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A concept of anti-satellite terminal guidance

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A CONCEPT OF ANTISATELLITE TERMINAL GUIDANCE

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by

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A CONCEPT OF ANTI-SATELLITE TERMINAL GUIDANCE

by

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ABSTRACT

Since earth satellites afford an offensive military potential to any of our nation's enemies it is essential that an effective countermeasure be developed. This paper analyzes the problem of terminal guidance for an anti-satellite missile and investigates some of the factors that would determine the feasibility of such a weapon if a nuclear warhead cannot be used.

Inherent errors of an anti-satellite weapon system necessitate terminal guidance. The kinematic relationship between the target and a weapon package which has been carried as the payload of an anti-satellite missile are investigated at the time when terminal guidance would have to be effected. Preliminary design criteria which could be derived from this relationship are discussed. A typical weapon package is then proposed in order to further investigate design parameters. Studies of some overall system errors are also included.

This paper proves that terminal guidance for an anti-satellite missile is both necessary and feasible.

Thesis Supervisor: Mr. H. Philip Whitaker

Title: Deputy Associate Director
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May 25, 1959

Professor Alvin Sloan
Secretary of the Faculty
Massachusetts Institute of Technology
Cambridge 39, Massachusetts

Dear Professor Sloan:

In accordance with the regulations of the faculty, we hereby submit a thesis entitled A Concept of Anti-Satellite Terminal Guidance in partial fulfillment of the requirements for the degree of Master of Science in Aeronautical Engineering.

Henry C. Arnold Jr.
Herbert J. Blaha
Delbert L. Nall

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The concepts, opinions, and conclusions of this thesis are solely those of the authors and in no way are to be construed as being those of the Bureau of Aeronautics, U. S. Navy, nor have they been approved by that agency.

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OBJECT

The object of this study is to present a solution to the problem of terminal guidance for an anti-satellite missile and to investigate some of the factors that would determine the feasibility of such a weapon.

CHAPTER 1

INTRODUCTION

Earth satellites are capable of performing a variety of missions which are of military value. Examples of such missions are:

1. Location and surveillance of strategic airfields, and marshalling yards.
2. Battle damage reconnaissance after the initial phases of a nuclear attack.
3. Service as a communication link.
4. Service as a navigation aid.
5. Possible bases for weapons carriers.

These and other satellite missions must be denied an enemy. An effective countermeasure to an earth satellite is therefore a necessity.

This paper assumes the existence of an anti-satellite weapon system. Its major components and their capabilities are described in a series of preliminary assumptions. The basis of the weapon system is an air-launched missile carrying a weapon package as a payload. Fig. 1-1 depicts the overall concept of the weapon system. The missile will be launched up a computer-determined local radial. At an altitude and velocity dependent on target altitude, the weapon package will be separated from the missile. This separation will occur such that the velocity imparted to the weapon

package will cause the peak of the trajectory to be at the predicted intersection of the radial and the satellite course. Furthermore, this peak will occur at the precise time that the satellite is at that predicted point in space.

Errors inherent in such a system can immediately be pointed out. It can also be immediately pointed out that a nuclear warhead would have a kill radius which is capable of offsetting these errors. However, due to either an international agreement or considerations of space contamination, it may be impossible to use a nuclear warhead. If faced with such a nuclear ban, and when confronted with the errors inherent in hitting a point in space, it becomes obvious that intercept probabilities remain extremely low unless some method of terminal guidance is an integral part of the weapon system.

Terminal guidance, as conceived by this paper, is to be effected by the weapon package. The weapon package includes a target seeker and a thrust unit capable of thrusting in any or all of four mutually orthogonal directions. The technique for utilization of this thrust is explained in detail in the body of the paper.

Note on Fig. 1-1 that the weapon package trajectory is hooked at its peak. The weapon package will of necessity remain in the satellite's cylinder of probable courses for a longer period of time (as compared to firing past the course while ascending and giving no thought to the descent) if this hookshot is used. If the exact intercept fails, probability of detection and the use of terminal guidance is enhanced since the seeker unit will be actuated during both the ascent and descent of the missile

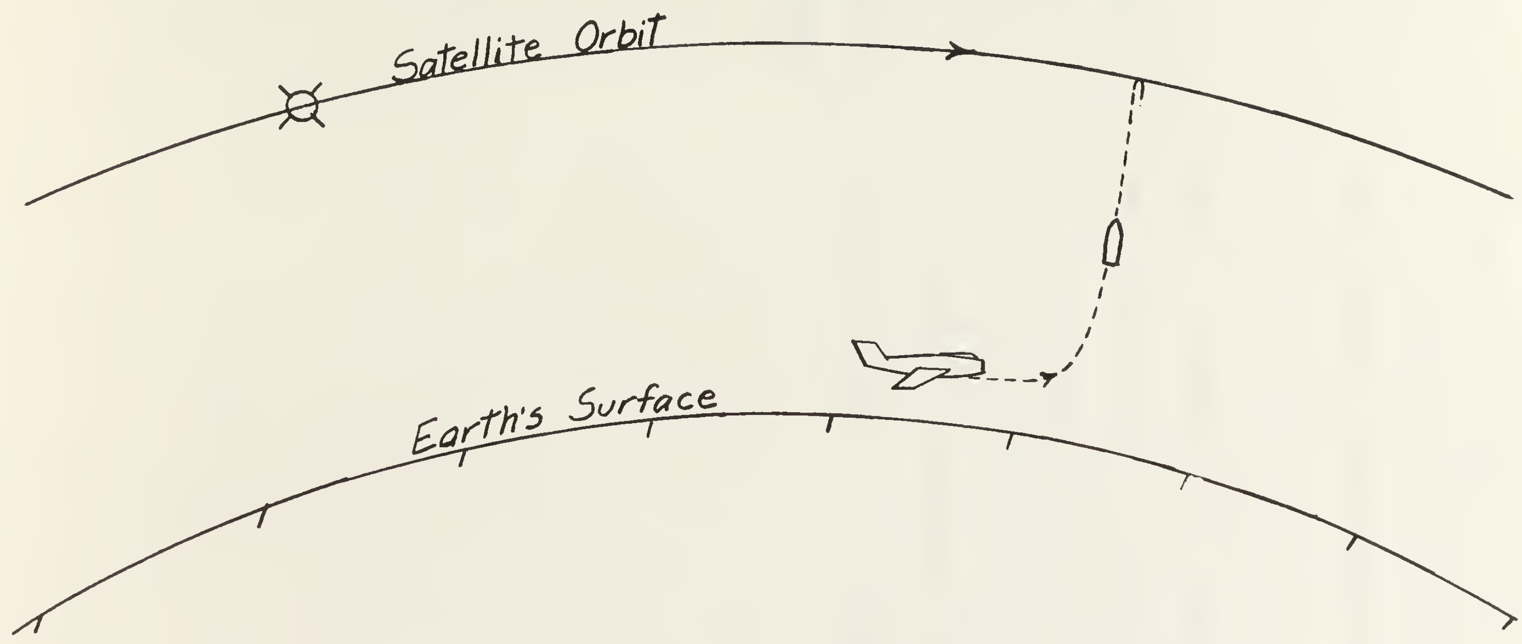


FIG. 1-1 HOOK SHOT WITH ANTI-SATELLITE MISSILE

while at the hook of the trajectory.

Derivation Summaries in Chapter 2 fully develop the geometrical relationships between satellite and weapon package during the terminal guidance phase of intercept. Chapter 3 points out some useful design parameters that result from a study of the equations of the Derivation Summaries.

Limited feasibility studies are carried out in Chapters 4, 5, and 6. In these chapters thrust requirements are analyzed in some detail. From the standpoint of thrust requirements, it is concluded that terminal guidance is possible if other system errors are reasonably controlled. The effect of increasing seeker capability over that of the basic assumptions is also studied. Finally, an analysis of some considerations which were found to have negligible effect on the terminal guidance problem is also included.

CHAPTER 2

SOLUTION OF THE GUIDANCE PROBLEM

A. Basic Assumptions

The introduction to this study of terminal guidance implied that an anti-satellite missile had been launched from an attack aircraft in such a manner that it would intercept and destroy an earth satellite. It is beyond the scope of this paper to investigate all of the concepts which would contribute to the success of a weapon system with such a capability. Brief mention of only a few of the factors which would contribute to a complex system of this type will illustrate this statement. Such factors are the determination of the satellite's exact position in space at any predicted time, the ability of the attacking aircraft to be at an exact launching point precisely at a specified time, the reliability demanded of the missile's primary propulsive installation, and the establishment of a reference system to enable the missile to determine its orientation in space.

Fig. 1-1 is a pictorial presentation of the ideal intercept using an air-launched system. To achieve the intercept as shown each phase of the system must function as planned. Prior to detailing the kinematics involved in the terminal guidance to be proposed by this paper, it would be useful to fully state the assumptions under which this terminal guidance

is to be applied. The role played by terminal guidance can then be more fully understood and appreciated.

It is to be assumed that the satellite's orbit can be determined with reasonable accuracy. At any time the satellite can be forecast to be within a cylinder in space whose diameter is 4 miles and whose length is 10 miles. Ideally, it would be at the center of this cylinder.

The existence of a missile capable of being fired from an aircraft to altitudes of 100 to 1000 miles is also assumed. If for no other reason than economic considerations, one missile should have a capability for any range within this spectrum. It necessarily follows from the missile assumption that there also be in existence an aircraft capable of launching the missile; one of the specifications of such an aircraft might be that it be able to attain speeds which complement initial missile velocities, and hence range, rather than increase missile size to achieve the same velocities.

Related to these aircraft factors is the assumption that there are in existence tracking and computing systems which monitor both the satellite and the launch aircraft and which furnish the necessary information to position the aircraft for missile launch. Included in this loop is a presentation enabling the pilot to maneuver the aircraft as necessary.

The departure point for implementation of terminal guidance is target acquisition by a detection system in the missile. An infra-red seeker with a 100 mile detection capability is assumed to be aboard the missile. This unit will yield information pertaining to the angular velocity of the line-of-sight between the missile and the target. However, intelligent

use of this information assumes that a reference system is being maintained, and the existence and stability of this reference system is a further assumption of the basic situation. Moreover, this same or a similar reference system will also be necessary to insure that the missile follows a precomputed local radial to the impact point in space.

Briefly summarizing, and with reference to Fig. 1-1, it will be assumed that the missile has been launched from the attack aircraft, has been programmed onto the local radial, and by accurate velocity cutoff techniques, has reached the peak of its trajectory at the precise time that the satellite reaches this same location in space.

The astute reader immediately can point out factors which make the probability of such an intercept almost zero. To mention a few will suffice:

1. The accuracy of determination of the satellite trajectory is the aforementioned space cylinder. Which of the points in this volume shall be the aiming point?
2. Any deviation from the exact launch point will cause a comparable miss at the peak of the missile's trajectory if the remainder of the system performs perfectly.
3. Are rocket thrusts so invariant that the missile will peak with no position error?

It may possibly be pointed out that the kill radius of a nuclear warhead will suffice to account for all inherent errors. With this thought in mind, one last assumption will be made. It will be assumed that the nuclear capability cannot be exploited due to either international agreement or military considerations of space contamination.

In view of the military advantages that satellites will afford, their destruction is necessary and will therefore become a task for conventional weapons. The inescapable errors mentioned above, and innumerable others, serve only to point out that terminal guidance aboard the missile will not be a luxury to merely refine the kill probability. Terminal guidance for an anti-satellite missile is a necessity if a nuclear weapon cannot be employed.

B. Kinematic Solution

The geometrical relationship between the satellite and the missile in the seconds just prior to the closest passage between the two bodies can be expressed in equation form. An earth-centered inertial coordinate frame can be used as a reference for physical quantities described by these equations. The angular velocity of the line-of-sight between the two bodies as seen by instrumentation aboard the missile can also be expressed with reference to this coordinate frame. Suitable combinations of the equations, coupled with a series of reasonable initial conditions, yield information pertinent to the feasibility of terminal guidance. The necessary equations are developed in Derivation Summaries on the pages that follow, and are summarized in Table 2-1.

Derivation Summary 2-1, in conjunction with Fig. 2-1, develops an expression for the angular velocity of the line-of-sight between the missile and the satellite. Fig. 2-1 is not to scale, but since the reference frame can be placed at any position the accuracy of the derivation is in no way destroyed due to this liberty taken with the figure. The result of Derivation Summary 2-1 will be recognized as one of the conventional fire control equations. It is derived in this space-intercept concept to stress

Derivation Summary 2-1

Angular Velocity of the Line-of-Sight

Reference Fig. 2-1

Angular velocity of the line-of-sight between the target and the missile, with respect to the inertial reference frame is given by

$$\bar{W}_{LS} = \frac{\bar{1}_{LS} \times \bar{V}_{M(T)}}{a} \qquad (1)$$

where

$$\bar{1}_{LS} = \frac{\bar{R}_{MT}}{R_{MT}}$$

$$a = R_{MT}$$

Now from the figure

$$\bar{R}_{IT} = \bar{R}_{IM} + \bar{R}_{MT}$$

$$\bar{V}_{I(T)} = \bar{V}_{I(M)} + \bar{V}_{M(T)}$$

Therefore

$$\bar{W}_{LS} = \frac{\bar{1}_{LS} \times (\bar{V}_{I(T)} - \bar{V}_{I(M)})}{a} \qquad (2)$$

And in the scaler form

$$W_{LS} = \frac{V_{I(T)} \sin \theta}{a} - \frac{V_{I(M)} \cos \beta}{a} \qquad (3)$$

the validity of its application.

