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A METHOD OF CHARACTERISTICS APPROACH TO THE PROBLEM OF SUPERSONIC FLOW PAST OSCILLATING CASCADES WITH FINITE BLADE THICKNESS

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A method of characteristics approach is described for the computation of supersonic flow past oscillating cascades having blades of finite thickness distribution. The transonic small disturbance equation is solved for cascades with subsonic leading-edge locus. Sample calculations are presented for the inlet and passage flow and comparisons are given with calculations for flat-plate cascades and available measured pressure distributions.

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1. INTRODUCTION

The demand for increasing performance and efficiency of turbines and compressors in jet engines forces the industry to advanced designs. This means that it is no longer acceptable to use blades in turbomachines which are loaded considerably below their mechanical limits. There are two major ways to improve the engine thrust-to-weight and thrust-to-volume characteristics:

- Reduced weight and size
- Increased massflow, temperature and pressure.

It is obvious that only a compromise can be successful, as advances in the aerothermodynamics oppose those of the structure.

These trends require the use of slender and thin blades which are increasingly susceptible to flutter and vibration problems. The most important ones are:

1. Supersonic unstalled flutter
2. Forced response
3. Subsonic stall flutter
4. Choke flutter
5. Supersonic stall flutter

While the latter three are quite difficult to describe in a fluid mechanical model, the unstalled supersonic blade flutter is amenable to analysis with reasonable effort. What makes it even more interesting and at the same time highly important is the possibility of its occurrence at the design condition of the engine. Especially, modern fans with large diameters operate with the outer part of their blades in the transonic flow region ($1. < M < 1.5$). Their flutter susceptibility therefore makes the analysis of supersonic unstalled flutter increasingly important. The problem has been attacked not only in the United States but in all major industrialized countries, which shows that it is a general one to be considered in the design of modern high-performance turbomachines.

During the past decade, several methods have been developed to predict supersonic blade flutter. All were based on the idealization of the actual flow by the planar flow through a staggered cascade of oscillating blades. However, the two-dimensional flow and the flat plate assumptions impose severe simplifications whose range of validity needs to be better understood.

Since the incorporation of three-dimensional flow effects is rather difficult, it seems logical to first explore the effect of blade thickness and shape on supersonic blade flutter while retaining the cascade concept. To this end, the nonlinear transonic small perturbation equation is adopted in this report as the governing equation. The use of this equation rather than the full potential equation or the Euler equations is suggested by Teipel's success to analyze the thickness effect of a single oscillating airfoil in low supersonic flow. Hence, the present work is an extension of Teipel's method to oscillating supersonic cascades for the purpose of determining the influence of steady non-uniform flow effects due to blade thickness, shape, camber or angle of attack on the oscillatory pressure distributions, forces and moments. The ultimate goal of this study is to replace the transonic small disturbance equation by the Euler equations so that the range of applicability of this simpler equation can be ascertained.

2. THE BASIC EQUATIONS

2.1 The Nonlinear Transonic Equation

Landahl presents in (1) the differential euqation which is valid for the transonic flow region

$$[M^2 - 1 + \frac{U\,(\kappa+1)}{a^2}\,\frac{\partial\Phi}{\partial X}]\,\frac{\partial^2\Phi}{\partial X^2} - \frac{\partial^2\Phi}{\partial Y^2} + \frac{1}{a^2} + \frac{\partial^2\Phi}{\partial T^2} + \frac{2M}{a}\,\frac{\partial^2\Phi}{\partial X\partial T} = 0 \qquad (1)$$

with U - Free stream velocity

a - Local velocity of sound

M - Free stream Mach number

Φ - Velocity potential (X,Y,T)

κ - Ratio of specific heats

Eq. (1) can be written non-dimensionally by using the terms

$$x = \frac{X}{c}\,,\ y = \frac{Y}{c}\,,\ t = \frac{TU}{c}\,,\ \phi = \frac{\Phi}{cU}\,,$$

where c is the chord.

Thus we get the new form

$$[M^2 - 1 + M^2\,(\kappa+1)\,\frac{\partial\phi}{\partial x}]\,\frac{\partial^2\phi}{\partial x^2} - M^2\,\frac{\partial^2\phi}{\partial t^2} + 2M^2\,\frac{\partial^2\phi}{\partial x\partial t} = 0 \qquad (2)$$

Following Teipel, who developed in (2) for single airfoils a method of characteristics using Eq. (1), in this work Eq. (2) is to be used. This has already successfully been done by Platzer, Chadwick and Strada (3,4,5,6,7) for a single oscillating airfoil and for oscillating cascades of wedges and thick blades with flat upper surfaces. Nevertheless, the basic steps, which lead to the solution of this problem shall be repeated in this report, to make it at the same time a summary of the work already done by the authors above.

It is an extension in so far as it introduces the thickness effect of the upper blade surface in an oscillating, staggered cascade and shows a method-of-characteristics-approach to the unsteady supersonic wake of not only a flat plate but also airfoils with small but finite thickness.

As we consider only small perturbations of the freestream flow values, the potential function ϕ of Eq. (2) can be split into a steady and an unsteady one. Furthermore, we assume only harmonic oscillations so that we can write

$$\phi(x,y,t) = \varphi(x,y) + \Psi(x,y) \cdot e^{ikt} \tag{3}$$

where $k = \frac{\omega \cdot c}{U}$ is the reduced frequency.

Introducing (3) in (2), we can separate the unsteady from the steady problem and we obtain a set of two differential equations

$$[M^2 - 1 + (\kappa+1)\, M^2\varphi_x]\, \varphi_{xx} - \varphi_{yy} = 0 \tag{4}$$

and

$$[M^2 - 1 + (\kappa+1)\, M^2\varphi_x]\, \Psi_{xx} - \Psi_{yy} + [M^2(\kappa+1)\varphi_{xx} + 2ik\, M^2]\, \Psi_x - M^2k^2\Psi = 0 \tag{5}$$

The boundary conditions for the flow over an oscillating airfoil can also be written as the sum of steady and unsteady influences:

$$h(x,t) = h_o(x) + h_1(x) \cdot e^{ikt} \tag{6}$$

$(h(x,t);\, x)$ is the true location of a surface point. Thus the boundary conditions for the steady and unsteady problem can be expressed

$$\varphi_y = \frac{\partial h_o}{\partial x} \equiv \text{slope of the surface} \tag{7a}$$

$$\Psi_y = \frac{\partial h_1}{\partial x} + \frac{\partial h_1}{\partial t} \equiv \text{the unsteady movement of a flat plate} \tag{7b}$$

Eq. (4) together with (7a) describes the transonic flow field over a fixed airfoil.

Following Sauer (8), we can attack the problem with the method of characteristics. The left- and right- running characteristics shall be indicated by α and β . We find for their slopes

$$\left(\frac{\partial y}{\partial x}\right)_{\alpha,\beta} = \pm \frac{1}{\sqrt{M^2-1 + (\kappa+1)\; M^2\varphi_x}} \tag{8}$$

or with

$$\lambda = M^2 - 1 + (\kappa+1)\; M^2\varphi_x \tag{9}$$

$$\left(\frac{\partial y}{\partial x}\right)_{\alpha,\beta} = \pm \frac{1}{\sqrt{\lambda}} \tag{10}$$

The upper sign indicates the α - direction. Introducing a second substitution

$$\mu = \frac{3}{2}\,(\kappa+1)\; M^2\varphi_y \tag{11}$$

The compatibility relation $\pm \sqrt{\lambda}\,\varphi_{xx} + \varphi_{yx} = 0$

can now be written as
$$\left(\frac{\partial \lambda^{3/2}}{\partial x}\right)_{\alpha,\beta} \mp \left(\frac{\partial \mu}{\partial x}\right)_{\alpha,\beta} = 0 \tag{12}$$

That Eq. (12) holds, can easily be verified by resubstituting (9) and (11) in (12) and executing the differentation. The result will be Eq. (4).

In (12) we find only derivatives in the x-direction along the characteristics. Therefore we can integrate (12) easily and obtain

$$\lambda^{3/2} \mp \mu = \text{const} = C_{\alpha,\beta} \tag{13}$$

We changed our variables from φ_x and φ_y to λ and μ. Consequently, we have to convert our boundary conditions for the steady problem:

$y = 0$:

$$\mu = \frac{3}{2} (\kappa+1) M^2 \frac{\partial h_o}{\partial x} \tag{14}$$

Eq. (13) says: as long as we move along one characteristic,

$$\left(\lambda^{3/2} \mp \mu\right)_{\alpha,\beta}$$

will not change. This makes (13) a tool to evaluate the original desired unknowns φ_x and φ_y in the field.

In the free-stream field φ_x and φ_y are zero. Hence, there we have

$$\lambda_\infty = M^2 - 1$$

and

$$\mu_\infty = 0$$

Therefore all the characteristics have here the slope

$$\left(\frac{\partial y}{\partial x}\right)_{\alpha,\beta} = \pm \frac{1}{\sqrt{M^2 - 1}}$$

and from (12) we obtain

$$C_\infty = \lambda_\infty^{3/2} = (M^2 - 1)^{3/2} \tag{15}$$

Fig. 1 shows a β -characteristic of the free stream hitting the surface of the airfoil.

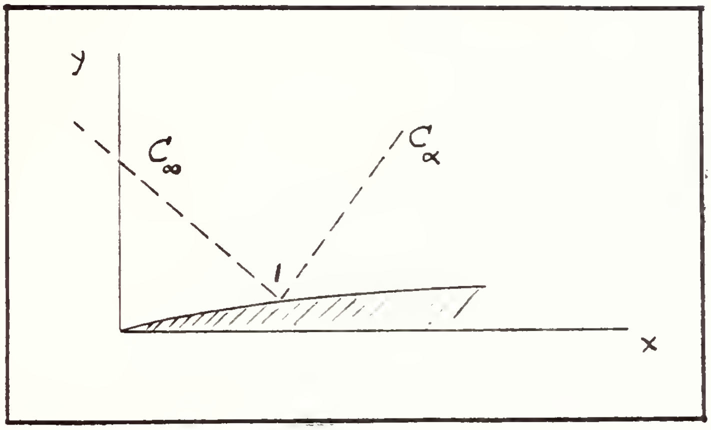


Fig. 1. Characteristics on the Airfoil Surface

In point 1 there has to be $C_\infty = C_\alpha$ with Eq. (9), (13) and (14) we now can find

$$\lambda_1 = \left[(M^2 - 1)^{3/2} \pm \frac{3}{2} (\kappa+1) M^2 \frac{\partial h_o}{\partial x}\right]^{2/3} \tag{16}$$

In the two-dimensional case there can be shown: for the characteristic leading away from the boundary there is not only

$$C_\alpha = \text{const}$$

but also

$$\mu_\alpha = \text{const} \quad \text{and}$$

$$\lambda_\alpha = \text{const}$$

With Eq. (5) through (16) the steady problem (4) can be solved. We get a net of characteristics. On the gridpoints the values of φ_x and φ_y are known. The computational procedure is shown later.

With the knowledge of the characteristic net and the values for φ_x , φ , λ and μ in each grid point we can now solve the unsteady part of the problem.

We put Eq. (9) and (11) into (5). The result is a new differential equation for the unsteady flow.

$$\lambda\Psi_{xx} - \Psi_{yy} + (\lambda_x + 2ik\,M^2)\,\Psi_x - M^2k^2\Psi = 0 \tag{17}$$

Further we assume irrotationality

$$\Psi_{xy} - \Psi_{yx} = 0 \tag{18}$$

Teipel shows explicitly how to derive from the system of Eq. (17) and (18) the compatibility relations for the unsteady characteristics.

$$\Psi_{xx} \mp \frac{1}{\sqrt{\lambda}}\,\Psi_{yx} + \frac{1}{\lambda}\,(\lambda_x + 2ik\,M^2)\,\Psi_x - \frac{1}{\lambda}\,k^2M^2\Psi = 0 \tag{19}$$

The geometry of the characteristic net is determined by the coefficients connected with the highest order terms. Thus we can see from Eq. (4) and (5) that the net remains the same for the unsteady flow problem.

To solve Eq. (17) via Eq. (19) by moving along the already known characteristics of the steady field, we need the unsteady boundary values along the airfoil and along the shock.

The first one is given with Eq. (7b).

$$\Psi_y = \frac{\partial h_1}{\partial x} + ikh_1$$

It reads for pitch - movement

$$\Psi_y = -\,[1 + ik(x - b)]$$

and for plunge - movement

$$\Psi_y = -\,ik$$

where b is the normalized location of the pitching axis. These expressions are derived in section 2.2. The boundary conditions along the oscillating shock are much more difficult to obtain. This is done in section 2.3.

After obtaining the solution for u_1, v_1 and Ψ the unsteady pressure coefficients can be computed from

$$c_{pl} = 2 \frac{p_1}{\rho_\infty U} \tag{20}$$

$$c_{pl} = -2(u_1 + ik\,\Psi)$$

2.2 Boundary Conditions Along the Airfoil

The general expression of the location of an oscillating airfoil is given by Eq. (6)

$$h(x,t) = h_o(x) + h_1(x) \cdot e^{ikt}$$

$h_o(x)$ represents the surface and we assume for now that it is described by an analytical function for which the second derivative exists. With Eq. (7a) we can calculate φ_y on the airfoil:

$$\varphi_y = \frac{\partial h_o}{\partial x}$$

However, as we only consider slender bodies, we project the point down to the x-axis $y = 0$, so that the boundary coordinates of the characteristic net are always $(x,0)$ instead of (x,y). This has two reasons

1. $y = 0$ makes the steady boundary step much less complicated, without introducing a considerable mistake;
2. The oscillating movement of the airfoil can be reduced to the movement of a flat plate.

We examine two moving modes. The pitch- and plunge- mode.

PITCH:

Fig. 2 shows the deflected airfoil (flat plate) in a system of coordinates

$$h_1 = (b - x) \cdot tg\alpha$$

As α is small and a harmonic motion, we can say

$$tg\alpha = \alpha_o \cdot e^{ikt}$$

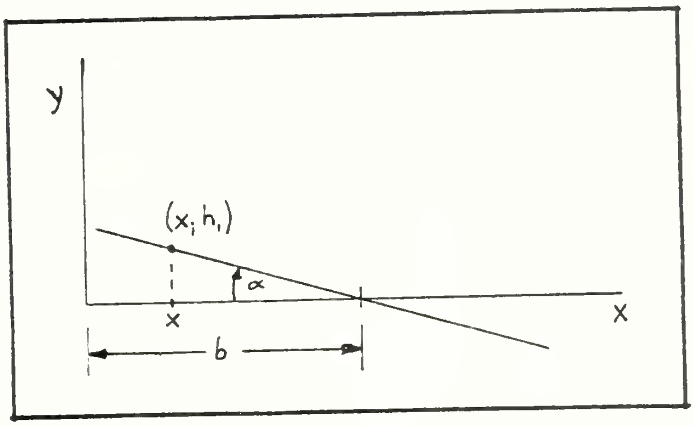


Fig. 2. Pitching Flat Plate

Thus we get

$$h_1 = (b - x)\ \alpha_o \cdot e^{ikt}$$

The unsteady boundary condition is with Eq. (7b)

$$\Psi_y = [-\ \alpha_o + \alpha_o\ ik(b - x)]\ e^{ikt}$$

or

$$\Psi_y = -\ \alpha_o\ [1 + ik(x - b)]$$

where the exp (ikt) - term is omitted. This can be normalized by $|\alpha_o|$. So the final expression is the well known form

$$\Psi_y = - [1 + ik(x - b)] \tag{21}$$

PLUNGE:

For the plunge mode the deflection h is no function of x . Again a harmonic motion is assumed.

$$h = - h_o \, e^{ikt}$$

Eq. (7b) gives us then the boundary value for

$$\Psi_y = - h_o \cdot ik \, e^{ikt}$$

To be consistent with the previous work (3 to 7), the downward deflection is defined as positive. Again the expression is normalized by $|h_o|$ and the

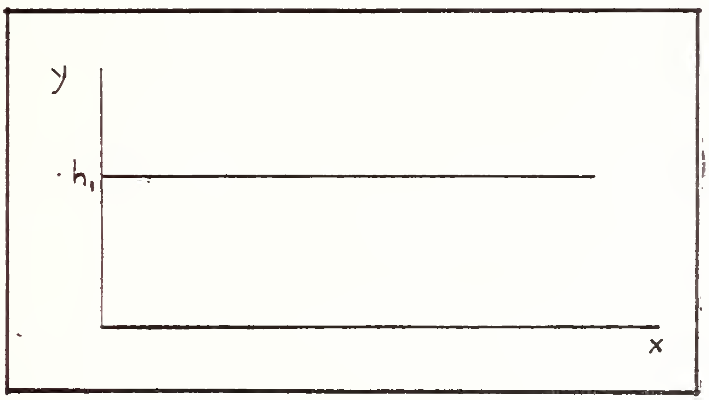


Fig. 3. Plunge Movement

exp(ikt) - term is omitted

$$\Psi_y = - ik \tag{22}$$

This approach to the boundary conditions is a rather physical one. A more rigorous derivation is shown by Bell in (9), including the derivation of Eq. (7b).

2.3 The Oscillating Shock Wave in an Oscillating Flow Field

The basic problem in using the method of characteristics is finding and introducing the proper boundary conditions. Section 2.2 gives us the influence of the moving airfoil into the field. The second boundary in the field between surface and shock are the unsteady flow properties immediately downstream of the shock. See Fig. 4.

It is

w - Velocity

u, v - x, y components of w

w_n, w_t - Normal and tangential components of

$\wedge$ - Indicates properties behind the shock

γ_o - Slope of the shock in the steady problem

γ' - Deflection of the shock due to oscillation of the airfoil

All the perturbation quantities are supposed to be very small.

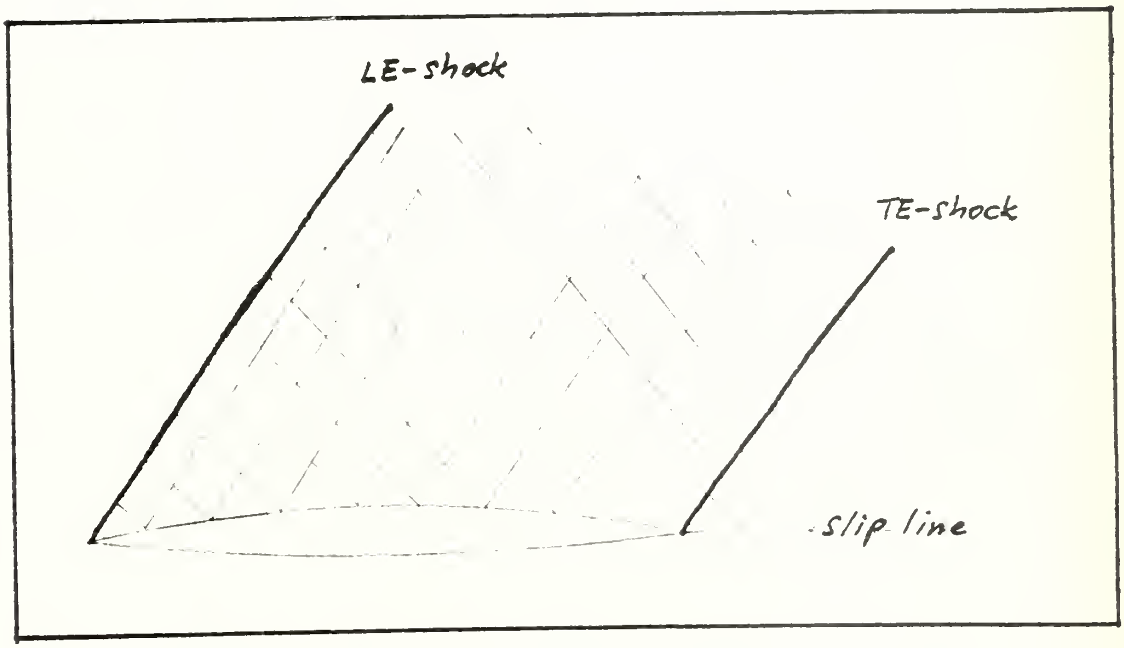


Fig. 4. Field of Characteristics Past an Airfoil

Teipel (10) found a way to obtain those properties for an airfoil in undisturbed supersonic flow. As the final object is to compute the flow in a staggered cascade, his work had to be extended. This was first done by Chadwick (4) who applied it to a cascade of wedges. Strada used Chadwick's equations in (5) to compute the inlet flow of a cascade with airfoils which have flat upper and curved lower surfaces.

Fig. 5 shows the velocities upstream and downstream of an arbitrary shock:

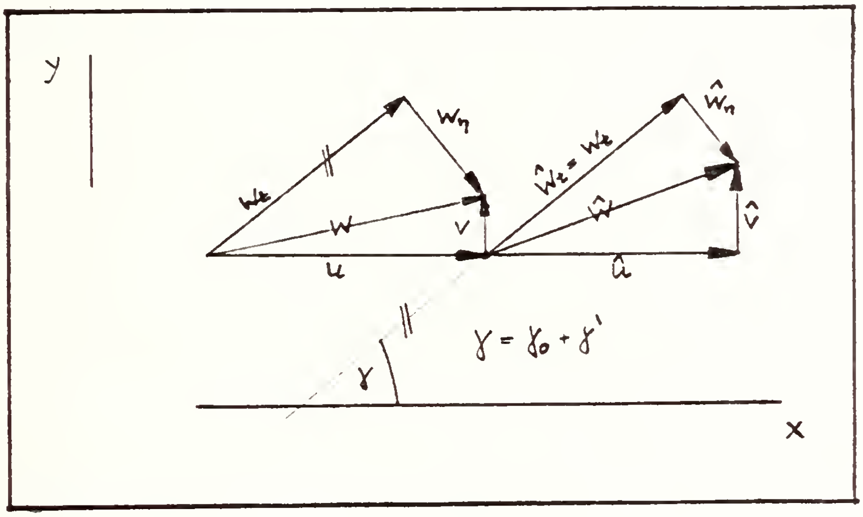


Fig. 5. Velocities at the Shock

The general case in the cascade is $w \neq u$. Thus we write

$$
\begin{aligned}
u &= 1 + u_o + u_1 \\
v &= v_o + v_1 \\
\hat{u} &= 1 + \hat{u}_o + \hat{u}_1 \\
\hat{v} &= \hat{v}_o + \hat{v}_1
\end{aligned}
\qquad (23)
$$

The only unknowns are $\hat{u}_1$ and $\hat{v}_1$, as the steady problem is already expected to be solved and the field in front of the shock is presumed to be well known. Then we can take from the geometry (Fig. 5).

$$
\begin{aligned}
w_n &= u \cdot \sin(\gamma_o + \gamma') - v \cos(\gamma_o + \gamma') \\
w_t &= u \cos(\gamma_o + \gamma') + v \sin(\gamma_o + \gamma')
\end{aligned}
\tag{24}
$$

If we apply Eq. (23) to (24) and neglect higher order terms like $u_o \gamma^1$, Eq. (24) can be rewritten. For convenience we use the abbreviation

$$
\begin{aligned}
\nu &= 1 + u_o \qquad \text{from eqn. (9)} \\
\nu &= 1 + \frac{\lambda - (M^2 + 1)}{(\kappa+1)\, M^2}
\end{aligned}
\tag{25}
$$

Eq. (24) becomes

$$
\begin{aligned}
w_n &= \nu \sin\gamma_o + u_1 \sin\gamma_o + \gamma' \cos\gamma_o - (v_o + v_1) \cos\gamma_o \\
w_t &= \nu \cos\gamma_o + u_1 \cos\gamma_o - \gamma' \sin\gamma_o + (v_o + v_1) \sin\gamma_o
\end{aligned}
\tag{26}
$$

Now we take a look at the equation for normal moving shocks (11).

$$
\hat{w}_n - w_n = \frac{2}{\kappa+1} \cdot W\left(1 - \frac{a^2}{W^2}\right), \tag{27}
$$

where W is the relative velocity of the shock with respect to the fluid. To transform from the system moving with the fluid into an airfoil - fixed system of coordinates, we find the velocity of a point oscillating with the shock as

$$
W^* = W + w_n \tag{28}
$$

See Fig. 6.

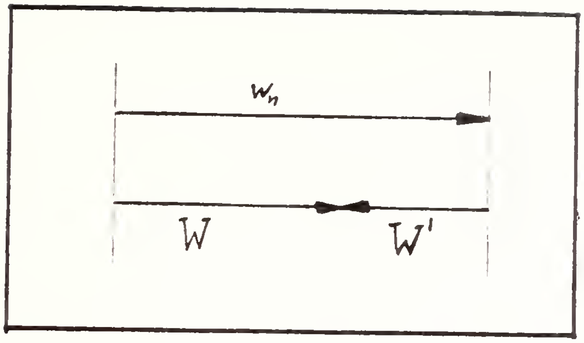


Fig. 6. Definition of Relative Velocities in a Shock-Point

Connecting Eq. (26) with Eq. (28) we obtain

$$W = W' - \nu \sin \gamma_o - u_1 \sin \gamma_o - \gamma' \cos \gamma_o + (v_o + v_1) \cos \gamma_o \qquad (29)$$

Forming W^2 and again neglecting all terms of higher order, recasting Eq. (29) leads to

$$\nu^2 \sin^2 \gamma_o = W(W + 2\nu \sin \gamma_o) \qquad (30)$$

Reintroducing this into the shockpolar Eq. (27), we get

$$w_n - w_n = \frac{2}{\kappa+1} W - \frac{2}{\kappa+1} \cdot \frac{a^2}{\nu^2 \sin^2 \gamma_o} (W + 2\nu \sin \gamma_o) \qquad (31)$$

By making use of Eq. (26), (29), (31) and the substitution

$$A = \frac{a_\infty^2}{\nu^2 \sin^2 \gamma_o} = \frac{1}{\nu^2 M^2 \sin^2 \gamma_o} = \frac{1}{\nu^2 M_n^2} \qquad (32)$$

We obtain, again neglecting all higher order terms

$$\hat{w}_n = \frac{\kappa-1}{\kappa+1}\, \nu \sin\gamma_o \left(1 + \frac{2A}{\kappa-1}\right) - \frac{\kappa-1}{\kappa+1}\, v_o \cos\gamma_o \left(1 - \frac{2A}{\kappa-1}\right) +$$

$$+ \frac{2}{\kappa+1}(1 + A)\, W' + \frac{\kappa-1}{\kappa+1}\left(1 - \frac{2A}{\kappa-1}\right)(\dot{\gamma}' \cos\gamma_o + u_1 \sin\gamma_o - v_1 \cos\gamma_o) \tag{33}$$

together with

$$\hat{w}_t = w_t \tag{34}$$

Out of Eq. (26) we have now expressions for the velocities behind the moving shock in terms of the known values and γ' and W'. In the next step we look at a point of the oscillating shock, Fig. 7.

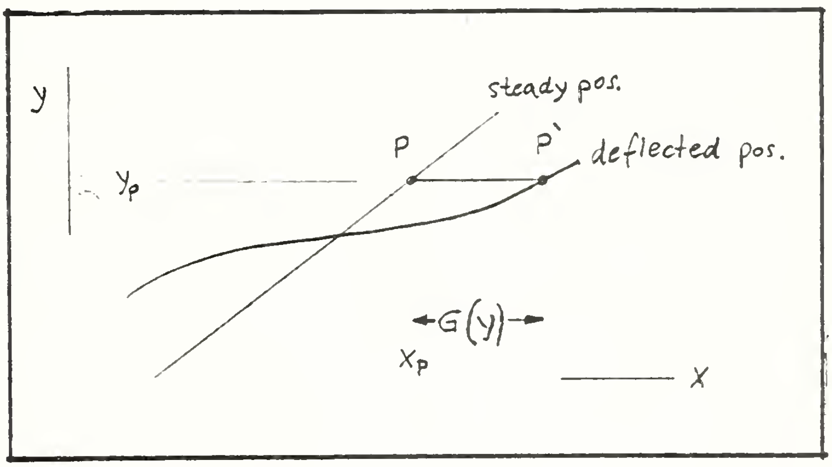


Fig. 7. Geometry of an Oscillating Shock

If we assume G to be a harmonic oscillation around the steady middle-position P, we can say

$$x = x_p + G(y) \cdot e^{ikt} \quad , \tag{35}$$

where $G(y)$ is the amplitude of the shock vibration and k the same reduced frequency as given for the airfoil.

The normal motion of P is

$$W' = \left(\frac{\partial x}{\partial t}\right)_y \cdot \sin\,(\gamma_o + \gamma)$$

$$W' = ik\,\sin\,\gamma_o\;G(y) \tag{36}$$

again the exponential term is omitted.

As we know $\gamma_o(y)$ from the solution of the steady problem, we can find the x-coordinates of P :

$$x_p = \int_o^y ctg\,(\gamma_o\;(y))\;dy$$

This completes Eq. (35) to

$$x = \int_o^y ctg\,(\gamma_o\;(y)) \cdot dy + G(y) \cdot e^{ikt} \tag{37}$$

The total differential gives us

$$dx = \left(\frac{\partial x}{\partial t}\right)_y \cdot dt + \left(\frac{\partial x}{\partial y}\right)_t \cdot dy$$

$$dx = ik\;G(y) \cdot e^{ikt} \cdot dt + \left[ctg\,(\gamma_o\;(y)) + \left(\frac{\partial G}{\partial y}\right)e^{ikt}\right] \cdot dy$$

with

$$\left(\frac{\partial x}{\partial y}\right)_t = ctg\,(\gamma_o\;(y) + \gamma'(y))$$

and now considering always a fixed y we can formulate

$$ctg\,(\gamma_o + \gamma') = ctg\;\gamma_o + G_y \cdot e^{ikt}$$

Trigonometric relations and higher order terms going to zero give us

$$\gamma' = - \sin^2 \gamma_o \; G_y \tag{38}$$

The transformation back to our Cartesian system of coordinates (Fig. 1) is done by

$$\begin{aligned} \hat{u} &= \hat{w}_t \cos (\gamma_o + \gamma') + \hat{w}_n \sin (\gamma_o + \gamma') \\ \hat{v} &= \hat{w}_t \sin (\gamma_o + \gamma') - \hat{w}_n \cos (\gamma_o + \gamma') \end{aligned} \tag{39}$$

The following steps have to be executed on Eq. (39):

- Introducing Eq. (33), (34)
- Applying trigonometric relations
- Neglecting all higher order products
- Introducing Eq. (36) and (38)
- Separating the purely steady expressions from the rest of the equations

The result is a substitute for the shock polar (27) which gives us the unsteady velocities u_1 and v_1 behind the shock.

$$\begin{aligned} \hat{u}_1 - m_1 G_y - i\, m_2 G - m_3 u_1 - m_4 v_1 &= 0 \\ \hat{v}_1 - n_1 G_y - i\, n_2 G - n_3 u_1 - n_4 v_1 &= 0 \end{aligned} \tag{40}$$

with A defined in Eq. (32) it is

$$\begin{aligned} m_1 &= \frac{2}{\kappa+1} \nu \sin 2\gamma_o \sin^2 \gamma_o \\ m_2 &= \frac{2k}{\kappa+1} (1 + A) \sin^2 \gamma_o \\ m_3 &= \sin^2 \gamma_o \frac{\kappa-1}{\kappa+1} (1 - \frac{2A}{\kappa-1}) + \cos^2 \gamma_o \\ m_4 &= \frac{\sin^2 \gamma_o}{\kappa+1} (1 + A) \end{aligned} \tag{41}$$

$$n_1 = -\frac{2\nu}{\kappa+1}(\cos\ \gamma_o + A)\sin^2\gamma_o$$

$$n_2 = -\frac{2k}{\kappa+1}(1+A)\cos\gamma_o\sin\gamma_o \tag{42}$$

$$n_3 = m_4$$

$$n_4 = \sin^2\gamma_o + \cos^2\gamma_o\frac{\kappa-1}{\kappa+1}\left(1-\frac{2A}{\kappa-1}\right)$$

These are exactly the coefficients Chadwick presented in (4). For the airfoil in undisturbed supersonic flow the perturbation quantities u_1 and v_1 in front of the shock are zero. With this in mind, Eq. (40) reduces to the shockpolar derived by Teipel in (10). It should be stated explicitly at this point that we followed Teipel very closely in this extension of his work and that Chadwick indicates the way in (4).

We have to do a last step, to make Eq. (40) a tool for computing the unsteady boundary values along the shock:

On the leading edge we know $G = 0$ because the shock is always attached. $\hat{v}_1\big|_{y=o}$ is known from the boundary conditions on the airfoil, Eq. (21). Now we can isolate $\hat{u}_1\big|_{y=o}$ in Eq. (40):

$$G_y\Big|_{y=o} = \frac{1}{n_1}\left[\hat{v}_1\Big|_{y=o} - (n_3\,u_1 + n_4\,v_1)\right] \tag{43}$$

and

$$\hat{u}_1\Big|_{y=o} = \frac{m_1}{n_1}\hat{v}_1\Big|_{y=o} + n_1\left(m_3 - \frac{m_1}{n_1}\cdot n_3\right) + v_1\left(m_4 - \frac{m_1}{n_1}n_4\right) \tag{44}$$

After this initial step we develop finite differences along the steady shock to solve gradually for the unsteady values of $\hat{u}_1$, $\hat{v}_1$ and G as shown later.

2.4 Connections Between the Linear and the Nonlinear System of Equations

In an earlier work Teipel developed in (12) a method of characteristics for an oscillating single flat plate. He derived an analytical solution for the unsteady boundary values along the shock. Using the perturbation velocity of sound rather than the perturbation potential, his concept was taken by Bell in (9) and by Platzer and associates in (3,13,14) to obtain results for a cascade of flat plates. They started from the Euler and continuity equations with the substitution

$$U(x,y) \cdot e^{ikt} = u_1 \tag{45a}$$

$$V(x,y) \cdot e^{ikt} = \frac{1}{\sqrt{M^2 - 1}} \cdot v_1 \tag{45b}$$

$$C(x,y) \cdot e^{ikt} = \frac{2}{\kappa-1} \frac{1}{M^2} \frac{a - a_\infty}{a_\infty} \tag{45c}$$

where U, V and C are complex nondimensional amplitudes. The result is a set of differential equations which reads

Continuity: $$\frac{\partial V}{\partial x} + \sqrt{M^2 - 1} \cdot \frac{\partial U}{\partial y} + M^2 \frac{\partial C}{\partial x} + ik\, M^2 C = 0 \tag{46}$$

Euler: $$\frac{\partial U}{\partial x} + \frac{\partial C}{\partial x} + ikU = 0 \tag{47}$$

Irrotationality: $$\frac{\partial U}{\partial x} - \sqrt{M^2 - 1}\, \frac{\partial V}{\partial x} = 0 \tag{48}$$

Furthermore it is shown by Bell in (9) that the pressure in terms of Eq. (45) can be expressed as

$$p - p_\infty = \frac{2}{\kappa-1} \rho_\infty a_\infty (a - a_\infty)$$

or

$$c_p = 2C$$

As $(p - p_\infty)$ is our unsteady pressure disturbance (the steady pressure disturbance is zero for a flat plate) we can say with Eq. (20)

$$2C = -2\,(u_1 + ik\,\Psi)$$

or

$$C = -\,(u_1 + ik\,\Psi) \tag{50}$$

Now we take Eq. (45a), (45b) and (50) and substitute into Eq. (46) and we get

$$(M^2 - 1)\,\frac{\partial u_1}{\partial x} - \frac{\partial v_1}{\partial y} + 2\,ik\,M^2\,u_1 - k^2 M^2\,\Psi = 0 \tag{51}$$

With $u_1 = \Psi_x$ and $v_1 = \Psi_y$ this is our Eq. (17), when we consider that for a flat plate the values for φ_x and λ_x are always zero.

$$\lambda = \sqrt{M^2 - 1}$$

Applying Eq. (50) on (47) gives us

$$u_1 = \frac{\partial \Psi}{\partial x}$$

which is one of our basic equations. Obviously, Eq. (48) can be transformed into Eq. (18). Hence it is shown that the basic equations previously used by Teipel and Platzer for the flat plate are equivalent to the perturbation potential equations used in this work, when these are reduced to the linear case.

The next step shows how to obtain an anlytical expression for the oscillating shock generated by a single flat plate from the rather complicated differential equation (40).

As there are no perturbations in front of the shock, Eq. (40) reduces to the Teipel-form

$$\hat{u}_1 = m_1 G_y + i\, m_2 G$$

$$\hat{v}_1 = n_1 G_y + i\, n_2 G$$

In addition the steady shock angle γ_o can be expressed as

$$\sin \gamma_o = \frac{1}{M} \tag{52}$$

and with this A from Eq. (41), defined in Eq. (32), becomes

$$A = \frac{1}{\nu^2 M^2 \sin^2\gamma_o} = 1 \tag{53}$$

$$(\nu = 1 + u_o = 1)$$

The shockpolar written down explicityly reads

$$\hat{u}_1 = \frac{2}{\kappa+1} \sin \gamma_o \sin^2\gamma_o G_y + i \frac{2k}{\kappa+1} 2 \sin^2\gamma_o G$$

$$\hat{v}_1 = - \frac{2}{\kappa+1} (\cos 2\gamma_o + 1) \sin^2\gamma_o G_y - i \frac{2k}{\kappa+1} 2 \cos\gamma_o \sin\gamma_o G$$

Using the trigonometrical relations and Eq. (52) we obtain after some adding and subtracting

$$\hat{u}_1 = - \frac{1}{\sqrt{M^2 - 1}} \hat{v}_1 \tag{54}$$

for the velocities immediately downstream of the shock. We now go into the compatibility relation Eq. (19). Linearized with $\lambda_x = 0$ it is for the upper side

$$\frac{\partial u_1}{\partial x} - \frac{1}{\sqrt{M^2 - 1}} \frac{\partial v_1}{\partial x} + \frac{2ik M^2}{M^2 - 1} u_1 - \frac{1}{M^2 - 1} k^2 M^2 \Psi = 0$$

Eq. (54) shows that

$$\frac{\partial u_1}{\partial x} - \frac{1}{\sqrt{M^2 - 1}} \frac{\partial v_1}{\partial x} = 2 \frac{\partial u_1}{\partial x}$$

Landahl (1) shows that $(\Psi + \varphi)$ should be continuous across a shock. As all the steady components are zero in this case his result applies here to the unsteady potential alone. On the other hand we consider the shock thickness as infinitely small. Assumming $d\hat{\Psi}$ to be nonzero,

$$\frac{\partial \hat{\Psi}}{\partial x}$$

would become infinitely large across the shock. Thus $\hat{\Psi}$ can only be zero along the unsteady shock of a single flat plate, or

$$\hat{\Psi} = \Psi$$

for a cascade.

The remaining steps are fairly straight forward. With the conditions shown above, Eq. (19) becomes

$$2 \frac{\partial u_1}{\partial x} + \frac{2ik\ M^2}{M^2 - 1} u_1 = 0$$

or

$$\frac{\partial u_1}{\partial x} = -\ ik \frac{M^2}{M^2 - 1} u_1$$

this can be integrated

$$u_1 = \text{const} \cdot e^{-i \frac{k\ M^2}{M^2 - 1} x}$$

for $x = 0$ we get the perturbation for the leading edge

$$\hat{u}_{1_{LE}} = - \frac{1}{\sqrt{M^2 - 1}} \cdot \hat{v}_{1_{LE}}$$

$\hat{v}_{1_{LE}}$ can be obtained from the unsteady boundary conditions. The final expression for the unsteady velocities behind the shock attached to an oscillating flat plate is now

$$\hat{u}_1\,(x) = \hat{u}_{1_{LE}} \cdot e^{-i \frac{k\,M^2}{M^2 - 1} x} \tag{55}$$

This result was also obtained by Bell (9) and considering the different systems of coordinates, it is equal to the solution found by Teipel in (12).

2.5 The Wake

Fig. 8 shows the characteristic net for the airfoil and wake regions. The fluid in the fields 2 and 11 has to adjust via the two trailing edge shocks in such a way, that the wake condition on the slip-line in the fields 4 and 13 is not violated. This condition requires flow tangency, continuity of pressure and normal velocity across the slip-line, i.e.,

$$\Theta_4 = \Theta_{13}$$

and

$$cp_4 = cp_{13} \tag{56}$$

$$w_{n_4} = w_{n_{13}}$$

w_n is the velocity component normal to the slip-line. Again this problem is split into steady and unsteady parts.

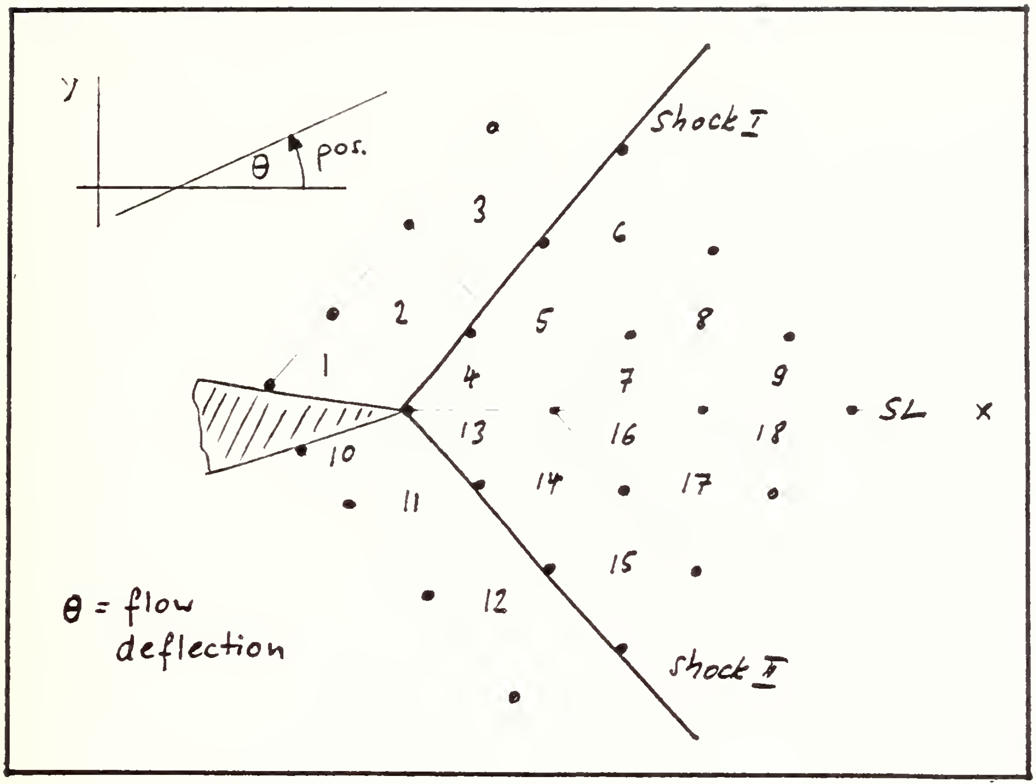


Fig. 8. The Net of Characteristics for the Steady Wake

Shapiro (11) gives a series solution for the pressure difference connected to a direction-change $\Delta\Theta = \Theta_4 - \Theta_2$ for supersonic flow.

$$2\,\frac{p_4 - p_2}{\kappa\, p_4\, M_2^2} = \pm\, C_{Iu}\cdot(\Theta_4 - \Theta_2) + C_{IIu}\cdot(\Theta_4 - \Theta_2)^2 \tag{57}$$

+ left running Mach lines

− right running Mach lines

with

$$C_{Iu} = \frac{2}{\sqrt{M^2 - 1}}$$

$$C_{IIu} = \frac{(M_2^2 - 2)^2 + \kappa M_2^4}{2 \cdot (M_2^2 - 1)^2} \tag{58}$$

The same equation is employed for the fields 11 and 13 where the indices 11 and 13 take the place of 2 or rather 4 in Eq. (57) and (58), resulting in the coefficients C_{IL} and C_{IIL}. Therefore, we need to know M_2 and M_{11}. We can determine M with Eq. (9) and

$$\frac{1}{2} c_p = - \varphi_x = \frac{p/p_\infty - 1}{\kappa M_\infty^2}$$

p/p_∞ can be expressed through the isentropic relations as

$$p/p_\infty = f\ (\kappa, M_\infty, M)$$

with this we obtain

$$M^2 = \frac{2}{\kappa-1} \left[\frac{1 + \frac{\kappa-1}{2} M_\infty^2}{\left[1 - \frac{\kappa}{\kappa+1} (\lambda - M_\infty^2 + 1)\right]^{\frac{\kappa-1}{\kappa}}} - 1 \right]$$

For a given point in which λ is known we can find the flow direction for $\Theta_\infty = 0$ as parallel to the x-axis.

$$\mathrm{tg}\ \Theta = \frac{v_o}{1 + u_o}$$

from Eq. (11)

$$v_o = \frac{2}{3} \frac{\mu}{(\kappa+1) M_\infty^2}$$

and (13)

$$\mu = \mp \left[(M_\infty^2 - 1)^{3/2} - \lambda^{3/2} \right]$$

we arrive through Eq. (25) at

$$\text{tg}\ \Theta = \mp \frac{2}{3} \cdot \frac{(M_\infty^2 - 1)^{3/2} - \lambda^{3/2}}{\lambda + M_\infty^2 \kappa + 1} \tag{60}$$

The upper sign indicates the left-running (= lower surface) and the lower sign the right running characteristic (= upper surface).

Now we have the coefficients C_{Iu} , C_{IIu} , C_{IL} and C_{IIL} . In addition it must be

$$p_4 = p_{13}$$

and

$$\Theta_4 = \Theta_{13}$$

Thus we can say

$$\frac{p_4}{p_\infty} = \frac{1}{2} \kappa M_\infty^2 \left[C_{Iu} \cdot (\Theta_4 - \Theta_2) + C_{IIu} \cdot (\Theta_4 - \Theta_2)^2 \right] + 1$$

$$\frac{p_{13}}{p_\infty} = \frac{1}{2} \kappa M_\infty^2 \left[- C_{IL} \cdot (\Theta_4 - \Theta_{11}) + C_{IIL} \cdot (\Theta_4 - \Theta_{11})^2 \right] + 1 \tag{61}$$

The equality of these two equations gives us a way to solve for Θ_4 . As we took only second order terms of $\Delta\Theta_1$ the result is $\Theta_4 = \Theta_{13} = 0$ for zero angle of attack. This is correct, because we assume isentropic flow across the shocks. The reason why it is done in this rather difficult way is that the upwash of the nonisentropic flow can be easily added to the program by simply adding the third coefficient C_{III} given by Shapiro. Actually, this is already done in the program. It has been reversed through setting $C_{III} = 0$ in order to be consistent with the assumption of isentropic flow across the weak shocks.

After we have determined the flow in field 4 and 13 we compute the trailing edge shocks and the rest of the wake field. The step from the slip-line is repeated from the fields 5 and 14 to 7 and 16 as described, etc. All other steps in the wake field are general characteristic steps like those over the airfoil.

It is now possible to solve for the unsteady flow field of the wake. This is again done on the same grid locations which are known from the steady solution.

The conditions for this part of the solution are

- No Pressure-Jump Across the Slip-Line

$$u_{12u} + ik\,\Psi_{2u} = u_{12L} + ik\,\Psi_{2L} \qquad (62)$$

(see Fig. 10).

- Tangential Velocity on the Slip-Line

$$\frac{v_o + v_1}{1 + u_o + u_1} = tg\,(\varepsilon_o + \varepsilon') \qquad (63)$$

(see Fig. 9)

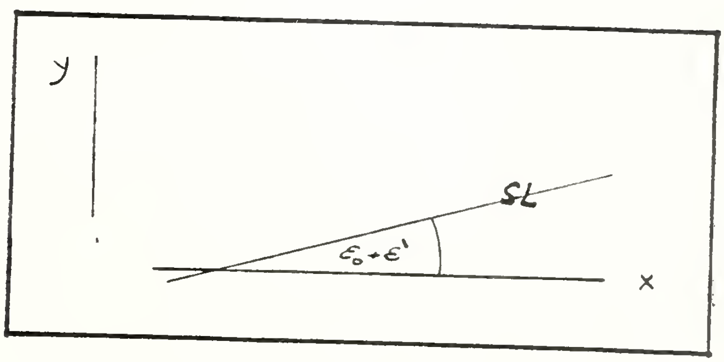


Fig. 9 Slip-Line Geometry

With trigonometric relations Eq. (63) becomes

$$\varepsilon' = v_o - (1 + u_o)\ \text{tg}\ \varepsilon_o + v_1 - u_1\ \text{tg}\ \varepsilon_o$$

from the steady solution we know

$$v_o - (1 + u_o)\ \text{tg}\ \varepsilon_o = 0$$

hence we obtain for the unsteady deflection of the slip-line

$$\varepsilon' = v_1 - u_1\ \text{tg}\ \varepsilon_o$$

as ε' has to be equal for the upper and the lower side of the flow field,

$$(v_1 - u_1 \cdot \text{tg}\ \varepsilon_o)_{2u} = (v_1 - u_1\ \text{tg}\ \varepsilon_o)_{2L} \qquad (64)$$

is the first equation in the system, which we want to solve for $(u_1 , v_1)_{2u}$ and $(u_1 , v_1)_{2L}$. Indices refer to Fig. 10.

