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THESIS

CONVOY SCREENING AGAINST A
MIXED SUBMARINE THREAT (U)

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and
Wayne P. Hughes, Jr.

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CONVOY SCREENING AGAINST A MIXED SUBMARINE THREAT (U)

by

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Submitted in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE
IN
OPERATIONS RESEARCH

United States Naval Postgraduate School
Monterey, California

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from the

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ABSTRACT

The problem of convoy defense against a mixed threat of conventional and nuclear submarines is posed. A general, conceptual method of solution is offered from which a simple analytical model is constructed. Use of the model for obtaining "optimal policies" of screening force disposition is demonstrated. Underlying assumptions of the model and related tactical problem areas are discussed and analyzed.

PREFACE

This thesis is intended to be read by both the operations analyst and the line officer. It also needs the critique of both. Chapters III and IV are primarily for the operations analyst, but the line officer should also read at least the summaries in Chapter III and the general formulation in Chapter IV. On the other hand, much of the analysis in Chapter V must be judged by the line officer, but is important to the operations analyst because it is vital to the analytical model.

A sample convoy screening problem was sent to Destroyer Squadron Commanders last fall when the study was begun. We wish to acknowledge a major debt to those Squadron Commanders listed in Appendix D who were kind enough to send solutions, and in many cases, offer further suggestions. Special appreciation is due to the other commands which made replies even though they were not requested. The ideas expressed permeate the entire thesis. Occasionally in the thesis, too, we have tried to correct what we regard as misconceptions, without reference to specific replies.

Mostly, however, we must thank our adviser, Professor W. P. Cunningham, whose counsel and encouragement have been so valuable during the past six months.

The thesis is CONFIDENTIAL because (1) reference is made in Chapter V to material in classified studies, and (2) the tactical concepts in Chapters V and VI, when taken together, would provide an unauthorized reader with insight into some aspects of current U. S. Naval tactics and thought. However, Chapters I through IV, and Appendices A, C, and D are unclassified.

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TABLE OF DEFINITIONS AND ABBREVIATIONS

Torpedo Danger Zone (TDZ) - The region around the convoy within which a sumbarine firing a torpedo has a positive probability of attaining a hit against the convoy.

Danger Zone - The area bounded by the detection circle, the torpedo danger zone, and the limiting lines of approach.

Detection Circle (R_d) - The expected distance from the convoy at which a submarine will have completed classification of the convoy and commenced closing.

Equi-threat Contour - The locus of points around the convoy at which a submarine, firing as many torpedoes as possible before neutralization, may be expected to achieve the same number of hits against the convoy.

Escort - A ship or helicopter of the screening force.

Iso-probability Contour - The locus of points around the convoy at which a submarine, firing a torpedo at the convoy, has the same probability of achieving a hit against the convoy. There are different sets of loci for aimed and random firings.

Limiting Lines of Approach -

1. LLA_o - The lines bounding the region ahead of the convoy within which conventional submarines have positive probability of closing the convoy and attaining a hit.
2. LLA - not uniquely defined. Loosely, the lines bounding the region ahead of the convoy within which conventional submarines represent a significant threat to the convoy.

Neutralize - To sink, cripple, drive off, or otherwise eliminate any further immediate threat of a submarine to the convoy.

Picket - An escort stationed outside the screen perimeter.

Pouncer - An escort stationed inside the screen perimeter.

RISK - The ratio of the expected number of hits which a submarine <u>attacking</u> the convoy will attain to the expected number of hits that a <u>conventional</u> submarine, having <u>penetrated</u> the screen, will attain.

Sonar Detection Range (R_w) - The detection range for which the expected number of submarine targets detected at a greater distance from the sonar is equal to the expected number of targets not detected at a lesser distance. It is roughly equivalent to the "effective sonar range", a term not use in the thesis. One-half the sweep width.

Split Bearings - The submarine tactic of attempting to pass between adjacent escorts at such distances as to increase its probability of undetected penetration of the screen.

Submarine Class - A "homogeneous class of submarines". A group of submarines for which the threat posed and the detection, tracking, and kill capabilities of the escorts for each is regarded as identical. In the thesis, two classes only are usually distinguished, conventional and nuclear.

Sweep Width (W) - The measure of detection capability for which the maximum detection range of a sweep is reduced so that the expected number of submarine targets detected beyond the sweep width is equal to the expected number of submarine targets missed inside the sweep width.

@ - (Read "circle a".) The probability that a submarine attacking the convoy is nuclear.

CHAPTER I

INTRODUCTION AND CONCLUSIONS

The thesis develops, in Chapter III, a basic concept for study of convoy defense problems when the threat to the convoy consists of submarines of widely different characteristics, and the defensive forces are composed of ships and aircraft, also of widely different characteristics.

Although the concept is applicable to nuclear as well as non-nuclear war, emphasis is on the latter. We have chosen to study the case in which enemy strategy is to attack merchant ships. Other strategies, such as weakening the screen by sinking escorts, are not specifically considered. We assume that the threat to the convoy from submarines predominates to the extent that the threat from other forces may be considered subordinately.

In Chapter IV the basic concept is applied to design an analytical model by which to study the disposition of a screen to best defend a convoy from a mixed threat of nuclear and conventionally powered enemy submarines. We attempt to choose and weigh appropriately the key parameters to arrive at an optimum allocation of forces inside and outside the effective limiting lines of approach, and at an approximate range from the convoy at which to set up the defenses in the two regions. The effectiveness of the screen in reducing the threat is also determined.

The model was used for calculations on a CDC 1604 computer and the results were analysed for various parameter changes. Examples are included to illustrate optimal tactical deployment of escorts under typical conditions.

In Chapter V ancillary problems that were studied in support of the analytical model are discussed.

The precept that guided us as we proceeded through the complexities of the problem to obtain some simple but useful results was this: certain key evaluations will greatly affect, or ought to affect, the escort commander's tactical decisions in deploying his escorts. The gross effects of his evaluations upon his decisions are amenable to mathematical analysis, and the result is precise and quantitative. The precision is an illusion, but the quantities serve as a point of departure -- a solid datum from which to combine calculations with the subtle information that experience and detailed intelligence provide.

Conclusions

1. Upon estimating the values of seven critical inputs, operationally useful values of the following quantities may be obtained:

a. The percentage of escort search effort to deploy within the region of threat from both conventional and nuclear submarines.

b. Within this region, the distance from the convoy at which to deploy the escorts.

c. In the region of exclusively nuclear submarine threat, the distance from the convoy at which to deploy the escorts.

d. The risk to the convoy under an optimal deployment.

The critical inputs are concerned with:

a. total escort sweep width.

b. comparativeescort screening effectiveness inside and outside of the limiting lines of approach

c. comparative threat of nuclear and conventional submarines.

d. fraction of nuclear submarines that will attack the convoy.

e. torpedo range.

f. outer limits of the convoy.

g. comparative threat of submarines penetrating the screen and submarines firing outside of the screen. (See Chapter IV-B)

2. Simple, concise optimal policies for escort commanders may be formulated as a function of the percentage of nuclear submarines expected to attack the convoy. (See Chapter IV-C)

3. The results are sufficiently insensitive to variations in the inputs that are the most difficult to estimate. Consequently considerable error may be absorbed without significantly affecting the optimal policies. (See Chapter IV-C)

4. Although some danger from conventional submarines exists as far aft as the limiting lines of approach, if submarines are forced to make a

submerged approach having once detected the convoy, the region of significant conventional submarine threat may be defined by arcs emanating from the center of the convoy, 60^{o} on either side of convoy track. This region may be regarded as independent of the ratio of convoy speed of advance to submarine speed for at least as long as convoy speed is as great as submarine speed. (See Chapter V-B)

5. There are important new weaknesses connected with using the sweep width concept as a basis for calculating screening effectiveness and stationing escorts. The "modified definite range law" affords a simple means of assessing operational detection probabilities that largely eliminates these weaknesses. (See Chapter V-D)

6. Because of the effect on the proper disposition of escorts for convoy defense, it is important to obtain additional experimental data comparing the effectiveness of sonars relative to various bearings of submarine approach on a convoy. (See Chapter V-E)

7. As a first approximation, escorts should be stationed a distance from the convoy at which the expected number of hits by a submarine firing from outside the screen is equal to the expected number of hits by a submarine attempting to penetrate the screen. Around the screen perimeter, adjustments inward and outward from that distance must be made to reflect submarine preference at the bearing concerned. (See Chapter V-F)

8. Because many aspects of convoy defense cannot be studied adequately either analytically or by exercises at sea, a necessary adjunct to investigations of convoy defense problems is war gaming, including computer war gaming. The concept developed in Chapter III may be adapted to the study of a wide variety of such problems. (See Chapter VI-A)

CHAPTER II

DISCUSSION OF THE PROBLEM

An important problem in ASW Warfare today is determining the optimal defense of a convoy or other screened force against a submarine threat consisting partly of conventional submarines and partly of nuclear submarines. Clearly, in the defense against conventional submarines, the traditional bent line screen concept remains valid, defenses being concentrated within the limiting lines of approach. Clearly, in the defense against only nuclear submarines, escorts must defend the flanks and rear as well as the van, and preliminary studies of this problem exist*. But the problem which is the most challenging and which remains unanswered is how to best defend against a mixed threat of both types.

A simple decision to defend all bearings equally is unsatisfactory. Weakening the van and spreading the defenses around all bearings for one or two nuclear submarines when there is a large number of conventional submarines threatening would be unsound. But what proportion of the threat should be at the flanks and rear before the defenses are changed? If experience has shown that, once it gets into the convoy, a nuclear submarine can be expected to sink twice as many merchant ships as will a conventional submarine, how, precisely, does this affect the problem? If the nuclear submarine is twice as likely to consummate a successful attack even if detected, how should the screen be modified? These and many more factors bear on the problem.

In the past, differences in detection ranges of the various types of escorts' sonars have been incremental, at least relative to the differences between sonar types today. The classical bent line screen tables assumed equivalent performance among all escort sonars, and spaced escorts on this assumption. Today wide differences between sonar detection ranges preclude such action. In this thesis part of the problem is to show how these differences may be accommodated in a simple fashion, and perhaps point the way for further study and analysis. Obviously, variations in sonar characteristics greatly affect any plan

*See [9], [14], and [15]

of escort deployment, which is the central problem of the thesis.

Longer range submarine torpedoes with greater accuracy add another dimension to the problem. It is optimistic to expect to have an escorting force large enough to protect the convoy at ranges which keep the submarine from firing from outside the screen without some probability of success. As submarine weapon ranges and accuracies become greater, the problem of stationing escorts at the proper distance from the convoy becomes more critical, and is an important element of the thesis.

At present, VS aircraft greatly assist in neutralizing a conventional submarine threat but are of lesser value against penetrating nuclear submarines. In general, aircraft effectivness against different submarine types may be expected to continue to differ widely. If VS aircraft are present and deployed optimally, how is their contribution to be accounted for by the escort commander? He can afford neither to ignore their presence nor exaggerate their role in convoy defense.

The escort commander may have very little intelligence on which to base his decision. Considering the probably heterogeneous nature of the ships in his force in wartime, he may even have very little informations about his own escorts -- who has the best sonar operators, whose sonar is likely to break down or operate at 50% effectiveness, who has a "hot" ECM team, and so forth. The thesis provides a framework on which to base a minimum-data solution. As more information devolves, incremental modifications can be built on the basic framework. At the same time, all information at hand can be weighed and fed into the decision process, either explicitly in a mathematical solution, or by modifications based on good judgment and experience.

Fundamental to the development of the thesis has been the authors' concept that a solution to a complicated tactical problem such as modern convoy defense must leave full power of decision with the tactical commander. Assumptions must be fully understood and procedures modified when the assumptions fail. The bent-line screen involves assumptions (for example, equivalent sonar performance, a specific effective torpedo range, straight-running torpedoes) which are not always appreciated. The procedure herein assists the tactical commander in reaching his decision, but requires him ultimately to make his own deployment. There is no alternative:

(1) Because of the complexity of the problem and the complexity of the possible situations, assumptions are frequently of fundamental importance and must hold. If, for example, the enemy's strategy is to sink escorts first, the situation is changed fundamentally.

(2) Tactical factors that are suppressed in a standard solution may become overriding, and the solution modified accordingly. If the opposing submariners prove to be excessively cautious, tending to fire from outside the screen, the protective belt should be moved out. If they demonstrate aggressiveness and consistently attempt penetration, the screen may be safely drawn in -- the belt tightened.

(3) Strategic considerations will affect a decision. If the escorts' primary mission is to destroy submarines rather than defend the convoy, the deployment must be reconsidered in that light.

To be useful in operational, tactical situations, any decision rule must be mathematically simple, and be independent of special shipboard equipment (we have computers particularly in mind). Therefore, precision must be sacrificed. But this sacrifice is easily justified on the grounds that computational procedures yield results no better than the worst of the input data, and input data is generally going to be very rough indeed.

Our objective then, is to provide a rapid, rough, analytical procedure which uses as inputs imprecise data (for some inputs, no data) to give the escort commander a useful means to deploy his heterogeneous force of escorts against a mixed submarine threat, in a manner more effective than now exists. The escort commander is confided with wide flexibility and full power to modify assignments to fit the circumstances.

Evolving the procedure has entailed:

(1) A careful review of assumptions, including an attempt to uncover all implicit assumptions.

(2) A comprehensive theoretical analysis of the problem, as

complete as possible, to provide a sound point of departure for simplification.

(3) Development of a relatively simple analytical method, with carefully considered and justified simplifications and approximations.

In judging the usefulness of the procedure developed in the thesis, it is important to retain in mind the concept of model building, along with the purpose and limitations of mathematical models. Although the model concept is thoroughly appreciated by operations analysts, it is not so well understood in the operating forces. A model is an attempt to simulate actual conditions (i.e., the natural environment) to the degree that it will yield predictions accurate enough to make application of the model worth-while. The more complex the actual situation, the more necessary it becomes to reduce the problem to what is viewed by the analyst as its essence, and the less accurate the prediction. The analyst's aspiration is to produce a method which will generate useful predictions. It is entirely possible that the prediction is valueless, not because of faulty mathematics or faulty logic, but because of a faulty conception of the problem's essentials. Usually the analyst can make a pretty shrewd estimate as to the model's effectiveness, that is to say, the precision of prediction (and the fact that the model's precision is low does not mean that its value may not still be high). But the interactions of nature are so complicated, and philosophically speaking, our perceptions of nature so artificial, that the accuracy of even relatively simple models must almost always be confirmed experimentally.

Finally, to those who are skeptical of the scientific study of war on the grounds that war is 5% science and 95% courage, sweat, and endurance, we say this: the margin of superiority which gives victory may be small indeed. We cannot afford to neglect the last 5% that science offers the soldier.

CHAPTER III

CONCEPT OF THE SOLUTION

In this chapter we present the fundamental characterization of the problem. First we review our basic assumptions. Second, we present in broad terms our concept of the solution.

A. Basic Assumptions

1. Escorts will be deployed to optimize ASW defense. We do not treat the effect of the air, missile, and surface threat on the problem.

2. We are analyzing convoy defense in non-nuclear war, although there is a carry-over to a nuclear war situation in which convoys are employed.

3. By leaving escort commanders wide latitude in the final assignment of ships and helicopters to screen stations, we assume that detection by an escort amounts to neutralization of the contact, or equivalently, that neutralization is directly proportional to detection.

4. Similarly, we do not consider the contribution of other sensors, radar, electronic countermeasures, etc., but leave the escort commander the problem (and the flexibility) to integrate their contribution.

5. Escorts are deployed for optimal detection, which is, roughly speaking, the greatest number of detections of the most dangerous submarines. We make no effort to measure the number of submarine sinkings, or the deterring effect of aggressive prosecution of contacts. Since these factors are closely linked to the detection rate, the distinction seems unimportant.

6. The attack of each submarine is treated as an independent event. We do not consider what the effect on defensive deployment might be of submarines concentrating tactically and conducting coordinated attacks.

7. The mission of the submarines is assumed to be convoy destruction. We attempt to maximize the defense of the convoy and never consider explicitly the security of the escorts. Again, however, the escort commander has wide latitude for providing for mutual support.

8. We use "hits on convoy shipping" as a measure of submarine threat to the convoy in preference to sinkings because (1) hits seem to be a better measure of effectiveness and (2) the probability of a hit is easier to determine than the probability of a sinking. We assume that no gross error results from treating the hit on one ship in the convoy as equivalent to a hit on another. In particular, we resist examining the intriguing problem of where best to put a CVS that is operating from within the screen. (But see Chapter VI-B for further comments).

9. We assume that an SSNK is not available to the defense.

B. Concept of the Solution

In outline, we first represent the threat to a convoy of a homogeneous class of submarines*. Second, the defensive capabilities of the escorting forces is determined for the class of submarines. Third, a deployment of forces to reduce the gross threat as much as possible is determined. Finally, the composite threat posed by all classes of submarines present, the capabilities of the escorts to reduce the composite threat, and the optimal deployment to minimize the composite threat is determined.

1. Characterizing the Gross Threat Represented by a Class of Enemy Submarines

SUMMARY: A given submarine which successfully reaches a convoy can expect to hit, on the average, a certain number of merchant ships before it is driven off or breaks off its attack. After the submarine completes its approach to the convoy, the submarine will not, in general, be equally likely to attack from any direction. Therefore it is important to consider the probability of its attack from each bearing. The probability that it will attack from a certain bearing times the number of hits it will achieve is our measure of the gross threat from that bearing.

(a) The gross threat represented by submarines of a homogeneous class is defined to be the number of hits E on convoy shipping a submarine will achieve if it reaches a convoy in the absence of a screen. See basic assumption 8 above.

*By a homogeneous class we mean a group of submarines for which the threat posed and the detection, tracking, and kill capabilities of the escorts for each submarine are essentially the same.

(b) Submarines detecting the convoy at their detection range will do so with some probability distribution $s_1(\theta)$ for each bearing θ.

(c) Part of the submarines detecting the convoy will be able to close successfully to the vicinity of the screen perimeter. Successful submarines will be distributed with some probability $s_2(\theta)$. Involved in the distribution is their ability to intercept the convoy from the point of detection, the probable closing paths, and attrition due to any cause: VS aircraft, picket ships, etc.

(d) There may be another, separate probability distribution $s_3(\theta)$, for the bearing of attack for submarines reaching the convoy vicinity. If we assume submarines essentially cannot shift their bearing after closing, then $s_3(\theta) = s_2(\theta)$. If we assume submarines have complete freedom to chose their attack bearing, and in addition, are able to detect the weakest point of the screen, then the distribution of attacking submarines would be the one which maximized their opportunity for success against the defensive disposition, in which case the minimax solution of a game theory problem is involved. In any case, we have made the important assumption that once the submarine has selected a bearing of attack, it is committed to closing the center of the convoy on an essentially constant bearing.

(e) The gross threat $E(\theta)$ from every bearing is therefore the probability $s_3(\theta)$ that a submarine of the class attacks on bearing θ times the expected number of hits E the submarine would achieve if it were to reach the convoy: $E(\theta) = E \cdot s_3(\theta)$.

2. Representing the Defensive Capabilities of the Screen

SUMMARY: Each escort has a certain probability of detecting a given submarine when the submarine passes near enough to the escort. The probability is usually characterized by a lateral range curve, which may be visualized as a quantity of detection capability spread on either side of the escort. This detection capability may be thought of as removing the threat of a proportion of submarines attempting to penetrate the screen within detection range of the escort -- screening out a fraction of submarines. The screening effectiveness of an escort depends upon a complex interaction of factors such as characteristics of the sonar, location of the escort, and tactics of the submarine. The detection capability under these varying conditions is our measure of effectiveness of each escort.

(a) The measure of defense is defined as the sonar detection capability of the escort screen consisting of surface ships and helicopters integrated into the screen. See basic assumptions 2 and 3 above. For each bearing θ and range r there is an applicable lateral range curve $y_k(\phi;\theta,r)$ for the k^{th} escort. A lateral range curve $y_k(\phi;\theta,r)$ depends on, among other things, submarine penetration depth and speed, water conditions, manner in which escort is patrolling, station sonar operator performance, and wake effect astern.

(b) Each escort will detect submarines attempting to penetrate in its vicinity according to its lateral range curve. An escort stationed at (θ_k,r_k) will detect a submarine attempting to penetrate the screen with probability

$$S_3(\phi) \cdot y_k(\phi;\theta_k,r_k)\, d\phi \quad .$$

3. Deploying Defenses to Reduce the Danger as Much as Possible

SUMMARY: To deploy the escorts most effectively against a given submarine, they should be stationed where they will achieve the most detections, which is a balance between where the escorts' screening effectiveness is greatest, and where the submarine is most likely to attack. In addition, the escorts must be stationed far enough from the convoy to offer the submarine no advantage by firing from beyond detection range, but not so far as to offer it an advantage by penetrating a screen which has been weakened too greatly by stationing escorts far from the convoy. The perimeter around the convoy which gives the submarine no advantage of choice is a line of defense along which to station the escorts. The escorts should be stationed along the perimeter so as to achieve the most detections. This is equivalent to reducing the threat of a given submarine by the greatest amount. For any screen disposition, the reduction of threat is our measure of effectiveness of that disposition.

(a) Ignore momentarily the danger from a submarine firing from outside the screen. Then for maximum detection, escorts will be stationed along a perimeter at the minimum range $r_m(\theta)$ from the convoy for which they can successfully close and neutralize a detected penetrator. Define <u>threat</u> $T(\theta)$ to be the expected number of hits a submarine on bearing θ will achieve when escorts are present. The escorts present provide some probability $Y(\theta)$ of detecting a submarine attempting penetration at bearing θ. Therefore $T(\theta) = E(\theta) \cdot (1-Y(\theta))$. Threat from all bearings, Th, is minimized by stationing escorts (by trial and error, dynamic programming, applying the theory of optimum allocation of search effort*, or any other procedure) at bearings such that the sum of the threats from every bearing, $Th = \sum_{\theta} T(\theta)$ is minimized. Since E is defined to be independent of θ, for this case it is equivalent to say that the threat Th is minimized when the overall probability of detection $P = \sum_{\theta} s_3(\theta) \cdot Y(\theta)$ is maximized.

(b) At every bearing θ and range r, a submarine can attain an expected number of hits $h(\theta,r)$ on the convoy if it chooses to fire from outside the screen. The threat from such a submarine may be characterized as the expected number of hits times the probability $w(\theta,r)$ that the submarine chooses to fire from outside rather than attempt penetration times the probability that the submarine is there:

$$T'(\theta,r) = h(\theta,r) \cdot w(\theta,r) \cdot s_3(\theta) .$$

The probability w is a highly subjective decision on the part of the submarine commanding officer that depends on such factors as the aggressiveness or timidity of the commanding officer, the value of the submarine compared with the value of the merchant ships he may sink, the risk of the submarine being sunk for each alternative, and the strength of the screen opposing.

(c) It may be assumed that if the submarine does not fire from outside the screen it will attempt penetration with probability $(1 - w)$. The threat of a submarine at (θ,r) which chooses to attempt penetration is therefore the probability that a submarine is on bearing θ times the expected number of hits that the submarine will achieve if it reaches the convoy times the probability that it evades detection by the screen times the probability that the submarine attempts penetration:

$$T(\theta,r) = s_3(\theta) \cdot E \cdot (1 - p(\theta)) \cdot (1 - w(\theta,r)) .$$

*See [5], or Section 3.3 of [4]

The threat $Th(\theta,r)$ for any bearing and range is then:

$$Th(\theta,r) = T(\theta,r) + T'(\theta,r) \quad .$$

(d) Conceptually, for every bearing there is a range from the convoy r_o at which to station escorts yielding $Th(\theta,r_o) \leq Th(\theta,r)$ for all $r \geq r_m$. A line connecting these ranges forms a perimeter around the convoy on which the escorts should be stationed. The solution to the problem of minimizing the threat of a single class of submarines over all bearing θ and ranges $r \geq rm$ is to assign stations on this perimeter which will minimize the threat over all bearings: the solution is to assign the K escorts to stations (θ_k, r_o) such that

$$Th = \sum_{\theta} Th(\theta,r_o) \text{ is minimized.}$$

4. Solution for Two or More Classes of Submarines

SUMMARY: Submarines of sufficiently similar tactical characteristics are treated as being of the same class. (In our analytical development we distinguish only between conventional and nuclear submarines.) The gross threat of each class and the escort detection capability against each class is determined. The perimeter must now be found that nullifies as much as possible the advantage a submarine of any class may have by attacking from outside or inside the screen. This should be done by weighting the decision in proportion to the likelihood that an attacking submarine is of one class or another and the relative threat of the class. Around the perimeter, escorts should next be deployed with regard for the relative threat of a successfully attacking submarine class and the proportion of submarines of that class that may be expected to attack on each bearing. Again, for any screen disposition, the reduction of threat is our measure of the effectiveness of that disposition, and the disposition which reduces the threat to a minimum is the solution to the problem.

(a) For each class of submarines, m, the fraction $@_m$ of submarines that will reach the perimeter of the convoy is determined. For each class, the probable bearing of attack $s_{3m}(\theta)$ is determined. Finally, for each bearing, the probability that an attacking submarine is of the m^{th} class is determined:

$$@_m(\theta) = s_{3m}(\theta) \cdot @_m \quad .$$

(b) For each class of submarines, the danger $E_m(\theta)$, the screen lateral range curves, $y_m(\emptyset;\theta,r)$, the expected hits from firing from outside the screen $h_m(\theta,r)$, and the aggressiveness factor $w_m(\theta,r)$ are determined.

(c) For each class of submarines there may be represented a threat from the fraction firing outside the screen $T'_m(\theta,r)$, a threat from the fraction attempting penetration $T_m(\theta,r)$, and a total threat for the class at the bearing θ:

$$Th_m(\theta,r) = T_m(\theta,r) + T'_m(\theta,r)$$

The threat for all classes on bearing θ is $Thr(\theta,r) = Th_m(\theta,r) \cdot @_m(\theta)$.

(d) Again we conceive of a range $r_o(\theta)$ at which to station the escorts to minimize the threat along bearing θ, and establish a perimeter line around the convoy. The concept of the solution of the problem of minimizing the threat of an attacking submarine of any class is to assign stations on this perimeter that will minimize the threat over all bearings: assign the K escorts to stations (θ_k,r_o) such that

$$Thr = \sum_{\theta} Thr(\theta,r_o) \text{ is minimized.}$$

CHAPTER IV

THE ANALYTICAL MODEL

We shall present two models. Model I is a simple first approximation which serves to introduce certain assumptions and illustrate the authors' method of evaluating screen disposition. It also provides a natural lead-in to Model II. Although some of the implications of Model I have a degree of validity, the conclusions in this paper are based upon the results of Model II.

A. Model I

Definitions

1. E_c...... the average (expected) number of hits which a conventional submarine, having penetrated the convoy screen, will score against the convoy. We assume that all conventional submarines may be treated as one class. E_c presumably will vary with convoy size, speed, disposition, and location; however, for a given set of conditions it represents a fixed (possibly unknown) number.

2. E_n...... as above, for a nuclear submarine.

3. k....... the ratio E_n / E_c . That is, $E_n = k \cdot E_c$.

4. LLA..... the limiting lines of approach. These are applicable to conventional submarines only and are defined by the ratio of submerged submarine speed to convoy speed. See Chapter V - B for a fuller discussion of the LLA.

5. U....... the angle at the front of the convoy subtended by the LLA. Typically, we consider U = 120 degrees. Again, see Chapter V-B.

6. sector 1.....the region bounded by and lying within the LLA.
   sector 2.....the region lying outside the LLA.

7. P_1...... the probability that the screen detects a submarine attempting penetration in sector 1.

   P_2...... as above, for sector 2.

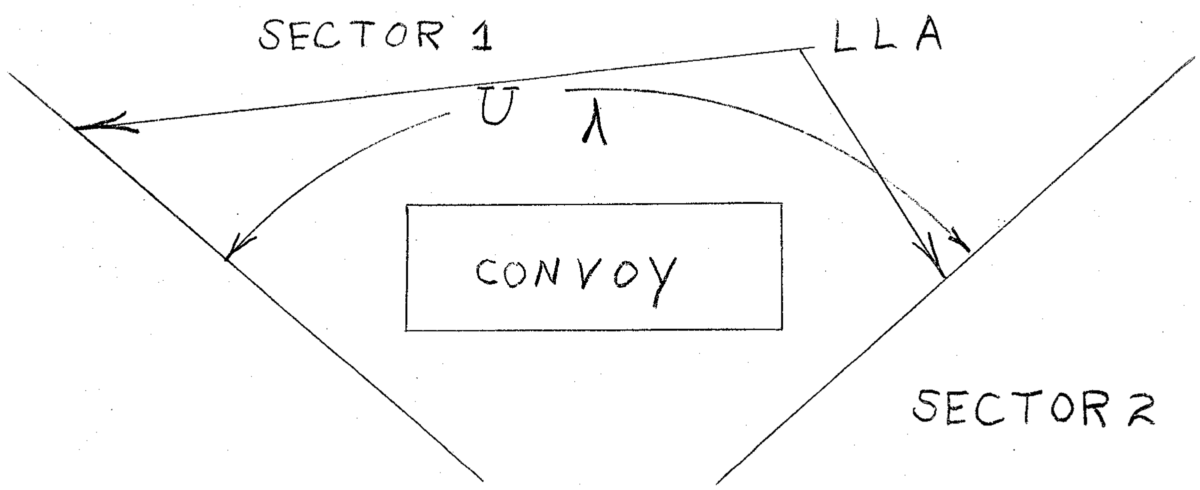


Figure 1

8. THR_1... the threat from sector 1 due to submarines attempting penetration. THR_1 = the expected number of hits suffered per submarine attacking via sector 1.

THR_2... the threat from sector 2.

THR.... the total threat. $THR = THR_1 + THR_2$.

9. @...... the probability that an attacking submarine is nuclear. In general, this is not simply the ratio of the nuclear submarines in the area to the total submarines in the area. @ depends not only upon that ratio, but also upon submarine disposition, convoy disposition and speed, the geography of the transit area, and other factors. See Chapter V-A.

10. RISK...THR / E_c .

Assumptions

These assumptions are in addition to the basic assumptions made in Chapter III.

1. P_1 is proportional to the amount of effective force assigned to sector 1: P_2 is proportional to the amount of effective force assigned to sector 2.

2. The attack bearing for a nuclear submarine is equally likely around the compass.

3. P_1 is not a function of submarine type. That is, a screen element is as likely to detect a conventional submarine as a nuclear submarine.

Analysis

In this model, the quantity RISK is our measure of screen effectiveness. For example, RISK = 1.00 says that the effectiveness of the screen is such that an attacking submarine will, on the average, inflict the same number of hits as would a conventional submarine which had successfully penetrated the screen. RISK is, among other things, a function of @, the probability that an attacking submarine is nuclear. It is to be expected that RISK increases as @ becomes larger. The rate of this increase depends upon the value of k, a measure of the nuclear submarines' superiority to the conventional submarine. RISK is reduced by the presence of defensive elements which either dissuade the submarine from pressing the attack or kill the submarine. As discussed in Chapter III, for the purposes of this paper we regard the detection of the attacking submarine as tantamount to the successful defense against that particular attack effort. Hence, RISK is directly affected by the values of P_1 and P_2. The purpose of this model, and in greater detail, Model II, is to investigate the dependence of RISK upon @, k, P_1, and P_2.

