NPS-57Hw73041A

NAVAL POSTGRADUATE SCHOOL
Monterey, California

NUMERICAL EXPERIMENTS WITH THE
TWO- AND THREE-DIMENSIONAL UNSTEADY
NAVIER-STOKES EQUATIONS

by

Gustave Hokenson

April 1973

NAVAL POSTGRADUATE SCHOOL

Monterey, California

Rear Admiral Mason Freeman, USN
Superintendent

Milton U. Clauser
Provost

ABSTRACT:

The two- and three-dimensional unsteady Navier-Stokes equations are solved numerically for the flow field about an impulsively started flat plate. In attempting to obtain an exact time dependent solution, several significant results were observed. First, with regard to the formulation of the differential equations themselves, it appears that Poisson's equation for the pressure field is a fundamental equation in as much as it allows us to solve for pressure most exactly at any given time. Secondly, the difference equations must be carefully and consistently formulated. In this research, a non-uniform lateral grid, a unique interpretation of the continuity equation, and "leap-frog" integration in time proved to be valuable techniques in obtaining an exact solution.

LIST OF FIGURES

Figure No.		Page
1.	Calculation Region for Flow Over a Finite Flat Plate	21
2.	Time-Dependent Solution for Flow Over a Finite Flat Plate . . .	22

TABLE OF CONTENTS

Section	Page
NOMENCLATURE	iv
INTRODUCTION	1
DIFFERENTIAL EQUATION FORMULATION	2
DIFFERENTIAL EQUATION APPLICATION	8
FINITE DIFFERENCE EQUATION FORMULATION	10
FINITE DIFFERENCE EQUATION APPLICATION	11
COMPUTER PROGRAM	14
RESULTS AND DISCUSSION	16
REFERENCES	19
APPENDICES	23
INITIAL DISTRIBUTION LIST	74

NOMENCLATURE

Δt = Finite difference time step

Δx = Finite difference grid spacing in x

Δy = Finite difference grid spacing in y

Δz = Finite difference grid spacing in z

η = Transformed lateral coordinate = e^{-y}

f = Arbitrary dummy dependent variable

L = Characteristic length

μ = Coefficient of molecular viscosity

ν = Kinematic viscosity

p = Pressure

ρ = Density

Re = Reynolds number = $\rho VL/\mu$

t = Time

u = Velocity in x-direction

v = Velocity in y-direction

V = Characteristic velocity

w = Velocity in z-direction

x = Axial coordinate

X = Body force vector

y = Lateral coordinate

z = Cross-streamwise coordinate

ω = Successive over-relaxation parameter

Subscripts

i,j = Indicates components of a vector

NUMERICAL EXPERIMENTS WITH THE TWO-AND THREE-DIMENSIONAL UNSTEADY NAVIER STOKES EQUATIONS

Gustave Hokenson

INTRODUCTION

Since Crocco (Ref. 1) suggested seeking asymptotic solutions to approximate forms of the parabolic unsteady Navier-Stokes equations, in lieu of attacking the elliptic steady equations, many researchers (Refs. 2, 3, and 4) have successfully applied the technique to solve problems ranging from real gas nozzle flow to the hypersonic blunt body problem. The attempts generally are aimed at retrieving only the steady solution and this limited goal allows for extensive approximation within the differential and difference equations as long as the steady flow is exactly represented by the equations. This approximation of the unsteady equations facilitates the numerical solution while supposedly not interfering with the uniqueness of the asymptotic solution. This technique, however, obviously negates the intermediate solutions from being actual representations of the time dependent flow field.

The essence of the numerical technique is to specify an "arbitrary" initial flow field, calculate the appropriate spacial derivatives in the Navier-Stokes equations, equate their sum to the time derivative of the respective dependent variable and step forward in time. Results of this investigation revealed that the combination of "arbitrary" initial conditions with the equation approximation previously mentioned creates some numerical problems which must be cloaked in various numerical subterfuges such as filters and arbitrary intermediate specifications of the velocity field. This is not meant to slight the technique for it does provide an easy method of obtaining apparently unique solutions for the steady flowfield. However it was discovered in this research that in order to guarantee an accurate time dependent solution, instantaneously consistent with the equations and boundary conditions, much more care is needed both in the formulation and the numerical implementation of the differential and difference equations.

The purpose of this investigation was to obtain the exact time dependent solution for the flow about a finite, infinitely thin flat plate impulsively started in its own plane into a uniform incompressible flow. Both the two- and three-dimensional Navier-Stokes equations were applied to the problem and the difficulties characteristic of both approaches are presented. A variety of techniques, all consistent with the equations and the boundary conditions, were employed to obtain exact time dependent solutions without recourse to non-rigorous numerical manipulations.

DIFFERENTIAL EQUATION FORMULATION

The Navier-Stokes equation for the flow of an incompressible, Newtonian fluid can be written in Cartesian tensor notation:

$$\frac{\partial u_i}{\partial x_i} = 0 \tag{1}$$

$$\rho \frac{Du_i}{Dt} = \rho X_i - \frac{\partial P}{\partial x_i} + \mu \nabla^2 u_i \tag{2}$$

where

$$\frac{D}{Dt} \equiv \frac{\partial}{\partial t} + u_j \frac{\partial}{\partial x_j} \tag{3}$$

With the aid of Equation 1, Equation 2 may be written in conservative form which is most amenable to exact numerical formulation (Ref. 5). In conservative form, Equation 2 becomes

$$\frac{\partial u_i}{\partial t} + \frac{\partial}{\partial x_j}(u_i u_j) = X_i - \frac{1}{\rho}\frac{\partial P}{\partial x_i} + \nu \nabla^2 u_i \tag{4}$$

Equations 4 and 1 form a closed system for the solution of the dependent variables u_i and p since, with the appropriate boundary and initial conditions, there are as many equations as there are unknowns. These equations have been applied directly (Ref. 6) to obtain steady solutions to the Navier-Stokes equations with one of the momentum equations serving as an equation for pressure. Early in the course of the investigation a variety of problems were discovered with this approach simply in retrieving steady solutions and its utilization to obtain the desired time dependent solutions proved completely unfeasible. In lieu of solving a momentum equation for pressure, Equation 4 may be differentiated and summed to obtain a time independent Poisson's equation for pressure in terms of the instantaneous flow field. If we differentiate Equation 4 with respect to x_i and apply the Einstein summing convention we get

$$\frac{\partial}{\partial x_i}\left[\frac{\partial u_i}{\partial t}\right] + \frac{\partial}{\partial x_i}\left[\frac{\partial}{\partial x_j}(u_i u_j)\right] = \frac{\partial X_i}{\partial x_i} - \frac{1}{\rho}\frac{\partial^2 P}{\partial x_i \partial x_i} + \frac{\partial}{\partial x_i}\left[\nu \nabla^2 u_i\right] \tag{5}$$

Assuming the functions to be continuous, we can interchange the order of differentiation in the temporal convective and diffusive terms and obtain

$$\frac{\partial}{\partial t}\left[\frac{\partial u_i}{\partial x_i}\right] + \frac{\partial}{\partial x_i}\left[\frac{\partial}{\partial x_j}(u_i u_j)\right] = \frac{\partial X_i}{\partial x_i} - \frac{1}{\rho}\frac{\partial^2 P}{\partial x_i \partial x_i} + \nu \nabla^2 \left[\frac{\partial u_i}{\partial x_i}\right] \tag{6}$$

for gravitational body forces we can write

$$\frac{\partial X_i}{\partial x_i} = 0 \tag{7}$$

and for incompressible flow, Equation 1 states

$$\frac{\partial u_i}{\partial x_i} = 0 \tag{8}$$

Applying these conditions to Equation 6, we obtain the desired Poisson's equation for pressure

$$\frac{\partial^2 P}{\partial x_i \partial x_i} = \nabla^2 P = - \rho \frac{\partial^2 u_i u_j}{\partial x_i \partial x_j} \tag{9}$$

With Equations 1, 4, and 9 we have a system of equations greater in number than the unknowns and we must choose the most fundamental set to solve. The specification of the apparent optimum choice was one of the goals of this research and is discussed in the next section.

For generality in problem solving, we non-dimensionalize Equations 1, 4, and 9 and for simplicity allow the new variables to have the same symbol as the terms in the original equations. It is understood from this point on that only the non-dimensional quantities will be discussed. The variable transformation is defined by

$$u_i = \frac{u_i}{V} \qquad x_j = \frac{x_j}{L} \qquad P = \frac{P}{\rho V^2} \qquad t = \frac{t}{(L/V)} \tag{10}$$

With this transformation the governing equations can be written

$$\frac{\partial u_i}{\partial x_i} = 0 \qquad (11)$$

$$\frac{\partial u_i}{\partial t} + \frac{\partial}{\partial x_j}(u_i u_j) = X_i - \frac{\partial P}{\partial x_i} + \frac{1}{Re}\nabla^2 u_i \qquad (12)$$

$$\nabla^2 P = -\frac{\partial^2 u_i u_j}{\partial x_i \partial x_j} \qquad (13)$$

where

$$Re \equiv \frac{\rho V L}{\mu}$$

For future use these equations are expanded for the three-dimensional case in rectangular Cartesian coordinates as follows

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \qquad (14)$$

$$\frac{\partial u}{\partial t} + \frac{\partial}{\partial x}(u^2) + \frac{\partial}{\partial y}(uv) + \frac{\partial}{\partial z}(uw) = -\frac{\partial P}{\partial x} + \frac{1}{Re}\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right] \qquad (15)$$

$$\frac{\partial v}{\partial t} + \frac{\partial}{\partial x}(uv) + \frac{\partial}{\partial y}(v^2) + \frac{\partial}{\partial z}(vw) = -\frac{\partial P}{\partial y} + \frac{1}{Re}\left[\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right] \qquad (16)$$

$$\frac{\partial w}{\partial t} + \frac{\partial}{\partial x}(uw) + \frac{\partial}{\partial y}(vw) + \frac{\partial}{\partial z}(w^2) = -\frac{\partial P}{\partial z} + \frac{1}{Re}\left[\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right] \qquad (17)$$

$$\frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2} + \frac{\partial^2 P}{\partial z^2} = -\left[\frac{\partial^2 u^2}{\partial x^2} + \frac{\partial^2 v^2}{\partial y^2} + \frac{\partial^2 w^2}{\partial z^2}\right] - 2\left[\frac{\partial^2 uv}{\partial x \partial y} + \frac{\partial^2 vw}{\partial y \partial z} + \frac{\partial^2 uw}{\partial x \partial z}\right] \qquad (18)$$

For the two-dimensional case we can simplify these equations by dropping all terms involving w and z and their derivatives. In this case, the equations become

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{19}$$

$$\frac{\partial u}{\partial t} + \frac{\partial}{\partial x}(u^2) + \frac{\partial}{\partial y}(uv) = -\frac{\partial P}{\partial x} + \frac{1}{Re}\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right] \tag{20}$$

$$\frac{\partial v}{\partial t} + \frac{\partial}{\partial x}(uv) + \frac{\partial}{\partial y}(v^2) = -\frac{\partial P}{\partial y} + \frac{1}{Re}\left[\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right] \tag{21}$$

$$\frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2} = -\left[\frac{\partial^2 u^2}{\partial x^2} + 2\frac{\partial^2 uv}{\partial x \partial y} + \frac{\partial^2 v^2}{\partial y^2}\right] \tag{22}$$

Just as the conservative form of the convective derivative is that which is most appropriate for numerical integration, the diffusive terms also can be reformulated to most exactly satisfy the equations when finite differenced (Ref. 5). The reformulation of the diffusive terms is accomplished by the following substitution in the three-dimensional equations

$$\frac{\partial^2 u}{\partial x^2} = -\frac{\partial^2 v}{\partial x \partial y} - \frac{\partial^2 w}{\partial x \partial z} \tag{23}$$

$$\frac{\partial^2 v}{\partial y^2} = -\frac{\partial^2 u}{\partial x \partial y} - \frac{\partial^2 w}{\partial y \partial z} \tag{24}$$

$$\frac{\partial^2 w}{\partial z^2} = -\frac{\partial^2 u}{\partial x \partial z} - \frac{\partial^2 v}{\partial y \partial z} \tag{25}$$

and for the two-dimensional system

$$\frac{\partial^2 u}{\partial x^2} = - \frac{\partial^2 v}{\partial x \partial y} \qquad (26)$$

$$\frac{\partial^2 v}{\partial y^2} = - \frac{\partial^2 u}{\partial x \partial y} \qquad (27)$$

where the appropriate continuity equation has been differentiated to obtain Equations 23-27. After performing these substitutions the 3-d equations become

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \qquad (28)$$

$$\frac{\partial u}{\partial t} + \frac{\partial}{\partial x}(u^2) + \frac{\partial}{\partial y}(uv) + \frac{\partial}{\partial z}(uw) = - \frac{\partial P}{\partial x} + \frac{1}{Re}\left[\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} - \frac{\partial^2 v}{\partial x \partial y} - \frac{\partial^2 w}{\partial x \partial z}\right] \qquad (29)$$

$$\frac{\partial v}{\partial t} + \frac{\partial}{\partial x}(uv) + \frac{\partial}{\partial y}(v^2) + \frac{\partial}{\partial z}(vw) = - \frac{\partial P}{\partial y} + \frac{1}{Re}\left[\frac{\partial^2 v}{\partial x^2} - \frac{\partial^2 u}{\partial x \partial y} - \frac{\partial^2 w}{\partial y \partial z} + \frac{\partial^2 v}{\partial z^2}\right] \qquad (30)$$

$$\frac{\partial w}{\partial t} + \frac{\partial}{\partial x}(uw) + \frac{\partial}{\partial y}(vw) + \frac{\partial}{\partial z}(w^2) = - \frac{\partial P}{\partial z} + \frac{1}{Re}\left[\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} - \frac{\partial^2 u}{\partial x \partial z} - \frac{\partial^2 v}{\partial y \partial z}\right] \qquad (31)$$

$$\frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2} + \frac{\partial^2 P}{\partial z^2} = - \left[\frac{\partial^2 u^2}{\partial x^2} + \frac{\partial^2 v^2}{\partial y^2} + \frac{\partial^2 w^2}{\partial z^2}\right] - 2\left[\frac{\partial^2 uv}{\partial x \partial y} + \frac{\partial^2 vw}{\partial y \partial z} + \frac{\partial^2 uw}{\partial x \partial z}\right] \qquad (32)$$

For the two-dimensional case all w and z terms and their derivatives may be dropped and we retain the following equations.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (33)$$

$$\frac{\partial u}{\partial t} + \frac{\partial}{\partial x}(u^2) + \frac{\partial}{\partial y}(uv) = -\frac{\partial P}{\partial x} + \frac{1}{Re}\left[\frac{\partial^2 u}{\partial y^2} - \frac{\partial^2 v}{\partial x \partial y}\right] \tag{34}$$

$$\frac{\partial v}{\partial t} + \frac{\partial}{\partial x}(uv) + \frac{\partial}{\partial y}(v^2) = -\frac{\partial P}{\partial y} + \frac{1}{Re}\left[\frac{\partial^2 v}{\partial x^2} - \frac{\partial^2 u}{\partial x \partial y}\right] \tag{35}$$

$$\frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2} = -\left[\frac{\partial^2 u^2}{\partial x^2} + 2\,\frac{\partial^2 uv}{\partial x \partial y} + \frac{\partial^2 v^2}{\partial y^2}\right] \tag{36}$$

DIFFERENTIAL EQUATION APPLICATION

As was mentioned earlier, there is a choice with regard to the differential equations which can be used to solve the problem. As a general rule, it was found necessary for both the continuity equation and Poisson's equation for pressure to be satisfied at each time step, including $t = 0$, to guarantee an accurate time dependent solution. Based on this principle each streamwise velocity component was calculated from its respective momentum equation, the cross-streamwise velocity from the continuing equation and pressure from Poisson's equation. If there is no distinct cross-streamwise direction, for example in a wake or cavity recirculation region, a momentum equation is solved for each velocity component, Poisson's equation is solved for the pressure, and continuity is checked and guaranteed by the use of correction factors (Ref. 7) in Poisson's equation in the terms

$$\frac{\partial}{\partial t}\left[\frac{\partial u_i}{\partial x_i}\right], \quad \frac{1}{Re}\nabla^2\left[\frac{\partial u_i}{\partial x_i}\right]$$

which cannot be arbitrarily set equal to zero if continuity is not being used to solve for one of the dependent variables.

For the particular problem we have attacked the differential equation application is as follows

$$
2\text{-d} \quad \begin{array}{l} u \text{ from } u \text{ momentum} \\ v \text{ from continuity} \\ P \text{ from Poisson's} \end{array} \tag{37}
$$

$$
3\text{-d} \quad \begin{array}{l} u \text{ from } u \text{ momentum} \\ w \text{ from } w \text{ momentum} \\ v \text{ from continuity} \\ P \text{ from Poisson's} \end{array} \tag{38}
$$

In order to obtain exact time dependent solutions, it was found necessary to specify physically meaningful and compatible initial conditions throughout the flow field, not "arbitrary initial conditions" as is so often quoted in the literature. The compatible and physically meaningful initial conditions for this investigation were specified as follows

$$
2\text{-d} \quad \begin{array}{l} u = 0 \text{ on the surface of the plate} \\ u = 1.0 \text{ everywhere except on the plate} \\ v = \text{calculated from continuity} \\ P = \text{calculated from Poisson's} \end{array} \tag{39}
$$

$$
3\text{-d} \quad \begin{array}{l} u = 0 \text{ on the plate} \\ u = 1.0 \text{ everywhere except on the plate} \\ v = \text{calculated from 2-d continuity} \\ w = \text{calculated from 3-d continuity} \\ P = \text{calculated from Poisson's} \end{array} \tag{40}
$$

There may be some question about the approach for the 3-d problem, however no inconsistent results were found using this approach. It appears that the calculation of w from 3-d continuity after calculating v from 2-d continuity would result in a null matrix for w. That this is not the case is observable from the way in which the differential continuity equations were implemented, using locally spacial averaged values of u, v, and w. This procedure is outlined in the next section and the appropriate difference equations are listed in the Appendices.

FINITE DIFFERENCE EQUATION FORMULATION

Based on the approach outlined in the previous section, the spacial partial derivatives of the dependent variables in each of the differential equations were finite differenced using three point central difference formulas. Special care was taken to avoid known inaccurate difference models (e.g. the Du-Fort-Frankel representation of diffusive terms) when we are seeking exact time dependent solutions. Substitution of the appropriate difference expressions into the momentum equations allows for the calculation of the instantaneous temporal derivatives of the dependent variables u and w based on knowledge of the current dependent variables. The differencing procedure for the momentum equations is outlined in Appendix A, followed by a listing of the respective calculations of the temporal derivatives as they appear in the computer program. In both the two and three-dimensional situations Poisson's equation was used for the calculation of pressure. Numerical experiments were conducted using the y-momentum equation for the solution of P, however, since we dealt with the exact equations the time derivative of v must be formed and added to the appropriate spacial derivatives in order to calculate the partial derivative of P with respect to y. With the pressure on the freestream boundary known, it was in theory possible to integrate the equation inward from the outer boundary. However, numerical instabilities arose from the inability to calculate physically compatible initial conditions since at this time v is known only at one time. In addition, this method of calculating P has inherent accuracy limitations and this study revealed the necessity of calculating P to a very high degree of accuracy, including the initial conditions. This discovery served as the impetus for using Poisson's equation which was solved using classical successive overrelaxation techniques with all derivatives in the equation being represented by three point central difference models. The finite differencing of Poisson's equation is presented in detail in Appendices B and C along with a listing of the encoded form as it appears in the full computer program.

The most crucial finite differencing formulation which was encountered was that of representing the continuity equation by a formula which is both accurate and physically meaningful. The method which was settled upon is equivalent to a third order Runge-Kutta approach and the details of its formulation along with the computer listing is presented in Appendices B and C.

In each of the equations which were finite differenced the cross-streamwise partial derivatives were formulated with the option of using a non-uniform grid spacing. This allows for the most efficient use of computer memory and concentrates the calculation grid in the regions where the dependent variables vary most rapidly. The central difference formulas for both the first and second order partial derivatives with a non-uniform grid are developed and presented in Appendix A.

FINITE DIFFERENCE EQUATION APPLICATION

The finite difference equations discussed in the previous section were applied to the solution of the incompressible flow about an impulsively started flat plate in the manner outlined in Equations 37, 38, 39, and 40. The physically compatible initial condition field was found and the appropriate equations were used to continue the velocity field downstream in time with Poisson's equation being solved for the new pressure field. When explicitly updating the velocity field in time, a stable solution is obtainable for a limited range of time step increments. Karplus (Ref. 8) obtained numerically stable solutions with a similar system of equations with a restriction for the time step which can be written in non-dimensional variables

$$\Delta t \le \frac{1}{\frac{u}{\Delta x} + \frac{v}{\Delta y} + \frac{w}{\Delta z} + \frac{2}{Re}\left(\frac{1}{\Delta x^2} + \frac{1}{\Delta y^2} + \frac{1}{\Delta z^2}\right)} \qquad (41)$$

Equation 41 proved to be a completely satisfactory stability criterion in this work.

In an effort to obtain the most exact solution (to second order in Δt) the mechanism of using effective central differencing in time was introduced and is included as an option in the computer program. In general, an independent variable f can be calculated at a future time from a second order Taylor series expansion in time as follows

$$f(x,y,z,t+\Delta t) = f(x,y,z,t) + \frac{\partial f}{\partial t}(x,y,z,t)(\Delta t) + \frac{\partial^2 f}{\partial t^2}(x,y,z,t)\frac{(\Delta t)^2}{2}$$

Since $f(x,y,z,t)$ is known and $\frac{\partial f}{\partial t}$ can be calculated from the differential equations, it only remains to calculate $\frac{\partial^2 f}{\partial t^2}$. For the Navier-Stokes equations $\frac{\partial f}{\partial t}$ is an explicit function of the dependent variables and their spacial derivatives and it therefore follows that $\frac{\partial^2 f}{\partial t^2}$ may be calculated by differentiating the governing equations with respect to time and interchanging spacial and temporal derivatives. However the implementation of these equations is lengthy and the need to solve a second Poisson's equation (for $\frac{\partial P}{\partial t}$) increased the computing time intolerably. Because of these factors, it was decided to attempt to obtain the same second order accuracy using the basic difference equations appropriate to a first order approach. This was accomplished by calculating the dependent variables at time $t + \Delta t/2$ with forward finite differencing and using these values to calculate the spacial derivatives in the Navier-Stokes equations at that time. The dependent variables are then stepped forward from time t to time $t + \Delta t$ based on the temporal derivatives evaluated at $t + \Delta t/2$. The implementation of this approach is most clearly understood from the flow chart of the computer program presented in Appendices D, E. The inherent susceptibility to numerical instabilities which this "leap-frog" technique exhibits should be damped by the strongly dissipative terms in the equations (Refs. 9, 10, and 11).

Finally, with respect to obtaining the solution throughout the flow field, some technique must be specified to meet the boundary conditions. Apart from setting $u = v = w = 0$ on the surface of the plate, some procedure is necessary for setting u, v, w, and p on the boundaries of the calculation region. Two methods are available and have been proven to be essentially equivalent. First, the values of the dependent variables which have been calculated in the interior of the region may be extrapolated to the boundaries using the arguments of symmetry and uniformity across the boundaries to generate the appropriate extrapolating function. Second, the equations themselves may be applied to the boundaries if special considerations are taken which give expression to those values needed outside of the calculation region, based again on symmetry and uniformity arguments. The latter method was chosen as the preferable from the standpoint of consistency of accuracy and error generation throughout the flow field.

COMPUTER PROGRAM

The computer programs which encoded the solution of the finite difference equations in Appendices B and C are listed in Appendices D and E, along with the flow charts appropriate to each. With regard to programming the system of finite difference equations and several mundane considerations such as the memory capacity of the computer and the accuracy required of the system vs. what the equations are demanding of it are deserving of some mention. First, with regard to the 3-d program (with the inherent 3-d arrays of dependent variables), is the question of computer core storage capability. The minimum number of 3-d arrays which must be dimensioned is equal to the number of dependent variables. Because the exact difference equations are being used, the calculations of new variables must be made in conjunction with the storage of old ones and it is apparently not possible to utilize only the minimum number of arrays. However in this program the temporal derivatives are written on discs immediately after being calculated, each as a dummy scalar. When the calculation of the dependent variables at a future time (either $t + \Delta t$ or $t + \Delta t/2$) is being initiated, the appropriate discs are rewound and the scalar strings are read in sequence to provide the temporal derivative of the proper dependent variable at that point in space.

