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THE EFFECTS OF DIFFERENT PRODUCTION RATE MEASURES AND COST STRUCTURES ON RATE ADJUSTMENT MODELS

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ABSTRACT

The effect of production rate on the cost of weapon systems has attracted much attention in the cost estimating community in recent years. A variety of adjustments to weapon systems cost models have been proposed to reflect the impact of different production rates. The most popular solution is to add a rate term to the traditional learning curve model. This paper examines the effects of different rate measures and cost structures on rate adjustment models. Numerical examples illustrate that the production rate term should be measured as a ratio and not as an absolute quantity of a production lot or a period. The paper also points out that a rate adjustment model is appropriate only with data collected from plants which have not undergone changes in cost structure.

THE EFFECTS OF DIFFERENT PRODUCTION RATE MEASURES AND COST STRUCTURES ON RATE ADJUSTMENT MODELS

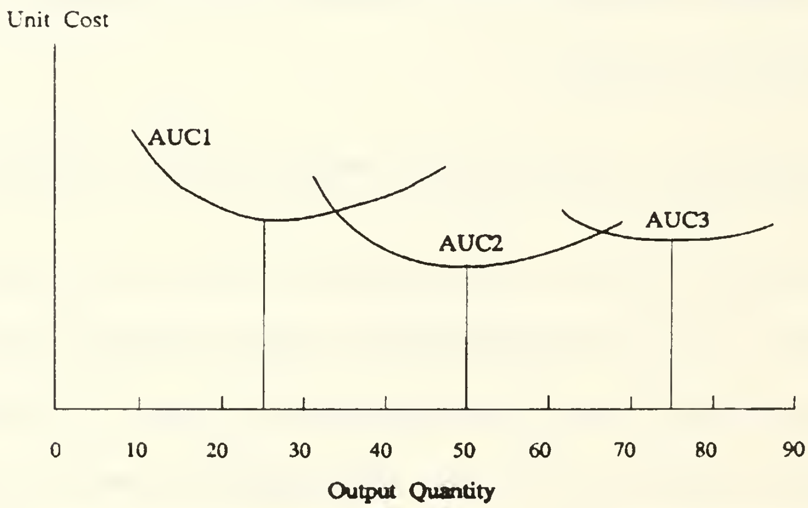
The effect of production rate on the cost of weapon systems has attracted much attention in the cost estimating community in recent years. A variety of adjustments to weapon systems cost models have been proposed to reflect the impact of different production rates. The most popular solution is to add a rate term to the traditional learning curve model. The resulting learning curve model augmented with the production rate variable is usually referred to as a rate adjustment model. The purpose of this paper is to examine the theoretical underpinning of the production rate effect on weapon system cost and illustrate that the popular solution to the rate problem may result in erroneous conclusions. Numerical examples will be used to illustrate the potential problems of the popular approach to production rate adjustment. The paper concludes with a discussion of the scenarios in which the rate adjustment models may be utilized.

CONCEPTUAL FOUNDATION OF PRODUCTION RATE EFFECT

The conceptual foundation of the production rate impact on cost is related to economies of scale. In many industries that effect is well understood. High production rates allow greater use of facilities and greater specialization of labor. The increased volume of materials purchased reduces their unit cost. The increased volume of production activities spreads fixed overhead costs over a larger quantity of products produced. Taken together, all these effects work to increase efficiency and lower production costs (Bemis, 1981; Large, et al., 1974; Linder and Wilbourn, 1973).

It should be noted, however, that a plant with a higher production rate does not necessarily produce at a lower unit cost when compared to another plant. This point is illustrated in Figure 1. Assume there are three plants capable of producing the same item, such as a missile. The Average Unit Cost curve for each plant is shown as AUC1, AUC2, and AUC3, respectively. If the output quantity were fixed at 25 units, then Plant 1 is the most efficient of the three plants. However, if the output level were fixed at the rate of 40 units per period, Plant 1's unit cost would be higher than that of Plant 2, which is the most efficient of the three at that production quantity. This is consistent with economic theory, which says that, in general, there are both economies and diseconomies of scale. This phenomenon is recognized by the above analysts and is reflected in their use of this familiar U-shaped average cost curve to incorporate the effect of production rate into weapon systems cost models.