The next step uses Eq. (64). Fig. 10 shows the simultaneous step from the upper and lower wake field to the slip-line

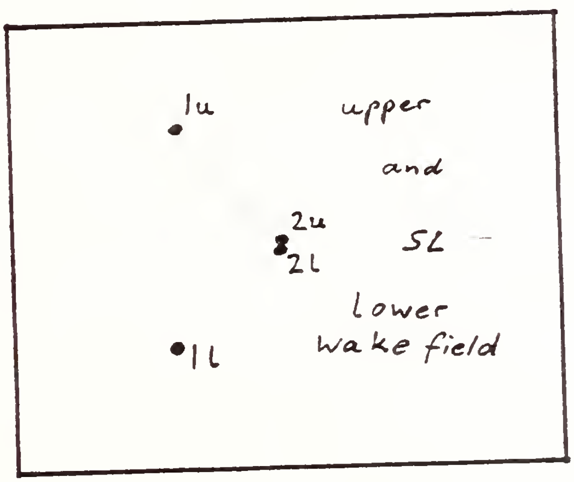


Fig. 10. Slip-Line Step

with

$$\Psi_2 = \Psi_1 + \frac{x_2 - x_1}{2}\left(u_{12} + u_{11} \pm 2\,\frac{v_{12} + v_{11}}{\sqrt{\lambda_1} + \sqrt{\lambda_2}}\right) \tag{65}$$

We can substitute ψ_2 by the known ψ_1 and the desired unknowns. This gives us the second equation of the system when properly recast in left- and right- hand sides.

The two missing statements which make the system solvable comes from the compatibility relation (19). Applied to the upper and lower slip-line steps, respectively, they complete the system.

The computational procedure is explained in more detail in chapter 3.

3. THE SINGLE OSCILLATING AIRFOIL: WING

3.1 The Program Organization Over the Airfoil

The name of the program for the single oscillating airfoil is W I N G. It was designed to use the subroutines as often as possible, that is over the surface as well as in the wake field. For this purpose a special notation was introduced which is shown in Fig. 11 and Fig. 12.

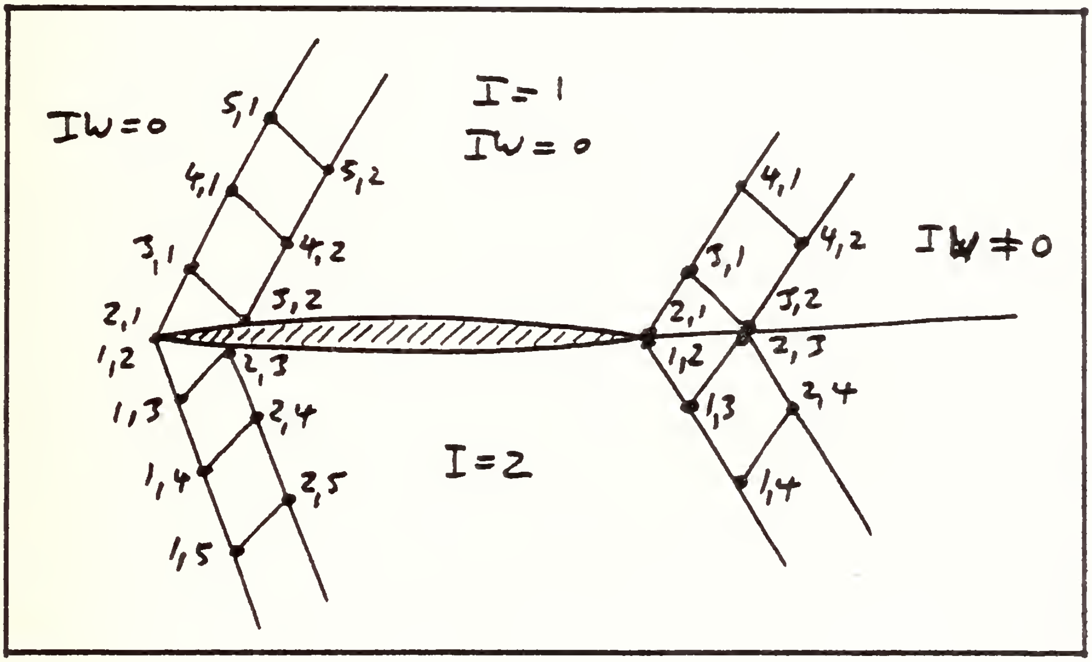


Fig. 11. Notation of the Fields in Wing

The arrays which contain properties of the flow field have the dimension (K , M , N) . M and N give the location of the gridpoint: Point M on the N-th characteristic for the upper field and vice versa for the lower field.

K and IW indicate where we are:

IW = 0	Not yet in the wake
IW = 0	Computation of the wake
K = 1	Field already known
K = 2	Field computed now

The upper and lower side is indicated by I = 1 and I = 2 respectively. Thus we can switch the changing sign of the characteristics with

$$(-1)^{(I+1)}$$

This notation has the advantage that we can compute the field on both sides of the wing (IW = 0) by switching I . For the wake we still can use the subroutines we used over the wing, when we change K . Proper combinations from IW , I , K , M and N cover the whole field. The main diagonal of (M , N) is not used, Fig. 12.

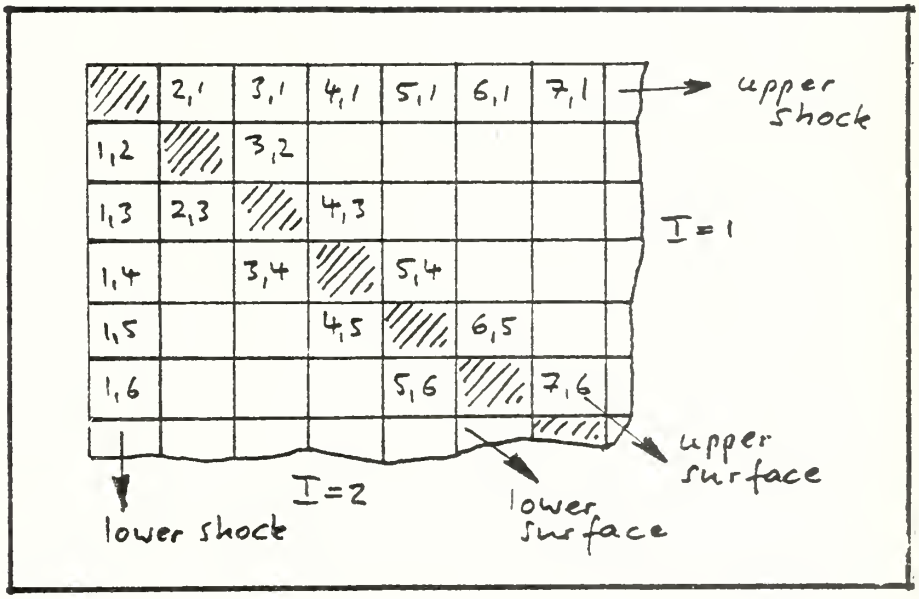


Fig. 12. Array Organization in Wing

Originally for the first shock we need only the Teipel - form of Eq. (2.40). But we have to use the whole expression in our step from the airfoil into the wake because the unsteady velocities over the airfoil are nonzero. So the subroutine R A N D S includes the extended shockpolar. To be consistent in our treatment of the shock at the leading- and trailing edge, an initial field in front of the airfoil is generated by the characteristics with the slope

$$\frac{dy}{dx} = \pm \frac{1}{\sqrt{M^2 - 1}}$$

all perturbation properties are set to zero. During the computation of the flow field over the airfoil (IW = 0) the index K has the meaning

K = 1 Initial field

K = 2 Field over the airfoil

After this calculation step we do not need the initial field any longer and we copy (2 , M , N) to (1 , M , N) . Now the K=2 - array is free for the wake field results and K indicates

K = 1 Field over the airfoil

K = 2 Wake field

In the shown listing of W I N G the arrays have the size (2, 25, 25) to keep the storage area reasonable. However, with 16 points on the airfoil the wake is only computed to a distance of 1.7 times the chordlength from the leading edge. If this is not enough, the program can be blown up by increasing the arrays to (2, KV , KV) .

The value of KV has to correspond with the first statement of the program. All the DO-loops are then dimensioned correctly. Another change has to be done in subroutine L I F T . Here all the arrays stay the same besides A . This is only a dummy-array to define the space between X, U, Q, P and PX, PS, PU . A must have the size

$$A(22 \cdot KV^2 + 4 \cdot KV - 1000)$$

The next page shows an approximate block diagram of the program. On the sides are the names of the subroutines involved in the step of the respective block. This diagram is only a summary of what is really done. A lot of details have been skipped in order to keep it clear. One of the most important routines does not even appear. F I N D is an orientation subprogram that works only on the field (1, M, N) . If the arbitrary coordinates (x,y) are the input of F I N D , it comes out with the indices (M2 , N2) which give us the mesh of the field in which (x,y) is positioned; see Fig. 13.

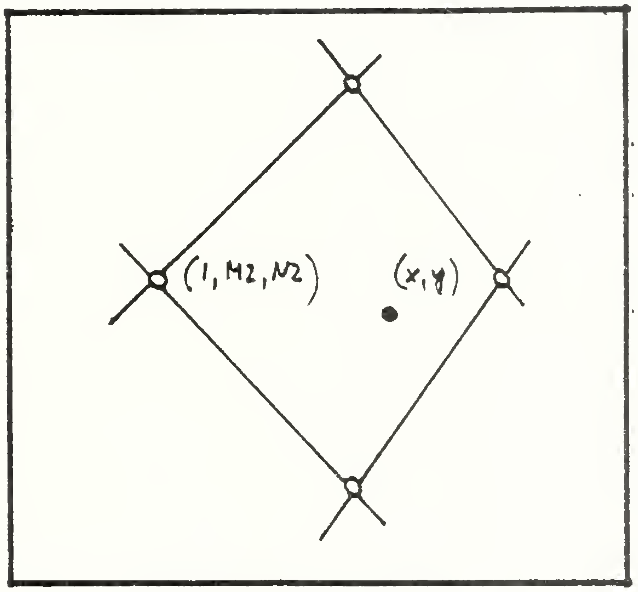
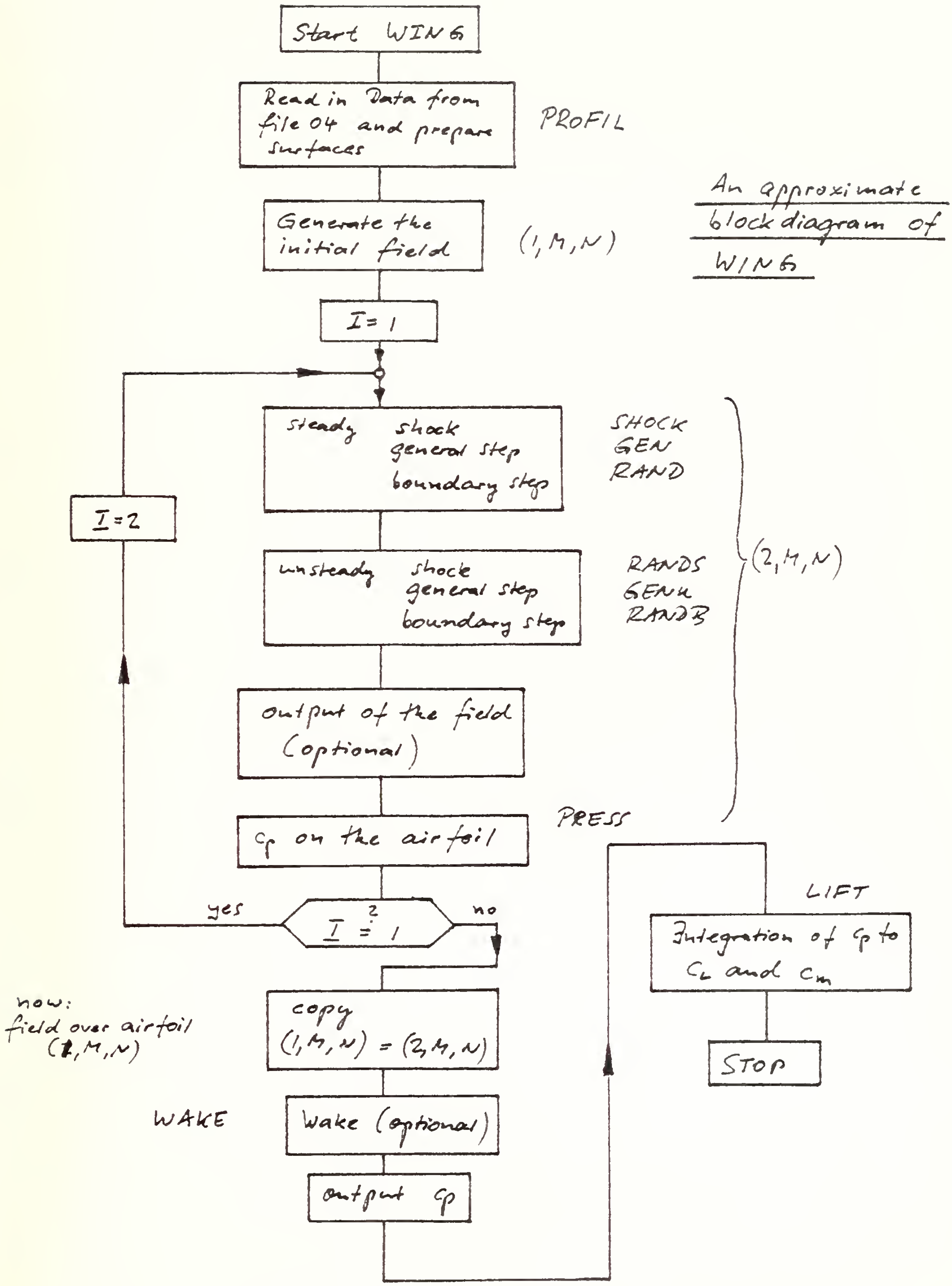


Fig. 13. A Characteristic Mesh

Again I is the switch for upper and lower sides. If (1, M, N) does not contain (x,y) , F I N D indicates this by setting IE = 1 , which is otherwise zero. Besides this there is a message in the output. IE = 1 causes for the LE-shock to assume free stream values in front of it. For the TE-shock it terminates further computations, because there would be no sense in it. If (1, M, N) contains (x,y) and (1, M2, N2) can not be found, this will be in the output also. Then we know, something went wrong. But this should not happen. It is only a precautionary feature.

Another important background - subroutine is S O L V E . During the run through W I N G , complex systems of linear equations have to be solved

TABLE I



AN APPROXIMATE BLOCK DIAGRAM OF WING

several times. This is done by S O L V E . It follows the Gaussian upper/ lower-concept.

In the next sections all examples and equations apply for the upper surface- or wake field. I = 2 switches indices and signs properly for the lower fields.

3.2 Shock Calculations

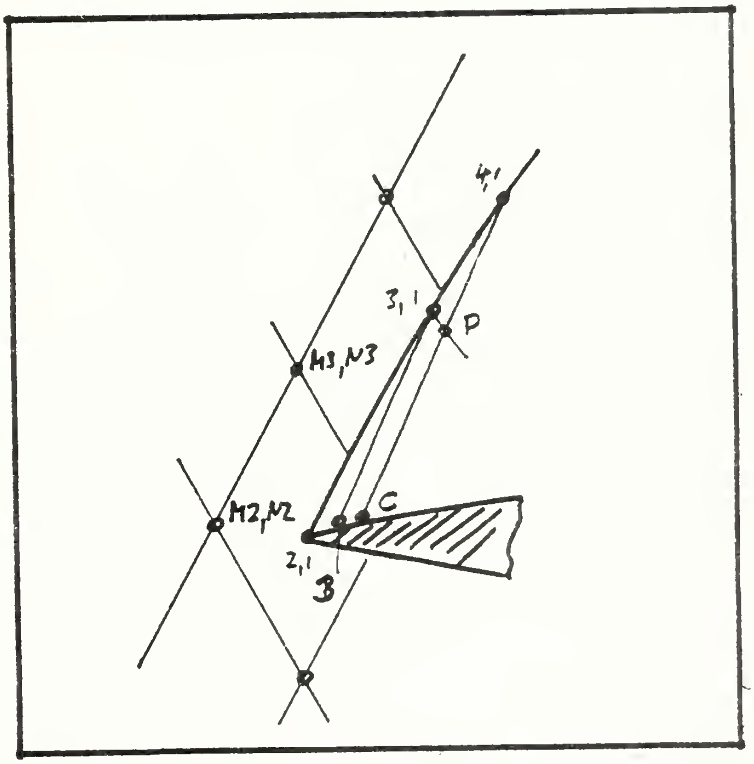


Fig. 14. Leading Edge Shock

The basis for the S T E A D Y shock computations is Eq. (2.13) in connection with Fig. 1. It is done in two steps:

1. Computation of the flow properties in point (2,1) with Eq. (2.16)

$$\lambda_{2,1} = \left[(M^2 - 1)^{3/2} - \frac{3}{2} (\kappa+1) M^2 \left. \frac{\partial h_o}{\partial x} \right|_{LE} \right]^{2/3} \tag{1}$$

which is all we want to know for a point in the steady field. The slope of the shock at the LE is taken as the average of the characteristic slopes upstream and downstream of the shock:

$$\operatorname{tg} \gamma_{o_{2,1}} = \frac{2}{\sqrt{\lambda_{2,1}} + \sqrt{\lambda_{M3,\ N2}}} \tag{2}$$

2. All the other points of the shock are found by the same step done repeatedly:

From the LE we make a step DX(I) to find B (Fig. 14). Employing $\left.\frac{\partial h_o}{\partial x}\right|_B$ in Eq. (2.16) gives us

$$\lambda_{3,1} = \lambda_B$$

and so the slope of the characteristics starting in B

$$\operatorname{tg} \alpha_B = \frac{1}{\sqrt{\lambda_B}}$$

The intersection of the shock with the slope $\operatorname{tg} \gamma_{o_{2,1}}$ and the B - characteristic is the point (3,1). Here the slope of the shock is changed to

$$\operatorname{tg} \gamma_{o_{3,1}} = \frac{2}{\sqrt{\lambda_{3,1}} + \sqrt{\lambda_{M3,\ N2}}}$$

we apply the same procedure for point C respectively (4,1) ; etc. for the whole shock.

The points (M2,N2) or rather (M3,N3) are part of the initial field for the LE-shock . In case of the TE-shock they are part of the field over the airfoil. They are spotted in F I N D .

All this is done in S H O C K .

The unsteady shock can be determined also with two basic steps:

1. The initial step was already described with Eq. (2.43) and (2.44). After this we have $\hat{u}_1$ and $\hat{v}_1$ at the origin of the shock

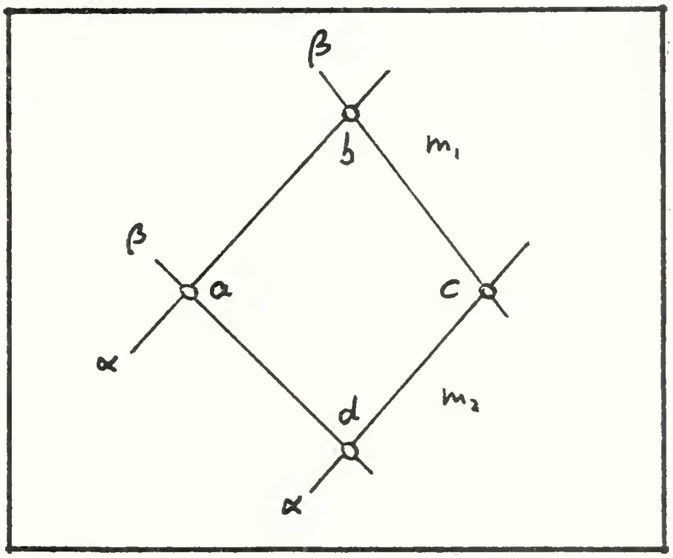


Fig. 15. Computation of λ_x

2. For the remaining points we take the two Eq. (2.40) (Shockpolar) and Eq. (2.19) (Compatibility Relations on the Characteristics Between (3,1) and (4,1) ; see fig. 14).

This gives us a complex system of three linear equations with the solution $\hat{u}_1$, $\hat{v}_1$ and G in each point. The system is solved by subroutine S O L V E . The coefficients of Eq. (2.40) are computed in C O E F F 1 .

However, to express Eq. (2.19) in finite difference form, we have some difficulties with λ_x . Consider Fig. 15.

If we are moving from a to b , $\Delta\lambda$ is zero because we are on a left-running characteristic.

λ_x is in point a

$$\lambda_x\big|_\alpha = \frac{\lambda_c - \lambda_a}{x_c - x_a}$$

through $\lambda_c = \lambda_\alpha$ and approximately $x_c - x_a = 2\,(x_c - x_d)$, we can say

$$\lambda_x\big|_\alpha = \frac{1}{2}\,\frac{\lambda_d - \lambda_a}{x_d - x_a}$$

or

$$\lambda_x\big|_\alpha = \frac{1}{2}\,\lambda_x\big|_\beta$$

We go now back to Fig. 14 and Eq. (2.19). Assuming that setting $\lambda_x(3,1) = \lambda_x(P)$ causes only a small error because both points are very close together, we can express $\lambda_x(P)$ as

$$\lambda_x(P) = \frac{1}{2}\,\frac{\lambda_{4,1} - \lambda_{3,1}}{x_p - x_{3,1}}$$

With this the finite difference form of Eq. (2.19) along the α - characteristic from P to (4,1) is no problem. It is written without the second index. Thus 3,1 becomes 3:

$$\frac{\hat{u}_{14} - \hat{u}_{13}}{x_4 - x_p} - \frac{\hat{v}_{14} - \hat{v}_{13}}{\sqrt{\lambda_4}} + \frac{\hat{u}_{14} + \hat{u}_{13}}{\sqrt{\lambda_4} + \sqrt{\lambda_3}} \cdot \left[\frac{1}{2}\,\frac{\lambda_4 - \lambda_3}{x_p - x_3} + 2ik\,M^2\right] - k^2M^2\,\frac{\hat{\Psi}_4 + \hat{\Psi}_3}{\sqrt{\lambda_4} + \sqrt{\lambda_3}} = 0 \qquad (4)$$

With Eq. (2.64) substituted for $\hat{\Psi}_4$ and the $\hat{u}_3$, $\hat{v}_3$ and $\hat{\Psi}_3$ already known, this is the third equation of the system after recasting into right and left hand sides.

After obtaining the results with S O L V E , $\hat{\Psi}_4$ is calculated separately. For the initial step at the shock origin we set

$$\hat{\Psi}_2 = \Psi_{M2,N2}$$

The procedure for the trailing edge shock is exactly the same. In the boundary values $\operatorname{tg} \varepsilon_o$ is substituted for

$$\frac{\partial h_o}{\partial x}$$

The unsteady boundary values of the field behind the shock are computed in subroutine R A N D S .

3.3 The Boundary Step

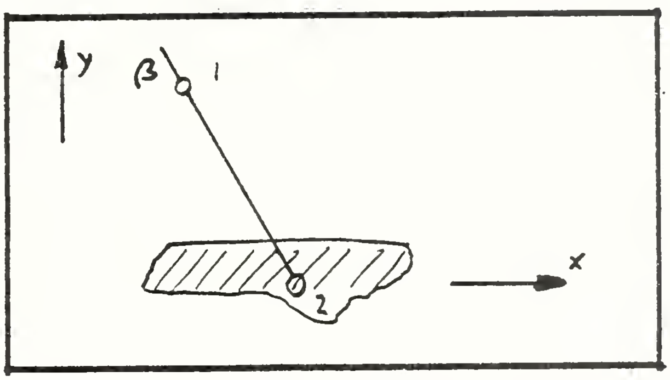


Fig. 16. Boundary Step

Fig. 16 shows the step from the field to the surface. It is done in R A N D . On the β - characteristic from 1 to 2 we have to satisfy two equations: Eq. (2.16)

$$\lambda_2 = \left[\lambda_\infty^{3/2} - \frac{3}{2} (\kappa+1) M^2 \left. \frac{\partial h_o}{\partial x} \right|_2 \right]^{2/3} \tag{5}$$

and

$$\frac{y_2 - y_1}{x_2 - x_1} = -\frac{2}{\sqrt{\lambda_2 + \lambda_1}}$$

To make it easier, we set y_2 to zero and get

$$\lambda_2 = \left(\frac{2(x_2 - x_1)}{\gamma_1} - \lambda_1\right)^2 \tag{6}$$

Setting Eq. (6) equal to (5), we can find by iteration the x_2 which matches both of them. With x_2 we then find λ_2. Here the steady problem is solved. The unsteady part is very straightforward. The known k gives us Ψ_1 this point via the unsteady boundary conditions Eq. (2.21) or (2.22). Now the only unknown left is Ψ_x. It can be separated from the compatibility form. Again Ψ_2 is obtained separately afterwards from Eq. (2.64). The unsteady step is computed in R A N D B .

3.4 The General Step

λ_b and λ_d are known (Fig. 15). From the α - characteristic we know:

$$\lambda_c = \lambda_d$$

Finding the point c for the steady case is only a geometrical problem. It is the intersection of the two lines with the slopes

$$m_1 = -\frac{2}{\sqrt{\lambda_b} + \sqrt{\lambda_c}} \qquad \text{and}$$

$$m_2 = \frac{1}{\sqrt{\lambda_c}}$$

they run through the points b and d . This is done in G E N .

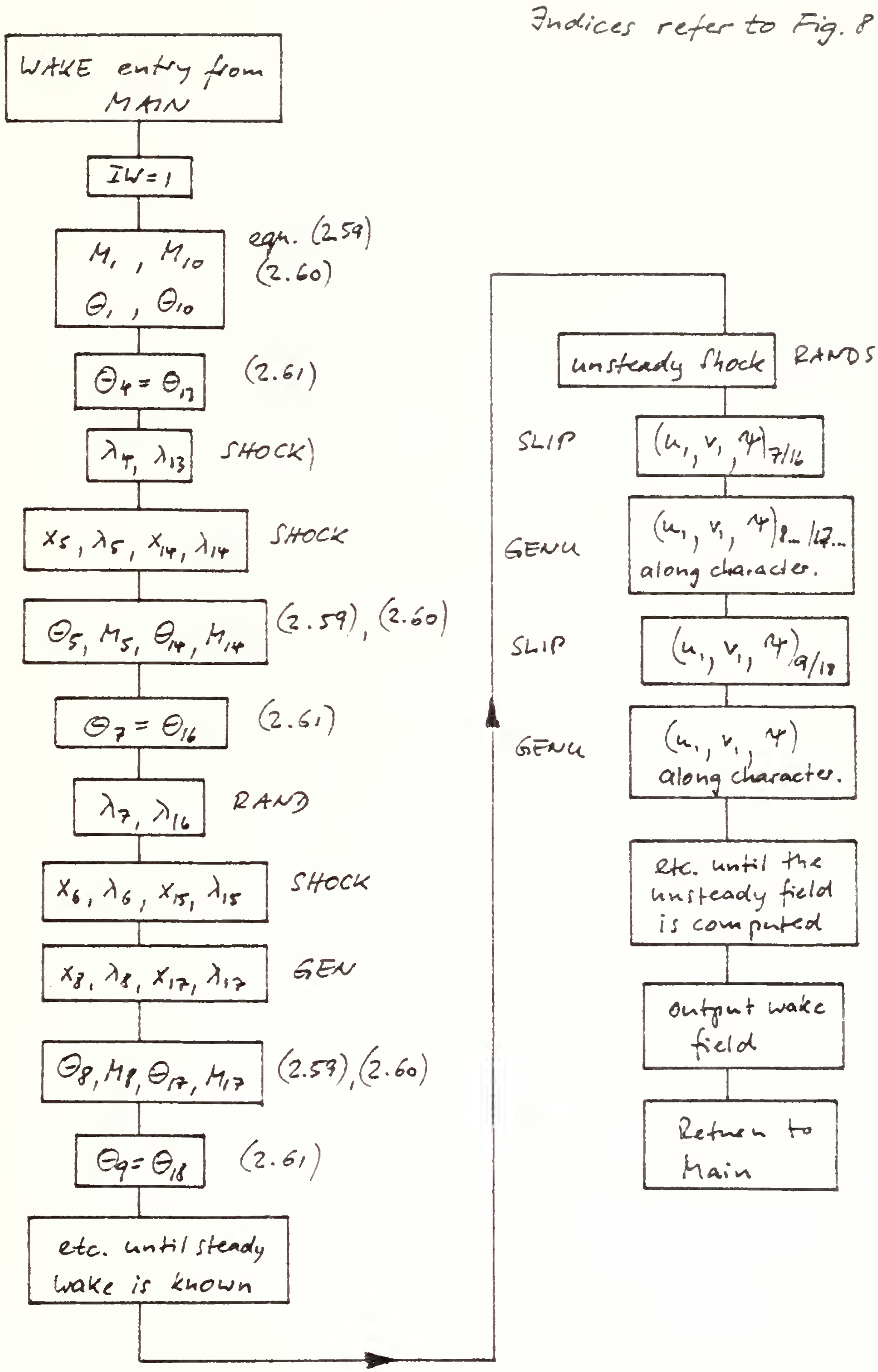
The unsteady part is done in G E N U . Here the compatibility relation (2.19) is applied for both characteristic directions from point b and d to the point c . The unknowns are u_{1c} and v_{1c} . We obtain them as results from a complex system of two linear equations which we form out of the two compatibility relations. The system is solved through S O L V E . As always Ψ_c is evaluated separately from Eq. (2.64).

3.5 Computation of the Wake

For the wake computation W I N G leaves the main program completely and works in a new organization program, called W A K E . This was mainly done to separate the entirely different computation sequence from the organization from the field above the surface. In section 2.5 it was already indicated how to solve for the slip-line and then for the wake field. The wake field computation is shown in the following flow diagram. The indices refer to Fig. 8. Attached to the blocks are the names of the subroutines or the equations involved in the step.

Once the steady field is known, the calculation becomes straightforward again. The conditions directly downstream of the TE are defined by the unsteady shockpolar (2.40) . Applied simultaneously for the upper and lower airfoil-side, they give the conditions for pressure continuity (2.62) and parallel flow on the slip-line (2.64). With these four equations we can solve for $(\hat{u}, \hat{v})_{up}$ and $(\hat{u}, \hat{v})_{lo}$. This is the slightly more difficult initial step for the TE-shock . After it is done, the computation runs along the shock or rather along the characteristics with the same subroutines as over the airfoil. The basic difference lies in the step to the slip-line. Here R A N D B can not be used any longer. S L I P does the simultaneous step from the upper and the lower wake field to the slip-line. The compatibility relation (2.19)

TABLE II



An Approximate Block Diagram for the Wake Computation gives two equations, one for each characteristic direction. Eq. (2.62) demands pressure continuity across the slip-line and Eq. (2.64) parallel flow on both sides. Recasting these expressions in terms of finite differences results in a system of four complex unknowns. It is solved by S O L V E .

3.6 Introduction of a Pointwise Given Surface

The surface of an airfoil is not always analytically given and therefore the possibility of a pointwise given airfoil should be included. This can be done with so called cubic splines. The spline function here used is described in (16). It reads

$$S_i\ (x) \equiv a_i + b_i\ (x - x_i) + c_i\ (x - x_i)^2 + d_i\ (x - x_i)^3$$

$$\text{for} \quad x \in [x_i\ ,\ x_{i+1}]\ ,\ i = 0\ (1)\ n - 1$$

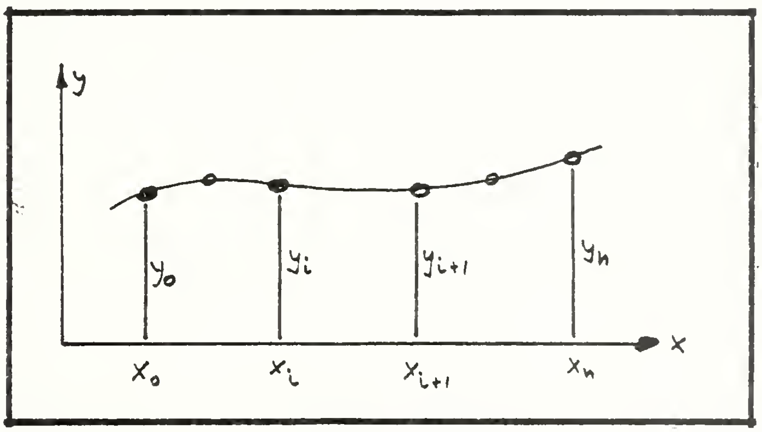


Fig. 17 The Spline Function

The demand for

$$S_i\ (x_i) = y_i \qquad\qquad i = 0\ (1)\ n$$

$$S_i\ (x_i) = S_{i-1}\ (x_i) \qquad i = 1\ (1)\ n$$

$$\left.\begin{array}{l} S_i'\ (x_i) = S_{i-1}'\ (x_i) \\ \\ S_i''\ (x_i) = S_{i-1}'' \end{array}\right\} \qquad i = 1\ (1)\ n-1$$

leads to a system of equations which give for each interval $(x_i\ ,\ x_{i+1})$ the coefficients of the spline $S_i(x)$. With

$$h_i = x_{i+1} - x_i$$

The coefficients can be expressed for known c_i as

$$a_i = y_i$$

$$b_i = \frac{1}{h_i} \quad (a_{i+1} - a_i) - \frac{h_i}{3} \quad (c_{i+1} + 2\ c_i) \tag{7}$$

$$d_i = \frac{1}{3h_i} \quad (c_{i+1} - c_i)$$

The c_i are the solution of the linear algebraic system

$$\begin{aligned} & h_{i-1} \cdot c_{i-1} + 2\ c_i\ (h_{i-1} + h_i) + h_i\ c_{i+1} = \\ & \frac{3}{h_i}\ (a_{i+1} - a_i) - \frac{3}{h_{i-1}}\ (a_i - a_{i-1}) \qquad i = (1)\ n-1 \end{aligned} \tag{8}$$

where c_o and c_n are presumed to be zero:

$$c_o = c_n = 0$$

Once the coefficients a_i through d_i are known, we have a set of polynomials which are excellent for interpolation between x_i and x_{i+1}. As in each x_i not only the value but also the slope and the curve of the two neighboring polynomials are identical, $S_i(x)$ is also applicable to determine $S_i'(x)$ and $S_i''(x)$ of a pointwise given function between those points:

$$S_i'(x) \equiv b_i + 2\, c_i\, (x - x_i) + 3\, d_i\, (x - x_i)^2$$

$$S_i''(x) \equiv 2\, c_i + 6\, d_i\, (x - x_i)$$

In W I N G the subroutine P R O F I L follows two options:

L04 = 1 Airfoil Analytical Given

L04 = 2 Pointwise Given

For L04 = 2 system (8) is solved. With (7) the coefficients can be recreated at any time. Thus it is possible to read in the geometry of any reasonable airfoil and obtain good approximations for position, slope and curve at any station of it. These values are needed for the steady boundary conditions in B O U N D called from R A N D and S H O C K .

3.7 RESULTS

In this chapter W I N G - results are compared with (2), Teipel's linear and nonlinear airfoil results, using the method of characteristic and with Verdon's (18) analytical results for the velocity and pressure distribution over a flat plate.

Figs. 18 through 23 provide a comparison of the Teipel - with the W I N G results. Although it can be said that the linear solutions ($\tau = 0$) generally agree quite well, this is not true for higher Mach numbers, as can be seen in

Fig. 20 for $M = 1.4$. For this case the real part of C_p shows considerable differences distributed toward the TE . For $\tau \neq 0$ there are differences disbuted over all cases. This was already noted by Chadwick (4) and Strada (5) who argued that the deviations could be caused by different difference equations, averaging procedures and number of grid points. However, this could not be proved because Teipel does not expose his complete set of finite difference equations.

This work does not show a comparison with Chadwick's results (4) obtained for airfoils $\tau \neq 0$ and wedges. Nevertheless this has been done. It shows excellent agreement in all cases no matter whether airfoil or wedge. Thus it can be said that W I N G is at least equivalent to Chadwick's solution over the airfoil.

Further W I N G - results are compared with Verdon's work (18), which provides analytically derived results over the flat plate and downstream of it into the wake.

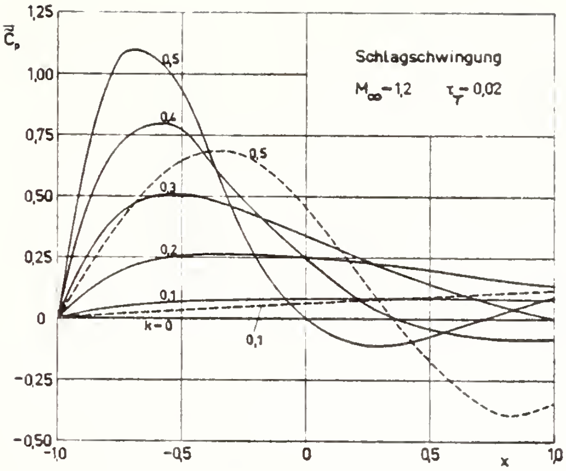
First it should be noted that the C_p - distribution for $0 \leq x \leq 1$ is very good. This can be seen in Fig. 24 and 25. The disagreement for the plunge motion in Fig. 25 looks appreciable only because of the extended scale. The original Verdon plots in this area are rather hard to read because of his desire to show the wake pressure distributions rather than those over the airfoil for the zero upwash. One of his results is that the assumption of $v_1 = 0$ on the slip-line is wrong.

It has to be $C_p = 0$. With this assumption he gets a v_1 - distribution on the slip-line shown in Fig. 26 and 27.

The system of equations used in W I N G does not presume $cp_u = 0$ and $cp_L = 0$ but only $cp_u = cp_L$. It turns out that $|c_p|$ on the slip-line is of the order 0.000001 . The upwash behind the airfoil obtained by W I N G is shown in Fig. 26 and 27 for $1 \leq x \leq 4.6$. For this range the agreement is considered to be excellent for pitch and plunge. The reason for not showing results in the area $x > 4.6$ is storage difficulties in the time sharing system of the used IBM 360/67.

If we proceed from the slip-line into the wake field, Verdon results versus W I N G - results are shown in Fig. 28 and 29. Again the agreement is very good. The total unsteady pressure along characteristics with origin $x = 1.08$ and $x = 2.0$ from the LE is shown. In this way the pressure field over the wake can be represented. Fig. 28 and 29 are very important, as the single airfoil is only the first step to the cascade.

The good agreement with Verdon's results justifies the method of characteristic approach for the problem of the oscillating cascade. The advantage of this approach is the flexibility with which characteristic computational procedures can be designed.



$\tau_T = 2\tau$

Abb. 8 *a*. — Druckverteilung $\tilde{C}_p$ (Realteil) für ein Parabelbogenprofil mit dem Dickenverhältnis $\tau = 0{,}02$ (Parameter : reduzierte Frequenz *k*).

Fig. 18A. From (2)

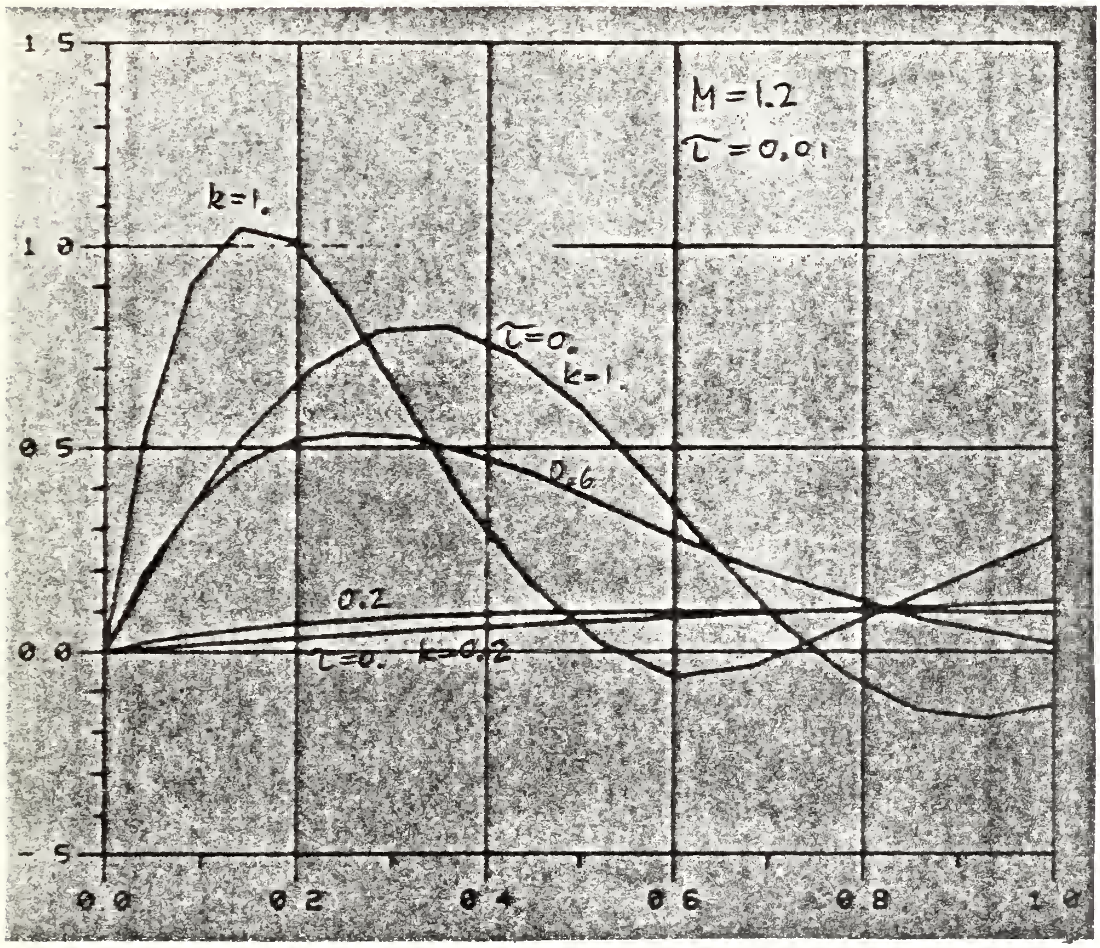


Fig. 18B. Pressure Distribution (Real Part) from W I N G , Plunge Motion

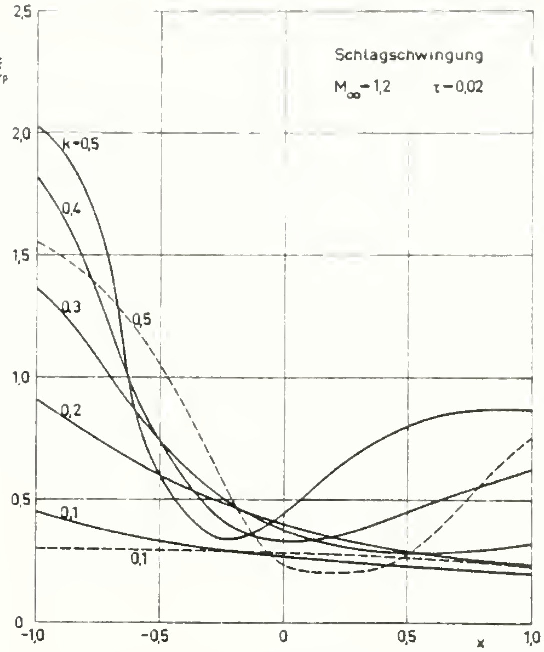


Abb. 8 b. — Druckverteilung $\tilde{C}_p$ (Imaginärteil) für ein Parabelbogenprofil mit dem Dickenverhältnis τ = 0,02 (Parameter : reduzierte Frequenz k).

Fig. 19A. From (2)

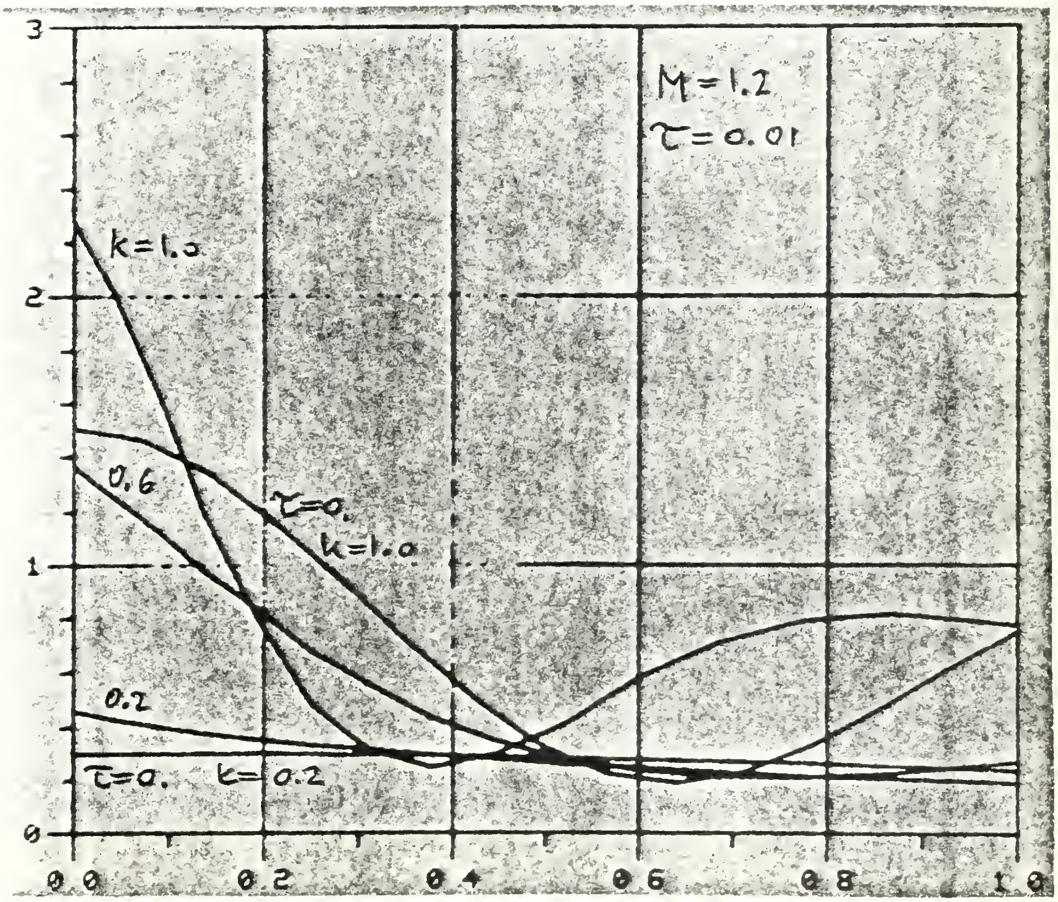


Fig. 19B. Pressure Distribution (Imaginary Part) from W I N G , Plunge Motion

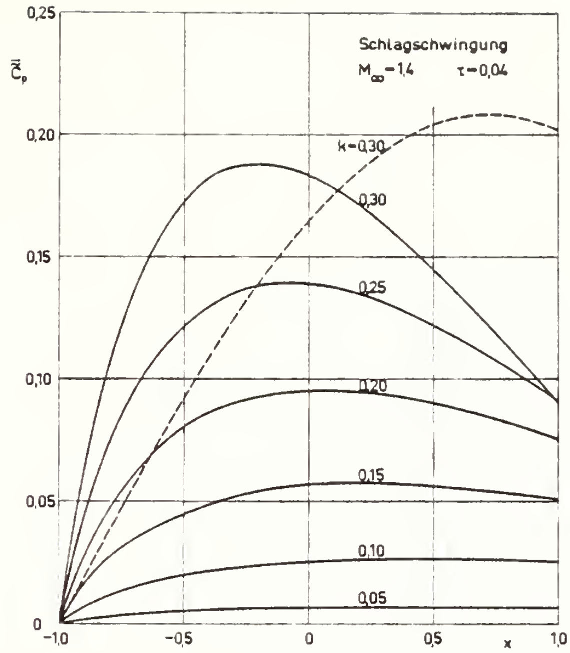


Abb. 9 *a*. — Druckverteilung $\bar{\tilde{C}}_p$ (Realteil) für ein Parabelbogenprofil mit dem Dickenverhältnis τ = 0,04 (Parameter : reduzierte Frequenz k).

Fig. 20A. From (2)

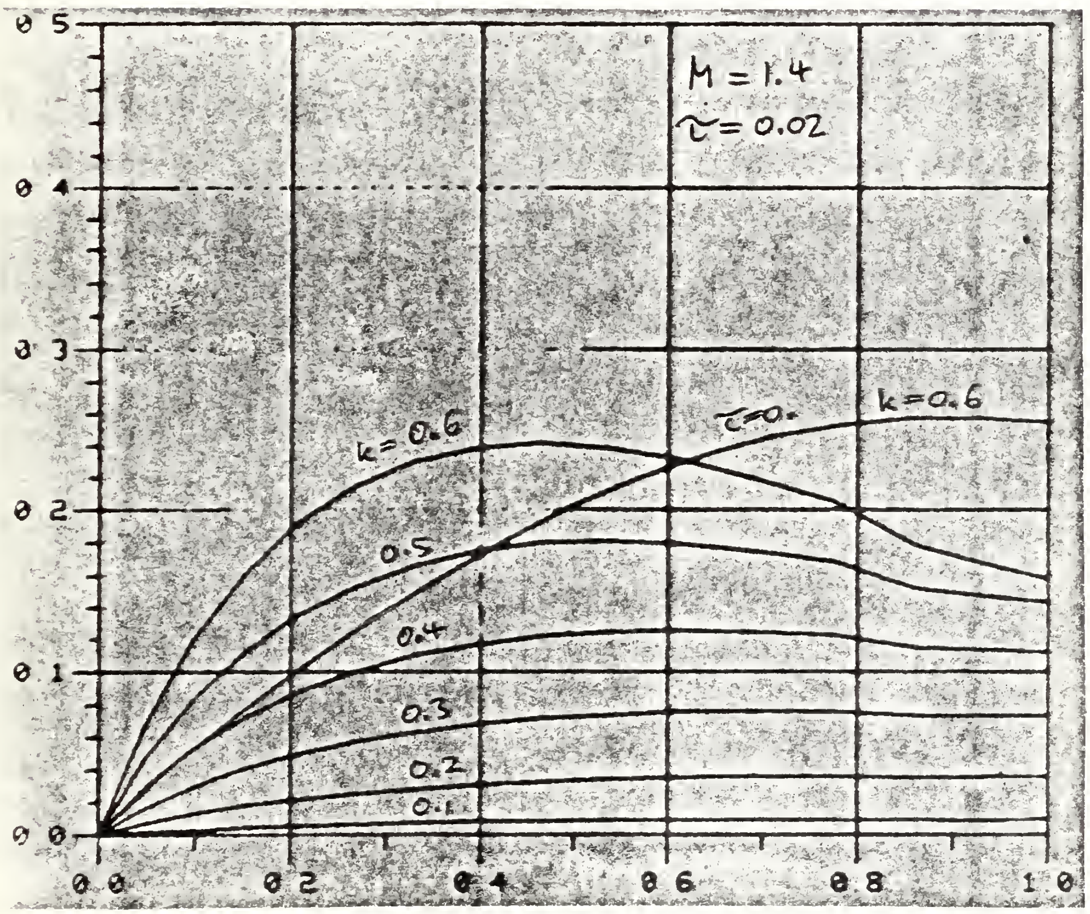


Fig. 20B. Pressure ~~Distribution~~ (Real Part) from W I N G , Plunge Motion

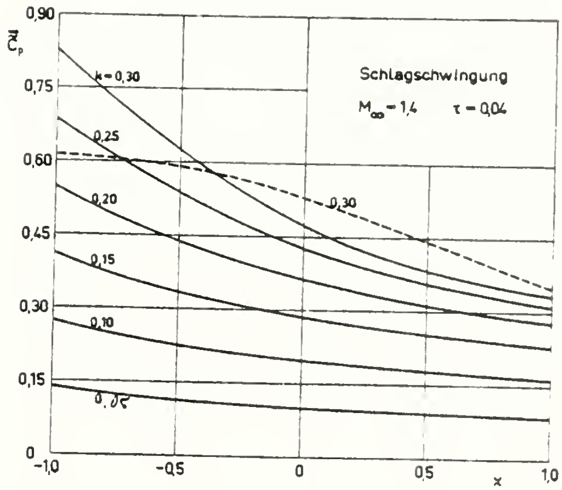


Abb. 9 b. — Druckverteilung $\tilde{C}_p$ (Imaginärteil) für ein Parabelbogenprofil mit dem Dickenverhältnis $\tau = 0{,}04$ (Parameter : reduzierte Frequenz k).

