NPS55ZG71061A

United States Naval Postgraduate School

SOME FINITE HORIZON DISPATCHING PROBLEMS

by

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June 1971

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ABSTRACT:

An arrival process $\{N(t),\ 0 \leq t \leq T\}$ is to be dispatched one or more times in the time interval $(0,T)$. The problem is to determine the optimal number of dispatches K given there are n available and to determine sequentially the epochs of dispatch $\tau_1,\ldots,\tau_K$. There are two trade off costs c_w and c_d, which are respectively the cost per unit time of a waiting customer and the cost of dispatching a single unit. A general result is found which gives us optimal $\tau_1,\ldots,\tau_K$ for fixed K (i.e. the K-optimal policy) under certain regularity conditions. This is used to obtain suboptimal policies for multiple dispatching of a Poisson process and single dispatching of a birth-death process. Applications to problems in transportation, repair facilities and insect-control are indicated.

Prepared by:

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1. Introduction and Background.

In this paper, the following general model is considered: Let $\{N(t), 0 \leq t \leq T\}$ denote an arrival process in the time interval $[0,T]$. Available to a central dispatcher are n dispatching units to be dispatched at his discretion during this time interval. If a unit is dispatched to the queue site at time t, then the queue instantly becomes either partially or totally diminished, the exact assumptions depending on the context of the problem. Introducing a lag time is also possible. Assume that there are two trade-off costs c_w and c_d, where c_w is the cost per unit time of a waiting arrival and c_d is the cost of dispatching a single unit. The problem of minimizing cost (expected cost) is twofold:

(1) Determine the number of units K to be dispatched in $[0,T]$,

(2) Determine the times $\tau_1, \tau_2, \ldots, \tau_K$ when these units should be dispatched.

The applications of such problems appear widespread. An example is the problem of dispatching buses to waiting commuters or generally that of dispatching a server (typically expensive) to a waiting line. A possibly important application of the latter might be to the problem of dispatching a complicated repair facility to sea for repair of ships with subcritical malfunctions. Still another possible application is to the problem of optimally dispatching insecticide-spraying units to field crops during a given season. In this problem c_w may be interpreted as the cost per unit time of an insect's damage to crops. It also appears that the choice of n enters here as a constraint dictated by the harmful side effects of insecticides.

Naturally, if the process $\{N(t), 0 \leq t \leq T\}$ is deterministic then the decision variables $K, \tau_1, \ldots, \tau_K$ will be explicit functions of c_w, c_d, T as well as a functional of the arrival process. More realistically, if this process is stochastic, then we have the added option of determining the dispatching times sequentially.

When the arrival process is deterministic with arrival epochs $\xi_1, \xi_2, \ldots, \xi_N$ $(N>n)$, the problem of finding optimal $\tau_1, \ldots, \tau_K$ for each $K < n$ reduces to finding which of $\binom{N}{K}$ allocations is optimal. This is so because a dispatch should always be made at the instant after an arrival. After this is done, one may then find which K from among $0,1,\ldots,n$ yields the optimal policy. It almost goes without saying that the above formulation may be handled more elegantly via dynamic programming. We will not enter into this here, for this author cannot foresee the development of any qualitative insights. For a solution to a related problem, where the cumulative arrival process is continuous and deterministic with $K \equiv n$, the reader is referred to Newell [2].

The simplest prototype of a dispatching problem with stochastic arrivals is discussed by Ross [3], and is one in which the arrival process is Poisson with known rate λ with one available dispatching unit. However, it is assumed that the dispatch must be used so that the decision variable K is excluded from the problem. This was then generalized to include nonhomogeneous Poisson arrivals with nonincreasing intensity function $\lambda(t)$. The basic tool used in solving these problems is the "monotone case" concept of optimal stopping as set forth by Chow and Robbins [1] and as extended to continuous time processes by Ross [4].

In section 2, we mention briefly a simple deterministic case. In section 3 we derive a general theorem for obtaining a K-optimal policy: that is, finding $\tau_1,\ldots,\tau_K$ for a given K. In section 4, the result of section 3 is applied to the problem of multiple dispatching of a Poisson process. In section 5, we introduce into consideration the decision variable K, and in section 6 the problem of dispatching a spatially homogeneous birth-death process is discussed.