In addition to the artifice of using virtual memory in the form of simple disc writing, an additional tack was tried. This was to map the infinite space in y into a finite space with the transformation

$$\eta = e^{-y}$$

if this is done, the appropriate partial derivatives become

$$\frac{\partial}{\partial y} = - \eta \frac{\partial}{\partial \eta}$$

$$\frac{\partial^2}{\partial y^2} = \eta^2 \frac{\eta^2}{\partial \eta^2} + \eta \frac{\partial}{\partial \eta}$$

and the lateral distances can be stretched such that there are fewer points laterally but there is denser data in the region of steepest gradients. This transformation also allows the external boundary conditions to be set somewhat more exactly and, in conjunction with the non-uniform grid size as established for y, should give the optimum use of total lateral space per point of computation. Because the results were adequate at the Reynolds numbers which we were studying, the transformed coordinate was considered an unnecessary complication for this problem. It is felt however that many of the computational difficulties associated with the exact numerical solution of lower Reynolds number problems may be alleviated by the use of this artifice.

With regard to the questions about the accuracy of the solutions, two specific areas of investigation were dealt with in this research. First was the utility of computing in double precision (real*8) and second was the use of higher than second order approximations to the spacial partial derivatives. The computations were all performed with a variety of grid sizes and accurate solutions were obtained with spacial and temporal grid sizes large enough to allow significant differences to be calculated in single precision (real*4). If the non-uniform lateral grid is set up with extremely small spacing near the plate, the ensuing small differences may force the use of double precision to retain the most accurate solution. In general it is felt that this is the only situation which should force one to use double precision. Experimentation with the use of higher order approximations to the spacial partial derivatives was carried out with the following two representations of the spacial derivatives

$$\left.\frac{\partial f}{\partial y}\right|_j = \frac{-f_{j+2} + 8 f_{j+1} - 8 f_{j-1} + f_{j-2}}{12 \Delta y}$$

$$\left.\frac{\partial^2 f}{\partial y^2}\right|_j = \frac{-f_{j+2} + 16 f_{j+1} - 30 f_j + 16 f_{j-1} - f_{j-2}}{12 \Delta y^2}$$

The results using these models were identical to those using the second order approximations with adequate grid size and the additional complication of increasing the thickness of the boundary on which the boundary conditions are to be set could be avoided by using the standard formulas. Therefore, neither the use of double precision nor the implementation of "more accurate" finite difference models was found to be of general use.

RESULTS AND DISCUSSION

The exact unsteady two and three-dimensional Navier-Stokes equations have been applied to the solution of the incompressible flow about an impulsively started flat plate. Some typical results are presented in Figure 2 which gives the transient development of both the downstream and upstream flow on the centerline and in the plane of the plate. For our purpose, it is not particularly startling that we can solve this specific problem but, more importantly, that we can numerically solve for the time dependent flow field using the two and three-dimensional Navier-Stokes equations. In so doing a set of non-rigorous rules has been obtained which can be used as guidelines in the application of the equations toward solution of more complex problems.

The Navier-Stokes equations written in strict conservative form and physical variables can be integrated to obtain accurate time dependent solutions providing care is taken in the formulation of the problem. At all times during the calculation, the continuity equation and Poisson's equation for pressure must be satisfied to ensure a valid solution while each streamwise velocity component must satisfy the appropriate momentum equation. In addition, the initial conditions must satisfy the continuity equation and Poisson's equation for pressure in order to allow a "natural" flow development. The boundary conditions can be satisfied either through application of the equations to the boundary with the use of symmetry and uniformity arguments or through the use of polynomial extrapolation from the interior calculation region.

In essence, Poisson's equation for pressure has been introduced as a fundamental equation in the numerical solution of the full equations because of the need for generating a compatible initial pressure field with a time independent equation and also because of the need for calculating the pressure field to an extremely accurate degree. This accurate pressure field calculation and the specification of physically compatible initial conditions eliminated the need for all numerical artifices such as filters to cover such purely numerical phenomena as velocity overshoot.

Second in importance to the adequate solution of Poisson's equation is the satisfaction of continuity equation at each point in time and space. Apparently, only one finite difference formulation of this equation, that presented in the Appendices, is entirely adequate in obtaining exact time dependent solutions of the problem.

The governing equations were finite differenced using a central difference approximation for all spacial derivatives. The DuFort-Frankel formulation of the diffusive terms was avoided in order to guarantee the most exact representation of the spacial derivatives at each instant in time. In order to obtain apparent central differencing in time the dependent variables at time $t + \Delta t$ were calculated using the spacial derivatives calculated at $t + \Delta t/2$ where the $t + \Delta/2$ values were obtained by pure forward differencing. To obtain stable magnitudes for these time steps, the approximate equation given by Karplus was used as a guide once accurate spacial grid sizes were chosen.

Beyond the formulation of the numerical problem, the operational problem of obtaining sufficient computer memory space was the most difficult to overcome. For both the two and three-dimensional problems, the use of disc storage was applied to help alleviate the problem. As is clear from the flow diagram of the program, the intermediate calculations were stored on discs as dummy scalars and re-read as matrices when the dependent variables were being updated. This technique allows the use of a minimum number of dimensional arrays. In addition, the use of a non-uniform grid and mapping the calculation region into a finite space were applied to help alleviate the space problem and optimize the number of calculation points at those places where they are needed.

Several further investigations are now underway to determine the effect of the initial conditions and the equation approximation on the uniqueness of the asympototic solution. In addition the two and three-dimensional instability problem is being studied from the standpoint of comparing the effect of two and three-dimensional disturbances and finite vs. infinitesimal disturbances on the classical instability maps. Finally the full compressible problem is being formulated in a manner identical to that presented here.

REFERENCES

1. Crocco, L., "A Suggestion for the Numerical Solution of the Steady Navier-Stokes Equations," AIAA J., Vol. 3, No. 10, pp. 1824-1832, 1965.
2. Benison, G. I. and Rubin, E. L., "A Time-Dependent Analysis for Quasi-One-Dimensional, Viscous, Heat Conducting Compressible Laval Nozzle Flows," PIBAL Report No. 70-12, March 1970.
3. Anderson, J. D., Jr., "A Time-Dependent Analysis for Quasi-One-Dimensional Nozzle Flows with Vibrational and Chemical Non-equilibrium," NDLTR 69-52, U.S. Naval Ordnance Laboratory, White Oak, Maryland.
4. Anderson, J. D., Jr., Albacete, L. M., and Winkelmann, A. E., "On Hypersonic Blunt Body Flow Fields Obtained with a Time-Dependent Technique," NOL-TR 68-129, U.S. Naval Ordnance Laboratory, White Oak, Maryland.
5. Welch, J. E., Harlow, F. H., Shannon, J. P., Daly, J. B., The MAC Method--A Computing Technique for Solving Viscous, Incompressible, Transient Fluid-Flow Problems Involving Free Surfaces. Rep. LA-3425, Los Alamos Scientific Laboratory.
6. Callens, E. E., Jr., "A Time-Dependent Approach to Fluid Mechanical Phenomenology," AIAA Paper No. 70-46.
7. Hirt, C. W. and Harlow, F. H., "A General Corrective Procedure for the Numerical Solution of Initial-Value Problems," J. of Computational Physics, Vol. 2, No. 2, pp. 114-119, 1967.
8. Karplus, W. J., "An Electric Circuit Theory Approach to Finite Difference Stability," Transactions of AIEE, Vol. 77, Part 1, May 1958, pp. 210-213.
9. Richtmyer, R. D. and Morton, K. W., Difference Methods for Initial Value Problems, Interscience, N. Y., 1967.

10. Lax, P. D., "Weak Solutions of Non-Linear Hyperbolic Equations and Their Numerical Computation," Comm. Pure and Applied Mathematics, Vol. 7, pp. 159-193, 1954.

11. Lax, P. D. and Wendroff, B., "Systems of Conservation Laws," Comm. Pure and Applied Mathematics, Vol. 13, pp. 217-237, 1960.

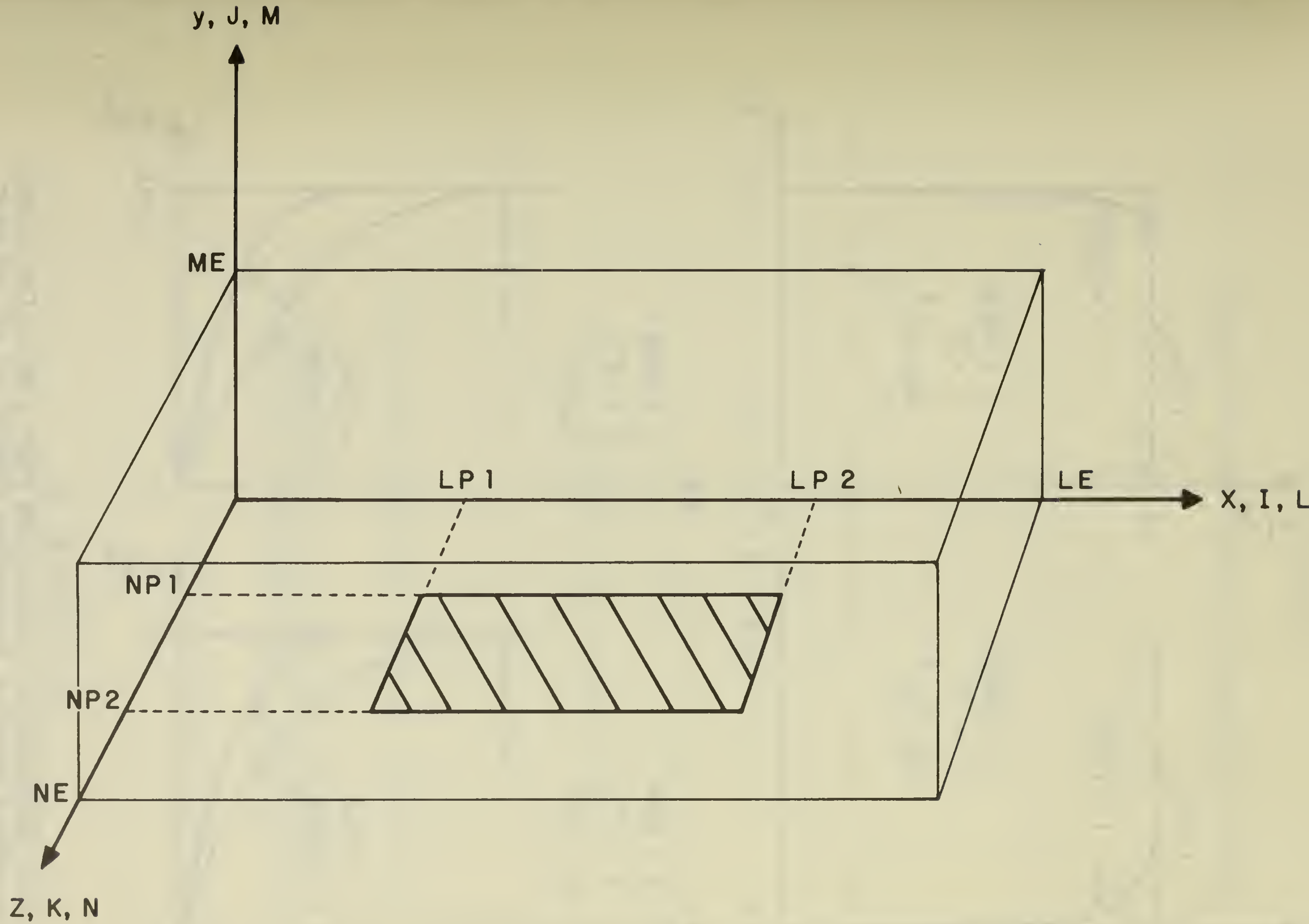


FIG. 1

CALCULATION REGION FOR FLOW OVER A FINITE FLAT PLATE.

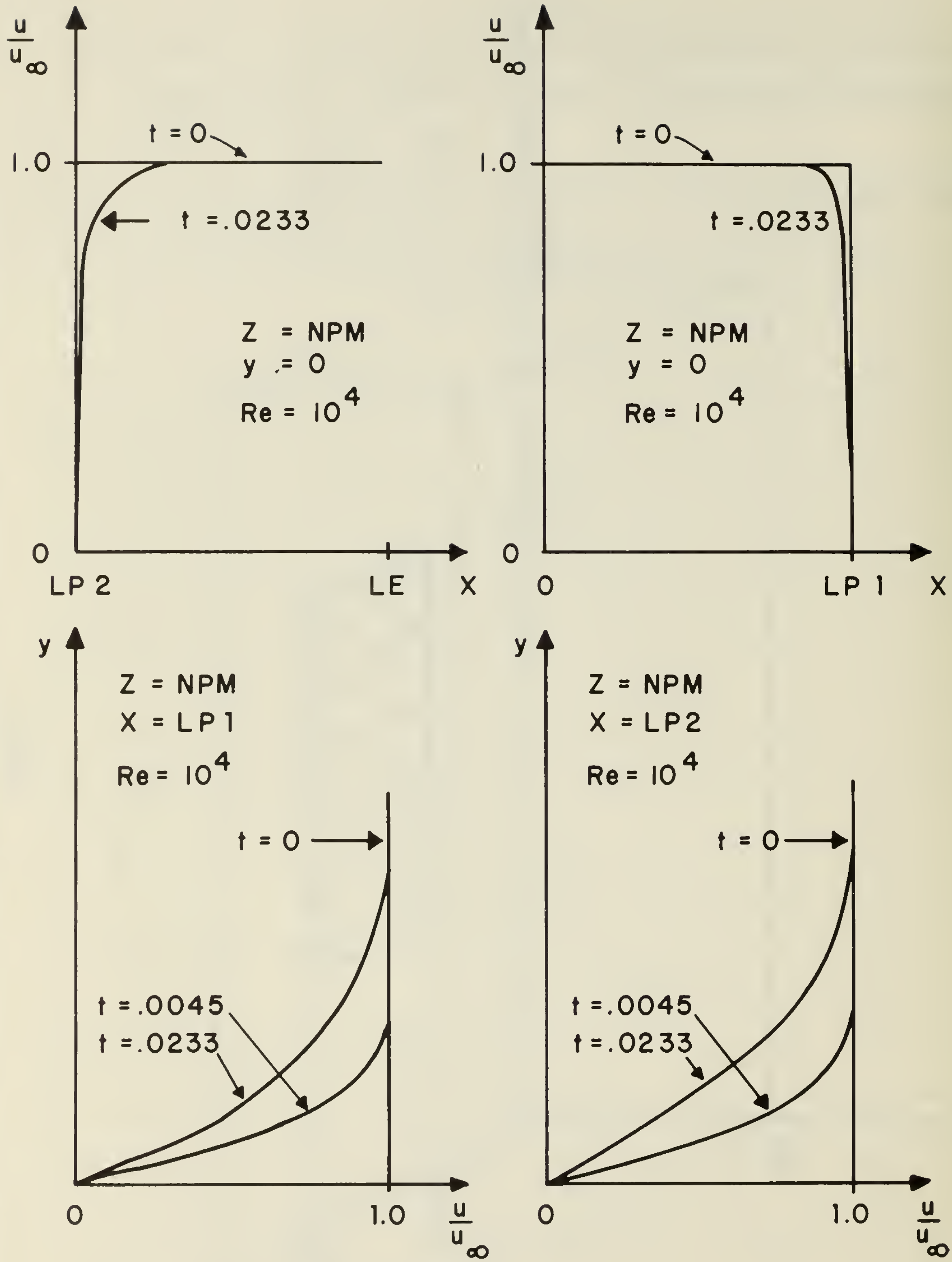
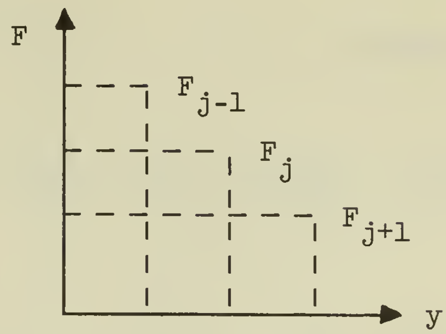


FIG. 2

TIME DEPENDENT SOLUTION FOR THE FLOW OVER AN IMPULSIVELY STARTED FINITE FLAT PLATE.

APPENDIX A

Finite Difference Formulas for Non-Uniform Spacing



We let $F = Ay^2 + By + C$ and expand around y_j to find A, B, and C.

$$F_{j+1} = A\ \mathrm{Dely}^2\ (J) + B\ \mathrm{Dely}\ (J) + C$$

$$F_j = C$$

$$F_{j-1} = A\ \mathrm{Dely}^2\ (J-1) + B\ \mathrm{Dely}\ (J-1) + C$$

Solving for A and B we get:

$$A = \frac{1}{\mathrm{Dely}\ (J) + \mathrm{Dely}\ (J-1)} \left[\frac{F_{j+1} - F_j}{\mathrm{Dely}\ (J)} + \frac{F_{j-1} - F_j}{\mathrm{Dely}\ (J-1)} \right]$$

$$B = \frac{\mathrm{Dely}\ (J)\ \mathrm{Dely}\ (J-1)}{\mathrm{Dely}\ (J) + \mathrm{Dely}\ (J-1)} \left[\frac{F_{j+1} - F_j}{\mathrm{Dely}^2\ (J)} - \frac{F_{j-1} - F_j}{\mathrm{Dely}^2\ (J-1)} \right]$$

Now that A, B, and C is known, we can differentiate F at $y = y_j$ and it is clear that

$$\frac{\partial F}{\partial y} = B \quad , \quad \frac{\partial^2 F}{\partial y^2} = 2A$$

APPENDIX B

Finite Difference Formulation of 3-d Navier-Stokes Equations

I. u Momentum Equation

$$\frac{\partial u}{\partial t} = -\left[\underset{A}{\frac{\partial u^2}{\partial x}} + \underset{B}{\frac{\partial uv}{\partial y}} + \underset{C}{\frac{\partial uw}{\partial z}} + \underset{D}{\frac{\partial P}{\partial x}} + \frac{1}{Re}\left\{\underset{E}{\frac{\partial}{\partial x}\left[\frac{\partial v}{\partial y}\right]} + \underset{F}{\frac{\partial^2 w}{\partial x \partial z}} - \underset{G}{\frac{\partial^2 u}{\partial y^2}} - \underset{H}{\frac{\partial^2 u}{\partial z^2}}\right\}\right]$$

```
      A=(U(I+1,J,K)**2-U(I-1,J,K)**2)/(2.0E0*DELX)
      B1=U(I,J+1,K)*V(I,J+1,K)/(DELY(J)**2)
      B2=U(I,J,K)*V(I,J,K)*(1.0E0/DELY(J-1)**2-1.0E0/DELY(J)**2)
      B3=U(I,J-1,K)*V(I,J-1,K)/DELY(J-1)**2
      B=DELY(J)*DELY(J-1)*(B1+B2-B3)/(DELY(J)+DELY(J-1))
      C=(U(I,J,K+1)*W(I,J,K+1)-U(I,J,K-1)*W(I,J,K-1))/(2.0E0*DELZ)
      D=(PHI(I+1,J,K)-PHI(I-1,J,K))/(2.0E0*DELX)
      E2A=V(I+1,J+1,K)/DELY(J)**2
      E2B=V(I+1,J,K)*(1.0E0/DELY(J-1)**2-1.0E0/DELY(J)**2)
      E2C=V(I+1,J-1,K)/DELY(J-1)**2
      E2=DELY(J)*DELY(J-1)*(E2A+E2B-E2C)/(DELY(J)+DELY(J-1))
      E1A=V(I-1,J+1,K)/DELY(J)**2
      E1B=V(I-1,J,K)*(1.0E0/DELY(J-1)**2-1.0E0/DELY(J)**2)
      E1C=V(I-1,J-1,K)/DELY(J-1)**2
      E1=DELY(J)*DELY(J-1)*(E1A+E1B-E1C)/(DELY(J)+DELY(J-1))
      E=(E2-E1)/(2.0E0*DELX)
      F=(W(I+1,J,K+1)-W(I+1,J,K-1)-W(I-1,J,K+1)+W(I-1,J,K-1))/
     1(4.0E0*DELX*DELZ)
      G1=U(I,J+1,K)/DELY(J)
      G2=U(I,J,K)*(1.0E0/DELY(J-1)+1.0E0/DELY(J))
      G3=U(I,J-1,K)/DELY(J-1)
      G=2.0E0*(G1-G2+G3)/(DELY(J)+DELY(J-1))
      H=(U(I,J,K+1)-2.0E0*U(I,J,K)+U(I,J,K-1))/DELZ**2
      UTEE=-(A+B+C+D+REI*(E+F-G-H))
```

II. w Momentum Equation

$$\begin{array}{cccccccc} A & B & C & D & E & F & G & H \end{array}$$

$$\frac{\partial w}{\partial t} = - \left[\frac{\partial uw}{\partial x} + \frac{\partial vw}{\partial y} + \frac{\partial w^2}{\partial z} + \frac{\partial P}{\partial z} + \frac{1}{Re} \left\{ \frac{\partial^2 u}{\partial x \partial z} + \frac{\partial}{\partial z} \left[\frac{\partial v}{\partial y} \right] - \frac{\partial^2 w}{\partial x^2} - \frac{\partial^2 w}{\partial y^2} \right\} \right]$$

```
 A=(U(I+1,J,K)*W(I+1,J,K)-U(I-1,J,K)*W(I-1,J,K))/(2.0E0*DELX)
 B1=V(I,J+1,K)*W(I,J+1,K)/DELY(J)**2
 B2=V(I,J,K)*W(I,J,K)*(1.0E0/DELY(J-1)**2-1.0E0/DELY(J)**2)
 B3=V(I,J-1,K)*W(I,J-1,K)/DELY(J-1)**2
 B=DELY(J-1)*DELY(J)*(B1+B2-B3)/(DELY(J)+DELY(J-1))
 C=(W(I,J,K+1)**2-W(I,J,K-1)**2)/(2.0E0*DELZ)
 D=(PHI(I,J,K+1)-PHI(I,J,K-1))/(2.0E0*DELZ)
 E=(U(I+1,J,K+1)-U(I+1,J,K-1)-U(I-1,J,K+1)+U(I-1,J,K-1))/
1(4.0E0*DELX*DELZ)
 F2A=V(I,J+1,K+1)/DELY(J)**2
 F2B=V(I,J,K+1)*(1.0E0/DELY(J-1)**2-1.0E0/DELY(J)**2)
 F2C=V(I,J-1,K+1)/DELY(J-1)**2
 F2=DELY(J)*DELY(J-1)*(F2A+F2B-F2C)/(DELY(J)+DELY(J-1))
 F1A=V(I,J+1,K-1)/DELY(J)**2
 F1B=V(I,J,K-1)*(1.0/DELY(J-1)**2-1.0/DELY(J)**2)
 F1C=V(I,J-1,K-1)/DELY(J-1)**2
 F1=DELY(J)*DELY(J-1)*(F1A+F1B-F1C)/(DELY(J)+DELY(J-1))
 F=(F2-F1)/(2.0E0*DELZ)
 G=(W(I+1,J,K)-W(I-1,J,K))/(2.0E0*DELX)
 H1=W(I,J+1,K)/DELY(J)
 H2=W(I,J,K)*(1.0E0/DELY(J-1)+1.0E0/DELY(J))
 H3=W(I,J-1,K)/DELY(J-1)
 H=2.0E0*(H1-H2+H3)/(DELY(J)+DELY(J-1))
 WTEE=-(A+B+C+D+REI*(E+F-G-H))
```

III. Poisson's Equation for Pressure

$$\frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2} + \frac{\partial^2 P}{\partial z^2} = - \left[\overset{B}{\frac{\partial^2 u^2}{\partial x^2}} + \overset{C}{\frac{\partial^2 v^2}{\partial y^2}} + \overset{D}{\frac{\partial^2 w^2}{\partial z^2}} + \left\{ \overset{E}{\frac{\partial}{\partial x}\left[\frac{\partial uv}{\partial y}\right]} + \overset{F}{\frac{\partial}{\partial z}\left[\frac{\partial vw}{\partial y}\right]} + \overset{G}{\frac{\partial^2 uw}{\partial x \partial z}} \right\} \right]$$

