NAVAL POSTGRADUATE SCHOOL

Monterey, California

HYPERSONIC LAMINAR HEAT TRANSFER

by

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NAVAL POSTGRADUATE SCHOOL

Monterey, California

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ABSTRACT:

The results of an analog computer study of hypersonic laminar heat transfer including the effects of pressure gradient, mass transfer, velocity slip, Prandtl number, and wall temperature are presented. Reynolds analogy factors and adiabatic wall enthalpy results are included for all combinations of the aforementioned effects. In addition, an auxiliary correlation is presented which simplifies the specification of the Reynolds analogy factor with velocity slip. All of the computations presented in this paper were performed on the EAI 580 analog computer in the Department of Aeronautics.

TABLE OF CONTENTS

TABLE OF FIGURES

NOMENCLATURE

β = Pressure gradient parameter in equation (1)

C_f = Skin friction coefficient $\quad \mu \frac{\partial U}{\partial y} \Big|_w / \frac{1}{2} \rho_e U_e^2$

C_p = Specific heat at constant pressure

η = Independent variable $\quad \frac{r^j \rho_e U_e}{\sqrt{2\zeta}} \int_o^y \frac{\rho}{\rho_e} dy$

f' = Normalized velocity $\quad U/U_e$

g = Normalized enthalpy $\quad H/H_e$

H = Stagnation enthalpy $\quad C_p T + U^2/2$

j = Dimensionality parameter 0 for two dimensions, 1 for axisymmetric

k = Coefficient of thermal conductivity

μ = Coefficient of viscosity

Pr = Prandtl number $\quad \mu C_p/k$

q = Heat transfer rate in BTU/ft^2-sec

r = Local body radius

ρ = Fluid density

S = Reynolds analogy fact defined in equations (6a) and (7a)

S* = Correlation defined in equation (8)

St = Stanton number defined in equation (4)

U = Streamwise velocity component

x = Streamwise coordinate

y = Lateral coordinate

ζ = Independent variable $\quad \int_o^x \rho_e \mu_e U_e r^{2j} dx$

SUBSCRIPTS

aw = Adiabatic wall conditions

e = Local external conditions

w = Wall conditions

SUPERSCRIPTS

' = Differentiation with respect to η

INTRODUCTION

Under the assumption of self-similarity, the equations of momentum and energy for a laminar boundary layer in a locally hypersonic flow can be written as

$$f''' + ff'' + \beta(g - f'^2) = 0 \quad (1)$$

$$g'' + Pr\, fg' + 2(Pr - 1)\,(f'f'')' = 0 \quad (2)$$

subject to the boundary conditions

$$@\ \eta = 0 \qquad f = f_w, \quad f' = f'_w, \quad g = g_w \text{ or } g' = g'_w \quad (3)$$

$$@\ \eta = \infty \qquad f' \to 1, \quad g \to 1$$

Equations (1) and (2) describe the flow of a constant property, perfect gas whose viscosity varies linearly with temperature. In addition, the restriction that the pressure gradient parameter, β, be a constant, implies that the pressure varies linearly with streamwise distance.

The results of an analog computer study of hypersonic laminar heat transfer involving some fifteen hundred solutions of equations (1) and (2) are presented. These calculations provide necessary data for making estimates of the convective heat transfer rates utilizing Reynolds analogy in situations involving mass transfer, velocity slip and strong pressure gradients.

A comparison of the analog solution of the Falkner-Skan equations with the numerical values of Cohen and Reshotko [1] indicated an error of less than one percent. Because of the computational complexities introduced by the additional nonlinearities of equations (1) and (2), the data presented is estimated to be accurate to within two percent.

DISCUSSION

Based on Reynolds analogy, the following equations relate the local convective heat transfer rate to the wall shear and the driving enthalpy difference

$$q_w = \rho_e U_e H_e \, (g_w - g_{aw}) \, St \tag{4}$$

$$St = C_f / 2S \tag{5}$$

In this formulation, g_{aw} represents the recovery factor which was calculated for all combinations of Pr, f_w, and β in the range of values studied. These results, presented in Fig. 1, reinforce the classical result that the laminar recovery factor varies as the square root of the Prandtl number for zero mass transfer. However with blowing or suction no such simple Prandtl number correlation is possible.

When calculating heat transfer using Reynolds analogy, the Reynolds analogy factor, S, which exhibits sensitivity to all the parameters of the problem, must also be specified. Therefore, calculated values of Reynolds analogy factors are presented for all combinations of g_w = 1.1, 1.0, .5, 0; Pr = 1.25, 1.0, .72, .5; f_w = 0, .25, -.25; β = -.2 → 1.0 for $f'_w = 0$. In these cases we can compute the Reynolds analogy factor from the calculated data with the equation

$$S = Pr \, f''_w \, \frac{(g_{aw} - g_w)}{g'_w} \tag{6a}$$

where, in conjunction with equation (4), we have utilized Fourier's law at the wall

$$q_w = -k_w \frac{\partial T}{\partial y}\Big|_w \tag{7a}$$

In addition to these situations, a case of large velocity slip ($f'_w = .1$) is included for Pr = .72 with all other parameters varied as before. In this case, equation (6a) must be modified to account for velocity slip and takes the form

$$S = \frac{(g_{aw} - g_w)\, f''_w\, Pr}{g'_w + 2f'_w f''_w\,(Pr - 1)} \tag{6b}$$

Which reduces to equation (6a) when $f'_w = 0$ or for a Prandtl number of one with an arbitrary f'_w. To obtain equation (6b) we have used the definition of H at the wall and Maslen's [2] results that the heat transfer at the wall must now be written

$$q_w = -\left(k_w \frac{\partial T}{\partial y}\Big|_w + \mu_w U_w \frac{\partial U}{\partial y}\Big|_w\right) \tag{7b}$$

The results of the computations of the Reynolds analogy factor without velocity slip are plotted in Figs. 2a - 2l. The data deviates from classical results for nonhypersonic boundary layers and the range of values which the Reynolds analogy factor encompasses illustrates the potential uncertainty in the calculation of heat transfer without knowledge of the dependence of S on the particular parameters of the problem.

The data was processed to evaluate various Pr - β correlations similar to that of Tifford and Chu [3] in an effort to coalesce the dependence of S on the variables of the problem. No satisfactory general correlation could be found and specific information on the appropriate Reynolds analogy factor must be obtained.

For the cases involving velocity slip without mass transfer and with Prandtl number different than unity, the data was processed with an auxiliary function

$$S^* = f''_w \, Pr \, (\frac{g_{aw} - g_w}{g'_w} + 2f'_w) \qquad (8)$$

The results of these computations are presented in Fig. 3. When these plots of S* are compared to the corresponding plots of Reynolds analogy factor without velocity slip for the same Prandtl number and wall enthalpy, it is seen that the curves agree to within a few percent. Therefore, from knowledge of S*, the Reynolds analogy factor with velocity slip may be obtained from equations (6) and (8)

$$\frac{1}{S} = \frac{1}{S^* - 2f'_w f''_w Pr} + \frac{2f'_w}{g_{aw} - g_w} \cdot \frac{Pr - 1}{Pr} \qquad (9)$$

where values for S* are taken from corresponding values of S for the appropriate Prandtl number and wall enthalpy but without velocity slip. Note that for a Prandtl number of one, the correlation is both inaccurate and unnecessary since equation (6b) says that S is unaffected by velocity slip in this case.

CONCLUSIONS

The wide range of values which the Reynolds analogy factor assumes in hypersonic flow with mass transfer and strong pressure gradients forces one to obtain data on the specific situation of interest. The analog computer has yielded fast, accurate solutions of the equations to provide this data and investigate all important effects. In addition an approximate correlation has been found which reduces the need for detailed information on Reynolds analogy factors in a flow with velocity slip.

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3. Tifford, A. N., and Chu, S. T., "Heat Transfer in Laminar Boundary Layers Subject to Surface Pressure and Temperature Distributions," Proceedings of the Second Midwestern Conference on Fluid Mechanics, Ohio State University, March 1952, pp. 363-377.