2. A Simple Deterministic Model.

Suppose $N(t) \equiv \lambda t$ for $0 \leq t \leq T$, and assume that a dispatching unit totally diminishes the queue. For fixed K, the cost c_d does not affect the choice of decision variables $\tau_1, \ldots, \tau_K$. Thus, we must find $\tau_K < \tau_{K-1} < \ldots < \tau_1$ to minimize the total waiting time

$$W_K = \sum_{j=0}^{K} \lambda(\tau_j - \tau_{j+1})^2/2$$

where $\tau_0 \equiv T$ and $\tau_{K+1} \equiv 0$. Elementary calculus yields

$$\tau^*_{K-j+1} = jT/(K+1) \ , \qquad j = 0,1,\ldots,K+1$$

and

$$W^*_K = \lambda T^2/2(K+1)$$

as the K-optimal values.

Thus for fixed K, the minimum cost of this K-optimal policy is

(1) $$C^*(K) = \lambda T^2 c_w/2(K+1) + Kc_d .$$

Minimizing (1) over the unrestricted variable K yields the value of K as one of the integers adjacent to

(2) $$-1 + (\lambda T^2 c_w/2\ c_d)^{1/2} .$$

Call this integer M^*. This may be seen by setting the derivative of $C^*(K)$ equal to zero and noting that $C^*(K)$ is convex in K.

It is thus clear that

$$K^* = \begin{cases} n & \text{if} \quad M^* > n \\ M^* & \text{if} \quad 0 \leq M^* \leq n \\ 0 & \text{if} \quad M^* < 0 \end{cases}$$

This together with τ_j^* $j = 1,\ldots,K^*$ yields the optimal policy.

It is worthwhile to note that (2) tells us that the interval between dispatches is inversely proportional to the square root of the arrival rate λ.

3. A General Theorem for the Stochastic Model.

In this section we consider the problem of obtaining sequentially the times of dispatch $\tau_K < \dots < \tau_1$ for a given K, i.e. the K-optimal policy. Assume without loss of generality that $c_w = 1$. Also assume total dispatching.

Consider an integrable stochastic process $\{N(t),\ 0 \leq t \leq T\}$, with $N(t) \uparrow$ a.e. Let $\bar{N}(t) \equiv N(t) - N(\gamma_t)$ where $\gamma_t \equiv$ epoch of most recent dispatch before time t; i.e. let $i(t) \equiv \min\{n : \tau_n \leq t\}$ whence $\gamma_t = \tau_{i(t)}$. Thus the process $\{\bar{N}(t),\ 0 \leq t \leq T\}$ depends on the dispatching policy $\tau_K < \dots < \tau_1$; in particular $\bar{N}(t)$ depends on $\{N(s),\ 0 \leq s \leq t\}$ and the particular sequential policy. Let F_t be the σ-field generated by $\{\bar{N}(s),\ 0 \leq s \leq t\}$ and let E_t denote conditional expectation given F_t. We shall also add the continuity condition

(3) $$E_t\bar{N}(s) \searrow \bar{N}(t) \text{ a.e. as } s \searrow t \text{ for every } t.$$

Let $w_j(t,T)$ be the j-optimal expected waiting time (assumed to exist) of only those arrivals in $[t,T]$ when j dispatching times remain. Thus,

$$w_0(t,T) = E\int_t^T [N(s) - N(t)]ds\,.$$

The following theorem, a dynamic programming extension of the infinitesimal look-ahead stopping rule of Ross [4], gives us the K-optimal policy under certain conditions.