The model

If there were no defensive force at all, the threat, THR, would be given by

THR = (probability sub is conventional) x (expected number hits due to conventional sub) + (probability sub is nuclear) x (expected number hits due to nuclear sub) or,

$$THR = (1 - @) \cdot E_c + @ \cdot E_n$$

$$= (1 - @) \cdot E_c + @ \cdot k \cdot E_c$$

$$= (1 - @ + @ \cdot k) \cdot E_c .$$

The event of a submarine attacking can be split into the two mutually exclusive events that it attacks via sector 1 or that it attacks via sector 2. THR can also be so split. THR_1, the threat in sector 1, is

$$THR_1 = (1 - @) \cdot E_c \cdot \text{(probability conventional sub attacks in sector 1)} + @ \cdot k \cdot E_c \cdot \text{(probability nuclear submarine attacks in sector 1)}$$

or

$$THR_1 = (1 - @) \cdot E_c \cdot 1 + @ \cdot k \cdot E_c \cdot (U/360) ,$$

where U is the angle in degrees subtended by the LLA, and the nuclear submarines have equal probability of attacking from any bearing. Similarly, for THR_2 we have,

$$THR_2 = @ \cdot k \cdot E_c \cdot (360 - U)/360 ,$$

since the probability of a conventional submarine attacking via sector 2 is zero.

Now if there is a defensive force, THR_1 and THR_2 must be reduced by the detection probabilities which the screen elements provide. Hence if the force assigned to sector 1 provides an over-all probability P_1 of detecting a submarine attempting penetration in sector 1, the probability that the submarine escapes detection is $(1 - P_1)$, and THR_1 becomes

$$THR_1 = (1 - P_1) \cdot (1 - @ + @ \cdot k \cdot U/360) \cdot E_c .$$

Similarly, if P_2 is the overall probability of detecting a submarine attempting penetration in sector 2, THR_2 becomes

$$THR_2 = (1 - P_2) \cdot @ \cdot k \cdot E_c \cdot (360 - U)/360 .$$

Using the fact that $THR = THR_1 + THR_2$, and that $RISK = THR/E_c$, we have

$$RISK = 1 - P_1 + @ \left[(1 - P_1) \cdot (Ak - 1) + k \cdot (1 - A) \cdot (1 - P_2) \right] ,$$

where $A = U/360$.

This is a linear function in @. Figure 2 is a plot of RISK versus @ for the case $k = 2$, and $A = 1/3$ (i.e., $U = 120$ degrees). The defensive force has been assumed to be of sufficient size so that when it is assigned in its entirety to sector 1, $P_1 = 1$. Since sector 2 is twice as large as sector 1 when $A = 1/3$, when all of the force is assigned to sector 2, $P_2 = 1/2$. (This is assuming that there is no change in the effectiveness of the defensive element due to station bearing.)

Figure 2 depicts case 1. In this case, the following relation holds:

$$P_1 + 2P_2 = 1, \quad 0 \leq P_1 \leq 1, \quad 0 \leq P_2 \leq 1/2.$$

The following plot shows that for case 1, minimum RISK occurs when $P_1 = 1$, (hence $P_2 = 0$ from the above equation) for all values of @. That is, if we define optimum policy as that disposition (as a function of @) of the defense force which minimizes RISK, then for case 1 the optimum policy is to always assign the entire defensive force to sector 1.

The dashed line in Fig. 2 represents the RISK when the entire

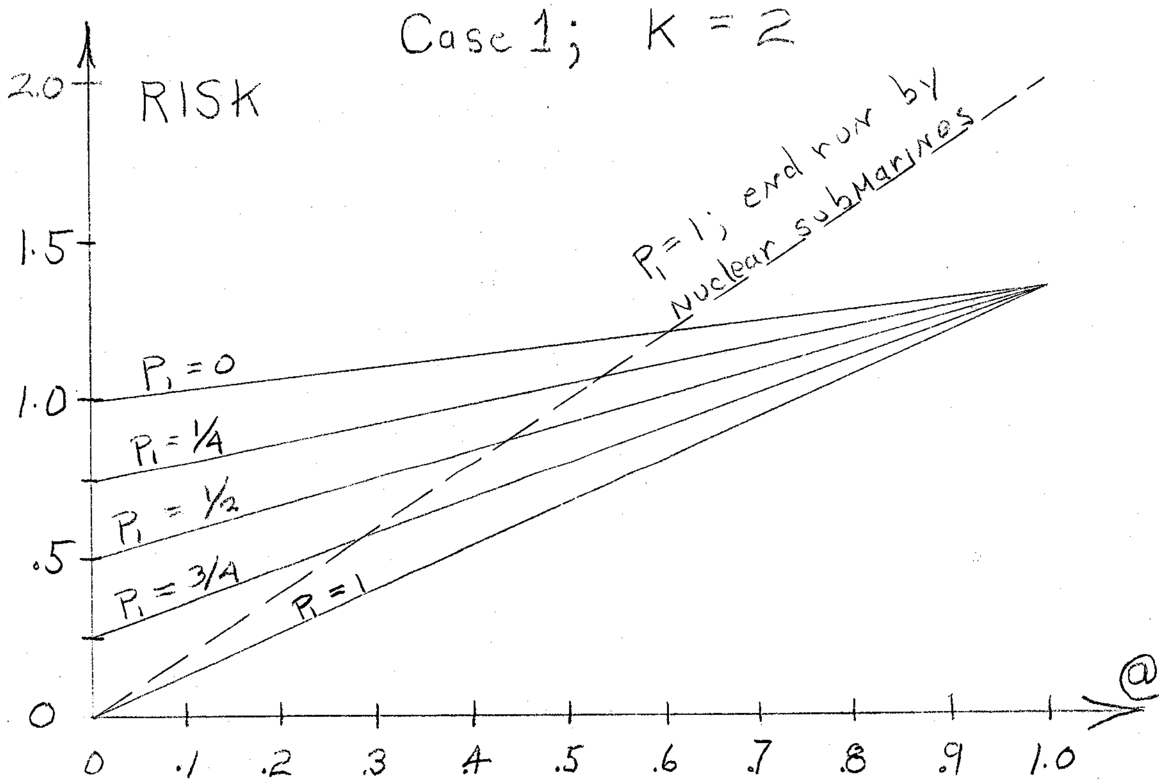


Figure 2

force is assigned to sector 1 and this fact is recognized by the nuclear submarines, hence they attack only from sector 2. (Hitherto, in accordance with Chapter III, we have assumed that the attacking submarines have no information as to the disposition of the defensive force.) Fig. 2 shows that even in this situation, for low values of @ the optimum policy is still to assign the entire defensive force to sector 1.

As an example, suppose the nuclear submarine can recognize when the entire force is protecting sector 1, and therefore it chooses to attack from sector 2. Suppose, however, that it can no longer make this distinction when 3/4 of the force is in sector 1 and 1/4 is in sector 2; in this event, it is equally likely to attack from any bearing. Then, according to Fig. 2, until we believe that the probability of an attacking submarine being nuclear is greater than .25, our optimum policy is to put our entire force in sector 1.

In Chapter V-C we discuss the effect of escorts patrolling stations upon their detection capabilities. It suffices for now to say that this effect is to increase an escort's capability, particularly when the escort is stationed on the beam or at the rear of the convoy. For simplicity, we assume there is number, which we call an effectiveness factor, which relates the effectiveness of escorts stationed in sector 2 to those stationed in sector 1, when the escorts are patrolling stations.

With this effectiveness factor in mind, consider the following examples, Cases 2 and 3.

Case 2: The overall effectiveness of an escort in sector 2 is twice that of an escort in sector 1.

Case 3: the overall effectiveness of an escort in sector 2 is three times that of an escort in sector 1.

For these cases, the following relationships hold:

Case 2: $P_1 + P_2 = 1, \; 0 \leq P_1 \, , P_2 \leq 1.0$

Case 3: $P_1 + P_2 \, / \, 2 = 1 \, , \; 0 \leq P_1 \, , P_2 \leq 1.0$.

Fig. 3 and Fig. 4 are plots of Cases 2 and 3 respectively.

Fig.'s 3 and 4 illustrate the effect of variation of effectiveness of escorts due to bearing of station assignment. In Fig. 2, all the lines for different values of P_1 intersected at a point for which @ = 1.0 . In Fig. 3, however, the point of intersection has moved to @ = .6; in Fig. 4, to @ = .3+ .

The points of intersections represent the boundaries of decision spaces. For example, in Case 2 (Fig. 3), to achieve optimal policy use the following rule:

@ $\leq$ .6, assign the entire defensive force to sector 1.

@ $>$ .6, assign the entire defensive force to sector 2.

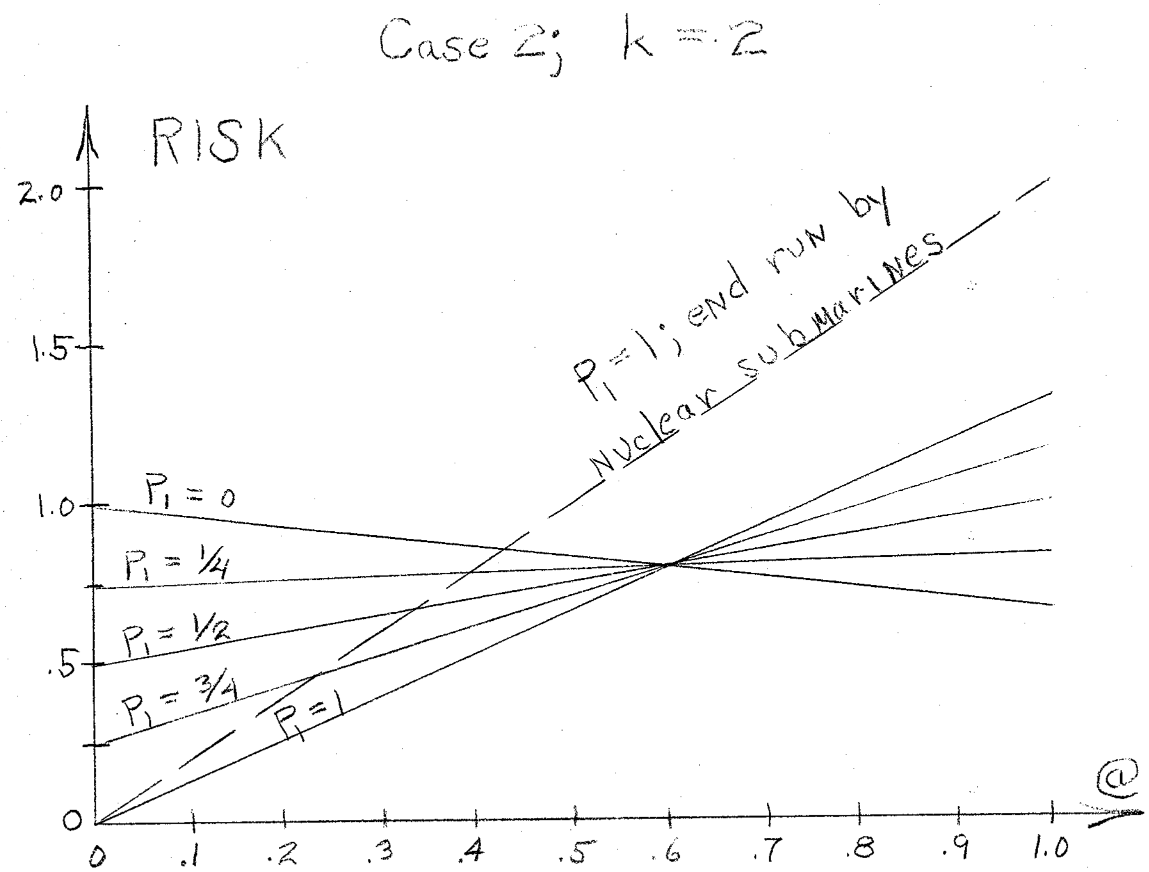


Figure 3

Again, the dashed lines represent the RISK when all the nuclear submarines attack via sector 2, given that the entire defensive force is stationed in sector 1. If we make the same assumptions as we did in Case 1 regarding the nuclear submarines' ability to discern the disposition of the defensive force, our rule for optimal policy becomes

for @ $\leq$ .22, put the entire force in sector 1.

.22 $<$ @ $\leq$ .6, put 3/4 of the force in sector 1, 1/4 in sector 2.

@ $>$ .6, put the entire force in sector 2.

Case 3; k = 2

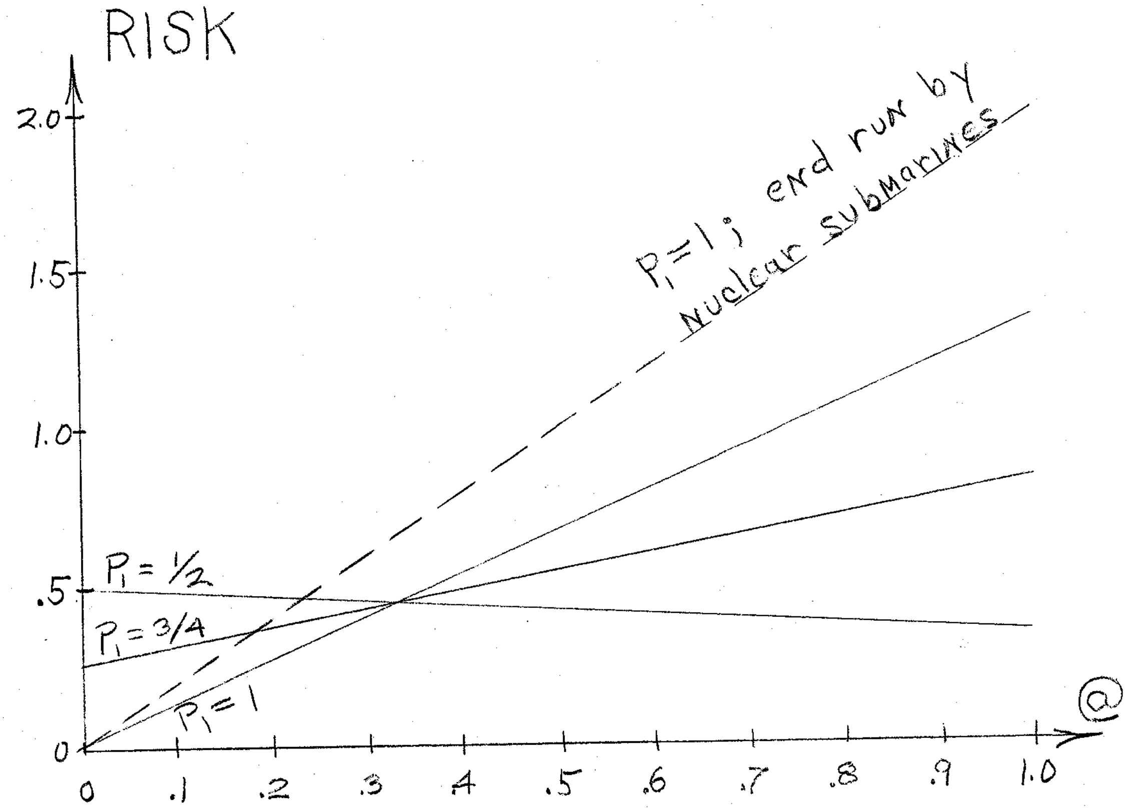


Figure 4

As noted above, the three cases plotted were for k = 2. The effect of increasing k is to shift the points of intersection to the left and to increase (for given @) RISK.

Conclusions

This model is too elementary to warrant any conclusions as to the optimal policy of convoy escort disposition. It does, however, demonstrate the utility of RISK (THR / E_c) as a measure of effectiveness for the assignment of forces. In particular, by minimizing RISK, we reduce the problem of force disposition to a function of @. It remains to construct a more detailed model, retaining this attribute, but including the other variables which vitally affect the solution.

B. Model II

Discussion

Model I was a method of evaluating RISK (THR / E_c) as a function of @, the probability of an attacking submarine being nuclear, given that the defending forces were deployed so that there was a probability P_1 of detecting an attacking submarine in sector 1 and a probability P_2 of detection in sector 2. The model did not specify how to divide forces (i.e., search effort) between sector 1 and sector 2, where to station them, or how to determine P_1 and P_2. Model II endeavors to answer these questions.

To achieve a model which yielded manageable mathematics the authors were obliged to make a number of simplifying assumptions. Several of these assumptions we were able to justify analytically. For others, the justification must be that the errors introduced by the simplifications are not so significant as to destroy the value of the model.

Before listing these assumptions, we introduce several definitions to supplement those presented in Model I.

Definitions

1. H_o..... a radius measured from the center of the convoy describing a circle about the convoy inside of which the defending forces will <u>not</u> be assigned. It is approximately the boundary of the convoy, perhaps several thousand yards larger.

2. RT..... the range of the torpedoes employed by the attacking submarines.

3. HL.....HL = H_o + RT. It is approximately the radius of the Torpedo Danger Zone as defined in NWP - 24A [8]. That is, it is the extreme limit of possible torpedo danger.

4. Y_i..... the sweep width of the ith defensive force element. (See Chapter V-C.)

5. Y.....Y = $\sum_i Y_i$ = the total sweep width available in the defending forces.

6. X..... the amount of total sweep width allocated to sector 1.

(Y - X is then allocated to sector 2.)

7. T..... T = X / Y · 100 = % of total sweep width available assigned to sector 1.

8. α .... detection effectiveness factor for forces patrolling station in sector 2 relative to sector 1. For example, if forces patrolling in sector 2 are twice as effective as if they were in sector 1, then α = 2. (See Chapter V-C for further discussion.)

9. R_1..... the distance from the center of the convoy at which forces assigned to sector 1 are stationed.

R_2..... the distance at which forces in sector 2 are stationed.

10. THR'.... the threat (expected number of hits suffered) due to a submarine firing at the convoy from outside the screen.

THR_1' .... as above, for sector 1 only.

THR_2' .... as above, for sector 2 only.

Assumptions

1. The convoy is circular with radius equal to H_0 .

2. The LLA emanate from the center of the convoy. This is a radical departure from the classical concept of limiting lines of approach. It is justified in detail in Chapter V-B.

3. Equi-threat contours for submarines firing from outside the screen are circular and concentric about the convoy. (See Chapter V-F.)

4. THR' does not depend on submarine type (i.e., nuclear or conventional). Since the same type of torpedoes will probably be used by both types of submarines, and since submarine maneuverability, speed, and underwater capability do not enter appreciably into this type threat, this assumption seems justified.

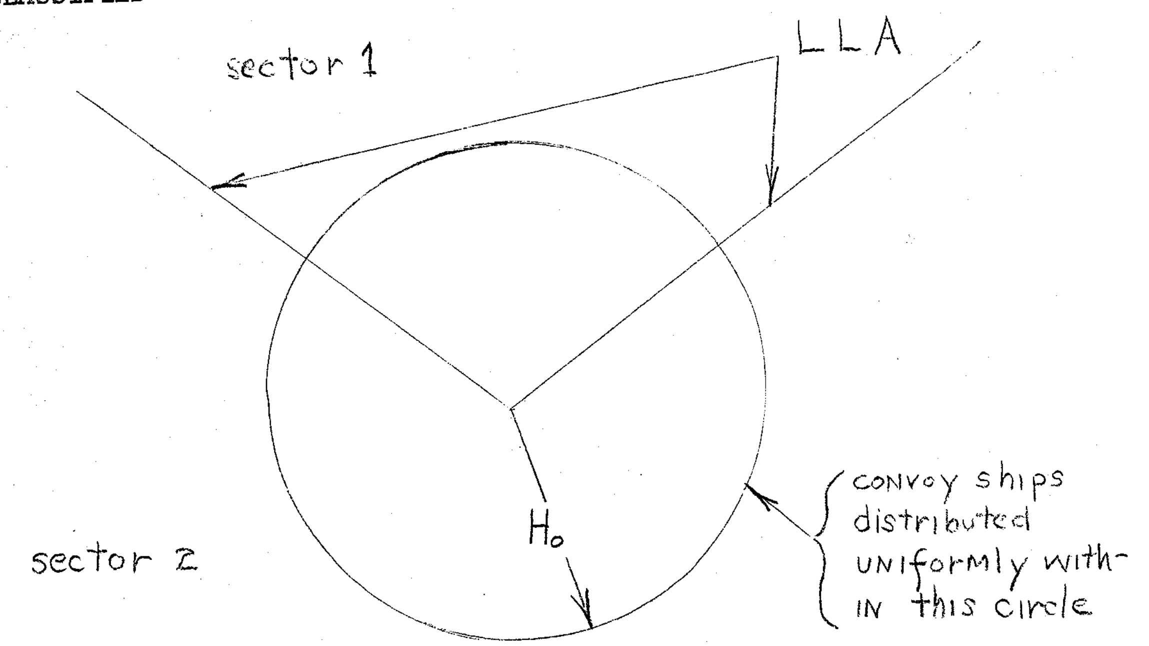


Figure 5

5. THR' varies linearly with R, the distance of the submarine from the center of the convoy. Specifically, we assume the relation

$$THR' = e \cdot E_c (1 - R/HL), \text{ for } H_o \leq R \leq HL$$

$$= 0, \quad \text{for } R > HL$$

and is undefined for $R < H_o$.

The scaling factor e is defined below.

Thus, when the submarine is at or beyond HL, it is out of range and THR' is equal to zero. At the perimeter of the convoy, $R = H_o$, and

$$THR' = e \cdot E_c \cdot (1 - H_o/HL)$$

$$= e \cdot E_c \cdot (HL - H_o)/HL$$

$$= e \cdot E_c \cdot RT / HL,$$

since $HL = H_o + RT$. The value of the scaling factor e represents an evaluation of the real threat due to a submarine firing from outside the screen. It is apparent that the maximum threat of this nature

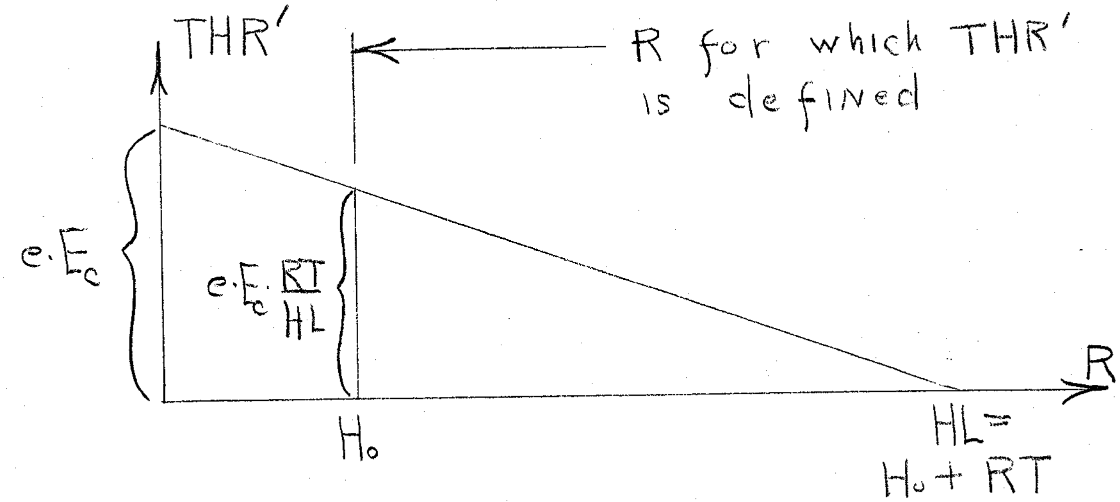


Figure 6

occurs when the submarine is at the perimeter of the convoy. It is equally apparent that a submarine firing random shots from outside the screen, even if at the convoy perimeter, cannot be more of a threat than if it were loose inside the convoy. Hence the upper bound on THR' is E_c, which occurs when e = HL / RT. For a torpedo range RT = 15000 yards, a convoy radius of 10000 yards, e = HL / RT = 1.4, which is the upper bound on e for that particular convoy size and torpedo range. The lower limit of e is of course zero.

Objective

We define an optimal policy to be that allocation and disposition of the available defensive force which minimizes RISK (THR / E_c). We wish our model to give us an optimal policy with respect to @ for any assigned force Y. That is, given a force having a total detection capability Y (yards of sweep width), the model should tell us what portion (X) to assign to sector 1 and at what distance (R_1), and at what distance (R_2) to assign the remaining force (Y - X) in sector 2, in order to suffer the smallest expected number of hits from attacking submarines.

Analysis

Since we have assumed the definite range law as characterizing the sonars of the defensive forces, an escort having a sonar of sweep width Y_i may be regarded as a barrier of length Y_i. Any submarine crossing this barrier will be detected with probability equal to one.

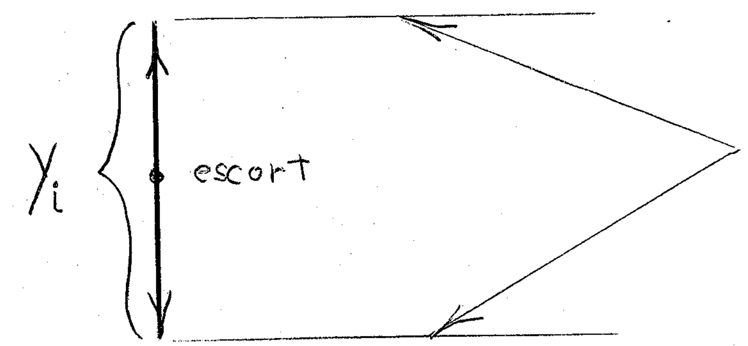


Figure 7

This barrier may also be regarded in an angular sense.

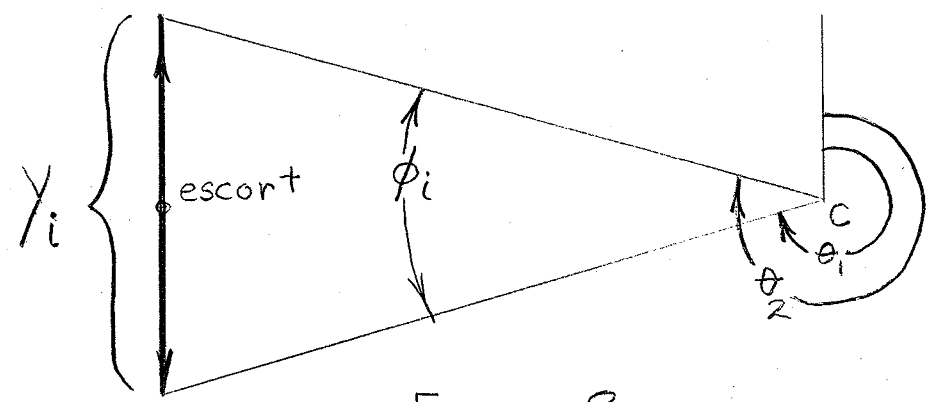


Figure 8

Any submarine approaching point C on a constant bearing between θ_1 and θ_2 is bound to cross the barrier and hence will be detected. If the escort is at a distance R from C, the angular width it protects with its linear barrier Y_i is

$$\emptyset_i = 2 \arctan (Y_i / 2R) .$$

Now if this one escort were screening in an angular sector U degrees wide, and the submarines were equally likely to approach C on any bearing within the sector U, the probability P_i of detecting a submarine approaching C on a constant bearing would be

$$P_i = \emptyset_i / U .$$

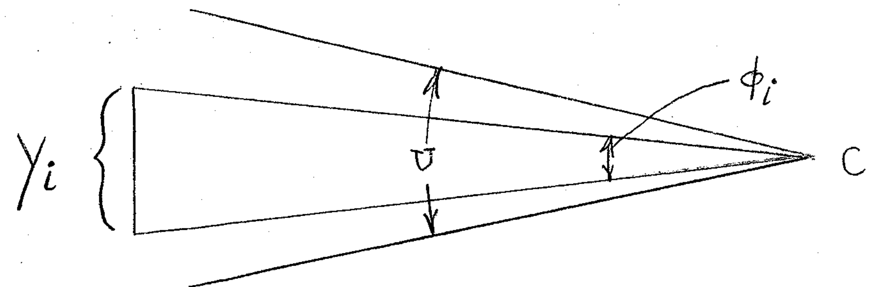


Figure 9

Now let there be N defensive units stationed at a distance R from C in the sector U. If we allow no overlap of their search effort and assume the independence of each unit from the others (in a probabilistic sense), the overall probability P of detecting a submarine approaching C on a constant bearing within the sector is the sum of the probabilities of each individual unit detecting the submarine. That is,

$$P \doteq \sum_{i=1}^{N} P_i = 1/U \cdot \sum_{i=1}^{N} \emptyset_i$$

If the Y_i are not too large with respect to R, we may make the approximation

$$2 \arctan (Y_i / 2R) \doteq Y_i / R \text{ (radians)}$$

$$\doteq 180 \cdot Y_i / \pi \cdot R \text{ (degrees)} .$$

For $Y_i = 10000$, $R = 10000$, the error is about 8%. For $Y_i = 6000$, $R = 10000$, the error is about 3%. (These values represent about the worst that might be encountered.)

Our probability is now

$$P = 180/(\pi R U) \cdot \sum_{i=1}^{N} Y_i$$

or

$$P = 180/(\pi R U) \cdot \text{(total sweep width available)} .$$

Thus, in sector 1, if X represents the total sweep width assigned to that sector, R_1 the distance from the center of the convoy at which the escorts are stationed, and U the angle subtended by the LLA, then P_1, the probability of detecting a submarine penetrating towards the center of the convoy on a constant bearing in sector 1 is

$$P_1 = \frac{180 X}{\pi R_1 U}$$

Define $A = U / 360$. Then

$$P_1 = X / (2\pi R_1 A) = B_o X / R_1$$

where $B_o = 1 / (2\pi A)$.

If X yards of defensive force are assigned to sector 1, then Y - X yards are assigned to sector 2. We must multiply the latter force by the effectiveness factor α in order to have the total effective screen width assigned to sector 2. We have then for P_2,

$$P_2 = \alpha \cdot (Y - X) / 2\pi R_2 (1 - A) = G_o (Y - X) / R_2 ,$$

where $G_o = \alpha / 2\pi (1 - A)$.

Using the expression for threat which we developed in Model I and substituting for P_1 and P_2, we have

$$THR_1 = \left[1 - @ (1 - A)\right] (1 - B_o X / R_1) E_c$$

$$THR_2 = @ k (1 - A) \left[1 - G_o (Y - X) / R_2\right] E_c .$$

Since $THR = THR_1 + THR_2$, we obtain

$$RISK = THR / E_c = 1 + @(k - 1) - \left[B_1 X / R_1 + G_1 (Y-X)/R_2\right] ,$$

where $B_1 = B_o \left[1 - @ (1 - A)\right]$,

$$G_1 = G_o @ k (1 - A) .$$

This is the quantity which we wish to minimize by suitable disposition of our available force. Mathematically, we wish to find that value of X which achieves $\min_X$ (RISK), with certain constraints on X, R_1, and R_2.

In order to solve this minimization problem we must first decide what values we should use for R_1 and R_2, or decide by what means we should determine R_1 and R_2.

For reasons discussed in Chapter V-F, the authors decided to determine R_1 and R_2 by balancing the threat due to submarines attempting penetration of the screen with the threat due to submarines firing from outside the screen.