After finite differencing $\frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2} + \frac{\partial^2 P}{\partial y^2}$, we find that the Laplacian can be written

$$\frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2} + \frac{\partial^2 P}{\partial z^2} = A1 + A2 + A3 - P(I,J,K)*H$$

combining equations and we can solve for P

$$P(I,J,K) = (A + B + C + D + 2.0\ (E + F + G))/H$$

where A = A1 + A2 + A3 and H are defined in the subsequent listing. Since equation is not an explicit solution for the pressure field, we must employ some iterative process such as successive over relaxation which is used here. With this approach, equation is written

$$P(I,J,K) = (1 - \omega)\ P(I,J,K) + \omega(A + B + C + D + 2.0\ (E + F + G))/H$$

```
A1=(PHI(I+1,J,K)+PHI(I-1,J,K))/DELX**2
A2=(PHI(I,J,K+1)+PHI(I,J,K-1))/DELZ**2
A3=2.0E0*(PHI(I,J+1,K)/DELY(J)+PHI(I,J-1,K)/DELY(J-1))/
```

```
1(DELY(J)+DELY(J-1))
 A=A1+A2+A3
 B=(U(I+1,J,K)**2-2.0E0*U(I,J,K)**2+U(I-1,J,K)**2)/DELX**2
 C1=V(I,J+1,K)**2/DELY(J)
 C2=(V(I,J,K)**2)*(1.0E0/DELY(J-1)+1.0/DELY(J))
 C3=V(I,J-1,K)**2/DELY(J-1)
 C=2.0E0*(C1-C2+C3)/(DELY(J)+DELY(J-1))
 D=(W(I,J,K+1)**2-2.0E0*W(I,J,K)**2+W(I,J,K-1)**2)/DELZ**2
 E2A=U(I+1,J+1,K)*V(I+1,J+1,K)/DELY(J)**2
 E2B=U(I+1,J,K)*V(I+1,J,K)*(1.0E0/DELY(J-1)**2-1.0E0/DELY(J)**2)
 E2C=U(I+1,J-1,K)*V(I+1,J-1,K)/DELY(J-1)**2
 E2=DELY(J)*DELY(J-1)*(E2A+E2B-E2C)/(DELY(J)+DELY(J-1))
 E1A=U(I-1,J+1,K)*V(I-1,J+1,K)/DELY(J)**2
 E1B=U(I-1,J,K)*V(I-1,J,K)*(1.0E0/DELY(J-1)**2-1.0E0/DELY(J)**2)
 E1C=U(I-1,J-1,K)*V(I-1,J-1,K)/DELY(J-1)**2
 E1=DELY(J)*DELY(J-1)*(E1A+E1B-E1C)/(DELY(J)+DELY(J-1))
 E=(E2-E1)/(2.0E0*DELX)
 F2A=V(I,J+1,K+1)*W(I,J+1,K+1)/DELY(J)**2
 F2B=V(I,J,K+1)*W(I,J,K+1)*(1.0E0/DELY(J-1)**2-1.0E0/DELY(J)**2)
 F2C=V(I,J-1,K+1)*W(I,J-1,K+1)/DELY(J-1)**2
 F2=DELY(J)*DELY(J-1)*(F2A+F2B-F2C)/(DELY(J)+DELY(J-1))
 F1A=V(I,J+1,K-1)*W(I,J+1,K-1)/DELY(J)**2
 F1B=V(I,J,K-1)*W(I,J,K-1)*(1.0E0/DELY(J-1)**2-1.0E0/DELY(J)**2)
 F1C=V(I,J-1,K-1)*W(I,J-1,K-1)/DELY(J-1)**2
 F1=DELY(J)*DELY(J-1)*(F1A+F1B-F1C)/(DELY(J)+DELY(J-1))
 F=(F2-F1)/(2.0E0*DELZ)
 G2=U(I+1,J,K+1)*W(I+1,J,K+1)+U(I-1,J,K-1)*W(I-1,J,K-1)
 G1=U(I-1,J,K+1)*W(I-1,J,K+1)+U(I+1,J,K-1)*W(I+1,J,K-1)
 G=(G2-G1)/(4.0E0*DELX*DELZ)
 H=(2.0E0/DELX**2+2.0E0/DELX**2+2.0E0/(DELY(J)*DELY(J-1)))
 APHI=(1.0E0-OMEGA)*PHI(I,J,K)+OMEGA*(A+B+C+D+2.0E0*(E+F+G))/H
 RESID=ABS(APHI-PHI(I,J,K))
 ERR=MAX1(ERR,RESID)
 PHI(I,J,K)=APHI
```

IV. 3-d Continuity Equation to Solve for w

$$\frac{\partial w}{\partial z} = - \left[\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right]$$

```
UEX2=U(I+1,J+1,K)+U(I+1,J-1,K)+U(I+1,J+1,K+1)+U(I+1,J-1,K+1)
UEX1=U(I-1,J+1,K)+U(I-1,J-1,K)+U(I-1,J+1,K+1)+U(I-1,J-1,K+1)
UEX=(UEX2-UEX1)/(8.0E0*DELX)
V1=(V(I+1,J+1,K)+V(I+1,J+1,K+1)+V(I-1,J+1,K)+V(I-1,J+1,K+1))/
14.0E0
V2=(V(I+1,J,K)+V(I+1,J,K+1)+V(I-1,J,K)+V(I-1,J,K+1))/4.0E0
V3=(V(I+1,J-1,K)+V(I+1,J-1,K+1)+V(I-1,J-1,K)+V(I-1,J-1,K+1))/
14.0E0
VY1=V1/DELY(J)**2
VY2=V2*(1.0E0/DELY(J)**2-1.0E0/DELY(J-1)**2)
VY3=V3/DELY(J-1)**2
VWY=DELY(J)*DELY(J-1)*(VY1+VY2-VY3)/(DELY(J)+DELY(J-1))
W(I,J,K+1)=W(I,J,K)-DELZ*(UEX+VWY)
```

To evaluate $\frac{\partial w}{\partial z}$ at I,J,K + 1/2 the following method was employed. First v at y_{j+1}, y_j, y_{j-1} was found by averaging the four v values in the x-z plane at a given y. With these average values, the non-uniform grid derivative formula was applied to form $\frac{\partial v}{\partial y}$ at I,J,K + 1/2. In a similar manner, an average u at I + 1 and I - 1 was found by averaging the four values in the y-z plane at each I. These average u's were then central differenced to form $\frac{\partial u}{\partial x}$ at I,J,K + 1/2. Finally, using these derivatives evaluated at I,J,K + 1/2, the w field is calculated using Equation using a central difference method.

V. 2-d Continuity Used to Solve for v

$$\frac{\partial v}{\partial y} = - \frac{\partial u}{\partial x}$$

The identical procedure was applied to the finite difference solution of this equation in two dimensions as was outlined for the 3-d continuity equation. The principle difference being that the appropriate averaging of the dependent variables now occurs on lines instead of planes in the 3-d matrix of grid points.

```
C1=U(I+1,J+1,K)+U(I+1,J,K)
C2=U(I-1,J+1,K)+U(I-1,J,K)
V(I,J+1,K)=V(I,J,K)-DELY(J)*(C1-C2)/(4.0E0*DELX)
```

VI. 3-d Continuity Used to Solve for v. Following the procedure outlined in Section IV of this Appendix, we can interpret the three-dimensional continuity equation to be an equation for v, here again using averaged values of the dependent variables in the difference formulation.

```
 UEX2=(U(I+1,J,K-1)+U(I+1,J+1,K-1) +U(I+1,J,K+1)+U(I+1,J+1,K+1))/
14.0E0
 UEX1=(U(I-1,J,K-1)+U(I-1,J+1,K-1)+U(I-1,J,K+1)+U(I-1,J+1,K+1))/
14.0E0
 UEX=(UEX2-UEX1)/(2.0E0*DELX)
 WZ2=(W(I+1,J,K+1)+W(I+1,J+1,K+1)+W(I-1,J,K+1)+W(I-1,J+1,K+1))/
14.0E0
 WZ1=(W(I+1,J,K-1)+W(I+1,J+1,K-1)+W(I-1,J,K-1)+W(I-1,J+1,K-1))/
14.0E0
 WZE=(WZ2-WZ1)/(2.0E0*DELZ)
 V(I,J+1,K)=V(I,J,K)-DELY(J)*(UEX+WZE)
```

APPENDIX C

Finite Differencing of 2-d Navier-Stokes Equations

I. u Momentum Equation

A B C D E

$$\frac{\partial u}{\partial t} = - \left[\frac{\partial u^2}{\partial x} + \frac{\partial uv}{\partial y} + \frac{\partial P}{\partial x} + \frac{1}{Re} \left\{ \frac{\partial}{\partial x} \left[\frac{\partial v}{\partial y} \right] - \frac{\partial^2 u}{\partial y^2} \right\} \right]$$

```
A=(U(I+1,J)**2-U(I-1,J)**2)/(2.0E0*DELX)
B1=U(I,J+1)*V(I,J+1)/DELY(J)**2
B2=U(I,J)*V(I,J)*(1.0E0/DELY(J-1)**2-1.0E0/DELY(J)**2)
B3=U(I,J-1)*V(I,J-1)/DELY(J-1)**2
B=DELY(J)*DELY(J-1)*(B1+B2-B3)/(DELY(J)+DELY(J-1))
C=(PHI(I+1,J)-PHI(I-1,J))/(2.0E0*DELX)
D2A=V(I+1,J+1)/DELY(J)**2
D2B=V(I+1,J)*(1.0E0/DELY(J-1)**2-1.0E0/DELY(J)**2)
D2C=V(I+1,J-1)/DELY(J-1)**2
D2=DELY(J)*DELY(J-1)*(D2A+D2B-D2C)/(DELY(J)+DELY(J-1))
D1A=V(I-1,J+1)/DELY(J)**2
D1B=V(I-1,J)*(1.0E0/DELY(J-1)**2-1.0E0/DELY(J)**2)
D1C=V(I-1,J-1)/DELY(J-1)**2
D1=DELY(J)*DELY(J-1)*(D1A+D1B-D1C)/(DELY(J)+DELY(J-1))
D=(D2-D1)/(2.0E0*DELX)
E1=U(I,J+1)/DELY(J)
E2=U(I,J)*(1.0E0/DELY(J-1)+1.0E0/DELY(J))
E3=U(I,J-1)/DELY(J-1)
E=2.0E0*(E1-E2+E3)/(DELY(J)+DELY(J-1))
UTEE=-(A+B+C+REI*(D-E))
```

II. Poisson's Equation for Pressure

B C D

$$\frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2} + - \left[\frac{\partial^2 u^2}{\partial x^2} + \frac{\partial^2 v^2}{\partial y^2} + 2 \frac{\partial}{\partial x} \left[\frac{\partial uv}{\partial y} \right] \right]$$

From finite differencing $\frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2}$, we can show that

$$\frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2} = A1 + A2 - P(I,J)*H$$

Substituting this into equation, we obtain

$$P(I,J) = (A + B + C + 2.0\ D)/H$$

To obtain a form appropriate to successive over-relaxation, we can write

$$P(I,J) = (1 - \omega)\ P(I,J) + \omega(A + B + C + 2.0 * D)/H$$

```
 A1=(PHI(I+1,J)+PHI(I-1,J))/DELX**2
 A2=2.0E0*(PHI(I,J+1)/DELY(J)+PHI(I,J-1)/DELY(J-1))/
1(DELY(J)+DELY(J-1))
 A=A1+A2
 B=(U(I+1,J)**2-2.0E0*U(I,J)**2+U(I-1,J)**2)/DELX**2
 C1=V(I,J+1)**2/DELY(J)
 C2=V(I,J)**2*(1.0E0/DELY(J-1)+1.0E0/DELY(J))
 C3=V(I,J-1)**2/DELY(J-1)
 C=2.0E0*(C1-C2+C3)/(DELY(J)+DELY(J-1))
 D2A=U(I+1,J+1)*V(I+1,J+1)/DELY(J)**2
 D2B=U(I+1,J)*V(I+1,J)*(1.0E0/DELY(J-1)**2-1.0E0/DELY(J)**2)
 D2C=U(I+1,J-1)*V(I+1,J-1)/DELY(J-1)**2
 D2=DELY(J)*DELY(J-1)*(D2A+D2B-D2C)/(DELY(J)+DELY(J-1))
 D1A=U(I-1,J+1)*V(I-1,J+1)/DELY(J)**2
 D1B=U(I-1,J)*V(I-1,J)*(1.0E0/DELY(J-1)**2-1.0E0/DELY(J)**2)
 D1C=U(I-1,J-1)*V(I-1,J-1)/DELY(J-1)**2
 D1=DELY(J)*DELY(J-1)*(D1A+D1B-D1C)/(DELY(J)+DELY(J-1))
 D=(D2-D1)/(2.0E0*DELX)
 H=2.0E0/DELX**2+2.0E0/(DELY(J)*DELY(J-1))
 APHI=(1.0E0-OMEGA)*PHI(I,J)+OMEGA*(A+B+C+2.0E0*D)/H
 RESID=ABS(APHI-PHI(I,J))
 ERR=MAX1(ERR,RESID)
 PHI(I,J)=APHI
```

III. 2-d Continuity, Solving for v

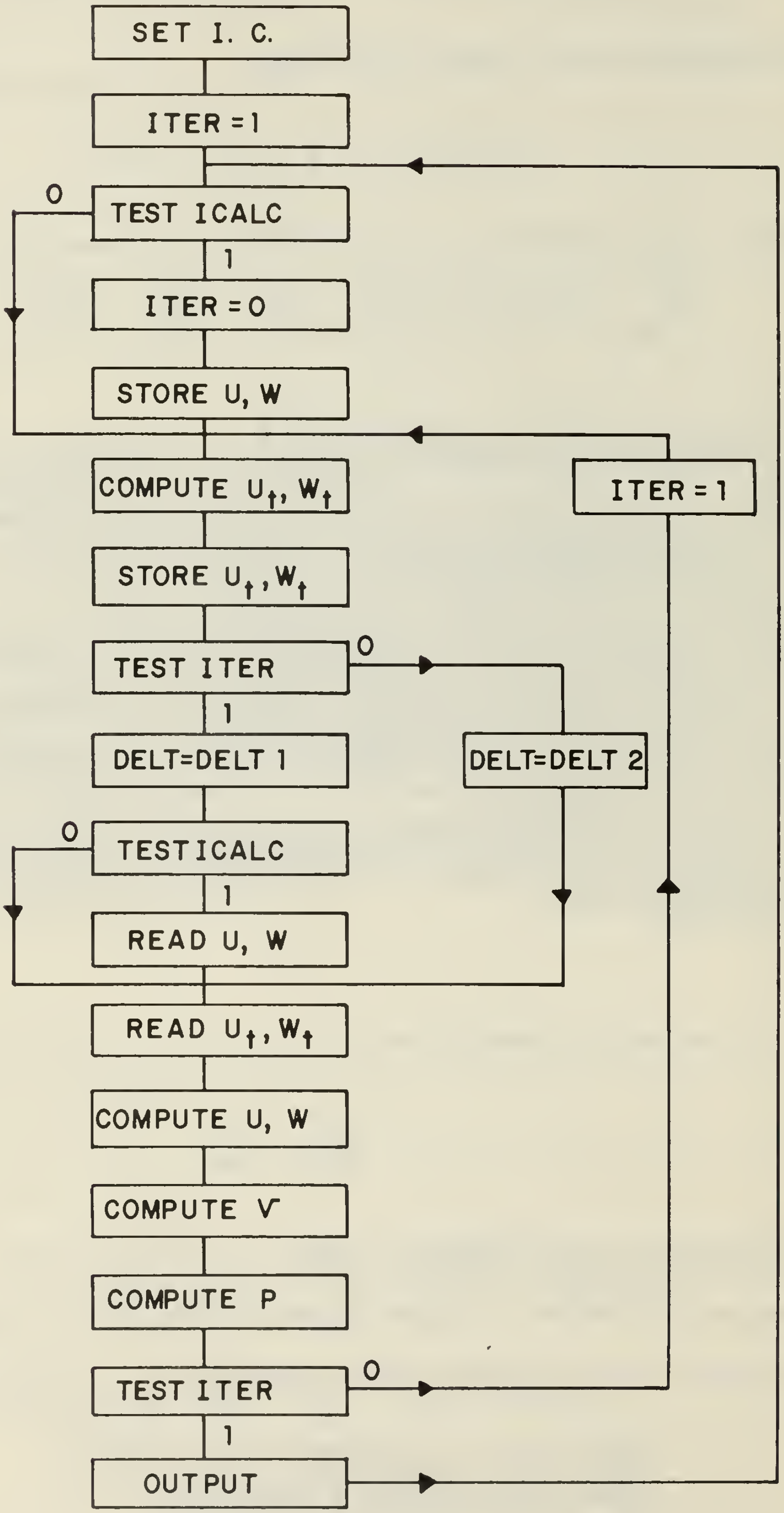
$$\frac{\partial v}{\partial y} = - \frac{\partial u}{\partial x}$$

To form $\frac{\partial v}{\partial y}$ at I,J + 1/2, average values of u at I + 1 and I - 1 were found by averaging the values of u along appropriate J, J + 1 lines. Finite differencing of these average values yields $\frac{\partial u}{\partial x}$ at I, J + 1/2.

```
UEX2=(U(I+1,J)+U(I+1,J+1))/2.0E0
UEX1=(U(I-1,J)+U(I-1,J+1))/2.0E0
UEX=(UEX2-UEX1)/(2.0E0*DELX)
V(I,J+1)=V(I,J)-DELY(J)*UEX
```

APPENDIX D

I. FLOW CHART FOR 3-D PROBLEM



APPENDIX D

II. LISTING OF COMPUTER PROGRAM FOR 3-D CASE

THE FOLLOWING FORTRAN CODED FINITE DIFFERENCE COMPUTER PROGRAM SOLVES THE FULL TIME DEPENDENT NAVIER STOKES EQUATIONS FOR THE THREE DIMENSIONAL FLOW ABOUT A FINITE, INFINITELY THIN FLAT PLATE IMPULSIVELY STARTED IN ITS OWN PLANE. "LEAP-FROG TIME-WISE INTEGRATION IS INCLUDED AS AN OPTION. EXTENSIVE DISC WRITING IS EMPLOYED TO SAVE COMPUTER CORE SPACE. ADDITIONALLY, THE EQUATIONS THEMSELVES ARE APPLIED TO THE BOUNDARIES OF THE CALCULATION REGION IN LIEU OF EXTRAPOLATING THE VARIABLES CALCULATED WITHIN THE REGION TO THE BOUNDARIES.