Equation (3) of Derivation Summary 2-1 is then an expression for the angular velocity of the line-of-sight between the missile and the satellite, referred to an inertial-space coordinate system. The reference axis parallel to the missile position vector $\bar{R}_{IM}$ and one of the reference axes normal to the missile position vector will form a plane parallel to the

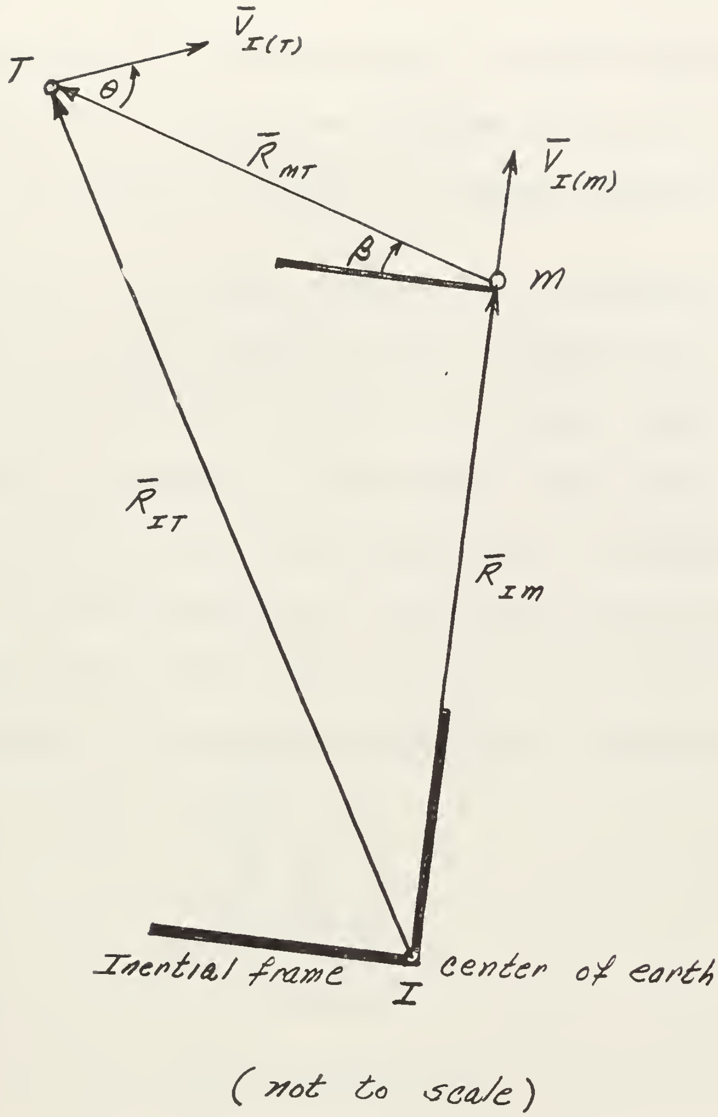


Fig. 2-1 Relationship of the angular velocity of the present line-of-sight to the present range, target velocity and the missile velocity.

satellite's orbital plane. The direction of the third reference vector will be such as to form a right-handed orthogonal reference frame. The same reference frame carried in the missile can be the basis for initial programming to the local radial and subsequent maneuvering during terminal guidance. The reference frame depicted in Fig. 2-1 is the reference frame which will be used as a basis for terminal guidance concepts of this paper.

In order to develop a solution for the terminal guidance problem in this paper, a uniform gravity field is assumed to be acting on both the missile and the satellite which is the target. Also the satellite as the target is treated as a point mass in space which is in a circular orbit about the earth. The velocity vector of the satellite is considered tangent to the circular orbit at any point and of a constant magnitude dependent on the height of the circular orbit above the earth. Therefore the magnitude of the target's velocity may be expressed as follows:

$$|\bar{V}_T| = \sqrt{g\,(R_o + h)}$$

where

$$g = g_o \left(\frac{R_o}{R_o + h} \right)^2$$

and

$$g_o = 32.17 \text{ ft/sec}^2$$

$$R_o = 3959 \text{ statute miles}$$

$$h = \text{height above earth's surface}$$

The direction of the target's velocity vector is assumed to be in the plane of the satellite's orbit and normal to the radial position vector of the target from the center of the earth at any instant of time. When the missile's position vector is assumed to be in the plane of the satellite's orbit a two-dimensional intercept problem is defined.

For the two-dimensional solution of the terminal guidance problem the initial conditions of the terminal phase of the intercept can be presented geometrically as shown in Fig. 2-2. Initial conditions are defined as the geometric and physical conditions at the instant of time when the missile seeker first detects the target. If at this instant the seeker also detects an angular velocity of the line-of-sight to the target, missile thrust is cut on - simultaneously with detection.

For time subsequent to the initial conditions, the geometric considerations of the intercept are shown in Fig. 2-3, where the missile thrust, if required, is directed along the local radial. With a uniform gravity field acting the target will follow the circular path; the missile (with some initial velocity, an added thrust program and gravity, all acting along the local radial) will proceed along the local radial to intercept the target at point G. If for the time subsequent to the initial conditions gravity is considered to influence both target and missile equally and hence considered to be absent, the target must follow a path tangent to its gravity-influenced orbit at its initial location. The missile will follow the local radial under the influence of its initial velocity and an added thrust program to intercept the target at point I.

Interception, of course, is only possible in both situations if the missile thrust program is properly devised. Consider that on initial seeker pickup an angular velocity of the line-of-sight will cause a simultaneous application of missile thrust of a constant magnitude directed along the local radial. This thrust is to act until the seeker detects zero angular velocity of the line-of-sight, at which time thrust is simultaneously cut off. For the gravity-free situation at the time of thrust

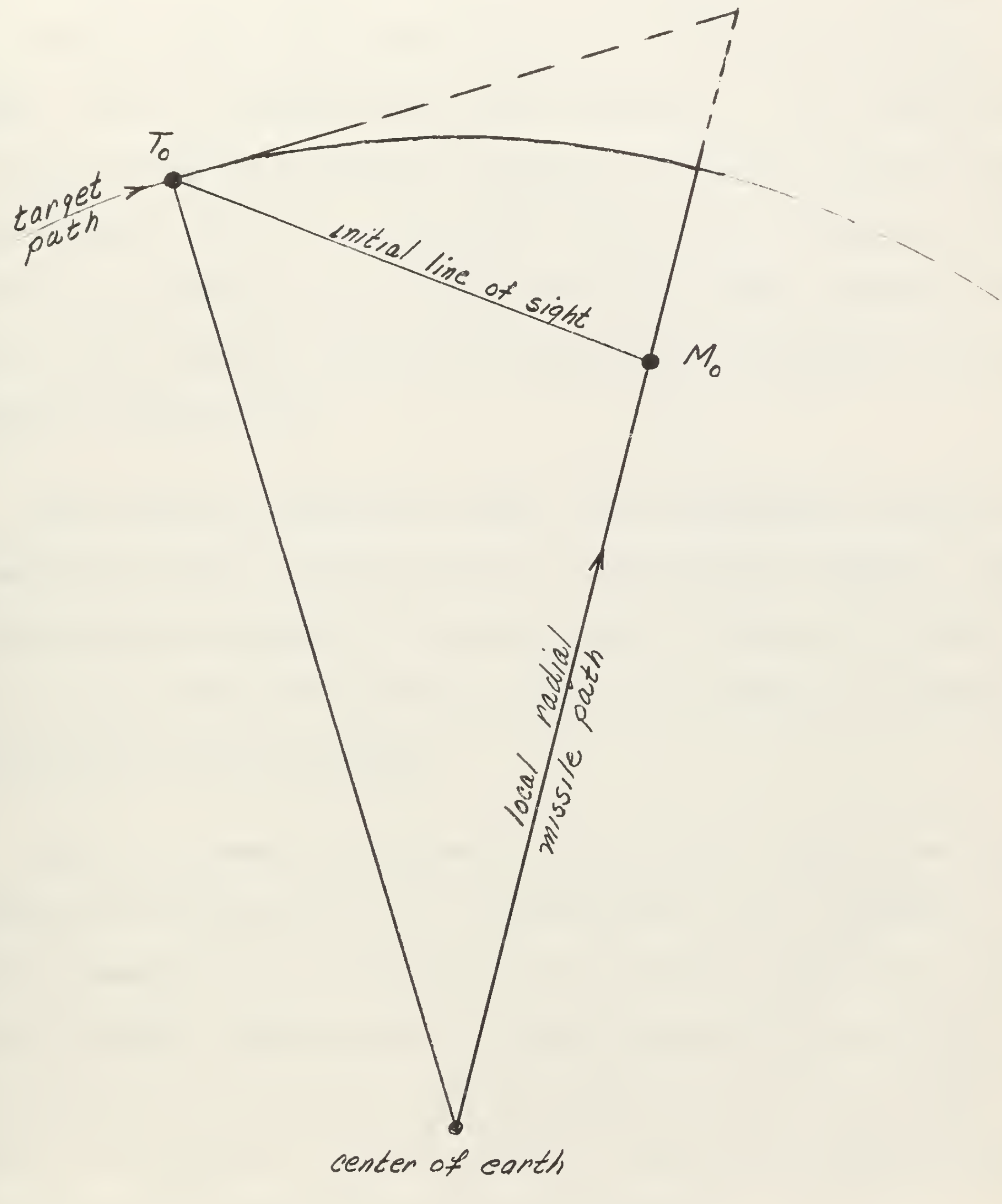


Fig. 2-2 Geometric Features of Initial Conditions

cutoff the missile and target both have constant velocities with the angular velocity of the line-of-sight zero, a constant bearing intercept must follow. For the gravity situation and the same thrust program there will develop a very small angular velocity of the line-of-sight after thrust cutoff. This is discussed more fully in Chapter 6. It is shown there that if thrust is applied at initial detection of an angular velocity of the line-of-sight and thrust cutoff simultaneously with zero angular velocity of the line-of-sight the miss distance is of very small magnitude. This miss distance approaches zero as the thrusting time approaches the total terminal intercept time.

The intercept equations are based on the gravity-free geometric considerations of Fig. 2-3 and developed in the Derivation Summaries. The three-dimensional aspects of the problem will be treated in a later section, as will some feasibility conclusions based on the presently discussed two-dimensional intercept equations.

Derivation Summary 2-2 presents the geometrical features and the geometrical relationships of the terminal phase of the interception. Derivation Summary 2-3 is a development of the missile equations for a gravity-free environment. Equations for the missile position, velocity, and acceleration during thrust as well as missile position and velocity after thrust cutoff are developed. These equations are later used to develop angular velocity of the line-of-sight and present range.

Derivation Summary 2-4 develops the equation for the angular velocity of the line-of-sight. The reference frame used in the development is an earth-centered inertial frame such that the equation for the angular

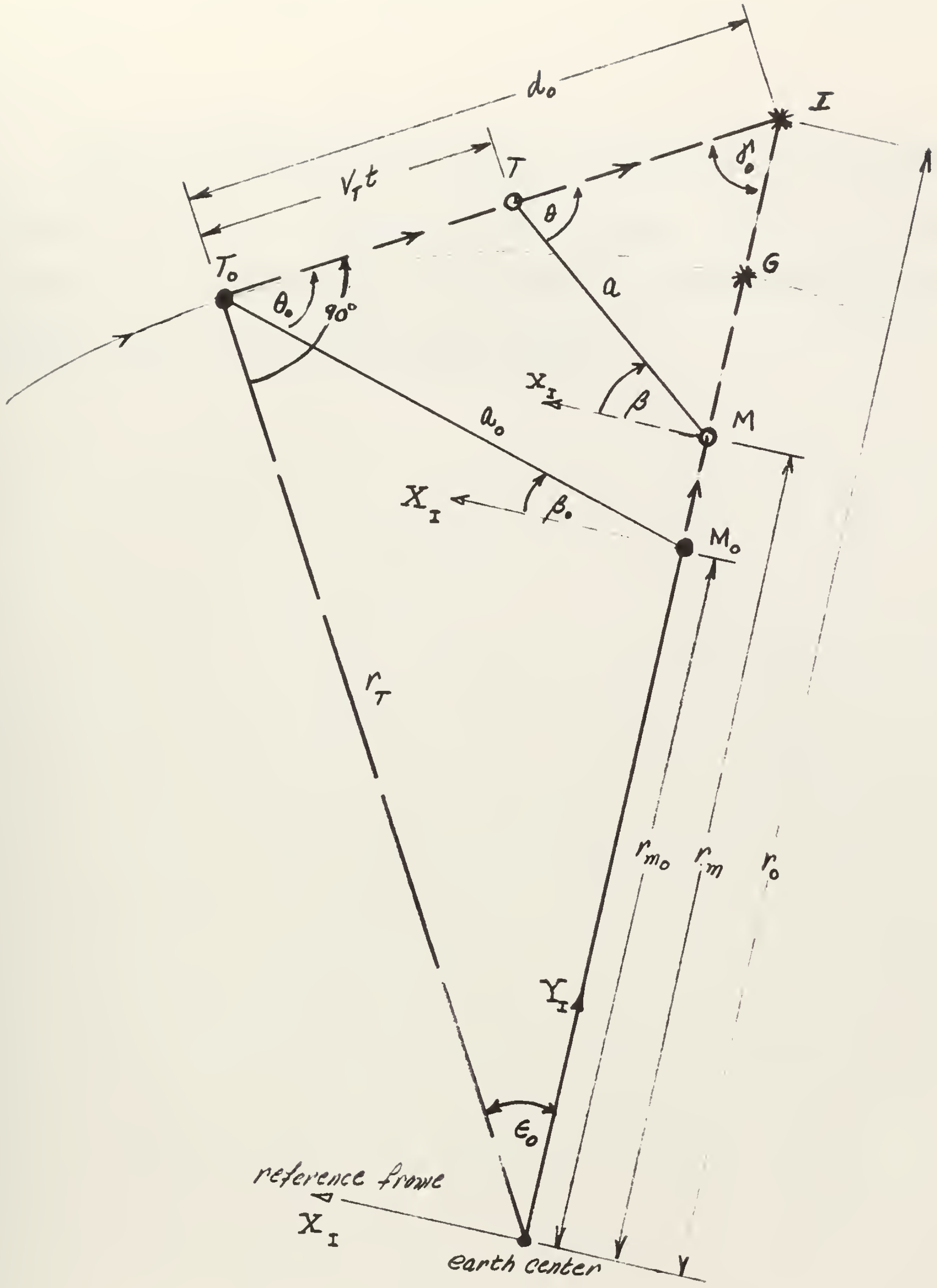


Fig. 2-3 Geometric Features of the Intercept

velocity of the present line-of-sight from Derivation Summary 2-1 is applicable.

Derivation Summary 2-5 develops the equation for the present range based on the rate of change of present range. The rate of change of present range is the vector sum of the components of target motion and missile motion resolved along the line-of-sight.

Derivation Summary 2-2

Geometrical Features

The initial conditions for the terminal guidance phase of the interception determine two sides and the included angle of the fire control triangle. The third side is the line-of-sight between the missile and target and a function of target and missile motion (see Fig. 2-4).

