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Optimal routing of military convoys through a road network

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THESIS

OPTIMAL ROUTING OF MILITARY CONVOYS
THROUGH A ROAD NETWORK

by

Dong Keun Lee

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Thesis Advisor R. Kevin Wood

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Optimal Routing of Military Convoys Through a Road Network

by

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Major, Republic of Korea Army
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Submitted in partial fulfillment of the
requirements for the degree of

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ABSTRACT

To wage a successful campaign, military units and materiel must be in position by the designated time. This thesis models the problem of moving military units and materiel in convoys through a road network as mathematical programming models. In particular, two models, linear and integer, are investigated. Both models belong to the class of multicommodity, dynamic transshipment network problems. Based on prototypic GAMS implementations, they provide essentially the same answer. However, the linear model is easier to construct, takes less time to solve and allows for more flexible convoy routing.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

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My parents who have stayed in my heart all the way through.

I. INTRODUCTION

During the initial phase of a war, a large number of troops and materiel must simultaneously move up to the front line in convoys. Routing of these convoys to their final destinations is inherently difficult because many possibilities exist. When routing is done nonoptimally, certain routes may become clogged while others are left unused and this may lead to delays detrimental to military objectives. Therefore, it is the goal of this thesis to develop and solve the problem of routing convoys to their destinations by designated times.

1. AIRLAND BATTLE

The concept of the Airland Battle in current military doctrine is a method to defeat a large armored force which attacks by echelon, through a narrow breach sector in an opponent's front. The Airland Battle requires complex integration of fire support weapons, Army and Air Force aviation assets and ground maneuver forces. Doctrine calls for the enemy to be defeated by first stopping his forward elements, second, by defeating his reserve echelons before their combat value can influence the battle, and third, by attacking his resources to prevent reconstitution of previously defeated forces.

Defeating the armored force echelons at the proposed breach point will require more troops and war materiel than in past conflicts. Most of the troops and materiel will arrive at the front by land convoys. Thus, the problem of effectively moving troops and war materiel through the available road network in a short period of time is even more important than before.

2. PROBLEM SCOPE AND GOAL

In this thesis, the term "unit" refers to a group of troops and materiel which will move through a road network as a single, coherent convoy. A convoy consists of at least 6 vehicles moving at the same time or 10 vehicles moving within a 1-hour period under a single commander, over the same route and in the same direction [Ref. 1 : p.5-1]. In practice, there can be several hundred vehicles in a single convoy and each of several convoys must arrive at its destination in a limited amount of time.

This thesis describes two mathematical programming models which determine routes for multiple convoys to arrive at their destination by the designated times. To determine the effectiveness of the models, they are implemented in GAMS (General Algebraic Modeling System) and their solutions are compared.

3. THE MODELS AND SOLUTION PROCEDURES

To simplify the presentation, a concrete scenario is used to demonstrate the models throughout this thesis. In the scenario, there are three units at three separate staging areas behind the battle front. Each unit must move through the road network to its own individual supply point to take on supplies and then move up to a common front-line location. A certain minimum amount of delay at supply points is incurred since some time is required to load supplies. The two models investigated in this thesis have the same underlying network structure in that the units movements are represented as flows through a road network. However, they have different assumptions on convoy movements.

The first model is an integer program which assumes that a unit consists of only one convoy and all convoys are of the same column length which is the length from the head of the convoy to the end of convoy. Moreover, all arcs in the network have the same capacity which allows only one convoy on any road link at any time. This simplifies the coordination of troop movements.

The second model is a linear program which allows units to move in convoys with different lengths. Each unit can be split up into smaller (sub)convoys, each taking different routes. Also, arcs can now have different capacities and can hold several (sub)convoys from different units. Although this model potentially uses the road network more efficiently, it is more complicated to administer and coordinate. This model can also be viewed as a multicommodity extension of the building evacuation models in which occupants of a large building must be evacuated in a short amount of time. [Refs. 2,3]

It is important to note that in both models units starting from a different staging area must be treated as separate commodities in a network flow formulation. Otherwise, it cannot be guaranteed that a complete unit will pass through its own supply point and end up at the correct destinations [Refs. 2,3,4,5]. In addition, the network must be dynamic in that the basic road network is expanded over time to represent the fact that a certain amount of time is necessary to transit various links in the network.

4. OUTLINE

Chapter II describes the concept of a multicommodity dynamic network model. Then, the two basic models are presented in Chapter III. Finally, the computational results, conclusions and recommendation for further research are given in Chapter IV. For completeness, the GAMS programs are listed in the appendices.

II. MULTICOMMODITY DYNAMIC ROAD NETWORK

This chapter describes how the basic structure of a road network is transformed into a multicommodity dynamic network for purposes of modeling convoy routing problems. In the basic road network, nodes represent road junctions, troop staging areas, supply points and the front line. Arcs represent segments of the road network. In the multi-commodity dynamic network, nodes, for the most part, correspond to nodes in the original network but are replicated over multiple time periods. Arcs in the dynamic network correspond to movements of units through time and space.

1. DIRECTED AND UNDIRECTED GRAPHS

Let $G = (V,E)$ be a graph which represents a road network. V is the set of nodes and E is the set of arcs where each arc $e \in E$ consists of a pair (u,v) where $u \in V$ and $v \in V$. A graph may either be directed or undirected [Ref. 6 : p.198, : p.230]. In an undirected graph any arc (u,v) is taken as an unordered pair while in a directed graph the pair is taken as ordered. In a directed graph an arc (u,v) is often expressed by $u \rightarrow v$ and u is called the *tail* of the arc and v is called the *head* of the arc. A road network can be represented by a directed graph with one-way road segments being represented by single arcs and two-way segments represented by two arcs in anti-parallel. While most roads networks consist mainly of two-way segments, for the purposes of moving units up to a front-line, most of the segments can be considered to be one-way, oriented in the direction of the front.

2. SINGLE AND MULTICOMMODITY DYNAMIC NETWORKS

Consider now a homogeneous, infinitely divisible commodity such as "materiel" measured in units of, say, tons moving through a road network $G = (V,E)$. G is called the *static network* since its structure does not include a reference to time. However, since each arc $e \in E$ in this network will require some finite time to transit it, an integer transit time $\tau(e)$ is associated with the arc.

Consider the network $G_T = (V_T,E_T)$, the *T-time expanded* network obtained from $G = (V, E)$ as follows:

$$V_T = \{u_t : u \in V,\ t = 1,2,\ldots,T\}$$

Here u_t is the t-th time copy of node $u \in V$. Similarly, the arc set E_T is given by

$$E_T = \{(u_t, v_s) : e = (u, v) \in E,\ s = t + \tau(e) \leq T,\ t = 1,2,\ldots,T-1,\}$$
$$\bigcup \{(u_t, u_{t+1}) : u \in V,\ t = 1,2\ldots T-1\}.$$

The network G_T is called the *dynamic network* associated with G . The arcs $(u_t, u_{t+1}) \in E_T$ are called the *holdover arcs.* Traversing such an arc represents materiel pausing at node u from time period t to time period $t+1$.

The arcs (u_t, v_s) are called the *movement arcs.* They represent the movement of materiel from one node in the road network to another. This movement starts at time t at node u, and terminates at node v at time s . Associated with each holdover arc (u_t, u_{t+1}) , is a capacity which represents, for example, the tons of materiel which can pause at node u in the road network. Associated with each movement arc (u_t, v_s) , is a capacity which represents the tons of materiel which can be moved from u to v in $(s-t)$ time periods.

Using standard flow balance constraints associated with the dynamic network [Ref. 3 : pp.98-99], and defining supplies and demands, it is then possible to model the flow of materiel through time and space through the road network. This is not sufficient for the purposes of this thesis, however, since multiple commodities, i.e., different military units, must be distinguished. Consequently, a multicommodity variant of the dynamic network model must be formulated.

In the multicommodity dynamic network, the basic dynamic network is replicated for each commodity. Each commodity network has its own supplies and demand. Then, as in standard multicommodity network flow problems [Ref. 5], joint capacity constraints are placed across the commodities for each set of analogous arcs. This then describes the setup necessary for the two models described in the next chapter. Some restrictions will have to be added however, since, in the first model, it will not be assumed that the commodities, i.e., military units, are infinitely divisible. Furthermore, certain modifications to the dynamic networks will be necessary to accommodate different objective functions and constraints.

3. TIME-EXPANDED NETWORKS

The following discussion describes some of the issues associated with practical use of the dynamic networks. Consider a road which is composed of an arc and two nodes. Figure 1 shows the static and dynamic network representation of this road network.

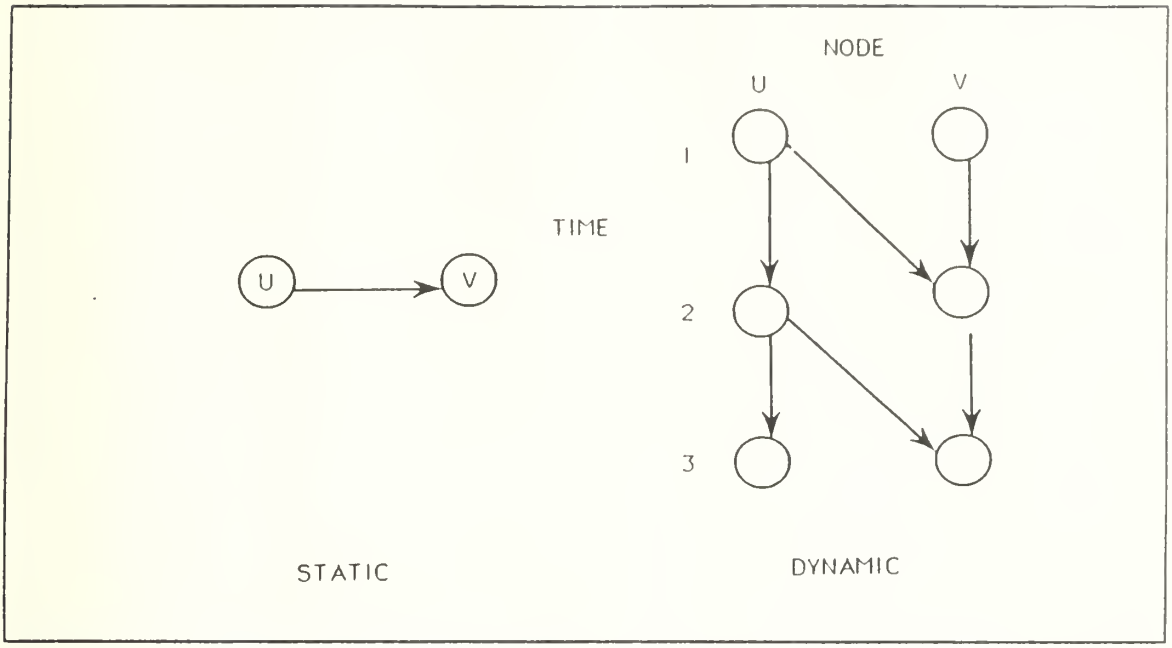


Figure 1. Static and Dynamic Networks

For each static directed arc, say from static node u to static node v , and having traversal time 1 and for every integer t between 0 and $T-1$ construct a (directed) movement arc in the dynamic network from copy t of node u to copy $t+1$ of node v . It is easy to verify that if the static model has n nodes and a arcs, and the dynamic model has T time periods, then $(n+a)T$ is an upper bound on the number of arcs in the dynamic model, and $n(T+1)$ is an upper bound on the number of nodes of the dynamic model. These upper bounds can often be decreased substantially by deleting "inessential" arcs and nodes in the dynamic model, i.e., arcs and nodes not lying in at least one directed path from copy 0 of some tail node to copy T of some head node.