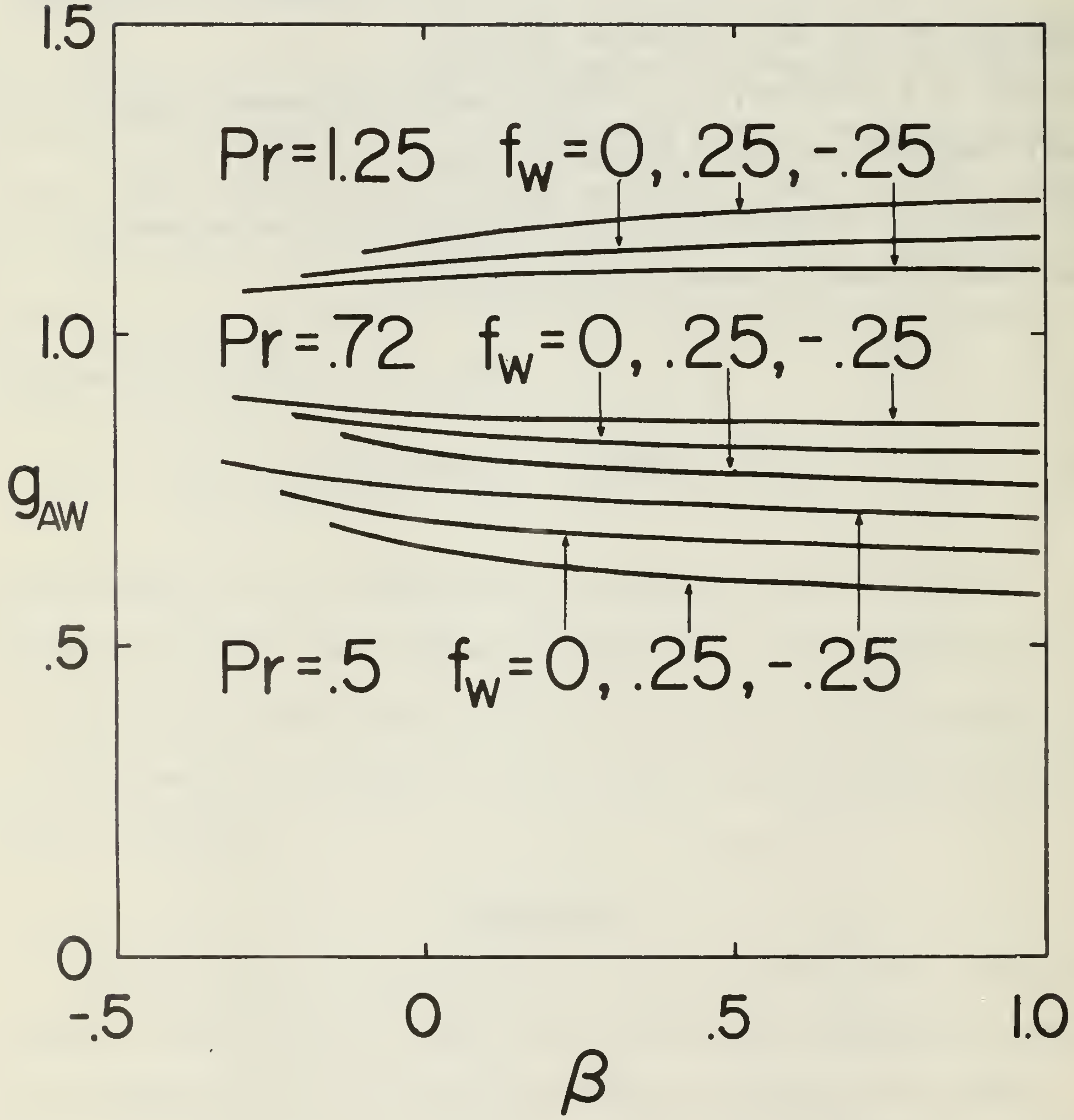

Fig. 1 Recovery Factor vs. β

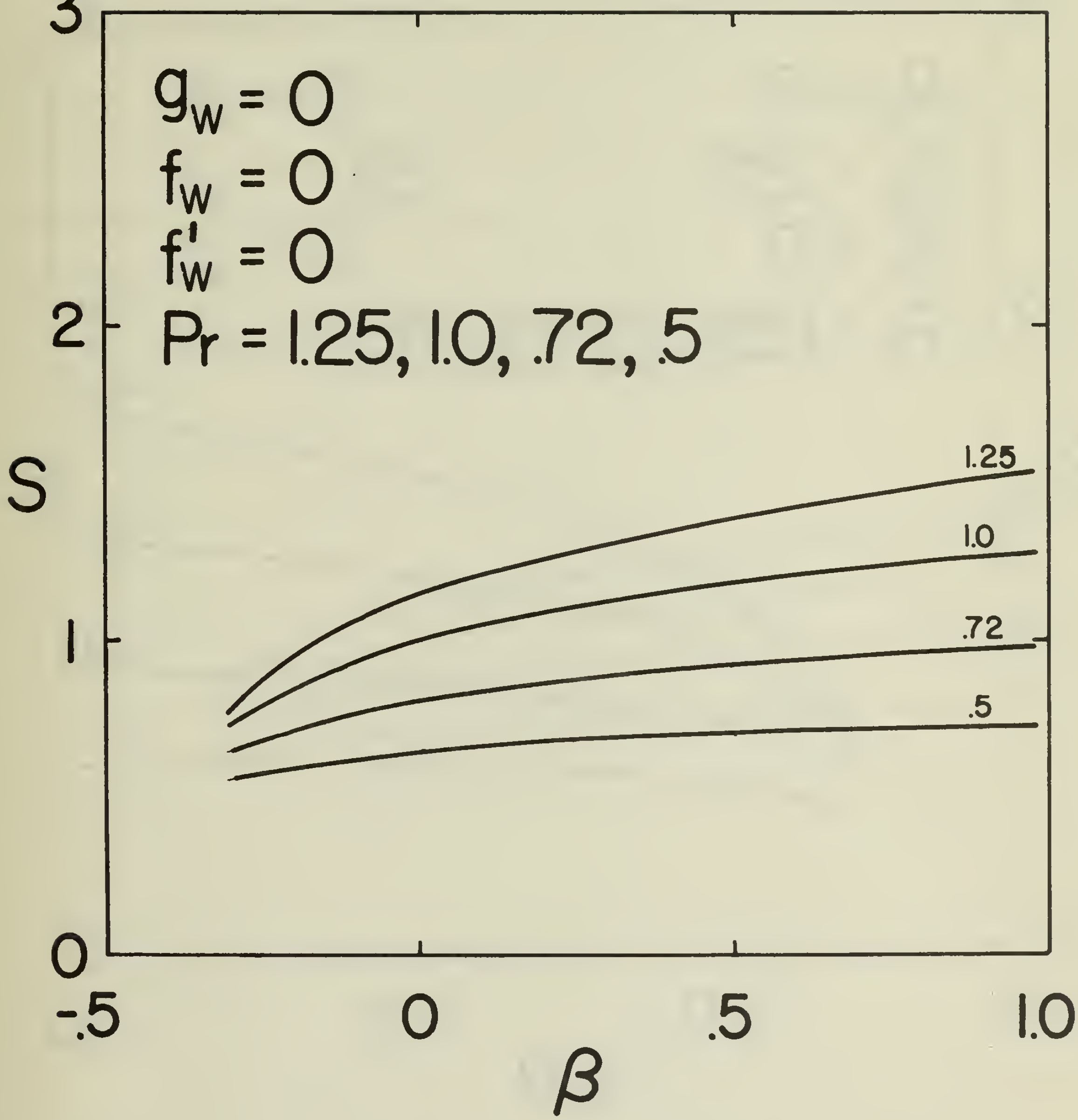

Fig. 2a Reynolds Analogy Factor vs. β for g_w = 0

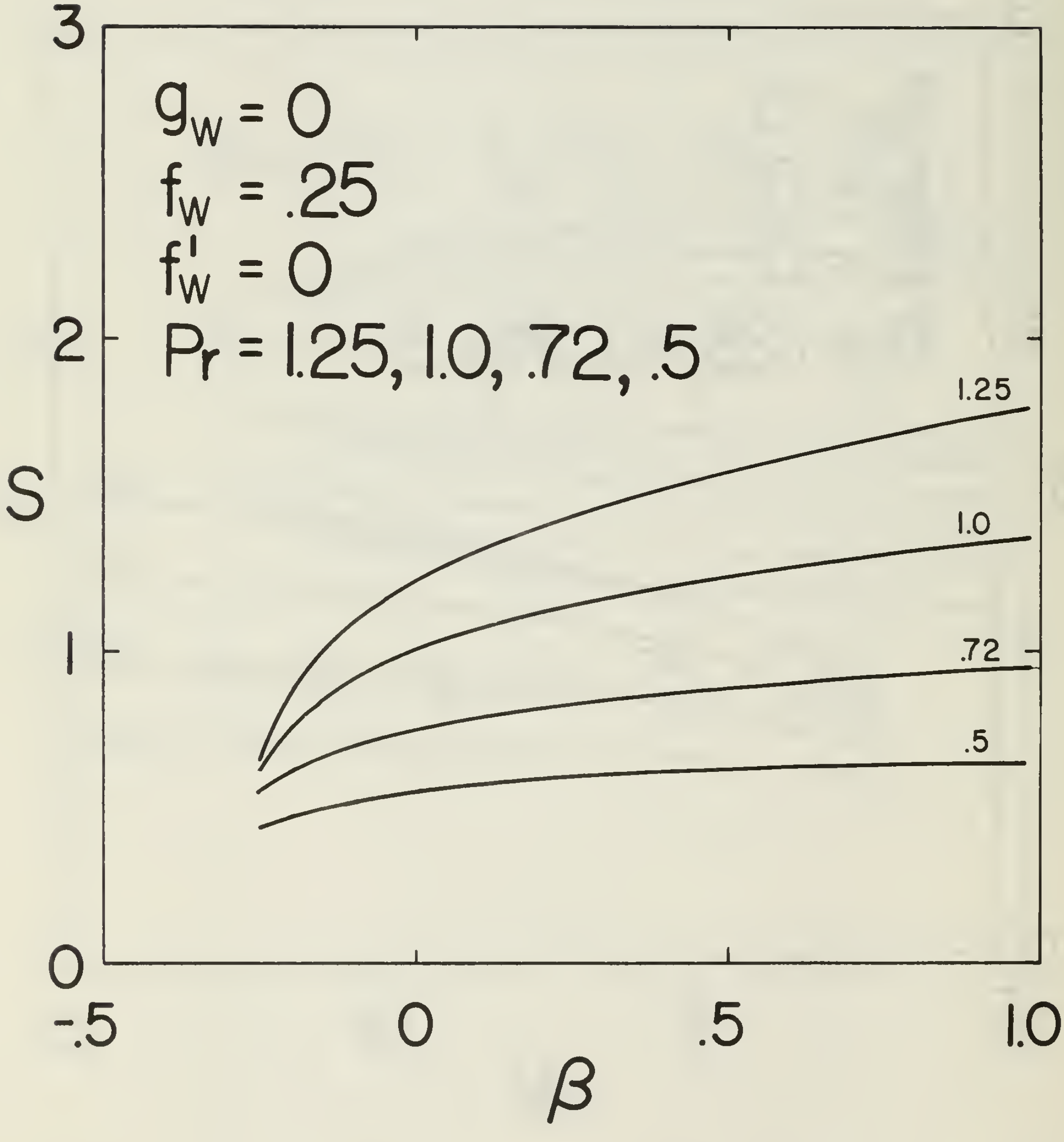