In assumption 4, Model II, we presumed the expression for the threat due to a submarine at a distance R from the center of the convoy firing from outside the screen to be

$$THR' = e \cdot E_c \cdot (1-R/HL) .$$

Then THR_1', the threat in sector 1 from submarines firing from outside the screen is

THR_1' = (threat from sub at R_1 firing from outside screen) x [Probability sub is conventional x probability conventional sub is in sector 1 + probability sub is nuclear x probability nuclear sub is in sector 1] or,

$$THR_1' = e \cdot E_c \cdot (1-R_1/HL)\ (1-@+@A)$$

recalling that A = U / 360.

Similarly, for sector 2,

$$THR_2' = e \cdot E_c \cdot (1 - R_2/HL) \cdot @ \cdot (1 - A)$$

For the condition of balanced threats,

$$THR_1 = THR_1' \quad \text{and} \quad THR_2 = THR_2'$$

Solving the above for R_1 and R_2, we have

$$R_1^* = \frac{HL}{2}\left(1 - \frac{B}{e}\right) + \sqrt{\left[\frac{HL}{2}\left(1 - \frac{B}{e}\right)\right]^2 + \frac{B \cdot HL \cdot X}{2\pi A \cdot e}},$$

where $$B = \frac{1 - @(1 - A \cdot k)}{1 - @(1 - A)},$$

$$R_2^* = \frac{HL}{2}\left(1 - \frac{k}{e}\right) + \sqrt{\left[\frac{HL}{2}\left(1 - \frac{k}{e}\right)\right]^2 + \frac{\alpha \cdot k \cdot HL \cdot (Y - X)}{2\pi(1-A) \cdot e}}$$

Notice that R_1^* and R_2^* are functions of X. That is, $R_1^*(X)$ is the distance at which X amount of force must be stationed in sector 1 to achieve equality of THR_1 and THR_1'.

Let us write R_1^* and R_2^* as

$$R_1^* = B_3 + \sqrt{B_3^2 + B_4 \cdot X}$$

$$R_2^* = G_3 + \sqrt{G_3^2 + G_4 \cdot (Y - X)}$$

and substitute in the equation for RISK. Then

$$\text{RISK} = 1 + @ \cdot (k - 1) - \left[\frac{B_1 \cdot X}{B_3 + \sqrt{B_3^2 + B_4 \cdot X}} + \frac{G_1 \cdot (Y - X)}{G_3 + \sqrt{G_3^2 + G_4 (Y - X)}} \right]$$

where

$$B_o = 1/2\pi A \qquad G_o = \alpha/2\pi(1 - A)$$

$$B_1 = B_o \left[1 - @ (1 - A \cdot k)\right] \qquad G_1 = G_o \cdot @ \cdot k (1 - A)$$

$$B_3 = \frac{HL}{2} \cdot (1 - B/e) \qquad G_3 = \frac{HL}{2} \cdot (1 - k/e)$$

$$B_4 = B_o \cdot B \cdot HL/e \qquad G_4 = G_o \cdot k \cdot HL/e$$

We are now in a position to find $\min_X$ (RISK). As a mathematical problem this poses no difficulty. It can be shown that

$$\frac{d^2}{dX^2} (\text{RISK}) \geqslant 0 \text{, for all } X,$$

hence a solution of

$$\frac{d}{dX} (\text{RISK}) = 0$$

is a solution which minimizes RISK. Rationalizing the denominators and differentiating, we have

$$\frac{d}{dX} (\text{RISK}) = -\frac{1}{2} \left[\frac{B_1}{\sqrt{B_3^2 + B_4 X}} - \frac{G_1}{\sqrt{G_3^2 + G_4 (Y - X)}} \right]$$

Set equal to zero and solve.

$$X = \frac{B_1^2 (G_3^2 + G_4 Y) - B_3^2 G_1^2}{B_1^2 G_4 + B_4 G_1^2}$$

Unfortunately, this solution is only valid for our problem if

1. $0 \leq X \leq Y$

2. $R_1 = B_3 + \sqrt{B_3^2 + B_4 \cdot X} \geq H_o$

3. $R_2 = G_3 + \sqrt{G_3^2 + G_4 \cdot (Y - X)} \geq H_o$

4. $0 \leq P_1 = B_o \cdot X / R_1 \leq 1$

5. $0 \leq P_2 = G_o \cdot (Y - X) / R_2 \leq 1$.

Because of these constraints, the authors found it expeditious to determine solutions by use of a computer. The program, written in FORTRAN 60 for the CDC 1604 computer, is included in Appendix C along with a sample output statement. The notation in the program conforms for the most part to that in this paper.

The logic of the program is this:

a. let X vary over a range from Y to 0 in increments of .01 Y;

b. calculate for each value of X the corresponding R_1 and R_2;

c. test R_1 and R_2 for being greater than or equal to H_o. Set R_1 (R_2) equal to H_o if test fails;

d. calculate the resulting P_1 and P_2;

e. test P_1 and P_2 for being between zero and one, discarding all results for that particular X if the test fails;

f. calculate the corresponding value of RISK;

g. select that value of RISK which is minimum along with the corresponding T (i.e., X / Y · 100), R_1 and R_2 .

The program was run many times over wide ranges of values for α, k, e, Y, and H_o. The calculus solution was verified for those cases where it is applicable, i.e., where the five constraints are satisfied. Results of the runs, their usefulness, and the conclusions which may be drawn are discussed in the next section of this chapter.

Results

To investigate the results of Model II it is best to consider an example. Suppose we have five escorts comprising the defensive force for the convoy. Let each escort be equipped with a sonar which may be characterized as having a below-the-layer definite range of 2500 yards. Table 1 below summarizes the situation.

α	Y, total sweep width (yards)	H_0, radius of convoy (yards)	RT, torpedo range (yards)	A = U/360
2	25000	10000	15000	1/3

Recall that A = 1/3 corresponds to U = 120 degrees, where U is the angle subtending the LLA. (See Chapter V-B). α has been chosen as 2 in this example and in all ensuing examples. This means that escorts patrolling in sector 2 are twice as effective as escorts patrolling in sector 1. We have given no increase in effectiveness for escorts patrolling station in sector 1 as compared with escorts maintaining fixed station in sector 1. An increase in this respect is tantamount to an increase in Y, (i.e., a larger defensive force). See Chapter V-C for further discussion.

For the above example, consider the case for which e = .5. From the relation,

$$THR' = e \cdot E_c \cdot (1 - R/HL) ,$$

at R = 10000 yards,

$$THR' = .5 \cdot E_c \cdot (1 - 10000/(10000 + 15000))$$

$$= .3 \cdot E_c .$$

That is, a submarine at the perimeter of the convoy firing from outside the screen (limiting case) is 3/10 as effective as a conventional submarine loose in the convoy - it will obtain about 1/3 as many hits.

Letting k = 2.5, (i.e., a nuclear submarine being 2½ times as effective as a conventional from the standpoint of scoring hits when within the convoy), Model II yields data presented in the table below. Table 2 shows the allocation of forces as a function of @, the probability that an attacking submarine is nuclear. T is the percent of defensive force assigned to sector 1 at distance R_1 from the center of the convoy. The percentage (1 - T) is assigned to sector 2 at distance R_2 from the center of the convoy. RISK is the measure of THR/E_c when this allocation and disposition have been made, and by the method of solution of Model II, represents the minimum attainable for the situation THR = THR' (balanced threat).

Table 2

@	T	R_1	R_2	RISK
.0	100	14900	-----	.20
.1	100	14800	-----	.36
.2	62	10100	10000	.44
.3	63	10100	10000	.52
.4	64	10100	10000	.60
.5	65	10000	10000	.69
.6	27	10000	10000	.74
.7	27	10000	10000	.75
.8	26	10000	10000	.76
.9	26	10000	10000	.77
1.0	26	10000	10000	.77

T = 62 implies that X (amount of force to be assigned to sector 1) = .62 · Y, or 15500 yards of defensive force. This corresponds to three escorts having the sonars stipulated. In this way we can construct the following optimal policy from Table 2. For

@ $<$ .2, put all the escorts in sector 1 at 15000 yards.

.2 ≤ @ ≤ .5, put three escorts in sector 1 at 10000 yards, two in sector 2 at 10000 yards.

.6 ≤ @ , put one escort in sector 1 at 10000 yards, four in sector 2 at 10000 yards.

For the same convoy situation, but with a defensive force of nine escorts equipped with 2500 yard sonars (definite range against submarine best depth), the Model II solution is presented in Table 3.

Table 3

@	T	R_1	R_2	RISK
.0	100	22600	-----	.05
.1	60	15700	10000	.18
.2	59	15400	10000	.20
.3	59	15200	10000	.21
.4	59	15000	10000	.22
.5	59	14800	10000	.23
.6	59	14700	10000	.24
.7	59	14500	10000	.25
.8	40	10200	10000	.25
.9	40	10000	14100	.24
1.0	41	10000	13900	.24

An optimal policy constructed from Table 3 is, for

@ = 0, put all nine escorts in sector 1 at R_1 = 22600 yards.

0 < @ ≤ .7 , put five escorts in sector 1 at 15700 yards, for lower values of @, moving R_1 in to 14500 yards for the higher values of @. Put four escorts in sector 2 at 10000 yards.

.8 ≤ @, put four escorts in sector 1 at 10000 yards, put five in sector 2 at 14000 yards.

The preceding examples illustrate the information Model II provides and show how it may be used to construct an optimal policy. Although in the examples all the escorts were assumed to have the same type sonar, this is not at all essential to the model. The only input regarding the defensive forces which the model requires is Y, the sum of the sweep widths of the individual force elements. (See Chapter V-C for discussion of sweep width determination.) It is, of course, necessary to know how Y is distributed among the force elements in order to make an optimal assignment of escorts to each sector in accordance with the policy deduced.

Validity of Model

For Model II to have validity (and usefulness) it must have three attributes:

1. It must indeed yield a solution which provides minimum RISK.

2. It must respond to changes of parameters (α, e, k, Y, H_0, RT) in a manner consistent with what experience tells us would happen for such changes. For example, we should expect RISK to increase as k increases. The model should reflect this. What our experience or intuition does not tell us is at what rate RISK increases with k. This is how the model may have usefulness.

3. With regard to allocation and disposition of the defensive force, (i.e., optimal policy), the model must not be overly sensitive to variations of certain parameters. We would expect changes of allocation when we have more or less defensive force, or when the convoy size changes significantly, or when there are no nuclear submarines present vice 90% present. Changing force disposition for these reasons is quite feasible, however, if the model tells us to shift our forces drastically for every change of @, or for a change of k from 1.5 to 2.5, its value is reduced substantially.

With respect to attribute 1, the method of solution (whether by calculus or by computer) guarantees minimum RISK for the condition of balanced threat. As an illustration, for the first convoy example above in which Y = 25000, Fig. 1 shows the results of plotting RISK vs @ using the optimal policy allocation and disposition. For comparison, the RISK for two other possible dispositions is also plotted. The first is for 50% of the force in sector 1, with $R_1 = R_2 = 10000$ yards. The second is for 50% of the force in sector 1, with $R_1 = R_2 = 12000$ yards. These two lines were plotted from the Model I expression for RISK,

$$RISK = 1 - P_1 + @ (1 - P_1) (Ak-1) + k(1-A) (1 - P_2) ,$$

where $P_1 = B_0 \cdot X/R_2$ and $P_2 = G_0 (Y - X)/R_2$.

Fig.'s 10 through 13 show the response of the model to certain parameter changes. Each is a plot of RISK vs k for various values of @. In all plots, the following parameter values were assigned:

H_0 = 10000 yards

RT = 15000 yards

A = 1/3

α = 2.

In Fig.'s 10 and 11, Y = 25000 and e = .5 and 1.0 respectively. In Fig.'s 12 and 13, Y = 45000 and e = .5 and 1.0 respectively. Thus a comparison of Fig.'s 10 and 12 (or 11 and 13) illustrates the effect of an increased defensive force, all else being constant. A comparison of Fig.'s 10 and 11 (or 12 and 13) illustrates the effect of a higher weighting of the threat due to submarines firing from outside the screen. All four plots show that RISK is an increasing function of both k and @, as was to be anticipated.

Also, as @ increases, in order to maintain a balance of the threats it is necessary to move the screen farther out, thereby decreasing the probability of detection and increasing RISK. As indicated in the graphs, this effect is much more pronounced for the larger value of Y. It is interesting to note that, in all cases, the rate of increase of RISK with respect to @ falls off sharply for values of @ larger than .5 .

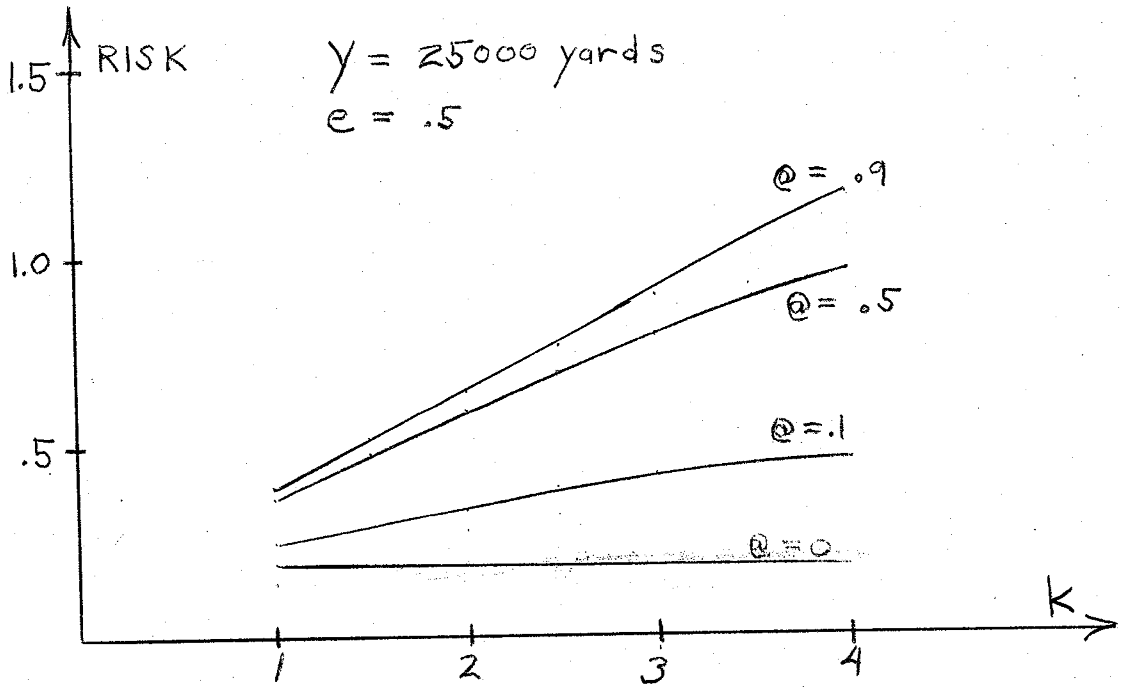


Figure 10

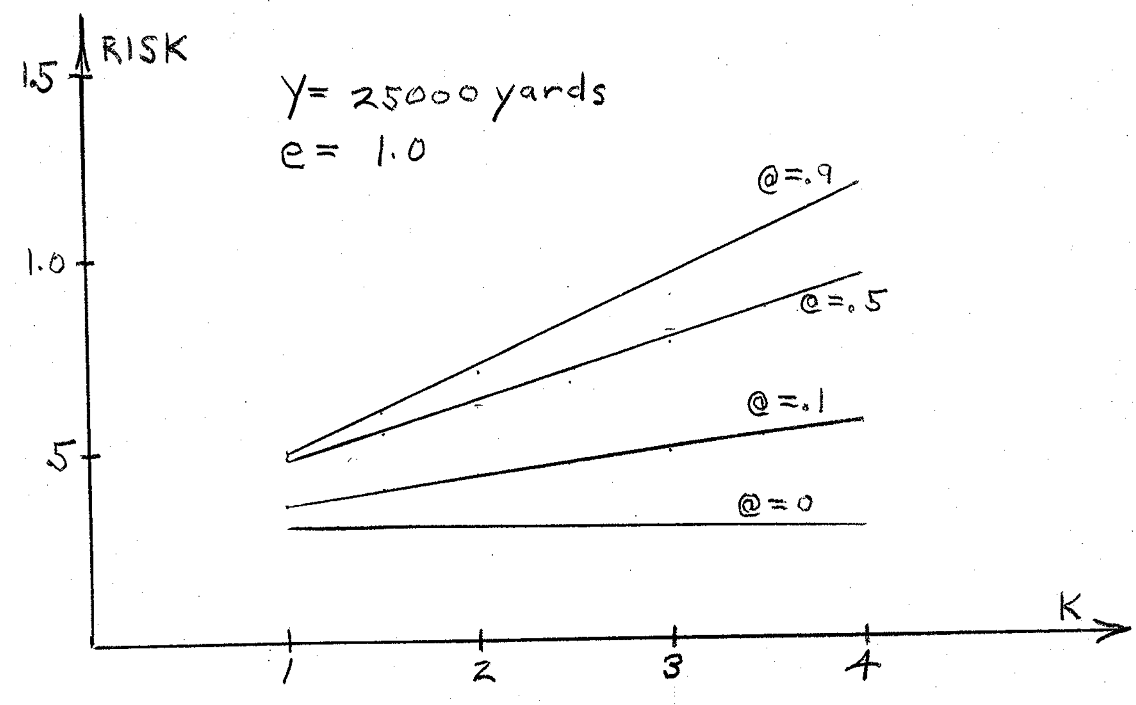


Figure 11

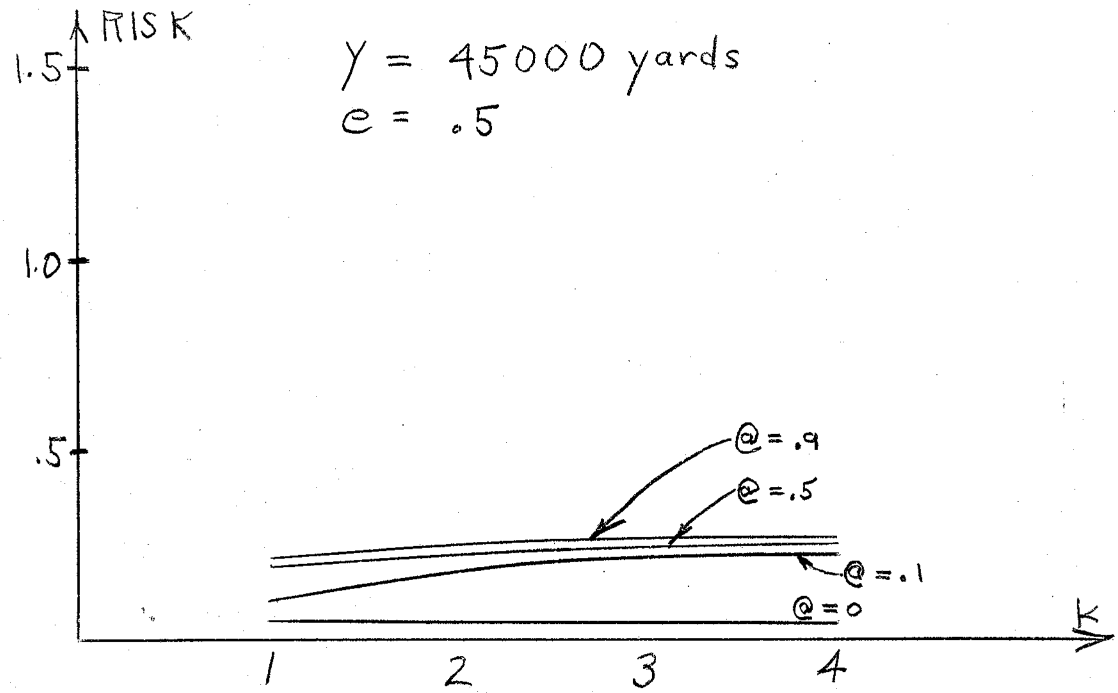


Figure 12

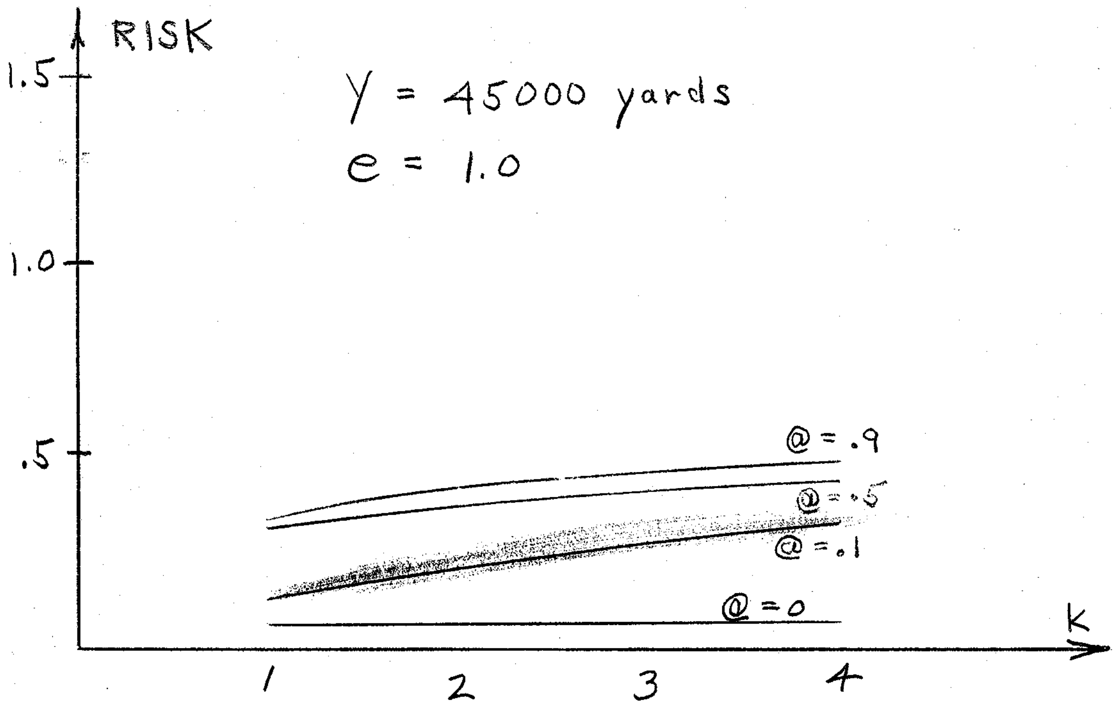


Figure 13

As was demonstrated earlier, the model yields data which may be used to form an "optimal policy" with regard to the assignment of the defensive force. T represents the percentage of the defensive force to be assigned to sector 1 at a distance R_1 from the center of the convoy; the remainder is to be assigned to sector 2 at a distance R_2. Let us investigate the sensitivity of this optimal assignment to parameter changes.

Fig.'s 14 and 15 are plots of RISK vs T for several values of @. The fixed parameters are again

$$H_0 = 10000 \text{ yards}$$

$$RT = 15000 \text{ yards}$$

$$\alpha = 2$$

$$A = 1/3$$

$$e = .5$$

$$k = 2.5 \quad .$$

In Fig. 14, Y = 25000 yards; in Fig. 15, Y = 45000 yards. The minima of the curves correspond to the optimal division of search effort, T, for the different values of @. It is apparent that the criticality of the minimum points varies considerably from case to case. That is, for some of the curves there is a wide band of values of T for which RISK is not appreciably greater than its minimum value.

With this in mind, let us assume that we are willing to accept an increase in the minimum value of RISK (for particular @)of .05 . That is, if T assures us a value of RISK which is no greater than minimum RISK + .05, we accept that T as optimal. Table 4 shows the results of this allowance. The left side of the table contains the same information as did the table shown earlier. The right side shows the range of T allowable in order to maintain RISK within .05 of its minimum value for that particular @. The spread of values for R_1 and R_2 corresponds to the choice of selections of T. For example, if the left end-point of the range of T is selected, the left end-points of R_1 and R_2 should be chosen. If the right end-point of T

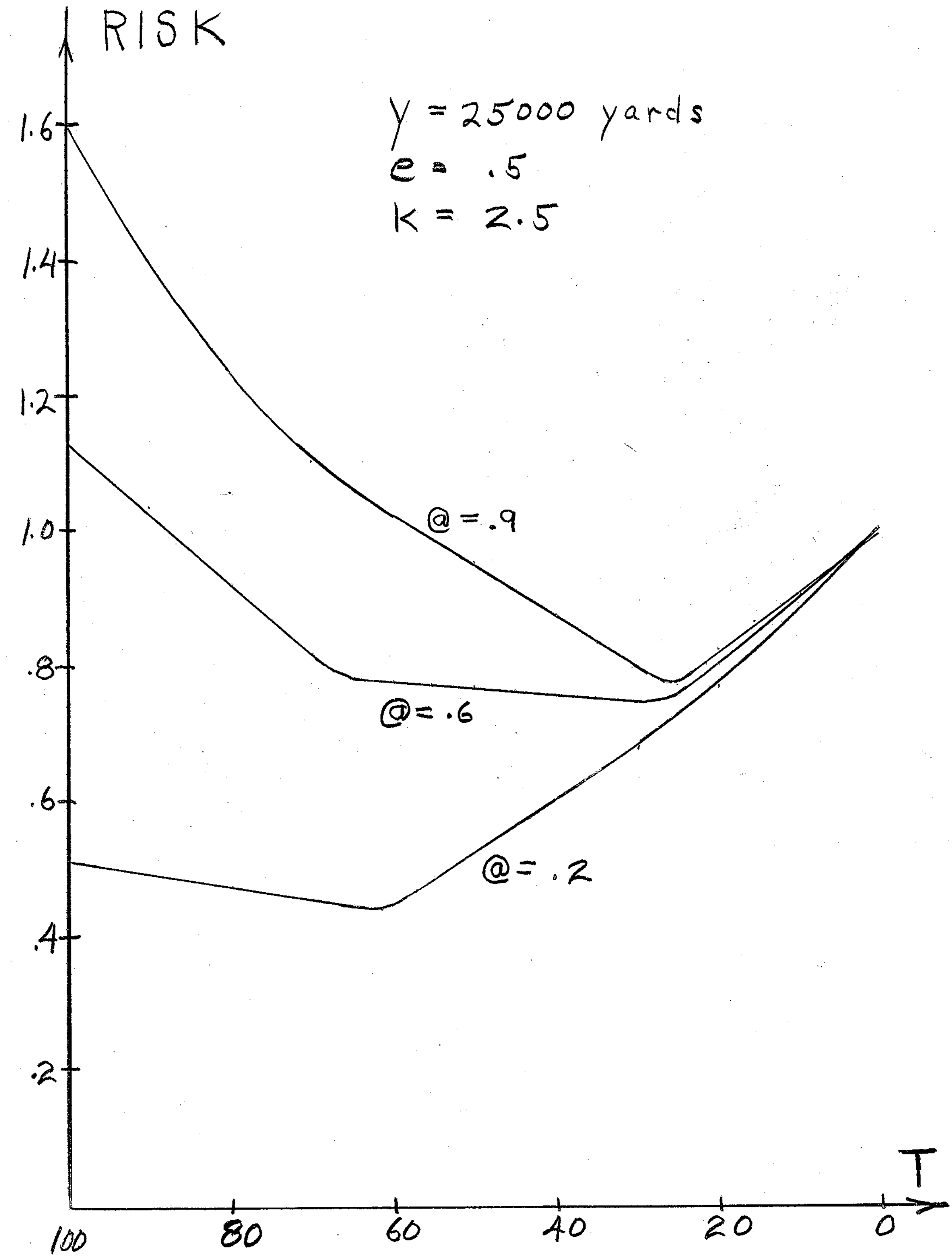


Figure 14

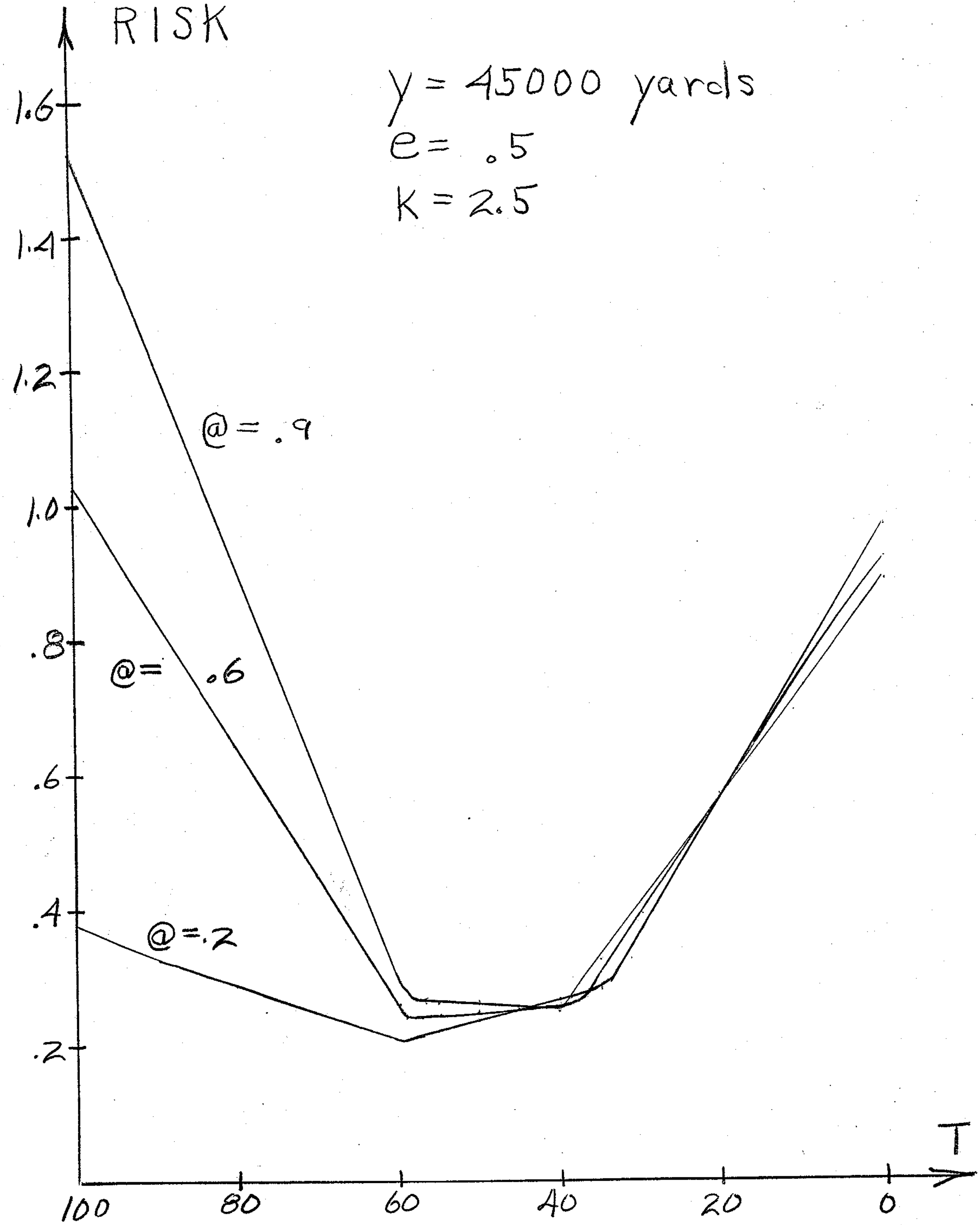


Figure 15

is selected, the right end-points of R_1 and R_2 should be chosen. For intermediate values of T, interpolate linearly to find the corresponding R_1 and R_2. Errors in R_1 or R_2 of less than 1000 yards do not significantly affect the value of RISK.