Let R be the maximum transportation planning time, and let τ be the length of a time period. Then, T, the number of time periods in the model is given by $T = R/\tau$ assuming exact divisibility. Thus, the dynamic model can have as many as $n(R/\tau + 1)$ nodes, in which case its computational tractability will be inversely proportional to the magnitude of τ.

Ideally, supposing all arc traversal times to be integers originally, a reasonable choice of τ is the greatest common divisor of all the traversal times. Unfortunately the greatest common divisor may be one, in which case it may well be necessary to alter some of the traversal times in order to find an acceptably large greatest common divisor.

In the view of the emergency nature of the convoy routing problem one may wish to make all such alterations larger, in order to be assured that the model will not underestimate the minimum transportation time. [Ref. 2 : pp.90-91.]

III. MODEL DEVELOPMENT AND FORMULATION

Time-minimizing transportation routes and schedules are concerned with determining the minimum time required for transporting a set of goods from given supply points to given demand points. The importance of such problems arises when certain consumers urgently require a set of commodities to be supplied from fixed distribution centers. The convoy routing problems are examples or at least modified examples of such time-minimizing problems. This chapter details two models for convoy routing problems.

1. SCENARIO

Consider a country under defense of a border point. The defense must be met by a given number of units and their equipment. It will be assumed that there are three units located at three separate points along with their equipment but not all of their supplies. Each unit must first move forward through the road network to its own unique supply point where it takes on supplies. A certain amount of time is necessary to take on supplies; if less time is spent a unit would have to move to the front line without its full complement of supplies making the unit less than 100% effective. After taking on supplies, the units must move up to a single front line location to arrive as early as possible or at least by a specified time. For this transportation operation to be successful, the sum of the time to move, the time to take on the equipment and unknown delay time should be less than the allowed operation time.

In these models, the arrival time at the destination and the sojourn time at the supply points are critical. If the units do not spend enough time at their respective supply points they will not have sufficient time to take on supplies. If the units do not move to the destination within the required time, a coordinated attack at the breach point cannot be launched and the operation may fail. Some conflict in these objectives may arise.

Consequently, two different objective types for Model 1 will be considered. In the one type, it is assumed that the units should arrive at the destination as early as possible so as to have as much time as possible at the front line to prepare for battle. However, they must spend a minimum amount of time at the supply points. In the other type, it is assumed that the units need only arrive at the destination by a specified time and they should spend as much time as possible at the supply points taking on supplies and perhaps making other preparations. It will be assumed that the length of a convoy, called its "column length" ,is fixed.

Two different column lengths for Model 2 will be considered with one objective, to minimize the arrival time at the destination. First, the number of vehicles in a convoy is equal to arc capacity in order to compare the solution to Model 1. Second, the number of vehicles in a convoy is made greater than arc capacity so that the convoy must be broken down into at least two small subconvoys. In addition, the capacity of one arc is reduced to 0 to represent a situation in which part of the road network is destroyed by enemy attack.

2. FIXED COLUMN LENGTH, MODEL 1

In the first model, Model 1, it is assumed that the column length of each unit corresponds to a fixed time period τ. Thus, if a particular arc in the road network requires one time period to traverse and the column length is two time periods, it will take two time periods for the unit to clear the arc. It is assumed that the joint capacity of any arc and node, except the staging nodes, supply point nodes and the destination nodes are 1. Decision variables in this model will be binary indicating whether or not a unit is transiting a particular arc or sojourning at a particular node. Thus, this model is an integer programming model (IP).

a. *Procedure*

Figure 2 shows the road network which is used to test Model 1. The number on each arc is its traversal time. To successfully model fixed column lengths it is necessary to split the arcs in the network into multiple arcs in series so that each has a length equal to a fixed fraction of the column length: 1, 1/2, 1/3, etc. Thus, if the length of each arc were 1/3 of the column length it would take 3 time periods for the column to clear the arc. Clearly, the shorter the length of each arc the better for accuracy but this, of course, leads to larger problems. For simplicity and computational tractability, it is assumed that each arc can be split into lengths which are equal to the column length (The column length is fairly short compared to actual arc lengths.). Figure 3 shows the modified static network in which each arc has the same length.

Once the static network is obtained it can be expanded into a dynamic network over a specified number of time periods. The number of time periods should be as few as possible to lead to a model which is as small as possible. Figure 4 displays the dynamic network.

It may be necessary for one or more units to wait to allow another unit to pass. Thus, it is sensible to allow holdover arcs on all nodes. However, this leads to a

