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FAMILIES OF COMPONENTS, AND SYSTEMS,

EXPOSED TO A COMPOUND POISSON DAMAGE PROCESS

by

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ABSTRACT*

A fairly common failure model in a wide variety of contexts is a cumulative damage process, in which shocks occur randomly in time and associated with each shock there is a random amount of damage which adds to previously incurred damage until a breaking threshold is reached. The multivariate life distributions that are induced when several "components," each with its own breaking threshold, are exposed to the same cumulative damage process are of interest in their own right, and are important examples in the general study of multivariate life distributions.

This paper is a summary of some results about the very special, but central, case in which the cumulative damage process is a compound Poisson process. It is focused on the multivariate life distributions that arise when the component breaking thresholds are random and have a Marshall-Olkin multivariate exponential distribution. There are two relevant multivariate life distributions that can be derived, an intermediate distribution for the number of shocks (cycles) to failure and the final distribution for the actual times to failure. The results have application to the life distribution of a coherent system whose components are exposed to the damage process.

Prepared by:

Families of Components, and Systems,
Exposed to a Compound Poisson Damage Process

J. D. Esary* and A. W. Marshall**

Abstract. A fairly common failure model in a wide variety of contexts is a cumulative damage process, in which shocks occur randomly in time and associated with each shock there is a random amount of damage which adds to previously incurred damage until a breaking threshold is reached. The multivariate life distributions that are induced when several "components," each with its own breaking threshold, are exposed to the same cumulative damage process are of interest in their own right, and are important examples in the general study of multivariate life distributions.

This paper is a summary of some results about the very special, but central, case in which the cumulative damage process is a compound Poisson process. It is focused on the multivariate life distributions that arise when the component breaking thresholds are random and have a Marshall-Olkin multivariate exponential distribution. There are two relevant multivariate life distributions that can be derived, an intermediate distribution for the number of shocks (cycles) to failure and the final distribution for the actual times to failure. The results

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have application to the life distribution of a coherent system whose components are exposed to the damage process.

1. Introduction. The broad aspects of a simple "failure" model, which has been of interest in the reliability and many other settings, are summarized in Figure 1.

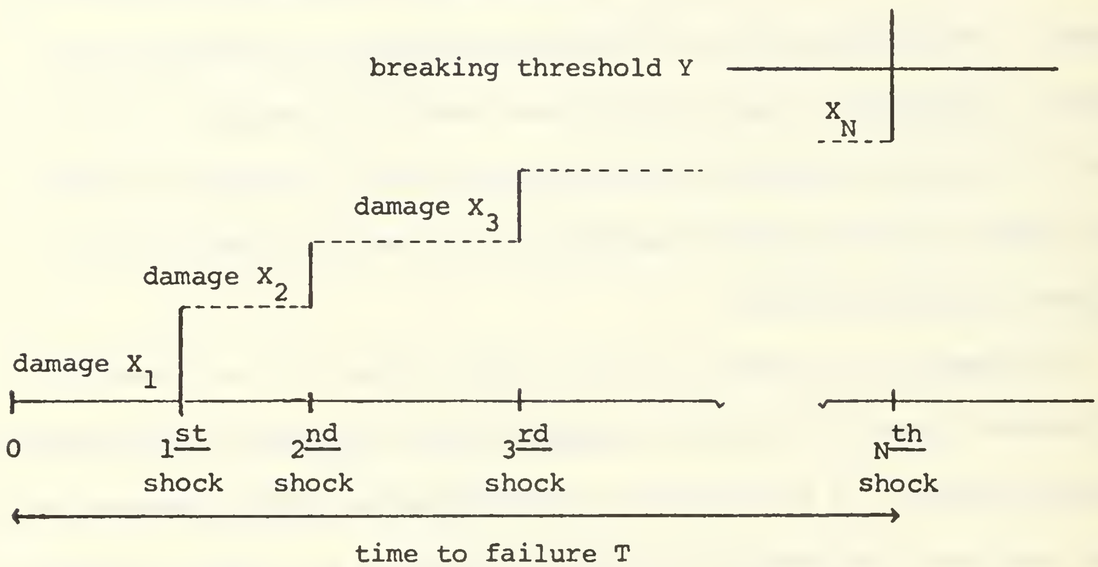

Figure 1

In the model a sequence of shocks occurs randomly in time. The i^{th} shock, $i = 1,2,\ldots,$ causes a random amount of damage X_i which adds to previously incurred damage until a possibly random breaking threshold Y is reached. The number of cycles (shocks) to failure is