Figure 1
Average Unit Cost and Production Capacity



The same theory of economies and diseconomies of scale is applicable to a single plant's expansion when it is operating beyond its efficient capacity level. This scenario has significant implications in weapon systems cost estimation. Recent experience has shown that production rates of major weapon systems are subject to continual adjustment, sometimes significantly. At the low end of the spectrum is the initial production rate. This is usually a function of early procurement funding constraints and the technical risk of building substantial numbers of newly developed items before the design has fully matured. Thus low rate initial production avoids the risk of incurring costly retrofits to early production units. During this early stage of production, the amount of fixed costs may vary from period to period because of the changing production setup. At the upper boundary is the limitation of available plant capacity and the requirement for additional investments in tooling and facilities for capacity expansion. Additional investments in tooling and facilities alter the cost structure of the plant. The unit cost curve of a plant expanding its investment in tooling and facilities is equivalent to changing from AUC1 to AUC2 as shown in Figure 1.

REVIEW OF RATE ADJUSTMENT MODELS

Although studies of the effect of production rate change on weapon systems cost began as early as the 1950s (Hirsch, 1952; Alchian, 1963), and various models had been proposed, the most widely used rate adjustment model in use today was developed by augmenting the traditional learning curve model with a production rate term:

$$Z = aX^bR^c = YR^c \tag{1}$$

where,

Z = unit cost of the item with production rate as well as learning considered,

X = cumulative quantity produced,

R = production rate measure,

Y = unit cost of the item with only learning considered,

a = a constant, usually called the theoretical first unit cost,

b = a parameter, usually called the slope of the learning curve,

c = a parameter, usually called the slope of the production rate curve.

Empirical work on this production rate/learning model was first conducted at RAND, but the model was later popularized by Bemis (1981). Large, et al. (1974) attempted to develop this model for various production cost elements. They were forced to conclude, however, that the production-rate/cost relationship could not be predicted with any reasonable degree of confidence. For production planning purposes, they recommended that production rate effects in aircraft production programs be ignored because they were dominated by other effects. They also suggested that production rate is subject to change and, hence, is difficult to predict.

Further work on the production rate/learning model was carried out by Smith (1976). He analyzed three aircraft programs for which a large number of data values were available due to long production periods. Where the data permitted, Smith applied his model separately to fabrication and assembly labor hours. He then compared his production rate/learning model to a reduced, learning-only model. Smith found that the rate term was an important contributor to the explanatory power of the model. However, he obtained a surprisingly large variation in parameter values for cases with similar production quantities and rates. Additional efforts using this approach were carried out by Bemis (1981), Cox and Gansler (1981), and others.

If one recognizes the inherent rate instability scenario of major weapon systems production and the resultant changing cost structure discussed in the preceding section, then none of the inconclusive findings discussed above would be surprising. In the following sections, we will examine the issues of alternative production rate measures and changing cost structures, and we will discuss other major considerations that must be addressed before one can use the rate adjustment model in weapon systems cost estimation.

ALTERNATIVE PRODUCTION RATE MEASURES

Although the concept of production rate is clear, its measurement is by no means unambiguous. Several alternatives have been used as surrogate measures of production rate. The two primary measures are lot size and annual/monthly production quantity. We will first discuss these two and related measures, along with the difficulties of their use. We then discuss a third alternative, a ratio measure which we believe will avoid some of the difficulties of the measures used to date.

Using Lot Size or Annual/Monthly Quantity As the Rate Measure

Hirsch (1952), Cox and Gansler (1981), and Bohn and Kratz (1984) all used lot size as their measure of production rate. Hirsch was careful to note that his lot intervals were fairly stable; however, this has not been the case with almost all more-recent aircraft programs. Since the time (and, hence, cost) required to produce sequential, similarly-sized lots often changes over the life of the program, it is unclear what is being measured by the lot size proxy.