The geometrical aspects of this triangle can be expressed in terms of the initial condition constants and the time dependent variables.

a. Initial Conditions

The initial range, a_0; the initial angle the line-of-sight makes with the reference frame, β_0; and the height of the target's circular orbit from the center of the earth, r_T, completely specify the initial conditions. All other geometrical initial value constants can be expressed in terms of these three.

$$\frac{a_0}{\sin\delta_0'} = \frac{d_0}{\sin(90-\beta_0)} = \frac{r_0 - r_{m_0}}{\sin\theta_0} \quad (1)$$

$$\sin\epsilon_0 = \sin(90-\delta_0') = \cos\delta_0' = \frac{d_0}{r_0} = \frac{a_0\cos\beta_0}{r_0\sin\delta_0'} \quad (2)$$

$$\cos\delta_0' = \frac{d_0}{r_0} = \frac{a_0\cos\beta_0}{r_0\sin\delta_0'} = \frac{a_0\cos\beta_0}{r_0\cos\epsilon_0} = \frac{a_0\cos\beta_0}{r_T} \quad (3)$$

From (3) it can be shown that,

$$\sin\delta_0' = \frac{\sqrt{r_T^2 - a_0^2\cos^2\beta_0}}{r_T} \quad (4)$$

from (2) and (3)

$$r_0 = \frac{r_T d_0}{a_0\cos\beta_0} = \frac{r_T}{\sin\delta_0'} = \frac{r_T^2}{\sqrt{r_T^2 - a_0^2\cos^2\beta_0}} \quad (5)$$

(Page 1 of 2)

from (1) and (3)

$$d_0 = \frac{a_0 \cos\beta_0}{\sin\delta_0'} = \frac{a_0 r_T \cos\beta_0}{\sqrt{r_T^2 - a_0^2\cos^2\beta_0}}$$

b. Time Dependent Variables

At some general time, t, after initial seeker pickup the following geometrical relationships are established by reference to Fig. 2-4. From triangle MIT,

$$\frac{a}{\sin\delta_0'} = \frac{d_0 - V_T t}{\sin(90-\beta)}$$

$$\cos\beta = \frac{(d_0 - V_T t)\sin\delta_0'}{a} \qquad (7)$$

$$\frac{a}{\sin\delta_0'} = \frac{(r_0 - r_m)}{\sin\theta}$$

$$\sin\theta = \frac{(r_0 - r_m)\sin\delta_0'}{a} \qquad (8)$$

$$\delta_0' + \theta + (90-\beta) = 180$$

$$\theta = 90 - (\delta_0' - \beta)$$

$$\cos\theta = \sin(\delta_0' - \beta) = \sin\delta_0'\cos\beta - \cos\delta_0'\sin\beta \qquad (9)$$

If a line is constructed from point T perpendicular to the line MI then MI will be divided into two parts as shown in Fig. 2-5. From the right triangle MPT formed by this construction, β can be expressed as a function of the initial value constants, the present range, and time.

$$\sin\beta = \frac{(r_0 - r_m) - V_T(t_h - t)\cos\delta_0'}{a} \qquad (10)$$

where $t_h = \dfrac{d_0}{V_T}$

time required for the target to travel from initial pickup to the impact point.

(Page 2 of 2)

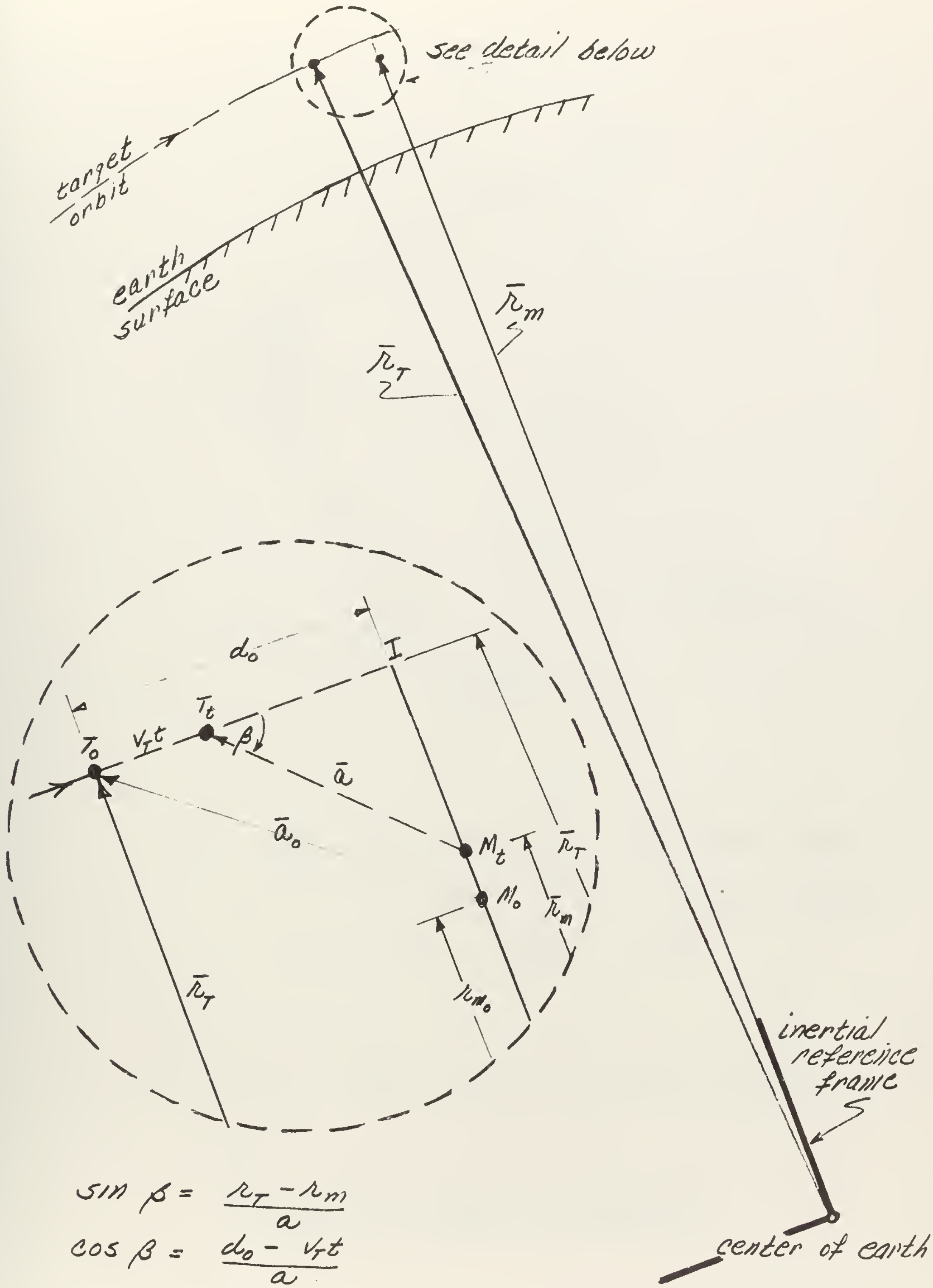
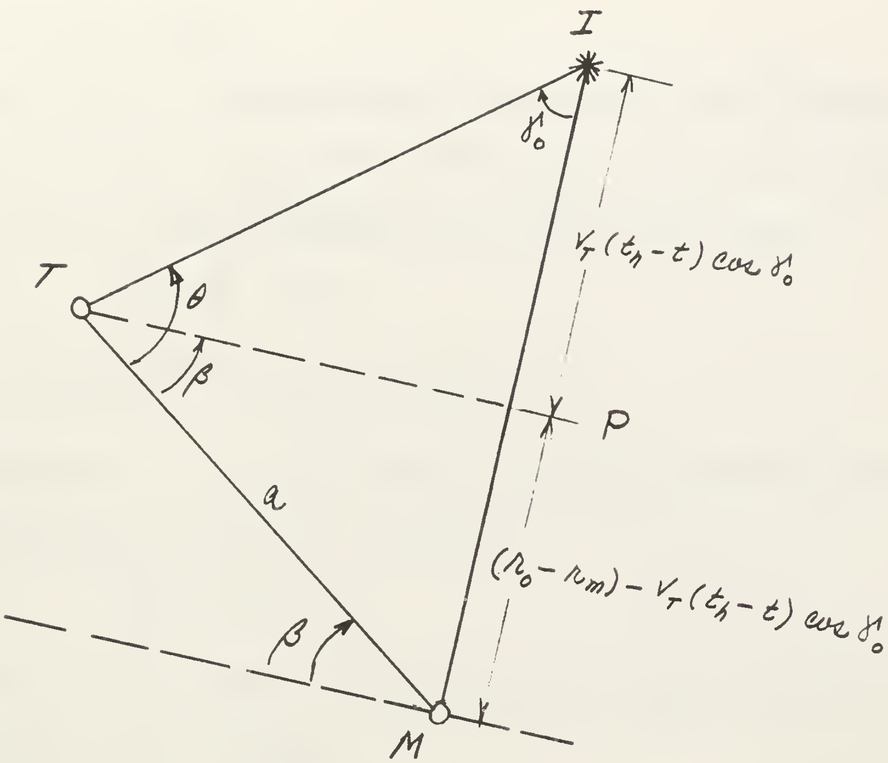


Fig. 2-4 Geometric Features of Terminal Guidance Phase



$t_h = \dfrac{d_o}{V_T}$ time required for the target to travel from initial pick up to the impact point

$$\sin\beta = \frac{(r_o - r_m) - V_T(t_h - t)\cos\gamma_0}{a}$$

Fig. 2-5 Expression for sin β as a Function of Geometric Parameters

Derivation Summary 2-3

Missile Equations

a. During Thrust

Assume that the acceleration of the missile in the reference frame is at every instant equal to the thrust force divided by the mass of the missile, gravity being neglected; that is for a point mass:

$$\ddot{r}_m = \frac{F}{m} = F' \qquad (1)$$

where F' is thrust force per unit mass.

Integration of the above equation yields the missile velocity at any time during which thrust is acting.

$$V_m = F't + V_{m_o}$$

Integration of this equation yields the missile position at any time during which thrust is acting.

$$r_m = r_{m_o} + V_{m_o} t + \frac{F't^2}{2}$$

or

$$(r_m - r_{m_o}) = V_{m_o} t + \frac{F't^2}{2} \qquad (2)$$

b. After Thrust Cutoff

Assume that the acceleration of the missile in the reference frame is at every instant equal to zero after thrust cutoff. The missile equation then becomes:

$$\ddot{r}_m = 0 \qquad (3)$$

$$V_m = F't_f + V_{m_o} \qquad (4)$$

$$r_m = r_{m_f} + V_{m_f} (t - t_f) \qquad (5)$$

(Page 1 of 2)

where t_f = the time at which thrust is cut off

t = any time after initial seeker pickup of the target and thrust cuton, which are considered to occur simultaneously

r_{m_f} = missile location at the time of thrust cutoff

V_{m_f} = missile velocity at the time of thrust cutoff (note that V_{m_f}, a constant, will define the missile velocity at any time, t, greater than t_f.

(Page 2 of 2)

Derivation Summary 2-4

Angular Velocity of the Line-of-Sight

From Derivation Summary 2-1, the angular velocity of the line-of-sight of the target from the missile may be expressed as:

$$W_{LS} = \frac{V_T \sin\theta}{a} - \frac{V_m \sin(90-\beta)}{a}$$

$$= \frac{1}{a}\left[V_T \sin\theta - V_m \cos\beta\right]$$

From equation (7) and equation (8) of Derivation Summary 2-2,

$$W_{LS} = \frac{1}{a}\left[V_T \frac{(r_0 - r_m)\sin\gamma_0}{a} - V_m \frac{(d_0 - V_T t)\sin\gamma_0}{a}\right]$$

$$= \frac{\sin\gamma_0}{a^2}\left[V_T r_0 - V_T r_m - V_m(d_0 - V_T t)\right]$$

where r_m and V_m are to be expressed as functions of initial conditions and time, depending on the thrust condition of the missile as shown in Derivation Summary 2-3.

a. Before Thrust Cutoff

$$W_{LS} = \frac{\sin\gamma_0}{a^2}\left[V_T r_0 - V_T\left(\frac{F' t^2}{2} + V_{m_0} t + r_{m_0}\right) - \left(F' t + V_{m_0}\right)\left(d_0 - V_T t\right)\right]$$

$$= \frac{\sin\gamma_0}{a^2}\left[V_T(r_0 - r_{m_0}) - d_0(V_{m_0} + F' t) + \frac{1}{2} F' V_T t^2\right]$$

where $\sin\gamma_0$, r_0 and d_0 can be expressed in terms of the initial values a_0, β_0, and r_T given by equations (4), (5), and (6) of Derivation Summary 2-2.

r_{m_0} = initial missile position from center of the earth along the missile path

V_{m_0} = initial missile velocity at seeker pickup

(Page 1 of 2)

b. After Thrust Cutoff

$$W_{LS} = \frac{\sin \delta_0}{a^2}\left\{ V_T r_0 - V_T\left[r_{mf} + V_{mf}(t - t_f)\right] - (F' t_f + V_{m_0})(d_0 - V_T t)\right\}$$

$$= \frac{\sin \delta_0}{a^2}\left\{ V_T (r_0 - r_{m_0}) - d_0 (V_{m_0} + F' t_f) + \frac{F'}{2} V_T t_f^2 \right\}$$

where t_f = time during which missile thrust acts.

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Derivation Summary 2-5

Present Range

The rate of change of present range is the vector sum of the components of target motion and missile motion resolved along the line-of-sight. From Fig. 2-4 the rate of change of range can be expressed as:

$$\frac{da}{dt} = -\left[V_T \cos\theta + V_m \cos(90-\beta) \right]$$

$$= -\left[V_T \cos\theta + V_m \sin\beta \right]$$

From equation (9) of Derivation Summary 2-2

$$\frac{da}{dt} = -\left[V_T (\sin\delta_0' \cos\beta - \cos\delta_0' \sin\beta) + V_m \sin\beta \right]$$

$$= -\left[V_T \sin\delta_0' \cos\beta + (V_m - V_T \cos\delta_0') \sin\beta \right]$$

From equation (7) and equation (10) of Derivation Summary 2-2

$$\frac{da}{dt} = -\left[\frac{V_T \sin\delta_0' (d_0 - V_T t) \sin\delta_0'}{a} + \frac{(V_m - V_T \cos\delta_0')\{(r_0 - r_m) - V_T(t_h - t)\cos\delta_0'\}}{a} \right]$$

$$a\,da = -\left[V_T d_0 \sin^2\delta_0' - V_T^2 t \sin^2\delta_0' + (V_m - V_T \cos\delta_0')(r_0 - r_m - V_T t_h \cos\delta_0' + V_T t \cos\delta_0' \right] dt$$

with $\quad t_h = \dfrac{d_0}{V_T}$

$$\int_{a_0}^{a} a\,da = -\int_0^t \left[V_T d_0 \sin^2\delta_0' - V_T^2 t \sin^2\delta_0' + (V_m - V_T \cos\delta_0')(r_0 - r_m - d_0 \cos\delta_0' + V_T t \cos\delta_0') \right] dt$$

where r_m and V_m are to be expressed as functions of initial conditions and time, depending on the thrust condition of the missile as shown in Derivation Summary 2-3.

a. Before Thrust Cutoff

With $\quad r_m = F' \dfrac{t^2}{2} + V_{m_0} t + r_{m_0}$

and $\quad V_m = F't + V_{m_0}$

(Page 1 of 2)

substituted into the equation above and both sides of the equation integrated, the result expresses the present range at any time $t < t_f$, where t_f is the time of missile thrust cutoff.