THE FOLLOWING PARAMETERS MUST BE SPECIFIED
ICALC LE ME NE LP1 LP2 NP1 NP2 RE DELX DELY DELZ OMEGA ERRTOL PE QE

```
********************************************************************
*                                                                  *
*     ICALC SPECIFES THE USE OF "LEAP-FROG" IN T                   *
*                                                                  *
*   LE IS THE LENGTH OF THE REGION IN DELX STEPS                   *
*                                                                  *
*   ME IS THE HEIGHT OF THE REGION IN DELY STEPS                   *
*                                                                  *
*   NE IS THE WIDTH OF THE REGION IN DELZ STEPS                    *
*                                                                  *
*           LP1 IS THE START OF THE PLATE                          *
*                                                                  *
*            LP2 IS THE END OF THE PLATE                           *
*                                                                  *
*           NP1 IS ONE SIDE OF THE PLATE                           *
*                                                                  *
*           NP2 IS ONE SIDE OF THE PLATE                           *
*                                                                  *
*     RE IS THE REYNOLDS NUMBER BASED ON U AND L                   *
*                                                                  *
*          DELX IS THE GRID SPACING IN X                           *
*                                                                  *
*          DELY IS THE GRID SPACING IN Y                           *
*                                                                  *
*          DELZ IS THE GRID SPACING IN Z                           *
*                                                                  *
*  OMEGA IS THE RELAXATION PARAMETER IN POISSONS                   *
*                                                                  *
*    ERRTOL IS THE ERROR TOLERANCE IN POISSONS                     *
*                                                                  *
*      PE IS THE FREESTREAM PRESSURE IN PSF                        *
```

```
C
C     ************************************************************
C     *  QE IS THE FREESTREAM DYNAMIC PRESSURE IN PSF            *
C     *                                                          *
C     *      NOTE THAT NE MUST BE AN ODD INTEGER                 *
C     *                                                          *
C     *      NOTE THAT DELY IS AN ARRAY DELY(ME)                 *
C     *                                                          *
C     *   ALL VARIABLES ARE DIMENSIONED F(LE,ME,NE)              *
C     *                                                          *
C     ************************************************************
C
      DIMENSION U(40,30,21),V(40,30,21),W(40,30,21),PHI(40,30,21)
      DIMENSION DELY(30)
C
      LE1=LE-1
      LE2=LE-2
      LE3=LE-3
C
      ME1=ME-1
      ME2=ME-2
      ME3=ME-3
C
      NE1=NE-1
      NE2=NE-2
      NE3=NE-3
C
      NPM=(NE+1)/2
      NPM1=NPM-1
      NPM2=NPM+1
C
      REI=1.0E0/RE
C
      PHIE=PE/(2.0E0*QE)
C
      A=1.0E0/DELX
      B=1.0E0/DELY(1)
      C=1.0E0/DELZ
      DELT=1.0E0/(A+0.1E0*B+0.1E0*C+2.0E0*REI*(A**2+B**2+C**2))
C
      DELT1=DELT
      DELT2=DELT1/2.0E0
C
      IC=0
C
C     INITIALIZE THE CALCULATION FIELD
      DO 2 I=1,LE
      DO 2 J=1,ME
      DO 2 K=1,NE
```

```
      U(I,J,K)=1.0E0
      V(I,J,K)=0.0E0
      W(I,J,K)=0.0E0
      PHI(I,J,K)=PHIE
    2 CONTINUE
C
C     SET PLATE TO ZERO VELOCITY
      DO 3 I=LP1,LP2
      DO 3 K=NP1,NP2
      U(I,1,K)=0.0E0
    3 CONTINUE
C
C     CALCULATE INITIAL V WITHIN THE REGION FROM 2-D CONTINUITY EQ
      DO 4 J=1,ME1
      DO 4 I=2,LE1
      DO 4 K=1,NE
      C1=U(I+1,J+1,K)+U(I+1,J,K)
      C2=U(I-1,J+1,K)+U(I-1,J,K)
      V(I,J+1,K)=V(I,J,K)-DELY(J)*(C1-C2)/(4.0E0*DELX)
    4 CONTINUE
C
C     SET UPSTREAM V FIELD
      DO 5 J=1,ME
      DO 5 K=1,NE
      V(1,J,K)=0.0E0
    5 CONTINUE
C
C     EXTRAPOLATE DOWNSTREAM V FIELD
      DO 6 J=1,ME
      DO 6 K=1,NE
      V(LE,J,K)=3.0E0*V(LE1,J,K)-3.0E0*V(LE2,J,K)+V(LE3,J,K)
    6 CONTINUE
C
C     CALCULATE W FIELD WITHIN CALCULATION REGION FROM 3-D CONTINUITY EQ
      DO 7 I=2,LE1
      DO 7 J=2,ME1
      DO 7 K=NPM,NE1
      UEX2=U(I+1,J+1,K)+U(I+1,J-1,K)+U(I+1,J+1,K+1)+U(I+1,J-1,K+1)
      UEX1=U(I-1,J+1,K)+U(I-1,J-1,K)+U(I-1,J+1,K+1)+U(I-1,J-1,K+1)
      UEX=(UEX2-UEX1)/(8.0E0*DELX)
      V1=(V(I+1,J+1,K)+V(I+1,J+1,K+1)+V(I-1,J+1,K)+V(I-1,J+1,K+1))/
     14.0E0
      V2=(V(I+1,J,K)+V(I+1,J,K+1)+V(I-1,J,K)+V(I-1,J,K+1))/4.0E0
      V3=(V(I+1,J-1,K)+V(I+1,J-1,K+1)+V(I-1,J-1,K)+V(I-1,J-1,K+1))/
     14.0E0
      VY1=V1/DELY(J)**2
      VY2=V2*(1.0E0/DELY(J)**2-1.0E0/DELY(J-1)**2)
      VY3=V3/DELY(J-1)**2
```

```
      VWY=DELY(J)*DELY(J-1)*(VY1+VY2-VY3)/(DELY(J)+DELY(J-1))
      W(I,J,K+1)=W(I,J,K)-DELZ*(UEX+VWY)
    7 CONTINUE
C
C     CALCULATE W IN THE PLANE OF THE PLATE
      DO 8 I=2,LE1
      DO 8 K=NPM,NE1
      J=1
      UEX2=U(I+1,J+1,K)+U(I+1,J+1,K)+U(I+1,J+1,K+1)+U(I+1,J+1,K+1)
      UEX1=U(I-1,J+1,K)+U(I-1,J+1,K)+U(I-1,J+1,K+1)+U(I-1,J+1,K+1)
      UEX=(UEX2-UEX1)/(8.0E0*DELX)
      V1=(V(I+1,J+1,K)+V(I+1,J+1,K+1)+V(I-1,J+1,K)+V(I-1,J+1,K+1))/
     14.0E0
      V2=(V(I+1,J,K)+V(I+1,J,K+1)+V(I-1,J,K)+V(I-1,J,K+1))/4.0E0
      V3=(V(I+1,J+1,K)+V(I+1,J+1,K+1)+V(I-1,J+1,K)+V(I-1,J+1,K+1))/
     14.0E0
      V3=-V3
      VY1=V1/DELY(J)**2
      VY2=V2*(1.0E0/DELY(J)**2-1.0E0/DELY(J)**2)
      VY3=V3/DELY(J)**2
      VWY=DELY(J)*DELY(J)*(VY1+VY2-VY3)/(DELY(J)+DELY(J))
      W(I,J,K+1)=W(I,J,K)-DELZ*(UEX+VWY)
    8 CONTINUE
C
C     CALCULATE W AT THE TOP OF THE CALCULATION REGION
      DO 9 I=2,LE1
      DO 9 K=NPM,NE1
      J=ME
      UEX2=U(I+1,J,K)+U(I+1,J-1,K)+U(I+1,J,K+1)+U(I+1,J-1,K+1)
      UEX1=U(I-1,J,K)+U(I-1,J-1,K)+U(I-1,J,K+1)+U(I-1,J-1,K+1)
      UEX=(UEX2-UEX1)/(8.0E0*DELX)
      V1=(V(I+1,J,K)+V(I+1,J,K+1)+V(I-1,J,K)+V(I-1,J,K+1))/
     14.0E0
      V2=(V(I+1,J,K)+V(I+1,J,K+1)+V(I-1,J,K)+V(I-1,J,K+1))/4.0E0
      V3=(V(I+1,J-1,K)+V(I+1,J-1,K+1)+V(I-1,J-1,K)+V(I-1,J-1,K+1))/
     14.0E0
      VY1=V1/DELY(J)**2
      VY2=V2*(1.0E0/DELY(J)**2-1.0E0/DELY(J-1)**2)
      VY3=V3/DELY(J-1)**2
      VWY=DELY(J)*DELY(J-1)*(VY1+VY2-VY3)/(DELY(J)+DELY(J-1))
      W(I,J,K+1)=W(I,J,K)-DELZ*(UEX+VWY)
    9 CONTINUE
C
C     INSERTION OF LEFT HALF REGION W VALUES BY SYMMETRY
      DO 10 I=2,LE1
      DO 10 J=2,ME1
      DO 10 K=NPM2,NE
      K1=NE+1-K
```

```
      W(I,J,K1)=W(I,J,K)
   10 CONTINUE
C
C     CALCULATE W ON THE FRONT OF THE CALCULATION REGION
      DO 11 J=1,ME
      DO 11 K=1,NE
      W(1,J,K)=0.0E0
   11 CONTINUE
C
C     CALCULATE W AT THE EXIT OF THE CALCULATION REGION
      DO 12 J=1,ME
      DO 12 K=1,NE
      W(LE,J,K)=3.0E0*W(LE1,J,K)-3.0E0*W(LE2,J,K)+W(LE3,J,K)
   12 CONTINUE
C
C     SET W ON THE PLATE SURFACE EQUAL TO ZERO
      DO 13 I=LP1,LP2
      DO 13 K=NP1,NP2
      W(I,1,K)=0.0E0
   13 CONTINUE
C
C     CALCULATE INITIAL P FIELD
C
      WRITE(6,810)
  810 FORMAT(1H0,20X,'INTO IC POISSONS')
      GO TO 200
  150 IC=1
      WRITE(6,811)
  811 FORMAT(1H0,20X,'OUT OF IC POISSONS')
      ITER=1
   14 CONTINUE
      IF(ICALC.EQ.0) GO TO 18
      ITER=0
      REWIND 16
      REWIND 18
C
C     STORE PREVIOUS FIELD ON TAPES
      WRITE(16) U
      WRITE(18) W
C
   18 CONTINUE
C
      REWIND 19
      REWIND 20
C     CALCULATION WITHIN THE REGION
      DO 19 I=2,LE1
      DO 19 J=2,ME1
      DO 19 K=2,NE1
```

```
C       CALCULATION OF UT FROM 3D MOMENTUM EQUATION
        A=(U(I+1,J,K)**2-U(I-1,J,K)**2)/(2.0E0*DELX)
        B1=U(I,J+1,K)*V(I,J+1,K)/(DELY(J)**2)
        B2=U(I,J,K)*V(I,J,K)*(1.0E0/DELY(J-1)**2-1.0E0/DELY(J)**2)
        B3=U(I,J-1,K)*V(I,J-1,K)/DELY(J-1)**2
        B=DELY(J)*DELY(J-1)*(B1+B2-B3)/(DELY(J)+DELY(J-1))
        C=(U(I,J,K+1)*W(I,J,K+1)-U(I,J,K-1)*W(I,J,K-1))/(2.0E0*DELZ)
        D=(PHI(I+1,J,K)-PHI(I-1,J,K))/(2.0E0*DELX)
        E2A=V(I+1,J+1,K)/DELY(J)**2
        E2B=V(I+1,J,K)*(1.0E0/DELY(J-1)**2-1.0E0/DELY(J)**2)
        E2C=V(I+1,J-1,K)/DELY(J-1)**2
        E2=DELY(J)*DELY(J-1)*(E2A+E2B-E2C)/(DELY(J)+DELY(J-1))
        E1A=V(I-1,J+1,K)/DELY(J)**2
        E1B=V(I-1,J,K)*(1.0E0/DELY(J-1)**2-1.0E0/DELY(J)**2)
        E1C=V(I-1,J-1,K)/DELY(J-1)**2
        E1=DELY(J)*DELY(J-1)*(E1A+E1B-E1C)/(DELY(J)+DELY(J-1))
        E=(E2-E1)/(2.0E0*DELX)
        F=(W(I+1,J,K+1)-W(I+1,J,K-1)-W(I-1,J,K+1)+W(I-1,J,K-1))/
       1(4.0E0*DELX*DELZ)
        G1=U(I,J+1,K)/DELY(J)
        G2=U(I,J,K)*(1.0E0/DELY(J-1)+1.0E0/DELY(J))
        G3=U(I,J-1,K)/DELY(J-1)
        G=2.0E0*(G1-G2+G3)/(DELY(J)+DELY(J-1))
        H=(U(I,J,K+1)-2.0E0*U(I,J,K)+U(I,J,K-1))/DELZ**2
        UTEE=-(A+B+C+D+REI*(E+F-G-H))
        WRITE(19,903) UTEE
  903   FORMAT(1PE14.7)
C
C       CALCULATION OF WT FROM 3D MOMENTUM EQUATION
        A=(U(I+1,J,K)*W(I+1,J,K)-U(I-1,J,K)*W(I-1,J,K))/(2.0E0*DELX)
        B1=V(I,J+1,K)*W(I,J+1,K)/DELY(J)**2
        B2=V(I,J,K)*W(I,J,K)*(1.0E0/DELY(J-1)**2-1.0E0/DELY(J)**2)
        B3=V(I,J-1,K)*W(I,J-1,K)/DELY(J-1)**2
        B=DELY(J-1)*DELY(J)*(B1+B2-B3)/(DELY(J)+DELY(J-1))
        C=(W(I,J,K+1)**2-W(I,J,K-1)**2)/(2.0E0*DELZ)
        D=(PHI(I,J,K+1)-PHI(I,J,K-1))/(2.0E0*DELZ)
        E=(U(I+1,J,K+1)-U(I+1,J,K-1)-U(I-1,J,K+1)+U(I-1,J,K-1))/
       1(4.0E0*DELX*DELZ)
        F2A=V(I,J+1,K+1)/DELY(J)**2
        F2B=V(I,J,K+1)*(1.0E0/DELY(J-1)**2-1.0E0/DELY(J)**2)
        F2C=V(I,J-1,K+1)/DELY(J-1)**2
        F2=DELY(J)*DELY(J-1)*(F2A+F2B-F2C)/(DELY(J)+DELY(J-1))
        F1A=V(I,J+1,K-1)/DELY(J)**2
        F1B=V(I,J,K-1)*(1.0/DELY(J-1)**2-1.0/DELY(J)**2)
        F1C=V(I,J-1,K-1)/DELY(J-1)**2
        F1=DELY(J)*DELY(J-1)*(F1A+F1B-F1C)/(DELY(J)+DELY(J-1))
        F=(F2-F1)/(2.0E0*DELZ)
        G=(W(I+1,J,K)-W(I-1,J,K))/(2.0E0*DELX)
```

```
      H1=W(I,J+1,K)/DELY(J)
      H2=W(I,J,K)*(1.0E0/DELY(J-1)+1.0E0/DELY(J))
      H3=W(I,J-1,K)/DELY(J-1)
      H=2.0E0*(H1-H2+H3)/(DELY(J)+DELY(J-1))
      WTEE=-(A+B+C+D+REI*(E+F-G-H))
      WRITE(20,904) WTEE
  904 FORMAT(1PE14.7)
   19 CONTINUE
C
C     CALCULATION ON THE SIDES
      DO 20 I=2,LE1
      DO 20 J=2,ME1
      K=1
C     CALCULATION OF UT FROM 3D MOMENTUM EQUATION
C     K-1 TERMS HAVE BEEN ADJUSTED BY UNIFORMITY ARGUMENT
      A=(U(I+1,J,K)**2-U(I-1,J,K)**2)/(2.0E0*DELX)
      B1=U(I,J+1,K)*V(I,J+1,K)/(DELY(J)**2)
      B2=U(I,J,K)*V(I,J,K)*(1.0E0/DELY(J-1)**2-1.0E0/DELY(J)**2)
      B3=U(I,J-1,K)*V(I,J-1,K)/DELY(J-1)**2
      B=DELY(J)*DELY(J-1)*(B1+B2-B3)/(DELY(J)+DELY(J-1))
      C=(U(I,J,K+1)*W(I,J,K+1)-U(I,J,K)*W(I,J,K))/(2.0E0*DELZ)
      D=(PHI(I+1,J,K)-PHI(I-1,J,K))/(2.0E0*DELX)
      E2A=V(I+1,J+1,K)/DELY(J)**2
      E2B=V(I+1,J,K)*(1.0E0/DELY(J-1)**2-1.0E0/DELY(J)**2)
      E2C=V(I+1,J-1,K)/DELY(J-1)**2
      E2=DELY(J)*DELY(J-1)*(E2A+E2B-E2C)/(DELY(J)+DELY(J-1))
      E1A=V(I-1,J+1,K)/DELY(J)**2
      E1B=V(I-1,J,K)*(1.0E0/DELY(J-1)**2-1.0E0/DELY(J)**2)
      E1C=V(I-1,J-1,K)/DELY(J-1)**2
      E1=DELY(J)*DELY(J-1)*(E1A+E1B-E1C)/(DELY(J)+DELY(J-1))
      E=(E2-E1)/(2.0E0*DELX)
      F=(W(I+1,J,K+1)-W(I+1,J,K)-W(I-1,J,K+1)+W(I-1,J,K))/
     1(4.0E0*DELX*DELZ)
      G1=U(I,J+1,K)/DELY(J)
      G2=U(I,J,K)*(1.0E0/DELY(J-1)+1.0E0/DELY(J))
      G3=U(I,J-1,K)/DELY(J-1)
      G=2.0E0*(G1-G2+G3)/(DELY(J)+DELY(J-1))
      H=(U(I,J,K+1)-2.0E0*U(I,J,K)+U(I,J,K))/DELZ**2
      UTEE=-(A+B+C+D+REI*(E+F-G-H))
      WRITE(19,903) UTEE
C
C     CALCULATION OF WT FROM 3D MOMENTUM EQUATION
      A=(U(I+1,J,K)*W(I+1,J,K)-U(I-1,J,K)*W(I-1,J,K))/(2.0E0*DELX)
      B1=V(I,J+1,K)*W(I,J+1,K)/DELY(J)**2
      B2=V(I,J,K)*W(I,J,K)*(1.0E0/DELY(J-1)**2-1.0E0/DELY(J)**2)
      B3=V(I,J-1,K)*W(I,J-1,K)/DELY(J-1)**2
      B=DELY(J-1)*DELY(J)*(B1+B2-B3)/(DELY(J)+DELY(J-1))
      C=(W(I,J,K+1)**2-W(I,J,K)**2)/(2.0E0*DELZ)
```

```
      D=(PHI(I,J,K+1)-PHI(I,J,K))/(2.0E0*DELZ)
      E=(U(I+1,J,K+1)-U(I+1,J,K)-U(I-1,J,K+1)+U(I-1,J,K))/
     1(4.0E0*DELX*DELZ)
      F2A=V(I,J+1,K+1)/DELY(J)**2
      F2B=V(I,J,K+1)*(1.0E0/DELY(J-1)**2-1.0E0/DELY(J)**2)
      F2C=V(I,J-1,K+1)/DELY(J-1)**2
      F2=DELY(J)*DELY(J-1)*(F2A+F2B-F2C)/(DELY(J)+DELY(J-1))
      F1A=V(I,J+1,K)/DELY(J)**2
      F1B=V(I,J,K)*(1.0/DELY(J-1)**2-1.0/DELY(J)**2)
      F1C=V(I,J-1,K)/DELY(J-1)**2
      F1=DELY(J)*DELY(J-1)*(F1A+F1B-F1C)/(DELY(J)+DELY(J-1))
      F=(F2-F1)/(2.0E0*DELZ)
      G=(W(I+1,J,K)-W(I-1,J,K))/(2.0E0*DELX)
      H1=W(I,J+1,K)/DELY(J)
      H2=W(I,J,K)*(1.0E0/DELY(J-1)+1.0E0/DELY(J))
      H3=W(I,J-1,K)/DELY(J-1)
      H=2.0E0*(H1-H2+H3)/(DELY(J)+DELY(J-1))
      WTEE=-(A+B+C+D+REI*(E+F-G-H))
      WRITE(20,904) WTEE
   20 CONTINUE
C
C     CALCULATION ON THE SIDES
C     K+1 TERMS HAVE BEEN ADJUSTED BY UNIFORMITY ARGUMENT
      DO 21 I=2,LE1
      DO 21 J=2,ME1
      K=NE
C     CALCULATION OF UT FROM 3D MOMENTUM EQUATION
      A=(U(I+1,J,K)**2-U(I-1,J,K)**2)/(2.0E0*DELX)
      B1=U(I,J+1,K)*V(I,J+1,K)/(DELY(J)**2)
      B2=U(I,J,K)*V(I,J,K)*(1.0E0/DELY(J-1)**2-1.0E0/DELY(J)**2)
      B3=U(I,J-1,K)*V(I,J-1,K)/DELY(J-1)**2
      B=DELY(J)*DELY(J-1)*(B1+B2-B3)/(DELY(J)+DELY(J-1))
      C=(U(I,J,K)*W(I,J,K)-U(I,J,K-1)*W(I,J,K-1))/(2.0E0*DELZ)
      D=(PHI(I+1,J,K)-PHI(I-1,J,K))/(2.0E0*DELX)
      E2A=V(I+1,J+1,K)/DELY(J)**2
      E2B=V(I+1,J,K)*(1.0E0/DELY(J-1)**2-1.0E0/DELY(J)**2)
      E2C=V(I+1,J-1,K)/DELY(J-1)**2
      E2=DELY(J)*DELY(J-1)*(E2A+E2B-E2C)/(DELY(J)+DELY(J-1))
      E1A=V(I-1,J+1,K)/DELY(J)**2
      E1B=V(I-1,J,K)*(1.0E0/DELY(J-1)**2-1.0E0/DELY(J)**2)
      E1C=V(I-1,J-1,K)/DELY(J-1)**2
      E1=DELY(J)*DELY(J-1)*(E1A+E1B-E1C)/(DELY(J)+DELY(J-1))
      E=(E2-E1)/(2.0E0*DELX)
      F=(W(I+1,J,K)-W(I+1,J,K-1)-W(I-1,J,K)+W(I-1,J,K-1))/
     1(4.0E0*DELX*DELZ)
      G1=U(I,J+1,K)/DELY(J)
      G2=U(I,J,K)*(1.0E0/DELY(J-1)+1.0E0/DELY(J))
      G3=U(I,J-1,K)/DELY(J-1)
```

```
      G=2.0E0*(G1-G2+G3)/(DELY(J)+DELY(J-1))
      H=(U(I,J,K)-2.0E0*U(I,J,K)+U(I,J,K-1))/DELZ**2
      UTEE=-(A+B+C+D+REI*(E+F-G-H))
      WRITE(19,903) UTEE
C
C     CALCULATION OF WT FROM 3D MOMENTUM EQUATION
      A=(U(I+1,J,K)*W(I+1,J,K)-U(I-1,J,K)*W(I-1,J,K))/(2.0E0*DELX)
      B1=V(I,J+1,K)*W(I,J+1,K)/DELY(J)**2
      B2=V(I,J,K)*W(I,J,K)*(1.0E0/DELY(J-1)**2-1.0E0/DELY(J)**2)
      B3=V(I,J-1,K)*W(I,J-1,K)/DELY(J-1)**2
      B=DELY(J-1)*DELY(J)*(B1+B2-B3)/(DELY(J)+DELY(J-1))
      C=(W(I,J,K)**2-W(I,J,K-1)**2)/(2.0E0*DELZ)
      D=(PHI(I,J,K)-PHI(I,J,K-1))/(2.0E0*DELZ)
      E=(U(I+1,J,K)-U(I+1,J,K-1)-U(I-1,J,K)+U(I-1,J,K-1))/
     1(4.0E0*DELX*DELZ)
      F2A=V(I,J+1,K)/DELY(J)**2
      F2B=V(I,J,K)*(1.0E0/DELY(J-1)**2-1.0E0/DELY(J)**2)
      F2C=V(I,J-1,K)/DELY(J-1)**2
      F2=DELY(J)*DELY(J-1)*(F2A+F2B-F2C)/(DELY(J)+DELY(J-1))
      F1A=V(I,J+1,K-1)/DELY(J)**2
      F1B=V(I,J,K-1)*(1.0/DELY(J-1)**2-1.0/DELY(J)**2)
      F1C=V(I,J-1,K-1)/DELY(J-1)**2
      F1=DELY(J)*DELY(J-1)*(F1A+F1B-F1C)/(DELY(J)+DELY(J-1))
      F=(F2-F1)/(2.0E0*DELZ)
      G=(W(I+1,J,K)-W(I-1,J,K))/(2.0E0*DELX)
      H1=W(I,J+1,K)/DELY(J)
      H2=W(I,J,K)*(1.0E0/DELY(J-1)+1.0E0/DELY(J))
      H3=W(I,J-1,K)/DELY(J-1)
      H=2.0E0*(H1-H2+H3)/(DELY(J)+DELY(J-1))
      WTEE=-(A+B+C+D+REI*(E+F-G-H))
      WRITE(20,904) WTEE
   21 CONTINUE
C
C     CALCULATION ON THE BOTTOM OF THE CALCULATION REGION
C     J-1 TERMS HAVE BEEN ADJUSTED ON SYMMETRY ARGUMENTS
      DO 22 I=2,LE1
      DO 22 K=2,NE1
      J=1
C     CALCULATION OF UT FROM 3D MOMENTUM EQUATION
      A=(U(I+1,J,K)**2-U(I-1,J,K)**2)/(2.0E0*DELX)
      B1=U(I,J+1,K)*V(I,J+1,K)/(DELY(J)**2)
      B2=U(I,J,K)*V(I,J,K)*(1.0E0/DELY(J-1)**2-1.0E0/DELY(J)**2)
      B3=U(I,J+1,K)*V(I,J+1,K)/DELY(J)**2
      B3=-B3
      B=DELY(J)*DELY(J)*(B1+B2-B3)/(DELY(J)+DELY(J))
      C=(U(I,J,K+1)*W(I,J,K+1)-U(I,J,K-1)*W(I,J,K-1))/(2.0E0*DELZ)
      D=(PHI(I+1,J,K)-PHI(I-1,J,K))/(2.0E0*DELX)
      F2A=V(I+1,J+1,K)/DELY(J)**2
```

```
      E2B=V(I+1,J,K)*(1.0E0/DELY(J)**2-1.0E0/DELY(J)**2)
      E2C=V(I+1,J+1,K)/DELY(J)**2
      E2C=-E2C
      E2=DELY(J)*DELY(J)*(E2A+E2B-E2C)/(DELY(J)+DELY(J))
      E1A=V(I-1,J+1,K)/DELY(J)**2
      E1B=V(I-1,J,K)*(1.0E0/DELY(J)**2-1.0E0/DELY(J)**2)
      E1C=V(I-1,J+1,K)/DELY(J)**2
      E1C=-E1C
      E1=DELY(J)*DELY(J)*(E1A+E1B-E1C)/(DELY(J)+DELY(J))
      E=(E2-E1)/(2.0E0*DELX)
      F=(W(I+1,J,K+1)-W(I+1,J,K-1)-W(I-1,J,K+1)+W(I-1,J,K-1))/
     1(4.0E0*DELX*DELZ)
      G1=U(I,J+1,K)/DELY(J)
      G2=U(I,J,K)*(1.0E0/DELY(J)+1.0E0/DELY(J))
      G3=U(I,J+1,K)/DELY(J)
      G=2.0E0*(G1-G2+G3)/(DELY(J)+DELY(J))
      H=(U(I,J,K+1)-2.0E0*U(I,J,K)+U(I,J,K-1))/DELZ**2
      UTEE=-(A+B+C+D+REI*(E+F-G-H))
      WRITE(19,903) UTEE
C
