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On the validity of the geostrophic approximation for the Florida Current

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THE UNIVERSITY OF MIAMI

ON THE VALIDITY OF THE GEOSTROPHIC APPROXIMATION FOR THE FLORIDA CURRENT

BY

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Subject

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ABSTRACT

The object of this thesis is an examination of the validity of geostrophic calculations of the downstream component of the time-averaged Florida Current in the Straits of Florida by comparison of calculated and directly measured current fields. The study is motivated by the assumption made in modern inertial current theory that downstream current speed is in geostrophic balance, plus evidence of recent studies indicating that this region of the Florida Current is primarily inertial in nature. The principal conclusion reached is that geostrophic calculations yield a valid first order approximation to the observed velocity fields, indicating that the assumption made in inertial current theory is valid. In addition, it has been shown that in geostrophic calculations in this region, the density field may be approximated as a parabolic function of temperature only. A general discussion of mass field adjustment to downstream speed changes is offered.

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Edward J. O'Brien III

Coral Gables, Florida
June, 1967

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF FIGURES--Continued

I. INTRODUCTION

The purpose of this thesis is the examination of the validity of the geostrophic approximation for the Florida Current (within the Straits of Florida). An experiment was conducted during 1965-1966 having as one objective the critical investigation of such an approximation. Earlier studies of the application of geostrophy in this area have been made in order to calculate the velocity field for the purpose of examining various features of the current. Since the development of the free instrument technique (Richardson and Schmitz, 1965), rapid, direct measurement of the current field is possible, and geostrophic calculations are no longer necessary for current field determination. In this thesis, interest is focused on the extent to which the geostrophic approximation describes the downstream component of the flow as postulated in contemporary inertial current theory (Robinson, 1965). Robinson (ibid.) further points out that use of space-temperature (x,y,T) coordinates simplifies the mathematics involved in developing models. The geostrophic equation as examined is unusual in its use of an equation of state with density expressed as a function of temperature only.

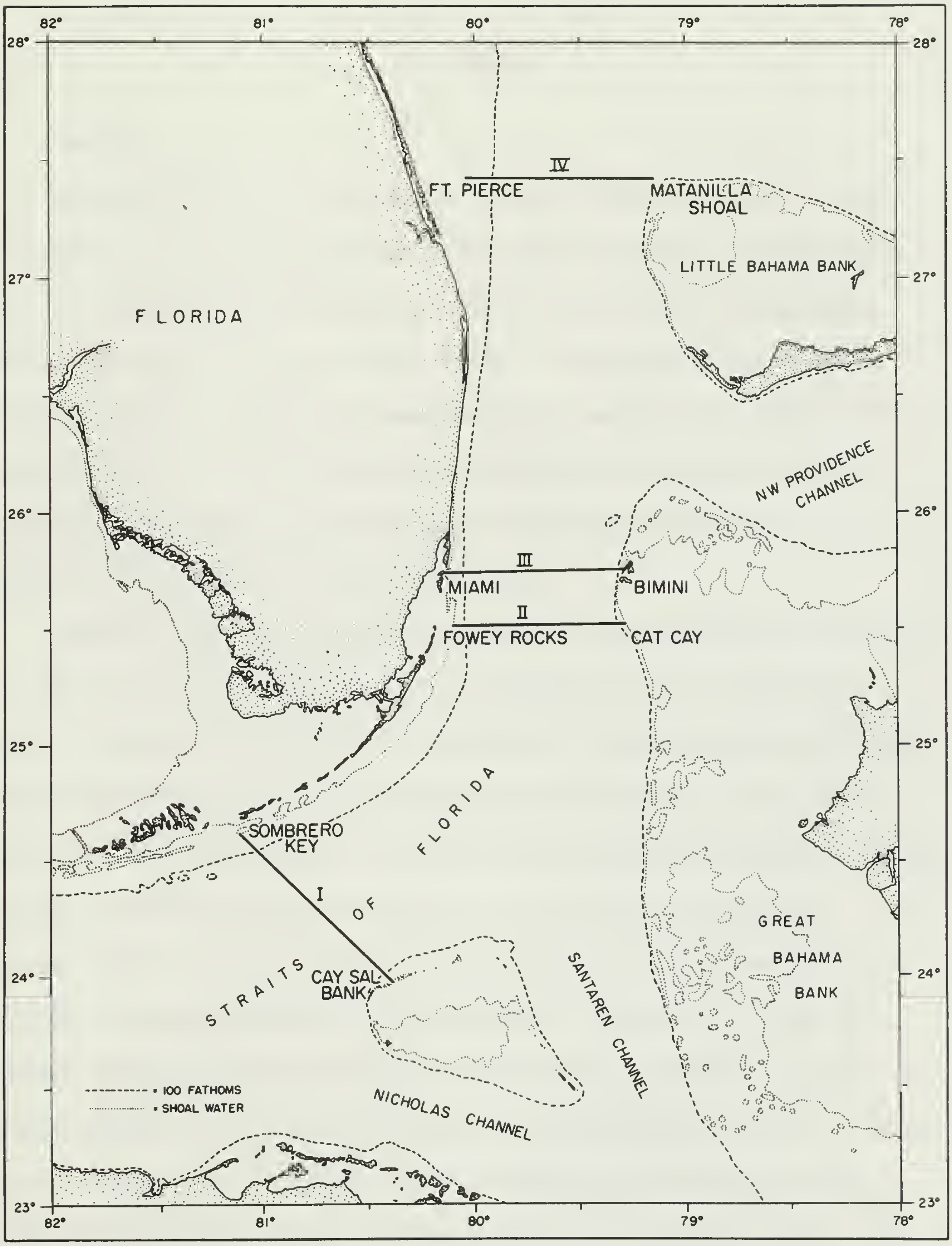
This thesis constitutes the first quantitative validity determination of geostrophic calculations based on simultaneous hydrographic data for a steady (time-averaged) current. The validity of the geostrophic equation will be determined by comparing calculated

downstream velocity fields with observed velocity fields obtained using the free instrument technique. These observed fields will represent a time-averaged Florida Current at four different sections across the Straits of Florida (Figure 1).

If the geostrophic approximation is valid for the Straits, it will give meaningful results throughout the area of investigation. Direct measurement of the current shows downstream acceleration (Clausner, 1967). For geostrophy to hold along the length of the current, the mass field structure must adjust as changes occur in the downstream velocity fields, maintaining the balance between pressure gradient and coriolis force. This mass field adjustment will be examined in conjunction with the changing velocity fields.

Several previous investigations have invoked geostrophy in the study of the Florida Current (see for example: Wüst, 1924; Parr, 1937). The extensiveness of the free instrument data available from this experiment (four cross-sections over a 225 Km. downstream scale) plus the quality of this data afford a good basis for a critique of earlier investigations. Such a critique will be included as a second objective of this thesis.

FIGURE 1: Chart of Section Locations.


82°
81°
80°
79°
78°
28°
27°
26°
25°
24°
23°
FLORIDA
FT. PIERCE
IV
MATANILLA
SHOAL
LITTLE BAHAMA BANK
NW PROVIDENCE
CHANNEL
III
MIAMI
BIMINI
II
FOWEY ROCKS
CAT CAY
SOMBRERO
KEY
I
STRAITS OF FLORIDA
CAY SAL
BANK
SANTAREN CHANNEL
GREAT
BAHAMA
BANK
NICHOLAS CHANNEL
= 100 FATHOMS
= SHOAL WATER

II. DATA

A. General

Four sections were selected at locations which provide various combinations of downstream scale and channel geography. It was felt that the geostrophic approximation could be thoroughly tested when applied in such a variety of situations. Furthermore, the Florida Straits over the length where sections were sampled is a closed channel, except for the Santaren and Northwest Providence Channel openings. This configuration permits only minimal distortion of the Florida Current by other sources of water.

Section I, between Marathon (Vaca Key), Florida and Cay Sal Bank, B.I. has a "V" shaped bottom profile reaching depths in excess of 1000 m. Section II is located about 100 Km. downstream between Fowey Rocks and Cat Cay, B.I. past the point where the axis of the stream has turned toward the north. At this section maximum depth has reduced to just over 800 m. and the bottom of the trough has flattened. Current width at the surface has been reduced from over 100 Km. at Section I to about 85 Km. at this section. Section III is 25 Km. further downstream and has much the same bottom profile as Section II. Current width at the surface, however, has decreased slightly to about 80 Km. Section IV is another 100 Km. downstream, from Fort Pierce, Florida to Matanilla Shoal, northwest of the Little Bahama Bank. The profile here has regained its sharp "V" profile while current width

at the surface has diverged to 85-90 Km., and maximum depth has decreased further to 700 m.