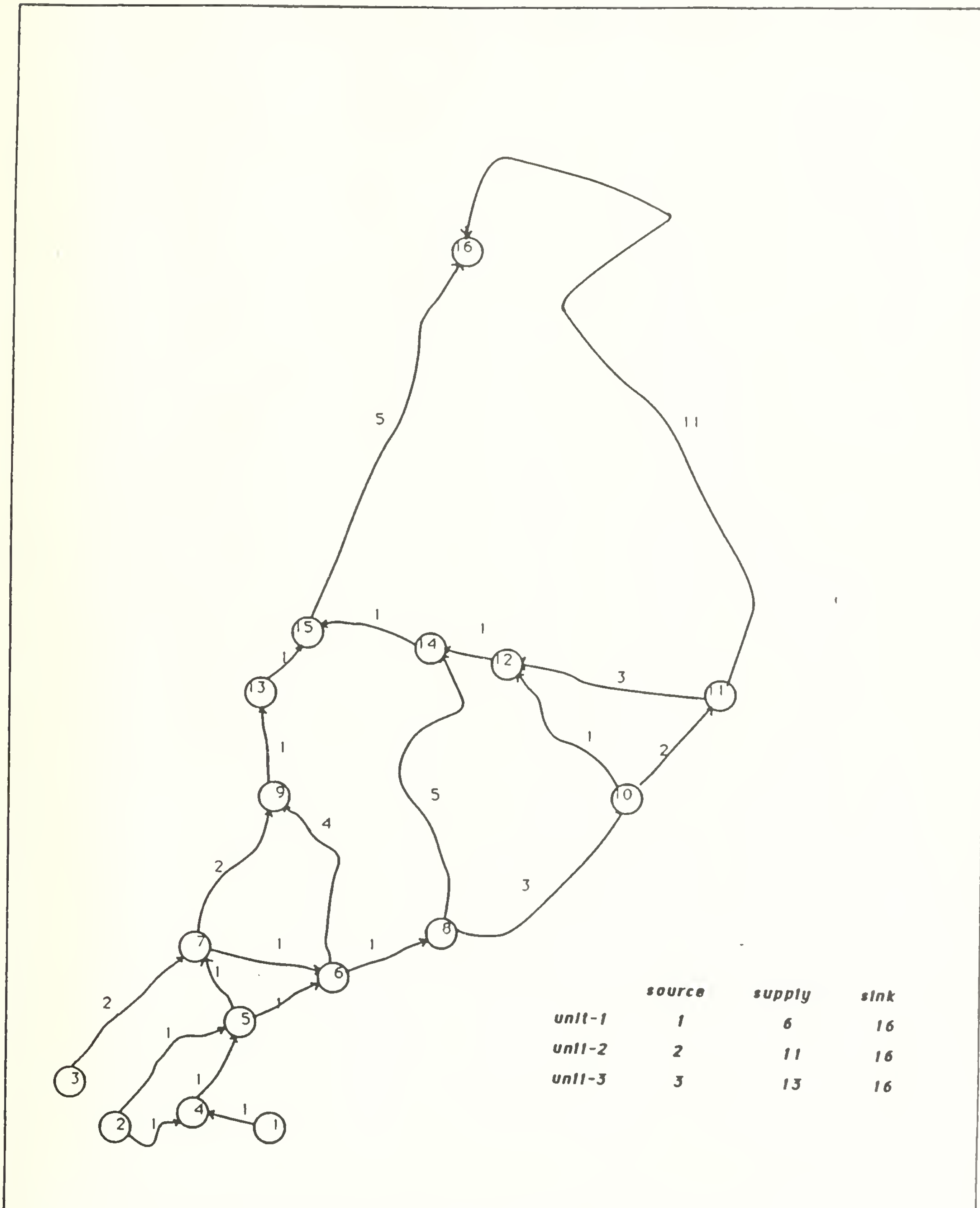


Figure 2. Road Network

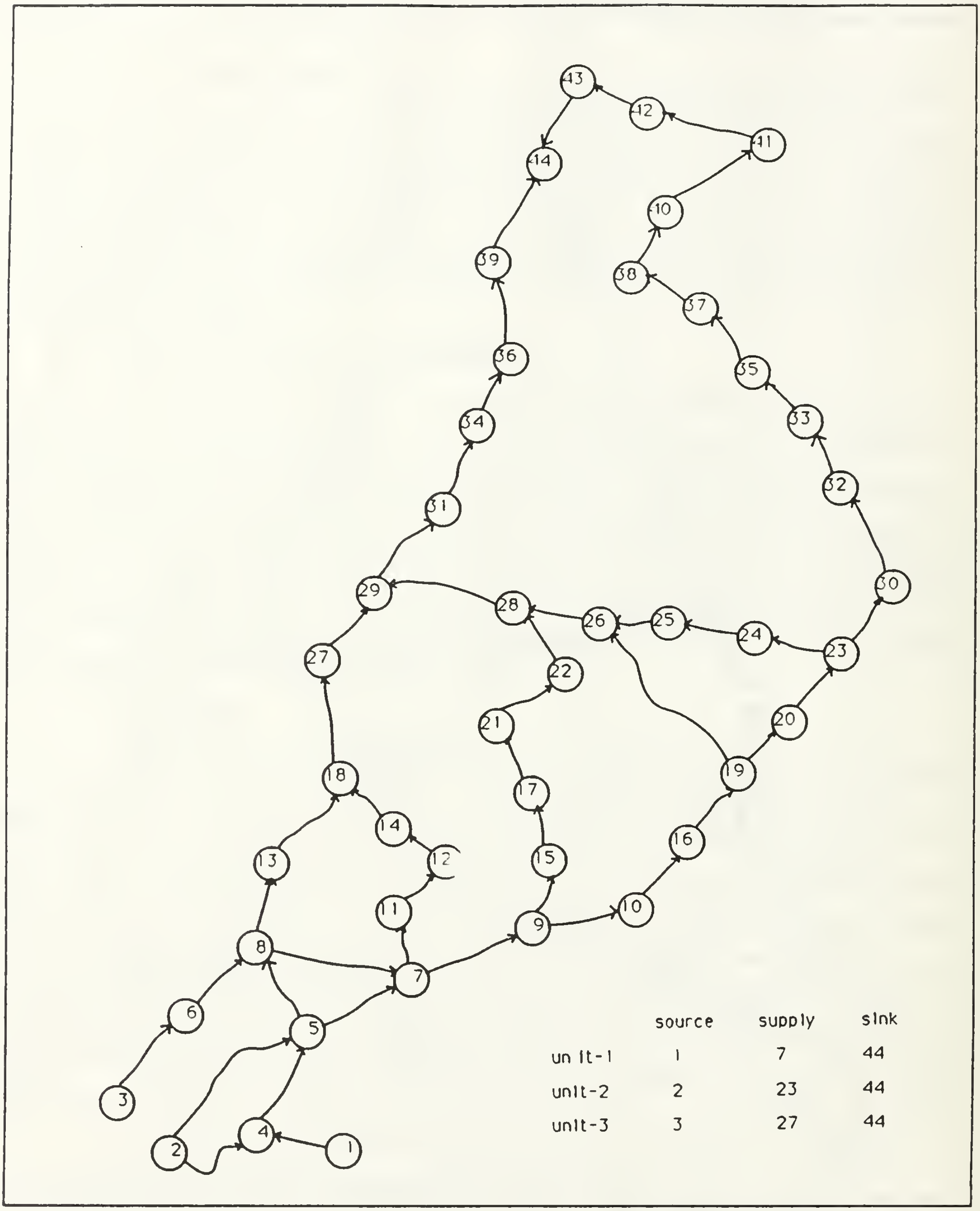


Figure 3. Modified Network

very large number of arcs and to avoid this it is assumed that any waiting will be done at the source nodes, the supply point nodes or at the destination node. Thus, only 7 nodes in the road network need holdover arcs.

The number of nodes and arcs in the dynamic network can be reduced further. Every node is expanded by time period. However, there can be no flow in the first time period on any arcs except those adjacent to the source nodes. Thus, many arcs can be deleted. Figure 5 shows the reduced dynamic network. It would be desirable to eliminate all arcs which can never have any flow on them because they are correspond to an early time period too far away from any source node. This shows on the right upper right of Figure 5. Also, there can be no flow on arcs too far away from the destination at later time periods. Thus, additional arcs could be deleted as shown in the lower left portion of Figure 5. The destination node does not need the holdover arc because its capacity is unlimited. The next step is to formulate this problem. The formulation assumes that the column length equals the arc lengths. The modifications necessary to handle longer columns are discussed in Chapter IV.

b. Model 1 Formulation

1. Indices
   - $u,v = 1,2,...,N$ nodes
   - $k = 1,2,...,K$ units
   - $t = 1,2,...,T$ time periods
2. Data
   - $a_{uvt} =$ 1 if there exists an arc between node u and node v at time period t; 0 otherwise
   - $mintime_{ku}$ minimum time to take on equipment at supply point u for unit k.
   - $arccapacity_{uvt}$ capacity of arc between u and v
   - $nodecapacity_{ut}$ capacity of node u at time period t
3. Decision Variables
   - $x_{kuvt} =$ 1 if unit k traverses arc (u,v) at time t; 0 otherwise
   - z total amount of time to transport
4. Formulation
   - The objective function is