Fig. 21A. From (2)

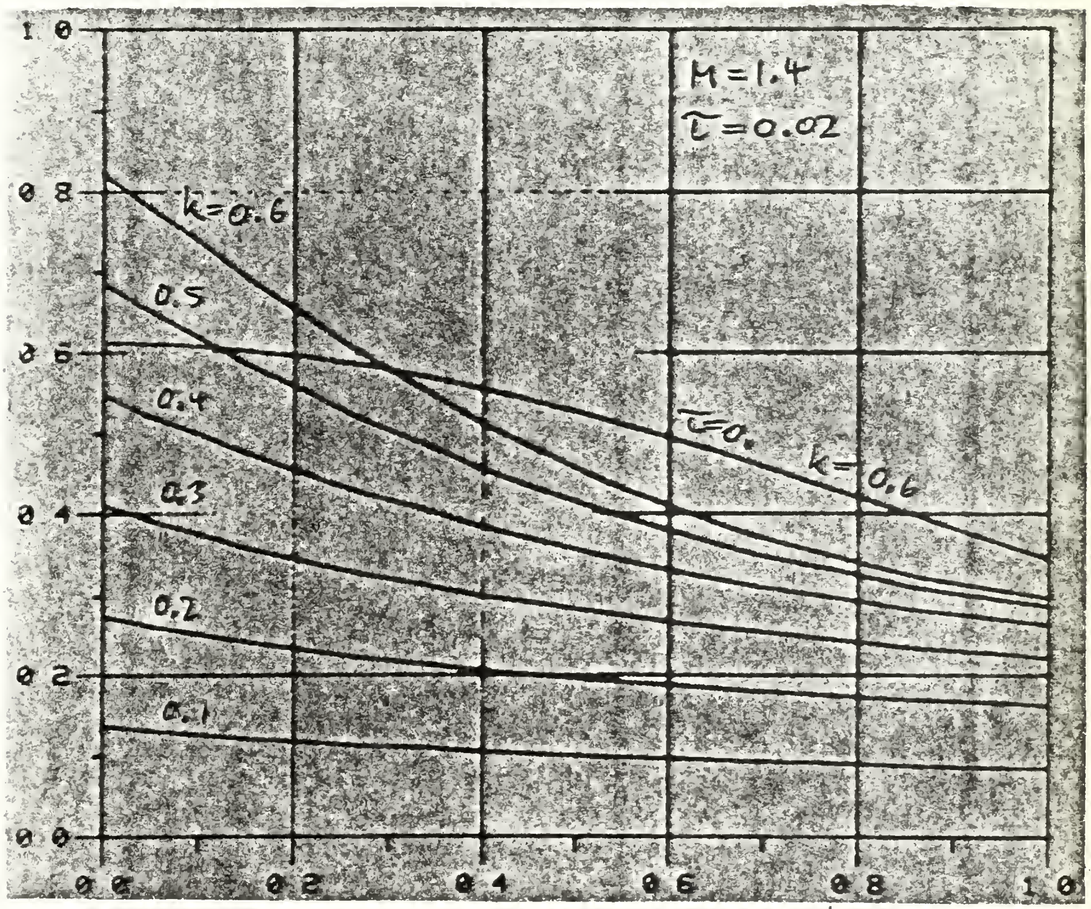


Fig. 21B. Pressure Distribution (Imaginary Part) from W I N G , Plunge Motion

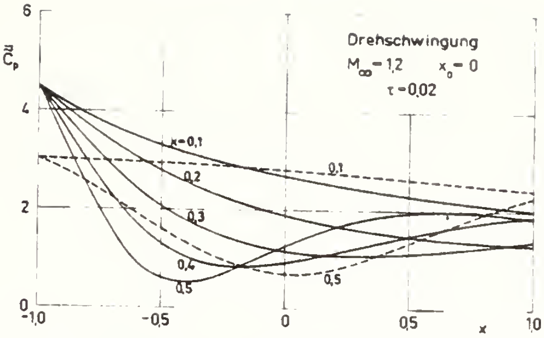


Abb. 13 a. — Druckverteilung $\tilde{\tilde{C}}_p$ (Realteil) für ein Parabelbogenprofil mit dem Dickenverhältnis $\tau = 0{,}02$ (Parameter : reduzierte Frequenz k).

Fig. 22A. From (2)

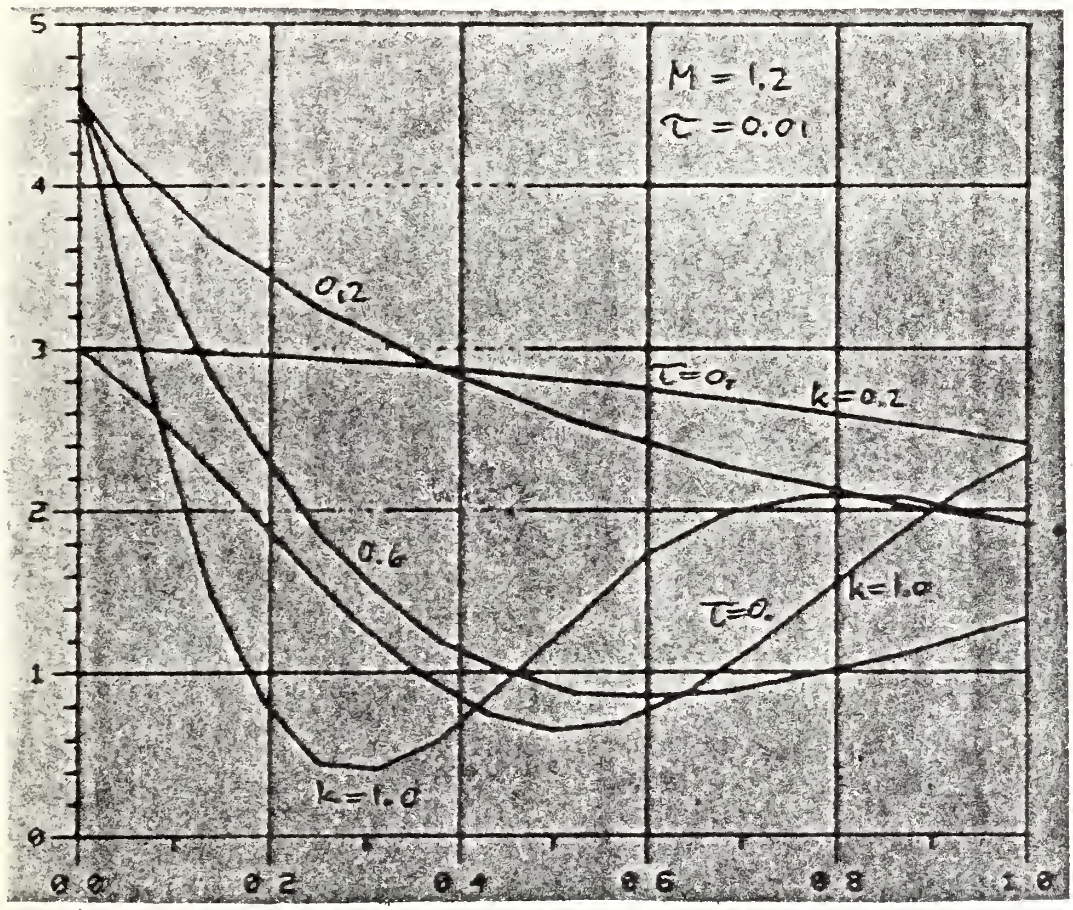


Fig. 22B. Pressure ~~Distribution~~ (Real Part) from W I N G , ~~Pitching~~ Motion

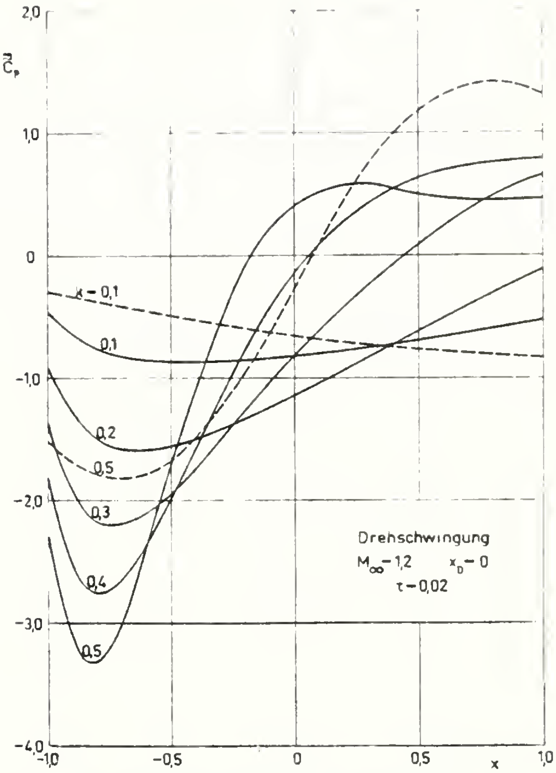


Fig. 23A. From (2)

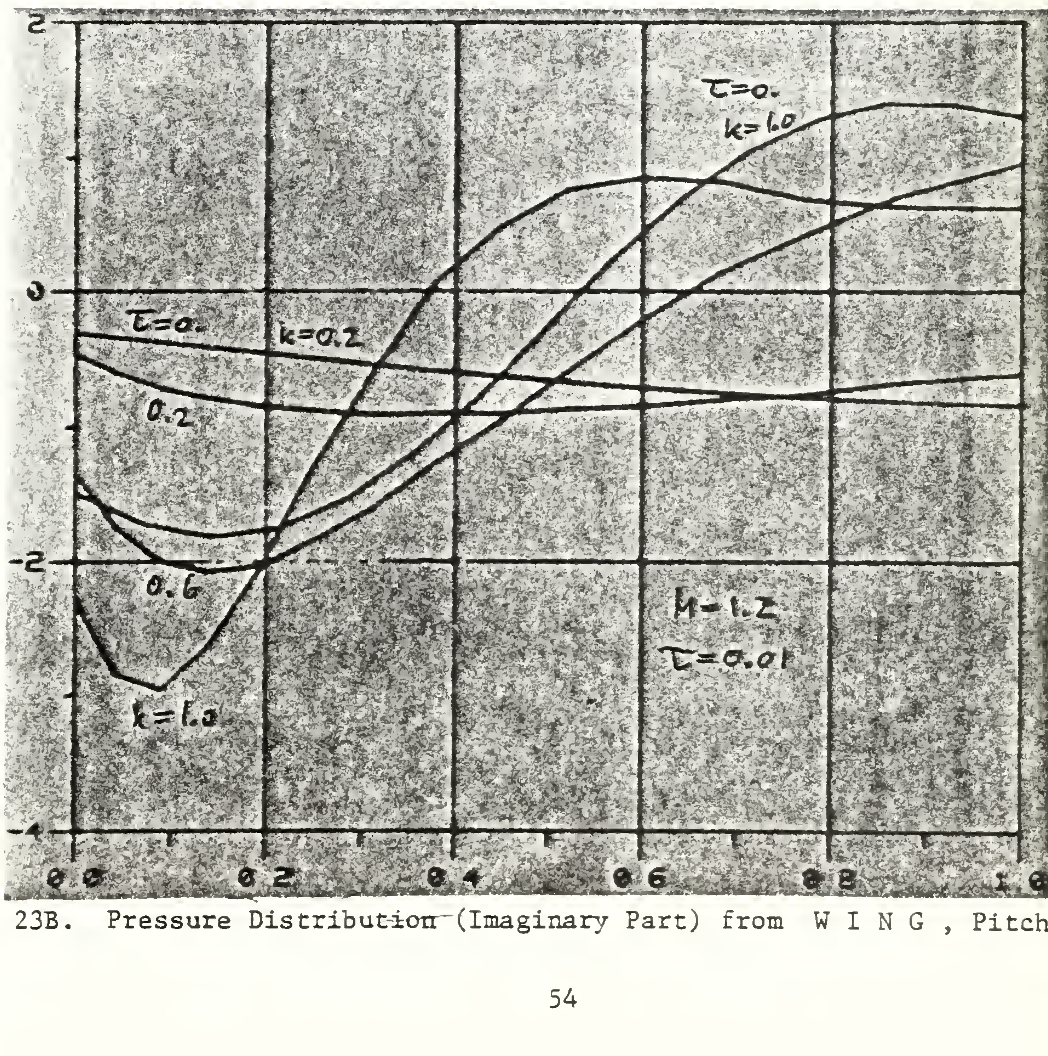


Fig. 23B. Pressure Distribution (Imaginary Part) from W I N G , Pitching Motion

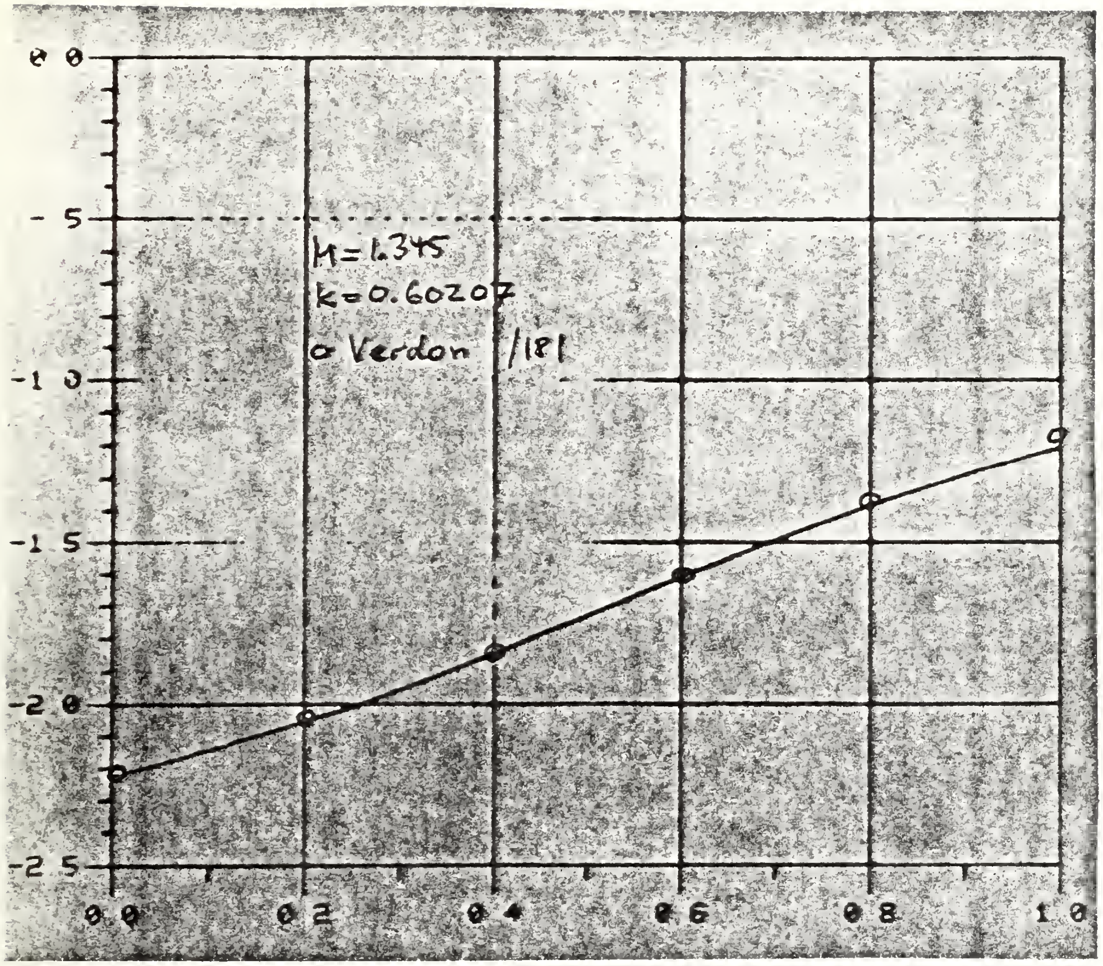


Fig. 24A. Pressure Distribution on Airfoil (Real Part) for Pitching Motion

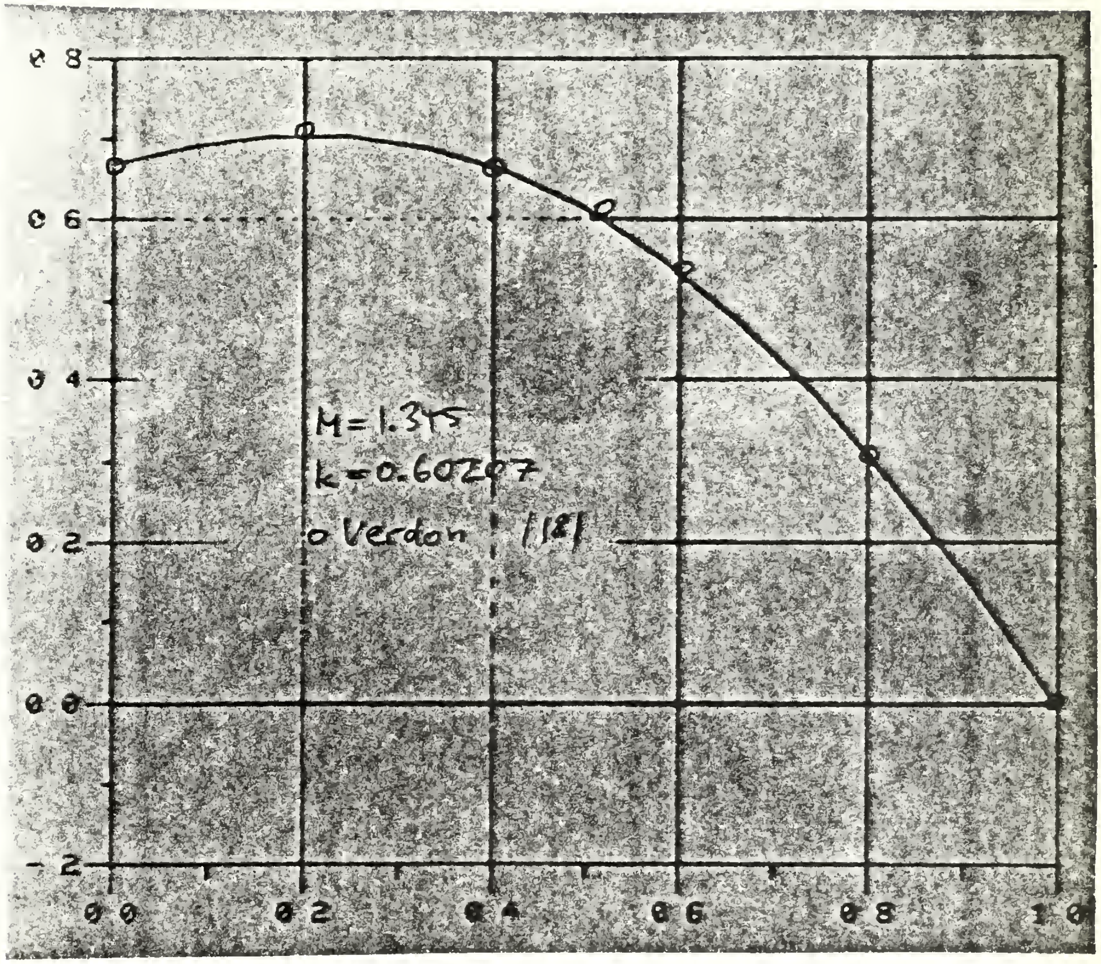


Fig. 24B. Pressure Distribution on Airfoil (Imaginary Part) for Pitching Motion

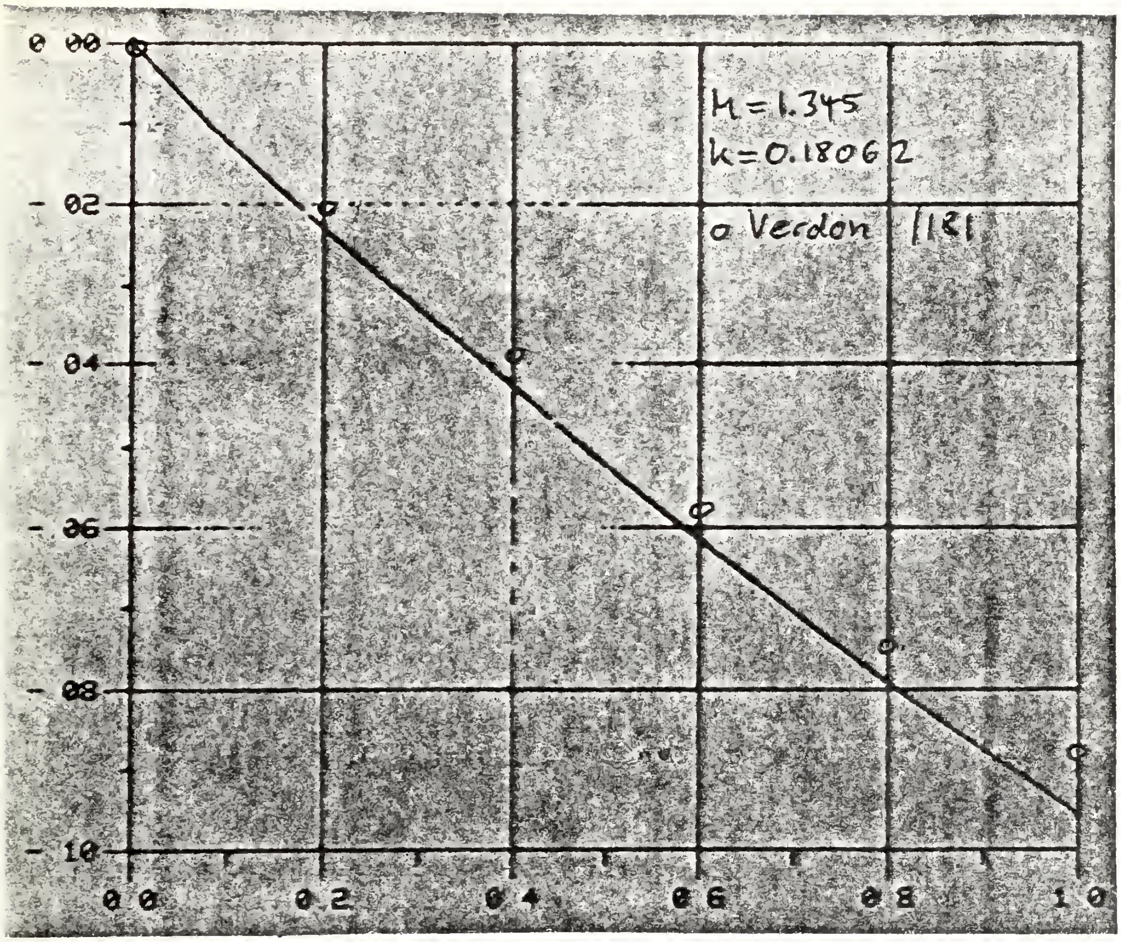


Fig. 25A. Pressure Distribution on Airfoil (Real Part) for Plunge Motion

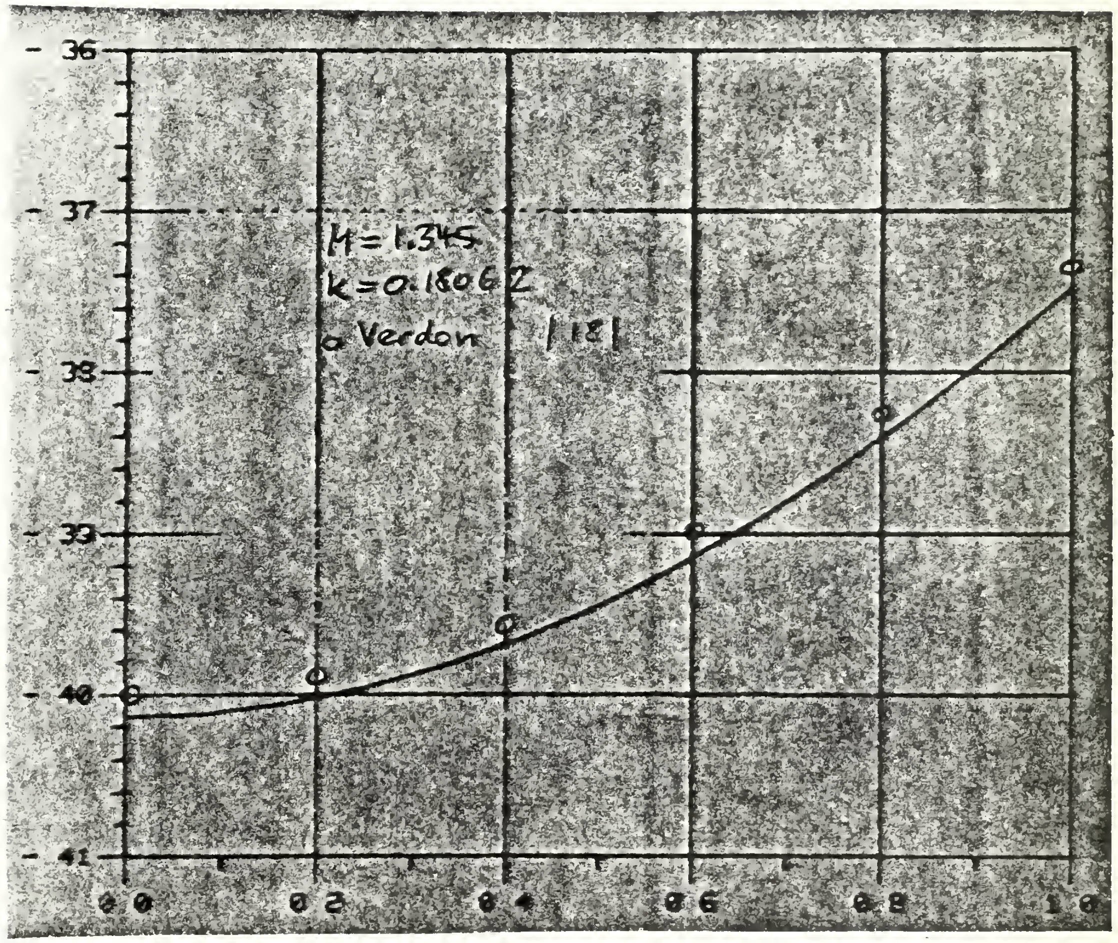


Fig. 25B. Pressure Distribution on Airfoil (Imaginary Part) for Plunge Motion

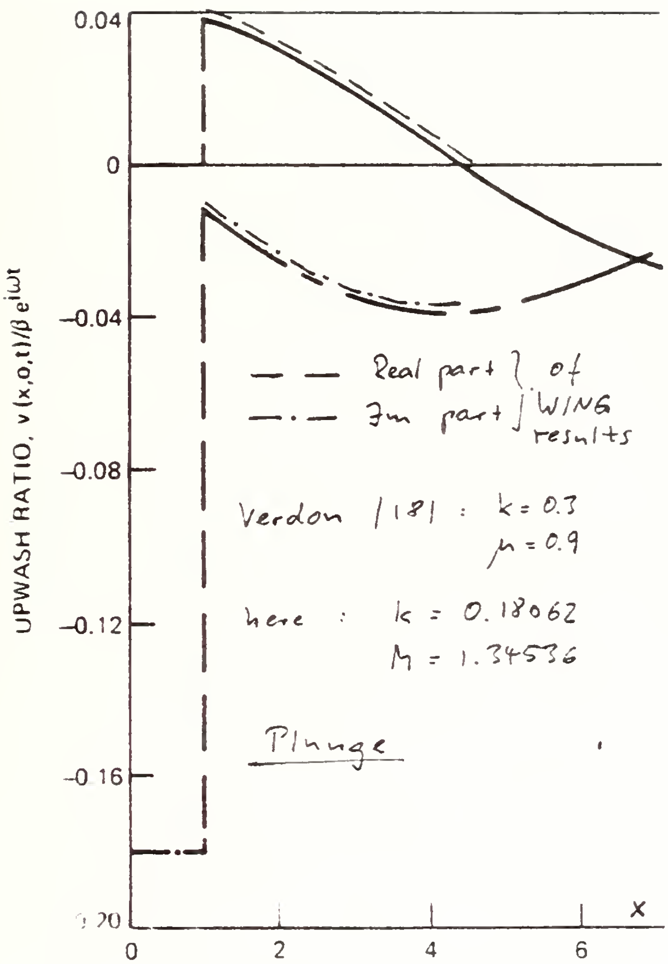


Fig. 26. Upwash on Airfoil and Wake for Plunge Motion from (18) Compared with W I N G -Results

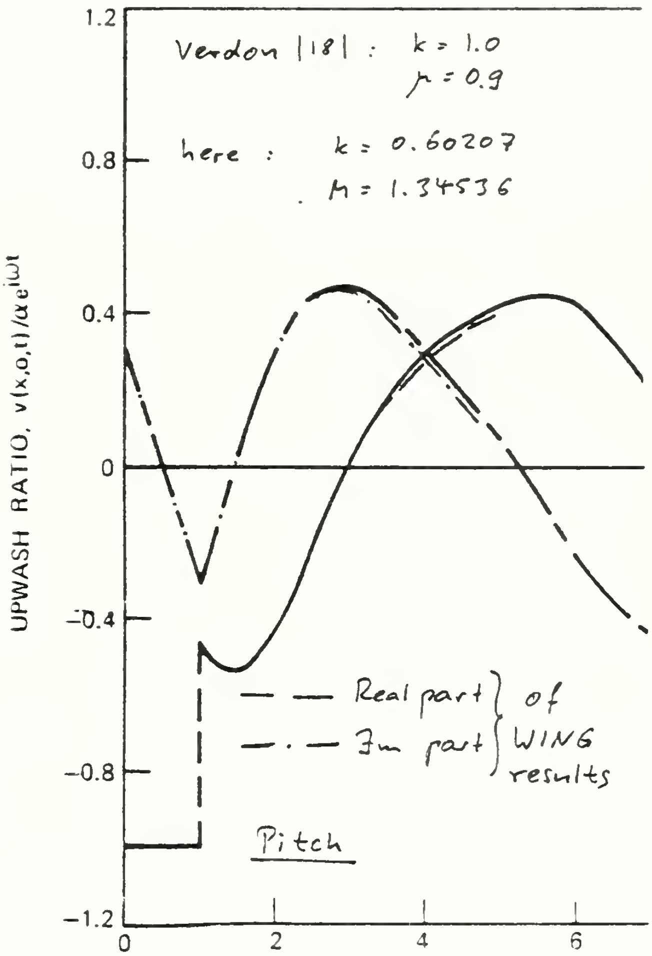


Fig. 27. Upwash on Airfoil and Wake for Pitching Motion from (18) Compared with W I N G -Results

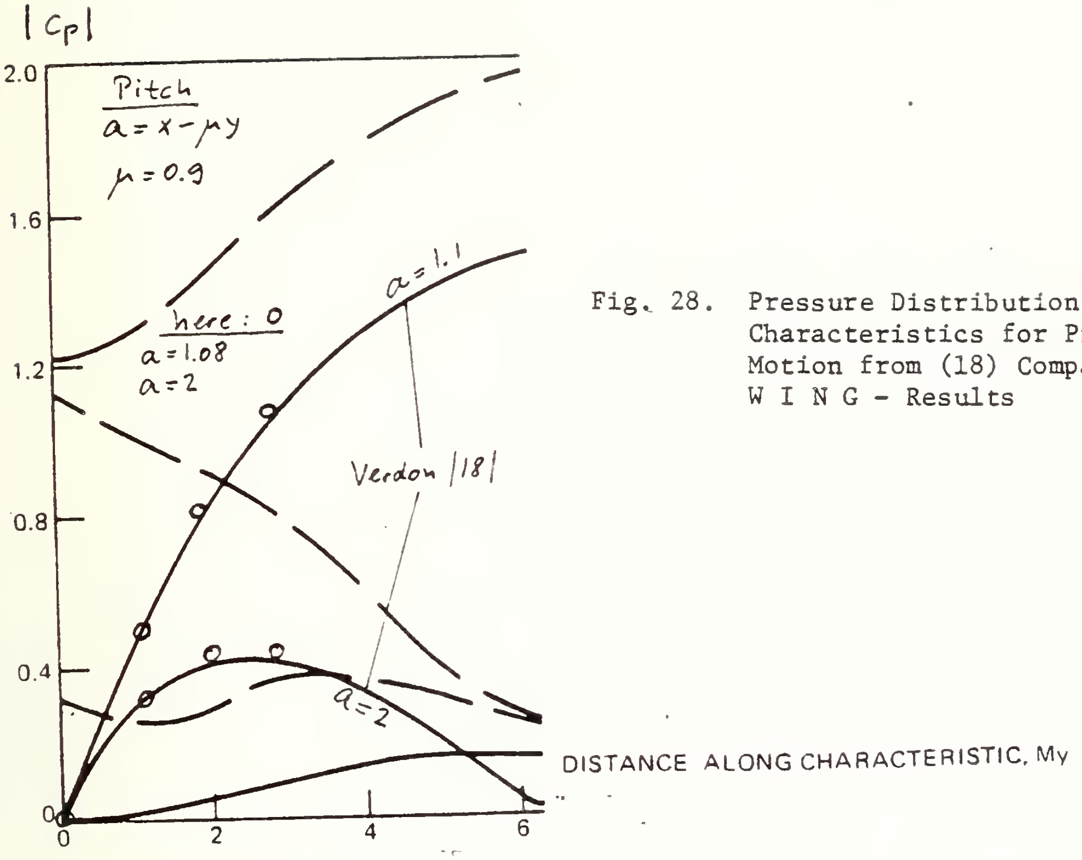


Fig. 28. Pressure Distribution Along Wake Characteristics for Pitching Motion from (18) Compared With W I N G - Results

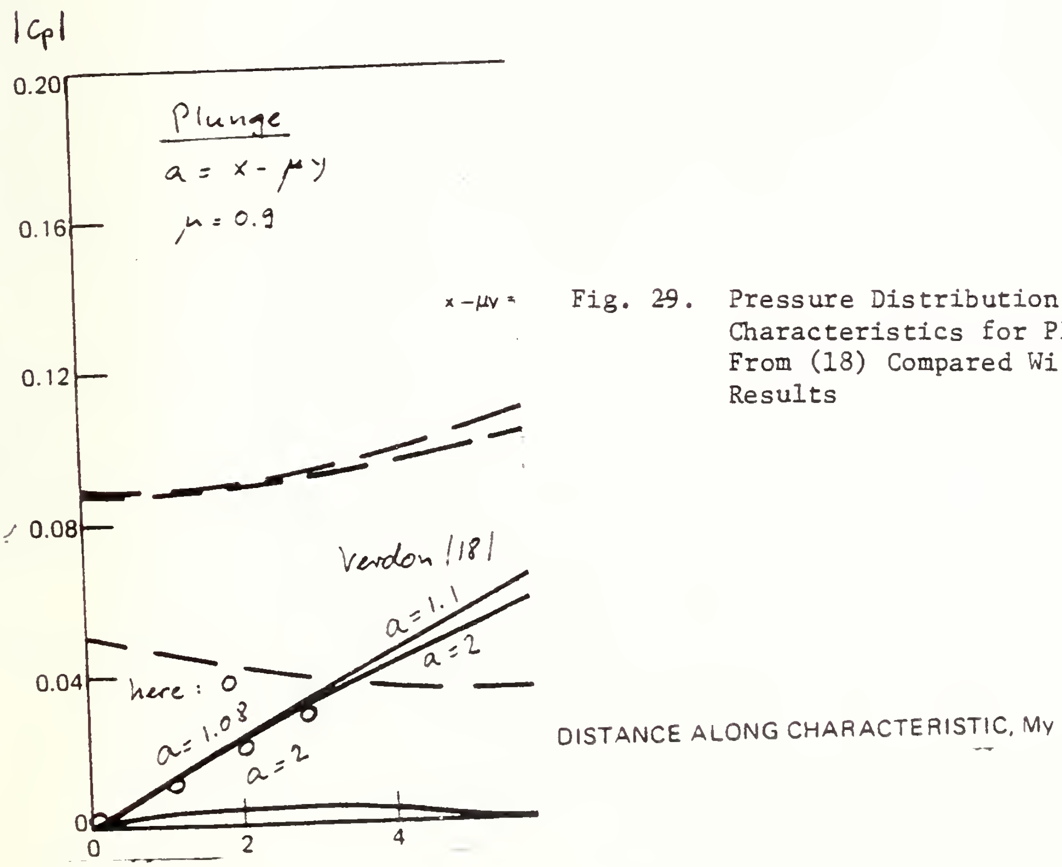


Fig. 29. Pressure Distribution Along Wake Characteristics for Plunge Motion From (18) Compared With W I N G - Results

3.8 Manual, Sample Data, Listing and Output of W I N G

```
CXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
CXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C     OSCILLATING SINGLE AIRFOIL FOR MACH.GT.1
C
      COMPLEX*8 E,V,AI,PSI,G,DYDXU,PU
      COMPLEX*16 EL,RI,ES
      COMMON/BA/ ALO,ALC,AM,C1,AK,I,IE,IW
      COMMON/BC/ T,B,DYDX,D2YDX2,DYDXU,AI,I1,T3
      COMMON/SP/ XS(2,50),YS(2,50),A(50,4),R(2,50)
      COMMON/SCL/ EL(4,4),RI(4),ES(4)
      COMMON V(2,25,25),X(2,25,25),P(2,25),U(2,25,25),PSI(2,25,25),
     FG(2,25,25),AL(2,25,25),Y(2,25,25),G(2,3),DX(2),IN(2,25),
     FPX(2,20),PS(2,20),PU(2,20),TET(2),TEE(20)
C
C     OPTIONS:
C     LO1 =0  : COMPLETE OUTPUT
C         =1  : ONLY PRESSURE DISTRIBUTION
C         =2  : STOP
C     LO2 =0  : PITCH
C         =1  : PLUNGE
C     LO3 =0  : NO WAKE
C         =1  : WAKE SHALL BE COMPUTED
C     LO4 =1  : SURFACES ARE ANALYTICALLY GIVEN
C         =2  : WEDGE (NO WAKE    )
C         =0  : SURFACES ARE POINTWISE GIVEN
C     MA = NUMBER OF POINTS ON THE SURFACE
C
      KV=25
      AI=CMPLX(0.,1.)
102   READ(4,999) LO1,LO2,LO3,LO4,MA
      IF(LO1.EQ.2) GOTO 101
      IF(LO4.EQ.2) LO3=0
      MAX=MA
      CALL PROFIL(WE,LO4,T1,T2,MX)
100   READ(4,1000) AK,AM,C,B,DX(1),DX(2)
      IF(AM.EQ.0.) GOTO 102
```

M A I N   P R O G R A M

```
      GOTO 100
101   K=0
      WRITE(1,1016) K
C
C     END OF MAIN PROGRAM
```

Above is shown the beginning and the final of the main program. The options LO1 through LO3 determine what the program does. LO4 gives the information how the surface is given, needed in P R O F I L . With MA we set how many gridpoin we want to distribute approximately equal spaced over the airfoil. W I N G has a built-in stepsize control, which changes the input-stepsize DX(I) , if it turns out to be too large or too small. However it is a good idea to chose DX correct because the iteration may increase the CPU - time severely, if DX is far away from the proper value.

For that purpose Fig. 30 gives an indication of the order of magnitude of DX for 14 points.

The next read - statements are in P R O F I L :

```
      SUBROUTINE PROFIL(WE,LC4,T1,T2,N)
      COMMON/SP/ XS(2,50),YS(2,50),A(50,4),B(2,5))
C
C     PREPARATION OF THE PROFIL - SURFACES
C
      READ(4,1000) T1 ,T2 ,NT,IKP
      IF(LC4.EQ.0) READ(4,1001) SP
      IF(LC4.EQ.2) READ(4,1001) WE
      DO 8 J=1,2
      IZ=0
      N=NT
      T=T1
      IF(J.EQ.2) T=T2
      IF(LC4.NE.0) GOTO 12
      DO 9 K=1,4
      M=(K-1)*5+1
      READ(4,1001) XS(J,M),XS(J,M+1) ,XS(J,M+2),XS(J,M+3), XS(J,M+4)
      READ(4,1001) YS(J,M),YS(J,M+1),YS(J,M+2),YS(J,M+3),YS(J,M+4)
      IF(YS(J,M+4).EQ.100.) GOTO 7
    9 CONTINUE
    7 CONTINUE
      ETC.
```

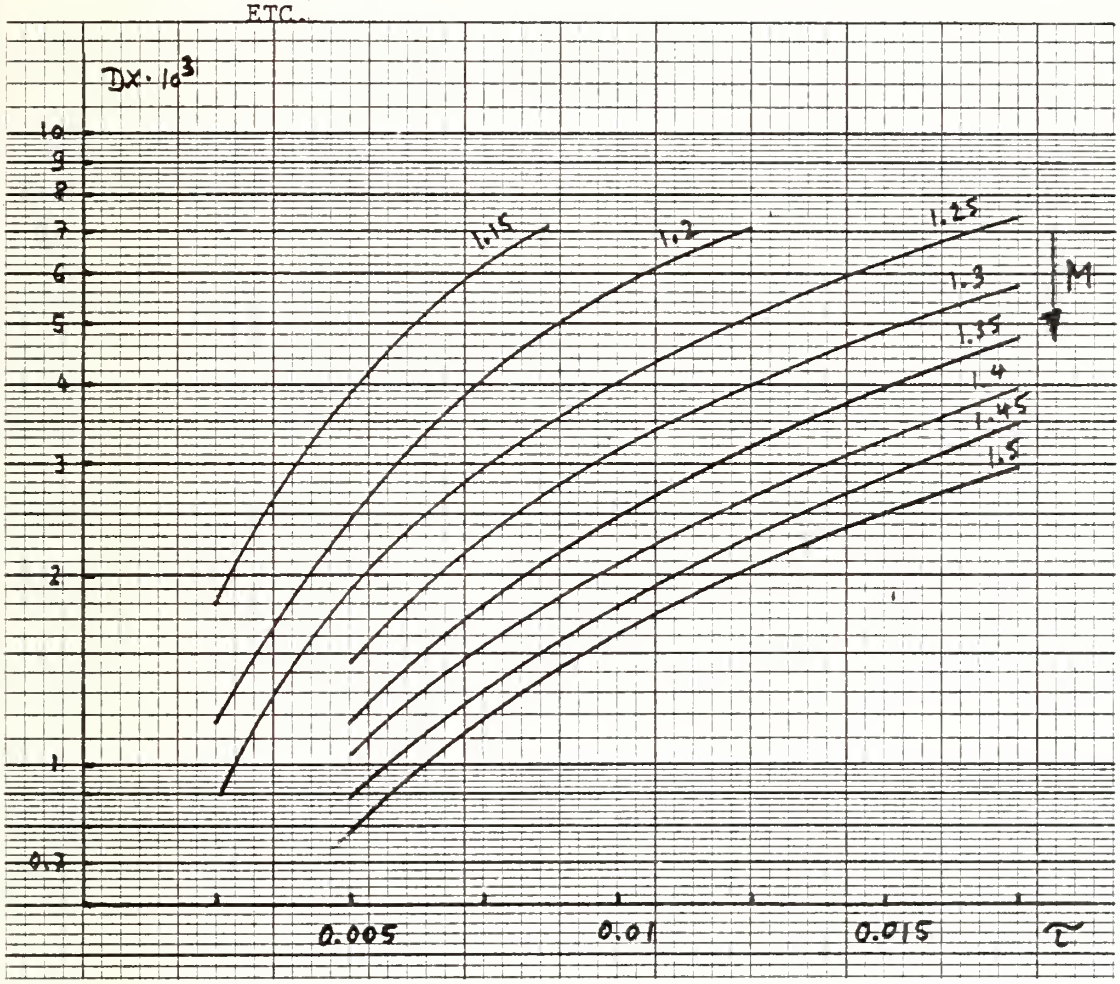


Fig. 30. Stepsize for N = 14 Points on the Airfoil

T1 and T2 represent the thickness of the airfoil. As W I N G is presented here, each surface is given by the parabola

$$y = 4\tau x\,(1 - x)$$

with

T = T1 for the upper

and

T = T2 for the lower side

For L04 = 0 NT determines the number of points on the blade and IKP sets a flag to print out the input data.

IKP = 0 NO OUTPUT OF (X,Y)

= 1 OUTPUT

SP is a scaling factor to make the input data non-dimensional. If the chord is already 1 , then SP = 1 . Otherwise SP is the characteristic length, normally the chord. L04 = 2 changes the airfoil into a symmetric wedge with the slope WE in degrees and unit chord, Fig. 31.

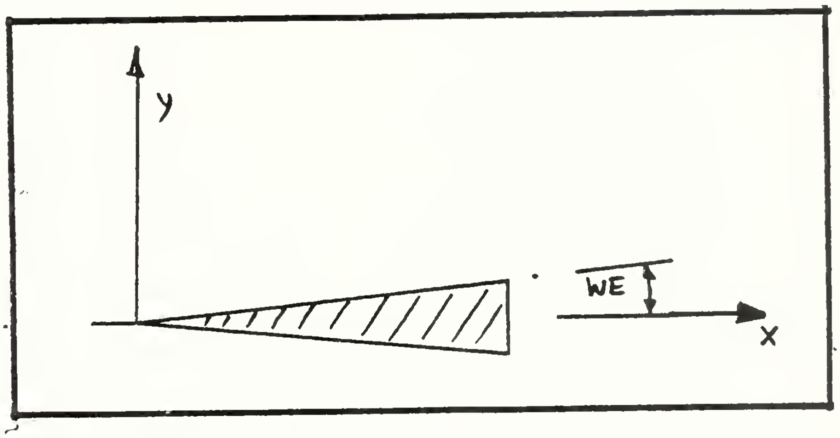


Fig. 31. Wedge Geometry

For L04 = 0 the x,y-coordinates of the airfoil are read in accordance to the format statement 1001. The airfoil shape can be examined prior to entering W I N G in the test program T E S T as shown in chapter 5. T E S T is a preparing program for the airfoil.

If the airfoil is given by another function than Eq. (9), one has to change the statements in B O U N D which compute y' = DYDX and y" = D2YDX2 .

After leaving P R O F I L there is another read for

$$AK = k$$

$$AM = M$$

$$C = ae$$

$$B = x_o = b$$

DX(1) = STEPSIZE ON THE UPPER SIDE

DX(2) = STEPSIZE ON THE LOWER SIDE

If the run is finished, W I N G jumps back to this line and expects changed data for this airfoil. For M = 0 it jumps to the start of the main program for mode and/or airfoil changes. However for L01 = 2 W I N G is finally terminated. This way a whole series of airfoil shapes and aerodynamic conditions can be examined in a single run. It should be noted that L01 = 0 produces a complete field - output which is normally not necessary and rather long.

For L03 = 1 the wake field output can not be suppressed as it is the only information which is printed about the wake. The whole wake may be skipped with L03 = 0 .

Besides the normal output file 06 W I N G has a second one: File 01 . Only the unsteady pressure distribution over the surfaces is written here. This file is used after the run by the plot program P L O T 1 to make plots of the pressure distribution on the Tektronix terminal in the computer center of the NPS. (S. Ch. 5).

W I N G contains still about 150 troubleshooting statements. All important subroutines start with

IKK = 0

The next statement is usually an if-statement converted into a comment card. If this is activated, one can choose here the conditions for a complete tracing of this particular subprogram. Thus

If (IW.NE.0) IKK = 1

would trace this program during the whole wake - computation. One can imagine that a general IKK = 1 will result in irresponsible amounts of paper.

