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## PROGRESS REPORT

## INVESTIGATION OF THE SPIN STABILITY OF THE NPS MINI-SATELLITE (ORION)

by
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Investigation of The Spin Stability of the The NPS Mini-Satellite (ORION)

Kilsoo Kim and Young S. Shin

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A preliminary design of the Naval Postgraduate School Mini-Satellite consists of a long cylindrical body and four radial booms and uses the single spin stabilization scheme for its attitude control. Stability conditions between design parameters for simple spinning motions of the satellite in space are investigated by applying Liapunov stability theory and using the natural modes as the admissible functions of internal coordinates. The stability diagram from this study shows that the internal stability conditions due to the flexibility of booms limit the angular velocity range of simple spins whlie the Maximum Axis Rule limit the minimum lengths of booms.

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## TABLE OF CONTENTS

LIST OF TABLES ..... 3
LIST OF FIGURES ..... 4
I. INTRODUCTION ..... 5
II. ANALYSIS ..... 6
II. 1 System Description ..... 6
II. 2 Stability Conditions ..... 8
II. 3 Stability of the NPS Mini-Satellite (ORION) ..... 12
III. CONCLUSIONS ..... 17
IV. REFERENCES ..... 18
FIGURES ..... 19
Distribution List ..... 27

## LIST OF TABLES

1. Parameters of NPS mini-satellite 13
2. Natural Frequencies of a Boom ....................... 14

## LIST OF FIGURES

1. The NPS Mini-Satellite (ORION) ..... 19
2. Random Vibration Levels in GAS Canisters ..... 20
3. Dynamic Model of The NPS Satellite During a Simple Spin Motion in Space ..... 21
4. The Finite Element Model of a Boom ..... 22
5. The Mode Shape of the First Mode ( $f=2.104 \mathrm{~Hz}$ ). ..... 23
6. The Second Mode $(f=4.097 \mathrm{~Hz})$ ..... 24
7. The Third Mode ( $f=23.43 \mathrm{~Hz}$ ) ..... 25
8. Stability Diagram of The NPS Mini-Satellite ..... 26

The Naval Postgraduate School has started a project to design and build a mini-satellite (ORION). This mini-satellite bus will support a small payload for various space science, technology and military missions. The first specific payload is not determined, yet, and therefore, its major design parameters, such as, launch vehicle, attitude stabilization scheme, are not known. A feasibility study and a preliminary design of the minisatellite were performed by Boyd (Ref. 1). In this preliminary design the launch of the satellite from the GAS (Get-Awayspecials) canister of a space Shuttle and single spin stabilization were assumed

This preliminary design, as defined in Ref. 1 , is investigated from the structural dynamics and attitude dynamics points of view. These include the survivability of the satellite under severe shock and vibration environment during launch periods, and the stability of the single spin attitude stabilization of the satellite. A rigorous investigation of the survivability satellite under shock and vibration environment is not possible in this preliminary design stage since the acceptible acceleration level of each electronic and mechanical components and detail design of connections and fasteners are not defined. However, it seems that the present design of bolts and nuts connection and no damping treatment (Figure 1) contains high failure probability areas considering the wide input frequency range $(20-2000 \mathrm{~Hz}$ ) of random vibration in GAS Canisters (Figure 2).

The main body of the NPS mini-satellite is long cylindrical shape (See Figure 1). The principal moment of inertia about the axial axis of symmetry is smaller than the principal moments of inertia about the radial axes of the main body without the four radial booms. It is well known that the attitude of a rigid body spinning about the axis of the minimum principal moment of inertia is unstable. Therefore, in the preliminary design of the NPS satellite, the lengths of booms and the masse of the magnetometer at the end of each boom are such that the total moment of inertia about the axial axis is the maximum, assuming the four radial booms are rigid. However, long booms are flexible and the flexibility of booms affects the attitude stability of a spinning body negatively. Therefore, each specific design of a spin stabilized satellite with flexible parts should be investigated to determine the stability conditions between design parameters.