Fig. 2b Reynolds Analogy Factor vs. β for $g_w = 0$

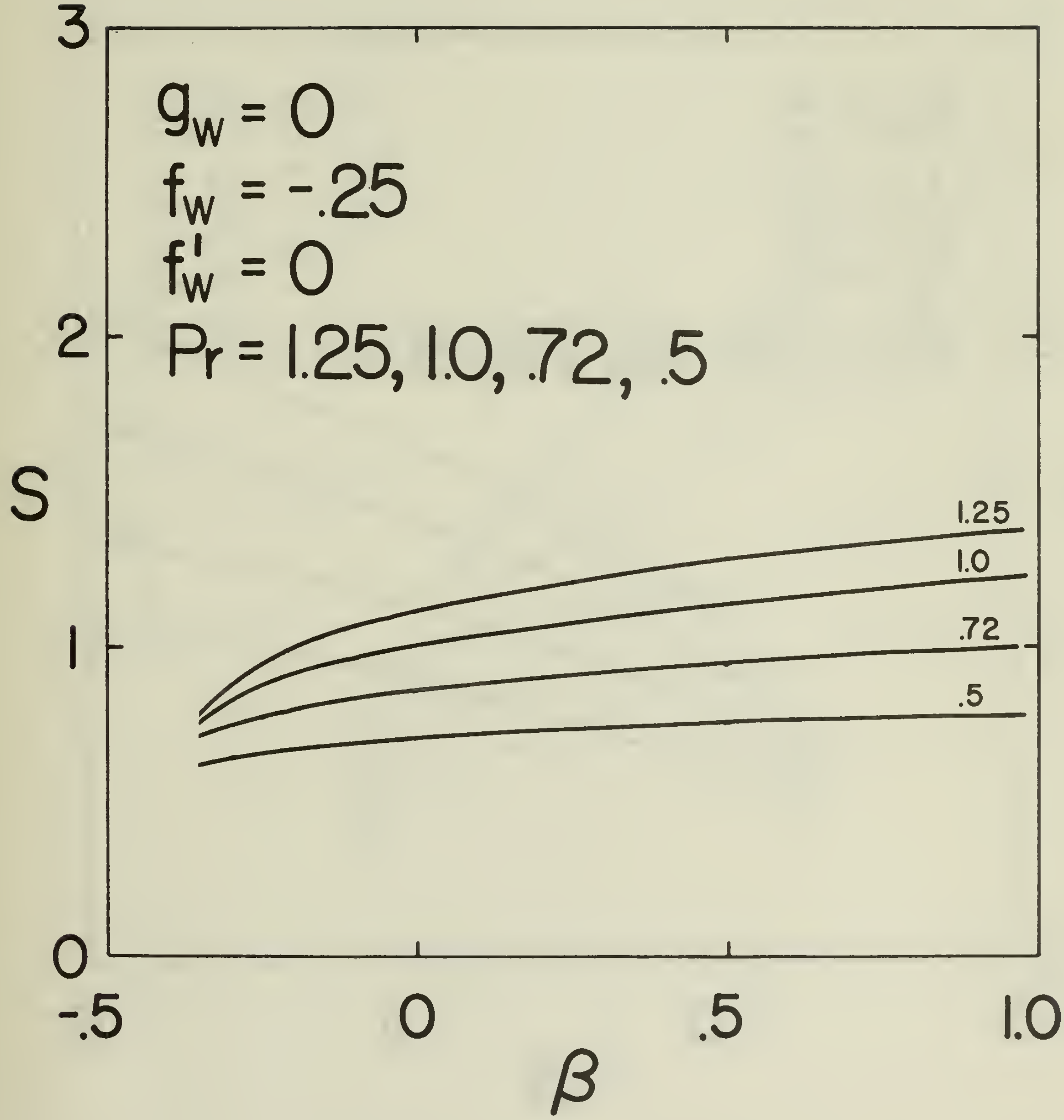

Fig. 2c Reynolds Analogy Factor vs. β for g_w = 0

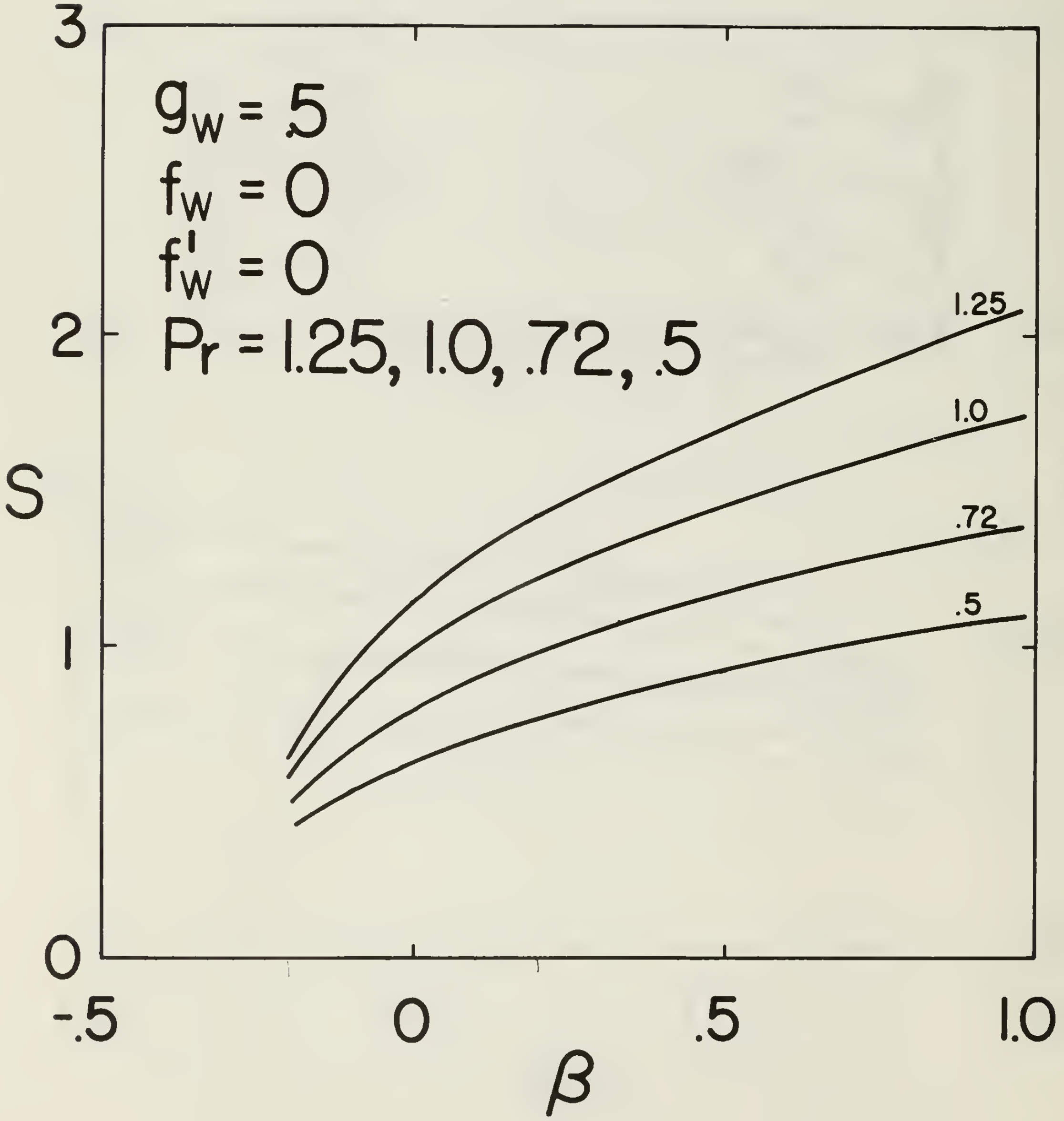

