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SOFTWARE FOR THE PARALLEL SOLUTION
OF SYSTEMS OF ORDINARY
DIFFERENTIAL EQUATIONS

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Software for the Parallel Solution of Systems of Ordinary Differential Equations

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Abstract

This report contains software for the solution of systems of ordinary differential equations on an INTEL iPSC/2 hypercube. A diskette is available upon request from the second author.

1. Introduction

In this report we supply software for the numerical solution of systems of ordinary differential equations (ODEs) on an INTEL iPSC/2 hypercube. The first program can only be used to solve *linear* initial or boundary value systems of ODEs and based on an algorithm developed by Katti and Neta (1989) and improved by Lustman *et al* (1990). The second program is based on polynomial extrapolation and Gragg's scheme and is useful for nonlinear ODEs as well. This algorithm is described in Lustman, Neta and Gragg (1991).

2. Linear Systems

In this section we give the software for the solution of *linear* systems of ODEs:

$$(1) \quad \begin{aligned} y'(x) &= Ay(x) + g(x), \quad a < x < b \\ y(a) &= y_a \end{aligned}$$

The algorithm used was developed by Katti and Neta (1989) and improved by Lustman *et al* (1990). The host and node program are given. The subroutines *sa*, *sf* and *putex* give the matrix *A*, the right hand side of (1) and the exact solution (for debugging purposes) respectively. An example of input and output corresponding to these subroutines are attached.

```

c
c
c
c          HOST
c solving initial value problems by multiple shooting
c on INTEL iPSC/2 hypercube having 8 (maxnp) processors
c
c          see Lustman, Neta & Katti
c
c change everywhere, in both node and host programs,
c          ndim=3
c to whatever value is appropriate.
c

```

```

    program mshivph
      integer intype,inlen,outype,outlen
      integer ymtype,ymlength
      integer n ,np,ndim,nin,nout, m , mnp
      integer allnodes,hostpid,nodepid
      parameter (nmax=100)
      parameter (ndim=3,nout=1)
      parameter(maxnp=8)
      parameter (nin=nmax*maxnp+ndim+10)
      parameter (intype=10,outype=20,inlen=4*nin
# ,ymtype=30,ymlength=ndim*(ndim+1)*4
# ,outlen=4*nout,allnodes= -1
# ,hostpid=8,nodepid=14)
      common/cin/n,ndimc,ninc,noutc
# ,m,mp,h,left,right,g,x
      real g (ndim) , x (0:nmax*maxnp) , vin (1)
      real vout (nout) , left , right
      equivalence
# (n,vin(1)),(ndimc,vin(2)),(ninc,vin(3))
# ,(noutc,vin(4)),(m,vin(5)),(mp,vin(6))
# ,(h,vin(7)),(left,vin(8)),(right,vin(9))
# ,(g(1),vin(10))
# ,(x(0),vin(10+ndim))
      ndimc=ndim
      ninc=nin
      noutc=nout
      call getcube('shoot',' ',' ',1)
      call setpid(hostpid)
      print*,' got the maximal cube,',numnodes(),' nodes'
      call load('node',allnodes,nodepid)
      print*,' after load'
      print*,' enter ',ndim,' initial values g'
      read*,(g(i),i=1,ndim)
      print*,' enter endpoints of interval'
      read*,left,right
      print*,'solve for ',left,' <x< ',right
      ,,' initially=',(g(i),i=1,ndim)
      print*,' enter number of points in interval, for each proc
      read*,m
      print*,m,' points for each processor'
      np=numnodes()
      mnp=m*np
      h=(right-left)/mnp