$$a^2 = a_0^2 - \Big[d_0 t \left\{ 2V_T - \cos\gamma_0 (2V_{m_0} + F't) \right\} + (r_0 - r_{m_0})(F't^2 + 2V_{m_0} t - 2V_T t \cos\gamma_0) - t^2 \Big\{ V_T^2 - V_T \cos\gamma_0 (2V_{m_0} + F't) + V_{m_0} t \left(F' + \tfrac{2}{3} V_{m_0}\right) - \left(\tfrac{F't}{2}\right)^2 \Big\} \Big]$$

b. **After Thrust Cutoff**

With $r_m = \dfrac{F't_f^2}{2} + V_{m_0} t_f + r_{m_0} + (F't_f + V_{m_0})(t - t_f)$

and $V_m = F't_f + V_{m_0}$

substituted into the basic equation and both sides of the equation integrated, the result expresses the present range at any time $t > t_f$.

$$a^2 = a_0^2 - \Big[2d_0 (t - t_f)(V_T - V_{m_0}\cos\gamma_0 - F't_f \cos\gamma_0) + (t^2 - t_f^2)(2V_{m_0} V_T \cos\gamma_0 - V_{m_0}^2 - V_T^2) + 2(t - t_f)(r_0 - r_{m_0})(F't_f + V_{m_0} - V_T \cos\gamma_0) + F'^2 t_f (2t_f^2 + 2t_f - 2t - t^2 - t_f t) + F'V_{m_0} t_f (t_f^2 + 2t_f t - 2t^2) + F'V_T t_f (2t^2 \cos\gamma_0 - t_f^2 \cos\gamma_0 - t_f t \cos\gamma_0) \Big]$$

(Page 2 of 2)

Table 2-1

Summary of Intercept Equations for a

Gravity-Free Intercept Space and Point Masses

During Thrust

$\ddot{r}_m$	F'
V_m	$F't + V_{m_0}$
r_m	$r_{m_0} + V_{m_0}t + \frac{F't^2}{2}$
W_{LS}	$\frac{\sin\gamma_0}{a^2}\left[V_T r_0 - V_T\left(\frac{F't^2}{2} + V_{m_0}t + r_{m_0}\right) - (F't + V_{m_0})(d_0 - V_T t)\right]$
a^2	$a_0^2 - \Big[d_0 t\{2V_T - \cos\gamma_0(2V_{m_0} + F't)\} + (r_0 - r_{m_0})$ $(F't^2 + 2V_{m_0}t - 2V_T t\cos\gamma_0) - t^2\{V_T^2 - V_T\cos\gamma_0$ $(2V_{m_0} + F't) + V_{m_0}t(F' + \frac{2}{3}V_{m_0}) - (\frac{F't}{2})^2\}\Big]$

After Thrust Cutoff

$\ddot{r}_m$	0 t_f = the time at which thrust is cut off
V_m	$F't_f + V_{m_0}$
r_m	$r_{m_f} + V_{m_f}(t - t_f)$
W_{LS}	$\frac{\sin\gamma_0}{a^2}\left[V_T(r_0 - r_{m_0}) - d_0(V_{m_0} + F't_f) + \frac{F'}{2}V_T t_f^2\right]$
a^2	$a_0^2 - \Big[2d_0(t - t_f)(V_T - V_{m_0}\cos\gamma_0 - F't_f\cos\gamma_0) + (t^2 - t_f^2)(2V_{m_0}V_T$ $\cos\gamma_0 - V_{m_0}^2 - V_T^2) + 2(t - t_f)(r_0 - r_{m_0})(F't_f +$ $V_{m_0} - V_T\cos\gamma_0) + F'^2 t_f(2t_f^2 + 2t_f - 2t - t^2 - t_f t)$ $+ F'V_{m_0}t_f(t_f^2 + 2t_f t - 2t^2) + F'V_T t_f(2t^2\cos\gamma_0$ $- t_f^2\cos\gamma_0 - t_f t\cos\gamma_0)\Big]$

C. Terminal Guidance Technique

As was indicated in the introduction and basic assumptions, terminal guidance must be used to make an anti-satellite weapons system effective. The anti-satellite missile will carry terminal guidance in the form of an infra-red target seeker and a thrust unit. As the missile approaches the peak of its trajectory the seeker will be turned on. Detection of the target by the seeker will undoubtedly be immediately followed by the sensing of an angular velocity of the line-of-sight between target and missile. This angular velocity signal will in turn actuate the thrust unit. Action between missile and target is as described by the equations of the preceeding section.

In order to produce a collision between the weapons package and the satellite, thrust must be applied to the weapon package in such a direction as to produce a velocity vector which will cause the angular velocity of the line-of-sight between the weapon package and the satellite to become nulled. Assume that the satellite or target - represented as a point mass in space with a given velocity vector - and the weapon package are being acted upon by a uniform gravity field. When the angular velocity of the line-of-sight has been nulled, thrust is cut off, target and missile are under the influence of the same gravity field, and a constant bearing intercept of the target by the weapon package will occur.

An intercept of a target will result, using a minimum of fuel, by applying a constant thrust in a fixed direction. For this study of terminal guidance, the minimum level of thrust required to effect an intercept is a fixed quantity determined by the initial acquisition slant range and the initial angular velocity of the line-of-sight.

The thrust direction for a minimum fuel intercept has been determined by Ref. 2 to be 90° from the initial line-of-sight. In vector form the expression for the direction of the applied thrust may be expressed as below:

$$\bar{F}/F = \left[\bar{1}_{\left[W_{(I)LS}\right]_o} \times \bar{1}_{\left[LS\right]_o} \right]$$

For the two-dimenionsal derivations developed in Derivation Summaries 2-3, 2-4, and 2-5, the direction of the applied thrust is $\bar{1}_{\bar{r}_m}$. In order for the thrust direction to be that for a minimum fuel intercept, the thrust direction should be $\left[\bar{1}_{\bar{W}_{I(LS)_o}} \times (\overline{LS})_o \right]$. Because the angle β, of the two-dimensional case, which is developed by the initial angular velocity of the line-of-sight, is a small angle, $\bar{1}_{\bar{r}_m}$ differs very little from $\left[\bar{1}_{\left[\bar{W}_{I(LS)}\right]_o} \times (\overline{LS})_o \right]$. Therefore, the thrust direction chosen for the derivations in this study differs very slightly from the optimum thrust direction for a minimum fuel intercept provided initial target acquisition is made at the extreme range of the weapon package seeker device capability.

In this study for terminal guidance it is envisaged that actual thrust direction for the weapon package will be generated by four thrust nozzles which are placed in perfect quadrature about the weapon package. Two of the nozzles will produce a thrust direction along the position vector for the weapon package. The other pair of nozzles will produce a thrust direction normal to the weapon package position vector, in a plane normal to the orbital plane of the satellite. For a situation which would require a thrust direction other than along or normal to the weapon package position vector, combinations of the four nozzles will be used to produce the required thrust direction.

The following chapters will consider some of the aspects of these concepts in further detail.

CHAPTER 3

PRELIMINARY STUDY OF FEASIBILITY

The two-dimensional equations of the terminal phase of the interception can be used to some advantage in a preliminary study of the feasibility of such a system. With any given set of initial conditions the equations will establish the minimum thrust required to reduce the angular velocity of the line-of-sight to zero within the total intercept time available (i.e., the time required for the target to move from its initial pickup point in orbit to the point where the local radial intersects the orbit). Under the assumptions of the derivation of the equations the missile is treated as a point mass with a constant acceleration during the thrust phase of the intercept.

With a given set of initial conditions, Fig. 3-1, Fig. 3-2, and Fig. 3-3 show the angular velocity of the line-of-sight as a function of time for various thrust levels. The initial conditions are based on a target predicted to be in a 100 mile orbit above the surface of the earth. The missile programmed flight is along the local radial to hook at the 100 mile orbit altitude at exactly the time the target passes the local radial. If the initial conditions were such that the target had no deviation from either its predicted path or its time passage of the local radial and the missile had no deviation from its programmed flight then a seeker pickup

at 100 miles range would detect no angular velocity of the line-of-sight. No terminal thrust would be initiated and the intercept would be completed without need for terminal guidance. If however, the initial conditions were such that a combination of the deviation of the target from its predicted path and time and the deviation of the missile from its programmed flight resulted in some error, terminal guidance would be required. This total deviation can be expressed in terms of displacement deviation alone if timing error is considered zero. Hence a 2 mile displacement error can be considered as pure displacement error with no time error or as a fictitious displacement error made up of displacement and time errors.

For the condition when target path is 2 miles above predicted and the missile has no deviation from programmed flight, seeker pickup at 100 miles range detects W_{LS} = 0.965 mr/sec, and terminal thrust is initiated until $W_{LS} = 0$. The result of these initial conditions is plotted in Fig. 3-1 for thrust levels of 100, 200, and 300 ft/sec^2 (lbs. of thrust/lbs. of mass). In a similar manner Fig. 3-2 and Fig. 3-3 are plotted for initial conditions with 4 and 6 mile total deviation from predicted and programmed conditions.

The feasibility of a system using this type of terminal guidance is defined in the assumptions and results of the time to null the angular velocity of the line-of-sight. The minimum required thrust in all initial conditions considered is 200 lbs. thrust per lb. of mass. The minimum required thrust for initial conditions up to 4 miles of deviation from predicted is 100 lbs. thrust per lb. of mass. For initial conditions of 2 miles deviation from predicted somewhat less than 100 lbs. thrust per lb. of mass is required.

Chapter 4 continues with some feasibility studies of a hypothetical missile introducing the concepts of a decreasing missile mass during thrust.

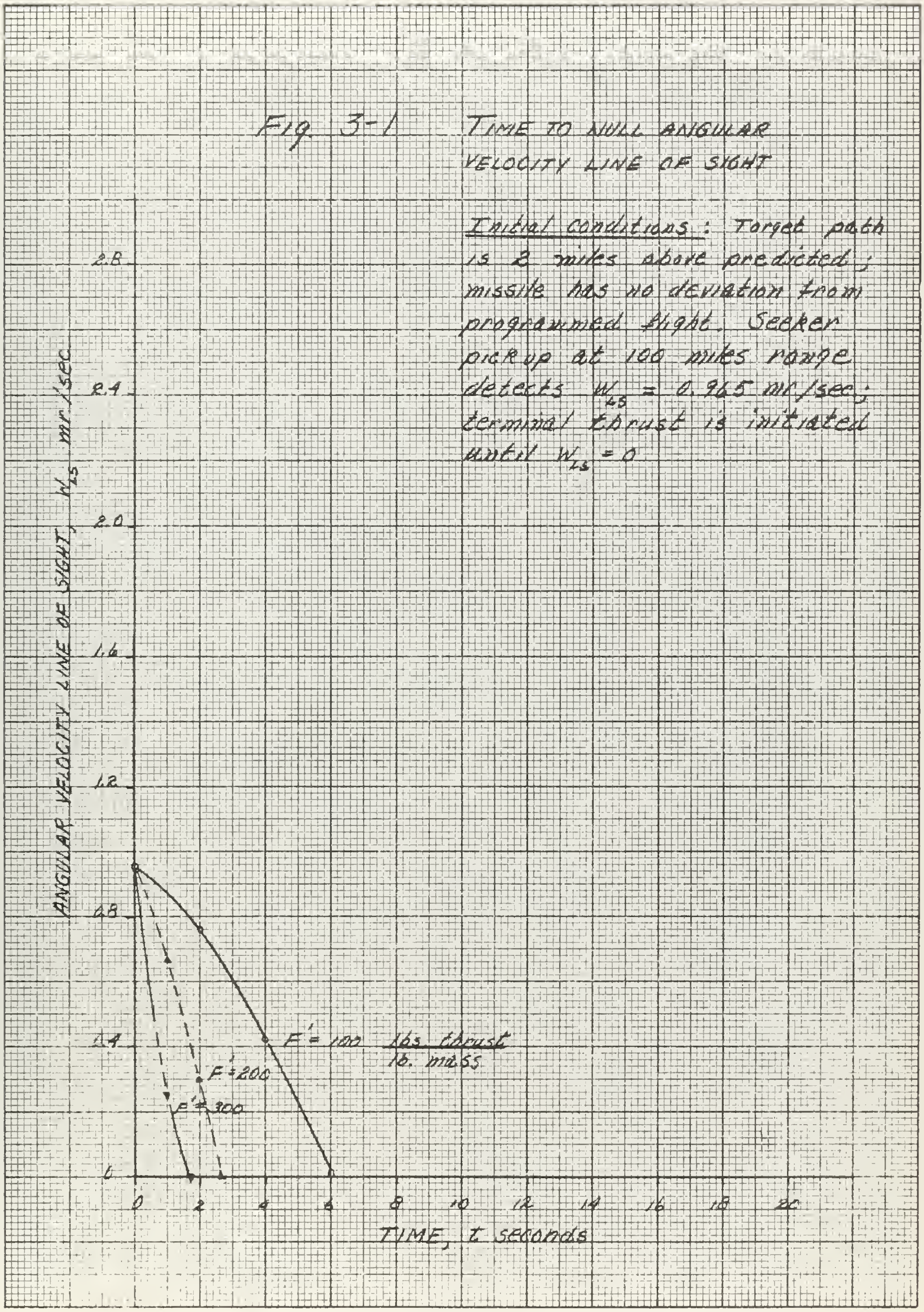
Fig. 3-1 TIME TO NULL ANGULAR VELOCITY LINE OF SIGHT
Initial conditions: Target path is 2 miles above predicted; missile has no deviation from programmed flight. Seeker pickup at 100 miles range detects W_{LS} = 0.965 mr/sec; terminal thrust is initiated until W_{LS} = 0
ANGULAR VELOCITY LINE OF SIGHT, W_{LS} mr/sec
2.8
2.4
2.0
1.6
1.2
0.8
0.4
0
F' = 100 lbs thrust / lb. mass
F' = 200
F' = 300
0
2
4
6
8
10
12
14
16
18
20
TIME, t seconds