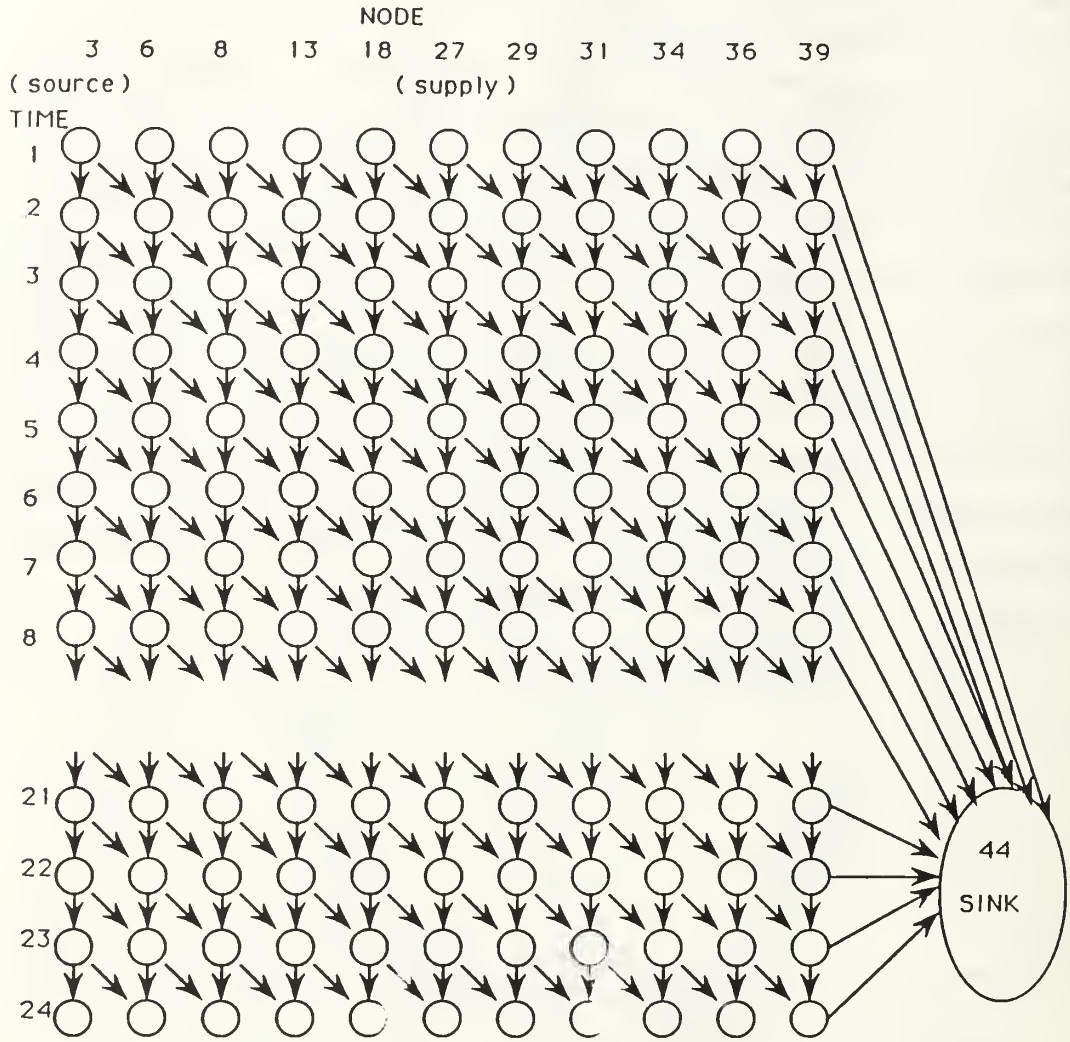


Figure 4. Dynamic Network for Model 1

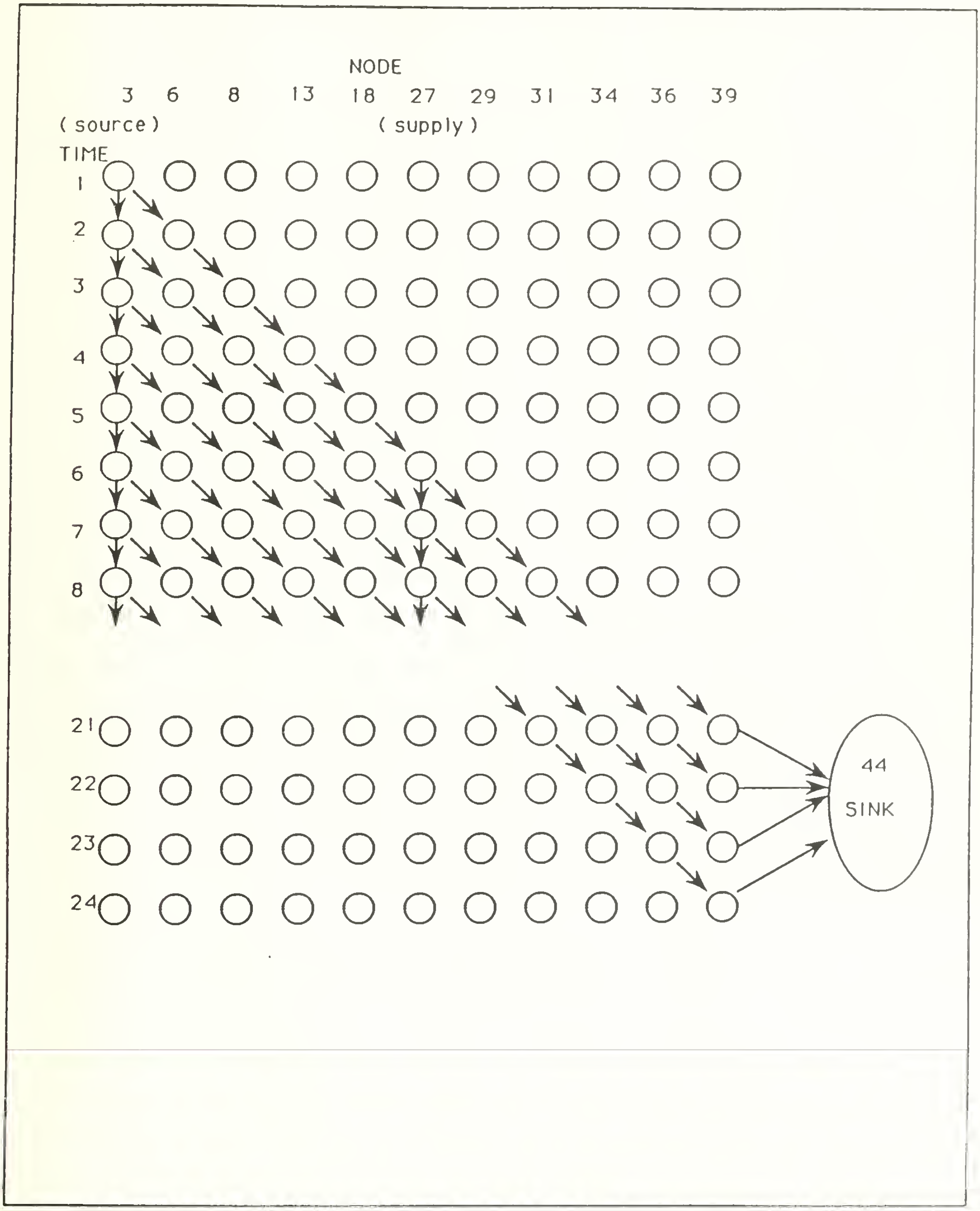


Figure 5. Modified Dynamic Network for Model 1

$$\min \sum_k \sum_t \sum_u t \; a_{uNt-1} \, x_{kuNt-1} \quad \textit{where } N \textit{ is the destination node} \tag{3.1.1}$$

- Subject to

$$\sum_u a_{vut} \, x_{kvut} = 1 \quad \textit{for } \forall \; k, \text{ and } v \; \textit{ the source node for } \; k, \; t = 1 \tag{3.1.2}$$

$$-\sum_u a_{uvt} \, x_{kuvt-1} + \sum_u a_{vut} \, x_{kvut} = 0 \quad \textit{for } \forall \;\; k, \, t, \text{ and } v \tag{3.1.3}$$

$$\sum_t \sum_u \; a_{uNt} \;\, x_{kuNt} = -\, 1 \quad \textit{for } \forall \;\; k \text{ and } \textit{where } N \textit{ is the destination node.} \tag{3.1.4}$$

$$\sum_k a_{uvt} \;\, x_{kuvt} \leq arccapacity_{uvt} \quad \textit{for } \forall \; u = v, \;\; t \tag{3.1.5}$$

$$\sum_u \sum_t \; a_{uut} \;\, x_{kuut} \geq mintime_{ku} \quad \textit{for } \forall \; k, \; \text{ and } u \textit{ the supply point for } k \tag{3.1.6}$$

$$\sum_k \sum_u \; a_{uvt} \;\, x_{kuvt} \leq nodecapacity_{vt} \quad \textit{for } \forall \; v, \, t \tag{3.1.7}$$

The GAMS code for Model 1 is given in appendix A.

The decision variable x_{kuvt} establish a flow amount from node u to v at time period t for unit k. If the objective is to minimize the average arrival time of the units at the destination, the objective function is 3.1.1. However, if the objective is to maximize the average time spent at the supply points , the objective function is

$$\max \sum_k \sum_{u \in U} \sum_t \; a_{uut} \;\, x_{kuut} \quad \textit{where } U \textit{ is the } \text{set} \quad \text{of } \textit{the supply points} \tag{3.1.8}$$

Constraints 3.1.2,3.1.3, and 3.1.4 are the flow balance equations. Constraints 3.1.2 specify one unit of "supply" at the source node for each unit k. Constraints 3.1.3 are balance equations for intermediate nodes,i.e. nodes other than the source and destination nodes. Constraints 3.1.4 for node N, the destination node, state that each unit must arrive at the destination in some allowable time period. Constraints 3.1.5 are joint capacities for each arc and time period. For the implementation of this thesis all capacities are 1. Constraints 3.1.6 enforce a minimum sojourn time at the supply point for each unit. Constraints 3.1.7 are the joint node capacity constraints corresponding to capacitated holdover arcs. The destination node has infinite capacity and is not included here.

3. VARIABLE COLUMN LENGTH : MODEL 2

In Model 2 it is assumed that any convoy may be stretched out and intermingled with other convoys subject to capacity limitations such as the maximum number of vehicles that can pass through an arc at any one time. However, the identity of a unit must be maintained since it has its own supply point which must be visited. In this model, the decision variables, the number of vehicles traversing an arc at any one time, are allowed to be continuous. Thus this model is a linear program (LP).

a. Procedure

Figure 2 shows the static road network with arc traversal times. In Model 1, the static network is first modified so that each arc has the same length as a column. Model 2 does not enforce a fixed column length so that the static network can be directly expanded into a dynamic network with some arcs spanning 1 unit of time, some 2 units of time, some 3 units of time and so on. This dynamic network is shown in figure 7.

In model 2 , any unit may be broken down into several smaller units which may use different routes. If the same constraints are used as in Model 1 to force a sojourn at the supply points some part of a unit may skip the supply point entirely and another part may spend extra time at supply point to satisfy the constraint. To avoid this, holdover arcs at the supply points are used which have the same length as the time needed to take on equipment. This is show is shown in figure 7.

In figure 8, nodes and arcs which can have no flow are eliminated. In addition, junction nodes have holdover arcs since some delay may occur there; such arcs have restricted capacity, too. The destination node doesn't need holdover arcs because its capacity is unlimited.

b. Model 2 Formulation

1. Indices
   - $u,v = 1,2,...,N$ nodes
   - $k = 1,2,...,K$ units
   - $t,s = 1,2,...,T$ time periods
2. Data
   - $a_{uvts} =$ 1 if there exists an arc between node u and node v from time period t to time period s; 0 otherwise
   - $vehicles_k$ vehicles in unit k