Table 4

H_0 = 10000 yards; RT = 15000 yards; A = 1/3; α = 2.

Y = 25000 yards; e = .5; k = 2.5 ; R_1 and R_2 x 1000 yards

	minimum RISK solution				allowable range for min RISK + .05		
@	T	R_1	R_2	RISK	T	R_1	R_2
.0	100	14.9	----	.20	78-100	12.4-14.9	10.0
.1	100	14.8	----	.36	56-100	10.0-14.8	10.0
.2	62	10.1	10.0	.44	55-89	10.0-13.4	10.0
.3	63	10.1	10.0	.52	53-75	10.0-11.6	10.0
.4	64	10.0	10.0	.60	48-72	10.0-11.0	10.0
.5	65	10.0	10.0	.69	26-71	10.0-10.1	10.0
.6	27	10.0	10.0	.74	21-66	10.0	10.1-10.0
.7	27	10.0	10.0	.75	21-42	10.0	10.1-10.0
.8	26	10.0	10.0	.76	21-35	10.0	10.1-10.0
.9	26	10.0	10.0	.76	21-33	10.0	10.1-10.0
1.00	26	10.0	10.0	.77	20-31	10.0	10.1-10.0

The spread of allowable values of T for each @ makes the selection of an optimal policy easier and permits much flexibility. For example, Fig. 16 is a plot of the spread of allowable values of T of Table 4 versus @. Indicated on the plot is one possible optimal policy, namely, for

@ $<$ .2, put 78-100% of the force in sector 1 ,

.2 $\leq$ @ $\leq$ .6, put 56-66% of the force in sector 1 ,

.6 $\leq$ @, put 21-31% of the force in sector 1.

Another possible optimal policy which can be formulated from the data in Table 4 is: for

@ $\leq$ .2, put 78-89% of the force in sector 1

.2 $\leq$ @ $\leq$ .4, put 56-72% of the force in sector 1

.5 $\leq$ @, put 26-31% of the force in sector 1.

Table 5 below contains the allowable spread of T for the case k = 3.5, all other parameters remaining constant. Comparison with Table 4 (where k = 2.5) shows that there are optimal policies common to both cases. This is an illustration of the relative insensitivity of the model to the input value for k, with respect to the model's specification of optimal policies.

Table 5

H_0 = 10000 yards; RT = 15000 yards; A = 1/3; α = 2.

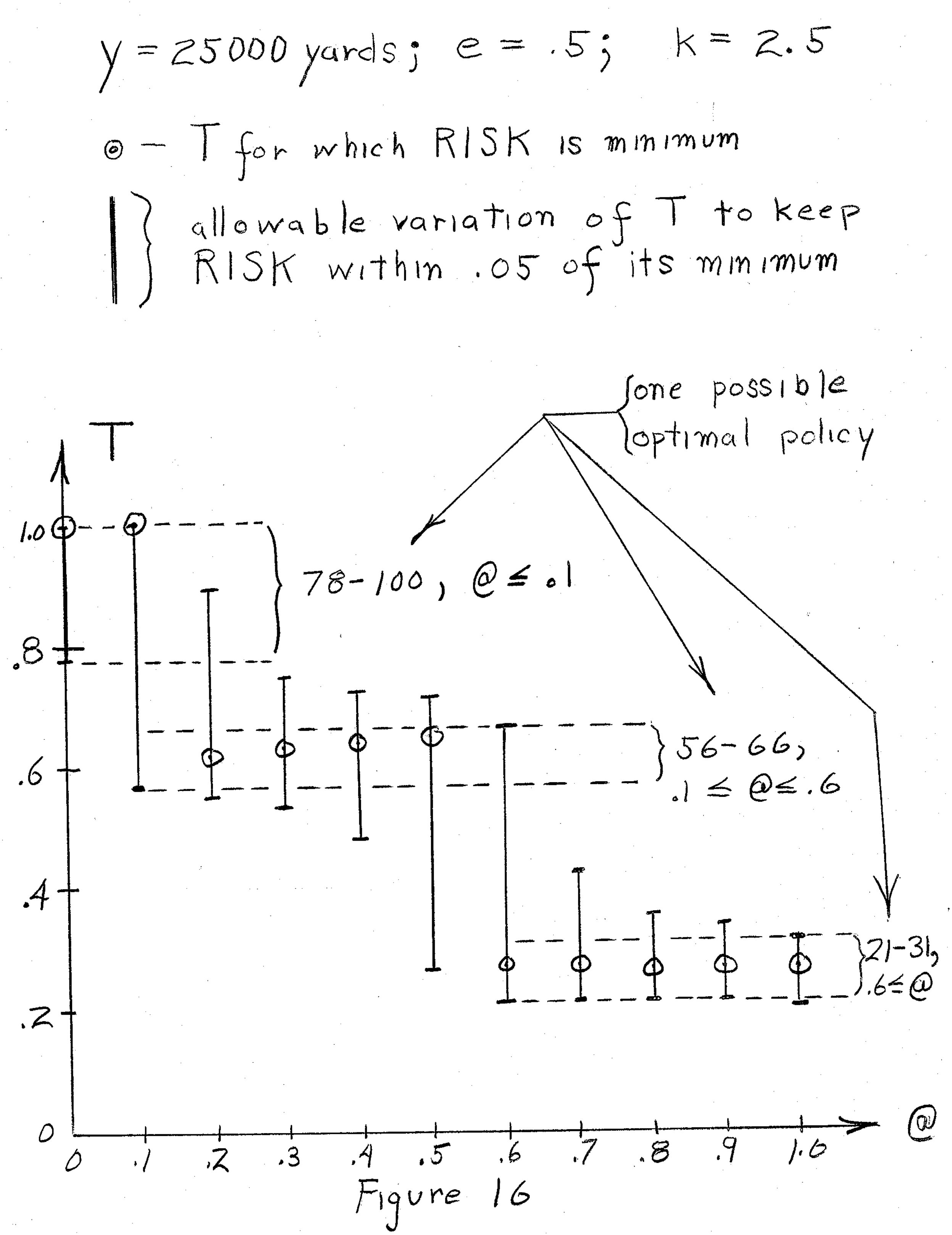
Y = 25000 yards; e = .5; k = 3.5; R_1 and R_2 x 1000 yards

	minimum RISK solution				allowable range for min RISK + .05		
@	T	R_1	R_2	RISK	T	R_1	R_2
.0	100	14.9	----	.20	78-100	12.4-14.9	10.0
.1	61	10.0	10.0	.40	56-100	10.0-14.7	10.0
.2	63	10.0	10.0	.52	56-77	10.0-11.8	10.0
.3	65	10.0	10.0	.65	52-73	10.0-11.1	10.0
.4	66	10.0	10.0	.78	35-72	10.0-10.7	10.0
.5	24	10.0	10.0	.88	20-68	10.0	10.4-10.0
.6	24	10.0	10.0	.91	20-38	10.0	10.5-10.0
.7	24	10.0	10.0	.95	19-32	10.0	10.5-10.0
.8	23	10.0	10.0	.98	19-29	10.0	10.5-10.0
.9	23	10.0	10.0	1.01	19-28	10.0	10.5-10.0
1.0	23	10.0	10.0	1.05	19-27	10.0	10.5-10.0

Conclusions

The model yields a minimum RISK solution for the balanced threat situation.

It reacts to parameter changes in a manner which is consistent with intuitive motions.


y = 25000 yards; e = .5; k = 2.5
⊙ – T for which RISK is minimum
allowable variation of T to keep RISK within .05 of its minimum
one possible optimal policy
T
1.0
.8
.6
.4
.2
0
78-100, @ ≤ .1
56-66, .1 ≤ @ ≤ .6
21-31, .6 ≤ @
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
@
Figure 16

It yields data which may be used to formulate optimal policies. These policies are sufficiently broad in nature to allow much flexibility in their implementation: they are not particularly sensitive to k, and they do not require shifts of force for small changes in @.

In Appendix I are ten tables, similar to Tables 4 and 5, in which the force level ranges in increments of 5000 yards from 25000 to 45000. For each force level shown there are two tables; one with e = .5, one with e = 1.0.

CHAPTER V

SUPPORTING ANALYSIS

A. Difference Between Submarines in the Area and Those That Can Intercept a Convoy

In the concept of the solution, Chapter III-B., the distinction is made between the number of submarines that detect a convoy and the number that are able to intercept it. In the analytical model, Chapter IV-B., the latter number is what determines @, the probability that an attacking submarine is nuclear. Because of the importance of distinguishing between the number of either class of submarines on patrol in an ocean area and the number that are able to intercept the convoy, this section will briefly discuss the factors that bring about the difference, although a quantitative investigation is not within the scope of the thesis.

Studies of the probable distribution of enemy submarines throughout distinguishable ocean regions have been made, and are available in classified documents. Calculations exist of detection ranges (the "detection circles") of passive sonars against large convoys, for various convoy speeds. In the following section, the manner in which enemy submarines will detect and attempt to close a convoy for a feasible detection range is studied in some detail. Our purpose there is to obtain a picture of the likely distribution of submarines as they reach the perimeter of the screen.

If the passive detection capabilities of nuclear and conventional submarines are identical, and the submarines are equally likely to be anywhere in the ocean area under consideration, then the fraction of submarines that detect the convoy will be the same for both classes of submarines. Theoretically the speed of the patrolling submarine increases its probability of detection (see Section 1.5, pg 7-9, of [3]) but for practical patrol speeds, the increase is negligible.

Essentially all nuclear submarines are able to intercept the convoy. For reasons which are detailed in the following chapter, not all conventional submarines will be able to intercept it, but only those within the effective limiting lines of approach (LLA). The limiting lines of approach are a function of the speed of the convoy, the amount of air cover, and the endurance of the submarine. Higher speed convoys result in narrower limiting lines of approach, (theoretically) but such convoys will also be detected at longer ranges, and the net effect is that a larger number of submarines will be able to detect and close the higher speed convoy. Aircraft patrolling in the Danger Zone* and preventing submarines from closing the convoy on the surface are a very powerful factor in reducing the number of conventional submarines that will be able to intercept the convoy. The submarine battery state at the time of detection and submarine submerged endurance play an important part in the ability of the conventional submarine to close.

Conventional submarines will be subject to greater attrition by patrolling aircraft or Hunter-Killer groups escorting the convoy than nuclear submarines, and this attrition must be taken into account in determining the probable fraction of each class of intercepting submarine.

Finally, nuclear submarines with their superior submerged speed will be better able to take advantage of tactical intelligence to place themselves athwart the track of approaching convoys.

The analytical model (Chapter IV) requires only an estimate of the probability that an attacking submarine is nuclear. It is unnecessary to estimate the actual numbers of conventional and nuclear submarines attacking.

B. The Distribution Of Submarines Around The Convoy

1. Conventional Submarines

The analysis that follows of the distribution of conventional submarines within the limiting lines of approach is in support of two

*The region between the detection circle and the torpedo danger zone (defined in Section F) within the limiting lines of approach.

simplifying assumptions of Model II, Chapter IV, namely that for purposes of dividing the search effort:

a. The limiting lines of approach may be treated as though emanating from the center of the convoy.

b. The region within which the conventional submarine threat should be presumed to lie is relatively insensitive to changes in the ratio of submarine speed to convoy speed, s/v, for $v \geqslant s$.

The analysis pertains to the situation in which convoy speed of advance, v, is at least equal to the submerged speed, s, of the approaching conventional submarines and, because of fixed-wing aircraft present or for any other reason, submarines detecting and intercepting the convoy must make a submerged approach, snorkeling or not.

Submarines with approach speed less than convoy speed are restricted to an approach arc ahead of the convoy delimited by $\theta = \sin^{-1} s/v$ on either side of convoy base course. Submarine speed is actually a variable, a function of submarine battery charge at the time it commences to close, and its ability to snorkel. The influence of these factors will be studied separately later, and for the moment s is treated as a constant.

As was discussed in Chapter III, a submarine represents a threat to the convoy from any point within torpedo range. The threat is a function of both bearing and range. The threat is usually represented graphically by drawing lines of equal probability of a hit around the convoy, called iso-probability curves or contours. [4]

The threat may also be represented by drawing lines for the expected number of hits from a salvo of two or more torpedoes, which we have chosen to call "equi-threat contour lines". Equi-threat curves might typically be drawn for expected number of hits of 1.5, 1.0, .5 and 0.0, which we have designated $h_{1.5}$, $h_{1.0}$, $h_{.5}$, and h_o in Figure 17. They may be determined for either aimed or unaimed* shots. In the studies we examined, two or three "Browning Shots" are the usual number assumed.

*A torpedo fired in the direction of the convoy, but not aimed at a specific target, sometimes called a "Browning shot".

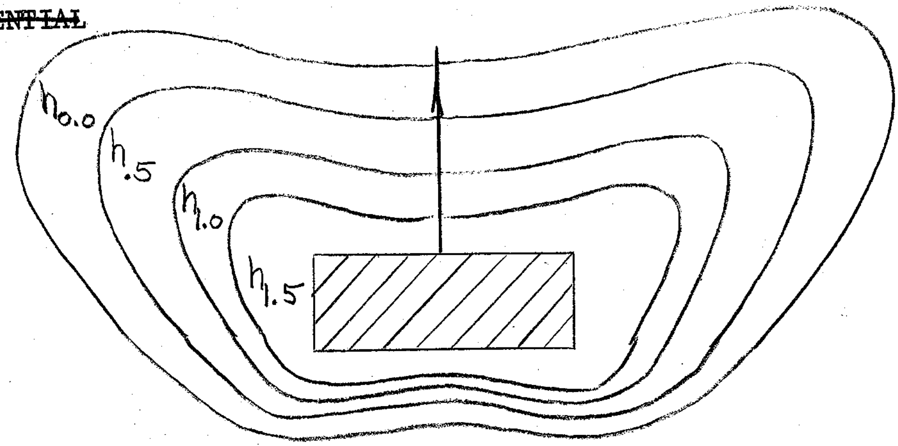


Figure 17

Submarines able to maintain the speed ratio s/v that are ahead of lines parallel to two radii on bearings $\pm \theta$ and tangent to the zero equi-threat contour line h_o theoretically represent a threat to the convoy. These lines at zero threat approximately correspond to the Limiting Lines of Approach, defined in NWP-24A [8] and will be designated LLA_o.

Bent line screens protecting a <u>point target</u> (e.g., a carrier) are formed along an iso-probability contour h_p with the wings of the screen being drawn back along a line perpendicular to the limiting line of approach and extending from a tangent to the curve h_p as far aft on the flanks as the line tangent to the iso-probability curve being guarded, LLA_p. See Fig. 18.

The iso-probability curve h_p selected is the one which equalizes the probability of a sinking or crippling number of hits by a submarine firing outside the screen with the probability of the submarine penetrating the screen [18] . The probability of penetration depends on the number of escorts, their sonar equipment, and the water conditions.

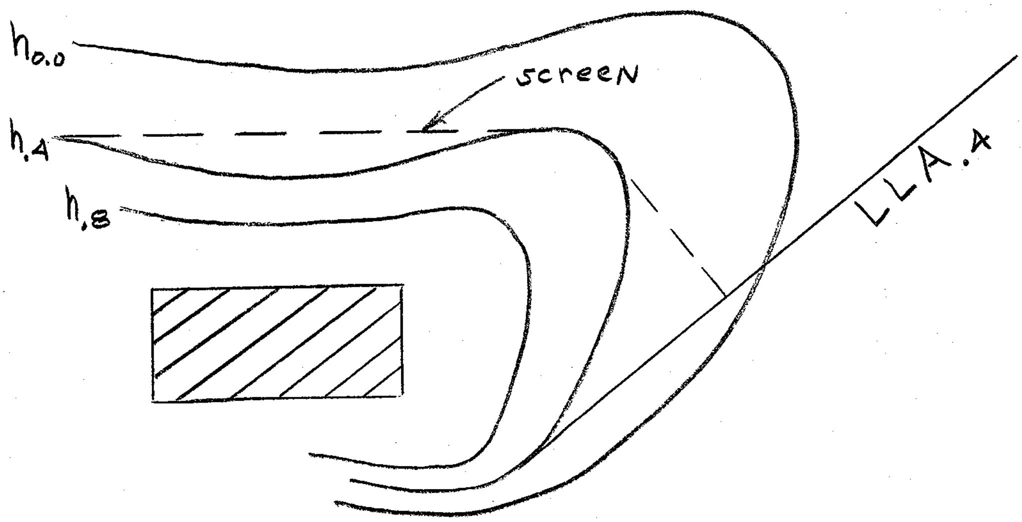


Figure 18

When a convoy is the target, a submarine approaching the convoy near the limiting lines of approach represents a significantly smaller threat than one approaching from ahead. A submarine tracking just inside the LLA_o cannot hope to get under the convoy or otherwise gain a firing position to obtain the maximum expected number of hits in the convoy. Furthermore, a small distraction by the protecting force which causes a course or speed change in the submarine would cause the submarine to fall outside the LLA_o. For these and other reasons that we will explore, the threat to the convoy diminishes near the LLA_o. In order to measure the relative threat to the convoy posed by conventional submarines at all bearings within the LLA_o, and in particular, to demark bearings from the convoy within which we should presume the significant conventional submarine threat to be confined, the following factors are considered:

a. Submarine bearing relative to convoy at time of detecting and classifying the convoy.

b. Other effects of air cover.

c. Decreased effectiveness of submarines approaching near the LLA_o.

d. Ability of the submarine to close on the relative track it desires.

e. Effect of convoy zig-zag.

f. Effect of escort station assignments.

a. Submarine bearing relative to convoy at time of detecting and classifying the convoy.

We assume conventional submarines attempting to intercept the convoy are distributed uniformly over the ocean ahead of the convoy (if the enemy should have intelligence allowing greater concentration along the convoy track, the effect will be to accentuate the results below). This assumption is equivalent to assuming a uniform distribution of submarines along a line just outside of submarine detection range on the convoy, R_d. The proportion of submarines that will detect the convoy over every interval of bearings ($\emptyset_1$, $\emptyset_2$) will be

$$P = \frac{\sin \phi_2 - \sin \phi_1}{2 \sin U/2}$$

where U is defined as follows: (See Figure 19). Extend the LLA_o lines until they intersect the detection circle, R_d. Draw lines connecting the center of the convoy and the points of intersection. The angle subtended by these two lines is U. For simplicity we have assumed that all noise from the convoy emanates from the center of the convoy, and detection and classification are completed at precisely range R_d from the convoy. Up to a submarine patrol speed of s = .5v, the effect of the motion of the submarine on patrol may be neglected. See Figure 25.

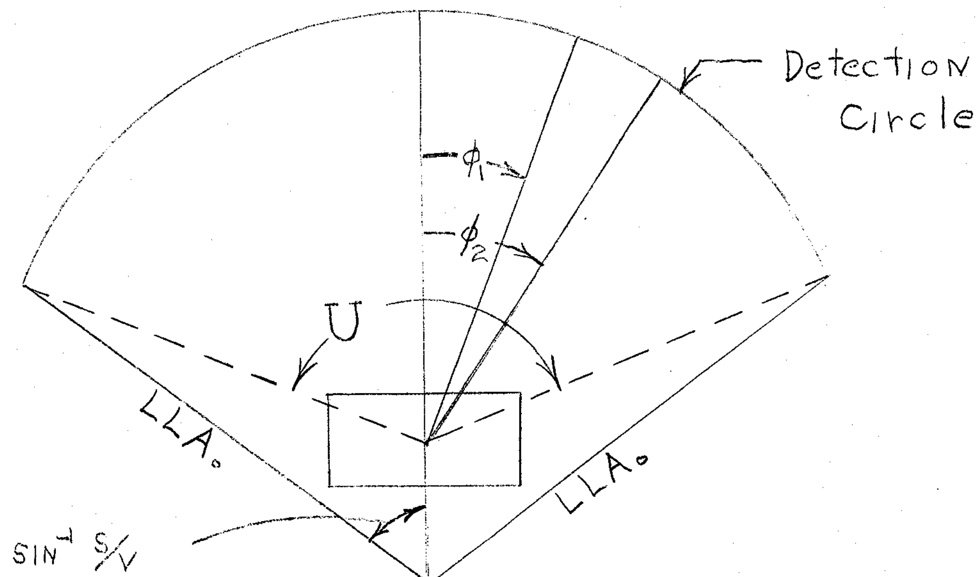


Figure 19

The density function for conventional submarines detecting the convoy at range R_d and able to close it within the LLA_o (i.e., submarines representing a threat) is then:

$$P(\phi) = \frac{\cos \phi}{2 \sin U/2} \qquad -U/2 \leq \phi \leq U/2$$

$$= 0 \qquad \text{elsewhere.}$$

Plots of a few illustrative densities for different values of $\frac{U}{2}$ are displayed in Figure 20.

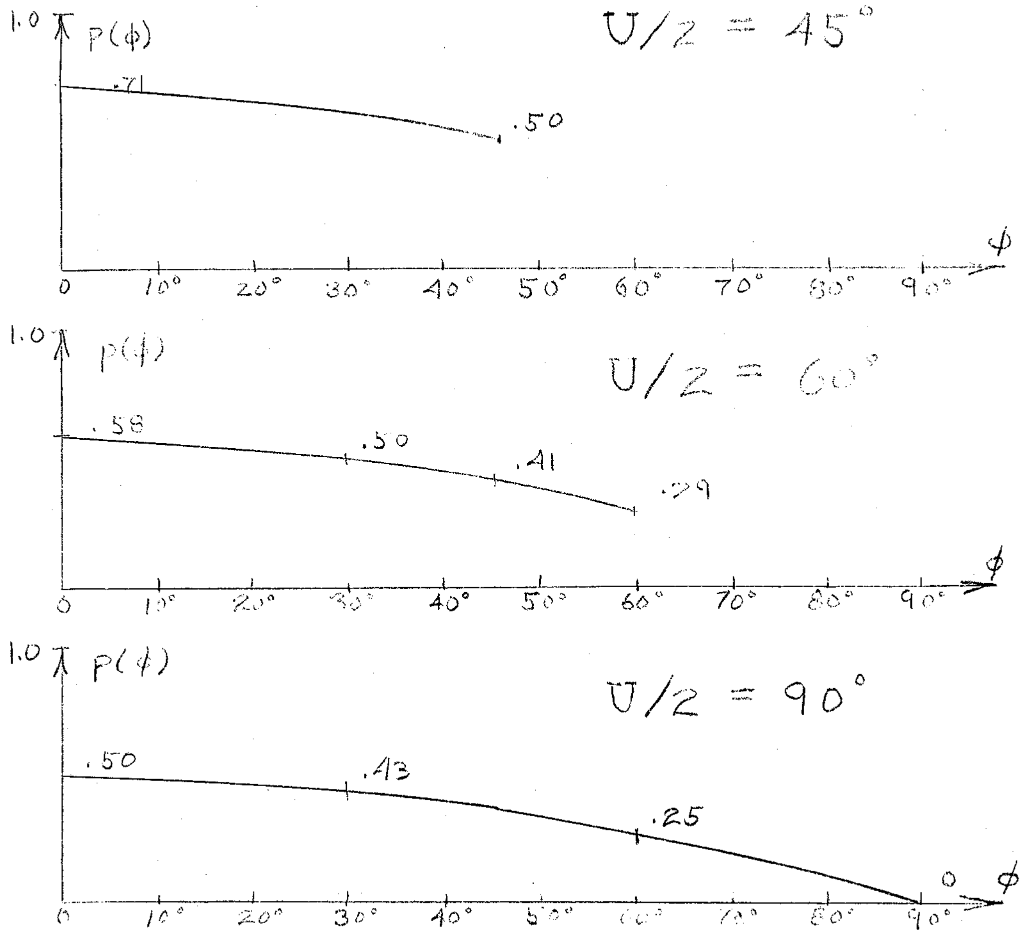


Figure 20

In general, the ratio of submarines which will detect the convoy near the limiting lines of approach to submarines which will detect the convoy dead ahead, for the above assumptions, is

$$\frac{P\ (U/2)}{P(0)} = \cos U/2 \quad .$$

If $U/2 = 60^{\circ}$, only half as many submarines will detect near the LLA_o as from dead ahead, and for $U/2 > 60^{\circ}$ the fraction rapidly approaches zero.

Now let us examine the distribution of submarines from the moment they start to close until they are in the vicinity of the convoy, under the minimum (not the most reasonable) assumption that a submarine closes on collision course whenever it is able. A typical case would look like Figure 21, for $8 \leq V \lesssim 12$ knots.

The expected fraction of conventional submarines in 30^{o} segments around the van of the convoy was found to be as follows for a typical detection range, R_d = 50 miles, <u>allowing the submarine to snorkel during approach</u>:

Table 6

Bearing Sector \ S/V	.6	.8	1.0
000 - 030 330 - 000	.70	.58	.51
030 - 060 300 - 330	.21	.33	.37
060 + 300 -	.09	.09	.12

at 10 miles from convoy center

Thus, the fraction of submarines that may be expected to attack from outside of an angle of 60^{o} from the track, measured from the center of the convoy, is small. Furthermore the fraction is surprisingly insensitive to s/v, which theoretically determines the limiting lines of approach.

At this juncture it is well to consider the extent to which a conventional submarine is able to choose its point of screen penetration within the limiting lines of approach. If there is not a strong connection between the submarine's approach track and its bearing of penetration, then the distribution of submarines after closing the convoy which we have depicted above is immaterial. Such a freedom of choice envisions the submarine choosing a position dead ahead of the convoy, locating say a weak right flank, outflanking the screen and slipping back into the convoy. The fact is, however, that at 10 miles from the convoy the conventional submarine is committed to a rather narrow range of penetration points, not more than one to three miles wide over a ten to fifteen mile screen front. Therefore an attacking conventional

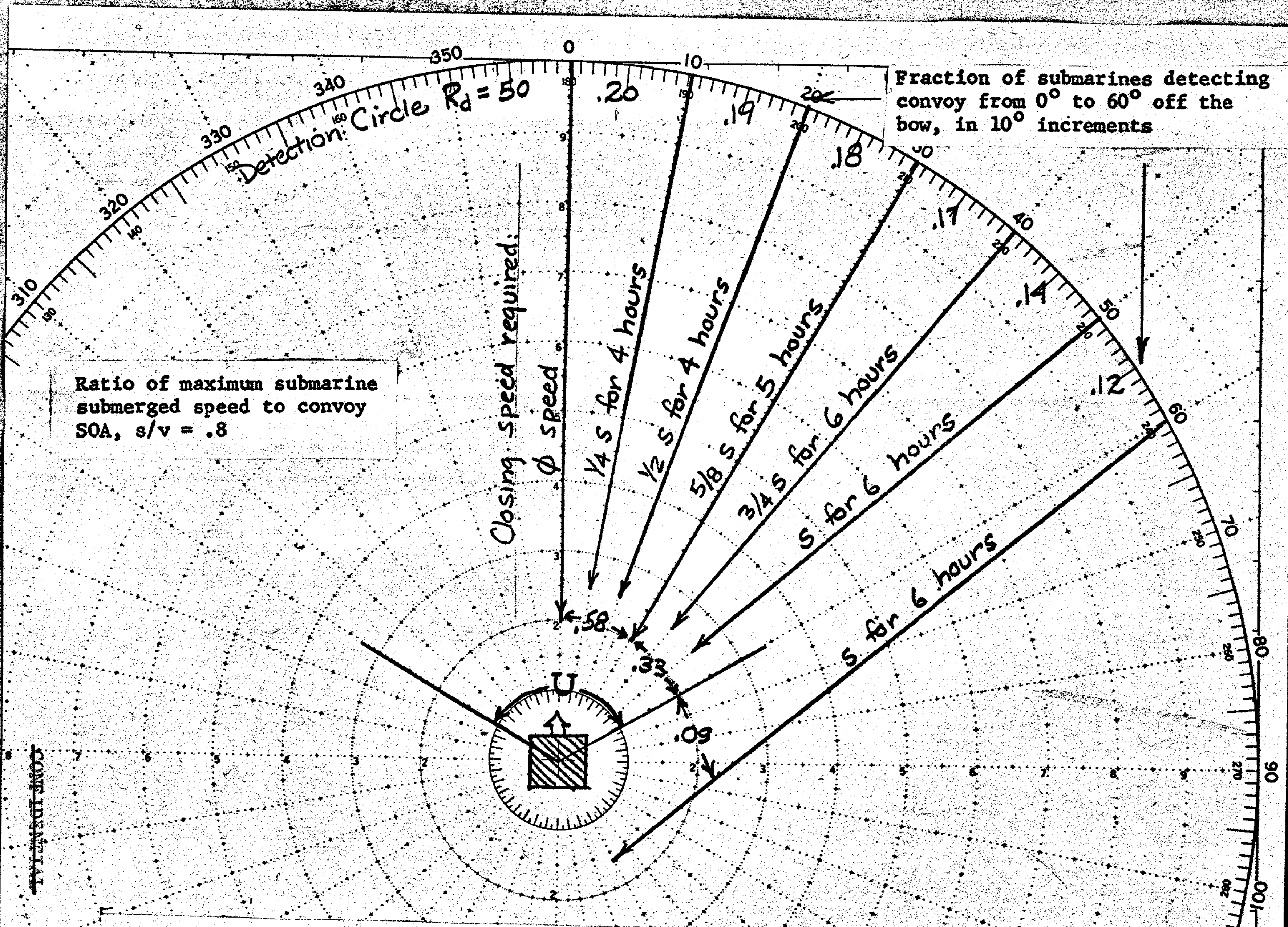


Figure 21. Typical Pattern of Submarine Approach on a Convoy, Submarine Closing on Collision Course Whenever Possible

submarine is unable to take advantage of tactical information it obtains of the screen disposition on the way in, with the possible exception of information concerning the location of a powerful long range sonar in the screen. Strategic information -- knowledge of our past screen stationing habits gleaned from previous attacks -- is another matter and is discussed in Section F of this chapter.

Now let us make the assumption that submarines, rather than close on a collision course, seek to gain a position athwart the convoy track at Rd/5 miles, perhaps 5-10 miles ahead. Submarines near convoy track ($\phi=0$) have no problem getting on the track. Submarines near LLA_o have a hopeless problem: at best they can barely intercept the convoy. But submarines increasingly far from the track (i.e., for increasing ϕ) must weigh the advantages of a more certain interception by getting onto the track against the disadvantage of depleting the battery excessively prior to attack, unless they are able to snorkel freely.

In a case for which submarines very aggressively try to get ahead of the convoy and are able to snorkel almost at will, the flow of submarines would look approximately like Figure 22.

