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# NAVAL POSTGRADUATE SCHOOL

# Monterey, California



CONTRACTOR REPORT

WAVE ROTOR RESEARCH: A COMPUTER CODE FOR PRELIMINARY DESIGN OF WAVE DIAGRAMS

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# LIST OF CONTENTS

1.	INTRO	DUCTION																		age 1
																				2
2.	иетно			• • •																3
	2.1.	Solution	n Procedu	re .	• •	• •	•	•	•	• •	•	•	•	•	•	٠	•	•	•	3
	2.2.	Boundary	y Conditi	ons .			•	•	•		•	•	•	٠	•	٠	•	•	•	8
		2.2.1.	Solid Wa	11 .				•	•		•	•	•		•	•	•	•		9
		2.2.2.	Outflow					•	•		•	•	•			•	•	•		10
		2.2.3.	'Piston'	Inflo	. WC				•		•	•	•		•	•	•		•	10
		2.2.4.	Isentrop	ic In	flow	from	n Re	eser	vo	ir			•	•		•	•	•		12
		2.2.5.	Special	Formu.	latic	n -	'Tu	ınir	ng '	Ро	rts	;	•			•	•	•	•	13
	2.3.	Example	Calculat	ions							•	•	•	•	•	•	•		•	14
		2.3.1.	Test Cas	e for	1-D	Rand	lom	Cho	oic	e M	etł	od		( R	RCM	1)	•	•		14
		2.3.2.	Wave Tur	bine	Exper	imer	nt .		•		•		•	•	•	•	•	•		15
		2.3.3.	General	Elect	ric b	lave	Eng	gine	9				•	•		•	•	•	•	18
		2.3.4.	Spectra	Techn	ology	Pre	essu	ıre	Ex	cha	nge	r					•	•		20
3.	DISCU	SSION ANI	D RECOMME	NDATI	ONS		•		•				•	•		•				24
	3.1.	Discuss	ion						•					•		•	•	•		24
	3.2.	Recommen	ndations						•											24
REF	ERENCE																			26
FIG	URES .																			27
			• • • •	• • •	• •	• •	•	•	•	• •	•	•	•	•	•	•	•	•	•	
APP	ENDIX	A. List:	ing of Pr	ogram	RCM	• •	•	•	•	• •	•	•	•	٠	٠	٠	•	•	•	A.l
APP	ENDIX	B. Progr	ram RCM				•	•	•		•	•	•	•	•	•	•	•	•	В.1
		В.1.	Program	Desc	ripti	.on	•		•		•	•	•		•		•		•	В.1
			B.1.1.	Comp	utati	ona.	l Gr	id	•						•	•	•	•	•	В.1
			B.1.2.	Data	Inpu	ıt.					•			•					•	В.1

			Pa	UP
		B.1.3.	Non-dimensionalization	
		B.1.4.	Structure	В.2
	B.2.	Example	Use of Program RCM	в.3
	в.3.	Execution	on	в.3
	В.4.	List of	Variables	В.4
	B.5.	List of	Subroutines and Function Subprograms	в. е
		B.5.1.	Subroutines	в. е
		B.5.2.	Function Subprograms	В.7
DICTPIBLITION	IICT			

# LIST OF FIGURES

	<u> </u>	Page
1.	The Riemann Problem in Gas Dynamics; Special Case of Shock-Tube Flow	27
2.	Cartesian Grid for 1-D Random Choice Method	28
3.	General Wave Structure Resulting from the Solution of a Riemann Problem	29
4.	Ideal Wave Diagram for Pressure Exchanger (Spectra Technology)	30
5.	Test Case for 1-D Random Choice Method	31
6.	Wave Diagram Computed by 1-D Random Choice Method	32
7.	Ideal Wave Diagram for General Electric Wave Engine	33
8.	Gas Exhaust and Fresh Air Induction Process in G. E. Wave Engine	34
9.	Ideal Performance Curves for G. E. Wave Rotor-as Gas Generator	35
10a.	Distribution of Flow Parameters in Rotor Passage Just as Inlet Port Opens	36
10b.	Distribution of Flow Parameters in Rotor Passage Just as Exhaust Port is Closed	36
10c.	Distribution of Flow Parameters in Rotor Passage Just as Inlet Port Closes	37
11.	Distribution of Flow Parameters in Rotor Passage When Port Closes Late, i.e., After Arrival of Shock	37
12a.	Distribution of Flow Parameters in Rotor Passage When the H. P. Air Outlet Port Opens on Time	38
12b.	Distribution of Flow Parameters in Rotor Passage for Condition of Mismatch: H. P. Air outlet Port Opens Prematurely	39
12c.	Distribution of Flow Parameters in Rotor Passage for Condition of Mismatch: H. P. Air Outlet Port Opens Late	39



#### 1. INTRODUCTION

Unsteady flow in the passages of wave rotor devices can adequately be modelled on a one-dimensional basis. However, this modelling can be quite involved due to the peculiar characteristics typical of wave rotor type flows. The numerical calculation has to provide approximate solutions of time-dependent compressible fluid flow problems which involve discontinuities and strong wave interactions. Ref. (1) lists three criteria which such approximate solutions should satisfy simultaneously: (i) the solution must be reasonably accurate in smooth regions of the flow. Continuous waves (rarefaction waves, compression waves) should propagate at the correct speed and should maintain the correct shape which involves steepening or spreading at the correct rate; (ii) discontinuities which are transported along characteristics (gradient discontinuities, contact surfaces), should be modelled by sharp and discrete jumps, and should be transported at the correct speed; and (iii) nonlinear discontinuities such as shocks should be computed stably and accurately.

In addition, the complex pattern of shock waves and contact surfaces that could evolve in wave rotor devices precludes the use of numerical methods which rely on either some type of artificial viscosity or a special treatment of discontinuities. Such methods would quickly become quite impractical for this application due to programming difficulties and cost of execution.

Computation of such solutions has generally been carried out by solving a set of finite difference equations which approximate the governing differential equations of flow. All such schemes inherently have a finite amount of dissipation as well as dispersion of the wave modes they model, and it is difficult to construct difference schemes which simultaneously satisfy the criteria given above. Stability problems may also be an added concern for

these schemes.

In view of the foregoing, an alternative approach to solving wave rotor type flows was sought, and the purpose of this report is to describe such a scheme along with some results. The scheme is known variously as Glimm's method, the Random Choice Method (RCM) or the piecewise sampling method. The method evolved from a constructive proof of the existence of solutions to systems of nonlinear hyperbolic conservation laws given by Glimm (Ref. 2). Chorin (Refs. 3 and 4) developed the scheme into an effective numerical tool for gas dynamic applications, with emphasis on detonation combustion problems and reacting gas flows. Although the RCM computes solutions on a fixed grid, it is not a difference scheme, utilizing solutions of locally defined Riemann problems as the basic building blocks for the global solution. Each of the local Riemann problems (defined in more detail in section 2) provides an analytically exact elementary similarity solution. By means of a suitable sampling procedure, usually of a pseudo-random or quasi-random nature, the similarity solutions are superposed to construct the approximate solution to the equations.

With an appropriate sampling technique, the RCM in one dimension is possibly superior to any finite difference scheme in meeting the criteria established above.

#### 2. METHOD

#### 2.1. Solution Procedure

The method models the one-dimensional, compressible, inviscid Euler equations, expressed in conservation form as

$$\frac{\partial U}{\partial t} + \frac{\partial F(U)}{\partial x} = 0$$
 , where

$$U(x,t) = \begin{cases} \rho \\ \rho u \\ E \end{cases} \quad \text{and} \quad F(U) = \begin{cases} \rho \\ \rho u^2 + p \\ (E + p)_{\rho} \end{cases}$$
 (1)

Here E is the total energy per unit volume and may be expressed as (for a polytropic gas)

$$E = \rho \varepsilon + \frac{1}{2} \rho u^2$$
 ,  $\varepsilon \stackrel{\triangle}{=}$  internal energy per unit mass 
$$= \frac{1}{\gamma - 1} \left( \frac{p}{\rho} \right)$$

p is the density, p is pressure and u is velocity in the one space dimension being considered here. With initial data specified in the form

$$U(x,0) = \varphi(x) ,$$

an initial value problem is defined for the Euler equations. The simplest initial value problem for which discontinuities appear is the Riemann problem: to find the gas flow resulting from an initial state in which the gas on the right of an 'origin' is in a constant state, and the gas on the left is in another constant state, i.e.,

$$\psi(x) = \begin{bmatrix} U_{L} & , & x < 0 \\ U_{R} & , & x > 0 \end{bmatrix}$$

with

$$U_{L,R} = \begin{cases} \rho L, R \\ (\rho u)_{L,R} \\ E_{L,R} \end{cases}$$

where the subsripts L and R denote the left and right sides of the 'origin', here arbitrarily prescibed at 0. That is, the Riemann problem consists of prescribing constant initial data on either side of an origin where a jump discontinuity exists. As mentioned before, the solution of the problem constitutes a basic building block of the random choice method. A special case of the Riemann problem in which  $u_L = u_R = 0$  is often referred to as the shock tube problem. The answer to the problem is that there are four possible types of subsequent flow, depending on the inequalities in the left and right side data prescribed. Thus, in both directions from the origin, a shock or a centered rarefaction wave may propagate, giving rise to the above mentioned four different possibilities. Fig. (1) illustrates the special case of shock tube type flow and the evolution of the wave pattern.

Fig. (2) shows the simple fixed Cartesian grid set up for the method. Let  $\Delta x$  be a spatial increment and  $\Delta t$  a time increment. The solution is to be evaluated at time  $(n+1)\Delta t$ , n being a non-negative integer, at spatial increments  $i\Delta x$ ,  $i=1,2,3,\ldots$  The initial data is prescribed for each time step at  $n\Delta t$  in a piecewise constant manner i.e., it consists of intervals of length  $\Delta x$  where the data is constant, separated by jump discontinuities:

$$U(x, n\Delta t) = U_i^n, (i\frac{1}{2})\Delta x < x < (i+\frac{1}{2})\Delta x$$

The solution at time  $(n+1)\Delta t$  then is required to have the same property, i.e., it is piecewise constant over an interval  $\Delta x$ , and it serves as the initial data for the next time step:

$$U(x,(n+1)\Delta t) = U_i^{n+1}, (i - \frac{1}{2})\Delta x < x < (i + \frac{1}{2})\Delta x$$

This procedure defines a sequence of local Riemann problems to be solved at each time level. On the grid shown in Fig. 2, for example, initial data would be specified at points 1, 3, 5 ...., setting up a succession of Riemann problems defined by each pair of states (1,3), (3,5), (5,7), with the discontinuities at the midpoint of each, i.e., at 2, 4, 6, .... etc. If the time step increment  $\Delta t$  is calculated such that

$$\Delta t < \sigma.(\Delta x).$$
  $\max_{i}$   $(\left|u_{i}^{n}\right|+a_{i}^{n})$  , with 
$$0 < \sigma < \frac{l}{2}$$

then the waves generated at the discontinuities of adjacent Riemann problems will not interact, as shown schematically in Fig. 2.

Each of the local Riemann problems yields an exact analytical solution, with the resulting wave structure a particular combination/variation of the general structure shown in Fig. 3.

In the x-t plane, the solution to a Riemann problem consists of essentially four regions connected by three waves. Thus states I and IV are the prescribed left and right states for the problem, and states II and III are the 'starred' middle states separated by a slip line or contact discontinuity  $\frac{dx}{dt} = u^*$ . The velocity, u, and pressure, p, are continuous across the contact, but p in general is not. Thus  $u_L^* = u_R^*$ ,  $p_L^* = p_R^*$  and  $p_L^* \neq p_R^*$ .  $S_{1,b}$ ,  $S_{2,b}$  and  $S_{1,f}$ ,  $S_{2,f}$  represent respectively the backward and forward facing waves generated at the point of discontinuity and may be either shocks or rarefaction waves.

Still referring to Fig. 3, it is seen that at a time  $n\Delta t < t < (n+1)\Delta t$ , the exact solution of the local Riemann problem for the interval  $[(i-1)\Delta x]$ ,  $[i\Delta x]$  may actually consist of several distinct states. Consider now a

translation of each interval  $[(i-1)\Delta x, i\Delta x]$  to  $\left[-\frac{\Delta x}{2}, +\frac{\Delta x}{2}\right]$  such that the discontinuity (i.e., the point from which the waves are generated) is centered at a zero origin. Let 0 be the value of a random variab<mark>le, defined</mark> over the interval  $\left[-\frac{1}{2}, +\frac{1}{2}\right]$ , and let

$$\xi = 0\Delta x$$
 , i.e.  $-\frac{\Delta x}{2} < \xi < +\frac{\Delta x}{2}$ 

Also, define  $U_{\text{exact}}^{n+}$  (x,t) ,  $n\Delta t < t < (n+1)\Delta t$  , to be the exact solution to each Riemann problem. Using the value of  $\xi$  to fix a point in the interval Ax of each Riemann problem, the exact solution at that point is determined and assigned to either the left or the right grid point, depending on whether  $\xi$  is  $\langle$  or  $\rangle$  0 . Thus, if the point fixed by  $\xi$  is P' (Fig. 3), the exact solution to the Riemann problem at that sampled location is assigned to the grid point on the right and if the sampled point is P", the solution at that location is assigned to the grid point on the left, i.e., for a typical interval [(i-1)  $\Delta x$ , i  $\Delta x$ ] ,  $if \quad \xi < 0 \quad , \quad U_{i-1}^{n+1} = U_{exact}^{n+} \quad (\xi \ , \ t)$  and if  $\xi > 0$  ,  $U_{i}^{n+1} = U_{exact}^{n+} \quad (\xi \ , \ t)$ 

if 
$$\xi < 0$$
 ,  $U_{i-1}^{n+1} = U_{exact}^{n+}$  ( $\xi$  , t and if  $\xi > 0$  ,  $U_{i}^{n+1} = U_{exact}^{n+}$  ( $\xi$  , t)

It is seen immediately that although the solutions are computed on a grid in this method, it is not a differencing scheme. Also, instead of using a weighted average of the Riemann problem solution to arrive at the solution for a grid point†, the RCM samples a particular value from an explicit wave † The Godunov method, for example implements

$$U_{i}^{n+1} = \frac{1}{\Delta x} \int_{(1-\frac{1}{2})\Delta x}^{(i+\frac{1}{2})\Delta x} U_{\text{exact}}^{n+} (x,t) dx$$

solution, thus eliminating the smoothing out of wave transport and interaction information inherent in averaging. This leads to the 'infinite' resolution of contact discontinuities and shocks that the scheme displays.

From the foregoing discussion, it is evident that the success of the scheme hinges, to a large extent, on the inexpensive and exact solution of Riemann problems and an appropriate sampling technique. Ref. (3) describes a modification to an iterative method due to Godunov (Ref. 5). Theoretical details for the Riemann problem solution are also given in Ref. (6).

The mathematical properties required in a sampling procedure applicable to this scheme are defined in Ref. (1). A brief description of the procedure is given below.