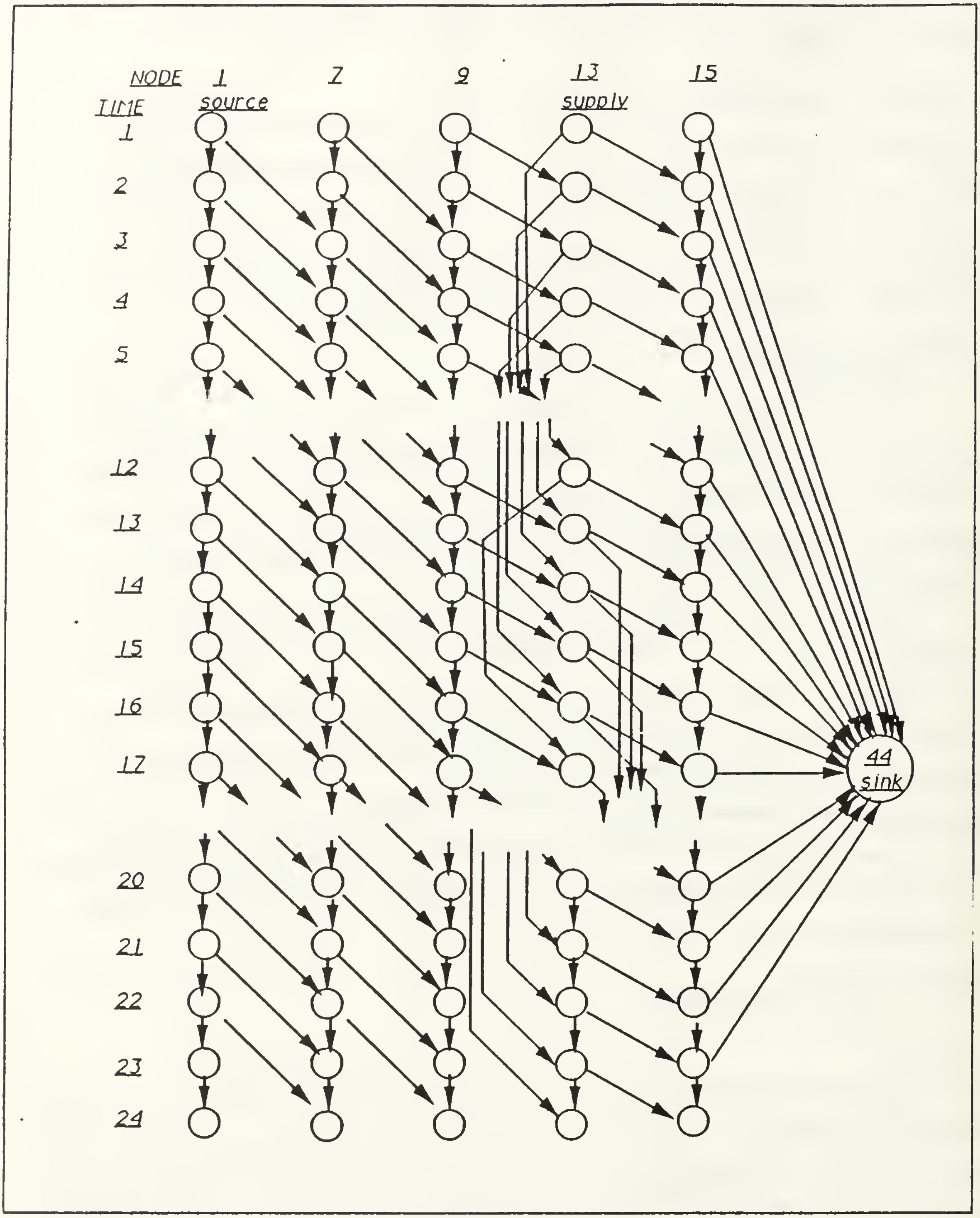


Figure 6. Dynamic Network for Model 2

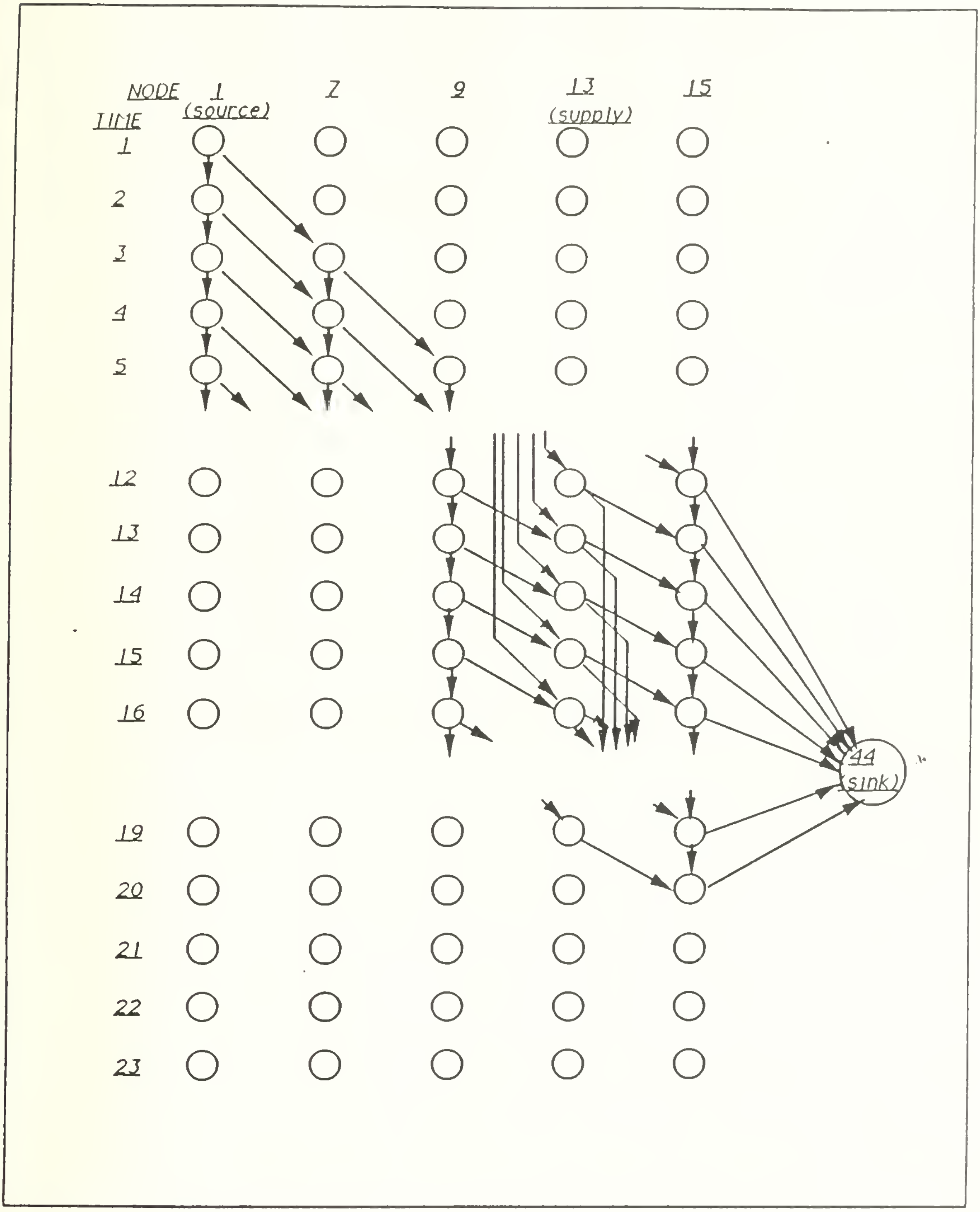


Figure 7. Dynamic Network

- T — maximum time for the operation
- $arccapacity_{uvst}$ — capacity of arc (u,v) between periods s and t
- $nodecapacity_{ut}$ — capacity of node u in time period t

3. Decision Variables

- x_{kuvts} = — number of vehicles for unit k traverses arc (u,v) from time period t to time period s
- z — total amount of time to transport

4. Formulation

- The objective function is

$$\min \sum_k \sum_t \sum_u \sum_s s \; a_{uNts} \; x_{kuNts} \quad \textit{where } N \textit{ is the destination node.} \tag{3.2.1}$$

- Subject to

$$\sum_s \sum_u a_{vuts} \; x_{kvuts} = vehicles_k \quad for \;\forall\; k,\; v \textit{ the source node for } k \text{ and } t = 1 \tag{3.2.2}$$

$$-\sum_u \sum_s a_{uvst} \; x_{kuvst} + \sum_u \sum_s a_{vuts} \; x_{kvuts} = 0 \quad for \;\forall\; k,\; v,\; t \tag{3.2.3}$$

$$\sum_u \sum_t \sum_s a_{uNst} \; x_{kuNst} = -\; vehicles_k \quad for \;\forall\; k,\; N \textit{ the destination node} \tag{3.2.4}$$

$$\sum_k a_{uvst} \; x_{kuvst} \leq arccapacity_{uvst} \quad for \;\forall\; u,\; v,\; s,\; t \tag{3.2.5}$$

$$\sum_{\substack{u \\ u \neq v}} \sum_s a_{uv_kst} \; x_{kuv_kst} \leq a_{v_kv_kt(t+r)} \; x_{kv_kv_kt(t+r)} \quad for \;\forall\; k,\; t \tag{3.2.6}$$

where r is sojourn time and v_k is supply point v for unit k

$$\sum_k \sum_u \sum_s a_{uvst} \; x_{kuvst} \leq nodecapacity_{vt} \quad for \;\forall\; v,\; t \tag{3.2.7}$$

$$\sum_{\substack{u \\ u \neq v}} \sum_t \sum_s a_{uvts} \; x_{kuvts} = vehicles_k \quad for \;\forall\; k,\; v \textit{ the supply point for } k \tag{3.2.8}$$

The objective function 3.2.1 minimizes the average arrival time to the destination node over all units. There is no variant of this model, as in Model 1, in which the objective is to maximize the sojourn times at the supply points. Constraints 3.2.2

are the balance equations in the first time period at the source nodes. Constraints 3.2.3 are balance equations for intermediate nodes and constraints 3.2.4 are for the destination node for each unit. Constraints 3.2.5 are the joint arc capacities for arcs (u,v) between time periods s and t. Constraints 3.2.6 enforce the necessary sojourn time at the supply point v for each unit. Constraints 3.2.7 enforce node capacities. Constraints 3.2.8 requires that for each unit all its vehicles pass through the unit's supply point.

IV. IMPLEMENTATION, RESULTS AND CONCLUSIONS

This chapter describes the results from the GAMS implementation of Models 1 and 2 on an IBM 3033AP at the Naval Postgraduate School(NPS). The GAMS complier at NPS uses BDMLP and ZOOM to solve linear and integer programs, respectively.

1. MODEL 1: FIXED COLUMN LENGTH

Two versions of this model, Models 1.1 and 1.2, were implemented. Model 1.1 minimizes the average arrival time of the units at the destination node and Model 1.2 maximizes the sojourn time at the supply points.

The road network contains 44 nodes and by solving three shortest path problems, one for each of the 3 units, it is determined that at least 24 time periods are needed for feasibility. In fact, 25 time periods were used and this proved sufficient. If all possible nodes and arcs are generated, the complete integer program would contain 145,200 decision variables (arcs) and 52,803 constraints. However, some of the variables (arcs) and flow balance constraints may discarded since they do not affect the optimal solution. The model statistics for models 1.1 and 1.2 are summarized in Table 1.

Model 1.1,1.2	Equations	2867	Variables	2023
	Non-zero elements	7912	0-1 variables	2022
	Continuous variables	1	Computer memory	3M
Model 1.1	Generation time	21.30 sec	Solution time	21.40 sec
Model 1.2	Generation time	21.35 sec	Solution time	21.45 sec

Table 1. MODEL STATISTICS FOR MODELS 1.1 AND 1.2

The optimal paths for Models 1.1 and 1.2 are shown in Figure 8 and 9, respectively. Model 1.1 has no delays since unit 1 arrives in 13 time periods, unit 2 in 15 time periods and unit 3 in 24 time periods all of which are the times which can be achieved if no joint capacity constraints are enforced. In the solution to Model 1.2, the units delay at the supply points as possible as and all arrive by time period 24 as required. The route taken by unit 2 is different than that taken in Model 1.1.

2. MODEL2: VARIABLE COLUMN LENGTH

Similar to the fixed convoy length model, two versions of this model, Models 2.1 and 2.2, were implemented. To compare Models 1 and 2, the capacities for all nodes and arcs in Model 2.1 are set to the length of the convoy, which is assumed to be 30., The capacities of nodes and arcs in Model 2.2 are the same as in Model 2.1; however, the convoy length is now 36. The road network for Model 2 is the same as the one in Model 1. The statistics for Models 2.1 and 2.2 are summarized in Table 2.

Model 2.1,2.2	Equations	600	Variables	814
	Non-zero elements	2931	Computer memory	3M
Model 2.1	Generation time	11.06 sec	Solution time	11.15 sec
Model 2.2	Generation time	11.07 sec	Solution time	11.16 sec

Table 2. MODEL STATISTICS FOR MODELS 2.1 AND 2.2

Figure 10 and 11 display the optimal solutions of Models 2.1 and 2.2, respectively. In the solution of Model 2.1, there is no delay experienced by any of the units because all arc and node capacities are sufficiently large to handle all the traffic. In the solution of Model 2.2, however, there is some delay. In particular, a fraction of unit 2 (6 vehicles) uses the arc between nodes 5 and 6 in period 3 while a fraction of unit 1 (24 vehicles) uses same arc at same time period. So, a fraction of unit 1 (24 vehicles) goes to node 5 and the other (6 vehicles) stays at node 4 for 1 time period. Unit 1 is then broken down into two components one of which contains 24 vehicles while the other contains 12 vehicles. The two components of unit 1 still use the same route to the destination node; part of the unit just lags behind the other. In fact, each unit uses a single route.

If some arc capacities are reduced, there might be more delays and the units might split into components following different routes. In fact, if the capacity of arc (9, 13) between time periods 14 and 16 is reduced to 0 then unit 1 splits up at node 6 and part of the unit reaches the destination via nodes 9,13,15, and 16 while the other part uses nodes 8,14,15, and 16; the other units use the same routes as in Figure 11.

3. COMPARISION OF BOTH MODELS

Below, we list the advantages and disadvantages of the two models.