(1.1) $$N = \min\{k: X_1+\ldots+X_k \geq Y\},$$

i.e. $N > k \Leftrightarrow X_1+\ldots+X_k < Y$, $k = 1,2,\ldots$. The <u>time to failure</u> is

(1.2) $$T = \inf\{t\colon K(t) \geq N\},$$

where $K(t)$ is the random number of shocks that occur in the time interval $[0,t]$, i.e. $T > t \Leftrightarrow K(t) < N$.

A basic case of the model arises when the process $\{K(t),\ t \geq 0\}$ for the incidence of shocks, the sequence of damages $X_1, X_2, \ldots$, and the breaking threshold Y are independent, and:

(a) $\{K(t),\ t \geq 0\}$ is a Poisson process.

(b) $X_1, X_2, \ldots$ are independent observations on a prototype damage variable X which is nonnegative and not degenerate at zero.

(c) Y has an exponential distribution.

In this case N has a geometric distribution, since

(1.3) $$P[N > k] = P[Y > X_1+\ldots+X_k] = E\,e^{-\lambda(X_1+\ldots+X_k)}$$
$$= \{E\,e^{-\lambda X}\}^k, \quad k = 0,1,\ldots,$$

where $P[Y > y] = e^{-\lambda y}$, $\lambda > 0$, $y \geq 0$, is the survival function for Y, and vacuous sums are taken to be zero. Then

(1.4) $$P[N > k] = \theta^k, \quad k = 0,1,\ldots,$$

where $\theta = E\,e^{-\lambda X}$. Since X is nonnegative and not degenerate at zero, and $\lambda > 0$, then $0 \leq \theta < 1$. Further, T has an exponential distribution, since

$$(1.5) \qquad P[T > t] = P[N > K(t)] = \sum_{k=0}^{\infty} \theta^k \frac{(\upsilon t)^k e^{-\upsilon t}}{k!}$$

$$= e^{-(1-\theta)\upsilon t}, \quad t \geq 0,$$

where $\upsilon > 0$ is the rate for the Poisson process $\{K(t), t \geq 0\}$. Since $\upsilon > 0$ and $\theta < 1$, then $(1-\theta)\upsilon > 0$.

A simplest multivariate generalization of the model would be to consider n "components," with different breaking thresholds $Y_1, \ldots$ $\ldots, Y_n$, which experience a common sequence of shocks and are damaged alike by any particular shock. The other elements of the model can be kept intact so that N_i, the number of cycles to failure for the $i^{\underline{th}}$ component, is related to Y_i by (1.1), and T_i, the time to failure for the $i^{\underline{th}}$ component, is related to N_i by (1.2). Distributional assumptions analogous to the basic case would require that the process for the incidence of shocks, the sequence of damages, and the vector of breaking thresholds be independent. Assumptions (a) and (b) can remain unaltered, and (c) can be replaced with the assumption that $Y_1, \ldots, Y_n$ are independent with possibly different exponential distributions.

This paper is a summary and synthesis of some results obtained from an investigation of the multivariate generalization described above. The main purpose is to trace an analogy to the chain of implications

$$Y \text{ exponential} \Rightarrow N \text{ geometric} \Rightarrow T \text{ exponential}$$

that holds for the univariate model. For this purpose it appears that the appropriate replacement for assumption (c) is not that $Y_1,\ldots,Y_n$ are independent and exponential, but rather that $Y_1,\ldots,Y_n$ have the multivariate exponential distribution introduced by Marshall and Olkin [6]. The investigation grew from work on the univariate model in the case that Y has an IHRA distribution(Esary, Marshall, and Proschan [5], Theorem 5.2). The relationship involves the life distribution attributable to a coherent system whose components are exposed to the multivariate damage process.

