NPS55-79-025

# NAVAL POSTGRADUATE SCHOOL
## Monterey, California

A STEADY STATE LONGITUDINAL
MANPOWER PLANNING MODEL
WITH SEVERAL CLASSES BY MANPOWER

by

Richard C. Grinold

August 1979

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NAVAL POSTGRADUATE SCHOOL
MONTEREY, CALIFORNIA

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This work was funded under Contract No. N62271-79-M-1981 in support of the Manpower Research Program of the Naval Postgraduate School. This program is funded by the Office of the Deputy Assistant Secretary of the Navy and by the Office of the Deputy Chief of Naval Operations (OP-01).



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# REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

| | | |
|---|---|---|
| 1. REPORT NUMBER<br>NPS55-79-025 | 2. GOVT ACCESSION NO. | 3. RECIPIENT'S CATALOG NUMBER |
| 4. TITLE (and Subtitle)<br>A Steady State Longitudinal Manpower Planning Model with Several Classes by Manpower | | 5. TYPE OF REPORT & PERIOD COVERED<br>Technical |
| | | 6. PERFORMING ORG. REPORT NUMBER |
| 7. AUTHOR(s)<br>Richard C. Grinold | | 8. CONTRACT OR GRANT NUMBER(s) |
| 9. PERFORMING ORGANIZATION NAME AND ADDRESS<br>Naval Postgraduate School<br>Monterey, California 93940 | | 10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS |
| 11. CONTROLLING OFFICE NAME AND ADDRESS<br>Naval Postgraduate School<br>Monterey, California 93940 | | 12. REPORT DATE<br>August 1979 |
| | | 13. NUMBER OF PAGES<br>67 |
| 14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) | | 15. SECURITY CLASS. (of this report)<br>Unclassified |
| | | 15a. DECLASSIFICATION/DOWNGRADING SCHEDULE |

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Manpower Planning
Linear Programming
Goal Programming
Quadratic Optimization
Longitudinal Models

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

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The report describes a model that constructs a personnel inventory by rank

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S/N 0102-014-6601

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20. Abstract continued

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The idea of the model is to allow a policy maker to quickly reconcile billet requirements with the reality of available accessions, job sharing targets between classes, and continuation rates of the different manpower classes.

Different allocations can be produced either by assuming the values of a few key variables, or they can be generated using an optimization scheme that sets "allowable" percentage errors. Four optimization variants based on this idea are described. The report contains some typical data, the results of calculations, and a description of computer programs used to solve the problem.

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ABSTRACT

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## 1. Introduction

This paper outlines the construction of a manpower planning model for a system in which each of several classes of manpower are assigned to several categories of jobs. The jobs are either specialized in that only one type manpower can perform the job or general in that more than one manpower type is qualified to do the job.

Both jobs and manpower are broken down by experience level. A job with a certain experience level must be filled with a person that has the same experience level.

The model assumes a steady state. In each year the job requirements are the same. In each year the same number of people enter the system (bottom level entry only), the same number leave the system from each experience level, and the same number move up from each one level so that the inventory of people at each exerpience level and in each manpower class remains the same.

There are several items represented in the model:

(i) The requirements for jobs by experience levels.

(ii) The requirements for people by experience levels.

(iii) The job sharing targets for common jobs; the fraction of common jobs we would like to see allocated to each manpower class.

(iv) The people sharing targets for manpower classes; the fraction of each manpower class that we would like to see allocated to each type of job.

(v) The rate of accessions in each manpower class.

(vi) The retention of manpower in each class; i.e. the relationship between accessions in each class and the inventory of people by experience level in each class.

The model is designed to study the interaction of these factors. It is easy to see from the list above that items (i) through (iv) are interrelated and unless care is taken in specifying these goals they will be inconsistent. Also items (ii), (v) and (vi) are strongly related in that (v) and (vi) determine the inventory of people in each class at each experience level. Only in the most fortunate circumstances will this be consistent with the personnel requirements (ii).

Items (i)-(iv) are targets. They may be unrealistic and inconsistent but they can be set by the manpower planner. Items (v) and (vi), however, depend to some extent on the behavior of personnel in the system (vi) and on the system's ability to attract qualified people to each class (v). The reader should keep in mind this caveat and be prepared for shifts in personnel behavior in response to changes in other system parameters or in external factors.

The model is a laboratory for testing the relationship between these factors. We can try one set of policies and look at the discrepancy between job requirements and actual requirements as well as the discrepancy between personnel requirement and the actual inventory in each class.

The model may be used in several ways. The easiest is for the planner to stipulate a policy and then examine its effects. A more difficult procedure is for the planner to stipulate a range of policies and then use some type of optimization scheme to select a policy within the range. This paper contains four examples of this type of optimization. Each is based on the notion of a penalty function the measures the discrepancy between desired job and personnel requirements and what is actually provided. We take two types of penalty functions: piecewise linear and quadratic, and we examine two broad sets of policies: allocating people to jobs using the job sharing rules ((iii) above), or allocating people to jobs using the people sharing rates ((iv) above). This gives us four combinations and thus four distinct optimization models. The piecewise linear penalty functions lead to linear programming models and the quadratic penalty functions to the minimization of a quadratic form subject to linear equality constraints.

The model is motivated by a study of the U. S. Navy's officer corps. We shall carry an example using that system throughout the text in order to illustrate each idea.

The model is based on a longitudinal manpower flow model. This type of system is described in depth in Grinold and Marshall [1], Chapter 3. The use of piecewise linear penalty functions to measure the discrepancy between actual performance and stated objectives is commonly called "goal programming." This idea has been extensively developed by Charnes and Cooper; a good review can be found in [3]. The use of quadratic penalty functions is quite common in the optimal control literature; its use in more behavioral settings was pioneered by Holt, Modigliani, Muth and Simon in [2].

The paper consists of several short sections each dedicated to a specific point and most illustrated by the example carried thoughout the text. The model's structure is given in Sections 1-14. Section 15 is a review that gathers all the definitions presented to that point. Sections 16 to 20 describe alternative ways to choose an allocation, the idea behind our use of penalty functions, and the specific construction of the piecewise linear and quadratic penalty functions.

Appendix A describes the two linear programming models that arise from the use of piecewise linear penalties and Appendix B the two models that stem from the use of quadratic penalties. Appendix C contains some sample solutions.

The organization reflects the relative importance of the topics. The structure of the model is the most important and it is stressed and reviewed in Sections 2-14. The use of optimization to select an allocation is, in the main, merely a device to circumvent the difficulty of having a wide range of policy choice. Once the optimization rule is set the planner has a direct route from policy to result. It is the variation in results that comes from changes in policy that will be of most interest to the planner. The optimization is a device to help make that connection.

One final point should be made before describing the model's structure. This is an aggregate planning model. It is intended to test policies that will, in turn, provide a foundation for the day-to-day operating of the system. This model certainly will not show us how to operate the system. No model can answer all questions simultaneously and ours is no exception.

2. Manpower

There are $K$ different classes of manpower indexed by $k = 1,2,\ldots,K$. The classification scheme is, of course, directly related to the objectives of the model builder. In general, the classification should be fine enough to capture the important substitution possibilities and economical in avoiding the listing of all possibilities in a futile attempt to replicate reality. In our example there are five manpower types.

| Manpower Type | Officer Designator | Description |
|---|---|---|
| 1 | 110x | Women |
| 2 | 111x (and 116x) | Surface warfare |
| 3 | 112x (and 117x) | Submarine warfare |
| 4 | 131x and 139x | Pilots |
| 5 | 132x and 137x | Naval flight officers |

Table 2-1: The manpower classes.

3. Stages

Each officer's career is broken down into I-stages indexed by $i = 1,2,\ldots, I$. The stages can be defined in many ways. The simplest is by period of service; if we track officers for 26 years then the index i would go from one to 26. In our example the stages roughly coincide with the time period in which officers hold a certain rank. In general for $i = 1,2,\ldots, I$, stage i will run from time of service $s_{i-1}$ to $s_i$. The maximum length of service is $s_I = S$ and $s_0 = 0$.