SAMPEL DATA SETS :

AIRFOIL ANALYTICALLY GIVEN:

```
  FILE: FILE        FT04F001 P1                                  NAVAL POSTGRADUATE
  1101  16
      0.01        0.01     20  0
      1.00        1.20         1.4         0.5         0.467E-02  0.467E-02
      0.60        1.20         1.4         0.5         0.467E-02  0.467E-02
      0.20        1.20         1.4         0.5         0.467E-02  0.467E-02
      0.30        0.00         1.4         0.5         0.467E-02  0.467E-02
  1101  16
      0.00        0.00     20  0
      1.00        1.20         1.4         0.5         0.467E-02  0.467E-02
      0.20        1.20         1.4         0.5         0.467E-02  0.467E-02
      0.0         0.0          0.0         0.0         0.0        0.0
  2222  22
```

OPTIONS:

(Refer to W I N G)

- Only Pressure Distribution (1)
- Plunge (1)
- No Wake (0)
- Analytical Surface (1)
- 16 Points on each side

AIRFOIL DATA

For

T1 = T2 = 0.01

M = 1.2

= 1.4

b = 0.5

DX (1) = DX (2) = 0.00467

The program runs through k = 1.0 , 0.6 and 0.2 . After M = 0 we want with the same options the linear case T1 = T2 = 0 for k = 1.0 and 0.2 . Finally we stop W I N G .

For the option-set 1001 the same data would be computed for the pitch mode. The sample output is the result for the first run of this data. Figs. 18b and 19b show the plots of the pressure distribution which we obtain with the data above. Figs. 22B and 23B show the same for the pitching mode.

AIRFOIL POINTWISE GIVEN :

```
FILE: FILE      FTC4F001 P1                              N      P
1000 14
  0.01          0.03       17.0
 100.
   0.0          5.21       11.53        17.84        24.14
  -0.26        -0.185      -0.110       -0.050       -0.005
  30.45        36.75       43.05        49.35        55.65
   0.029        0.056       0.074        0.087        0.097
  61.95        68.25       74.55        80.88        87.21
   0.098        0.095       0.08         0.0516       0.016
  93.53       100.00        0.0          0.0          0.0
  -0.035       -0.10      100.         100.         100.
   0.0          6.29       12.59        18.90        25.21
  -0.26        -1.38       -1.65        -2.12        -2.49
  31.53        37.85       44.18        50.51        56.83
  -2.76        -2.93       -3.00        -2.97        -2.84
  63.15        69.47       75.73        82.06        88.36
  -2.60        -2.29       -1.94        -1.57        -1.17
  94.68       100.00        0.0          0.0          0.0
  -0.70        -0.10      100.         100.         100.
   1.5          1.55        1.4          0.5          0.544E-03   0.454E-02
   0.0          0.0         0.0          0.0          0.0         0.0
2222 22
```

OPTIONS:

- ONLY PRESSURE DISTRIBUTION
- NO PITCH
- NO WAKE
- POINTWISE GIVEN SURFACES
- 16 POINTS ON EACH SIDE

IN PROFIL :

- T1 = 0.01

  NOT IMPORTANT
- T2 = 0.03
- N = 17 POINTS GIVEN FOR EACH SIDE
- IKP = 0 NO READ-BACK OF DATA
- SP = CHORD = 100

This example shows the different input-type for L04 = 0 . The profil thickness T1 and T2 is not important any longer. But the variables have to be defined because they appear in the output. They can be used for identifying purposes.

Here x-location and y-value are given in percent of chord, therefore SP = 100 .

PLUNGE - MODE

W*C/U= 1.000, M= 1.20, K= 1.40, B/C= 0.50, DX= 0.467E-02, T/C= 0.0100

PRESSURE-DISTRIBUTION UPPER SURFACE:

POINT	X	CPS	RCPU	ICPU
2, 1	0.0	0.143E 00	0.0	-0.455E 01
3, 2	0.043	0.128E 00	-0.111E 01	-0.402E 01
4, 3	0.090	0.113E 00	-0.181E 01	-0.326E 01
5, 4	0.140	0.968E-01	-0.209E 01	-0.235E 01
6, 5	0.193	0.807E-01	-0.203E 01	-0.159E 01
7, 6	0.251	0.643E-01	-0.169E 01	-0.957E 00
8, 7	0.312	0.475E-01	-0.123E 01	-0.621E 00
9, 8	0.378	0.304E-01	-0.725E 00	-0.492E 00
10, 9	0.448	0.128E-01	-0.332E 00	-0.604E 00
11,10	0.522	-0.531E-02	-0.381E-01	-0.821E 00
12,11	0.601	-0.239E-01	0.133E 00	-0.116E 01
13,12	0.685	-0.429E-01	0.787E-01	-0.141E 01
14,13	0.774	-0.625E-01	-0.112E 00	-0.158E 01
15,14	0.869	-0.826E-01	-0.297E 00	-0.163E 01
16,15	0.969	-0.103E 00	-0.501E 00	-0.157E 01
17,16	1.000	-0.110E 00	-0.564E 00	-0.155E 01

****

W*C/U= 1.000, M= 1.20, K= 1.40, B/C= 0.50, DX= 0.467E-02, T/C= 0.0100

PRESSURE-DISTRIBUTION LOWER SURFACE:

POINT	X	CPS	RCPU	ICPU
1, 2	0.0	0.143E 00	0.0	0.455E 01
2, 3	0.043	0.128E 00	0.111E 01	0.402E 01
3, 4	0.090	0.113E 00	0.181E 01	0.326E 01
4, 5	0.140	0.968E-01	0.209E 01	0.235E 01
5, 6	0.193	0.807E-01	0.203E 01	0.159E 01
6, 7	0.251	0.643E-01	0.169E 01	0.957E 00
7, 8	0.312	0.475E-01	0.123E 01	0.621E 00
8, 9	0.378	0.304E-01	0.725E 00	0.492E 00
9,10	0.448	0.128E-01	0.332E 00	0.604E 00
10,11	0.522	-0.531E-02	0.381E-01	0.821E 00
11,12	0.601	-0.239E-01	-0.133E 00	0.116E 01
12,13	0.685	-0.429E-01	-0.787E-01	0.141E 01
13,14	0.774	-0.625E-01	0.112E 00	0.158E 01
14,15	0.869	-0.826E-01	0.297E 00	0.163E 01
15,16	0.969	-0.103E 00	0.501E 00	0.157E 01
16,17	1.000	-0.110E 00	0.564E 00	0.155E 01

****

MOMENTUM- AND LIFT - COEFFICIENTS FOR A SINGLE AIRFOIL WITH A SURFACE DESCRIBED ABOVE:

	RCL	ICL	RCM	ICM
UNSTEADY	-3.0585	1.3519	0.2039	-0.2964
STEADY	0.0	0.0	0.0	0.0

4. The Oscillating Finite Cascade: C A S C A D E

4.1 The Physical Difference of the Two Problems

The name of the program, developed to calculate the inlet flow of a staggered finite cascade with thick airfoils is C A S C A D E .

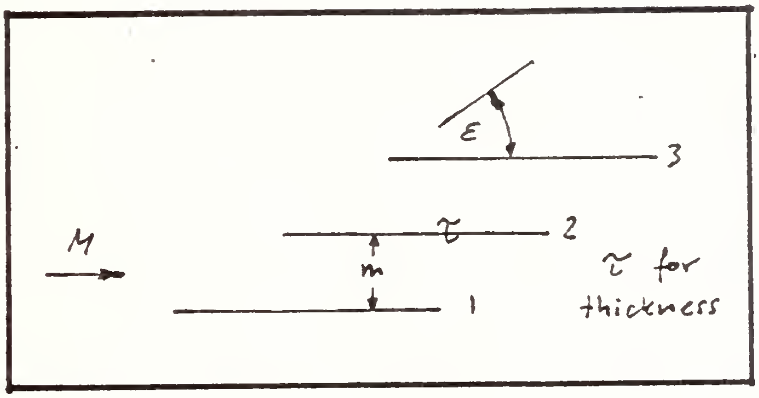


Fig. 32. Staggered Cascade of Airfoils

Nearly all basic steps to simulate this problem are already done in W I N G and therefore described in Chapter 3. However, there are two major differences which have to be considered in its mathematical treatment.

1. Only the first blade is exposed to the free stream flow without any disturbances. This means for the cascade of flat plates already that the perturbations of $(n - 1)$ blades hit the n-th blade and have influence on the development of the flow and the shocks at this airfoil. For the cascade of curved blades there is an additional consequence:

The constant value

$$C_\infty = (M_\infty^{\,2} - 1)^{3/2}$$

from Eq. (2.15) can not be used any longer, as in general the steady flow will not have the free stream velocity.

Similarly, the wake slip line will not be parallel to the x-axis but it will rather have the direction of the field in front of that particular blade. This is the reason why the wake slope in W I N G was not simply set to zero. Because of these difficulties the first approach to the cascade including a thickness effect was made with airfoils whose upper surface were flat. This way the steady field in front of each new blade was identical with the free-stream field (see /5/).

C A S C A D E , however, permits curved lower and upper sides and is able to consider these differences.

2. As in actual turbomachines blade flutter is often observed with a phase lag from blade to blade, the mathematical treatment has to permit this. The phase lag is called μ and has the range

$$-180^\circ \leq \mu \leq +180^\circ$$

which covers all cases.

The introduction of the phase lag can be done relatively easily and is shown in Section 4.3.

4.2 The Constant Value Along the Characteristics

We consider Eq. (2.13) :

$$\lambda^{3/2} \mp \mu = C_{\alpha,\beta}$$

(where in this case μ is not the phase lag)

as long as the incoming characteristics originate in the freestream, μ is zero and therefore

$$C_{\alpha,\beta} = (M^2 - 1)^{3/2}$$

(Fig. 1)

This still holds in a cascade of flat plates, because here the steady field is identical with the freestream field. This is also true for a cascade with blades which have flat upper surfaces.

It is not valid any longer when we permit curved surfaces on both sides, as the steady inlet flow field for the blade ($n \geq 2$) will in general be deflected. Therefore μ on the characteristics is not zero and has influence on $C_{\alpha,\beta}$.

Fig. 33 shows the geometry of an incoming characteristic which is reflected into the cascade.

In A it has to be

$$C_\infty = \lambda_\infty^{3/2} = (\lambda_A^{3/2} + \mu_A)\, B \tag{1}$$

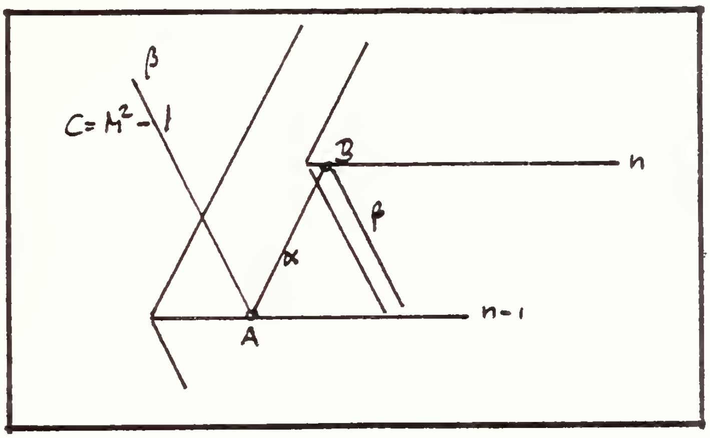


Fig. 33, Reflection of Characteristics

Thus we get λ_A by solving for it, as μ_A is known from the steady boundary conditions.

The next step provides the new constant along the α- characteristic.

$$C_{A\alpha} = \lambda_A^{3/2} - \mu_A \tag{2}$$

This is now the value which has to be applied in B in order to solve for λ_B .

$$C_{A\alpha} = C_{B\beta}$$

$$\lambda_A^{3/2} - \mu_A = \lambda_B^{3/2} = \mu_B$$

$$\lambda_B = [\lambda_A^{3/2} - \mu_A + \mu_B]^{2/3} \qquad (3)$$

with

$$\lambda_A^{3/2} - \mu_A = C_\infty - 2\ \mu_A$$

From Eq. (1) we obtain

$$\lambda_B = [C_\infty - 2\ \mu_A + \mu_B]^{2/3} \qquad (4)$$

In the case of $\mu_A = \mu_B = 0$, Eq. (4) reduces to the flat-plate solution $\lambda_B = \lambda_\infty$.

Eq. (4) contains some useful physical information. The maximum deflection δ_{max} which is connected to each Mach number M cannot be used for the leading-edge-slope of an airfoil in a cascade. Instead it has to be considerably smaller due to the double deflection in A and B . Only in case of a flat upper surface (5), the slope in B could be δ_{max} . The general case, μ_A and μ_B not zero, demands smaller deflections if detached leading edge shocks shall be avoided. This is a necessity according to our earlier assumption of weak shocks.

The calculation of the particular constant value, valid along the respective considered characteristic is done in C O N S T 1 .

4.3 The Phase Lag

Each time the unsteady properties u_1 , v_1 or Ψ are computed, the result is always the amplitude of the oscillating function

$$F(x,y,t) = A(x,y) \cdot e^{ikt}$$

Therefore, if we consider the two neighbouring blades (n-1) and n , where n leads the oscillation with the phase lag μ , we can express this for the time t as follows

$$F_n = A_n \cdot e^{ikt}$$

$$F_{n-1} = A_{n-1} \cdot e^{i(kt-\mu)}$$

For the single airfoil we did not need to write the exponent expression because it was always a common factor.

We reconsider this. For an example we take Eq. (2.40) which is the used unsteady shock polar. The first of those two formulas reads now, if written explicitly

$$\hat{u}_1 \cdot e^{ikt} = (m_1 G_y + i\, m_2 G) \cdot e^{ikt} + (m_3 u_1 + m_4 v_1)\, e^{i(kt-\mu)}$$

$$\hat{u}_1 = m_1 G_y + i\, m_2 G + (m_3 u_1 + m_4 v_1)\, e^{-i\mu} \qquad (5)$$

We see that the phase lag does not cancel out. If we do this for all the equations used in W I N G , we find as a general rule: the term exp (ikt) always can be dropped. Each time when properties of the previous blade appear in the equation, they have to be multiplied by exp(- iμ) . Thus we reduce the magnitude of the (n-1)st -blade-values with the phase lag to the actual size they have when they influence the nth -blade-properties. These are the unknown amplitudes of the oscillating functions connected to the nth -blade.

Eq. (5) gives an example how the equations change from W I N G to C A S C A D E . It is not necessary to show the whole set of finite difference formulas because they can be generated simply by using the concept outlined above.

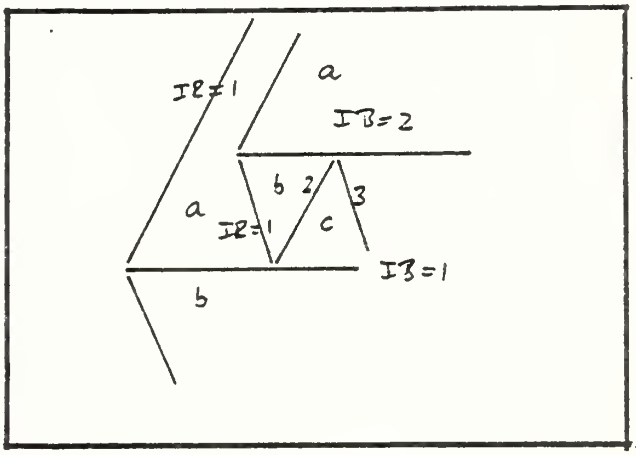


Fig. 34. Reflected Shocks in the Passage

The subroutines for the general and the boundary step can be taken as they were used in W I N G for the unsteady field. R A N D S is changed in so far, as the phase lag has to be added to the system of equations. The steady field is not affected.

For the field behind the reflected shocks in the passage one has to be careful, see Fig. 34.

It has not only to be considered that the oscillation of field b is ahead of that in field a , according to the phase lag μ . But also it is ahead of field c , which is again connected to the movement of blade 1 . This is done by reversing the sign of μ in R A N D S for this case.

4.4 The Organization of the Program

As the computation of the reflected shocks and the fields behind them in the passages of a cascade is considered a rather complex procedure, the goal for the design of the program was to keep it as straightforward as possible. Therefore the convenient array organization from W I N G was adopted and extended. The main variable fields have the form

X(IB,IR,M,N)

The part (...,M,N) corresponds to Fig. 11 and 12. It allows the separation of fields on the upper and lower surfaces. IB indicates the blade, which is connected to the respective field and IR counts the shock in the passage, Fig. 34. This notation allows again repeated use of the same routines when only IB and IR are set correctly. Again throughout the whole program upper and lower sides are indicated by I=1 and I=2 . The index I is used to control the M,N-notation and the sign like it was done in W I N G .

In C A S C A D E there appear three different kinds of flow fields which are shown in Fig. 34.

Type a is the field over the whole upper surface. The procedure is here the same as in W I N G . Computation of the b -field follows also W I N G , but it is stopped when the shock crosses blade 1 . At this point the shock is reflected and the main difference in the subroutines is a transformed system of coordinates, because the origins of the shock and the system of coordinates have to be identical. This transformation causes considerable changes of the statements in S H O C K and F I N D , compared to their counterparts in W I N G . However, the computational sequence is not really changed, but the extension of those subroutines enables them to identify the type of the field and to make all necessary changes for signs, coordinates and termination points.

C A S C A D E starts with blade 2 (IB=2) exposed to an initial field where all perturbations are zero. After evaluating the complete upper field, the b -field is computed. This is terminated when the shock hits blade 1 , which is assumed to be a fixed flat plate for this first step. So here blade 1 could be considered a wind tunnel wall. When all the reflections in this first channel are done, the IB=2-field is copied to the IB=1-field . Now the former can be used again. From this point on blade 1 is an oscillating airfoil with the given shape.

C A S C A D E has three output files:

- File 07 for complete field output and/or the final pressure distribution
- File 06 is a documentation which allows to follow the iterations and the main steps from blade to blade. Like W I N G , C A S C A D E contains trouble shooting statements which can be activated by IKK=1 for the desired subprogram. For a general IKK=0 File 06 will be limited to a few pages. IKK=1 will print a complete tracing of the particular routine, which increases File 06 significantly.
- File 01 contains geometric data of the cascade and shock configuration. It is used as an input file for P L O T 2 which produces plots like Fig. 35 on the Tektronix - terminal. P L O T 2 is described in Section 5.

The input data have to be in File 03. They are shown in Section 4.6.

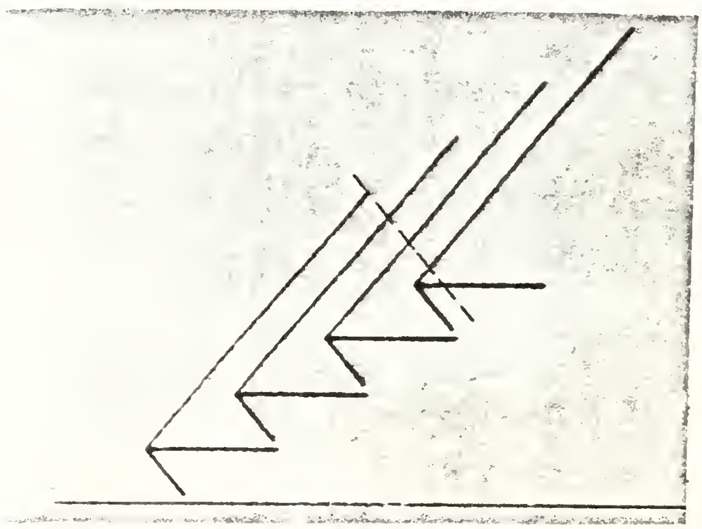


Fig. 35. Cascade A and B Shock Geometry [15]

Finally, it should be said that the organization of the storage arrays is actually a waste of memory space because fields of the b - and c - types (Fig. 34) need only small areas whereas a -fields demand large ones. This expensive way was chosen in order to make a general program clear. If one considers that the next step is the addition of the wake fields for each blade and that these have to be much larger than those in the passages, this solution gives an approach which can be extended analogous to W I N G . The desire to save storage place would certainly complicate the main organization and increase the number of subroutines significantly.

The main disadvantage, besides a slow-down of the computer is the limitation of the fields. The dashed line in Fig. 35 was not produced by P L O T 2 but it was added to show the limit of C A S C A D E . Beyond the end of the headshock for the first blade its influence has vanished. This results in completely wrong fields behind the dashed line. Hence the fields in Fig. 35 are usable only up to blade 3. The arrays in C A S C A D E used in this work have the size

X(2,3,50,20)

They can be extended to

X(2,3,KV,20)

where KV has to correspond with the first FORTRAN-STATEMENT of C A S C A D E . Then all the counters and Do-loops are dimensioned properly. The number of possible blade-computation is thus given by the limitation of the computer.

4.5 Results

As a test for C A S C A D E three different supersonic cascades are checked for which results already exist. Cascade A and B are those introduced by Verdon (15;17) and cascade T is a model for an experimental cascade used by Fleeter (19) for which Strada (5) gives computer results for the inlet flow.

Cascade Data:

	A	B	T
M	1.345	1.281	1.550
m	0.4	0.301	0.335
τ_{up}	0.0	0.0	0.0
τ_{Lo}	0.0	0.0	0.03
ε	30.5	26.6	23.9
k	0.90226	0.75054	0.28

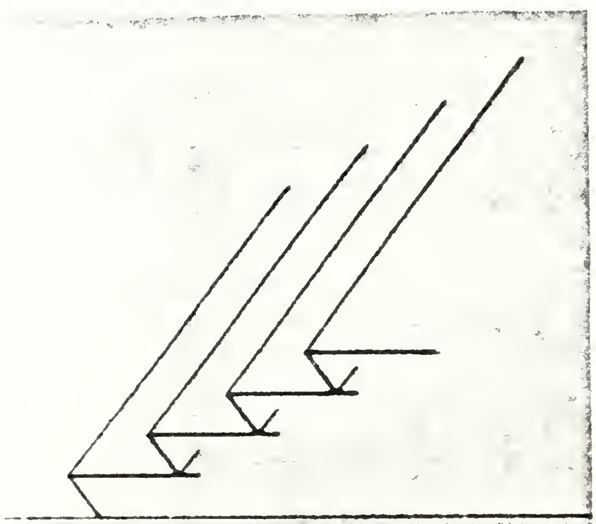


Fig. 36. Cascade B (15)

Fig. 35 shows the geometry of cascade A . Cascade B and T are shown in Fig. 36 and 37. As C A S C A D E is not completely programmed, it is only possible to show results of the inlet flow. We compare with theoretical aerodynamic data obtained by Verdon (15) for the linear cases, which are identical with those computed by Bell (9) and with results for the cascade T , given by Strada in (5).

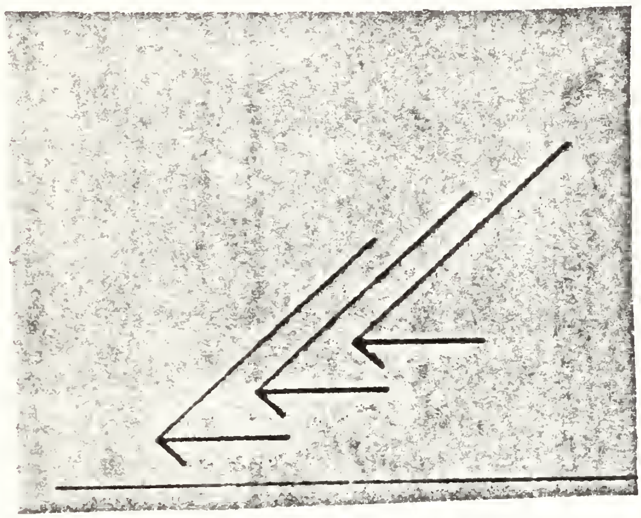


Fig. 37. Cascade T (5)

Fig. 38 and 39 show pressure difference distributions calculated by C A S C A D E for the third blade. Considering that Verdon introduced an infinite cascade and that Bell gave results for the 14th blade, the inlet flow computed with C A S C A D E looks very promising. The differences are not too significant and can be explained by the low number of blades which were examined.

As C A S C A D E and Verdon-results agree sufficiently, it could be expected that the linear data given by Strada and Platzer (P-) results would coincide very well with C A S C A D E computations. Actually, as both programs examine the second blade, the distribution of the total pressure coefficient agrees exactly for both sides of the A blade.

This is not true for the included slope effect. Fig. 40 through 43 show computations of this study in direct comparison with Strada (5). The upper surface represents the linear case and shall not be discussed because of good agreement. However, the lower surface is shaped and the agreement is good only at the leading edge. In all cases results of C A S C A D E for the total pressure distribution have the tendency to be considerably smaller than those presented by Strada. There is no real progress compared to the Fleeter Experiments (19) but with some imagination one could say that C A S C A D E meets the tendency of those data slightly better. It is remarkable that in spite of significant differences in the magnitude, the phase angle between imaginary and real part of the pressure coefficients agrees very well with Strada. The difference between the Strada results and those obtained here may be caused by the different treatment of the shock and remains a problem to be investigated more thoroughly in the near future.

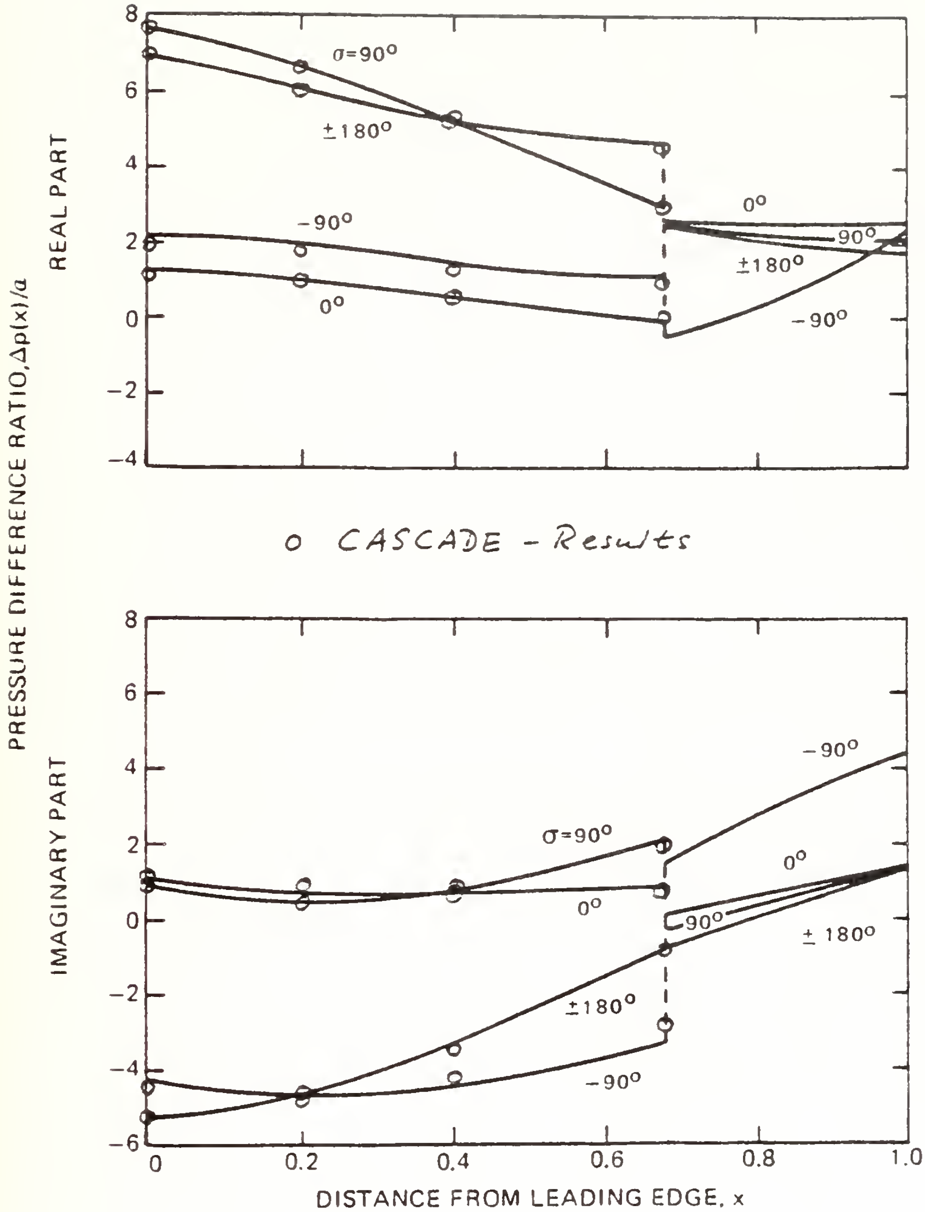


Fig. 38. From (15) Comparison of Linear Results Casc. A

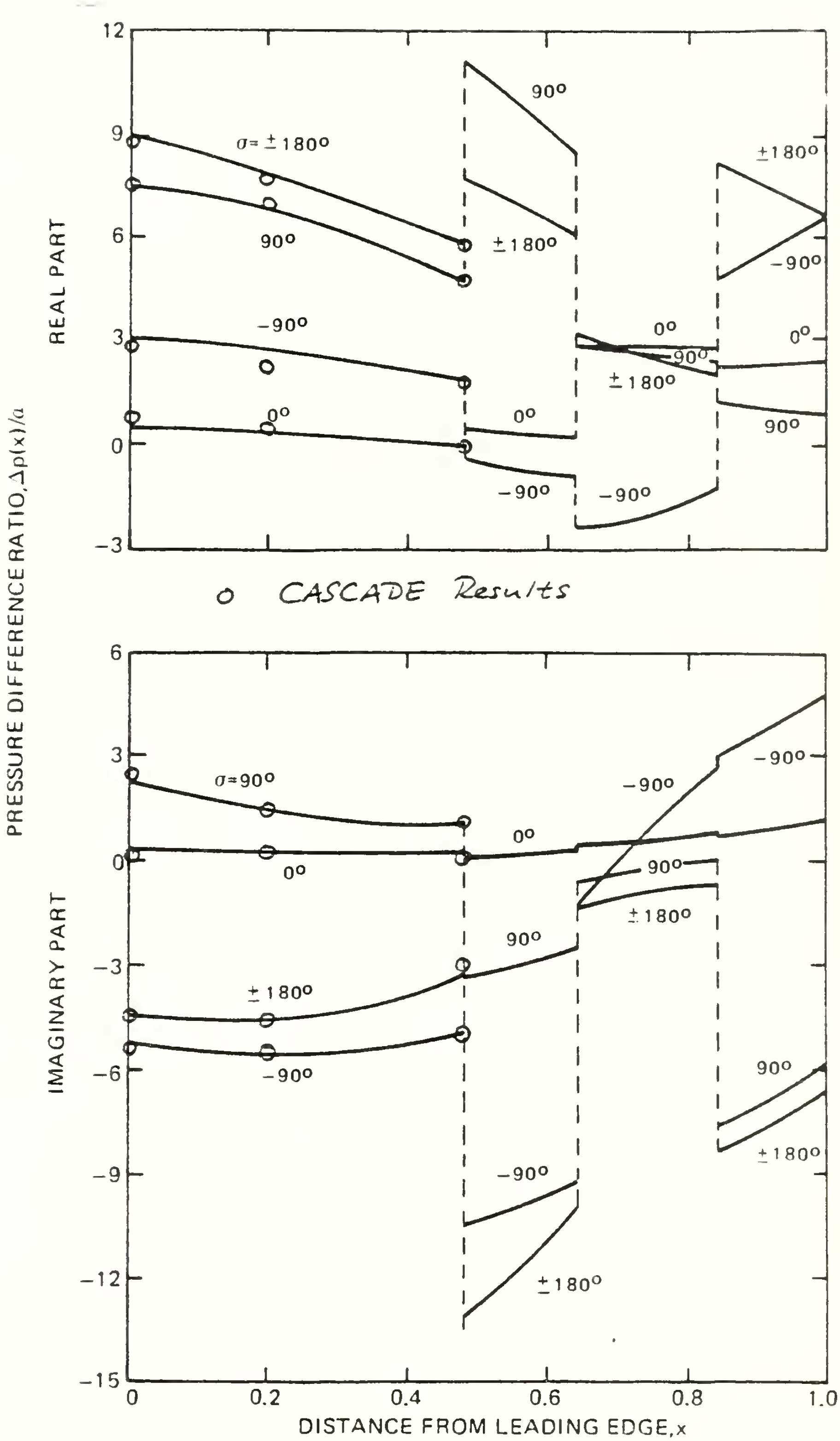


Fig. 39. From (15) Comparison of Linear Results Casc. B

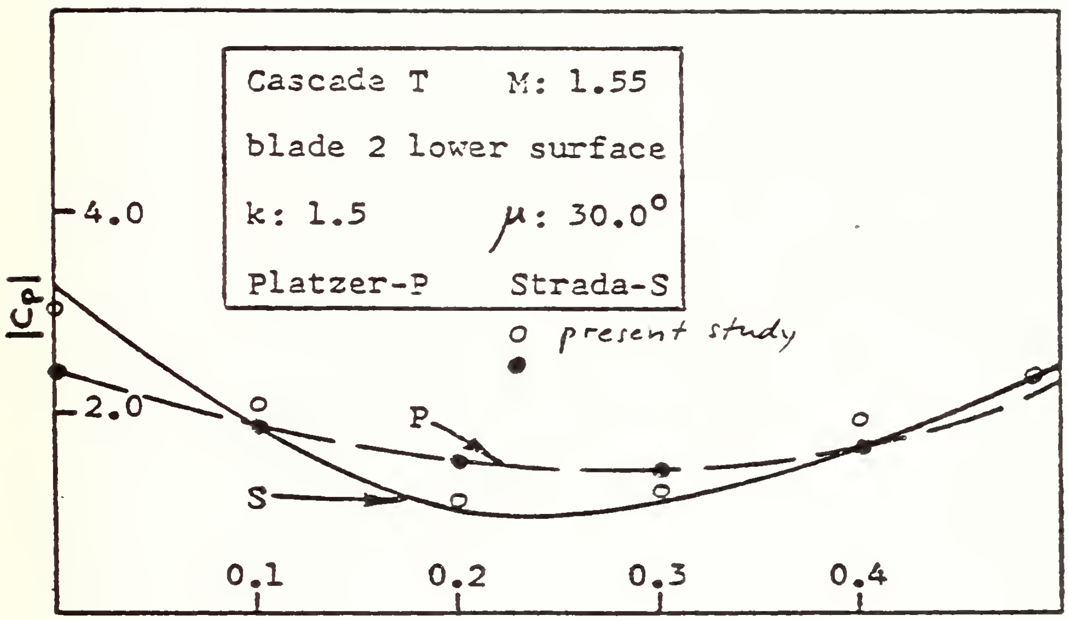


Figure 3.10.2 High-k comparison of the linear and non-linear solutions.

Fig. 40. From (15) Comparison with Strada

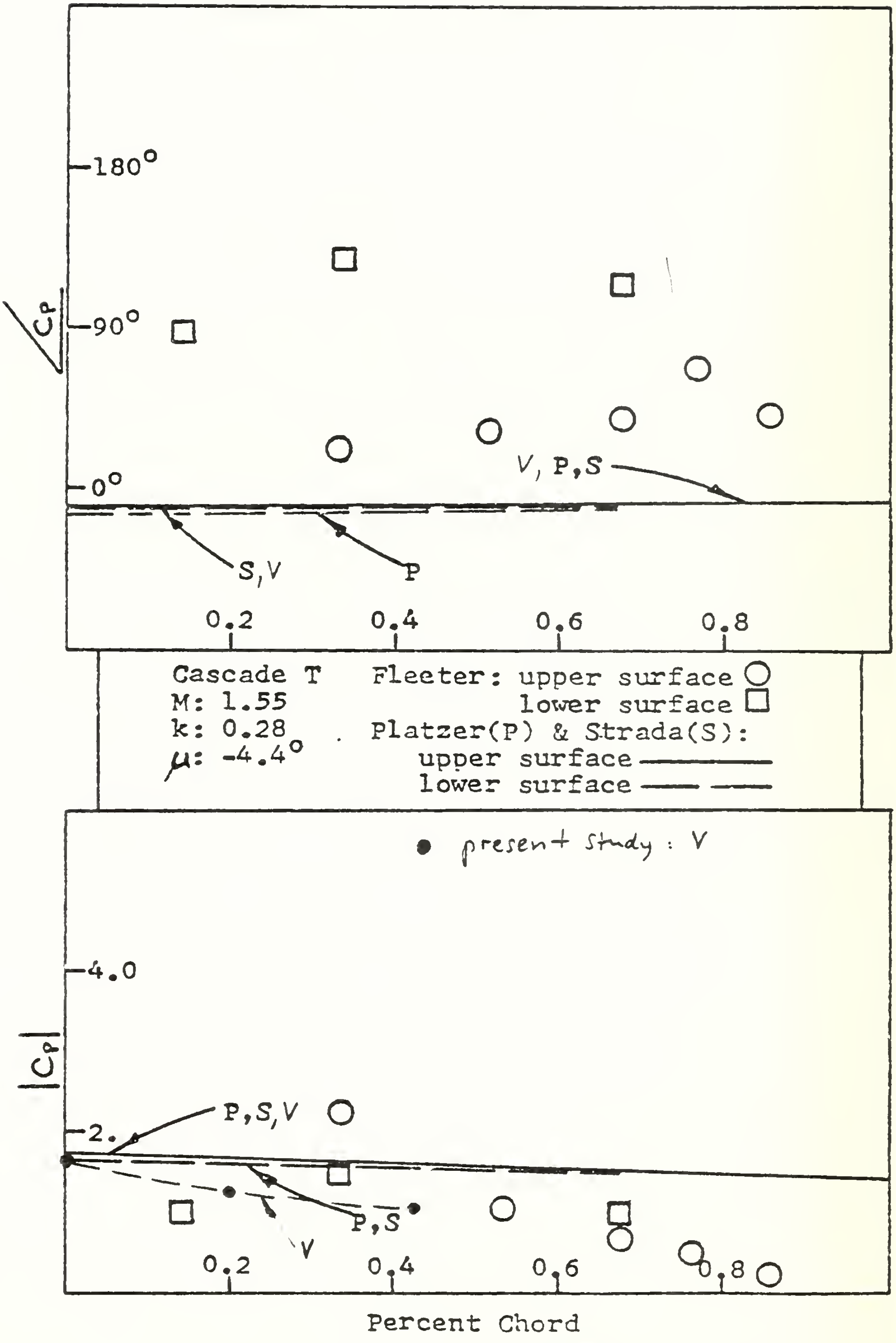


Fig. 41. From (5) Comparison with Strada

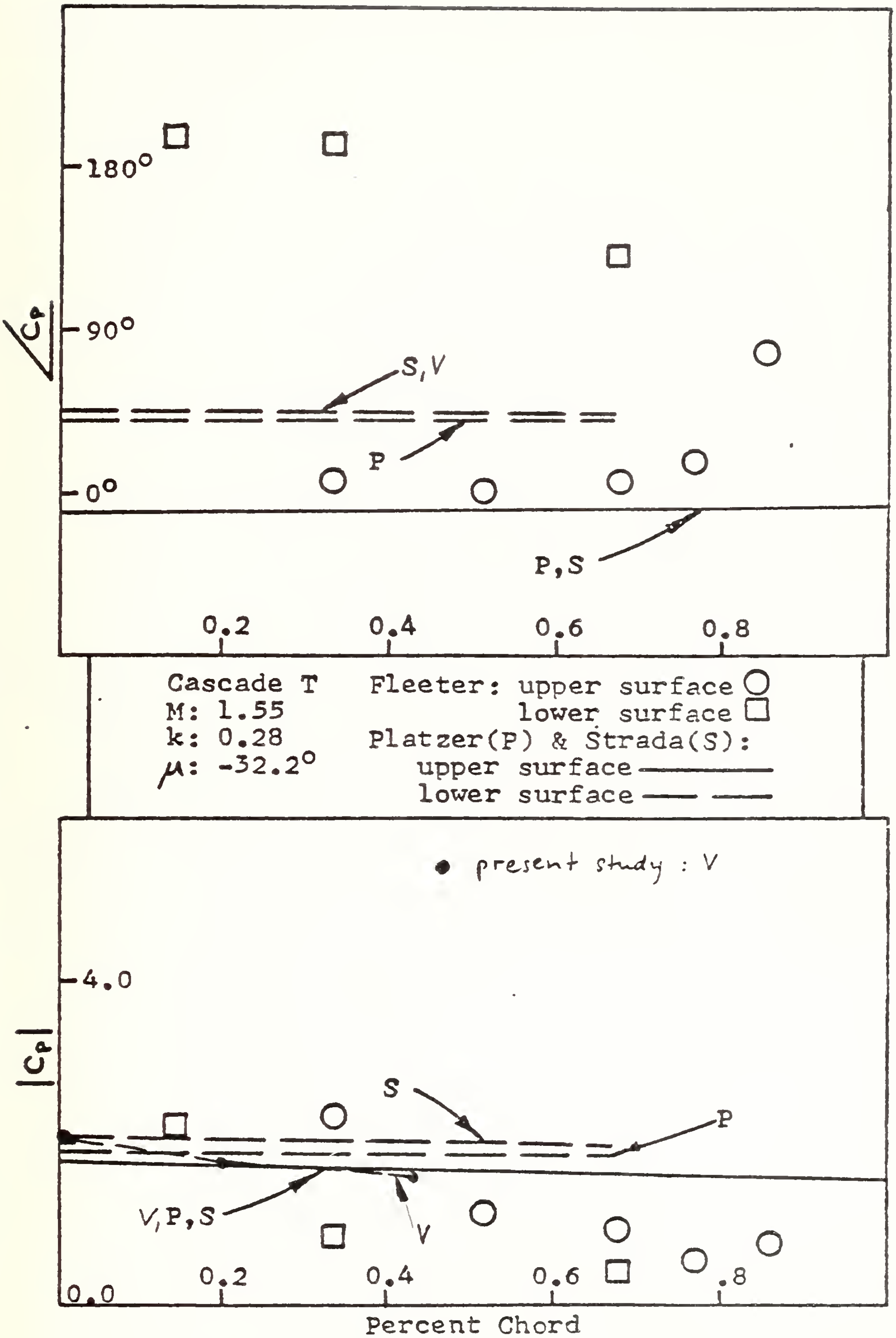


Fig. 42. From (5) Comparison with Strada

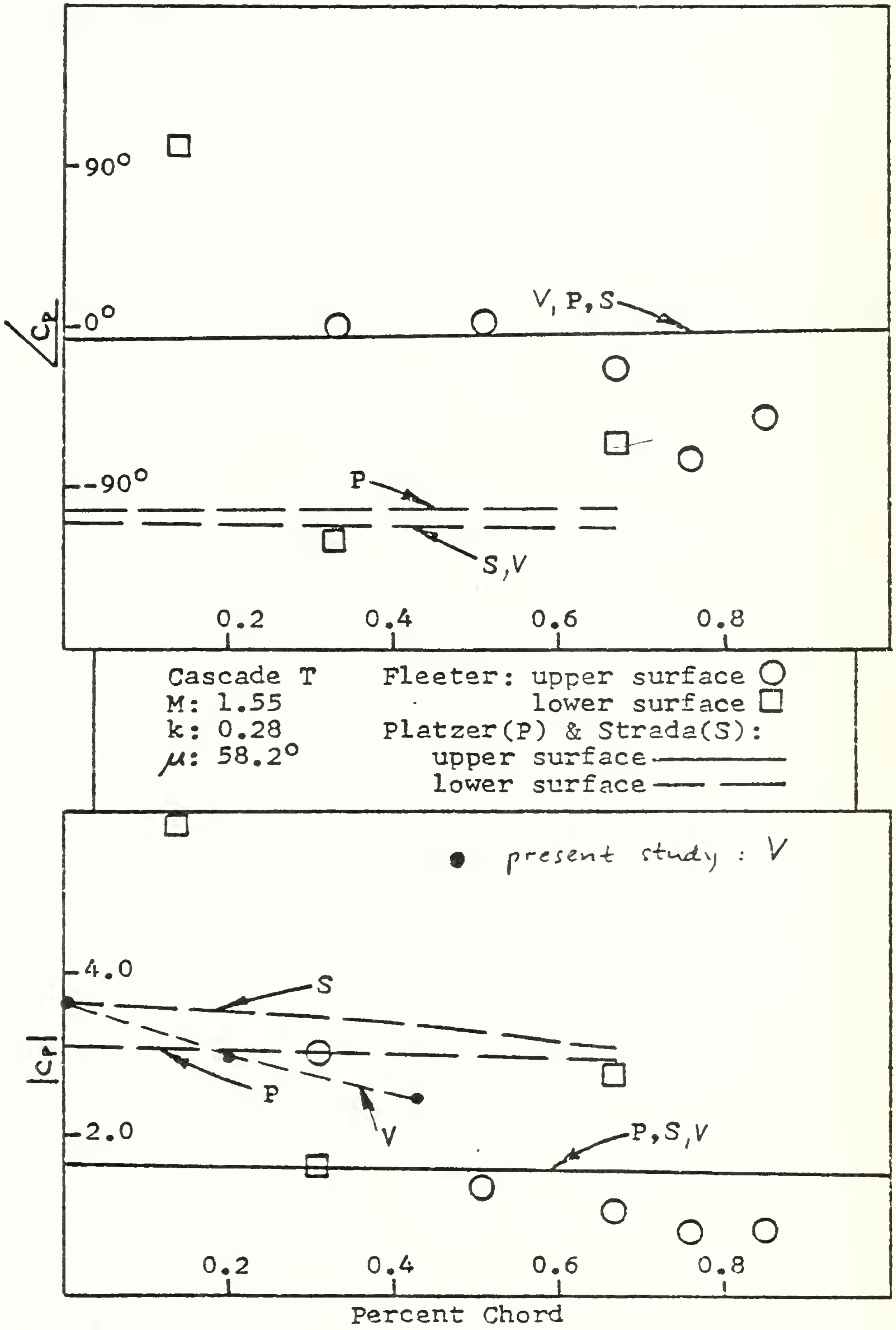


Fig. 43. From (5) Comparison with Strada

4.6 Updated Listing, Data and Output of C A S C A D E

If one refers to the comment cards at the beginning of C A S C A D E , it is obvious that this program is a derivation of W I N G .

```
C       CASCADE OF OSCILLATING AIRFOILS FOR  MACH.GT.1
C
C
        COMPLEX*8 U,V,PSI,G,DYDXU
        COMPLEX*8 AI,PU,BI
        COMPLEX*16 BL,RI,BS
        REAL*4 TX,TY,XX,YY,X,Y,AL,D1,D2,D3,XP,ALD,P,C3,ALZ
        COMMON/BA/ ALO,AM,ALC,C1,AK,I,IM,TX,TY,IW,MA,I ,ST,XX,YY
        COMMON/BC/ T1,T2,T,B,DYDX,D2YDX2,DYDXU,AI,BI,I1,IBACK,IE
        COMMON/SP/ XS(2,50),YS(2,50),A(2,50,4),R(2,50)
        COMMON/SCL/ BL(4,4),RI(4),BS(4)
        COMMON V(2,3,50,20),X(2,3,50,20),P(2,50),U(2,3,50,20),
       FPSI(2,3,50,20),G(2,3,50,20),AL(2,3,50,20),Y(2,3,50,20),
       FG(2,50),DX(2),IN(2,50),IN2(2,50),PX(2,30),PS(2,30),PL(2,30)
C
C       OPTIONS:
C       LO1 =0  : COMPLETE OUTPUT
C           =1  : ONLY PRESSURE DISTRIBUTION
C           =2  : STOP
C       LO2 =0  : PITCH
C           =1  : PLUNGE
C       LO3 =1  : DO THE 1. PASSAGE
C           =0  : DO THE 1. PASSAGE NOT
C       LO4 =1  : SURFACES ARE ANALYTICALLY GIVEN
C           =0  : SURFACES ARE POINTWISE GIVEN
C       MA = MAXIMUM NUMBER OF POINTS ON THE SURFACE .LT.20
C       MAXS = BLADE,WHERE THE PRESSURE DISTRIBUTION IS COMPUTED
C       IBLA = FIRST BLADE OF FIELD - OUTPUT
C
C
        KV=50
        AI=CMPLX(0.,1.)
 102    READ(3,999) LO1,LO2,LO3,LO4,MA,MAXS,IBLA
        IF(LO1.EQ.2) GOTO 101
        MAXS=MAXS+1
        MAX=MA
        CALL PROFIL(LO4,T1,T2,NX)
 100    READ(3,1000) AK,AM,C,DX(1),DX(2)
        IF(AM.EQ.0.) GOTO 102
        READ(3,1020) AMY,B,B3,B4
```

The data are set up in the same way. There are five additional input variables. Maxs and IBLA are explained above. IBLA is only meaningful for LO1=0 . Then for example IBLA=3 starts the field output only from the third blade.

The other three variables define the Cascade:

ET = ε Stagger Angle

EM = m Distance Between the Blades

AMY = μ Phase Lag

See Fig. 32.

The rest of the input data have the same definition as those in W I N G .

Two sample data sets shall be shown:

1. Analytically Given Blades

```
FILE: FILE      FTC35C01 P1                         NAVAL P
10C1160301
   0.66          0.03      200
   0.75054       1.281       1.4     0.3927-02 0.3927-02
  90.0           0.5        26.0       0.301
   0.28000       0.000       1.4     0.3927-02 0.3927-02
2222222222
```

This would cause a run as follows.

1st Row : No field output, pitch motion, no computation between the first blade and the imaginary wall (first channel), 16 grid points on each surface, they are analytically given, the third blade shall be examined.