2. Distributions with exponential or geometric minimums. A set of nonnegative random variables $T_1,\ldots,T_n$ (or $Y_1,\ldots,Y_n$) has a joint distribution with exponential minimums if $\min_{i\in I} T_i$ has an exponential distribution for each nonempty $I \subset \{1,\ldots,n\}$. A set of positive integer valued random variables $N_1,\ldots,N_n$ has a joint distribution with geometric minimums if $\min_{i\in I} N_i$ has a geometric distribution for each nonempty $I \subset \{1,\ldots,n\}$. It is automatic that the univariate marginals of joint distributions with exponential(geometric) minimums are exponential(geometric). In addition, these classes of distributions have other properties(Esary and Marshall [3], Section 2) that justify regarding them as very comprehensive classes of multivariate exponential(geometric) distributions. Here, the reason for considering them is to note for subsequent reference how they are propogated through the multivariate damage process.

Theorem 2.1. $Y_1,\ldots,Y_n$ have exponential minimums $\Rightarrow$ $N_1,\ldots,N_n$

<u>have geometric minimums</u> $\Rightarrow$ $T_1,\ldots,T_n$ <u>have exponential minimums</u>.

<u>Proof</u>. Let I be a nonempty subset of $\{1,\ldots,n\}$. To show the first implication note that $N_i > k \Leftrightarrow Y_i > X_1+\ldots+X_k$, $i\epsilon I$. Then

(2.1) $$\min_{i\epsilon I} N_i > k \Leftrightarrow \min_{i\epsilon I} Y_i > X_1+\ldots+X_k,$$

and since $\min_{i\epsilon I} Y_i$ is exponential, it follows from (1.3) and (1.4) that $\min_{i\epsilon I} N_i$ is geometric. To show the second implication note that $T_i > t \Leftrightarrow N_i > K(t)$, $i\epsilon I$. Then

(2.2) $$\min_{i\epsilon I} T_i > t \Leftrightarrow \min_{i\epsilon I} N_i > K(t),$$

and since $\min_{i\epsilon I} N_i$ is geometric, it follows from (1.5) that $\min_{i\epsilon I} T_i$ is exponential. □

<u>3. A multivariate exponential distribution and its discrete analogues</u>. Nonnegative random variables $T_1,\ldots,T_n$ (or $Y_1,\ldots,Y_n$) have the multivariate exponential distribution considered by Marshall and Olkin [6] if

(3.1) $$T_i = \min_{\{J\epsilon\mathcal{J}: i\epsilon J\}} S_J, \quad i = 1,\ldots,n,$$

where $\mathcal{J}$ is a class of nonempty subsets of $\{1,\ldots,n\}$ such that each $i \epsilon \{1,\ldots,n\}$ is an element of at least one of the sets $J \epsilon \mathcal{J}$, and the S_J, $J\epsilon\mathcal{J}$, are independent, exponentially distributed random variables. The requirement on the class $\mathcal{J}$ insures that the joint distribution of $T_1,\ldots,T_n$ is proper. For convenience the term <u>multivariate exponential distribution</u> (MVE) will mean a distribution in this class. By contrast with joint distributions with exponential minimums, multivariate exponential distributions are highly structured, and stand much closer to the joint distributions for which $T_1,\ldots,T_n$ are independent and exponential([3], Section 2).

The class of multivariate exponential distributions has several characterizations. One of these([3], Section 5.1), which is of special interest here, is that $T_1,\ldots,T_n$ have a multivariate exponential distribution if, and only if:

(3.2)

(a) $T_1,\ldots,T_n$ have exponential minimums.

(b) On each simplex $0 \le t_{i_1} \le t_{i_2} \le \ldots \le t_{i_n}$

$$P[T_1 > t_1,\ldots,T_n > t_n] = \prod_{j=1}^{n} P[\min_{i\epsilon I_j} T_i > t_{i_j} - t_{i_{j-1}}],$$

where $I_1 = \{i_1,\ldots,i_n\}$, $I_2 = \{i_2,\ldots,i_n\},\ldots,$ $I_n = \{i_n\}$ depend on the simplex, and $t_{i_0} = 0$.