Dynamic equations governing 3-dimensional motions of a body (especially a flexible body) in space are highly nonlinear and complicated. Therefore, a free or forced motion and the
stability of the motion of a body in 3-dimensional space can be best investigated by the computer simulation for each specific case using a highly capable computer code such as DISCOS (Ref. 2), or TREETOPS (Ref. 3). In this preliminary design stage, and without an available code, the stability of the simple spinning motion of the NPS mini-satellite about the axial axis of symmetry is investigated analytically in Chapter II. The analytical study of the spin stability of free motions of elastic disspative systems was performed previously by Teixeira-Filho and Kane (Ref. 4). The results of this study were applied to the stability studies of a spinning rigid satellite that carries four elastically mounted rigid antennas (Ref. 5) and of a spacecraft consisting of a rigid body and four elastic booms using finite number of beam elements (Ref. 6). In the present investigation of the stability of free spinning motion of the NPS minisatellite the basic results of the Ref. 4 are applied, using natural vibration modes of the booms as the admissble functions of internal coordinates.

## II. Analysis

II-1 system Description
The NPS mini-satellite $S$ to be analyzed consists of a rigid body $B$ of cylindrical shape and four flexible booms $C, D, E, F$, as shown in Figure 3 . $B^{*}$ is the mass center of $B$ and $X_{1}, X_{2}, X_{3}$ are the principal axes of inertia of $B$. When undeformed, boom $C$ and D lie on the opposite side of $B$ in the $X_{1}-X_{2}$ plane and are parallel to $X_{2}$ while booms $E$ and $F$ lie on opposite sides of $B$ in the $X_{1}-X_{3}$ plane and are parallel to $X_{3}$. The four booms are assumed to be rigidly attached on the cyclindrical surface of radius $R$ of body $B$ in the radial direction and are on the $X_{2}-X_{3}$ plane. Each boom has length $L$ and rectangle-tubular cross section of flexural rigidities $E I_{1}$ about axis $X_{1}$ and $E I_{2}$ about axis $X_{3}$ (for $C, D$ ) or about $X_{2}$ (for $E, F$ ).
$B$ has mass $M$ and the mass distribution in $B$ is assumed to be axisymmetric about $X_{1}$. Therefore among the principal moments of inertia $B_{1}, B_{2}, B_{3}$ of $B$ for the mass center $B *$ associated with $X_{1}, X_{2}, X_{3}$, respectively, $B_{2}$ and $B_{3}$ are the same. Each boom has
mass $m_{b}$ and the mass of a magnetometer attached at the tip of each boom is $m$.

The orientation of $B$, or the $X_{1}, X_{2}, X_{3}$, in a Newtonian reference frame $N$ can be specified in terms of three external coordinates, $\theta_{1}, \theta_{2},{ }_{3}$, Let $\theta_{1}, \theta_{2}, \theta_{3}$ be Euler angles such that $X_{1}, X_{2}, X_{3}$ coordinates system is brought into an orientation in $N$ by successive rotations $\theta_{1}, \theta_{2}, \theta_{3}$ about $X_{1}, X_{2}, X_{3}$ from the $\mathrm{N}_{1}, \mathrm{~N}_{2}, \mathrm{~N}_{3}$ direction.

The deformed configuration of the satellite is described using $4 n$ internal coordinates $q_{1}, q_{2}, \ldots q_{4 n}$ in addition to the external coordinates. The general deformation of a boom $C$ is described using the $n$ internal coordinates.

$$
\begin{equation*}
\mathrm{W}_{\mathrm{c}}(\mathrm{x}, \mathrm{t})=\sum_{\mathrm{i}=1}^{\mathrm{n}} \phi_{\mathrm{i}}(\mathrm{x}) \mathrm{q}_{\mathrm{i}}(\mathrm{t}) \tag{1}
\end{equation*}
$$

where $W_{C}(x, t)$ is the deflection of a boom cross-section at position $x$ from the boom root in $X_{1} X_{2} X_{3}$ coordinates, and $i_{i}$ $i=1,2, \ldots n$ are admissible functions chosen to be the same as the normal modes of the boom as a cantilever beam with an end mass in this analysis. The general deformation of booms $D, E, F$, are described using the same form of equation (I) with $q_{i}$, $i=$ $n+1, \ldots, 2 n$ for boom $D$, with $q_{i}, i=2 n+1, \ldots, 3 n$ for boom $E$, with $q_{i}, i=3 n+1, \ldots, 4 n$ for boom $F$, respectively.

