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PERIODIC AND LOG PERIODIC COUPLING

OP MODES OF PROPAGATION

by

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ABSTRACT:

This report summarizes an investigation of the applicability of the log periodic concept to coupled microwave transmission lines. It is shown that active coupling at a given frequency may be achieved between two lines supporting waves with different phase constants provided they are coupled periodically with a periodicity determined by the difference $\beta_2 - \beta_1$ at that frequency. Since $\beta_2 - \beta_1$ will, in general, vary with frequency, coupling over a broad frequency range requires a variable period. This may be realized by scaling the structure period, thereby giving rise to a log periodic mode coupler. This technique appears to hold promise for achieving the same wideband performance that has been attained with log periodic radiators.

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The assistance of LT A. E. Whitehead in providing the preliminary experimental results is also acknowledged. More complete data will appear in his Master of Science thesis which will be published in December 1971.

LIST OF FIGURES

- Figure 1. Periodically coupled lines.
- Figure 2. Unit cell of periodically coupled lines.
- Figure 3. ω - β diagram for uncoupled waveguide and coaxial line.
- Figure 4. ω - β diagram of coupled lines: the limiting case $k\rightarrow 0$.
- Figure 5. ω - β diagram of coupled lines: finite k.
- Figure 6. Backward wave coupler.
- Figure 7. ω β diagram for log periodic coupler.
- Figure 8. Experimental coupler.
- Figure 9 Power transfer in experimental coupler.

TABLE OF CONTENTS

ABSTRACT

ACKNOWLEDGEMENTS

LIST OP FIGURES

I. INTRODUCTION

Since the publication of Pierce's theory of coupled modes of propagation [1] in 1954, that approach has been used to explain the operation of many microwave devices. Currently this theory is well understood and has received subsequent treatment by many authors [2] - [6]. Certainly the most exhaustive discussion of the theory is that presented by Louisell [3].

The theory that has been developed is applicable to situations where there is uniform coupling between propagating waves with phase constants which are approximately equal. When there is a significant difference in the phase constants, no interaction will occur if the coupling is uniform. Thus, if it is desired that waves with different phase constants be coupled together, uniform coupling cannot be employed

Several recent papers have dealt with the subject of periodic coupling $[6]$ - $[8]$. The authors have shown that it is possible to achieve coupling between waves with different phase constants β_1 , $\beta \neq \beta_1$ provided the waves are coupled by functions C(z) which are periodic with period 2 $\pi/\Delta\beta$ where $\Delta\beta$ = β $_2$ - β $_1.$ Only passive interactions and applications have been considered, however. The purpose of this report is to consider active coupling and to show how this type of interaction may be used in wideband applications.

II. PERIODIC COUPLING

In the papers previously mentioned $\lceil 6 \rceil$ - $\lceil 8 \rceil$, the authors have considered solution of the coupled line equations

$$
-j \frac{dA_1(z)}{dz} = \beta_1 A_1(z) + C(z) A_2(z)
$$
 (1a)

$$
-j \frac{dA_2(z)}{dz} = C(z) A_1(z) + \beta_2 A_2(z)
$$
 (1b)

for a variety of coupling functions $C(z)$. Although it is not explicitly pointed out, the exact functional dependence of $C(z)$ is unimportant so long as the period is adjusted in accordance with the difference $\beta_2-\beta_1$. This will be shewn later. Additionally, coupling coefficients suoh as $C(z)$ = a sin kz can be realized in practice only approximately at best. The solution of equations (1), therefore, seems to be an unnecessarily difficult exercise. A more practical approach appears to result from treating the coupled lines as a periodic structure and determining the propagation constants from the eigenvalues of a coupling matrix. Wave amplitudes at points separated by an integral number of structure periods may then be easily related.