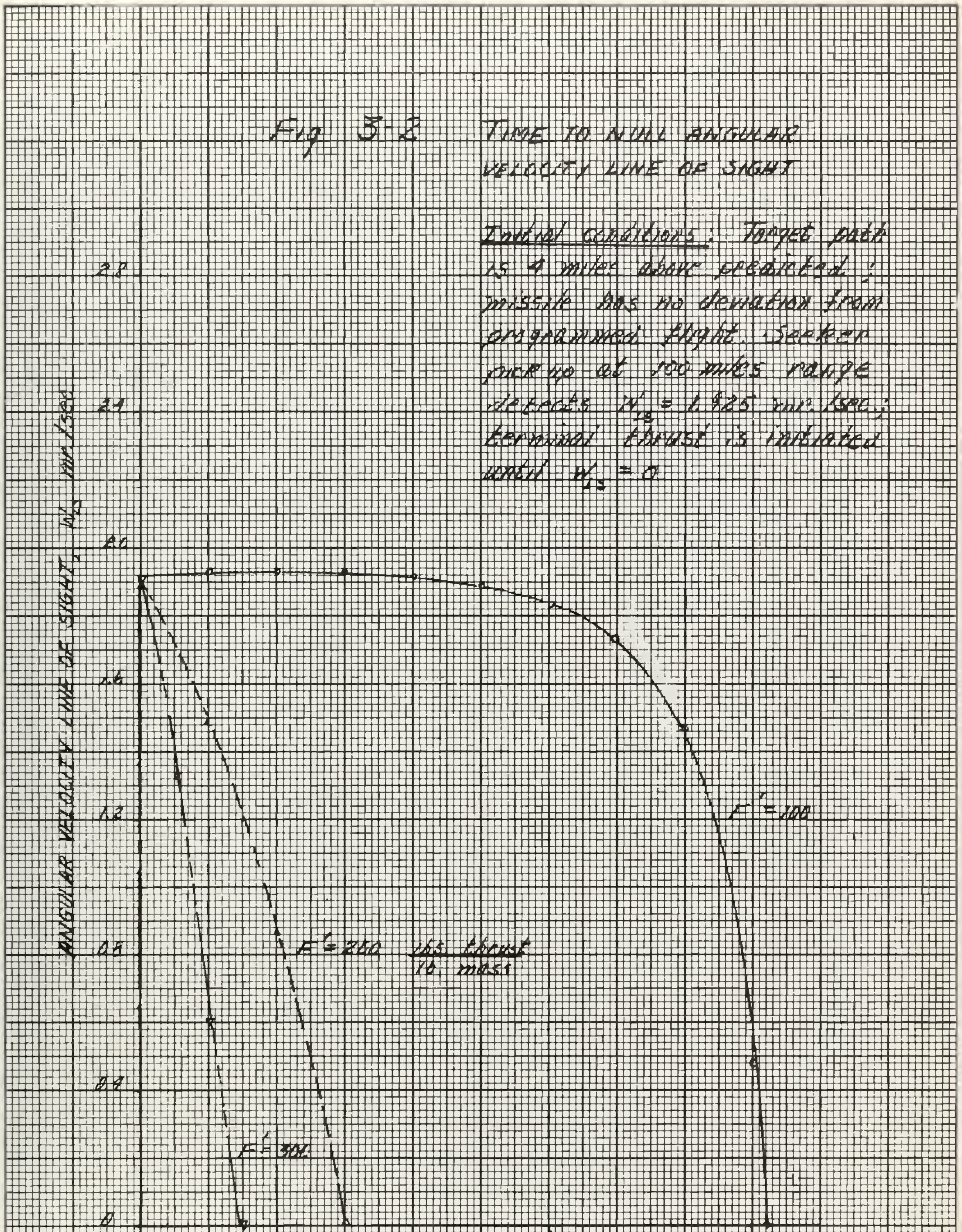
Fig 3-2 TIME TO NULL ANGULAR VELOCITY LINE OF SIGHT
Initial conditions: Target path is 4 miles above predicted; missile has no deviation from programmed flight. Seeker pick up at 100 miles range detects W_{LS} = 1.925 mr/sec; terminal thrust is initiated until W_{LS} = 0
ANGULAR VELOCITY LINE OF SIGHT, W_{LS} mr/sec
2.8
2.4
2.0
1.6
1.2
0.8
0.4
0
F' = 100
F' = 200 lbs. thrust / lb. mass
F' = 300
0
2
4
6
8
10
12
14
16
18
20
TIME, t seconds

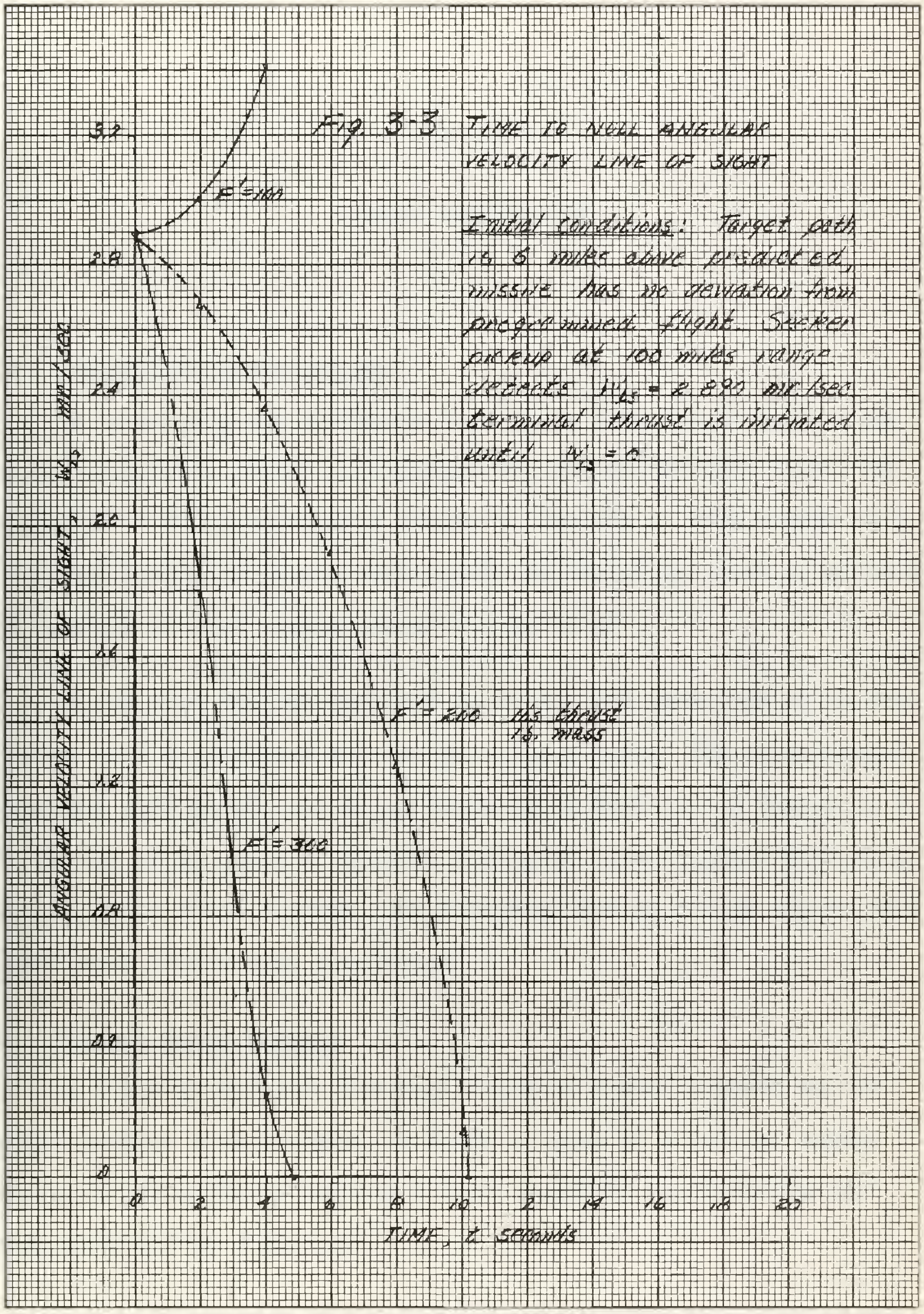

Fig. 3-3 TIME TO NULL ANGULAR VELOCITY LINE OF SIGHT
Initial conditions: Target path is 6 miles above predicted, missile has no deviation from programmed flight. Seeker pickup at 100 miles range detects w'_{LS} = 2.880 mr/sec terminal thrust is initiated until w'_{LS} = 0
F' = 100
F' = 200 lbs thrust / lb. mass
F' = 300
3.2
2.8
2.4
2.0
1.6
1.2
0.8
0.4
0
0
2
4
6
8
10
12
14
16
18
20
TIME, t, seconds
ANGULAR VELOCITY LINE OF SIGHT, w'_{LS}, mr/sec

CHAPTER 4

THRUST REQUIREMENTS

A. Preliminary Considerations

An interesting and productive feasibility study can be made of the thrust required to effect an intercept if a missile is fired within the framework of the assumptions of Chapter 2, Section A. This chapter will show that insofar as thrust requirements are concerned, terminal guidance is feasible if other system assumptions can be met.

As was done with the intercept equations, the essentials of the thrust problem will be developed on a two-dimensional basis. The three-dimensional intercept will be effected as was explained in Section C of Chapter 2, i.e., by the use of two pairs of mutually orthogonal thrust nozzles exhausting from a single thrust chamber. The following section of this chapter will again touch briefly upon this design.

B. Typical Weapon Package

In order to arrive at numerical values of thrust required for the weapon package it is necessary to think in terms of a definite weapon. Purely for illustrative purposes a basic weapon of the following configuration can be visualized:

Unit		Weight, lbs.
Motor, dry		400
Rated Sea Level Thrust, 2710 lbs.		
Sea Level Exhaust Velocity, 8700 ft/sec		
Propellant Consumption, 10 lb/sec		
$k = 1.27$		
Propellant Loading Factor, $\delta = 0.2735$		
Warhead		150
Propellants		235
Seeker		75
	TOTAL	860

Brief justification of these values can be made. The Snarler aircraft rocket engine developed in England had a dry weight of approximately 275 lbs. and a thrust of 2630 lbs. Engineering advances since its development should permit the increased thrust rating; the additional weight should be adequate to account for the four nozzle configurations mentioned when discussing the three-dimensional intercepts. The engineering details of such a power plant will not be considered since it is felt that such a discussion is not fully in keeping with the objective of the paper. If it is felt that such a power plant is not a practical scheme, four prepackaged liquid rocket motors can be used in its place. The latter propulsion scheme for the weapons package would increase its overall weight but would be an engineering fact at this time. The method to be used to arrive at feasibility conclusions can be applied to either power plant by merely substituting the correct numbers. Since the single thrust chamber power plant results in a decided savings in weight, thrust studies will be made assuming that it is the power plant aboard the missile package.

Fuel consumption of the Snarler engine was 10 lbs. of propellant per

second. The flight time to intercept, after detection, was used in conjunction with this fact as a basis for propellant weight. Seeker weight represents an estimate for a complete infra-red seeker unit, including power supply, capable of 100 mile detection ranges. The warhead is a government-furnished high explosive of a design and size suitable for its intended mission.

C. Shots Short of Satellite Trajectory

Equation 2 of Derivation Summary 2-3 can be solved for F' to give the thrust required, per pound of mass, to null the angular velocity of the line-of-sight between target and missile at any time. If thrust is found by this method for various sets of initial conditions it should be noted that the results will be based on a constant acceleration.

The actual problem however, with a constant thrust motor during the brief terminal guidance phase, would result in increasing accelerations as fuel is burned and the weight of the missile package decreases. This in turn means that higher errors could be rectified by the terminal guidance system for the same thrust rating than is indicated by Equation 2 of Derivation Summary 2-3. In view of this consideration the following equation, developed in Chapter 12 of Ref. 1 of the Bibliography can be used to find relevant miss distance and thrust information:

$$\Delta h = V_o t + I_s g \frac{t_B}{\mathcal{S}}\left[\left(1-\frac{\mathcal{S}}{t_B}t\right)\ln\left(1-\frac{\mathcal{S}}{t_B}t\right)+\frac{\mathcal{S}}{t_B}t\right]$$

where Δh is equivalent to the quantity $r_m - r_{m_o}$ of Fig. 2-3

V_o is missile velocity at time of target detection

I_s is specific impulse, a variable with altitude

$\mathcal{S}$ is propellant loading factor

(The component of Δh due to gravity has been omitted in this application of the equation. This is in keeping with the principles discussed in Chapter 2.)

With reference to Fig. 4-1, assume that the target is detected at T_o when the missile is at M_o. If the shot in progress at this time is a perfect shot, intercept will be made at point K, the missile reaching the peak of its trajectory at the same time the satellite passes through this peak point. In such a case no terminal guidance is required. However, if the satellite is instead on course A, Fig. 4-1, one mile above the predicted trajectory, it will pass one mile above point K when the missile peaks if nothing is done to correct this error. In accordance with the technique described in Section C of Chapter 2, when missile position, M_{o_A}, and target position, T_{o_A}, are such that the slant range is 100 miles, detection will occur and some value of thrust will be actuated to effect the intercept. By assuming a series of initial conditions for various probable errors all of the terms of the above equation except I_s can be fixed. The equation can be solved for I_s, and since the propellant flow rate has been fictitiously established, necessary thrust can be found. If this thrust is less than the available rated thrust of the engine for the particular altitude an intercept is possible. This difference in thrust values means that the angular velocity of the line-of-sight would be nulled more quickly and thrust would then be cut off in accordance with principles already outlined. The limiting error for which an intercept is possible is established when the thrust determined exactly equals the thrust available. It should be noted that thrust from the engine which is rated at 2710 pounds at sea level increases as altitude increases.

Fig. 4-2 is the result of taking various initial conditions at 100, 500, and 1000 mile altitudes in which a miss would occur due to the missile's falling short of the satellite track. Initially it was assumed that the

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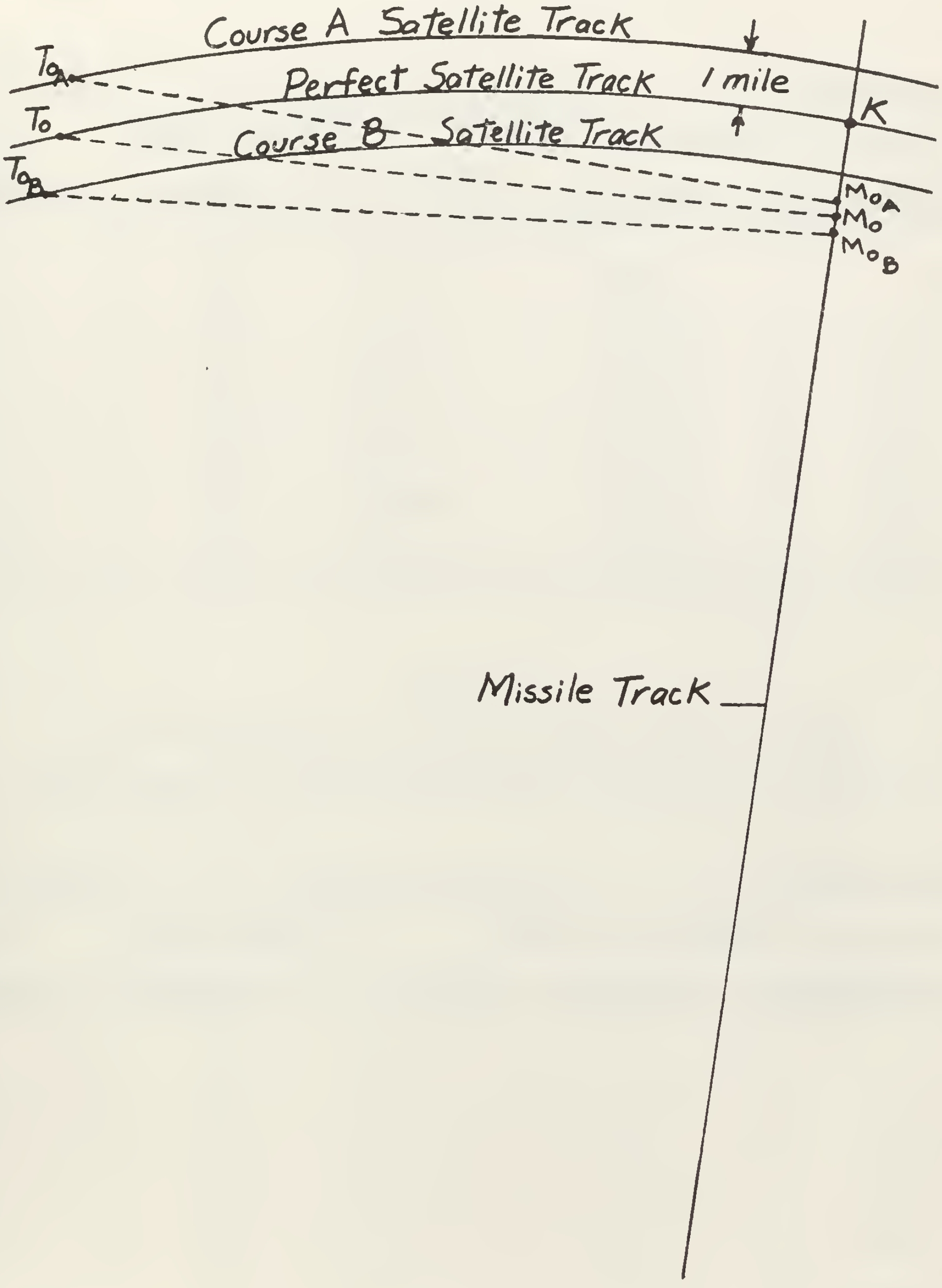