A person in manpower class k and in stage i of their career will be called a type (i,k) person.

| Stage | LOS | Description |
|---|---|---|
| 1 | 0-2 | Ensign |
| 2 | 2-4 | Lieutenant--J.G. |
| 3 | 4-9 | Lieutenant |
| 4 | 9-14 | Lieutenant Commander |
| 5 | 14-19 | Commander |
| 6 | 19-26 | Captain |

Table 3-1. Stages or experience levels.

## 4. Manpower Flows

We assume a steady state model. In each year the same number of people will enter each manpower class, and the inventory of people in each year of service and in each manpower class will remain constant.

For class $k = 1,2,\ldots,K$ let $y_k$ be the number of people entering manpower class $k$, and let $\alpha(k,s)$ be the fraction of these accessions that remain in the system for $s$ years. The index $s$ runs from 0 to $S$ years and the function $\alpha(k,s)$ is decreasing in $s$. The function $\alpha(k,s)$ is frequently called the survivor curve.

Stage i of a person's career runs from length of service $s_{i-1}$ to $s_i$ $(s_0 = 0)$. The number of people in stage k is $w_{ik}y_k$ where $w_{ik}$ is defined by

$$w_{ik} = \int_{s_{i-1}}^{s_i} \alpha(k,s)ds \ .$$

In our example, let's take a hypothetical survivor curve for pilots (class 4). We follow these officers for 26 years. Figure 4-1 shows the survivor curve. The shaded areas are integrals under this curve for the duration of each stage.

The calculation of $w_{ik}$ can be approximated using a discrete form of the survivor curve.

The coefficients $w_{ik}$ can be interpreted as the amount of time a person in manpower class $k$ is expected to spend in stage i. A crude way to view this is to say that $w_{ik}$ is the product of the probability of reaching stage i, and the length of stage i. The probability will be $w_{ik}/(s_i - s_{i-1})$.

| | i | $w_{ik}$ | $s_i - s_{i-1}$ | Prob. of reaching stage i |
|---|---|---|---|---|
| S | 1 | 1.97 | 2 | 0.985 |
| T | 2 | 1.85 | 2 | 0.925 |
| A | 3 | 2.50 | 5 | 0.5 |
| G | 4 | 1.40 | 5 | 0.28 |
| E | 5 | 1.22 | 5 | 0.25 |
| S | 6 | 0.54 | 7 | 0.77 |

Table 4-1. The expected waiting time.

5. Jobs

There are $J$ different types of jobs indexed by $j = 1,2,...,J$. In our example we consider seven job types:

| Job No. | Billet Code | Description |
|---|---|---|
| 1 | 1000 | General, nonwarfare billets |
| 2 | 1050 | General, warfare billets |
| 3 | 1110 and 1160 | Surface |
| 4 | 1120 and 1170 | Subsurface billets |
| 5 | 1310 and 1390 | Pilots |
| 6 | 1320 and 1370 | Naval Flying Officer |
| 7 | 1300 | General Aviation |

Table 5-1. Definition of Jobs

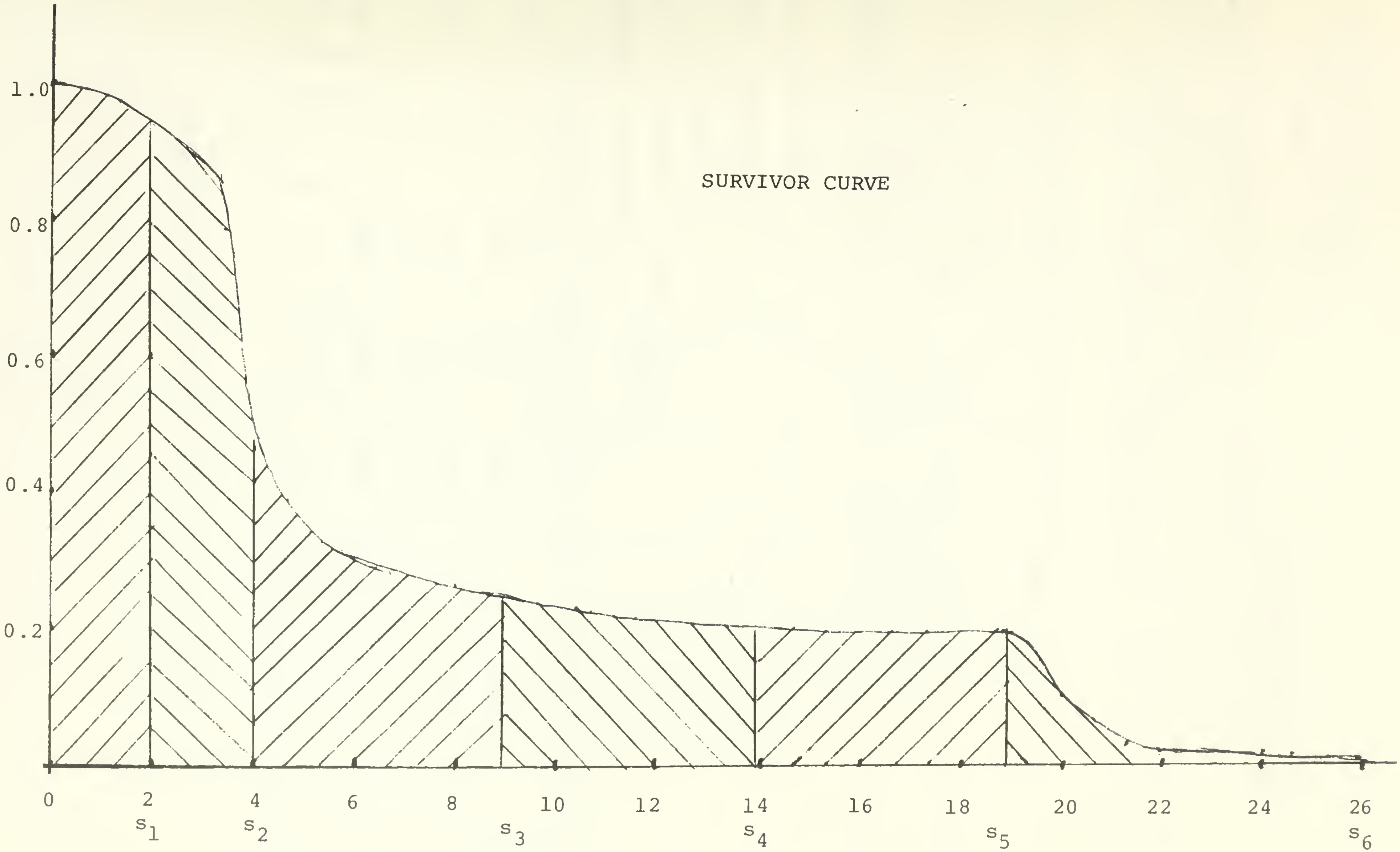



Figure 4-1. Length of service

## 6. Billet Requirements

The I by J matrix B contains the billet requirements. Since the career stage is a measure of experience (e.g. length of service or rank), the requirements are broken down by stage.

$b_{ij}$ the number of stage i people need for jobs type j

The billet requirements matrix for our example is shown below.

The billets will be identified by the index pair (i,j). Thus we shall speak of type (i,j) billets and type (i,k) people.

| | | JOBS 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| STAGE | 1 | 409 | 0 | 2095 | 688 | 961 | 521 | 0 |
| | 2 | 1008 | 0 | 1883 | 734 | 1998 | 1212 | 1 |
| | 3 | 1806 | 378 | 2080 | 844 | 3572 | 1399 | 636 |
| | 4 | 1495 | 599 | 1464 | 810 | 1780 | 547 | 726 |
| | 5 | 1031 | 470 | 926 | 501 | 678 | 55 | 902 |
| | 6 | 592 | 490 | 321 | 149 | 0 | 0 | 387 |

Table 6. The billet requirements matrix B.