A. COUPLING MATRIX ANALYSIS

Consider two lines which are discretely coupled at points separated by a distance L as shown in Figure 1. Such a system is periodic and the unit cell has the form shown in Figure 2. The $A_1^{\{1\}}$'s are the wave amplitudes on the lines and are chosen such that time average power

 \overline{c}

flow is given by $A^{(i)}_j A^{(i) \; *}_{j}.$ Assuming linearity and conservation of energy, it can be shown [2] that we may write

$$
\begin{pmatrix}\nA_{n+1}^{(p)} \\
A_{n+1}^{(q)}\n\end{pmatrix} = \begin{pmatrix}\n\sqrt{1+k^2} e^{-j\theta} + ke^{-j\theta} \\
-j\theta_p\n\end{pmatrix} \begin{pmatrix}\nA_n^{(p)} \\
n\end{pmatrix}
$$
\n(2)

for active coupling where

 $k = coupling coefficient$ $\theta_{\rm p}$ = phase shift across unit cell on p-line = phase shift across unit cell on q-line q i

and where it has been assumed that no phase shift is introduced by the coupling mechanism. If this last assumption is not valid a slight modification of tne equations results but there is no change in the conclusions below. For brevity, equation (2) may be written

$$
A_{n+1} = CA_n \tag{3}
$$

where C is a square matrix and the A's are column vectors.

Since the structure under consideration is periodic, we may apply Floquet's theorem to obtain

$$
A_{n+1} = A_n e^{-\gamma L} \tag{4}
$$

and upon substituting (4) in (3) we have

$$
CA_n = e^{-\gamma L}A_n \t\t(5)
$$

The permissible values of the propagation constant are, therefore, given by the eigenvalues of C. We thus find that

$$
e^{-\gamma L} = \left\{ \sqrt{1+k^2} \cos \left(\frac{\theta_q - \theta_p}{2} \right) \pm \sqrt{(1+k^2) \cos^2 \left(\frac{\theta_q - \theta_p}{2} \right) - 1} \right\} e^{-j \left(\frac{\theta_p + \theta_q}{2} \right)}.
$$
\n(6)

Examination of equation (6) reveals that the propagation constant may take either of two forms:

Case 1.
$$
(1+k^2) cos^2(\frac{\theta_q-\theta_p}{2}) < 1
$$

In this case, the bracketed term in equation (6) is a complex number with modulus unity and we have

$$
e^{-\gamma L} = \left\{ e^{\pm} \, \frac{j\alpha}{2} \right\} \, e^{-\frac{j(\frac{\beta}{2} + \theta_q)}{2}} \tag{7a}
$$

where

$$
\cos \alpha = \sqrt{1+k^2} \cos \left(\frac{\theta_q - \theta_p}{2} \right) \tag{7b}
$$

Since $k \ll 1$ we may write

$$
\alpha = \left(\frac{\theta_{q} - \theta_{p}}{2}\right) \tag{8}
$$

and hence,

$$
e^{-\gamma L} = e^{\frac{\theta_q - \theta_p}{2}} e^{-j(\frac{\theta_p + \theta_q}{2})}
$$
 (9)

Therefore,

$$
\gamma_1 = \mathbf{j}\theta_q / \mathbf{L} \tag{10a}
$$

$$
\gamma_2 = j\theta_p/L \t\t(10b)
$$

 \bar{t}

There are two waves which propagate and they are, for all practical purposes, the waves of the unperturbed lines. The lines behave as if there were no coupling since the period is not adjusted so that a

cumulative interaction may occur.