It is assumed that no external forces and moments are applied. Due to the internal forces of interaction between the particles of systems, it is presumed that energy dissipation can be described by functions $D_{r}(q, \dot{q})$ which are functions of $q_{1}, \ldots, q_{4 n}, \dot{q}_{1}, \ldots, \dot{q}_{4 n}$ such that

$$
\begin{array}{ll}
D_{r}(q, 0)=0, & r=1,2, \ldots, 4 n \\
D_{r}(q, \dot{q}) \dot{q}_{r} \leqq 0 & \tag{3}
\end{array}
$$

where the equality sign of equation (3) holds only when $\dot{q}_{1}, \ldots, \dot{q}_{4 n}$ all vanish.

## I-2 Stability Conditions

In torque-free motion, the satellite system $S$ can move such that the angular velocity of $B$ in the Newtonian reference frame $N$ has a constant magnitude $\Omega$ and is parallel to $X_{1}$ which remains fixed in $N$ while the booms remain normal to $X_{1}$. This motion is defined as a simple spin.

A simple spin is called stable if the system resumes this motion, or acquires a motion closely resembling it, subsequent to every sufficiently small disturbance that leaves the angular momentum unaltered. If the system returns to the original simple spin state after a disturbance as time $t$ approaches infinity, then the simple spin is said to be asymptotically stable.

Stability conditions can be established by stating conditions under which the function $F(\theta, q, \dot{q})$ defined as

$$
\begin{equation*}
F \triangleq 2\left(E-E_{0}\right) \tag{4}
\end{equation*}
$$

is a Liapunov function (Ref. 7), where $E$ is the total energy of the system and $E_{0}$ is the total energy of the system during the simple spin. It was shown in Ref. 4 that for a simple spin of an axisymmetric system,

$$
\begin{equation*}
F=\frac{H^{2}}{I_{s}^{o}}\left\{\frac{I_{s}^{o}-I_{t}^{o}}{I_{t}^{o}}\left(\theta_{2}^{2}+\theta_{3}^{2}\right)\right\}+Z_{, r s}^{o} q_{r} q_{s}+M_{r s}^{o} q_{r} q_{s}+0_{3}(\theta, q, \dot{q}) \tag{5}
\end{equation*}
$$

where $H$ is the magnitude of angular momentum, $I_{s}$ and $I_{t}$ are the principal moment of inertia of the system about the axial axis and about the transverse axis, respectively, $0_{3}(\theta, q, \dot{q})$ indicates terms of third or higher degree in its arguments, and superscript o means its value at the simple spin state.
$M_{r s}$ is an element of an inertia like matrix defined as

$$
\begin{gather*}
\mathrm{M}_{\mathrm{rs}}=<\mathrm{mp},_{\mathrm{r}} \cdot \mathrm{p},,_{\mathrm{s}}>-<m p \times p,>\cdot \mathrm{I}^{-1} \cdot<\mathrm{mp} \times \mathrm{p},>  \tag{6}\\
\mathrm{r}, \mathrm{~s}=1,2, \ldots \ldots \ldots \ldots, 4 \mathrm{n}
\end{gather*}
$$

where the symbol < > denote summation over all particles of the system $S, p$ is the position vector of a particle relative to the mass center of $F_{, ~} p_{r}$ indicates its derivative about $q_{r}, m$ is the mass of the particle, and $I^{-1}$ is the inverse of the inertia dyadic I.