Fig. 2d Reynolds Analogy Factor vs. β for g_w = .5

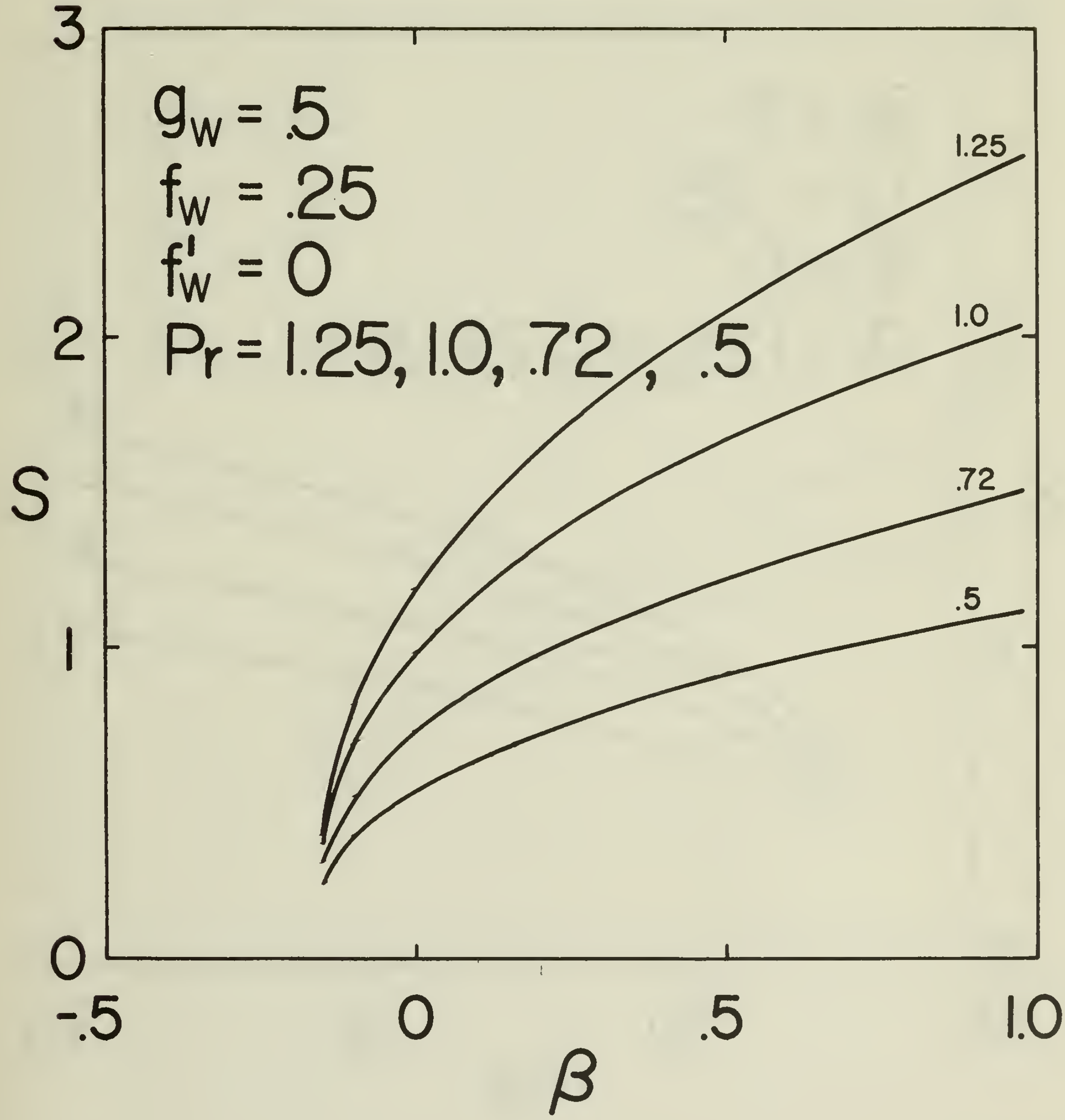

Fig. 2e Reynolds Analogy Factor vs. β for g_w = .5

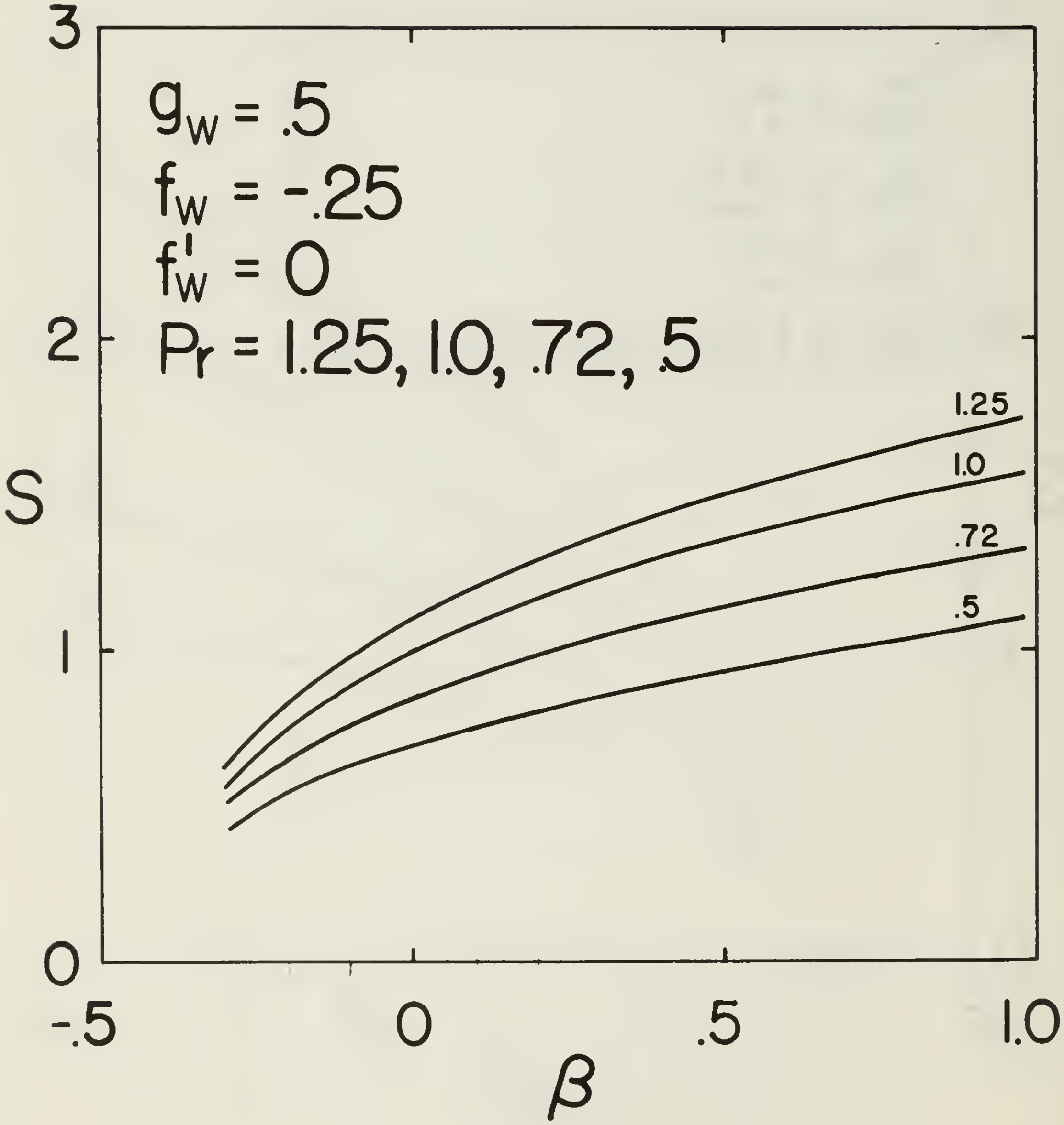