In previous computations using the RCM, random sampling with some variance reduction technique (stratified sampling), was used, i.e., the values were taken from the random number generator installed in the computer (Ref. 3). It was shown in Ref. (1) that a more accurate form of sampling is a technique due to van der Corput (Ref. 7). The sequence generated is, strictly speaking, non-random, but has particular statistical properties that are suitable to the application. The sequence is referred to as quasirandom and is generated as follows:

The binary expansion of natural numbers may be expressed as (with R=2):  $n=A_0R^0+A_1R^1+A_2R^2+\ldots +A_mR^m \quad , \quad (0\leqslant A_k\leqslant R)$  i.e.  $n=\sum_{k=0}^m A_k.2^k \quad , \text{ with } A_k=0 \text{ or } 1 \quad , \quad n=1,\ 2,\ 3,\ \ldots$ 

Next, the digits of the binary numbers are reversed and a decimal point is put preceding the number; this gives the numbers

$$\phi_n = A_0 R^{-1} + A_1 R^{-2} + \dots + A_m R^{-(m+1)}$$

or, 
$$\phi_n = \sum_{k=0}^{m} A_k \cdot 2^{-(k+1)}$$
, again with  $A_k = 0$  or 1

Conversion to the decimal scale of these numbers yields the required sequence of quasirandom numbers defined over the interval [0,1], i.e.,

$$\phi_n \text{ (decimal)} = \Theta_n + \frac{1}{2}$$
or 
$$\Theta_n = \phi_n \text{ (decimal)} - \frac{1}{2}$$

and  $\xi_n = O_n \cdot \Delta x$  as defined earlier.

The first few elements of the sequence given below illustrate the construction =

The van der Corput sequence is 'equidistributed', and yields better results than those obtained using a 'stratified' random sampling technique.

The subroutine employed in the program to compute the random numbers is described in Appendix B.

### 2.2. Boundary Conditions

In general, the implementation of boundary conditions in the RCM is quite straightforward, but does require some thought. Referring to Fig. 2, the b.c.'s are specified at points 1 and N for the left and right boundary

respectively. Note that if the sampled solution at  $(n+1)\Delta t$  corresponds to a random number  $\xi_n < 0$ , the solution is assigned to the grid point on the left. For the Riemann problem defined by points 1 and 3, the sampled solution would then be assigned to grid point 1 at  $(n+1)\Delta t$ ; however this is overridden by assigning the proper boundary condition at 1 again, and there is no contradiction. A similar procedure is adopted at the right hand boundary when  $\xi_n > 0$ .

The subroutines for the boundary conditions are named in the format BCxn , BC standing for Boundary Condition, x being either L (for Left), or R (for Right boundary) and n being a number from 1 to 5 with the following designations:

- l solid wall condition
- 2 outflow at constant static pressure
- 3 special formulation ('piston' inflow)
- 4 isentropic inflow from reservoir
  - 5 special formulation (rarefaction wave cancellation)

#### 2.2.1. Solid Wall Conditions

The solid wall boundary condition requires a zero normal velocity at the wall for inviscid flow computations. Due to the random sampling involved in the method and the lateral movement of the sampled solution  $\frac{\Delta x}{2}$  to the left or right of the discontinuity, the condition is difficult to implement uniquely. However, the procedure adopted here is found to yield reasonably accurate results for the applications intended. (Note that the difficulty is not unique to this method only. The implementation of zero mass flux through a surface is difficult per se for the Euler equations).

Referring to Fig. 2, let the physical boundaries be at point 2 and

point (N-1) for the left and right sides respectively. However, the boundary conditions are specified at point 1 (point N) for the left (right) side as a fictitious 'mirror' state of the conditions at point 3 (point (n-2)) respectively, but with the reverse sign taken for the velocity component. Thus, for the left hand boundary Riemann problem,

$$P_L = p(3)$$
 ,  $\rho_L = \rho(3)$  ,  $u_L = -u(3)$   
 $p_R = p(3)$  ,  $\rho_R = \rho(3)$  ,  $u_R = u(3)$ 

and, analogously, for the right hand boundary Riemann problem.

$$p_L = p(N-2)$$
 ,  $\rho_L = \rho(N-2)$  ,  $u_L = u(N-2)$   
 $p_R = p(N-2)$  ,  $\rho_R = \rho(N-2)$  ,  $u_R = -u(N-2)$ 

The solutions are then sampled in the manner outlined earlier.

#### 2.2.2. Outflow Conditions

For subsonic outflow, only the static pressure p is defined, with the continuation condition being applied to the rest of the variables. Thus, for the right hand boundary for example, the Riemann problem is defined as follows:

$$p_L = p(N-2)$$
 ,  $\rho_L = \rho(N-2)$  ,  $u_L = u(N-2)$   
 $p_R = p_{Out}$  ,  $\rho_R = \rho(N-2)$  ,  $u_R = -u(N-2)$ 

where p<sub>out</sub> is the specified outlet pressure. If the flow going out is supersonic, there can be no propagation of disturbances upstream, and the continuation condition is implemented for all the variables, i.e., the Riemann problem now is the trivial case defined by

$$p_L = p(N-2)$$
 ,  $\rho_L = \rho(N-2)$  ,  $u_L = u(N-2)$    
 $p_R = p(N-2)$  ,  $\rho_R = \rho(N-2)$  ,  $u_R = -u(N-2)$ 

# 2.2.3. Special Formulation of 'Piston' Inflow

In general, for idealized wave rotor flows, hot combustion gases are

'slip onto' the rotor, i.e., without incurring incidence or deviation angle losses. Also, in the ideal treatment, the air in the passages of a wave rotor is exposed to the hot gas at high pressure instantaneously. The idealizations allow for uniform conditions to be prescribed at the hot gas inlet port. Thus, a 'special' form of inflow boundary condition needs to be specified here, namely, the static pressure, the velocity and the density of the incoming hot gas. Although equivalent to specifying the total pressure and temperature in the usual inflow boundary condition treatment, some thought is required in wave rotor type flows when specifying  $m p_{gas}$  ,  $m p_{gas}$  and  $m u_{gas}$  . This is because only a shock wave needs to be generated, with no waves travelling opposite to the direction of flow. The solution to the Riemann problem would then consist of just two states connected by a single shock wave. The flow is equivalent to that generated when a piston is pushed instantaneously into a gas at rest. In general, the state of the air inside the rotor passage is known; explicit relations for two states connected through a shock wave are given in Ref. (6). These so-called transition functions help in specifying the boundary conditions for the incoming flow properly.

introduced into the rotor through nozzles angled such as to allow the flow to

If we consider the left boundary for this inflow, the Riemann problem is set up as:

PL = Phot gas , 
$$\rho$$
L =  $\rho$ hot gas ,  $u$ L =  $u$ hot gas 
$$p_R = p(3) , \rho_R = \rho(3) , u_R = u(3)$$

with  $p_L$  ,  $\rho_L$  and  $u_L$  having been chosen in accordance with the considerations discussed above.

#### 2.2.4. Isentropic Inflow From Reservoir

The induction of fresh charge or air onto the rotor usually corresponds to an isentropic inflow situation. The flow in the vicinity of the passage end can be treated as quasi-steady, with the assumption that no flow separation takes place when the flow enters. Two boundary conditions are required for this type of inflow; these are provided by the conservation of energy in the flow from the external region to the inlet (assumed to be steady), and by the prescibed entropy level of the gas in the external region.

The boundary conditions may thus be expressed as

$$u_{in}^{2} + \frac{2}{\gamma - 1} a_{in}^{2} = \frac{2}{\gamma - 1} a_{tot}^{2}$$

$$S_{in} = S_{tot}$$

where the subscripts 'in' and 'tot' apply to conditions at the inlet of the passage and external reservoir respectively. The sonic velocity is denoted by a, and flow velocity by u. Note that knowledge of the Riemann variable arriving at the passage end from within the passage is required to be able to solve the energy equation above for  $a_{\rm in}$  and  $u_{\rm in}$ . For the left end, for example,

$$Q_{in} = \frac{2}{\gamma - 1} a_{in} - u_{in}$$

which together with the energy equations yields

$$a_{in} = \underbrace{Q_{in} + \sqrt{\frac{\gamma+1}{\gamma-1}} a_{tot}^2 - \frac{\gamma-1}{2} Q_{in}^2}_{\frac{\gamma+1}{\gamma-1}}$$

and subsequently the other variables.

The simple analytical treatment given above has to be modified somewhat if a contact discontinuity is formed when the inflow begins. This is due to the fact that the value of the arriving Riemann variable is changed across such a discontinuity, which thus leads to an additional unknown. Procedures for solving the inflow for these situations are given in Ref. (8). In the program developed here, reasonably good results are obtained by setting the velocity at the boundary point equal to the velocity at the point nearest the physical boundary. For the left end e.g., the variables for the left state of the Riemann problem are obtained as follows:

$$u(1) = u(3)$$
,

a reasonably accurate assumption just at the point of inlet opening.

Then, from the 'energy ellipse',

$$a(1) = \sqrt{a_{tot}^2 - \frac{\gamma - 1}{2} u(1)^2}$$

 $M(1) = \frac{u(1)}{a(1)}$ , incoming Mach number

$$p(1) = \frac{p_{tot}}{[1+\frac{\gamma-1}{2}M(1)^2]} \gamma/(\gamma-1)$$
,

with similar isentropic relations to compute other flow variables. Note that once the interface or contact discontinuity has moved a certain distance inside the passage, the simple analytical expressions given earlier in the section can be used, since now the value of the arriving Riemann variable would be known at the boundary.

# 2.2.5. Special Formulation for Rarefaction Wave Cancellation

The spreading of rarefaction fans leads to unwanted wave reflections

which occupy large zones in the passages of wave rotors. Fig. (4) shows a wave diagram proposed by Spectra Technology, Inc., which incorporates so-called 'wave management' or 'tuning' ports to ideally cancel (and otherwise attenuate) impinging rarefaction fans. The physical boundary conditions are thus dictated by the flow developing in the passage, i.e., the port has non-uniform flow conditions in it, which at each point match those of the flow at the end of the passage so as to disallow any reflections to take place. Numerically, this condition is achieved by implementing the continuity condition across the boundary for all the flow variables involved. For the left boundary, thus, the Riemann problem is defined by:

$$p_L = p(3)$$
 ,  $\rho_L = \rho(3)$  ,  $u_L = u(3)$ 

$$p_R = p(3)$$
 ,  $\rho_R = \rho(3)$  ,  $u_R = u(3)$ 

and analogously for the right boundary. Note that these boundary conditions may involve either inflow or outflow.

#### 2.3 Example Calculations

The listing of the program is included in Appendix A, and the various names for the variables are listed in Appendix B, along with some instructions on how to use the program. No effort as yet has been made to optimize the code either for storage requirements or for execution efficiency.

In this section, some sample calculations are carried out using the code, to illustrate its usefulness in constructing idealized design point wave diagrams which can serve as the starting configuration for detailed construction of diagrams incorporating real flow effects.

# 2.3.1. Test Case for 1-D, Inviscid, Unsteady, Compressible Flow

Fig. (1) illustrates the initial conditions in a shock tube, with the diaphragm at  $x_0$ . Sod (Ref. 9) suggested a test case for hyperbolic

conservation laws with the following conditions as initial states in the shock tube:

$$p_1 = 1.0$$
 ,  $p_1 = 1.0$  ,  $u_1 = 0.0$   $p_5 = 0.1$  ,  $p_5 = 0.125$  ,  $u_5 = 0.0$ 

i.e., the gas on either side of the diaphragm is in a quiescent state initially. The ratio of specific heats is chosen to be 7/5, and  $\Delta x$  is chosen to be 0.01.

The solution (before any wave has reached either the left or right end) is shown in Fig. (5). The squares shown at locations  $x_1$ ,  $x_2$ ,  $x_3$  and  $x_4$  in the density plot give the analytically calculated amplitude and location of the head — and tail waves of the left-running rarefaction, the contact surface moving to the right and the shock wave moving at supersonic velocity to the right respectively. The solid lines are the solutions obtained by the RCM at different time levels; the zero numerical diffusion feature of the method is evident in the 'infinite' resolution of the contact discontinuity and the shock, and the dispersion (phase error) is within one grid spacing. The constant states are perfectly realized.

It is these features of the method that make it very attractive for application to wave rotor type flows, since the successful design of the device is predicated on being able to accurately compute wave arrival times at the various ports.

# 2.3.2. Wave Turbine Experiment

Ref. (10) describes the wave rotor experimental set up at the Turbopropulsion Laboratory. Initial tests being carried out currently are with the wave rotor in a turbine mode, i.e., one side of the rotor is blocked off, and high pressure air is brought onto the rotor and taken off again from

the other side. The passages of the rotor being angled at 60° to the axis, the 180° reversal in the direction of the fluid flow creates an angular momentum change, in turn generating large turbomachinery work coefficients. Fig. (6) shows the wave diagram computed using the code. The movement of the rotor is from top to bottom. At t=0, the high pressure air is brought into contact with quiescent atmospheric air in the rotor passages, at point a. This corresponds to the 'piston' inflow boundary condition described in section 2.2.3.. A shock, S , is generated immediately, (idealized case of instantaneous cell opening), which travels from the right to the left, and strikes the solid wall at the left end. The reflection of the shock takes place at point b according to the solid wall boundary condition described in section 2.2.1.. Behind this shock, and moving at a slower velocity is the contact surface, I , which penetrates into the passage only a fractional distance before encountering the reflected shock, RS, at point c. The reflected shock is transmitted through the contact surface, (bringing the flow to a near halt), and reaches the right side at point d, whereupon the inlet port is closed. The air trapped in the rotor passages is now at a high pressure and in a quiescent state. When this air is released at point e to a low pressure region, a rarefaction wave is generated, R, which travels to the left, spreading out in the process. It interacts with the stationary contact surface, I , setting it into motion again, and reflects off the solid wall at the left as RR . The boundary condition imposed at point e is the outflow at constant static pressure condition described in section 2.2.2.. The outlet port is closed at a time when the exit velocity falls to about half its initial value.

This experiment embodies two fundamental processes in wave rotors: those of cell filling and cell emptying. Almost all the other processes

typical to wave rotors are combinations of the cell filling and cell emptying unit processes. Comparison of the ideal computed numbers obtained here with experimental data will provide information helpful in the identification and sources of losses.

The program is set up to start at t=0 in this case, with initial data provided along the entire passage, i.e., from x=0 to x=0.1863m (the actual length of the wave rotor being tested). Since the passages have quiescent atmospheric air in them at t=0 , the initial data, of course, describes these conditions. Switches for the left and right boundaries describe what type of boundary conditions prevail and direct the program to the appropriate subroutines. These switches, designated SWL and SWR, for left and right respectively, are assigned integer number values which correspond to the numeric value of the particular boundary condition they represent. Thus, if the left boundary is a solid wall, SWL=1 , corresponding to the boundary condition subroutine BCLl . In this example then, the initial switch settings at t=0 are SWL=1 and SWR=3, corresponding to a solid wall at the left and a 'piston' inflow at the right (which starts at t=0 at point a). At point d , the switches are reset to SWL=1 and SWR=1 due to the closure of the inlet port. At point e , the switches are SWL=1 , SWR=2 , signifying opening of the exhaust port with outflow at a constant static pressure. The whole wave diagram can thus be packaged into a 'module' subroutine and called from the main program with a single call statement. This type of modularity allows for wave diagrams of different 'families' to be developed by simply calling the right 'module' subroutine.