Z,rs is a second derivative of an energylike function $Z(q)$ with respect to $q_{r}$ and $q_{s}$, which is defined as

$$
\begin{equation*}
\mathrm{Z}(\mathrm{q})=\mathrm{V}(\mathrm{q})+\frac{1}{2} \mathrm{H}^{2} \mathrm{I}^{-1}(\mathrm{q}) \tag{7}
\end{equation*}
$$

where $V(q)$ is the potential energy function of $S$.
From the Liapunov stability theory, the stability conditions are

$$
\begin{equation*}
\dot{\mathrm{F}} \leqq 0 \tag{8}
\end{equation*}
$$

with equality sign holds only when $\dot{q}_{1}, \ldots, \dot{q}_{4 n}$ all vanish,

$$
\begin{equation*}
I_{s}^{0}-I_{t}^{0}>0 \tag{9}
\end{equation*}
$$

and

$$
\begin{equation*}
\mathrm{z}^{\circ} \mathrm{rs} \text { is positive definite. } \tag{10}
\end{equation*}
$$

The first condition, equation (8) is satisfied from the energy dissipation, equations (2) and (3). The second condition, equation (9), is the well known "Maximum Axis Rule" of the stability condition of a rigid body, and can be called the "external stability condition" of a deformable body system. This condition can be satisfied when the boom length $L$ is longer than the critical length $L_{\text {min }}$, which is about 60 inches for the NPS mini-satellite (ORION).

The third stability condition, equation (10), is due to the
flexible deformation of booms and can be called the "internal stability condition". It is shown in Ref. 4 that for a stressfree spin about axis $X_{1}$, that is,

$$
\begin{equation*}
\mathrm{V}^{\mathrm{o}}{ }_{\mathrm{r}}=0 \quad ; \quad \mathrm{r}=1,2, \ldots \ldots \ldots, 4 \mathrm{n} \tag{11}
\end{equation*}
$$

or

$$
\begin{equation*}
I_{11, r}^{0}=0 \quad ; \quad r=1,2, \ldots \ldots \ldots, 4 n \tag{12}
\end{equation*}
$$

from the associated eqations governing the internal coordinates, $Z_{i}^{\circ}$ rs can be expressed as follows with $H$ replaced by $I_{11}{ }^{\circ} \Omega$

$$
\begin{equation*}
Z_{r s}=V^{\mathrm{o}},{ }_{\mathrm{rs}}-\Omega^{2}\left\{\frac{1}{\mathrm{I}_{\mathrm{s}}^{\mathrm{o}}-\mathrm{I}_{\mathrm{t}}^{\mathrm{o}}}\left(\mathrm{I}_{12, \mathrm{r}}^{\mathrm{o}} \mathrm{I}_{12, \mathrm{~s}}^{\mathrm{o}}+\mathrm{I}_{13, \mathrm{r}}^{\mathrm{o}} \mathrm{I}_{13, \mathrm{~s}}^{\mathrm{o}}\right)+\frac{1}{2} \mathrm{I}_{11, \mathrm{rs}}^{\mathrm{o}}\right\} ; \quad \mathrm{r}=1,2, \ldots 4 n \tag{13}
\end{equation*}
$$

The simple spin motion of the NPS satelite system $S$ is a stress-free spin, since it satisfies the condition, equation (11) or equation (12), by choosing the internal coordinates as in equation (1). Moreover, by this choice of internal coordinates using the normal vibration modes of the cantilevered boom, the components of $\mathrm{Z}^{\circ}$ rs in equation (13) can be easily calculated, using the orthonormal relations between normal modes of the boom:

$$
\begin{align*}
& m \phi_{r}(\mathrm{~L}) \phi_{s}(\mathrm{~L})+\int_{\mathrm{L}} \phi_{\mathrm{r}}(\mathrm{x}) \phi_{\mathrm{s}}(\mathrm{x}) \mu \mathrm{dx}=\delta_{\mathrm{rs}} \mathrm{~m}_{\mathrm{r}}  \tag{14}\\
& \int_{\mathrm{L}} E I \phi_{\mathrm{r}}^{\prime \prime}(\mathrm{x}) \phi_{\mathrm{s}}^{\prime \prime}(\mathrm{x}) \mathrm{dx}=\delta_{\mathrm{r}} \mathrm{k}_{\mathrm{rr}} \tag{15}
\end{align*}
$$

where $\delta_{r s}$ is the Kronecker delta of $r, s=1,2, \ldots, n$, and $m_{r r}$ and $k_{r r} r=1,2, \ldots, n$, are modal masses and modal stiffnesses of the vibration modes, $\mu$ is the mass per unit boom length.