This vigorous attempt by submarines to get in the van results, quite naturally, in making the fluctuation of the density of submarines quite insensitive to s/v:

Table 7

Bearing Sector \ S/V	.6	.8	1.0
000 - 030 330 - 000	.75	.75	.71
030 - 060 300 - 330	.15	.15	.19
060 + 300 -	.10	.10	.12

Distribution at 10 miles from convoy center

b. <u>Other effects of air cover.</u>

The range that a submarine with a given passive sonar will detect a convoy depends on convoy size and speed. If the submarine's ability to snorkel is inhibited by patrolling aircraft, the effective

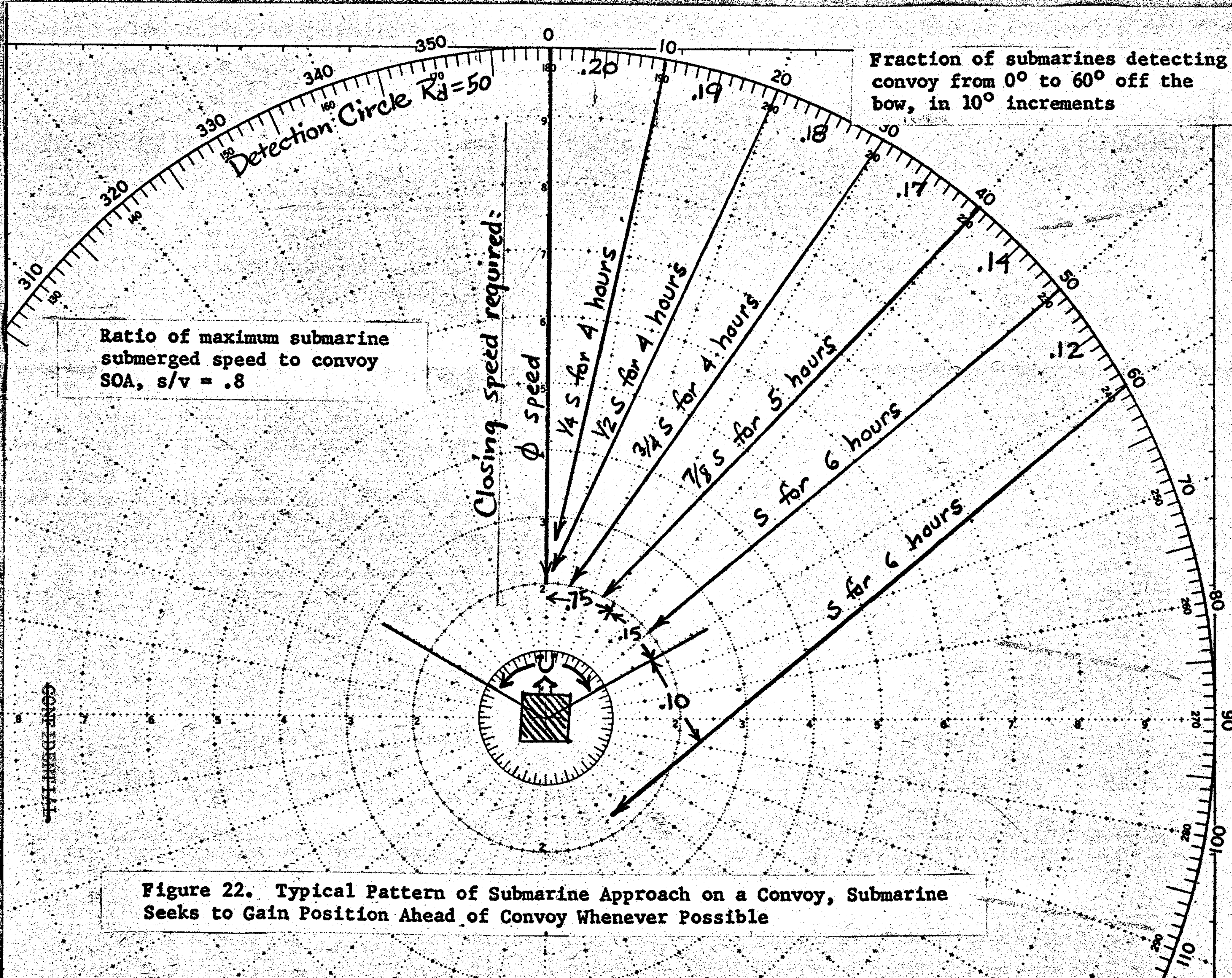


Figure 22. Typical Pattern of Submarine Approach on a Convoy, Submarine Seeks to Gain Position Ahead of Convoy Whenever Possible

limiting lines of approach of submarines attempting to close from long range is drastically reduced. For example, if R_d = 100 miles, a mean speed ratio s/v = .8 would be reduced in effect to s/v = .6. See Figure 14-4 of [7]. Data on probable passive sonar detection ranges for various sizes and speeds of convoys is available in Figure 20 of reference [19]. The effect of long detection range coupled with air cover is to increase the likelihood that intercepting conventional submarines will be within 60° on either side of the convoy track.

ATP-1(A) Vol. 1 [7] shows the limiting lines of approach based on 60% battery depletion during the approach. The battery state which the submarine commanding officer will accept and still attack is a variable that must be estimated. In addition, submarine battery state at the time of detection is also a variable. If the submarine is forced to patrol at much under 90% battery charge, the effect will be another significant reduction in the number of submarines that will approach from near the LLA_o.

Figure 23 exhibits typical speed ratios and times required to close for the case in which submerged speed = convoy SOA and detection range R_d = 50 miles. Observe that submarines approaching from 50° or more off the bow have an impossible task without snorkeling even if they start with a full battery charge. When aircraft radar flood the zone between the submerged LLA_o and the snorkeling LLA_o, the reduction of submarines intercepting the convoy is considerable. If s/v = 1.0, c ompletely preventing submarines from snorkeling during approach will reduce the number reaching the convoy by about 25%.

Even among the conventional submarines that are able to make an interception, air operations will have an effect. Diesel submarines will have had to make a 40 to 100 mile approach submerged. Coordinated attacks will be greatly complicated because of the limited communications and difficulty of conjoint effort after the long approach.

c. <u>Decreased threat represented by submarines approaching near the LLA_o.</u>

Figure 24 illustrates how the threat by a conventional submarine decreases as it is forced to approach from farther and farther aft.

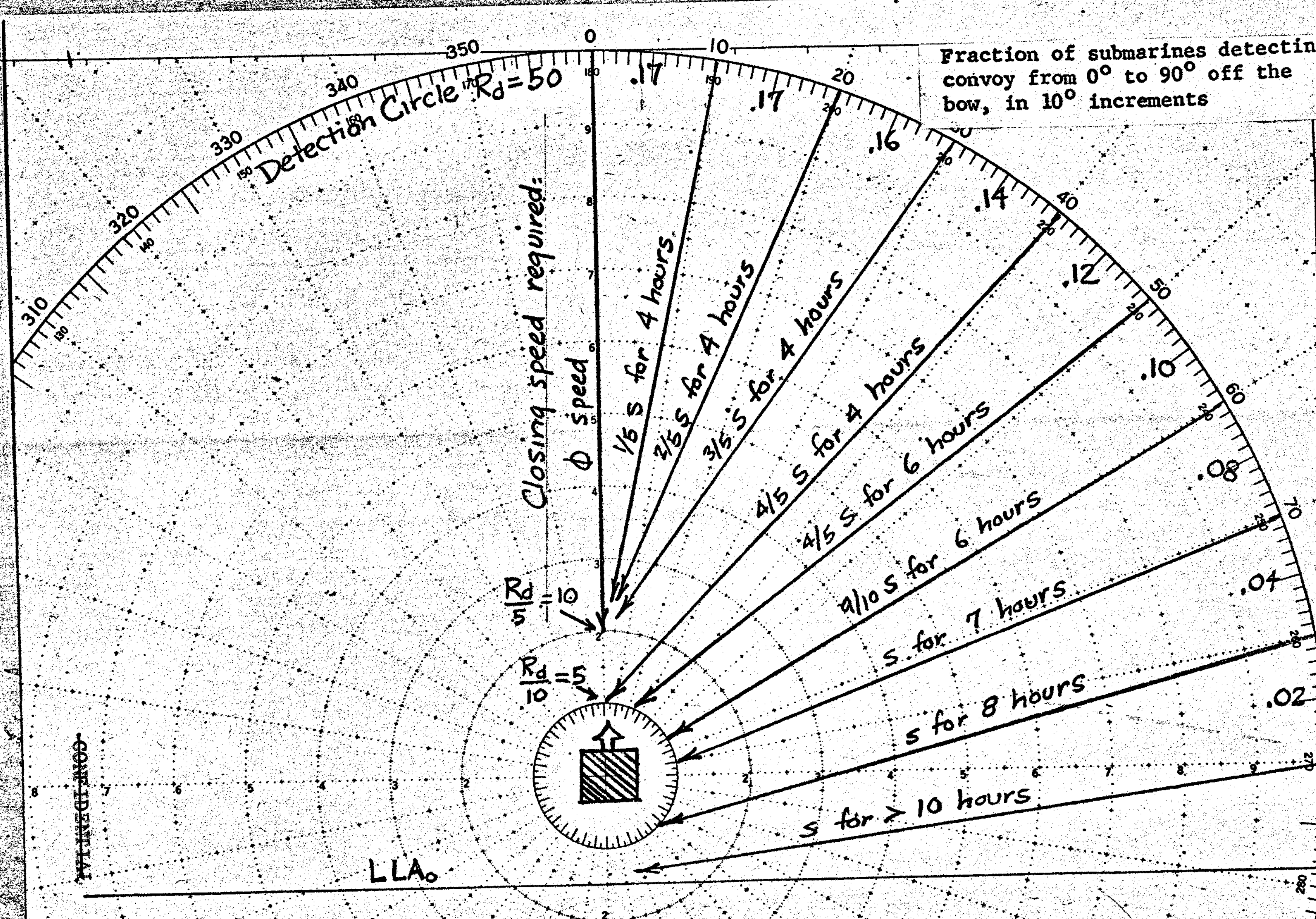


Figure 23. Typical Pattern of Submarine Approach on a Convoy, Ratio of Maximum Submarine Submerged Speed to Convoy SOA, s/v = 1.0

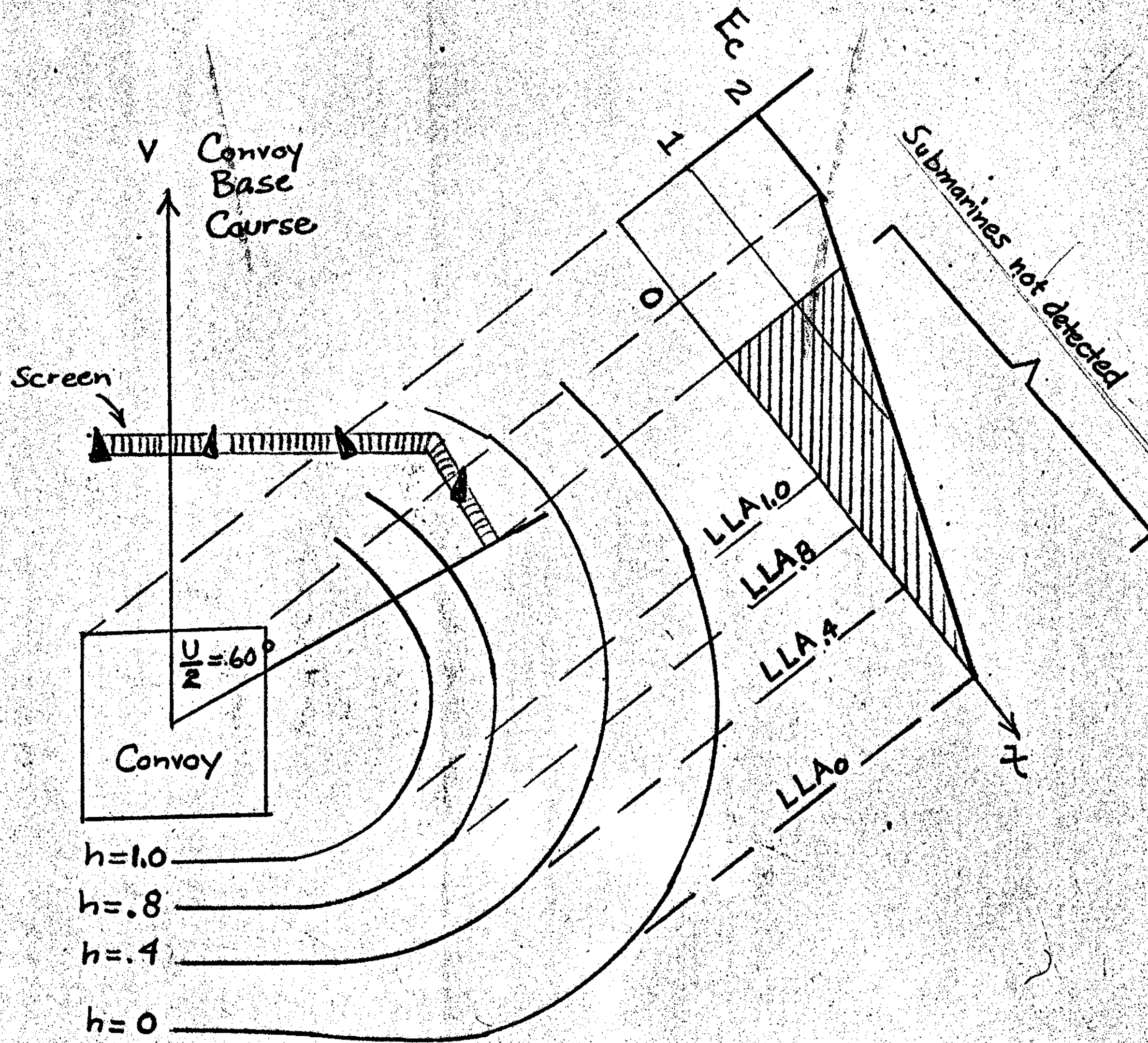


Figure 24. The threat of submarines approaching a convoy with their best relative motion versus the distance x the submarine must approach aft of the front row of the convoy.

Each line labeled LLA_p represents the best relative motion a submarine can achieve when closing with speed s/v = .8. Hypothetical values of expected hits relative to E_c are plotted against the distance x abaft the ideal entry area, the leading row of the convoy. In this illustration, E_c = 2 for an optimal entry, and h_p lines represent distances from the convoy from which the submarine may be expected to attain p hits by firing into the convoy from line h_p.

In this example, if attacking submarines are equally likely to approach in the region aft of the screen's right extremity and forward of LLA_o, the threat in expected hits per submarine there is about 1/3 that of submarines approaching in the screened region. If, in addition, it may be assumed that only 10% of submarines intercepting the convoy are likely to approach in the region aft of the screen, then the threat in the unprotected region is only .03 of the total threat from conventional submarines.

Factors (a) and (b) tend to cause the preponderence of conventional submarines to approach the convoy well forward of the LLA_o. Factor (c) reduces the threat from submarines approaching from near the LLA_o. The following two factors, however, will tend to decrease the density of submarines ahead of the convoy and disperse them toward either beam, and the last factor may also have this effect.

d. <u>Inability of submarine to close on exact relative track desired</u>.

Erroneous closing courses and speeds will disperse submarines on either side of their intended tracks. However, the dispersal of submarines gaining initial detection within 30° of convoy track will be small since the submarine commanding officer has ample time to correct any initial errors. Beyond 30°, the number of submarines that by mistake drift out to higher relative bearings should be roughly compensated for by submarines at higher bearings making similar mistakes, the percentage of errors being greater at the greater bearings, where the relative motion problem is more acute and the margin for error smaller.

e. <u>Effect of convoy zig-zag.</u>

The convoy zig-zag plan may be thought of as dispersing the submarines to the right and left by an amount equal to the deviation of the zig-zag plan from the track made good. The amount of deviation should not be large in proportion to the three to five mile front of a large convoy.

Thus, neither factor (d) or (e) would appear to add much to the proportion of submarines approaching from $U/2 > 60^\circ$.

f. Effect of escort station assignments.

A third and more serious influence that will tend to disturb the natural concentration of submarines in the van may be the presence of powerful sonar on or near the convoy track. An AN/SQS-23 sonar positioned 10 or 15 miles in the van is a persuasive argument to the submariner to choose one side or the other for his approach. If the escort commander has reason to believe that such is the case, he must of course strengthen the flanks. (See Section G of this chapter.)

Summary

1. If conventional submarines are forced to make a submerged approach and if $v \geqslant s$, then the submarines able to intercept the convoy will tend to be concentrated within $U/2 = \pm 60^\circ$. The fraction of submarines approaching from abaft of $U/2 = \pm 60^\circ$ is nearly constant over a wide range of speed ratios, and at $R_d/5$, is about .12 of all submarines closing.

2. Conventional submarines approaching from abaft of $U/2 = \pm 60^\circ$ are less of a threat than those approaching from forward of 60°, by a factor of 1/4 to 1/2, in terms of expected hits.

3. The escort commander must appreicate that a slight conventional submarine threat exists beyond the 60° radii. The two factors which probably influence the amount of this threat the most are known to him, namely:

(a) The vigor with which fixed wing aircraft search just forward of the snorkeling LLA.

(b) The disposition of his own long range sonars.

4. The use of $U/2 = \pm 60^{\circ}$ to define sector 1, the region of conventional submarine threat, is appropriate.

2. <u>The Distribution of Nuclear Submarines Around the Convoy</u>

The probability $S(\theta)$ that nuclear submarines will detect the convoy at a given bearing θ relative to the convoy base course depends on the patrol speed of the submarine relative to the speed of advance of the convoy. The solution for $S(\theta)$ is given on page 8 of OEG Report 56 [4]. For ratios of patrol speed to convoy speed of .5 and 1, the distribution of detecting submarines by bearing is as shown in Figure 25.

Thus, for a .5 ratio (e.g., submarine search speed of 5 knots and convoy speed of 10 knots) the proportion of submarines detecting the convoy in 30° sectors is as follows:

$0-30^{\circ}$	.47
$30-60^{\circ}$	.34
$60-90^{\circ}$	.15
$90^{\circ}+$	.04

If the ratio is 1.0 then the proportion by sector becomes:

$0-30^{\circ}$	.42
$30-60^{\circ}$	.31
$60-90^{\circ}$	.20
$90^{\circ}+$	.07

It is instructive to observe that like conventional submarines, the preponderence of nuclear submarines which detect and classify the convoy will approach the convoy from ahead. However, this observation is not very helpful in deploying escorts. We can inconvenience and delay the nuclear submarine by concentrating escorts in the van and forcing him to penetrate from the rear. But essentially the nuclear submarine has the ability to probe for a screen weakness and exploit it, <u>if the weakness can be found</u>. How obvious the weakness must be is a question we do not examine further, but it is an important question to be answered.

Clearly if given a choice, the nuclear submarine would most prefer

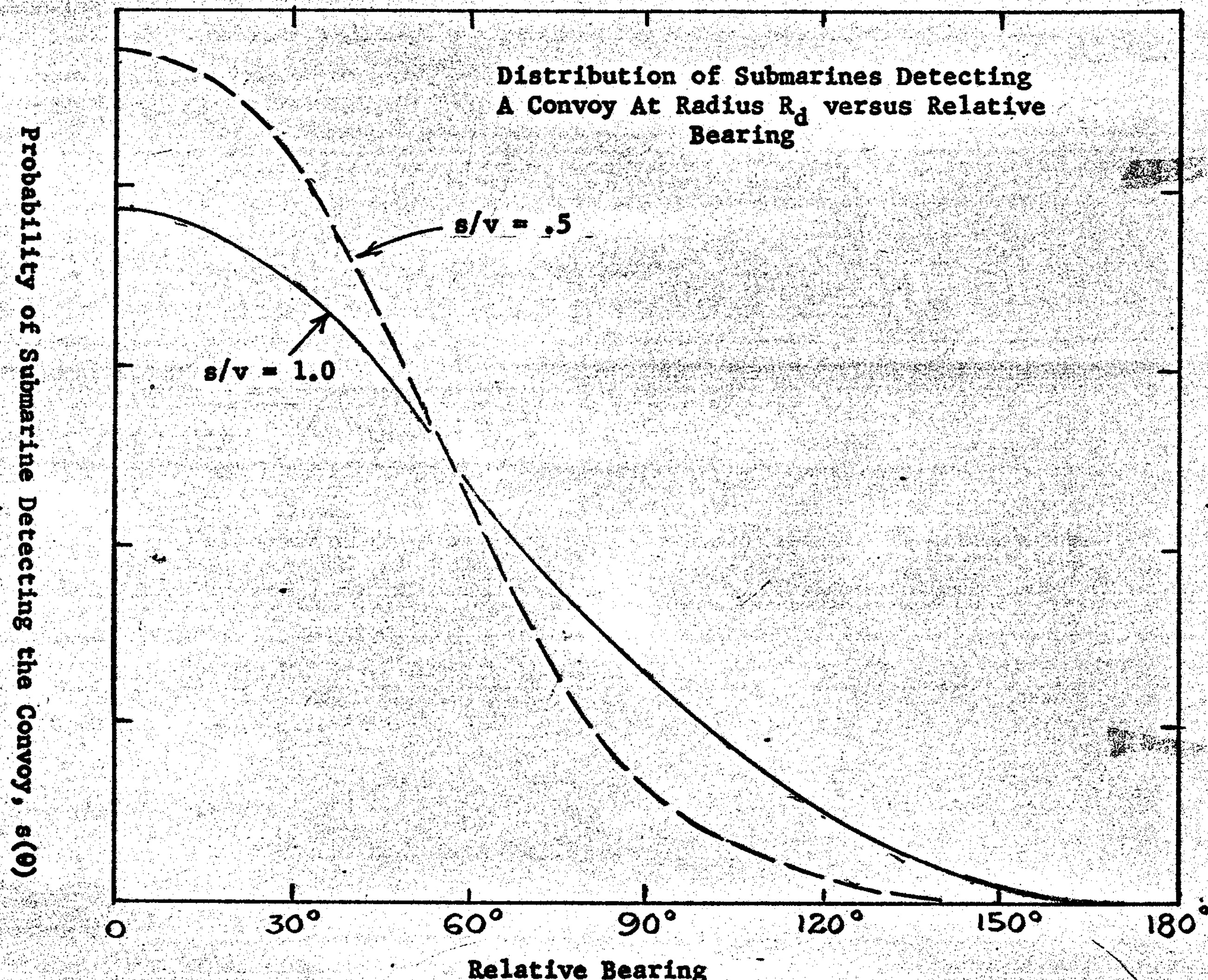

Distribution of Submarines Detecting
A Convoy At Radius R_d versus Relative
Bearing
s/v = .5
s/v = 1.0
Probability of Submarine Detecting the Convoy, s(θ)
0
30°
60°
90°
120°
150°
180°
Relative Bearing

Figure 25.

to close from ahead. Arguments have been advanced both that a beam approach and a stern approach would be the next preference. We will only say that, as between the two choices, the submarine commanding officer will attack where he thinks the screen is weakest. However, against a rectangular convoy, an approach from either quarter is sufficiently undesirable that these regions may be given somewhat less attention, since:

(1) The submarine has a difficult relative motion problem.

(2) The submarine must make almost as much speed to attain the same closing rate as it does from astern. (Closing from 120° relative, about 85% as much.)

(3) The submarine cannot take advantage of the convoy wake. The net effect is that the escort commander must assume equal likelihood for all bearings outside of sector 1, and distribute his search appropriately, except perhaps, for some lessening of effort on the quarters.

The nuclear submarine must get into the convoy to fully exploit its technological advantages. In the convoy is the most secure place to carry out its attacks. Any time the escorts can compel it to fire from outside the screen, even though it secures hits, (1) the nuclear submarine's effectiveness has been reduced and (2) it is much more liable to counteraction. Therefore, aft of the LLA for conventional submarines, escorts should generally accept the threat of hits from firings outside the screen to reduce the more serious danger of the submarine getting into the convoy unless the screening force is very large. In other words, when a solid screen cannot be presented, the escorts should patrol as close to the convoy as possible, within limitations imposed by the time required to attack an approaching submarine, once detected, and the aforementioned wake interference.

C. Determining Escort Detection Ranges for Computational Purposes

In the conceptual solution, Chapter III, employment of the most correct representation of escort operational detection capability was envisioned. In Model I of Chapter IV, the overall detection probabilities P_1 and P_2 in sectors 1 and 2 respectively were assumed to be known. How they were to be computed was not considered. In Model II of Chapter IV, the computational model, a definite range law was taken as a sufficiently accurate approximation of the escorts' true detection capabilities for the purposes of the calculations. Furthermore, computations were based on the premise that the degradation resulting from overlapping of the true detection capabilities would have an inconsequential effect on the results. Further discussion of the significance of this assumption is deferred to Section D of this Chapter. It is also assumed that the average sonar search effectiveness α of an escort stationed in sector 2 relative to its effectiveness in sector 1 is known. A brief discussion of this topic may be found in Section E, as well as in sub-section 2 below.

Proper characterization of operational sonar performance is naturally the subject of much study. The accuracy of the results of our analysis naturally depends on the accuracy of the model of escort detection; the same may be said of any model using sonar performance. Although it is not our purpose to study the problem here, two remarks are in order concerning a practical procedure for determining the proper values of detection ranges to be used for computational purposes.

1. Choosing Between Best Depth Range and Periscope Depth Range

The two sonar ranges that are generally known to an operational commander are the best depth range (BDR) and the periscope depth range (PDR) for each escort. The difference between them is so great that a choice between the two would make a fundamental difference in the results. A not unreasonable procedure would be to offer no guidance but to leave entirely to the discretion of the escort commander the choice of either, or a range in between, based on his knowledge of the tactical situation.

An alternative would be to offer the escort commander a simple rule-of-thumb, such as the following reasonable but arbitrary procedure: If the escort commander believes that at least half of all submarines approaching the convoy will come to periscope depth within PDR of an escort, he is to use 1/2 (PDR + BDR) as his sonar range. Otherwise he is to use BDR.

At first glance it might appear to the analyst that as a second alternative, a simple linear combination of ranges could be easily determined. Reflection on the problem will reveal however that a number of factors serve to complicate theproblem to the point that, in view of the lack of precise information, the effort would not be worthwhile. To be considered would be:

(1) The proportion of nuclear submarines.

(2) The relative threat of each class of submarines.

(3) The probability that each class would come to periscope depth within range of an escort in each sector.

(4) The assignment of escorts with various BDR and PDR between the two sectors.

(5) The fact that the mean ranges calculated for each sector would be different.

We believe that the most effective procedure is the simple use of escorts' BDR for all computations with the model. Unqualified use of BDR is, of course, pessimistic. This pessimism is offset by two overly optimistic factors concerning escort performance that exist in Model II. The first is the assumption that submarines are unable to split bearings or take advantage of gaps in the screen. The second is that the model does not provide for the aforementioned degradation of performance resulting when detection capabilities overlap. We regard the consistent use of escorts' best depth ranges for all calculations to be the optimum procedure.

2. Effect of Escort Patrolling Station of Detection Capability

For convoy speeds up to twelve knots or so, that is, where escorts have a sufficient speed margin, the escorts can increase their effectiveness by patrolling stations. A mathematical model for determining the amount of improvement is described in detail in Appendix A of OEG Study 575, "Force Requirements for Anti-submarine Sonar Screens to Protect Convoys" [14]. In particular (neglecting other factors discussed in Section E) an escort which patrols its station will, with the same probability of detecting submarines attempting penetration, screen a larger area at the rear of the convoy than in the van.

In our model, α represents the patrolling effectiveness factor of escorts assigned to sector 2 with respect to escorts assigned to sector 1. That is, if α_1 is the patrolling effectiveness factor in sector 1 and α_2 is the patrolling effectiveness factor in sector 2, then

$$\alpha = \alpha_2 / \alpha_1 .$$

In the model, for simplicity of presentation, we have chosen $\alpha_1 = 1$. Hence, $\alpha = \alpha_2$. If α_1 is greater than one, as it normally is for escorts patrolling in the van, the effect is equivalent to an increase of Y, the total (summed) sweep width of the available defensive force. For example, a force of Y = 25,000 yards with $\alpha_1 = 1.2$ is the equivalent when patrolling station of a force for which Y = 30,000 with $\alpha_1 = 1.0$ (i.e., maintaining fixed station).

In the model, $\alpha = 2.0$ is the representative value we have chosen to use. Since we have also chosen $\alpha_1 = 1.0$, letting $\alpha = 1.0$ corresponds closely with a fixed station, non-patrolling screen.

D. The Model of Escort Detection Capability -- A Discussion of Lateral Range Curves and Their Approximations

1. The Lateral Range Curve, the Definite Range Law, and Sweep Width

When a sonar and its submarine target are on straight, reciprocal courses at constant speeds for a long time before and after CPA, the probability of detection p(x) is a function of the lateral range x. The graph of p (x) is called a Lateral Range Curve. In Figure 26, the escort is at the origin and the curve p(x) is the probability of detecting a submarine passing at any range x from the escort.

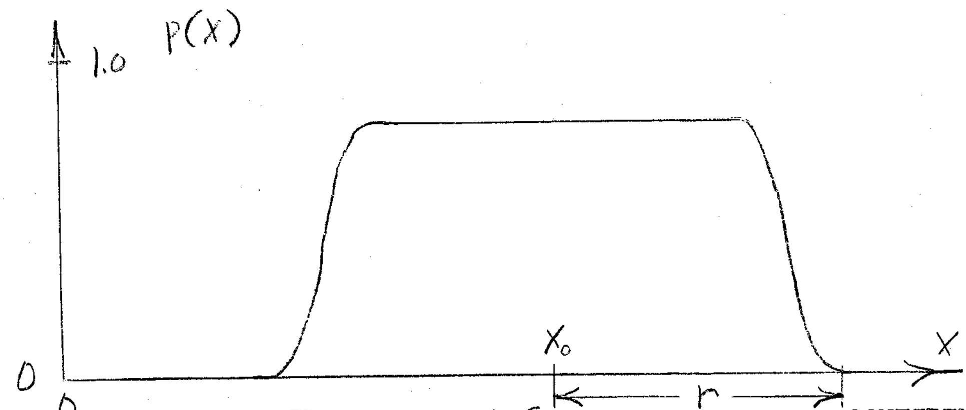


Figure 26

A sonar's lateral range curve varies with every change of parameter, more drastically than a radar's lateral range curve, and much more drastically than the lateral range curve for visual search. Submarine type, speed, depth, and aspect, sonar performance, and water conditions all combine to make p(x) fluctuate drastically.

A simple form of Lateral Range Curve that is frequently assumed in analyzing sonar performance is known as the "Definite Range Law". Because it is popular and mathematically simple, and because of its similarity to the widely known Sweep Width concept, we adopted the Definite Range Law in Model II. The Definite Range Law assumes that if a submarine passes within a certain range R characteristic of the sonar, detection always occurs. If the submarine passes outside that range, detection never occurs. The Definite Range Law is represented graphically in Figure 27.