1. Model 1: Advantages
   a. The objective function is straightforward.

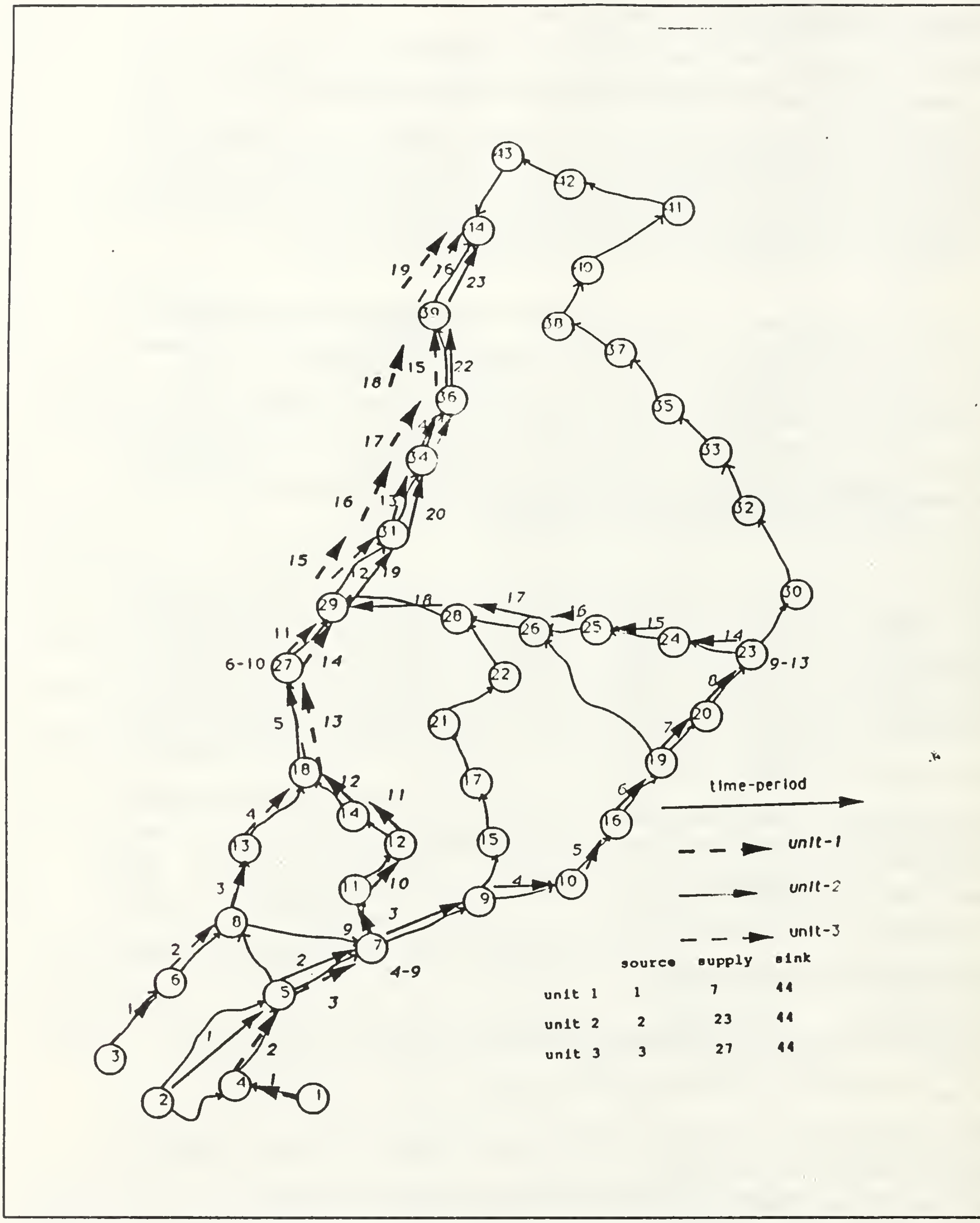


Figure 8. Optimal Paths For Model 1.1

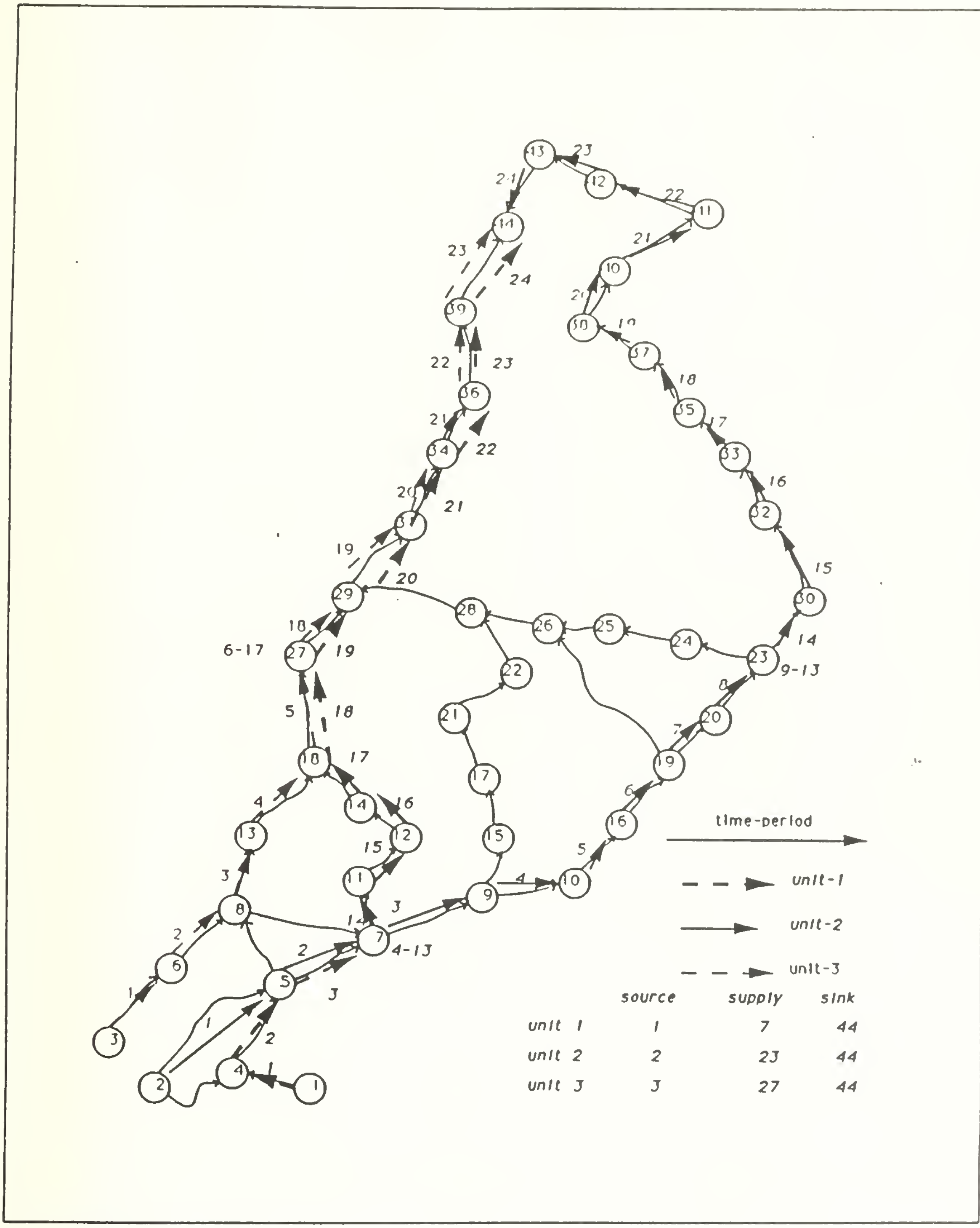


Figure 9. Optimal Paths For Model 1.2

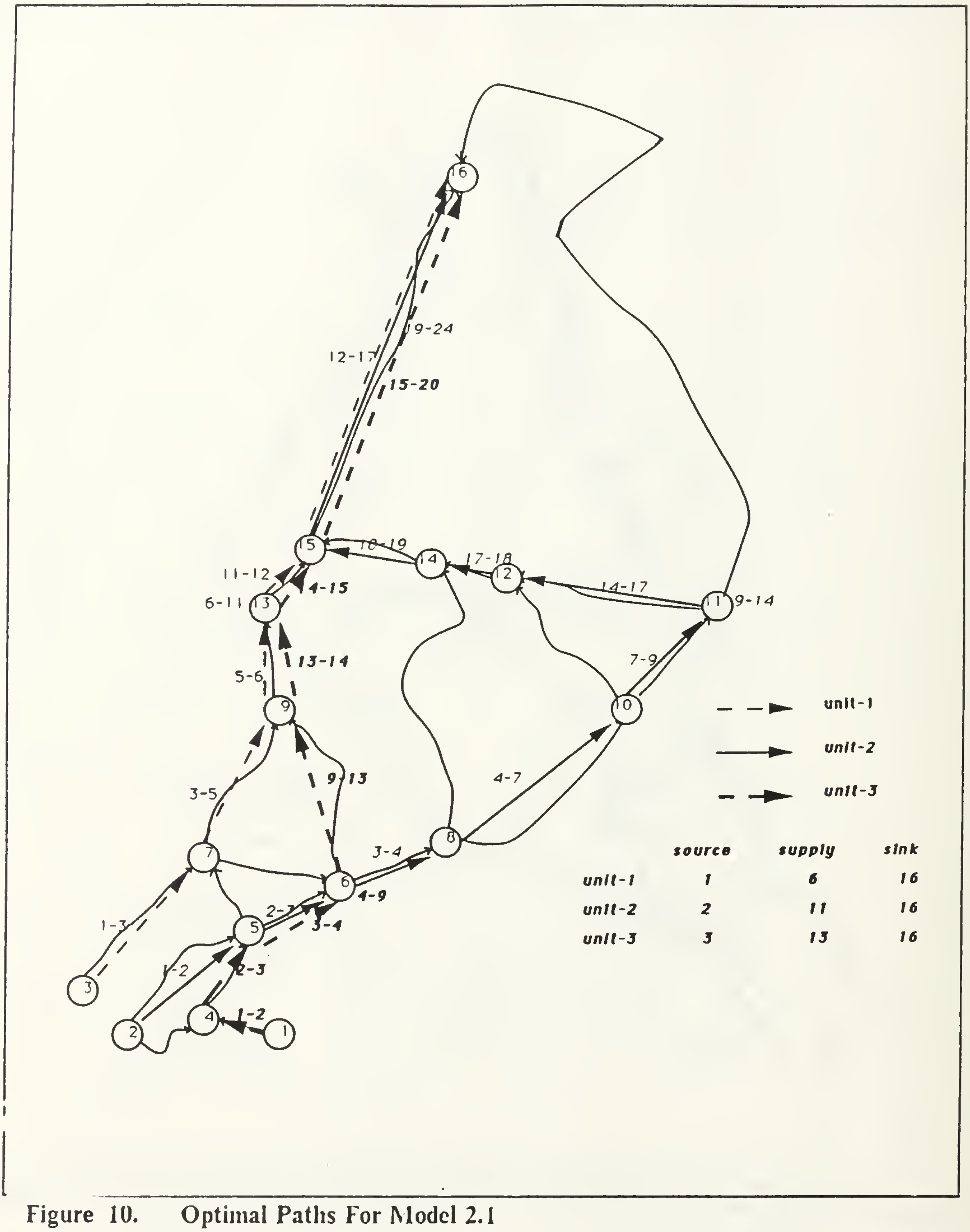
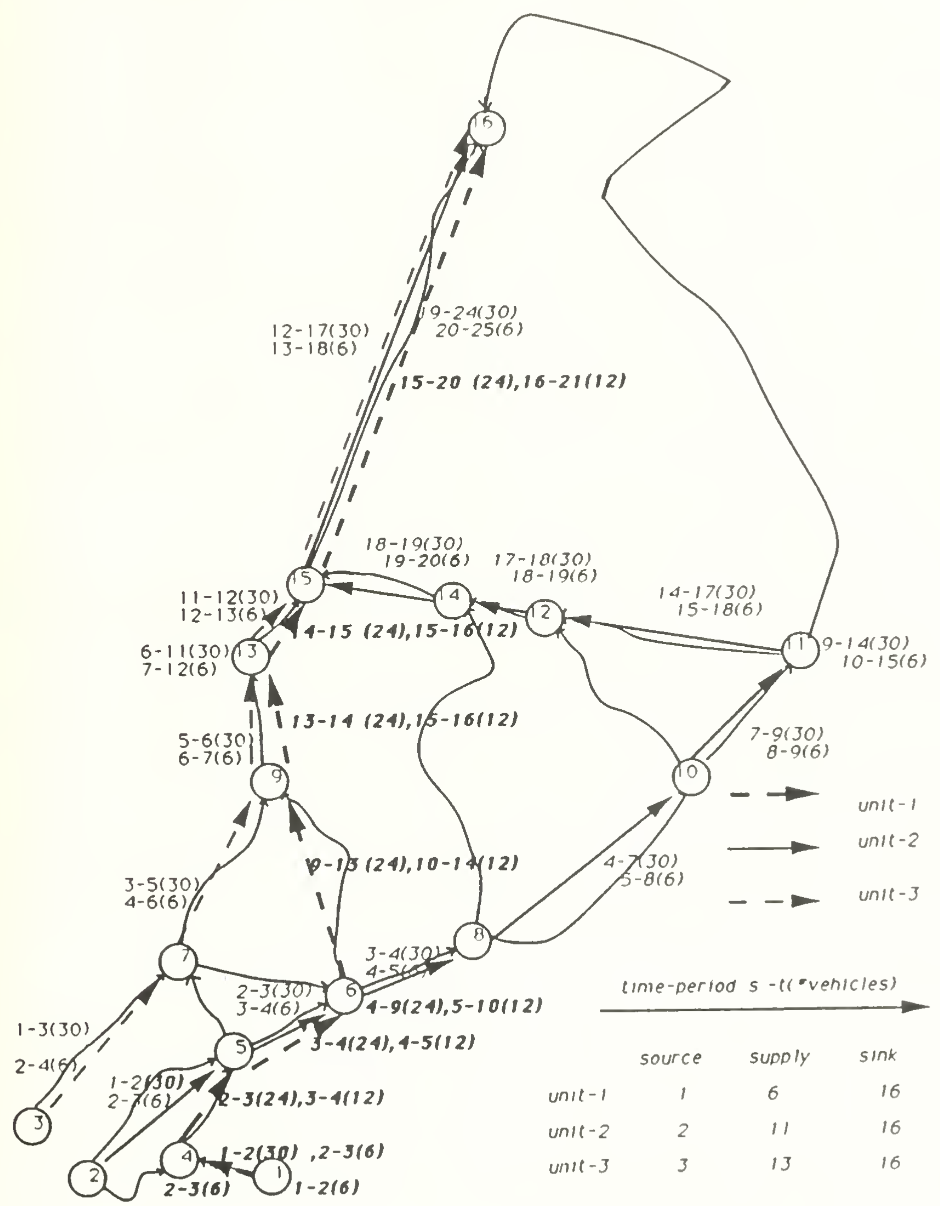


Figure 10. Optimal Paths For Model 2.1



	source	supply	sink
unit-1	1	6	16
unit-2	2	11	16
unit-3	3	13	16

Figure 11. Optimal Paths For Model 2.2

b. It is easy to modify the minimum time required to take on supplies at the supply points: Just change the data element $mintime_k$.

2. Model 1: Disadvantages

a. The length of all arcs should be the same and must be related to the column length. This decreases the accuracy of model if there is a large difference between column length and the length of the shortest arc.

b. The number of nodes and arcs in the static network is increased by the necessity to modify the length of all arcs to be the same.

c. It is hard to change the column length. For instance, suppose that the length of each arc remains one time unit but that the column length is two time units. Then, x_{kuvt} becomes x_{kuvwt}. That is, we must consider that a unit is covering two arcs (u,v) and (v,w) at any one time. This leads to many more variables in the formulation. Furthermore, the balance constraints become much more complicated and numerous.

d. Because Model 1 is an integer program, not an LP as in Model 2, computational times are long.

e. Model 1 has more variables and equations than Model 2 after applying reductions.

3. Model 2 : Advantages

a. The dynamic network for this model is easier to construct than the analogous network for Model 1 since it is not necessary to modify the initial static network so that all arc lengths are equal to the column length.

b. This model has more flexibility in that it is easy to change the column length without modification of network structure. (Just change the supplies and demands.)

c. Model 2 has fewer variables and equations than Model 1 after reductions.

d. Model 2 is an LP and computational time is much shorter t an for Model 1 which is an IP.

4. Model 2: Disadvantages

a. It is hard to modify the time to take on equipment since it is given by network structure.

b. There does not appear to be a variant of Model 2 analogous to Model 1.2 in which the average time spent at the supply points is maximized.

4. CONCLUSIONS AND RECOMMENDATION

Based on the GAMS implementation using the example scenario, Model 2 provides essentially the same answer as Model 1 using considerably less cpu time. In addition, Model 2 is more flexible and can handle larger problems. On the other hand, Model 2 has one disadvantage in that it can not maximize the spent at supply points. However, one could use Model 2 to find the maximum time by varying the length of sojourn at supply points.

Future work in this area should include applying Model 2 to larger scenarios and extending the model to the case which includes the effects of congestion. This has been done, for instance, in building evacuation models where the flow of people through an exit route can be reduced when the density of people becomes too high. This non-linear effect can be included, at least approximately, in a modified linear programming model [Ref. 3].

APPENDIX A. GAMS PROGRAM FOR MODEL 1

```
$TITLE MIXED INTEGER PROGRAM FOR A TIME PERIOD COLUMN LENGTH
*THESIS MODEL
*MAJ DONG KEUN, LEE
*MODEL: MULTICOMMODITY DYNAMIC TRANSPORTATION NETWORK TO MINIMIZE THE
*       TRANSPORTING TIME.

SETS
  N  NODES         /N1*N44/
  T  TIME PERIODS /T1*T25/
  K  # OF TROOPS   /K1*K3/  ;
ALIAS(N,M);

PARAMETERS
  A(N,M,T) ROAD NETWORKBETWEEN N AND M AT TIME T FOR TROOP K
