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The design, fabrication and analysis of a torsional-vibration inducer

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THE DESIGN, FABRICATION AND ANALYSIS OF A TORSIONAL-VIBRATION INDUCER

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PREFACE

The problem of developing a torsional vibration inducing unit arose from a class room discussion during which Dr. E. K. Gatcombe pointed out the unavailability of such a unit and the effort that had been expended by certain manufacturers in attempts at the development of such a unit.

In 1944 while attached to the staff of Cornell University Messrs. E. K. Gatcombe and W. A. Johnson were engaged by the Scintilla Magneto Company to develop a machine to simulate the vibration encountered in a magneto armature while revolving in its magnetic field. The underlying reason for this request was that the Company had experienced failure of armature shafts, and had been unable to determine the exact cause of these failures by means of testing equipment then available. An electrical machine was devised which, it was hoped, would simulate the required condition but for various reasons was found unsatisfactory. Attention was turned to a mechanical unit but due to more pressing problems of production during the war, this mechanical unit was not produced.

The authors wish to express their appreciation to personnel of the Naval Gun Factory and the Engineering Experiment Station who made possible the fabrication of the machine herein described, to the machine shop personnel of the U. S. Naval Postgraduate School who contributed their skill and constructive suggestions and especially to Dr. E.

K. Gatombe for his guidance and valuable suggestions in the preparation of this paper.

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TABLE OF SYMBOLS

C - a constant

F_c - centrifugal force (lb)

G - modulus of rigidity of test shaft

I_g - centroidal moment of inertia (slug-in^2)

I_b - I_g of test shaft

I_{fb} - I_g of test shaft flange

I_{fg} - I_g of inducer shaft flanges

I_{fm} - I_g of magnet flange

I_p - I_g of pinions

I_r - I_g of inducer shaft

I_s - I_g of inducer spider

I_w - I_g of unbalanced weights

J - polar moment of inertia (in^4)

K.E. - kinetic energy (lb-in)

l - length of test shaft (in)

M - mass (slugs)

M_t - torque (in-lb)

r - eccentricity of unbalanced weight (in)

R - distance from center of shaft to center of pinion(in)

S_s - shear stress (lb/sq in)

V - linear velocity (in/sec)

W - weight (lb)

w - unbalanced weight (lb)

α - angular acceleration (rad/sec/sec)

θ - angular displacement of spider from vertical (degrees)

ϕ - angle through which a radius of a right section, at end of test shaft, is turned (degrees)

ω - angular velocity (rad/sec)

I. INTRODUCTION

The problem of unbalanced forces and vibratory motion induced by them is present in every piece of rotating machinery built. In the present stage of machinery development where rotative speeds are becoming greater it becomes increasingly more important for designers to understand vibration, its cause and effect and methods for eliminating or controlling vibrations. In order to carry out the study of vibratory motion so necessary to the above requirements, investigators have developed many and varied devices for simulating the type of vibration to be studied under controlled and known conditions.

The most usual method for studying the effects of torsional vibration involves a test specimen mounted with one end rigidly held in a solid structure while the other end is oscillated in torsion by means of some device capable of applying the desired torque.

In the case of the shaft of a magneto, an electric motor or the crankshaft of an internal combustion engine, the shaft is in a state of rotation while oscillating torques are superimposed upon the rotation. It is generally assumed that the rotative motion may be disregarded and test equipment such as described in the preceding paragraph applied to one of these shafts in order to determine the effects of the torsional vibration. This assumption is probably true in many cases; however in the case of magneto shaft failure encountered by the Scintilla Magneto Co. it was found that data obtained by the ordinary test methods could not be correlated with the actual conditions.

In 1944, two consulting engineers, E. K. Gatcombe and W. A. Johnson were retained by the Scintilla Company for the purpose of developing a test machine whose purpose would be to simulate the actual conditions obtained while the magneto was in service. They attempted to simulate service conditions by applying magnetic torque but found that the time required for build-up and collapse of the magnetic field was of such magnitude that the condition of loading was not sufficiently similar to that required, and hence was not considered entirely satisfactory. Attention was then given to some form of mechanical device whereby the required motion might be obtained. The feasability of a mechanical unit was studied but the unit was not actually constructed.

The authors consider that such a machine should be of value to designers in that it would provide means for studying the effects of torsional vibration under service conditions in almost any kind of rotating machinery. For purposes of test, a crankshaft or geared system may be reduced to a single equivalent shaft (1)* and where large machinery is involved, scale models may be substituted and the results evaluated in terms of the actual machine by dimensional comparison.

It is believed that such a device will provide means for investigating the effects of torsional vibration in

*Parenthesized numbers refer to Bibliography.

conjunction with resonant speeds.

It is considered by the authors that by suitable modifications in design, a device of this basic type could be applied as a vibration absorber on any rotating engine, even though vibrations in all phases were not equal. Present absorbers act to absorb only a single, usually the predominate, vibration. The unit under consideration here could be made to damp out different torques at particular phases of its rotation by proper combinations of the unbalanced weights and their relative phase positions.

A further practical application of this unit would be in the field of the analysis of the tuned pendulum type of vibration absorber. By proper modification of the unit, any reasonable variable torque could be induced in rotating equipment.

The device could also be employed to advantage in the fatigue testing of shafts which were subjected to a pulsating torque in actual service.

The project of this paper is to design, fabricate and analyse this machine. Due to time limitations, it must be left to future investigators to accomplish refinements in instrumentation and develop further uses for the machine herein presented.

II. DESIGN CONSIDERATIONS

It was assumed from conditions existing in motors and engines that the most general case of torque application to rotating machinery would be that wherein the torque is applied in the form of a harmonic vibration. Accordingly, an effort was made to create this effect.

In determining the amplitude of the torque harmonic it was assumed that sufficient torque should be applied to stress a one-half inch, mild-steel shaft to its yield strength.

Determination of the frequency of oscillation rests upon two considerations. Firstly, it is desirable to cause a torsional oscillation more than once during each revolution. It was decided to make this factor four since this might best simulate conditions in an engine having four crank throws or any multiple of this number. A similar argument holds for an electric motor shaft where the number of poles is often some multiple of four. Secondly, the nominal speed of rotation is arbitrarily set at 1750 rpm. Reasons for this decision are: (a) the most easily obtainable driving motor can reasonably be expected to operate at or near synchronous speed of 1800 rpm; (b) this speed may be considered predominant in the case of electric motors and would be applicable to testing of their shafts at "normal" speed; (c) this speed is well within the range of speeds applicable to high speed internal combustion engines. Where it might be desirable or advisable to operate at reduced speed this could be arranged by some form of speed control applied to the driving motor.

Several methods of obtaining the desired oscillation were considered. The best and most easily applied method appeared to be one which takes advantage of the centrifugal force generated by an unbalanced rotating mass.

An effort was made to keep the size of the vibration inducer as small as possible, an eight inch cube being considered the limiting volume. The size consideration was essential from the viewpoint of obtaining materials for fabrication, and desirable from the viewpoint of installation and convenience in use.

III. DESIGN OF TORSIONAL VIBRATION INDUCER

Several basic ideas were investigated and many arrangements sketched before the rough outline of the final machine was decided upon. Most of the early designs required the use of two independent sources of rotating power. The use of an internal gear eliminated this necessity and the sketch shown in fig. I was accepted as the starting point. The ring gear A is supported externally and provides a mounting for the machine. Pinion gears B are carried in bearings in the spider C in such manner as to engage the ring gear. The spider is keyed to and driven by the central shaft D. Each pinion is loaded eccentrically by a known weight at a known eccentricity. The centrifugal force generated by each unbalanced pinion may be calculated from the usual relationship:

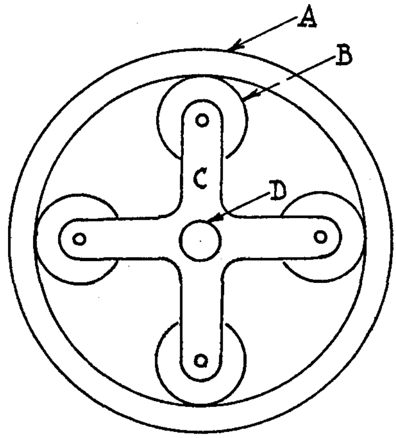

Figure I

$$F_c = M r \omega^2 = \frac{W}{g} r \omega^2$$

where: F_c = centrifugal force (lb)

W = unbalanced weight (lbs)

g = 386.4 in/sec/sec

ω = angular velocity (rad/sec)

r = eccentricity (in)

The amount of total torque output desired from the machine is determined from that torque necessary to stress a one-half inch bar of mild steel to its yield point in shear. The following relation is used: (4)

$$M_t = \frac{S_s J}{r} = \frac{S_s \pi d^3}{16}$$

Using 20,000 lbs. per sq. in. as the maximum shear stress allowable, M_t is found to be 490 lb.-in. This figure is the torque that must be superimposed on the constant rotation of the shaft.

From fig. I it is seen that the unbalanced weights are so oriented that during rotation each weight generates a centrifugal force in a radial direction with respect to the pinion axis. When this force is related to the axis of the whole machine it is seen to have both radial and tangential components. Since the spider is so arranged as to carry pinions diametrically opposite one another, the radial components always cancel one another while the tangential components are clearly additive. This fact makes possible the use of multiple pinions in order to increase the torque output or, conversely, reduces the torque required from each individual pinion in order to achieve any given total torque.

The problem of obtaining four impulses per revolution was solved by simply making the ratio of gear to pinions 4:1. This consideration is the major factor in determining the machine proportions.

By cut and try methods it was found that a pinion diameter of $1\frac{1}{2}$ inches and internal gear diameter of 6 inches would be most adaptable to this machine (3). Investigation into machine proportions revealed that the necessary unbalance could not be accommodated on the four pinions originally proposed. For this reason a six-pinion arrangement was adopted.