Case 2.
$$
(1+k^2) cos^2(\frac{\theta_q-\theta_p}{2}) > 1
$$

In this case, the bracketed term in equation (6) is real and γ must θ - θ be complex. If we examine the interaction for $\frac{q-1}{q-1} = n\pi$ (n an integer) we find that

$$
e^{-\gamma L} = \left\{ \sqrt{1 + k^2} \pm k \right\} e^{-j\left(\frac{\theta_p + \theta_q}{2}\right)}.
$$
 (11)

Since $k^2 \ll k$, we may write

$$
e^{-\gamma L} = e^{-(\alpha + j\beta)L} = \left\{1+k\right\} e^{-j\left(\frac{\beta + \theta_q}{2}\right)}
$$
(12)

and, therefore,

 α L $\dot{=} + k$ (13a)

$$
\beta L = \left(\frac{\theta_p + \theta_q}{2}\right) \tag{13b}
$$

Here, there are again two waves which may propagate but they are significantly different from the waves of the unperturbed or uncoupled lines. Now, we see that the amplitude of one wave decays while that of the other grows. Physically, we have waves travelling in opposite directions on the two lines. The amplitude of the wave on the driven line will decay as its energy is transferred to the wave on the undriven line.

We note at this point that an exponentially decaying driven wave amplitude is one of the conditions that must be satisfied on a log periodic structure. This is necessary if the structure is to be truncated (as it must be) without causing a disturbance.

The results of this section show that active coupling may occur

between waves having unequal phase constants and carrying energy in opposite directions provided the waves are periodically coupled and the period is properly adjusted. The condition for synchronism, θ - θ - $(\frac{q}{2}^{\mu}) = n\pi$, is equivalent to

$$
\beta_{q}L - \beta_{p}L = n2\pi . \qquad (14)
$$

Choosing $n = + 1$, the required structure period is

$$
L = \frac{2\pi}{\beta_p - \beta_d} = \frac{2\pi}{\Delta \beta} \tag{15}
$$

where $\Delta \beta = | \beta_p - \beta_q |$.

B. SYMMETRY ANALYSIS

In order to gain a more complete picture of the various interaction points for periodically coupled lines, one must have more information than that gained from only the equations of the previous section. The greatest shortcoming of these equations is that we have considered only two waves when actually there are four present. The best approach is to examine the lines from the point of view of the ω - β diagram.

Initially, let us consider the ω - β diagrams of the uncoupled lines. For purposes of illustration, suppose that one line is a rectangular waveguide which has the TE_{10} as its only propagating mode and let the second line be coaxial with only a TEM mode. We further assume that both lines are air insulated and lossless. The ω - β branches for these lines are plotted together in Figure 3 and numbered, 1-4, to indicate that there are four linearly independent waves which correspond to these branches.

Now suppose that we wish to couple wave 1 to wave 4 by periodically coupling the two lines. According to equation (15), we find the difference in the phase constants at the frequency of interest and then couple the lines periodically with $L = 2\pi/\Delta\beta$. However, when the lines are coupled periodically the whole system becomes a periodic structure and the form of the ω - β diagram is completely altered. One can show by symmetry arguments that the ω - β diagram must have periodicity $2\pi/L$ and therefore it must repeat itself in a distance $\Delta\beta$. Furthermore, all relevant information is contained in the range $-\Delta\beta/2$ to + $\Delta\beta/2$ which is known as the Brilluoin zone.

A first approximation to the correct ω - β diagram for the coupled lines may be obtained by translating the ω - β diagram of Figure 3 by $n\Delta\beta$, where n is an integer, and retaining only that part for which $-\Delta\beta/2 < \beta < \Delta\beta/2$. This procedure results in the ω - β diagram shown in Figure 4. It may be thought of as being the correct ω - β diagram for the limiting case of vanishing periodicity $(k\rightarrow 0)$.

For k finite, modifications of the ω - β diagram of Figure 4 may occur at any point where there are branch crossings in that diagram. These are the points where the coupling is such that a cumulative interaction may occur.

The interaction which occurs at point 2 in Figure 4 was described by the equations of the previous section. At the crossing point and for frequencies in the immediate vicinity of the crossing point, the propagation constant is complex. The real part of the propagation constant has been plotted in Figure ⁵ where the appropriate modifications at point 2 are shown. As we move away from point 2, it is evident that the real part of γ goes to zero and thereafter there are two waves which propagate almost as if there were no coupling present.