The next two examples illustrate this concept as they deal with two very different types of wave diagrams.

#### 2.3.3. General Electric Wave Engine

Fig. (7) shows a schematic of the wave diagram constructed for the G.E. wave engine. Briefly, the device is configured for a gas generator mode of operation, with counterflow scavenging, and is capable of producing net shaft power. For a fuller description of the machine, see Ref. (11). In this example, fresh charge (air) is induced into the rotor (from an external reservoir) through the wave action of the rarefaction fan originating at the exhaust port opening. The usefulness of the rotor is gauged by the net pressure rise across the machine, i.e., the ratio of the total exhaust pressure to total (fresh air) inlet pressure.

For performance estimation purposes, it is sufficient to investigate only the exhaust and induction processes as shown in Fig. (8). The initial data specified is as follows: the exhausting pressure ratio  $p_{\rm e}/p_{\rm o}$ , the total pressure ratio across the rotor  $p_{\rm te}/p_{\rm ta}$  and an assumed total temperature ratio  $T_{\rm te}/T_{\rm ta}$ . In this particular cycle, the amount of fresh charge induced in is ideally equal to the gases exhausted out, i.e.,  $m_{\rm in}$  =  $m_{\rm out}$ , and this mass balance is carried out after each computation to correct the assumed temperature ratio  $T_{\rm te}/T_{\rm ta}$  (which otherwise constitutes overspecification of the initial conditions).

The calculation starts at t=0, with initial data consistent with the chosen pressure and temperature ratios specified along the passage length. Initial switch settings are SWL=1 and SWR=2 for the solid wall boundary at the left and the exhaust to a constant pressure at the right. As shown in the figure, a rarefaction fan is generated, propagating to the left and reflecting off the solid wall. At time  $t=\tau_1$ , the pressure at the wall has been reduced to that outside the passage,  $p_{ta}$ , which is when the inlet port is opened. The switches are now set to SWL=4 and SWR=2 for isentropic inflow

from an external reservoir at the left, and still outflow at a constant pressure at the right. The exhaust port is closed at time  $t=\tau_2$  which corresponds to the exit velocity having dropped off to approximately half its steady state value at the beginning of the exhaust process. Now the switches are set to SWL=4 and SWR=1 , for the solid wall condition at the right. The sudden closure of the exhaust port generates a 'hammer' shock travelling to the left, interacting with the incoming interface (shown by dashed line), and reaching the passage end at  $t=\tau_4$  at which time the inlet port is closed, with the switches being reset to SWL=1 and SWR=1 . Note the reflected shock travelling from left to right generated at the interaction of the contact surface and the hammer shock.

Once this solution is obtained, integration of the mass flux through the inlet and exhaust ports is carried out and if the two numbers do not match, the assumed temperature ratio  $T_{te}/T_{ta}$  is adjusted in the initial data, till such time as  $m_{in} = m_{out}$ .

This calculation is sufficient for performance analyses: if the entire wave diagram has to be worked out, then at a time t > 13, hot gas from the combustion chamber is brought onto the rotor (the boundary condition corresponding to 'piston' inflow) on the right hand side. This would generate the shock to compress the induced air and when this shock reached the left end, the transfer port (see Fig. 7) would be opened for such time it takes for the compressed air to be completely scavenged out of the rotor.

Fig. (9) shows some performance curves obtained using the procedure outlined above. In Figs. (10a, b, c) are shown three sets of flow parameters at different time steps corresponding to the inlet port just opening, the exhaust port closing and the inlet port closing; the qualitative distributions of the flow parameters in the passage are immediately seen to be accurate when

compared with the wave diagram shown in Fig. (8). Of interest is the set of plots for the time step when the inlet port has just been closed. The flow between the end of the passage and the location of the interface is seen to be quite non-uniform in the density plot. At the same time, the shock reflected from the interface has reached the right side and reflected off the solid wall. These considerations help to decide optimum port opening and closing times. For example, Fig. (11) shows what happens if the inlet port is not closed at just the time the shock reaches the end, but rather at some short time later. The shock now sees an open boundary and reflects off as an expansion to match the high pressure behind it with the incoming total pressure which is at a lower value. This reflected expansion is manifested in the pressure, density and velocity plots of the figure.

The entire sequence of wave interactions of this example is computed by the RCM without the implementation of artificial viscosity or artificial compression methods, or tracking and capturing schemes. This 'hands off' feature of the method renders it eminently useful for fast preliminary evaluations of complex wave diagrams for the application at hand.

The next example computes an idealized wave diagram for the nine-port pressure exchanger concept proposed by Spectra Technology, Ref. (12).

## 2.3.4. Spectra Technology Pressure Exchanger

Fig. (4) shows the ideal wave diagram for the nine-port pressure exchanger. This configuration is a good case example to compute with the RCM because of the different types of boundary conditions that need to be dealt with in the evaluation of the cycle. The computation is started at t=0, at the point of high pressure hot gas inlet (driver gas inlet). In the manner described in the G.E. wave engine example, the initial data is prescribed for

the entire passage at this time step and the boundary condition switches are initially set at SWL=1 and SWR=3 for the solid wall at the left, and the 'piston' inflow at the right hand end. Since there is a multiplicity of types of boundary conditions, e.g., three outflow ports, an index, JCOUNT, is used to ensure proper sequencing of the switches. The following table is presented as an example of the settings of the switches to carry out calculations for one cycle. The inflow and outflow port conditions are those proposed by Spectra Technology for their idealized diagram.

TIME STEP, N	JCOUNT	SWL	SWR	REMARKS
0	0	1	3	CYCLE STARTS. HP GAS INLET PORT OPENS
500	1	2	3	HP AIR OUTLET PORT OPENS
1408	2	2	1	HP GAS INLET PORT CLOSES
1765 -	3	5	1	HP AIR OUTLET PORT CLOSES. TUNING PORT L1 OPENS
1816	4	2	1	TUNING PORT L1 CLOSES. IP GAS OUTLET PORT OPENS (PORT E1)
2069	5	2	5	TUNING PORT R1 OPENS
2261	6	2	1	TUNING PORT R1 CLOSES
2595	7	5	1	IP GAS OUTLET PORT CLOSES. TUNING PORT L2 OPENS
2636	8	2	1	TUNING PORT L2 CLOSES. LP GAS OUTLET PORT OPENS (PORT E2)
3029	9	2	5	TUNING PORT R2 OPENS
3237	10	2	4	TUNING PORT R2 CLOSES. LP AIR INLET PORT OPENS
4961	11	1	4	LP GAS OUTLET PORT CLOSES
5529,0	0	1	3	LP AIR INLET PORT CLOSES. CYCLE COMPLETED

The total cycle time as calculated by the RCM is 3.0676 mseconds, which compares well with the time computed by Spectra Technology (using the FCT-SHASTA algorithm) of 3.07 mseconds. The execution time on an IBM 370-3033AP for the 5529 steps computed in the example above was 3 minutes 38 seconds, including the I/O operations and the graphics.

Figs. (12a, b, c) show three sets of plots of the flow parameters for the following cases: a) the H.P. air outlet port opens on time, i.e., just as the shock reaches the left end of the passage, b) the port opens before the shock has reached the end, and c) the port opens after the shock has reached the end. The constant pressure and velocity states that prevail in the passage just after the shock has reached the left end (time 'section' line T) in wave diagram), are perfectly realized in Fig. (12a), while the contact surface is at the location shown by the sharp discontinuities in the density and entropy plots. Should the inlet port be opened earlier, e.g., at the time level shown by  $\tau_{1-}$  in the wave diagram, what happens is as follows: pressure in the passage is still at the pre-compressed level and this comes into contact with the pressure level in the port which is considerably higher, resulting in a shock propagating into the passage, colliding with the left moving shock and raising the overall pressure level to ~3.0 as shown in Fig. (12b). However, as soon as the left moving shock reaches the end, it now encounters an open boundary with conditions that do not match those behind the shock, resulting in a rarefaction fan being generated, which propagates to the right. This expansion fan, travelling at sonic velocity relative to the gas into which it is propagating, soon overtakes the right moving shock which is travelling at a subsonic velocity relative to the same gas. This interaction results in an attenuation of both the rarefaction as well as the shock wave. Note that the overall pressure and velocity levels behind the rarefaction are about the same as for case a), i.e., the effects of the mismatch are not very significant at the outlet port. However, should the right moving pressure perturbations of case b) not attenuate each other significantly before they reach the right hand end, the consequences could be severe for the overall wave diagram, since this will lead to further (unwanted) wave reflections.

Fig. (12c) shows what occurs if the outlet port is opened too late, corresponding to time level  $\tau_{1+}$  on the wave diagram. Now the left travelling shock encounters a wall boundary condition on reaching the left end and reflects off as a shock, effectively doubling the pressure level behind it (>3.5 in pressure plot of Fig. (12c)). When the outlet port opens, there is again a mismatch of conditions in the port and in the passage, with the pressure level in the passage being considerably higher than that prescribed for the outlet port. A rarefaction wave is generated which propagates to the right and overtakes the reflected shock. The same criterion holds for this case too, i.e., the ensuing attenuation of these pressure pulses should occur before they reach the right hand end, preferably even before they reach the interface still propagating towards the left at the flow velocity.

The considerations above give a preview of the nature of decisions required in the successful design of a wave rotor device. It is clear that quite a few iterations are involved in the process of designing a viable wave diagram for a particular application, and each iteration entails calculating two or more complete cycles to ensure 'closure' or repeatability of the cycle. A fast solver like the RCM allows reaching an idealized 'base' design quickly and inexpensively.

Appendix A is a listing of the program in its present development stage. As mentioned earlier, no attempt has been made to optimize the program, either for storage requirements or for execution.

Appendix B gives a description of the structure of the program, a listing of the important variables, the subroutines and the function subprograms. A step by step guide is also included to set up and run the program.

#### 3. DISCUSSION AND RECOMMENDATIONS

#### 3.1. Discussion

For meeting the criteria listed in the Introduction, in one dimension, Glimm's method or the RCM appears to be superior to any difference method.

For wave rotor type applications, where discontinuities need to be computed with sharpness, the 'infinite' resolution of such discontinuities inherent in the RCM make it a natural choice to carry out ideal flow calculations for preliminary design purposes. Boundary conditions can be implemented quite easily and do not require information from points outside the domain of dependance as is the case in some finite difference schemes. The van der Corput sampling technique results in the best possible representation of the wave propagation, which is essential for the correct representation of continuous waves, particularly those produced by nonlinear interactions.

The method, however, is not recommended to solve for flows with real effects such as friction, heat transfer and area change, or to be extended to multi-dimensional flows. Although considerable research is being done to rigorously extend the method to such flows, with some degree of success (see Refs. 1, 4, 13), the present state of development is not mature enough to ensure a useful practical code as the outcome.

#### 3.2. Recommendations

Many options are available for one wishing to develop either a 1-D code with real effects and/or a multi-dimensional code for wave rotor type applications. The author prefers to recommend numerical formulations which are dependent on the solution of Riemann problems, such as the Godunov method; the motivating reason for this preference is that a Riemann problem constitutes the solution of a discontinuity in the flow in terms of other

discontinuities (if any are, indeed, present), and the scheme is thus intrinsically suited for solving such flows; on the other hand, the other schemes, in general, require to be made aware of discontinuities in the flow through some external device, and then treat them through other artificial devices.

A second-order, quasi one-dimensional (variable cross-sectional area) scheme has recently been developed by Ben-Artzi and Falcovitz (Ref. 14). The method is based on the exact solution of 'generalized Riemann propblems', and has demonstrated very good results; it's least accurate approximation is equivalent to Godunov's first order method (Ref. 9). The resolution of shocks and other disconitinuities and singularities of the flow field is also high. Extension to more than one dimension appears to be straightforward through the use of operator splitting techniques, but has as yet not been tried extensively.

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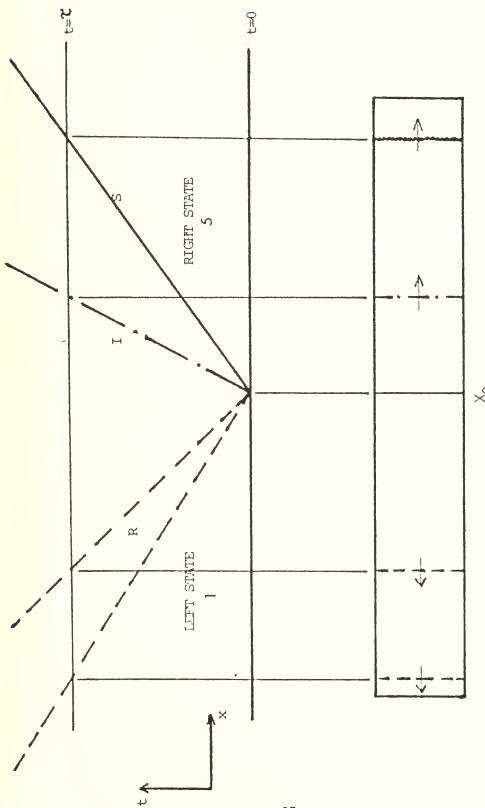


FIG.1: THE RIEMANN PROBLEM IN GAS DYNAMICS SPECIAL CASE OF SHOCK-TUBE FLOW

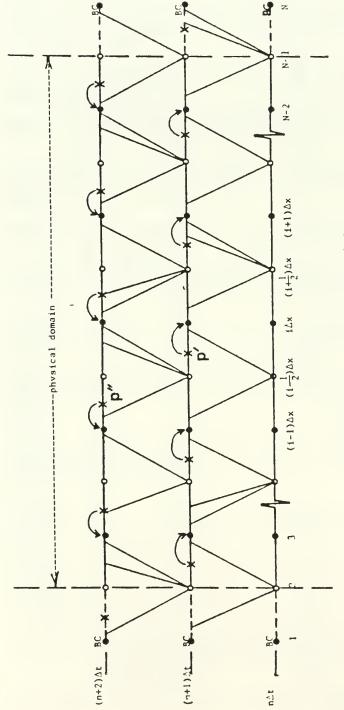


Fig.2 : Cartesian Grid for 1-D Random Choice Method.