$$
\begin{align*}
V^{0}{ }_{r \mathrm{rs}} & =\mathrm{k}_{\mathrm{rr}} \delta_{\mathrm{rs}}+\mathrm{K}_{\mathrm{G}} \\
& =m_{\mathrm{rr}} \omega_{\mathrm{r}}^{2} \delta_{\mathrm{rs}}+\Omega^{2} \int_{\mathrm{R}}^{\mathrm{R}+\mathrm{L}} \frac{\mathrm{x}}{2}\{\mathrm{~m}+(\mathrm{R}+\mathrm{L}-\mathrm{x})\} \dot{\phi}_{\mathrm{r}}^{\prime}(\mathrm{x}) \dot{\phi}_{\mathrm{s}}^{\prime}(\mathrm{x}) \mathrm{dx} \tag{16}
\end{align*}
$$

where $K_{G}$ is the geometric stiffness of the boom due to the centrifugal force of the spin, and $\omega_{r}$ is the natural frequency of the $r$ mode.

$$
\begin{equation*}
\mathrm{I}_{12, \mathrm{r}}^{0}=\int_{\mathrm{R}}^{\mathrm{R}+\mathrm{L}} \mathrm{x} \phi_{\mathrm{r}}(\mathrm{x}) \mu \mathrm{dx}+\mathrm{L} \phi_{\mathrm{r}}(\mathrm{~L}) \mathrm{m} \tag{17}
\end{equation*}
$$

$r=$ vibration modes in $X_{1}-X_{2}$ plane for booms $C$ and $D$

$$
\begin{equation*}
\mathrm{I}_{13, \mathrm{~s}}^{0}=\int_{\mathrm{R}}^{\mathrm{R}+\mathrm{L}} \mathrm{x} \phi_{\mathrm{s}}(\mathrm{x}) \mu \mathrm{dx}+\mathrm{L} \phi_{\mathrm{s}}(\mathrm{~L}) \mathrm{m} \tag{18}
\end{equation*}
$$

$s=$ vibration modes in $X_{1}-X_{3}$ plane for booms $E$ and $F$

$$
\begin{equation*}
\mathrm{I}_{11, \mathrm{rs}}^{\mathrm{o}}=\mathrm{m}_{\mathrm{r}} \delta_{\mathrm{rs}} \tag{19}
\end{equation*}
$$

$r=$ vibration modes in $X_{2}-X_{3}$ plane for booms $C, D, E$, and $F$

When all ingredients required for the evaluation of $z$ rirs are available using equations (13), (16-19), the internal stability conditions can be examinded as followes:

$$
\begin{align*}
& \Delta_{1} \triangleq Z_{, 11}^{0}  \tag{20,a}\\
& \Delta_{2} \triangleq\left|\begin{array}{ll}
Z^{0}{ }_{11} & Z^{\circ}{ }_{12} \\
Z^{\circ}{ }^{\circ}{ }_{21} & Z^{0}{ }_{22}
\end{array}\right|
\end{align*}
$$

and so on.
The internal stability condition is

$$
\begin{equation*}
\Delta_{i}>0 \quad \text { for all } i=1,2,3, \ldots \ldots, 4 n \tag{21}
\end{equation*}
$$

II-3 Stability of the NPS Mini-Satellite (ORION)
The stability conditions of the simple spinning motion of the NPS mini-satellite between the angular velocity $\Omega$ and the length of a boom $L$ for other parameters defined in the preliminary design (See Ref. 1). The numerical value of these parameters are shown in Table 1.