Determination of the required unbalance was conducted as shown in Appendix I and from these calculations it is shown that the total unbalanced weight per pinion must be .0617 lbs. at an eccentricity of 0.4 inches. Since the pinions are relatively small it was found difficult to connect the required weight at the proper eccentricity without causing the pinions to be structurally unsound. For this reason it was decided to add as much of the weight as possible on one radius of the pinions and to accomplish the remaining unbalance by lightening the pinion along the opposite radius. The required computations are shown in Appendix I.

Since the nominal speed of the vibration inducer is 1750 rpm and by virtue of the 4:1 gear ratio the pinions revolve at about 7000 rpm, it was decided that forced lubrication would be required both to assure lubrication at all bearings and to provide for heat removal. Accordingly, oil passages to each bearing and pinion were provided which are fed from a central bore in the main shaft. This bore is in turn supplied through a radial hole in one of its main journals. A similar arrangement is provided at the other main journal so as to act as an overflow. By maintaining a discharge from the overflow it is assured that the entire lubrication system is receiving oil service. A small vane-pump is used to circulate the oil. A crude cooling system is provided so as to maintain oil service at a reasonably constant temperature. No attempt was made to control this temperature closely, although this would be a desirable feature.

Calculations indicated that full depth gear teeth would not be sufficiently sturdy to carry the load (2). For this reason the 20 degree stub tooth system was adopted (3).

With the foregoing considerations in mind, plans were drawn for construction of the vibration inducer as shown in figures II, III and IV.

IV. FABRICATION

The U. S. Naval Engineering Experiment Station was commissioned to manufacture the vibration inducer in accordance with the plans shown in Appendix II. Shortly thereafter it was found that they were unable to cut the gears required and the job of gear production was therefore turned over to the U. S. Naval Gun Factory. In designing the gears it was determined from Buckingham's Formula for Wear Load (2) that the hardness of the pinions and gear should be 400 and 350 Brinell respectively. The Gun Factory was unable to produce gears of this hardness but was able to achieve a final hardness in all parts of slightly less than 300 Brinell.

The job of fabrication, including assembly, was originally estimated as about a 30 day job. It finally consumed about two and a half months.

The machine appears to embody first-class workmanship. During run-in and preliminary trials it seemed to give excellent performance. Lack of time precludes exhaustive test and only future runs of the machine can demonstrate whether or not it is of sufficiently rugged design.

V. THEORETICAL CONSIDERATIONS

The theoretical development is found in Appendix I. This development is derived on the theory that the total kinetic energy within a system remains constant so long as no energy is added to or removed from the system. The analysis is idealized to the extent that frictional losses and internal damping losses are neglected. The constant is determined by setting up the equation for kinetic energy and evaluating it for a position where all quantities are known. In this analysis, all factors are known under the condition where the centers of unbalance on the pinion gears are directly radial to the central shaft along a diameter through the pinion axes and are at the maximum distance from the machine center.

The general equation for kinetic energy as applied to this analysis is:

$$K.E. = \frac{1}{2 \times 12}\left[\sum I_z \omega_i^2 + M_p V_p^2 + M_w V_w^2\right] = C \qquad (1)$$

where: K.E. - kinetic energy (in.-lb.)

$\sum I_z \omega_i^2$ - sum of products of the centroidal moments of inertia (slug-in^2) and corresponding ω^2 (rad/sec)^2

V_p - linear velocity of pinions (in/sec)

M_p - mass of pinions (slugs)

ω_i - angular velocity (rad/sec)

M_w - mass of unbalanced weights (slugs)

V_w - linear velocity of unbalanced weights (in/sec)

By evaluating the constant of Eq. 1 and by solving this general equation for C , the value of ω for any angular displacement of the spider can be obtained.

By differentiating the general equation for ω , the angular acceleration for any position of the spider can be found.

Curves of ω and α for angular displacements of the spider, are included in Appendix II.

From the plotted curve for α, it is seen that the unit will induce four pulses for each revolution of the spider - each pulse consisting of an acceleration and a deceleration caused by the unbalanced weights on the pinions.

Using the information obtainable from the angular acceleration curve, the magnitude of torque exerted on the unit or component parts of the unit can be calculated. Knowing these values of torque for the various spider positions, the angular twist which will be exerted on the test bar can be computed from the equation:

$$\phi = \frac{M_t \, l}{J \, G} \times 57.3$$

where: ϕ - angle through which a radius, of a right section at end of shaft, is turned (degrees)

M_t - torque exerted by shaft of inducer unit (in-lb)

l - length of test bar (in)

J - polar moment of inertia of test bars (in^4)

G - modulus of rigidity of test bar material (lb/in^2)

From the above considerations, the test bar will be twisted through an angle of 2.7 degrees on each side of neutral. This twist will be a repeating one in that the unbalance inherent in the unit will cause the test bar to be twisted from $+2.7^\circ$ to -2.7° four times for each revolution of the spider, or approximately 7000 times per minute.

The results of the above calculations for obtaining angle of twist (ϕ) and torque (M_t) should compare quite favorably with the respective values obtained from test data.

VI. MEASURING INSTRUMENTS

Measuring instruments for determining the amount of torsional deflection occurring in the test specimen while rotating at 1750 rpm presents one of the most difficult problems encountered in this development. The instruments used are described below but it is strongly recommended that further effort be made to provide more adequate and more easily interpreted instruments.

In order to measure the torsional deflection of the specimen at any phase of its rotation, it is necessary to be able to read the relative positions of the two ends of the shaft. Identifying scribe marks are provided on the flanges at both ends of the test specimen. A protractor is mounted adjacent to the flanges so that the position of the scribe mark may be defined. A stroboscopic light source is used to make this observation possible while the shaft is in motion. Both ends of the shaft must be located simultaneously and these relative locations noted. Since it is known that a complete torque oscillation occurs during each 90^{o} of rotation, it follows that only a 90^{o} sector must be searched in order to find the points of maximum deformation. This search is accomplished by operating the stroboscope slightly slower or faster than the speed of shaft rotation. A plot of the relative positions of the two ends vs. the angle at which observation was made will serve to define the rate and manner of loading. This, however, is a difficult procedure since the actual deflections obtainable are small. For this reason a second method of torque appraisal is being developed which, it is hoped, will provide a better means of measurement.

The second device provided for indicating torque application depends upon an electrical pickup. A permanent bar magnet is fastened into the coupling at each end of the test specimen and a Gramme wound ring is mounted in such a way as to compose the elements of a simple generator. The driven end of the specimen shaft is assumed to rotate at constant speed due to the flywheel effect of the large driving motor. The other end of the specimen is expected to rotate at driven speed and also to oscillate in accordance with the torque applied to it by the vibration inducer unit. Since the voltage produced by a generator is a direct function of the speed of the generator then a comparison of the amplitude of voltage curves at any point should give a direct comparison of the angular velocities of the two ends of the shaft throughout rotation. Comparison of the two voltage curves may be accomplished in several ways. A multiple element cathode ray oscilloscope may be used to display the two resultant curves simultaneously or a single element oscilloscope may be used in conjunction with an electronic switch. The first method is considered best since in that case the two signals may be focussed independently and by causing the zero voltage points to coincide, a better and more direct comparison may be accomplished.

The above methods of measurement still leave a great deal to be desired. It is hoped that some future investigator will be able to further develop the measurement methods herein outlined.

The arrangement of instruments is included in the sketch of the test setup to be found in Appendix II.

VII RESULTS

The design of the vibration inducer presented little difficulty once the basic form had been decided upon. The original premises outlined under Design Considerations were accomplished by fairly straight forward design technique. Analyses of the machine were made by both graphical and mathematical means.

In fabrication it was found expedient to make several alterations to the original design. The nature of these alterations was minor, consisting of such changes as the addition of oil seals, changing the spider material to steel with bronze sleeve bearings for the pinions, revision of the keying arrangement, etc. None of these alterations had any effect upon the operating characteristics of the machine. The major difficulty in production was the fabrication of gears. The U. S. Naval Gun Factory manufactured the internal gear and pinions required but was unable to attain the desired hardness. Because of the reduced hardness that was accepted as a necessity, it is anticipated that the operating lifetime of the machine will be reduced because of excessive wear. It is further anticipated that, if excessive wear occurs, the increased backlash between gear and pinions will be such as to introduce extraneous vibrations, thus reducing the effectiveness of the machine.

The analysis of this machine was first conducted by graphical means as a rough indication of expected performance. As a check on this method an attempt was made to utilize the theory of conservation of momentum. A considerable amount of work was done on this premise with negative results. The theory is considered valid but difficulties were encountered in evaluating some of the components because of the intricate shapes involved. This analysis was abandoned and attention then turned to analysis from consideration of the total kinetic energy present in the system. This analysis is shown in Appendix I and agrees qualitatively with the experimental data as is shown in Appendix II where the computed curves may be compared with the curves obtained experimentally.

Experimental evaluation of the machine posed the most difficult problem. The authors hoped to be able to devise an electrical pickup system whose output would be susceptible to ready interpretation. The basic idea was to utilize the rotation of a permanent magnet attached to each end of the test shaft to cause a voltage to be generated in a Gramme-wound ring. It was assumed that the driven end would rotate at constant speed due to the inertia of the large driving motor and that the end connected to the vibration inducer would have oscillation superimposed on its rotation. It was anticipated that comparison of the differing voltage waves would be a simple method of determining instantaneous relative velocities; i.e., amplitude of voltage wave is a direct function of velocity and where the value of the wave on the driven end is known, the value of the other wave may be defined by a simple proportion.