Each section consists of twelve (Sections I, IV) or thirteen (Sections II, III) stations. Measurements were taken at these stations using the free instrument technique. This technique yields data in the form of down and cross-stream current fields and vertical temperature profiles (see Tables I-IV). Each of the 50 stations was sampled over the full surface to bottom range an average of six to twelve times; each sampling consisted of between one and six instrument drops (depending on station depth), plus a surface current measurement.

Horizontal station spacing never exceeds 10 Km., and the spacing is reduced near boundaries and in the cyclonic zone where there is intensified velocity shear. Sampling of each section was conducted over a period of at least three weeks (except for Section IV, where only seventeen days of sampling were conducted due to equipment difficulties). It was hoped, by judicious sampling over this period, to minimize tidal biasing of the data and construct a time-averaged representation of the mean, steady state Florida Current at the selected stations.

B. Analysis

The free instrument method yields data in a form which can be applied in geostrophic calculations in a space-temperature (x,y,T) reference frame. The x coordinate is horizontal, has its origin at the left shore of the section (looking downstream) and increases cross-stream. The y coordinate is horizontal, parallels the axis of the stream, and is positive in the downstream direction, with its zero line along the section. The T coordinate is vertical,

positive upward.

Of the raw data obtained from the free instrument measurements, the downstream component of the time-averaged velocity ($\bar{V}obs$) and the temperature profile for each station, depths of selected isotherms were obtained. These have been used to construct smooth isotherm profiles for the four sections. Perusal of the isotherm profiles suggested selection of the 26° isotherm as the upper bound, as it stays fairly shallow (never deeper than 130 m.) and rises to the surface only at the extreme left hand edge of the stream. Likewise, the 8° isotherm was chosen as the lower bound because it is present across all of the sections at depths sufficient to include areas of relatively low velocity (less than 20 cm/sec). Intermediate isotherms of 10, 14, 18 and 22°C were chosen for convenience.

Profiles of $\bar{V}obs$ have been constructed in the same general way as the isotherms, but from vertical velocity profiles for individual stations. These contours are seen as dashed lines in all figures showing the geostrophically calculated isotachs (Figures 2-7).

A detailed derivation of the form of the geostrophic approximation used is given in Appendix A. In brief, one starts with the geostrophic equation for the downstream velocity component, (v):

$$\rho_o fv = \partial P/\partial x$$

where f is the coriolos parameter ($2\Omega\sin\phi$) and P is pressure. Ω is the angular speed of the earth, and ϕ is latitude. Taking the derivative of both sides with respect to depth, and applying the hydrostatic approximation:

$$\rho_o f\partial v/\partial z = g\partial\rho/\partial x.$$

Transforming to (x,y,T) coordinates and transposing,

$$\partial v/\partial T = g/\rho_o f\ (\partial \rho/\partial T)(\partial D/\partial x)$$

where D is isotherm depth.

Isotherm slope $\partial D/\partial x$ is computed for each of the six chosen isotherms at each station. Next the thermal shear $\partial v/\partial T$ is plotted along the selected isotherms, and velocity differences between isotherms determined planimetrically. Starting from an isotherm along which downstream component of velocity is known from direct measurements (usually the 8°C isotherm is chosen), the downstream component of the geostrophic velocity field is calculated by numerical integration of $\partial v/\partial T$ values.

At this point a density-temperature relationship must be determined. It is desired to construct a relationship of the simplest form capable of producing an acceptable reproduction of the existing velocity field when used in the geostrophic calculations. Three equations of state are compared in the extent to which they achieve this end. One equation of state used is of the form:

$$\sigma_t = \sum_{K=0}^{5} A_K T^K,$$

where the coefficients

$$A_K = A_K(x).$$

This equation will be called the variable coefficient polynomial equation (herein abbreviated VCP). The coefficients (A_K) have been determined from hydrographic data by fitting the data to a least squares polynomial of order K, where K is the smallest integer such that the RMS deviation of the fit has a maximum of .1 σ_t. The hydrographic data used was taken by University of Miami Marine Laboratory personnel at 8 stations in the Fowey Rocks - Gun Cay region,

and has been presented in the form of a table of coefficients by Schmitz and Richardson (1966). Interpolation has been made where necessary to account for differences between the locations of the hydrographic and the free instrument stations.

Another ρ-T relationship used is a parabloic equation:

$$\rho = \rho_o \, [1+\alpha(T-T_o)],$$

where

$$\alpha = \alpha_o \, [1+\kappa(T-T_o)].$$

The zero subscript denotes reference quantities (i.e. based on the 8°C isotherm). The constants α_o and κ are determined on the basis of mean density characteristics of Florida Current water.

A linear equation of state was applied in the calculations at one section.

$$\rho = \rho_o \, [1+\gamma(T-T_o)]$$

The constant γ was determined as were α_o and κ.

C. Errors

All error estimates discussed in this section are considered upper bounds. It has been shown (Clausner, op. cit.) that errors in observed velocity are up to 5%, and errors in isotherm depth are also up to 5%. Since the error in x is of the order of tens of meters and isotherm slopes are calculated over a minimum distance of 5 Km., errors in isotherm slope $\partial D/\partial x$ due to errors in x are negligible. Isotherm slope errors are, then twice the error in isotherm depth or 10%.

Errors in the $\partial\rho/\partial T$ calculation depend upon the equation of state. For the VCP equation, a 3-5% error is introduced; the parabolic equation yields errors up to about 10%, and; the linear equation gives

typical errors up to about 50%. Conditions are considerably worse at the left edge of the stream than in the middle or right sections, but this area of extreme errors is disregarded in the above estimates.

Errors in thermal shear are merely accumulations of the $\partial\rho/\partial T$ and $\partial D/\partial x$ errors. These values would be 15% for the polynomial equations, 20% for the parabolic equation, and 60% for the linear equation. Final geostrophic velocity is estimated to be in error by about the same amount as the thermal shear, with systematic errors increasing away from the reference isotherm.

The validity of the geostrophic approximation is determined by calculating the mean and root mean square deviations of the geostrophic velocity from the observed velocity at intervals along the observed isotachs. The calculations are of the form:

$$\bar{\Delta} = 1/N \overset{N}{\Sigma}(\Delta_i) \qquad , \qquad \Delta_{rms} = [1/N\ \Sigma(\Delta_i^2)]^{\frac{1}{2}}$$

where Δ_i indicates the derivation at the selected points. The deviations will be expressed as percentages.

In relation to any discussion of the Florida Current as a geostrophic current, one should be cognizant that Webster (1962) has shown that horizontal gradients of Reynolds stresses produce non-geostrophic components of velocity which average 10% of the velocity, and may reach a maximum of 25% of the velocity.

TABLE I: Section I

A. Cross stream distance

B. Isotherm depth

C. Observed velocity along isotherms

D. Coriolis parameter

Grid # → Isotherms	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
A. X TABLE															
26	*	10	15	20	30	40	50	60	70	80	90	100	103	105	**
22	6	10	15	20	30	40	50	60	70	80	90	100	103	104	**
18	**	13	15	20	30	40	50	60	70	80	90	100	103	104	**
14	**	**	18	20	30	40	50	60	70	80	90	100	103	104	**
10	**	**	**	**	**	**	51	60	70	80	90	100	102	**	**
8	**	**	**	**	**	**	57	60	70	80	90	100	**	**	**
	**	**	**	**	**	**	**	**	**	**	**	**	**	**	**
B. D TABLE															
26	*	0	17	26	41	53	69	84	91	99	110	112	112	112	**
22	20	39	59	72	87	100	111	128	143	155	166	170	170	170	**
18	**	100	108	119	138	157	177	193	217	233	243	257	260	261	**
14	**	**	170	175	189	213	260	293	320	348	380	408	418	420	**
10	**	**	**	**	**	**	393	442	495	545	589	623	630	**	**
8	**	**	**	**	**	**	550	569	628	680	729	770	**	**	**
	**	**	**	**	**	**	**	**	**	**	**	**	**	**	**
C. $\bar{V}$ TABLE															
26	*	0	32	47	65	99	116	113	105	99	96	93	67	0	**
22	0	25	42	48	53	77	97	99	93	88	85	83	61	0	**
18	**	0	25	37	40	53	72	81	79	75	72	70	52	0	**
14	**	**	0	17	23	27	44	62	64	57	49	47	37	0	**
10	**	**	**	**	**	**	0	38	47	38	23	9	0	0	**
8	**	**	**	**	**	**	0	30	35	26	14	0	**	**	**
	**	**	**	**	**	**	**	**	**	**	**	**	**	**	**
D. $f(x10^5)$ TABLE															
ALL	**	6.07	6.05	6.00	5.98	5.93	5.89	5.86	5.82	5.79	5.75	5.72	5.70	**	**