## 7. Billet Sharing Array

For each $i$ the $K$ by $J$ matrix $F^i$ gives the share (fraction) of the type $(i,j)$ billets that should be performed by manpower type k. The elements of $F^i$ are denoted $f^i_{kj}$, they are nonnegative, and the rows sum to one, $\sum_k f^i_{kj} = 1$.

If $f^i_{kj} > 0$, we interpret the fraction as a goal; we would like that fraction of the $(i,j)$ billets filled from manpower class k. However, we interpret $f^i_{kj} = 0$ to mean that manpower class k is not qualified to fill billet $(i,j)$.

There is also possible confusion between the idea of job sharing and a people sharing concept. A manpower class may perform two types of jobs: the jobs for which it is uniquely qualified and other jobs that are shared among several manpower classes. Another way to look at the allocation of the common jobs is to stipulate a fraction of the inventory of $(i,k)$ people that should be assigned to job j. That idea will be considered in Section 12.

To save space, we shall only give one of the matrices $F^i$ (for $i = 3$) used in our example.

JOBS

| MANPOWER CLASSES | | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| | 1 | 0.7 | | | | | | |
| | 2 | 0.15 | .5 | 1 | | | | |
| | 3 | 0.06 | .2 | | 1 | | | |
| | 4 | 0.05 | .18 | | | 1 | | .57 |
| | 5 | 0.04 | .12 | | | | 1 | .43 |

Table 7-1. The job sharing matrix $F^i$ for $i = 3$.

## 8. Target Allocations

The number of (i,k) people desired in job (i,j) is given by

$$t_{kj}^{i} = f_{kj}^{i} b_{ij} \; .$$

Recall that $b_{ij}$ is the (i,j) billet requirement and $f_{kj}^{i}$ is the fraction of those billets to be filled by manpower class k.

The target allocations used in our example (for i = 3) are shown in Table 8-1.

| | | JOBS | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 | 5 | 6 | |
| MANPOWER CLASSES | 1 | 1264.2 | | | | | | |
| | 2 | 270.9 | 189 | 2080 | | | | |
| | 3 | 108.36 | 75.6 | | 844 | | | |
| | 4 | 90.3 | 68.04 | | | 3572 | | 362.52 |
| | 5 | 72.24 | 45.36 | | | | 1399 | 273.48 |

Table 8-1. The target allocation $t_{kj}^{3}$

## 9. Actual Allocation

Let the variable $a_{kj}^{i}$ give the actual number of (i,k) people assigned to billet (i,j). If it was possible we would like $a_{kj}^{i} = t_{kj}^{i}$. That would ensure that all targets are met. That, however, is usually impossible. Indeed, reconciling the $a_{kj}^{i}$ and $t_{kj}^{i}$ is the purpose of this model.

The variables $a_{kj}^{i}$ do not mean that $a_{kj}^{i}$ individuals from class k are locked into type (i,j) billets when they are in stage i. It does mean that at any time $a_{kj}^{i}$ people from class k are filling type (i,j) billets. Any particular (i,k) person, may spend stage i in several billets.

If $f_{kj}^{i} = 0$, then an (i,k) person is not qualified to fill an (i,j) billet. So if $f_{kj}^{i} = 0$, we shall require that $a_{kj}^{i} = 0$.

10. Conservation

Recall that $w_{ik}y_k$ is the number of type (i,k) people. The allocation must satisfy the following conservation relation

$$\sum_{j=1}^{J} a_{kj}^{i} = w_{ik}y_k \qquad \text{for all } (i,k)$$

This says that all manpower fills some job.

## 11. Excess and Deficit

One of our main objectives in constructing this model is to compare the allocation and target. We could look at all the discrepancies $a^i_{jk} - t^i_{jk}$. That would be very general, but it may not be comprehensible. We have chosen to concentrate on more aggregate measures of excess and deficit: the actual number of (i,k) people and the actual number of (i,j) billets filled as compared with the targets. Our notation for these measures is

| | |
|---|---|
| $x_{ij} = \sum_{k=1}^{K} a^i_{kj}$ | the number of poeple assigned to billet (i,j) |
| $x_{ij} - b_{ij}$ | the discrepancy between billet (i,j) assignment and requirements |
| $z_{ik} = \sum_{j=1}^{J} a^i_{kj}$ | the number of (i,k) people |
| $p_{ik} = \sum_{j=1}^{J} t^i_{kj}$ | the total requirement for type (i,k) people |
| $z_{ik} - p_{ik}$ | the discrepancy between the inventory of (i,k) people and requirements. |

12. People Sharing

Recall that we defined our targets from the viewpoint of billet sharing. Given any allocation we can calculate the people sharing fractions. Let $g^i_{kj}$ be the fraction of type (i,k) people that are assigned to job j, thus billet (i,j). The $g^i_{kj}$ are given implicitly by the equation

$$a^i_{kj} = g^i_{kj} z_{ik}$$

As mentioned above, an alternative model could be constructed using the people sharing fractions $g^i_{kj}$ as input targets and the billet sharing fractions $f^i_{kj}$ as outputs. In that case we would start with people requirement $p_{ik}$, and the people sharing rules $g^i_{kj}$ and define target allocations by

$$t^i_{kj} = g^i_{kj} p_{ik}.$$

Then we can calculate $b_{ij}$ from

$$b_{ij} = \sum_k g^i_{kj} p_{ik}.$$

In any case, for consistency, the following relations should hold

(i) $g^{i}_{kj} p_{ik} = f^{i}_{kj} b_{ij}$

(ii) $\sum_{j=1}^{J} g^{i}_{kj} = 1 \implies p_{ik} = \sum_{j=1}^{J} f^{i}_{kj} b_{ij}$

(iii) $\sum_{k=1}^{K} f^{i}_{kj} = 1 \implies b_{ij} = \sum_{k=1}^{K} g^{i}_{kj} p_{ik}$ .

To assure consistency we should either start with $f$ and $b$ and then calculate $g$ and $p$, or start with $g$ and $p$ and calculate $f$ and $b$. If we have a desire to start with $g$ and $b$, then one suggestion is to guess $p$, see if it works, and then do some revision.

From the values of $b$ and $f$ (for all i) used in this paper, we can compute the personnel requirements, shown below.

| | | Manpower Class | | | | |
|---|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 | 5 |
| S | 1 | 286 | 2156 | 713 | 981 | 537 |
| T | 2 | 706 | 2034 | 794 | 2049 | 1253 |
| A | 3 | 1264 | 2540 | 1028 | 4093 | 1790 |
| G | 4 | 1046 | 1988 | 1019 | 2376 | 991 |
| E | 5 | 722 | 1316 | 657 | 1328 | 540 |
| | 6 | 414 | 655 | 283 | 338 | 249 |

Table 12-1. The personnel inventories, $p_{ik}$.

13. <u>Stage Substitution</u>

The astute reader will notice that we have not allowed substitution between stages. Thus a type (i,k) person is not allowed to fill a type (i+1, j) billet or a type (i-1, j) billet.

Our formuation actually allows such assignments, however, we are assuming that they net out to zero. That is for each (i,k) person filling a stage i+1 billet, there is an (i+1,k) person filling a stage i billet.

One way to study the possibility of stage substitution is to vary the length of the intervals $(s_{i-1}, s_i)$ that define the stages. However, the billet requirement are presumably set with some experience level in mind. Therefore major shifts in the definitions of stages would require adjustments in the billet requirements.

## 14. Cost

The cost of any allocation can be calculated using

$$\sum_i \sum_k c_{ik} w_{ik} y_k ,$$

where $c_{ik}$ is the annual cost of an (i,k) person and $w_{ik} y_k = z_{ik}$ is the number of (i,k) people. The cost data $c_{ik}$ should include salary, benefits, retirement; training, promotion, and recruitment costs. Notice we can define $c_k$ as $\sum_k c_{ik}\,{}_{ik}$, and then rewrite the cost as $\sum_k c_k y_k$. We interpret $c_k$ as the average career cost of each accession into manpower class k.

## 15. Review

We have constructed our model. The remainder of the paper is devoted to ways of calculating particular assignments and contains a sample calculation.