Theorem 1. *Assume that* $w_j(t,T)$ *are differentiable and convex functions of* t *in* $[0,T]$ *for* $j = 0,1,\ldots,K-1$. *Then the (sequential) dispatching times* $\tau_K<\tau_{K-1}<\ldots<\tau_1$ *defined by* $\tau_{j+1} = \inf\{t:\bar{N}(t) \geq -\frac{d}{dt} w_j(t,T)\}$ *constitute a* K-*optimal policy.*

Proof. Suppose we are at time t with $j + 1$ dispatches remaining $(j = 0,1,\ldots,K-1)$. If we dispatch at t, our "loss" from time t under a j-optimal policy is $w_j(t,T)$, since the cost of dispatching is irrelevant. If instead we go on to dispatch at $t + \varepsilon$ ($\varepsilon > 0$ arbitrary), our optimal expected loss from t is

$$(4) \qquad \bar{N}(t) \cdot \varepsilon + E_t \int_t^{t+\varepsilon} [\bar{N}(s) - \bar{N}(t)]ds + w_j(t+\varepsilon,T)$$

We now compare (4) with $w_j(t,T)$ noting that $(4) \geq w_j(t,T)$ iff

$$(5) \quad \bar{N}(t) \geq \varepsilon^{-1}[w_j(t,T)-w_j(t+\varepsilon,T)] - \varepsilon^{-1} \int_t^{t+\varepsilon} E_t\bar{N}(s)ds - \bar{N}(t).$$

Thus dispatching at t is better than dispatching at $t + \varepsilon$ iff (5) holds. By (3) as well as the convexity and differentiability of $w_j(t,T)$, the right-hand side of (5) $\nearrow -\frac{d}{dt} w_j(t,T) = -w_j'(t,T)$ as $\varepsilon \searrow 0$. Thus if $\bar{N}(t) \geq -w_j'(t,T)$ then dispatching now (at t) compares favorably with dispatching at $t + \varepsilon$ for any $\varepsilon > 0$. Not only is this the case, but if we go on from t and refrain from dispatching we should note that since $-w_j'(t,T)$ is nonincreasing in t and $\bar{N}(s)$ nondecreasing (a.e.) for $s > t$, our fortunes will not change. That is to say, $\bar{N}(t) \geq -w_j'(t,T)$ implies $\bar{N}(s) \geq w_j'(s,T)$ for all $s > t$.

We thus find ourselves in the monotone case (Chow and Robbins [1]) and hence with $j+1$ dispatches remaining τ_{j+1} is optimal. q.e.d.

We may extend this theorem to include certain types of partial dispatching. Let $Z_1, Z_2, \ldots, Z_K$ be i.i.d. and concentrated on the interval $(0,1]$ with $EZ_i = p$. Here Z_i represents the proportion of units dispatched at τ_i. Thus we may define $\bar{N}(t)$, the number waiting at time t, in the intervals $\tau_j < t \leq \tau_{j-1}$ $(j = K+1, K, \ldots, 1)$ as follows: $\bar{N}(t) = N(t)$ for $\tau_{K+1} < t \leq \tau_K$ and recursively $\bar{N}(t) = N(t) - \sum_{i=j}^{K} Z_i \bar{N}(\tau_i)$ for $\tau_j < t \leq \tau_{j-1}$ for $j = K, \ldots, 1$. Note that we have implicitly assumed that $Z_1, \ldots, Z_K$ is also independent of $\{N(t),\ 0 \leq t \leq T\}$.

Let $w_0(t,T)$ be the cost of waiting due to those customers arriving in (t,T). In the one-stage problem, if we dispatch at t, our expected loss from t is

$$w_0(t,T) + \bar{N}(t)(1-p)(T-t).$$

If we go on to dispatch at $t+\varepsilon$, the expected loss is

$$\bar{N}(t)\cdot\varepsilon + w_0(t+\varepsilon,T) + E_t[\bar{N}(t+\varepsilon)](1-p)(T-t-\varepsilon) + E_t \int_t^{t+\varepsilon} [\bar{N}(s) - \bar{N}(t)]ds.$$

The same argument as in Theorem 1 yields $\tau_1 \equiv \inf\{t: \bar{N}(t) \geq -w_0'(t,T)\}$ to be 1-optimal provided $w_0(t,T)$ is differentiable and convex. In fact, $\tau_k \equiv \inf\{t: \bar{N}(t) \geq -w_k'(t,T)\}$ $(k = 0,1,\ldots,j-1)$ is j-optimal by the same reasoning provided $w_k(t,T)$ is differentiable and convex for $k = 0, \ldots, j-1$.