TABLE II: Section II

A. Cross stream distance

B. Isotherm depth

C. Observed velocity along isotherms

D. Coriolis parameter

Grid # → Isotherms	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
A. X TABLE															
26	5	10	15	20	25	30	35	45	55	65	75	80	83	87	90
22	6	10	15	20	25	30	35	45	55	65	75	80	83	87	89
18	8	10	15	20	25	30	35	45	55	65	75	80	83	87	88
14	9	10	15	20	25	30	35	45	55	65	75	80	83	87	**
10	**	10	15	20	25	30	35	45	55	65	75	80	83	84	**
8	**	12	15	20	25	30	35	45	55	65	75	80	81	**	**
	**	**	**	**	**	**	**	**	**	**	**	**	**	**	**
B. D TABLE															
26	0	22	40	51	62	70	78	86	90	90	90	90	90	90	90
22	44	61	80	94	105	115	123	138	150	160	166	168	169	170	170
18	80	90	108	124	141	156	170	198	230	265	289	300	304	308	308
14	103	110	140	167	191	218	240	290	337	384	421	446	458	470	**
10	**	146	192	232	273	316	357	421	490	549	599	618	628	629	**
8	**	250	281	333	376	412	447	510	580	661	728	752	756	**	**
	**	**	**	**	**	**	**	**	**	**	**	**	**	**	**
C. $\bar{V}$ TABLE															
26	0	80	116	131	139	142	140	134	124	105	84	69	57	37	0
22	0	62	94	114	126	129	129	122	111	94	77	66	57	37	0
18	0	49	78	99	111	113	113	107	94	78	63	53	47	32	0
14	0	39	61	78	88	90	91	86	74	60	49	39	27	0	**
10	**	0	26	43	49	52	57	58	49	38	30	24	17	0	**
8	**	0	0	4	13	23	35	40	34	23	15	8	0	**	**
	**	**	**	**	**	**	**	**	**	**	**	**	**	**	**
D. $f(x10^5)$ TABLE															
ALL	**	6.28	6.28	6.28	6.28	6.28	6.28	6.28	6.23	6.28	6.28	6.28	6.28	6.28	**

TABLE III: Section III

A. Cross stream distance

B. Isotherm depth

C. Observed velocity along isotherms

D. Coriolis parameter

Grid # → Isotherms	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
A. X TABLE															
26	8	10	15	20	25	30	35	45	55	65	70	75	80	83	86
22	8	10	15	20	25	30	35	45	55	65	70	75	80	83	85
18	9	10	15	20	25	30	35	45	55	65	70	75	80	83	84
14	**	10	15	20	25	30	35	45	55	65	70	75	80	83	**
10	**	10	15	20	25	30	35	45	55	65	70	75	79	**	**
8	**	13	15	20	25	30	35	45	55	65	70	75	77	**	**
	**	**	**	**	**	**	**	**	**	**	**	**	**	**	**
B. D TABLE															
26	0	12	38	49	52	55	60	65	70	74	75	76	77	78	78
22	46	60	81	90	92	94	96	106	120	135	140	142	146	147	147
18	80	83	100	109	120	133	151	180	210	239	251	262	271	278	279
14	**	105	121	143	168	192	220	284	328	362	381	397	416	421	**
10	**	135	171	203	234	284	323	397	461	527	555	586	610	**	**
8	**	252	270	315	355	400	438	508	561	621	650	677	687	**	**
	**	**	**	**	**	**	**	**	**	**	**	**	**	**	**
C. $\bar{V}$ TABLE															
26	0	70	105	127	148	157	160	159	144	123	111	102	94	68	0
22	0	49	85	112	130	138	137	131	124	115	108	101	94	68	0
18	0	45	75	98	114	119	117	112	105	98	91	85	81	60	0
14	**	0	55	83	90	87	81	79	79	78	71	63	44	0	**
10	**	0	39	53	47	41	43	50	55	52	43	27	0	**	**
8	**	0	9	12	12	12	17	29	39	38	29	17	0	**	**
	**	**	**	**	**	**	**	**	**	**	**	**	**	**	**
D. $f(x10^5)$ TABLE															
ALL	**	6.32	6.32	6.32	6.32	6.32	6.32	6.32	6.32	6.32	6.32	6.32	6.32	6.32	**

TABLE IV: Section IV

A. Cross stream distance

B. Isotherm depth

C. Observed velocity along isotherms

D. Coriolis parameter

Grid # → Isotherms	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
A. X TABLE															
26	*	34	35	40	45	50	55	65	75	85	95	105	115	**	**
22	*	30	35	40	45	50	55	65	75	85	95	105	112	**	**
18	29	30	35	40	45	50	55	65	75	85	95	105	109	**	**
14	**	**	36	40	45	50	55	65	75	85	95	105	**	**	**
10	**	**	**	42	45	50	55	65	75	85	95	101	**	**	**
8	**	**	**	**	48	50	55	65	75	85	92	**	**	**	**
	**	**	**	**	**	**	**	**	**	**	**	**	**	**	**
B. D TABLE															
26	*	0	11	50	80	99	107	120	125	127	130	131	134	**	**
22	*	0	47	85	117	139	151	161	166	172	185	200	220	**	**
18	0	18	68	112	150	176	192	212	223	240	265	295	310	**	**
14	**	**	110	140	173	212	243	284	323	355	380	405	**	**	**
10	**	**	**	210	230	270	304	354	390	428	466	484	**	**	**
8	**	**	**	**	310	325	359	430	497	567	615	**	**	**	**
	**	**	**	**	**	**	**	**	**	**	**	**	**	**	**
C. $\bar{V}$ TABLE															
26	0	0	61	93	109	128	147	146	136	123	98	59	0	**	**
22	0	0	48	67	77	101	127	139	138	122	93	56	0	**	**
18	0	17	33	42	52	75	102	122	127	111	85	53	0	**	**
14	**	**	0	18	33	54	78	102	109	93	59	0	**	**	**
10	**	**	**	0	9	27	51	81	92	81	54	0	**	**	**
8	**	**	**	**	0	11	28	57	64	37	0	**	**	**	**
	**	**	**	**	**	**	**	**	**	**	**	**	**	**	**
D. $f(x10^5)$ TABLE															
ALL	**	6.71	6.71	6.71	6.71	6.71	6.71	6.71	6.71	6.71	6.71	6.71	6.71	6.71	**

III. RESULTS

A. Geostrophic Approximation

Results of the geostrophic calculations are presented as isotachs drawn on cross-stream bathymetric profiles. Observed isotachs are superimposed on the profiles as dashed lines. Calculations of percent deviation of calculated from observed velocities disregard the area of large error which occurs at the far left (looking downstream) of the current.

The calculated downstream component of the velocity field has been computed for comparison on three different bases: first, three different equations of state have been used; second, calculations have been based on three different levels (isotherms) as reference, and; third, calculations were made at the four cross-stream sections.

In comparing velocity fields calculated on the basis of the linear, parabolic and VCP equations, the 8°C isotherm has been used as reference (Figures 2-3) and Section II was chosen as the comparison section. The linear equation yields an extremely poor approximation to the observed field: in fact, calculated surface velocities are nearly double those observed. This equation of state applied in the geostrophic approximation is clearly unacceptable. Isotachs constructed from calculations based on the parabolic equation of state tend to skew to the right, producing deviations which increase as one approaches the left extremity of the channel. This indicates that the "mean" parabolic equation is more appropriate to waters in

mid-channel and to the right thereof. Mean errors using the parabolic are 10-20%. The VCP equation of state produces geostrophically calculated isotachs in generally good agreement with observed conditions. Again, there is a tendency for the calculated isotachs to shift to the right though the effect is less pronounced than in the parabolic calculations. One cause may be the different water mass on the left side of the channel. Differences between VCP calculated and observed isotachs are quite close, quantitatively to those between parabolic calculated and observed isotachs, except, as pointed out, on the far left of the channel.