• solution grid points

- O- intermediate grid points BC- points where boundary conditions are
  - specified
- X- location where local Riemann Problem solution is sampled

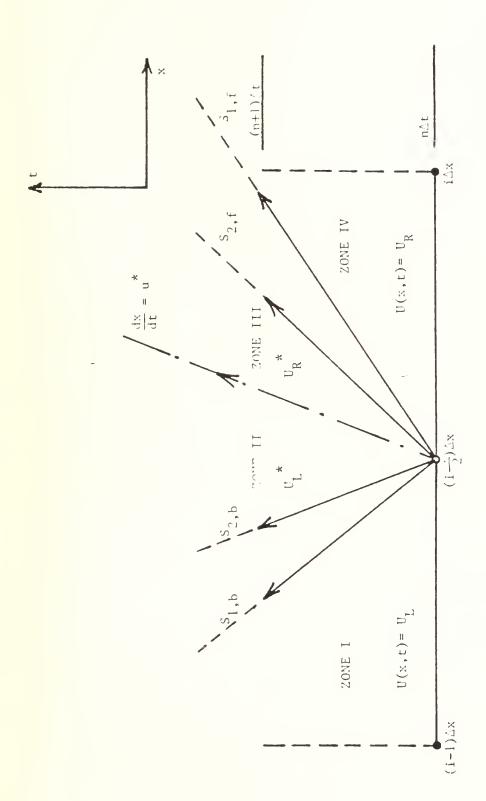


Fig. 3 : General Wave Structure Resulting From the Solution of a Riemann Problem

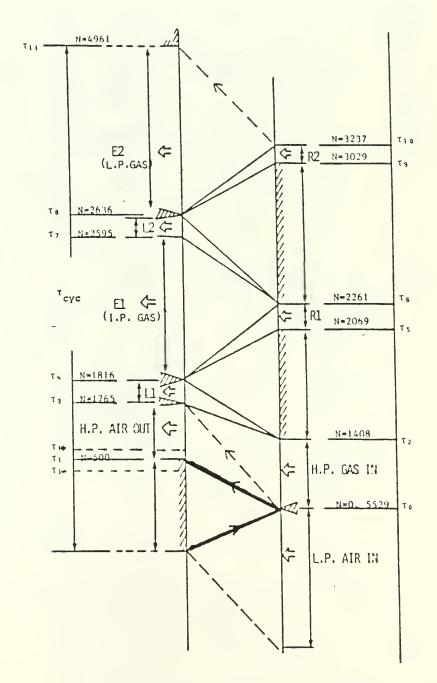
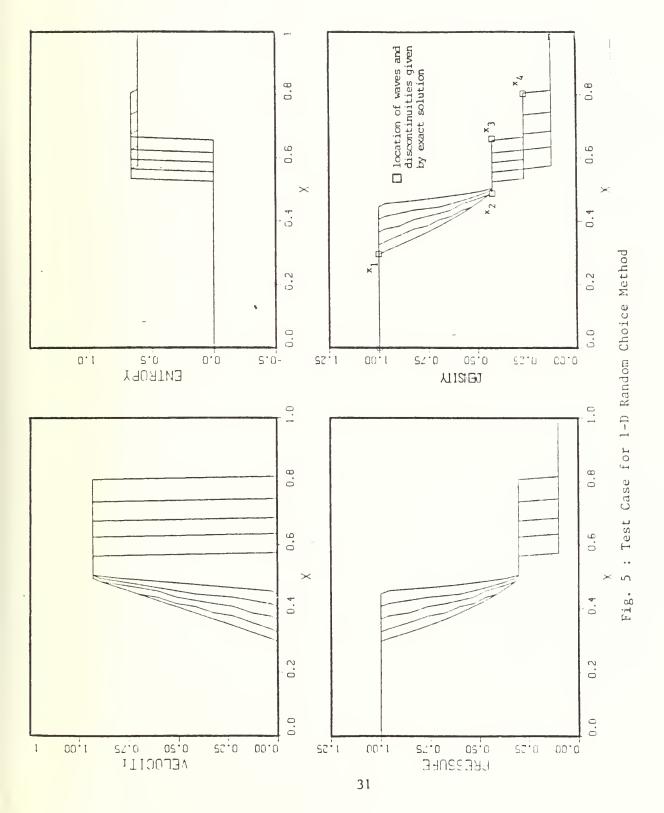
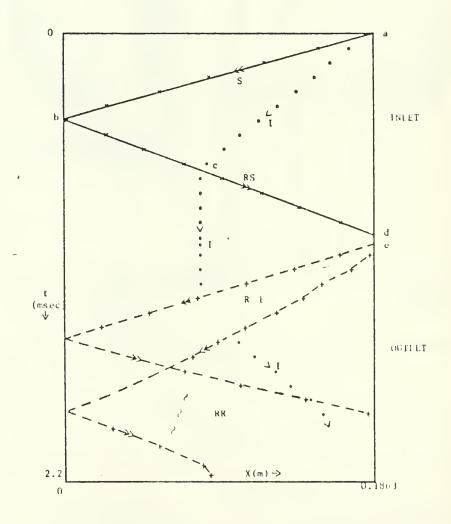


Fig.4: Ideal Nave Diagram for Pressure Exchanger (Spectra Technology).
LP.(P.,HP: Low, Intermediate, High Pressure





Wave Diagram Computed by 1-D Random Choice Method. S--Shock; RS--Reflected Shock; R--Rarefaction Fan; RR--Reflected Rarefaction; I--Interface;

Figure 6.

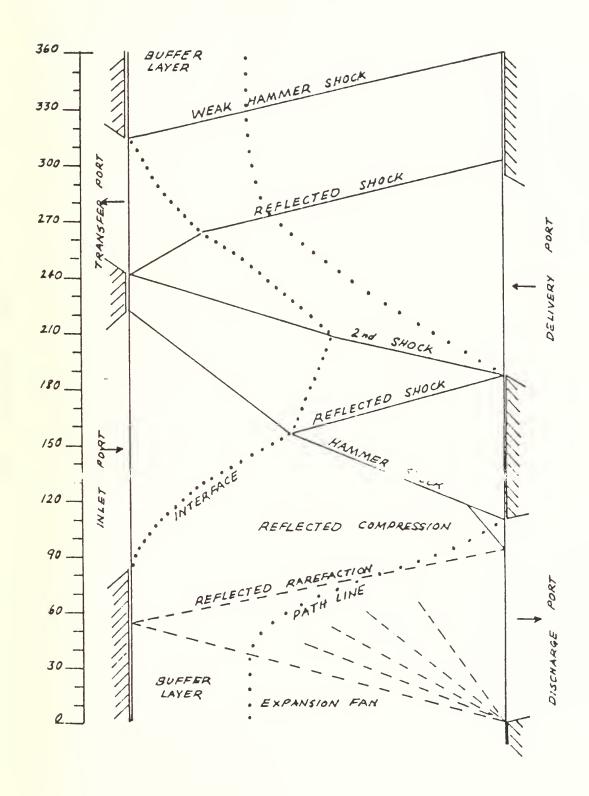


Fig.7: Ideal Wave Diagram for General Electric Wave Engine

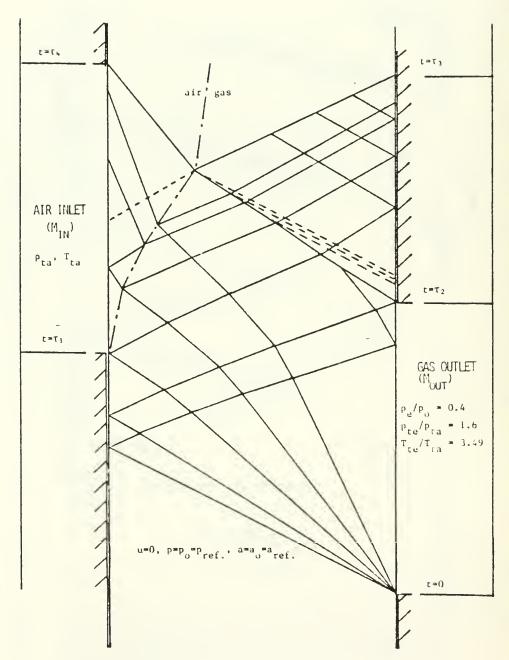


Fig.8 : Gas Exhaust and Fresh Air Induction Process in G.E. Wave Engine

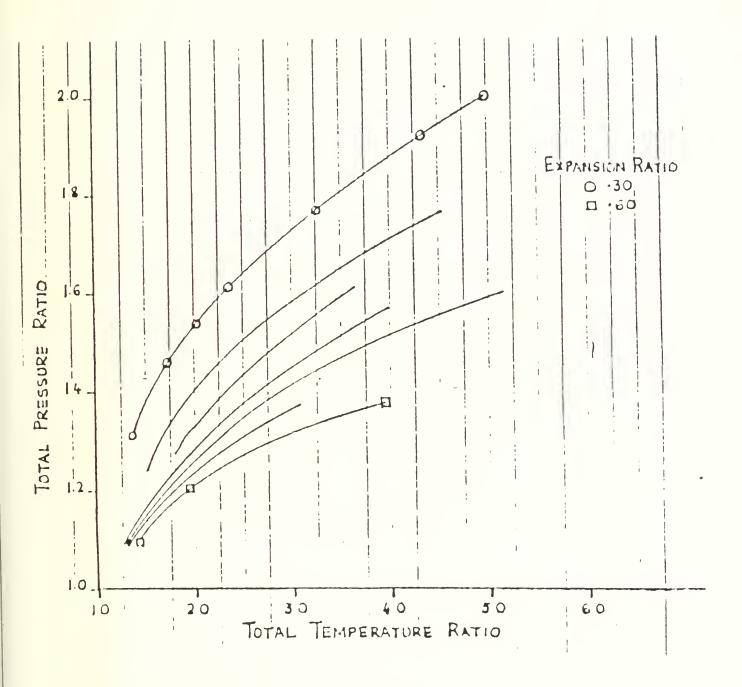


Fig.9: Ideal Performance Curves for G.E. Wave Rotor as Gas Generator

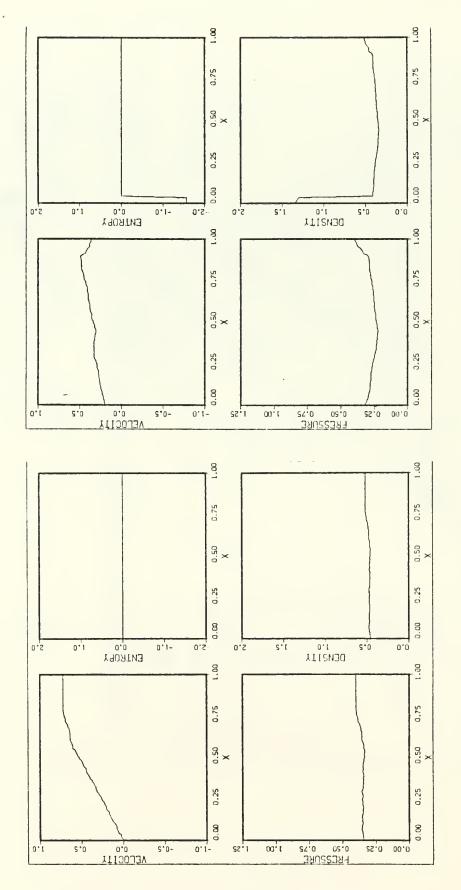


Fig.10 b; Distribution of Flow Parameters in Rotor Passage Just as Exhaust Port is Closed. N= 998. Fig.10 a : Distribution of Flow Parameters in Rotor Passage Just as Inlet Port

Opens. N= 793.

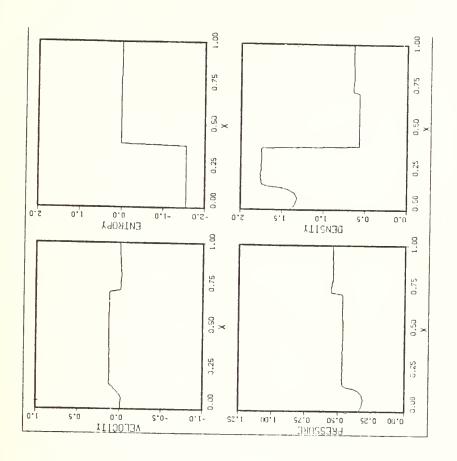


Fig.11: Distribution of Flow Parameters in Rotor Passage When Inlet Port Closes Late, i.e. After Arrival Of Shock. N= 1700.

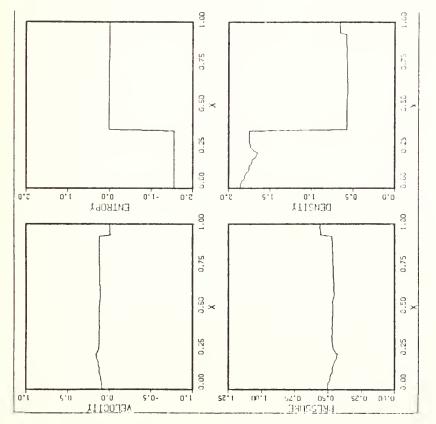


Fig.10 c: Distribution of Flow Parameters in Rotor Passage Just as Inlet Port Closes. N= 1617.

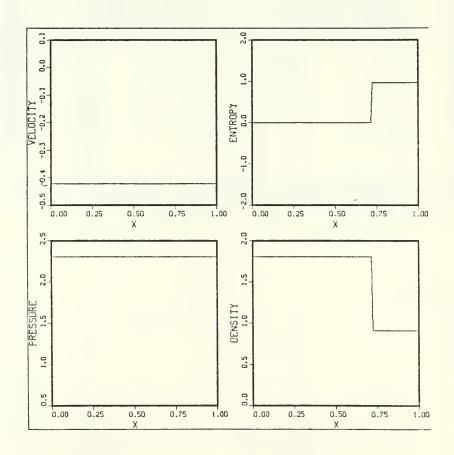


Fig.12 a: Distribution of Flow Parameters in Rotor Passage when the H.P. Air Outlet Port Opens on Time.