We summarize here the notations and definitions presented to this point.

| | |
|---|---|
| manpower class | indexed by $k = 1,2,\ldots,K$ |
| stages of a career | indexed by $i = 1,2,\ldots,I$ defined by length of service $(s_1, s_2, \ldots, s_I)$ |
| jobs | indexed by $j = 1,2,\ldots,J$ |
| people | indexed by stage and class $(i,k)$ |
| accessions | $y_k$, the number of people entering class k per year |
| stage inventory | $w_{ik}y_k$ is the number of $(i,k)$ people; $y_k$ is the accession rate and $w_{ik}$ is obtained from the survivor curve and the stage definition. Note $z_{ik} = w_{ik}y_k$. |
| billets requirements | indexed by stage and job $(i,j)$; $b_{ij}$ is the billet requirements |
| billet sharing | indexed by $(i,j,k)$; $f^i_{kj}$ is the fraction of $(i,j)$ billets that should be filled from class k. |

| | |
|---|---|
| target allocation | $t^i_{kj}$. The number of (i,k) people desired in billet (i,j) |
| actual allocation | $a^i_{kj}$ the number of (i,k) people in billet (i,j) |
| class sums | $x_{ij}$ and $b_{ij}$ are respectively the number assigned to billet (i,j) and the requirement for billet (i,j) |
| job sums | $z_{ik}$ and $p_{ik}$ are respectively the number of (i,k) people, and the target for (i,k) people. |
| people sharing | $g^i_{kj}$, the fraction of (i,k) people assigned to billet (i,j); $a^i_{kj} = g^i_{kj}\ z_{ik}$. |
| cost | $c_{ik}$ is the annual cost of an (i,k) person. The cost of a particular allocation is $\sum_i \sum_k c_{ik} z_{ik}$. |

## 16. Choice of an Allocation

Up to this point we have set out a model structure. The next task is to devise one or two more procedures for selecting allocations. There are two general approaches to this task.

On approach is to use an ad hoc rule. For example, we could fix accessions at projected rates for each class. That would fix the variables $y_k$ and thus $z_{ik} = w_{ik}y_k$. Next we could specify the people sharing rules $g^i_{kj}$. That would give us the allocation $a^i_{kj} = g^i_{kj}z_{ik}$.

The second approach is to use some type of optimization to select a policy. This optimization is either based on a trade-off between the cost of any allocation and some measure of its quality, or simply a measure of the quality of the allocation.

## 17. Quality of an Allocation

In order to choose an allocation using optimization we need some measure of the quality of that allocation.

In our model we have already decided to focus on the discrepancy between people inventory and targets $(z_{ik} - p_{ik})$ and the discrepancy between billet assignments and requirements $(x_{ij} - b_{ij})$.

We have selected the simplest form of quality measure that makes sense. It is a penalty function that is zero for a perfect allocation and positive for others. Thus it measures the lack of quality. The penalty function has three properties.

(1) The penalty can be written as

$$\sum_i \sum_k h_{ik}(z_{ik}) + \sum_i \sum_j \ell_{ij}(x_{ij})$$

where

(2) $h_{ik}(z_{ik})$ and $\ell_{ij}(x_{ij})$ are nonnegative, convex and equal to zero if $z_{ik} = p_{ik}$ and $x_{ij} = b_{ij}$ respectively.

(3) Parameters $\theta_{ij}$ and $\psi_{ij}$ measure a unit upper and lower percentage error in meeting billet requirements

$$\ell_{ij}(x_{ij}) = 1 \quad \begin{array}{l} \text{if } x_{ij} = (1 + \theta_{ij})b_{ij} \, , \\ \text{or } x_{ij} = (1 - \psi_{ij})b_{ij} \, . \end{array}$$

Similar parameters $\phi_{ik}$ and $\delta_{ik}$ are used for discrepancies between actual inventories of people and requirements.

$$h_{ik}(z_{ik}) = 1 \quad \begin{array}{l} \text{if } z_{ik} = (1 + \phi_{ik})p_{ik} \ , \\ \text{or } z_{ik} = (1 - \delta_{ik})p_{ik} \ . \end{array}$$

Item (1) above says that the quality measure is separable, item (2) that is nonnegative, convex, and zero if the assignment is exactly on target. The third item requires more discussion. Item (3) is designed to answer the question: How do we compare a 4 percent shortfall in meeting a critical target with a 10 percent short fall in meeting a less critical target? Our answer to this question is to take a single target as a benchmark and to say arbitrarily for that target that a certain percent over and a certain percent under the target yields an error of one. Then for any other target we can compare the seriousness of deviations with our benchmark. We say a fraction $\theta_{ij}$ over target or $\psi_{ij}$ under target is as serious as the deviations we have established for our benchmark.

For example, we could take stage 3 pilots as our benchmark ($i = 3$, $j = 5$), and take the percentage under as 4% ($\psi_{35} = 0.04$) and percentage over as 10% ($\theta_{35} = 0.10$) as the unit serious deviation for our benchmark. Now take any other category, for example, stage 4 general warfare billets ($i = 4$, $j = 2$). Then we ask how much under target would the assignment to these billets have to be in order to be as serious as a 4% shortage of stage 3 pilots. In this way we can try to make the essential judgments about the trade-offs between different categories.

We shall now give two examples of penalty functions and then give some practical examples of how such a criterion could be used.

## 18. Piecewise Convex Penalties

One of the simplest ways to construct the penalty measure for deviations from target is to use a piecewise convex function. To simplify notation we should consider the case of a discrepancy between the inventory of people and desired inventories and we shall drop the (i,k) subscripts.

We need a piecewise linear convex function $h(z)$ that satisfies

(i) $h(z) = 0$ if $x = p$

(ii) $h(z) = 1$ if $x = (1 + \phi)p$

(iii) $h(z) = 1$ if $x = (1 - \delta)p$

Such a function is shown in Figure 2. This function can be represented in several ways. One of the simplest is as the maximum of two linear functions

$$h(z) = \max \left[ \frac{1}{\phi} \left(\frac{z-p}{p}\right) , \frac{1}{\delta} \left(\frac{p-z}{p}\right) \right]$$

This functional form is flexible and easy to manipulate. We shall see below, that it can be used to obtain an allocation using a linear program. Two examples of this are contained in the appendix.