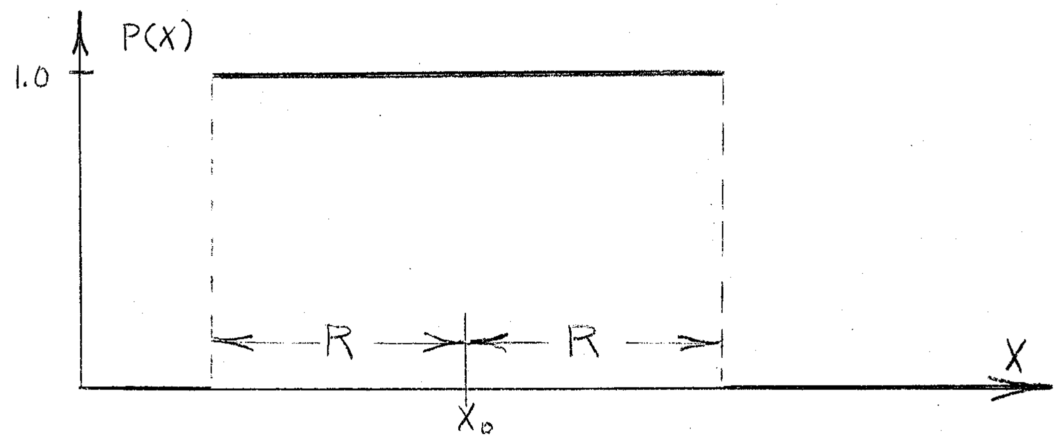


Figure 27

The Definite Range Law is identical mathematically with the Sweep Width approximation of any Lateral Range Curve p(x). As ATP-1(A) Vol 1 [7] states on page 8-4, the Sweep Width expresses the measure of detection capability in which the maximum detection range of a sweep is reduced so that the number of targets detected beyond the sweep width W is equal to the number of targets inside W that are missed. Mathematically,

$W = \int_{-\infty}^{\infty} p(x)dx$ is the area under the lateral range curve, and

$W = 2R_W$ is the equivalent expression for sweep width, so

$R_W = 1/2 \int_{-\infty}^{\infty} p(x)dx$. Graphically:

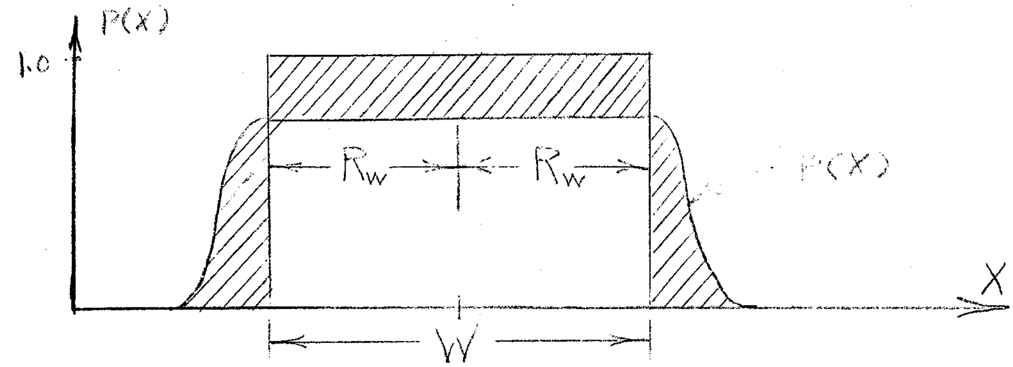


Figure 28

In Figure 28, R_W is such that the shaded area under the Lateral Range Curve p(x) and beyond R_W from x_o (representing extra targets detected) is equal to the shaded area above the curve p(x) and within a distance R_W of x_o (representing short-range targets missed). For any given W, the range R of the Definite Range Law equals R_W, the effective sweep width, and the two concepts are mathematically equivalent.

The similarity between the two concepts is more apparent than the distinction. Conceptually, the Definite Range Law purports to represent the actual sonar detection "curve". The Sweep Width is only supposed to be a simple but useful measure of effectiveness of a true lateral range detection curve, p(x). Both have been referred to as approximations of lateral range curves.

The Sweep Width (or Definite Range Law) approximation of sonar detection capability was used in our computations with reluctance. It was attractive mathematically of course, but the decisive factor in its favor was the desirability of using as many familiar and commonly accepted concepts as possible, without significantly compromising accuracy. The Definite Range Law was found repeatedly being used in a wide variety of analyses. We felt that we would be well within our goal of keeping the inaccuracies of the model less drastic than the inaccuracies of the input data.

In retrospect, we are not sure that our concession to tradition was wise. In the first place, because of the nature of the problem (or at least because of the nature of our model of the problem) our solutions frequently involve putting escorts' detection zones adjacent to each other. More precisely stated, we occasionally had a solution with escorts stationed $R_i + R_j$ yards apart (R_i and R_j being the half-sweep widths of the ith and jth escorts), resulting in a calculated probability of detecting 100% of submarines passing between them. The result was a slightly less accurate "optimal" division of forces.

In the second place, we believe we may have foresaken a good opportunity to join in a coming battle to modify the Definite Range Law into a more valuable form. The remainder of this section is devoted to presenting an argument for more widespread use of the simple, more conservative, and more accurate Modified Definite Range Law.

2. <u>How Adding Sweep Widths Introduces Error into Overall Detection Probabilities</u>

The Sweep Width concept (or alternatively, the Definite Range Law used as an approximation) is a simple, one-number way of describing the effectiveness of a sonar. It has proven to be convenient for computation and accurate enough for most situations. As long as sonar search paths are far enough apart that regions of positive detection probability do not overlap, accurate results are obtained.

In Figure 29 below, escorts are stationed at x_1 and x_2, a distance d apart.

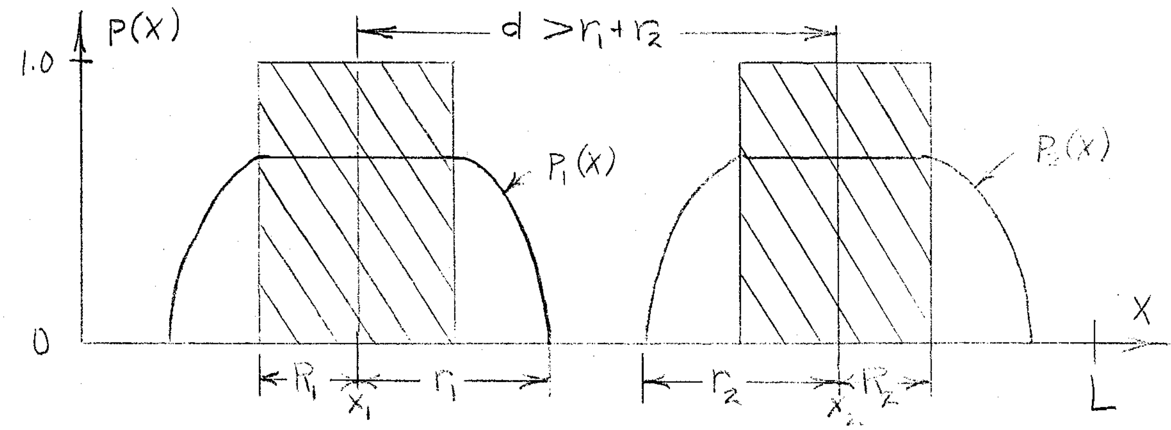


Figure 29

The heavy bell-shaped curves are the lateral range curves $p_1(x)$ and $p_2(x)$ for the escorts. They show a positive probability of detection out to a distance r_1 on either side of the escort at x_1, and out to a distance r_2 on either side of the escort at x_1. The shaded rectangles represent the equivalent Definite Range Laws. If the two escorts are screening an area of width L, as shown, the overall probability of detecting a submarine attempting to penetrate the screen on the reciprocal of their track (assuming the escorts are not patrolling station and the submarines are equally likely to pass at any point in the interval from 0 to L) is:

$$P = 1/L \int_0^L p_1(x)dx + 1/L \int_0^L p_2(x)dx,$$ or equivalently, the sum of the two sweep widths divided by the width of interval being screened:

$$P = \left[2R_1 + 2R_2\right] 1/L$$

as long as the distance between escorts is at least $r_1 + r_2$.

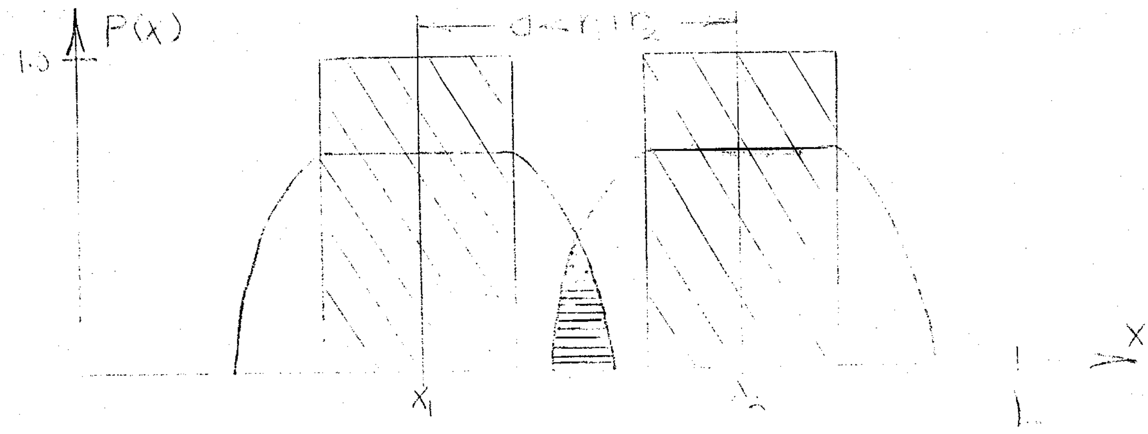


Figure 30

However, if the bell-shaped detection curves p(x) overlap, i.e., if $d < r_1 + r_2$, then the overall probability of detection is no longer simply the sum of the detection probabilities but somewhat less, because the horizontally cross-hatched area in Figure 30 is being searched twice by the escorts, and probabilities in this region are not additive.

If one continues to evaluate the overall detection probability P using the Definite Range Law approximation, he is likely to add Sweep Widths after the lateral range curves have started to overlap. That is precisely the error we introduce in some of our Model II calculations when we assume a Definite Range Law. A rule of thumb has been that in calculating the overall probability P of detecting a submarine attempting penetration of a screen such as a bent line screen, no significant error will result from adding sweep widths until the probability of detection exceeds .7. In Model II, the optimal solution occasionally exceeds that figure in one sector or the other. We have not analyzed the amount of error we thereby introduce but we believe it is small compared to the accuracy of such estimates as @, the fraction of submarines attacking that are nuclear.

3. Why the Error Involved in Using Sweep Widths is More Serious Than In the Past

A number of new factors have caused an increase in the likelihood that serious error will be introduced by summing Sweep Widths in order to determine overall screen detection probabilities.

(1) Patrolling is, or ought to be, employed in most escorting situations. Patrolling station results in more frequent overlap of lateral range detection curves, and hence, more error.

(2) Evidence is accumulating that the operational lateral range curves are lower than was often assumed in the past, more like Figure 32 than Figure 31.

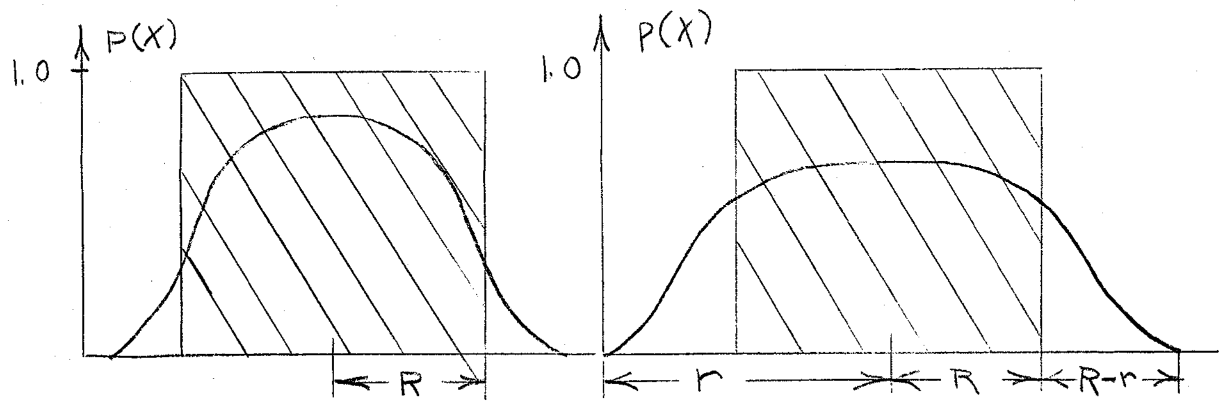


Figure 31 Figure 32

The equivalent Sweep Width, $W = 2R_W$, becomes more hazardous to use as the maximum height of p(x) becomes lower. If Figure 32 represents the true lateral range curve, the distance r - R that the true curve projects beyond the Sweep Width approximation is much greater than in Figure 31. Although the data we have seen are skimpy, we believe the curve showing operational detection curves with a maximum probability in the neighborhood of .6 - .8 will appeal more to the intuition of officers with extensive ASW experience than a curve with a maximum in the neighborhood of .8 - .9. See pages 7pp of reference [17] and Appendix A of reference [3] for recent evidence supporting the lower figures.

(3) The AN/SQS-26 sonar has a detection pattern in deep water under certain sound conditions that cannot be approximated in even the crudest way by a Sweep Width.

(4) The modern tactical situation is such that frequent overlapping of in-layer detection ranges (Periscope Depth Ranges) may occur. Modern sonars have much longer in-layer detection ranges than below-layer ranges (Best Depth Ranges), compared with World War II sonars. When screen spacing is guided by below layer ranges as one would always prefer, appreciable overlapping of in-layer detection regions will result.

These four factors combine to make a powerful argument to abandon the Sweep Width system of calculating sonar performance, other than as a rough figure of merit. This is true even when one distinguishes separate Sweep Width figures for PDR and BDR.

4. Certain Weaknesses in the Lateral Range Curves

From an operational point of view, the best study with which the authors are familiar of broadly applicable sonar detection characteristics is OEG Study 69, Characteristics of Sonar Performance as Indicated by Analysis of 1951 OPDEVFOR Trial Data, [11]. Although published in 1953, it is as pertinent today as when written. The study was prepared by B. O. Koopman, a leading analyst in the field of search theory.

One conclusion of the study was that under a set of ascertainably similar conditions there is a range R outside of which detection is unlikely and within which it is fairly certain. Contact solidity (i.e., the tendency for contact once gained to be held) will be high within R and very low beyond; and the value of R will be subject to only minor dispersion -- dispersion attributable to small variation in conditions or data taking and not to the nature of the detection. "The practical issue is whether there is a set of observable conditions which, when kept fixed, leads to an approximate definite range law of detection."

Koopman concluded that once the conditions are sufficiently constant, a Definite Range Law of detection is obeyed with good approximation.* His conditional conclusion has been used, unfortunately, all too often as a justification for the erroneous assumption that the Definite Range Law could be used for calculating probabilities in operational situations in a changing environment. This was not what Koopman intended to imply.

In the first place, for the Definite Range Law to apply, a necessary condition is a continuously alerted sonar operator. In the second place, using the Definite Range Law as the most accurate model of a sonar's performance in an operational environment assumes that the variables can continually be measued and accounted for in the formulation of the detection range R; but the fact is that R fluctuates in and out as water conditions, operator alertness, and even target aspect change! Koopman's investigation of the validity of the Definite Range law was for other purposes, two of which are discussed below.

He then undertook the problem of how to handle the effect of variable conditions, and hence variable range, in practice: "The second method for dealing with range variability is statistical. It assumes that on each approach the definite range law is valid but that the detection range is chosen at random from a set of values distributed statistically according to some ascertainable law." Instead of attempting to discover all the underlying conditions and how they affect R, one tries to measure their total effect and the modification which they induce on the lateral range probability p(x). The result is the familiar lateral range curve, p(x) -- experimentally determined and far more apt to fit the situation encountered in naval operations, with all the variability of conditions that are encountered.

*But he added that for a given sonar and target, traveling at given speeds, the four easily observable conditions -- submarine depth, sonar message, sea state, and target aspect -- did not, when held constant, ensure a Definite Range Law.

Koopman points out two important implications:

(a) The Lateral Range Curve, p(x), is the product of its environment.

There is no one p(x) applicable to a given sonar. The lateral range curve p(x) is also a product of its geographic environment. If it is derived from data taken off Key West, it is, strictly speaking, only applicable in waters off Key West, or waters which have similar acoustic characteristics to those off Key West. If the lateral range curve is derived near Norfolk, or in the Mediterranean, or the South Pacific, the same reservations holds.

While conducting ASW operations, our practical solution is to try to eliminate the water-condition variable by bathythermograph reading and by correcting our estimated range accordingly. But we should not visualize that eliminating this one variable transforms the bell-shaped p(x) into anything approaching a rectangular-shaped definite range law.

In calculating overall screen detection probabilities, such as the probability that a submarine will penetrate, say, a 5C6 bent line screen, analysts are fairly smug about the results if they use the sonars' published lateral range curves in calculating the detection probabilities of the screen. We do regard these values as high because they are taken under experimental conditions with alerted operators, and we do attempt to compensate for this. But we seldom consider, or even mention, the fact that even the most precise Lateral Range Curve is applicable only for a particular water environment.

The practical significance of the geographic variability of p(x) is this: a close approximation to a p(x) such as the Modified Definite Range Law probably loses very little in accuracy from its mathematically much more difficult parent, the Lateral Range Curve.

(b) Adjacent ship detection probabilities are statistically non-independent

If the overall screen effectiveness is being calculated using appropriate lateral range curves $p_i(x)$, usually the assumption is made that the individual detection probabilities are statistically independent. Suppose that several escorts are in line abreast facing a submarine attempting penetration on an anti-parallel course. Fixing attention on any two, separated a distance S (see Figure 33) consider a submarine

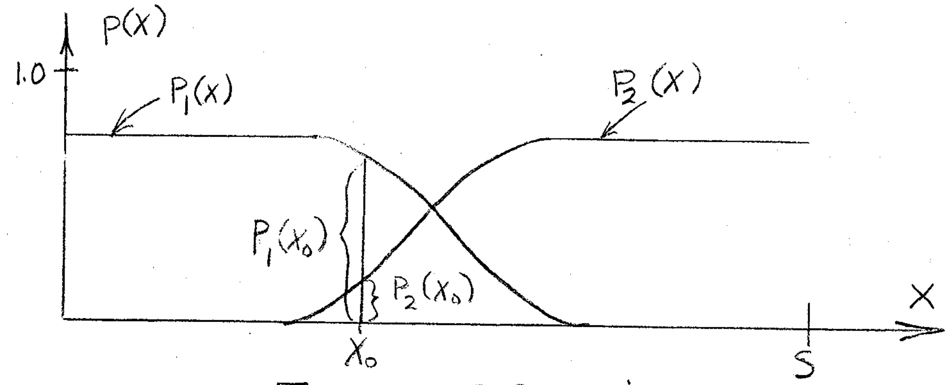


Figure 33

passing at distance x_o from escort Nr. 1 and a distance $S - x_o$ from escort Nr. 2. Let $P(x_o)$ be the probability that the submarine is detected by either ship. The usual method of evaluating $P(x_o)$ is to regard $(1 - p_1(x_o))\ (1 - p_2(x_o))$ as the probability that both ships fail to make detection. Then the probability that one or both detect the submarine is:

$$P(x_o) = 1 - \left[1 - p_1(x_o)\right] \left[1 - p_2(x_o)\right]$$

$$= p_1(x_o) + p_2(x_o) - p_1(x_o) \cdot p_2(x_o)$$

But this expression assumes that the detections by the escorts are independent events.

As Koopman points out on page 58, "Since a greater definite range R means a greater chance of detection on the part of each surface craft (supposed to be operating sufficiently close together under sufficiently homogeneous sound conditions so that they are both subjected to approximately the same R), the events of their detection are dependent and in fact positively correlated. Consequently the assumptions underlying the screen formula are false." In summary, if water conditions are such that one escort misses the submarine, the probability is greater than the formula indicates that the other escort will also miss the submarine. Averaging over a wide variety of sonar conditions such as would be experienced in an ocean transit, the number of detections is less by a significant amount than would be calculated assuming statistical independence. As an example, suppose that water conditions are such that two escorts stationed adjacently in the screen have detection ranges of 3000 yards for half the transit and 2000 yards for half the transit. Consider the probability of detecting a submarine penetrating exactly half way between the escorts stationed at a distance of 5000 yards apart. Using a single lateral range curve for the whole transit, the probability of detection would be calculated as .75. Computing the probabilities of detection for each half of the transit separately, the overall probability of detection is only .5. And the latter calculation assumes that as many submarines will attack when water conditions are unfavorable to them as will attack when water conditions are favorable.

The foregoing amounts to another argument favoring the use of a Lateral Range Curve that is truncated at the extreme detection range R for use in calculating screen effectiveness, i.e., a Modified Definite Range Law.

One final caution regarding OEG Report 69 must be added. The investigation was conducted as a statistical analysis of experimental data obtained with AN/SQS-4 sonar at detection ranges on the order of 3,000 yards. The physics of the situation was treated as incidental. Anyone familiar with the complex factors involved in an analysis based on acoustic principles will appreciate the advantage of the statistical approach. At the same time he will understand why Koopman was forced to conclude that all important variables had not been included among the four that were held constant. What is remarkable is that at relatively short ranges, the physical variables such as surface reflection, reverberation, transmission loss, and roll and pitch of the escort may be disregarded to the extent that, for all practical purposes, a definite range law applies.

Only in the most idealistic sense, however, could detection at ranges longer than 3,000 to 5,000 yards be said to obey a definite range law. Beyond some such range, signal distortion and scattering, the interference of multiple surface and bottom reflections, and other effects of the physical environment, have so powerful and variable an influence on sonar signal attenuation, that any attempt to identify and fix them, even for experimental purposes, would probably fail. In any case, bottom bounce and convergence properties of acoustic signals compel the abandonment of the Definite Range Law for characterizing either experimental or operational detection properties of long range sonars such as the AN/SQS-26.

E. The Modified Definite Range Law

The Modified Definite Range Law was conceived as a measure of operational effectiveness of any sensor.* The Modified Definite Range Law assumes that everywhere within a range R, targets are detected with probability p (rather than probability one) and outside of R, no targets are detected. See Figure 34.

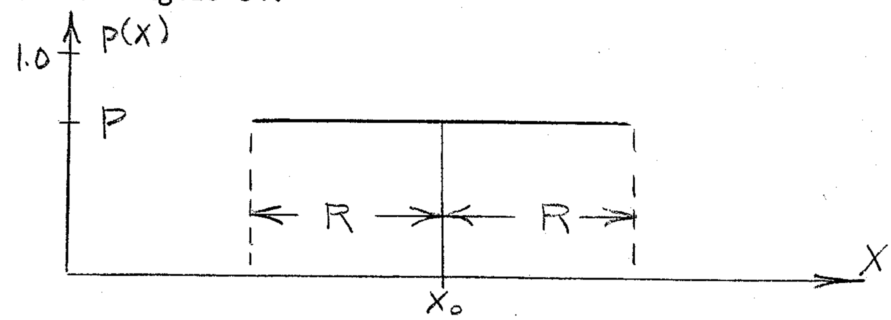


Figure 34

The Sweep Width for the Modified Definite Range Law is therefore: $W = 2pR$ (instead of 2R as for the Definite Range Law).

The Modified Definite Range Law has two advantages over the true lateral range curve:

1. It is far easier to use mathematically.
2. It is easier to obtain data on which to determine it. See [3] for the procedure for calculating from operational exercises.

The disadvantage is some loss of accuracy, but Section D points out two reasons why the loss is less than might otherwise be supposed.

*It came to the attention of the authors through the thesis of a previous Operations Analysis student, LT. W. E. Clark, USN [2]

The Modified Definite Range Law has two advantages over the Definite Range Law.

1. The Modified Definite Range Law more closely approximates the sensor's actual performance, a particularly important factor when the maximum probability of detection is considerably less than one.

2. The Modified Definite Range Law serves as an alert to an erroneous estimate of detection performance resulting when two or more sensors search on parallel tracks with overlapping lateral range curves.

The disadvantages of the Modified Definite Range Law compared with the Definite Range Law are:

1. It is slightly more difficult to grasp conceptually.
2. It is slightly more difficult to manipulate mathematically.

The advantage of the Modified Definite Range Law over the Definite Range Law may be seen graphically if one compares the form of a recent lateral range curve determined experimentally against nuclear submarine targets with, first the Definite Range Law, and second, the Modified Definite Range Law. See Figures 35 and 36 on the following pages. The experimental data is taken from [17].

One final advantage of the Modified Definite Range Law over the Definite Range Law as an approximation of the true Lateral Range Distribution might be mentioned. The height p or the width R may be changed if there is some good reason for doing so. The only criterion is that p, or equivalently R, is chosen to most usefully approximate the true lateral range curve. For example, looking at Figure 36, $p = .8$ might have been chosen giving, $R = 2120$ yards; or $R = 3400$ yards might have been chosen, giving $p = .5$. Normally the choice would be the one which would minimize the amount of shaded area, but there might be reasons for doing otherwise. From this point of view, the Definite Range Law is merely an extreme case of the Modified Definite Range Law.

F. Effect of Submarine Bearing of Approach on Detection Probability

A number of interrelated factors make the relative probabilities of detecting a given submarine with a given sonar, for different relative

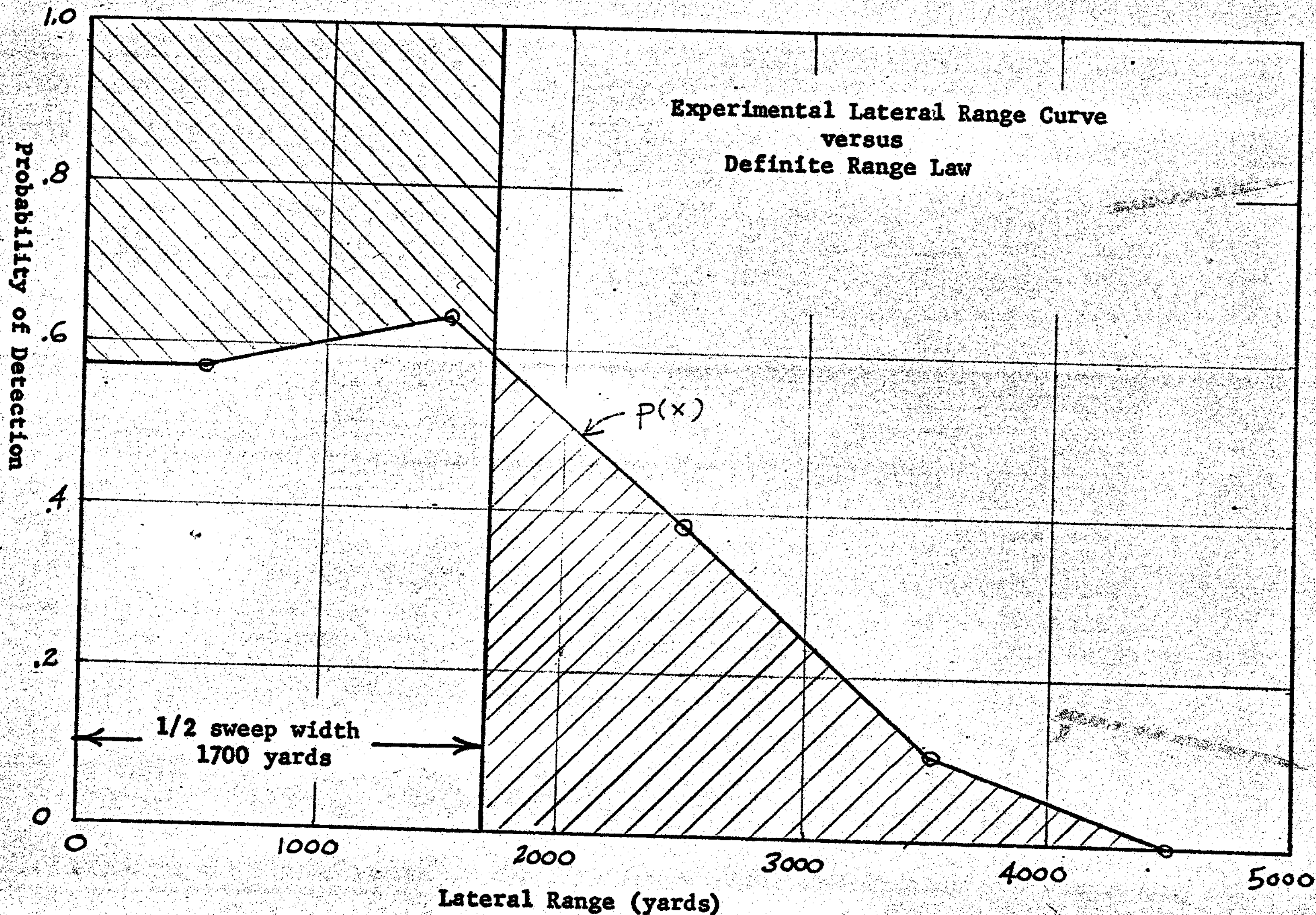

Experimental Lateral Range Curve
versus
Definite Range Law
Probability of Detection
1.0
.8
.6
.4
.2
0
0
1000
2000
3000
4000
5000
Lateral Range (yards)
P(x)
1/2 sweep width
1700 yards

Figure 35.

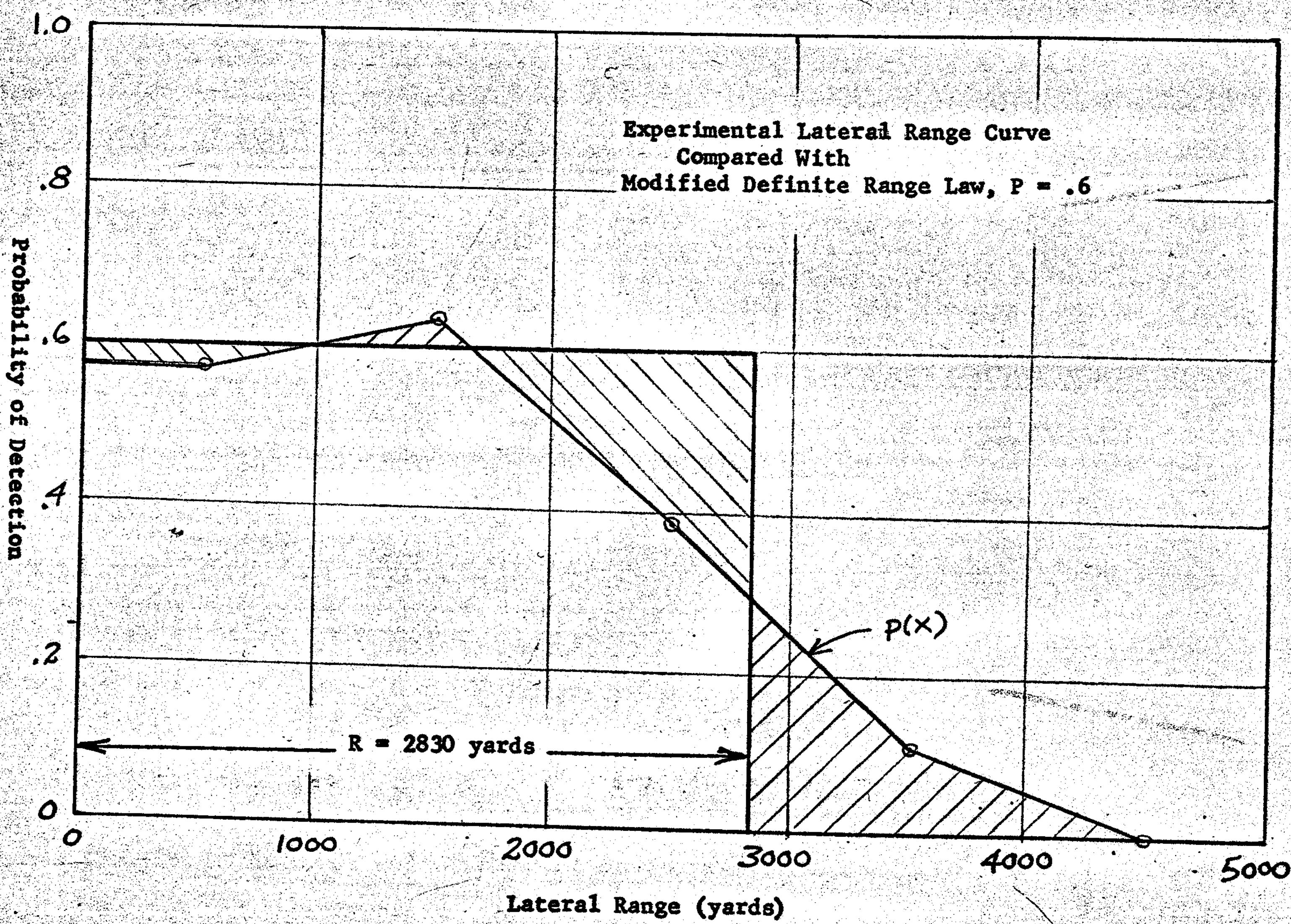

Experimental Lateral Range Curve
Compared With
Modified Definite Range Law, P = .6
Probability of Detection
1.0
.8
.6
.4
.2
0
R = 2830 yards
p(x)
0
1000
2000
3000
4000
5000
Lateral Range (yards)

Figure 36.

bearings of approach and different approach speeds, very difficult to estimate analytically and impossible to measure with any precision. Specifically these factors are:

1. Relative speed of approach:

(a) Patrolling escorts can screen greater areas in submarine-space with the same probability of detection (See Appendix A of [14]).