Perhaps the most common measure of rate is that of production quantity in some time interval. The time period involved is usually selected as a function of data availability. Most studies use annual quantities as a measure of production rates. An inverse of the quantity-per-unit-time measure has also been used; Large, et al. (1974) used the number of months required to reach a certain cumulative production quantity as their inverse measure. Some studies, such as Womer (1984), use monthly data. Womer notes that if there is substantial work-in-progress and the production period is long compared to the period of observation, then units produced in the following time period actually reflect work performed in the preceding time period, and this can result in substantial bias in estimation. Since this problem is especially critical for monthly data, Womer used a lagged model of production to obtain his estimates.

When analyzing a cross-section of programs, it is possible to use an average rate for each program. Because the production rate may change in a typical production run, an average rate for an individual program is usually used in these cross-section analyses. Use of an average may understate the effects of these disruptive rate changes, but we do not expect it to mask the effect of production rate itself. Large, et al. (1974) used this approach in their examination of several programs.

Gulledge and Womer (1986) noted that cumulative quantity is highly correlated with any of the production rate measures discussed above. Hence, using either the lot size or monthly/annual quantities as the measure of R in Equation (1) will produce unreliable models due to this collinearity of the cumulative quantity measure of learning (X) and the measure of production rate (R). The presence of this collinearity has resulted in the inability of analysts to separate statistically the effects of learning and production rate. For example, Large et al. (1974) concluded that the influence of production rate could not be estimated with confidence.

Using a Ratio as the Rate Measure

An alternative to the above measures which will tend to mitigate the multicollinearity problem is that of a ratio of the above production rate measures. This use of a ratio, if keyed to a base production rate, as the rate curve measure appears to be an innovation in the literature. Bemis (1981) uses the ratio of new rate to present rate as the rate measure, which is more a measure of rate change than a measure of the rate per se. A similar measure was adopted by Balut (1981) and Balut, et al. (1989); they used a ratio of old-to-new lot sizes to account for rate effects in an aircraft repricing model which also included a learning curve. On the other hand, Boger and Liao (1988) proposed using a standard, base, or predetermined rate as the denominator in the ratio and either lot sizes or annual/monthly quantities as the numerator. The advantage of using a base rate is that if one uses the rate to which the manufacturer has tooled the production facility as the base rate, then ratios greater than unity would indicate decreasing returns to variable inputs and ratios lesser than unity would indicate increasing returns to variable inputs.

In addition to the mitigation of statistical problems, the use of a ratio as the rate measure has some intuitive advantages for cost estimating purposes. While the general formulation shown in Equation (1) for production rate is widely used, little has been done to examine the empirical implications of adding the production rate factor to the well known learning curve model. The definition of the parameter **a** of Equation (1) (referred to as the theoretical first unit cost in learning curve theory) is the unit cost when $X=1$ and $R=1$. While this interpretation seems logical, it does result in some awkward numbers because $R=1$ is not close to the relevant production range for most of the production rate measures used in practice. It is, however, for our proposed measure. This issue can be illustrated with a simple example. This example will use a minimum of data points since this is the typical situation faced by cost analysts.

An Illustrative Example

Assume that the data for the first two production contracts for a new weapon system are as follows:

Lot #	Quantity	Unit Price	Algebraic Lot Midpoint
1	100	$43,773	33.9
2	100	31,035	147.0

The algebraic lot midpoint is that quantity on the learning curve which corresponds to the average cost for that entire lot. Liao (1988 and 1989) provides detailed discussions of this concept and its measurement.