C     CALCULATION OF WT FROM 3D MOMENTUM EQUATION
      A=(U(I+1,J,K)*W(I+1,J,K)-U(I-1,J,K)*W(I-1,J,K))/(2.0E0*DELX)
      B1=V(I,J+1,K)*W(I,J+1,K)/DELY(J)**2
      B2=V(I,J,K)*W(I,J,K)*(1.0E0/DELY(J)**2-1.0E0/DELY(J)**2)
      B3=V(I,J+1,K)*W(I,J+1,K)/DELY(J)**2
      B3=-B3
      B=DELY(J)*DELY(J)*(B1+B2-B3)/(DELY(J)+DELY(J))
      C=(W(I,J,K+1)**2-W(I,J,K-1)**2)/(2.0E0*DELZ)
      D=(PHI(I,J,K+1)-PHI(I,J,K-1))/(2.0E0*DELZ)
      E=(U(I+1,J,K+1)-U(I+1,J,K-1)-U(I-1,J,K+1)+U(I-1,J,K-1))/
     1(4.0E0*DELX*DELZ)
      F2A=V(I,J+1,K+1)/DELY(J)**2
      F2B=V(I,J,K+1)*(1.0E0/DELY(J)**2-1.0E0/DELY(J)**2)
      F2C=V(I,J+1,K+1)/DELY(J)**2
      F2C=-F2C
      F2=DELY(J)*DELY(J)*(F2A+F2B-F2C)/(DELY(J)+DELY(J))
      F1A=V(I,J+1,K-1)/DELY(J)**2
      F1B=V(I,J,K-1)*(1.0/DELY(J)**2-1.0/DELY(J)**2)
      F1C=V(I,J+1,K-1)/DELY(J)**2
      F1C=-F1C
      F1=DELY(J)*DELY(J)*(F1A+F1B-F1C)/(DELY(J)+DELY(J))
      F=(F2-F1)/(2.0E0*DELZ)
      G=(W(I+1,J,K)-W(I-1,J,K))/(2.0E0*DELX)
      H1=W(I,J+1,K)/DELY(J)
      H2=W(I,J,K)*(1.0E0/DELY(J)+1.0E0/DELY(J))
      H3=W(I,J+1,K)/DELY(J)
      H=2.0E0*(H1-H2+H3)/(DELY(J)+DELY(J))
      WTEE=-(A+B+C+D+REI*(E+F-G-H))
```

```
      WRITE(20,904) WTEE
   22 CONTINUE
C
C     CALCULATION ON THE TOP OF THE CALCULATION REGION
      DO 23 I=2,LE1
      DO 23 K=2,NE1
      J=ME
C     J+1 TERMS HAVE BEEN ADJUSTED WITH UNIFORMITY ARGUMENTS
C     CALCULATION OF UT FROM 3D MOMENTUM EQUATION
      A=(U(I+1,J,K)**2-U(I-1,J,K)**2)/(2.0E0*DELX)
      B1=U(I,J,K)*V(I,J,K)/(DELY(J)**2)
      B2=U(I,J,K)*V(I,J,K)*(1.0E0/DELY(J-1)**2-1.0E0/DELY(J)**2)
      B3=U(I,J-1,K)*V(I,J-1,K)/DELY(J-1)**2
      B=DELY(J)*DELY(J-1)*(B1+B2-B3)/(DELY(J)+DELY(J-1))
      C=(U(I,J,K+1)*W(I,J,K+1)-U(I,J,K-1)*W(I,J,K-1))/(2.0E0*DELZ)
      D=(PHI(I+1,J,K)-PHI(I-1,J,K))/(2.0E0*DELX)
      E2A=V(I+1,J,K)/DELY(J)**2
      E2B=V(I+1,J,K)*(1.0E0/DELY(J-1)**2-1.0E0/DELY(J)**2)
      E2C=V(I+1,J-1,K)/DELY(J-1)**2
      E2=DELY(J)*DELY(J-1)*(E2A+E2B-E2C)/(DELY(J)+DELY(J-1))
      E1A=V(I-1,J,K)/DELY(J)**2
      E1B=V(I-1,J,K)*(1.0E0/DELY(J-1)**2-1.0E0/DELY(J)**2)
      E1C=V(I-1,J-1,K)/DELY(J-1)**2
      E1=DELY(J)*DELY(J-1)*(E1A+E1B-E1C)/(DELY(J)+DELY(J-1))
      E=(E2-E1)/(2.0E0*DELX)
      F=(W(I+1,J,K+1)-W(I+1,J,K-1)-W(I-1,J,K+1)+W(I-1,J,K-1))/
     1(4.0E0*DELX*DELZ)
      G1=U(I,J,K)/DELY(J)
      G2=U(I,J,K)*(1.0E0/DELY(J-1)+1.0E0/DELY(J))
      G3=U(I,J-1,K)/DELY(J-1)
      G=2.0E0*(G1-G2+G3)/(DELY(J)+DELY(J-1))
      H=(U(I,J,K+1)-2.0E0*U(I,J,K)+U(I,J,K-1))/DELZ**2
      UTEE=-(A+B+C+D+REI*(E+F-G-H))
      WRITE(19,903) UTEE
C
C     CALCULATION OF WT FROM 3D MOMENTUM EQUATION
      A=(U(I+1,J,K)*W(I+1,J,K)-U(I-1,J,K)*W(I-1,J,K))/(2.0E0*DELX)
      B1=V(I,J,K)*W(I,J,K)/DELY(J)**2
      B2=V(I,J,K)*W(I,J,K)*(1.0E0/DELY(J-1)**2-1.0E0/DELY(J)**2)
      B3=V(I,J-1,K)*W(I,J-1,K)/DELY(J-1)**2
      B=DELY(J-1)*DELY(J)*(B1+B2-B3)/(DELY(J)+DELY(J-1))
      C=(W(I,J,K+1)**2-W(I,J,K-1)**2)/(2.0E0*DELZ)
      D=(PHI(I,J,K+1)-PHI(I,J,K-1))/(2.0E0*DELZ)
      E=(U(I+1,J,K+1)-U(I+1,J,K-1)-U(I-1,J,K+1)+U(I-1,J,K-1))/
     1(4.0E0*DELX*DELZ)
      F2A=V(I,J,K+1)/DELY(J)**2
      F2B=V(I,J,K+1)*(1.0E0/DELY(J-1)**2-1.0E0/DELY(J)**2)
      F2C=V(I,J-1,K+1)/DELY(J-1)**2
```

```
      F2=DELY(J)*DELY(J-1)*(F2A+F2B-F2C)/(DELY(J)+DELY(J-1))
      F1A=V(I,J,K-1)/DELY(J)**2
      F1B=V(I,J,K-1)*(1.0/DELY(J-1)**2-1.0/DELY(J)**2)
      F1C=V(I,J-1,K-1)/DELY(J-1)**2
      F1=DELY(J)*DELY(J-1)*(F1A+F1B-F1C)/(DELY(J)+DELY(J-1))
      F=(F2-F1)/(2.0E0*DELZ)
      G=(W(I+1,J,K)-W(I-1,J,K))/(2.0E0*DELX)
      H1=W(I,J,K)/DELY(J)
      H2=W(I,J,K)*(1.0E0/DELY(J-1)+1.0E0/DELY(J))
      H3=W(I,J-1,K)/DELY(J-1)
      H=2.0E0*(H1-H2+H3)/(DELY(J)+DELY(J-1))
      WTEE=-(A+B+C+D+REI*(E+F-G-H))
      WRITE(20,904) WTEE
   23 CONTINUE
C
C      CALCULATION ON THE CORNER
C     CALCULATION OF UT FROM 3D MOMENTUM EQUATION
      DO 24 I=2,LF1
      J=1
      K=1
C     J-1 AND K-1 TERMS HAVE BEEN ADJUSTED
      A=(U(I+1,J,K)**2-U(I-1,J,K)**2)/(2.0E0*DELX)
      B1=U(I,J+1,K)*V(I,J+1,K)/(DELY(J)**2)
      B2=U(I,J,K)*V(I,J,K)*(1.0E0/DELY(J)**2-1.0E0/DELY(J)**2)
      B3=U(I,J+1,K)*V(I,J+1,K)/DELY(J)**2
      B3=-B3
      B=DELY(J)*DELY(J)*(B1+B2-B3)/(DELY(J)+DELY(J))
      C=(U(I,J,K+1)*W(I,J,K+1)-U(I,J,K)*W(I,J,K))/(2.0E0*DELZ)
      D=(PHI(I+1,J,K)-PHI(I-1,J,K))/(2.0E0*DELX)
      E2A=V(I+1,J+1,K)/DELY(J)**2
      E2B=V(I+1,J,K)*(1.0E0/DELY(J)**2-1.0E0/DELY(J)**2)
      E2C=V(I+1,J+1,K)/DELY(J)**2
      E2C=-E2C
      E2=DELY(J)*DELY(J)*(E2A+E2B-E2C)/(DELY(J)+DELY(J))
      E1A=V(I-1,J+1,K)/DELY(J)**2
      E1B=V(I-1,J,K)*(1.0E0/DELY(J)**2-1.0E0/DELY(J)**2)
      E1C=V(I-1,J+1,K)/DELY(J)**2
      E1C=-E1C
      E1=DELY(J)*DELY(J)*(E1A+E1B-E1C)/(DELY(J)+DELY(J))
      E=(E2-E1)/(2.0E0*DELX)
      F=(W(I+1,J,K+1)-W(I+1,J,K)-W(I-1,J,K+1)+W(I-1,J,K))/
     1(4.0E0*DELX*DELZ)
      G1=U(I,J+1,K)/DELY(J)
      G2=U(I,J,K)*(1.0E0/DELY(J)+1.0E0/DELY(J))
      G3=U(I,J+1,K)/DELY(J)
      G=2.0E0*(G1-G2+G3)/(DELY(J)+DELY(J))
      H=(U(I,J,K+1)-2.0E0*U(I,J,K)+U(I,J,K))/DELZ**2
      UTEE=-(A+B+C+D+REI*(E+F-G-H))
```

```
      WRITE(19,903) UTEE
C
C     CALCULATION OF WT FROM 3D MOMENTUM EQUATION
      A=(U(I+1,J,K)*W(I+1,J,K)-U(I-1,J,K)*W(I-1,J,K))/(2.0E0*DELX)
      B1=V(I,J+1,K)*W(I,J+1,K)/DELY(J)**2
      B2=V(I,J,K)*W(I,J,K)*(1.0E0/DELY(J)**2-1.0E0/DELY(J)**2)
      B3=V(I,J+1,K)*W(I,J+1,K)/DELY(J)**2
      B3=-B3
      B=DELY(J)*DELY(J)*(B1+B2-B3)/(DELY(J)+DELY(J))
      C=(W(I,J,K+1)**2-W(I,J,K)**2)/(2.0E0*DELZ)
      D=(PHI(I,J,K+1)-PHI(I,J,K))/(2.0E0*DELZ)
      E=(U(I+1,J,K+1)-U(I+1,J,K)-U(I-1,J,K+1)+U(I-1,J,K))/
     1(4.0E0*DELX*DELZ)
      F2A=V(I,J+1,K+1)/DELY(J)**2
      F2B=V(I,J,K+1)*(1.0E0/DELY(J)**2-1.0E0/DELY(J)**2)
      F2C=V(I,J+1,K+1)/DELY(J)**2
      F2C=-F2C
      F2=DELY(J)*DELY(J)*(F2A+F2B-F2C)/(DELY(J)+DELY(J))
      F1A=V(I,J+1,K)/DELY(J)**2
      F1B=V(I,J,K)*(1.0/DELY(J)**2-1.0/DELY(J)**2)
      F1C=V(I,J+1,K)/DELY(J)**2
      F1C=-F1C
      F1=DELY(J)*DELY(J)*(F1A+F1B-F1C)/(DELY(J)+DELY(J))
      F=(F2-F1)/(2.0E0*DELZ)
      G=(W(I+1,J,K)-W(I-1,J,K))/(2.0E0*DELX)
      H1=W(I,J+1,K)/DELY(J)
      H2=W(I,J,K)*(1.0E0/DELY(J)+1.0E0/DELY(J))
      H3=W(I,J+1,K)/DELY(J)
      H=2.0E0*(H1-H2+H3)/(DELY(J)+DELY(J))
      WTEE=-(A+B+C+D+REI*(E+F-G-H))
      WRITE(20,904) WTEE
   24 CONTINUE
C
C     CALCULATION ON A CORNER
      DO 25 I=2,LF1
      J=1
      K=NE
C     J-1 AND K+1 TERMS HAVE BEEN ADJUSTED
C     CALCULATION OF UT FROM 3D MOMENTUM EQUATION
      A=(U(I+1,J,K)**2-U(I-1,J,K)**2)/(2.0E0*DELX)
      B1=U(I,J+1,K)*V(I,J+1,K)/(DELY(J)**2)
      B2=U(I,J,K)*V(I,J,K)*(1.0E0/DELY(J)**2-1.0E0/DELY(J)**2)
      B3=U(I,J+1,K)*V(I,J+1,K)/DELY(J)**2
      B3=-B3
      B=DELY(J)*DELY(J)*(B1+B2-B3)/(DELY(J)+DELY(J))
      C=(U(I,J,K)*W(I,J,K)-U(I,J,K-1)*W(I,J,K-1))/(2.0E0*DELZ)
      D=(PHI(I+1,J,K)-PHI(I-1,J,K))/(2.0E0*DELX)
      E2A=V(I+1,J+1,K)/DELY(J)**2
```

```
      E2B=V(I+1,J,K)*(1.0E0/DELY(J)**2-1.0E0/DELY(J)**2)
      E2C=V(I+1,J+1,K)/DELY(J)**2
      E2C=-E2C
      E2=DELY(J)*DELY(J)*(E2A+E2B-E2C)/(DELY(J)+DELY(J))
      E1A=V(I-1,J+1,K)/DELY(J)**2
      E1B=V(I-1,J,K)*(1.0E0/DELY(J)**2-1.0E0/DELY(J)**2)
      E1C=V(I-1,J+1,K)/DELY(J)**2
      E1C=-E1C
      E1=DELY(J)*DELY(J)*(E1A+E1B-E1C)/(DELY(J)+DELY(J))
      E=(E2-E1)/(2.0E0*DELX)
      F=(W(I+1,J,K)-W(I+1,J,K-1)-W(I-1,J,K)+W(I-1,J,K-1))/
     1(4.0E0*DELX*DELZ)
      G1=U(I,J+1,K)/DELY(J)
      G2=U(I,J,K)*(1.0E0/DELY(J)+1.0E0/DELY(J))
      G3=U(I,J+1,K)/DELY(J)
      G=2.0E0*(G1-G2+G3)/(DELY(J)+DELY(J))
      H=(U(I,J,K)-2.0E0*U(I,J,K)+U(I,J,K-1))/DELZ**2
      UTEE=-(A+B+C+D+REI*(E+F-G-H))
      WRITE(19,903) UTEE
C
C     CALCULATION OF WT FROM 3D MOMENTUM EQUATION
      A=(U(I+1,J,K)*W(I+1,J,K)-U(I-1,J,K)*W(I-1,J,K))/(2.0E0*DELX)
      B1=V(I,J+1,K)*W(I,J+1,K)/DELY(J)**2
      B2=V(I,J,K)*W(I,J,K)*(1.0E0/DELY(J)**2-1.0E0/DELY(J)**2)
      B3=V(I,J+1,K)*W(I,J+1,K)/DELY(J)**2
      B3=-B3
      B=DELY(J)*DELY(J)*(B1+B2-B3)/(DELY(J)+DELY(J))
      C=(W(I,J,K)**2-W(I,J,K-1)**2)/(2.0E0*DELZ)
      D=(PHI(I,J,K)-PHI(I,J,K-1))/(2.0E0*DELZ)
      E=(U(I+1,J,K)-U(I+1,J,K-1)-U(I-1,J,K)+U(I-1,J,K-1))/
     1(4.0E0*DELX*DELZ)
      F2A=V(I,J+1,K)/DELY(J)**2
      F2B=V(I,J,K)*(1.0E0/DELY(J)**2-1.0E0/DELY(J)**2)
      F2C=V(I,J+1,K)/DELY(J)**2
      F2C=-F2C
      F2=DELY(J)*DELY(J)*(F2A+F2B-F2C)/(DELY(J)+DELY(J))
      F1A=V(I,J+1,K-1)/DELY(J)**2
      F1B=V(I,J,K-1)*(1.0/DELY(J)**2-1.0/DELY(J)**2)
      F1C=V(I,J+1,K-1)/DELY(J)**2
      F1C=-F1C
      F1=DELY(J)*DELY(J)*(F1A+F1B-F1C)/(DELY(J)+DELY(J))
      F=(F2-F1)/(2.0E0*DELZ)
      G=(W(I+1,J,K)-W(I-1,J,K))/(2.0E0*DELX)
      H1=W(I,J+1,K)/DELY(J)
      H2=W(I,J,K)*(1.0E0/DELY(J)+1.0E0/DELY(J))
      H3=W(I,J+1,K)/DELY(J)
      H=2.0E0*(H1-H2+H3)/(DELY(J)+DELY(J))
      WTEE=-(A+B+C+D+REI*(E+F-G-H))
```

```
      WRITE(20,904) WTEE
   25 CONTINUE
C
C     CALCULATION OF UT FROM 3D MOMENTUM EQUATION
C     CALCULATION ON A CORNER
C     J+1 AND K-1 TERMS HAVE BEEN ADJUSTED
      DO 26 I=2,LE1
      J=ME
      K=1
      A=(U(I+1,J,K)**2-U(I-1,J,K)**2)/(2.0E0*DELX)
      B1=U(I,J,K)*V(I,J,K)/(DELY(J)**2)
      B2=U(I,J,K)*V(I,J,K)*(1.0E0/DELY(J-1)**2-1.0E0/DELY(J)**2)
      B3=U(I,J-1,K)*V(I,J-1,K)/DELY(J-1)**2
      B=DELY(J)*DELY(J-1)*(B1+B2-B3)/(DELY(J)+DELY(J-1))
      C=(U(I,J,K+1)*W(I,J,K+1)-U(I,J,K)*W(I,J,K))/(2.0E0*DELZ)
      D=(PHI(I+1,J,K)-PHI(I-1,J,K))/(2.0E0*DELX)
      E2A=V(I+1,J,K)/DELY(J)**2
      E2B=V(I+1,J,K)*(1.0E0/DELY(J-1)**2-1.0E0/DELY(J)**2)
      E2C=V(I+1,J-1,K)/DELY(J-1)**2
      E2=DELY(J)*DELY(J-1)*(E2A+E2B-E2C)/(DELY(J)+DELY(J-1))
      E1A=V(I-1,J,K)/DELY(J)**2
      E1B=V(I-1,J,K)*(1.0E0/DELY(J-1)**2-1.0E0/DELY(J)**2)
      E1C=V(I-1,J-1,K)/DELY(J-1)**2
      E1=DELY(J)*DELY(J-1)*(E1A+E1B-E1C)/(DELY(J)+DELY(J-1))
      E=(E2-E1)/(2.0E0*DELX)
      F=(W(I+1,J,K+1)-W(I+1,J,K)-W(I-1,J,K+1)+W(I-1,J,K))/
     1(4.0E0*DELX*DELZ)
      G1=U(I,J,K)/DELY(J)
      G2=U(I,J,K)*(1.0E0/DELY(J-1)+1.0E0/DELY(J))
      G3=U(I,J-1,K)/DELY(J-1)
      G=2.0E0*(G1-G2+G3)/(DELY(J)+DELY(J-1))
      H=(U(I,J,K+1)-2.0E0*U(I,J,K)+U(I,J,K))/DELZ**2
      UTEE=-(A+B+C+D+REI*(E+F-G-H))
      WRITE(19,903) UTEE
C
C     CALCULATION OF WT FROM 3D MOMENTUM EQUATION
      A=(U(I+1,J,K)*W(I+1,J,K)-U(I-1,J,K)*W(I-1,J,K))/(2.0E0*DELX)
      B1=V(I,J,K)*W(I,J,K)/DELY(J)**2
      B2=V(I,J,K)*W(I,J,K)*(1.0E0/DELY(J-1)**2-1.0E0/DELY(J)**2)
      B3=V(I,J-1,K)*W(I,J-1,K)/DELY(J-1)**2
      B=DELY(J-1)*DELY(J)*(B1+B2-B3)/(DELY(J)+DELY(J-1))
      C=(W(I,J,K+1)**2-W(I,J,K)**2)/(2.0E0*DELZ)
      D=(PHI(I,J,K+1)-PHI(I,J,K))/(2.0E0*DELZ)
      E=(U(I+1,J,K+1)-U(I+1,J,K)-U(I-1,J,K+1)+U(I-1,J,K))/
     1(4.0E0*DELX*DELZ)
      F2A=V(I,J,K+1)/DELY(J)**2
      F2B=V(I,J,K+1)*(1.0E0/DELY(J-1)**2-1.0E0/DELY(J)**2)
      F2C=V(I,J-1,K+1)/DELY(J-1)**2
```

```
      F2=DELY(J)*DELY(J-1)*(F2A+F2B-F2C)/(DELY(J)+DELY(J-1))
      F1A=V(I,J,K)/DELY(J)**2
      F1B=V(I,J,K)*(1.0/DELY(J-1)**2-1.0/DELY(J)**2)
      F1C=V(I,J-1,K)/DELY(J-1)**2
      F1=DELY(J)*DELY(J-1)*(F1A+F1B-F1C)/(DELY(J)+DELY(J-1))
      F=(F2-F1)/(2.0E0*DELZ)
      G=(W(I+1,J,K)-W(I-1,J,K))/(2.0E0*DELX)
      H1=W(I,J,K)/DELY(J)
      H2=W(I,J,K)*(1.0E0/DELY(J-1)+1.0E0/DELY(J))
      H3=W(I,J-1,K)/DELY(J-1)
      H=2.0E0*(H1-H2+H3)/(DELY(J)+DELY(J-1))
      WTEE=-(A+B+C+D+REI*(E+F-G-H))
      WRITE(20,904) WTEE
   26 CONTINUE
C
C     CALCULATION OF UT FROM 3D MOMENTUM EQUATION
C     CALCULATION ON A CORNER
C     J+1 AND K+1 TERMS HAVE BEEN ADJUSTED
      DO 27 I=2,LE1
      J=ME
      K=NE
      A=(U(I+1,J,K)**2-U(I-1,J,K)**2)/(2.0E0*DELX)
      B1=U(I,J,K)*V(I,J,K)/(DELY(J)**2)
      B2=U(I,J,K)*V(I,J,K)*(1.0E0/DELY(J-1)**2-1.0E0/DELY(J)**2)
      B3=U(I,J-1,K)*V(I,J-1,K)/DELY(J-1)**2
      B=DELY(J)*DELY(J-1)*(B1+B2-B3)/(DELY(J)+DELY(J-1))
      C=(U(I,J,K)*W(I,J,K)-U(I,J,K-1)*W(I,J,K-1))/(2.0E0*DELZ)
      D=(PHI(I+1,J,K)-PHI(I-1,J,K))/(2.0E0*DELX)
      E2A=V(I+1,J,K)/DELY(J)**2
      E2B=V(I+1,J,K)*(1.0E0/DELY(J-1)**2-1.0E0/DELY(J)**2)
      E2C=V(I+1,J-1,K)/DELY(J-1)**2
      E2=DELY(J)*DELY(J-1)*(E2A+E2B-E2C)/(DELY(J)+DELY(J-1))
      E1A=V(I-1,J,K)/DELY(J)**2
      E1B=V(I-1,J,K)*(1.0E0/DELY(J-1)**2-1.0E0/DELY(J)**2)
      E1C=V(I-1,J-1,K)/DELY(J-1)**2
      E1=DELY(J)*DELY(J-1)*(E1A+E1B-E1C)/(DELY(J)+DELY(J-1))
      E=(E2-E1)/(2.0E0*DELX)
      F=(W(I+1,J,K)-W(I+1,J,K-1)-W(I-1,J,K)+W(I-1,J,K-1))/
     1(4.0E0*DELX*DELZ)
      G1=U(I,J,K)/DELY(J)
      G2=U(I,J,K)*(1.0E0/DELY(J-1)+1.0E0/DELY(J))
      G3=U(I,J-1,K)/DELY(J-1)
      G=2.0E0*(G1-G2+G3)/(DELY(J)+DELY(J-1))
      H=(U(I,J,K)-2.0E0*U(I,J,K)+U(I,J,K-1))/DELZ**2
      UTEE=-(A+B+C+D+REI*(E+F-G-H))
      WRITE(19,903) UTEE
C
C     CALCULATION OF WT FROM 3D MOMENTUM EQUATION
```

```
      A=(U(I+1,J,K)*W(I+1,J,K)-U(I-1,J,K)*W(I-1,J,K))/(2.0E0*DELX)
      B1=V(I,J,K)*W(I,J,K)/DELY(J)**2
      B2=V(I,J,K)*W(I,J,K)*(1.0E0/DELY(J-1)**2-1.0E0/DELY(J)**2)
      B3=V(I,J-1,K)*W(I,J-1,K)/DELY(J-1)**2
      B=DELY(J-1)*DELY(J)*(B1+B2-B3)/(DELY(J)+DELY(J-1))
      C=(W(I,J,K)**2-W(I,J,K-1)**2)/(2.0E0*DELZ)
      D=(PHI(I,J,K)-PHI(I,J,K-1))/(2.0E0*DELZ)
      E=(U(I+1,J,K)-U(I+1,J,K-1)-U(I-1,J,K)+U(I-1,J,K-1))/
     1(4.0E0*DELX*DELZ)
      F2A=V(I,J,K)/DELY(J)**2
      F2B=V(I,J,K)*(1.0E0/DELY(J-1)**2-1.0E0/DELY(J)**2)
      F2C=V(I,J-1,K)/DELY(J-1)**2
      F2=DELY(J)*DELY(J-1)*(F2A+F2B-F2C)/(DELY(J)+DELY(J-1))
      F1A=V(I,J,K-1)/DELY(J)**2
      F1B=V(I,J,K-1)*(1.0/DELY(J-1)**2-1.0/DELY(J)**2)
      F1C=V(I,J-1,K-1)/DELY(J-1)**2
      F1=DELY(J)*DELY(J-1)*(F1A+F1B-F1C)/(DELY(J)+DELY(J-1))
      F=(F2-F1)/(2.0E0*DELZ)
      G=(W(I+1,J,K)-W(I-1,J,K))/(2.0E0*DELX)
      H1=W(I,J,K)/DELY(J)
      H2=W(I,J,K)*(1.0E0/DELY(J-1)+1.0E0/DELY(J))
      H3=W(I,J-1,K)/DELY(J-1)
      H=2.0E0*(H1-H2+H3)/(DELY(J)+DELY(J-1))
      WTEE=-(A+B+C+D+REI*(E+F-G-H))
      WRITE(20,904) WTEE
   27 CONTINUE
C
C CALCULATION AT ENDS OF THE REGION
      DO 28 J=1,ME
      DO 28 K=1,NE
      I=1
      UTEE=0.0E0
      WRITE(19,903) UTEE
      WTEE=0.0E0
      WRITE(20,903) WTEE
   28 CONTINUE
C
      DO 29 J=1,ME
      DO 29 K=1,NE
      I=LE
      UTEE=0.0E0
      WRITE(19,903) UTEE
      WTEE=0.0E0
      WRITE(20,903) WTEE
   29 CONTINUE
C
      IF(ITER.EQ.0) DELT=DELT2
      IF(ITER.EQ.1) DELT=DELT1
```

```
      IF(ITER.EQ.0) GO TO 402
      IF(ICALC.EQ.0) GO TO 402
      REWIND 16
      REWIND 18
      READ(16) U
      READ(18) W
  402 CONTINUE
      REWIND 19
      REWIND 20
C
C     READ VALUES OF UTEE AND WTEE AND COMPUTE NEW U AND W
C
      DO 410 I=2,LE1
      DO 410 J=2,ME1
      DO 410 K=2,NE1
      READ(19,903) UTEE
      U(I,J,K)=U(I,J,K)+UTEE*DELT
      READ(20,904) WTEE
      W(I,J,K)=W(I,J,K)+WTEE*DELT
  410 CONTINUE
C
      DO 411 I=2,LE1
      DO 411 J=2,ME1
      K=1
      READ(19,903) UTEE
      U(I,J,K)=U(I,J,K)+UTEE*DELT
      READ(20,904) WTEE
      W(I,J,K)=W(I,J,K)+WTEE*DELT
  411 CONTINUE
C
      DO 412 I=2,LE1
      DO 412 J=2,ME1
      K=NE
      READ(19,903) UTEE
      U(I,J,K)=U(I,J,K)+UTEE*DELT
      READ(20,904) WTEE
      W(I,J,K)=W(I,J,K)+WTEE*DELT
  412 CONTINUE
C
      DO 413 I=2,LE1
      DO 413 K=2,NE1
      J=1
      READ(19,903) UTEE
      U(I,J,K)=U(I,J,K)+UTEE*DELT
      READ(20,904) WTEE
      W(I,J,K)=W(I,J,K)+WTEE*DELT
  413 CONTINUE
C
```

```
      DO 414 I=2,LE1
      DO 414 K=2,NE1
      J=ME
      READ(19,903) UTEE
      U(I,J,K)=U(I,J,K)+UTEE*DELT
      READ(20,904) WTEE
      W(I,J,K)=W(I,J,K)+WTEE*DELT
  414 CONTINUE
C
      DO 415 I=2,LE1
      J=1
      K=1
      READ(19,903) UTEE
      U(I,J,K)=U(I,J,K)+UTEE*DELT
      READ(20,904) WTEE
      W(I,J,K)=W(I,J,K)+WTEE*DELT
  415 CONTINUE
C
      DO 416 I=2,LE1
      J=1
      K=NE
      READ(19,903) UTEE
      U(I,J,K)=U(I,J,K)+UTEE*DELT
      READ(20,904) WTEE
      W(I,J,K)=W(I,J,K)+WTEE*DELT
  416 CONTINUE
C
      DO 417 I=2,LE1
      J=ME
      K=1
      READ(19,903) UTEE
      U(I,J,K)=U(I,J,K)+UTEE*DELT
      READ(20,904) WTEE
      W(I,J,K)=W(I,J,K)+WTEE*DELT
  417 CONTINUE
C
      DO 418 I=2,LE1
      J=ME
      K=NE
      READ(19,903) UTEE
      U(I,J,K)=U(I,J,K)+UTEE*DELT
      READ(20,904) WTEE
      W(I,J,K)=W(I,J,K)+WTEE*DELT
  418 CONTINUE
C
      DO 419 J=1,ME
      DO 419 K=1,NE
      I=1
```

```
      U(I,J,K)=1.0E0
      W(I,J,K)=0.0E0
  419 CONTINUE
C
      DO 420 J=1,ME
      DO 420 K=1,NE
      U(LE,J,K)=3.0E0*U(LE1,J,K)-3.0E0*U(LE2,J,K)+U(LE3,J,K)
      W(LE,J,K)=3.0E0*W(LE1,J,K)-3.0E0*W(LE2,J,K)+W(LE3,J,K)
  420 CONTINUE
C
      DO 421 I=LP1,LP2
      DO 421 K=NP1,NP2
      J=1
      U(I,J,K)=0.0E0
      W(I,J,K)=0.0E0
  421 CONTINUE
C
C     CALCULATE V WITHIN REGION FROM 3D CONTINUITY
      DO 30 I=2,LE1
      DO 30 J=1,ME1
      DO 30 K=2,NE1