Again with Section II as the comparison section, isotachs were constructed based on the VCP equation of state using the 8°C, 18°C, and 26°C isotherms as reference (Figures 3-4). As expected the error increases as one moves away from the reference level. Largest percentage errors occur around the 20 cm/sec isotach when the 26°C isotherm is the reference. Minimum overall discrepancies exist when the 18°C isotherm is reference.

Examining the validity of the geostrophically constructed $\bar{v}$ fields at the different sections (Figures 2, 3, 5-7), the parabolic and VCP equations of state were used with the 8°C isotherm as reference. Best agreement is found at Sections I, II and III. On the average, isotachs constructed on the basis of both the VCP and parabolic equations agree to about 10-20% with observed values.

At Section IV, the largest discrepancy occurs, about 15-20%. This could be due to the influx of water of different characteristics through the Northwest Providence Channel (Finlen, 1966). In Sections II and IV the parabolic equation has given better results than the

TABLE V: Percent Deviations Between Observed and Calculated Downstream Speeds. Sections I and II.

SECTION I

8°C Reference Isotherm

Observed Velocity (cm/s)→ Equation of State ↓		140	100	60	20
Parabolic	$\bar{\Delta}$	*	10.0	7.0	3.2
	Δrms	*	17.1	13.5	9.1
VCP	$\bar{\Delta}$	*	4.2	-1.8	6.8
	Δrms	*	6.1	11.8	46.0

* Indeterminate

SECTION II

8°C Reference Isotherm

Observed Velocity (cm/s)→ Equation of State ↓		140	100	60	20
Linear	$\bar{\Delta}$	*	90.0	76.0	17.5
	Δrms	*	90.5	88.5	49.4
Parabolic	$\bar{\Delta}$	5.0	8.8	-8.3	-3.1
	Δrms	5.0	17.5	16.0	8.8
VCP	$\bar{\Delta}$	5.0	6.3	-10.4	0.0
	Δrms	5.0	13.5	15.7	7.1
	18°C Reference Isotherm				
VCP	$\bar{\Delta}$	10.0	-2.0	-16.7	-28.5
	Δrms	10.3	5.5	19.8	35.0
	26°C Reference Isotherm				
VCP	$\bar{\Delta}$	0.0	-27.0	-21.5	-37.5
	Δrms	0.0	31.0	24.6	41.5

TABLE VI: Percent Deviations Between Observed and Calculated Downstream Speeds. Sections III and IV.

SECTION III

8°C Reference Isotherm

Observed Velocity (cm/s)→ Equation of State ↓		140	100	60	20
Parabolic	$\bar{\Delta}$	-1.0	1.3	-3.3	-2.5
	Δrms	16.4	15.0	6.6	7.9
VCP	$\bar{\Delta}$	-3.6	-4.0	-9.5	-5.0
	Δrms	30.0	15.8	12.6	11.2

SECTION IV

8°C Reference Isotherm

Observed Velocity (cm/s)→ Equation of State ↓		140	100	60	20
Parabolic	$\bar{\Delta}$	-0.7	-7.5	2.8	13.5
	Δrms	3.8	17.4	14.4	23.0
VCP	$\bar{\Delta}$	-6.4	-14.0	-14.7	15.0
	Δrms	7.9	16.6	24.8	19.6

FIGURE 2. a. GEOSTROPHIC ISOTACHS - Section II

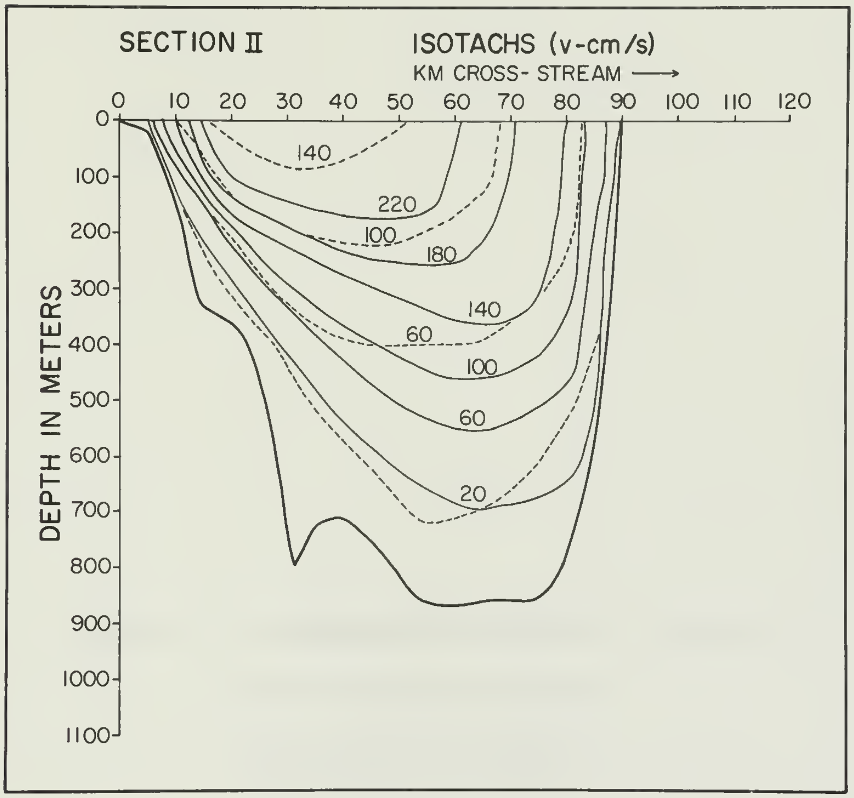
8°C reference isotherm

Linear equation of state

b. GEOSTROPHIC ISOTACHS - Section II

8°C reference isotherm

Parabolic equation of state


SECTION II
ISOTACHS (v-cm/s)
KM CROSS- STREAM
0 10 20 30 40 50 60 70 80 90 100 110 120
DEPTH IN METERS
0 100 200 300 400 500 600 700 800 900 1000 1100
140
220
100
180
140
60
100
60
20

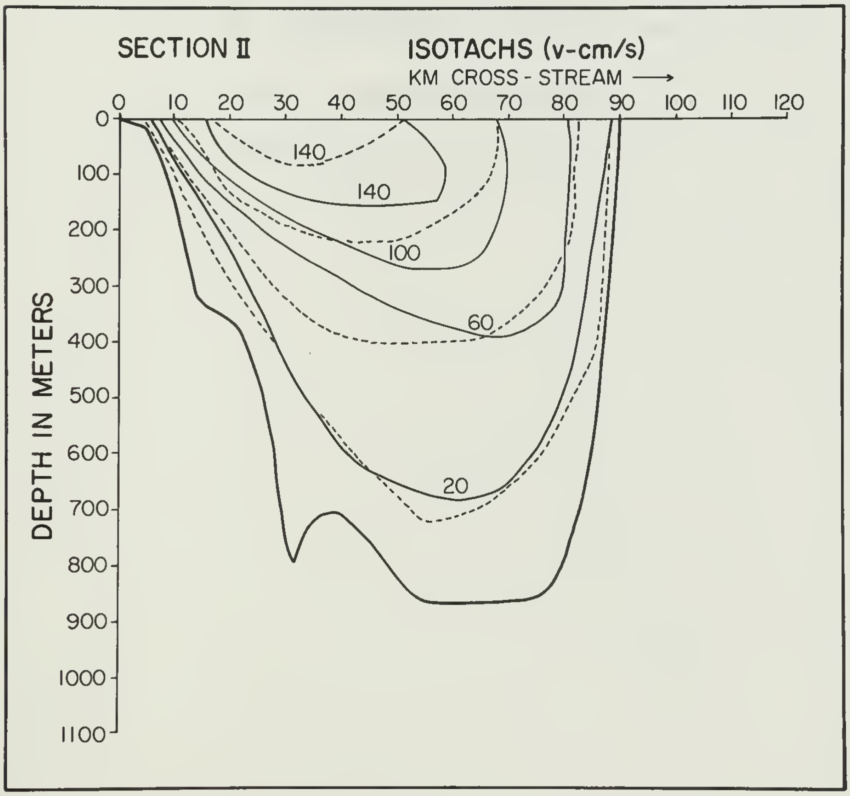

SECTION II
ISOTACHS (v-cm/s)
KM CROSS - STREAM
0 10 20 30 40 50 60 70 80 90 100 110 120
DEPTH IN METERS
0 100 200 300 400 500 600 700 800 900 1000 1100
140
140
100
60
20