A. Ratio Rate Measure -- Since there are only two data points, only the learning curve slope may be estimated at this point. We may use the following formula to determine the learning curve slope:

$$b = \frac{\text{Log } (Y_2 / Y_1)}{\text{Log } (M_2 / M_1)} \qquad (2)$$

where Y_i and M_i represent the unit price and the algebraic midpoint of each lot respectively. The slope of the learning curve for our illustrative data may now be determined as follows:

$$b = \frac{\text{Log } \frac{31,035}{43,773}}{\text{Log } \frac{147.0}{33.9}} = -0.234422 \text{ or } 85\% \text{ curve}$$

The first unit cost can be readily obtained by substituting the value of b into the basic learning curve equation:

$$43,773 = a\ (33.9)^{-0.234422}$$

$$a = 100,000$$

Note that **implicit** in the above computation is the production rate of 100 units. In other words, the $100,000 represents the cost of producing the first unit when the rate is 100 units per year.

Let us assume that for year 3 requirements the government solicits step-ladder quotes from a potential contractor for this system. Step-ladder quotes are the quotes in a schedule of bids from a potential contractor for varying percentages of the government's planned total requirement for that year. (A full set of quotes, using a 10% step, would give the potential contractor's prices for 10%, 20%,. . ., and 100% of the government's requirement.) The differences in the prices quoted by a single contractor for various quantity levels during this single year, in principle, should reflect only the production rate effect. Let us further assume that the slope for the rate curve is 80%. If we want to evaluate the reasonableness of quotes at different production rate levels, the most logical approach is to anchor the rate measure at a given level within the relevant rate range, e.g., 100 units (base rate = 100), and measure different quantity levels as a ratio of that base rate. If the rate curve is known or agreed upon by both parties, the reasonable quotes for various quantity levels may be directly calculated by using the following formula:

$$Z = Yr^{d} \qquad (3)$$

where,

r = the slope of the production rate curve, and

d = the logarithm of R (the ratio measure of rate) divided by the logarithm of 2.

For example, with the assumed 80% rate curve, 85% learning curve, and a=100,000, the reasonable quote for 300 units may be computed as follows:

$$Z_{300} = 25{,}554(0.8)^{\log(3)/\log(2)} = 25{,}554(0.8)^{1.585} = 17{,}942$$

If the parameter value of the rate term is unknown, it can be estimated from annual step ladder quotes as follows. Since we define $Z = aX^bR^c$ or YR^c, the ratio of reasonable bid prices at various quantity levels as a function of the long-term learning curve may be determined as follows:

$$R^c = Z/Y, \text{ or } Z/aX^b \tag{4}$$

We may use the computed ratios for various quantity levels to determine the parameter value for the rate term. Table 1 shows the procedures described above.

Table 1

Estimating Rate Effect from Year 3 Step-Ladder Quotes

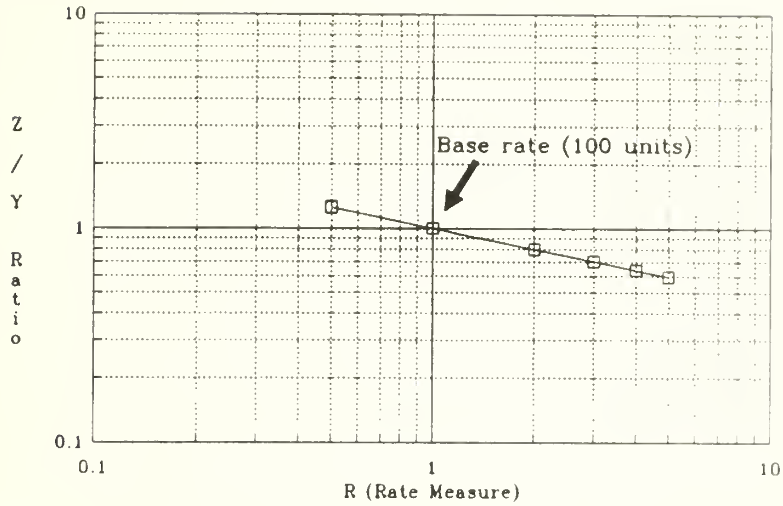
(a = 100,000, Total Previous Quantity = 200 units)