2nd Row : The blades have flat upper and thick lower sides

3rd Row : k, M, x , Dx

4th Row : μ, b, ε , m

5th and 6th Row : STOP

2. Pointwise Given Blades

```
FILE: FILE     FT03F001 P1                                NAVAL
1000160201
   0.01        0.03    17 1
 100.
   0.0         5.21       11.53       17.84       24.14
  -0.26       -0.165      -0.110      -0.050      -0.005
  30.45       36.75       43.05       49.35       55.65
   0.029       0.056       0.074       0.087       0.097
  61.95       68.25       74.55       80.88       87.21
   0.098       0.095       0.08        0.0516      0.016
  93.53      100.00        0.0         0.0         0.0
  -0.035      -0.10      100.        100.        100.
   0.0         6.29       12.59       18.92       25.21
  -0.26       -1.08       -1.65       -2.12       -2.45
  31.53       37.85       44.18       50.51       56.83
  -2.76       -2.93       -3.03       -2.97       -2.84
  63.15       69.47       75.78       82.16       88.36
  -2.60       -2.29       -1.94       -1.57       -1.17
  94.66      100.00        0.0         0.0         0.0
  -0.70       -0.10      100.        100.        100.
   0.28        1.55        1.4     0.544E-03   0.454E-02
  -4.4         0.5        23.9         0.335
   0.28        1.55        1.4     0.544E-03   0.454E-02
 -32.2         0.5        23.9         0.335
   0.28        1.55        1.4     0.544E-03   0.454E-02
  53.2         0.5        23.9         0.335
   0.28        0.00        1.4     0.544E-03   0.454E-02
 2222222222
```

1st Row : No field output, pitch motion, no first passage, pointwise given blades, 16 gridpoints on the surface, 2nd blade shall be examined;

2nd Row : Identification for upper and lower side (no real importance here), 17 points given as input for each side, the geometry of the blades shall be printed;

3rd Row : Chord is 100

46h to 19th Row : Input geometry of blades, according to subprogram P R O F I L ;

20th Row : k , m , , Dx

21st Row : μ , b , m , ε

Two other cases and STOP

The output of the pressure distribution for the first case is shown next, followed by a listing of C A S C A D E in Appendix B.

PITCH - XOCE

PHASE= -4.40 / 2. BLADE

W*C/U= 0.28000, M= 1.550, K= 1.40, B/C= 0.50, T/C= 0.0100

PRESSURE-COEFFICIENTS UPPER SURFACE:

POINT	X	CPS	RCPU	ICPU
2, 1	0.0	0.255E-01	-0.169E 01	0.364E 00
3, 2	0.083	0.201E-01	-0.167E 01	0.351E 00
4, 3	0.167	0.149E-01	-0.164E 01	0.334E 00
5, 4	0.251	0.995E-02	-0.162E 01	0.317E 00
6, 5	0.335	0.738E-02	-0.160E 01	0.302E 00
7, 6	0.419	0.418E-02	-0.158E 01	0.262E 00
8, 7	0.503	0.325E-02	-0.157E 01	0.265E 00
9, 8	0.568	0.870E-03	-0.155E 01	0.248E 00
10, 9	0.629	-0.282E-03	-0.138E 01	0.689E 00
11,10	0.691	-0.252E-02	-0.202E 01	-0.654E 00
12,11	0.752	-0.646E-02	-0.134E 01	0.650E 00
13,12	0.814	-0.851E-02	-0.202E 01	-0.695E 00
14,13	0.876	-0.115E-01	-0.130E 01	0.611E 00
15,14	0.938	-0.157E-01	-0.201E 01	-0.728E 00
16,15	1.000	-0.174E-01	-0.126E 01	0.568E 00

****

W*C/U= 0.28000, M= 1.550, K= 1.40, B/C= 0.50, T/C= 0.0300

PRESSURE-COEFFICIENTS LOWER SURFACE:

POINT	X	CPS	RCPU	ICPU
1, 2	0.0	0.306E 00	-0.148E 01	0.440E 00
2, 3	0.028	0.290E 00	-0.141E 01	0.468E 00
3, 4	0.058	0.235E 00	-0.123E 01	0.349E 00
4, 5	0.089	0.180E 00	-0.112E 01	0.334E 00
5, 6	0.124	0.154E 00	-0.111E 01	0.285E 00
6, 7	0.160	0.144E 00	-0.110E 01	0.311E 00
7, 8	0.200	0.120E 00	-0.106E 01	0.257E 00
8, 9	0.242	0.979E-01	-0.104E 01	0.211E 00
9,10	0.253	0.839E-01	-0.102E 01	0.220E 00
10,11	0.286	0.722E-01	-0.102E 01	0.233E 00
11,12	0.341	0.442E-01	-0.998E 00	0.192E 00
12,13	0.398	0.192E-01	-0.987E 00	0.111E 00
13,14	0.462	-0.168E-01	-0.980E 00	0.337E-01
14.15	0.491	0.486E 00	0.157E 01	-0.217E 00

****

5. The Programs as a Working System

5.1 T E S T

T E S T was written to examine the blade geometry if it is pointwise given. C A S C A D E and W I N G are not prepared to handle shocks which are caused by a changing slope over the airfoil. Therefore it has to be checked that the surface has no turning point.

For a given set of points one has to deform the PROFIL in such a way that the curvature of it never changes the sign. This would be indicated by a changing sign for the second derivative of the surface function.

An example for T E S T - input data is shown below:

0.01	0.05	17 0		
100.				
0.0	6.21	11.53	17.84	24.14
-0.26	-0.185	-0.110	-0.050	-0.005
30.45	36.75	43.05	49.35	55.65
0.029	0.056	0.074	0.087	0.097
61.95	68.25	74.55	80.85	87.21
0.093	0.095	0.08	0.0516	0.016
93.52	100.00	0.0	0.0	0.0
[illegible]	[illegible]	100.	100.	100.
0.0	6.29	12.59	18.9	25.21
-0.26	-1.03	-1.65	-2.12	-2.49
31.53	37.85	44.18	50.51	56.83
-2.76	-2.93	-3.00	-2.97	-2.84
63.15	69.47	75.79	82.16	88.36
-2.60	-2.29	-1.94	-1.57	-1.17
94.66	100.00	0.0	0.0	0.0
-0.70	-0.10	100.	100.	100.
-0.71	-0.10	100.	100.	100.

T E S T reads from File 05 and writes the result x, y'_L, y'_u, y''_L and y''_u on File 06 and 01. File 06 can be printed and shows the original input data, the interpolated surfaces and besides the derivatives mentioned above, the coefficients of the cubic splines for each side. File 06 contains only x, y, y' and y''. It is used as an input file for P L O T 1 which produces plots like Fig. 44 and Fig. 45 with it. Fig. 44 shows y' of the two PROFIL sides and Fig. 45 shows y'. P L O T 1 is explained in Section 5.2.

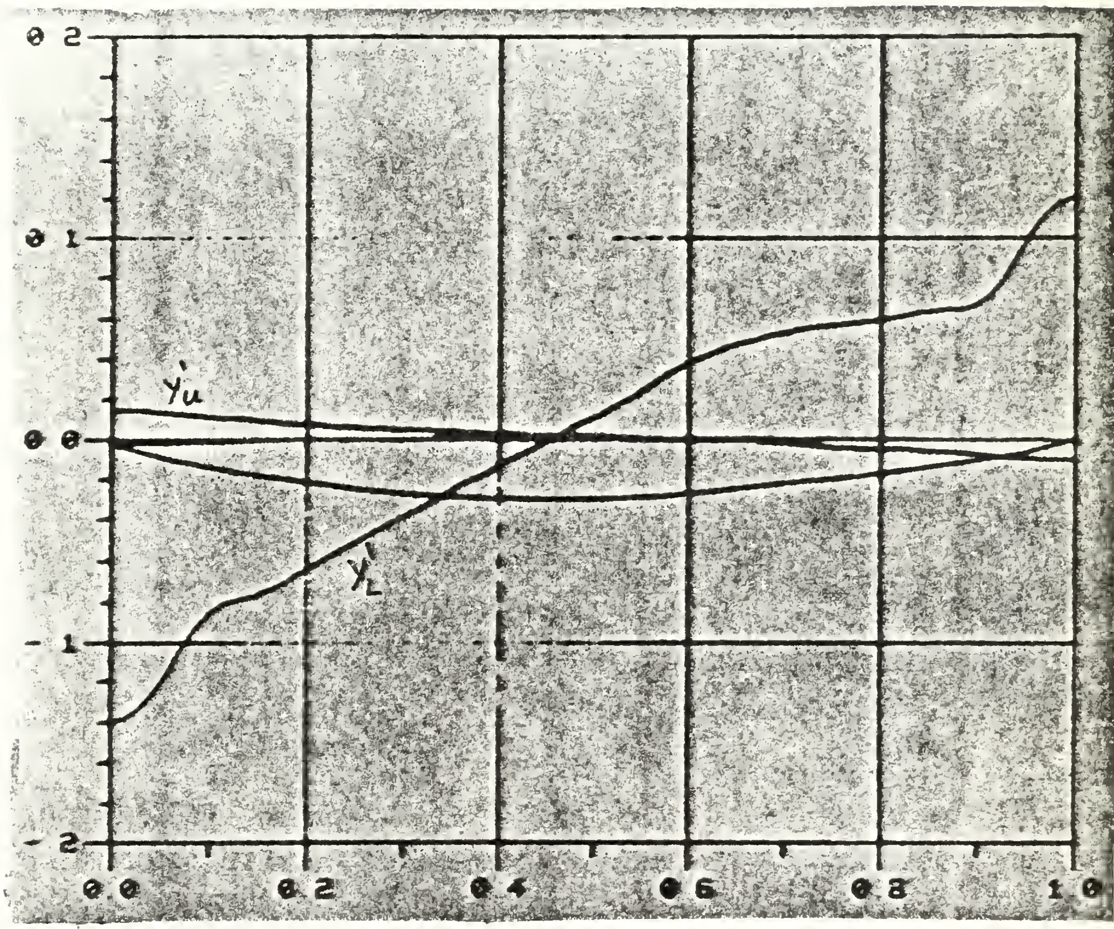


Fig. 44. Blade Geometry and y'

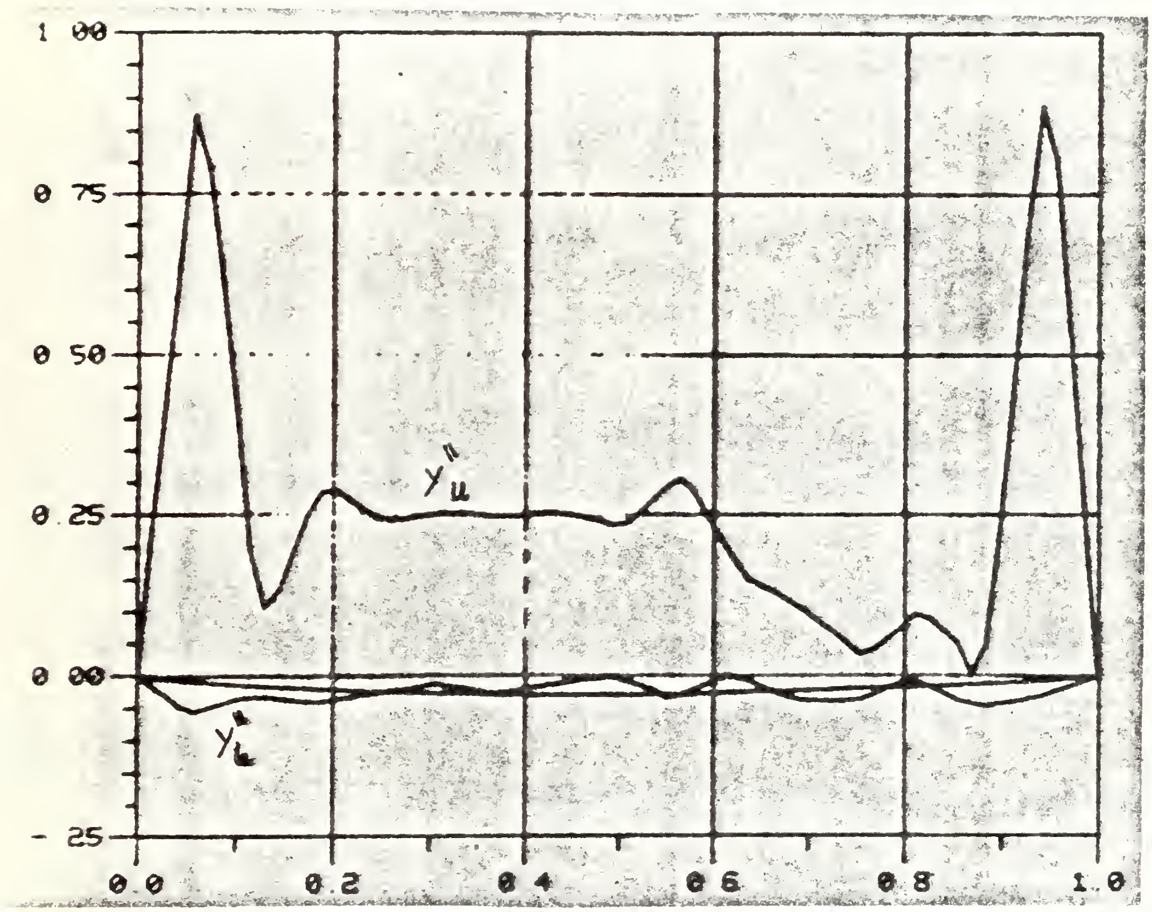


Fig. 45. Blade Geometry and y''

T E S T LISTING :

```
C
C
C     TEST
      COMMON/SP/ XS(2,50),YS(2,50),A(50,4),R(2,50)
C
C     TEST OF THE SPLINEFUNCTIONS FOR THE PROFILSURFACES
C     LC4 = 1  ANALYTICALLY GIVEN
C         = 0  POINTWISE GIVEN
C
      CALL PROFIL(LO4,T1,T2,NX)
C
C     COMPUTATION OF THE SPLINE - COEFFICIENTS
C
      DO 30 I=1,2
      IF(I.EQ.1) WRITE(6,1002)
      IF(I.EQ.2) WRITE(6,1003)
      DO 49 K=1,NX
      A(K,1)=YS(I,K)
      A(K,3)= R(I,K)
      H=XS(I,K+1)-XS(I,K)
      A(K,2)=(YS(I,K+1)-YS(I,K))/H-H*(R(I,K+1)+2.*R(I,K))/3.
49    A(K,4)=(R(I,K+1)-R(I,K))/(3.*H)
      WRITE(6,1001)
      DX=1./69.
      MM=70
      WRITE(1,1004) MM
1004  FORMAT(I3)
      DO 10 M=1,70
      X=DX*(M-1)
      DO 5 K=2,50
      J=K-1
      IF(XS(I,K).GE.X) GOTO 6
5     CONTINUE
6     H=X-XS(I,J)
      DYDX=A(J,2)+2.*A(J,3)*H+3.*A(J,4)*H*H
      D2YDX2=2.*A(J,3)+6.*A(J,4)*H
      Y=A(J,1)+A(J,2)*H+A(J,3)*H*H+A(J,4)*H*H*H
      WRITE(6,1000) X,Y,DYDX,D2YDX2,A(J,1),A(J,2),A(J,3),A(J,4),J
      WRITE(1,1005) X,DYDX,D2YDX2
10    CONTINUE
      MM=0
30    WRITE(1,1004) MM
1000  FORMAT(1X,8(2X,E12.5),I4)
1001  FORMAT(1X,7X,'X',13X,'Y',9X,'DYDX',8X,'D2YDX2',10X,
     F'SPLINECOEFFICIENTS :',/)
1002  FORMAT(1H1,1X,'PROFIL UPPER SURFACE: ',//)
1003  FORMAT(1H1,1X,'PROFIL LOWER SURFACE: ',//)
1005  FORMAT(3F10.5)
      END
CXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
CXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
      SUBROUTINE PROFIL(LO4,T1,T2,N)
      COMMON/SP/ XS(2,50),YS(2,50),A(50,4),R(2,50)
C
C     PREPARATION OF THE PROFIL - SURFACES
C
      READ(5,1000) T1,T2,NT,LO4
1000  FORMAT(2F10.5,2I2)
      IF(LO4.EQ.0) READ(5,1001) SP
      DO 8 J=1,2
      IZ=0
      N=NT
      T=T1
      IF(J.EQ.2) T=T2
      IF(LO4.NE.0) GOTO 12
      DO 9 K=1,4
      M=(K-1)*5+1
      READ(5,1001) XS(J,M),XS(J,M+1),XS(J,M+2),XS(J,M+3),XS(J,M+4)
      READ(5,1001) YS(J,M),YS(J,M+1),YS(J,M+2),YS(J,M+3),YS(J,M+4)
      WRITE(6,2001)XS(J,M),XS(J,M+1),XS(J,M+2),XS(J,M+3),XS(J,M+4)
      WRITE(6,2001)YS(J,M),YS(J,M+1),YS(J,M+2),YS(J,M+3),YS(J,M+4)
2001  FORMAT(1X,5F10.5)
1001  FORMAT(5F10.5)
```

```
      IF(YS(J,M+4).EQ.100.) GOTO 7
9     CONTINUE
7     CONTINUE
      DO 2 LL=1,N
      XS(J,LL)=XS(J,LL)/SP
2     YS(J,LL)=YS(J,LL)/SP
      WRITE(1,1004) NT
1004  FORMAT(I3)
      DO 16 KV=1,NT
      WRITE(1,1005) XS(J,KV),YS(J,KV),YS(J,KV)
16    CONTINUE
      IF(LO4.EQ.0) GOTO 3
12    DX=1.0/(N-1)
      I1=(-1)**(J+1)
      DO 20 K=1,N
      XS(J,K)=(K-1)*DX-0.25
20    YS(J,K)=4.*T*XS(J,K)*(1.-XS(J,K))*I1
C
C     TO ENTER THIS PART,THE SURFACES SHOULD ALREADY BE
C     GIVEN POINTWISE
C
3     CONTINUE
C
C     INTERPOLATION THROUGH CUBIC SPLINES
C
      IF(T.NE.0. .OR. LO4.EQ.0) GOTO 51
      DO 52 I=1,N
52    A(I,1)=0.
      GOTO 53
51    CONTINUE
      DO 10 I=1,N
      A(I,3)=XS(J,I)
10    A(I,4)=YS(J,I)
C
C     MATRIX OF COEFFICIENTS AND RIGHT SIDES
C
      K=N-2
      DO 25 I=1,K
      A(I,1)=A(I+1,3)-A(I,3)
      A(I,3)=A(I+2,3)-A(I+1,3)
      A(I,2)=2.*(A(I,1)+A(I,3))
      A(I,4)=3.*(A(I+2,4)-A(I+1,4))/A(I,3)-
     F3.*(A(I+1,4)-A(I,4))/A(I,1)
25    CONTINUE
      A(1,1)=0.0
      A(N-2,3)=0.0
C
C     THE STEP OF GAUSS
C
      K=N-3
      DO 30 I=1,K
      DO 35 M=3,4
35    A(I,M)=A(I,M)*A(I+1,1)/A(I,2)
      A(I,1)=0.0
      A(I,2)=A(I+1,1)
      A(I+1,2)=A(I+1,2)-A(I,3)
      A(I+1,4)=A(I+1,4)-A(I,4)
30    CONTINUE
C
C     SOLUTION
C
      A(1,1)=0.
      A(N,1)=0.
      A(N-2,1)=0.0
      L=N-1
      DO 40 I=2,L
      K=N-I
      M=K+1
40    A(M,1)=(A(K,4)-A(K,3)*A(M+1,1))/A(K,2)
53    CONTINUE
      DO 11 M=1,N
11    R(J,M)=A(M,1)
      IF(IZ.EQ.1) GOTO 8
      KV=N-1
```

```
      DC 49  K=1,KV
      A(K,1)=YS(J,K)
      A(K,3)= R(J,K)
      H=XS(J,K+1)-XS(J,K)
      A(K,2)=(YS(J,K+1)-YS(J,K))/H-H*(R(J,K+1)+2.*R(J,K))/3.
49    A(K,4)=(R(J,K+1)-R(J,K))/(3.*H)
      WRITE(6,1001)
      DX=1./49.
      DC 4  M=1,50
      X=DX*(M-1)
      DC 5  K=2,50
      I=K-1
      IF(XS(J,K).GE.X) GOTO 6
5     CONTINUE
6     H=X-XS(J,I)
      Y=A(I,1)+A(I,2)*H+A(I,3)*H*H+A(I,4)*H*H*H
      R(J,M)=X
      YS(J,M)=Y
4     CCNTINUE
      IZ=1
      N=50
      DC 13  I=1,50
13    XS(J,I)=R(J,I)
      GCTO 3
8     CCNTINUE
      WRITE(6,1003) N
      DC 15  M=1,N
15    WRITE(6,1002) XS(1,M),YS(1,M),XS(2,M),YS(2,M)
      N=N-1
1002  FCRMAT(5X,4(E12.5,2X))
1003  FORMAT( 1H1,5X,'THE COMPLETE EXTRAPOLATED SURFACES N=',I3,//)
1005  FCRMAT(3F10.5)
      RETURN
      END
```

5.2 P L O T 1

P L O T 1 was written to produce plots from two pointwise given functions

$$Y1 = F1(x)$$

$$Y2 = F2(x)$$

The x -stations are for both functions identical. This is useful to make diagrams for real and imaginary part of pressure distributions (see Fig. 18 through 25) and to visualize the y' and y" values of the blade geometry (Fig. 44 and 45). The input is read from File 01. This was produced from the respective program whose results shall be plotted. That is either W I N G or T E S T in this work.

The next step is to look in File 01 for the highest and lowest values of the functions and to decide the scale of the diagram. The last line of File 01 is a zero. Behind this zero there has to be inserted now the coordinates of Points 1 to 4 from Fig. 46.

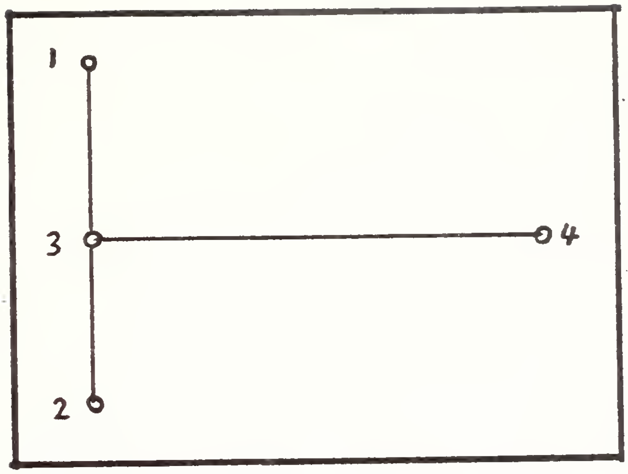


Fig. 46. Dimensioning of the Plot

This must be done for both expected diagrams. An example for the final lines of File 01 is shown below.

This file produced Fig. 44 and 45.

x	y1	y2
:	:	:
0.89855	0.06626	0.21406
0.91304	[illegible]	[illegible]
0.92754	0.07913	0.69786
0.94203	0.09115	0.89099
0.95652	0.10344	[illegible]
0.97101	0.11344	0.54123
0.98551	[illegible]	0.25650
1.00000	0.120[illegible]9	0.00000
0		
0.	0.15	
0.	-0.15	
0.	0.	
1.	0.	
0.	1.	
0.	-0.1	
0.	0.	
1.	0.	

One can see the inserted eight additional lines behind the zero.

P L O T 1 works only on the Tektronix-terminal. After unpacking and compiling, it can be called on this device with

$$ PLOT 1

After the first plot is done, the terminal makes a tone. Then P L O T 1 hits a pause-statement. By striking an arbitrary key on the keyboard and CTRLS , it produces the second plot.

```
C
      P L O T 1   LISTING  :
C
C
      DIMENSION X(100),Y(100),X1(10,100),Y1(10,100),Y2(10,100),
     FA(4),B(4),N1(11)
      M=1
1     READ(1,1000) N1(M)
      IF(N1(M).EQ.0) GOTO 2
      N2=N1(M)
      DO 10 I=1,N2
10    READ(1,1001) X1(M,I),Y1(M,I),Y2(M,I)
      M=M+1
      GOTO 1
2     M=M-1
      DO 14 I=1,4
14    READ(1,1001) A(I),B(I)
      CALL INIT
      CALL NPTS(I)
      CALL PLOT(B,A)
      DO 3 I=1,M
      N=N1(I)
      DO 4 J=1,N
      X(J)=X1(I,J)
4     Y(J)=Y1(I,J)
      CALL NPTS(N)
      CALL CPLOT(X,Y)
3     CONTINUE
      CALL PAUSE
      DO 15 I=1,4
15    READ(1,1001) A(I),B(I)
      CALL INIT
      CALL NPTS(I)
      CALL PLOT(B,A)
      DO 5 I=1,M
      N=N1(I)
      DO 6 J=1,N
      X(J)=X1(I,J)
6     Y(J)=Y2(I,J)
      CALL NPTS(N)
      CALL CPLOT(X,Y)
5     CONTINUE
      CALL FIN
1000  FORMAT(I3)
1001  FORMAT(3F10.5)
      STOP
      END
```

5.3 P L O T 2

P L O T 2 is a special program which graphs the shock and cascade geometry on the Tektronix terminal. It is called with

$$ PLOT 2_

Input is read from File 01 which is prepared by C A S C A D E in each run. The resulting plots are for example Fig. 35, 36 and 37. P L O T 2 asks for the size of the diagram by SCALE:

By enterine 1.5, 1., 0.5 or any other digital number from the keyboard, one may change the graph to the desired size. Each time the program has to be started again with the $$ - command.

```
      P L C T 2    LISTING  :
C     PLOT2 READS THE DATA FRCM FILE FT01F001
C     AND PRCDUCES ON THE TEKTRONIX-TERMINAL PLCTS
C     OF THE GEOMETRY OF THE HEADSHOCKS AND THE SHOCKS
C     IN THE PASSAGES
C
      DIMENSION X(50),Y(50)
      WRITE(6,998)
      READ(5,999) ST
      CALL INIT
      CALL NPTS(2)
      X(1)=20.
      Y(1)=20.
      X(2)=1000.
      Y(2)=20.
      CALL CPLOT(X,Y)
15    CONTINUE
      READ(1,1000) X(1),Y(1),IM
      IF(IM.EQ.-1) GOTO 25
      IF(IK.EQ.IM) GOTO 30
      IK=IM
      X(2)=X(1)+1.
      Y(2)=Y(1)
      GOTO 35
30    READ(1,1000) X(2),Y(2),IM
35    CONTINUE
      DO 20 I=1,2
      X(I)=X(I)*ST*200.+20.
20    Y(I)=Y(I)*ST*200.+20.
      CALL CPLOT(X,Y)
      GOTO 15
25    CONTINUE
      CALL PAUSE
      CALL FIN
998   FORMAT(1X,'GIVE SCALE :')
999   FORMAT(F10.5)
1000  FORMAT(2F10.3,I3)
      END
```

5.4 Sample Sessions

The programs are too large to fit into the two cylinder disk space which is available to the time sharing user. Therefore only so-called packed versions are present on the private disk.

To log in an additional temporty T-Disk, execute the program S T A R T by calling it from the CMS-Level. The T-Disk is then logged in and eventually files are read in which are in the virtual card reader of the system.

The execution of one of the programs is prepared by

G E T T E S T

G E T W I N G

or

G E T C A S

This means for T E S T , W I N G or C A S C A D E to be copied to the T-Disk, unpacked, altered to a FORTRAN file and compiled. After this, the input/output files are defined correctly. Finally the execution can be initialized by

$ FILE NAME

If a compiled version of the desired program is already on the T-Disk,

D O T E S T

D O W I N G

or

D O C A S

will also define the input/output files. Again $ starts the execution.

See the three sample sessions on the next pages.

CONTENTS OF THE DISK:

```
 1
FILENAME FILETYPE MODE NO.REC. DATE
TEST     DATA     P1      19    9/30
FILE4    DATA     P1       3    9/29
PLOT1    PACKED   P5       2    9/19
FILE     FT03F001 P1       1    9/30
FILE     FT04F001 P1       2    9/29
DOTEST   EXEC     P1       1    9/29
START    EXEC     P1       1    9/29
DOWING   EXEC     P1       1    9/29
CLOSE    EXEC     P1       1   10/01
TEST     PACKED   P5       8    9/29
PLOT2    PACKED   P5       2    9/28
FILE3    DATA     P1       3    9/29
WING     PACKED   P5      77    9/26
FILE     FT05F001 P1       2    9/29
CASCADE  PACKED   P5      81    9/28
GETWING  EXEC     P1       1    9/29
GETCAS   EXEC     P1       1    9/29
DOCAS    EXEC     P1       1    9/29
GETTEST  EXEC     P1       1    9/29
```

```
 this is a sampel session for  t e s t .
condition  :   you have just logged in.


start
17.30.06 VSET RDYMSG OFF
17.30.07 VSET BLIP /
17.30.08 CP DEFINE T2314 192 10
17.30.09 FORMAT T
** "FORMAT T" WILL ERASE ALL YOUR T-DISK (192) FILES **
**DO YOU WISH TO CONTINUE?  ENTER "YES" OR "NO":
>yes
FORMATTING T-DISK (2314)...
T (192): 010 CYL
17.30.30 OFFLINE READ *
READER EMPTY OR NOT READY.
 !!! E(00009) !!!
17.30.33 OFFLINE READ *
READER EMPTY OR NOT READY.
 !!! E(00009) !!!
R;
>settest
17.30.42 VSET BLIP /
17.30.43 COMBINE TES PACKED T5 TEST PACKED P5
17 30.45 UNPACK TES
17.30.46 ALTER TES UNPACKED T5 TEST FORTRAN T1
17.30.47 ERASE TES PACKED T5
17.30.49 F TEST
////17.31.02 FILEDEF FT05F001 DSK
17.31.03 FILEDEF FT01F001 DSK-T1
17.31.05 FILEDEF FT06F001 DSK-T1
R;
>$ test
/EXECUTION BEGINS...
/R;
>
```

```
this is a samrel session for  w i n g  .

condition  :  t - disk is already logged in




setwing
17.34.46 VSET BLIP /
17.34.47 COMBINE WIN PACKED T5 WING PACKED P5
17.34.53 UNPACK WIN
/17.35.01 ALTER WIN UNPACKED T5 WING FORTRAN T1
17.35.03 ERASE WIN PACKED T5
17.35.04 F WING
////////////////////////////////////////////////////17.37.09 FILEDEF FT01F001 DSK-T1
17.37.10 FILEDEF FT04F001 DSK
17.37.12 FILEDEF FT06F001 DSK-T1
R;
>$ wing
/EXECUTION BEGINS...
////////////////R;
>
```

this is a sampel session for c a s c a d e .

condition : you have already logged in the t disk

```
setcas
14.12.23 VSET BLIP /
14.12.24 COMBINE CAS PACKED T5 CASCADE PACKED P5
14.12.35 UNPACK CAS
/14.12.46 ALTER CAS UNPACKED T5 CASCADE FORTRAN T1
14.12.48 ERASE CAS PACKED T5
14.12.49 F CASCADE
//////////////////////////////////////////////////////////////14.16.35 FILEDEF FT01F001 DSK T1
14.16.37 FILEDEF FT03F001 DSK
14.16.38 FILEDEF FT06F001 DSK T1
14.16.39 FILEDEF FT07F001 DSK T1 RECFM F BLKSIZE 133
R; T=122.98/127.06 14.16.39

>$ cascade
/EXECUTION BEGINS...
/////////////////////////////////////////R; T=70.81/73.66 14.18.45

>
```

6. Critique, Conclusion and Outlook

A systems of programs was written, which allows a systematic approach to the problem of oscillating shaped airfoils in a staggered cascade. T E S T is only a geometrical examination of the PROFIL. W I N G is a test for the theoretical behaviour of the blade, if exposed oscillating to a supersonic stream. Finally, C A S C A D E is a computer model for a finite cascade of airfoils which may oscillate with a phase lag from blade to blade in a supersonic flow. C A S C A D E is not yet finished. Results are available up to now only for the inlet flow, as the wake could not be added because of lack of time.

All three programs do not only give numerical but also graphical results. The latter can be obtained by the two plot codes P L O T 1 and P L O T 2 .

C A S C A D E - and W I N G -results are compared in this report with solutions of Teipel (2), Verdon (15,17,18) and Platzer and collaborators (3,4,5, 6,7,9,13,14). For the linear cases the agreement is considered to be very good. The included thickness effect gives different results to those from Strada (5). This has to be investigated more closely in the near future.

It is clearly understood that C A S C A D E is a rather expensive approach to the problem. This is considered worth while due to the complexity of the problem. After the model of shaped, oscillating airfoils is better understood, it would be desirable to write a more efficient computer code.

To finish this work, there are two more steps to do:

1. Adding the procedure of wake computation to C A S C A D E and then obtaining results over the whole blades.
2. Optimizing the program in this form.

This will be done in the near future.