An attempt was made to compare the resultant output waves on a single-beam, cathode-ray oscilloscope by using an electronic switch to alternate the input signals. This attempt was unsuccessful because the two signals could not be focussed independently and the resultant display could not be interpreted. The next step was to try a dual-beam, cathode-ray oscilloscope in order to avoid the troubles encountered in the single-beam unit. This effort was more successful and gave fair results at reduced machine speeds. However, as the operating speed of the vibration inducer was increased, it was found that unexplained signals appeared on the voltage curves and at speeds of about 1400 rpm and upwards, the curve was indistinguishable because of the "grass" appearing on the oscilloscope screen. It was impossible, in the allotted time, to deviate from the main investigation long enough to ascertain definitely the cause of the unexplained signal. It is considered likely that some undesired vibration is introduced by reason of imperfections in the gears and by the fact that spur gearing was used instead of some smoother type such as herringbone or helical gearing. Another probable cause for irregularity in the resultant wave is that the pickup coils were hand wound and are therefore not as accurately wound as would be desirable. Further, it is possible that the individual turns are caused to vibrate either by the rotating field or by vibrations conducted to them through the machine frame. The latter explanation seems more probable since the effect increases with machine speed.

The next effort at measuring the effect of the vibration inducer was by use of a Brush recording oscillograph. The curves obtained by this method bore out the general shape of the predicted output curves but could not be obtained in sufficient magnitude to permit analysis. Examples of the curves obtained by this method are shown in fig. IX. These oscillograms show very definitely that the end of the specimen attached to the vibration inducer is torsionally oscillated with respect to the angular velocity at which it is being driven. As may be seen on these curves, the voltage wave taken from the driven end is nearly sinusoidal except for small imperfections due to induced vibration which is not prevented by inertia of the drive motor. The curve taken from the end attached to the vibration inducer exhibits deviations from the sinusoidal wave shape indicative of velocity changes superimposed on the steady rotation.

Attention was next directed to obtaining the output curves from a string-type oscilloscope. It was noted that, when using the Brush instrument, the undesired vibrations found in using the cathode-ray oscilloscope were minimized. It was assumed that these vibrations were damped out by reason of the inertia of moving parts in the instrument. The string oscilloscope, depending as it does on mechanical movements, was expected to have a similar effect upon the unwanted vibrations. Early trials bore out this expectation. In fact, the damping in the oscilloscope was so great that only a negligible response could be observed. This difficulty was overcome by using an electronic amplifier on the signal input. Photographs of curves resulting from this procedure are shown in fig. X.

The results obtained from the electrical devices mentioned above indicate qualitatively that an oscillatory torque does exist and is superimposed upon the rotation of the specimen. The manner of torque application closely approximates the performance characteristics for which the machine was designed.

Visual evidence of the oscillation produced in the specimen by the vibration inducer may be seen by using a stroboscopic light. If the light is focussed on the overhung flange of the inducer unit and operated at a speed slightly less or slightly greater than the speed of inducer rotation, oscillation is readily apparent. By mounting a protractor adjacent to each end of the specimen and providing an identifying mark at the specimen ends, the stroboscopic light may be used to determine visually the angle of specimen twist produced by the applied torque. In order to determine the manner of torque application by this method it is necessary to take readings of twist at intervals through about 90° of shaft rotation. A plot of angular twist against the angle at which the reading is taken results in a curve which represents one cycle of torque application. The magnitude of torque applied may be calculated from knowledge of the physical constants of the specimen material. This method of torque appraisal is considered adequate for the purpose of this paper but too difficult for routine usage. A plat of values obtained by this method is shown in fig. XI.

The principle used in attempting electrical-pickup measurements points the way to development of a simplified torsion measuring device. It is considered that more accurately constructed pickup coils would materially increase the accuracy of information obtainable by this method.

It was noted throughout this investigation that the intentionally induced vibrations were reflected at the driven end of the specimen. This objectionable effect indicates that the inertia of the drive motor is insufficient to maintain the desired constant angular velocity at the driven end. It is expected that this difficulty might be overcome by inserting a heavy flywheel between the drive motor and the driven end of the specimen.

The project herein outlined may be considered successful in that a machine has been designed and manufactured to produce torsional vibration superimposed on a constantly rotating member. The theoretical analysis has been carried out in considerable detail and is confirmed by experimental data. The usual difficulties in original design work are met in instrumentation: This project was no exception. It is felt by the authors that the subject of more satisfactory instrumentation will provide a field for investigation of sufficient scope to be considered a suitable future thesis project. Although measurements of performance have been accomplished, the methods used are susceptible to further improvement.

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APPENDIX I

DETERMINATION OF UNBALANCED LOADING REQUIRED TO PRODUCE DESIRED TORQUE

From design considerations it was determined that a total torque output of 490 lb-in is required. It was further determined that it would be necessary to distribute the unbalance over six pinions. Equal distribution therefore requires that each pinion be responsible for 81.666 lb-in of torque.

From the configuration of the pinions and gear it may be seen that the distance from gear center to pinion center must be 2.25 inches. This, then, is the moment arm of the unbalanced force generated by the pinions. The centrifugal force required to be developed on each pinion by its unbalance is then:

$$\frac{81.666 \text{ lb-in}}{2.25 \text{ in.}} = 36.3 \text{ lb.}$$

In order for the pinions to cause four impulses per revolution of the main shaft the gear ratio must be 4 : 1. Since the pinions must then rotate at four times the angular velocity of the central member their velocity is then calculated to be 754 rad./sec.

Determination of the weight to be added to each pinion is accomplished by transposing the equation for centrifugal force:

$$F_c = Mr\omega^2 = \frac{W}{g} r \omega^2$$

to give:

$$W = \frac{F_c\, g}{r\, \omega^2}$$

Substituting the known values and choosing the eccentricity r, as 0.4 inches,

$$W = \frac{(36.3 \text{ lb.})\ (386.4 \text{ in/sec./sec.})}{(754 \text{ rad./sec.})^2\ (0.4 \text{ in.})}$$

$$= 0.0617 \text{ lbs.}$$

The configuration of the pinion gears restricted the space available for adding the unbalancing masses. The best shape that could be fitted into the available space was found to be the segment of a circle whose radius would be 0.55 inches. The height of the segment was determined on the basis that the center of gravity must fall at 0.4 in. eccentricity. Thus, the area above and below this point were made equal. The height of the segment was found to be 0.278 in. and the area 0.1555 sq. in.

The pinions were designed to carry a one-half inch thick weight in the annular space on each side of the web. Hence the volume of weight that can be added to each pinion is 0.1555 cu. in. If brass weights are used and the density of the brass is 0.3048 lb. per cu. in. (3), then these weights account for 0.04735 lb. unbalance on each pinion.

Since, as shown previously, each pinion must carry 0.0617 lb. unbalance at an eccentricity of 0.4 inches, the difference between this figure and that accounted for by adding weights was effectively added by removing weight from the diametrically opposite radius of the pinion. The configuration of the pinion would not allow a single round hole of sufficient diameter to account for the remaining unbalance. Hence, the following calculations were made to ascertain the required size and location of multiple holes which would create the unbalance still required. The effect of adding the weights is:

$$F_c = \frac{(0.04735)x(0.4)x(754)^2}{386.4} = 27.9 \text{ lb.}$$

The force remaining to be generated is the difference between this and the 36.3 lb, already found to be required of each pinion. The remainder will then be 8.4 lb.

Assuming that two holes would suffice to effect the remaining weight correction, a scale drawing of the pinion was consulted. From this study it was determined that the maximum diameter of hole that could be tolerated was 0.28 in. In displacing the holes from the diameter on which the weights are located it was found that the holes must be located as shown in the plans to be found in Appendix II. Determination of the diameter required of the holes was made as follows:

$$W_h = \frac{(8.4)x(386.4)}{(0.37)x(754)^2} = 0.01542 \text{ lb.}$$

Since two holes are to be used, each hole will account for one-half the weight or 0.00771 lb. The material to be removed is steel whose density is 0.2817 lb./cu. in. The thickness of the pinion web is 0.5 in. Hence the volume to be removed by each hole is:

$$V = \frac{0.00771}{0.2817} = 0.0274 \text{ cu. in.}$$

The area of the hole is:

$$A = \frac{0.0274}{0.5} = 0.0658 \text{ sq. in.}$$

and the diameter is:

$$D = \frac{0.0548}{0.7854} = 0.264 \text{ inches.}$$

The combination of lightening holes and additive weights calculated above are sufficient to create the unbalance required.

THEORETICAL ANALYSIS

The total energy in the system to be analysed herein is considered to be the sum of the kinetic energy K.E., and the potential energy. From observation of the symmetry of the unit it may be determined that the potential energy stored in the system is always zero. Neglecting frictional effects and internal damping, the kinetic energy alone is a constant quantity.