      UEX2=(U(I+1,J,K-1)+U(I+1,J+1,K-1) +U(I+1,J,K+1)+U(I+1,J+1,K+1))/
     14.0E0
      UEX1=(U(I-1,J,K-1)+U(I-1,J+1,K-1)+U(I-1,J,K+1)+U(I-1,J+1,K+1))/
     14.0E0
      UEX=(UEX2-UEX1)/(2.0E0*DELX)
      WZ2=(W(I+1,J,K+1)+W(I+1,J+1,K+1)+W(I-1,J,K+1)+W(I-1,J+1,K+1))/
     14.0E0
      WZ1=(W(I+1,J,K-1)+W(I+1,J+1,K-1)+W(I-1,J,K-1)+W(I-1,J+1,K-1))/
     14.0E0
      WZE=(WZ2-WZ1)/(2.0E0*DELZ)
      V(I,J+1,K)=V(I,J,K)-DELY(J)*(UEX+WZE)
   30 CONTINUE
C
C     CALCULATE V ON A SIDE FROM 3-D CONTINUITY EQUATION
C     K-1 TERMS HAVE BEEN ADJUSTED
      DO 31 I=2,LE1
      DO 31 J=1,ME1
      K=1
      UEX2=(U(I+1,J,K)+U(I+1,J+1,K) +U(I+1,J,K+1)+U(I+1,J+1,K+1))/
     14.0E0
      UEX1=(U(I-1,J,K)+U(I-1,J+1,K)+U(I-1,J,K+1)+U(I-1,J+1,K+1))/
     14.0E0
      UEX=(UEX2-UEX1)/(2.0E0*DELX)
      WZ2=(W(I+1,J,K+1)+W(I+1,J+1,K+1)+W(I-1,J,K+1)+W(I-1,J+1,K+1))/
     14.0E0
      WZ1=(W(I+1,J,K)+W(I+1,J+1,K)+W(I-1,J,K)+W(I-1,J+1,K))/
     14.0E0
```

```
      WZE=(WZ2-WZ1)/(2.0E0*DELZ)
      V(I,J+1,K)=V(I,J,K)-DELY(J)*(UEX+WZE)
   31 CONTINUE
C
C     CALCULATE V ON A SIDE FROM 3-D CONTINUITY EQUATION
C     K+1 TERMS AHVE BEEN ADJUSTED
      DO 32 I=2,LE1
      DO 32 J=1,ME1
      K=NE
      UEX2=(U(I+1,J,K-1)+U(I+1,J+1,K-1) +U(I+1,J,K)+U(I+1,J+1,K))/
     14.0E0
      UEX1=(U(I-1,J,K-1)+U(I-1,J+1,K-1)+U(I-1,J,K)+U(I-1,J+1,K))/
     14.0E0
      UEX=(UEX2-UEX1)/(2.0E0*DELX)
      WZ2=(W(I+1,J,K)+W(I+1,J+1,K)+W(I-1,J,K)+W(I-1,J+1,K))/
     14.0E0
      WZ1=(W(I+1,J,K-1)+W(I+1,J+1,K-1)+W(I-1,J,K-1)+W(I-1,J+1,K-1))/
     14.0E0
      WZE=(WZ2-WZ1)/(2.0E0*DELZ)
      V(I,J+1,K)=V(I,J,K)-DELY(J)*(UEX+WZE)
   32 CONTINUE
C
C     SET V EQUAL TO ZERO IN THE PLANE OF THE PLATE
      DO 34 I=1,LE
      DO 34 K=1,NE
      V(I,1,K)=0.0E0
   34 CONTINUE
C
C     SET UPSTREAM V EQUAL TO ZERO
      DO 35 J=1,ME
      DO 35 K=1,NE
      V(1,J,K)=0.0E0
   35 CONTINUE
C
C     EXTRAPOLATE DOWNSTREAM V FIELD
      DO 36 J=1,ME
      DO 36 K=1,NE
      V(LE,J,K)=3.0E0*V(LE1,J,K)-3.0E0*V(LE2,J,K)+V(LE3,J,K)
   36 CONTINUE
C
      WRITE(6,812)
  812 FORMAT(1H0,20X,'INTO CALCULATION POISSON')
  200 CONTINUE
C     CALCULATE PRESSURE FIELD FROM POISSONS EQUATION
      ERR=0.0E0
C
C     CALCULATION OF PRESSURE FROM 3D POISSON'S EQUATION
C     CALCULATION WITHIN REGION
```

```
      DO 37 I=2,LE1
      DO 37 J=2,ME1
      DO 37 K=2,NE1
      A1=(PHI(I+1,J,K)+PHI(I-1,J,K))/DELX**2
      A2=(PHI(I,J,K+1)+PHI(I,J,K-1))/DELZ**2
      A3=2.0E0*(PHI(I,J+1,K)/DELY(J)+PHI(I,J-1,K)/DELY(J-1))/
     1(DELY(J)+DELY(J-1))
      A=A1+A2+A3
      B=(U(I+1,J,K)**2-2.0E0*U(I,J,K)**2+U(I-1,J,K)**2)/DELX**2
      C1=V(I,J+1,K)**2/DELY(J)
      C2=(V(I,J,K)**2)*(1.0E0/DELY(J-1)+1.0/DELY(J))
      C3=V(I,J-1,K)**2/DELY(J-1)
      C=2.0E0*(C1-C2+C3)/(DELY(J)+DELY(J-1))
      D=(W(I,J,K+1)**2-2.0E0*W(I,J,K)**2+W(I,J,K-1)**2)/DELZ**2
      E2A=U(I+1,J+1,K)*V(I+1,J+1,K)/DELY(J)**2
      E2B=U(I+1,J,K)*V(I+1,J,K)*(1.0E0/DELY(J-1)**2-1.0E0/DELY(J)**2)
      E2C=U(I+1,J-1,K)*V(I+1,J-1,K)/DELY(J-1)**2
      E2=DELY(J)*DELY(J-1)*(E2A+E2B-E2C)/(DELY(J)+DELY(J-1))
      E1A=U(I-1,J+1,K)*V(I-1,J+1,K)/DELY(J)**2
      E1B=U(I-1,J,K)*V(I-1,J,K)*(1.0E0/DELY(J-1)**2-1.0E0/DELY(J)**2)
      E1C=U(I-1,J-1,K)*V(I-1,J-1,K)/DELY(J-1)**2
      E1=DELY(J)*DELY(J-1)*(E1A+E1B-E1C)/(DELY(J)+DELY(J-1))
      E=(E2-E1)/(2.0E0*DELX)
      F2A=V(I,J+1,K+1)*W(I,J+1,K+1)/DELY(J)**2
      F2B=V(I,J,K+1)*W(I,J,K+1)*(1.0E0/DELY(J-1)**2-1.0E0/DELY(J)**2)
      F2C=V(I,J-1,K+1)*W(I,J-1,K+1)/DELY(J-1)**2
      F2=DELY(J)*DELY(J-1)*(F2A+F2B-F2C)/(DELY(J)+DELY(J-1))
      F1A=V(I,J+1,K-1)*W(I,J+1,K-1)/DELY(J)**2
      F1B=V(I,J,K-1)*W(I,J,K-1)*(1.0E0/DELY(J-1)**2-1.0E0/DELY(J)**2)
      F1C=V(I,J-I,K-1)*W(I,J-1,K-1)/DELY(J-1)**2
      F1=DELY(J)*DELY(J-1)*(F1A+F1B-F1C)/(DELY(J)+DELY(J-1))
      F=(F2-F1)/(2.0E0*DELZ)
      G2=U(I+1,J,K+1)*W(I+1,J,K+1)+U(I-1,J,K-1)*W(I-1,J,K-1)
      G1=U(I-1,J,K+1)*W(I-1,J,K+1)+U(I+1,J,K-1)*W(I+1,J,K-1)
      G=(G2-G1)/(4.0E0*DELX*DELZ)
      H=(2.0E0/DELX**2+2.0E0/DELX**2+2.0E0/(DELY(J)*DELY(J-1)))
      APHI=(1.0E0-OMEGA)*PHI(I,J,K)+OMEGA*(A+B+C+D+2.0E0*(E+F+G))/H
      RESID=ABS(APHI-PHI(I,J,K))
      ERR=MAX1(ERR,RESID)
      PHI(I,J,K)=APHI
   37 CONTINUE
C
C     CALCULATION OF PRESSURE FROM 3D POISSON'S EQUATION
C     CALCULATION ON A SIDE
C     K-1 TERMS HAVE BEEN ADJUSTED
      DO 38 I=2,LE1
      DO 38 J=2,ME1
      K=1
```

```
      A1=(PHI(I+1,J,K)+PHI(I-1,J,K))/DELX**2
      A2=(PHI(I,J,K+1)+PHI(I,J,K))/DELZ**2
      A3=2.0E0*(PHI(I,J+1,K)/DELY(J)+PHI(I,J-1,K)/DELY(J-1))/
     1(DELY(J)+DELY(J-1))
      A=A1+A2+A3
      B=(U(I+1,J,K)**2-2.0E0*U(I,J,K)**2+U(I-1,J,K)**2)/DELX**2
      C1=V(I,J+1,K)**2/DELY(J)
      C2=(V(I,J,K)**2)*(1.0E0/DELY(J-1)+1.0/DELY(J))
      C3=V(I,J-1,K)**2/DELY(J-1)
      C=2.0E0*(C1-C2+C3)/(DELY(J)+DELY(J-1))
      D=(W(I,J,K+1)**2-2.0E0*W(I,J,K)**2+W(I,J,K)**2)/DELZ**2
      E2A=U(I+1,J+1,K)*V(I+1,J+1,K)/DELY(J)**2
      E2B=U(I+1,J,K)*V(I+1,J,K)*(1.0E0/DELY(J-1)**2-1.0E0/DELY(J)**2)
      E2C=U(I+1,J-1,K)*V(I+1,J-1,K)/DELY(J-1)**2
      E2=DELY(J)*DELY(J-1)*(E2A+E2B-E2C)/(DELY(J)+DELY(J-1))
      E1A=U(I-1,J+1,K)*V(I-1,J+1,K)/DELY(J)**2
      E1B=U(I-1,J,K)*V(I-1,J,K)*(1.0E0/DELY(J-1)**2-1.0E0/DELY(J)**2)
      E1C=U(I-1,J-1,K)*V(I-1,J-1,K)/DELY(J-1)**2
      E1=DELY(J)*DELY(J-1)*(E1A+E1B-E1C)/(DELY(J)+DELY(J-1))
      E=(E2-E1)/(2.0E0*DELX)
      F2A=V(I,J+1,K+1)*W(I,J+1,K+1)/DELY(J)**2
      F2B=V(I,J,K+1)*W(I,J,K+1)*(1.0E0/DELY(J-1)**2-1.0E0/DELY(J)**2)
      F2C=V(I,J-1,K+1)*W(I,J-1,K+1)/DELY(J-1)**2
      F2=DELY(J)*DELY(J-1)*(F2A+F2B-F2C)/(DELY(J)+DELY(J-1))
      F1A=V(I,J+1,K)*W(I,J+1,K)/DELY(J)**2
      F1B=V(I,J,K)*W(I,J,K)*(1.0E0/DELY(J-1)**2-1.0E0/DELY(J)**2)
      F1C=V(I,J-1,K)*W(I,J-1,K)/DELY(J-1)**2
      F1=DELY(J)*DELY(J-1)*(F1A+F1B-F1C)/(DELY(J)+DELY(J-1))
      F=(F2-F1)/(2.0E0*DELZ)
      G2=U(I+1,J,K+1)*W(I+1,J,K+1)+U(I-1,J,K)*W(I-1,J,K)
      G1=U(I-1,J,K+1)*W(I-1,J,K+1)+U(I+1,J,K)*W(I+1,J,K)
      G=(G2-G1)/(4.0E0*DELX*DELZ)
      H=(2.0E0/DELX**2+2.0E0/DELX**2+2.0E0/(DELY(J)*DELY(J-1)))
      APHI=(1.0E0-OMEGA)*PHI(I,J,K)+OMEGA*(A+B+C+D+2.0E0*(E+F+G))/H
      RESID=ABS(APHI-PHI(I,J,K))
      ERR=MAX1(ERR,RESID)
      PHI(I,J,K)=APHI
   38 CONTINUE
C
C     CALCULATION OF PRESSURE FROM 3D POISSON'S EQUATION
C     CALCULATION ON A SIDE
C     K+1 TERMS HAVE BEEN ADJUSTED
      DO 39 I=2,LE1
      DO 39 J=2,ME1
      K=NE
      A1=(PHI(I+1,J,K)+PHI(I-1,J,K))/DELX**2
      A2=(PHI(I,J,K)+PHI(I,J,K-1))/DELZ**2
      A3=2.0E0*(PHI(I,J+1,K)/DELY(J)+PHI(I,J-1,K)/DELY(J-1))/
```

```
     1(DELY(J)+DELY(J-1))
      A=A1+A2+A3
      B=(U(I+1,J,K)**2-2.0E0*U(I,J,K)**2+U(I-1,J,K)**2)/DELX**2
      C1=V(I,J+1,K)**2/DELY(J)
      C2=(V(I,J,K)**2)*(1.0E0/DELY(J-1)+1.0/DELY(J))
      C3=V(I,J-1,K)**2/DELY(J-1)
      C=2.0E0*(C1-C2+C3)/(DELY(J)+DELY(J-1))
      D=(W(I,J,K)**2-2.0E0*W(I,J,K)**2+W(I,J,K-1)**2)/DELZ**2
      E2A=U(I+1,J+1,K)*V(I+1,J+1,K)/DELY(J)**2
      E2B=U(I+1,J,K)*V(I+1,J,K)*(1.0E0/DELY(J-1)**2-1.0E0/DELY(J)**2)
      E2C=U(I+1,J-1,K)*V(I+1,J-1,K)/DELY(J-1)**2
      E2=DELY(J)*DELY(J-1)*(E2A+E2B-E2C)/(DELY(J)+DELY(J-1))
      E1A=U(I-1,J+1,K)*V(I-1,J+1,K)/DELY(J)**2
      E1B=J(I-1,J,K)*V(I-1,J,K)*(1.0E0/DELY(J-1)**2-1.0E0/DELY(J)**2)
      E1C=U(I-1,J-1,K)*V(I-1,J-1,K)/DELY(J-1)**2
      E1=DELY(J)*DELY(J-1)*(E1A+E1B-E1C)/(DELY(J)+DELY(J-1))
      E=(E2-E1)/(2.0E0*DELX)
      F2A=V(I,J+1,K)*W(I,J+1,K)/DELY(J)**2
      F2B=V(I,J,K)*W(I,J,K)*(1.0E0/DELY(J-1)**2-1.0E0/DELY(J)**2)
      F2C=V(I,J-1,K)*W(I,J-1,K)/DELY(J-1)**2
      F2=DELY(J)*DELY(J-1)*(F2A+F2B-F2C)/(DELY(J)+DELY(J-1))
      F1A=V(I,J+1,K-1)*W(I,J+1,K-1)/DELY(J)**2
      F1B=V(I,J,K-1)*W(I,J,K-1)*(1.0E0/DELY(J-1)**2-1.0E0/DELY(J)**2)
      F1C=V(I,J-1,K-1)*W(I,J-1,K-1)/DELY(J-1)**2
      F1=DELY(J)*DELY(J-1)*(F1A+F1B-F1C)/(DELY(J)+DELY(J-1))
      F=(F2-F1)/(2.0E0*DELZ)
      G2=U(I+1,J,K)*W(I+1,J,K)+U(I-1,J,K-1)*W(I-1,J,K-1)
      G1=U(I-1,J,K)*W(I-1,J,K)+U(I+1,J,K-1)*W(I+1,J,K-1)
      G=(G2-G1)/(4.0E0*DELX*DELZ)
      H=(2.0E0/DELX**2+2.0E0/DELX**2+2.0E0/(DELY(J)*DELY(J-1)))
      APHI=(1.0E0-OMEGA)*PHI(I,J,K)+OMEGA*(A+B+C+D+2.0E0*(E+F+G))/H
      RESID=ABS(APHI-PHI(I,J,K))
      ERR=MAX1(ERR,RESID)
      PHI(I,J,K)=APHI
   39 CONTINUE
C
C     CALCULATION OF PRESSURE FROM 3D POISSON'S EQUATION
C     CALCULATION ON TOP
C     J+1 TERMS HAVE BEEN ADJUSTED
      DO 40 I=2,LE1
      DO 40 K=2,NE1
      J=ME
      A1=(PHI(I+1,J,K)+PHI(I-1,J,K))/DELX**2
      A2=(PHI(I,J,K+1)+PHI(I,J,K-1))/DELZ**2
      A3=2.0E0*(PHI(I,J,K)/DELY(J)+PHI(I,J-1,K)/DELY(J-1))/
     1(DELY(J)+DELY(J-1))
      A=A1+A2+A3
      B=(U(I+1,J,K)**2-2.0E0*U(I,J,K)**2+U(I-1,J,K)**2)/DELX**2
```

```
      C1=V(I,J,K)**2/DELY(J)
      C2=(V(I,J,K)**2)*(1.0E0/DELY(J-1)+1.0/DELY(J))
      C3=V(I,J-1,K)**2/DELY(J-1)
      C=2.0E0*(C1-C2+C3)/(DELY(J)+DELY(J-1))
      D=(W(I,J,K+1)**2-2.0E0*W(I,J,K)**2+W(I,J,K-1)**2)/DELZ**2
      E2A=U(I+1,J,K)*V(I+1,J,K)/DELY(J)**2
      E2B=U(I+1,J,K)*V(I+1,J,K)*(1.0E0/DELY(J-1)**2-1.0E0/DELY(J)**2)
      E2C=U(I+1,J-1,K)*V(I+1,J-1,K)/DELY(J-1)**2
      E2=DELY(J)*DELY(J-1)*(E2A+E2B-E2C)/(DELY(J)+DELY(J-1))
      E1A=U(I-1,J,K)*V(I-1,J,K)/DELY(J)**2
      E1B=U(I-1,J,K)*V(I-1,J,K)*(1.0E0/DELY(J-1)**2-1.0E0/DELY(J)**2)
      E1C=U(I-1,J-1,K)*V(I-1,J-1,K)/DELY(J-1)**2
      E1=DELY(J)*DELY(J-1)*(E1A+E1B-E1C)/(DELY(J)+DELY(J-1))
      E=(E2-E1)/(2.0E0*DELX)
      F2A=V(I,J,K+1)*W(I,J,K+1)/DELY(J)**2
      F2B=V(I,J,K+1)*W(I,J,K+1)*(1.0E0/DELY(J-1)**2-1.0E0/DELY(J)**2)
      F2C=V(I,J-1,K+1)*W(I,J-1,K+1)/DELY(J-1)**2
      F2=DELY(J)*DELY(J-1)*(F2A+F2B-F2C)/(DELY(J)+DELY(J-1))
      F1A=V(I,J,K-1)*W(I,J,K-1)/DELY(J)**2
      F1B=V(I,J,K-1)*W(I,J,K-1)*(1.0E0/DELY(J-1)**2-1.0E0/DELY(J)**2)
      F1C=V(I,J-1,K-1)*W(I,J-1,K-1)/DELY(J-1)**2
      F1=DELY(J)*DELY(J-1)*(F1A+F1B-F1C)/(DELY(J)+DELY(J-1))
      F=(F2-F1)/(2.0E0*DELZ)
      G2=U(I+1,J,K+1)*W(I+1,J,K+1)+U(I-1,J,K-1)*W(I-1,J,K-1)
      G1=U(I-1,J,K+1)*W(I-1,J,K+1)+U(I+1,J,K-1)*W(I+1,J,K-1)
      G=(G2-G1)/(4.0E0*DELX*DELZ)
      H=(2.0E0/DELX**2+2.0E0/DELX**2+2.0E0/(DELY(J)*DELY(J-1)))
      APHI=(1.0E0-OMEGA)*PHI(I,J,K)+OMEGA*(A+B+C+D+2.0E0*(E+F+G))/H
      RESID=ABS(APHI-PHI(I,J,K))
      ERR=MAX1(ERR,RESID)
      PHI(I,J,K)=APHI
   40 CONTINUE
C
C     CALCULATION OF PRESSURE FROM 3D POISSON'S EQUATION
C     CALCULATION ON THE BOTTOM
C     J-1 TERMS AHVE BEEN ADJUSTED
      DO 41 I=2,LE1
      DO 41 K=2,NE1
      J=1
      A1=(PHI(I+1,J,K)+PHI(I-1,J,K))/DELX**2
      A2=(PHI(I,J,K+1)+PHI(I,J,K-1))/DELZ**2
      A3=2.0E0*(PHI(I,J+1,K)/DELY(J)+PHI(I,J+1,K)/DELY(J))/
     1(DELY(J)+DELY(J))
      A=A1+A2+A3
      B=(U(I+1,J,K)**2-2.0E0*U(I,J,K)**2+U(I-1,J,K)**2)/DELX**2
      C1=V(I,J+1,K)**2/DELY(J)
      C2=(V(I,J,K)**2)*(1.0E0/DELY(J)+1.0/DELY(J))
      C3=V(I,J+1,K)**2/DELY(J)
```

```
      C3=-C3
      C=2.0E0*(C1-C2+C3)/(DELY(J)+DELY(J))
      D=(W(I,J,K+1)**2-2.0E0*W(I,J,K)**2+W(I,J,K-1)**2)/DELZ**2
      E2A=U(I+1,J+1,K)*V(I+1,J+1,K)/DELY(J)**2
      E2B=U(I+1,J,K)*V(I+1,J,K)*(1.0E0/DELY(J)** 2-1.0E0/DELY(J)**2)
      E2C=U(I+1,J+1,K)*V(I+1,J+1,K)/DELY(J)**2
      E2C=-E2C
      E2=DELY(J)*DELY(J)*(E2A+E2B-E2C)/(DELY(J)+DELY(J))
      E1A=U(I-1,J+1,K)*V(I-1,J+1,K)/DELY(J)**2
      E1B=U(I-1,J,K)*V(I-1,J,K)*(1.0E0/DELY(J)**2-1.0E0/DELY(J)**2)
      E1C=U(I-1,J+1,K)*V(I-1,J+1,K)/DELY(J)**2
      E1C=-E1C
      E1=DELY(J)*DELY(J)*(E1A+E1B-E1C)/(DELY(J)+DELY(J))
      E=(E2-E1)/(2.0E0*DELX)
      F2A=V(I,J+1,K+1)*W(I,J+1,K+1)/DELY(J)**2
      F2B=V(I,J,K+1)*W(I,J,K+1)*(1.0E0/DELY(J)**2-1.0E0/DELY(J)**2)
      F2C=V(I,J+1,K+1)*W(I,J+1,K+1)/DELY(J)**2
      F2C=-F2C
      F2=DELY(J)*DELY(J)*(F2A+F2B-F2C)/(DELY(J)+DELY(J))
      F1A=V(I,J+1,K-1)*W(I,J+1,K-1)/DELY(J)**2
      F1B=V(I,J,K-1)*W(I,J,K-1)*(1.0E0/DELY(J)**2-1.0E0/DELY(J)**2)
      F1C=V(I,J+1,K-1)*W(I,J+1,K-1)/DELY(J)**2
      F1C=-F1C
      F1=DELY(J)*DELY(J)*(F1A+F1B-F1C)/(DELY(J)+DELY(J))
      F=(F2-F1)/(2.0E0*DELZ)
      G2=U(I+1,J,K+1)*W(I+1,J,K+1)+U(I-1,J,K-1)* W(I-1,J,K-1)
      G1=U(I-1,J,K+1)*W(I-1,J,K+1)+U(I+1,J,K-1)*W(I+1,J,K-1)
      G=(G2-G1)/(4.0E0*DELX*DELZ)
      H=(2.0E0/DELX**2+2.0E0/DELX**2+2.0E0/(DELY(J)*DELY(J)))
      APHI=(1.0E0-OMEGA)*PHI(I,J,K)+OMEGA*(A+B+C+D+2.0E0*(E+F+G))/H
      RESID=ABS(APHI-PHI(I,J,K))
      ERR=MAX1(ERR,RESID)
      PHI(I,J,K)=APHI
   41 CONTINUE
C
C     CALCULATION OF PRESSURE FROM 3D POISSON'S EQUATION
C     CALCULATION ON A CORNER
C     J-1 AND K-1 TERMS HAVE BEEN ADJUSTED
      DO 42 I=2,LE1
      J=1
      K=1
      A1=(PHI(I+1,J,K)+PHI(I-1,J,K))/DELX**2
      A2=(PHI(I,J,K+1)+PHI(I,J,K))/DELZ**2
      A3=2.0E0*(PHI(I,J+1,K)/DELY(J)+PHI(I,J+1,K)/DELY(J))/
     1(DELY(J)+DELY(J))
      A=A1+A2+A3
      B=(U(I+1,J,K)**2-2.0E0*U(I,J,K)**2+U(I-1,J,K)**2)/DELX**2
      C1=V(I,J+1,K)**2/DELY(J)
```

```
      C2=(V(I,J,K)**2)*(1.0E0/DELY(J)+1.0/DELY(J))
      C3=V(I,J+1,K)**2/DELY(J)
      C3=-C3
      C=2.0E0*(C1-C2+C3)/(DELY(J)+DELY(J))
      D=(W(I,J,K+1)**2-2.0E0*W(I,J,K)**2+W(I,J,K)**2)/DELZ**2
      E2A=U(I+1,J+1,K)*V(I+1,J+1,K)/DELY(J)**2
      E2B=U(I+1,J,K)*V(I+1,J,K)*(1.0E0/DELY(J)**2-1.0E0/DELY(J)**2)
      E2C=U(I+1,J+1,K)*V(I+1,J+1,K)/DELY(J)**2
      E2C=-E2C
      E2=DELY(J)*DELY(J)*(E2A+E2B-E2C)/(DELY(J)+DELY(J))
      E1A=U(I-1,J+1,K)*V(I-1,J+1,K)/DELY(J)**2
      E1B=U(I-1,J,K)*V(I-1,J,K)*(1.0E0/DELY(J)**2-1.0E0/DELY(J)**2)
      E1C=U(I-1,J+1,K)*V(I-1,J+1,K)/DELY(J)**2
      E1C=-E1C
      E1=DELY(J)*DELY(J)*(E1A+E1B-E1C)/(DELY(J)+DELY(J))
      E=(E2-E1)/(2.0E0*DELX)
      F2A=V(I,J+1,K+1)*W(I,J+1,K+1)/DELY(J)**2
      F2B=V(I,J,K+1)*W(I,J,K+1)*(1.0E0/DELY(J)**2-1.0E0/DELY(J)**2)
      F2C=V(I,J+1,K+1)*W(I,J+1,K+1)/DELY(J)**2
      F2C=-F2C
      F2=DELY(J)*DELY(J)*(F2A+F2B-F2C)/(DELY(J)+DELY(J))
      F1A=V(I,J+1,K)*W(I,J+1,K)/DELY(J)**2
      F1B=V(I,J,K)*W(I,J,K)*(1.0E0/DELY(J)**2-1.0E0/DELY(J)**2)
      F1C=V(I,J+1,K)*W(I,J+1,K)/DELY(J)**2
      F1C=-F1C
      F1=DELY(J)*DELY(J)*(F1A+F1B-F1C)/(DELY(J)+DELY(J))
      F=(F2-F1)/(2.0E0*DELZ)
      G2=U(I+1,J,K+1)*W(I+1,J,K+1)+U(I-1,J,K)*W(I-1,J,K)
      G1=U(I-1,J,K+1)*W(I-1,J,K+1)+U(I+1,J,K)*W(I+1,J,K)
      G=(G2-G1)/(4.0E0*DELX*DELZ)
      H=(2.0E0/DELX**2+2.0E0/DELX**2+2.0E0/(DELY(J)*DELY(J)))
      APHI=(1.0E0-OMEGA)*PHI(I,J,K)+OMEGA*(A+B+C+D+2.0E0*(E+F+G))/H
      RESID=ABS(APHI-PHI(I,J,K))
      ERR=MAX1(ERR,RESID)
      PHI(I,J,K)=APHI
   42 CONTINUE
C
C     CALCULATION OF PRESSURE FROM 3D POISSON'S EQUATION
C     CALCULATION ON A CORNER
C     J-1 AND K+1 TERMS HAVE BEEN ADJUSTED
      DO 43 I=2,LE1
      J=1
      K=NE
      A1=(PHI(I+1,J,K)+PHI(I-1,J,K))/DELX**2
      A2=(PHI(I,J,K)+PHI(I,J,K-1))/DELZ**2
      A3=2.0E0*(PHI(I,J+1,K)/DELY(J)+PHI(I,J+1,K)/DELY(J))/
     1(DELY(J)+DELY(J))
      A=A1+A2+A3
```

```
      B=(U(I+1,J,K)**2-2.0E0*U(I,J,K)**2+U(I-1,J,K)**2)/DELX**2
      C1=V(I,J+1,K)**2/DELY(J)
      C2=(V(I,J,K)**2)*(1.0E0/DELY(J)+1.0/DELY(J))
      C3=V(I,J+1,K)**2/DELY(J)
      C3=-C3
      C=2.0E0*(C1-C2+C3)/(DELY(J)+DELY(J))
      D=(W(I,J,K)**2-2.0E0*W(I,J,K)**2+W(I,J,K-1)**2)/DELZ**2
      E2A=U(I+1,J+1,K)*V(I+1,J+1,K)/DELY(J)**2
      E2B=U(I+1,J,K)*V(I+1,J,K)*(1.0E0/DELY(J)**2-1.0E0/DELY(J)**2)
      E2C=U(I+1,J+1,K)* V(I+1,J+1,K)/DELY(J)**2
      E2C=-E2C
      E2=DELY(J)*DELY(J)*(E2A+E2B-E2C)/(DELY(J)+DELY(J))
      E1A=U(I-1,J+1,K)*V(I-1,J+1,K)/DELY(J)**2
      E1B=U(I-1,J,K)*V(I-1,J,K)*(1.0E0/DELY(J)**2-1.0E0/DELY(J)**2)
      E1C=U(I-1,J+1,K)*V(I-1,J+1,K)/DELY(J)**2
      E1C=-E1C
      E1=DELY(J)*DELY(J)*(E1A+E1B-E1C)/(DELY(J)+DELY(J))
      E=(E2-E1)/(2.0E0*DELX)
      F2A=V(I,J+1,K)*W(I,J+1,K)/DELY(J)**2
      F2B=V(I,J,K)*W(I,J,K)*(1.0E0/DELY(J)**2-1.0E0/DELY(J)**2)
      F2C=V(I,J+1,K)*W(I,J+1,K)/DELY(J)**2
      F2C=-F2C
      F2=DELY(J)*DELY(J)*(F2A+F2B-F2C)/(DELY(J)+DELY(J))
      F1A=V(I,J+1,K-1)*W(I,J+1,K-1)/DELY(J)**2
      F1B=V(I,J,K-1)*W(I,J,K-1)*(1.0E0/DELY(J)**2-1.0E0/DELY(J)**2)
      F1C=V(I,J+1,K-1)*W(I,J+1,K-1)/DELY(J)**2
      F1C=-F1C
      F1=DELY(J)*DELY(J)*(F1A+F1B-F1C)/(DELY(J)+DELY(J))
      F=(F2-F1)/(2.0E0*DELZ)
      G2=U(I+1,J,K)*W(I+1,J,K)+U(I-1,J,K-1)*W(I-1,J,K-1)
      G1=U(I-1,J,K)*W(I-1,J,K)+U(I+1,J,K-1)*W(I+1,J,K-1)
      G=(G2-G1)/(4.0E0*DELX*DELZ)
      H=(2.0E0/DELX**2+2.0E0/DELX**2+2.0E0/(DELY(J)*DELY(J)))
      APHI=(1.0E0-OMEGA)*PHI(I,J,K)+OMEGA*(A+B+C+D+2.0E0*(E+F+G))/H
      RESID=ABS(APHI-PHI(I,J,K))
      ERR=MAX1(ERR,RESID)
      PHI(I,J,K)=APHI
   43 CONTINUE
C
C     CALCULATION OF PRESSURE FROM 3D POISSON'S EQUATION
C     CALCULATION ON A CORNER
C     J+1 AND K-1 TERMS HAVE BEEN ADJUSTED
      DO 44 I=2,LE1
      J=ME
      K=1
      A1=(PHI(I+1,J,K)+PHI(I-1,J,K))/DELX**2
      A2=(PHI(I,J,K+1)+PHI(I,J,K))/DELZ**2
      A3=2.0E0*(PHI(I,J,K)/DELY(J)+PHI(I,J-1,K)/DELY(J-1))/
```