4. Multiple Dispatching of a Poisson Process.

In this section we apply the theorem of section 3 to the problem of finding the K-optimal policy $\tau_1, \ldots, \tau_K$ when $\{N(t),\ 0 \le t \le T\}$ is a Poisson process with known rate λ. See Ross [3] for the 1-optimal policy.

For this problem $EN(s) = \lambda s$, whence

$$w_0(t,T) = \int_t^T \lambda(s-t)ds = \lambda(T-t)^2/2 \quad \text{and} \quad -w_0'(t,T) = \lambda(T-t).$$

Therefore, by Theorem 1, $\tau_1 \equiv \inf\{t : N(t) \ge \lambda(T-t)\}$ is the 1-optimal policy. It remains to find $w_j(t,T)$ $(j = 1,\ldots,K-1)$ with the hope they are everywhere differentiable and convex.

We make the simple observation that $w_j(t,T)$ depends only on $T - t$ since $N(t)$ has stationary independent increments. Thus we may write $w_j(t,T) \equiv w_j(0,T-t)$ and proceed to investigate $w_j(0,T)$ as a function of T, noting that $w_j(0,T)$ is convex in T iff $w_j(t,T)$ is convex in t.

From Ross [3], we find that

$$\text{(6)} \quad \lambda T^2/4 - T/2 - 1/4\lambda \le w_1(0,T) \le \lambda T^2/4 \quad \text{for all} \quad T \ge 0.$$

Now it remains to ponder whether $w_1(0,T)$ itself is convex in T.

Even if this were so, it would not then follow that the inequality in (6) would be preserved under differentiation. It seems prudent to forego rigor for convenience and heurism. Since the approximation in (6) results only from a possible excess over the boundary at τ_1, it would seem that when this excess is small relative to λT, that $w_1(0,T) \approx \lambda T^2/4$ is a close approximation. In fact, for T large, $\lambda T^2/4$ dominates the left hand side of (6). In any event we should have

(7) $$-w_1'(t,T) \approx \lambda(T-t)/2.$$

If we "approximate" $w_1(t,T)$ by the left hand side of (6), then

(8) $$-w_1'(t,T) \approx [\lambda(T-t) - 1]/2$$

Since the three stage (three dispatch) problem depends on which boundary is used in the two stage problem, we would do well to show that the difference between the upper and lower approximations of $w_2'(t,T)$ is "small." In fact we shall show by induction that the differences between successive upper and lower approximations of $w_n'(t,T)$ are for all n bounded by 1. Suppose that

(9) $$w_{n-1}(0,T) \geq \lambda T^2/2n - \beta_{n-1}T + \beta'_{n-1} \quad \text{for some } n,$$

with $0 \leq \beta_{n-1} \leq 1$. Thus (9) holds for $n = 1,2,$ with $\beta_0 = \beta_0' = 0$ and $\beta_1 = 1/2$, $\beta_1' = 1/4\lambda$. Thus the "lower approximation" to $-w'_{n-1}(t,T)$ is

(10) $$\lambda(T-t)/n - \beta_{n-1}$$

and constitutes the boundary at the n^{th} stage. Since we use (10) to define the n stage dispatching time τ_n, we must investigate $w_n(0,T)$. We proceed to find a lower bound for $w_n(0,T)$:

(11) $w_n(0,T) \geq E\{[\lambda(T-\tau_n)/n - \beta_{n-1} - 1]\tau_n/2 + w_{n-1}(0,T-\tau_n)\}$

Since $\{N(t) - \lambda t\}$ is a zero mean martingale, a simple martingale systems theorem yields

(12) $$\lambda E\tau_n = EN(\tau_n) \leq E\lambda(T-\tau_n)/n - \beta_{n-1} + 1$$

whence

(13) $$E\tau_n \leq T/(n+1) + n(1-\beta_{n-1})/\lambda(n+1)$$

Combining (9) with (11), (13) and much tedious algebra yields

(14) $$w_n(0,T) \geq \lambda T^2/2(n+1) - \beta_n T + \beta_n'$$

where

(15) $$\beta_n = \beta_{n-1} + (1-\beta_{n-1})/(n+1) \quad \text{for} \quad n = 1,2,\ldots$$