FIGURE 3. a. GEOSTROPHIC ISOTACHS - Section II

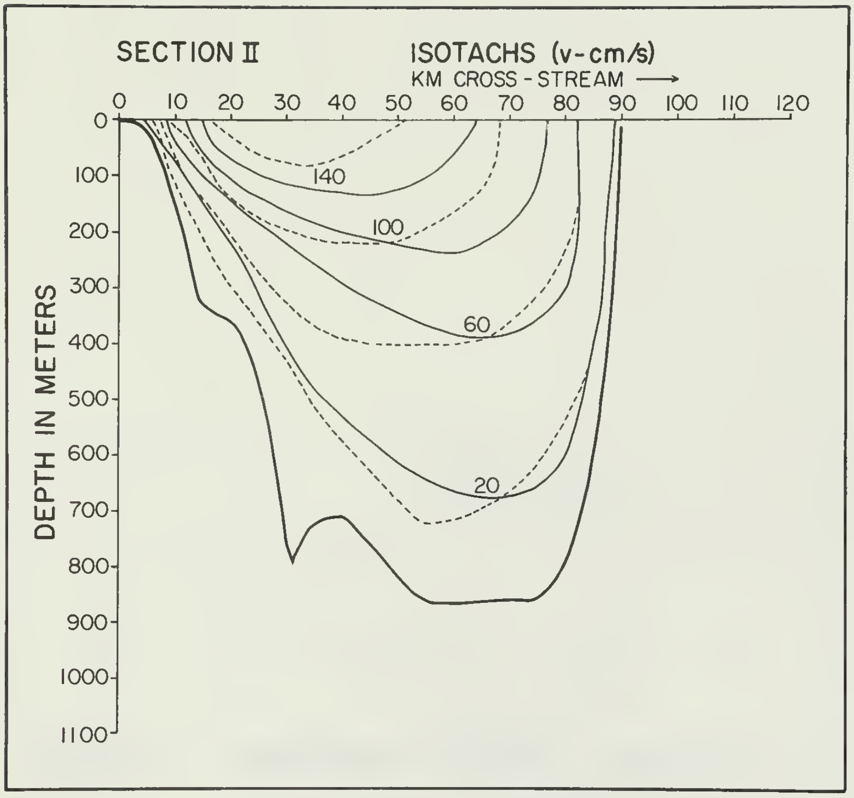
8°C reference isotherm

VCP equation of state

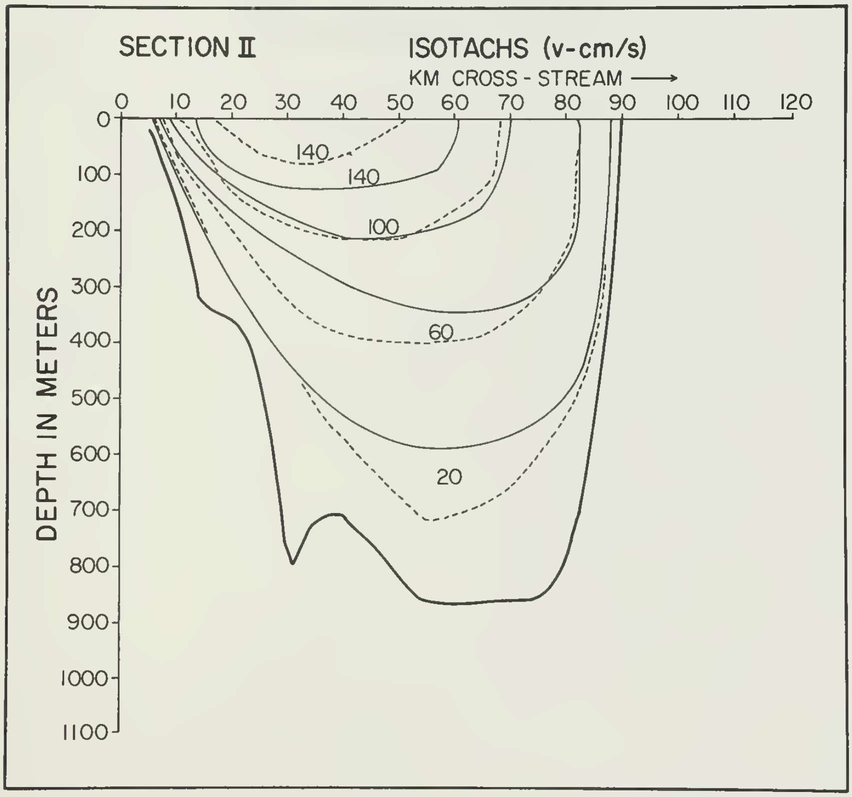
b. GEOSTROPHIC ISOTACHS - Section II

18°C reference isotherm

VCP equation of state



SECTION II
ISOTACHS (v-cm/s)
KM CROSS-STREAM →
0 10 20 30 40 50 60 70 80 90 100 110 120
DEPTH IN METERS
0 100 200 300 400 500 600 700 800 900 1000 1100
140
100
60
20

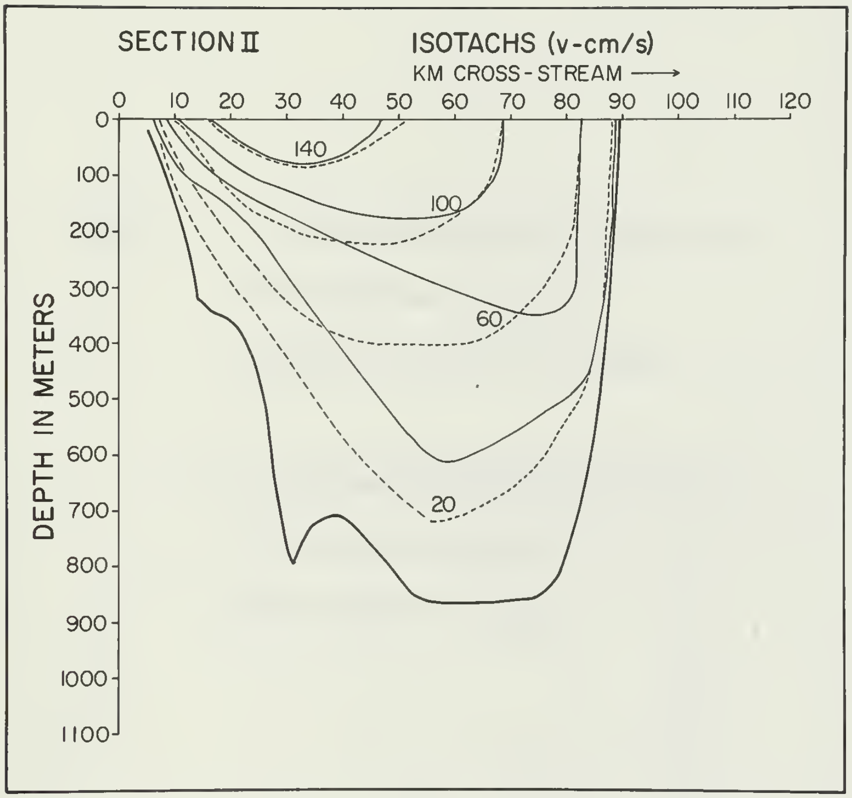


SECTION II
ISOTACHS (v-cm/s)
KM CROSS-STREAM →
0 10 20 30 40 50 60 70 80 90 100 110 120
DEPTH IN METERS
0 100 200 300 400 500 600 700 800 900 1000 1100
140
140
100
60
20

FIGURE 4. GEOSTROPHIC ISOTACHS - Section II

26°C reference isotherm

VCP equation of state



SECTION II
ISOTACHS (v-cm/s)
KM CROSS-STREAM
0 10 20 30 40 50 60 70 80 90 100 110 120
DEPTH IN METERS
0 100 200 300 400 500 600 700 800 900 1000 1100
140
100
60
20