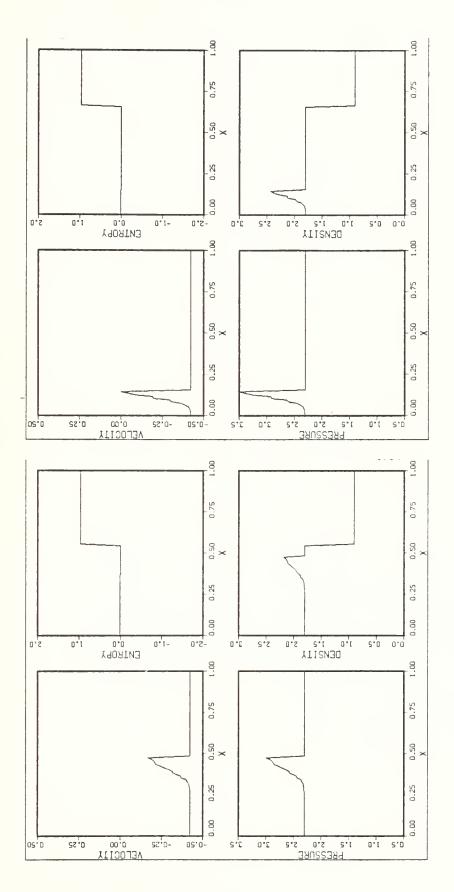


Fig.12 b,c : Distribution of Flow Parameters in Rotor Passage for Two Conditions of Mismatch

: H.P. Air Outlet Port Opens Prematurely

Р

: H.P. Air Outlet Port Opens Late

# APPENDIX A

Listing of Program RCM

```
PROGRAM RCM WITH VAN DER CORPUT SAMPLING AND SINGLE TIME STEP
                                                                           RCM00030
   INTEGER QPRINT, QSTOP, SWL, SWR
                                                                           RCM00040
   DIMENSION XX(6), YY(6)
                                                                           RCM00050
   DIMENSION XARRAY(100)
                                                                           RCM00060
   DIMENSION WNORM(12), IDIGT(12)
                                                                            RCM00070
   DIMENSION P(203), R(203), U(203), A(203), S(203), X(203)
                                                                            RCM00080
   COMMON/SUBS/P.R.U.A.S.X
                                                                           RCM00090
   COMMON/GLIMM1/PGLIM, RGLIM, UGLIM, PL, RL, UL, PR, RR, UR, AL, AR, GL, GR, EPS RCM00100
   COMMON/GLIMM2/DT,DX,XI
                                                                            RCM00110
   COMMON/FUN1/G, PA, RA, UA, RB, RMU
                                                                            RCM00120
   COMMON/SAMPLE/WNORM.IDIGT
                                                                            RCM00130
   COMMON XARRAY, N1
                                                                           RCM00140
   CALL COMPRS
                                                                           RCM00150
   CALL BLOWUP (0.5)
                                                                           RCM00160
   CALL PAGE (11.0.8.5)
                                                                           RCM00170
   CALL HWSCAL ('SCREEN')
                                                                           RCM00180
   DATA K, SWL, SWR / 500, 1, 3 /
                                                                           RCM00190
   DATA N, CFLNUM, TTOTAL/0,0.60,0.0/
                                                                           RCM00200
   DATA PSEXIT, PSINL, PSINR, RINL, RINR/116954., 3770000., 3819952.50, 6.8, RCM00210
  *6.800/
                                                                            RCM00220
   DATA PSOUT1, PSOUT2, PSOUT3/3819952.5, 2431800.0, 1530007.5/
                                                                           RCM00230
   DATA PTOTIN, RTOTIN/1656663.8, 7.486/
                                                                           RCM00240
   DATA PREF, RREF, XREF/1656663.8,7.486,0.1800/
                                                                           RCM00250
   G = 1.4
                                                                            RCM00260
   GL=1.4
                                                                            RCM00270
   GR = 1.4
                                                                            RCM00280
   EPS=1.E-06
                                                                            RCM00290
   OSTOP = 20
                                                                            RCM00300
   N1 = 0
                                                                            RCM00310
   JCOUNT=0
                                                                            RCM00320
                                                                            RCM00330
   KCOUNT=0
   UEXMAX=0.
                                                                            RCM00340
   DX = 0.01
                                                                            RCM00350
   AREF = SORT (PREF / RREF)
                                                                           RCM00360
                                                                            RCM00370
   TIMREF=XREF/AREF
   RMU=SQRT((G-1.)/(G+1.))
                                                                            RCM00380
                                                                            RCM00390
   X(1) = -0.5 *DX
   ZETA=WDP(1)
                                                                            RCM00400
   XII=DX*(WDP(0)-0.5)
                                                                            RCM00410
   DO 25 I=2,203
                                                                            RCM00420
   X(I) = X(I-1) + 0.5 *DX
                                                                            RCM00430
25 CONTINUE
                                                                            RCM00440
   DO 35 I=1,100
                                                                            RCM00450
   XARRAY(I)=X(I*2+1)
                                                                            RCM00460
                                                                            RCM00470
35 CONTINUE
                                                                            RCM00480
   INITIAL DATA
   CALL INIT1
                                                                            RCM00490
   CALL INIT2L(PSEXIT)
                                                                            RCM00500
   CALL INIT2R(PSEXIT, PREF, RREF)
                                                                            RCM00510
   CALL INIT3L(PSINL, RINL)
                                                                            RCM00520
   CALL INIT3R(PSINR, RINR)
                                                                            RCM00530
   NONDIMENSIONALIZATION
                                                                            RCM00540
   DO 30 I=1,203,2
                                                                            RCM00550
   P(I) = P(I) / PREF
                                                                            RCM00560
```

```
R(I)=R(I)/RREF
   U(I)=U(I)/AREF
   A(I)=A(I)/AREF
   S(I)=ALOG(P(I)/R(I)**G)
30 CONTINUE
   CALL PLOT1(K)
   DO 40 J=1,K
   N = N + 1
   XII=DX*(WDP(0)-0.5)
   QPRINT=N/50
   DT = 100.
   DO 50 I=1,203,2
   DTT=CFLNUM*DX/(2.*AMAX1(ABS(A(I)+U(I)),ABS(A(I)-U(I))))
   DT=AMIN1(DTT,DT)
50 CONTINUE
   TTOTAL=TTOTAL+DT
   TIME=TTOTAL*TIMREF
   XI = - XII
   DO 60 I=1,201,2
   PL=P(I)
   RL=R(I)
   UL=U(I)
   PR=P(I+2)
   RR = R(I + 2)
   UR=U(I+2)
   XITEMP=XI
   IF(I.EQ.1) XI=ABS(XI)
   IF((I.EQ.201).AND.(XI.GT.0.0)) XI=-XI
   CALL GLIMM(QSTOP, PSTAR, USTAR, ASTAR)
   XI=XITEMP
   P(I+1)=PGLIM
   R(I+1)=RGLIM
   U(I+1)=UGLIM
60 CONTINUE
   DO 70 I=1,201,2
   IF(XI.LT.O.) GOTO 80
   P(I+2)=P(I+1)
   R(I+2)=R(I+1)
   U(I+2)=U(I+1)
   A(I+2) = SQRT(G*P(I+2)/R(I+2))
   S(I+2)=ALOG(P(I+2)/R(I+2)**G)
   GOTO 70
80 P(I) = P(I+1)
   R(I) = R(I+1)
   U(I)=U(I+1)
   A(I) = SQRT(G*P(I)/R(I))
   S(I)=ALOG(P(I)/R(I)**G)
70 CONTINUE
   CALL GE(SWL, SWR, N, TTOTAL, TIME, UEXMAX, PTOTIN, PREF)
   CALL DETON(SWL, SWR, N, QPRINT, TTOTAL, TIME)
   CALL SPCTRA(N, SWL, SWR, TIME, UEXMAX, PSEXIT, PSOUT1, PSOUT2, PSOUT3, J
  *COUNT, QPRINT, TTOTAL, KCOUNT)
   IF(SWL.EQ.1) CALL BCL1
   IF(SWL.EQ.2) CALL BCL2(PSEXIT, PREF)
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IF(SWL.EO.3) CALL BCL3(PSINL, RINL, PREF, RREF)
                                                                         RCM01110
   IF(SWL.EQ.4) CALL BCL4(PTOTIN, RTOTIN, PREF, RREF)
                                                                         RCM01120
   IF(SWL.EO.5) CALL BCL5
                                                                         RCM01130
   IF(SWR.EQ.1) CALL BCR1
                                                                         RCM01140
   IF(SWR.EQ.2) CALL BCR2(PSEXIT, PREF)
                                                                         RCM01150
   IF(SWR.EO.3) CALL BCR3(PSINR,RINR,PREF,RREF)
                                                                         RCM01160
   IF(SWR.EQ.4) CALL BCR4(PTOTIN,RTOTIN,PREF,RREF)
                                                                         RCM01170
   IF(SWR.EO.5) CALL BCR5
                                                                         RCM0118C
   IF ((N.EO.(50*OPRINT)).AND.(N.GE.O)) CALL PLOT2(N.K)
                                                                         RCM01190
40 CONTINUE
                                                                         RCM01200
   CALL ENDPL(0)
                                                                         RCM01210
   CALL DONEPL
                                                                         RCM01220
   STOP
                                                                         RCM01230
   END
                                                                         RCM01240
   SUBROUTINE GLIMM(QSTOP, PSTAR, USTAR, ASTAR)
                                                                         RCM01250
   INTEGER Q,QSTOP
                                                                         RCM01260
   REAL ML, MR, MLN, MRN
                                                                         RCM01270
   COMMON/GLIMM1/PGLIM, RGLIM, UGLIM, PL, RL, UL, PR, RR, UR, AL, AR, GL, GR, EPS RCM01280
   COMMON/GLIMM2/DT, DX, XI
                                                                         RCM01290
   DATA 0,ML,MR/0,100.,100./
                                                                         RCM01300
   PSTAR=0.5*(PL+PR)
                                                                         RCM01310
   COEFL=SQRT(PL*RL)
                                                                         RCM01320
   COEFR = SQRT_(PR*RR)
                                                                         RCM01330
   ALPHA=1.
                                                                         RCM01346
   BEGIN GODUNOV ITERATION
                                                                         RCM01350
30 Q=Q+1
                                                                         RCM01360
   IF(PSTAR.LT.EPS) PSTAR=EPS
                                                                         RCM01370
   COMPUTE NEXT ITERATION FOR ML AND MR
                                                                         RCM01380
   MLN=COEFL*PHI(PSTAR, PL)
                                                                         RCM01390
   MRN=COEFR*PHI (PSTAR, PR)
                                                                         RCM01400
   DIFML=ABS (MLN-ML)
                                                                         RCM01410
   DIFMR=ABS (MRN-MR)
                                                                         RCM01420
   ML=MLN
                                                                         RCM01430
   MR=MRN
                                                                         RCM01440
   COMPUTE NEW PSTAR
                                                                         RCM01450
   PTIL=PSTAR
                                                                         RCM01460
   PSTAR=(UL-UR+PL/ML+PR/MR)/(1./ML+1./MR)
                                                                         RCM01470
   PSTAR=ALPHA*PSTAR+(1.-ALPHA)*PTIL
                                                                         RCM01480
   IF(Q.LE.QSTOP) GOTO 10
                                                                         RCM01490
   IF(ABS(PSTAR-PTIL).LT.EPS) GOTO 20
                                                                         RCM01500
   COMPUTE NEW ALPHA
                                                                         RCM01510
   ALPHA=0.5*ALPHA
                                                                          RCM01520
                                                                         RCM01530
   0 = 0
   IF((1.-ALPHA).LT.EPS) GOTO 20
                                                                          RCM01540
10 IF(DIFML.GE.EPS) GOTO 30
                                                                          RCM01550
   IF(DIFMR.GE.EPS) GOTO 30
                                                                          RCM01560
   END OF GODUNOV ITERATION; COMPUTE USTAR
                                                                          RCM01570
20 USTAR=(PL-PR+ML*UL+MR*UR)/(ML+MR)
                                                                          RCM01580
   BEGIN SAMPLING PROCEDURE
                                                                          RCM01590
   IF (XI.LT.USTAR*DT) GO TO 40
                                                                          RCM01600
   RIGHT SIDE; SELECT CASE OF SHOCK OR EXPANSION
                                                                          RCM01610
   IF (PSTAR.LT.PR) GO TO 50
                                                                          RCM01620
   RIGHT WAVE IS A SHOCK WAVE
                                                                          RCM01630
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WR=UR+MR/RR

RCM01640

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IF (XI.LT.WR*DT) GO TO 60
                                                                             RCM0
C
      RIGHT OF RIGHT SHOCK CASE
                                                                             RCM0
      RGLIM=RR
                                                                             RCM0
      PGLIM=PR
                                                                             RCM0
      UGLIM=UR
                                                                             RCM0
      RETURN
                                                                             RCM0
C
      LEFT OF RIGHT SHOCK CASE
                                                                             RCM0
   60 RGLIM=-MR/(USTAR-WR)
                                                                             RCM0
      PGLIM=PSTAR
                                                                             RCM0
      UGLIM=USTAR
                                                                             RCM0
      RETURN
                                                                             RCM0
C
      RIGHT WAVE IS A RAREFACTION WAVE
                                                                             RCM0
   50 CONST=PR/RR**GR
                                                                             RCM0
      RSTAR = (PSTAR/CONST) **(1./GR)
                                                                             RCM0
      ASTAR = SQRT (GR*PSTAR/RSTAR)
                                                                             RCM0
      AR=SQRT(GR*PR/RR)
                                                                             RCM0
      IF (XI.GE. (USTAR+ASTAR)*DT) GO TO 70
                                                                             RCM0
C
      LEFT OF RIGHT FAN CASE
                                                                             RCM0
      RGLIM=RSTAR
                                                                             RCM0
      UGLIM=USTAR
                                                                             RCMO
      PGLIM=PSTAR
                                                                             RCM01
      RETURN
                                                                             RCM0
C
      SELECT RIGHT OF FAN OR IN FAN
                                                                             RCM0
   70 IF (XI.GE.(UR+AR)*DT) GO TO 80
                                                                             RCM0
C
      IN RIGHT FAN CASE
                                                                             RCM0
      UGLIM=2./(GR+1.)*(XI/DT-AR+(GR-1.)/2.*UR)
                                                                             RCMO!
      RGLIM=(((AR+(GR-1.)/2.*(UGLIM-UR))**2.)/(GR*CONST))**(1./(GR-1.)) RCMO
      PGLIM=CONST*RGLIM**GR
                                                                             RCM0
      RETURN
                                                                             RCM0
C
      RIGHT OF RIGHT FAN CASE
                                                                             RCM0
   80 RGLIM=RR
                                                                             RCM0
      PGLIM=PR
                                                                             RCMO!
      UGLIM=UR
                                                                             RCMO!
      RETURN
                                                                             RCMO!
С
      LEFT SIDE; SELECT CASE OF SHOCK OR RAREFACTION
                                                                             R CMC
   40 IF (PSTAR.LT.PL) GO TO 90
                                                                             RCMC(
C
      LEFT WAVE IS A SHOCK WAVE
                                                                             RCMC (
      WL=UL-ML/RL
                                                                             RCMC
      IF (XI.GE.WL*DT) GO TO 100
                                                                             RCMC
      LEFT OF LEFT SHOCK CASE
C
                                                                             R CMC
      RGLIM=RL
                                                                             RCMC
      PGLIM=PL
                                                                             RCM(
      UGLIM=UL
                                                                             RCMC
      RETURN
                                                                             RCM(
      RIGHT OF LEFT SHOCK CASE
                                                                             RCM(
  100 RGLIM=ML/(USTAR-WL)
                                                                             RCM(
      PGLIM=PSTAR
                                                                             RCM(
      UGLIM=USTAR
                                                                             RCM(
      RETURN
                                                                             RCM(
C
      LEFT WAVE IS A RAREFACTION WAVE
                                                                             RCM(
   90 CONST=PL/RL**GL
                                                                             RCM(
      RSTAR=(PSTAR/CONST)**(1./GL)
                                                                             RCM(
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ASTAR = SQRT (GL\*PSTAR/RSTAR)

AL=SQRT(GL\*PL/RL)

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IF (XI.LT. (USTAR-ASTAR)*DT) GO TO 110
                                                                             RCM02190
C
      RIGHT OF LEFT FAN CASE
                                                                             RCM02200
      RGLIM=RSTAR
                                                                             RCM02210
      PGLIM=PSTAR
                                                                             RCM02220
      UGLIM=USTAR
                                                                             RCM02230
      RETURN
                                                                             RCM02240
C
      SELECT LEFT OF FAN OR IN FAN CASE
                                                                             RCM02250
  110 IF (XI.LT.(UL-AL)*DT) GO TO 120
                                                                             RCM02260
C
      IN LEFT FAN CASE
                                                                             RCM02270
      UGLIM=2./(GL+1.)*(AL+(GL-1.)/2.*UL+XI/DT)
                                                                             RCM02280
      RGLIM=(((AL+(GL-1.)/2.*(UL-UGLIM))**2.)/(GL*CONST))**(1./(GL-1.)) RCM02290
      PGLIM=CONST*RGLIM**GL
                                                                             RCM02300
      RETURN
                                                                             RCM02310
      LEFT OF LEFT FAN CASE
C
                                                                             RCM02320
 120 RGLIM=RL
                                                                             RCM02330
      PGLIM=PL
                                                                             RCM02340
      UGLIM=UL
                                                                             RCM02350
                                                                             RCM02360
      RETURN
                                                                             RCM02370
      END
                                                                             RCM02380
      FUNCTION PHI(Y,Z)
                                                                             RCM02390
      REAL RMU
                                                                             RCM02400
      COMMON/FUN1/G, PA, RA, UA, RB, RMU
      EPS=1.E-06-
                                                                             RCM02410
                                                                             RCM02420
      PARAM=Y/Z
      IF (ABS(1.-PARAM).GE.EPS) GO TO 10
                                                                             RCM02430
      PHI=SQRT(G)
                                                                             RCM02440
                                                                             RCM02450
      RETURN
   10 IF (PARAM.GE.1.) GO TO 20
                                                                             RCM02460
      PHI=(G-1.)/2.*(1.-PARAM)/(SORT(G)*(1.-PARAM**((G-1.)/(2.*G))))
                                                                             RCM02470
                                                                             RCM02480
      RETURN
   20 PHI=SORT((G+1.)/2.*PARAM+(G-1.)/2.)
                                                                             RCM02490
                                                                             RCM02500
      RETURN
                                                                             RCM02510
      END
      FUNCTION PHIL(PB)
                                                                             RCM02520
                                                                             RCM02530
      REAL RMU
      COMMON/FUN1/G, PA, RA, UA, RB, RMU
                                                                             RCM02540
      PHI1=(PB-PA)*SORT((1.-RMU**2.)/(RA*(PB+RMU**2.*PA)))
                                                                             RCM02550
                                                                             RCM02560
      RETURN
      END
                                                                             RCM02570
      FUNCTION PSI(PB)
                                                                             RCM02580
                                                                             RCM02590
      REAL RMU
                                                                             RCM02600
      COMMON/FUN1/G, PA, RA, UA, RB, RMU
      PSI=SORT(1.-RMU**4.)/RMU**2./SQRT(RA)*PA**(1./(2.*G))*(PB**((G-1.)RCM02610
     */(2.*G))-PA**((G-1.)/(2.*G)))
                                                                             RCM02620
      RETURN
                                                                             RCM02630
      END
                                                                             RCM02640
                                                                             RCM02650
      SUBROUTINE INIT1
      DIMENSION P(203), R(203), U(203), A(203), S(203), X(203)
                                                                             RCM02660
                                                                             RCM02670
      COMMON/FUN1/G, PA, RA, UA, RB, RMU
                                                                             RCM02680
      COMMON/SUBS/P,R,U,A,S,X
      DO 10 I=1,9,2
                                                                             RCM02690
                                                                             RCM02700
      P(I) = 810600.00
      R(I) = 0.7132
                                                                             RCM02710
```