```

```
'      do 400 i=0, mnp
400    x(i)=left+(i)*h
      call csend(intype, vin, inlen, allnodes, nodepid)
411    continue
      call waitall(allnodes, nodepid)
      call relcube('shoot')
      stop
      end
```



```

c initialization
c
      call init(ndim,ucphi,ytilde)
      xme=jh*h+left
cdebug call putex(xme,phiex,g)
      do 100 j=jl,jh-1
      xx=x(j)+0.5*h
c
c get A
c
      call sa(ndim,xx,a)
c
c get B=I - h/2 A
c
      call sb(h,ndim,a,b)
c
c evaluate B inverse
c
      call sbinv(b,binv,ndim)
c
c evaluate D = Binv *(I + h/2 A)
c
      call sd(binv,h,ndim,a,b)
c
c multiply ucphi*B
c
      call smult(ucphi,b,ndim)
c
c get right hand side
c
      call sf(ndim,xx,f)
c
c get phi
c
      call sphi(b,ytilde,h,binv,f,ndim,phi)
c
c copy phi to ytilde
c
      if(j.lt.jh-1) then
      call scopy(phi,ytilde,ndim)
      endif
100 continue
c
c the following starts with initial conditions
c
      if(me.eq.0) call sma(ucphi,g,phi,ndim)
c
c here the process of recursive doubling
c
      jq=me+1
      iq=1
1132 continue
c
c send to some node after me
c

```

```

      if(jq+iq.le.numno) then
c
c make a list of data to send in the buffer vym0
c
      call enlist(me,phi,ucphi,vym0,ndim)
      call csend(ymtype+me,vym0,ymlen,iq+me,nodepid)
      endif
c
c y1j = bj =phi j
c m1j = phi j
c
1133 continue
      if(me.ge.iq) then
c
c me requires data from me-iq
c
c
      call crecv (ymtype+me-iq,vym0,ymlen)
      do 58 i=1,ndim+ndim*ndim
58      vym(i,1)=vym0(i)
c
c y1 =y1 + M * y0
c
      call defy(ndim,phi,ucphi,vym(1,1))
c
c M = M * M0
c
      call defm(ndim,ucphi,vym(ndim+1,1))
      endif
      iq=2*iq
      if(iq.lt.numno) goto 1132
c
c end of processing
c
c iunit=10+me
cdebug do 1001 i=1,ndim
cdebug 1001 er(i)=abs(phi(i)-phiex(i))
      print1000,xme,phi
      1000 format('x=',f6.2,' phi=',3f6.2)
cdebug print1001,er
cdebug 1001 format(8x,' err=',3f6.2)
      stop
      end
c
c makes a list of values to send in the buffer v
c
      subroutine enlist(me,phi,ucphi,v,n)
      dimension v(0:1),phi(n),ucphi(n,n)
      v(0)=me
      l=1
      do 1 i=1,n
      v(l)=phi(i)
      l=l+1
1      continue
      do 2 j=1,n

```



```

      do 2 i=1,n
        v(l)=ucphi(i,j)
        l=l+1
2      continue
        return
      end

c
c computes B= I - h/2 A
c
      subroutine sb(h,ndim,a,b)
c evaluate b=i-h/2*a
      real a(ndim,ndim),b(ndim,ndim)
      do 10 i=1,ndim
        do 10 j=1,ndim
          r=0
          if(i.eq.j) r=1
          b(i,j)=r-0.5*h*a(i,j)
10      continue
        return
      end

c
c computes D= Binv * ( I + h/2 A )
c
      subroutine sd(binv,h,ndim,a,b)
      real a(ndim,ndim),b(ndim,ndim),binv(ndim,ndim)
      do 10 i=1,ndim
        do 10 j=1,ndim
          b(i,j)=0
          do 10 k=1,ndim
            r=0
            if(k.eq.j) r=1
            b(i,j)=b(i,j)+binv(i,k)*(r+0.5*h*a(k,j))
10      continue
          return
        end

c
c evaluate b*ucphi into ucphi
c
      subroutine smult(ucphi,b,idim)
      parameter (ndim=3)
      real ucphi(idim,idim),b(idim,idim)
      real temp(ndim)
      do 100 j=1,idim
        do 10 i=1,idim
          temp(i)=0
          do 10 k=1,idim
            temp(i)=temp(i)+b(i,k)*ucphi(k,j)
10      continue
          do 20 k=1,idim
            ucphi(k,j)=temp(k)
20      continue
100     continue
        return
      end

c
c evaluate d*ytilde + h*binv*f