FIGURE 5. a. GEOSTROPHIC ISOTACHS - Section I

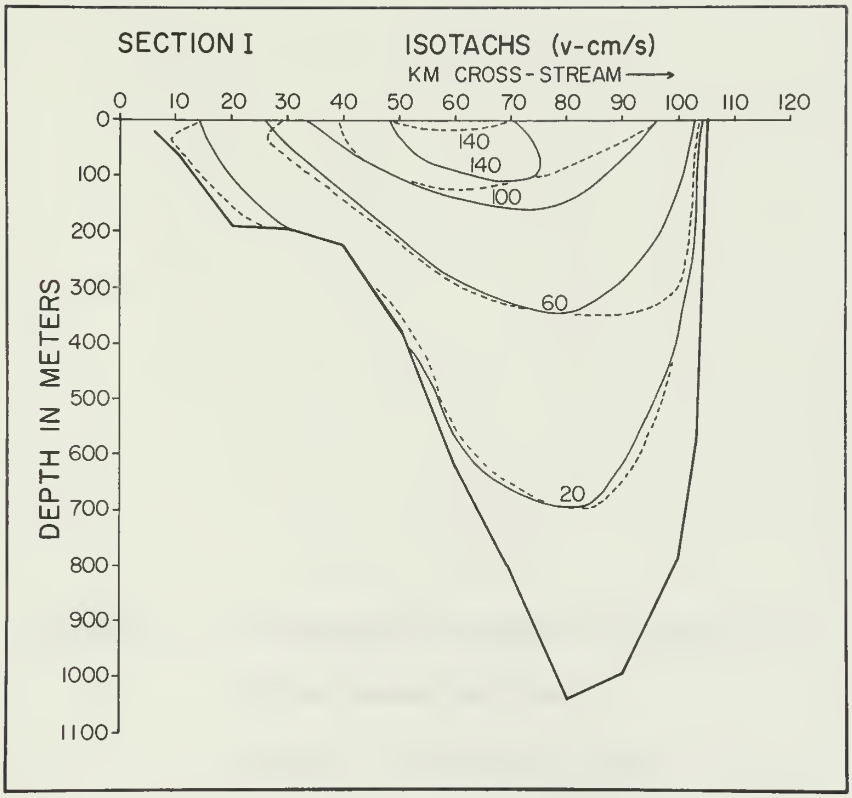
8°C reference isotherm

Parabolic equation of state

b. GEOSTROPHIC ISOTACHS - Section I

8°C reference isotherm

VCP equation of state


SECTION I
ISOTACHS (v-cm/s)
KM CROSS-STREAM
0 10 20 30 40 50 60 70 80 90 100 110 120
DEPTH IN METERS
0 100 200 300 400 500 600 700 800 900 1000 1100
140
140
100
60
20

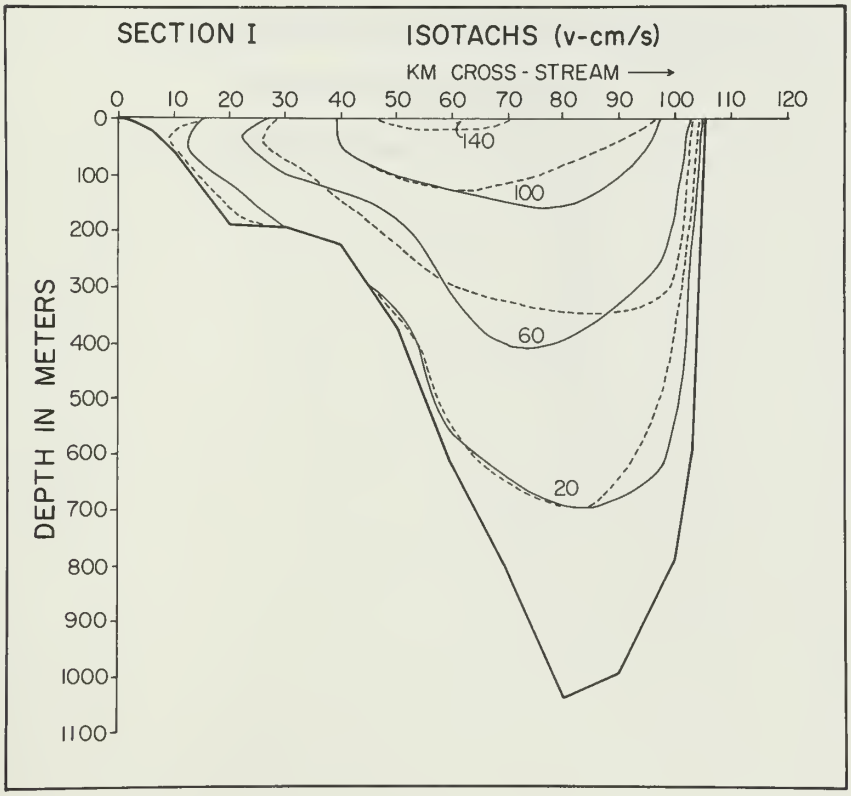

SECTION I
ISOTACHS (v-cm/s)
KM CROSS-STREAM
0 10 20 30 40 50 60 70 80 90 100 110 120
DEPTH IN METERS
0 100 200 300 400 500 600 700 800 900 1000 1100
140
100
60
20

FIGURE 6. a. GEOSTROPHIC ISOTACHS - Section III

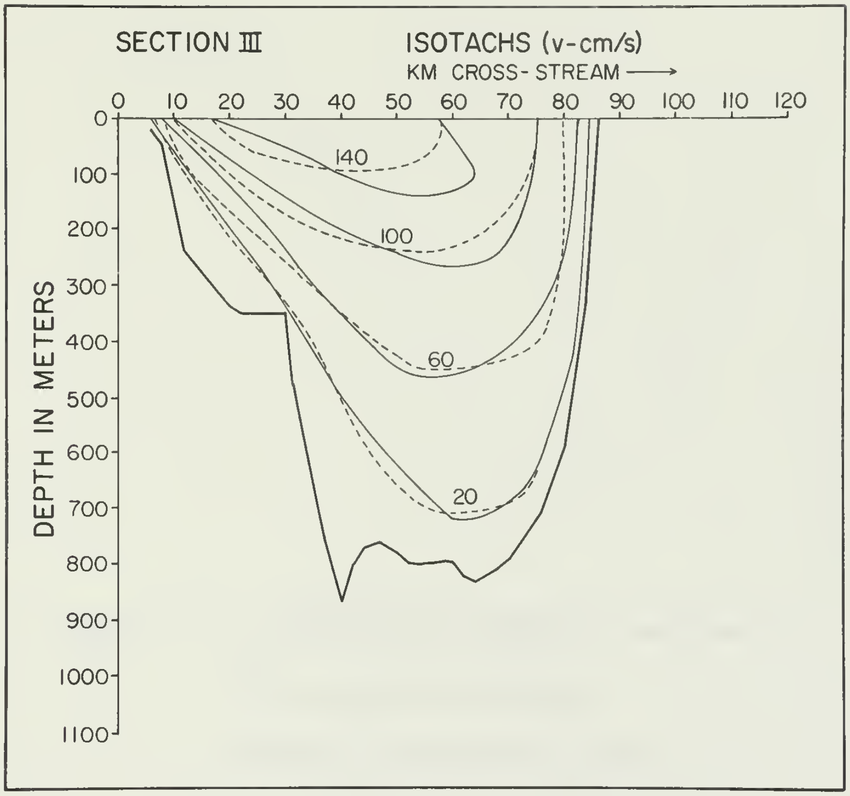
8°C reference isotherm

Parabolic equation of state

b. GEOSTROPHIC ISOTACHS - Section III

8°C reference isotherm

VCP equation of state


SECTION III
ISOTACHS (v-cm/s)
KM CROSS-STREAM ⟶
0 10 20 30 40 50 60 70 80 90 100 110 120
DEPTH IN METERS
0 100 200 300 400 500 600 700 800 900 1000 1100
140
100
60
20

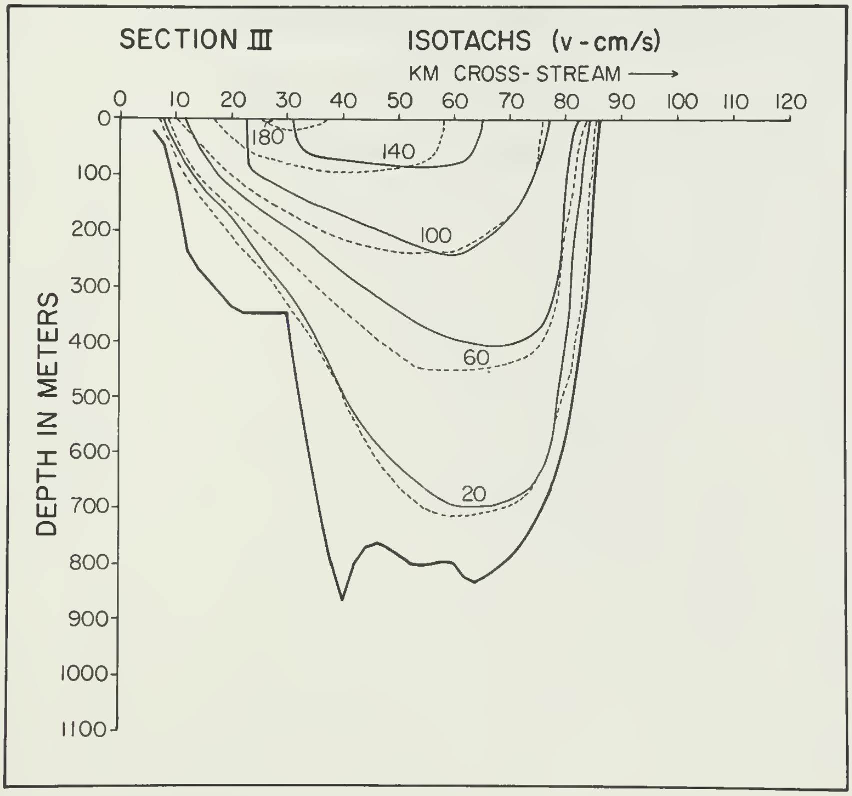

SECTION III
ISOTACHS (v-cm/s)
KM CROSS-STREAM ⟶
0 10 20 30 40 50 60 70 80 90 100 110 120
DEPTH IN METERS
0 100 200 300 400 500 600 700 800 900 1000 1100
180
140
100
60
20