Fig. 2f Reynolds Analogy Factor vs. β for g_w = .5

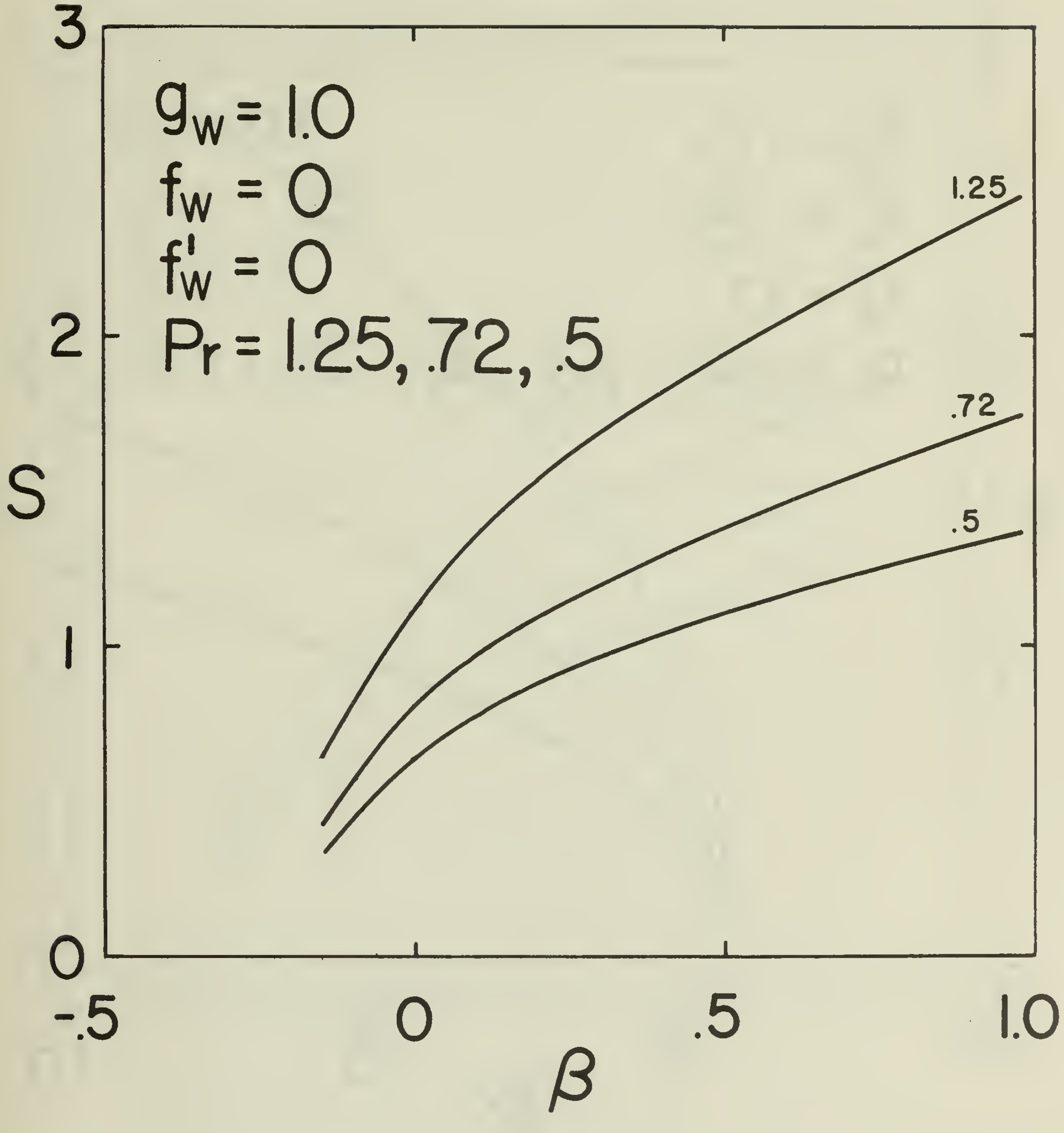

Fig. 2g Reynolds Analogy Factor vs β for $g_w = 1.0$

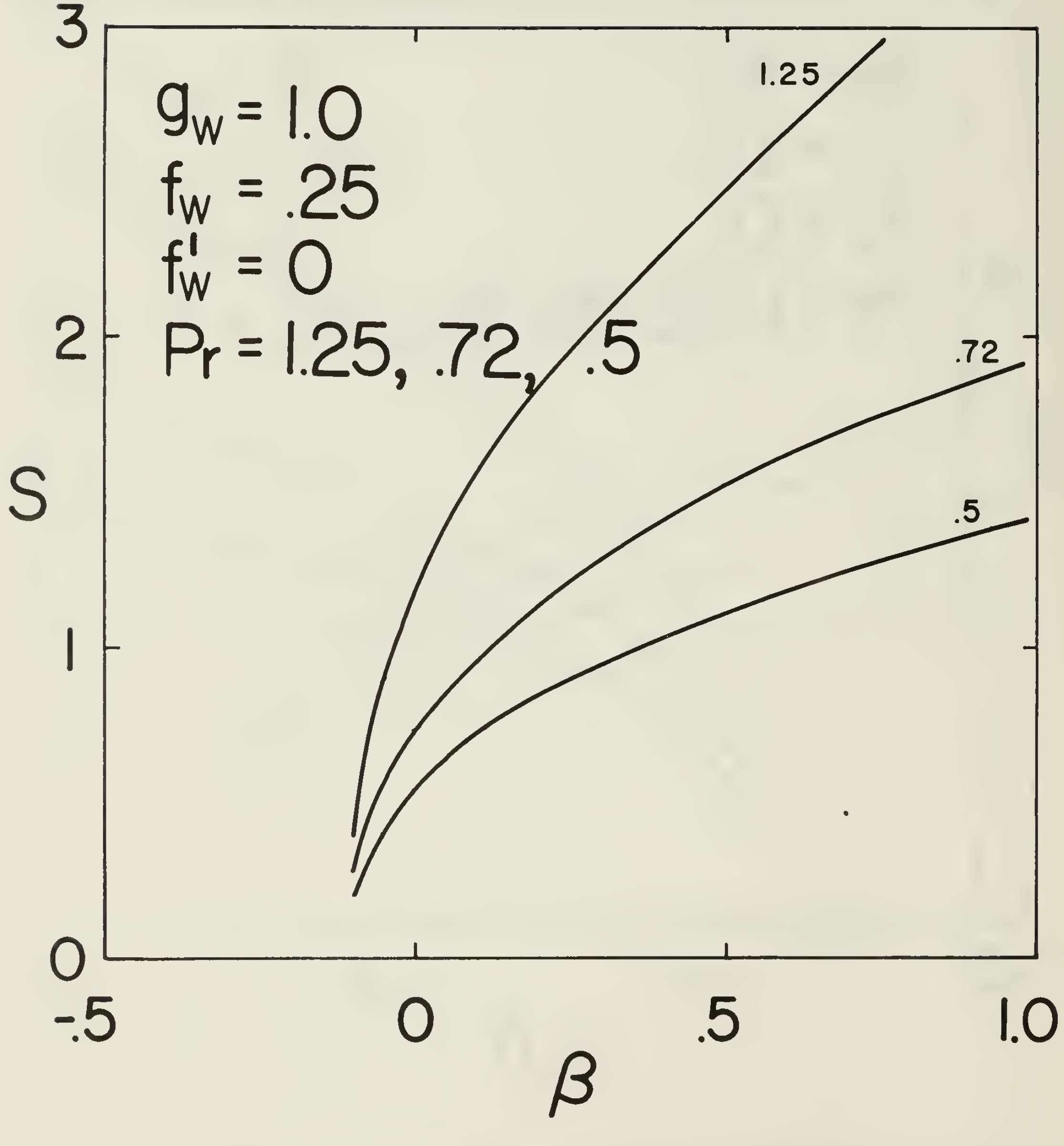