For the system under consideration, the general expression for kinetic energy is:

$$K.E. = \frac{1}{2 \cdot 12}\left[\sum I_z \omega_i^2 + M_p V_p^2 + M_w V_w^2\right] = C \tag{1}$$

Expanded, this equation becomes:

$$K.E. = \frac{1}{2 \times 12}\left[+I_r \omega_r^2 + I_s \omega_s^2 + I_{fg} \omega_{fg}^2 + I_{fm} \omega_{fm}^2 + I_{fb} \omega_{fb}^2 + I_b \omega_b^2 + I_p \omega_p^2 + M_p V_p^2 + I_w \omega_w^2 + M_w V_w^2\right] = C \tag{1-a}$$

The following relationships are derived from geometric considerations:

$$\omega_r = \omega_s = \omega_{fg} = \omega_{fm} = \omega_{fb} = \omega_b = \frac{\omega_p}{3} = \frac{\omega_w}{3} = \omega$$

$$\bar{V}_p = R\omega = 2.25\,\omega$$

$$\bar{V}_w = \left[R\omega - 4r\omega\cos 4\theta\right] = \omega(2.25 - 4 \times 0.4\cos 4\theta)$$

Substitution of these values and those from Table I into equation (1-a), gives:

$$K.E. = \frac{\omega^2}{24}\left[0.01228 + 0.7484 + 0.04426 + 0.06082 + 0.10025 + 0.00467 + 0.02862(3)^2 + 0.0906(2.25)^2 + 0.00087(3)^2 + 0.02038(2.25 - 1.6\cos 4\theta)^2\right] = C \tag{1-b}$$

The constant, C, can be evaluated for the position of the spider for which the angular velocity is known, i.e., when $\theta = 0^{\circ}, \omega = 183.26$ rad./sec. Inserting these values and collecting terms :

$$K.E. = \frac{(183.26)^2}{24}\left[1.69405 + 0.02038 + 0.4225\right] = C \qquad (2)$$

$$= 2383.5 = C \qquad (2\text{-a})$$

Transposing equation (2) and inserting the value obtained for C gives the following general equation for angular velocity at any point in rotation, ω_θ :

$$\omega_\theta = \left[\frac{24 \times 2385.3}{1.69405 + 0.02038\,(2.25 - 1.6\cos 4\theta)^2}\right]^{\frac{1}{2}} \qquad (3)$$

$$= \left[\frac{28602}{0.847 + 0.01019\,(2.25 - 1.6\cos 4\theta)^2}\right]^{\frac{1}{2}} \qquad (4)$$

Substituting various values of θ into equation (4) gives ω_θ in Table II, a plot of which is included in Appendix II. Taking the first derivative of equation (4) and rearranging terms, we obtain the expression for angular acceleration at any point in rotation, α_θ :

$$\frac{d\omega_\theta}{dt} = \alpha_\theta = \frac{1}{2}\left\{\frac{3730.616 \sin 4\theta\,(2.25 - 1.6\cos 4\theta)\,\omega_\theta}{\omega_\theta\left[0.847 + 0.01019\,(2.25 - 1.6\cos 4\theta)^2\right]^2}\right\} \qquad (5)$$

$$= -\frac{1865.308 \sin 4\theta\,(2.25 - 1.6\cos 4\theta)}{\left[0.847 + 0.01019\,(2.25 - 1.6\cos 4\theta)^2\right]^2} \qquad (5\text{-a})$$

Substituting various values of θ into equation 5a gives α_θ in Table II, a plot of which appears in Appendix II.

To find the angle ϕ through which a radius of a right section of the test shaft will twist with respect to a similar radius at its other end, the following relation is used:

$$\phi = \frac{M_t l}{JG} \times 57.3 \text{ degrees.} \tag{6}$$

$$= \frac{I \alpha_e l}{\pi/2 \times r^4 \times G} \times \frac{57.3}{12} \tag{6-a}$$

Where: I - sum of $I_r + I_s + 2I_{fg} + I_{fm} + I_{fb} + I_b$ (slug-in.^{2})

l - length of specimen shaft (in.)

J - polar moment of inertia of specimen, (in.^{4})

G - modulus of rigidity of specimen material (lb./in.^{2})

α_e instantaneous angular acceleration of the specimen due to oscillatory torque, (rad./sec.^{2})

Substituting known values for the constant terms of equation (6-a), it is apparent that the instantaneous angle of shaft twist varies directly with instantaneous values of angular acceleration :

$$\phi_e = 472.213 \times 10^{-6} \alpha_e \tag{6-b}$$

The values of ϕ_e found in Table II are calculated from this relation and a plot of the results is included in Appendix II.

TABLE I

Centroidal Moments of Inertia and Masses of Component Parts of the Vibration Inducer Unit.

Part	Identifying Subscript *	Moment of Inertia ($slug\text{-}in^2$)	Mass (slugs)
Unbalanced Weights(6)	w	0.0008680	0.00204
Pinions (6)	p	0.0286163	0.09060
Spider	s	0.7484000	0.24530
Central Shaft	r	0.0122800	0.07770
Shaft Flanges (2)	fg	0.0442640	0.02688
Magnet Flange	fm	0.0608245	0.05887
Specimen Flange	fb	0.1002500	0.04709
Specimen	b	0.0046664	0.00747

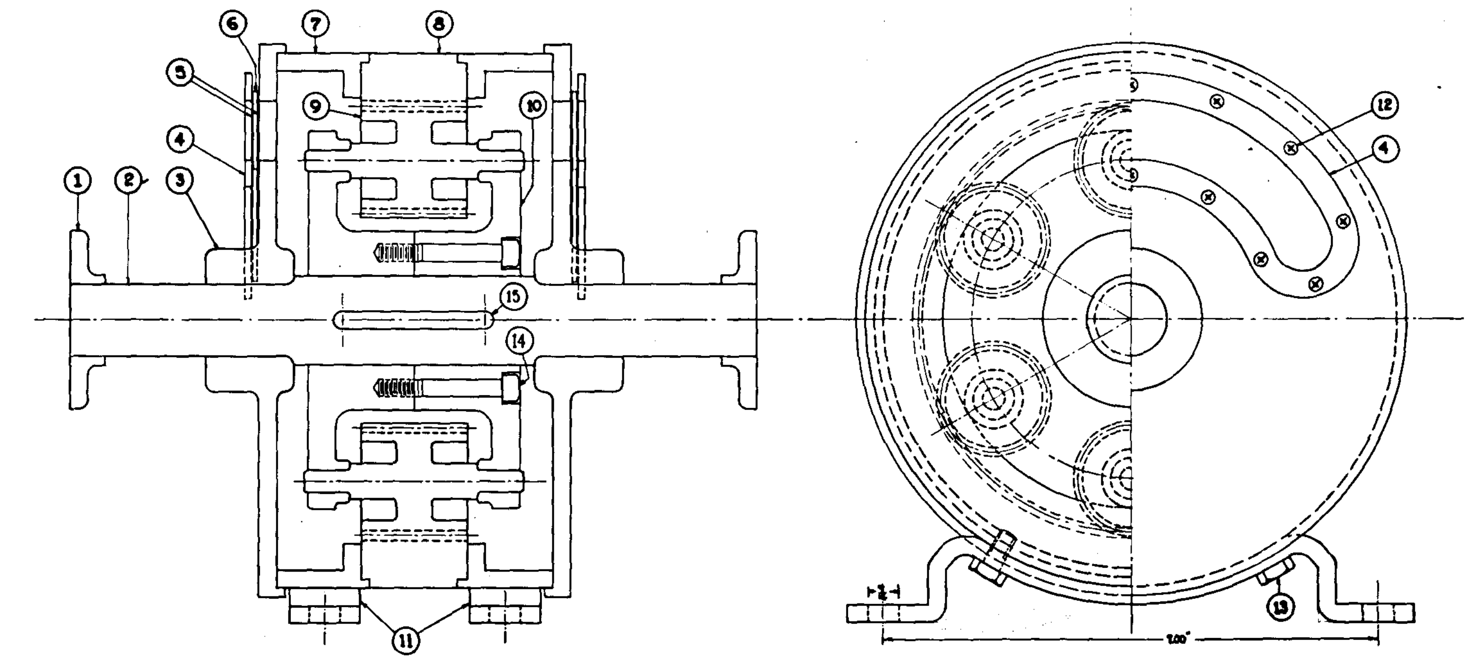
*Note: Subscripts are to be applied to symbols for moment of inertia and mass as:- I_w, M_w, etc.

TABLE II

This table summarizes the results obtained by substituting various values for the displacement angle θ, into equations (4), (5a) and (6b).

θ degrees	ω_θ (rad./sec.) Eq.(4)	α_θ (rad./sec.^{2}) Eq.(5a)	ϕ degrees Eq.(6b)
0.0	183.260	0.000	0.000000
7.5	182.942	-1103.945	0.521297
15.0	181.481	-3106.093	1.466737
22.5	178.409	-5198.255	2.454684
25.0	177.081	-5581.874	2.635833
28.0	175.674	-5716.525	2.699417
30.0	174.269	-5555.186	2.623231
35.0	171.707	-4428.485	2.091188
38.0	170.517	-3315.130	1.565447
40.0	169.923	-2438.721	1.151596
45.0	169.285	0.000	0.000000
50.0	169.923	2438.721	1.151596
52.0	170.517	3315.130	1.565447
55.0	171.707	4428.485	2.091188
60.0	174.269	5555.186	2.623231
62.0	175.674	5716.525	2.699417
65.0	177.081	5581.874	2.635833
67.5	178.409	5198.255	2.454684
75.0	181.481	3106.093	1.466737
82.5	182.942	1103.945	0.521297
90.0	183.260	0.000	0.000000

APPENDIX II



PC. NO.	REQ'D	PART	MAT'L	REMARKS
1	2	FLANGE	ST.	ALT. SEE SHEETS
2	1	CENTER SHAFT	ST	" " " "
3	2	END PLATE	[illegible]	
4	2	SIGHTGLASS RET'N	ST	
5	4	GASKETS		OIL PROOF
6	2	SIGHT GLASS		
7	2	GEAR SPACER	C.I.	
8	1	INTERNAL GEAR	2315	BRINELL 350
9	6	PINION GEAR	3250	BRINELL 400
10	2	SPIDER HALF	BRNZ	
11	2	BRACKET	ST.	1"×¼" STRAP
12	25	SCREWS,CAP 8-32×¼"		HEX-SOCKET HEAD
13	4	CAPSCREW ¼"-20×⅝"		HEX-HEAD
14	6	CAPSCREW ¼"-20×1¼"		HEX-SOCKET HEAD
15	2	KEY	ST	
16	12	PINION WEIGHT	[illegible]	
	48	CAPSCREW 8-32×½	HEX	SKT (FOR ASSY [illegible])
	2	WOODRUFF KEYS		#2 (⅛"×½")