Quote Quantity	Midpoint	aX^b (Y)	aX^bR^c (Z)	R^c Z/Y	R Q/100
50	224.9	28,088	35,111	1.250	0.5
100	248.4	27,442	27,442	1.000	1
200	293.5	26,390	21,112	0.800	2
300	336.7	25,554	17,942	0.702	3
400	378.6	24,861	15,911	0.640	4
500	419.5	24,269	14,456	0.576	5

Figure 2 shows the relationship between Z/Y and the rate measure, R. Note that the reasonable quotes should reflect a straight line on a log-log graph as shown in Figure 2. The slope of the rate curve can be derived from the values of the last two columns of Table 1 in the same way that the learning curve slope is usually derived (by using the log-linear regression method). In our case, the regression yields the exponent, **c**, -0.3218, which represents an 80% curve, the slope we used to generate the hypothetical data.

B. Absolute Size Rate Measure -- If we use the lot size or annual/monthly quantity

Figure 2
Production Rate Curve (80%)



directly as the measure of the production rate, the definition of **a** is necessarily changed to the theoretical first unit cost in the learning curve when X=1 and R=1. Since the rates for the first two buys of our illustrative example are not unity, it is impossible to determine the parameter value of the rate term unless there are at least three, and preferably more, data points.

By combining all available price data when year 3 quotes become available, we can derive the parameter values for the Z equation as shown below:

	Ratio Rate Measure	Absolute Size Rate Measure	
a =	$100,000	$440,352	
b =	-0.23445	-0.23445	(85% learning curve)
c =	-0.321915	-0.321915	(80% rate curve)

The only difference in results is the first unit cost, **a**. The high value of the first unit cost when using the absolute size rate measure is due to the implicit assumption that it is for X=1 and R=1, which is outside the relevant production rate range and, therefore, is not a meaningful number.

CHANGING COST STRUCTURE

The second major issue facing the use of rate adjustment model for weapon systems cost estimation is the changing cost structure as a result of changes in production setup. Any additional investments in a plant's facilities, whether for capacity expansion or for more efficient production methods, alter the cost structure. This change of cost structure does not create a significant problem for the X term in Equation (1), since it captures the effect of cumulative production experience (a continuous phenomenon). The changing cost structure, however, poses a serious question about the suitability of using multi-year cost data for cost models involving rate adjustments. The production rate term captures the effect of spreading fixed costs over varying numbers of units. During the early stages of production, the amount of fixed costs may vary from period to period because of the changing production setup. Therefore, the effect of production rate on unit costs may not stabilize until after the production setup and its inherent cost structure is stabilized. Trying to derive a rate curve with historical data from only the early stages of production is probably unreliable.

Let us extend the previous example by assuming that the plant capacity is expanded in year 3 to accommodate the higher quantity required. The resultant higher fixed costs push up the total production curve for any given quantity level from TC1 to TC2, as shown in Figure 3. TC3 represents the total cost curve if the capacity is further expanded. Figure 4 depicts the cost reduction curves under different production rates after the learning curve effect has been considered (see Column 3 in Table 1).

Figure 3
Total Cost vs Production Rate

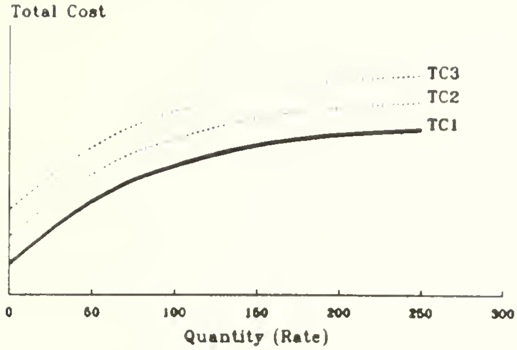
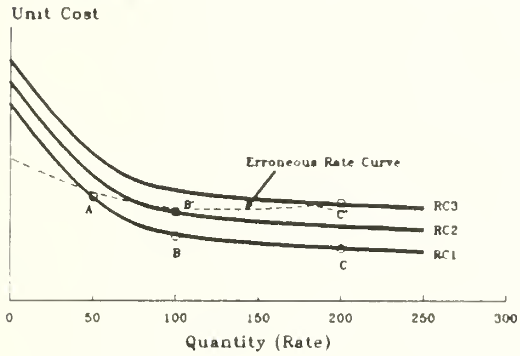