When this is accomplished, a rather flexible logic structure for the problem of an oscillating supersonic cascade is available. This could be used to examine different systems of basic equations and their influence on the results with the final purpose to substitute the potential equations by those of Euler. Thus we could get rid of the assumption of constant entropy in the field which would be a new step forward.

```
CXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C       OSCILLATING SINGLE AIRFOIL FOR MACH.GT.1
C
        COMPLEX*8 U,V,AI,PSI,G,DYDXU,PU
        COMPLEX*16 EL,RI,ES
        COMMON/BA/ ALO,ALD,AM,C1,AK,I,IE,IW
        COMMON/BC/ T,B,DYDX,D2YDX2,DYDXU,AI,I1,T3
        COMMON/SF/ XS(2,50),YS(2,50),A(50,4),R(2,50)
        COMMON/SOL/ EL(4,4),RI(4),ES(4)
        COMMON V(2,25,25),X(2,25,25),P(2,25),U(2,25,25),PSI(2,25,25),
       FG(2,25,25),AL(2,25,25),Y(2,25,25),C(2,3),DX(2),IN(2,25),
       FPX(2,20),PS(2,20),PU(2,20),TET(2),TED(20)
C
C       OPTIONS:
C       LO1 =0   : COMPLETE OUTPUT
C           =1   : ONLY PRESSURE DISTRIBUTION
C           =2   : STOP
C       LO2 =0   : PITCH
C           =1   : PLUNGE
C       LO3 =0   : NO WAKE
C           =1   : WAKE SHALL BE COMPUTED
C       LO4 =1   : SURFACES ARE ANALYTICALLY GIVEN
C           =2   : WEDGE (NO WAKE   )
C           =0   : SURFACES ARE POINTWISE GIVEN
C       MA = NUMBER OF POINTS ON THE SURFACE
C
        KV=25
        AI=CMPLX(0.,1.)
102     READ(4,999) LO1,LO2,LO3,LO4,MA
        IF(LO1.EQ.2) GOTO 101
        IF(LO4.EQ.2) LO3=0
        MAX=MA
        CALL PROFIL(WE,LO4,T1,T2,NX)
100     READ(4,1000) AK,AM,C,B,DX(1),DX(2)
        IF(AM.EQ.0.) GOTO 102
        AX=AM
        AM=AM*AM
        ALO=AM-1.
        C1=C+1.
        ALD=ALO**1.5
        C3=C1*AM*1.5
C
C       INITIAL FIELD
C
        XM=1./SQRT(ALO)
        D=3.1/9.
        DO 8 N=1,10
        M=N+1
        Y(1,M,N)=0.
        Y(1,N,M)=0.
        X(1,M,N)=D*(N-1)-2.
8       X(1,N,M)=X(1,M,N)
        DO 36 M=3,KV
        X(1,M,1)=D*(M-2)/2.-2.
        X(1,1,M)=X(1,M,1)
        Y(1,M,1)=XM*(2.+X(1,M,1))
36      Y(1,1,M)=-Y(1,M,1)
        DO 37 N=2,10
        L=N+2
        DO 37 M=L,KV
        K=M-1
        X(1,M,N)=X(1,K,N)+0.5*D
        X(1,N,M)=X(1,M,N)
        Y(1,M,N)=Y(1,K,N)+XM*D/2.
37      Y(1,N,M)=-Y(1,M,N)
        DO 45 M=1,KV
        DO 45 N=1,KV
        PSI(1,M,N)=0.
        AL(1,M,N)=ALO
        U(1,M,N)=0.
        V(1,M,N)=0.
45      G(1,M,N)=0.
C
C       THE STEADY FLOW FIELD
```

```
C       COMPUTATION OF THE SPLINE - COEFFICIENTS
C
        IW=0
        IE=0
        T3=0.
        I=0
30      I=I+1
        T=T1
        IF(I.EQ.2) T=T2
        DO 46 K=1,NX
        A(K,1)=YS(I,K)
        A(K,3)= R(I,K)
        H=XS(I,K+1)-XS(I,K)
        A(K,2)=(YS(I,K+1)-YS(I,K))/H-H*(R(I,K+1)+2.*R(I,K))/3.
46      A(K,4)=(R(I,K+1)-R(I,K))/(3.*H)
        I1=(-1)**(I+1)
16      MA=MAX
        MZ=1
C
C       THE STEADY BOUNDERY-PROPERTIES BEHIND THE SHOCK
C
        DX1=1.07/(MA-1)
        DO 9 J=2,KV
        KT=J
        XP=DX(I)*(J-2)
        IF(T.EQ.0.) XP=0.
        IM=1
        CALL SWITCH(J,IM,M,N,I)
9       CALL SHOCK(KV,WE,LO4,M,N,XP,C3,MA,DX1)
C
C       ALL OTHER STEPS OF THE STEADY FLOW FIELD
C
        KA=KT
        DO 1 J=2,KV
        J1=J
        IM=J+1
        CALL SWITCH(IM,J,M,N,I)
        CALL RAND(WE,LO4,M,N,C3)
        IF(X(2,M,N).GT.1.) MZ=0
        KB=KA+1
        DO 3 J2=J,KB
        IF(J2.EQ.KB) GOTO 3
        IM=J-1
        CALL SWITCH(J2,IM,K1,K2,I)
        IF(AL(2,K1,K2).GT.AL(2,M,N)) GOTO 6
3       CONTINUE
6       J2=J2-1
        IN(I,J-1)=J2
        L=J+2
        DO 7 K=L,J2
        CALL SWITCH(K,J,M,N,I)
7       CALL GEN(M,N,I,IW)
        IF(MZ.EQ.0) GOTO 15
        IF(J2.EQ.KA) GOTO 1
        J3=M+1
        J4=N-1
        J5=M
        J6=N-1
        IF(I.NE.2) GOTO 12
        J3=M-1
        J4=N+1
        J5=M-1
        J6=N
12      D1=(Y(2,J3,J4)-Y(2,J5,J6))/(X(2,J3,J4)-X(2,J5,J6))*I1
        D2=1./SQRT(AL(2,M,N))
        K1=M+1
        K2=J
        IF(I.NE.2) GOTO 14
        K1=J
        K2=N+1
14      AL(2,K1,K2)=AL(2,M,N)
        X(2,K1,K2)=(Y(2,J5,J6)-Y(2,M,N))*I1
        X(2,K1,K2)=-D2*X(2,M,N)+D1*X(2,J5,J6)-X(2,K1,K2)
        X(2,K1,K2)=X(2,K1,K2)/(D1-D2)
```

```
      Y(2,K1,K2)=C2*(X(2,K1,K2)-X(2,M,N))*I1+Y(2,M,N)
      IF(I.EQ.2) J3=J4
      DO 11 K=J3,KA
      IF(K.EQ.KV) GOTO 11
      IM=K+1
      CALL SWITCH(IM,J,M,N,I)
      IM=J-1
      CALL SWITCH(K,IM,K1,K2,I)
      X(2,M,N)=X(2,K1,K2)
      Y(2,M,N)=Y(2,K1,K2)
      AL(2,M,N)=AL(2,K1,K2)
11    CONTINUE
      IF(KA.LT.KV) KA=KA+1
      IF(MA.GT.IN(I,J-1)) MA=MA+1
1     CONTINUE
15    CONTINUE
      IF(T.EQ.0.) GOTO 47
C
C     AUTOMATIC STEP - SIZE CONTROL
C
      IF(J1.GT.MA) DX(I)=DX(I)*1.05
      IF(J1.LT.MA) DX(I)=DX(I)*0.96
      IF(J1.NE.MA) GOTO 16
C
C     THE UNSTEADY FLOW FIELD
C     THE UNSTEADY BOUNDERY PROPERTIES BEHIND SHOCK
C
47    KA=KT
      IF(LO1.EQ.0) WRITE(6,1007)
      DO 18 J=2,KT
      IM=1
      CALL SWITCH(J,IM,M,N,I)
18    CALL RANDS(KV,M,N,DX(I),LO2,M2,N2)
      X(2,1,2)=0.
      Y(2,1,2)=0.
      X(2,2,1)=0.
      Y(2,2,1)=0.
C
C     ALL OTHER STEPS OF UNSTEADY FLOW FIELD
C
      J5=J1-1
      DO 20 J=2,J5
      IM=J+1
      CALL SWITCH(IM,J,M,N,I)
      CALL RANDB(M,N,AM,AK,I,LO2)
      L=J+2
      J2=IN(I,J-1)
      DO 22 K=L,J2
      CALL SWITCH(K,J,M,N,I)
22    CALL GENU(M,N,I,AK,AM)
      IF(J2.EQ.KA) GOTO 20
      K1=M+1
      K2=J
      K3=K1
      IF(I.NE.2) GOTO 24
      K1=J
      K2=N+1
      K3=K2
24    P(I,K3)=X(2,M,N)
      CALL RANDS(KV,K1,K2,DX(I),LO2,M2,N2)
      J3=J2+1
      DO 29 K=J3,KA
      IF(K.EQ.KV) GOTO 20
      IM=K+1
      CALL SWITCH(IM,J,M,N,I)
      IM=J-1
      CALL SWITCH(K,IM,K1,K2,I)
      U(2,M,N)=U(2,K1,K2)
      V(2,M,N)=V(2,K1,K2)
      G(2,M,N)=G(2,K1,K2)
29    PSI(2,M,N)=PSI(2,K1,K2)
      IF(J2.LT.KA .AND. KA.LT.KV) KA=KA+1
20    CONTINUE
C
```

```
C     OUTPUT STEADY AND UNSTEADY FIELD
C
      IF(LO1.NE.0) GOTO 38
      WRITE(6,1004)
      IF(I.EQ.1) WRITE(6,1002)
      IF(I.EQ.2) WRITE(6,1003)
      J5=J1+1
      DO 25 J=2,J5
      K=J-1
      LL=IN(I,K)
      WRITE(6,1007)
      WRITE(6,1008)
      N=K
      IF(I.NE.2) GOTO 26
      M=K
      DO 31 N=J,LL
31    WRITE(6,1009) M,N,X(2,M,N),Y(2,M,N),AL(2,M,N),U(2,M,N),
     FV(2,M,N),PSI(2,M,N),G(2,M,N)
      GOTO 17
26    CONTINUE
      DO 32 M=J,LL
32    WRITE(6,1009) M,N,X(2,M,N),Y(2,M,N),AL(2,M,N),U(2,M,N),
     FV(2,M,N),PSI(2,M,N),G(2,M,N)
17    WRITE(6,1005)
25    CONTINUE
C
C     COMPUTATION OF THE PRESSURE - COEFFICIENTS
C
38    K=J1+1
      DO 27 J=2,K
      IM=J-1
      CALL SWITCH(J,IM,M,N,I)
27    CALL PRESS(M,N,J1)
      IF(I.EQ.1) IUS=K-1
      IF(I.EQ.2) ILS=K-1
      IF(I.EQ.1) GOTO 30
C
C     CHANGING THE FIELDS AND WAKE - COMPUTATION
C
      DO 48 M=1,KV
      DO 48 N=1,KV
      X(1,M,N)=X(2,M,N)
      Y(1,M,N)=Y(2,M,N)
      U(1,M,N)=U(2,M,N)
      V(1,M,N)=V(2,M,N)
      G(1,M,N)=G(2,M,N)
      AL(1,M,N)=AL(2,M,N)
48    PSI(1,M,N)=PSI(2,M,N)
      CALL WAKE(KV,WE,T1,T2,LO4,IUS,ILS,LO3,NX,MA)
C
C     CORRECTION OF THE INDEX NUMBERS
C
      DO 35 I=1,2
      K=IUS
      IF(I.EQ.2) K=ILS
      KA=K
      DO 43 J=1,K
      IF(IN(I,J).GT.K) GOTO 35
      KA=KA-1
      J1=IN(I,J)
      DO 44 L=J1,KA
      M=L+1
      PX(I,L)=PX(I,M)
      PU(I,L)=PU(I,M)
44    PS(I,L)=PS(I,M)
      IF(I.EQ.1) IUS=KA
43    IF(I.EQ.2) ILS=KA
35    CONTINUE
C
C     OUTPUT PRESSURE DISTRIBUTION
C
      WRITE(6,1004)
      IF(LO2.EQ.0) WRITE(6,996)
      IF(LO2.EQ.1) WRITE(6,995)
```

```
      WRITE(6,1001) AK,AX,C,B,DX(1),T1
      WRITE(6,1010)
      WRITE(6,1011)
      DO 33 I=1,2
      IF(I.EQ.2) WRITE(6,1001) AK,AX,C,B,DX(2),T2
      IF(I.EQ.2) WRITE(6,1012)
      IF(I.EQ.2) WRITE(6,1011)
      K=IUS
      IF(I.EQ.2) K=ILS
      WRITE(1,1016) K
      DO 34 J=1,K
      IM=J+1
      CALL SWITCH(IM,J,M,N,I)
      WRITE(6,1013) M,N,PX(I,J),PS(I,J),PL(I,J)
34    WRITE(1,1017) PX(I,J),PL(I,J)
33    WRITE(6,1014)
C
C     INTEGRATION OF THE MOMENTUM AND FORCES,POINTWISE GIVEN,
C     OVER THE SURFACES OF THE AIRFOIL
C
      CALL LIFT(IUS,ILS,LO2)
      GOTO 100
101   K=0
      WRITE(1,1016) K
C
C     END OF MAIN PROGRAM
995   FORMAT(1X,'PLUNGE - MODE',/)
996   FORMAT(1X,'PITCH - MODE',/)
999   FORMAT(4I1,I3)
1000  FORMAT(4F10.5,2E10.3)
1001  FORMAT(1X,/, 1X,'W*C/U= ',F5.3,', M= ',F4.2,', K= ',F4.2,
     F', B/C= ',F4.2,', CX= ',E9.3,', T/C= ',F6.4,//)
1002  FORMAT(1X,'   THE PROPERTIES AT THE MESHPOINTS OF THE FLOW',
     F' FIELD',/,1X,'   FOR THE UPPER SURFACE:',//)
1003  FORMAT(1X,'   THE PROPERTIES AT THE MESHPOINTS OF THE FLOW',
     F' FIELD',/,1X,'   FOR THE LOWER SURFACE:',//)
1004  FORMAT(1H1)
1005  FORMAT(1X,/)
1007  FORMAT(1X,//)
1008  FORMAT(2X,'POINT',7X,'X',10X,'Y',7X,'LAMBDA',7X,'RU',9X,'IU',
     F9X,'RV',9X,'IV',8X,'RPSI',7X,'IPSI',8X,'RG',9X,'IG',/)
1009  FORMAT(2X,I2,',',I2,11(3X,F8.5))
1010  FORMAT(1X,'PRESSURE-DISTRIBUTION UPPER SURFACE:',//)
1011  FORMAT(2X,'POINT',4X,'X',8X,'CPS',8X,'RCPU',8X,'ICPU',/)
1012  FORMAT(1X,'PRESSURE-DISTRIBUTION LOWER SURFACE:',//)
1013  FORMAT(1X,I2,',',I2,4X,F5.3,3(2X,E10.3))
1014  FORMAT(//,1X,'*********************************************',
     F'******',//)
1016  FORMAT(I3)
1017  FORMAT(3F10.5)
      END
CXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
CXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
      SUBROUTINE PROFIL(WE,LO4,T1,T2,N)
      COMMON/SP/ XS(2,50),YS(2,50),A(50,4),R(2,50)
C
C     PREPARATION OF THE PROFIL - SURFACES
C
      READ(4,1000) T1,T2,NT,IKP
      IF(LO4.EQ.0) READ(4,1001) SP
      IF(LO4.EQ.2) READ(4,1001) WE
      DO 8 J=1,2
      IZ=0
      N=NT
      T=T1
      IF(J.EQ.2) T=T2
      IF(LO4.NE.0) GOTO 12
      DO 9 K=1,4
      M=(K-1)*5+1
      READ(4,1001) XS(J,M),XS(J,M+1),XS(J,M+2),XS(J,M+3),XS(J,M+4)
      READ(4,1001) YS(J,M),YS(J,M+1),YS(J,M+2),YS(J,M+3),YS(J,M+4)
      IF(YS(J,M+4).EQ.100.) GOTO 7
9     CONTINUE
7     CONTINUE
```

```
      DC 2 LL=1,N
      XS(J,LL)=XS(J,LL)/SP
2     YS(J,LL)=YS(J,LL)/SP
      IF(L04.EG.0) GOTO 3
12    DX=1.0/(N-1)
      I1=(-1)**(J+1)
      DC 20 K=1,N
      XS(J,K)=(K-1)*CX-0.25
20    YS(J,K)=I1*4.*T*XS(J,K)*(1.-XS(J,K))
C
C     TC ENTER THIS PART,THE SURFACES SHCULD ALREADY BE
C     GIVEN PCINTWISE
C     EXTRAPOLATING POINTS
C
3     CCNTINUE
C
C     INTERPOLATION THROUGH CUBIC SPLINES
C
      IF(T.NE.0. .OR. L04.EG.C) GCTC 51
      DC 52 I=1,N
52    A(I,1)=0.
      GCTO 53
51    CCNTINUE
      DO 10 I=1,N
      A(I,3)=XS(J,I)
10    A(I,4)=YS(J,I)
C
C     MATRIX CF COEFFICIENTS AND RIGHT SICES
C
      K=N-2
      DC 25 I=1,K
      A(I,1)=A(I+1,3)-A(I,3)
      A(I,3)=A(I+2,3)-A(I+1,3)
      A(I,2)=2.*(A(I,1)+A(I,3))
      A(I,4)=3.*(A(I+2,4)-A(I+1,4))/A(I,3)-
     F3.*(A(I+1,4)-A(I,4))/A(I,1)
25    CCNTINUE
      A(1,1)=C.0
      A(N-2,3)=0.0
C
C     THE STEP OF GAUSS
C
      K=N-3
      DC 30 I=1,K
      DC 35 M=3,4
35    A(I,M)=A(I,M)*A(I+1,1)/A(I,2)
      A(I,1)=C.0
      A(I,2)=A(I+1,1)
      A(I+1,2)=A(I+1,2)-A(I,3)
30    A(I+1,4)=A(I+1,4)-A(I,4)
C
C     SCLUTION
C
      A(1,1)=0.
      A(N,1)=C.
      A(N-2,1)=0.C
      L=N-1
      DC 40 I=2,L
      K=N-I
      M=K+1
40    A(M,1)=(A(K,4)-A(K,3)*A(M+1,1))/A(K,2)
53    CCNTINUE
      DO 11 M=1,N
11    R(J,M)=A(M,1)
      IF(IZ.EQ.1) GOTC 8
      KV=N-1
      DC 49 K=1,KV
      A(K,1)=YS(J,K)
      A(K,3)= R(J,K)
      H=XS(J,K+1)-XS(J,K)
      A(K,2)=(YS(J,K+1)-YS(J,K))/H-H*(R(J,K+1)+2.*R(J,K))/3.
49    A(K,4)=(R(J,K+1)-R(J,K))/(3.*H)
      DX=1./49.
      DC 4 M=1,50
```

```
      X=DX*(M-1)
      DC 5 K=2,50
      I=K-1
      IF(XS(J,K).GE.X) GOTO 6
5     CCNTINUE
6     H=X-XS(J,I)
      Y=A(I,1)+A(I,2)*H+A(I,3)*H*H+A(I,4)*H*H*H
      R(J,M)=X
      YS(J,M)=Y
4     CONTINUE
      IZ=1
      N=50
      DC 13 I=1,50
13    XS(J,I)=R(J,I)
      GCTO 3
8     CCNTINUE
      IF(IKP.EQ.0) GOTO 17
      DC 15 M=1,N
15    WRITE(6,1002) XS(1,M),YS(1,M),XS(2,M),YS(2,M)
17    N=N-1
1000  FCRMAT(2F10.5,2I2)
1001  FCRMAT(5F10.5)
1002  FORMAT(5X,4(E12.5,2X))
      RETURN
      END
CXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
CXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
      SLBROUTINE FIND(KV,IW,XS,YS,M,N,IE,I)
      CCMPLEX*8 U,V,AI,PSI,G,DYDXU
      CCMMON/BC/ T,B,DYDX,D2YDX2,DYDXU,AI,I1
      CCMMON V(2,25,25),X(2,25,25),P(2,25),U(2,25,25),
     FPSI(2,25,25),G(2,25,25),AL(2,25,25),Y(2,25,25),
     FG(2,3),DX(2),IN(2,25)
C
C     FIND LOOKS FOR THE MESHINDEX,RESPCNSIBEL FCR
C     THE PCINT  XS;YS
C
      IF(IE.EQ.1) GOTO 19
      IKK=0
C     IF(IW.GT.5 .AND. XS.GE.1. .AND. XS.LE.1.10) IKK=1
      IF(IKK.EQ.1) WRITE(6,2011) I,IE,I1
2011  FCRMAT(1X,'FIND ENTRY: ',3I4)
      IZ=0
      KA=0
      L1=1
      L2=1
21    I9=21
      IF(IKK.EQ.1) WRITE(6,2000) I9,I,IE,XS,YS
      DO 22 I2=L1,20
      IM=I2+L2
      CALL SWITCH(IM,I2,M,N,I)
      L3=I2+L2
      I9=26
      IF(IKK.EQ.1) WRITE(6,2CCC) I9,M,N,X(1,M,N),Y(1,M,N)
      IF(L3.GE.KV) GOTO 13
      J1=M+1
      J2=N+1
24    KM=J1-J2
      IF(I.EQ.2) KM=J2-J1
      I9=24
      IF(IKK.EQ.1) WRITE(6,20CC) I9,M,N,X(1,M,N),Y(1,M,N)
      IF(XS.GE.X(1,M,N) .AND. XS.LT.X(1,J1,J2)) GCTC 23
      IF(X(1,J1,J2).GT.1. .AND. KM.EQ.1) KA=J2
      IF(I.EQ.2 .AND. KA.EQ.J2) KA=J1
      IF(I.EQ.1 .AND. KA.NE.J2) GOTO 22
      IF(I.EQ.2 .AND. KA.NE.J1) GOTC 22
      IF(I.EQ.1) J1=J1+1
      IF(I.EQ.2) J2=J2+1
      IF(J1.EQ.KV .OR. J2.EQ.KV) GOTO 13
      GCTO 24
22    I9=22
      IF(IKK.EQ.1) WRITE(6,2CCC) I9,M,N,X(1,M,N),Y(1,M,N)
23    I9=23
      IF(IKK.EQ.1) WRITE(6,2000) I9,M,N,X(1,M,N),Y(1,M,N)
```

```
      IF(I.EQ.2) GOTO 27
      IF(YS.LT.Y(1,J1,N)) GOTO 1
      IF(N.EQ.1) GOTO 13
      L2=L2+1
      L1=N-1
      GOTO 21
27    I9=27
      IF(IKK.EQ.1) WRITE(6,2000) I9,M,N,X(1,M,N),Y(1,M,N)
      IF(YS.GT.Y(1,M,J2)) GOTO 1
      IF(M.EQ.1) GOTO 13
      L2=L2+1
      L1=M-1
      GOTO 21
1     I9=1
      IF(IKK.EQ.1) WRITE(6,2000) I9,M,N,X(1,M,N),Y(1,M,N)
      M2=M
      N2=N
      GOTO 14
28    I9=28
      IF(IKK.EQ.1) WRITE(6,2000) I9,M,N,X(1,M,N),Y(1,M,N)
      IZ=IZ+1
      IF(IZ.EQ.5) GOTO 29
      M=M2
      N=N2
18    I9=18
      IF(IKK.EQ.1) WRITE( 6,2000) I9,M,N,X(1,M,N),Y( 1,M,N)
      IF(IZ.EQ.0) GOTO 28
      IF(IZ.EQ.1 .AND. I.EQ.1) N=N-1
      IF(IZ.EQ.1 .AND. I.EQ.2) M=M-1
      IF(M.EQ.0 .OR. N.EQ.0) GOTO 28
      IF(IZ.EQ.2 .AND. I.EQ.1) M=M+1
      IF(IZ.EQ.2 .AND. I.EQ.2) N=N+1
      IF(M.EQ.KV .OR.  N.EQ.KV) GOTO 28
      IF(IZ.EQ.3 .AND. I.EQ.1) N=N+1
      IF(IZ.EQ.3 .AND. I.EQ.2) M=M+1
      IF(IZ.NE.3) GOTO 3
      IF(N.EQ.M) GOTO 28
3     IF(IZ.EQ.4 .AND. I.EQ.1) M=M-1
      IF(IZ.EQ.4 .AND. I.EQ.2) N=N-1
      IF(IZ.NE.4) GOTO 2
      IF(M.EQ.N) GOTO 28
2     CONTINUE
      GOTO 14
29    WRITE(6,1000) KK,M,N,XS,YS,X(1,M,N),Y(1,M,N)
13    IE=1
      WRITE(6,1001) M,N,XS,YS,IE
      GOTO 19
C
C     CONTROL - STEP
C
14    J1=M+1
      J2=N+1
      K1=M+1
      K2=N
      K3=M
      K4=N+1
      IF(I.NE.2) GOTO 20
      K1=M
      K2=N+1
      K3=M+1
      K4=N
20    I9=20
      IK1=0
      IF(IKK.EQ.1 .AND. M.EQ.14 .AND. N.EQ.13) IK1=1
      IF(IK1.EQ.1) WRITE(6,2000) I9,M,N,X(1,M,N),Y(1,M,N)
      IF(IK1.EQ.1) WRITE(6,2000) I9,J1,J2,X(1,J1,J2),X(1,J1,J2)
      IF(IK1.EQ.1) WRITE(6,2000) I9,K1,K2,X(1,K1,K2),Y(1,K1,K2)
      IF(IK1.EQ.1) WRITE(6,2000) I9,K3,K4,X(1,K3,K4),Y(1,K3,K4)
      IF(XS.EQ.X(1,M,N) .AND. YS.EQ.Y(1,M,N)) GOTO 19
      D1=I1*(Y(1,K1,K2)-Y(1,M,N))/(X(1,K1,K2)-X(1,M,N))
      IF(K4.EQ.K3) GOTO 11
      D2=I1*(Y(1,K3,K4)-Y(1,M,N))/(X(1,K3,K4)-X(1,M,N))
11    IF(K4.EQ.K3) D2=0.
      D12=I1*(YS-Y(1,M,N))/(XS-X(1,M,N))
```

```
      IF(K4.EQ.K3) GOTO 25
      D3=I1*(Y(1,J1,J2)-Y(1,K3,K4))/(X(1,J1,J2)-X(1,K3,K4))
25    IF(K4.EQ.K3) D3=0.
      D4=I1*(Y(1,J1,J2)-Y(1,K1,K2))/(X(1,J1,J2)-X(1,K1,K2))
      D34=I1*(Y(1,J1,J2)-YS )/(X(1,J1,J2)-XS)
      IF(D1.LT.D12 .OR. D2.GT.D12) GOTO 18
      IF(D3.LT.D34 .OR. D4.GT.D34) GOTO 18
      IF(XS.LT.X(1,M,N) .OR. XS.GE.X(1,J1,J2)) GOTO 18
1000  FORMAT(1X,'NO-FIND : ',I2,',',I2,4(2X,F8.3))
1001  FORMAT(1X,'FIND: ',I2,',',I2,2(2X,E10.3),'  IE= ',I2)
2000  FORMAT(1X,3I4,2X,F8.3,2X,F8.3)
19    RETURN
      END
CXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
CXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
      SUBROUTINE BOUND(WE,LO4,I,X,IW)
      COMPLEX*8 U,V,AI,PSI,G,DYDXU
      COMMON/BC/ T,B,DYDX,D2YDX2,DYDXU,AI,I1,T3
      COMMON/SP/ XS(2,50),YS(2,50),A(50,4),R(2,50)
      IF(IW.NE.0) GOTO 1
C
C     STEADY BOUNDERY CONDITIONS ALONG THE AIRFOIL
C
      IF(LO4.EQ.1) GOTO 3
      IF(LO4.EQ.2) GOTO 4
      DO 5 K=2,50
      J=K-1
      IF(XS(I,K).GE.X) GOTO 6
5     CONTINUE
6     H=X-XS(I,J)
      DYDX=A(J,2)+2.*A(J,3)*H+3.*A(J,4)*H*H
      D2YDX2=2.*A(J,3)+6.*A(J,4)*H
      GOTO 2
3     DYDX=I1*4.*T*(1.-2.*X)
      D2YDX2=-I1*8.*T
      GOTO 2
4     W=WE*4.*ATAN(1.)/180.
      DYDX=I1*TAN(W)
      D2YDX2=0.
      GOTO 2
C
C     STEADY BOUNDERY CONDITIONS ALONG WAKE-SLIP-LINE
C
1     DYDX=TAN(T3)
      D2YDX2=0.
2     RETURN
      END
CXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
CXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
      SUBROUTINE SHOCK(KV,WE,LO4,M,N,XP,C3,MA,DX1)
      COMPLEX*8 U,V,AI,PSI,G,DYDXU
      COMMON/BA/ ALO,ALD,AM,C1,AK,I,IE,IW
      COMMON/BC/ T,B,DYDX,D2YDX2,DYDXU,AI,I1,T4
      COMMON V(2,25,25),X(2,25,25),P(2,25),U(2,25,25),
     FPSI(2,25,25),G(2,25,25),AL(2,25,25),Y(2,25,25),
     FQ(2,3),DX(2),IN(2,25)
C
C     COMPUTATION OF THE FIELD NEAR BEHIND THE SHOCK
C
      ITE=0
      IKK=0
C     IF(IW.NE.0) IKK=1
      IF(IKK.EQ.1) WRITE(6,2000) M,N,I,IE
2000  FORMAT(1X,'SHOCK ENTRY: ',4I4)
      TX=0.
      IF(IW.NE.0) TX=1.
      J=M-1
      K=N
      L=M
      IF(I.NE.2) GOTO 5
      J=M
      K=N-1
      L=N
5     CALL BOUND(WE,LO4,I,XP,IW)
```

```
      AL(2,M,N)=(ALD-C3*DYDX*I1)**(2./3.)
      IF(L.EQ.2) GOTO 17
      D1=I1/SQRT(AL(2,M,N))
      X5=X(2,J,K)
      Y5=Y(2,J,K)
      CALL FIND(KV,IW,X5,Y5,M2,N2,IE,I)
9     IK=9
      IF(IKK.EQ.1) WRITE(6,1000) IK,IE,AL(2,M,N),X5,Y5
      IL=9
      IF(IE.EQ.1 .OR. M2.GE.KV) GOTO 16
      L1=M2+1
      L2=N2
      IF(I.NE.2) GOTO 7
      L1=M2
      L2=N2+1
7     X1=X(1,M2,N2)
      Y1=Y(1,M2,N2)
      X2=X(1,L1,L2)
      Y2=Y(1,L1,L2)
      X3=X(1,M2+1,N2+1)
      Y3=Y(1,M2+1,N2+1)
      IZ=0
14    D2=SQRT(AL(1,M2,N2))+SQRT(AL(2,J,K))
      D2=2.*I1/D2
      IK=7
      IF(IKK.EQ.1) WRITE(6,1000) IK,M2,X1,X2,X3
      IF(T.NE.0. .AND. IW.EQ.C .AND. ITE.EQ.0) GOTO 2
      IF(T4.NE.0. .AND. IW.NE.C .AND. ITE.EQ.0) GOTO 2
      X4=X(2,J,K)+DX1/2.
      Y4=Y(2,J,K)+(X4-X(2,J,K))*D2
      IK=14
      IF(IKK.EQ.1) WRITE(6,1000) IK,IE,X4,Y4,X5,Y5
      GOTO 13
2     X4=(D1*XP-D2*X5+Y5)/(D1-D2)
      Y4=(X4-XP)*D1
      IF(X4.LE.TX) ITE=1
      IF(ITE.EQ.1) GOTO 14
      IK=2
      IF(IKK.EQ.1) WRITE(6,1000) IK,IE,X4,Y4,X5,Y5
13    CALL FIND(KV,IW,X4,Y4,M1,N1,IE,I)
      IF(M1.EQ.M2 .AND. N1.EQ.N2 .AND. IE.EQ.0) GOTO 8
      IF(IW.NE.0 .AND. IE.EQ.1) GOTO 11
      X6=(Y2-Y5)/D2+X5
      IF(X6.GE.X2) GOTO 10
      D3=(Y2-Y1)/(X2-X1)
      X5=(Y1-Y5+D2*X5-D3*X1)/(D2-D3)
      Y5=D3*(X5-X1)+Y1
      IL=131
      IF(N2.EQ.1) GOTO 16
      N2=N2-1
      IF(I.EQ.2) GOTO 19
      IL=132
      IF(AL(1,M2,N2).NE.AL(1,M2-1,N2)) GOTO 16
      GOTO 20
19    IL=19
      IF(AL(1,M2,N2).NE.AL(1,M2,N2-1)) GOTO 16
20    CONTINUE
      IF(I.NE.2) GOTO 9
      M2=M2-1
      N2=N2+1
      GOTO 9
10    D3=(Y3-Y2)/(X3-X2)
      X5=(Y2-Y5+D2*X5-D3*X2)/(D2-D3)
      Y5=D3*(X5-X2)+Y2
      M2=M2+1
      IL=10
      IF(M2.EQ.KV) GOTO 16
      IF(I.NE.2) GOTO 9
      N2=N2+1
      M2=M2-1
      GOTO 9
8     X(2,M,N)=X4
      Y(2,M,N)=Y4
      GOTO 11
```

```
16     IE=1
       D2=2.*SQRT(ALO)+SQRT(AL(2,J,K))+SQRT(AL(2,M,N))
       D2=4.*I1/D2
       IK=16
       IF(IKK.EQ.1) WRITE(6,1000) IK,IL,XP,X5,Y5,X(2,M,N)
       IF(IKK.EQ.1) WRITE(6,1000) M,N,D1,D2,DX1
       IF(T.EQ.0.) GOTO 1
       X(2,M,N)=(D1*XP-D2*X5+Y5)/(D1-D2)
       Y(2,M,N)=(X(2,M,N)-XP)*D1
       GOTO 11
1      X(2,M,N)=X5+DX1/2.
       Y(2,M,N)=D2*DX1/2.+Y5
C
C      COMPUTATION OF THE ADDITIONAL POINTS FOR THE UNSTEADY
C      FLOW FIELD
C
11     D3=1./SQRT(AL(2,M,N))
       D4=1./SQRT(AL(2,J,K))
       P(I,L)=(D4*X(2,J,K)+D3*XP+Y(2,J,K)*I1)/(D3+D4)
       IF(L.EQ.3) P(I,L)=XP
       IF(T.EQ.0. .OR. IW.NE.0) P(I,L)=X(2,J,K)
       GOTO 12
17     X(2,M,N)=0.
       Y(2,M,N)=0.
       IF(IW.EQ.0) GOTO 12
       X(2,M,N)=1.
12     IF(IKK.EQ.1) WRITE(6,1000) M,N,X(2,M,N),Y(2,M,N),AL(2,M,N),
      FP(I,L)
1000   FORMAT(1X,2I4,4(2X,F8.3))
       RETURN
       END
CXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
CXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
       SUBROUTINE RAND(WE,LO4,M,N,C1)
       COMPLEX*8 U,V,AI,PSI,G,DYDXU
       COMMON/BA/ ALO,ALC,AM,C3,AK,I,IE,IW
       COMMON/BC/ T,B,DYDX,D2YDX2,DYDXU,AI,I1
       COMMON V(2,25,25),X(2,25,25),P(2,25),U(2,25,25),PSI(2,25,25),
      FG(2,25,25),AL(2,25,25),Y(2,25,25),G(2,25)
C
C      BOUNDERYSTEP OF THE STEADY FIELD TO THE AIRFOIL
C
       IKK=0
C      IF(LO4.EQ.0) IKK=1
       IF(IKK.EQ.1) WRITE(6,1002)
1002   FORMAT(1X,'RAND-ENTRY')
       A=-C1*I1
       J1=M
       J2=N-1
       IF(I.NE.2) GOTO 5
       J1=M-1
       J2=N
5      X(2,M,N)=X(2,J1,J2)
       DO 11 KX=1,15
       CALL BOUND(WE,LO4,I,X(2,M,N),IW)
       IF(IKK.EQ.1) WRITE(6,1001) M,N,KX,X(2,M,N),DYDX,D2YDX2
1001   FORMAT(1X,3I3,3E12.5)
       AL(2,M,N)=(ALO+A*DYDX)**(2./3.)
       IF(IW.NE.0) GOTO 2
       IF(DYDX.EQ.0. AND. D2YDX2.EQ.0.) GOTO 2
       F=2.*(X(2,M,N)-X(2,J1,J2))*I1/Y(2,J1,J2)
       F=F-(SQRT(AL(2,J1,J2))+SQRT(AL(2,M,N)))
       FS=2.*I1/Y(2,J1,J2)-A*D2YDX2/(3.*AL(2,M,N))
       D=F/FS
       X(2,M,N)=X(2,M,N)-D
       IF(ABS(D).LE.0.000001) GOTO 10
11     CONTINUE
       WRITE(6,1000) M,N,D
1000   FORMAT(1X,'BOUNDERYSTEP DID NOT CONVERGE: ',I2,',',I2,
      F'  D= ',E10.3)
10     Y(2,M,N)=0.
       CALL BOUND(WE,LO4,I,X(2,M,N),IW)
       AL(2,M,N)=(ALO+A*DYDX)**(2./3.)
       GOTO 1
```

```
2     Y(2,M,N)=0.
      D=-2.*I1/(SQRT(AL(2,J1,J2))+SQRT(AL(2,M,N)))
      X(2,M,N)=X(2,J1,J2)-Y(2,J1,J2)/D
1     RETURN
      END
CXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
CXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
      SUBROUTINE GEN(M,N,I,IW)
      COMPLEX*8 U,V,AI,PSI,G,DYDXU
      COMMON/BC/ T,B,DYDX,D2YDX2,DYDXU,AI,I1
      COMMON V(2,25,25),X(2,25,25),P(2,25),U(2,25,25),PSI(2,25,25),
     FG(2,25,25),AL(2,25,25),Y(2,25,25),C(2,25)
C
C     GENERAL STEP OF STEADY FIELD
C
      IKK=0
      IF(IKK.EQ.1) WRITE(6,1000)
1000  FORMAT(1X,'GEN - ENTRY')
      J1=M
      J2=N-1
      K1=M-1
      K2=N
      IF(I.NE.2) GOTO 5
      J1=M-1
      J2=N
      K1=M
      K2=N-1
5     D1=1./SQRT(AL(2,K1,K2))
      D2=2./(SQRT(AL(2,J1,J2))+SQRT(AL(2,K1,K2)))
      XY=D2*X(2,J1,J2)+D1*X(2,K1,K2)+(Y(2,J1,J2)-Y(2,K1,K2))*I1
      X(2,M,N)=XY/(D1+D2)
      Y(2,M,N)=D1*I1*(X(2,M,N)-X(2,K1,K2))+Y(2,K1,K2)
      AL(2,M,N)=AL(2,K1,K2)
      IF(IKK.EQ.1) WRITE(6,1001) M,N,X(2,M,N),Y(2,M,N),AL(2,M,N)
1001  FORMAT(1X,2I3,3(2X,E12.5))
      RETURN
      END
CXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
CXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
      SUBROUTINE RANDS(KV,M,N,X1,L,M2,N2)
      COMPLEX*8 U,V,AI,PSI,G,DYDXU,DU,DV,C2,C6
      COMPLEX*16 A,RI,ES
      COMMON/BA/ ALO,ALD,AM,C,AK,I,IE,IW
      COMMON/BC/ T,B,DYDX,D2YDX2,DYDXU,AI,I1,T4
      COMMON/SOL/ A(4,4),RI(4),ES(4)
      COMMON V(2,25,25),X(2,25,25),P(2,25),U(2,25,25),
     FPSI(2,25,25),G(2,25,25),AL(2,25,25),Y(2,25,25),
     FQ(2,3),DX(2),IN(2,25)
C
C     COMPUTATION OF UNSTEADY BOUNDERY-PROPERTIES ALONG SHOCK
C
      IKK=0
C     IF(IW.NE.0) IKK=1
      J =M
      IF(I.NE.2) GO TO 5
      J =N
5     IF(IKK.EQ.1) WRITE(6,2000)
2000  FORMAT (1X,'RANDS-ENTRY')
      CALL COEF1(KV,I1,M,N,M2,N2,AM1,AM2,AM3,AM4,AN1,AN2,AN3,
     FAN4,S)
      IK=5
      IF(IKK.EQ.1) WRITE(6,1001) IK,M,N,U(1,M2,N2),V(1,M2,N2),
     FAL(1,M2,N2)
      TX=1.
      DV=V(1,M2,N2)
      DU=U(1,M2,N2)
      ALT=AL(1,M2,N2)
      IF(IW.EQ.0.) TX=0.
      IF(IE.EQ.1) ALT=ALO
      IF(IE.EQ.1) DV=0.
      IF(IE.EQ.1) DU=0.
      IF(J.GT.2) GOTO 6
      CALL BOUNDU(IW,TX,I,AK,L)
      V(2,M,N)=DYDXU
```

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      D2=AM1*V(2,M,N)/AN1+DU*(AM3-AM1*AN3/AN1)
      U(2,M,N)=D2+DV*(AM4-AM1*AN4/AN1)
      K1=M+1
      K2=N
      IF(I.NE.2) GOTO 4
      K1=M
      K2=N+1
4     D1=(AL(2,M,N)+AL(2,K1,K2))/2.
      IK=4
      IF(IKK.EQ.1) WRITE(6,1001) IK,M2,N2,C1,X1,TX,S
      D1=I1/SQRT(D1)
      D3=I1/SQRT(AL(2,K1,K2))
      XX=-Y(2,K1,K2)/D3+X(2,K1,K2)-TX
      IF(T.EQ.0.) XX=0.
      X(2,M,N)=D1*XX/(TAN(S)+D1)+TX
      Y(2,M,N)=(X(2,M,N)-TX)*TAN(S)
      PSI(2,M,N)=PSI(1,M2,N2)
      P(I,3)=XX
      GOTO 9
6     J1=M-1
      J2=N
      LP=M
      IF(I.NE.2) GOTO 8
      J1=M
      J2=N-1
      LP=N
8     CONTINUE
      IF(AL(2,J1,J2).LT.AL(2,M,N) .OR. T.EQ.0.) GOTO 12
      IF(IW.NE.0) GOTO 12
      J2=N-1
      IF(I.NE.2) GOTO 12
      J1=M-1
12    DO=X(2,M,N)-P(I,LP)
      DAL=AL(2,M,N)
      IE=0
      CALL FIND(KV,IW,X(2,J1,J2),Y(2,J1,J2),M3,N3,IE,I)
      DU=U(1,M2,N2)+U(1,M3,N3)
      DV=V(1,M2,N2)+V(1,M3,N3)
      MP=1000000*(P(I,LP)-X(2,J1,J2))
      IF(MP.LT.0) WRITE(6,1002) M,N,P(I,LP),X(2,J1,J2)
1002  FORMAT(1X,'P-X.LT.0 :',2I4,2(2X,E12.5))
      IF(MP.NE.0) GOTO 3
      D1=0.
      GOTO 2
3     D1=(AL(2,M,N)-AL(2,J1,J2))*DO
      IK=3
      IF(IKK.EQ.1) WRITE(6,1001) IK,IW,IE,DU,DV,DAL
      C1=D1/(4.*DAL*(P(I,LP)-X(2,J1,J2)))
2     D2=AI*AK*AM*DO/DAL
      D3=-0.25*AM*(AK*DO)**2/DAL
      D4=1./SQRT(AL(2,M,N))
      D5=-D3*D4
      D6=1.*AK*AK*AM*PSI(2,J1,J2)*DO/DAL
      D7=Y(2,M,N)-Y(2,J1,J2)
      A(1,1)=1.+D1+D2+D3
      A(1,2)=-(D4+D5)*I1
      A(1,3)=C.
      RI(1)=U(2,J1,J2)*(2.-A(1,1))-V(2,J1,J2)*(D4-D5)*I1+D6
      A(2,3)=-2.*AM1/D7-AI*AM2
      RI(2)=-(2.*AM1/D7-AI*AM2)*G(2,J1,J2)-U(2,J1,J2)
      RI(2)=RI(2)+(AM3*DU+AM4*DV)
      A(2,1)=1.
      A(2,2)=0.
      A(3,3)=-2.*AN1/D7-AI*AN2
      RI(3)=-(2.*AN1/D7-AI*AN2)*G(2,J1,J2)-V(2,J1,J2)
      RI(3)=RI(3)+(AN3*DU+AN4*DV)
      A(3,1)=0.
      A(3,2)=1.
C
      NC=3
      CALL SOLVE(NO)
      U(2,M,N)=ES(1)
      V(2,M,N)=ES(2)
      G(2,M,N)=ES(3)
```

```
7       DU=U(2,M,N)+U(2,J1,J2)+(V(2,M,N)+V(2,J1,J2))*D4*I1
        PSI(2,M,N)=DU*0.5*D0+PSI(2,J1,J2)
9       RETURN
1001    FORMAT(1X,3I4,5(2X,E11.4))
        END
CXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
CXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
        SUBROUTINE BOUNDU(IW,X,I,AK,L)
        COMPLEX*8 U,V,AI,PSI,G,DYDXU
        COMMON/BC/ T,B,DYDX,D2YDX2,DYDXU,AI,I1
C
C       UNSTEADY BOUNDERY - CONDITIONS ALONG THE AIRFOIL
C
        IF(L.EQ.0) GOTO 10
        IF(L.EQ.1) GOTO 11
10      DYDXU=-1.-(X-B)*AK*AI
        GOTO 13
11      DYDXU=-AI*AK
13      RETURN
        END
CXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
CXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
        SUBROUTINE RANDB(M,N,AM,AK,I,L)
        COMPLEX*8 U,V,AI,PSI,G,DYDXU,D2,A,D
        COMMON/BC/ T,B,DYDX,D2YDX2,DYDXU,AI,I1
        COMMON V(2,25,25),X(2,25,25),P(2,25),U(2,25,25),PSI(2,25,25),
       FG(2,25,25),AL(2,25,25),Y(2,25,25)
C
C       BOUNDARY STEP OF THE UNSTEADY FIELD TO THE AIRFOIL
C
        CALL BOUNDU(IW,X(2,M,N),I,AK,L)
        V(2,M,N)=DYDXU
        J1=M
        J2=N-1
        IF(I.NE.2) GOTO 5
        J1=M-1
        J2=N
5       D1=0.5*(AL(2,M,N)-AL(2,J1,J2))
        D0=X(2,M,N)-X(2,J1,J2)
        D6=V(2,M,N)+V(2,J1,J2)
        D2=AI*AK*AM*D0*2.
        D3=-0.5*(AK*D0)**2*AM
        D4=AL(2,M,N)+AL(2,J1,J2)
        D5=SQRT(AL(2,M,N))+SQRT(AL(2,J1,J2))
        A=(D1+D2+D3)/D4
        U(2,M,N)=U(2,J1,J2)*(1.-A)-2.*(V(2,M,N)-V(2,J1,J2))*I1/D5
        D=2.*PSI(2,J1,J2)-D0*D6*I1/D5
        U(2,M,N)=U(2,M,N)+AK*AK*AM*D0*D/D4
        U(2,M,N)=U(2,M,N)/(1.+A)
        PSI(2,M,N)=U(2,M,N)+U(2,J1,J2)-2.*D6*I1/D5
        PSI(2,M,N)=PSI(2,J1,J2)+D0*PSI(2,M,N)/2.
        RETURN
        END
CXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
CXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
        SUBROUTINE GENU(M,N,I,AK,AM)
        COMPLEX*8 U,V,AI,PSI,G,DYDXU,D2,D6,B2,B6
        COMPLEX*16 A,RI,ES
        COMMON/BC/ T,B,DYDX,D2YDX2,DYDXU,AI,I1
        COMMON/SOL/ A(4,4),RI(4),ES(4)
        COMMON V(2,25,25),X(2,25,25),P(2,25),U(2,25,25),PSI(2,25,25),
       FG(2,25,25),AL(2,25,25),Y(2,25,25)
C
C       GENERAL STEP OF UNSTEADY FIELD
C
        J1=M
        J2=N-1
        J3=M-1
        J4=N
        IF(I.NE.2) GOTO 5
        J1=M-1
        J2=N
        J3=M
        J4=N-1
```

```
5     DO=X(2,M,N)-X(2,J1,J2)
      D1=0.5*(AL(2,M,N)-AL(2,J1,J2))
      D2=AI*2.*AK*AM*DO
      D3=-0.5*AM*(AK*DO)**2
      D4=AL(2,M,N)+AL(2,J1,J2)
      D5=SQRT(AL(2,M,N))+SQRT(AL(2,J1,J2))
      D6=(D1+D2+D3)/D4
      A(1,1)=1.+D6
      A(1,2)=2.*(1.-D3/D4)*I1/D5
      RI(1)=U(2,J1,J2)*(1.-D6)+V(2,J1,J2)*I1*2.*(1.+D3/D4)/D5
      RI(1)=2.*AK*AK*AM*PSI(2,J1,J2)*DO/D4+RI(1)
      BO=X(2,M,N)-X(2,J3,J4)
      B1=(AL(2,M,N)-AL(2,J1,J2))*BO/(2.*DO)
      B2=2.*AI*AK*AM*BO
      B3=-0.5*AM*(AK*BO)**2
      B4=1./SQRT(AL(2,M,N))
      B5=-0.5*B3*B4/AL(2,M,N)
      B6=0.5*(B1+B2+B3)/AL(2,M,N)
      A(2,1)=1.+B6
      A(2,2)=-(B4+B5)*I1
      RI(2)=U(2,J3,J4)*(1.-B6)+V(2,J3,J4)*I1*(-B4+B5)
      RI(2)=PSI(2,J3,J4)*AM*BO*AK*AK/AL(2,M,N)+RI(2)
C
      NO=2
      CALL SOLVE(NO)
      U(2,M,N)=ES(1)
      V(2,M,N)=ES(2)
      D2=V(2,J1,J2)+V(2,M,N)
      D2=(U(2,J1,J2)+U(2,M,N)-I1*2.*D2/D5)*DO/2.
      D6=(V(2,J3,J4)+V(2,M,N))*B4
      D6=(U(2,J3,J4)+U(2,M,N)+I1*D6)*BO/2.
      D2=PSI(2,J1,J2)+PSI(2,J3,J4)+D2+D6
      PSI(2,M,N)=D2/2.
      RETURN
      END
CXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
CXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
      SUBROUTINE PRESS (M,N,J1)
      COMPLEX*8 U,V,AI,PSI,G,DYDXU,PU
      COMMON/BA/ ALO,ALC,AM,C1,AK,I,IE,IW
      COMMON/BC/ T,B,DYDX,D2YDX2,DYDXU,AI,I1
      COMMON V(2,25,25),X(2,25,25),P(2,25),U(2,25,25),PSI(2,25,25),
     FG(2,25,25),AL(2,25,25),Y(2,25,25),Q(2,3),DX(2),IN(2,25),
     FPX(2,20),PS(2,20),PU(2,20)
C
C     COMPUTATION OF THE PRESSURE - COEFFICIENTS ALONG
C     THE AIRFOIL
C     PU=CPU,   PS=CPS
C
      K=N
      IF(I.EQ.2) K=M
      IF(K.EQ.J1) GOTO 5
      PS(I,K)=-2.*(AL(2,M,N)-ALC)/(C1*AM)
      PU(I,K)=-2.*(U(2,M,N)+AI*AK*PSI(2,M,N))
      PX(I,K)=X(2,M,N)
      GOTO 6
5     K1=K-1
      K2=K-2
      D2=PX(I,K1)-PX(I,K2)
      D1=(1.-PX(I,K1))/D2
      PS(I,K)=PS(I,K1)+(PS(I,K1)-PS(I,K2))*D1
      PU(I,K)=PU(I,K1)+(PU(I,K1)-PU(I,K2))*D1
      PX(I,K)=1.
6     RETURN
      END
CXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
CXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
      SUBROUTINE LIFT(IUS,ILS,LC2)
      COMPLEX*8 U,AI,CL,CM,DCL,DCM,PU,Q
      COMPLEX*8 P,DYDXU,CLS,DCLS,CMS
      COMMON/BC/ T,B,DYDX,D2YDX2,DYDXU,AI,I1
      COMMON X(2,100),U(2,100),Q(2,100),P(2,2),A(12850),
     FPX(2,20),PS(2,20),PU(2,20)
C
```

```
C       COMPUTATION OF THE LIFT- AND MOMENTUM - COEFFICIENTS
C       FOR BOTH OF THE SURFACES AND FOR THE COMPLETE AIRFOIL
C       LIFT: CL,CLS   ;   MOMENTUM: CM,CMS
C
        DO 4 I=1,2
        I1=(-1)**(I+1)
        K=IUS
        IF(I.EQ.2) K=ILS
        DO 5 J=1,K
        L=2*J-1
        X(I,L)=PX(I,J)
        U(I,L)=PU(I,J)
5       Q(I,L)=PS(I,J)
        M=K-1
        DO 7 J=1,M
        N=2*J
        L1=N-1
        L2=N+1
        X(I,N)=(X(I,L1)+X(I,L2))/2.
        U(I,N)=(U(I,L1)+U(I,L2))/2.
7       Q(I,N)=(Q(I,L1)+Q(I,L2))/2.
        CL=0.
        CM=0.
        CLS=0.
        CMS=0.
        N=2*K-2
        DO 8 J=1,N
        D1=X(I,J+1)-X(I,J)
        D2=((X(I,J+1)+X(I,J))/2.-B)*I1
        DCL =0.5*D1*(U(I,J)+U(I,J+1))
        DCLS=0.5*D1*(Q(I,J)+Q(I,J+1))
        CM =CM -DCL *D2
        CMS=CMS-DCLS*D2
        CL =CL -I1*DCL
8       CLS=CLS-I1*DCLS
        P(I,1)=CLS
        Q(I,1)=CMS
        P(I,2)=CL
4       U(I,1)=CM
        CLS=(P(2,1)+P(1,1))
        CL =(P(2,2)+P(1,2))
        CM =(U(2,1)+U(1,1))
        CMS=(Q(2,1)+Q(1,1))
        IF(LO2.EQ.1) CM=AI*CM
        IF(LO2.EQ.1) CL=AI*CL
        WRITE(6,1000)
        WRITE(6,1001)
        WRITE(6,1002) CL,CM
        WRITE(6,1003) CLS,CMS
1000    FORMAT(2X,'MOMENTUM- AND LIFT - COEFFICIENTS FOR A SINGLE'
       F,/,2X, 'AIRFOIL WITH A SURFACE DESCRIBED ABOVE:',//)
1001    FORMAT(14X,'RCL',6X,'ICL',6X,'RCM',6X,'ICM',/)
1002    FORMAT(2X,'UNSTEADY',4(2X,F7.4))
1003    FORMAT(2X,'  STEADY',4(2X,F7.4))
        RETURN
        END
CXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
CXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
        SUBROUTINE WAKE(KV,WE,TA,TB,LO4,IUS,ILS,LO3,NX,MA)
        COMPLEX*8 L,V,AI,PSI,CL,G,DYDXU,PL,AUU,ALL
        COMMON/BA/ ALO,ALD,AM2,E1,AK,I,IE,IW
        COMMON/BC/ T,Z,DYDX,D2YDX2,DYDXL,AI,I1,T4
        COMMON/SP/ XS(2,50),YS(2,50),A(50,4),R(2,50)
        COMMON V(2,25,25),X(2,25,25),P(2,25),U(2,25,25),PSI(2,25,25),
       FG(2,25,25),AL(2,25,25),Y(2,25,25),Q(2,3),DX(2),IN(2,25),
       FPX(2,20),PS(2,20),PU(2,20),TET(2),TED(20)
C
C       COMPUTATION OF THE FIELD BEHIND THE AIRFOIL
C
        IKK=0
        IF(LO3.EQ.0) GOTO 1
        DXS=0.5*(DX(1)+DX(2))
        EX=E1*AM2*1.5
        E=E1-1.
```

```
      E5=E-1.
      IW=0
      IE=0
C
C     COMPUTATION OF THE SPLINE COEFFICIENTS
C
      DO 2 I=1,2
      T=TA
      IF(I.EQ.2) T=TB
      I1=(-1)**(I+1)
      DO 46 K=1,NX
      A(K,1)=YS(I,K)
      A(K,3)= R(I,K)
      H=XS(I,K+1)-XS(I,K)
      I9=46
      IF(IKK.EQ.1) WRITE(6,2000) I9,K,I,JX,H,XS(I,K)
      A(K,2)=(YS(I,K+1)-YS(I,K))/H-H*(R(I,K+1)+2.*R(I,K))/3.
46    A(K,4)=(R(I,K+1)-R(I,K))/(3.*H)
C
C     THE STEADY WAKE
C
      XXS=1.
      YYS=0.
      CALL FIND(KV,IW,XXS,YYS,M,N,IE,I)
      I9=2
      IF(IKK.EQ.1) WRITE(6,2000) I9,IW,M,N,X(1,M,N)
      TE=2.*I1*(ALD-AL(1,M,N)**1.5)/3.
      TE=TE/(AL(1,M,N)+E*AM2+1.)
2     TET(I)=ATAN(TE)
      I9=2
      IF(IKK.EQ.1) WRITE(6,2000) I9,IW,M,N,DYDX
      M=2
      N=1
      XP1=1.
      XP2=1.
4     I3=0
      I9=4
      IF(IKK.EQ.1) WRITE(6,2000) I9,IW,M,N,TET(1),TET(2)
      DX1=DXS
      DX2=DXS
      DXU=1./(MA-1)
      DXL=DXU
      DO 6 I4=1,30
      XP3=XP1+DX1
      XP4=XP2+DX2
      IF(IW.EQ.0) GOTO 36
      N=1
      IF(IW.LE.2) T4=TED(1)
      IF(IW.LE.2) GOTO 28
      II=IW-1
      DO 29 I2=1,II
      IF(X(2,I2+1,I2).GT.XP3) GOTO 30
29    I9=29
      IF(IKK.EQ.1) WRITE(6,2000) I9,IW,M,N,XP3
30    T4=TED(I2-1)
      I9=30
      IF(IKK.EQ.1) WRITE(6,2000) I9,IW,M,N,T4
28    I=1
      I9=28
      IF(IKK.EQ.1) WRITE(6,2000) I9,IW,M,N,XP3,XP4
      T=TA
      I1=1
      CALL SHOCK(KV,WE,LO4,M,N,XP3,EX,MA,DXU)
      IF(IE.EQ.1) GOTO 17
      I=2
      I1=-1
      T=TB
      IF(IW.LE.2) GOTO 35
      DO 33 I2=2,II
      IF(X(2,I2,I2+1).GT.XP4) GOTO 34
33    I9=33
      IF(IKK.EQ.1) WRITE(6,2000) I9,IW,M,N,XP4
34    T4=TED(I2-1)
35    I9=35
```

```
      IF(IKK.EQ.1) WRITE(6,2000) I9,IW,M,N,T4
      IF(IKK.EQ.1) WRITE(6,2000) I,I1,IW,M,EX
      CALL SHOCK(KV,WE,L04,N,M,XP4,EX,MA,DXL)
      IF(IKK.EQ.1) WRITE(6,2000) I9,IW,M,N,X(2,1,M)
      IF(IE.EQ.1) GOTO 17
      IF(IW.GT.1) GOTO 12
      IW=IW+1
      M=M+1
      GOTO 4
12    I9=12
      IF(IKK.EQ.1) WRITE(6,2000) I9,IW,M,N,X(2,M,1)
      IF(IW.EQ.2) GOTO 7
      IX=IW-1
      DO 5 N=2,IX
      I=1
      I1=1
      CALL GEN(M,N,I,IW)
      I=2
      I1=-1
5     CALL GEN(N,M,I,IW)
      I9=5
      IF(IKK.EQ.1) WRITE(6,2000) I9,IW,M,N,X(2,M,N)
      DO 8 I2=1,2
      IM=N+1
      CALL SWITCH(IM,N,L1,L2,I2)
      I9=11
      IF(IKK.EQ.1) WRITE(6,2000) I9,IW,L1,L2,AL(2,L1,L2)
      I1=(-1)**(I2+1)
      TE=2.*I1*(ALD-AL(2,L1,L2)**1.5)/3.
      TE=TE/(AL(2,L1,L2)+E*AM2+1.)
8     TET(I2)=ATAN(TE)
36    I9=36
      IF(IKK.EQ.1) WRITE(6,2000) I9,IW,M,N,TET(1),TET(2)
      T1=TET(1)
      T2=TET(2)
      T3=0.5*(T1+T2)
      DO 47 IA=1,2
      IM=N+1
      CALL SWITCH(IM,N,L1,L2,IA)
      IF(N.GT.1) GOTO 53
      I1=(-1)**(IA+1)
      XXS=1.
      YYS=0.
      IE=0
      CALL FIND(KV,IW,XXS,YYS,M2,N2,IE,IA)
53    IF(N.EQ.1) ALT=AL(1,M2,N2)
      IF(N.GT.1) ALT=AL(2,L1,L2)
      XM=(1.-(ALT-ALO)*E/E1)**(E5/E)
      XM=2.*((1.+E5*AM2/2.)/XM-1.)/E5
      I9=47
      IF(IKK.EQ.1) WRITE(6,2000) I9,M,N,IA,XM,ALT
      IF(IA.EQ.1) XM1=XM
47    IF(IA.EQ.2) XM2=XM
      PII=((1.+E5*XM1/2.)/(1.+E5*XM2/2.))**(E/E5)
      CALL KONST(Q(1,1),Q(1,2),Q(1,3),XM1,E1)
      CALL KONST(Q(2,1),Q(2,2),Q(2,3),XM2,E1)
      DO 9 K=1,20
      T4=T3-T1
      T5=T3-T2
      F=E*XM1*(Q(1,1)*T4+Q(1,2)*T4*T4+Q(1,3)*T4**3)/2.+1.
      FS=E*XM2*(-Q(2,1)*T5+Q(2,2)*T5*T5-Q(2,3)*T5**3)/2.+1.
      F=F-FS*PII
      FS=E*XM2*(Q(2,1)-2.*Q(2,2)*T5+3.*Q(2,3)*T5*T5)/2.*PII
      FS=FS+E*XM1*(Q(1,1)+2.*Q(1,2)*T4+3.*Q(1,3)*T4*T4)/2.
      D=F/FS
      T3=T3-D
      IF(ABS(D).LE.0.000001) GOTO 10
9     CONTINUE
      WRITE(6,1000) M,N,T3,D
10    I9=10
      IF(IKK.EQ.1) WRITE(6,2000) I9,IW,M,N,T3
      IF(IW.EQ.0) IW=1
      TED(IW)=T3
      IF(IW.EQ.1) GOTO 4
```

```
7       N=N+1
        I9=7
        IF(IKK.EQ.1) WRITE(6,2000) I9,IW,M,N,X(2,M,N)
        T4=TED(IW)
        I=1
        T=TA
        I1=1
        CALL RAND(WE,L04,M,N,E1)
        I1=-1
        I=2
        T=TB
        CALL RAND(WE,L04,N,M,E1)
        D=X(2,M,N)-X(2,N,M)
        IF(ABS(D).LE.0.0001) GOTO 14
        IF(X(2,M,N).LT.X(2,N,M) .OR. I3.EQ.1) GOTO 15
16      I3=2
        I9=16
        IF(IKK.EQ.1) WRITE(6,2000) I9,IW,M,N,X(2,M,N),D
        DX1=DX1*((X(2,N,M)-1.)/(X(2,M,N)-1.))**(1.+0.08*(N-2))
        GOTO 6
15      I9=15
        IF(IKK.EQ.1) WRITE(6,2000) I9,IW,M,N,X(2,N,M),D
        IF(I3.EQ.2) GOTO 16
        I3=1
        DX2=DX2*((X(2,M,N)-1.)/(X(2,N,M)-1.))**(1.+0.08*(N-2))
6       CONTINUE
        WRITE(6,1000) M,N,X(2,M,N),X(2,N,M),D
14      I9=14
        IF(IKK.EQ.1) WRITE(6,2000) I9,IW,M,N,X(2,M,N)
        X(2,M,N)=(X(2,M,N)+X(2,N,M))/2.
        X(2,N,M)=X(2,M,N)
        XP1=XP3
        XP2=XP4
        IW=IW+1
        M=M+1
        GOTO 4
C
C       THE UNSTEADY WAKE
C
C       THE INITIAL STEP
17      IW=IW-1
        I=1
        M=2
        N=1
        I1=1
        CALL COEF1(KV,I1,M,N,M2,N2,AM1,AM2,AM3,AM4,AN1,AN2,AN3,
     FAN4,S1)
        IF(IKK.EQ.1)WRITE(6,1009)M2,N2,AM1,AM2,AM3,AM4,AN1,AN2,
     FAN3,AN4,S1
        I=2
        I1=-1
        CALL COEF1(KV,I1,N,M,M3,N3,AM5,AM6,AM7,AM8,AN5,AN6,AN7,
     FAN8,S2)
        IF(IKK.EQ.1)WRITE(6,1009)M3,N3,AM5,AM6,AM7,AM8,AN5,AN6,
     FAN7,AN8,S2
        AUU=U(1,M2,N2)*(AM3-AM1*AN3/AN1)+V(1,M2,N2)*(AM4-AM1*AN4/AN1)
        ALL=U(1,M3,N3)*(AM7-AM5*AN7/AN5)+V(1,M3,N3)*(AM8-AM5*AN8/AN5)
        PSI(2,M,N)=PSI(1,M2,N2)
        PSI(2,N,M)=PSI(1,M3,N3)
        T4=TED(1)
        CL=(PSI(2,N,M)-PSI(2,M,N))*(1.-AM1*TAN(T4)/AN1)*AI*AK
        CL=ALL-AUU+CL
        V(2,N,M)=CL/(AM1/AN1-AM5/AN5)
        U(2,N,M)=ALL+AM5*V(2,N,M)/AN5
        U(2,M,N)=U(2,N,M)+AI*AK*(PSI(2,N,M)-PSI(2,M,N))
        V(2,M,N)=V(2,N,M)+TAN(T4)*(U(2,M,N)-U(2,N,M))
        G(2,M,N)=0.
        G(2,N,M)=0.
        DO 24 I=1,2
        I1=(-1)**(I+1)
        IM=M+1
        CALL SWITCH(M,N,L1,L2,I)
        CALL SWITCH(IM,N,K1,K2,I)
        S=S1
```