```

```

c
    subroutine sphl(b,ytilde,h,binv,f,ndim,phi)
    real b(ndim,ndim),ytilde(ndim),binv(ndim,ndim)
    real f(ndim),phi(ndim)
    do 10 i=1,ndim
    phi(i)=0
    do 10j=1,ndim
10    phi(i)=phi(i)+b(i,j)*ytilde(j)+h*binv(i,j)*f(j)
    return
    end

c
c moves phi to ytilde
c
    subroutine scopy(phi,ytilde,ndim)
    real ytilde(ndim),phi(ndim)
    do 10 i=1,ndim
10    ytilde(i)=phi(i)
    return
    end

c
c evaluate ucphi*g +phi and put into phi
c
    subroutine sma(ucphi,g,phi,ndim)
    real phi(ndim),ucphi(ndim,ndim),g(ndim)
    do 10 i=1,ndim
    do 10 j=1,ndim
10    phi(i)=phi(i)+ucphi(i,j)*g(j)
    return
    end

c
c initialize ucphi and ytilde
c
    subroutine init(ndim,ucphi,ytilde)
    real ytilde(ndim),ucphi(ndim,ndim)
    do 10 i=1,ndim
    ytilde(i)=0
    do 20 j=1,ndim
    ucphi(i,j)=0
20    continue
    ucphi(i,i)=1
10    continue
    return
    end

c
c inverts b into binv . b is destroyed
c
c =====
c
    subroutine sbinv(b,binv,ndim)
    real b(ndim,ndim),binv(ndim,ndim)
    do 20 i=1,ndim
    do 10 j=1,ndim
10    binv(i,j)=0
20    binv(i,i)=1
    do 2 j=1,ndim
    z=1/b(j,j)

```



```

do 30 k=1,ndim
b(j,k)=z*b(j,k)
binv(j,k)=z*binv(j,k)
30 continue
do 1 i=1,ndim
if(i.eq.j) goto 1
z=b(i,j)
do 3 k=1,ndim
b(i,k)=b(i,k)-z*b(j,k)
binv(i,k)=binv(i,k)-z*binv(j,k)
3 continue
1 continue
2 continue
return
end

c
c evaluates Y1=Y1+M*Y0
c
subroutine defy(ndim,y1,em,y0)
dimension y1(ndim),em(ndim,ndim),y0(ndim)
do 1 i=1,ndim
do 1 j=1,ndim
y1(i)=y1(i)+em(i,j)*y0(j)
1 continue
return
end

c
c evaluates M=M*M0
c
subroutine defm(ijmax,em,em0)
parameter (ndim=3)
dimension row(ndim)
dimension em(ijmax,ijmax),em0(ijmax,ijmax)
do 1 i=1,ijmax
do 3 j=1,ijmax
row(j)=em(i,j)
3 continue
do 1 j=1,ijmax
s=0
do 2 k=1,ijmax
s=s+row(k)*em0(k,j)
2 continue
em(i,j)=s
1 continue
return
end

cdebugc
cdebugc given x, and initial values g, computes v=exact(x)
cdebugc
cdebugc subroutine putex(x,v,g)
cdebugc parameter (ndim=3)
cdebugc parameter(e=2.718281828,ei=1./e)
cdebugc dimension v(ndim),g(ndim)
cdebugc dimension v(3)
cdebugc real l0g

```

```

cdebug  ex=exp(x)
cdebug  log=log(x)
cdebug  a=(g(1)-1)*ei
cdebug  b=(g(2)-e)*ei
cdebug  c=(g(3)-ei)*ei
cdebug  v(1)=ex*(a+log*(b+c/2*log))+1
cdebug  v(2)=ex*(b+c*log)+ex
cdebug  v(3)=ex*c+1/ex
cdebug  return
cdebug  end

```

```

c
c evaluate right hand side f(x)
c

```

```

subroutine sf(idim,x,f)
parameter (ndim=3)
real x, f(idim)
ex=exp(x)
f(1)=-1-ex/x
f(2)=-1/x/ex
f(3)=-2/ex
return
end