Fig. 2h Reynolds Analogy Factor vs. β for $g_w = 1.0$

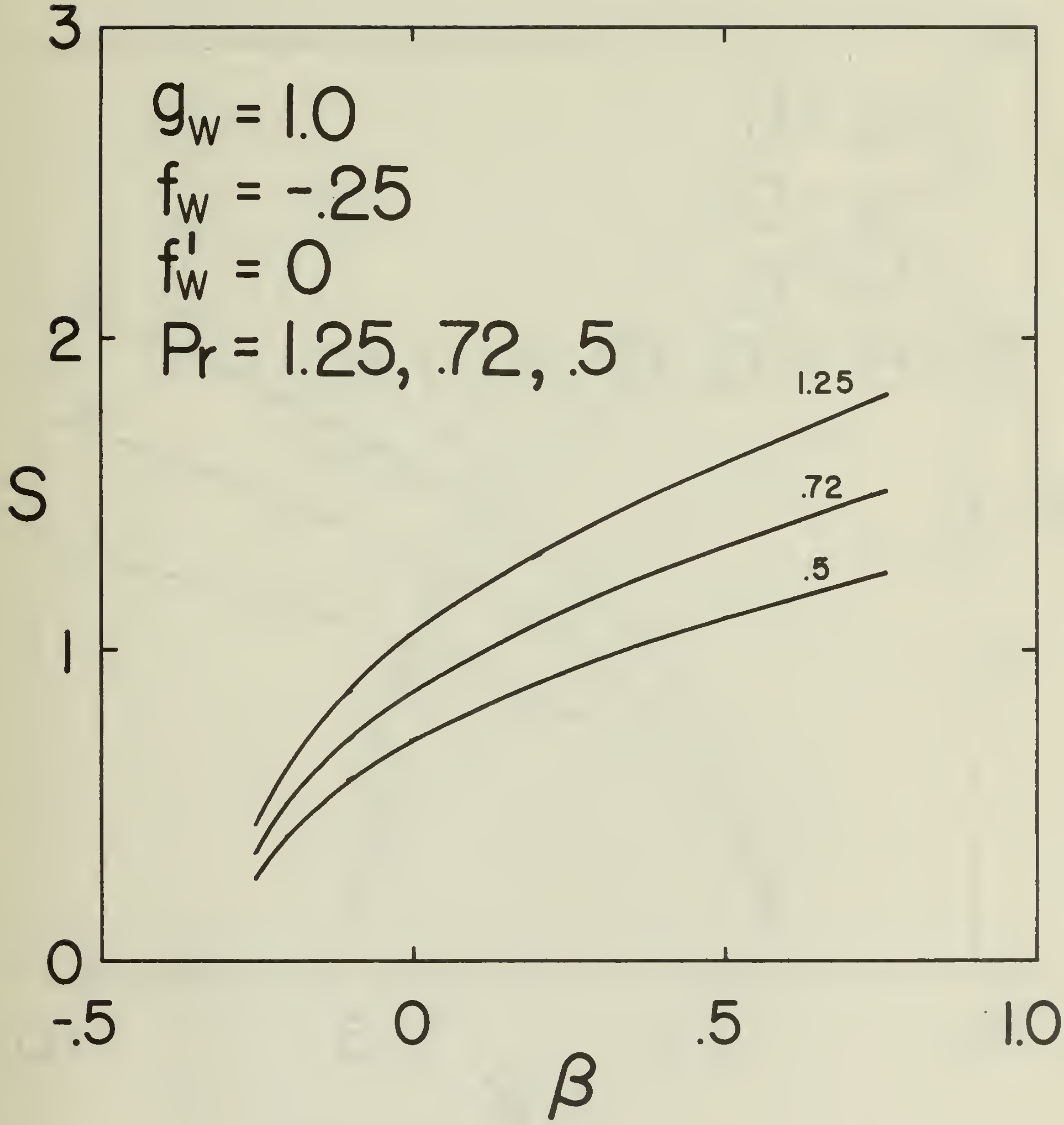

Fig. 2i Reynolds Analogy Factor vs. β for $g_w = 1.0$

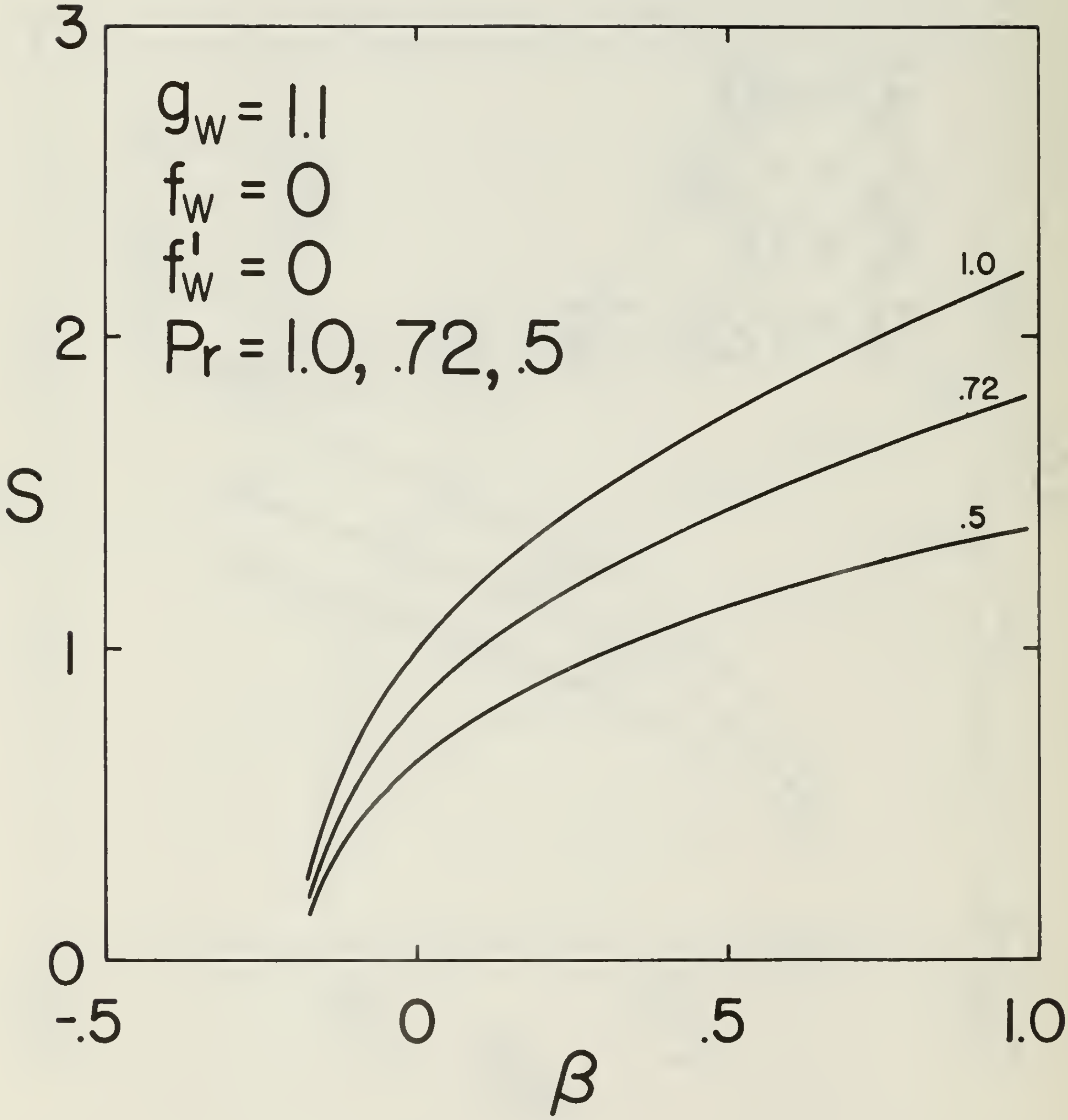