   /N1.(N1,N4).T1*T11              1
    N2.(N2,N4,N5).T1*T11           1
    N3.(N3,N6).T1*T11              1
    N4.N5.T2*T12                   1
    N5.(N7,N8).T2*T12              1
    N6.N8.T2*T12                   1
    N7.(N7,N11,N9).T3*T15          1
    N8.(N13   ).T3*T16             1
    N9.(N15,N10).T3*T16            1
    N10.N16.T4*T18                 1
    N11.N12.T4*T16                 1
    N12.N14.T5*T16                 1
    N13.N18.T4*T14                 1
    N14.N18.T6*T18                 1
    N15.N17.T5*T18                 1
    N16.N19.T6*T18                 1
    N17.N21.T6*T20                 1
    N18.N27.T5*T15                 1
    N19.(N20,N26).T7*T20           1
    N20.N23.T8*T20                 1
    N21.N22.T7*T20                 1
    N22.N28.T8*T20                 1
    N23.(N24,N30,N23).T9*T19       1
    N24.N25.T10*T20                1
    N25.N26.T11*T21                1
    N26.N28.T12*T22                1
    N27.(N29,N27).T6*T20           1
    N28.N29.T9*T20                 1
    N29.N31.T7*T23                 1
    N30.N32.T10*T20                1
    N31.N34.T8*T23                 1
    N32.N33.T11*T21                1
    N33.N35.T12*T22                1
    N34.N36.T9*T24                 1
    N35.N37.T13*T23                1
    N36.N39.T14*T24                1
    N37.N38.T13*T23                1
```

```
    N38.N40.T13*T23              1
    N39.N44.T15*T24              1
    N40.N41.T12*T22              1
    N41.N42.T13*T23              1
    N42.N43.T14*T24              1
    N43.N44.T15*T24              1 /

    MP(M) SOURCE SUPPLY POINT AND SINK NODE
       /N1   1
        N2   1
        N3   1
        N7   1
        N23  1
        N27  1
        N44  1/


    MID1(K,N,N) SUPPLY POINT NFOR TROOP K
               /K1.N27.N27       1
                K2.N23.N23       1
                K3.N7.N7         1/

    S(K,N,T) SOURCE AND SINK NODE FOR UNIT K AT TIME T
        /K1.N3.T1    1
         K2.N2.T1    1
         K3.N1.T1    1 /;

SCALAR   MINTIM  TIME TO TAKE ON EQUIPMENT AT SUPPLY POINT /5/;
SCALAR   ARCCAP  ARC CAPACITY /1/;
SCALAR   NODECAP NODE CAPACITY /1/;
SCALAR   REQTIM  MAXIMUM TIME FOR THE OPERATION /25/;

VARIABLES
    X(K,N,M,T)  FLOW AMOUNT FROM N TO M FOR UNIT K AT TIME T
      Z         TOTAL AMOUNT TIME TO TRANSPORT FROM SOURCE TO SINK NODE;
BINARY VARIABLES X;

EQUATIONS
    COST              DEFINE OBJECTIVE FUNCTION
    MIDTIM(K)         TIME TO STAY AT MIDPOINT FOR UNIT K
    MID(K,M,T)        INTERMEDIATE NODE FOR UNIT K AT TIME T
    START(K,M,T)      SOURCE NODE N FOR UNIT K AT TIME T
    TSINK(K,N)         SINK NODE N FOR UNIT K
    CAP(M,T)          NODE CAPACITY FROM NODE N TO NODE M AT TIME T
    CAPACITY(N,M,T) ARC CAPACITY FROM NODE N TO NODE M AT TIME T;

START(K,M,T)$(S(K,M,T) EQ 1)..  SUM(N,A(M,N,T)*X(K,M,N,T)) =E= 1;

MID(K,M,T)$(S(K,M,T) EQ 0 AND ORD(M) NE CARD(M))..
  SUM(N,A(M,N,T)*X(K,M,N,T))$(ORD(T) NE REQTIM)
  -SUM(N,A(N,M,T-1)*X(K,N,M,T-1)) =E= 0 ;

TSINK(K,N)$(ORD(N) EQ CARD(N)).. -SUM((M,T),A(M,N,T-1)*X(K,M,N,T-1))
=E= -1;

CAPACITY(N,M,T)$(ORD(M) NE ORD(N)).. SUM(K,X(K,N,M,T)*A(N,M,T)) =L=
```

```
                                                    ARCCAP;

CAP(M,T)$(CARD(M) NE ORD(M)).. SUM((K,N),A(N,M,T)*X(K,N,M,T)) =L=
                                                     NODECAP;

MIDTIM(K)..   SUM((N,M,T)$(MID1(K,N,M) EQ 1),X(K,N,M,T)*A(N,M,T))
                                                     =G= MINTIM;

COST..      SUM((K,N,M,T)$(ORD(M) EQ CARD(M) )
        ,ORD(T)*X(K,N,M,T-1)*A(N,M,T-1)) =E= Z;

MODEL RT1 /ALL/;
OPTION LIMROW =0, LIMCOL = 0 , SOLPRINT = OFF ,ITERLIM = 10000;
SOLVE RT1 USING MIP MINIMIZING Z;
DISPLAY X.L,Z.L;
```