By analogy with definition (3.1) positive integer valued random variables $N_1,\ldots,N_n$ can be said to have a multivariate geometric distribution in the narrow sense(MVG-N) if

$$N_i = \min_{\{J\epsilon\mathcal{J}:i\epsilon J\}} M_J, \quad i = 1,\ldots,n, \tag{3.3}$$

where the class $\mathcal{J}$ has the same property as in (3.1), and the M_J, $J\epsilon\mathcal{J}$, are independent, geometrically distributed random variables. By analogy with the characterization (3.2), $N_1,\ldots,N_n$ can be said to have a multivariate geometric distribution in the wide sense(MVG-W) if:

(a) $N_1,\ldots,N_n$ have geometric minimums.

(3.4)

(b) On each simplex $0 \le k_{i_1} \le k_{i_2} \le \ldots \le k_{i_n}$

$$P[N_1 > k_1, \ldots, N_n > k_n] = \prod_{j=1}^{n} P[\min_{i \in I_j} N_i > k_{i_j} - k_{i_{j-1}}],$$

where $I_1, \ldots, I_n$ are as in (3.2,b), and $k_{i_0} = 0$.

It is easy to see that the class of MVG-N distributions is contained in the class of MVG-W distributions. That the two definitions produce distinct classes of distributions is shown in Esary and Marshall [4], Example 2.1.

These classes of distributions are also preserved when propogated through the multivariate damage process, provided the weaker multivariate geometric concept is employed.

Theorem 3.1. $Y_1, \ldots, Y_n$ MVE $\Rightarrow$ $N_1, \ldots, N_n$ MVG-W $\Rightarrow$ $T_1, \ldots, T_n$ MVE.

Proof. In the proof MVE distributions are described by the characterization (3.2). Then since $Y_1, \ldots, Y_n$ have exponential minimums, it follows from Theorem 2.1 that $N_1, \ldots, N_n$ satisfy (3.4,a) and that $T_1, \ldots, T_n$ satisfy (3.2,a). Thus it remains to show that $N_1, \ldots, N_n$ satisfy (3.4,b) and that $T_1, \ldots, T_n$ satisfy (3.2,b).

To show the first implication note from (3.2) that on the simplex $0 \le y_1 \le y_2 \le \ldots \le y_n$

$$P[Y_1 > y_1, \ldots, Y_n > y_n] = \prod_{j=1}^{n} e^{-\lambda_{I_j}(y_j - y_{j-1})}, \tag{3.5}$$

where $I_1 = \{1,\dots,n\}$, $I_2 = \{2,\dots,n\},\dots,$ $I_n = \{n\}$, and λ_{I_j} is the parameter in the exponential distribution for $\min_{i\in I_j} Y_i$. Then recall that $N_i > k_i \Leftrightarrow Y_i > X_1+\dots+X_{k_i}$, $i = 1,\dots,n$. Thus on a simplex, which without loss of generality can be assumed to be $0 \le k_1 \le k_2 \le \dots \le k_n$,

$$P[N_1 > k_1,\dots,N_n > k_n] = P[Y_1 > X_1+\dots+X_{k_1},\dots,Y_n > X_1+\dots+X_{k_n}]$$

$$= E\left(e^{-\lambda_{I_1}(X_1+\dots+X_{k_1})} e^{-\lambda_{I_2}(X_{k_1+1}+\dots+X_{k_2})} \cdots \right.$$

$$\left. \cdots e^{-\lambda_{I_n}(X_{k_{n-1}+1}+\dots+X_{k_n})}\right)$$

$$= \left(E e^{-\lambda_{I_1} X}\right)^{k_1} \left(E e^{-\lambda_{I_2} X}\right)^{k_2-k_1} \cdots \left(E e^{-\lambda_{I_n} X}\right)^{k_n-k_{n-1}}$$

$$= \prod_{j=1}^{n} P[\min_{i\in I_j} N_i > k_j - k_{j-1}].$$

Thus $N_1,\dots,N_n$ satisfy (3.4,b).