## Table 1 - Parameters of NPS mini-sátellite

Radius of $B$
Mass of $B$
Axial moment of inertia of $B$ Transverse moment of inertia of $B$

Area of a boom cross-section
Area moment of inertia of a boom about $X_{2}-X_{2}$ plane

Area moment of inertia of a boom about $X_{1}-X_{2}$ plane

Young's modulus of the boom Mass of a boom per unit length

Mass of a magnetometer at the end of a boom

$$
\begin{aligned}
& \mathrm{R}=9.5 \text { inch } \\
& \mathrm{M}=7.542 \text { slug } \\
& \mathrm{B}_{1}=2.18 \text { slug-ft } \\
& \mathrm{B}_{2}=\mathrm{B}_{3}=6.78 \text { slug-ft } \\
& \mathrm{A}=0.212 \mathrm{in} \\
& I_{1}=0.01668 \mathrm{in} \\
& I_{2}=0.06328 \mathrm{in} \\
& E=10.3 \mathrm{X} \mathrm{10} \mathrm{Psi} \\
& \mu=0.000415 \text { (Slug/in) } \\
& m=0.0621 \mathrm{slug}
\end{aligned}
$$

For the design value of $L=78.5$ inches the cantilever beam model of a boom with an end mass was analyzed using the MSC/NASTRAN code for the natural frequencies and mode shapes. The natural frequencies are shown in Table 2 , and the finite element model and the first 3 mode shapes are shown in Figures 4 to 7 .

Mode Natural Frequency (Hz) Mode Shape

1
2.104
4.097
23.426
45.569
73.656
142.980

1-2 plane bending
2-3 plane bending
1-2 plane bending
2-3 plane bending
1-2 plane bending
2-3 plane bending

When the angular velocity of simple $\operatorname{spin} \Omega$ is increased from very small value, it can be seen from stability conditions, equations (20-21) and from equations (13), (16-19) that the stability condition will be violated first when $\Omega$ approaches the first natural frequency ${ }^{4}{ }^{\prime}{ }^{\prime}$, since for higher modes $w{ }_{i}$, $i=$ $2,3, \ldots$, are much greater than $\Omega^{2}$ and the matrix $Z^{0}$ irs become diagonally dominant. Therefore, the stability conditions can be examined simply by reducing the number of internal coordinates $q_{i}$ to the first mode of each boom, $q_{1}, q_{2}, q_{3}, q_{4}$.

Inserting the expressions of its components, equations (16) - (19) into equation (13), the matrix $Z_{\text {, rs }}^{0}$ becomes the following form:

$$
Z_{\text {,rs }}^{0}=\left[\begin{array}{cccc}
\omega_{1}^{2}-\alpha \Omega^{2} & 0 & -\beta \Omega^{2} & 0  \tag{22}\\
0 & \omega_{1}^{2}-\alpha \Omega^{2} & 0 & -\beta \Omega^{2} \\
-\beta \Omega^{2} & 0 & \omega_{1}^{2}-\alpha \Omega^{2} & 0 \\
0 & -\beta \Omega^{2} & 0 & \omega_{1}^{2}-\alpha \Omega^{2}
\end{array}\right]
$$

where

$$
\begin{equation*}
\alpha=\frac{1}{I_{s}^{o}-I_{t}^{o}}\left\{\int_{R}^{R+L} x \phi_{1}(x) \mu d x+L \phi_{1}(L) m\right\}^{2}-\int_{R}^{R+L} \frac{x}{2}\{m+\mu(R+L-x)\}\left\{\phi_{1}^{\prime}(x)\right\}^{2} d x \tag{23}
\end{equation*}
$$

and

$$
\begin{equation*}
\beta=\frac{1}{\mathrm{I}_{\mathrm{s}}^{\mathrm{o}}-\mathrm{I}_{\mathrm{t}}^{\mathrm{o}}}\left\{\int_{\mathrm{R}}^{\mathrm{R}+\mathrm{L}} \mathrm{x} \phi_{1}(\mathrm{x}) \mu \mathrm{dx}+\mathrm{L} \phi_{1}(\mathrm{~L}) \mathrm{m}\right\}^{2} \tag{24}
\end{equation*}
$$