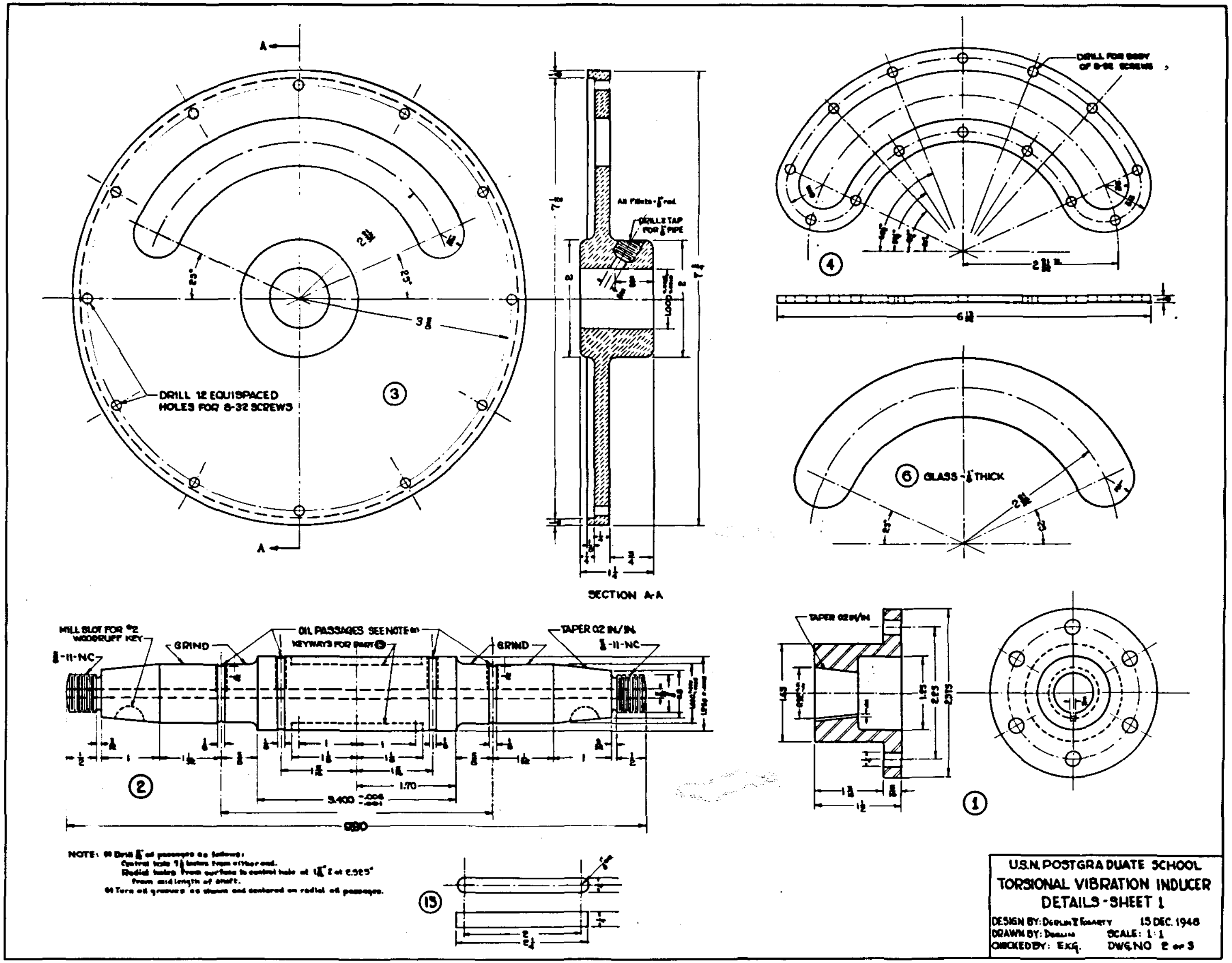
U.S.N. POSTGRADUATE SCHOOL

TORSIONAL VIBRATION INDUCER

ASSEMBLY

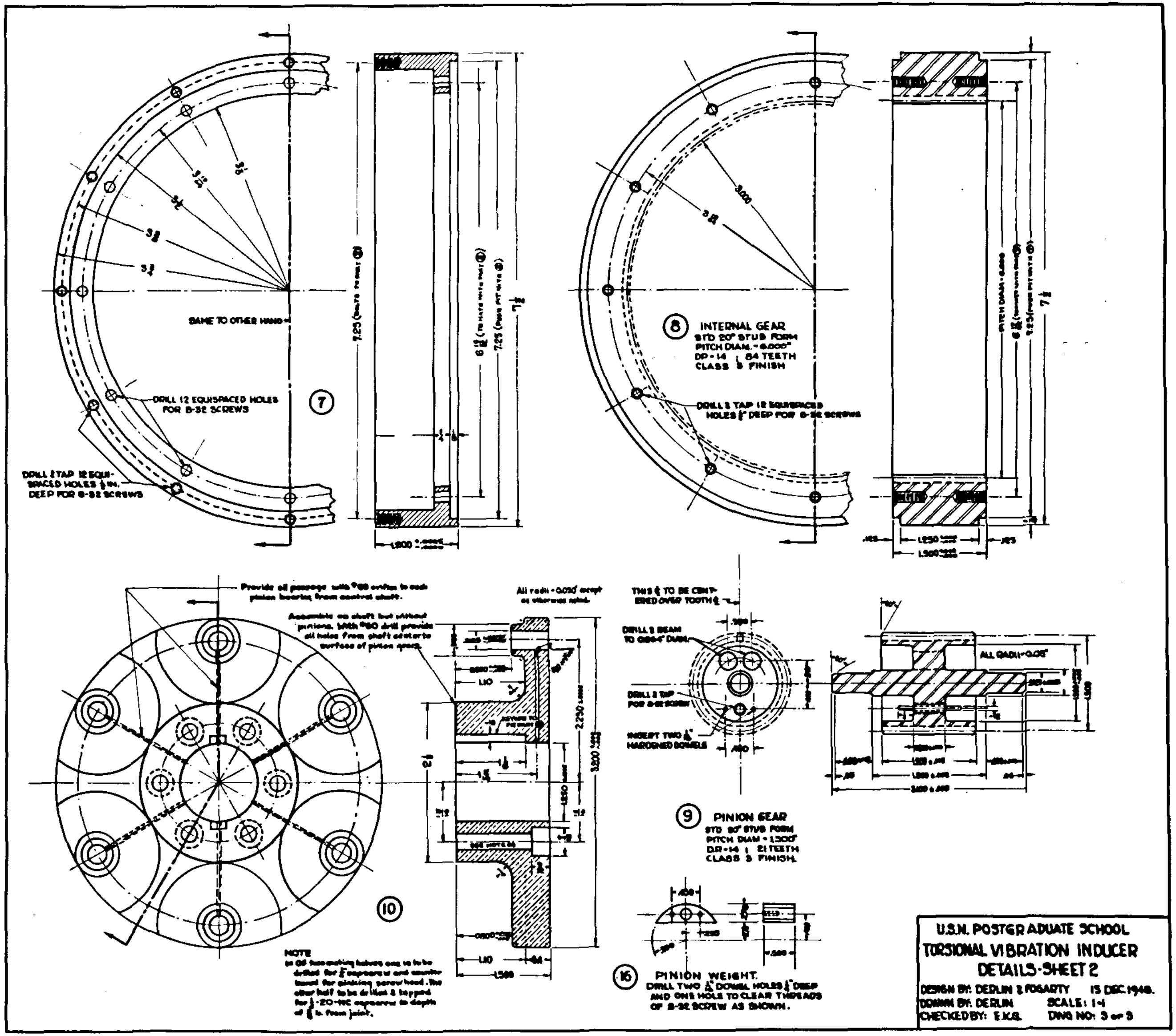
DESIGN BY: DERLIN & FOGARTY
DRAWN BY: DERLIN 15 DEC. 1948
CHECKED BY: E.K.G. DWG.NO. 1 OF 2

Figure II



DRILL 12 EQUISPACED HOLES FOR 8-32 SCREWS
SECTION A-A
DRILL FOR BODY OF 8-32 SCREWS
GLASS - 1/8 THICK
MILL SLOT FOR #2 WOODRUFF KEY
GRIND
OIL PASSAGES SEE NOTE
KEYWAYS FOR PART
TAPER .02 IN/IN.
TAPER .02 IN/IN
U.S.N. POSTGRADUATE SCHOOL
TORSIONAL VIBRATION INDUCER
DETAILS - SHEET 1
DESIGN BY:
15 DEC. 1948
DRAWN BY:
SCALE: 1:1
CHECKED BY: E.K.G.
DWG.NO 2 of 3

Figure III



SAME TO OTHER HAND
DRILL 12 EQUISPACED HOLES FOR 8-32 SCREWS
DRILL & TAP 12 EQUI-SPACED HOLES DEEP FOR 8-32 SCREWS
7
8
INTERNAL GEAR
STD 20° STUB FORM
PITCH DIAM. - 6.000"
DP - 14 ; 84 TEETH
CLASS 3 FINISH
DRILL & TAP 12 EQUISPACED HOLES DEEP FOR 8-32 SCREWS
10
9
PINION GEAR
STD 20° STUB FORM
PITCH DIAM - 1.500"
DP-14 ; 21 TEETH
CLASS 3 FINISH.
16
PINION WEIGHT.
DRILL TWO DOWEL HOLES DEEP AND ONE HOLE TO CLEAR THREADS OF 8-32 SCREW AS SHOWN.
NOTE
U.S.N. POSTGRADUATE SCHOOL
TORSIONAL VIBRATION INDUCER
DETAILS-SHEET 2
DESIGN BY: DERLIN & FOGARTY 15 DEC. 1948.
DRAWN BY: DERLIN SCALE: 1-1
CHECKED BY: E.K.G. DWG NO: 3 of 3