To show the second implication note from (3.4) that on the simplex $0 \le k_1 \le k_2 \le \dots \le k_n$

$$P[N_1 > k_1,\dots,N_n > k_n] = \prod_{j=1}^{n} \theta_{I_j}^{k_j - k_{j-1}}, \tag{3.6}$$

where θ_{I_j} is the parameter in the geometric distribution for $\min_{i\in I_j} N_i$. Then recall that $T_i > t_i \Leftrightarrow N_i > K(t_i)$, $i = 1,\dots,n$. Thus on a simplex, which without loss of generality can be assumed to be $0 \le t_1 \le t_2 \le \dots \le t_n$,

$$P[T_1 > t_1, \dots, T_n > t_n] = P[N_1 > K(t_1), \dots, N_n > K(t_n)]$$

$$= \sum_{k_1=0}^{\infty} \sum_{k_2=k_1}^{\infty} \cdots \sum_{k_n=k_{n-1}}^{\infty} \Bigg(\theta_{I_1}^{k_1} \theta_{I_2}^{k_2-k_1} \cdots \theta_{I_n}^{k_n-k_{n-1}}$$

$$\frac{(\upsilon t_1)^{k_1} e^{-\upsilon t_1}}{k_1!} \frac{\{\upsilon(t_2-t_1)\}^{k_2-k_1} e^{-\upsilon(t_2-t_1)}}{(k_2-k_1)!} \cdots$$

$$\cdots \frac{\{\upsilon(t_n-t_{n-1})\}^{k_n-k_{n-1}} e^{-\upsilon(t_n-t_{n-1})}}{(k_n-k_{n-1})!} \Bigg)$$

$$= e^{-(1-\theta_{I_1})\upsilon t_1} e^{-(1-\theta_{I_2})\upsilon(t_2-t_1)} \cdots$$

$$\cdots e^{-(1-\theta_{I_n})\upsilon(t_n-t_{n-1})}$$

$$= \prod_{j=1}^{n} P[\min_{i\in I_j} T_i > t_j - t_{j-1}].$$

Thus $T_1, \dots, T_n$ satisfy (3.2,b). □

Theorem 3.1 requires no hypothesis on the damage variable X, except that X not be degenerate at zero to insure that the distributions for $N_1, \dots, N_n$ and $T_1, \dots, T_n$ are proper. However([4], Theorem 5.4) _if_ X _is infinitely divisible, then_ $N_1, \dots, N_n$ _have a_ MVG-N _dis-_

tribution. The situation is summarized in Figure 2.

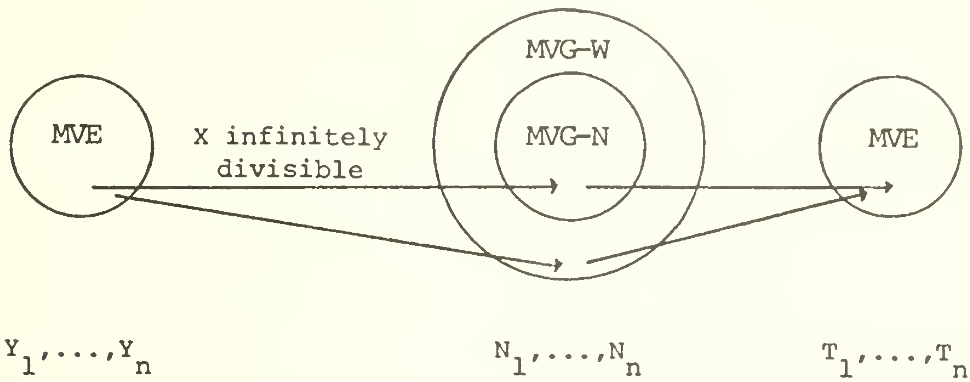

Figure 2

A converse result([4], Theorem 5.5) is that if $N_1,\ldots,N_n$ are MVG-N for all n and for all $Y_1,\ldots,Y_n$ which are independent and exponential, then X is infinitely divisible.

The bivariate damage process is a special case([4], Theorem 3.1) in that for it the chain of implications is

$$Y_1,Y_2 \text{ BVE } \Rightarrow N_1,N_2 \text{ BVG-N } \Rightarrow T_1,T_2 \text{ BVE}$$

without special hypotheses on X.