Fig 4-1 Representation of Displacement Errors

Table 4-1

Initial Conditions and Thrust Requirements For Shots Falling Short of Perfect Trajectory, Satellite at 100 Miles

Unguided Miss Distance (miles)	Initial Conditions			Thrust Required at Intercept Altitude (pounds)
	Pickup Range (miles)	V_{mo} ft/sec	Time to Intercept (seconds)	
0	100	634.3	20.75	0
1.0	100	634.0	20.742	723
2.0	100	633.7	20.734	1450
3.0	100	633.4	20.726	2158
4.0	100	633.1	20.718	2852
5.0	100	632.8	20.710	3568
5.29	100	632.7	20.708	3800

Table 4-2

Initial Conditions and Thrust Requirements For Shots Falling Short of Perfect Trajectory, Satellite at 500 Miles

Unguided Miss Distance (miles)	Initial Conditions			Thrust Required at Intercept Altitude (pounds)
	Pickup Range (miles)	V_{mo} ft/sec	Time to Intercept (seconds)	
0	100	550.1	21.83	0
1.0	100	549.7	21.815	745
2.0	100	549.3	21.80	1484
3.0	100	548.9	21.785	2244
4.0	100	548.5	21.77	2990
5.0	100	548.1	21.755	3740
5.59	100	547.9	21.747	4180

Table 4-3

Initial Conditions and Thrust Requirements For Shots Falling Short of Perfect Trajectory, Satellite at 1000 Miles

Unguided Miss Distance (Miles)	Initial Conditions			Thrust Required for Intercept (pounds)
	Pickup Range (Miles)	V_{mo} ft/sec	Time to Intercept (seconds)	
0	100	470.2	23.05	0
1.0	100	470.0	23.04	767
2.0	100	469.8	23.03	1532
3.0	100	469.6	23.02	2301
4.0	100	469.4	23.01	3070
5.0	100	469.2	23.00	3845
6.0	100	469.0	22.99	4620
6.01	100	469.0	22.99	4630

missile shot was on schedule for a perfect intercept but the satellite was either one or two miles above its predicted track, this error going to the maximum allowable satellite error of the basic assumptions. The total intercept error was then extended to the maximum allowable error, based on the thrust available, assuming that these additional errors were due to the missile's falling short of the perfect intercept point.

This chapter studies only those errors which are due to physical departures from the perfect satellite or missile courses. It is also possible to have errors in time in which either or both the satellite or missile are on perfect tracks but are following time schedules which would not result in an intercept. However cases such as these, in which for instance the satellite will reach the impact point earlier than predicted, are less critical than errors due to trajectory or orbital position errors. This is shown in Chapter 6. Since thrust requirements will be

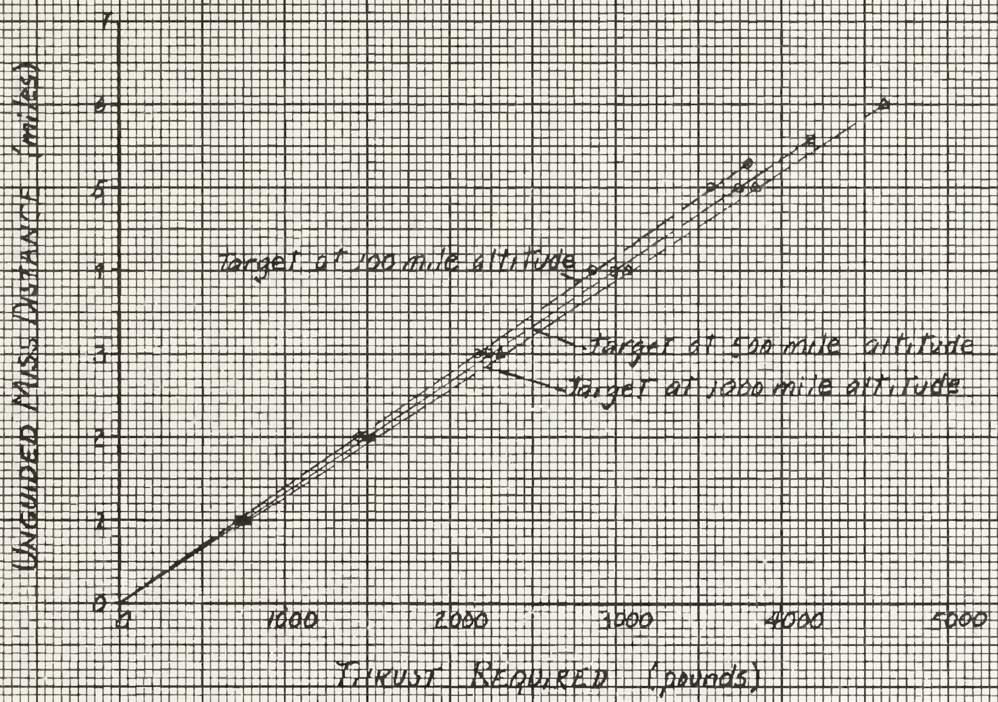

FIG. 4-2 THRUST REQUIREMENTS FOR SHOTS FALLING SHORT OF PERFECT TRAJECTORY

governed by the more dominant or more critical errors this chapter analyzes only those errors due to trajectory or orbital displacement inaccuracies. Pertinent data and results are tabulated in Tables 4-1, 4-2, and 4-3. Both Fig. 4-2 and the tables are carried only to the value of thrust available at the altitude being analyzed.

Fig. 4-2 immediately shows that for the spectrum of targets from 100 to 1000 miles in altitude errors at low altitude are more detrimental to overall success. A shot which would result in a 5.5 mile short if no terminal guidance were used could not be salvaged if the target were at 100 miles altitude and the assumed propulsion system aboard. The same propulsion system would be capable of effecting terminal guidance against a 500 or 1000 mile altitude target with a 5.5 mile miss distance.

The fact that less error is correctable at lower altitudes is offset however by two factors:

1. Satellite tracking is more accurate at the lower altitudes, therefore errors due to the determination of the satellite's track should be smaller.
2. Errors due to missile misalignment, thrust variation, etc., do not have the opportunity to be compounded to the same extent in a 100 mile shot as they would in a 1000 mile shot.

Both of these factors will result in a smaller total error at lower altitudes. Therefore, while the system has a lesser capability at lower altitudes, the error situations at lower altitudes should be less severe.

In order to obtain Fig. 4-2, errors were assigned to missile to fully exploit the capability of the assumed power plant. If the cumulative

effect of missile errors can also be given an upper bound as was done with satellite track determination, Fig. 4-2 may be used to find thrust required of a power plant. For instance, if the missile could definitely be placed within one half mile of a desired point in space, an error no greater than two and one half miles could be assigned to the entire shot. From Fig. 4-2, a thrust of 1800 pounds would effect a kill against a 100 mile target. The design problem could then be worked in reverse to find sea level thrust, exhaust velocity, and other power plant parameters.

D. Shots Over Satellite Trajectory

Situations could also be presented to the weapon package in which it would peak over the precomputed satellite track. Such a situation would develop if the satellite track were actually below the perfect or precomputed track, or if missile performance caused the package to peak at a higher altitude than predicted. Course B of Fig. 4-1 depicts such a situation with a one mile overshoot. In this case the weapon package is at M_{o_B} and the satellite at T_{o_B} when detection occurs. The same techniques used in the previous section can then be used to find thrust requirements to effect terminal guidance. Tables 4-4, 4-5, and 4-6, and Fig. 4-3 present the results of assuming the same series of unguided miss distances for shots over the perfect satellite track that were used for shots short of the perfect satellite track.

A comparison of the data of Tables 4-1 to 4-6 yields the maximum error capability of the assumed weapon package. As can be seen from the data, at all altitudes shots that fall short of the trajectory have lower salvageable errors than shots that go over the trajectory. Since there is no way of determining whether a shot will be short or over when it is

Table 4-4

Initial Conditions and Thrust Requirements For Shots Going Over Perfect Trajectory, Satellite at 100 Miles

Unguided Miss Distance (miles)	Initial Conditions			Thrust Required at Intercept Altitude (pounds)
	Pickup Range (miles)	V_{mo} ft/sec	Time to Intercept (seconds)	
0	100	634.3	20.75	0
1	100	634.6	20.758	556
2	100	634.9	20.766	1112
3	100	635.2	20.774	1669
4	100	635.5	20.782	2223
5	100	635.8	20.790	2772
6	100	636.1	20.798	3333
6.83	100	636.4	20.805	3800

Table 4-5

Initial Conditions and Thrust Requirements For Shots Going Over Perfect Trajectory, Satellite at 500 Miles

Unguided Miss Distance (miles)	Pickup Range (miles)	V_{mo} ft/sec	Time to Intercept (seconds)	Thrust Required at Intercept Altitude (pounds)
0	100	550.1	21.83	0
1	100	550.5	21.845	640
2	100	550.9	21.86	1279
3	100	551.2	21.875	1917
4	100	551.6	21.89	2553
5	100	552.0	21.905	3189
6	100	552.4	21.920	3834
6.54	100	552.6	21.928	4180

Table 4-6

Initial Conditions and Thrust Requirements For Shots Going Over Perfect Trajectory, Satellite at 1000 Miles

Unguided Miss Distance (miles)	Initial Conditions			Thrust Required at Intercept Altitude (pounds)
	Pickup Range (miles)	V_{mo} ft/sec	Time to Intercept (seconds)	
0	100	470.2	23.05	0
1	100	470.4	23.06	744
2	100	470.6	23.07	1492
3	100	470.8	23.08	2237
4	100	471.0	23.09	2989
5	100	471.2	23.10	3728
6	100	471.4	23.11	4488
6.19	100	471.4	23.112	4630

launched, the shots short of the trajectory determine the limit of the terminal guidance capability for the assumed weapon package. For instance, if it were known that the total error against a 100 mile target could be as high as 6 miles, the assumed weapon package would not be capable of correcting this error if it occurred due to a short shot. To use the assumed weapon package errors against 100 mile targets cannot exceed 5.29 miles. Similar limits of 5.59 miles and 6.01 miles can be set for the 500 and 1000 mile altitude targets respectively.

A comparison of the thrust requirements reveals that less thrust is needed to effect terminal guidance if the shot is over the perfect satellite track. This is because gravity augments the power plant thrust in effecting closure to the target. This latter statement is true in spite of the fact that the missile's upward velocity must first be stopped and then the closure to the target effected.

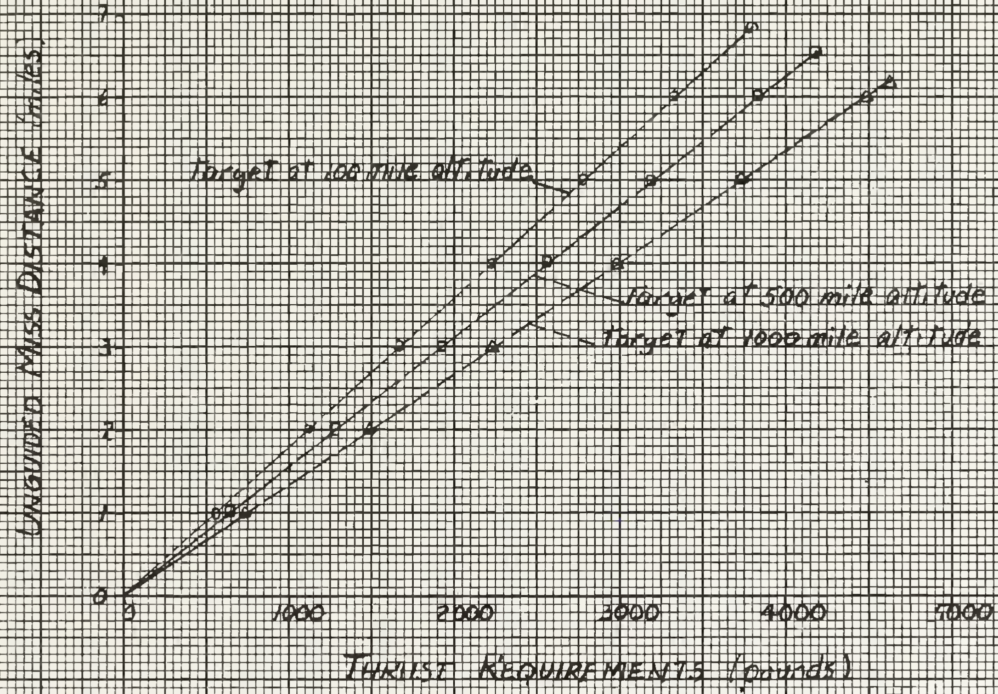

FIG 4-3 THRUST REQUIREMENTS FOR SHOTS GOING OVER PERFECT TRAJECTORY

E. <u>Time Lead Technique</u>

A missile shot which peaked the weapon package so that it is exactly on the satellite course at detection time would require only a holding thrust to counter gravity. A guidance thrust of 692.0 pounds would hold the simulated weapon package at a satellite altitude of 100 miles until the satellite arrived. This technique of leading the satellite in time instead of attempting to make the exact intercept of the basic assumptions would result in lower thrust requirements for the weapon package. It can be shown, for example, that at the same 100 mile target altitude with an overshoot of one mile, 135 pounds of thrust is sufficient to effect terminal guidance. In this case the weapon package was at the peak of its trajectory with zero velocity when target detection occurred. This thrust, plus the gravity effect, will then effect terminal guidance.