```
      IF(I.NE.2) GOTO 27
      S=S2
27    IF(IKK.EQ.1) WRITE(6,1009) L1,L2,X(2,L1,L2),Y(2,L1,L2),
     FX(2,K1,K2),Y(2,K1,K2),AL(2,K1,K2),T4,S
      D1=1./SQRT(AL(2,L1,L2))
      D1=-I1*D1
      D3=I1/SQRT(AL(2,K1,K2))
      XX=-Y(2,K1,K2)/D3+X(2,K1,K2)
      IF(T4.EQ.O.) XX=1.
      X(2,L1,L2)=(D1*XX-TAN(S))/(D1-TAN(S))
      Y(2,L1,L2)=(X(2,L1,L2)-1.)*TAN(S)
      P(I,3)=XX
      X(2,L1,L2)=1.
      Y(2,L1,L2)=0.
24    P(I,3)=1.
C
C     FURTHER STEPS
C
      LO2=0
      DO 22 I=1,2
      IE=0
      I1=(-1)**(I+1)
      T=TA
      IF(I.EQ.2) T=TB
      DO 22 J1=3,IW
      IM=1
      CALL SWITCH(J1,IM,M,N,I)
22    CALL RANDS(KV,M,N,DX(I),LO2,M2,N2)
      X(2,2,1)=1.
      Y(2,2,1)=0.
      X(2,1,2)=1.
      Y(2,1,2)=0.
      M2=3
      N2=2
      CALL SLIP(M2,N2)
      DO 21 J1=4,IW
      K=J1-2
      DO 23 J2=2,K
      I1=1
      IM=1
      CALL GENU(J1,J2,IM,AK,AM2)
      I1=-1
      IM=2
23    CALL GENU(J2,J1,IM,AK,AM2)
      K=J1-1
21    CALL SLIP(J1,K)
C
C     OUTPUT OF THE WAKE FIELD
C
      DO 51 I=1,20
51    WRITE(6,1012) TED(I)
      J1=IW
      DO 18 I=1,2
      WRITE(6,1004)
      IF(I.EQ.1) WRITE(6,1010)
      IF(I.EQ.2) WRITE(6,1011)
      DO 25 J=2,J1
      WRITE(6,1007)
      WRITE(6,1008)
      K=J-1
      N=K
      IF(I.NE.2) GOTO 26
      M=K
      DO 31 N=J,J1
31    WRITE(6,1009) M,N,X(2,M,N),Y(2,M,N),AL(2,M,N),U(2,M,N)
     F,V(2,M,N),PSI(2,M,N),G(2,M,N)
      GOTO 25
26    CONTINUE
      DO 32 M=J,J1
32    WRITE(6,1009) M,N,X(2,M,N),Y(2,M,N),AL(2,M,N),U(2,M,N)
     F,V(2,M,N),PSI(2,M,N),G(2,M,N)
25    CONTINUE
18    WRITE(6,1004)
1000  FORMAT(1X,' WAKE BAD: ',2I4,2(2X,F8.3),2X,E12.5)
```

```
2000  FCRMAT(1X,4I4,2(2X,F8.3))
1004  FCRMAT(1H1)
1005  FCRMAT(1X,/)
1007  FCRMAT(1X,//)
1008  FCRMAT(2X,'POINT',7X,'X',10X,'Y',7X,'LAMBDA',7X,'RU',9X,'IU',
     F9X,'RV',9X,'IV',8X,'RPSI',7X,'IPSI',8X,'RG',9X,'IG',/)
1009  FCRMAT(2X,I2,',',I2,11(3X,F8.5))
1010  FCRMAT(1X,'UPPER WAKE FIELD :')
1011  FCRMAT(1X,'LOWER WAKE FIELD :')
1012  FCRMAT(1X,F10.5)
1     RETURN
      END
CXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
CXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
      SUBROUTINE KONST(C1,C2,C3,AM2,E1)
      ALO=AM2-1.
      E=E1-1.
      E5=E-1.
      E2=E*E
      E3=E2*E
      E4=E2*E2
      AM4=AM2*AM2
      C1=2./SQRT(ALO)
      C2=((AM2-2.)**2+E*AM4)/(2.*ALO**2)
      C3=E1*(AM2-(5.+7.*E-2.*E2)/(2.*E1))**2/6.
      C3=C3+(-4.*E4+28.*E3+11.*E2-8.*E-3.)/(24.*E1)
      C3=C3-E1*((5.-3.*E)*AM4-(12.-4.*E)*AM2+8.)/48
      C3=(C3*AM4+3.*(AM2-4./3.)**2/4.)/(ALO**3.5)
      C3=0.
      RETURN
      END
CXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
CXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
      SUBROUTINE SLIP(M3,N3)
      CCMPLEX*8 U,V,G,PSI,PU,AI,DYDXU,DA
      CCMPLEX*16 A,RI,ES
      CCMMON/BA/ ALO,ALC,AM,C1,AK,I,IE,IW
      CCMMON/BC/ T,B,DYDX,D2YDX2,DYDXU,AI,I1,T3
      CCMMON/SOL/ A(4,4),RI(4),ES(4)
      CCMMON V(2,25,25),X(2,25,25),P(2,25),U(2,25,25),PSI(2,25,25),
     FG(2,25,25),AL(2,25,25),Y(2,25,25),Q(2,25),DX(2),IN(2,25),
     FIN2(2,25),PX(2,20),PS(2,20),PU(2,20),TET(2),TED(20)
C
C     SLIP DOES THE SIMULTANEOUS STEPS FROM BOTH SIDES
C     OF THE WAKE TO THE SLIP - LINE
C
      M2=M3-1
      N2=N3-1
      M4=N3
      N4=M3
      M13=M3
      N13=N3-1
      M14=M4-1
      N14=N4
      DC3=X(2,M3,N3)-X(2,M13,N13)
      DAL3=AL(2,M3,N3)+AL(2,M13,N13)
      D13=SQRT(AL(2,M3,N3))+SQRT(AL(2,M13,N13))
      D23=AL(2,M3,N3)-AL(2,M13,N13)
      A(2,1)=1.+0.5*D23/DAL3+2.*AI*AK*AM*DC3/DAL3
      A(2,1)=A(2,1)-AM*(AK*DC3)**2/DAL3*0.5
      A(2,2)=2./D13+AM*(AK*DC3)**2/(DAL3*D13)
      RI(2)=(2.-A(2,1))*U(2,M13,N13)+(4./D13-A(2,2))*V(2,M13,N13)
      RI(2)=RI(2)+2.*AK*AK*AM*PSI(2,M13,N13)*DC3/DAL3
      A(2,3)=0.
      A(2,4)=C.
      DC2=X(2,M3,N3)-X(2,M2,N2)
      A(4,1)=-TAN(TED(N3))
      A(4,2)=1.
      A(4,3)=-A(4,1)
      A(4,4)=-1.
      RI(4)=0.
      DC4=X(2,M4,N4)-X(2,M14,N14)
      D14=SQRT(AL(2,M4,N4))+SQRT(AL(2,M14,N14))
      A(1,1)=1.+AI*AK*DC3/2.
```

```
      A(1,2)=-AI*AK*D03/D13
      A(1,3)=-1.-AI*AK*D04/2.
      A(1,4)=-AI*AK*D04/D14
      RI(1)=PSI(2,M13,N13)-PSI(2,M14,N14)
      RI(1)=RI(1)-D04*(U(2,M14,N14)+2.*V(2,M14,N14)/D14)/2.
      RI(1)=-(RI(1)+D03*(U(2,M13,N13)-2.*V(2,M13,N13)/D13)/2.)
      RI(1)=RI(1)*AI*AK
      DAL4=AL(2,M4,N4)+AL(2,M14,N14)
      A(3,1)=0.
      A(3,2)=0.
      A(3,3)=1.+0.5*(AL(2,M4,N4)-AL(2,M14,N14))/DAL4
      A(3,3)=A(3,3)+2.*AI*AK*AM*D04/DAL4-AM*(AK*D04)**2/DAL4*0.5
      A(3,4)=-(2./D14+AM*(AK*D04)**2/(DAL4*D14))
      RI(3)=U(2,M14,N14)*(2.-A(3,3))+V(2,M14,N14)*(-4./D14-A(3,4))
      RI(3)=RI(3)+2.*AK*AK*AM*PSI(2,M14,N14)*D04/DAL4
C
      NO=4
      CALL SOLVE(NO)
      U(2,M3,N3)=ES(1)
      V(2,M3,N3)=ES(2)
      U(2,M4,N4)=ES(3)
      V(2,M4,N4)=ES(4)
      DA=0.5*(U(2,M3,N3)+U(2,M13,N13))
      DA=(DA-(V(2,M3,N3)+V(2,M13,N13))/D13)*D03
      PSI(2,M3,N3)=PSI(2,M13,N13)+DA
      DA=0.5*(U(2,M4,N4)+U(2,M14,N14))
      DA=(DA+(V(2,M4,N4)+V(2,M14,N14))/D14)*D04
      PSI(2,M4,N4)=PSI(2,M14,N14)+DA
      RETURN
      END
CXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
CXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
      SUBROUTINE COEF1(KV,I1,M,N,M2,N2,A1,A2,A3,A4,B1,B2,B3,B4,S)
      COMPLEX*8 U,V,PSI,G
      COMMON/BA/ ALO,ALC,AM,C,AK,I,IE,IW
      COMMON V(2,25,25),X(2,25,25),P(2,25),U(2,25,25),PSI(2,25,25)
     F,G(2,25,25),AL(2,25,25),Y(2,25,25)
C
C     COEF1 DETERMINES THE COEFFICIENTS FOR THE  UNSTEADY
C     TEIPEL - CHADWICK SHOCKPOLAR
C
      IE=0
      CALL FIND(KV,IW,X(2,M,N),Y(2,M,N),M2,N2,IE,I)
      CX=C-2.
      ALT=AL(1,M2,N2)
      IF(IE.EQ.1) ALT=ALO
      ANY=(ALT-AM+1.)/(C*AM)+1.
      ANY2=ANY*ANY
      L1=M+1
      L2=N
      IF(I.NE.2) GOTO 1
      L1=M
      L2=N+1
1     IF(L1.GT.KV .OR. L2.GT.KV) IE=1
      IF(IE.EQ.1) GOTO 2
      S=(Y(2,L1,L2)-Y(2,M,N))/(X(2,L1,L2)-X(2,M,N))
2     IF(IE.EQ.1) S=2.*I1/(SQRT(ALT)+SQRT(AL(2,M,N)))
      S=ATAN(S)
      CY=SIN(S)**2
      CZ=COS(S)**2
      AN=1./(AM*CY*ANY2)
      A1=2.*CY*ANY*SIN(2.*S)/C
      A2=2.*AK*CY*(1.+AN)/C
      A3=CX*CY*(1.-2.*AN/CX)/C+CZ
      A4=SIN(2.*S)*(1.+AN)/C
      B1=-2.*CY*ANY*(COS(2.*S)+AN)/C
      B2=-2.*AK*SIN(S)*COS(S)*(1.+AN)/C
      B3=A4
      B4=CY+CZ*CX*(1.-2.*AN/CX)/C
      RETURN
      END
CXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
CXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
      SUBROUTINE SOLVE(N)
```

```
      COMPLEX*16 G,C,X
      COMMON/SOL/ G(4,4),C(4),X(4)
C
C     SOLVE GIVES THE SOLUTION FOR A COMPLEX SYSTEM OF
C     LINEAR EQUATIONS  G*X=C
C
      IKK=0
      IF(IKK.NE.1) GOTO 2
      WRITE(6,1001)
      DO 1 M=1,N
1     WRITE(6,1000) (G(M,L),L=1,N),C(M)
2     MN=N-1
      DO 10 M=1,MN
      K=M+1
      DO 10 J=K,N
      G(M,J)=G(M,J)/G(M,M)
      IF(M.EQ.1) GOTO 14
      MA=M-1
      DO 13 L=1,MA
13    G(M,J)=G(M,J)-G(M,L)*G(L,J)/G(M,M)
14    CONTINUE
      DO 15 L=1,M
15    G(J,K)=G(J,K)-G(L,K)*G(J,L)
10    CONTINUE
      C(1)=C(1)/G(1,1)
      DO 12 I=2,N
      C(I)=C(I)/G(I,I)
      MA=I-1
      DO 12 M=1,MA
12    C(I)=C(I)-G(I,M)*C(M)/G(I,I)
      X(N)=C(N)
      DO 11 I=1,MN
      NA=N-I
      X(NA)=C(NA)
      NB=NA+1
      DO 11 J=NB,N
11    X(NA)=X(NA)-G(NA,J)*X(J)
1001  FORMAT(1X,'SOLVE-ENTRY:')
1000  FORMAT(1X,10(2X,E10.3))
      RETURN
      END
CXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
CXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
      SUBROUTINE SWITCH(J,K,M,N,I)
C     INDEX - SWITCH
      M=J
      N=K
      IF(I.NE.2) GOTO 1
      M=K
      N=J
1     RETURN
      END
CXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
CXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
```

```
CXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
CXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C      CASCADE OF OSCILLATING AIRFOILS FOR  MACH.GT.1
C
C
      COMPLEX*8 L,V,PSI,G,DYDXU
      COMPLEX*8 AI,PU,EI
      COMPLEX*16 EL,RI,ES
      REAL*4 TX,TY,XX,YY,X,Y,AL,D1,D2,D3,XP,ALD,P,C3,ALZ
      COMMON/BA/ ALO,AM,ALD,C1,AK,I,IM,TX,TY,IW,MA,IC,ET,XX,YY
      COMMON/BC/ T1,T2,T,B,DYDX,D2YDX2,DYDXU,AI,EI,I1,IBACK,IE
      COMMON/SP/ XS(2,50),YS(2,50),A(2,50,4),R(2,50)
      COMMON/SCL/ EL(4,4),RI(4),ES(4)
      COMMON V(2,3,50,20),X(2,3,50,20),P(2,50),L(2,3,50,20),
     FPSI(2,3,50,20),G(2,3,50,20),AL(2,3,50,20),Y(2,3,50,20),
     FQ(2,50),DX(2),IN(2,50),IN2(2,50),PX(2,30),PS(2,30),PL(2,30)
C
C      OPTIONS:
C      LC1 =0   : COMPLETE OUTPUT
C          =1   : ONLY PRESSURE DISTRIBUTION
C          =2   : STOP
C      LC2 =0   : PITCH
C          =1   : PLUNGE
C      LC3 =1   : DO THE 1. PASSAGE
C          =0   : DO THE 1. PASSAGE NOT
C      LC4 =1   : SURFACES ARE ANALYTICALLY GIVEN
C          =0   : SURFACES ARE POINTWISE GIVEN
C      MA = MAXIMUM NUMBER OF POINTS ON THE SURFACE .LT.20
C      MAXS = BLADE,WHERE THE PRESSURE DISTRIBUTION IS COMPUTED
C      IBLA = FIRST BLADE OF FIELD - OUTPUT
C
C
      KV=50
      AI=CMPLX(0.,1.)
102   READ(3,999) LO1,LC2,LC3,LO4,MA,MAXS,IBLA
      IF(LO1.EQ.2) GOTO 101
      MAXS=MAXS+1
      MAX=MA
      CALL PRCFIL(LO4,T1,T2,NX)
100   READ(3,1000) AK,AM,C,DX(1),DX(2)
      IF(AM.EQ.0.) GOTO 102
      READ(3,1020) AMY,B,ET,EM
      PI=4.*ATAN(1.)
      ET=ET*PI/180.
      TX1=1.+EM/TAN(ET)
      AMY=AMY*PI/180.
      AX=AM
      AM=AM*AM
      ALO=AM-1.
      C1=C+1.
      ALD=ALO**1.5
      C3=C1*AM*1.5
      IM=1
      NT=NX+1
      DO 74 I=1,2
      DO 74 J=1,NT
74    WRITE(1,1026) XS(I,J),YS(I,J),NT
C
C      INITIAL FIELD
C
      XM=1./SQRT(ALO)
      D=4./9.
      DO 46 N=1,10
      M=N+1
      Y(1,1,M,N)=0.
46    X(1,1,M,N)=D*(N-1)-2.
      DO 47 M=3,KV
      X(1,1,M,1)=D*(M-2)/2.-2.
47    Y(1,1,M,1)=XM*(2.+X(1,1,M,1))
      DO 48 N=2,20
      L=N+2
      DO 48 M=L,KV
      K=M-1
      X(1,1,M,N)=X(1,1,K,N)+0.5*D
```

```
48      Y(1,1,M,N)=Y(1,1,K,N)+XM*D/2.
        DO 44 M=1,KV
        DO 44 N=1,20
        AL(1,1,M,N)=ALO
        PSI(1,1,M,N)=0.
        G(1,1,M,N)=0.
        U(1,1,M,N)=0.
44      V(1,1,M,N)=0.
C
C       COMPUTATION OF THE SPLINE - COEFFICIENTS
C
        DO 49 I=1,2
        DO 49 K=1,NX
        A(I,K,1)=YS(I,K)
        A(I,K,3)= R(I,K)
        H=XS(I,K+1)-XS(I,K)
        A(I,K,2)=(YS(I,K+1)-YS(I,K))/H-H*(R(I,K+1)+2.*R(I,K))/3.
49      A(I,K,4)=(R(I,K+1)-R(I,K))/(3.*H)
C
        EI=COS(AMY)+AI*SIN(AMY)
55      TX=EM/TAN(ET)
        TY=EM
        IR=1
        IB=2
        M1=0
        N1=0
        WRITE(6,1021) IM
        XPL=IM*TX
        YPL=IM*TY
        WRITE(1,1026)XPL,YPL,IM
        I=0
        IE=0
        CALL FIND(KV,IM,IB,IR,TX,TY,M1,N1)
30      I=I+1
        IF(I.EQ.1) GOTO 67
        T2Y=2.*EM
        DO 56 J=2,KV
        IF(Y(2,1,J,1).GT.T2Y) GOTO 57
56      CONTINUE
57      MAX2=J
C
C       THE STEADY FLOW FIELD
C
67      I2=0
16      I2=I2+1
        IF(I2.GT.30) GOTO 101
        WRITE(6,1024) I2,MA,J1,DX(I)
1024    FORMAT(1X,I3,'. ITERATION ALONG THE SURFACE',2I4,2X,E9.3)
        MA=MAX
        XX=EM*(IB-1)/TAN(ET)
        YY=EM*(IB-1)
        IF(I.EQ.2 .OR. IR.GT.1) MA=MAX2
        IF(IR.GT.1) I=IB
        I1=(-1)**(I+1)
        MZ=1
        IBACK=0
        IQ=1
        KC=0
        IF((I*IM).EQ.1 .AND. IR.GT.1) IQ=0
        T=T1
        IF(I.EQ.2) T=T2
        MM=KV
        IF(I.EQ.2. OR. IR.GT.1) MM=20
        DO 5 K=1,20
        DO 5 J=K,MM
        CALL SWITCH(J,K,M,N,I)
5       AL(IB,IR,M,N)=0.
        M2=M1
        N2=N1
C
C       THE STEADY BOUNDARY-PROPERTIES BEHIND THE SHOCK
C
        IE=0
        KW=MM+1
```

```
      DC 9 J=2,KW
      KT=J-1
      XP=DX(I)*(J-2)+TX-XX
      IF(T.EQ.0. .OR. IQ.EQ.0) XP=TX-XX
      IF(IQ.NE.0 .AND. XP.GT.1. .AND. I.EQ.1) GOTO 8
      IF(J.GT.MM) GOTO 8
      LI=1
      CALL SWITCH(J,LI,M,N,I)
      CALL SHOCK(MM,LO4,EM,IR,IB,M,N,XP,M2,N2,C3)
      IF(IR.GT.1 .AND. IB.EQ.1 .AND.Y(IB,IR,M,N).EQ.EM) GOTO 63
      IF(I.EQ.1 .AND.IR.EQ.1) GOTO 9
      IF(IB.EQ.2 .AND. Y(IB,IR,M,N).EQ.C.) GOTO 63
      IF(X(IB,IR,M,N).LE.1.) GOTO 9
      ALO=-Y(IB,IR,M,N)/(1.-X(IB,IR,M,N))
      ALO=1./(ALO*ALO)
      IF(ALO.GT.ALO .AND. IM.NE.1) GOTO 8
      IF(X(IB,IR,M,N).GT.1.) GOTO 8
      GOTO 9
63    KT=KT+1
      GOTO 8
9     CONTINUE
8     CONTINUE
      IF(X(IB,IR,2,1).GE.1.) GOTO 38
      IF(I.EQ.2 .AND. X(IB,IR,M,N).GE.1.) KC=2
C
C     ALL OTHER STEPS OF THE STEADY FLOW FIELD
C
      KA=KT
      DC 1 J=2,KA
      J1=J
      IF(J.EQ.KA .AND. I.EQ.2 .AND. IR.EQ.1) GOTO 15
      IF(J.EQ.KA .AND. IR.GT.1) GOTO 15
      LI=J+1
      CALL SWITCH(LI,J,M,N,I)
      CALL RANC(LO4,IB,IR,M,N,C3)
      IF(X(IB,IR,M,N).GT.TX1) MZ=0
      IF(IR.GT.1.AND.I.EQ.1.AND.X(IB,IR,M,N).GT.1..AND.IC.NE.C) MZ=0
      IF(MZ.EQ.0 .AND. IR.GT.1) KC=1
      KB=KA+1
      DC 3 J2=J,KB
      IF(J2.EQ.KB) GOTO 3
      LI=J-1
      CALL SWITCH(J2,LI,K1,K2,I)
      IF(AL(IB,IR,K1,K2).GT.AL(IB,IR,M,N)) GOTO 6
3     CONTINUE
6     J2=J2-1
      IN(I,J-1)=J2
      L=J+2
      DC 7 K=L,J2
      CALL SWITCH(K,J,M,N,I)
7     CALL GEN(IB,IR,M,N,I)
      IF(MZ.EQ.0) GOTO 15
      IF(J2.EQ.KA) GOTO 1
C
C     CHARACTERISTICS OVERTAKE THE SHOCK
C
      J3=M+1
      J4=N-1
      J5=M
      J6=N-1
      IF(I.NE.2) GOTO 12
      J3=M-1
      J4=N+1
      J5=M-1
      J6=N
12    D1=Y(IB,IR,J3,J4)-Y(IB,IR,J5,J6)
      D1=D1/(X(IB,IR,J3,J4)-X(IB,IR,J5,J6))
      D2=I1/SQRT(AL(IB,IR,M,N))
      K1=M+1
      K2=J
      IF(I.NE.2) GOTO 14
      K1=J
      K2=N+1
14    AL(IB,IR,K1,K2)=AL(IB,IR,M,N)
```

```
      D3=-D2*X(IB,IR,M,N)+D1*X(IB,IR,J5,J6)
      D3=D3-Y(IB,IR,J5,J6)+Y(IB,IR,M,N)
      X(IB,IR,K1,K2)=D3/(D1-D2)
      Y(IB,IR,K1,K2)=D2*(X(IB,IR,K1,K2)-X(IB,IR,M,N))+Y(IB,IR,M,N)
      IF(I.EQ.2) J3=J4
      DO 11 K=J3,KA
      IF(K.EQ.MM) GOTO 11
      LI=K+1
      CALL SWITCH(LI,J,M,N,I)
      LI=J-1
      CALL SWITCH(K,LI,K1,K2,I)
      X(IB,IR,M,N)=X(IB,IR,K1,K2)
      Y(IB,IR,M,N)=Y(IB,IR,K1,K2)
      AL(IB,IR,M,N)=AL(IB,IR,K1,K2)
11    CONTINUE
      IF(KA.LT.MM) KA=KA+1
      IF(MA.GT.IN(I,J-1)) MA=MA+1
1     CONTINUE
C
C     AUTOMATIC STEP-SIZE CONTROL
C
15    J2=J1
      IF(KC.EQ.1) J2=KT
      IF(T.EQ.0. .OR. IC.EQ.0) GOTO 28
      IF(J2.EQ.MA) GOTO 28
      IF(J2.GT.MA) GOTO 68
      DIF=(J2*1.)/(MA*1.)
      IF(DIF.GE.0.85) GOTO 69
      GOTO 72
68    DIF=(J2*1.)/(MA*1.)
      IF(DIF.LE.1.15) GOTO 69
72    DX(I)=DX(I)*DIF
      GOTO 73
69    IF(J2.GT.MA) DX(I)=DX(I)*1.05
      IF(J2.LT.MA) DX(I)=DX(I)*0.96
73    CONTINUE
      IF(J2.NE.MA) GOTO 16
C
28    CONTINUE
      IF(IR.EQ.1 .AND. I.EQ.1) GOTO 59
      IF(KC.NE.0) GOTO 59
C
C     ADDITIONAL POINTS FOR THE FIELDS IN THE PASSAGE
C
      DO 33 LY=2,20
      LI=LY-1
      CALL SWITCH(LY,LI,M,N,I)
      IF(AL(IB,IR,M,N).EQ.0.) GOTO 35
C     WRITE(6,1009) M,N,X(IB,IR,M,N),Y(IB,IR,M,N),AL(IB,IR,M,N)
33    CONTINUE
35    KD=LY
      LI=KD-1
      LZ=2
      CALL SWITCH(LI,LZ,M,N,I)
      LE=1
      CALL SWITCH(KD,LE,L1,L2,I)
      X(IB,IR,L1,L2)=X(IB,IR,M,N)
      CALL SWITCH(LI,LE,M,N,I)
      LZ=KD-2
      CALL SWITCH(LI,LZ,L3,L4,I)
      Y1=I1*(X(IB,IR,L1,L2)-X(IB,IR,M,N))/SQRT(AL(IB,IR,M,N))+EM*(IB-2
      X1=X(IB,IR,L1,L2)
      Y(IB,IR,L1,L2)=Y1
      D1=(Y(IB,IR,L3,L4)-Y(IB,IR,M,N))/(X(IB,IR,L3,L4)-X(IB,IR,M,N))
      D1=D1/1.5
      DO 43 JY=2,LZ
      CALL SWITCH(KD,JY,M,N,I)
      CALL SWITCH(LI,JY,L1,L2,I)
      D2=I1/SQRT(AL(IB,IR,L1,L2))
      X2=D1*X1-Y1+Y(IB,IR,L1,L2)-D2*X(IB,IR,L1,L2)
      X(IB,IR,M,N)=X2/(D1-D2)
43    Y(IB,IR,M,N)=D1*(X(IB,IR,M,N)-X1)+Y1
      CALL SWITCH(KD,LI,M,N,I)
      Y(IB,IR,M,N)=EM*(IB-1)
```

```
      X(IB,IR,M,N)=(Y(IB,IR,M,N)-Y1)/C1+X1
      DC 65  JY=1,LZ
      CALL  SWITCH(LI,JY,M,N,I)
      CALL  SWITCH(KD,JY,L1,L2,I)
C     WRITE(6,1009)  JY,KD,X(IB,IR,M,N),Y(IB,IR,M,N),X(IB,IR,L1,L2),
C    FY(IB,IR,L1,L2)
65    CCNTINUE
      CALL  SWITCH(KC,LI,M,N,I)
C     WRITE(6,1CCS)  M,N,X(IB,IR,M,N) ,Y(IB,IR,M,N)
C
C     THE UNSTEADCY FLOW FIELC
C     THE UNSTEADY BOUNDARY PRCPERTIES BEHINC THE SHCCK
C
59    KA=KT
      LZ=2
      LI=1
      CALL  SWITCH(LZ,LI,I7,I8,I)
      XPL=X(IB,IR,I7,I8)+(IM-1)*EM/TAN(ET)
      YPL=Y(IB,IR,I7,I8)+(IM-1)*EM
      WRITE(1,1026)  XPL,YPL,IM
      CALL  SWITCH(KT,LI,I7,I8,I)
      XPL=X(IB,IR,I7,I8)+(IM-1)*EM/TAN(ET)
      YPL=Y(IB,IR,I7,I8)+(IM-1)*EM
      WRITE(1,1026)  XPL,YPL,IM
      WRITE(6,1025)  IR
1025  FCRMAT(1X,I3,'. UNSTEACY FIELC')
      IE=0
      DC 18  J=2,KA
      CALL  SWITCH(J,LI,M,N,I)
      IF(X(IB,IR,M,N).LT.0.)  GCTO 18
      CALL  RANCS(MM,IB,IR,M,N,LC2,AMY,M2,N2)
18    CCNTINUE
      IF(I.EC.2)  X(IB,IR,1,2)=TX
      IF(I.EQ.2)  Y(IB,IR,1,2)=TY
      IF(I.EQ.1)  X(IB,IR,2,1)=TX
      IF(I.EQ.1)  Y(IB,IR,2,1)=TY
C
C     ALL OTHER STEPS OF THE UNSTEADY FIELD
C
      IF(J1.EC.2)  GOTO 38
      J5=J1-1
      DC 20  J=2,J5
      LI=J+1
      CALL  SWITCH(LI,J,M,N,I)
      CALL  RANCE(IB,IR,M,N,LC2)
      IF(J.EQ.J5 .AND. I.EQ.2 .AND. IR.EC.1)  GCTC 2C
      IF(J.EC.J5 .AND. IR.GT.1 .AND. MZ.NE.0)  GCTC 2C
      L=J+2
      J2=IN(I,J-1)
      DC 22  K=L,J2
      CALL  SWITCH(K,J,M,N,I)
22    CALL  GENU(IM,IB,IR,M,N,I,AK,AM)
      IF(J2.EC.KA)  GOTO 20
      IE=0
      K1=M+1
      K2=J
      K3=K1
      IF(I.NE.2)  GOTO 24
      K1=J
      K2=N+1
      K3=K2
24    P(I,K3)=X(IB,IR,M,N)
      CALL  RANDS(MM,IB,IR,K1,K2,LO2,AMY,M2,N2)
      J3=J2+1
      DC 29  K=J3,KA
      IF(K.EQ.MM)  GCTC 20
      LI=K+1
      CALL  SWITCH(LI,J,M,N,I)
      LI=J-1
      CALL  SWITCH(K,LI,K1,K2,I)
      U(IB,IR,M,N)=U(IB,IR,K1,K2)
      V(IB,IR,M,N)=V(IB,IR,K1,K2)
      G(IB,IR,M,N)=G(IB,IR,K1,K2)
29    PSI(IB,IR,M,N)=PSI(IB,IR,K1,K2)
```

```
      IF(J2.LT.KA .AND. KA.LT.MM) KA=KA+1
20    CONTINUE
C
C     OUTPUT STEADY AND UNSTEADY FIELD
C
      IF(IM.LT.IBLA) GOTO 38
      IF(LO1.NE.0) GOTO 38
      WRITE(7,1004)
      IF(I.EQ.1 .AND. IR.EQ.1) WRITE(7,1002) IM
      IF(I.EQ.2 .OR. IR.GT.1) WRITE(7,1003) IR,IM
      WRITE(7,1007)
      WRITE(7,1001) AK,AX,C,B,CX(I),T
      DO 25 J=2,J1
      K=J-1
C     LL=IN(I,K)+1
      LL=IN(I,K)
      WRITE(7,1007)
      WRITE(7,1008)
      N=K
      IF(I.NE.2) GOTO 26
      M=K
      DO 31 N=J,LL
31    WRITE(7,1009) M,N,X(IB,IR,M,N),Y(IB,IR,M,N),AL(IB,IR,M,N)
     F,U(IB,IR,M,N),V(IB,IR,M,N),PSI(IB,IR,M,N),G(IB,IR,M,N)
      GOTO 17
26    CONTINUE
      DO 32 M=J,LL
32    WRITE(7,1009) M,N,X(IB,IR,M,N),Y(IB,IR,M,N),AL(IB,IR,M,N)
     F,U(IB,IR,M,N),V(IB,IR,M,N),PSI(IB,IR,M,N),G(IB,IR,M,N)
17    WRITE(7,1005)
25    CONTINUE
      WRITE(7,1004)
38    CONTINUE
      IF(IR.GT.1) GOTO 60
      IF(I.EQ.1) IUS=K-1
      IF(I.EQ.2) ILS=K-1
      IF(I.EQ.1) GOTO 30
      IF(IM.EQ.1 .AND. LO3.EQ.0) GOTO 62
C
C     CHANGING THE COUNTERS FOR THE COMPUTATION OF
C     THE FIELDS BEHIND THE REFLECTED SHOCKS
C
60    IR=IR+1
      IF(IR.GE.4) GOTO 62
      IF(KC.NE.0) GOTO 62
      WRITE(6,1023) IR
      LI=1
      CALL SWITCH(KT,LI,M,N,I)
      CALL SWITCH(KT,LI,M1,N1,I)
      TX=X(IB,IR-1,M,N)
      TY=Y(IB,IR-1,M,N)
      IF(IR.EQ.2 .OR. IR.EQ.4) IB=1
      IF(IR.EQ.3 .OR. IR.EQ.5) IB=2
      GOTO 67
62    CONTINUE
      IF(KC.EQ.0) GOTO 77
C     COMPUTATION OF THE BLADE - WAKE IN
C     SUBROUTINE   *WAKE*
C
C
      CALL WAKE
C     NEXT BLADE
77    IM=IM+1
      IF(IM.LE.MAXS) GOTO 58
      IR=IR-1
      CALL PRESS(AMY,IR,EM,LC2,AX,C)
      IM=0
      WRITE(1,1026) XPL,YPL,IM
      GOTO 100
C
C     TRANSFORMATION TO PROFIL 1
C
58    XX=EM/TAN(ET)
      DO 70 IL=1,3
      MM=20
```

```
      IF(IL.EQ.1) MM=KV
      DO 51 M=1,MM
      DO 51 N=1,20
      X(1,IL,M,N)=X(2,IL,M,N)-XX
      Y(1,IL,M,N)=Y(2,IL,M,N)-EM
      PSI(1,IL,M,N)=PSI(2,IL,M,N)
      U(1,IL,M,N)=U(2,IL,M,N)
      V(1,IL,M,N)=V(2,IL,M,N)
      G(1,IL,M,N)=G(2,IL,M,N)
51    AL(1,IL,M,N)=AL(2,IL,M,N)
70    CONTINUE
      GOTO 55
101   IM=-1
      WRITE(1,1026) XPL,YPL,IM
999   FORMAT(4I1,3I2)
1000  FORMAT(3F10.5,2E10.3)
1001  FORMAT(1X,//, 1X,'W*C/U= ',F5.3,', M= ',F4.2,', K= ',F4.2,
     F', B/C= ',F4.2,', DX= ',E9.3,', T/C= ',F6.4,//)
1002  FORMAT(1X,'   THE PROPERTIES AT THE MESHPOINTS OF THE FLOW FIELD',
     F/,1X,'   FOR THE UPPER SURFACE OF THE ',I2,'. BLADE',//)
1003  FORMAT(1X,'   THE PROPERTIES AT THE MESHPOINTS OF THE FLOW FIELD',
     F/,1X,'   BEHIND THE ',I2,'. SHOCK IN THE ',I2,'. PASSAGE',//)
1004  FORMAT(1H1)
1005  FORMAT(1X,/)
1007  FORMAT(1X,//)
1008  FORMAT(2X,'POINT',7X,'X',10X,'Y',7X,'LAMBDA',7X,'RU',9X,'IU',
     F9X,'RV',9X,'IV',8X,'RPSI',7X,'IPSI',8X,'RG',9X,'IG',/)
1009  FORMAT(2X,I2,',',I2,11(3X,F8.5))
1020  FORMAT(4F10.5)
1021  FORMAT(2X,I2,'. BLADE')
1023  FORMAT(1X,I2,'. REFLECTION',/)
1026  FORMAT(2F10.3,I3)
      END
CXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
CXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
      SUBROUTINE PROFIL(LO4,T1,T2,N)
      COMMON/SP/ XS(2,50),YS(2,50),A(2,50,4),B(2,50)
C
C     PREPARATION OF THE PROFIL - SURFACES
C
      READ(3,1000) T1,T2,NT,IKP
      IF(LO4.EQ.0) READ(3,1001) SP
      DO 8 J=1,2
      I2=0
      N=NT
      T=T1
      IF(J.EQ.2) T=T2
      IF(LO4.NE.0) GOTO 12
      DO 9 K=1,4
      M=(K-1)*5+1
      READ(3,1001) XS(J,M),XS(J,M+1),XS(J,M+2),XS(J,M+3),XS(J,M+4)
      READ(3,1001) YS(J,M),YS(J,M+1),YS(J,M+2),YS(J,M+3),YS(J,M+4)
      IF(YS(J,M+4).EQ.100.) GOTO 7
9     CONTINUE
7     CONTINUE
      DO 2 LL=1,N
      XS(J,LL)=XS(J,LL)/SP
2     YS(J,LL)=YS(J,LL)/SP
      IF(LO4.EQ.0) GOTO 3
12    DX=1.0/(N-1)
      I1=(-1)**(J+1)
      DO 20 K=1,N
      XS(J,K)=(K-1)*DX-0.25
20    YS(J,K)=I1*4.*T*XS(J,K)*(1.-XS(J,K))
C
C     TO ENTER THIS PART,THE SURFACES SHOULD ALREADY BE
C     GIVEN POINTWISE
3     CONTINUE
C
C     INTERPOLATION THROUGH CUBIC SPLINES
C
      IF(T.NE.0. .OR. LO4.EQ.0) GOTO 51
      DO 52 I=1,N
52    A(J,I,1)=0.
```

```
      GOTO 53
51    CONTINUE
      DO 10 I=1,N
      A(J,I,3)=XS(J,I)
10    A(J,I,4)=YS(J,I)
C
C     MATRIX OF COEFFICIENTS AND RIGHT-HAND SIDES
C
      K=N-2
      DO 25 I=1,K
      A(J,I,1)=A(J,I+1,3)-A(J,I,3)
      A(J,I,3)=A(J,I+2,3)-A(J,I+1,3)
      A(J,I,2)=2.*(A(J,I,1)+A(J,I,3))
      A(J,I,4)=3.*(A(J,I+2,4)-A(J,I+1,4))/A(J,I,3)-
     F3.*(A(J,I+1,4)-A(J,I,4))/A(J,I,1)
25    CONTINUE
      A(J,1,1)=0.0
      A(J,N-2,3)=0.0
C
C     THE STEP OF GAUSS
C
      K=N-3
      DO 30 I=1,K
      DO 35 M=3,4
35    A(J,I,M)=A(J,I,M)*A(J,I+1,1)/A(J,I,2)
      A(J,I,1)=0.0
      A(J,I,2)=A(J,I+1,1)
      A(J,I+1,2)=A(J,I+1,2)-A(J,I,3)
30    A(J,I+1,4)=A(J,I+1,4)-A(J,I,4)
C
C     SOLUTION
C
      A(J,1,1)=0.
      A(J,N,1)=0.
      A(J,N-2,1)=0.0
      L=N-1
      DO 40 I=2,L
      K=N-I
      M=K+1
40    A(J,M,1)=(A(J,K,4)-A(J,K,3)*A(J,M+1,1))/A(J,K,2)
53    CONTINUE
      DO 11 M=1,N
11    R(J,M)=A(J,M,1)
      IF(IZ.EQ.1) GOTO 8
      KV=N-1
      DO 49 K=1,KV
      A(J,K,1)=YS(J,K)
      A(J,K,3)= R(J,K)
      H=XS(J,K+1)-XS(J,K)
      A(J,K,2)=(YS(J,K+1)-YS(J,K))/H-H*(R(J,K+1)+2.*R(J,K))/3.
49    A(J,K,4)=(R(J,K+1)-R(J,K))/(3.*H)
      DX=1./49.
      DO 4 M=1,50
      X=DX*(M-1)
      DO 5 K=2,50
      I=K-1
      IF(XS(J,K).GE.X) GOTO 6
5     CONTINUE
6     H=X-XS(J,I)
      Y=A(J,I,1)+A(J,I,2)*H+A(J,I,3)*H*H+A(J,I,4)*H*H*H
      R(J,M)=X
      YS(J,M)=Y
4     CONTINUE
      IZ=1
      N=50
      DO 13 I=1,50
13    XS(J,I)=R(J,I)
      GOTO 3
8     CONTINUE
      IF(IKP.EQ.0) GOTO 17
      DO 15 M=1,N
15    WRITE(6,1002) XS(1,M),YS(1,M),XS(2,M),YS(2,M)
17    N=N-1
1000  FORMAT(2F10.5,2I2)
```

```
1001  FCRMAT(5F10.5)
1002  FCRMAT(5X,4(E12.5,2X))
      RETURN
      END
CXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
CXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
      SUBROUTINE FIND(KV,IM,IB,IR,XS,YS,M,N)
      CCMPLEX*8 U,V,PSI,G,DYDXL
      CCMPLEX*8 AI,EI
      REAL*4 X,Y,AL,XS,YS,P,X9
      CCMMON/BC/ T1,T2,T,B,DYDX,D2YDX2,DYDXU,AI,EI,IY,IBACK,IE
      CCMMON V(2,3,50,20),X(2,3,50,20),P(2,50),U(2,3,50,2C),
     FPSI(2,3,50,20),G(2,3,50,20),AL(2,3,50,20),Y(2,3,50,20),
     FQ(2,50),DX(2),IN(2,50),IN2(2,50)
C
C     FIND LOOKS FOR THE MESHINDEX,RESPCNSIBEL FCR
C     THE POINT  XS;YS
C
      NX7=10000*XS
      NY7=10000*YS
      ID=1
      IC=1
      IF(IR.EQ.1) GOTO 26
      ID=IR-1
      IF(IB.EQ.2) IC=1
      IF(IB.EQ.1) IC=2
26    I1=(-1)**(IC+1)
      IF(IE.EQ.1) GOTO 19
      IKK=0
C     IF(I.EQ.2) IKK=1
      IF(IKK.EQ.1) WRITE(6,1004)
1004  FCRMAT(1X,'FIND-ENTRY')
      KA=0
      L1=1
      L2=1
21    IK=21
      IF(IKK.EQ.1) WRITE(6,1003) IK,M,N,NX7,NY7,NX9,NY9
      DC 22 I2=L1,20
      L3=I2+L2
      CALL SWITCH(L3,I2,M,N,IC)
      IK=22
      IF(IKK.EQ.1) WRITE(6,1003) IK,M,N,NX8,NY8,NX9,NY9
      IF(IC.EQ.1 .AND. L3.GE.KV) GOTO 13
      IF(IC.EQ.2 .AND. L3.GT.19) GOTO 13
      J1=M+1
      J2=N+1
24    KM=J1-J2
      IK=24
      IF(IKK.EQ.1) WRITE(6,1003) IK,M,N,NX8,NY8,NX9,NY9
      IF(IC.EQ.2) KM=J2-J1
      NX8=10000*X(IC,ID,M,N)
      NX9=10000*X(IC,ID,J1,J2)
      NY8=10000*Y(IC,ID,M,N)
      NY9=10000*Y(IC,ID,J1,J2)
      X9=X(IC,ID,J1,J2)
      IF(NX7.GE.NX8 .AND. NX7.LT.NX9) GCTC 23
      IF(X9.GE.1. .AND. KM.EQ.1 .AND. IR.EQ.1) KA=J2
      IF(IC.EQ.2 .AND. KA.EQ.J2) KA=J1
      IF(IC.EQ.1 .AND. KA.NE.J2) GOTO 22
      IF(IC.EQ.2 .AND. KA.NE.J1) GOTO 22
      IF(IC.EQ.1) J1=J1+1
      IF(IC.EQ.2) J2=J2+1
      IF(J1.EQ.KV .OR. J2.EQ.20) GOTO 13
      GCTO 24
22    CCNTINUE
23    CCNTINUE
      IF(IC.EQ.2) GOTO 28
      IK=23
      IF(IKK.EQ.1) WRITE(6,1003) IK,M,N,NX8,NY8,NX9,NY9
      NY8=10000*Y(IC,ID,J1,N)
      IF(NY7.LT.NY8) GOTO 1
      IF(N.EQ.1) GOTO 13
      L2=L2+1
      L1=N-1
```

```
      GCTO 21
28    IK=28
      IF(IKK.EQ.1) WRITE(6,1CC3) IK,M,N,NX8,NY8,NX9,NY9
      NY8=10000*Y(IC,ID,M,J2)
      IF(NY7.GT.NY8) GOTO 1
      IF(M.EQ.1) GOTO 13
      L2=L2+1
      L1=M-1
      GCTO 21
1     IK=1
      IF(IKK.EQ.1) WRITE(6,1003) IK,M,N,NX8,NY8,NX9,NY9
      IF(M.GE.KV) GOTO 13
      J=M+1
      K=N+1
      NX8=10000*X(IC,ID,M,N)
      NY8=1000C*Y(IC,ID,M,N)
      IF(NX7.EQ.NX8 .AND. NY7.EQ.NY8) GCTC 14
      IF(IR.GT.1) GOTO 29
C
C     CRIENTATION IN THE FIELC OVER THE UPPER BLADE
C
      IF(NX7.NE.NX8) GOTO 5
      IF(NY7.GT.NY8 .AND. N.EQ.1) GOTO 13
      IF(NY7.GT.NY8) N=N-1
      IF(NY7.LT.NY8) M=M-1
      GCTO 1
5     IK=5
      IF(IKK.EQ.1) WRITE(6,1CC3) IK,M,N,NX8,NY8,NX9,NY9
      I=M+2
      IF(I.GT.KV) GOTO 7
      NX8=10000*X(IC,ID,J,N)
      NX9=10000*X(IC,ID,I,K)
      NY8=10000*Y(IC,ID,J,N)
      NY9=10000*Y(IC,ID,I,K)
      IF(NX8.EQ.NX9 .AND. NY8.EQ.NY9) GCTC 6
      GOTO 7
6     IK=6
      IF(IKK.EQ.1) WRITE(6,1003) IK,M,N,NX8,NY8,NX9,NY9
      D1=(Y(IC,ID,J,N)-Y(IC,IC,M,N))/(X(IC,ID,J,N)-X(IC,IC,M,N))
      D2=(Y(IC,ID,M,K)-Y(IC,IC,M,N))/(X(IC,ID,M,K)-X(IC,IC,M,N))
      D12=(YS-Y(IC,ID,M,N))/(XS-X(IC,ID,M,N))
      D5=(Y(IC,ID,J,K)-YS)/(X(IC,ID,J,K)-XS)
      D3=(Y(IC,ID,J,K)-Y(IC,IC,M,K))/(X(IC,ID,J,K)-X(IC,IC,M,K))
      IF(D12.LE.D1.AND.D12.GE.D2.AND.D5.GE.D1.AND.D5.LT.D3) GOTO 14
      N=N+1
      GCTO 1
7     IK=7
      IF(IKK.EQ.1) WRITE(6,1003) IK,M,N,NX8,NY8,NX9,NY9
      D1=(Y(IC,ID,J,N)-Y(IC,IC,M,N))/(X(IC,ID,J,N)-X(IC,IC,M,N))
      IF(K.EQ.M) GOTO 4
      D2=(Y(IC,ID,M,K)-Y(IC,IC,M,N))/(X(IC,ID,M,K)-X(IC,IC,M,N))
4     IF(K.EQ.M) D2=0.
      D5=(Y(IC,ID,J,K)-Y(IC,IC,M,N))/(X(IC,ID,J,K)-X(IC,ID,M,N))
      D12=(YS-Y(IC,ID,M,N))/(XS-X(IC,ID,M,N))
      IF(D12.LE.D1 .AND. D12.GE.D2 .AND. XS.GT.X(IC,IC,M,N)) GCTC 8
      NX8=1000C*X(IC,ID,M,N)
      IF(NX7.GT.NX8) GOTO 2
      IF(N.EC.1) GOTO 13
      N=N-1
      M=M-1
      GCTO 3
2     IF(D12.GT.D1 .AND. N.EQ.1) GOTC 13
      IK=2
      IF(IKK.EQ.1) WRITE(6,1003) IK,M,N,NX8,NY8,NX9,NY9
      IF(D12.GT.D1) N=N-1
      IF(D12.LT.D2) M=M-1
3     IBACK=IBACK+1
      J=M+1
      IF(AL(IC,ID,M,N).NE.AL(IC,ID,J,N)) GCTO 13
      GCTO 1
8     IK=8
      NX8=1000C*X(IC,ID,J,K)
      NY8=100CC*Y(IC,ID,J,K)
      IF(IKK.EQ.1) WRITE(6,1003) IK,M,N,NX8,NY8,NX9,NY9
```

```
      IF(NX7.EQ.NX8 .AND. NY7.EQ.NY8) GOTO 12
      IF(NX7.NE.NX8) GOTO 15
      IF(NY7.GT.NY8) M=M+1
      IF(NY7.LT.NY8) N=N+1
      GOTO 1
12    IK=12
      IF(IKK.EQ.1) WRITE(6,1003) IK,M,N,NX8,NY8,NX9,NY9
      M=J
      N=K
      GOTO 14
15    IK=15
      IF(IKK.EQ.1) WRITE(6,1003) IK,M,N,NX8,NY8,NX9,NY9
      IF(NX7.GE.NX8) GOTO 16
      IF(K.EQ.M) GOTO 10
      D3= (Y(IC,ID,J,K)-Y(IC,ID,M,K))/(X(IC,ID,J,K)-X(IC,ID,M,K))
10    IF(K.EQ.M) D3=0.
      D4= (Y(IC,ID,J,K)-Y(IC,ID,J,N))/(X(IC,ID,J,K)-X(IC,ID,J,N))
      D34= (Y(IC,ID,J,K)-YS )/(X(IC,IC,J,K)-XS)
      IF(D3.GE.D34 .AND. D4.LE.D34) GOTO 17
      GOTO 16
17    IF(D4.EQ.D34) M=M+1
      IF(D3.EQ.0. .AND. D34.EQ.0.) GOTO14
      IF(D3.EQ.D34) N=N+1
      IK=17
      IF(IKK.EQ.1) WRITE(6,1003) IK,M,N,NX8,NY8,NX9,NY9
      GOTO 14
16    IK=16
      IF(IKK.EQ.1) WRITE(6,1003) IK,M,N,NX8,NY8,NX9,NY9
      IF(D12.GT.D5) M=M+1
      IF(D12.LT.D5) N=N+1
      IF(D12.EQ.D5) GOTO 9
      GOTO 1
9     M=M+1
      IK=9
      IF(IKK.EQ.1) WRITE(6,1003) IK,M,N,NX8,NY8,NX9,NY9
      N=N+1
      GOTO 1
13    IE=1
      GOTO 19
C
C     ORIENTATION IN THE PASSAGE
C
29    IK=29
      IF(IKK.EQ.1) WRITE(6,1003) IK,M,N,NX8,NY8,NX9,NY9
      L1=M+1
      L2=N
      L3=M
      L4=N+1
      IF(IC.NE.2) GOTO 30
      L1=M
      L2=N+1
      L3=M+1
      L4=N
30    L5=M+1
      L6=N+1
      NX8=10000*X(IC,ID,L5,L6)
      NX9=10000*X(IC,ID,L1,L2)
      NY8=10000*Y(IC,ID,L5,L6)
      NY9=10000*Y(IC,ID,L1,L2)
      IK=30
      IF(IKK.EQ.1) WRITE(6,1003) IK,M,N,NX8,NY8,NX9,NY9
      IF(NX7.GE.NX8 .AND. NY7.EQ.NY8) GOTO 31
      IF(NX7.LT.NX8 .AND. NY7.EQ.NY8) GOTO 14
      IF(NX7.EQ.NX9 .AND. NY7.EQ.NY9) GOTO 36
      IF(NX7.GT.NX9) GOTO 32
      D1=I1*(Y(IC,ID,L1,L2)-Y(IC,ID,M,N))/(X(IC,IC,L1,L2)-X(IC,ID,M,N))
      IF(L3.EQ.L4) GOTO 33
      D2=I1*(Y(IC,ID,L3,L4)-Y(IC,ID,M,N))/(X(IC,IC,L3,L4)-X(IC,ID,M,N))
33    IF(L3.EQ.L4) D2=0.
      D12=I1*(Y(IC,ID,M,N)-YS)/(X(IC,ID,M,N)-XS)
      IF(D12.LE.D1 .AND. D12.GE.D2) GOTO 14
      NY8=10000*Y(IC,ID,M,N)
      IF(NY7.GT.NY8) N=N-1
      IF(NY7.LT.NY8) M=M-1
```