(b) Submarines with slower relative speeds (closing rates) will be within the detection zone of an escort longer, whether the escort patrols or not.

2. True speed of approach:

(a) A submarine may be significantly noisier when approaching from astern, in which case it will be more liable to passive detection.

(b) At high speeds, the submarine will be able to gain less listening information for splitting bearings, etc.

3. A submarine has less control over the aspect it will present when approaching from abeam or astern.

4. Wake effect degrades hull-mounted sonar performance astern of a convoy.

Wartime data will not give the answer: in wartime we will seldom know how (from what direction primarily) a submarine that gets into the convoy evaded the screen. We will only know how submarines did <u>not</u> penetrate successfully.

Yet the <u>relative</u> probabilities of detection will be fairly easy to learn experimentally. We need not worry, at least as a first approximation, about the difference between experimental and operational performance and other factors which are important in determining actual operational lateral range curves. By making the reasonable assumption that operational performance is degraded proportionately for all approach bearings, knowing the detection probabilities relative to each other will satisfy the need. These may be determined by experiment in which the emphasis is on uniformity of all conditions (water, submarine depth, sonar performance, etc.) except bearing of approach. Since these relative probabilities have such an important effect on the proper disposition of escorts it is important to obtain data as soon as possible.

G. The Best Range at Which to Station Escorts

Introduction

In developing the analytical model (Chapter IV, Model II) we assumed that regions of equal torpedo threat to the convoy were circularly distributed and concentric with the center of the convoy. We assumed that the threat varied linearly with range from the center of the convoy, having a maximum value at the perimeter of a circle, H_0, established as the minimum feasible at which to station the escorts, to a value of zero at the maximum range of the enemy torpedo. A scaling factor was introduced to allow for varying the estimate of the difference between the number of hits, h_{max}, a submarine would be expected to achieve by firing at H_o and the number of hits it would be expected to achieve once it penetrated the screen.

One procedure that may be used for establishing the escort perimeter has been to station screening ships and helicopters near the limit of the torpedo danger zone (abbreviated TDZ. See Figure 2-6, [8]). A more realistic procedure, even with a powerful screening force, would be to station the escorts where the probability of a hit is small, say .2. This procedure has the advantage of simplicity, but allows no provision for screens so weakened by deploying them at the required range that submarines would have a considerable opportunity to split bearings and penetrate between escorts.

Bent line screens are based on the concept of stationing escorts at the range from the convoy at which the probability of a submarine sinking or severely damaging a single major target by firing at that range equals the probability that the submarine can penetrate the existing screen successfully [18]. The most important weakness in this approach for establishing convoy screens stems from the fact that in a convoy there is not one but a multiplicity of targets. Submarines firing from long range and submarines electing to penetrate may be able to attain hits on several merchant ships.

Balancing the threat

A first approximation of the proper distance from the convoy to station escorts (for a given bearing) is one that balances the threat in expected number of hits by a submarine firing at a given range before it is neutralized by the screen, with the expected number of hits by a submarine which attempts to penetrate the screen. We used this concept to develop the much simplified method we used in Model II.

An example of balancing the threat at a particular bearing for a particular class of submarines will illustrate the principles involved. Fixing attention at one bearing, the threat from outside is

$$T' = N' \cdot H'(r)$$

where N' is the number of torpedoes the submarine may be expected to fire before neutralization and H'(r) is the probability of a hit from range r.

The threat from a penetrating submarine is

$$T = N \cdot H \cdot (1 - P) \qquad \text{where}$$

N is the number of torpedoes the submarine may be expected to fire once it penetrates the screen, before it breaks off its attack or is neutralized.

H is the probability of a hit from in or near the convoy.

P is the probability that the submarine is detected during penetration.

Equating the threats:

$$N \cdot H \cdot (1 - P) = N' \cdot H'(r)$$

or

$$H'(r) = \frac{N}{N'} \cdot H \cdot (1 - P)$$

If $N = 5$, $N' = 2$, $H = .8$, and $P = .5$

$$H'(r) = (5/2)\,(.8)\,(1 - .5) = 1.0 \text{ hits.}$$

Thus, the solution is that escorts should be stationed at the range which prevents the submarine from attaining one hit by firing at the convoy from outside the screen. However, in this example, the screen detection probability at the bearing was assumed to be constant. Typically P is also a function of r, and another method, such as that used in reference [18] would have to be employed to actually determine the perimeter.

The probability of an unaimed straight running torpedo achieving a hit in a convoy has been investigated in references [12] and [13] for various convoy sizes, speeds, and ship spacing, and various torpedo ranges. Against the traditional broad front convoy formation, contour lines of equal hit probability ("iso-probability contours") are roughly as depicted in Figure 37.

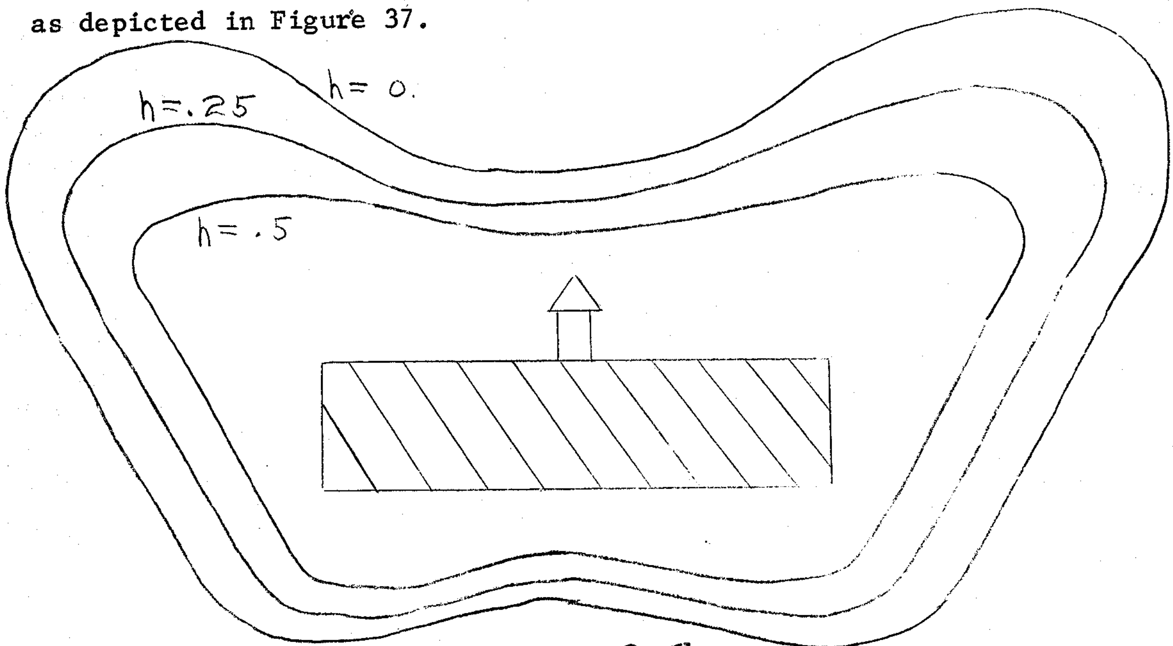


Figure 37

The iso-probability contours are characterized by:

a. A dip in the van attributable to the narrower target aspect.

b. A bulge on the bows attributable to the broad target aspect and short torpedo run.

c. A gradually decreasing distance from the convoy beam aft to the quarter attributable to successively longer torpedo runs.

d. A dip astern attributable to the narrow target aspect and maximum torpedo run.

The authors did not investigate whether similar data was available for acoustic homing torpedoes.

The single most important factor in varying the distance from the convoy and shape of the iso-probability contours was the convoy spacing.

The authors compare the difference between the calculated contours in reference [12] and those in Model II for an appropriate selection of

h_{max}. We assumed that the probabilities of hits for multiple shots were independent, so that the total expected number of hits would be merely the product of the single shot probability and the number of torpedoes fired. As would be expected, the model contour lines were, by comparison, too far from the convoy in the van, too short on either bow, approximately correct on the flanks, and too far astern in the rear. Our assumption of a linear decrease in hit probabilities with range seemed to be as accurate as any other. Variations in ship spacing may be accounted for by proper variation of h_{max}. For reasons we will now undertake to explain, we believe that any attempt to attain more detail in the analytical model would serve no purpose.

The incompleteness of the "balanced threat" concept

If two assumptions held, the correct action would be the procedure we have just presented: to station escorts around a perimeter which balances the threat of the submarine firing outside or inside the screen (the "equi-threat contour line" previously defined) thereby nullifying the submarine's advantage of choice. The first assumption is that a submarine can and will fire right up to the line of screening ships. Longer ranges of detection and stand-off weapons make this assudption increasingly poor. In addition, when we are able to take advantage of a submarine's inability to determine which form of attacking the defenses are optimizing against, we should do so.

The second assumption concerns the probability that the submarine will be sunk during an attack. In effect we have assumed above that the same number of submarines will be sunk regardless of how they attack; or alternatively, we have assumed that the importance of the submarine's surviving is small compared to the importance of its carrying out its mission to sink merchant ships.

An example of how the second assumption fails is as follows: Assume that doctrine in the escort forces is to balance the inside/outside threats to shipping (i.e., letting $T = T'$). For values of N, N', P, and H used in the above example, the solution was to place escorts at range r so as to make the expected number of hits from firing outside the screen $h(r) = 1.0$. In this example, the probability that a penetrating submarine is detected is $P = .5$. After attack and during its escape, the submarine again runs the risk of detection. Let us say for purposes of illustration that the probability of detection is $P_e = .3$. Suppose that, for the bearing we are considering, the probability that a submarine which fires from outside the screen is detected is $P' = .4$. Let us further suppose that any time a submarine is detected, the escorts have a probability of sinking it before it escapes of $D = .6$.

For a submarine penetrating, the rate of sinking per attack is

$$P \cdot D + P_e \cdot D \cdot (1 - P) = (.5 \times .6) + (.3 \times .6 \times .5) = .39$$

For a submarine firing from outside the screen, the rate of sinking per attack is $P' \cdot D = .4 \times .6 = .24$

The submarine can attain the same number of expected hits in either case, because the screen was deployed to that end. Therefore if the submarine commanding officer recognizes the situation, he would always choose to fire from outside the screen.

Suppose now that the escort commander decides to take the risk to the submarine into consideration, and extends the screen until the value of lost merchantmen is weighed against the value of sunk submarines. Another element enters the problem: submarine attrition other than in attacks on convoys may influence the submarine decision. Sinkings due to Hunter-Killer Groups, VS patrols around the convoy, barrier operations, or any other source, may be high compared to attrition by the screen against penetrating submarines. The correct action by the submarine would then be to attempt screen penetration (especially if the number of convoys the submarine may expect to intercept during a patrol is small) in order to achieve the greatest number of sinkings per convoy.

We conclude, therefore, that determining the best distance from the convoy on the basis of equating $T = T'$ is satisfactory only as a first approximation, and that both tactical and strategic considerations must govern the escort commanders' exact disposition. Simple and direct information as to which choice submarines actually make will be the most important wartime basis for adjusting the screen out or in from the

T = T' line. Therefore we believe that any attempt at greater detail in Model II would be fruitless.

Considerations affecting escort deployment at various relative bearings

We conclude this section by offering opinions as to the probable courses of action by submarines attacking from various relative bearings around the convoy.

a. Conventional Submarines: A conventional submarine in advance of the convoy front has little choice but to be over-run by the screen and will prefer to penetrate the screen safely before attacking. On the other hand, a conventional submarine attacking from either bow aft to the extreme limiting lines of approach has a superior position for obtaining hits in the convoy by firing at long range, but it is on a less favorable bearing for penetration, because of the relative motion problem. There will be a preference to fire from outside the screen in this region.

b. Nuclear submarines: The above considerations apply, but to a lesser extent, to nuclear submarines. A noisy, high speed approach tactic would be advantageous only in the van. The nuclear submarine has its longest, slowest approach from abaft of 135° relative to the convoy track, but it is able to take advantage of wake effect directly astern of the convoy. (See Section B of this chapter for further information.) The two most important factors in determining the range for defending against a nuclear submarine are the comparative damage it will achieve by attacking from inside or outside the screen, and the ability of the escorts, including aircraft present, to counterattack in each case. Therefore nuclear submarines should have a strong preference for screen penetration.

We reemphasize the importance that convoy spacing has on the optimum distance from the convoy to station the escorts. The wider the spacing of merchant ships within the convoy, the smaller the probability of a hit from a "Browning shot" fired outside the screen, and at the same time, the longer will be the perimeter around the convoy. Unless the screening force is very strong, it will almost always be advantageous to place the escorts at minimum distance (distance at which they can effectively neutralize penetrating submarines) if merchant ship spacing is 2000 yards x 2000 yards or greater.

H. Single Perimeter vs. Defense in Depth

1. Basic Reasons for Perimeter Defense

Implicit in our approach to the convoy defense problem has been the assumption that a single perimeter at an optimal range is superior to a defense in depth. The justification has been:

a. The fact that the farther from the convoy an escort is stationed, the smaller the angle it can defend. For a circular defense, doubling the distance from the center of the convoy halves the angle screened.

b. The assumption that the submarine cannot detect weak points in the screen. (Although the reader may recall that we have investigated the consequences if a nuclear submarine is able to detect gross weaknesses and exploit them. See Chapter IV.)

2. Basic Reasons for Defense in Depth

A major argument advanced in favor of a defense in depth has been to provide insurance that the assumption (para. b above) holds. Stationing escorts with powerful in-layer (PDR) detection capabilities at longer ranges in the van is intended to deprive the submarine of the opportunity to come to periscope depth to detect screen weaknesses. Better yet, there is the strong belief current that submarines will come to periscope depth in spite of the presence of the sonar, in which case the escort would have a good detection opportunity against it.

A defense in depth is also envisioned as depriving the submarine of tactical intelligence about the overall strength of the screen, the primary information it desires to base a decision whether or not to attempt screen penetration.

In a similar vein, it is argued that the submarine must have visual or radar intelligence of not only the screen, but of the course, speed, and composition of the convoy as well. Placing pickets at ranges from the convoy beyond the normal screen in the region at which the submarine must come to periscope depth to observe the convoy magnifies its problem and its risk.

Stationing pickets or pouncers (i.e., providing for a second line of defense) in the van also affords additional protection against the danger of a high speed penetration by a nuclear submarine. The high relative speeds involved in such attacks from ahead of the convoy make it difficult, to put it conservatively, for an escort to detect, classify, and react effectively to neutralize such a target. An alerted back up ship will be in a better position to take successful action.

Finally, a long range sonar stationed directly ahead of the convoy may induce the submarine to avoid it by sheering to one side, thereby missing the convoy or at least reducing the danger of a successful attack.

3. Disadvantages of a Defense in Depth

However, there are serious disadvantages to a defense in depth. The fundamental weakness is that the farther an escort is from the convoy the smaller the arc it defends. To put an escort with a powerful sonar in a picket station is to remove the most effective detector from the area where it will be most useful. Furthermore it will in all likelihood remove the best weapon carrier from the most probable scene of action. On the other hand, putting an escort with a short range sonar in a picket station is of little value for inhibiting the submarine from coming to periscope depth at long range.

Placing one screening ship behind another as a second line of defense reduces the effectiveness of the second. For example, suppose two escorts patrol zones in tandem with respect to the direction of an approaching submarine, and each escort has a .7 probability of detecting a submarine passing through its zone. The second escort will have an opportunity to gain initial detection on only the .3 of the submarines that evaded the first. If we were to assume statistical independence of the detection probabilities, then the second escort would detect only .7 of the remaining submarines, (i.e., .7 X .3 = .21 of submarines coming through the sector). If there is an unguarded gap in the screen, the escort would probably be more effective in the gap.

Two modifying remarks are pertinent:

a. The performance of the second escort is not independent of the presence of the first. A submarine which has been preoccupied with evading the first escort ought to be somewhat more vulnerable to detection by the second.

b. If each escort is patrolling a wide sector, on the order of perhaps three or four times its effective sweep width, the probability of it detecting a penetrating submarine (in the van) would be only about .3. The second escort would then still play an important role: crediting it with the same sonar, a smaller arc to patrol, and the advantage of a less wary submarine, its probability of detection might be as high as .5.

However, the effectiveness of the "pouncer" is dependent on its <u>not</u> being directly behind the first screening ship as much of the time as possible. Rather than assigning escorts to patrol areas as depicted in Figure 38 it would be wiser to station them as in Figure 39, if the need for defense in depth is imperative.

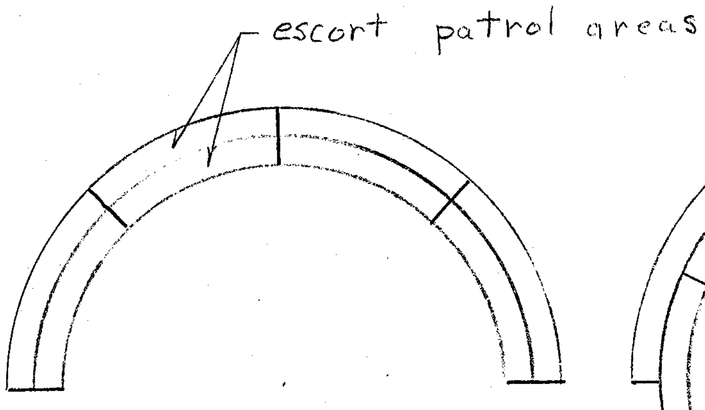


Figure 38

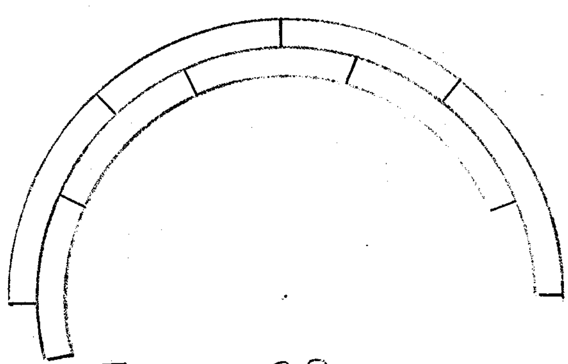


Figure 39

Regarding the need to deprive a conventional submarine of tactical intelligence, the need is probably less important than some may have believed. We have argued (see Section B of this chapter) that a conventional submarine is largely committed to the relative bearing of its attack by the time it is close enough to obtain tactical information about the screen disposition. Although a nuclear submarine has much greater flexibility in selecting its bearing of attack, it will not probe for weaknesses indefinitely. It is doubtful that, under the stress of the situation, it will be able to determine any but the grossest weaknesses in a screen, whether the screen is deployed in a single perimeter or a defense in depth.

Placing a long-range sonar directly in advance of the convoy has another disadvantage. The less aggressive submarine commanding officer who is most likely to stand clear of the sonar is probably the same one who would like to fire from outside the screen on either bow. By avoiding the sonar he will place himself, barring a fortuitous zig-zag, in just the position he desires to achieve.

The argument favoring a second line of defense in the van to react to nuclear submarines penetrating at high relative speed is a persuasive one. One comment might be made. Escorts located in pouncer station to combat this threat will not be very effective if they are equipped with short range sonar and weapons.

4. Conclusion

There is no simple answer to the question of whether a perimeter defense or a defense in depth is better. An estimate of the probable aggressiveness or lack of it on the part of the enemy is the most important single consideration. Fleet experience with exercise submarines in peacetime obviously cannot be a guide in this respect. Peacetime intelligence when known, and wartime intelligence when available as to enemy submarine aggressiveness will be the single most important consideration.

The next most important factor is the strength of the screen. If water conditions are such that operational sweep widths (including correction for patrolling) total greater than .6 of the length of the optimum defense perimeter, overlapping detection regions will result anyway, and a partial defense in depth without loss of sonar search effectiveness is feasible. If total sweep width is less than .6, careful consideration must be given to the danger of creating large gaps in the screen by any form of defense in depth.

The third most important factor is what the enemy <u>believes</u> is the strength of the screen. If the escort commander can persuade the enemy that he faces a tighter screen than usual, the enemy will be more inclined to attack at long range. If he may be induced to believe he faces a tight screen in the van, he will be inclined to fire from off the bows. On the other hand, if he believes he faces a thin screen or a screen deployed at too great a distance from the convoy, he will naturally be tempted to attempt screen penetration. Facing a staggered screen, as shown in Figure 39 above, the enemy may observe only the outer ring of escorts and choose to split bearing and pass between a pair of them, only to find himself facing a strong pouncer on the inner ring that he is unable to evade.

The recommended ranges at which to station escorts in Appendix I are based on a perimeter defense. However, some of the advantages of staggering escorts accrue from deploying escorts on the bows at longer ranges from the convoy.

A. Computer War Gaming

Computer war gaming would be of great benefit either in verifying the conclusions of the thesis or in studying other convoy defense problems. It is likely belaboring the obvious to say parts of the analysis need substantiation. Fleet exercises can shed very little light on such matters as the probability distribution of submarines approaching a convoy in wartime. Nor does our experience in World War II, such as the preference of U-Boats to attack convoys from the beams, seem to be very helpful. The concept developed in Chapter III should be adaptable to the study of a wide variety of convoy defense problems by computer war gaming.

Sections of the thesis which we would particularly suggest for study:

1. The analytical model, Chapter IV-B

2. The difference between submarines on patrol in an area and those that can intercept a convoy, Chapter V-A.

3. The distribution of submarines around a convoy, Chapter V-B.

4. Defense in depth vs. a perimeter defense, Chapter V-G.

Other convoy problems suggested for study:

1. Proper disposition of escorts when the primary mission of the submarines is to sink escorts.

2. Convoy defense in the face of coordinated simultaneous attacks by two or more submarines.

3. Convoy defense in a nuclear war and/or in the face of submarine launched missile attacks.

B. The Aircraft Platform

Throughout most of the analysis the presence of fixed wing aircraft and helicopters has been assumed. No investigation was made of the problem of how best to provide air support, but of course a CVS is the most likely source. The advantages and disadvantages of integrating a CVS with the convoy are well known. However one aspect of the problem has an important bearing on the analysis in the thesis.

If a CVS is located in the convoy in a box at the rear, the expected number of hits among merchant ships is no longer a valid measure of screening effectiveness, because the CVS is so much more important a target. The small fraction of conventional submarines approaching from bearings greater than 60°, or a handful of nuclear submarines able to approach from astern, present a disproportionately greater threat to the carrier.

There are other important considerations in determining where to station the CVS, but this much is clear: for situations in which the nuclear submarine threat is small, the screen is frequently most effective if all or nearly all of it is stationed in the van. Stationing the CVS inside the convoy screen entails an effective screen all around the convoy. Added to the convoy screen will be the escorts that were with the CVS, but these additional escorts will probably not be sufficient to provide the all-around protection now needed. The protection in the van may be so reduced as to result in much easier screen penetration from ahead, and an overall increase in the submarine threat to the convoy.

C. Helicopters in Convoy Defense

In the thesis we have had very little to say about the best way to employ helicopters as screening units, other than to reaffirm the special advantages helicopters have for searching astern of the convoy. We have simply assumed that the effective sweep width of the helicopters could be characterized and that their unique capabilities and limitations would be provided for by the escort commander. One method for determining helicopter sonar search effectiveness may be found in [14].

D. Long Range Sonars

If it develops that the AN/SQS-26 sonar has a dignificant detection capability by bottom bounce or in the convergence zone, annuli of positive detection probability will exist at 35 miles or more from the escorts equipped with that sonar. No matter how high the detection probability is in those regions, however, unless an effective kill capability accompanies the detection capability, such detection cannot be regarded as equivalent to the type of detection hitherto considered. It would be more appropriate to treat these long range detections and the corresponding kill probabilities as part of the <u>approach</u> attrition. That is, a long range detection would not be treated as a screen detection, but rather would be taken into consideration in determining $s_3(\theta)$, the probability that a submarine would attack from bearing θ (see Chapter III).

Remaining to be considered is the optimum station for an escort with an AN/SQS-26 type sonar. In the conceptual framework of Chapter III, op timization would have to be taken over not only the screen perimeter but over the region of approach as well. The importance of solving this problem will depend on how effective the long range capability of the AN/SQS-26 sonar proves to be and on the effectiveness of the weapon that is mated with it.

E. Advantageous Escort Stations for Counterattack

An escort stationed ahead of the convoy is able to move much more rapidly relative to the convoy than one stationed astern, and therefore is better positioned to counterattack inside the screen. This is a factor that we have ignored in our model, because the solutions always call for a greater concentration of escorts in the van.

In heavy weather, it is advantageous for submarines to retreat into the wind after attack. Escorts may move downwind much more efficiently when high seas are running. Therefore, all else being equal, the escort commander should favor a slightly heavier concentration of escorts to windward in severe weather.

F. Convoy Shape

Much of the analysis herein has assumed a square convoy shape only because among rectangular column-and-row configurations, a square has the smallest perimeter.

The broad-front World War II convoy was justified because submarines preferred to attack from the bows and beams of the convoy, and shortening the fore-and-aft dimension reduced the convoy target length.

It is not obvious that a broad front convoy is justified today, particularly against an aggressive enemy. Submarines penetrating the screen from ahead will have an easier task against a broader front. Submarines approaching in the convoy wake astern will also benefit from a broad front.

We believe Model II may be used without significant error with convoys of length:breadth dimension ratios up to 1:2.

G. The SSNK as a Convoy Escort

The authors are not entirely familiar with the limitations of a nuclear submarine for ASW. However it is apparent that the factors prohibiting its use in the defense of a carrier are much less important for convoy defense, namely, high carrier speed and poor surface-to-submarine communications.

It is envisioned that an SSNK would be stationed directly under the convoy. Escorts would be prohibited from pursuing an enemy submarine into the convoy and the responsibility would devolve on the SSNK to counter the threat. The premise is that surface escorts will have an extremely difficult problem in detecting and attacking any submarine, especially a nuclear submarine, once it gets under the closely spaced merchant ships. There would be no communication between SSNK and surface ships except, perhaps, for an alerting explosive signal.

The SSNK would cruise with its sonar in passive. Although the listening would be degraded by its speed and convoy noise, we conjecture that it would retain the capability of detecting high speed approaches from astern of the convoy. The <u>minimum</u> requirement is that the SSNK have an appreciable capability of detecting and sinking the enemy submarine after it has attacked the convoy.

The slower the SOA of the convoy, the greater the value of the SSNK. On the one hand, enemy nuclear submarines would be more likely to approach from astern. On the other hand, the SSNK passive sonar would be more effective.

It is envisioned that all surface escorts would be stationed ahead of the convoy, with responsibility of detecting submarines waiting quietly to be over-run by the convoy. The SSNK would be responsible for the defense on the beams and aft.

The SSNK would be uniquely effective countering the threat of multiple-submarine attacks. The enemy would no longer know whether another submarine was friend or foe; the SSNK would know that any other submarine was an enemy.

The basic principle justifying the assignment of an SSNK is that it must be employed where the greatest number of enemy submarines may be expected. If a real threat to convoys exists at all, then convoys will be focal points of enemy submarine action.

The knowledge that an SSNK might be operating with a convoy, even with low probability, would force upon the enemy a considerable measure of additional caution. It might even be profitable to "reveal" our intention of using SSNK's with convoys, whether we actually do so or not.

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APPENDIX A

SAMPLE RESULTS

The tables in this appendix are for a convoy described by the following parameter values:

1. H_o = 10000 yards.
2. A = 1/3.
3. RT = 15000 yards.

Force level ranges from 25000 to 45000 yards of sweep width. For each force level there is a table with e = .5 and e = 1.0 . In all cases, α is taken to be 2. and k is taken to be 2.5 .

The left side of the table gives the values of T, R_1 , and R_2 corresponding to the minimum RISK solution. The right side shows the allowable spread of T in order to keep RISK within .05 of its minimum value. If T is chosen within the limits specified, interpolate to find the corresponding R_1 and R_2. Variations of R_1 or R_2 of less than 1000 yards are insignificant.

The method discussed in Chapter IV-C may be used to establish optimal policies for the particular situation.