Figure IV

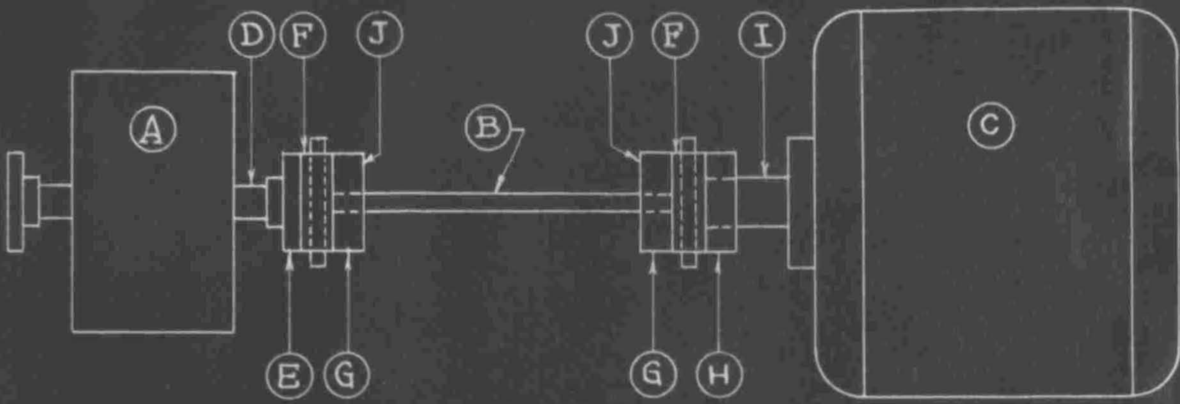


Figure V

ARRANGEMENT OF TEST SETUP

A.- Torsional Vibration Inducer.

B.- Specimen under test.

C.- Drive motor.

D - Vibration inducer shaft.

E - Vibration inducer shaft flange.

F - Magnet flange carrying permanent magnet as field for the pick-up coils. The coils (not shown) are mounted so as to surround the magnets as they rotate.

G - Specimen flange, (split so as to form a clamp).

H - Motor flange.

I - Motor shaft.

J - Location of protractors for visual measurement of specimen deformation.

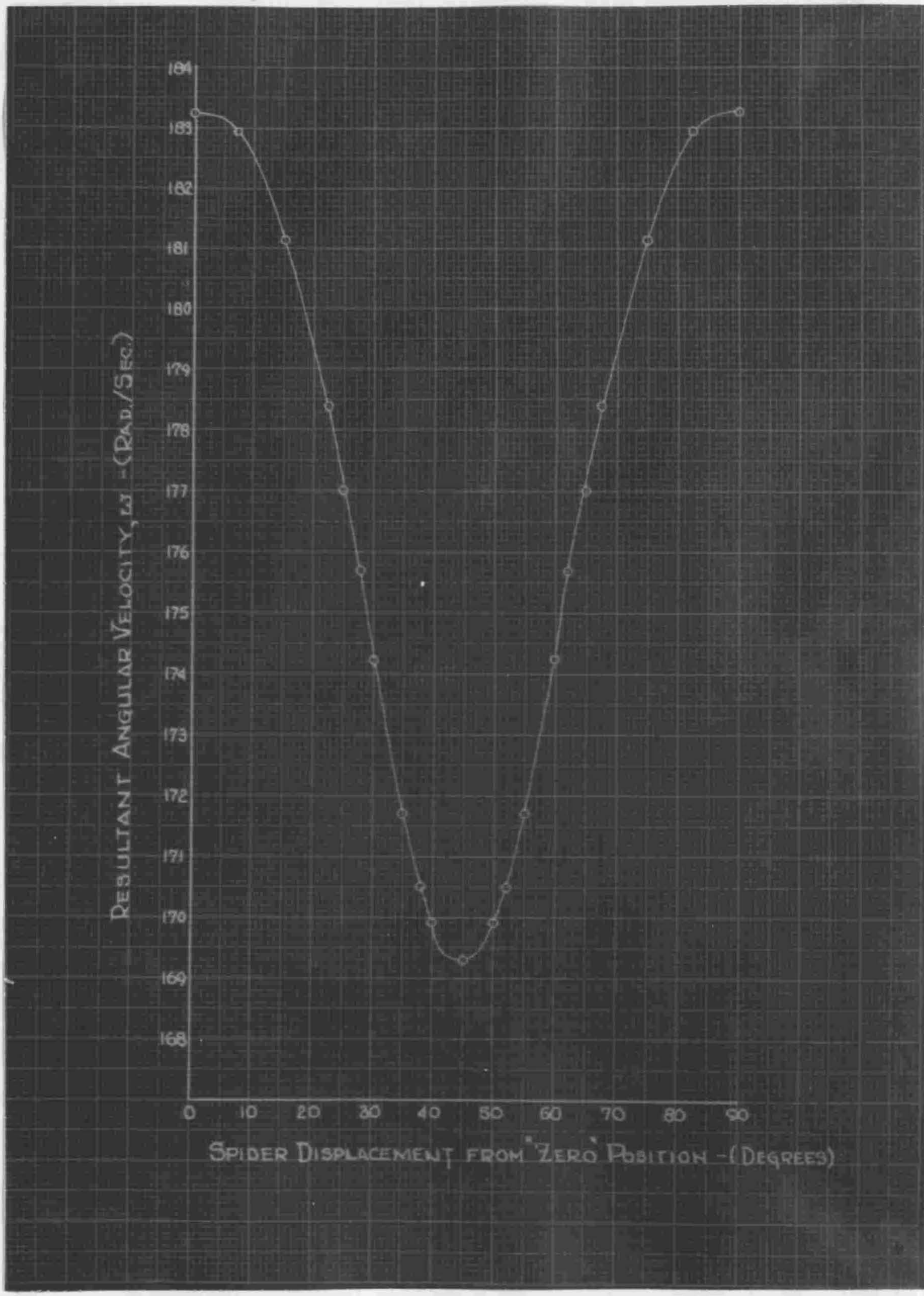

184
183
182
181
180
179
178
177
176
175
174
173
172
171
170
169
168
RESULTANT ANGULAR VELOCITY, ω - (RAD./SEC.)
0
10
20
30
40
50
60
70
80
90
SPIDER DISPLACEMENT FROM "ZERO" POSITION - (DEGREES)