FIGURE 7. a. GEOSTROPHIC ISOTACHS - Section IV

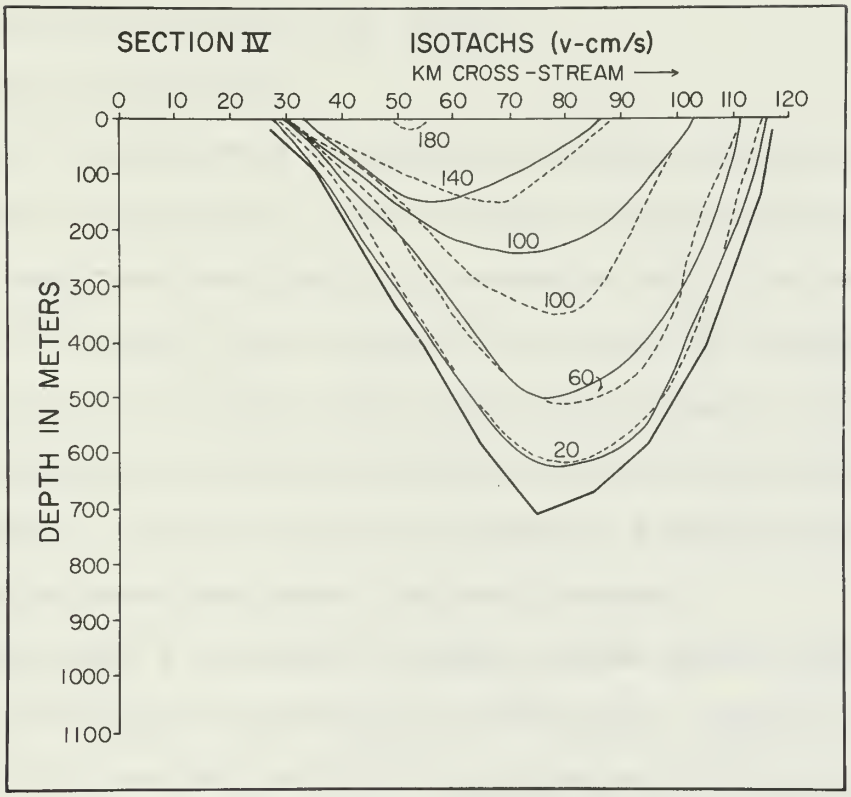
8°C reference isotherm

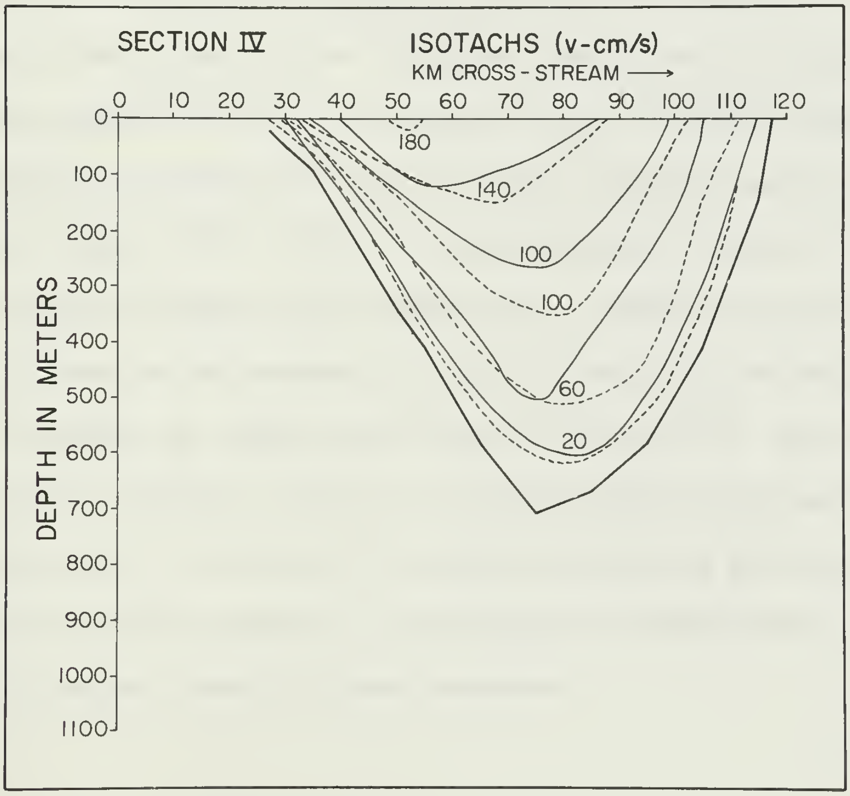
Parabolic equation of state

b. GEOSTROPHIC ISOTACHS - Section IV

8°C reference isotherm

VCP equation of state


SECTION IV
ISOTACHS (v-cm/s)
KM CROSS-STREAM
DEPTH IN METERS
180
140
100
100
60
20


SECTION IV
ISOTACHS (v-cm/s)
KM CROSS-STREAM
DEPTH IN METERS
180
140
100
100
60
20

VCP equation for the bulk of the current.