It may be seen from (15) that $\{\beta_n\}$ is an increasing sequence bounded by 1. The conclusion we may draw from this is that for $\lambda T/n$ fairly "large" it will not matter a great deal which approximations are used. Thus, an approximate n-optimal policy is given by

(16) $$\tau_j = \inf\{t:\bar{N}(t) \geq \lambda(T-t)/j\} \quad \text{for} \quad j = 1,2,\ldots,n$$

See Figure 1. This same policy is "almost" optimal under the partial dispatching of section 3.

As a final comment, we mention that if $\{N(t),\ 0 \le t \le T\}$ is nonhomogeneous Poisson with continuous intensity function $\lambda(t)$, that

$$w_0(t,T) = \int_t^T \lambda(u)(T-u)\,du$$

whence $-w_0'(t,T) = \lambda(t)(T-t)$. Thus if $\lambda(t)(T-t)$ is nonincreasing, $\tau_1 \equiv \inf\{t: N(t) \ge \lambda(t)(T-t)\}$ is 1-optimal. The problem of finding the K-optimal policy for $K > 1$ is more difficult and will not be pursued.

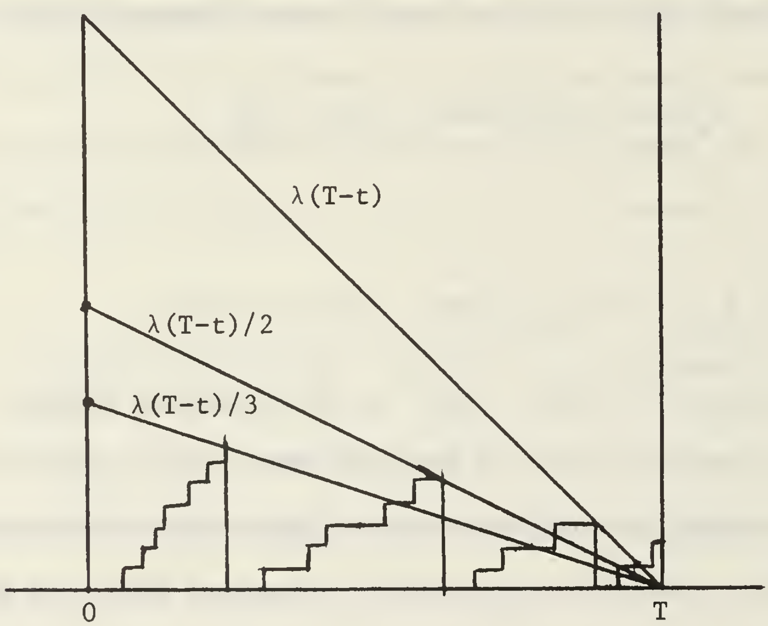

Figure 1. The approximate K-optimal policy for Poisson arrivals and $K = 3$.

5. <u>The Decision Variable K.</u>

We begin this section by discussing the simple problem where the dispatcher has exactly one unit which he has the option not to dispatch if he chooses. Thus, K may be either 0 or 1. The assumptions of Theorem 1 are assumed throughout.

Consider the general stochastic model of section 3. If the dispatcher is contemplating use of his single unit, Theorem 1 says that he should dispatch at time $\tau_1 = \inf\{t:N(t) \geq -w_0'(t,T)\}$. Suppose, however, that at time τ_1 he reconsiders his decision and compares the expected losses associated with dispatching at τ_1 and not dispatching at all in $(0,T)$. Clearly, he should not dispatch at τ_1 if the former is greater; that is, if

$$c_d + c_w \cdot w_0(\tau_1,T) > c_w N(t)(T-t) + c_w \cdot w_0(\tau_1,T)$$

or equivalently

$$\text{(17)} \qquad N(t) < c_d/(T-t)c_w$$

Evidently, should an unexpected throng arrive shortly thereafter, the dispatcher could again reconsider. It then follows that the optimal policy is:

(18)

(i) Dispatch at $\bar{\tau}_1 = \inf\{t:N(t) \geq \max[-w_0'(t,T), c_d/(T-t)]$,

(ii) Do not dispatch at all if $N(t) < \max[-w_0'(t,T), c_d/(T-t)]$ for all $0 \leq t \leq T$.