APPENDIX B. GAMS PROGRAM FOR MODEL 2

```
* LP PROGRAM FOR FLEXIBLE COLUMN LENGTH
* maj Dong Keun, Lee
* Model : Multicommodity dynamic transportation network to minimize
*         the transportation time
SET M NODE            /M1*M16/
    T TIME PERIODS    /T1*T25/
    K UNIT troops     /K1*K3/;
ALIAS(M,N);
ALIAS(T,S);
TABLE   A(M,N,T,S)  ROAD NETWORKBETWEEN NODE M AND N AT TIME T TO S
                 T1.T2 T2.T3 T3.T4 T4.T5 T5.T6 T6.T7 T7.T8 T8.T9 T9.T10
 M1.M1             1     1     1     1     1     1     1      1
 M2.(M2,M4,M5)     1     1     1     1     1     1     1      1
 M3.(M3,M4)         1     1     1     1     1     1    1      1
 M4.(M5,M4)              1     1     1     1     1    1      1     1
 M5.(M7,M5,M6)           1     1     1     1     1    1      1     1
 M6.M8                         1     1     1     1    1      1     1
 M7.(M6,M7)                     1     1     1     1     1    1     1
 M9.(M9,M13)                                1     1     1    1     1
     +             T10.T11
 M5.(M7,M5,M6)        1
 M6.M8                1
 M7.(M6,M7)           1
 M9.(M9,M13)          1
    +            T11.T12 T12.T13 T13.T14
 M6.M8             1        1       1
 M7.(M6,M7)        1        1       1
 M10.(M12,M10)      1       1       1
 M13.M15            1       1       1
 M9.(M9,M13)       1        1       1
    +            T14.T15 T15.T16 T16.T17 T17.T18 T18.T19  T19.T20
 M10.(M12,M10)     1        1       1       1
 M12.(M12,M14)      1       1       1       1       1
 M13.M15            1       1       1       1       1         1
 M14.(M14,M15)      1       1       1       1       1         1
   +    T1.T3 T2.T4 T3.T5 T4.T6 T5.T7 T6.T8 T7.T9 T8.T10 T9.T11
M1.M7    1     1     1     1     1     1     1     1      1
M10.M11                                      1     1      1
M7.M9                1     1     1     1     1     1      1
    +    T10.T12 T11.T13 T12.T14 T13.T15
M10.M11     1       1       1       1
M7.M9        1      1
   +     T4.T7 T5.T8 T6.T9 T7.T10 T8.T11 T9.T12 T10.T13
 M8.M10     1    1     1     1      1      1       1
   +     T14.T17 T15.T18
 M11.M12     1      1
   +   T8.T12 T9.T13 T10.T14 T11.T15 T12.T16 T13.T17 T14.T18
M6.M9    1      1       1        1      1       1        1
  +     T3.T8 T4.T9 T5.T10 T6.T11 T7.T12 T8.T13 T9.T14
M6.M6    1     1     1      1      1      1      1
M8.M14                                           1
```

```
M11.M11                                                     1
M13.M13                          1      1      1      1
  +     T10.T15 T11.T16 T12.T17 T13.T18 T14.T19
M8.M14      1       1       1       1       1
M11.M11     1
M15.M16                     1       1       1
M13.M13     1        1      1       1       1
   +      T15.T20 T16.T21 T17.T22 T18.T23 T19.T24 T20.T25
M15.M16       1       1       1       1       1       1
    +    T13.T24 T14.T25
M11.M16     1       1                  ;

PARAMETERS

    SR(K,M,T,S)  SOURCE NODE FOR UNIT K AT NODE M FROM T1 TO T2
         /K1.M1.T1.T2  1
          K2.M2.T1.T2  1
          K3.M3.T1.T2  1/
    NARCCAP(M,N) ARC CAPACITYBETWEEN M AND N
         /M1.M7        60
          M1.M1        60
          M2.(M2,M4)   30
          M2.M5        30
          M3.(M3,M4)   30
          M4.(M4,M5)   30
          M5.(M5,M7,M6)  30
          M6.(   M8)   30
          M6.M6        150
          M6.M9        120
          M7.(M9)      60
          M7.M7        30
          M7.M6        30
          M8.M8        30
          M8.M14        150
          M8.M10         90
          M9.(   M11) 30
          M9.M9        150
          M9.M13       30
          M10.(    M11) 60
          M10.(M10,M12) 30
          M11.(M11)     150
          M11.M12        90
          M11.M16       330
          M12.M12        30
          M12.M14        30
          M13.M13       150
          M13.M15        30
          M14.(M14,M15) 30
          M15.M15        30
          M15.M16       150/


    MP(N)    MIDPOINT NODE
        /M6   1
         M11 1
         M13 1/
```

```
     S1(K,M,T)  SOURCE NODE FOR UNIT K  AT TIME T1
          /K1.M1.T1  1
           K2.M2.T1  1
           K3.M3.T1  1/
   NODE1(M)   M IS SOURCE  SUPPLY POINT  SINK NODE
          /M1  1
           M2  1
           M3  1
           M6  1
           M11 1
           M13 1
           M16 1 /

    MIDPT(K,M) MIDPOINT NODE FOR UNIT K
            /K1.M13 1
             K2.M11 1
             K3.M6  1/;
SCALAR TROOP NUMBER OF VEHICLE PER TROOP /30/;
SCALAR NNODECAP CAPACITY OF NODE         /30/;
VARIABLES
  X(K,M,N,T,S)  FLOW AMOUNT FROM M TO N TIME BETWEEN T TO S FOR UNIT K

      Z  TOTAL AMOUNT FLOW * TIME                   ;
POSITIVE VARIABLE X;
EQUATIONS
 COST                OBJECTIVE FUNCTION
 START(K,N,T,S)      SOURCE NODE FOR UNIT K AT NODE N TIME T TO S
 INTERMID(K,N,T)     INTERMID NODE FOR UNIT K AT NODE N TIME T
 TSINK(K,N)          SINK NODE N FOR UNIT K
 MIDPOINT(K,N)       MIDPOINT NODE N FOR UNIT K
 STAY(K,N,T)         TIME TO STAY AT MIDPOINT NODE N FOR UNIT K AT TIME T
 ARCCAP(M,N,T,S )    ARC CAPACITY M TO N TIME T TO S
 NODECAP(M,T)        NODE CAPACITY M AT TIME T;


START(K,N,T,S )$(SR(K,N,T,S ) EQ 1).. SUM(M,A(N,M,T,S)*X(K,N,M,T,S)) =E=
                                       TROOP;

INTERMID(K,N,T)$(S1(K,N,T) NE 1 AND ORD(N) NE CARD(N))..
   -SUM((M,S),A(M,N,S,T)*X(K,M,N,S,T)) + SUM((M,S),A(N,M,T,S)*X(K,N,M,T,S
    )) =E= 0 ;

TSINK(K,N)$(ORD(N) EQ CARD(N)).. -SUM((M,S,T),A(M,N,S,T)*X(K,M,N,S,T))
    =E= -TROOP;

ARCCAP(M,N,T,S)$(MP(N) NE 1).. SUM(K,A(M,N,T,S)*X(K,M,N,T,S)) =L=
                               NARCCAP(M,N);

MIDPOINT(K,N)$(MIDPT(K,N) EQ 1).. SUM((M,T,S)$(ORD(M) NE ORD(N))
,A(M,N,T,S)*X(K,M,N,T,S))=G= TROOP;

NODECAP(M,T)$(NODE1(M) EQ 0).. SUM((K,S,N), A(N,M,S,T)*X(K,N,M,S,T))
    =L= NNODECAP;

STAY(K,N,T)$(MIDPT(K,N) EQ 1).. SUM((M,S)$(ORD(M) NE ORD(N)),A(M,N,S,T)
```

```
          *X(K,M,N,S,T)) =L=   SUM(S,A(N,N,T,S)*X(K,N,N,T,S));

COST.. SUM((K,N,M,T,S)$(ORD(M) EQ CARD(M)),ORD(S)*X(K,N,M,T,S)*A(N,M,
                                                          T,S)) =E= Z ;

MODEL CH3 /ALL/;
OPTION  ITERLIM = 10000 ,LIMROW = 0 ,LIMCOL = 0 , SOLPRINT = OFF ;
SOLVE CH3 USING LP MINIMIZING Z ;
DISPLAY X.L,Z.L;
```

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Thesis
L3838 Lee
c.1 Optimal routing of
military convoys through
a road network.