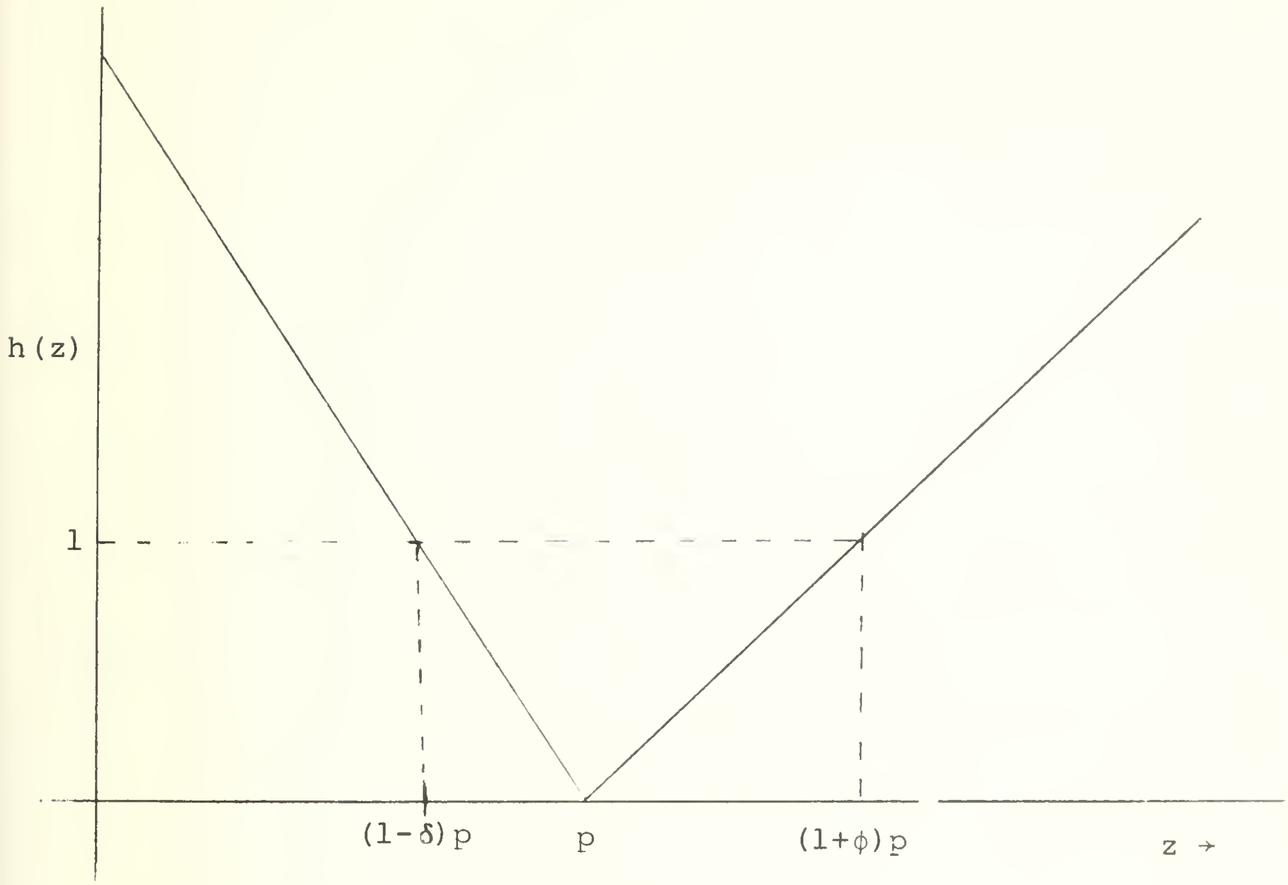



Figure 18-1. Piecewise linear convex function.

## 19. Quadratic Penalty Functions

Quadratic penalty functions are also quite easy to use. Unfortunately, we cannot get a quadratic function to satisfy all of our conditions. For example let us write

$$\ell(x) = \alpha(x - b)^2 + \beta(x - b) + \gamma.$$

We want to have

(i) $\ell(x) = 0$ if $x = b$

(ii) $\ell(x) = 1$ if $x = (1 + \theta)b$

(iii) $\ell(x) = 1$ if $x = (1 - \psi)b$

(iv) $\ell(x) \geq 0$

where item (iv) implies $\ell(x)$ has its minimum at $x = b$.

These four conditions cannot be met by a quadratic function unless $\theta = \psi$; that is unless there is a symmetry between being under and over target. In the symmetric case we can write $\ell(x)$ as

$$\ell(x) = \left(\frac{x - b}{\theta b}\right)^2 \qquad \text{when} \quad \theta = \psi.$$

When the penalties are not symmetric, we must relax one of our four conditions. If we relax condition (iv) then $\ell(x)$ becomes

$$h(x) = \frac{1}{\theta\psi}\left(\frac{x - b}{b}\right)^2 - \frac{1}{2}\,\frac{(\theta - \psi)}{\theta\psi}\,\left(\frac{x - b}{b}\right) .$$

This function has its minimum midway between $(1 + \theta)b$ and $(1 - \psi)b$ and it has a value of

$$- \frac{(\theta - \psi)^2}{2\theta\psi}$$

at the minimum.

This approximation is useful if $\theta$ and $\psi$ are similar. However, the approximation becomes much worse when $\theta$ and $\psi$ are much different.

Two quadratic penalty models are described in the appendix.

20. <u>Cost Quality Trade-Off</u>

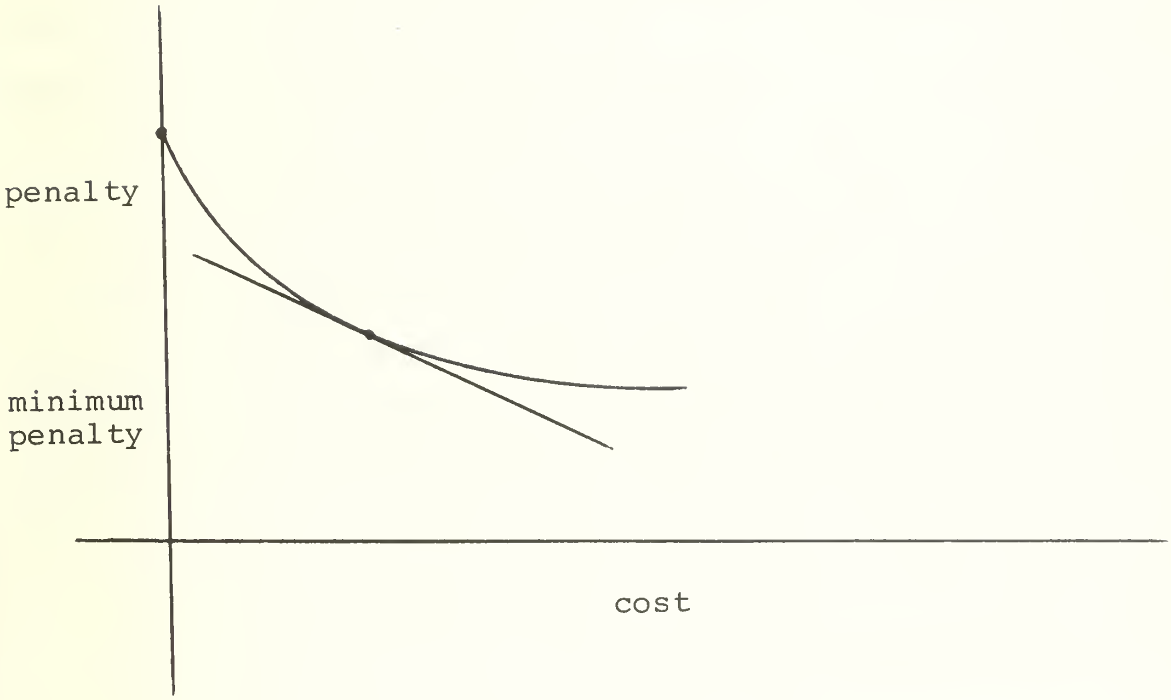
We saw above that any particular allocation has an annual cost and also a penalty associated with its deviation from targets for billet requirements and personnel inventories. By placing different weights on the cost and penalty we can obtain a family of objectives that will lead to efficient allocations.

The cost-penalty frontier is shown below. Notice the minimum cost solution is zero, since we would not have any accessions; i.e. y = 0, thus no cost.

We write our objective as

$$\lambda \sum_k c_k y_k + (1-\lambda) \{ \sum_i \sum_k h_{ik}(z_{ik}) + \sum_i \sum_j \ell_{ij}(x_{ij}) \} ,$$

As the parameter $\lambda$ increases from 0 to 1 more emphasis is placed on the cost and less on the penalty. At $\lambda = 0$, we minimize the penalty and ignore the cost; at $\lambda = 1$ we minimize cost and ignore the penalty. For intermediate values of $\lambda$ we establish a trade-off between cost and penalty. This curve should be used with some caution since the costs are expressed in real units (dollars per year), but the penalty is expressed in ad-hoc units.


penalty
minimum
penalty
cost

# APPENDIX A

## LINEAR PROGRAMMING MODELS

This appendix describes two linear programming approaches for selecting an allocation. The discussion is brief and is intended for those familiar with linear programming. The first model is called LPB for billet sharing and the second is called LPP for people sharing.

In both models the idea is to choose the number of people assigned to billet (i,j), and the number of accessions in manpower class k; these are respectively denoted $x_{ij}$ and $y_k$. The allocation $a^i_{kj}$ is then fixed using an ad hoc rule. We have selected two rules and thus get two different linear programming models.