4. Coherent systems. It can be supposed that $Y_1,\ldots,Y_n$ are the breaking thresholds of the components in a coherent system. In this context a coherent system can be conveniently described by a form of its life function based on its minimal path sets. The minimal path sets $P_1,\ldots,P_p$ are set minimal combinations of components that by all functioning can cause the system to function. The number of cycles un-

til a minimal path set P ceases to "function" is $\min_{i\epsilon P} N_i$, and the time until the path set ceases to function is $\min_{i\epsilon P} T_i$. The number of cycles to failure for a <u>coherent</u> system is

$$(4.1) \qquad \tau(N_1,\ldots,N_n) = \max_{j=1,\ldots,p} \min_{i\epsilon P_j} N_i,$$

and the time to failure is

$$(4.2) \qquad \tau(T_1,\ldots,T_n) = \max_{j=1,\ldots,p} \min_{i\epsilon P_j} T_i,$$

where $\tau(t_1,\ldots,t_n) = \max_{j=1,\ldots,p} \min_{i\epsilon P_j} t_i$, $t_i \geq 0$, $i = 1,\ldots,n$, is the <u>life function</u> of the system [2].

Relationships similar to (1.1) and (1.2) hold for coherent systems. From (4.1) and (2.1) it is immediate that

$$(4.3) \qquad \tau(N_1,\ldots,N_n) > k \iff \tau(Y_1,\ldots,Y_n) > X_1+\ldots+X_k,$$

and from (4.2) and (2.2)

$$(4.4) \qquad \tau(T_1,\ldots,T_n) > t \iff \tau(N_1,\ldots,N_n) > K(t).$$

Then, similarly to (1.3),

$$(4.5) \qquad P[\tau(N_1,\ldots,N_n) > k] = E\,\bar{G}_\tau(X_1+\ldots+X_k),$$

provided that $\bar{G}_\tau(0) = 1$, where $\bar{G}_\tau(y) = P[\tau(Y_1,\ldots,Y_n) > y]$, $y \geq 0$, is the survival function for $\tau(Y_1,\ldots,Y_n)$, and similarly to (1.5),

$$(4.6) \qquad P[\tau(T_1,\ldots,T_n) > t] = \sum_{k=0}^{\infty} E\,\bar{G}_\tau(X_1+\ldots+X_k) \frac{(\upsilon t)^k e^{-\upsilon t}}{k!}.$$

Thus the experience of a coherent system whose components are exposed to a multivariate damage process can be represented by a univariate damage process with $Y = \tau(Y_1,\ldots,Y_n)$, $N = \tau(N_1,\ldots,N_n)$, and $T = \tau(T_1,\ldots,T_n)$.

A nonnegative random variable T(or Y) has an _increasing hazard rate average_(IHRA) distribution if $\frac{R(t)}{t}$ is increasing, where $R(t) = -\log P[T > t]$, $t \geq 0$. These distributions have been considered in the connection that if $T_1,\ldots,T_n$ are independent and IHRA(in particular if $T_1,\ldots,T_n$ are exponential), then $\tau(T_1,\ldots,T_n)$ is IHRA, where τ is the life function of a coherent system(Birnbaum, Esary, and Marshall [1], Theorem 4.2). A partial extension is that _if_ $T_1,\ldots,T_n$ _have a joint distribution with exponential minimums, then_ $\tau(T_1,\ldots,T_n)$ _has an_ IHRA _distribution_([3], Application 5.3).

The time to failure of a coherent system exposed to the multivariate damage process has an IHRA distribution under relatively weak assumptions on $Y_1,\ldots,Y_n$.

Theorem 4.1. $Y_1,\ldots,Y_n$ _have exponential minimums_(in particular $Y_1,\ldots,Y_n$ MVE) $\Rightarrow$ $\tau(T_1,\ldots,T_n)$ _is_ IHRA, _where_ τ _is a coherent life function._

Proof. The result follows from Theorem 2.1 and the preceeding remarks. □

For the basic case of the univariate damage process, but with Y assumed to be IHRA rather than exponential, it is shown in [5](Theorem 5.2,a) that T is IHRA. The proof uses Theorem 4.1 in conjunction with (4.6) and certain properties of IHRA distributions. An application of the result is that $\tau(T_1,\ldots,T_n)$ is IHRA for any joint distribution of $Y_1,\ldots,Y_n$ such that $\tau(Y_1,\ldots,Y_n)$ is IHRA.

It is also shown in [5](Theorem 5.2,b) that if Y has an increasing hazard rate(IHR) distribution, then N has a discrete analogue of an IHRA distribution(referred to as a D-IHRA distribution subsequently), and conversely(Theorem 5.2,c) that if N has a D-IHRA distribution, then Y must have an IHRA distribution. Assuming Y is IHR is more restrictive than assuming that Y is IHRA, and the question of whether Y is IHRA implies N is D-IHRA is unresolved. The remaining results in this section have a potential bearing upon, but do not resolve, this question.