 $\overline{7}$

Points ¹ and 3 represent regions where waves traveling in opposite directions on the same line may be coupled together. If such coupling were to occur at these points this would result in reflection of the energy on the line or a stopband. Whether or not a stopband will occur depends upon the translational symmetry of the structure. A detailed analysis is beyond the scope of this report and thus only a statement of sufficient conditions will be given here. The proof of the theorem may be found elsewhere [9].

Theorem: If the space group of a periodic structure contains the glide or second order screw element, then branches of the ω - β

diagram will stick together in pairs at $\beta = \pi/L$.

For the problem under consideration, this simply means that if the coupled lines have glide or second order screw symmetry, then degeneracies will occur at points 1 and ³ and there will be no further modification of the ω - β diagram at those points. If, on the other hand, the coupled lines have only pure translational symmetry, then this will induce stopbands at these points. Both possibilities are indicated in Figure 5

The particular realization of the coupling function determines which of the above situations will occur. If one is interested in single frequency operation around point 2, then the Interaction or lack thereof at points ¹ and 3 is of no concern. If, however, it is desired to operate the device over a broad frequency range then these regions must be carefully examined. As we shall see shortly, region 1 has profound consequences for the log periodic case.

This section has discussed the ω - β diagram for a rectangular waveguide operating in the TE_{10} mode and periodically coupled to a coaxial line operating in the TEM mode. Proper selection of line

dimensions with respect to operating frequency will ensure that these conditions will be realized. To operate as a backward wave coupler, the device must be operated in the frequency range where γ is complex. Then, if one of the lines is driven, the wave on that line will decay exponentially as it progresses and the energy will be transferred to the other line. The wave on the other line will leave the coupler at the same end to which the excitation was applied. This is shown schematically in Figure 6. Evidently, if the coupling region is extended far enough to allow for sufficient decay of wave amplitude, then both lines may be terminated without causing any appreciable disturbance.

III. LOG PERIODIC COUPLING

The limited bandwidth of the periodic coupler described in the previous section is evident from the ω - β diagram of Figure 5. The active coupling occurs over only a small frequency range. In this section we will describe a modification of the periodic structure which permits broadband operation.

In the general case, the phase constant of a traveling wave will vary with frequency and hence the difference between phase constants on two lines will vary with frequency. At any given frequency, the lines may be periodically coupled so as to achieve interaction by adjusting the period according to the difference in phase constants. The problem is that no fixed period can be right for interaction at all frequencies. The log periodic solution to this dilemma is to allow the period to change by a scale factor. A wave then should travel along a line until it reaches a region where the "local" period is approximately right for interaction. In this region the energy of the wave would be transferred to a wave on the other line. The extent of this active region should be sufficiently great to allow for essentially complete exchange of energy. Any physical structure must be finite in extent and hence the scaled structure must be twice truncated. The frequency band of the truncated structure should then be approximately determined by the local period at either end.

Currently employed mathematical approaches to the analysis of log periodic structures lack the rigor and precision of the methods which may be applied to periodic structures. In particular, no

rigorous counterpart to the ω - β diagram has yet been described for log periodic structures. The ω - β diagram of a periodic structure provides information on wave propagation and has characteristics which may be rigorously described using group theory. What one would like is something of a similar nature for log periodic structures. In the absence of any such development, the intuitive approach has been taken.

For log periodic structures, the intuitive approach is to say that if the variation of the structure period Is slow enough then any given section of the structure behaves as if it were (in that locality) periodic. On the basis of this assumption, it is then possible to use a normalized version of the ω - β diagram of the periodic circuit from which the log periodic structure is derived. For a given frequency, the behavior of waves on a particular part of the log periodic structure may be determined by examining the appropriate region of the diagram. Such a region might be a passband, a stopband or a coupling region.