```

```

c
c evaluate the matrix A(x)
c

```

```

subroutine sa(ndim,x,a)
real a(ndim,ndim),x
do 10 i=1,ndim
do 10 j=1,ndim
a(i,j)=0
10 continue
a(1,1)=1
a(2,2)=1
a(3,3)=1
a(1,2)=1/x
a(2,3)=1/x
return
end

```

```
# This file is used to compile and link the host.f, node.f
#
# The command "make all" causes compilation and linking.
```

```
all : host node
```

```
host: host.o
      f77 -o host host.o -host
```

```
node: node.f
      f77 -o node node.f -node
```

```
*****
      example of an input file
      for the subroutine sa, sf, putex
      currently in node.f
```

```
*****
0,0,0  initial values
1,2    endpoints
5      subintervals for each processor
```

```
*****
      example of output file for the above
*****
```

```
got the maximal cube,          8 nodes
after load
enter          3 initial values g
enter endpoints of interval
solve for      1.000000      <x<      2.000000
initially=     0.0000000E+00  0.0000000E+00  0.0000000E+00
enter number of points in interval, for each processor
      5 points for each processor
x=  1.13 phi= -0.50 -0.05 -0.09
x=  1.25 phi= -1.07 -0.11 -0.19
x=  1.38 phi= -1.74 -0.17 -0.28
x=  1.50 phi= -2.52 -0.25 -0.38
x=  1.63 phi= -3.42 -0.33 -0.49
x=  1.75 phi= -4.46 -0.44 -0.61
x=  1.88 phi= -5.67 -0.55 -0.73
x=  2.00 phi= -7.08 -0.69 -0.86
```

(may appear in a different order, each line written by a different processor, when it is ready)

3. Nonlinear Systems

The algorithm used is based on Gragg's Method (1964,1965) and polynomial extrapolation as described by Lustman, Neta and Gragg (1991). One can solve

$$(2) \quad \begin{aligned} y'(x) &= f(x, y(x)) \\ y(a) &= y_a \end{aligned}$$

where y and f are vector valued functions and y_a is a vector of initial values.

The host and node programs are supplied along with `exa.f` file containing subroutines for the evaluation of the exact solution (`putex`) and the right hand side (`rhs`) of (2). The `make` file to compile and link these programs is given at the end followed by an example of input and output files for the given `putex` and `rhs`.

```

c
c          HOST
c          program for the solution of nonlinear systems
c          based on Gragg's method and polynomial extrapolation
c          on INTEL iPSC/2 having 8 (maxproc) processors
c
c          see Lustman, Neta and Gragg
c
c          leny0   = length of vector of initial values
c          nptmax  = maximum number of points in common to all processors
c
c          implicit double precision (a-h,o-z)
c          parameter(leny0=20,nptmax=100)
c          parameter(maxproc=8,iv=5)
c          parameter(initype=1000,inilen=4*(iv+leny0)
c          , ,nodes=-1,idhost=2,nodepid=3)
c          dimension y0(leny0),sendata(iv+leny0)
c          call getcube('extrap',' ',' ',1)
c          call setpid(idhost)
c          nproc=numnodes()
c          print*,' got the maximal cube,',nproc,' nodes'
c          call load('node',nodes,nodepid)
c
c          xmin, xmax = the interval of integration
c
c          print*,'Enter xmin,xmax'
c          read*,xmin,xmax
c          print*,'How many result points (excluding xmin)?'
c          read*,npt
c          print*,'Enter dimension of solution vector'
c          read*,leny
c          if(leny.gt.leny0) then
c          print*,'dimension=',leny,'>',leny0
c          stop
c          endif
c          print*,'Enter ',leny,' initial values'
c          read*,(y0(i),i=1,leny)
cdebugc if debugging, replace the two lines above by
cdebugc call putex(xmin,leny,y0)
c          print*,'How many processors will be used?'
c          read*,nn
c          if(nn.gt.nproc.or.nn.lt.1) then
c          print*,nn,' is unreasonable. '
c          nn=nproc
c          endif
c          nproc=nn
c          print*,' will use ',nproc,' processors'
c          sendata(1)=xmin
c          sendata(2)=xmax
c          sendata(3)=leny
c          sendata(4)=npt
c          sendata(5)=nproc
c          do 1 j=1,leny
c          1 sendata(iv+j)=y0(j)
c          call csend(initype,sendata,inilen,nodes,nodepid)