The first rule is for sharing billets

Rule B (billet share)

(A-1) $$a^i_{kj} = f^i_{kj} x_{ij}$$

Recall that $f^i_{kj}$ is the fraction of type (i,j) billets that we desire to have satisfied from manpower class k. Rule B allocates the error in meeting billet requirements among manpower classes so that there is a constant percentage error in each class's allocation. If there is a percentage error of $\eta_{ij}$ in meeting the billet (i,j) requirement, i.e.

$$\eta_{ij} = \left( \frac{x_{ij} - b_{ij}}{b_{ij}} \right) ,$$

then the billet sharing rule means that the percentage error in the assignment of type k manpower to billet (i,j) will also be $\eta_{ij}$.

The people sharing rule is

Rule P

(A-2) $$a^i_{kj} = g^i_{kj} z_{ik}$$

where $z_{ik} = \alpha_{ik} y_k$ is the number of type (i,k) people. The type (i,k) people are allocated among different jobs using the people sharing parameters $g^i_{kj}$. If there is an error of $\eta_{ik}$ in the inventory of type (i,k)

$$\eta_{ik} = \frac{(z_{ik} - p_{ik})}{p_{ik}}$$

then the percentage error in the assignment of type (i,k) people to job j will be $\eta_{ik}$ (recall $t^i_{kj} = g^i_{kj} p_{ik}$). The billet sharing rule leads to a larger linear program than the people sharing rule.

The constraints of the linear program for billet sharing are given below. The objective is the same for both programs; it will be discussed after we examine the constraints.

| Dual Variable | Constraint | Explanation |
| --- | --- | --- |
| $u_{ik}$ | $\sum_{j=1}^{J} f_{kj}^{i} x_{ij} - w_{ik} y_k = 0$ for all $(i,k)$ | conservation of people |
| $v_{ik}$ | $w_{ik} y_k + d_{ik} - e_{ik} = p_{ik}$ for all $(i,k)$ | discrepancy between type $(i,k)$ people inventory and requirement |
| $q_{ij}$ | $x_{ij} + m_{ij} - n_{ij} = b_{ij}$ for all $(i,j)$ | discrepancy between type $(i,j)$ billet assignments and requirements |
| | $x_{ij}, m_{ij}, n_{ij}, d_{ik}, e_{ik}, y_k \geq 0$ | |

Table A-1. Constraints for LP-B.

The first set of constraints is simply the conservation of people. There will be $w_{ik}y_k$ type (i,k) people and under the billet sharing rule $f^i_{kj}x_{ij}$ of them will be assigned to billet (i,j).

The second set of constraints measures the difference between actual inventories of type (i,k) people $w_{ik}y_k$ and the requirement $p_{ik}$. The objective is selected so that the solution to the linear program will have $e_{ik}$ positive and $d_{ik}$ equal to zero if $w_{ik}y_k$ exceed $p_{ik}$ and $e_{ik}$ equal to zero and $d_{ik}$ positive if $w_{ik}y_k$ is less than $p_{ik}$. The $e_{ik}$ and $d_{ik}$ thus measure the excess and deficit in the type (i,k) people account.

The variables $m_{ij}$ and $n_{ij}$ play a similar excess and deficit role in the type (i,j) billet account.

The first set of constraints for the LP-P program is

(A-3) $$x_{ij} - \sum_{k=1}^{K} g^i_{kj} w_{ik} y_k = 0 \quad \text{for all } (i,j) .$$

This again is a conservation constraint. There are $w_{ik}y_k$ type (i,k) people and $g^i_{kj}$ of them are assigned to job j. Thus these constraints assure that the assignments to billets, $x_{ij}$, are consistent with actual manpower available.

The second and third sets of constraints in LP-P are identical with the second and third sets of constraints in LP-B. They define the excess and deficit in people and billet accounts.

The form of the conservation constraint in LP-P allow for some simplifications. Notice that if all the $y_k$ are nonnegative then (A-3) implies that all the $x_{ij}$ will be nonnegative. Thus we can use (A-3) to eliminate $x_{ij}$ from the problem. With this simplification the constraints of LP-P are shown in Table A-2.

The objective for both linear programs will typically be a compromise between minimizing costs and minimizing the penalty that measures our departure from people and billet targets. The cost objective is

$$\text{(A-4)} \qquad \sum_{k=1}^{K} c_k y_k \, .$$

The penalty measure is

$$\text{(A-5)} \qquad \sum_{i=1}^{I} \sum_{j=1}^{J} \pi_{ij} n_{ij} + \mu_{ij} m_{ij} + \sum_{i=1}^{I} \sum_{k=1}^{I} \gamma_{ik} e_{ik} + \sigma_{ik} d_{ik}$$

where

(A-6)

$$\text{(i)} \quad \pi_{ij} = (\theta_{ij} b_{ij})^{-1}$$

$$\text{(ii)} \quad \mu_{ij} = (\psi_{ij} b_{ij})^{-1}$$

$$\text{(iii)} \quad \gamma_{ik} = (\phi_{ik} p_{ik})^{-1}$$

$$\text{(iv)} \quad \sigma_{ik} = (\delta_{ik} p_{ik})^{-1}$$

| Dual Variable | Constraint | | Explanation |
|---|---|---|---|
| $v_{ik}$ | $w_{ik}y_k + d_{ik} - e_{ik} = p_{ik}$ | for all (i,k) | Type (i,k) people discrepancy |
| $q_{ij}$ | $\sum_k g^i_{kj} w_{ik} y_k + m_{ij} - n_{ij} = b_{ij}$ | for all (i,j) | Type (i,j) billet discrepancy |
| | $y_k, d_{ik}, e_{ik}, m_{ij}, n_{ij} \geq 0$ | | |

Table A-2. Constraints for LP-P

Recall that $\theta$, $\psi$, $\phi$, and $\delta$ are the percentage errors that lead to a unit loss.

A balance between these two objectives is obtained by taking a weighted combination.

$$\text{(A-7)} \quad \lambda \sum_{k=1}^{K} c_k y_k + (1 - \lambda)$$

$$\times \{ \sum_{i=1}^{I} \sum_{j=1}^{J} \pi_{ij} n_{ij} + \mu_{ij} m_{ij} + \sum_{i=1}^{I} \sum_{k=1}^{K} \gamma_{ik} e_{ik} + \sigma_{ik} d_{ik} \}$$

The problem LP-B has $I \times (J + 2K)$ constraints and $K + I \times (2K + 3J)$ variables. For our example with $I = 6$, $J = 7$ and $K = 5$ this means 102 constraints and 191 variables. The problem LP-P has $I \times (J + K)$ constraints and $K + I \times (2K + 2J)$ variables. For our example this works out to 72 constraints and 149 variables. LP-P is particularly easy to solve since one can always get a reasonable first basic solution by guessing the $y_k$ and then choosing the $d_{ik}$, $e_{ik}$, $m_{ij}$, and $n_{ij}$ to satisfy the constraints.

The duals of both LP-B and LP-P appear to be easier to solve. The dual variables for each problem and each set of constraints is shown in Tables (A-1) and (A-2) respectively. The dual program for LP-B is

(A-8) maximize $\sum_{i=1}^{I} \sum_{k=1}^{K} v_{ik}p_{ik} + \sum_{i=1}^{I} \sum_{j=1}^{J} q_{ij}b_{ij}$

subject to $\sum_{k=1}^{K} u_{ik}f_{kj}^{i} + q_{ij} \leq 0$ for all $(i,j)$

$- \sum_{i=1}^{I} u_{ik}w_{ik} + \sum_{i=1}^{I} v_{ik}w_{ik} \leq \lambda c_k$ for all $k$

$- (1 - \lambda)\gamma_{ik} \leq v_{ik} \leq (1 - \lambda)\sigma_{ik}$ for all $(i,k)$

$- (1 - \lambda)\pi_{ij} \leq q_{ij} \leq (1 - \lambda)\mu_{ij}$ for all $(i,j)$

Dual of LP-B

The dual of LP-B has $K + I \times J$ constraints, $I \times (2K + J)$ variables, and $I \times (K + J)$ of these variables have upper and lower bounds. In our example this works out to 47 constraints, 102 variables, and 72 variables subject to upper and lower bound constraints.