Fig. 2j Reynolds Analogy Factor vs. β for $g_w = 1.1$

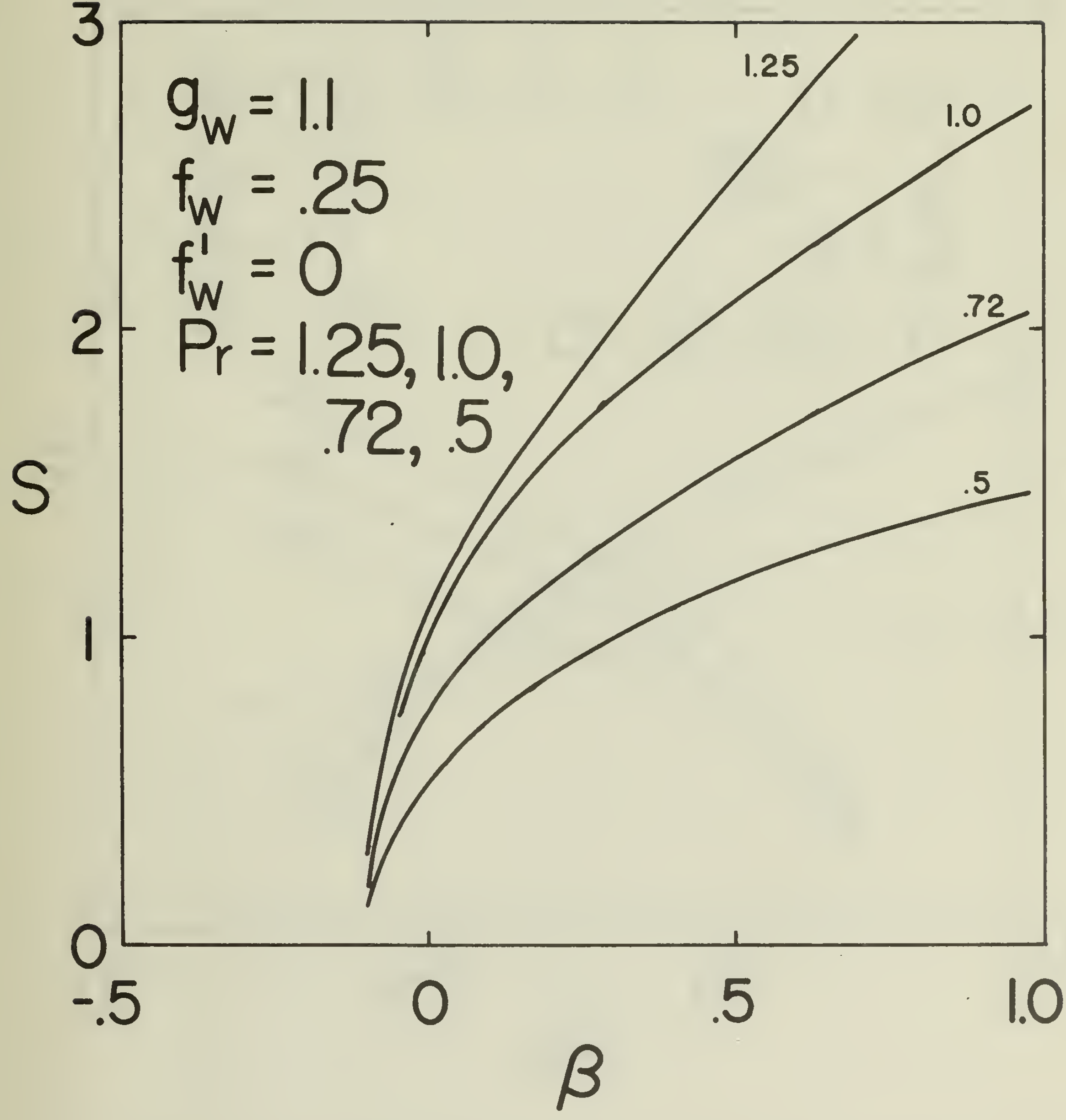

Fig. 2k Reynolds Analogy Factor vs. β for $g_w = 1.1$

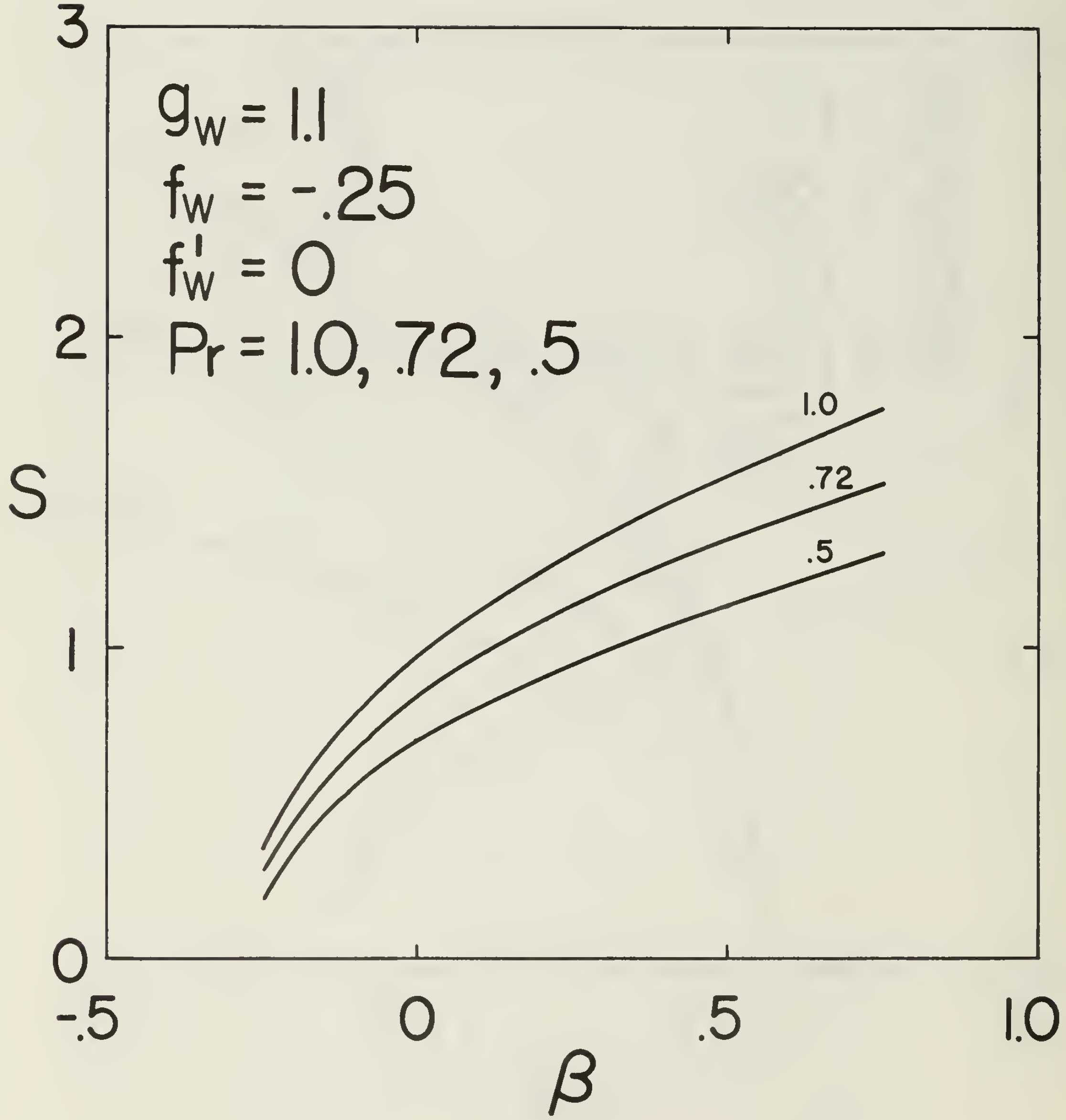