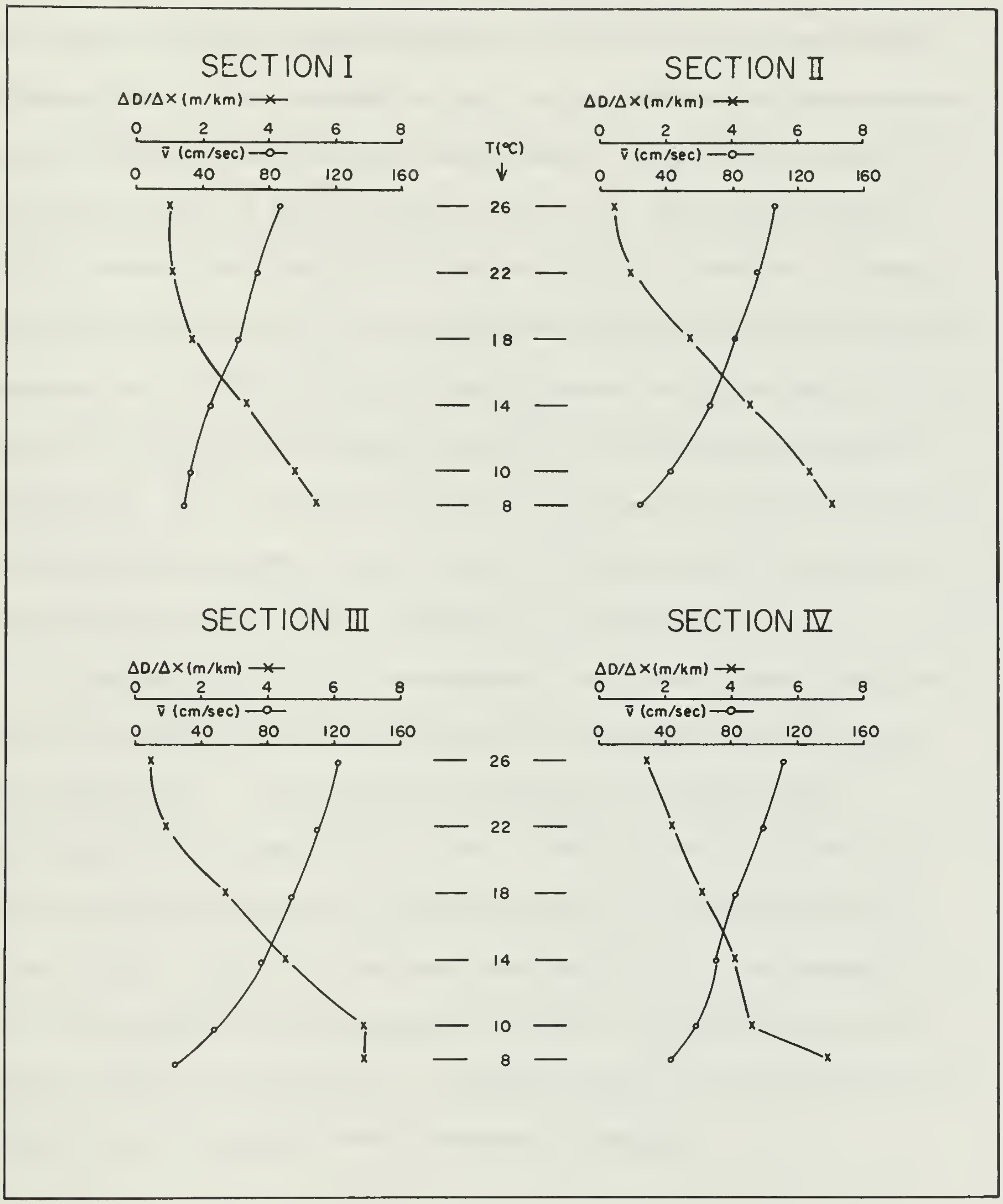
B. Mass Field Adjustments

Mass field adjustment in response to changing downstream velocity components is displayed as a plot of average isotherm slope and average downstream speed along the isotherms at each of the sections (Figure 8). Average slope of each of the six selected isotherms is plotted, as is the average velocity along each isotherm. The average isotherm slope was determined graphically from isotherm profiles for the sections. Average velocity was computed as a number average of the observed downstream speeds along each isotherm.

From Section I to Section II speeds increase from the upper waters (26°C region) down through the 10°C region. Isotherm slope, though less at the 26°C and 22°C lines, increases markedly below the 22°C level indicating, for reasons of continuity, deepening of the isotachs as the isotherms rise. From Sections II to III little difference appears in the plots, except for the increased slope of the 10°C isotherm at Section III, which reflects moderate speed increase in the mid layers (22°C - 10°C). Between Sections III and IV the velocity profile changes quite differently. Speeds are lower at Section IV down to the vicinity of the 14°C isotherm, and below this, Section IV speeds are higher than those of Section III. Down to just above the 14°C isotherm, Section IV isotherm slopes are greater. Below this point, the Section III isotherm slopes are greater, especially the 10°C isotherm. This reflects the first marked increase in speeds along the lower isotherms.

FIGURE 8. Mass field - velocity adjustment

Sections I-IV


SECTION I
SECTION II
SECTION III
SECTION IV
ΔD/ΔX (m/km)
v̄ (cm/sec)
T(°C)
0 2 4 6 8
0 40 80 120 160
26
22
18
14
10
8

C. Earlier Work

The geostrophic calculations of Wüst (op. cit.) and the agreement of his results with the direct measurements of Pillsbury (1890) have done much to convince oceanographers of the value of the geostrophic approximation. It is remarkable that, using temperature measurements taken in 1878-81, some salinity measurements taken in 1914 and a T-S correlation based on North Atlantic waters (to cover zones where no salinity values were available) Wüst was able to construct a density field which yielded such good results when compared with the Pillsbury measurements of 1885-1886. His results agree (in the vicinity of our Section II) with those obtained using the free instrument method to roughly 20%. It is interesting that the isotachs resulting from his calculations are skewed to the right (as compared with the observed isotachs) much as are those calculated in this thesis.

In his analysis of five hydrographic stations between Miami and Bimini, B.I., Parr (op. cit.) showed the variations in the density field with time. In particular he described the cross-stream oscillations of a high salinity core. Broida (1966) has shown how this variation in the mass field can result in a distorted picture of the current field as calculated geostrophically. One example of such a distortion is the appearance of a bi- or multi-axial surface current profile cross-stream, a phenomenon which is not evident using the more nearly synoptic free instrument technique.

IV. SUMMARY AND CONCLUSIONS

During the summer months of 1965 and 1966, four sections across the Florida Straits (over a 225 Km. downstream distance) were sampled using the free instrument technique, over time and space scales designed to yield a picture of a time-averaged, steady state Florida Current. From this series of measurements profiles were constructed for each section of the downstream component of velocity and of depth of selected isotherms to provide a basis for evaluation of geostrophic velocity fields.

Based on hydrographic data obtained in 1962-63 in the Fowey Rocks - Gun Cay area, three equations of state were developed in which density was expressed as a function of temperature only. A fifth order polynomial equation was used with coefficients a function of cross-stream distance; a simple parabolic equation was applied, and; a linear equation was employed at one section. Each of these equations was used in the geostrophic equation for downstream velocity in a space-temperature coordinate system, to determine the validity of this form of the geostrophic equation. Three different levels were tested as reference for the calculations. These were the 8°C, 18°C and 26°C isotherms, along which velocity was accurately known from the direct measurements. Velocity fields from the geostrophic calculations have been compared with those obtained by direct measurement.

Lastly, an attempt has been made to show the adjustment of the

mass field (specifically the temperature field) to changes in the downstream component of the velocity field as the current is accelerated downstream.

Results of the experiment discussed in this thesis point to the conclusion that, under certain conditions, the geostrophic approximation yields a model of the downstream component of the velocity field which is a first order approximation to the directly observed field. For most of the current, geostrophically calculated velocities agree within experimental error with directly measured velocities. Necessary conditions for such a conclusion include the use of an accurately known reference level and use of an equation of state in which density is expressed in terms of temperature by a second (or higher) order equation of state. Constants for the equation must be determined on the basis of local hydrographic conditions. These results confirm the validity of using the geostrophic approximation in theoretical models of inertial flow.

Furthermore, it has been shown that there is a direct relationship between the mass distribution (specifically the temperature) and the downstream component of the velocity field.

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APPENDIX A: DERIVING THE PARTICULAR FORM OF THE GEOSTROPHIC EQUATION

Robinson (ibid.) has stated that the downstream component of an inertial flow is in geostrophic balance. In (x,y,z) coordinates,

$$\rho_o f v = \partial P/\partial x$$

where the use of a constant ρ_o is an approximation resulting in less than 1% error. Taking the derivative with respect to depth of the above equation and applying the hydrostatic approximation;

$$\rho_o f(\partial v/\partial z) = -g(\partial \rho/\partial x).$$

One may transform to (x,y,T) coordinates by:

$$(\partial v/\partial z)_{x,y} = (\partial v/\partial T)_{x,y}(\partial T/\partial z)_{x,y}$$

and;

$$(\partial \rho/\partial x)_{y,z} = -(\partial \rho/\partial T)(\partial D/\partial x)_{y,T}(\partial T/\partial z)_{x,y}$$

which yields:

$$\partial v/\partial T = g/f\rho_o(\partial \rho/\partial T)(\partial D/\partial x)$$

where D is the depth of an isotherm T.

It now remains to take the temperature derivative of the three equations of state, which is straightforward and gives the results shown below.

Linear equation;

$$\partial \rho/\partial T = \rho_o \gamma$$

Parabolic equation;

$$\partial \rho/\partial T = \rho_o \alpha_o [1+2\kappa(T-T_o)]$$

VCP equation;

$$\partial \rho/\partial T = \Sigma\ [K A_K T^{(K-1)}]$$

VITA

LT. Edward J. O'Brien III, USN, was born in Pittsburgh, Pennsylvania, on November 25, 1939. His parents are Edward J. O'Brien Jr. and Barbara B. O'Brien. He received his elementary education at St. Mary School, Baltimore, Maryland, and his secondary education at Loyola High School, Towson, Maryland.

In July, 1957, he entered the U.S. Naval Academy, Annapolis, Maryland. Upon graduation in June, 1961, with a B.S., he was commissioned an Ensign in the U.S. Navy. Subsequent Naval service included three years in the Destroyer Force of the Pacific Fleet followed by a year as Instructor of Naval Science at College of the Holy Cross, Worcester, Massachusetts.

He was admitted to the Graduate School of the University of Miami in September, 1965. He was granted the degree of Master of Science in June, 1967.

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