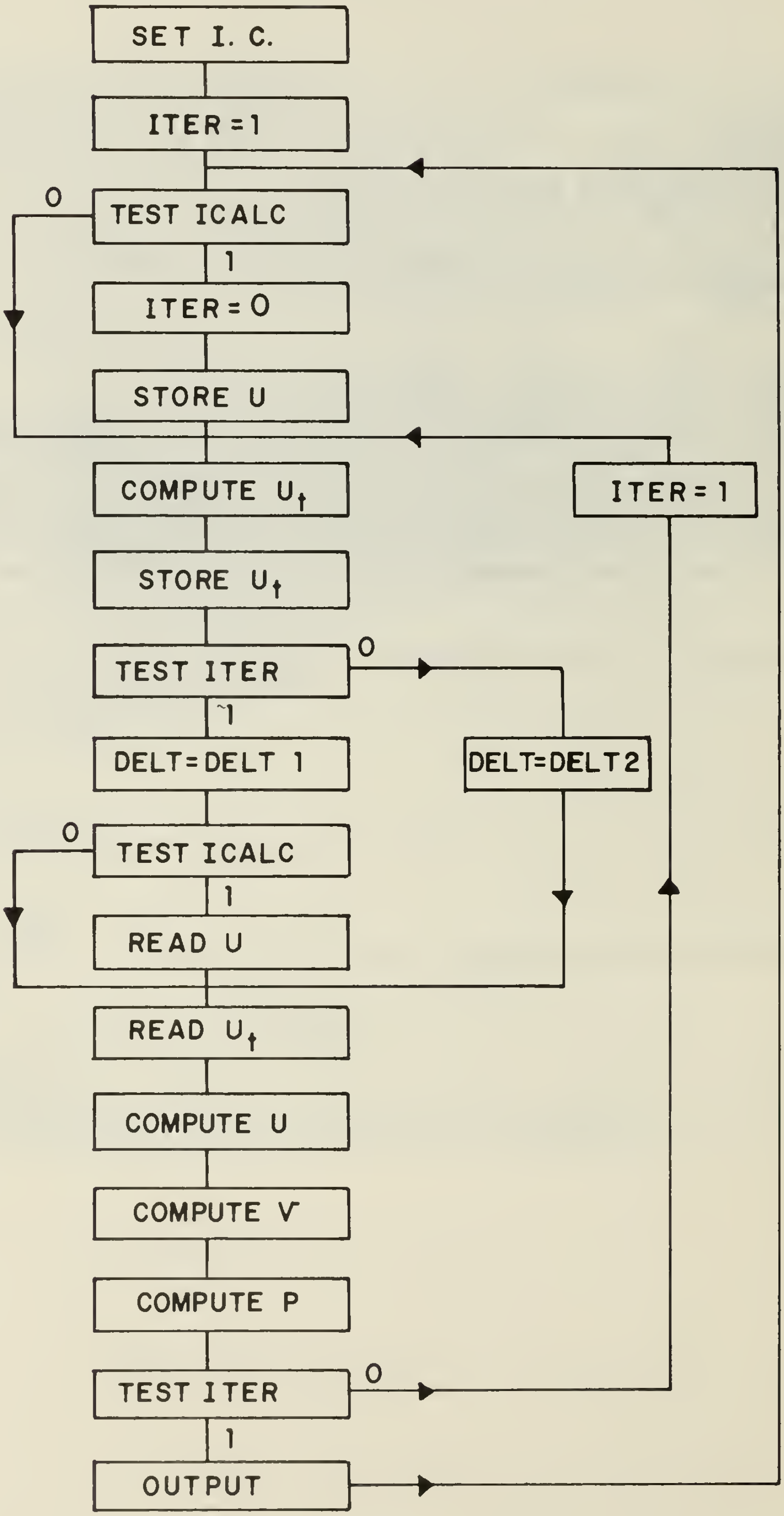
```
     1(DELY(J)+DELY(J-1))
      A=A1+A2+A3
      B=(U(I+1,J,K)**2-2.0E0*U(I,J,K)**2+U(I-1,J,K)**2)/DELX**2
      C2=(V(I,J,K)**2)*(1.0E0/DELY(J-1)+1.0/DELY(J))
      C3=V(I,J-1,K)**2/DELY(J-1)
      C=2.0E0*(C1-C2+C3)/(DELY(J)+DELY(J-1))
      D=(W(I,J,K+1)**2-2.0E0*W(I,J,K)**2+W(I,J,K)**2)/DELZ**2
      E2A=U(I+1,J,K)*V(I+1,J,K)/DELY(J)**2
      E2B=U(I+1,J,K)*V(I+1,J,K)*(1.0E0/DELY(J-1)**2-1.0E0/DELY(J)**2)
      E2C=U(I+1,J-1,K)*V(I+1,J-1,K)/DELY(J-1)**2
      E2=DELY(J)*DELY(J-1)*(E2A+E2B-E2C)/(DELY(J)+DELY(J-1))
      E1A=U(I-1,J,K)*V(I-1,J,K)/DELY(J)**2
      E1B=U(I-1,J,K)*V(I-1,J,K)*(1.0E0/DELY(J-1)**2-1.0E0/DELY(J)**2)
      E1C=U(I-1,J-1,K)*V(I-1,J-1,K)/DELY(J-1)**2
      E1=DELY(J)*DELY(J-1)*(E1A+E1B-E1C)/(DELY(J)+DELY(J-1))
      E=(E2-E1)/(2.0E0*DELX)
      F2A=V(I,J,K+1)*W(I,J,K+1)/DELY(J)**2
      F2B=V(I,J,K+1)*W(I,J,K+1)*(1.0E0/DELY(J-1)**2-1.0E0/DELY(J)**2)
      F2C=V(I,J-1,K+1)*W(I,J-1,K+1)/DELY(J-1)**2
      F2=DELY(J)*DELY(J-1)*(F2A+F2B-F2C)/(DELY(J)+DELY(J-1))
      F1A=V(I,J,K)*W(I,J,K)/DELY(J)**2
      F1B=V(I,J,K)*W(I,J,K)*(1.0E0/DELY(J-1)**2-1.0E0/DELY(J)**2)
      F1C=V(I,J-1,K)*W(I,J-1,K)/DELY(J-1)**2
      F1=DELY(J)*DELY(J-1)*(F1A+F1B-F1C)/(DELY(J)+DELY(J-1))
      F=(F2-F1)/(2.0E0*DELZ)
      G2=U(I+1,J,K+1)*W(I+1,J,K+1)+U(I-1,J,K)*W(I-1,J,K)
      G1=U(I-1,J,K+1)*W(I-1,J,K+1)+U(I+1,J,K)*W(I+1,J,K)
      G=(G2-G1)/(4.0E0*DELX*DELZ)
      H=(2.0E0/DELX**2+2.0E0/DELX**2+2.0E0/(DELY(J)*DELY(J-1)))
      APHI=(1.0E0-OMEGA)*PHI(I,J,K)+OMEGA*(A+B+C+D+2.0E0*(E+F+G))/H
      RESID=ABS(APHI-PHI(I,J,K))
      ERR=MAX1(ERR,RESID)
      PHI(I,J,K)=APHI
   44 CONTINUE
C
C     CALCULATION OF PRESSURE FROM 3D POISSON'S EQUATION
C     CALCULATION ON A CORNER
C     J+1 AND K+1 TERMS HAVE BEEN ADJUSTED
      DO 45 I=2,LE1
      J=ME
      K=NE
      A1=(PHI(I+1,J,K)+PHI(I-1,J,K))/DELX**2
      A2=(PHI(I,J,K)+PHI(I,J,K-1))/DELZ**2
      A3=2.0E0*(PHI(I,J,K)/DELY(J)+PHI(I,J-1,K)/DELY(J-1))/
     1(DELY(J)+DELY(J-1))
      A=A1+A2+A3
      B=(U(I+1,J,K)**2-2.0E0*U(I,J,K)**2+U(I-1,J,K)**2)/DELX**2
      C1=V(I,J,K)**2/DELY(J)
```

```
      C2=(V(I,J,K)**2)*(1.0E0/DELY(J-1)+1.0/DELY(J))
      C3=V(I,J-1,K)**2/DELY(J-1)
      C=2.0E0*(C1-C2+C3)/(DELY(J)+DELY(J-1))
      D=(W(I,J,K)**2-2.0E0*W(I,J,K)**2+W(I,J,K-1)**2)/DELZ**2
      E2A=U(I+1,J,K)*V(I+1,J,K)/DELY(J)**2
      E2B=U(I+1,J,K)*V(I+1,J,K)*(1.0E0/DELY(J-1)**2-1.0E0/DELY(J)**2)
      E2C=U(I+1,J-1,K)*V(I+1,J-1,K)/DELY(J-1)**2
      E2=DELY(J)*DELY(J-1)*(E2A+E2B-E2C)/(DELY(J)+DELY(J-1))
      E1A=U(I-1,J,K)*V(I-1,J,K)/DELY(J)**2
      E1B=U(I-1,J,K)*V(I-1,J,K)*(1.0E0/DELY(J-1)**2-1.0E0/DELY(J)**2)
      E1C=U(I-1,J-1,K)*V(I-1,J-1,K)/DELY(J-1)**2
      E1=DELY(J)*DELY(J-1)*(E1A+E1B-E1C)/(DELY(J)+DELY(J-1))
      E=(E2-E1)/(2.0E0*DELX)
      F2A=V(I,J,K)*W(I,J,K)/DELY(J)**2
      F2B=V(I,J,K)*W(I,J,K)*(1.0E0/DELY(J-1)**2-1.0E0/DELY(J)**2)
      F2C=V(I,J-1,K)*W(I,J-1,K)/DELY(J-1)**2
      F2=DELY(J)*DELY(J-1)*(F2A+F2B-F2C)/(DELY(J)+DELY(J-1))
      F1A=V(I,J,K-1)*W(I,J,K-1)/DELY(J)**2
      F1B=V(I,J,K-1)*W(I,J,K-1)*(1.0E0/DELY(J-1)**2-1.0E0/DELY(J)**2)
      F1C=V(I,J-1,K-1)*W(I,J-1,K-1)/DELY(J-1)**2
      F1=DELY(J)*DELY(J-1)*(F1A+F1B-F1C)/(DELY(J)+DELY(J-1))
      F=(F2-F1)/(2.0E0*DELZ)
      G2=U(I+1,J,K)*W(I+1,J,K)+U(I-1,J,K-1)*W(I-1,J,K-1)
      G1=U(I-1,J,K)*W(I-1,J,K)+U(I+1,J,K-1)*W(I+1,J,K-1)
      G=(G2-G1)/(4.0E0*DELX*DELZ)
      H=(2.0E0/DELX**2+2.0E0/DELX**2+2.0E0/(DELY(J)*DELY(J-1)))
      APHI=(1.0E0-OMEGA)*PHI(I,J,K)+OMEGA*(A+B+C+D+2.0E0*(E+F+G))/H
      RESID=ABS(APHI-PHI(I,J,K))
      ERR=MAX1(ERR,RESID)
      PHI(I,J,K)=APHI
   45 CONTINUE
C
      DO 46 J=1,ME
      DO 46 K=1,NE
      PHI(1,J,K)=3.0E0*PHI(2,J,K)-3.0E0*PHI(3,J,K)+PHI(4,J,K)
   46 CONTINUE
C
      DO 47 J=1,ME
      DO 47 K=1,NE
      PHI(LE,J,K)=3.0E0*PHI(LE1,J,K)-3.0E0*PHI(LE2,J,K)+PHI(LE3,J,K)
   47 CONTINUE
C
      IF(ERR.GT.ERRTOL) GO TO 200
C
      IF(IC.EQ.0) GO TO 150
      WRITE(6,813)
  813 FORMAT(1H0,20X,'OUT OF CALCULATION POISSON')
      IF(ITER.EQ.1) GO TO 49
```

```
      ITER=1
      GO TO 18
   49 T=T+DELT
      WRITE(6,51) T
   51 FORMAT(1H0,20X,'TIME =',E15.5)
      DO 52 J=1,50
      I=LE+1-J
      WRITE(6,50) U(LP1,J),U(LP2,J),U(J,1),U(I,1)
   50 FORMAT(1H0,4E20.5)
   52 CONTINUE
      GO TO 14
      END
C     /* CARD MUST APPPEAR HERE
//GO.FT06F001 DD SYSOUT=A,SPACE=(CYL,(5,1))
//GO.FT16F001 DD UNIT=SYSDA,SPACE=(CYL,(5,1),RLSE),
//   DCB=(RECFM=VS,LRECL=3504,BLKSIZE=3508),DISP=NEW,DSN=KOREAN1
//GO.FT18F001 DD UNIT=SYSDA,SPACE=(CYL,(5,1),RLSE),
//   DCB=(RECFM=VS,LRECL=3504,BLKSIZE=3508),DISP=NEW,DSN=KOREAN3
//GO.FT19F001 DD UNIT=SYSDA,SPACE=(CYL,(5,1),RLSE),
//   DCB=(RECFM=FB,LRECL=14,BLKSIZE=3500),DISP=NEW,DSN=KOREAN4
//GO.FT20F001 DD UNIT=SYSDA,SPACE=(CYL,(5,1),RLSE),
//   DCB=(RECFM=FB,LRECL=14,BLKSIZE=3500),DISP=NEW,DSN=KOREAN5
```

