. 2. Unperturbed Keplerian orbit.

FIG. 3. Satellite variations δw out of the unperturbed orbit plane and δw^* out of the precessing perturbed orbit plane.

FIG. 4. Relation between $r^{-1}\delta w^*$ and α.

FIG. 5. Regression $\delta\varphi(2\pi)$ of the ascending node in the equatorial plane and precession $\Delta\psi(2\pi)$ of the perturbed orbit.

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This paper presents an elementary treatment of the first order differential effects of the earth's oblateness on a close satellite using the simple notion of a varied orbit. The usual result of this approach is that the radial variation is unbounded for infinite time. The standard method of celestial mechanics for removing this difficulty is to analyze the perturbed orbit as an ellipse whose shape and space orientation are functions of time. It is shown here that the secular term may be avoided more simply by relating points on the varied and unvaried orbits by a type of radial velocity correspondence instead of the usual time correspondence, and by making the varied orbit osculate at a latus rectum chord end point of the unvaried orbit.

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