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U(I)=644.4

```
A(I) = SQRT(G*P(I)/R(I))
10 CONTINUE
   DO 20 I=11,203,2
   P(I) = 101325.0
   R(I) = 1.22
   U(I) = 0.0
   A(I) = SQRT(G*P(I)/R(I))
20 CONTINUE
   RETURN
   END
   SUBROUTINE INIT2R(PSEXIT, PREF, RREF)
   DIMENSION P(203), R(203), U(203), A(203), S(203), X(203)
   COMMON/FUN1/G, PA, RA, UA, RB, RMU
   COMMON/SUBS/P,R,U,A,S,X
   DO 10 I=3,201,2
   P(I) = PREF
   R(I) = RREF
   U(I) = 0.0
   A(I) = SQRT(P(I)*G/R(I))
10 CONTINUE
   P(1)=P(3)
   R(1)=R(3)
   U(1) = -U(3)
   A(1) = SQRT(G*P(1)/R(1))
   P(203) = PSEXIT
   R(203)=R(201)
   PA=P(201)
   RA = R(201)
   UA = U(201)
   PB = P(203)
   RB = R(203)
   IF(PA.GT.PB) GO TO 20
   U(203) = UA - PHI1(PB)
   GO TO 30
20 U(203)=UA-PSI(PB)
30 A(203) = SQRT(G*P(203)/R(203))
   RETURN
   END
   SUBROUTINE INIT2L(PSEXIT)
   DIMENSION P(203), R(203), U(203), A(203), S(203), X(203)
   COMMON/FUN1/G, PA, RA, UA, RB, RMU
   COMMON/SUBS/P,R,U,A,S,X
   DO 10 I=3,201,2
   P(I) = 285080.0
   R(I)=0.897
   U(I) = 0.0
   A(I) = SQRT(G*P(I)/R(I))
10 CONTINUE
   RETURN
   END
   SUBROUTINE INIT3L(PSINL, RINL)
   DIMENSION P(203), R(203), U(203), A(203), S(203), X(203)
   COMMON/FUN1/G, PA, RA, UA, RB, RMU
   COMMON/SUBS/P,R,U,A,S,X
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DO 10 I=3,201,2
                                                                              RCM03270
   P(I) = 2390000.0
                                                                              RCM03280
   R(I) = 9.787
                                                                              RCM03290
   U(I) = 0.0
                                                                              RCM03300
   A(I) = SQRT(G*P(I)/R(I))
                                                                              RCM03310
10 CONTINUE
                                                                              RCM03320
   P(1)=PSINL
                                                                              RCM03330
   R(1) = RINL
                                                                              RCM03340
   PA=P(3)
                                                                              RCM03350
   RA=R(3)
                                                                              RCM03360
   UA=U(3)
                                                                              RCM03370
   PB=P(1)
                                                                              RCM03380
   U(1) = UA + PHI1(PB)
                                                                              RCM03390
   A(1) = SORT(G*P(1)/R(1))
                                                                              RCM03400
   P(203) = P(201)
                                                                              RCM03410
   R(203) = R(201)
                                                                              RCM03420
   U(203) = -U(201)
                                                                              RCM03430
   A(203) = SQRT(G*P(203)/R(203))
                                                                              RCM03440
   RETURN
                                                                              RCM03450
   END
                                                                              RCM03460
   SUBROUTINE INIT3R(PSINR, RINR)
                                                                              RCM03470
   DIMENSION P(203), R(203), U(203), A(203), S(203), X(203)
                                                                              RCM03480
   COMMON/SUBS/P.R.U.A.S.X
                                                                              RCM03490
   COMMON/FUN1/G, PA, RA, UA, RB, RMU
                                                                              RCM03500
   DO 10 I=3,201,2
                                                                              RCM03510
   P(I) = 2421667.5
                                                                              RCM03520
   R(I) = 9.787
                                                                              RCM03530
   U(I) = 0.0
                                                                              RCM03540
   A(I) = SORT(G*P(I)/R(I))
                                                                              RCM03550
10 CONTINUE
                                                                              RCM03560
   P(203) = PSTNR
                                                                              RCM03570
   PA=P(201)
                                                                              RCM03580
   RA=R(201)
                                                                              RCM03590
   UA=U(201)
                                                                              RCM03600
   PB = P(203)
                                                                              RCM03610
   U(203) = UA - PHI1(PB)
                                                                              RCM03620
   R(203) = RINR
                                                                              RCM03630
   A(203) = SQRT(G*P(203)/R(203))
                                                                              RCM03640
   P(1) = P(3)
                                                                              RCM03650
   R(1) = R(3)
                                                                              RCM03660
   U(1) = -U(3)
                                                                              RCM03670
   A(1) = SQRT(G*P(1)/R(1))
                                                                              RCM03680
   RETURN
                                                                              RCM03690
   END
                                                                              RCM03700
   SUBROUTINE BCL1
                                                                              RCM03710
   DIMENSION P(203), R(203), U(203), A(203), S(203), X(203)
                                                                              RCM03720
   COMMON/SUBS/P,R,U,A,S,X
                                                                              RCM03730
   COMMON/FUN1/G, PA, RA, UA, RB, RMU
                                                                              RCM03740
   P(1) = P(3)
                                                                              RCM03750
   R(1) = R(3)
                                                                              RCM03760
   U(1) = -U(3)
                                                                              RCM03770
   A(1) = SQRT(G*P(1)/R(1))
                                                                              RCM03780
   RETURN
                                                                              RCM03790
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END

```
SUBROUTINE BCR1
   DIMENSION P(203), R(203), U(203), A(203), S(203), X(203)
   COMMON/SUBS/P,R,U,A,S,X
   COMMON/FUN1/G, PA, RA, UA, RB, RMU
   P(203)=P(201)
   R(203)=R(201)
   U(203) = -U(201)
   A(203) = SQRT(G*P(203)/R(203))
   RETURN
   END
   SUBROUTINE BCL2(PSEXIT, PREF)
   DIMENSION P(203), R(203), U(203), A(203), S(203), X(203)
   COMMON/SUBS/P,R,U,A,S,X
   COMMON/FUN1/G, PA, RA, UA, RB, RMU
   P(1)=PSEXIT/PREF
   R(1)=R(3)
   U(1)=U(3)
   A(1)=SQRT(G*P(1)/R(1))
   RETURN
   END
   SUBROUTINE BCR2(PSEXIT, PREF)
   DIMENSION P(203), R(203), U(203), A(203), S(203), X(203)
   COMMON/SUBS/P,R,U,A,S,X
   COMMON/FUN1/G, PA, RA, UA, RB, RMU
   P(203)=PSEXIT/PREF
   R(203)=R(201)
   PA = P(201)
   RA = R(201)
   UA=U(201)
   PB = P(203)
   RB = R(203)
   IF(PA.GT.PB) GO TO 10
   U(203)=UA-PHI1(PB)
   GO TO 20
10 U(203) = UA - PSI(PB)
20 A(203) = SQRT(G*P(203)/R(203))
   RETURN
   SUBROUTINE BCL3(PSINL, RINL, PREF, RREF)
   DIMENSION P(203), R(203), U(203), A(203), S(203), X(203)
   COMMON/SUBS/P,R,U,A,S,X
   COMMON/FUN1/G, PA, RA, UA, RB, RMU
   P(1)=PSINL/PREF
   R(1)=RINL/RREF
   PA=P(3)
   RA=R(3)
   UA=U(3)
   PB=P(1)
   U(1)=UA+PHI1(PB)
   A(1) = SQRT(G*P(1)/R(1))
   RETURN
   END
   SUBROUTINE BCR3 (PSINR, RINR, PREF, RREF)
   DIMENSION P(203), R(203), U(203), A(203), S(203), X(203)
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COMMON/SUBS/P,R,U,A,S,X
                                                                           RCM04350
   COMMON/FUN1/G.PA.RA.UA.RB.RMU
                                                                           RCM04360
   P(203)=PSINR/PREF
                                                                           RCM04370
   R(203)=RINR/RREF
                                                                           RCM04380
   PA = P(201)
                                                                           RCM04390
   RA = R(201)
                                                                           RCM04400
   UA=U(201)
                                                                           RCM04410
   PB = P(203)
                                                                           RCM04420
   U(203)=UA-PHI1(PB)
                                                                           RCM04430
   A(203) = SORT(G*P(203)/R(203))
                                                                           RCM04440
   RETURN
                                                                           RCM04450
   END
                                                                           RCM04460
   SUBROUTINE BCL4 (PTOTIN.RTOTIN.PREF.RREF)
                                                                           RCM04470
   INTEGER QOUT
                                                                           RCM04480
   DIMENSION P(203), R(203), U(203), A(203), S(203), X(203)
                                                                           RCM04490
   DIMENSION XARRAY(100)
                                                                           RCM04500
   COMMON/SUBS/P,R,U,A,S,X
                                                                           RCM04510
   COMMON/FUN1/G, PA, RA, UA, RB, RMU
                                                                           RCM04520
   COMMON XARRAY, N1
                                                                           RCM04530
   N1 = N1 + 1
                                                                           RCM04540
   OOUT=N1/5
                                                                           RCM04550
   PTOT=PTOTIN/PREF
                                                                           RCM04560
   RTOT=RTOTIN/RREF
                                                                           RCM04570
   ATOT=SQRT (G*PTOT/RTOT)
                                                                           RCM04580
   STOT=ALOG(PTOT/RTOT**G)
                                                                           RCM04590
   U(1)=U(3)
                                                                           RCM04600
   A(1) = SORT(ATOT**2. - (G-1.)/2.*ABS(U(1))**2.)
                                                                           RCM04610
   AMACH=U(1)/A(1)
                                                                           RCM04620
   IF(AMACH.LT.0.0) GO TO 60
                                                                           RCM04630
   P(1) = PTOT/(1. + (G-1.)/2.*AMACH**2.)**(G/(G-1.))
                                                                           RCM04640
   R(1) = RTOT/(1. + (G-1.)/2. *AMACH**2.) **(1./(G-1.))
                                                                           RCM04650
   S(1)=ALOG(P(1)/R(1)**G)
                                                                           RCM04660
   GO TO 50
                                                                           RCM04670
60 P(1)=PTOT/(1.+(G-1.)/2.*ABS(AMACH)**2.)**(G/(G-1.))
                                                                           RCM04680
   R(1) = RTOT/(1. + (G-1.)/2. *ABS(AMACH) **2.) **(1./(G-1.))
                                                                           RCM04690
   S(1) = ALOG(P(1)/R(1) **G)
                                                                           RCM04700
50 RETURN
                                                                           RCM04710
   END
                                                                           RCM04720
   SUBROUTINE BCR4 (PTOTIN, RTOTIN, PREF, RREF)
                                                                           RCM04730
                                                                           RCM04740
   INTEGER QOUT
   DIMENSION P(203), R(203), U(203), A(203), S(203), X(203)
                                                                           RCM04750
   DIMENSION XARRAY(100)
                                                                           RCM04760
   COMMON/SUBS/P,R,U,A,S,X
                                                                           RCM04770
   COMMON/FUN1/G, PA, RA, UA, RB, RMU
                                                                           RCM04780
   COMMON XARRAY, N1
                                                                           RCM04790
   N1 = N1 + 1
                                                                           RCM04800
                                                                           RCM04810
   QOUT=N1/25
   PTOT=PTOTIN/PREF
                                                                           RCM04820
   RTOT=RTOTIN/RREF
                                                                           RCM04830
   ATOT = SQRT (G*PTOT / RTOT)
                                                                           RCM04840
   STOT=ALOG(PTOT/RTOT**G)
                                                                           RCM04850
   U(203)=U(201)
                                                                           RCM04860
   A(203) = SQRT(ATOT**2.-(G-1.)/2.*ABS(U(203))**2.)
                                                                           RCM04870
   AMACH=U(203)/A(203)
                                                                           RCM04880
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. 18