```

```
call waitall(nodes,nodepid)
call relcube('extrap')
stop
end
```



```

c
c          NODE
c      program for the solution of nonlinear systems of ODEs
c      based on Gragg's method and polynomial extrapolation
c      on INTEL iPSC/2 having 8 (maxproc) processors
c
c      see Lustman, Neta and Gragg
c
c      implicit double precision (a-h,o-z)
c      parameter(leny0=20,nptmax=100)
c      parameter(maxproc=8,iv=5)
c      parameter(iii=5,jdata=iii+leny0+nptmax*leny0)
c      parameter(initype=1000,inilen=4*(iv+leny0)
,,nodes=-1,idhost=2,nodepid=3)
c      dimension y0(leny0),dataini(iv+leny0)
c      dimension ysave(leny0,0:nptmax)
,,y(leny0),yexa(leny0),hlfway(leny0)
c      dimension data(jdata)
c      dimension hvec(0:maxproc)
c      me=mynode()
c      iam=me
c      call crecv(initype,dataini,inilen)
c      xmin= dataini(1)
c      xmax= dataini(2)
c      leny= dataini(3)
c      npt= dataini(4)
c      nproc= dataini(5)
c      lastproc=(nproc-1)
c      if(iam.gt.lastproc) stop
c      jdta=iii+leny+npt*leny
c
c ABSOLUTELY ESSENTIAL: 8 bytes per double precision item
c
c      lendta=8*jdta
c
c      message length in bytes
c
c      ne=nproc-me
c
c      save results every ne steps
c
c      do 1 j=1,leny
1      y0(j) = dataini(iv+j)
c      ipow=1
c
c all the h's must be known to all the processors
c
c      do 10 i=0,nproc-1
c      hvec(i)=(xmax-xmin)/(npt*(nproc-i))
10      continue
c      h=hvec(me)
c
c fixes the size for integration.

```

```

c'
      jindex=0
      do 2 j=1, leny
      ysave(j, jindex)=y0(j)
2      y(j)=y0(j)
      do 3 index=1, npt*(ne)
      x=xmin+h*(index-1)
      call odestep(h, x, y, index, hlfway, leny)
c
c advances the solution
c in this form, it is a two step method, i.e.
c   h, x, y(x) and y(x-h/2) is what you need to obtain y(x+h)
c
      if(mod(index, ne).eq.0) then
c
c save this result, it belongs to a common point
c
      jindex=jindex+1
      do 4 j=1, leny
      ysave(j, jindex)=y(j)
4      continue
      endif
3      continue

      if(me.ne.lastproc) then
c
c send my saved data to lastproc (who probably is done by now)
c
      l=iii
      if(jindex.ne.npt) then
      print*, ' i am ', me, ' jindex=', jindex
      , , ' .ne. npt=', npt
      stop
      endif
      do 6 j=0, npt
      do 6 i=1, leny
      l=l+1
      data(l)=ysave(i, j)
6      continue
      call csend(me, data, lendta, lastproc, nodepid)
      endif
c
c i am waiting for data to do extrapolations on
c
      level=nproc-me
c
c the new data will be sent to me-1 with superscript level
c
c
      msgtyp=(me)
      if(me.eq.lastproc) msgtyp=(me-1)
134      continue
      call crecv(msgtyp, data, lendta)
      if(msgtyp.eq.me) then