FIG. VI

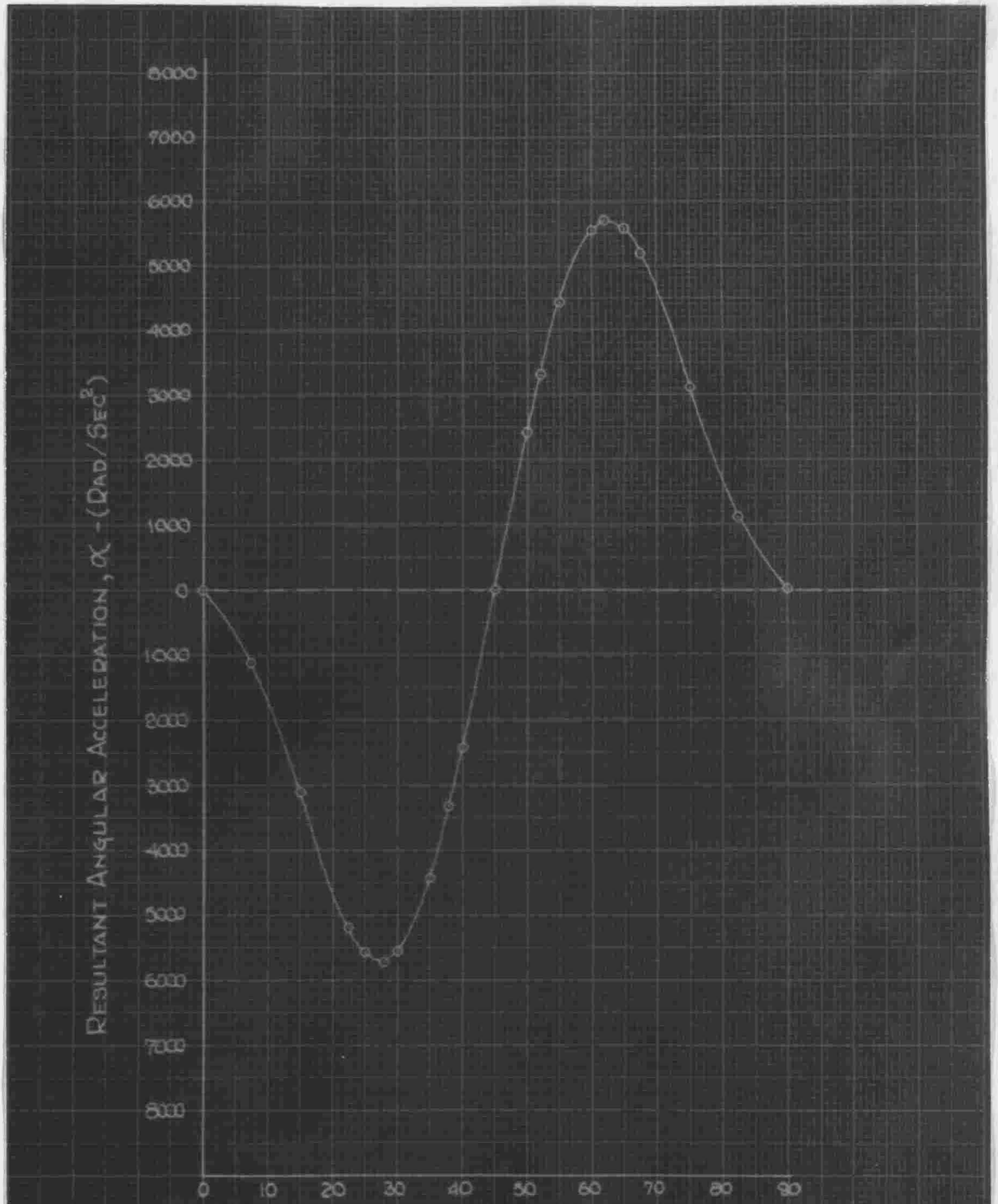
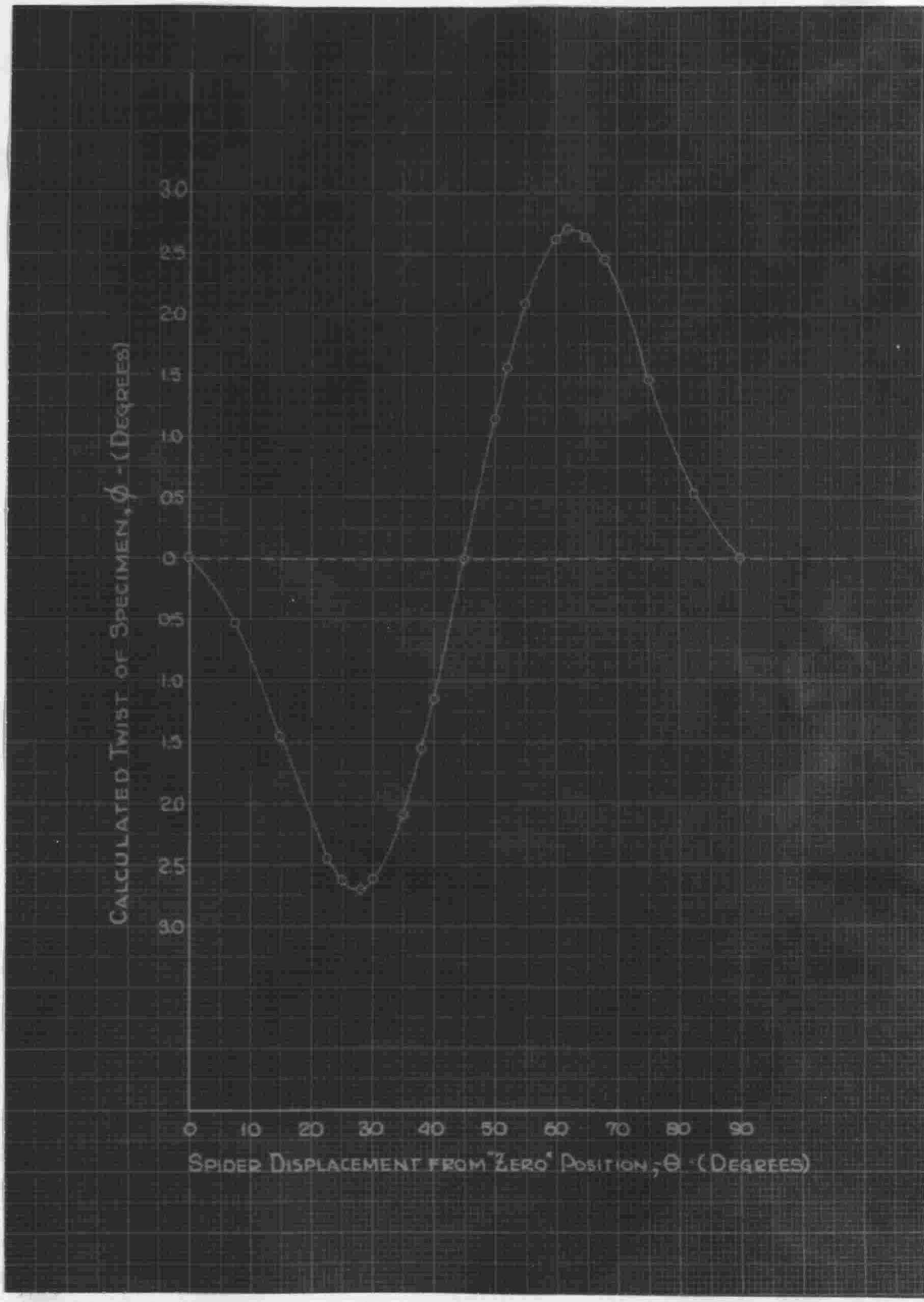
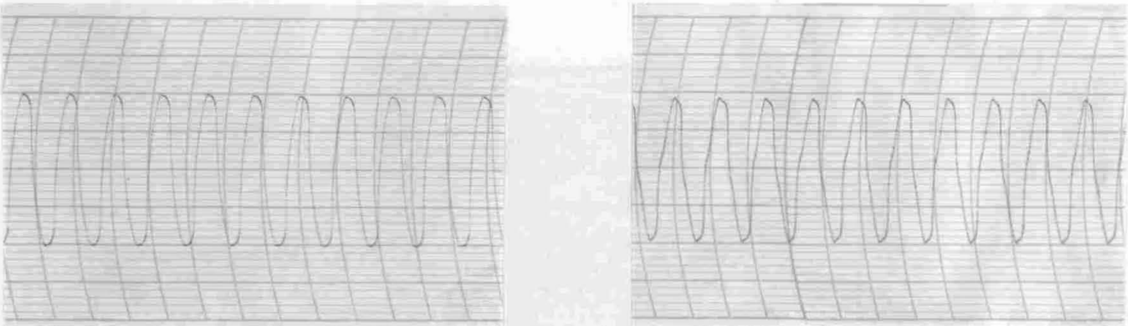

8000
7000
6000
5000
4000
3000
2000
1000
0
1000
2000
3000
4000
5000
6000
7000
8000
RESULTANT ANGULAR ACCELERATION, α ~ (RAD/SEC^2)
0
10
20
30
40
50
60
70
80
90
SPIDER DISPLACEMENT FROM "ZERO" POSITION, θ ~ (DEGREES)

FIG. VII

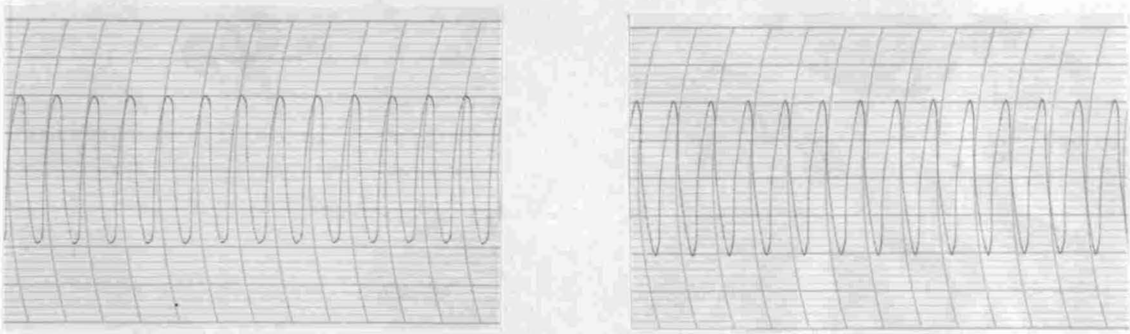


Calculated Twist of Specimen, ϕ - (Degrees)
3.0
2.5
2.0
1.5
1.0
0.5
0
0.5
1.0
1.5
2.0
2.5
3.0
0
10
20
30
40
50
60
70
80
90
Spider Displacement from "Zero" Position, θ - (Degrees)

FIG. VIII



(a) Vibration Inducer Speed = 1200 rpm



(b) Vibration Inducer Speed = 1500 rpm

Fig. IX

OSCILLOGRAMS OF OUTPUT FROM PICKUP COILS

The left-hand oscillograms were taken from the pickup attached to the driven end of the specimen. The right-hand oscillograms were taken from the end attached to the vibration inducer. Note that a definite deformation of the normal wave is caused by the velocity changes at the vibrated end due to the superimposed oscillating torque. A Brush recording oscillograph and amplifier were used to obtain these curves.