In this sense, the results of section II may be applied directly to the log periodic version of the coupled lines described in this section. The ω - β diagram of Figure 5 is normalized by the period L and becomes a plot of ω vs β L. This is called an $\widetilde{\omega}$ - $\widetilde{\beta}$ diagram as shown in Figure $7.$ For a given frequency of operation and a given region of the line, one computes $\widetilde{\omega}$ and then enters the $\widetilde{\omega}$ - $\widetilde{\beta}$ diagram to find β . The real, imaginary or complex nature of β describes the way in which waves propagate in that neighborhood.

Let us consider the movement of the operating point with position for the TEM-waveguide mode coupler that has been proposed. Assume that the waveguide is excited above its TE_{10} cutoff frequency and that the local period at the driving point is such that the operating point is ¹ in Figure 7. Assume also, that the structure period becomes

greater as distance from the driving point increases. In that case, $\widetilde{\omega}$ will increase with distance (ω constant) and as the wave moves down the structure the operating point will move in the direction indicated by the arrow at point 1. Ultimately the wave will move into a region where the operating point lies in the active coupling region as indicated by 2. In this region, the TEM wave will be excited and transfer of energy will occur. If the region is of sufficient extent, then this transfer of energy will be essentially complete. The TEM wave then travels away from the coupling region in the opposite direction (direction of decreasing structure period) . The operating point moves as indicated to point 3. Note the negative group velocity for this branch.

In order for the TEM wave to return to the driving point position, it must be possible for the operating point to move to position 4 . The significance of the structure symmetry is now apparent. As mentioned previously, glide or screw symmetry results in a degeneracy at βL = π for periodic circuits. If this similarity were not present on the log periodic structure, the operating point could not move from 3 to 4-. We would thus have a stopband or reflection region on the log periodic circuit with the obvious result that the operating point would proceed back to 1. In that case we would not have an operable device.

Upon reaching the position corresponding to point 4 , the TEM wave may be extracted as an output or guided away from the log periodic coupling region for utilization at another location. From the previous discussion, it should be evident that as operating frequency is increased, the active region moves toward the driving point while if ω is decreased it moves away from the driving point. Since the structure must be finite in extent, the upper and lower frequency limits are

determined by the local period at either end of the structure. Exponential decay of wave amplitude through the active region guarantees the absence of end effects due to truncation so long as the active region is sufficiently long to allow for essentially complete transfer of energy.

It should further be noted that the assumption of a TEM wave and only a \mathbb{TE}_{10} waveguide mode also places bandwidth restrictions upon this particular realization of log periodically coupled propagating modes.

IV. EXPERIMENTAL COUPLER

To verify the theoretical predictions of Section II, a waveguide to coaxial periodic mode coupler was constructed by LT A. E. Whitehead and a test program was begun in the Microwave Laboratory of the Department of Electrical Engineering. The prototype structure consisted of a length of RG8U (ϵ_n =2.25) coaxial cable which was coupled to X-band waveguide $(0.4" \times 0.9")$ by a series of 21 probes. The probes were connected to the center conductor of the coaxial cable and extended 0.125" into the center of the waveguide as shown in Figure 8. The periodicity of 0.618 inches resulted in a crossing of the ω - β curves of the uncoupled lines at 8.75 ghz.

Preliminary testing of the prototype structure was conducted to determine where the input power went as a function of frequency over the range of 8 - 10.5 ghz. The waveguide was driven at one end and the other three device ports were terminated in matched loads. The results of the experimental measurements are shown in Figure 9. It may be seen that the device behaved as predicted with 92% of the power transferred to the coaxial line at the interaction frequency. The interaction frequency of 9.45 ghz, however, was considerably greater than the 8.75 ghz crossover point for the ω - β branches of the uncoupled lines. This shift may be attributed to the loading of the lines which was caused by the coupling technique. As studies progress this will be investigated further, both experimentally and theoretically.