The dual of LP-P is shown below.

$$\text{(A-9)} \qquad \text{maximize} \quad \sum_{i=1}^{I} \sum_{k=1}^{K} v_{ik} p_{ik} + \sum_{i=1}^{I} \sum_{j=1}^{J} q_{ij} b_{ij}$$

$$\text{subject to} \quad \sum_{i=1}^{I} v_{ik} w_{ik} + \sum_{i=1}^{I} \sum_{j=1}^{J} q_{ij} g_{kj}^{i} w_{ik} \leq \lambda c_k \quad \text{for all } k$$

$$-(1 - \lambda)\gamma_{ik} \leq v_{ik} \leq (1 - \lambda)\sigma_{ik} \qquad \text{for all } (k,i)$$

$$-(1 - \lambda)\pi_{ij} \leq q_{ij} \leq (1 - \lambda)\mu_{ij} \qquad \text{for all } (i,j)$$

Dual of LP-P

This problem has $K$ constraints and $I \times (J + K)$ variables with upper and lower bounds. In our example, this would be 5 constraints and 72 variables with upper and lower bounds. This problem may be quite easy to solve.

## APPENDIX B

### UNCONSTRAINED QUADRATIC MODELS

This appendix outlines two methods for selecting an allocation using the quadratic penalty functions. The two models are similar to the two linear programming models; they use the billet sharing or people sharing rules to go from an aggregate problem to a detailed allocation.

The models do not have any inequality contraints. This is to insure that the solution can be obtained quickly. A full model with inequality constraints might appear to be more appropriate, however, we must recall our ultimate objective is to calculate allocations in a relatively simple way. The model is not built on exact premises and it does not use precise data. It is therefore not terribly important to be exact in choosing an allocation. We hope to have an easy and consistent way of choosing allocations so we can compare the effects of changing assumptions on the allocations.

Both models calculate the billet assignments $x_{ij}$ and the accession rules $y_k$. The actual allocation is determined by the billet sharing rule in model UQ-B

<u>Rule B</u>.

(B-1) $$a^i_{kj} = f^i_{kj} x_{ij} \ .$$

In model UQ-P, the people sharing rule is used

<u>Rule P</u>.

(B-2) $$a^i_{kj} = g^i_{kj} w_{ik} y_k$$

These rules are discussed in Appendix A.

The objective in these quadratic models is based on symmetric penalties; i.e. $\theta_{ij} = \psi_{ij}$ and $\phi_{ik} = \delta_{ik}$. The objective is a combination of two terms: the cost term and a penalty term

(B-3) $$\lambda \sum_{k=1}^{K} c_k y_k + (1 - \lambda)$$

$$\times \{ \sum_{i=1}^{I} \sum_{j=1}^{J} \mu_{ij} (x_{ij} - b_{ij})^2 + \sum_{i=1}^{I} \sum_{k=1}^{K} \gamma_{ik} (w_{ik} y_k - p_{ik})^2 \}$$

where

(B-4) (i) $$\mu_{ij} = (\theta_{ij} b_{ij})^{-2}$$

(ii) $$\gamma_{ik} = (\phi_{ik} p_{ik})^{-2} .$$

In the UQ-B model we minimize (B-3) subject to the conservation constants.

(B-5) $$\sum_{j=1}^{J} f^i_{kj} x_{ij} - w_{ik} y_k = 0 \quad \text{for each } (i,k).$$

The people sharing model, UQ-P, minimizes (B-3) subject to the constraints

(B-6)
$$x_{ij} - \sum_{k=1}^{K} g_{kj}^{i} w_{ik} y_k = 0$$

In UQ-P we can substitute (B-6) directly into (B-3) and thus solve a completely unconstrained problem. In UQ-B we minimize (B-3) subject to (B-5).

APPENDIX C

SAMPLE SOLUTION

This section contains some sample solutions using the unconstrained minimization described in Appendix B.

The data, $b_{ij}$, $f^i_{kj}$, $w_{ik}$, $p_{ik}$, are described in the paper. The $p_{ik}$ and $g^i_k$, are calculated in the manner suggested in Section 12.

The solutions are based on data that is largely subjective and does not, in any way, pretend to capture the situation that exists in the Navy. The intent of this section is to demonstrate the feasibility of the scheme.

The target errors are expressed in percentage terms. The 10,000% target error indicates that we do not care very much about meeting billit or personnel targets in the first two stages.

| Stage \ Job | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| ENS | 10000 | 10000 | 10000 | 10000 | 10000 | 10000 | 10000 |
| LTJG | 10000 | 10000 | 10000 | 10000 | 10000 | 10000 | 10000 |
| LT | 10 | 8 | 6 | 4 | 4 | 5 | 6 |
| LTCDR | 8 | 6.4 | 4.8 | 3.2 | 3.2 | 4 | 4.8 |
| CDR | 7 | 5.6 | 4.2 | 2.8 | 2.8 | 3.5 | 4.2 |
| CAPT | 6 | 4.8 | 3.6 | 2.4 | 2.4 | 3 | 3.6 |

Table C-1. The inputs $\theta_{ij}$ in %; for billet requirements

| Stage \ Job | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| ENS | 0 | 10000 | 10000 | 10000 | 10000 |
| LTJG | 10000 | 10000 | 10000 | 10000 | 10000 |
| LT | 10 | 7.5 | 4.5 | 5 | 6 |
| LTCDR | 8 | 6 | 3.6 | 4 | 4.8 |
| CDR | 7 | 5.25 | 3.15 | 3.5 | 4.2 |
| CAPT | 6 | 4.5 | 2.7 | 3 | 3.6 |

Table C-2/ The inputs $\phi_{ik}$ in %; for manpower inventories

The billet assignments and the actual percentage errors using the people share rule are

| STAGE/JOB | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| ENS | 1444 | 0 | 2231 | 1000 | 1791 | 972 | 0 |
| LTJG | 1498 | 0 | 1989 | 895 | 1669 | 908 | 1 |
| LT | 2164 | 383 | 2380 | 1976 | 2021 | 992 | 399 |
| LTCDR | 1242 | 444 | 1246 | 586 | 977 | 395 | 452 |
| CDR | 1108 | 482 | 1000 | 489 | 578 | 63 | 884 |
| CAPT | 540 | 529 | 308 | 150 | 0 | 0 | 510 |

Table C-3. The billet assignment, $x_{ij}$

| STAGE/JOB | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| ENS | -253 | 0 | -6 | -45 | -86 | -87 | 0 |
| LTJG | -47 | 0 | -6 | -22 | -16 | 25 | 20 |
| LT | -20 | -1 | -14 | -27 | 43 | 29 | 37 |
| LTCDR | 17 | 26 | 18 | 28 | 45 | 28 | 38 |
| CDR | -7 | -3 | -8 | 2 | 15 | -15 | 2 |
| CAPT | 9 | -8 | 4 | 0 | 0 | 0 | -32 |

Table C-4. The percentage error in meeting billet requirements

The accessions to the five classes are:

| Class | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| | 645 | 1162 | 524 | 926 | 508 |

Table C-5. Annual rate of accessions, $y_k$

These accessions produce the actual personnel inventories and the precentage error in meeting inventories are given below.

| Class | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| ENS | 1274 | 2296 | 1035 | 1829 | 1003 |
| LTJG | 1192 | 2149 | 969 | 1712 | 938 |
| LT | 1613 | 2906 | 1311 | 2316 | 1269 |
| LTCDR | 909 | 1637 | 738 | 1304 | 715 |
| CDR | 789 | 1421 | 641 | 1132 | 621 |
| CAPT | 349 | 629 | 284 | 501 | 275 |

Table C-6. Personnel inventory by class and stage, $z_{ik}$.

| Class | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| ENS | -345 | -6 | -45 | -86 | -87 |
| LTJG | -67 | -6 | -22 | 16 | 25 |
| LT | -28 | -14 | -27 | 43 | 29 |
| LTCDR | 13 | 18 | 28 | 45 | 27 |
| CDR | -9 | -8 | 2 | 15 | -15 |
| CAPT | 16 | 4 | 0 | -48 | -10 |

Table C-7. Percentage errors in meeting personnel inventory budgets.