Figure 4
Changing Cost Structure & Rate Curve



If the government procured 50 units in year 1 under the cost structure labeled TC1 and RC1, 100 units in year 2 under TC2 and RC2, and 200 units in year 3 under TC3 and RC3, the unit costs to the government, after considering the learning curve effect, would be Points A, B', and C'. Deriving a rate curve using A, B', and C' would result in an erroneous rate curve, as shown in Figure 4. The slope of the erroneous rate curve is biased by the changing cost structure.

On the other hand, if there is no change in the plant's cost structure, the same cost curve (TC1 or RC1) applies to years 1 through 3, and the three data points (A, B, and C in

Figure 4) would all fall on the same curve (RC1). Therefore, the data would be appropriate for estimating the parameter value for the rate term. The same is also true for step-ladder quotes for any particular year, which reflect the spreading of fixed costs in a particular year (Points A, B, and C) and, therefore, are also appropriate for estimating the parameter values using Equation (1).

Table 2

The Effect of Changing Cost Structure on Unit Costs
(a = 100,000, LC = 85%, R = 80%)

Lot #	Quantity	Total Cost	Unit Cost
A. Same Cost Structure:			
1	50	$3,191,511	$63,830
2	100	3,437,523	34,375
3	200	5,601,184	18,671
B. Changing Cost Structure:			
1	50	$3,191,511	$63,830
2	100	3,837,523	38,375
3	200	6,401,184	21,337

The issue discussed above can be illustrated with a numerical example as shown in Table 2. Data for Scenario A are constructed by assuming that there was no change in the cost structure in the contractor's plant. Data for Scenario B are constructed by adding $400,000 and $800,000 of additional fixed costs to year 2 and year 3 total costs respectively. Using the three data points under each scenario to derive the parameters for Equation (1) results in the following:

	Scenario A	Scenario B:
a =	$100,000	$72,227
b =	-0.2344 (85%)	-0.1389 (91%)
c =	-0.3219 (80%)	-0.3959 (76%)

It can be seen clearly that analysis of data from Scenario A results in correct parameters, while analysis of data from Scenario B distorts all three parameters. What we can conclude is that using cost data obtained from a plant which has experienced a changing cost structure violates the statistical requirement of drawing samples from a homogeneous population. The consequence of sampling from different populations is the distortion of all parameters, as shown above.

CONCLUSIONS

In this paper, we examined the conceptual underpinning of the production rate effect on weapon system costs as well as various production rate measures for rate adjustment models. The first conclusion is that the production rate term should be measured as a ratio, not as an absolute quantity of lot size or annual/monthly quantity. Expressing the production rate as a function of a base rate within the relevant range allows the analyst to estimate the learning curve from scanty historical data with more confidence as well as adjust costs for the applicable rate effect. It also facilitates the comparison of current step-ladder quotes with the historical contract awards.

There are several other practical considerations that favor the use of a ratio as the rate measure. The data base available for learning curve and rate curve determination is typically scanty. Using unity as the rate base requires both X and R as the independent variables in parameter determination. Having to use two independent variables reduces the degrees of freedom and increases the estimating error accordingly.

The second conclusion is that a stringent condition must be met before an analyst can use multi-year cost data to derive parameter values for the widely used rate adjustment model (Equation 1). The condition is that the underlying cost structure (variable/fixed cost mix and direct/indirect cost mix) must remain the same for all time periods covered by the data. This condition is met by step-ladder quotes for various quantities within the same period or by a plant that has stabilized its production capacity and setup. Unless this condition is fulfilled, the rate adjustment cost model may significantly distort the parameters. We believe that the inconclusive findings of prior research regarding production rate impact on weapon systems cost can be partially attributed to this problem.

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