Fig. 21 Reynolds Analogy Factor vs. β for g_w = 1.1

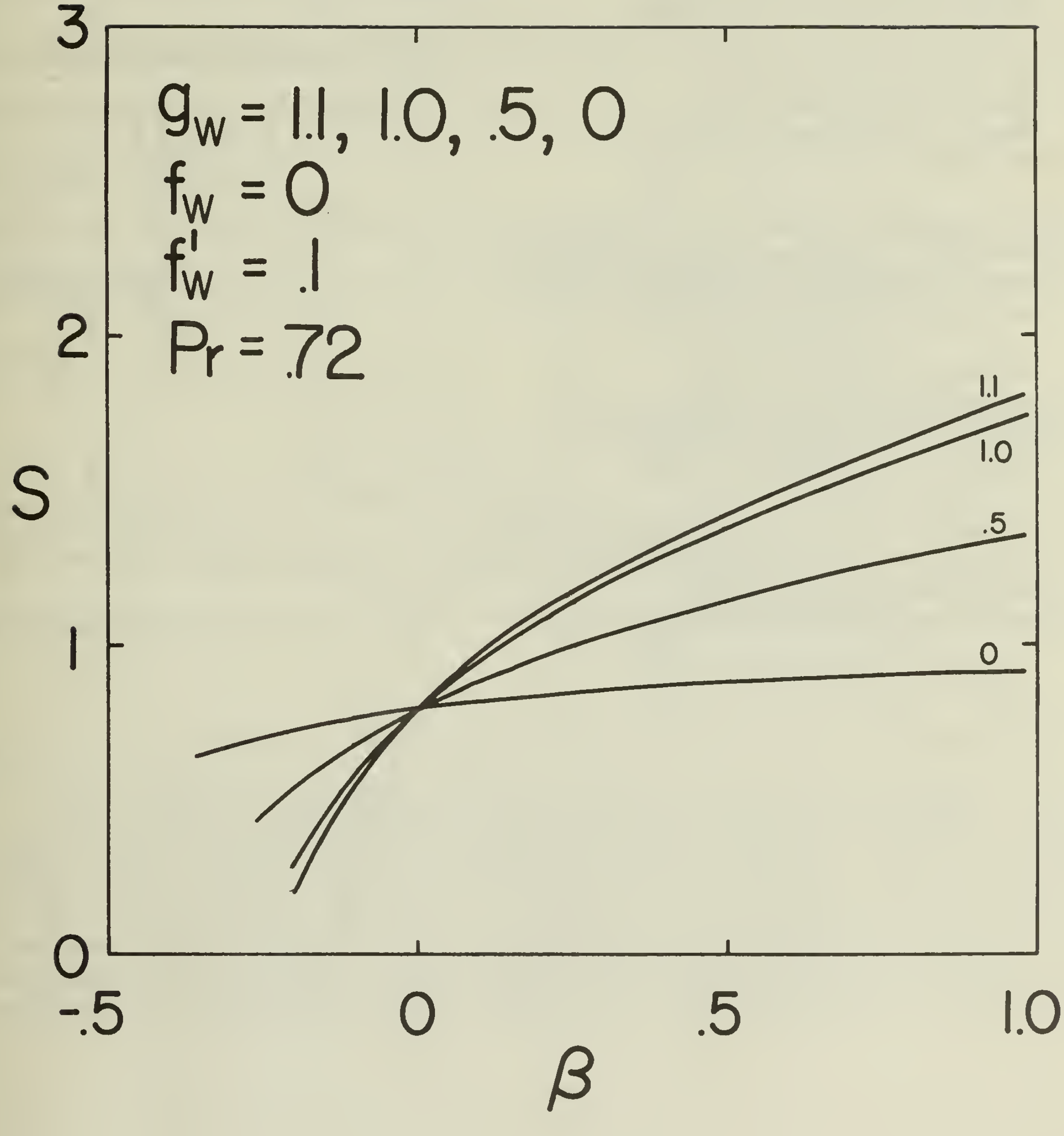

Fig. 3 Reynolds Analogy Factor vs. β with Velocity Slip

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