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IF(AMACH.LT.0.0) GO TO 60
   P(203)=PTOT/(1.+(G-1.)/2.*AMACH**2.)**(G/(G-1.))
   R(203) = RTOT/(1.+(G-1.)/2.*AMACH**2.)**(1./(G-1.))
   S(203) = ALOG(P(203)/R(203)**G)
   GO TO 50
60 P(203) = PTOT/(1.+(G-1.)/2.*ABS(AMACH)**2.)**(G/(G-1.))
   R(203)=RTOT/(1.+(G-1.)/2.*ABS(AMACH)**2.)**(1./(G-1.))
   S(203) = ALOG(P(203)/R(203)**G)
50 RETURN
   END
   SUBROUTINE BCL5
   DIMENSION P(203), R(203), U(203), A(203), S(203), X(203)
   COMMON/SUBS/P,R,U,A,S,X
   P(1)=P(3)
   R(1)=R(3)
   U(1)=U(3)
   A(1)=A(3)
   RETURN
   END
   SUBROUTINE BCR5
   DIMENSION P(203), R(203), U(203), A(203), S(203), X(203)
   COMMON/SUBS/P,R,U,A,S,X
   P(203)=P(201)
   R(203) = R(201)
   U(203)=U(201)
   A(203)=A(201)
   RETURN
   END
   FUNCTION WDP(II)
   DIMENSION WNORM(12), IDIGT(12)
   COMMON/SAMPLE/WNORM, IDIGT
   IF (II.EQ.0) GO TO 10
   L1=2
   L2 = 1
   DO 20 JJ=1,12
   IDIGT(JJ)=0
   WNORM(JJ)=1./FLOAT(L1**JJ)
20 CONTINUE
   WDP=0.
   RETURN
10 DO 40 JJ=1,12
   L1=2
   L2=1
   KJ0=IDIGT(JJ)
   KJN=MOD((KJO+1),L1)
   IDIGT(JJ)=KJN
   IF (KJO.LT.KJN) GO TO 50
40 CONTINUE
50 SUM=0.
   DO 60 JJ=1,12
   KNEW=MOD(IDIGT(JJ)*L2,L1)
   SUM=SUM+FLOAT(KNEW)*WNORM(JJ)
60 CONTINUE
   WDP=SUM
```

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RETURN
                                                                            RCM05430
    END
                                                                            RCM05440
    SUBROUTINE PLOT1(K)
                                                                            RCM05450
    DIMENSION XORG(4), YORG(4), YMAX(4), YMIN(4)
                                                                            RCM05460
    DATA XORG/0.5,4.75,0.5,4.75/
                                                                            RCM05470
    DATA YORG/0.5,0.5,4.75,4.75/
                                                                            RCM05480
    DATA YMAX/3.50,3.0,0.5,2.0/
                                                                            RCM05490
    DATA YMIN/0.5,0.0,-0.5,-2.0/
                                                                            RCM05500
    DO 10 I=1.4
                                                                            RCM05510
    CALL PHYSOR(XORG(I), YORG(I))
                                                                            RCM05520
    CALL AREA2D(3.5,3.5)
                                                                            RCM05530
    CALL FRAME
                                                                            RCM05540
    CALL GRAF(0., 'SCALE', 1.0, YMIN(I), 'SCALE', YMAX(I))
                                                                            RCM05550
    CALL ENDGR (0)
                                                                            RCM05560
10 CONTINUE
                                                                            RCM05570
    CALL PHYSOR (8.5,0.5)
                                                                            RCM05580
    CALL AREA2D(2.25,7.75)
                                                                            RCM05590
    CALL FRAME
                                                                            RCM05600
    CALL GRAF(0., 'SCALE', 1., 0, 'SCALE', K)
                                                                            RCM05610
    CALL ENDGR (0)
                                                                            RCM05620
    RETURN
                                                                            RCM05630
    END
                                                                            RCM05640
    SUBROUTINE_PLOT2(N.K)
                                                                            RCM05650
    DIMENSION XORG(4), YORG(4), YMAX(4), YMIN(4), KNT(4), IYNAM(10)
                                                                            RCM05660
    DIMENSION PARRAY(100), RARRAY(100), UARRAY(100), SARRAY(100), XARRAY(1RCM05670
   *00)
                                                                            RCM05680
    DIMENSION P(203), R(203), U(203), A(203), S(203), X(203)
                                                                            RCM05690
    COMMON/SUBS/P,R,U,A,S,X
                                                                            RCM05700
    COMMON XARRAY
                                                                            RCM05710
    DATA XORG/0.5,4.75,0.5,4.75/
                                                                            RCM05720
    DATA YORG/0.5,0.5,4.75,4.75/
                                                                            RCM05730
    DATA YMAX/3.50,3.0,0.5,2.0/
                                                                            RCM05740
    DATA YMIN/0.5,0.0,-0.5,-2.0/
                                                                            RCM05750
    DATA KNT/1,4,6,9/
                                                                            RCM05760
    DATA IYNAM/'PRES', 'SURE', '$ ', 'DENS', 'ITY$', 'VELO', 'CITY', '$
                                                                            'RCM05770
   *, 'ENTR', 'OPY$'/
                                                                            RCM05780
    DO 200 I=1,100
                                                                            RCM05790
    PARRAY(I) = P(I * 2 + 1)
                                                                            RCM05800
    RARRAY(I) = R(I * 2 + 1)
                                                                             RCM05810
    UARRAY(I)=U(I*2+1)
                                                                            RCM05820
    SARRAY(I) = S(I*2+1)
                                                                            RCM05830
200 CONTINUE
                                                                             RCM05840
                                                                            RCM05850
    DO 300 I=1,4
    CALL PHYSOR(XORG(I), YORG(I))
                                                                            RCM05860
    CALL AREA2D(3.5,3.5)
                                                                             RCM05870
    CALL XNAME ('X',1)
                                                                            RCM05880
    CALL YNAME(IYNAM(KNT(I)), 100)
                                                                            RCM05890
    CALL GRAF(0., 'SCALE', 1.0, YMIN(I), 'SCALE', YMAX(I))
                                                                             RCM05900
    IF(I.EQ.1) CALL SETCLR('YELLOW')
                                                                             RCM05910
    IF(I.EQ.2) CALL SETCLR('CYAN')
                                                                            RCM05920
    IF(I.EQ.3) CALL SETCLR('RED')
IF(I.EQ.4) CALL SETCLR('MAGENTA')
                                                                             RCM05930
                                                                            RCM05940
    IF(N.EQ.K) CALL SETCLR('WHITE')
                                                                            RCM05950
    IF(I.EQ.1) CALL CURVE (XARRAY, PARRAY, 100,0)
                                                                             RCM05960
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IF(I.EQ.2) CALL CURVE (XARRAY, RARRAY, 100,0)
                                                                               RCM(
      IF(I.EQ.3) CALL CURVE (XARRAY, UARRAY, 100,0)
                                                                               RCM(
      IF(I.EQ.4) CALL CURVE (XARRAY, SARRAY, 100,0)
                                                                               RCM(
      CALL ENDGR (0)
                                                                               RCM(
  300 CONTINUE
                                                                               RCM(
      RETURN
                                                                               RCM(
      END
                                                                               RCM(
      SUBROUTINE GE(SWL, SWR, N, TTOTAL, TIME, UEXMAX, PTOTIN, PREF)
                                                                               RCM(
      INTEGER SWL, SWR
                                                                               RCM(
      DIMENSION P(203), R(203), U(203), A(203), S(203), X(203)
                                                                               RCM(
      COMMON/SUBS/P,R,U,A,S,X
                                                                               RCM(
C**CALCULATION STARTS AT EXHAUST PORT OPENING. SUBROUTINE STRUCTURED
                                                                               RCM(
C**ACCORDINGLY.
                                                                               RCM(
      IF((SWL.EQ.1).AND.(SWR.EQ.2)) GO TO 10
                                                                               RCM(
      IF((SWL.EQ.4).AND.(SWR.EQ.2)) GO TO 30
                                                                               RCM(
      IF((SWL.EQ.4).AND.(SWR.EQ.1)) GO TO 50
                                                                               RCM(
      IF((SWL.EQ.1).AND.(SWR.EQ.1)) RETURN
                                                                               RCM(
   10 \text{ PWALL} = P(2)
                                                                               RCM(
      IF(PWALL.LE.(PTOTIN/PREF)) GO TO 20
                                                                               RCM(
      RETURN
                                                                               RCM(
   20 SWL=4
                                                                               RCM(
      WRITE(6,74)
                                                                               RCM(
      WRITE(6,75) N,TTOTAL,TIME
                                                                               RCM(
      RETURN
                                                                               RCM(
   30 UEXIT=U(202)
                                                                               RCM(
      IF (UEXMAX.LT.UEXIT) UEXMAX=UEXIT
                                                                               RCM(
      IF(UEXIT.LT.UEXMAX/2.) GO TO 40
                                                                               RCM(
      RETURN
                                                                               RCM(
   40 SWR=1
                                                                               RCM(
                                                                               RCM(
      WRITE(6,76)
      WRITE(6,75) N,TTOTAL,TIME
                                                                               RCM(
                                                                               RCM(
      RETURN
   50 \text{ P1SHOK} = P(2)
                                                                               RCM(
                                                                               RCM(
      IF(P1SHOK.GT.PTOTIN/PREF) GO TO 60
                                                                               RCM(
      RETURN
   60 SWL=1
                                                                               RCM(
      WRITE(6,77)
                                                                               RCM(
      WRITE(6,75) N,TTOTAL,TIME
                                                                               RCM(
   74 FORMAT(5X, 'INLET PORT OPENS AT: ')
                                                                               RCM(
   75 FORMAT(5X, 14, 5X, 2F14.7)
                                                                               RCM(
   76 FORMAT (5X, 'EXHAUST PORT CLOSES AT: ')
                                                                               RCM(
   77 FORMAT(5X, 'INLET PORT CLOSES AT: ')
                                                                               RCM(
      RETURN
                                                                               RCM(
      END
                                                                               RCM(
      SUBROUTINE SPCTRA(N, SWL, SWR, TIME, UEXMAX, PSEXIT, PSOUT1, PSOUT2, PSOUTRCM(
     *3, JCOUNT, QPRINT, TTOTAL, KCOUNT)
                                                                               RCM(
      INTEGER SWL, SWR, QPRINT
                                                                               RCM()
      DIMENSION P(203), R(203), U(203), A(203), S(203), X(203)
                                                                               RCM(
      COMMON/SUBS/P,R,U,A,S,X
                                                                               RCM(
C*CALCULATION STARTS AT HP GAS IN PORT. JCOUNT IS NUMBERED ACCORDINGLY**RCM(
      IF((SWL.EQ.1).AND.(SWR.EQ.3)) GO TO 10
                                                                               RCM(
      IF((SWL.EQ.2).AND.(SWR.EQ.3)) GO TO 20
                                                                               RCM(
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RCM(

RCM(

IF((SWL.EQ.2).AND.(SWR.EQ.1)) GO TO 30

IF((SWL.EQ.5).AND.(SWR.EQ.1)) GO TO 40

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IF((SWL.EQ.2).AND.(SWR.EQ.5)) GO TO 50
                                                                           RCM06510
   IF((SWL.EQ.2).AND.(SWR.EQ.4)) GO TO 60
                                                                           RCM06520
   IF((SWL.EQ.1).AND.(SWR.EQ.4)) GO TO 70
                                                                           RCM06530
10 IF(U(3).LT.0.0) GO TO 11
                                                                           RCM06540
   RETURN
                                                                           RCM06550
11 JCOUNT=JCOUNT+1
                                                                           RCM06560
   PSEXIT=PSOUT1
                                                                           RCM06570
   SWI.=2
                                                                           RCM0658C
   WRITE(6,12)
                                                                           RCM06590
   WRITE(6,13)N, TIME, SWL, SWR, JCOUNT
                                                                           RCM06600
12 FORMAT (5X, 'H.P. AIR OUT PORT OPENS AT')
                                                                           RCM06610
13 FORMAT (5X, I4, 5X, F9.7, 5X, 3I3)
                                                                           RCM06620
   RETURN
                                                                           RCM06630
20 DO 26 I=5,199,2
                                                                           RCM06640
   IF((R(I)-R(I+2)).GT.0.1) GO TO 22
                                                                           RCM06650
   GO TO 26
                                                                           RCM06660
22 \times CNTCT = X(I)
                                                                           RCM06670
   UCNTCT=U(I)
                                                                           RCM06680
   TCNTCT=XCNTCT/ABS(UCNTCT)
                                                                           RCM06690
   AHEAD=A(199)
                                                                           RCM06700
   THEAD=1.0/A(199)
                                                                           RCM06710
   IF(TCNTCT.LE.THEAD) GO TO 23
                                                                           RCM06720
   RETURN
                                                                           RCM06730
23 JCOUNT=JCOUNT+1
                                                                           RCM06740
   SWR = 1
                                                                           RCM06750
                                                                           RCM06760
   WRITE(6,24)
   WRITE (6,25)N, TIME, SWL, SWR, JCOUNT
                                                                           RCM06770
24 FORMAT (5X, 'H.P. GAS IN PORT CLOSES AT')
                                                                           RCM06780
25 FORMAT (5X, I4, 5X, F9.7, 5X, 3I3)
                                                                           RCM06790
   RETURN
                                                                           RCM06800
26 CONTINUE
                                                                           RCM06810
30 IF(JCOUNT.EQ.4) GO TO 80
                                                                           RCM06820
   IF(JCOUNT.EQ.6) GO TO 90
                                                                           RCM06830
                                                                           RCM06840
   IF(JCOUNT.EO.8) GO TO 100
   IF((R(2)-R(4)).GT.0.1) GO TO 31
                                                                           RCM06850
   RETURN
                                                                           RCM06860
31 JCOUNT=JCOUNT+1
                                                                           RCM06870
                                                                           RCM06880
   SWL=5
   WRITE(6,32)
                                                                           RCM06890
   WRITE(6,33)N, TIME, SWL, SWR, JCOUNT
                                                                           RCM06900
32 FORMAT (5X, 'HP AIR OUT PORT CLOSES AND TUNING PORT L1 OPENS AT')
                                                                           RCM06910
33 FORMAT(5X,14,5X,F9.7,5X,313)
                                                                           RCM06920
                                                                           RCM06930
   RETURN
40 IF(JCOUNT.EQ.7) GO TO 110
                                                                           RCM06940
   IF(U(3).GE.O.O) GO TO 41
                                                                           RCM06950
   RETURN
                                                                           RCM06960
41 JCOUNT=JCOUNT+1
                                                                           RCM06970
                                                                           RCM06980
   PSEXIT=PSOUT2
                                                                           RCM06990
   SWL=2
                                                                           RCM07000
   WRITE(6,42)
   WRITE(6,43)N, TIME, SWL, SWR, JCOUNT
                                                                           RCM07010
42 FORMAT (5X, 'TUNING PORT L1 CLOSES AND EXHAUST PORT E1 OPENS AT')
                                                                           RCM07020
43 FORMAT(5X, I4, 5X, F9.7, 5X, 3I3)
                                                                           RCM07030
```