Tables 4-7 and 4-8 present thrust requirements for an overall system based on such a time lead technique. The same increments of error from the aiming point as were used previously were used for these tables. The aiming point for these cases however is a lead point such that the weapon package peaks on the satellite track with zero velocity when the satellite is at the detection range of the seeker.

As would be expected due to the influence of gravity, overshoots with this technique are more salvageable than undershoots. Since the lower miss distance determines the system capability however, the system has a maximum error capability of 5.57 miles at this altitude.

While it is evident that thrust requirements are lower it should be noted that in the example used the maximum error capability was increased

Table 4-7

Initial Conditions and Thrust Requirements For Shots

Short of Perfect Trajectory, Time Lead Technique, Satellite at 100 Miles

Unguided Miss Distance (miles)	Initial Conditions			Thrust Required at Intercept Altitude (pounds)
	Pickup Range (miles)	V_{mo} ft/sec	Time to Intercept (seconds)	
0	100	0	20.75	692.0
1	100	0	20.742	1248.0
2	100	0	20.734	1805.0
3	100	0	20.726	2360.0
4	100	0	20.718	2920.0
5	100	0	20.710	3480.0
5.57	100	0	20.705	3800.0

Table 4-8

Initial Conditions and Thrust Requirements For Shots

Going Over Perfect Trajectory, Time Lead Technique, Satellite at 100 Miles

Unguided Miss Distance (miles)	Initial Conditions			Thrust Required at Intercept Altitude (pounds)
	Pickup Range (miles)	V_{mo} ft/sec	Time to Intercept (seconds)	
0	100	0	20.75	692.0
1	100	0	20.742	135.0
2	100	0	20.734	420.0
3	100	0	20.726	975.0
4	100	0	20.718	1532.0
5	100	0	20.710	2082.0
6	100	0	20.702	2638.0
7	100	0	20.694	3182.0
8	100	0	20.686	3740.0
8.1	100	0	20.685	3800.0

by only 0.28 miles. It should also be kept in mind that disadvantages can be inherent in the time lead technique. One such disadvantage is the fact that predicting the necessary time lead can introduce further errors into an overall system which already has many error possibilities. If satellite prediction and missile performance prove to give errors of only two to three miles, the thrust requirements for the system originally assumed are not unreasonable. Use of the original system to avoid time lead errors would therefore be justified.

F. Conclusions

The thrust levels discussed in this chapter are attainable as is evidenced by achievements in the aircraft rocket engine field. Firing errors to be corrected are felt to be above those predicted by authorative sources. In view of these considerations and the results of previous sections of this chapter, terminal guidance for an anti-satellite missile is feasible insofar as thrust requirements are concerned.

The numerical results of the chapter apply only to the simulated weapon package. However the method of arriving at these results is valid for any weapon package and unless a unit were used which departed radically from the one simulated, thrust requirements would be of comparable magnitude.

CHAPTER 5

EFFECT OF SEEKER PERFORMANCE

A. Purpose of Analysis

The chapter immediately preceding developed the thrust requirement and allowable errors for a simulated missile package. Changes in the capability of this weapon package will probably occur if the performance of its components were changed. This chapter will show the major effects of an increase in the seeker's detection capability.

This change in seeker range will primarily change the amount of thrust required to achieve satisfactory terminal guidance for various error conditions. It is recognized that a compounding of effects occurs. For instance, if as a result of increasing seeker range, reduced thrust levels may be used to achieve terminal guidance, the lower thrust would permit a weight savings on the motor. A further slight reduction in thrust is then possible because of the decreased weight. However, the effect of increasing seeker capability will not be pursued to this extent. Only the major effect of a change in seeker performance will be analyzed.

B. Considerations and Results of Analysis

The analysis was made using the simulated missile package of Chapter 4. Seeker detection capability was changed however from 100 to 125 miles. One

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

immediate effect of this increase in detection range is that longer flight times after detection are involved. An investigation of the three altitudes previously analyzed shows that the longest flight time occurs with a 1000 mile target altitude. Missile thrust for a maximum period of 28.8 seconds may be necessary in this case. Since the same missile package will be used over the entire altitude spectrum from 100 to 1000 miles, fuel must be provided for this flight time. Accordingly, in the simulated package, fuel is increased to 290 pounds; it follows that $\mathcal{f}$ becomes 0.317.

The same techniques used to find thrust requirements in Chapter 4 can then be used. Shots short of a 100 mile satellite trajectory were arbitrarily chosen as a basis for comparing the thrust required of a weapon package with either of the seekers aboard. Initial missile velocities and time to intercept are higher because of the greater detection range. Table 5-1 presents the results of this analysis. For comparative purposes, Table 4-1 presents thrust requirements for a missile with a seeker detection capability of only 100 miles.

As is immediately evident, the weapon package can effect terminal guidance for shots which would normally have missed by 8.02 miles. This represents an increase of 51% in terminal guidance capability for the same motor. The possibility or method of increasing the seeker detection capability by 25% is not within the scope of this paper; the effect of such an increase in detection capability is to vastly improve terminal guidance capabilities.

If it were possible to put limits on probable errors another interesting comparison is available from the data. A 2 mile maximum error would

Table 5-1

Initial Conditions and Thrust Requirements For Shots Short of Perfect Trajectory, Satellite at 100 Miles

Unguided Miss Distance (miles)	Initial Conditions			Thrust Required at Intercept Altitude (pounds)
	Pickup range (miles)	V_{mo} ft/sec	Time to Intercept (seconds)	
0	125	793.6	25.97	0
1	125	793.3	25.962	473
2	125	793.0	25.954	946
3	125	792.7	25.946	1420
4	125	792.4	25.938	1895
5	125	792.1	25.930	2370
6	125	791.8	25.922	2840
7	125	791.5	25.914	3310
8	125	791.2	25.906	3790
8.02	125	791.2	25.9058	3800

mean that 1450 pounds of thrust is the maximum required if the seeker has a 100 mile detection capability. With a 125 mile detection range a motor capability of only 946 pounds of thrust is required for the same maximum error. A 34.7% reduction in thrust can thus be effected if seeker detection range is increased by 25%. The remarks in the initial paragraphs of this chapter pertaining to compounding of effects would be applicable in this case.

C. <u>Conclusions</u>

Only one set of initial conditions has been used for comparative purposes. Other initial conditions will yield similar information. Design changes in the missile package, or entire weapon system, due to an increase in seeker range, could be based on the results of a comprehensive analysis

which would yield the limiting conditions on which to base a design change.

It is evident that an increase in seeker detection range increases the terminal guidance capability. For the simulated weapon package used in this paper, an increase of 51% in terminal guidance capability results from a 25% increase in detection range.

CHAPTER 6

EFFECT OF SYSTEM ERRORS

A. Actual Target Path Versus Simplified Concept

In Derivation Summaries 2-2 and 2-3, the intercept of the satellite is to be accomplished by a constant bearing intercept. The mathematical development of the intercept equations uses a fictitious line-of-sight with an angular velocity which will remain zero for the remaining time of the problem after it has been nulled. Fig. 6-1 is a diagram, which is not to scale, for the two-dimensional intercept problem with the satellite or target actually travelling along a curved path and the interceptor missile travelling along a radial. It is necessary to know whether or not there will be a miss-distance or error produced in the actual intercept case by assuming that once the angular velocity of the line-of-sight has been nulled it will remain nulled.

Referred to an inertial coordinate system, the angular velocity of the line-of-sight between the target and the weapon package of the anti-satellite missile may be expressed as below.

$$\bar{\omega}_{LS} = \frac{\bar{a} \times \bar{V}_T}{|\bar{a}|^2} - \frac{\bar{a} \times \bar{V}_m}{|\bar{a}|^2}$$

The above vector expression for the angular velocity of the line-of-sight will produce the following magnitude expression for the two-dimensional conditions shown in Fig. 6-1.

$$\omega_{LS} = \frac{1}{a}\left[V_T \sin\alpha - V_m \sin\gamma\right] \qquad (1)$$

Therefore in order for ω_{LS} to become zero a value of V_m must be obtained which is equal to $V_T \frac{\sin\alpha}{\sin\gamma}$.

In this paper it is proposed that the guidance thrust will be applied when the acquisition device acquires the target and develops an angular velocity of the line-of-sight. The thrust will then be terminated at such time when the angular velocity of the line-of-sight between the target and weapon package has been nulled.

In Fig. 6-1 it is assumed that the angular velocity of the line-of-sight has been nulled with the target and weapon package in the positions shown. Therefore for the conditions in Fig. 6-1

$$V_{m_o} = V_T \frac{\sin\alpha}{\sin\gamma} = V_T \frac{\sin(\theta+\beta)}{\cos\beta}$$

where $\sin(\theta+\beta) = \sin\theta\cos\beta + \sin\beta\cos\theta$

$$\therefore V_{m_o} = V_T \sin\theta + V_T \tan\beta\cos\theta \qquad (2)$$

In order to determine the distance which the weapon package must travel in the time, t, remaining to complete the intercept, the magnitude of $\bar{r}_m$ is obtained using the law of sines and it is then subtracted from r_T.

$$\frac{r_T}{\sin(90+\beta)} = \frac{r_m}{\sin(90-\alpha)} = \frac{r_m}{\cos(\theta+\beta)}$$

$$r_m = r_T \frac{\cos\alpha}{\cos\beta} = \frac{r_T}{\cos\beta}\left[\cos\beta\cos\theta - \sin\beta\sin\theta\right]$$

$$\therefore r_m = r_T\left[\cos\theta - \tan\beta\sin\theta\right]$$

$$\therefore r_T - r_m = r_T\left[1 - \cos\theta + \tan\beta\sin\theta\right] \qquad (3)$$

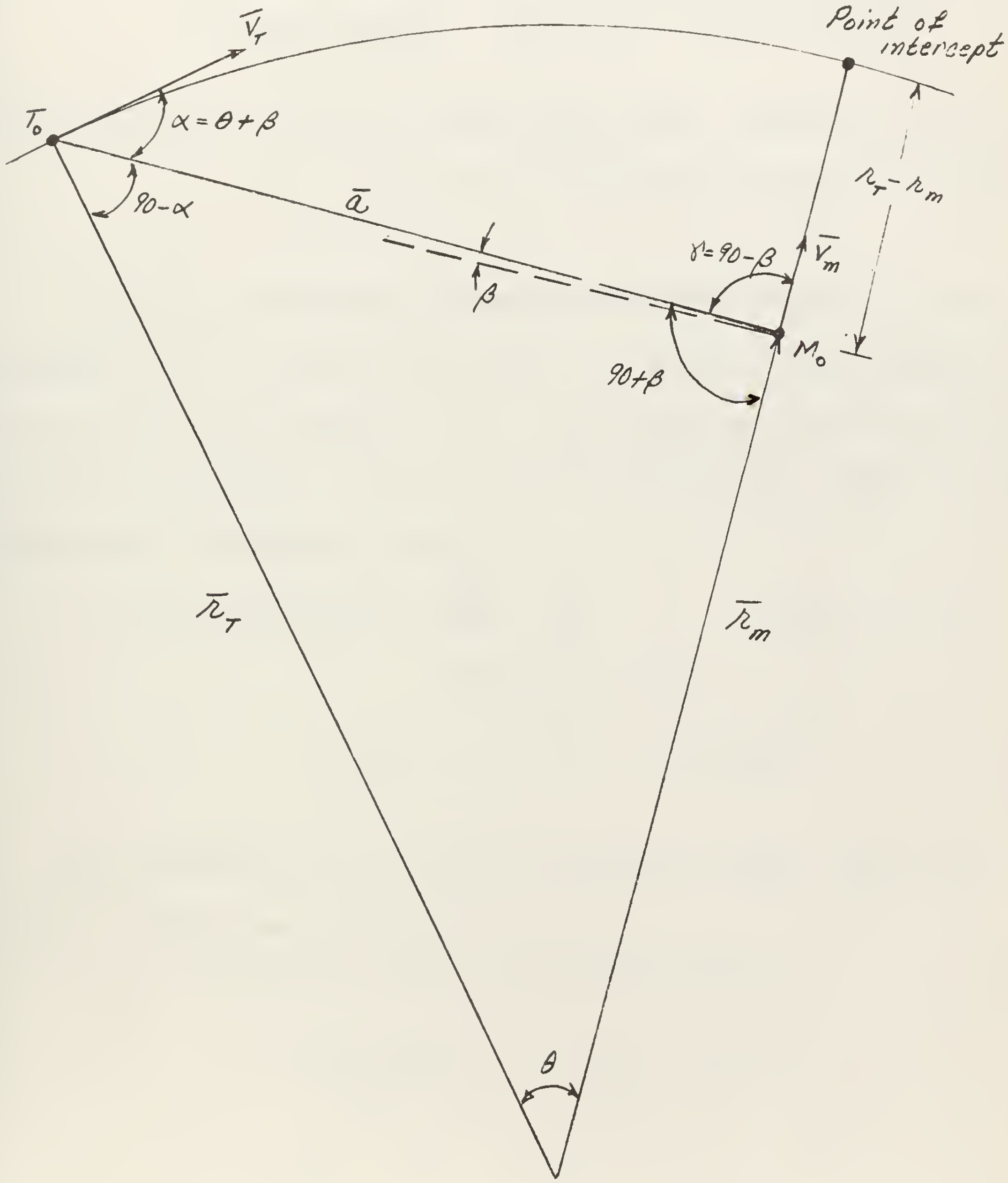

Fig. 6-1 Geometric Features of the Actual Intercept

The error or miss-distance which could result if the guidance thrust is not actuated after once nulling the angular velocity of the line-of-sight will become the difference between the actual distance, $r_T - r_m$, and the distance actually covered in the time remaining by the weapon package.

$$\therefore \epsilon \triangleq \text{Miss Distance} = [r_T - r_m] - [V_{m_0} t - \tfrac{1}{2} g t^2] \qquad (4)$$

Using equations (2) and (3), miss-distance may be expressed as below.

$$\epsilon = r_T[1 - \cos\theta + \tan\beta \sin\theta] - [V_T(\sin\theta + \tan\beta\cos\theta)t - \tfrac{1}{2} g t^2] \qquad (5)$$

In order to obtain any information from equation (5), the small angle assumption will be used for the angle θ. The small angle assumption may also be used with the angle β, but it will only ease the actual calculations and not actually bring out any vital information. Therefore the angle θ will be defined as below:

For a circular orbit $\frac{d\theta}{dt} = \frac{V_T}{r_T}$ where $V_T^2 = g r_T$

$$\therefore \theta \triangleq \frac{V_T}{r_T} t = \sin\theta$$

$$\cos\theta = \sqrt{1 - \sin^2\theta} \doteq 1 - \tfrac{1}{2}\left(\frac{V_T}{r_T} t\right)^2$$

Taking equation (5) and setting the angle β equal to zero, the following results are obtained for ϵ.

$$\epsilon = r_T[1 - \cos\theta] - [V_T t \sin\theta - \tfrac{1}{2} g t^2] \qquad (6)$$

$$= r_T\left[\tfrac{1}{2}\left(\frac{V_T}{r_T} t\right)^2\right] - \left[V_T\left(\frac{V_T}{r_T}\right)t^2 - \tfrac{1}{2} g t^2\right]$$

$$\therefore \epsilon = \tfrac{1}{2} g t^2 - g t^2 + \tfrac{1}{2} g t^2 = 0$$

or $$\epsilon = 0 = r_T[1 - \cos\theta] - [V_T t \sin\theta - \tfrac{1}{2} g t^2] \qquad (6a)$$

Therefore it may be seen when equations (6a) and (5) are compared, any miss-distance which will be produced in the actual intercept problem

will be the result of the terms which contain the angle β.