The coupling coefficient of the structure may be computed from equation (13a). Since 92% of the power is transferred in 21 periods of the device we have

$$
e^{-2\alpha(21L)} = .08 \tag{16}
$$

and taking the logarithm of both sides,

$$
42\alpha L = 2.52
$$
 (17)

Since $k = \alpha L$, we have

$$
k=.06.
$$
 (18)

This justifies the assumption $\kappa^2<\!\!<$ k which was made in deriving equation (12).

Preliminary tests with $1/16$ " and $3/16$ " probes show that both coupling and bandwidth vary as the probe depth. This is in accordance with the equations of Section IIA.

V. SUMMARY

This report has presented the theory of periodic coupling of modes of propagation. It has been shown that energy may be transferred between lines supporting waves with unequal phase constants. Attention was focused upon the active interaction which occurs when the period of the coupling function is $L = 2\pi/\Delta\beta$. This interaction is characterized by an exponential decay of the wave amplitude on the driven line and transfer of energy to a wave with oppositely directed group velocity on the auxiliary line.

This interaction is further shown to have all the properties necessary for use in a log periodic coupler. Primarily, the exponential wave decay ensures lack of end effect due to truncation. Tt has also been shown that the log periodic structure must be glide or screw similar in order to ensure that there will be no reflection region along the wave path. Log periodic coupling appears to hold promise for achieving the same wideband performance that has been realized with antennas designed using this principle.

Preliminary measurements on a prototype periodic coupler have demonstrated the practicality of this type of coupling. As work progresses this structure will be further investigated both experimentally and analytically. The ultimate goal will be to construct a log periodic coupler. Realization of such a structure is of interest not only because of the new devices which may result but also because it will result in a better understanding of the log periodic concept itself.

REFERENCES

- [l] J. R. Pierce, "Coupling of Modes of Propagation", Journal of Applied Physics., vol. 25, pp 179-183, February 1954.
- [2] D. A. Watkins, Topics in Electromagnetic Theory. New York: Wiley, 1958, Chapter 3.
- [3] W. H. Louisell, Coupled Mode and Parametric Electronics. New York: Wiley, i960.
- [4] C. W. Barnes, "Conservative Coupling Between Modes of Propagation", Proc. IEEE, vol 52, pp 64-73, January 1964, pp 295-299, March 1964.
- [5] C. C. Johnson, Field and Wave Electrodynamics. New York: McGraw Hill, 1965, Chapter 9.
- [6] S. E. Miller, "On Solutions for Two Waves with Periodic Coupling", Bell System Technical Journal, pp 1801-1822, October 1968.
- [7] S. E. Miller, "Some Theory and Applications of Periodically Coupled Waves", Bel^l System Technical Journal , pp 2189-2219, September 1969.
- [8] A. W. Snyder and O.J. Davies, "Asymptotic Solution of Coupled Mode Equations for Sinusoidal Coupling", Proc . IEEE (Letter), pp 168-169, January 1970.
- [9] J. B. Knorr, "Electromagnetic Applications of Group Theory", Ph.D. Thesis, Cornel University, June 1970.

FIGURE 2. UNIT CELL OF PERIODICALLY COUPLED LINES

 $\mathcal{L}^{\text{max}}_{\text{max}}$, where $\mathcal{L}^{\text{max}}_{\text{max}}$

FIGURE 3. ω - β DIAGRAM FOR UNCOUPLED WAVEGUIDE AND COAXIAL LINE

FIGURE 4. ω - β DIAGRAM OF COUPLED LINES: THE LIMITING CASE K->0

FIGURE 5. ω - β DIAGRAM OF COUPLED LINES: FINITE K

FIGURE 6. BACKWARD WAVE COUPLER

FIGURE 7. $\widetilde{\omega}$ - $\widetilde{\beta}$ DIAGRAM FOR LOG PERIODIC COUPLER

SCALE APPROXIMATELY 1.5:1

POWER TRANSFER IN EXPERIMENTAL COUPLER ∞ FIGURE

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