RETURN

RCM07040

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80 IF(ABS(U(201)).GT..0001) GO TO 81
                                                                           RCM07(
    RETURN
                                                                           RCM07(
 81 JCOUNT=JCOUNT+1
                                                                           RCM07(
    SWR = 5
                                                                           RCM070
    WRITE(6,82)
                                                                           RCM07(
    WRITE(6,83)N,TIME,SWL,SWR,JCOUNT
                                                                           RCM07:
82 FORMAT(5X, 'TUNING PORT R1 OPENS AT')
                                                                           RCM07
83 FORMAT(5X,14,5X,F9.7,5X,313)
                                                                           RCM071
    RETURN
                                                                           RCM07:
 50 IF(JCOUNT.EQ.9) GO TO 120
                                                                           RCM07:
    IF(ABS(U(201)-U(3)).LE.0.001) GO TO 51
                                                                           RCM07
    RETURN
                                                                           RCM07:
                                                                           RCM07:
 51 JCOUNT=JCOUNT+1
                                                                           RCM07
    SWR=1
                                                                           RCM07
    WRITE(6,52)
    WRITE(6,53)N,TIME,SWL,SWR,JCOUNT
                                                                           RCM07:
 52 FORMAT(5X, 'TUNING PORT R1 CLOSES AT')
                                                                           RCM07:
 53 FORMAT(5X, I4, 5X, F9.7, 5X, 3I3)
                                                                           RCM07
                                                                           RCM07:
    RETURN
 90 THEAD1=X(201)/(ABS(U(3))+A(3))
                                                                           RCM07
                                                                           RCM07:
    KCOUNT=KCOUNT+1
    IF (KCOUNT.EQ.1) TTOT1=TTOTAL
                                                                           RCM07:
    IF(TTOTAL.GE.(TTOT1+THEAD1)) GO TO 91
                                                                           RCM07:
    RETURN
                                                                           RCM07:
 91 JCOUNT=JCOUNT+1
                                                                           RCM07
    SWL=5
                                                                           RCM07
    WRITE(6,92)
                                                                           RCM07
    WRITE(6,93)N,TIME,SWL,SWR,JCOUNT
                                                                           RCM07
 92 FORMAT(5X, 'EXHAUST PORT E1 CLOSES AND TUNING PORT L2 OPENS AT')
                                                                           RCM07
 93 FORMAT(5X,14,5X,F9.7,5X,3I3)
                                                                           RCM07
    RETURN
                                                                           RCM07
110 IF(U(3).GE.O.O) GO TO 111
                                                                           RCM07
    RETURN
                                                                           RCM07
111 JCOUNT=JCOUNT+1
                                                                           RCM07
    PSEXIT=PSOUT3
                                                                           RCM07
    SWL=2
                                                                           RCM07
    WRITE(6,112)
                                                                           RCM07
    WRITE (6, 113)N, TIME, SWL, SWR, JCOUNT
                                                                           RCM07
112 FORMAT(5X, 'TUNING PORT L2 CLOSES AND EXHAUST PORT E2 OPENS AT')
                                                                           RCM07
113 FORMAT(5X,14,5X,F9.7,5X,3I3)
                                                                           RCM07
    RETURN
                                                                           RCM07
100 IF(ABS(U(201)).GT..0001) GO TO 101
                                                                           RCM07
                                                                           RCM07
    RETURN
101 JCOUNT=JCOUNT+1
                                                                           RCM07
    SWR=5
                                                                           RCM07
    WRITE(6,102)
                                                                           RCM07
    WRITE(6,103)N, TIME, SWL, SWR, JCOUNT
                                                                           RCM07
102 FORMAT(5X, 'TUNING PORT R2 OPENS AT')
                                                                           RCM07
103 FORMAT(5X,14,5X,F9.7,5X,313)
                                                                           RCM07
    RETURN
                                                                           RCM07
120 IF(ABS(U(201)-U(3)).LE.0.0001) GO TO 121
                                                                           RCM07
    RETURN
                                                                           RCM07
121 JCOUNT=JCOUNT+1
                                                                           RCM07
    SWR=4
                                                                           RCM07
```

	WRITE(6,122) WRITE(6,123)N,TIME,SWL,SWR,JCOUNT	RCM07590 RCM07600
	FORMAT(5X, 'TUNING PORT R2 CLOSES AND L.P. AIR INLET OPENS AT')	RCM07610
123	FORMAT(5X,14,5X,F9.7,5X,313)	RCM07620
	RETURN	RCM07630
60	IF((R(4)-R(2)).GT.0.1) GO TO 61	RCM07640
	RETURN	RCM07650
61	JCOUNT=JCOUNT+1	RCM07660
	SWL=1	RCM07670
	WRITE(6,62)	RCM07680
	WRITE(6,63)N, TIME, SWL, SWR, JCOUNT	RCM07690
62	FORMAT(5X, 'EXHAUST PORT E2 CLOSES AT')	RCM07700
63	FORMAT(5X, 14, 5X, F9.7, 5X, 313)	RCM07710
	RETURN	RCM07720
70	IF(U(201).GE.0.0) GO TO 71	RCM07730
	RETURN	RCM07740
71	JCOUNT=0	RCM07750
	SWR=1	RCM07760
	WRITE(6,72)	RCM07770
	WRITE(6,73)N, TIME, SWL, SWR, JCOUNT	RCM07780
72	FORMAT(5X, 'CYCLE COMPLETED.')	RCM07790
73	FORMAT(5X,14,5X,F9.7,5X,313)	RCM07800
	RETURN -	RCM07810
	END	RCM07820

### PROGRAM RCM

### B.l. Program Description

### B.1.1. Computational Grid

The computational region is divided into 100 cells; the solution grid points are odd numbered, e.g., 3, 5, 7 ..., 201 with 1 and 203 being the points where the boundary conditions are specified. The even numbered points, 2, 4, 6 ..., 202 are intermediate locations where solutions are stored before being assigned to the solution grid points. See Fig. (2).

### B.1.2. Data Input

Data for various ports (exhaust, inlet, etc.) is specified in dimensional form in S.I. units (Pascal ( $N/m^2$ ) for pressure,  $kg/m^3$  for density, m/s for velocity etc.). Reference values are also specified in like manner. See lines RCM00210 through 00250.

Initial data is specified through a call to an appropriate subroutine, depending on where the calculation is started for a particular wave diagram. For the example given in section II on the Spectra Technology wave diagram, the computation is started at the point when the high pressure gas inlet port just opens. The call for initial data is made to subroutine INIT3R, which prescribes data consistent with a solid wall boundary at the left and a 'piston' inflow boundary at the right.

### B.1.3. Non-dimensionalization

Non-dimensionalization is carried out in lines 00540 through 00610 with entropy defined as

$$S = \ln \left(\frac{p}{\rho \gamma}\right)$$

Note that velocities are all referred to a reference sonic velocity defined by

$$a_{ref} = \frac{p_{ref}}{p_{ref}}$$

## B.1.4. Structure

The main program loop starts at line 00630, for the number of time steps specified. The time step is computed according to the appropriate CFL condition for the method, and a random number for the time step is generated by a call to the function subroutine WDP.

A secondary loop to define the sequence of local Riemann problems for the time step is set up at line 00750. For each Riemann problem defined, a call is made to subroutine GLIMM which i) solves the Riemann problem, and ii) samples the solution using the random number generated. The subroutine then returns the sampled solution as the parameters PGLIM, RGLIM, UGLIM for the pressure, density and velocity respectively. These solutions are initially stored in the even numbered intermediate locations on the grid, and are then assigned to either the left or the right solution grid point depending on whether the random number was in the negative or the positive half of the interval respectively.

A call is then made to one of the modular subroutines structured for particular types of wave diagrams, lines 01050-01080, and the others are commented out.

Boundary conditions are invoked after the call to the modular subroutines which return the proper values of the switches SWL and SWR. The structure of the boundary condition subroutines is described in section II. This sequence completes one pass through the main loop and the process is repeated for the number of time steps specified.

## B.2. Example Use of Program RCM

The program is set up in the following steps:

- i) Line 00150 output device designation. See B.3.
- ii) Line 00190 specify the number of time steps, k , and the switches SWL and SWR consistent with where the computation is to be started.
- iii) Lines 00210 prescribe flow data for various ports in through 00250 dimensional form. See list of variables for explanation of variable names.
- iv) Lines 00490 invoke the proper initial data subroutine and through 00530 comment out the rest. See list of subroutines for explanation of subroutine, function subprogram names.
- v) Line 00660 set the interval for number of time steps at which a plot of the flow parameters is required.
- vi) Lines 01050 user supplied modular subroutine for particular through 01080 wave diagram to be computed. Comment out the rest.
- vii) Line 01190 call to plotting routine should be consistent with interval specified in line 0660.
- viii) Lines 02650 identify proper subroutine to prescribe initial through 03700 data (consistent with iv), and specify the data in the subroutine in dimensional form.
- ix) Lines 05470 specify plotting parameters, viz., origins of through 05990 plots, scales, number of points to be plotted, color of plots, etc. Facility dependent.

The subroutines PLOT1 and PLOT2 given in the listing are structured for DISSPLA software installed in the facility at NPGS.

x) Lines 06040 - user supplied modular subroutine for wave diagram through 07820 to be computed.

### B.3. Execution

The program is run in an interactive mode and is invoked through a call to DISSPLA, available on most mainframes. After compiling the program

(FORTRAN H Extended compiler), the following command executes it:

#### DISSPLA filename

If working at stations equipped with dual screens, the command on line 150 can be of the type

CALL TEK618 + Tektronix screen

If working on a non-graphics terminal, or a single screen station, this should be changed to

#### CALL COMPRS

which generates a 'DISSPLA METAFILE' to be routed later to either a screen or a plotter, e.g., VRSTEC, IBM79, TEK618, etc. Once generated, the metafile can be accessed and routed by the command

DISSPOP device designation

These are facility dependent commands and should be modified accordingly.

## B.4. List of Important Variables (In Alphabetical Order)

A - sonic velocity

AHEAD - sonic speed of head wave of rarefaction fan

AMACH - Mach number

AL - left side sonic speed value for RP
AR - right side sonic speed value for RP

AREF - reference speed of sound

ASTAR - speed of sound in 'starred' state of RP solution (see Fig. 3)

CFLNUM - CFL number for time step determination

DT - time step

DX - grid cell width

EPS - small number for pressure iteration in RP solver

G - ratio of specific heats, Y

IDIGT - see WNORM

II - argument used in function subprogram PHI equal to either 0 or 1

JCOUNT - counter

K - number of time steps

KCOUNT - counter

N - counter for time steps

N1 - counter

P - pressure

PA - flow parameter describing 'a' state in transition functions

PGLIM - pressure value returned by subroutine GLIMM

PL - left side pressure value for RP
PR - right side pressure value for RP

PREF - reference pressure

PSEXIT - static pressure at exit or outlet port

PSINL - static pressure for incoming 'piston' flow on left side
PSINR - static pressure for incoming 'piston' flow on right side
PSOUTn - n = 1,2,3 - exit static pressures for cycles with more than
one exhaust port

PSTAR - pressure in 'starred' state of RP solution (see Fig. 3)

PTOTIN - total pressure for isentropic inflow

QPRINT - specification of interval size for output

QSTOP - maximum number of iterations for solution of Riemann

problem, (RP)

R - density

RA,RB - flow parameters describing 'a' and 'b' states in transition functions

RGLIM - density returned by subroutine GLIMM

RINL - static density for incoming 'piston' flow on left side
RINR - static density for incoming 'piston' flow on right side

RL - left side density for RP

RMU - function of  $\gamma$ 

RR - right side density for RP

RREF - reference density

RTOTIN - total density for isentropic inflow

S - entropy

SWL - switch for left boundary
SWR - switch for right boundary

TCNTCT - time taken by contact surface to travel a certain distance
THEAD - time taken by head wave of expansion to travel a certain distance

TIME - real time in seconds

TIMEREF - reference time

TTOTAL - cumulative non-dimensional time for number of time steps

U - velocity

UA - flow parameter for 'a' state in transition functions

UCNTCT - velocity of contact surface

UEXMAX - maximum velocity occurring at an outflow boundary

UGLIM - velocity returned by subroutine GLIMM

UL - left side velocity for RP
UR - right side velocity for RP

USTAR - velocity in 'starred' state of RP solution (see Fig. 3)
WDP - value returned by random number generator subprogram

WL - left shock wave velocity
WR - right shock wave velocity

WNORM - variable used in random number generator subprogram

X - space dimension

XCNTCT - location of contact surface

XI,XII - random numbers scaled to grid cell

XREF - reference length

Y - argument used in function subprogram PHI equal to PSTAR

Z - argument used in function subprogram PHI equal to either PL or PR

PL or PR

# B.5. List of Subroutines, Function Subprograms

## B.5.1. Subroutines

INIT1 - prescribes initial data corresponding to SWL=1, SWR=1;
 e.g., shock-tube problem

INIT2L - prescribes initial data corresponding to SWL=2, SWR=1

INIT2R - prescribes initial data corresponding to SWL=1, SWR=2

INIT3L - prescribes initial data corresponding to SWL=3, SWR=1

INIT3R - prescribes initial data corresponding to SWL=1, SWR=3

PLOT1,2 - graphics subroutines

GLIMM - solves the Riemann problem, samples the solution and returns values for flow parameters

GE - modular user supplied subroutine to simulate wave diagram of General Electric Wave Engine

DETON - modular user supplied subroutine to simulate evacuation of detonation chamber

SPCTRA - modular user supplied subroutine to simulate wave diagram of Spectra Technology's Pressure Exchanger

BCLl - prescribes boundary conditions (BC's) corresponding to SWL=1, i.e., solid wall on left side

BCL2 - prescribes BC's corresponding to SWL=2, i.e., outflow at constant static pressure on left side

BCL3 - prescribes BC's corresponding to SWL=3, i.e., 'piston' inflow on left side

BCL4 - prescribes BC's corresponding to SWL=4, i.e., isentropic inflow from reservoir on left side

BCL5 - prescribes BC's corresponding to SWL=5, i.e., wave 'tuning' on left side

BCR1, BCR2, BCR3, BCR4, BCR5 - prescribe BC's corresponding to SWR=1,2,3,4,5 respectively on right side

## B.5.2. Function Subprograms

- PHI(y,z) required in iteration procedure for solution of RP
- PHII(PB) describes shock transition function,  $\psi_a(p_b)$ , for two states a and b connected by a shock wave (see Ref. 6, Ch. III)
- PSI(PB) describes rarefaction transition function,  $\psi_a(p_b)$ , for two states a and b connected by a rarefaction wave (see Ref. 6, Ch. III)
- WDP(II) generates a random number in a van der Corput sequence each time it is invoked. Note that it needs to be called once from outside the main loop by specifying an argument II=1 to initialize IDIGT and WNORM, returning a value of 0 for the dummy variable ZETA, and then a second time from within the main loop with an argument II=0 to return a value which is the random number.

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