APPENDIX E

I. FLOW CHART FOR 2-D PROBLEM



```
C                              APPENDIX E
C
C  II. LISTING OF COMPUTER PROGRAM FOR 2-D CASE
C
C   THE FOLLOWING FORTRAN CODED FINITE DIFFERENCE COMPUTER PROGRAM
C  SOLVES THE FULL TIME DEPENDENT NAVIER STOKES EQUATIONS FOR THE
C  TWO DIMENSIONAL FLOW ABOUT A FINITE, INFINITELY THIN FLAT PLATE
C  IMPULSIVELY STARTED IN ITS OWN PLANE. "LEAP-FROG TIME-WISE INTEG-
C  RATION IS INCLUDED AS AN OPTION. EXTENSIVE DISC WRITING IS EMPLOY-
C  ED TO SAVE COMPUTER CORE SPACE. ADDITIONALLY, THE EQUATIONS
C THEMSELVES ARE APPLIED TO THE BOUNDARIES OF THE CALCULATION REGION
C  IN LIEU OF EXTRAPOLATING THE VARIABLES CALCULATED WITHIN THE
C  REGION TO THE BOUNDARIES.
C
C        THE FOLLOWING PARAMETERS MUST BE SPECIFIED
C         ICALC LE ME LP1 LP2 RE DELX DELY OMEGA ERRTOL PE QE
C
C         *************************************************
C         *                                               *
C         *  ICALC SPECIFES THE USE OF "LEAP-FROG" IN T   *
C         *                                               *
C         * LE IS THE LENGTH OF THE REGION IN DELX STEPS  *
C         *                                               *
C         * ME IS THE HEIGHT OF THE REGION IN DELY STEPS  *
C         *                                               *
C         *         LP1 IS THE START OF THE PLATE         *
C         *                                               *
C         *          LP2 IS THE END OF THE PLATE          *
C         *                                               *
C         *  RE IS THE REYNOLDS NUMBER BASED ON U AND L   *
C         *                                               *
C         *        DELX IS THE GRID SPACING IN X          *
C         *                                               *
C         *        DELY IS THE GRID SPACING IN Y          *
C         *                                               *
C         * OMEGA IS THE RELAXATION PARAMETER IN POISSONS *
C         *                                               *
C         *   ERRTOL IS THE ERROR TOLERANCE IN POISSONS   *
C         *                                               *
C         *     PE IS THE FREESTREAM PRESSURE IN PSF      *
C         *                                               *
C         * QE IS THE FREESTREAM DYNAMIC PRESSURE IN PSF  *
C         *                                               *
C         *      NOTE THAT DELY IS AN ARRAY DELY(ME)      *
C         *                                               *
C         *     ALL VARIABLES ARE DIMENSIONED F(LE,ME)    *
C         *                                               *
C         *************************************************
```

```
C
      DIMENSION U(100,50),V(100,50),PHI(100,50),DELY(50)
C
      T=0.0E0
C
      LE1=LE-1
      LE2=LE-2
      LE3=LE-3
C
      ME1=ME-1
      ME2=ME-2
      ME3=ME-3
C
      REI=1.0E0/RE
C
      PHIE=PE/(2.0E0*QE)
C
      A=1.0E0/DELX
      B=1.0E0/DELY(1)
      DELT=1.0E0/(A+0.1E0*B+2.0E0*REI*(A**2+B**2))
C
      DELT1=DELT
      DELT2=DELT1/2.0E0
C
      IC=0
C
C     INITIALIZE THE VARIABLES WITHIN THE CALCULATION REGION
      DO 2 I=1,LE
      DO 2 J=1,ME
      U(I,J)=1.0E0
      V(I,J)=0.0E0
      PHI(I,J)=PHIE
    2 CONTINUE
C
C     SET PLATE TO ZERO VELOCITY
      DO 3 I=LP1,LP2
      U(I,1)=0.0E0
    3 CONTINUE
C
C     CALCULATE V FROM CONTINUITY EQUATION
      DO 4 J=1,ME1
      DO 4 I=2,LE1
      C1=U(I+1,J+1)+U(I+1,J)
      C2=U(I-1,J+1)+U(I-1,J)
      V(I,J+1)=V(I,J)-DELY(J)*(C1-C2)/(4.0E0*DELX)
    4 CONTINUE
C
C     SET UPSTREAM VALUES FOR V VELOCITY
```

```
      DO 5 J=1,ME
      V(1,J)=0.0E0
    5 CONTINUE
C
C     EXTRAPOLATE DOWNSTREAM VALUES FOR V VELOCITY
      DO 6 J=1,ME
      V(LE,J)=3.0E0*V(LE1,J)-3.0E0*V(LE2,J)+V(LE3,J)
    6 CONTINUE
C
      WRITE(6,810)
  810 FORMAT(1H0,20X,'INTO IC POISSONS')
C
C     CALCULATE INITIAL P FIELD FROM POISSONS EQUATION
      GO TO 200
C
  150 IC=1
C
      WRITE(6,811)
  811 FORMAT(1H0,20X,'OUT OF IC POISSONS')
C
      ITER=1
   14 CONTINUE
      IF(ICALC.EQ.0) GO TO 18
      ITER=0
C
      REWIND 16
C     STORE VALUES OF PRESENT TIME ON TAPE
      WRITE(16) U
C
   18 CONTINUE
      REWIND 19
C
C     CALCULATE TIME DERIVATIVE OF U FROM MOMENTUM EQUATION
      DO 19 I=2,LE1
      DO 19 J=2,ME1
      A=(U(I+1,J)**2-U(I-1,J)**2)/(2.0E0*DELX)
      B1=U(I,J+1)*V(I,J+1)/DELY(J)**2
      B2=U(I,J)*V(I,J)*(1.0E0/DELY(J-1)**2-1.0E0/DELY(J)**2)
      B3=U(I,J-1)*V(I,J-1)/DELY(J-1)**2
      B=DELY(J)*DELY(J-1)*(B1+B2-B3)/(DELY(J)+DELY(J-1))
      C=(PHI(I+1,J)-PHI(I-1,J))/(2.0E0*DELX)
      D2A=V(I+1,J+1)/DELY(J)**2
      D2B=V(I+1,J)*(1.0E0/DELY(J-1)**2-1.0E0/DELY(J)**2)
      D2C=V(I+1,J-1)/DELY(J-1)**2
      D2=DELY(J)*DELY(J-1)*(D2A+D2B-D2C)/(DELY(J)+DELY(J-1))
      D1A=V(I-1,J+1)/DELY(J)**2
      D1B=V(I-1,J)*(1.0E0/DELY(J-1)**2-1.0E0/DELY(J)**2)
      D1C=V(I-1,J-1)/DELY(J-1)**2
```

```
      D1=DELY(J)*DELY(J-1)*(D1A+D1B-D1C)/(DELY(J)+DELY(J-1))
      D=(D2-D1)/(2.0E0*DELX)
      E1=U(I,J+1)/DELY(J)
      E2=U(I,J)*(1.0E0/DELY(J-1)+1.0E0/DELY(J))
      E3=U(I,J-1)/DELY(J-1)
      E=2.0E0*(E1-E2+E3)/(DELY(J)+DELY(J-1))
      UTEE=-(A+B+C+REI*(D-E))
      WRITE(19,903) UTEE
  903 FORMAT(1PE14.7)
   19 CONTINUE
C
C     CALCULATE TIME DERIVATIVE OF U FROM MOMENTUM EQUATION
      DO 22 I=2,LE1
      J=1
      A=(U(I+1,J)**2-U(I-1,J)**2)/(2.0E0*DELX)
      B1=U(I,J+1)*V(I,J+1)/DELY(J)**2
      B2=U(I,J)*V(I,J)*(1.0E0/DELY(J)**2-1.0E0/DELY(J)**2)
      B3=U(I,J+1)*V(I,J+1)/DELY(J)**2
      B3=-B3
      B=DELY(J)*DELY(J)*(B1+B2-B3)/(DELY(J)+DELY(J))
      C=(PHI(I+1,J)-PHI(I-1,J))/(2.0E0*DELX)
      D2A=V(I+1,J+1)/DELY(J)**2
      D2B=V(I+1,J)*(1.0E0/DELY(J)**2-1.0E0/DELY(J)**2)
      D2C=V(I+1,J+1)/DELY(J)**2
      D2C=-D2C
      D2=DELY(J)*DELY(J)*(D2A+D2B-D2C)/(DELY(J)+DELY(J))
      D1A=V(I-1,J+1)/DELY(J)**2
      D1B=V(I-1,J)*(1.0E0/DELY(J)**2-1.0E0/DELY(J)**2)
      D1C=V(I-1,J+1)/DELY(J)**2
      D1C=-D1C
      D1=DELY(J)*DELY(J)*(D1A+D1B-D1C)/(DELY(J)+DELY(J))
      D=(D2-D1)/(2.0E0*DELX)
      E1=U(I,J+1)/DELY(J)
      E2=U(I,J)*(1.0E0/DELY(J)+1.0E0/DELY(J))
      E3=U(I,J+1)/DELY(J)
      E=2.0E0*(E1-E2+E3)/(DELY(J)+DELY(J))
      UTEE=-(A+B+C+REI*(D-E))
      WRITE(19,903) UTEE
   22 CONTINUE
C
C     CALCULATE TIME DERIVATIVE OF U FROM MOMENTUM EQUATION
      DO 23 I=2,LE1
      J=ME
      A=(U(I+1,J)**2-U(I-1,J)**2)/(2.0E0*DELX)
      B1=U(I,J)*V(I,J)/DELY(J)**2
      B2=U(I,J)*V(I,J)*(1.0E0/DELY(J-1)**2-1.0E0/DELY(J)**2)
      B3=U(I,J-1)*V(I,J-1)/DELY(J-1)**2
      B=DELY(J)*DELY(J-1)*(B1+B2-B3)/(DELY(J)+DELY(J-1))
```

```
      C=(PHI(I+1,J)-PHI(I-1,J))/(2.0E0*DELX)
      D2A=V(I+1,J)/DELY(J)**2
      D2B=V(I+1,J)*(1.0E0/DELY(J-1)**2-1.0E0/DELY(J)**2)
      D2C=V(I+1,J-1)/DELY(J-1)**2
      D2=DELY(J)*DELY(J-1)*(D2A+D2B-D2C)/(DELY(J)+DELY(J-1))
      D1A=V(I-1,J)/DELY(J)**2
      D1B=V(I-1,J)*(1.0E0/DELY(J-1)**2-1.0E0/DELY(J)**2)
      D1C=V(I-1,J-1)/DELY(J-1)**2
      D1=DELY(J)*DELY(J-1)*(D1A+D1B-D1C)/(DELY(J)+DELY(J-1))
      D=(D2-D1)/(2.0E0*DELX)
      E1=U(I,J)/DELY(J)
      E2=U(I,J)*(1.0E0/DELY(J-1)+1.0E0/DELY(J))
      E3=U(I,J-1)/DELY(J-1)
      E=2.0E0*(E1-E2+E3)/(DELY(J)+DELY(J-1))
      UTEE=-(A+B+C+REI*(D-E))
      WRITE(19,903) UTEE
   23 CONTINUE
C
      DO 28 J=1,ME
      I=1
      UTEE=0.0E0
      WRITE(19,903) UTEE
   28 CONTINUE
C
      DO 29 J=1,ME
      I=LE
      UTEE=0.0E0
      WRITE(19,903) UTEE
   29 CONTINUE
C
      IF(ITER.EQ.0) DELT=DELT2
      IF(ITER.EQ.1) DELT=DELT1
      IF(ITER.EQ.0) GO TO 402
      IF(ICALC.EQ.0) GO TO 402
C
      REWIND 16
C     READ VALUES OF U FROM TAPE
      READ(16) U
C
  402 CONTINUE
C
      REWIND 19
C     READ VALUES OF UTEE FROM TAPE AND COMPUTE NEW U
      DO 410 I=2,LE1
      DO 410 J=2,ME1
      READ(19,903) UTEE
      U(I,J)=U(I,J)+UTEE*DELT
  410 CONTINUE
```

```
C
      DO 413 I=2,LE1
      J=1
      READ(19,903) UTEE
      U(I,J)=U(I,J)+UTEE*DELT
  413 CONTINUE
C
      DO 414 I=2,LE1
      J=ME
      READ(19,903) UTEE
      U(I,J)=U(I,J)+UTEE*DELT
  414 CONTINUE
C
      DO 419 J=1,ME
      I=1
      U(I,J)=1.0E0
  419 CONTINUE
C
      DO 415 I=LP1,LP2
      J=1
      U(I,J)=0.0E0
  415 CONTINUE
C
      DO 420 J=1,ME
      I=LE
      U(I,J)=3.0E0*U(LE1,J)-3.0E0*U(LE2,J)+U(LE3,J)
  420 CONTINUE
C
      DO 30 I=2,LE1
      DO 30 J=1,ME1
      UEX2=(U(I+1,J)+U(I+1,J+1))/2.0E0
      UEX1=(U(I-1,J)+U(I-1,J+1))/2.0E0
      UEX=(UEX2-UEX1)/(2.0E0*DELX)
      V(I,J+1)=V(I,J)-DELY(J)*UEX
   30 CONTINUE
C
      DO 34 I=1,LE
      V(I,1)=0.0E0
   34 CONTINUE
C
      DO 35 J=1,ME
      V(1,J)=0.0E0
   35 CONTINUE
C
      DO 36 J=1,ME
      I=LE
      V(I,J)=3.0E0*V(LE1,J)-3.0E0*V(LE2,J)+V(LE3,J)
   36 CONTINUE
```

```
C
      WRITE(6,812)
  812 FORMAT(1H0,20X,'INTO CALCULATION POISSON')
C
  200 CONTINUE
C
      ERR=0.0E0
C
C     CALCULATION OF PRESSURE FROM 2D POISSON'S EQUATION
      DO 37 I=2,LE1
      DO 37 J=2,ME1
      A1=(PHI(I+1,J)+PHI(I-1,J))/DELX**2
      A2=2.0E0*(PHI(I,J+1)/DELY(J)+PHI(I,J-1)/DELY(J-1))/
     1(DELY(J)+DELY(J-1))
      A=A1+A2
      B=(U(I+1,J)**2-2.0E0*U(I,J)**2+U(I-1,J)**2)/DELX**2
      C1=V(I,J+1)**2/DELY(J)
      C2=V(I,J)**2*(1.0E0/DELY(J-1)+1.0E0/DELY(J))
      C3=V(I,J-1)**2/DELY(J-1)
      C=2.0E0*(C1-C2+C3)/(DELY(J)+DELY(J-1))
      D2A=U(I+1,J+1)*V(I+1,J+1)/DELY(J)**2
      D2B=U(I+1,J)*V(I+1,J)*(1.0E0/DELY(J-1)**2-1.0E0/DELY(J)**2)
      D2C=U(I+1,J-1)*V(I+1,J-1)/DELY(J-1)**2
      D2=DELY(J)*DELY(J-1)*(D2A+D2B-D2C)/(DELY(J)+DELY(J-1))
      D1A=U(I-1,J+1)*V(I-1,J+1)/DELY(J)**2
      D1B=U(I-1,J)*V(I-1,J)*(1.0E0/DELY(J-1)**2-1.0E0/DELY(J)**2)
      D1C=U(I-1,J-1)*V(I-1,J-1)/DELY(J-1)**2
      D1=DELY(J)*DELY(J-1)*(D1A+D1B-D1C)/(DELY(J)+DELY(J-1))
      D=(D2-D1)/(2.0E0*DELX)
      H=2.0E0/DELX**2+2.0E0/(DELY(J)*DELY(J-1))
      APHI=(1.0E0-OMEGA)*PHI(I,J)+OMEGA*(A+B+C+2.0E0*D)/H
      RESID=ABS(APHI-PHI(I,J))
      ERR=MAX1(ERR,RESID)
      PHI(I,J)=APHI
   37 CONTINUE
C
C     CALCULATION OF PRESSURE FROM 2D POISSON'S EQUATION
      DO 40 I=2,LE1
      J=ME
      A1=(PHI(I+1,J)+PHI(I-1,J))/DELX**2
      A2=2.0E0*(PHI(I,J)/DELY(J)+PHI(I,J-1)/DELY(J-1))/
     1(DELY(J)+DELY(J-1))
      A=A1+A2
      B=(U(I+1,J)**2-2.0E0*U(I,J)+U(I-1,J)**2)/DELX**2
      C1=V(I,J)**2/DELY(J)
      C2=V(I,J)**2*(1.0E0/DELY(J-1)+1.0E0/DELY(J))
      C3=V(I,J-1)**2/DELY(J-1)
      C=2.0E0*(C1-C2+C3)/(DELY(J)+DELY(J-1))
```

```
      D2A=U(I+1,J)*V(I+1,J)/DELY(J)**2
      D2B=U(I+1,J)*V(I+1,J)*(1.0E0/DELY(J-1)**2-1.0E0/DELY(J)**2)
      D2C=U(I+1,J-1)*V(I+1,J-1)/DELY(J-1)**2
      D2=DELY(J)*DELY(J-1)*(D2A+D2B-D2C)/(DELY(J)+DELY(J-1))
      D1A=U(I-1,J)*V(I-1,J)/DELY(J)**2
      D1B=U(I-1,J)*V(I-1,J)*(1.0E0/DELY(J-1)**2-1.0E0/DELY(J)**2)
      D1C=U(I-1,J-1)*V(I-1,J-1)/DELY(J-1)**2
      D1=DELY(J)*DELY(J-1)*(D1A+D1B-D1C)/(DELY(J)+DELY(J-1))
      D=(D2-D1)/(2.0E0*DELX)
      H=2.0E0/DELX**2+2.0E0/(DELY(J)*DELY(J-1))
      APHI=(1.0E0-OMEGA)*PHI(I,J)+OMEGA*(A+B+C+2.0E0*D)/H
      RESID=ABS(APHI-PHI(I,J))
      ERR=MAX1(ERR,RESID)
      PHI(I,J)=APHI
   40 CONTINUE
C
C     CALCULATION OF PRESSURE FROM 2D POISSON'S EQUATION
      DO 41 I=2,LE1
      J=1
      A1=(PHI(I+1,J)+PHI(I-1,J))/DELX**2
      A2=2.0E0*(PHI(I,J+1)/DELY(J)+PHI(I,J+1)/DELY(J))/
     1(DELY(J)+DELY(J))
      A=A1+A2
      B=(U(I+1,J)**2-2.0E0*U(I,J)+U(I-1,J)**2)/DELX**2
      C1=V(I,J+1)**2/DELY(J)
      C2=V(I,J)**2*(1.0E0/DELY(J)+1.0E0/DELY(J))
      C3=V(I,J+1)**2/DELY(J)
      C3=-C3
      C=2.0E0*(C1-C2+C3)/(DELY(J)+DELY(J))
      D2A=U(I+1,J+1)*V(I+1,J+1)/DELY(J)**2
      D2B=U(I+1,J)*V(I+1,J)*(1.0E0/DELY(J)**2-1.0E0/DELY(J)**2)
      D2C=U(I+1,J+1)*V(I+1,J+1)/DELY(J)**2
      D2C=-D2C
      D2=DELY(J)*DELY(J)*(D2A+D2B-D2C)/(DELY(J)+DELY(J))
      D1A=U(I-1,J+1)*V(I-1,J+1)/DELY(J)**2
      D1B=U(I-1,J)*V(I-1,J)*(1.0E0/DELY(J)**2-1.0E0/DELY(J)**2)
      D1C=U(I-1,J+1)*V(I-1,J+1)/DELY(J)**2
      D1C=-D1C
      D1=DELY(J)*DELY(J)*(D1A+D1B-D1C)/(DELY(J)+DELY(J))
      D=(D2-D1)/(2.0E0*DELX)
      H=2.0E0/DELX**2+2.0E0/(DELY(J)*DELY(J))
      APHI=(1.0E0-OMEGA)*PHI(I,J)+OMEGA*(A+B+C+2.0E0*D)/H
      RESID=ABS(APHI-PHI(I,J))
      ERR=MAX1(ERR,RESID)
      PHI(I,J)=APHI
   41 CONTINUE
C
      DO 46 J=1,ME
```

```
      I=1
      PHI(I,J)=3.0E0*PHI(2,J)-3.0E0*PHI(3,J)+PHI(4,J)
   46 CONTINUE
C
      DO 47 J=1,ME
      I=LE
      PHI(I,J)=3.0E0*PHI(LE1,J)-3.0E0*PHI(LE2,J)+PHI(LE3,J)
   47 CONTINUE
C
      IF(ERR.GT.ERRTOL) GO TO 200
      IF(IC.EQ.0) GO TO 150
C
      WRITE(6,813)
  813 FORMAT(1H0,20X,'OUT OF CALCULATION POISSON')
C
      IF(ITER.EQ.1) GO TO 49
      ITER=1
C
      GO TO 18
C
   49 T=T+DELT
      WRITE(6,51) T
   51 FORMAT(1H0,20X,'TIME =',E15.5)
C
      DO 52 J=1,50
      I=LE+1-J
      WRITE(6,50) U(LP1,J),U(LP2,J),U(J,1),U(I,1)
   50 FORMAT(1H0,4E20.5)
   52 CONTINUE
C
      GO TO 14
      END
C     /* CARD MUST APPPEAR HERE
//GO.FT06F001 DD SYSOUT=A,SPACE=(CYL,(5,1))
//GO.FT16F001 DD UNIT=SYSDA,SPACE=(CYL,(5,1),RLSE),
//   DCB=(RECFM=VS,LRECL=3504,BLKSIZE=3508),DISP=NEW,DSN=KOREAN1
//GO.FT19F001 DD UNIT=SYSDA,SPACE=(CYL,(5,1),RLSE),
//   DCB=(RECFM=FB,LRECL=14,BLKSIZE=3500),DISP=NEW,DSN=KOREAN4
```

INITIAL DISTRIBUTION LIST

		No. Copies
1.	Defense Documentation Center Cameron Station Alexandria, Virginia 22314	20
2.	Library Naval Postgraduate School Monterey, California 93940	2
3.	Dr. John M. Wozencraft Dean of Research Naval Postgraduate School Monterey, California 93940	2
4.	Dr. Richard W. Bell Chairman, Department of Aeronautics Naval Postgraduate School Monterey, California 93940	1
5.	Professor Allen E. Fuhs Department of Aeronautics Naval Postgraduate School Monterey, California 93940	1
6.	Professor Gustave Hokenson Assistant Professor, Department of Aeronautics Naval Postgraduate School Monterey, California 93940	5

UNCLASSIFIED

Security Classification

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

ORIGINATING ACTIVITY (Corporate author) Naval Postgraduate School Monterey, California 93940	2a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED 2b. GROUP None

REPORT TITLE

Numerical Experiments with the Two- and Three-Dimensional Unsteady Navier-Stokes Equations

DESCRIPTIVE NOTES *(Type of report and, inclusive dates)*

Technical Report, 1973

AUTHOR(S) *(First name, middle initial, last name)*

Gustave J. Hokenson

REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO OF REFS
April 1973	81	11

a. CONTRACT OR GRANT NO.	9a. ORIGINATOR'S REPORT NUMBER(S) NPS-57Hw73041A
b. PROJECT NO.	
c.	9b. OTHER REPORT NO(S) *(Any other numbers that may be assigned this report)*
d.	

DISTRIBUTION STATEMENT

This document has been approved for public release and sale; its distribution is unlimited.

SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY

ABSTRACT

The two- and three-dimensional unsteady Navier-Stokes equations are solved numerically for the flow field about an impulsively started flat plate. In attempting to obtain an exact time dependent solution, several significant results were observed. First, with regard to the formulation of the differential equations themselves, it appears that Poisson's equation for the pressure field is a fundamental equation in as much as it allows us to solve for pressure moat exactly at any given time. Secondly, the difference equations must be carefully and consistently formulated. In this research, a non-uniform lateral grid, a unique interpretation of the continuity equation, and "leap frog" integration in time proved to be valuable techniques in obtaining an exact solution.

DD FORM 1 NOV 65 1473 (PAGE 1)

S/N 0101-807-6811

UNCLASSIFIED

Security Classification

A-31408

14 KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Numerical Analysis						
Fluid Flow						
Navier-Stokes Equation						