A positive integer valued random variable N can be said to have a <u>discrete increasing hazard rate average</u>(D-IHRA) distribution if $\frac{R(k)}{k}$ is increasing, $k = 1,2,\ldots,$ where $R(k) = -\log P[N > k]$.

<u>Lemma 4.2</u>. $N_1,\ldots,N_n$ MVG-N $\Rightarrow$ $\tau(N_1,\ldots,N_n)$ D-IHRA, <u>where</u> τ <u>is a coherent life function</u>.

<u>Proof</u>. Suppose that $N_1,\ldots,N_n$ satisfy (3.3) with $P[M_J > k] = \theta_J^k$, $0 \le \theta_J < 1$, $J\in\mathcal{J}$. Let $\lambda_J = -\log\theta_J$, $J\in\mathcal{J}$. Since $\theta_J < 1$, then $\lambda_J > 0$. Let S_J, $J\in\mathcal{J}$, be independent random variables with the exponential survival functions $P[S_J > s] = e^{-\lambda_J s}$, $s \ge 0$. Let $U_1,\ldots,U_n$ be related to the S_J, $J\in\mathcal{J}$, by (3.1). Then $U_1,\ldots,U_n$ are MVE, and

$$P[\min_{i\in I} N_i > k] = P[\min_{\{J\in\mathcal{J}:I\cap J\neq\emptyset\}} M_J > k]$$

$$= \prod_{\{J\in\mathcal{J}:I\cap J\neq\emptyset\}} \theta_J^k = \prod_{\{J\in\mathcal{J}:I\cap J\neq\emptyset\}} e^{-\lambda_J k}$$

$$= P[\min_{\{J \in \mathcal{J}: I \cap J \neq \emptyset\}} S_J > k] = P[\min_{i \in I} U_i > k]$$

for each nonempty $I \subset \{1,\ldots,n\}$. From the form of τ it follows by the standard inclusion-exclusion argument that

$$P[\tau(N_1,\ldots,N_n) > k] = P[\tau(U_1,\ldots,U_n) > k].$$

From Application 5.3 of [3], $\tau(U_1,\ldots,U_n)$ is IHRA, so that

$$P[\tau(U_1,\ldots,U_n) > t] = e^{-R(t)}$$

where $\frac{R(t)}{t}$ is increasing. Then

$$P[\tau(N_1,\ldots,N_n) > k] = e^{-R(k)}$$

where $\frac{R(k)}{k}$ is increasing, $k = 1,2,\ldots$. □

<u>Theorem 4.3</u>. $Y_1,\ldots,Y_n$ MVE <u>and</u> X <u>infinitely divisible</u> $\Rightarrow$ $\tau(N_1,\ldots,N_n)$ D-IHRA, <u>where</u> τ <u>is a coherent life function</u>.

<u>Proof</u>. The result follows from Lemma 4.2 and the observation in Section 2 that the hypotheses imply that $N_1,\ldots,N_n$ are MVG-N. □

Theorem 4.3 and (4.5) can be used in an argument(similar to the proof of Theorem 5.2,a in [5]) to show for the univariate damage process that

$$Y \text{ IHRA and } X \text{ infinitely divisible} \Rightarrow N \text{ D-IHRA}.$$

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A fairly common failure model in a wide variety of contexts is a cumulative damage process, in which shocks occur randomly in time and associated with each shock there is a random amount of damage which adds to previously incurred damage until a breaking threshold is reached. The multivariate life distributions that are induced when several "components," each with its own breaking threshold, are exposed to the same cumulative

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damage process are of interest in their own right, and are important examples in the general study of multivariate life distributions.

This paper is a summary of some results about the very special, but central, case in which the cumulative damage process is a compound Poisson process. It is focused on the multivariate life distributions that arise when the component breaking thresholds are random and have a Marshall-Olkin multivariate exponential distribution. There are two relevant multivariate life distributions that can be derived, an intermediate distribution for the number of shocks (cycles) to failure and the final distribution for the actual times to failure. The results have application to the life distribution of a coherent system whose components are exposed to the damage process.