```



```

c
c just save the message in ysave
c
      l=iii
      do 69 j=0,npt
      do 69 i=1,leny
      l=l+1
      ysave(i,j)=data(l)
69      continue
      else
c
c extrapolate incoming data and ysave
c
      it=      data(1)
      itsne=   data(2)
      itspow=  data(4)
      hish=    data(5)
c
c because the error goes in powers of h**2
c
      w=1/(      (hvec(msgtyp)/hvec(msgtyp+level))**2      -1)
      l=iii
      do 7 j=0,npt
      do 7 i=1,leny
      l=l+1
      z=data(l)
      data(l)= ysave(i,j)+w*(ysave(i,j)-data(l))
      ysave(i,j)=z
c
c This prepares extrapolated data to send and saves
c the data received to extrapolate with other message data
c
      7      continue

      call csend(msgtyp,data,lendta,me-1,nodepid)
      endif
      msgtyp=msgtyp-1
      if(msgtyp.ge.0) goto 134
      if(me.ne.0) goto 1512
c
c everything done, report results
c
      hout=(xmax-xmin)/npt
      orm=0
      er=0
      do 9 j=0,npt
      x=xmin+j*hout
cdebug  call putex(x,leny,yexa)
      print900,j,x
      900  format(i5,f10.3)
      do 8 i=1,leny
      print800,ysave(i,j)
cdebug  ,,yexa(i),abs(ysave(i,j)-yexa(i))
cdebug  orm=orm+yexa(i)**2

```

```

cdebug  er=er+(ysave(i,j)-yexa(i))**2
      800  format(2f10.3,1pe10.2)
      8    continue
      9    continue
cdebug  print900, -999,-999.
cdebug  orm=sqrt(orm)
cdebug  er=sqrt(er)
cdebug  reler=er/orm
cdebug  print800, orm,er,reler
      1512 continue
      end

c
c      subroutine for ode stepping using Gragg's method
c
c      subroutine odestep(h,x,y0,index,hlfway,l)
c
c  y0,hlfway are input and output. the step is from x=x to x=x+h
c
c      implicit double precision (a-h,o-z)
c      parameter(leny0=20,nptmax=100)
c      dimension y0(1),hlfway(1),r(leny0)
c      if(index.eq.1) then
c
c  this is the first step
c
c      call rhs(x,y0,l,r)
c      do 61 i=1,l
61      hlfway(i)=y0(i)+h/2*r(i)
c      else
c
c  the general step : hlfway is at x-h/2, y0 at x
c      they advance to x+h/2, x+h correspondingly
c
c      call rhs(x,y0,l,r)
c      do 661 i=1,l
661      hlfway(i)=hlfway(i)+h*r(i)
c      endif
c      call rhs(x+h/2,hlfway,l,r)
c      do 662 i=1,l
662      y0(i)=y0(i)+h*r(i)
c
c
c  Gragg formula. the errors go in powers of h**2
c
c      return
c      end

```

```

C
C           EXA.F
C
C putex evaluates the exact solution
C for this examples y(i) exact = x **i
C
      subroutine putex (x,l,y)
      implicit double precision (a-h,o-z)
      dimension y(1)
      y(1)=x
      do 1 j=2,l
      y(j)=x*y(j-1)
1      continue
      return
      end

C
C evaluates the right hand side for the above system
C
      subroutine rhs (x,y,l,r)
      implicit double precision (a-h,o-z)
      dimension y(1),r(1)
      x2=x*x
      div=x2*x
      do 1 i=1,l-1
      r(i)=i*y(i)*y(i+1)/div
      div=div*x
1      continue
      r(l)=l*y(l)*y(l)/x2
      return
      end

```

```
#
#           this is the makefile
# this file is used to compile and link the host.f, node.f
#
# the command "make all" causes compilation and linking.
```

```
all :   exa.o host node
```

```
exa.o:  exa.f
```

```
host:   host.f exa.o
        f77 -o host exa.o host.f -host
```

```
node:   node.f  exa.o
        f77 -o node exa.o node.f -node
```

```
*****
                example of input file for
                the subroutines in exa.f
*****
```

```
1,2
2
4
1,1,1,1
5
```

```
*****
                example of output file for
                the above input
*****
```

```
got the maximal cube,           8 nodes
Enter xmin,xmax
How many result points (excluding xmin)?
Enter dimension of solution vector
Enter           4 initial values
How many processors will be used?
will use           5 processors
```

```
0      1.000
  1.000
  1.000
  1.000
  1.000
  1.000
1      1.500
  1.500
  2.250
  3.375
  5.062
```

2 2.000
2.000
4.000
8.000
15.999

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