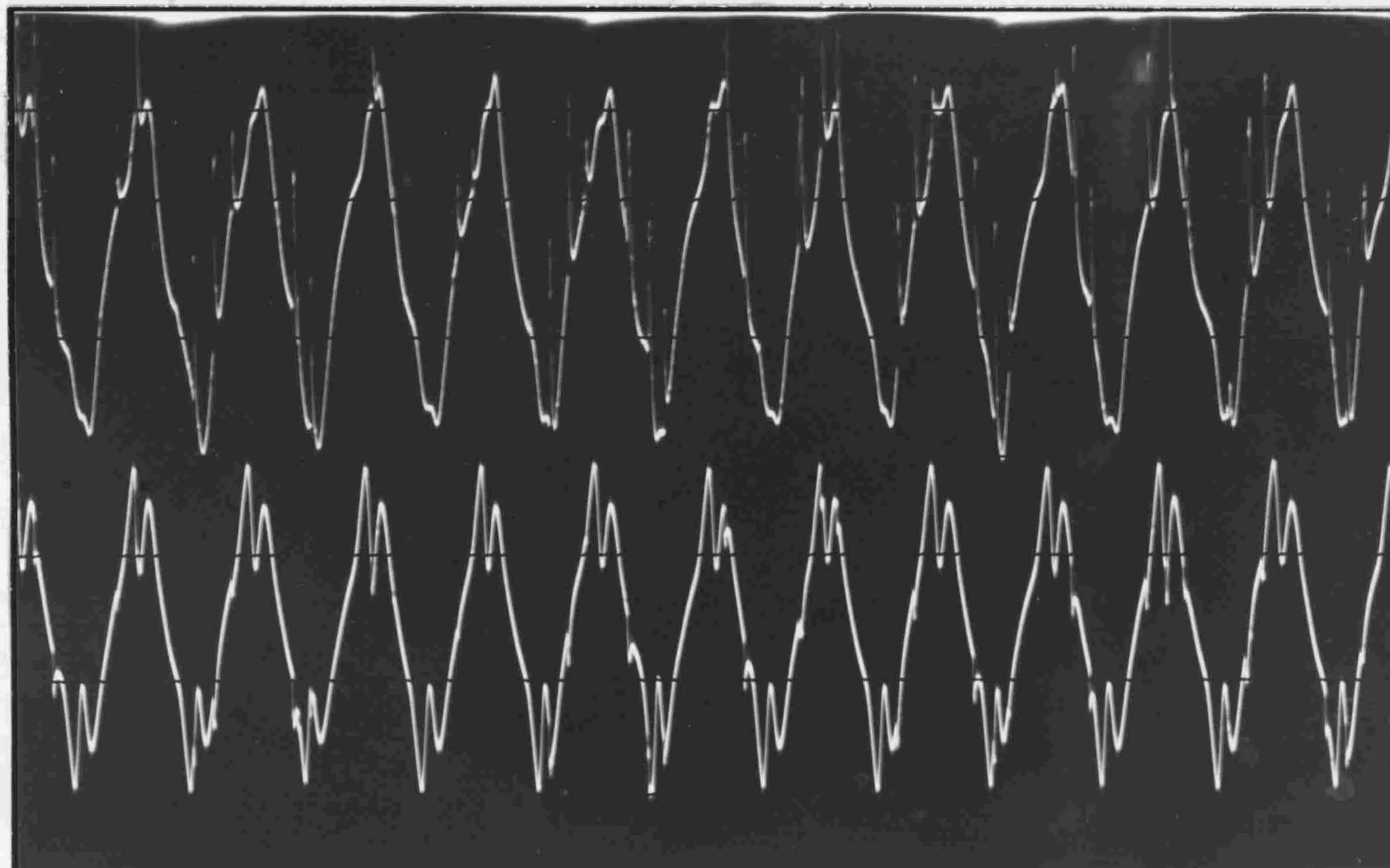


FIG. X-a OSCILLOGRAM OF VOLTAGE WAVES FROM ELECTRICAL PICKUPS

Oscillogram from Westinghouse Multi-element Oscillograph. Top curve is output from the driven end of specimen; lower curve is output from the vibrator end of specimen. Reading taken at 1800 rpm.

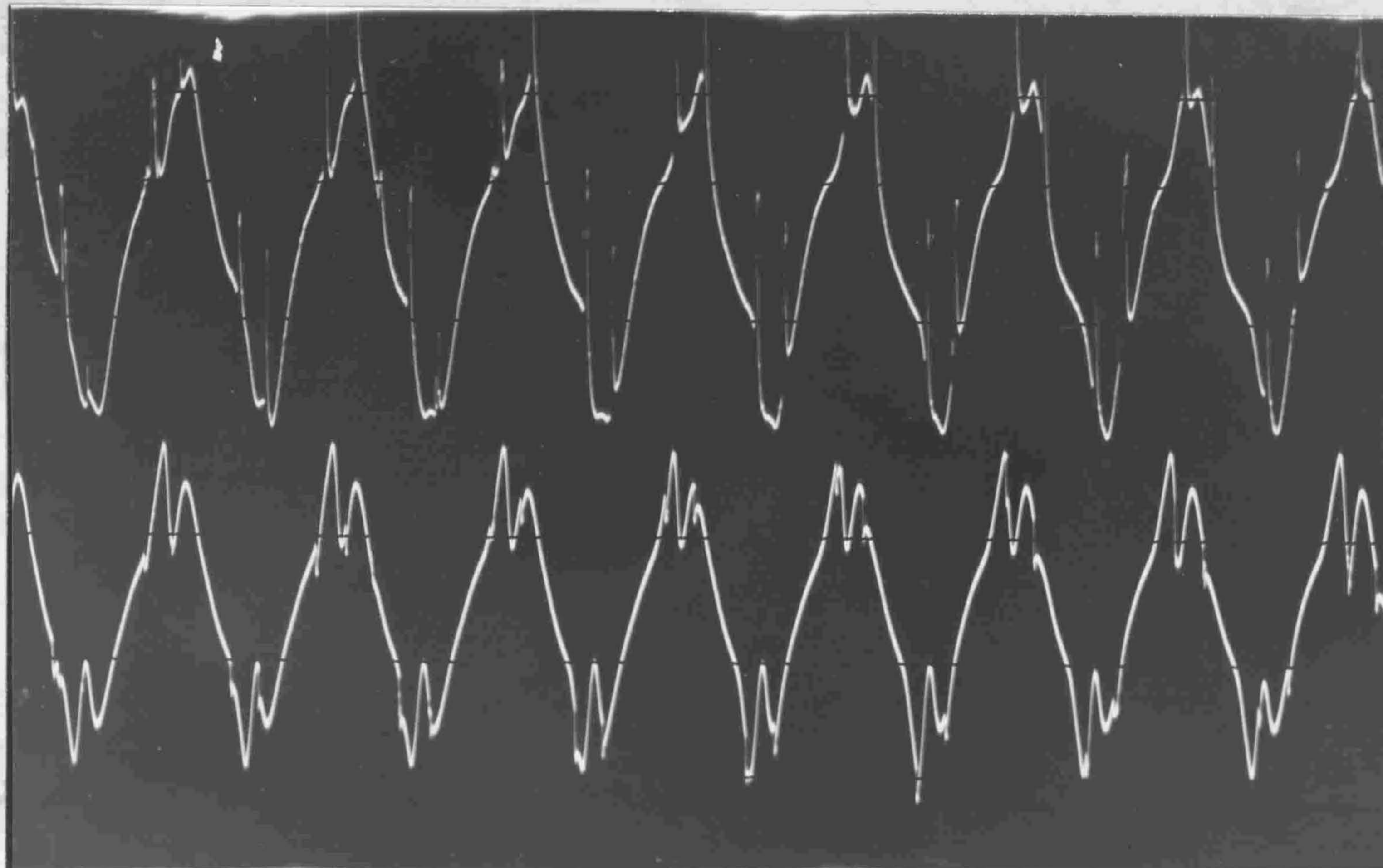


FIG. X-b OSCILLOGRAM OF VOLTAGE WAVES FROM ELECTRICAL PICKUPS

Oscillogram from Westinghouse Multi-element Oscillograph. Top curve is output from the driven end of specimen; lower curve is output from the vibrator end of specimen. Reading taken at 1500 rpm.

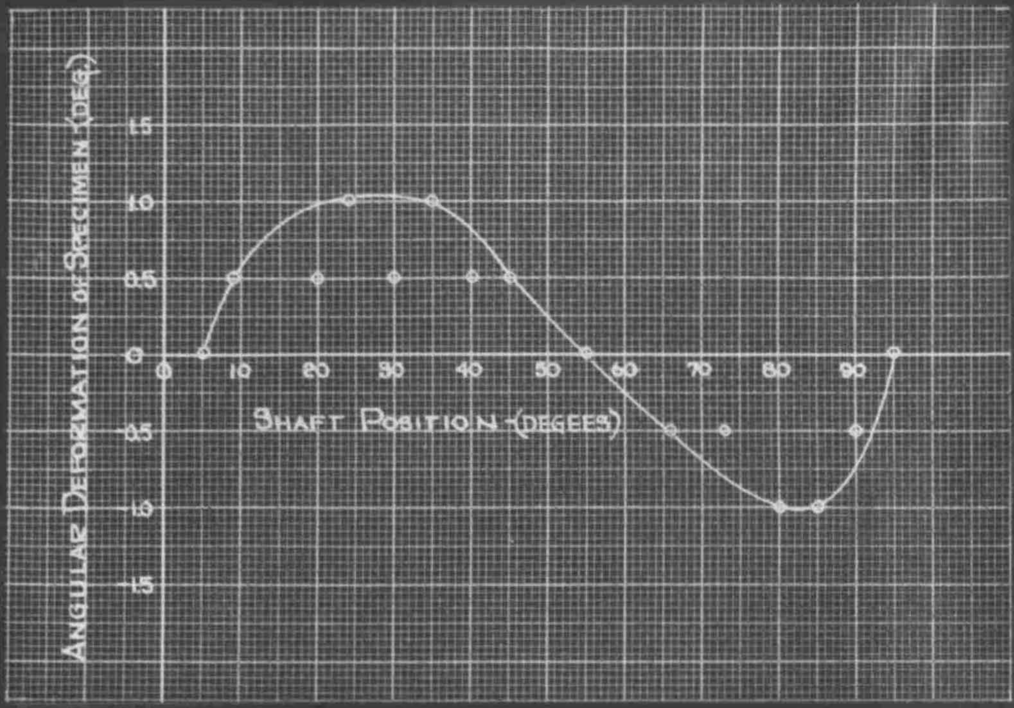


Figure XI

PLOT OF ANGULAR TWIST VS SHAFT POSITION

The readings from which this curve is derived are made visually by comparing pointers mounted at each end of the specimen with protractors mounted adjacent to each pointer. A stroboscopic light was used in order to see the pointers and an automatic contactor was devised to predetermine the position at which readings would be made. The accuracy of these readings is dependent on the accuracy of the protractors and the ability of the operator to interpolate readings between the actual lines appearing on the protractors. These are 1° apart and are read at a radius of 3 inches. Hence, the readings obtained are only approximate. This curve agrees with the predicted deformation curve except in amplitude. This is explained from the fact that the sample used was a high-carbon steel while the prediction was based on mild steel.