The solution using the billet sharing rule produced similar results. The billet assignments and % error in billet assignments were

| STAGE/JOB | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| ENS | 1801 | 0 | 2074 | 940 | 1388 | 1015 | 0 |
| LTJG | 1685 | 0 | 1941 | 879 | 1298 | 949 | 1 |
| LT | 2280 | 405 | 2423 | 1109 | 1399 | 1021 | 499 |
| LTCDR | 1284 | 370 | 1294 | 596 | 676 | 493 | 432 |
| CDR | 1115 | 496 | 1036 | 483 | 101 | 64 | 1173 |
| CAPT | 493 | 554 | 291 | 147 | 0 | 0 | 492 |

Table C-8

| STAGE/JOB | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| ENS | ¯340 | 0 | 1 | ¯37 | ¯44 | ¯95 | 0 |
| LTJG | ¯67 | 0 | ¯3 | ¯20 | 35 | 22 | 0 |
| LT | ¯26 | ¯7 | ¯17 | ¯31 | 61 | 27 | 21 |
| LTCDR | 14 | 38 | 12 | 26 | 62 | 10 | 40 |
| CDR | ¯8 | ¯5 | ¯12 | 4 | 85 | ¯17 | ¯30 |
| CAPT | 17 | ¯13 | 9 | 2 | 0 | 9 | ¯27 |

Table C-9

The accessions using billet sharing were

| Class | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| | 638 | 1187 | 530 | 748 | 550 |

Table C-10

These accession rates give rise to the personnel inventory and percentage error in personnel levels

| Class | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| ENS | 1261 | 2345 | 1048 | 1478 | 1087 |
| LTJG | 1180 | 2194 | 981 | 1383 | 1017 |
| LT | 1596 | 2968 | 1326 | 1871 | 1376 |
| LTCDR | 899 | 1672 | 747 | 1054 | 775 |
| CDR | 780 | 1451 | 649 | 915 | 673 |
| CAPT | 345 | 642 | 287 | 405 | 298 |

Table C-11

| Class | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| ENS | ¯340 | ¯9 | ¯47 | ¯51 | ¯102 |
| LTJG | ¯67 | ¯8 | ¯23 | 33 | 19 |
| LT | ¯26 | ¯17 | ¯29 | 54 | 23 |
| LTCDR | 14 | 10 | 1 | 31 | ¯24 |
| CDR | 17 | 2 | ¯2 | ¯20 | ¯20 |

Table C-12

The analysis of this output should be directed toward constructive changes in the input data. Can we shift some billet assignments from LT to LTJG and LTCDR? Should we tighten up more on the pilot inventory? Should we change the sharing rules? Is it possible to alter the survivor fractions, and thereby improve the solution?

We see that there are a number of potential questions we can answer. The model is flexible and the calculations are rapid. It should be an excellent tool for analyzing manpower policy.

# APPENDIX D

## ORGANIZATION OF PROGRAMS

This appendix shows how the programs and files for the optimization problem are set up.

FILE STRUCTURE

| Component Number | Dimension | Symbol | Description |
|---|---|---|---|
| 1 | (K,I,J) | G | People sharing array. +/G is an (I,K) element matrix of ones. G is nonnegative: $g^i_{kj}$ is the fraction of type (i,k) people assigned to job(i,j) |
| 2 | (I,J) | B | The billet requirements. $b_{ij}$ is the number of type(i,j) jobs to be filled. |
| 3 | (I,J) | θ | The range for a unit error in overfilling billets. If $x_{ij} = (1 + 0.01\ _{ij})b_{ij}$ there is a unit error. $\theta_{ij} > 0$. Expressed as %. |
| 4 | (I,K) | φ | The range for a unit error in overfilling personnel inventories $z_{ik} = (1\|0.0\|\phi_{ik})p_{ik}$ a unit error is counted. $\phi_{ik} > 0$. Expressed as %. |
| 5 | (K,I,J) | F | The job sharing array. F is nonnegative and +/[1]F is an (I,J) matrix of ones: $f^i_{kj}$ is the fraction of type(i,j) jobs to be filled by type(i,k) people. |
| 6 | (I,K) | P | The desired manpower inventory: $p_{ik}$ is the number type(i,k) people desired. |

FILE STRUCTURE CONT.

| Component Number | Dimensional | Symbol | Description |
|---|---|---|---|
| 7 | (N,K) | $\alpha$ | The survivor fractions for each of the K classes. N is the maximum LOS. |
| 8 | I | S | The last year in each stage. Increasing positive numbers with $S_I = N$. |
| 9 | (I,K) | W | The element $w_{ik}$ is the number of years a class k input expects to spend in stage i. |
| 10 | K | y | The calculated accessions rate; $y_k$ is the number of accessions in manpower class k. |
| 11 | (I,K) | z | The calculated inventory of each type of person. There are $z_{ik}$ type (i,k) people. |
| 12 | (K,I,J) | A | The calculated allocation: $a^i_{kj}$ is the number of type (i,k) people filling type(i,j) jobs. |
| 13 | (I,J) | X | The calculated billets filled. There are $x_{ij}$ people in job(i,j). |
| 14 | (K,I,J) | T | The target allocation. We want $t^i_{kj}$ type(i,k) people in type(i,j) jobs. If things are consistent +/T equals P and +/[1]T equals B. |
| 15 | (I,J) | $\psi$ | The range for a unit error in under filling billets. If $x_{ij} = (1-\psi_{ij})b_{ij}$ then an error of 1 is counted. $\psi_{ij} \geq 0$. |

| Component Number | Dimensional | Symbol | Description |
|---|---|---|---|
| 16 | (I,K) | $\delta$ | The range for a unit error in underfilling personnel inventories. If $z_{ik} = (1-\delta_{ik})p_{ik}$ then a unit error is counted. $\delta_{ik} \geq 0$. |
| 17 | (K,I,J) | C | The element $c^{i}_{kj}$ is the total cost of having a type (i,k) person in job(i,j) for one year. |

FUNCTIONS

| Function | Uses | Computes | Syntax and Description |
|---|---|---|---|
| BILQUAD | B, θ, φ, F, P, W | Y, X, Z, A | BILQUAD 'FILENAMS'<br>Calculates "optimal" accessions y, personnel inventory (z), and allocation (A), given the data, objective, and the billet share rule (F). |
| BILSIM | F,Y,X | Z,A | BILSIM'FILENAMS'<br>Given Y and X (which presumably satisfy the conservation constraint), calculates allocation (A) and personnel inventory (Z), using the billet sharing rule. |
| PGREC | P, G | B, F, T | PGREC 'FILENAMS'<br>reconciles the file elements B, G, and T with P and G |
| BFREC | B, F | P, G, T | BFREC'FILENAMS'<br>reconciles the file elements P, G, and T with the file elements B and F in the same file. |
| PEOQUAD | G, B, θ, φ, F, P, W | Y, X, Z,A | PEOQUAD 'FILENAMS'<br>calculates optimal accessions (y), personnel inventory (z), billet staffing (x), and allocation (A), given the data, and use of the people share rule. |

FUNCTIONS CONT.

| FUNCTION | USES COMPONENTS | CALCULATES COMPONENT | SYNTAX AND DESCRIPTION |
|---|---|---|---|
| PEOSIM | G, Y, W | Z, X, A | PEOSIM'FILENAMS' calculates inventory z, staffing x and allocation A, where accessions and people share rule are given. |
| WAITS | α, S | W | WAITS 'FILENAMS' Given the survivor fractions (α) and stage definitions (s), calculates w the expected waiting time in each state. |
| PEOPRONT | P, Z | 100×(D-z)÷P | PEOPRONT 'FILENAMS' calculates the percentage error in inventory. Compare with ϕ. |
| BILPRONT | B, X | 100×(B-x)÷B | BILPRONT 'FILENAMS' calculates the percentage error in meeting billet requirements. Compare with θ. |
| ROUND | | | ROUND XXX takes any array and rounds elements to integers. |

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