Y = 25000 yards; R_1 and R_2 x 1000 yards

e = .5

	minimum RISK solution				allowable range for min RISK + .05		
@	T	R_1	R_2	RISK	T	R_1	R_2
.0	100	14.9	----	.20	78-100	12.4-14.9	10.0
.1	100	14.8	----	.36	56-100	10.0-14.8	10.0
.2	62	10.1	10.0	.44	55-89	10.0-13.4	10.0
.3	63	10.1	10.0	.52	53-75	10.0-11.6	10.0
.4	64	10.0	10.0	.60	48-72	10.0-11.0	10.0
.5	65	10.0	10.0	.69	26-71	10.0-10.1	10.0
.6	27	10.0	10.0	.74	21-66	10.0	10.1-10.0
.7	27	10.0	10.0	.75	21-42	10.0	10.1-10.0
.8	26	10.0	10.0	.76	21-35	10.0	10.1-10.0
.9	26	10.0	10.0	.76	21-33	10.0	10.1-10.0
1.00	26	10.0	10.0	.77	20-31	10.0	10.1-10.0

e = 1.0

	minimum RISK solution				allowable range for min RISK + .05		
@	T	R_1	R_2	RISK	T	R_1	R_2
.0	100	17.3	----	.31	86-100	16.0-17.3	10.0
.1	100	17.1	---	.46	70-100	14.2-17.1	10.0
.2	63	13.1	10.0	.60	35-100	10.0-16.9	10.2-10.0
.3	42	10.1	10.0	.63	35-67	10.0-13.2	10.2-10.0
.4	44	10.0	10.0	.66	34-56	10.0-11.6	10.3-10.0
.5	47	10.0	10.0	.71	33-55	10.0-11.1	10.4-10.0
.6	37	10.0	10.0	.75	31-54	10.0-10.5	10.7-10.0
.7	37	10.0	10.0	.78	30-51	10.0	10.9-10.0
.8	36	10.0	10.0	.81	30-45	10.0	10.9-10.0
.9	36	10.0	10.0	.84	29-43	10.0	10.9-10.0
1.0	36	10.0	10.0	.87	29-41	10.0	10.9-10.0

Y = 30000 yards; R_1 and R_2 x 1000 yards

e = .5

@	minimum RISK solution T	R_1	R_2	RISK	allowable range for min RISK + .05 T	R_1	R_2
.0	100	17.0	----	.16	80-100	14.5-17.0	10.0
.1	90	15.7	10.0	.32	47-100	10.0-16.9	10.0
.2	51	10.0	10.0	.36	46-73	10.0-13.2	10.0
.3	52	10.0	10.0	.40	45-62	10.011.5	10.0
.4	53	10.0	10.0	.44	40-60	10.0-11.0	10.0
.5	54	10.0	10.0	.49	36-59	10.0-10.1	10.4-10.0
.6	39	10.0	10.0	.52	35-58	10.0-10.3	10.5-10.0
.7	39	10.0	10.0	.54	34-51	10.0	10.7-10.0
.8	39	10.0	10.0	.55	34-46	10.0	10.7-10.0
.9	39	10.0	10.0	.56	34-44	10.0	10.7-10.0
1.0	39	10.0	10.0	.58	33-43	10.0	10.8-10.0

e = 1.0

@	minimum RISK solution T	R_1	R_2	RISK	allowable range for min RISK + .05 T	R_1	R_2
.0	100	18.9	----	.24	88-100	17.8-18.9	10.0
.1	100	18.8	----	.40	69-100	15.5-18.8	10.0
.2	52	13.0	10.0	.52	37-100	10.8-18.6	10.5-10.0
.3	47	12.0	10.0	.54	35-64	10.1-14.3	11.8-10.0
.4	47	11.6	10.0	.55	36-56	10.0-12.9	11.7-10.0
.5	47	11.2	10.0	.57	37-53	10.0-12.1	11.5-10.0
.6	47	10.8	10.0	.58	38-51	10.0-11.4	11.4-10.0
.7	46	10.2	10.1	.59	40-50	10.0-10.9	11.0-10.0
.8	47	10.0	10.0	.60	42-50	10.0-10.4	10.9-10.0
.9	47	10.0	10.0	.64	41-51	10.0	10.9-10.0
1.0	47	10.0	10.0	.67	41-51	10.0	10.9-10.0

Y = 35000 yards; R_1 and R_2 x 1000 yards

e = .5

	minimum RISK solution				allowable range for min RISK + .05		
@	T	R_1	R_2	RISK	T	R_1	R_2
.0	100	19.0	----	.12	82-100	16.5-19.0	10.0
.1	77	15.6	10.0	.28	42-100	10.0-18.9	10.9-10.0
.2	47	10.6	10.1	.29	42-66	10.0-13.8	10.9-10.0
.3	47	10.4	10.1	.29	42-56	10.0-12.0	10.9-10.0
.4	47	10.2	10.1	.30	43-53	10.0-11.3	10.8-10.0
.5	47	10.0	10.1	.30	44-51	10.0-10.8	10.6-10.0
.6	48	10.0	10.0	.31	44-51	10.0-10.6	10.6-10.0
.7	48	10.0	10.0	.33	44-51	10.0-10.4	10.6-10.0
.8	47	10.0	10.1	.35	44-51	10.0-10.1	10.6-10.0
.9	47	10.0	10.1	.36	43-51	10.0-10.1	10.8-10.0
1.0	47	10.0	10.1	.38	43-51	10.0	10.8-10.0

e = 1.0

	minimum RISK solution				allowable range for min RISK + .05		
@	T	R_1	R_2	RISK	T	R_1	R_2
.0	100	20.4	----	.18	87-100	19.3-20.4	10.0
.1	100	20.3	----	.34	68-100	16.6-20.3	10.0
.2	55	14.6	10.0	.44	44-95	12.9-19.6	10.0-11.9
.3	55	14.3	10.0	.47	41-68	12.1-16.2	10.0-12.4
.4	54	13.9	10.1	.49	38-62	11.2-15.1	10.0-12.9
.5	54	13.5	10.1	.50	34-59	10.1-14.3	10.0-13.5
.6	54	13.2	10.0	.52	34-58	10.0-13.8	10.0-13.5
.7	54	12.8	10.1	.54	35-57	10.0-13.3	13.4-10.0
.8	41	10.1	12.4	.55	36-56	10.0-12.7	13.2-10.0
.9	43	10.1	12.0	.55	38-56	10.0-12.3	12.9-10.0
1.0	46	10.1	11.5	.56	40-55	10.0-11.7	12.5-10.0

Y = 40000 yards; R_1 and R_2 x 1000 yards

e = .5

	minimum RISK solution				allowable range for min RISK + .05		
@	T	R_1	R_2	RISK	T	R_1	R_2
.0	100	20.8	----	.08	83-100	18.3-20.8	10.0
.1	67	15.6	10.0	.24	40-100	10.5-20.8	12.7-10.0
.2	54	13.1	10.0	.25	38-69	10.0-15.8	13.1-10.0
.3	54	12.9	10.0	.25	39-61	10.0-14.2	12.9-10.0
.4	54	12.8	10.0	.26	39-59	10.0-13.7	12.9-10.0
.5	54	12.6	10.0	.27	39-57	10.0-13.2	12.9-10.0
.6	53	12.2	10.2	.27	40-56	10.0-12.8	12.7-10.0
.7	53	12.0	10.2	.28	40-56	10.0-12.6	12.7-10.0
.8	44	10.0	11.9	.28	41-55	10.0-12.2	12.3-10.0
.9	45	10.0	11.7	.28	42-55	10.0-12.0	12.3-10.0
1.0	46	10.0	11.6	.28	43-55	10.0-11.7	12.1-10.0

e = 1.0

	minimum RISK solution				allowable range for min RISK + .05		
@	T	R_1	R_2	RISK	T	R_1	R_2
.0	100	21.9	----	.13	89-100	20.6-21.9	10.0
.1	100	21.8	----	.29	66-100	17.6-21.8	10.0
.2	60	16.5	10.0	.37	49-89	14.8-20.4	12.2-10.0
.3	60	16.2	10.0	.40	47-70	14.1-17.7	12.6-10.0
.4	60	16.0	10.0	.42	43-66	13.1-17.1	13.4-10.0
.5	60	15.7	10.0	.45	38-64	11.8-16.3	14.3-10.0
.6	60	15.4	10.0	.47	31-63	10.0-15.9	15.5-10.0
.7	55	14.2	11.1	.49	31-62	10.0-15.4	15.5-10.0
.8	39	10.7	14.1	.50	32-61	10.0-14.9	15.4-10.0
.9	38	10.1	14.3	.50	34-60	10.0-14.2	15.0-10.0
1.0	40	10.0	13.9	.49	36-59	10.0-13.7	14.6-10.2

Y = 45000 yards; R_1 and R_2 x 1000 yards

e = .5

	minimum RISK solution				allowable range for min RISK + .05		
@	T	R_1	R_2	RISK	T	R_1	R_2
.0	100	22.6	----	.05	85-100	20.2-22.6	10.0
.1	60	15.7	10.0	.20	45-100	12.6-22.5	13.1-10.0
.2	59	15.3	10.0	.21	43-71	12.0-17.6	13.5-10.0
.3	59	15.2	10.0	.22	39-65	11.0-16.3	14.3-10.0
.4	59	15.0	10.0	.23	36-63	10.1-15.8	15.0-10.0
.5	59	14.9	10.0	.24	36-62	10.0-15.5	15.0-10.0
.6	59	14.7	10.0	.24	36-61	10.0-15.1	15.0-10.0
.7	59	14.5	10.0	.25	36-60	10.0-14.9	15.0-10.0
.8	40	10.2	14.1	.25	36-60	10.0-14.5	15.0-10.0
.9	40	10.0	14.1	.25	37-60	10.0-14.3	14.7-10.0
1.0	41	10.0	13.9	.25	38-59	10.0-13.9	14.5-10.0

e = 1.0

	minimum RISK solution				allowable range for min RISK + .05		
@	T	R_1	R_2	RISK	T	R_1	R_2
.0	100	23.2	----	.07	90-100	22.0-23.2	10.0
.1	100	23.1	----	.24	64-100	18.4-23.1	10.0
.2	65	18.3	10.0	.31	53-87	16.4-21.4	12.6-10.0
.3	64	18.0	10.1	.34	50-73	15.7-19.4	13.2-10.0
.4	64	17.8	10.1	.37	46-70	14.6-18.7	14.1-10.0
.5	64	17.6	10.1	.40	40-68	13.1-18.2	15.3-10.0
.6	64	17.3	10.1	.42	30-67	10.5-17.8	17.2-10.0
.7	54	15.2	12.4	.44	28-66	10.0-17.4	17.7-10.0
.8	38	11.5	15.7	.45	29-65	10.0-16.9	17.4-10.0
.9	34	10.1	16.4	.44	30-63	10.0-16.2	17.2-10.4
1.0	36	10.1	16.0	.44	32-55	10.0-14.3	16.8-12.2

APPENDIX B

DIAGRAM OF A TYPICAL SCREEN PERIMETER

Fig. 40 illustrates how ranges R_1 and R_2 may be modified to provide for the shape of the convoy, and submarine preference for attempting screen penetration or attacking from outside the screen. (See Chapter V-F.)

In ZONE I (Ahead): All submarines will prefer to penetrate the screen. Station escorts at the minimum distance from the convoy at which the neutralization of a detected contact is still practical. With very strong screening forces, stagger escorts alternately on lines A and B.

In ZONES II and III (Bows): Conventional submarines may prefer to fire from outside unless the screen is deployed to discourage them. Station escorts with strong in-layer detection capabilities in these zones. With very strong screening forces, consider increasing the distance of the perimeter from the convoy. Without VS aircraft patrolling the limiting lines of approach, extend the screen arc aft to ± 80 degrees relative.

In ZONE IV (Flanks and rear): Nuclear submarines will prefer to penetrate the screen before attacking. Station escorts at the minimum distance from the convoy at which neutralization of a detected contact is still practical. Ships or helicopters with the ability to search beneath the convoy wakes should be stationed in the rear. Width of patrol sectors should be proportioned according to existing doctrine.

The diagram was prepared for the following situation:

H_o = 10000 yards

Y = 35000 yards.

A = 1/3.

k = 2.5.

RT = 15000 yards.

$e = 1.0$

@ $= .3$.

$= 2$.

From the appropriate table in Appendix A we obtain the following:

T = 55% or 19000 yards of sweep width (spread: 41 to 68%, or 14000 to 23500 yards of sweep width).

R_1 = 14300 yards (spread: 12100 to 16200) .

R_2 = 10000 yards (spread: 10000 to 12400) .

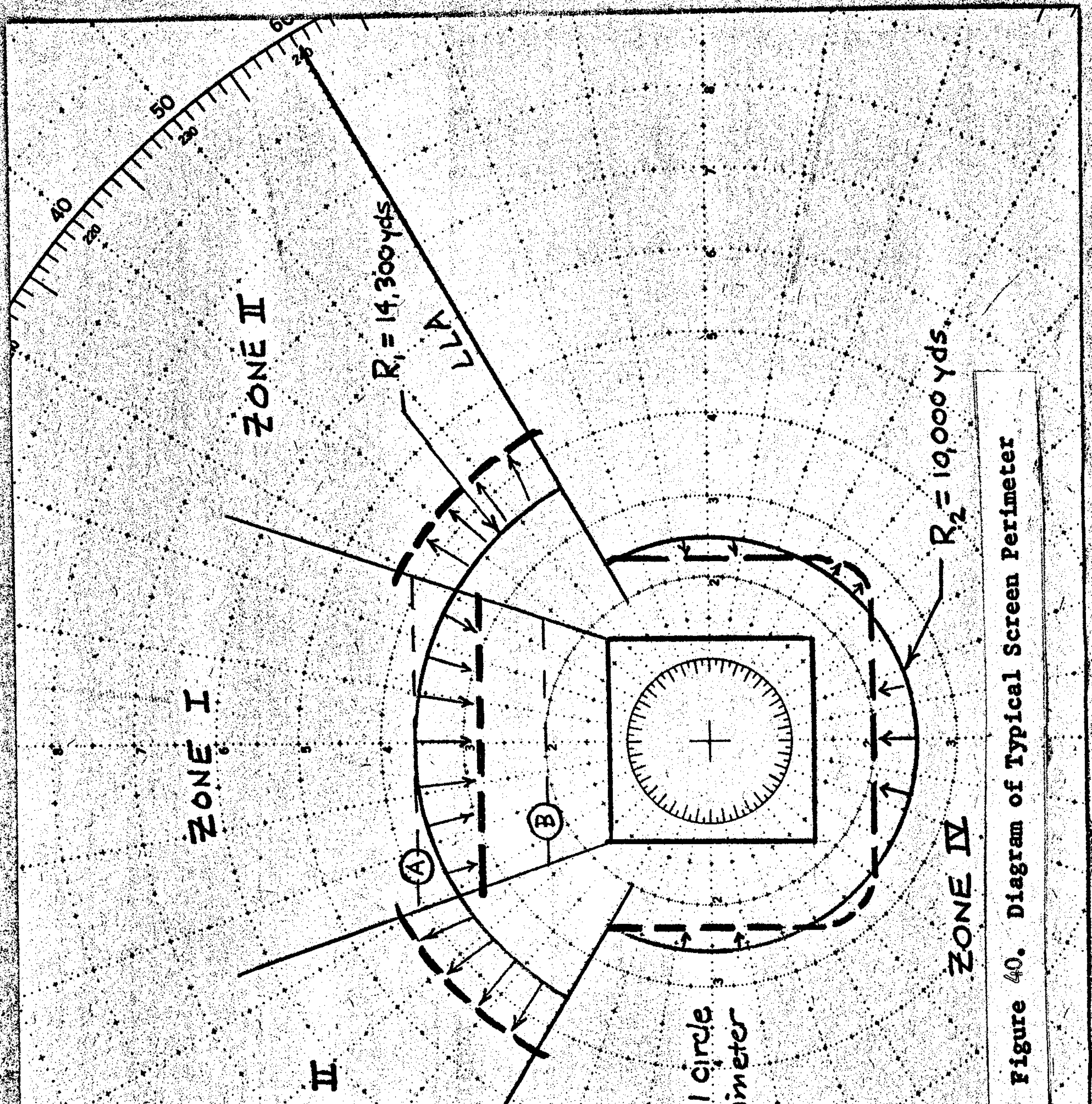

Figure 40. Diagram of Typical Screen Perimeter

APPENDIX C

COMPUTER PROGRAM AND SAMPLE OUTPUT

The program below is written in FORTRAN 60 for use on the CDC 1604 computer. The data input statements have been omitted. Data inputs necessary are A, H_o, RT, e, $\propto$, and Y. The symbols correspond to those in the thesis with the following exceptions and modifications:

Thesis	Program
A	AA
@	A
k	CK
e	E
$\propto$	ALPHA

The program as reproduced here prints out only those values corresponding to the minimum RISK solution (see sample output sheet following program). In order to examine the rate of change of RISK with respect to T in the neighborhood of the minimum RISK solution, it is necessary merely to insert an additional PRINT statement into the program so that the desired values are printed out.

```
      PROGRAM ONE

     (  DATA CARDS HERE FOR  AA, HO, RT, E, ALPHA, Y )

      PI = 3.1415926536
      HL = RT + HO
      A1= 1. -AA
      BO = 1./(2.*PI*AA)
      GO = ALPHA/(2.*PI*A1)
      PRINT 1000,AA,Y,HO,RT,ALPHA,E
 1000 FORMAT (6H1AA = F10.4,4X,4HY = F10.4,4X,5HHO = F10.4,4X,
     1 5HRT = F10.4,4X,8HALPHA = F10.4,4X,4HE = F10.4/)
      DO 100 J = 1,7
      CK = .5 + FLOATF(J)/2.
      PRINT 1001,CK
 1001 FORMAT (1HO,15X,4HK = F10.4///12X,1HA,17X,1HT,16X,2HR1,16X,
     1  2HR2,18X,4HRIS</)
      A2=1.0-(AA*CK)
      G3=HL*(1.-CK/E)/2.
      G4=CK*HL*GO/E
      DO 100 I =1,11
      A=(FLOATF(I-1))/10.0
      B = (1. -A*A2) /(1.-A*A1)
      B1=BO*(1.-A*A2)
```

```
      B3=HL*(1.-B/E)/2.
      B4=B0*B*HL/E
      G1=G0*A*CK*A1
      D=1.-A*(1.-CK)
      RISK = 10.
      R1 = 0.
      R2 = 0.
      T = 0.
      DO 500 KK = 1,101
      XT = Y*(1.-FLOATF(KK-1)*.01)
      R1T = B3 + SQRTF(B3*B3 + B4*XT)
      IF (R1T) 10,10,11
   11 IF (R1T - H0) 12,13,13
   12 R1T = H0
   13 P1T = B0*XT/R1T
      IF(1.-P1T) 500,14,14
   14 R2T = G3 + SQRTF( G3*G3 + G4*(Y-XT))
      IF (R2T) 15,15,16
   16 IF(R2T -H0)17,18,18
   17 R2T = H0
   18 P2T = G0*(Y-XT)/R2T
      IF(1.-P2T) 500,19,19
   10 R1T = 0.
      R2T = G3+SQRTF(G3*G3+G4*Y)
      IF(R2T-H0)30,31,31
   30  R2T = H0
   31 P2T = G0*Y/R2T
      IF (1.-P2T) 500,20,20
```

```
 20 RISKT = D-G1*Y/R2T
    TT = 0.
    XT = 0.
    GO TO 150
 15 R2T = 0.
    R1T = B3+SQRTF(B3*B3 + B4*Y)
    IF(R1T-H0) 40,41,41
 40  R1T = H0
 41  P1T = B0*Y/R1T
    IF(1.-P1T) 500,21,21
 21 RISKT = D-B1*Y/R1T
    TT = 1.0
    XT = 1.
    GO TO 150
 19 RISKT = D -(B1*XT/R1T  + G1*(Y-XT)/R2T)
    TT = XT/Y
150  IF(RISK - RISKT) 500,500,151
151 RISK = RISKT
    R1 = R1T
    R2 = R2T
    T = TT
500 CONTINUE
160 PRINT 1002,A,T,R1,R2,RISK
1002 FORMAT (5(8X,F10.4))
100 CONTINUE
    END
    END
```

AA = .3333 Y = 30.0000 HO = 10.0000 RT = 15.0000 ALPHA = 2.0000 E = .5000

K = 1.5000

A	T	R1	R2	RISK
.0000	1.0000	17.0372	.0000	.1593
.1000	1.0000	16.9968	.0000	.2494
.2000	.5000	10.0562	10.0000	.3158
.3000	.5000	10.0000	10.0000	.3264
.4000	.5100	10.0607	10.0000	.3384
.5000	.5100	10.0000	10.0000	.3512
.6000	.5200	10.0217	10.0000	.3672
.7000	.4500	10.0000	10.0000	.3796
.8000	.4400	10.0000	10.0229	.3816
.9000	.4400	10.0000	10.0229	.3831
1.0000	.4400	10.0000	10.0229	.3846

K = 2.0000

A	T	R1	R2	RISK
.0000	1.0000	17.0372	.0000	.1593
.1000	1.0000	16.9575	.0000	.2835
.2000	.5100	10.1000	10.0000	.3378
.3000	.5100	10.0000	10.0000	.3618
.4000	.5200	10.0000	10.0000	.3878
.5000	.5300	10.0000	10.0000	.4185
.6000	.5200	10.0000	10.0000	.4541
.7000	.4100	10.0000	10.0000	.4610
.8000	.4100	10.0000	10.0000	.4679
.9000	.4100	10.0000	10.0000	.4748
1.0000	.4100	10.0000	10.0000	.4817

APPENDIX D

THE QUESTIONNAIRE SENT TO DESTROYER SQUADRON COMMANDERS

Attached is a copy of the convoy screening problem which the authors mailed to all Destroyer Squadron Commanders in October 1963. Copies were also sent to 23 other commands for information.

Solutions to the problem were returned by the following commands:

Commander Destroyer Squadron TWO
Commander Destroyer Squadron FOUR (two solutions)
Commander Destroyer Squadron TWELVE
" " " FOURTEEN
" " " EIGHTEEN
" " " TWENTY-TWO
" " " THIRTY-TWO
Commander Escort Squadron TEN
Commander Destroyer Squadron THREE
" " " FIVE
" " " SEVEN
" " " FIFTEEN
" " " TWENTY-ONE
" " " TWENTY-THREE
" " " TWENTY-FIVE
Commanding Officer and Director, U. S. Atlantic Fleet ASW Tactical School
Commander Cruiser Destroyer Flotilla SIX
Commander Cruiser Destroyer Flotilla NINE
Commanding Officer, U. S. Fleet Sonar School, Key West (two solutions)

Copies of this thesis are being sent to commands requesting them.

The authors had originally intended to include a synopsis of the solutions. However, we concluded that most of our comments would be more misleading than helpful. Such wide differences of command experience in ASW and care in preparation were evident that any simple tabulation or summarization would not have been meaningful. Furthermore, the reader, with justice, might have been suspicious that any statements we made were colored by our conception of the important elements of the problem. We have given our views in the thesis: there is little to be gained by reframing them in the context of the solutions received.

One fact, which became apparent from the variety of the solutions and accompanying comments, was that there was no common method of attacking the problem of convoy defense against a mixed submarine threat. Indeed, a wide diversity of opinion was expressed regarding almost every aspect of the problem. Perhaps no questionnaire was needed to persuade the reader that such would be the case.

We have declined to explicitly "solve" our own problem. There is a gulf separating our analytical solution from the solution that places specific ships with specific attributes at specific stations. We have been away from fleet operations too long to presume to bridge this gap. It is only the commander on the scene, intimately knowledgeable about antisubmarine warfare, who is qualified to deploy and fight his force. We hope we have helped him.

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U. S. NAVAL POSTGRADUATE SCHOOL
Monterey, California

21 October 1963

Commander Destroyer Squadron ZERO
Fleet Post Office
New York, New York

Dear Captain Smith:

For a master's degree thesis in the Operations Analysis Curriculum at the U. S. Naval Postgraduate School I am undertaking a study of the following ASW problem:

> Conjecture a convoy of given dimensions transitting an area in which opposing submarines of various characteristics and/or weapons are stationed. The fractions of each type of submarine can be estimated. The convoy is defended by surface escorts that have a variety of sonars, other sensors, and ASW weapons. In addition, the convoy may be defended by VS and HS aircraft. The problem is to deploy the escorts for optimum defense of the convoy under the conditions given.

I am writing you, as well as all other destroyer squadron commanders, soliciting your solution to a particular problem of this nature, the details of which are contained in enclosure (1). The solution consists of positioning each of eight escorts on the maneuvering board plot of the convoy vicinity, enclosure (2). Your answer will assist in evaluating my own analysis and solution of the general problem.

The plot alone will be completely adequate. Supporting arguments for your decision are unnecessary, but are invited, as well as other comments you may consider helpful. Copies of squadron doctrine on the subject and reference to recent studies of the problem among the operating forces which might not otherwise be available would be welcome.

I shall feel free to use your data in the thesis unless you specify to the contrary. However, no identification of source by name or title will be made unless you request credit. The classification of your reply will of course be a matter under your cognizance.

The Office of the Chief of Naval Operations (Op-32) has interposed no objection to my conducting this inquiry.

Your reply will be more helpful if mailed not later than 31 December 1963. My address is LCDR Wayne P. Hughes, Jr., USN, c/o Department of

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Operations Research (Code 5510), U. S. Naval Postgraduate School, Monterey, California.

I shall be happy to send you a copy of the completed thesis if you wish.

Very respectfully,

Wayne P. Hughes, Jr.
Lieutenant Commander, USN

Enclosure (1) Screening Problem Data
(2) Maneuvering Board Plot

Distribution:
COMDESRONS LANT/PAC

Copies to:
SUPT USNPGS
CNO (Op-32)
COMCRUDESLANT/PAC
COMASWFORLANT/PAC
COMCRUDESFLOTS LANT/PAC
Operations Evaluation Group
Fleet Sonar School, KWEST
ASW Tactical School, NORVA
COMDESDEVGRU TWO, NWPT

SCREENING PROBLEM DATA

1. Except for the number of escorts or ships in the convoy, the parameters following may be changed if desired. Although a change of inputs will not detract from the usefulness of your solution, bear in mind that the situation is deliberately artificial in order to introduce a variety of complicating elements.

2. The inference should not be drawn that all data given will explicitly affect your solution.

3. Assume the current state-of-the-art, except consider that all new sensors and weapons are in fully operational status. It is intended that operational, rather than theoretical or test performance, be considered.

4. Consider a non-nuclear war.

5. Escort data:

Escort Number	Sonar	Weapons
1	AN/SQS-26	ASROC, DASH, Mark 44 torpedo
2	SQS-23	ASROC, DASH, Mark 44 torpedo
3	SQS-23	DASH, Mark 44 t/o
4	SQS-29	WEAPON ALFA, Mark 35 t/o
5	VDS and SQS-30	Mark 44 t/o, Hedgehogs
6	VDS and SQS-32	Mark 43 t/o, Hedgehogs
7	SQS-4	Mark 32 t/o, Hedgehogs
8	SQS-4	Mark 32 t/o, Hedgehogs

Radar, ecm equipment, speed, and endurance are compatible with above characteristics. Assume depth charges carried in any or all escorts if desired. Assume weapons expenditure does not enter into the problem, except in the case of the aircraft.

6. Convoy data: 99 ships, 1000 x 1000 yard spacing. CVS in box in rear center (CVS-10 class). Dimensions: 16,000 yard front, 6,000 yard depth. Speed: 10 knots.

7. Weather: Sea state two. Wind from ahead at 12 knots. Daylight conditions. Water temperature: 77^o from surface to 150 feet; thence linear gradient to 52^o at 600 feet.

8. Aircraft data: two S-2A (S2F-3) on station in ATP-1A Airplan 7.
three H-3 (HSS-2) to be assigned as desired.

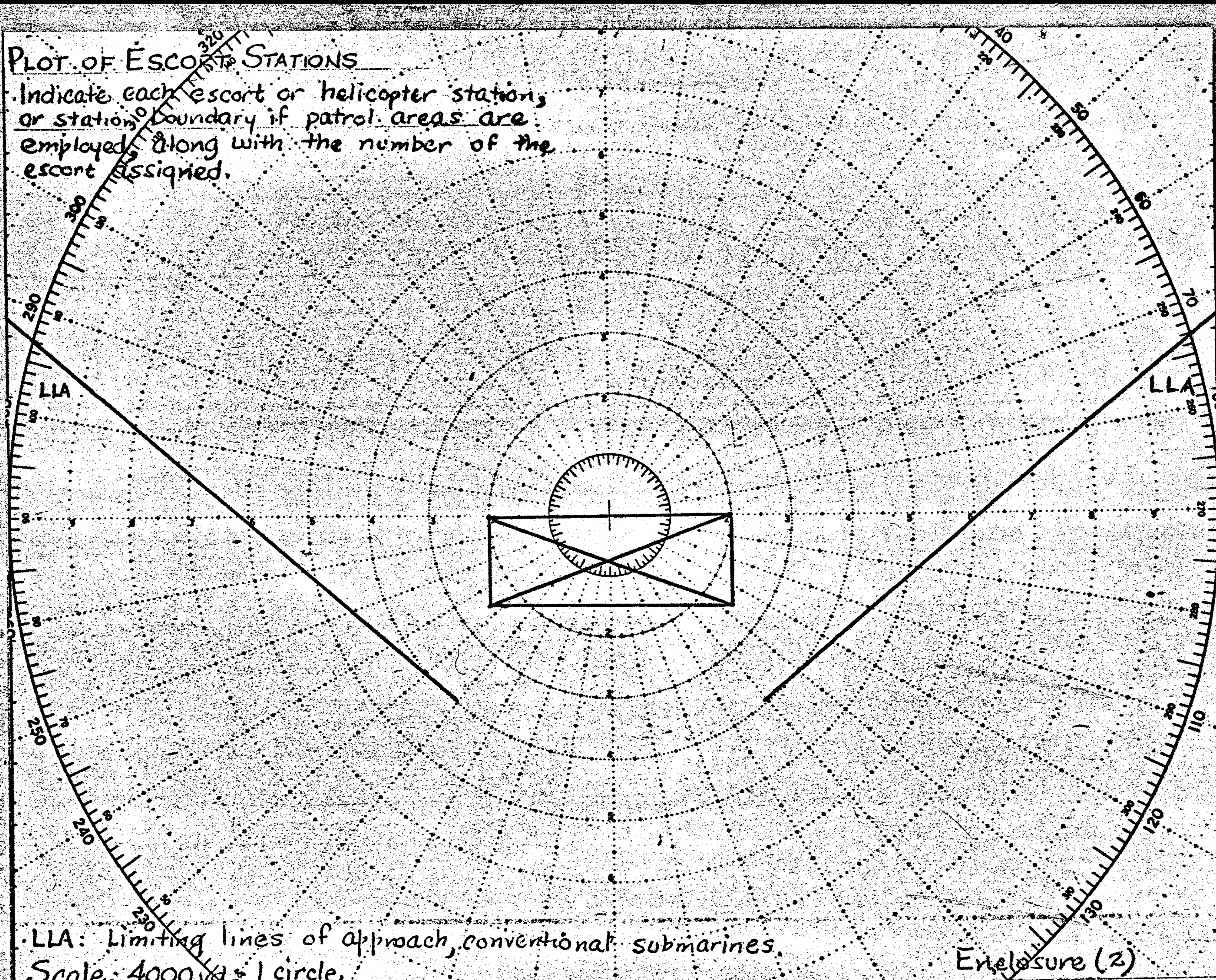
ENCLOSURE (1)

9. Submarine intelligence: Totals unknown, but estimated 80% conventional (USS TANG class characteristics) and 20% nuclear (USS SKATE class characteristics). Submarines will be stationed at intervals of not less than 50 miles, and normally attack independently. Their mission is convoy destruction. Weapon: 15,000 yard straight-running torpedoes. They will probably counterattack upon assurance of detection and use passive acoustic homing torpedoes for defense. They may be expected to attempt penetration "in the dark" 50% of the time, and come to periscope depth at unknown distances before penetrating 50% of the time. Assume the limiting lines of approach for the conventional submarines that are plotted on enclosure (2).

10. It will be assumed that appropriate evasive steering plans for the convoy and the escorts are in effect, and other conventional defensive measures such as antiacoustic torpedo noisemakers are provided for. Special or unusual tactical procedures may be noted if desired.

PLOT OF ESCORT STATIONS

Indicate each escort or helicopter station, or station boundary if patrol areas are employed, along with the number of the escort assigned.



LLA: Limiting lines of approach, conventional submarines.

Scale: 4000 yds = 1 circle.

Enclosure (2)