Rewriting equation (5) and omitting the terms that add to zero, the following results are obtained for error, ϵ .

$$\epsilon = r_T \tan\beta \sin\theta - V_T t \tan\beta \cos\theta \tag{7}$$

$$= r_T \tan\beta \frac{V_T}{r_T} t - V_T t \tan\beta \cos\theta$$

$$\therefore \epsilon = V_T t \tan\beta [1 - \cos\theta] \tag{7a}$$

$$\text{or } \epsilon = \frac{1}{2} g t^2 \left[\frac{V_T}{r_T} t\right] \tan\beta \tag{7b}$$

Examining equation (7a) it is apparent that there is a miss-distance produced in the actual, problem, if the guidance thrust is not actuated after the angular velocity of the line-of-sight has been nulled. However the actual magnitude of the error is very small as is shown in the example below.

Assume: $t = 10$ sec.

$\beta = 3° = 0.0524$ rad.

$V_T = 4.8493$ mile/sec

$R_T = 4059$ statute miles

$g = 30.59$ ft/sec^2

$$\therefore \epsilon = \frac{1}{2}\left(\frac{30.59}{5280}\right)(10)^2\left[\frac{4.8493}{4059}(10)\right](0.05241)$$

$= 0.00015254$ miles

$= 0.8054$ ft.

From the above example it may be assumed that unless large values of problem time are remaining when the angular velocity of the line-of-sight is nulled, and unless the angle β, generated by the angular velocity of the line-of-sight, is large, the error produced by assuming that the angular

velocity of the line stays nulled can be ignored.

B. Misalignment of Weapon Package Reference Axes

In this paper it has been assumed that there is an inertial reference frame in the weapon package of the anti-satellite missile. The alignment of the reference frame of the weapon package was discussed in Chapter 2, Section B.

The seeker device of the weapon package will initially be aligned along the axis of the onboard reference system which is normal to the radial position vector of the anti-satellite missile and in the orbital plane of the satellite. When the seeker device acquires the satellite, an angular velocity of the line-of-sight between the missile weapon package and the satellite will be generated as the seeker moves away from its initial position and commences to track the satellites.

Misalignment of the seeker reference axis within the orbital plane of the satellite will cause the magnitude of the initial angular velocity of the line-of-sight between the weapon package and the satellite to be larger or smaller than the ideal case where no misalignment of the reference axis exists. Because in this paper the initial angular velocity of the line-of-sight is used only to actuate the thrust mechanism the magnitude of the initial angular velocity of the line-of-sight has no significance. Only if the misalignment of the seeker's reference axis is greater than the half angle of the seeker's cone of acquisition will there be any reduction of time available to complete the intercept. Any appreciable reduction of available intercept time will invalidate the thrust requirements specified in Chapter 4.

Misalignment of the seeker reference axis normal to the orbital plane and/or misalignment of the other two onboard reference axes, which are used as a reference for guidance thrust direction, will cause the initial thrust direction to be in error. Again this paper has stated that the thrust direction for the terminal guidance phase will be in such a direction as to null the angular velocity of the line-of-sight between the weapon package and the target satellite. Therefore because the seeker device continues to generate an angular velocity of the line-of-sight, a misaligned initial thrust direction will be corrected in order to produce a velocity direction for the weapon package which will null the angular velocity of the line-of-sight.

C. Premature Thrust Termination

In section A of this chapter an investigation was conducted to determine the miss distance which would be produced in the actual intercept problem if, after the angular velocity of the line-of-sight between the anti-satellite missile's weapon package and the satellite was nulled, the guidance thrust was not actuated again. Now using the equations developed in section A of this chapter an investigation will be made of miss distance resulting from termination of the guidance thrust before the angular velocity of the line-of-sight between the weapon package and the satellite is completely nulled and failure of the guidance thrust to be re-applied to the weapon package. Early termination of thrust could result from a failure in the thrust system itself or because the seeker could no longer distinguish an angular velocity of the line-of-sight.

In order to null the angular velocity of the line-of-sight between the weapon package and the satellite, the weapon package must have a

velocity, V_m, equal to $V_T \sin \theta + V_T \tan \beta \cos \theta$ (Equation 2 of section A). For this investigation let the weapon package velocity equal 99% of the value required to null the angular velocity of the line-of-sight.

$$\therefore V_m = 0.99 \quad V_{m_{null}} = 0.99 \, [V_T \sin \theta + V_T \tan \beta \cos \theta] \qquad (8)$$

Using equations (3) and (4) from section A for the miss distance produced and the value of V_m above, the following expression may be written for miss distance, ϵ ,

$$\epsilon = r_T \, [1 - \cos \theta + \tan \beta \sin \theta] - [0.99 \, V_T \, (\sin \theta + \tan \beta \cos \theta)t - \frac{1}{2} \, gt^2] \qquad (9)$$

Taking equation (8) above and setting the angle β equal to zero, the following results are obtained for .

$$\epsilon = r_T \, [1 - \cos \theta] - [0.99 \, V_T t \sin \theta - \frac{1}{2} gt^2] \qquad (10)$$

$$\therefore \epsilon = \frac{1}{2} gt^2 - 0.99 \, gt^2 + \frac{1}{2} gt^2 \cong 0.01 \, gt^2 \qquad (11)$$

Rewriting equation (9) and substituting the value of ϵ from equation (11) for those terms which do not contain the angle β, the following results may be obtained for miss distance.

$$\epsilon = 0.01 \, gt^2 + [r_T \tan \beta \sin \theta - 0.99 \, V_T t \tan \beta \cos \theta] \qquad (12)$$

$$\epsilon = 0.01 \, gt^2 + V_T t \tan \beta \, [1 - 0.99 \cos \theta] \qquad (13)$$

For the example worked in section A, equation (13) gives a value of 31.4 ft. for miss distance. A comparison of equations (7) and (13) shows that the dominant term when the angular velocity of the line-of-sight is not nulled is the difference between the velocity actually acquired by the weapon package and the velocity required of the weapon package in order to

null the angular velocity of the line-of-sight. By keeping the time short between thrust termination and intercept, miss distance produced by failure to null the angular velocity of the line-of-sight may be kept to a negligible value.

Performance equations developed in Chapter 3 indicate that the angular velocity of the line-of-sight decreases as the intercept nears completion. It is also true that the seeker will have some minimum threshold value at which detection of an angular velocity of the line-of-sight will no longer be possible. Because of this dead zone terminal guidance thrust will be terminated prematurely. Equation (13) shows that if the seeker threshold is reached early in the problem, or if thrust is terminated prematurely for any other reason, it is imperative that thrust be re-activated.

D. <u>Satellite or Missile Time Errors</u>

In Chapter 4 thrust requirements are developed for the terminal guidance phase of the satellite intercept problem based on errors which produce physical displacements between the satellite and the weapon package of the anti-satellite missile. In all the calculations of Chapter 4, it was assumed that if no thrust were applied to the weapon package the satellite would pass through the missile's position radial either above or below the weapon package's position when the weapon package reached the peak of its trajectory along the radial. For this look at thrust requirements it will be assumed that if the weapon package were on time a perfect intercept would be accomplished.

For the situation in which the anti-satellite missile is early, the thrust requirement will only be that which is needed to counter the effects

of gravity acting on the weapon package after it has reached the peak of the trajectory.

In the case in which the anti-satellite missile is late, the procedure for determining the thrust requirement applies the same equations utilized in Chapter 4 for determination of thrust requirements. The position of the weapon package of the anti-satellite missile is determined by having the weapon package acquire the satellite from a position along the radial trajectory path which corresponds to the number of seconds that the weapon package is late. Fig. 6-2 shows a plot of thrust requirement for the weapon package versus late time of the weapon package.

Table 6-1 shows the same breakdown that is utilized in the tables of Chapter 4 for the cases when the weapon package is 20 seconds late and 40 seconds late. For the maximum available thrust used in this paper, it can be shown that at an altitude of 100 miles the weapon package could be approximately 48 seconds late and still complete an intercept if there were no error caused by physical displacement between theoretical and actual locations of the satellite and weapon package.

Table 6-1

Initial Conditions and Thrust Requirements For a Late Weapon Package

Seconds Late	Target Altitude (miles)	Initial Conditions			Thrust Required (pounds)
		Pickup Range (miles)	V_{m_o} (ft/sec)	Time to Intercept (seconds)	
20	100	100	1245.00	20.60	654.6
20	500	100	1047.00	21.55	619.0
20	1000	100	873.70	22.62	590.0
40	100	100	1856.81	20.33	2672.5
40	500	100	1551.10	21.55	2476.7
40	1000	100	1283.71	22.62	2358.2

A comparison of the thrust requirement tables in Chapter 4 and those above in Table 6-1 indicates that a late arrival of the weapon package by 40 seconds calls for a thrust requirement comparable to a 3 - 4 mile physical displacement error. It is much easier to envision a physical displacement error of 3 - 4 miles between satellite and weapon package positions than a late arrival of the weapon package of 40 seconds. Therefore if the terminal guidance phase thrust requirements are based on anticipated physical displacement errors, the thrust available would be sufficient to take care of any errors resulting from satellite or weapon package time errors. Displacement errors are therefore more dominant than errors in time.

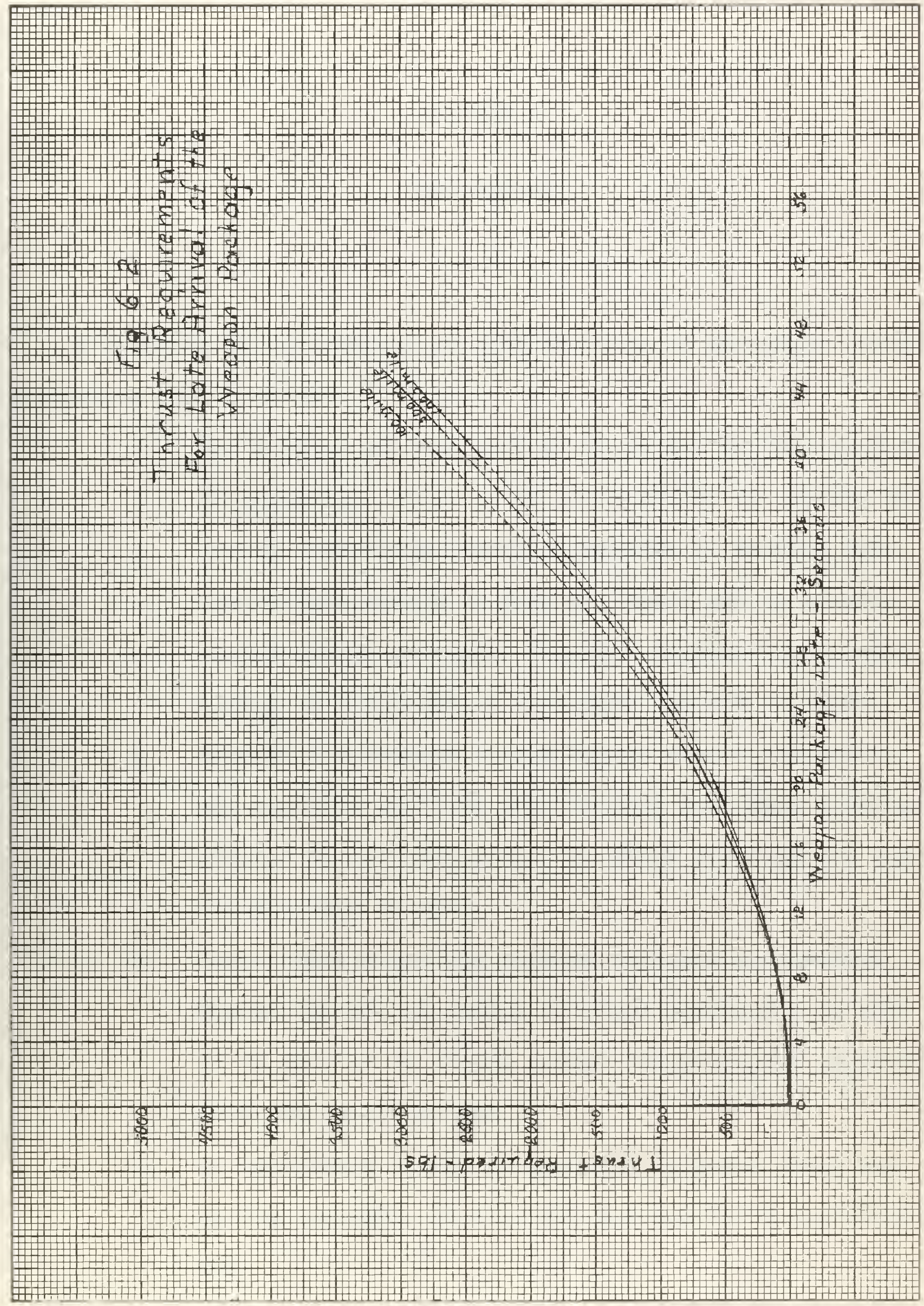
Fig 6.2
Thrust Requirements
For Late Arrival of the
Weapon Package
Weapon Package Late - Seconds
Thrust Required - lbs

CHAPTER 7

CONCLUSIONS

As a result of the analysis and considerations of this paper certain conclusions may be reached:

1. The terminal guidance problem can be defined with geometrical relationships. Solution of the equations for results which outline feasibility limits is enhanced by assuming that gravity acts equally on both the target and the missile. General thrust and seeker threshold criteria for a weapon package as visualized by this paper can then be found.
2. Physical displacement errors between missile trajectory and satellite orbit are more critical than other possible errors. Thrust requirements to effect intercept are higher for this type of error.
3. Thrust requirements for terminal guidance are feasible. Minimum thrusts are needed if target acquisition is made at the peak of the weapon package trajectory and physically above the satellite path.
4. An improvement of approximately 25% in seeker range capability increases terminal guidance capability approximately 50%.

CHAPTER 8

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