When $\{N(t), t \geq 0\}$ is a Poisson Process with rate λ, then the "dispatching region" is shown in Figure 2.

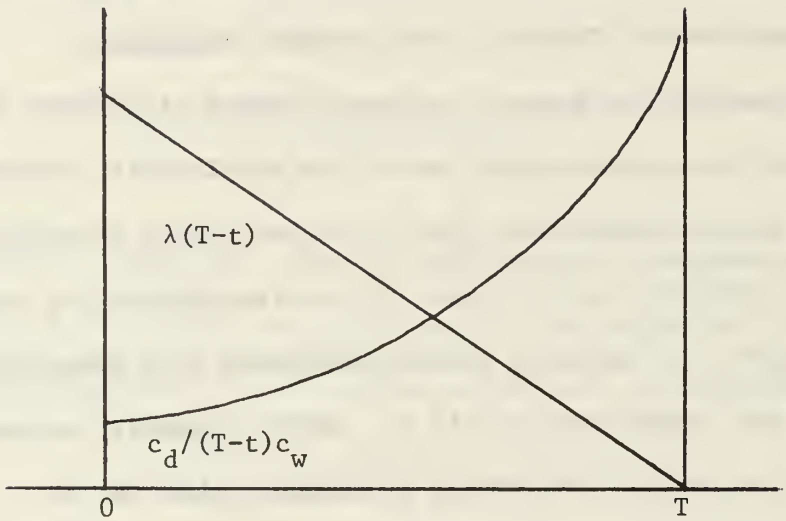

Figure 2: Dispatch when process enters shaded region. Otherwise do not dispatch.

We now define $\bar{c}_j(t,T)$ as the optimal total expected cost, with j available dispatches, of all arrivals in (t,T). The difference here from section 3, is that we have the option to use a subset of these j dispatches. In principle, we would proceed as follows:

(i) Determine $\bar{c}_1(t,T)$ from (18).

(19) (ii) For the 2 stage problem, wait until τ_2, as given by Theorem 1, and then dispatch only if $c_d < \bar{N}(t)E_t(\bar{\tau}_1 - t) \cdot c_w$.

The iteration continues, but tells us nothing of how to compute $E[\bar{\tau}_1 | E_t \bar{\tau}_1]$. This is a complex problem as it involves investigation of first passage times to boundaries of a very intractable nature. We do not enter into such a discussion here.

6. Dispatching a Birth-Death Process.

Suppose $\{N(t), 0 \leq t \leq T\}$ is a spatially homogeneous birth-death process, with state space consisting of all integers both positive and negative, and with birth parameter λ and death parameter μ. Actually this model allows for a negative queue size, and will serve only as an approximation to the usual birth-death process with reflecting barrier at 0. Thus we allow the possibility of dispatching a negative queue although this is never optimal. The reason we do this is that for the latter process, the derivations of $w_j(t,T)$ are intractable by virtue of the impenetrable barrier at the origin. However, we should note that if $\lambda >> \mu$, which in queueing parlance means that the traffic intensity $\rho >> 1$, then the dispatching rule should be "very nearly" optimal. In fact, it is this author's conjecture that the same rule is optimal for both problems.

We can no longer use Theorem 1 because we are not in the monotone case. This is so because $\{N(t)\}$ is not nonincreasing. However, intuition suggests that because the "drift" of the process is $\lambda - \mu > 0$, we ought to use the same type rule as that dictated by Theorem 1.