```
      GCTO 1
32    D3=(Y(IC,IC,L5,L6)-Y(IC,ID,L1,L2))/(X(IC,ID,L5,L6)-X(IC,ID,L1,L2
      D3=D3*I1
      IK=32
      IF(IKK.EQ.1) WRITE(6,1003) IK,M,N,NX8,NY8,NX9,NY9
      IF(L3.EQ.L4) GOTO 34
      D4=(Y(IC,ID,L5,L6)-Y(IC,ID,L3,L4))/(X(IC,IC,L5,L6)-X(IC,IC,L3,L4
      D4=D4*I1
34    IF(L3.EQ.L4) D4=0.
      D34=I1*(YS-Y(IC,ID,L5,L6))/(XS-X(IC,ID,L5,L6))
      IF(D34.GT.D3 .AND. D34.LT.D4) GCTC 14
      NY8=10000*Y(IC,ID,L5,L6)
      IF(NY7.GT.NY8) M=M+1
      IF(NY7.LT.NY8) N=N+1
      GCTO 1
31    M=L5
      N=L6
      IK=31
      IF(IKK.EQ.1) WRITE(6,1003) IK,M,N,NX8,NY8,NX9,NY9
      GCTO 14
36    M=L3
      N=L4
      IK=35
      IF(IKK.EQ.1) WRITE(6,1003) IK,M,N,NX8,NY8,NX9,NY9
14    IK=14
      IF(IKK.EQ.1) WRITE(6,1003) IK,M,N,NX8,NY8,NX9,NY9
C
C     CONTROL - STEP
C
      K1=M+1
      K2=N
      K3=M
      K4=N+1
      IF(IC.NE.2) GOTO 35
      K1=M
      K2=N+1
      K3=M+1
      K4=N
35    J1=M+1
      J2=N+1
      D1 =99.
      D2 =99.
      D3 =99.
      D4 =99.
      D12=99.
      D34=99.
      NX8=10000*X(IC,ID,M,N)
      NX9=10000*X(IC,ID,J1,J2)
      NY8=10000*Y(IC,ID,M,N)
      NY9=10000*Y(IC,ID,J1,J2)
      IF(NX7.EQ.NX8 .AND. NY7.NE.NY8) GCTC 18
      IF(NX7.EQ.NX8 .AND. NY7.EQ.NY8) GCTC 19
      GCTO 20
18    WRITE(6,1000) M,N,NX7,NY7,NX8,NY8,D1,D2,D3,D4,D12,D34
      GCTO 19
20    D1=I1*(Y(IC,ID,K1,K2)-Y(IC,ID,M,N))/(X(IC,IC,K1,K2)-X(IC,ID,M,N)
      IF(K4.EQ.K3) GOTO 11
      D2=I1*(Y(IC,ID,K3,K4)-Y(IC,ID,M,N))/(X(IC,IC,K3,K4)-X(IC,ID,M,N)
11    IF(K4.EQ.K3) D2=0.
      D12=I1*(YS-Y(IC,IC,M,N))/(XS-X(IC,IC,M,N))
      IF(K4.EQ.K3) GOTO 25
      D3=I1*(Y(IC,ID,J1,J2)-Y(IC,ID,K3,K4))
      D3=D3/(X(IC,ID,J1,J2)-X(IC,ID,K3,K4))
25    IF(K4.EQ.K3) D3=0.
      D4=I1*(Y(IC,ID,J1,J2)-Y(IC,ID,K1,K2))
      D4=D4/(X(IC,ID,J1,J2)-X(IC,ID,K1,K2))
      D34=I1*(Y(IC,ID,J1,J2)-YS)/(X(IC,IC,J1,J2)-XS)
      IF(D1.LT.D12 .OR. D2.GT.D12) GOTO 18
      IF(D3.LT.D34 .OR. D4.GT.D34) GOTC 18
      IF(NX7.LT.NX8 .OR. NX7.GE.NX9) GCTC 18
1000  FCRMAT(1X,'FIND : ',I2,',',I2,4(1X,I8),10(1X,F8.3))
1003  FCRMAT(1X,3I3,4(2X,I8))
1005  FCRMAT(1X,'FIND-EXIT')
19    IF(IKK.EQ.1) WRITE(6,1005)
```

```
      RETURN
      END
CXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
CXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
      SUBROUTINE BOUND(LO4,IB,IR,IM,I,X)
      COMPLEX*8 DYDXU
      COMPLEX*8 AI,EI
      REAL*4 X
      COMMON/BC/ T1,T2,T,B,DYDX,D2YDX2,DYDXU,AI,EI
      COMMON/SP/ XS(2,50),YS(2,50),A(2,50,4),R(2,50)
C
C     STEADY BOUNDERY CONDITIONS ALONG BODY
C
      T3=T1
      IF(I.EQ.2) T3=T2
      IF((I*IM).EQ.1 .AND. IR.GT.1) GOTO 2
      IF(LO4.EQ.1) GOTO 3
      DO 5 K=2,50
      J=K-1
      IF(XS(I,K).GE.X) GOTO 6
5     CONTINUE
6     H=X-XS(I,J)
      DYDX=A(I,J,2)+2.*A(I,J,3)*H+3.*A(I,J,4)*H*H
      D2YDX2=2.*A(I,J,3)+6.*A(I,J,4)*H
      GOTO 1
3     I1=(-1)**(I+1)
      DYDX=I1*4.*T3*(1.-2.*X)
      D2YDX2=-I1*8.*T3
      GOTO 1
2     DYDX=0.
      D2YDX2=0.
1     RETURN
      END
CXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
CXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
      SUBROUTINE SHOCK(KV,LO4,EM,IR,IB,M,N,XO,M2,N2,C3)
      COMPLEX*8 U,V,PSI,G,DYDXU
      COMPLEX*8 AI,EI
      REAL*4 X,Y,AL,D1,D3,D,X1,X2,X3,X4,X5,Y1,Y2,Y3,Y4,Y5,Y6,X6
      REAL*4 TX,TY,XX,YY,XP,YP,ALD,D2,P,C3,XO,XT,ALZ,DX1
      COMMON/BA/ ALO,AM,ALD,C1,AK,I,IM,TX,TY,IW,MA,IQ,ET,XX,YY
      COMMON/BC/ T1,T2,T,B,DYDX,D2YDX2,DYDXU,AI,EI,I1,IBACK,IS
      COMMON V(2,3,50,20),X(2,3,50,20),P(2,50),U(2,3,50,20),
     FPSI(2,3,50,20),G(2,3,50,20),AL(2,3,50,20),Y(2,3,50,20),
     FQ(2,50),DX(2),IN(2,50),IN2(2,50)
C
C     COMPUTATION OF THE FIELD NEAR BEHIND THE SHOCK
C
      IKK=0
C     IF(I.EQ.2) IKK=1
      IF(IKK.EQ.1) WRITE(6,1003)
      MAX=1
      ID=1
      IC=1
      IF(IR.EQ.1) GOTO 19
      IC=IR-1
      IF(IB.EQ.1) IC=2
      IF(IB.EQ.2) IC=1
19    DX1=DX(I)
      IF(T.EQ.0. .OR. IQ.EQ.0) GOTO 17
      GOTO 21
17    D=EM*2.*SQRT(ALO)
      IF(IB.EQ.2 .AND. T.EQ.0. .AND. I.EQ.1) D=1.05
      DX1=D/(MA-1)
21    IK=21
      IF(IKK.EQ.1) WRITE(6,1000) M,N,IS,IK,D,DX1,XO,TX,TY
      J=M-1
      K=N
      L=M
      IF(I.NE.2) GOTO 5
      J=M
      K=N-1
      L=N
5     CALL CONST1(LO4,IR,IB,ALZ,C3,XO)
```

```
      CALL BOUND(LO4,IB,IR,IM,I,XO)
      AL(IB,IR,M,N)=(ALZ-C3*DYCX*I1)**(2./3.)
      IF(L.NE.2) GOTO 3
      WRITE(6,1001) M,N,AL(IC,ID,M2,N2),M2,N2
      GOTO 6
3     IK=3
      IF(IKK.EQ.1) WRITE(6,1000) M2,N2,IR,IK,DYCX,XO,AL(IB,IR,M,N)
      D1=I1/SQRT(AL(IB,IR,M,N))
      XP=XO+XX
      YP=YY
      X5=X(IB,IR,J,K)
      Y5=Y(IB,IR,J,K)
9     IK=9
      IF(IKK.EQ.1) WRITE(6,1000) J,K,I1,IK,D1,XP,YP,X5,Y5
      IF(IE.EQ.1 .OR. M2.GE.KV) GOTO 16
      L1=M2+1
      L2=N2
      L3=M2+1
      L4=N2+1
      IF(I.NE.2) GOTO 7
      L1=M2
      L2=N2+1
7     CONTINUE
      IF(IR.EQ.1) GOTO 22
      L1=M2
      L2=N2+1
      IF(IC.NE.2) GOTO 22
      L1=M2+1
      L2=N2
22    X1=X(IC,ID,M2,N2)
      Y1=Y(IC,ID,M2,N2)
      X2=X(IC,ID,L1,L2)
      Y2=Y(IC,ID,L1,L2)
      X3=X(IC,ID,L3,L4)
      Y3=Y(IC,ID,L3,L4)
      IZ=0
      XT=XO
      IK=22
      IF(IKK.EQ.1) WRITE(6,1000) M2,N2,IZ,IK,X5,Y5,X1,Y1,X2,Y2
14    D2=SQRT(AL(IC,ID,M2,N2))+SQRT(AL(IB,IR,J,K))
      D2=2.*I1/D2
      IF(T.NE.0. .AND. IQ.NE.0) GOTO 2
      X4=(L-2)*DX1/2.+TX
      Y4=D2*(X4-TX)+TY
      GOTO 1
2     X4=(D1*XP-D2*X5+Y5-YP)/(D1-D2)
      Y4=(X4-XP)*D1+YP
1     IK=1
      IF(IKK.EQ.1) WRITE(6,1000) IC,ID,IE,IK,X4,Y4,XP,XX,D2
      IF(I.EQ.1 .AND. IR.GT.1) GOTO 20
      IF(Y4.LE.0.0001 .AND. Y4.GE.0.) GOTO 8
      IF(Y4.GT.0. .AND. IZ.EQ.0) GOTO 13
      IF(T.NE.0.) GOTO 4
      Y4=0.
      X4=-EM/D2+TX
      GOTO 8
20    CONTINUE
      IF(Y4.LE.EM .AND. Y4.GE.(0.999*EM)) GOTO 8
      IF(Y4.LT.EM .AND. IZ.EQ.0) GOTO 13
      IF(T.NE.0. .AND. IQ.NE.0) GOTO 4
      Y4=EM
      X4=TX+EM/D2
      GOTO 8
4     X5=X(IB,IR,J,K)
      Y5=Y(IB,IR,J,K)
      X6=(EM*(2-IB)-Y5)/D2+X5
      XT=(XP-X4)*EM/(I1*Y4+EM*(IB-1))+X6-XX
      IK=4
      IF(IKK.EQ.1) WRITE(6,1000) IC,ID,IE,IK,X6,XT,D1,X5,Y5
      CALL CONST1(LO4,IR,IB,ALZ,C3,XT)
      CALL BOUND(LO4,IB,IR,IM,I,XT)
      AL(IB,IR,M,N)=(ALZ-C3*DYCX*I1)**(2./3.)
      D1=I1/SQRT(AL(IB,IR,M,N))
      IZ=IZ+1
```

```
      XP=XT+XX
      IF(IZ.LT.30) GOTO 14
      IF(IKK.EQ.1) WRITE(6,1000) M,N,IZ,IE,X4,Y4,X5,Y5
      X4=X6
      Y4=EM*(2-IB)
      AL(IB,IR,M,N)=AL(IB,IR,J,K)
      GOTO 8
13    CALL FIND(KV,IM,IB,IR,X4,Y4,M1,N1)
      IK=13
      IF(IKK.EQ.1) WRITE(6,1000) M1,N1,IE,IK,X4,Y4
     F,X(IC,ID,M1,N1),Y(IC,ID,M1,N1)
      IF(IE.EQ.1) WRITE(6,1002) M,N,M1,N1,X4,Y4,X(IC,ID,M1,N1)
     F,Y(IC,ID,M1,N1)
      IF(M1.EQ.M2 .AND. N1.EQ.N2 .AND. IE.EQ.0) GOTO 8
      IF(T.NE.0. .AND. IQ.NE.0) GOTO 15
      IF(IE.EQ.1) GOTO 16
      M2=M1
      N2=N1
      GOTO 8
15    X6=(Y2-Y5)/D2+X5
      IF(X6.GE.X2) GOTO 10
      D3=(Y2-Y1)/(X2-X1)
      X5=(Y1-Y5+D2*X5-D3*X1)/(D2-D3)
      Y5=D3*(X5-X1)+Y1
      IK=15
      IF(IKK.EQ.1) WRITE(6,1000) M1,N1,IE,IK,X5,Y5,X6,D3
      IF(IC.EQ.2) GOTO 23
      N2=N2-1
      IF(IR.GT.1) GOTO 26
      IF(N2.EQ.0) GOTO 16
      IF(AL(IC,ID,M2,N2).NE.AL(IC,ID,M2-1,N2) .AND. IR.EQ.1) GOTO 16
      IF(I.NE.2) GOTO 9
26    M2=M2-1
      N2=N2+1
      GOTO 9
23    N2=N2-1
      GOTO 9
10    D3=(Y3-Y2)/(X3-X2)
      X5=(Y2-Y5+D2*X5-D3*X2)/(D2-D3)
      Y5=D3*(X5-X2)+Y2
      IK=10
      IF(IKK.EQ.1) WRITE(6,1000) M1,N1,IE,IK,X5,Y5,X6,D3
      IF(IC.EQ.2) GOTO 24
      M2=M2+1
      IF(IR.GT.1) GOTO 25
      IF(M2.EQ.KV) GOTO 16
      IF(I.NE.2) GOTO 9
25    N2=N2+1
      M2=M2-1
      GOTO 9
24    M2=M2+1
      GOTO 9
8     X(IB,IR,M,N)=X4
      Y(IB,IR,M,N)=Y4
      IF(Y4.LE.0.0001 .AND. Y4.GE.0. .AND. IB.EQ.2) Y(2,IR,M,N)=0.
      IF(Y4.LE.EM .AND. Y4.GE.(0.999*EM) .AND. IB.EQ.1) Y(1,IR,M,N)=EM
      IK=8
      IF(IKK.EQ.1) WRITE(6,1000) M,N,IE,IK,X4,Y4,X1,Y1
      GOTO 11
6     X(IB,IR,M,N)=TX
      Y(IB,IR,M,N)=TY
      GOTO 12
16    IE=1
      D2=SQRT(AL(IB,IR,J,K))+SQRT(AL(IB,IR,M,N))
      D2=4.*I1/(2.*SQRT(ALO)+D2)
      IF(T.NE.0. .AND. IQ.NE.0) GOTO 18
      X(IB,IR,M,N)=(L-2)*DX1/2.+TX
      Y(IB,IR,M,N)=D2*(X(IB,IR,M,N)-TX)+TY
      GOTO 11
18    X(IB,IR,M,N)=(D1*XP-D2*X5+Y5-YP)/(D1-D2)
      Y(IB,IR,M,N)=(X(IB,IR,M,N)-XP)*D1+YP
C
C     COMPUTATION OF THE ADDITIONAL POINTS FOR THE UNSTEADY
C     FLOW FIELD
```

```
C
11    D3=1./SQRT(AL(IB,IR,M,N))
      D4=1./SQRT(AL(IB,IR,J,K))
      P(I,L)=(D4*X(IB,IR,J,K)+D3*XP+(Y(IB,IR,J,K)-YP)*I1)/(D3+D4)
      Q(I,L)=D3*(P(I,L)-XP)*I1+YP
      IF(T.EQ.0.) P(I,L)=X(IB,IR,J,K)
      IF(L.NE.3) GOTO 12
      P(I,L)=XP
      Q(I,L)=0.
12    CONTINUE
1000  FORMAT(1X,4I4,2X,6(2X,F8.3))
      IN2(1,L)=M2
      IN2(2,L)=N2
1001  FORMAT(1X,I2,',',I2,' LAMBDA= ',F9.7,' MESH: ',I2,',',I2)
1002  FORMAT(1X,'SHOCK RUNS OUT: ',2(I2,',',I2,2X),4(F8.3,2X))
1003  FORMAT(1X,'SHOCK - ENTRY')
      RETURN
      END
CXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
CXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
      SUBROUTINE RAND(L04,IB,IR,M,N,C1)
      COMPLEX*8 L,V,PSI,G,DYDXU
      COMPLEX*8 AI,EI
      REAL*4 TX,TY,XX,YY,X,Y,AL,F,D,X2,X1,P,ALC,C1,A,Y1,FS,ALZ
      COMMON/BA/ ALO,AM,ALD,C3,AK,I,IM,TX,TY,IW,MA,IG,ET,XX,YY
      COMMON/BC/ T1,T2,T,B,DYDX,D2YDX2,DYDXU,AI,EI,I1
      COMMON V(2,3,50,20),X(2,3,50,20),P(2,50),U(2,3,50,20),
     FPSI(2,3,50,20),G(2,3,50,20),AL(2,3,50,20),Y(2,3,50,20),
     FQ(2,50),DX(2),IN(2,50),IN2(2,50)
C
C     BOUNDERY STEP OF THE STEADY FIELD TO THE BODY
C
      IK=0
C     IF(IR.GT.1) IK=1
      IF(IK.EQ.1) WRITE(6,1003)
      A=-C1*I1
      J1=M
      J2=N-1
      IF(I.NE.2) GOTO 5
      J1=M-1
      J2=N
5     X1=X(IB,IR,J1,J2)-XX
      Y1=Y(IB,IR,J1,J2)-YY
      X2=X1
      DO 11 KX=1,30
      CALL CONST1(L04,IR,IB,ALZ,C1,X2)
      CALL BOUND(L04,IB,IR,IM,I,X2)
      IF(IK.EQ.1) WRITE(6,1002) IB,IR,M,N,DYDX,ALZ,X1,Y1
      AL(IB,IR,M,N)=(ALZ+A*DYDX)**(2./3.)
      IF(T.EQ.0. .OR. IQ.EQ.0) GOTO 2
      IF(IK.EQ.1) WRITE(6,1002) IB,IR,M,N,DYDX,AL(IB,IR,M,N),Y1,D2YDX
1002  FORMAT(1X,4I4,4(F10.3))
      F=2.*(X2-X1)*I1/Y1
      F=F-(SQRT(AL(IB,IR,J1,J2))+SQRT(AL(IB,IR,M,N)))
      FS=2.*I1/Y1-A*D2YDX2/(3.*AL(IB,IR,M,N))
      D=F/FS
      X2=X2-D
      IF(ABS(D).LE.0.000001) GOTO 10
11    CONTINUE
      WRITE(6,1000) M,N,D
10    CALL CONST1(L04,IR,IB,ALZ,C1,X2)
      CALL BOUND(L04,IB,IR,IM,I,X2)
      IF(IK.EQ.1) WRITE(6,1002) IB,IR,M,N,DYDX,ALZ,X2,D
      IF(IK.EQ.1) WRITE(6,1004)
      AL(IB,IR,M,N)=(ALZ+A*DYDX)**(2./3.)
      Y(IB,IR,M,N)=YY
      X(IB,IR,M,N)=X2+XX
      GOTO 1
2     Y(IB,IR,M,N)=YY
      D=-2.*I1/(SQRT(AL(IB,IR,J1,J2))+SQRT(AL(IB,IR,M,N)))
      X(IB,IR,M,N)=X(IB,IR,J1,J2)-(Y(IB,IR,J1,J2)-YY)/D
1     IF(IK.EQ.1) WRITE(6,1002) IB,IR,M,N,X(IB,IR,M,N),AL(IB,IR,M,N)
1000  FORMAT(1X,'RAND : ',I2,',',I2,'   D= ',E10.3)
1003  FORMAT(1X,'RAND-ENTRY :')
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1004  FORMAT(1X,'RAND-ITERATION-EXIT.')
      RETURN
      END
CXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
CXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
      SUBROUTINE GEN(IB,IR,M,N,I)
      COMPLEX*8 U,V,PSI,G,DYDXU
      COMPLEX*8 AI,EI
      REAL*4 X,Y,AL,D1,D2,X1,P
      COMMON/BC/ T1,T2,T,B,DYDX,D2YDX2,DYDXU,AI,EI,I1
      COMMON V(2,3,50,20),X(2,3,50,20),P(2,50),U(2,3,50,20),
     FPSI(2,3,50,20),G(2,3,50,20),AL(2,3,50,20),Y(2,3,50,20),
     FQ(2,50),DX(2),IN(2,50),IN2(2,50)
C
C     GENERAL STEP OF STEADY FIELD
C
      J1=M
      J2=N-1
      K1=M-1
      K2=N
      IF(I.NE.2) GOTO 5
      J1=M-1
      J2=N
      K1=M
      K2=N-1
5     D1=1./SQRT(AL(IB,IR,K1,K2))
      D2=2./(SQRT(AL(IB,IR,J1,J2))+SQRT(AL(IB,IR,K1,K2)))
      X1=D2*X(IB,IR,J1,J2)+D1*X(IB,IR,K1,K2)
      X1=X1+(Y(IB,IR,J1,J2)-Y(IB,IR,K1,K2))*I1
      X(IB,IR,M,N)=X1/(D1+D2)
      Y(IB,IR,M,N)=D1*I1*(X(IB,IR,M,N)-X(IB,IR,K1,K2))+Y(IB,IR,K1,K2)
      AL(IB,IR,M,N)=AL(IB,IR,K1,K2)
      RETURN
      END
CXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
CXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
      SUBROUTINE BOUNDU(IM,IR,IB,X,I,AK.L)
      COMPLEX*8 DYDXU
      COMPLEX*8 AI,EI
      REAL*4 X
      COMMON/BC/ T1,T2,T,B,DYDX,D2YDX2,DYDXU,AI,EI,I1
C
C     UNSTEADY BOUNDERY - CONDITIONS ALONG BODY
C
      DYDXU=0.
      IF((I*IM).EQ.1 .AND. IR.GT.1) GOTO 13
      IF(L.EQ.1) GOTO 11
      DYDXU=-1.-(X-B)*AK*AI
      GOTO 13
11    DYDXU= AI*AK
13    RETURN
      END
CXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
CXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
      SUBROUTINE RANDS(KV,IB,IR,M,N,L,ANY,M2,N2)
      COMPLEX*8 U,V,PSI,G,DYDXU,D2,D6,PII,CL,DL,DV
      COMPLEX*8 AI,EI,EI2
      COMPLEX*16 A,RI,ES
      REAL*4 X,Y,AL,ANY,S,AMN,AM3,AM4,AN3,AN4,D0,D1,D3,D4,D5,D7,XE
      REAL*4 P,ALD,CY,CZ,Z1,Z2,W,AA1,AA2,AA3,AA4,AA5,AM1,AM2,AN1,AN2
      REAL*4 TX,TY,XX,YY,DAL,ANY2,B1
      COMMON/BA/ AL0,AM,ALD,C,AK,I,IM,TX,TY,IW,MA,IQ,ET,XX,YY
      COMMON/BC/ T1,T2,T,B,DYDX,D2YDX2,DYDXU,AI,EI,I1,IBACK,IE
      COMMON/SCL/ A(4,4),RI(4),ES(4)
      COMMON V(2,3,50,20),X(2,3,50,20),P(2,50),U(2,3,50,20),
     FPSI(2,3,50,20),G(2,3,50,20),AL(2,3,50,20),Y(2,3,50,20),
     FQ(2,50),DX(2),IN(2,50),IN2(2,50)
C
C     COMPUTATION OF UNSTEADY BOUNDERY-PROPERTIES ALONG SHOCK
C
      IKK=0
C     IF(IM.EQ.4 .AND. IR.GT.1) IKK=1
      IF(IKK.EQ.1) WRITE(6,1001)
1001  FORMAT(1X,'RANDS-ENTRY')
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      IK=0
      IF(IKK.EQ.1) WRITE(6,1CC2) M,N,IK,X(IB,IR,M,N),Y(IB,IR,M,N)
1002  FCRMAT(3I4,5(2X,E11.4))
      ID=1
      IC=1
      IF(IR.EQ.1) GOTO 1
      ID=IR-1
      IF(IB.EQ.1) IC=2
      IF(IB.EQ.2) IC=1
1     EI2=1./EI
      IF(IC.EQ.2) EI2=EI
      J =M
      IF(I.NE.2) GOTO 5
      J =N
5     IE=0
      CALL FIND(KV,IM,IB,IR,X(IB,IR,M,N),Y(IB,IR,M,N),M2,N2)
      ALT=AL(IC,ID,M2,N2)
      DV=V(IC,ID,M2,N2)
      DU=U(IC,ID,M2,N2)
      IF(IE.EQ.1) ALT=ALO
      IF(IE.EQ.1)  DV=0.
      IF(IE.EC.1)  DU=0.
      IK=5
      IF(IKK.EQ.1) WRITE(6,1CC2) IC,ID,IK,X(IC,ID,M2,N2),Y(IC,ID,M2,N
      CX=C-2.
      ANY=(ALT-AM+1.)/(C*AM)+1.
      ANY2=ANY*ANY
      L1=M+1
      L2=N
      IF(I.NE.2) GOTO 22
      L1=M
      L2=N+1
22    IF(L1.GT.KV .OR. L2.GT.KV) IE=1
C     IF(IE.EC.1) GOTO 23
C     S=Y(IB,IR,L1,L2)-Y(IB,IR,M,N)
C     S=S/(X(IB,IR,L1,L2)-X(IB,IR,M,N))
      S=2.*I1/(SQRT(ALT)+SQRT(AL(IB,IR,M,N)))
      S=ATAN(S)
      CY=SIN(S)**2
      CZ=COS(S)**2
      AMN=1./(AM*CY*ANY2)
      AM1=2.*CY*ANY*SIN(2.*S)/C
      AM2=2.*AK*CY*(1.+AMN)/C
      AM3=CX*CY*(1.-2.*AMN/CX)/C+CZ
      AM4=SIN(2.*S)*(1.+AMN)/C
      AN1=-2.*CY*ANY*(COS(2.*S)+AMN)/C
      AN2=-2.*AK*SIN(S)*COS(S)*(1.+AMN)/C
      AN3=AM4
      AN4=CY+CZ*CX*(1.-2.*AMN/CX)/C
      IF(J.GT.2) GOTO 6
      XE=X(IB,IR,M,N)-XX
      CALL BCUNDU(IM,IR,IB,XE,I,AK,L)
      V(IB,IR,M,N)=DYDXU
      CL=AM1*V(IB,IR,M,N)/AN1+DU*(AM3-AM1*AN3/AN1)*EI2
      U(IB,IR,M,N)=CL+DV*(AM4-AM1*AN4/AN1)*EI2
      K1=M+1
      K2=N
      IF(I.NE.2) GOTO 4
      K1=M
      K2=N+1
4     D3=(SQRT(AL(IB,IR,M,N))+SQRT(AL(IE,IR,K1,K2)))/2.
      D3=-I1/D3
      D1=I1/SQRT(AL(IB,IR,K1,K2))
      IK=4
      IF(IKK.EQ.1) WRITE(6,1CC2) M,N,IK,D3,D1,S
      D4=(YY-Y(IB,IR,K1,K2))/D1+X(IB,IR,K1,K2)
      X(IB,IR,M,N)=(D3*D4-TX*TAN(S))/(D3-TAN(S))
      IF(T.EQ.0. .OR. IQ.EQ.C) X(IB,IR,M,N)=TX
      Y(IB,IR,M,N)=(X(IB,IR,M,N)-TX)*TAN(S)+YY
6     J1=M-1
      J2=N
      LF=M
      IF(I.NE.2) GOTO 8
      J1=M
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      J2=N-1
      LP=N
8     CONTINUE
      IF(AL(IB,IR,J1,J2).LT.AL(IB,IR,M,N) .OR. T.EQ.0.) GOTO 12
      IF(IQ.EQ.0) GOTO 12
      J2=N-1
      IF(I.NE.2) GOTO 12
      J1=M-1
12    AMN=AM*CY
C     AA1=-2.*(C-1.)*SIN(2.*S)*AMN*ANY2/C
C     AA2=-4.*(C-1.)*AK*ANY*AMN/C
C     AA5=(C-1.)*AM*(CX-2.*AMN*ANY2)/C
C     AA3=4.*(C-1.)*ANY*AMN/C+AA5
C     AA4=-4.*(C-1.)*ANY*AMN*COS(S)/(C*SIN(S))
C     AA5=AK*AA5
      IF(J.EQ.2) GOTO 7
      DO=X(IB,IR,M,N)-P(I,LP)
C     WRITE(6,4711) M,N,LP,X(IB,IR,M,N),X(IB,IR,J1,J2),P(I,LP)
C4711  FORMAT(1X,3I4,3(2X,E12.5))
      DAL=AL(IB,IR,M,N)
      IE=0
      CALL FIND(KV,IM,IB,IR,X(IB,IR,J1,J2),Y(IE,IR,J1,J2),M3,N3)
      DU=U(IC,ID,M3,N3)+U(IC,ID,M2,N2)
      DV=V(IC,ID,M3,N3)+V(IC,ID,M2,N2)
      IF(IE.EQ.1) DU=0.
      IF(IE.EQ.1) DV=0.
      MP=1000000.*(P(I,LP)-X(IB,IR,J1,J2))
      IF(MP.LE.0 .AND. T.NE.0.) WRITE(6,1003) M,N,P(I,LP),X(IB,IR,J1,J2
1003  FORMAT(1X,'P-X.LE.0. :',2I4,2(2X,E12.5))
      IF(MP.NE.0) GOTO 3
      IF(T.NE.0. .AND. IQ.NE.0) GOTO 3
      D1=0.
      GOTO 2
3     D1=(AL(IB,IR,M,N)-AL(IB,IR,J1,J2))*DO
      D1=D1/(4.*DAL*(P(I,LP)-X(IB,IR,J1,J2)))
      IK=3
      IF(IKK.EQ.1) WRITE(6,1002) M2,N2,IK,DO,D1,DU,DV,DAL
2     D2=AI*AK*AM*DO/DAL
      D3=-AM*(AK*DO)**2/(4.*DAL)
      D4=1./SQRT(AL(IB,IR,M,N))
      D5=-D3*D4
      D6=AK*AK*AM*PSI(IB,IR,J1,J2)*DO/DAL
      D7=Y(IB,IR,M,N)-Y(IB,IR,J1,J2)
      A(1,1)=1.+D1+D2+D3
      A(1,2)=-(D4+D5)*I1
      A(1,3)=0.
      RI(1)=U(IB,IR,J1,J2)*(1.-D1-D2-D3)-V(IB,IR,J1,J2)*(D4-D5)*I1+D6
      A(2,1)=1.
      A(2,2)=0.
      A(2,3)=-2.*AM1/D7-AI*AM2
      RI(2)=-(2.*AM1/D7-AI*AM2)*G(IB,IR,J1,J2)-U(IB,IR,J1,J2)
      RI(2)=RI(2)+(AM3*DU+AM4*DV)*EI2
      A(3,1)=0.
      A(3,2)=1.
      A(3,3)=-2.*AN1/D7-AI*AN2
      RI(3)=-(2.*AN1/D7-AI*AN2)*G(IB,IR,J1,J2)-V(IB,IR,J1,J2)
      RI(3)=RI(3)+(AN3*DU +AN4*DV )*EI2
C
      NO=3
      CALL SOLVE(NO)
      U(IB,IR,M,N)=ES(1)
      V(IB,IR,M,N)=ES(2)
      G(IB,IR,M,N)=ES(3)
C7    PII=V(IB,IR,M,N)-AI*AN2*G(IB,IR,M,N)
CYY   PII=PII-(AN3*U(IC,ID,M2,N2)-AN4*V(IC,ID,M2,N2))*EI2
CYY   PII=AA1*PII/AN1+AI*AA2*G(IB,IR,M,N)+AA3*U(IC,ID,M2,N2)*EI2
CYY   PII=PII+(AA4*V(IC,ID,M2,N2)+AA5*AI*PSI(IC,ID,M2,N2))*EI2
CYY   AT=AK
CYY   IF(AK.EQ.0.) AT=1.
CYY   PSI(IB,IR,M,N)=AI*(U(IB,IR,M,N)+PII/((C-1.)*AM))/AT
7     IF(J.GT.2) GOTO 20
      PSI(IB,IR,M,N)=PSI(IC,ID,M2,N2)*EI2
      GOTO 21
20    PII=(V(IB,IR,M,N)+V(IB,IR,J1,J2))*D4*I1
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      PII=(PII+U(IB,IR,M,N)+U(IB,IR,J1,J2))*0.5*DO
      PSI(IB,IR,M,N)=PII+PSI(IB,IR,J1,J2)
21    RETURN
      END
CXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
CXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
      SUBROUTINE GENU(IM,IB,IR,M,N,I,AK,AM)
      COMPLEX*8 U,V,PSI,G,DYDXU,D2,D6,B6,CL,B2,CM
      COMPLEX*8 AI,EI
      COMPLEX*16 A,RI,ES
      REAL*4 X,Y,AL,Z1,Z2,Z3,P,DO,D1,D3,D4,D5,BO,B1,B3,B4,B4,F1,F2
      COMMON/BC/ T1,T2,T,B,DYDX,D2YDX2,DYDXU,AI,EI,I1
      COMMON/SCL/ A(4,4),RI(4),ES(4)
      COMMON V(2,3,50,20),X(2,3,50,20),P(2,50),U(2,3,50,20),
     FPSI(2,3,50,20),G(2,3,50,20),AL(2,3,50,20),Y(2,3,50,20),
     FQ(2,50),DX(2),IN(2,50),IN2(2,50)
C
C     GENERAL STEP OF UNSTEADY FIELD
C
      IKK=0
C     IF(IR.EQ.2) IKK=1
      IF(IKK.EQ.1) WRITE(6,1002)
      J1=M
      J2=N-1
      J3=M-1
      J4=N
      IF(I.NE.2) GOTO 5
      J1=M-1
      J2=N
      J3=M
      J4=N-1
5     DO=X(IB,IR,M,N)-X(IB,IR,J1,J2)
      D1=0.5*(AL(IB,IR,M,N)-AL(IB,IR,J1,J2))
      D2=AI*2.*AK*AM*DO
      D3=-0.5*AM*(AK*DO)**2
      D4=AL(IB,IR,M,N)+AL(IB,IR,J1,J2)
      D5=SQRT(AL(IB,IR,M,N))+SQRT(AL(IB,IR,J1,J2))
      IF(IKK.EQ.1) WRITE(6,1001) M,N,AL(IB,IR,M,N),D4,DO
      D6=(D1+D2+D3)/D4
      A(1,1)=1.+D6
      A(1,2)=2.*(1.-D3/D4)*I1/D5
      RI(1)=U(IB,IR,J1,J2)*(1.-D6)+V(IB,IR,J1,J2)*I1*2.*(1.+D3/D4)/D5
      RI(1)=2.*AK*AK*AM*PSI(IB,IR,J1,J2)*DO/D4+RI(1)
      BO=X(IB,IR,M,N)-X(IB,IR,J3,J4)
      B1=(AL(IB,IR,M,N)-AL(IB,IR,J1,J2))*BO/(2.*DO)
      B2=2.*AI*AK*AM*BO
      B3=-0.5*AM*(AK*BO)**2
      B4=1./SQRT(AL(IB,IR,M,N))
      B5=-0.5*B3*B4/AL(IB,IR,M,N)
      B6=0.5*(B1+B2+B3)/AL(IB,IR,M,N)
      A(2,1)=1.+B6
      A(2,2)=-(B4+B5)*I1
      RI(2)=U(IB,IR,J3,J4)*(1.-B6)+V(IB,IR,J3,J4)*I1*(-B4+B5)
      RI(2)=PSI(IB,IR,J3,J4)*AM*BO*AK*AK/AL(IB,IR,M,N)+RI(2)
C
      NO=2
      CALL SOLVE(NO)
      U(IB,IR,M,N)=ES(1)
      V(IB,IR,M,N)=ES(2)
      CL=V(IB,IR,J1,J2)+V(IB,IR,M,N)
      CL=(U(IB,IR,J1,J2)+U(IB,IR,M,N)-I1*2.*CL/D5)*DO/2.
      CM=(V(IB,IR,J3,J4)+V(IB,IR,M,N))*B4
      CM=(U(IB,IR,J3,J4)+U(IB,IR,M,N)+I1*CM)*BO/2.
      CL=PSI(IB,IR,J1,J2)+PSI(IB,IR,J3,J4)+CL+CM
      PSI(IB,IR,M,N)=CL/2.
1001  FORMAT(1X,2I4,3(2X,F8.3))
1002  FORMAT(1X,'GENU-ENTRY')
      RETURN
      END
CXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
CXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
      SUBROUTINE RANDB(IB,IR,M,N,L)
      COMPLEX*8 U,V,PSI,G,DYDXU,D2,A,D,CL,D6
      COMPLEX*8 AI,EI
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      REAL*4 TX,TY,XX,YY,X,Y,AL,D0,D1,D4,D5,ALD,D3,P,XT
      COMMON/BA/ ALO,AM,ALD,C1,AK,I,IM,TX,TY,IW,MA,IC,ET,XX,YY
      COMMON/BC/ T1,T2,T,B,DYCX,D2YDX2,DYDXU,AI,EI,I1
      COMMON V(2,3,50,20),X(2,3,50,20),P(2,50),U(2,3,50,20),
     FPSI(2,3,50,20),G(2,3,50,20),AL(2,3,50,20),Y(2,3,50,20),
     FQ(2,50),DX(2),IN(2,50),IN2(2,50)
C
C     COMPUTATION OF UNSTEADY BOUNDERY-PROPERTIES ON BODY
C
      XT=X(IB,IR,M,N)-XX
      CALL BOUNDU(IM,IR,IB,XT,I,AK,L)
      V(IB,IR,M,N)=DYDXU
      J1=M
      J2=N-1
      IF(I.NE.2) GOTO 5
      J1=M-1
      J2=N
5     D1=0.5*(AL(IB,IR,M,N)-AL(IB,IR,J1,J2))
      D0=X(IB,IR,M,N)-X(IB,IR,J1,J2)
      D6=V(IB,IR,M,N)+V(IB,IR,J1,J2)
      D2=AI*AK*AM*D0*2.
      D3=-0.5*(AK*D0)**2*AM
      D4=AL(IB,IR,M,N)+AL(IB,IR,J1,J2)
      D5=SQRT(AL(IB,IR,M,N))+SQRT(AL(IB,IR,J1,J2))
      A=(D1+D2+D3)/D4
      U(IB,IR,M,N)=(V(IB,IR,M,N)-V(IB,IR,J1,J2))*I1/D5
      U(IB,IR,M,N)=U(IB,IR,J1,J2)*(1.-A)-2.*U(IB,IR,M,N)
      D=2.*PSI(IB,IR,J1,J2)-D0*D6*I1/D5
      U(IB,IR,M,N)=U(IB,IR,M,N)+AK*AK*AM*D0*D/D4
      U(IB,IR,M,N)=U(IB,IR,M,N)/(1.+A)
      CL=U(IB,IR,M,N)+U(IB,IR,J1,J2)-2.*D6*I1/D5
      PSI(IB,IR,M,N)=PSI(IB,IR,J1,J2)+D0*CL/2.
      RETURN
      END
CXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
CXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
      SUBROUTINE CONST1(L04,IR,IB,ALZ,C3,XP)
      COMPLEX*8 L,V,PSI,G,DYDXU
      COMPLEX*8 AI,EI
      REAL*4 X,Y,AL,D1,D2,D3,XP,ALD,P,C3,TX,TY,XX,YY,ALZ,X1,Y1,X2,X3
      COMMON/BA/ ALO,AM,ALD,C1,AK,I,IM,TX,TY,IW,MA,IC,ET,XX,YY
      COMMON/BC/ T1,T2,T,B,DYCX,D2YDX2,DYDXU,AI,EI,I1
      COMMON V(2,3,50,20),X(2,3,50,20),P(2,50),U(2,3,50,20),
     FPSI(2,3,50,20),G(2,3,50,20),AL(2,3,50,20),Y(2,3,50,20),
     FQ(2,50),DX(2),IN(2,50),IN2(2,50)
C
C     DETERMINATION OF THE CONSTANT VALUE ALONG THE CHARACTERISTIC
C
      IF(IR.EQ.1 .AND. IM.EQ.1) GOTO 1
      IF(IR.EQ.1 .AND. I.EQ.1) GOTO 1
      IK=0
C     IF(IR.GT.1) IK=1
      IC=1
      ID=1
      IF(IR.EQ.1) GOTO 2
      ID=IR-1
      IC=2
      IF(IB.EQ.2) IC=1
2     CONTINUE
      DO 3 J=3,20
      M=J
      N=J-1
      IF(IC.NE.2) GOTO 4
      M=J-1
      N=J
4     X1=X(IC,ID,M,N)
      Y1=Y(IC,ID,M,N)
      IF(IK.EQ.1) WRITE(7,1000) IC,ID,M,N,X1,Y1,AL(IC,ID,M,N)
1000  FORMAT(1X,'CONST1: ',4I4,3F8.3)
      D1=-I1/SQRT(AL(IC,ID,M,N))
      X2=(YY-Y1)/D1+X1
      X3=XP+XX
      IF(X2.GE.X3) GOTO 5
3     CONTINUE
```

```
5      M=M-1
       N=N-1
       X1=X(IC,ID,M,N)-X(IC,1,2,1)
       CALL BOUND(LO4,IC,ID,IM,IC,X1)
       ALZ=AL(IC,IC,M,N)**1.5+I1*C3*DYDX
       GOTO 6
1      ALZ=ALD
6      RETURN
       END
CXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
CXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
       SUBROUTINE PRESS(AMY,IR,EM,LO2,AX,C)
       COMPLEX*8 U,V,PSI,G,DYDXU
       COMPLEX*8 AI,EI,PU
       REAL*4 TX,TY,XX,YY,X,Y,AL,D1,D2,D3,XP,ALD,F,C3,ALZ
       COMMON/BA/ ALO,AM,ALD,C1,AK,I,IM,TX,TY,IW,MA,IQ,ET,XX,YY
       COMMON/BC/ T1,T2,T,B,DYDX,D2YDX2,DYDXU,AI,EI,I1
       COMMON V(2,3,50,20),X(2,3,50,20),P(2,50),U(2,3,50,20),
      FPSI(2,3,50,20),G(2,3,50,20),AL(2,3,50,20),Y(2,3,50,20),
      FQ(2,50),DX(2),IN(2,50),IN2(2,50),PX(2,30),PS(2,30),PU(2,30)
C
C      COMPUTATION AND OUTPUT OF THE PRESSURE - COEFFICIENTS
C      ALONG THE CHOSEN BLADE
C      PU=CPU , PS=CPS
C
       YN=-EM
       YM=AMY*180./(4.*ATAN(1.))
       IM2=IM-2
       DO 7 I=1,2
       K1=0
       DO 1 K=1,IR
       K3=2
       IF(I.EQ.1 .AND. K.EQ.1) K3=1
       K3=K+K3
       DO 3 J1=1,20
       LI=J1+1
       CALL SWITCH(LI,J1,M,N,I)
       IF(K3.GT.IR) GOTO 10
       LI=1
       LZ=2
       CALL SWITCH(LZ,LI,M1,N1,I)
       IF(X(1,K,M,N).GT.X(1,K3,M1,N1)) GOTO 6
       GOTO 3
10     D1=REAL(U(1,K,M,N))**2+AIMAG(U(1,K,M,N))**2
       IF(D1.EQ.0.) GOTO 6
3      CONTINUE
6      IF(J1.EQ.1) GOTO 1
       J1=J1-1
       K1=K1+J1
       DO 4 J2=1,J1
       LI=J2+1
       CALL SWITCH(LI,J2,M,N,I)
       K3=J2+K1-J1
       PS(I,K3)=-2.*(AL(1,K,M,N)-ALO)/(C1*AM)
       PU(I,K3)=-2.*(U(1,K,M,N)+AI*AK*PSI(1,K,M,N))
4      PX(I,K3)=X(1,K,M,N)
1      CONTINUE
7      IN2(I,1)=K1
C
C      OUTPUT
C
       WRITE(7,1015)
       IF(LO2.EQ.0) WRITE(7,996)
       IF(LO2.EQ.1) WRITE(7,995)
       WRITE(7,1016) YM, IM2
       WRITE(7,1001) AK,AX,C,B,T1
       WRITE(7,1010)
       WRITE(7,1011)
       DO 8 I=1,2
       IF(I.EQ.2) WRITE(7,1001) AK,AX,C,B,T2
       IF(I.EQ.2) WRITE(7,1012)
       IF(I.EQ.2) WRITE(7,1011)
       K=IN2(I,1)
       DO 9 J=1,K
```

```
      LI=J+1
      CALL SWITCH(LI,J,M,N,I)
9     WRITE(7,1013) M,N,PX(I,J),PS(I,J),PU(I,J)
8     WRITE(7,1014)
995   FORMAT(1X,/,1X,'PLUNGE - MODE',/)
996   FORMAT(1X,/,1X,'PITCH - MODE',/)
1001  FORMAT(1X,//,1X,'W*C/U= ',F7.5,', M= ',F5.3,', K= ',F4.2,
     F', B/C= ',F4.2,', T/C= ',F6.4,////)
1010  FORMAT(1X,'PRESSURE-COEFFICIENTS UPPER SURFACE:',//)
1011  FORMAT(2X,'POINT',4X,'X',8X,'CPS',8X,'RCPU',8X,'ICPU',/)
1012  FORMAT(1X,'PRESSURE-COEFFICIENTS LOWER SURFACE:',//)
1013  FORMAT(1X,I2,',',I2,4X,F5.3,3(2X,E10.3))
1014  FORMAT(//,1X,'***********************************************
     F,//)
1015  FORMAT(1H1)
1016  FORMAT(1X,'PHASE= ',F7.2,'  / ',I2,'. BLADE')
      RETURN
      END
CXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
CXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
      SUBROUTINE SWITCH(J,K,M,N,I)
C     INDEX - SWITCH
      M=J
      N=K
      IF(I.NE.2) GOTO 1
      M=K
      N=J
1     RETURN
      END
CXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
CXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
      SUBROUTINE SOLVE(N)
      COMPLEX*16 G,C,X
      COMMON/SOL/ G(4,4),C(4),X(4)
C
C     SOLVE GIVES THE SOLUTION FOR A COMPLEX SYSTEM OF
C     LINEAR EQUATIONS   G*X=C
C
      IKK=0
      IF(IKK.NE.1) GOTO 2
      WRITE(6,1001)
      DO 1 M=1,N
1     WRITE(6,1000) (G(M,L),L=1,N),C(M)
2     MN=N-1
      DO 10 M=1,MN
      K=M+1
      DO 10 J=K,N
      G(M,J)=G(M,J)/G(M,M)
      IF(M.EQ.1) GOTO 14
      MA=M-1
      DO 13 L=1,MA
13    G(M,J)=G(M,J)-G(M,L)*G(L,J)/G(M,M)
14    CONTINUE
      DO 15 L=1,M
15    G(J,K)=G(J,K)-G(L,K)*G(J,L)
10    CONTINUE
      C(1)=C(1)/G(1,1)
      DO 12 I=2,N
      C(I)=C(I)/G(I,I)
      MA=I-1
      DO 12 M=1,MA
12    C(I)=C(I)-G(I,M)*C(M)/G(I,I)
      X(N)=C(N)
      DO 11 I=1,MN
      NA=N-I
      X(NA)=C(NA)
      NB=NA+1
      DO 11 J=NB,N
11    X(NA)=X(NA)-G(NA,J)*X(J)
1001  FORMAT(1X,'SOLVE-ENTRY:')
1000  FORMAT(1X,10(2X,E10.3))
      RETURN
      END
CXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
```

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