with $\phi_{1}(x)$ is the mode shape of the first mode.
The stability conditions, equations (20) - (21), become

$$
\begin{align*}
& \omega_{1}^{2}-\alpha \Omega^{2}>0  \tag{25,a}\\
& \left(\omega_{1}^{2}-\alpha \Omega^{2}\right)^{2}>0  \tag{25,6}\\
& \left(\omega_{1}^{2}-\alpha \Omega^{2}\right)\left\{\left(\omega_{1}^{2}-\alpha \Omega^{2}\right)^{2}-\beta^{2} \Omega^{4}\right\}>0  \tag{25,c}\\
& \left\{\left(\omega_{1}^{2}-\alpha \Omega^{2}\right)^{2}-\beta^{2} \Omega^{4}\right\}^{2}>0 \tag{25,d}
\end{align*}
$$

Therefore, the internal stability conditions is reduced to the condition

$$
\begin{equation*}
\Omega^{2}<\frac{\omega_{1}^{2}}{\alpha+\beta} \tag{26}
\end{equation*}
$$

from the condition $(25, c)$.

As the length of a boom $L$ increases, the first natural frequency $\omega_{1}$ decreases as follows;

$$
\begin{equation*}
\omega_{1}=\sqrt{\frac{3 E I}{(m+0.23 \mu L) L^{3}}} \tag{27}
\end{equation*}
$$

The denominator in equations (23), (24), $I_{s}^{0}-I_{t}^{0}$ is proportional to ( $L^{2}-L_{m i n}^{2}$ ) and increases from zero for $L>L_{m i n}$, which is the external stability condition. Therefore, the stability boundary in the stability diagram varies as shown in Figure 8. For the designed value of $L=78.5$ inches the stability condition is

$$
\begin{equation*}
\Omega<8.658 \quad(\mathrm{rad} / \mathrm{sec}) \tag{28}
\end{equation*}
$$

However, it was assumed that each boom is a single piece and rigidly connected to the rigid main body B. Since each boom consist of three beam and the connection between the boom and the body $B$ is not ideally rigid the critical angular velocity $\Omega_{C}$ may be much lower than the value calculated, equation (28), depending on the design of joints of booms.

## III. Conclusions

The stability conditions of a simple spin motion of the NPS mini-satellite (ORION) are derived analytically using Liapunov stability theory and by choosing the natural vibration modes as admissible functions of internal coordinates of the system. By applying this conditions to the preliminary design of the satellite it is shown that the angular velocity of the simple spin should be limited to the value of $8.685 \mathrm{rad} / \mathrm{sec}$ for the boom length $L=78.5$ inches. If the boom length is changed the critical angular velocity $\Omega_{c}$ also changes and the stability conditions should be re-examined using the results derived in this study.

It should be recognized that the stability conditions were derived for the simplified ideal model under the following assumptions; 1) the mass distribution of the cylindrical main body is axisymmetric; 2) the connections between the booms and the main body and the main body itself are rigid; 3) each boom acts as a single piece and the locking mechanism of the boom should be ideal.

Due to the highly nonlinear behavior of a three dimensional motion of a flexible body, the stability analysis is valid for only small disturbances. Therefore, stability of dynamic motion of a satellite for general environments should be investigated by dynamic simulation using computers and proper dynamic models.

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Figure 1. The NPS Mini-Satellite (ORION)


Figure 2. Random Vibration Level in GAS Canisters.
(NASA G. A. S. Manual, 1984, P. 57)


Figure 3. Dynamic Model of The NPS Satellite During a Simple Spin Motion in Space.


Figure 4. The Finite Element Model of a Boom $\left(X_{1}, X_{2}, X_{3}\right.$ show the Orientation of the Satellite)


Figure 5. The Mode Shape of the First Mode ( $f=2.104 \mathrm{~Hz}$ )


Figure 6. The Second Mode $(f=4.097 \mathrm{~Hz})$


Figure 7. The Third Mode ( $f=23.43 \mathrm{~Hz}$ )


Figure 8. Stability Diagram of The NPS Mini-Satellite.

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