Consider then the rule:

$$\tau_1 \equiv \min[T, \inf\{t:N(t) \geq (\lambda-\mu)(T-t)\}]$$

Theorem 2. For the birth-death process mentioned at the beginning of this section τ_1 *is 1-optimal.*

Proof. Let α be any policy. We shall show that τ_1 is good as α and hence optimal. If there is any $t < T$ with $0 \leq N(t)<(\lambda-\mu)(T-t)$ where α tells us to dispatch, then let us modify α by choosing instead to dispatch at $t + [(\lambda-\mu)(T-t) - N(t)]/(\lambda-\mu) \equiv t + h_1$. If $N(t) < 0$, we shall dispatch at T (i.e. $h_1 = T - t$). Call this modified policy α_1 and note that α_1 is as good as α since

$$(\lambda-\mu)(T-t)^2/2 \geq N(t) \cdot h_1 + (\lambda-\mu)h_1^2/2 + (\lambda-\mu)(T-t-h_1)^2/2.$$

Since $N(t)$ may go up or down we distinguish between two cases. Let

$$A_1 \equiv \{\omega:\tau_1 \leq t + h_1\} \text{ with } A_1^c \equiv \{\omega:\tau_1 > t + h_1\}$$

Naturally $P(A_1^c) = 0$ if this t is such that $N(t) < 0$. On A_1, $t + h_1 = \tau_1 + \delta$ where δ is nonnegative and random. Since the expected wait from τ_1 is at least

$$N(\tau_1) \cdot \delta + (\lambda-\mu)(T-\tau_1-\delta)^2/2 > (\lambda-\mu)(T-\tau_1)^2/2,$$

we conclude that on A_1 we should dispatch at τ_1. Thus on A_1, τ_1 is as good as α_1. On A_1^c, if it is nonempty, we have $t + h_1 < \tau_1$ so that $N(t+h_1) < (\lambda-\mu)(T-t-h_1)$. Recall, however, that α_1 tells us to dispatch at $t + h_1$. Let us modify α_1 by choosing to dispatch at

$$t + h_1 + h_2 \equiv t + h_1 + [(\lambda-\mu)(T-t-h_1) - N(t+h_1)]/(\lambda-\mu).$$

If $N(t+h_1) < 0$, let $h_2 = T - t - h_1$. Call this policy α_2 and note that α_2 is as good as α_1. Such an iterative scheme will lead us eventually to a sequence of nested sets

$$A_j^c \equiv \{\omega:\tau_1 > t + h_1 + \ldots + h_j\}.$$

For some j $N(t+h_1+\ldots+h_{j-1}) < 0$ whence $P[A_j^c] = 0$ in which case having shown that τ_1 is as good as α_j on $A_1, A_2, \ldots, A_j$ proves that τ_1 is as good as α.

q.e.d.

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DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) Naval Postgraduate School Monterey, California	2a. REPORT SECURITY CLASSIFICATION Unclassified	
	2b. GROUP	
3. REPORT TITLE Some Finite Horizon Dispatching Problems		
4. DESCRIPTIVE NOTES (Type of report and inclusive dates) Technical Report		
5. AUTHOR(S) (First name, middle initial, last name) Edward A. Brill		
6. REPORT DATE June 1971	7a. TOTAL NO. OF PAGES 25	7b. NO. OF REFS 4
8a. CONTRACT OR GRANT NO.	9a. ORIGINATOR'S REPORT NUMBER(S)	
b. PROJECT NO.		
c.	9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)	
d.		
10. DISTRIBUTION STATEMENT Approved for public release; distribution unlimited.		
11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY	

13. ABSTRACT

An arrival process $\{N(t), 0 \le t \le T\}$ is to be dispatched one or more times in the time interval $(0,T)$. The problem is to determine the optimal number of dispatches K given there are n available and to determine sequentially the epochs of dispatch $\tau_1,\ldots,\tau_K$. There are two trade off costs c_w and c_d, which are respectively the cost per unit time of a waiting customer and the cost of dispatching a single unit. A general result is found which gives us optimal $\tau_1,\ldots,\tau_K$ for fixed K (i.e. the K-optimal policy) under certain regularity conditions. This is used to obtain suboptimal policies for multiple dispatching of a Poisson process and single dispatching of a birth-death process. Applications to problems in transportation, repair facilities and insect-control are indicated.

14 KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Dispatch						
Optimal stopping						
Monotone case						
Dynamic programming						
Poisson process						
Birth-Death process						