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by

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SUMMARY

The simulation of a turbojet engine on a standard electronic analogue computer is presented. The non-linear equation for the static and transient behavior of the engine was developed and solved on the computer without reducing the equation to a linear form. A typical control was simulated and used to operate the engine under transient and steady state conditions.

The results indicate that a reasonably accurate simulation of an actual engine on a standard computer without special equipment is possible. Control studies may be made using a variety of engine parameters which are continuously generated during transient as well as equilibrium operation. The control studied indicates that the control coefficients should be made functions of engine speed to obtain maximum acceleration. It is also shown that fuel flow can be positively limited to maintain a given maximum turbine inlet temperature during acceleration in order to obtain maximum acceleration.

INTRODUCTION

In a turbojet engine the speed responds compara-
tively slowly to changes in fuel flow. A control charged
with the responsibility of accelerating the engine must
change the fuel flow at a rate that will bring the engine
to the new speed as quickly as possible, but not so fast
that excessive turbine temperatures are encountered. Thus
to design an optimum control the designer must know the
response rate of the engine variables that he is control-
ling. This requires a knowledge of the dynamic behavior
of the engine.

One approach to the problem has been to predict the
engine dynamics from steady state performance charts of the
engine components using the hypothesis that the thermody-
namics and flow processes are quasi-static; that is, they
act like a continuous series of equilibrium states (Ref. 1).
Then further, the non-linear differential equation for
engine acceleration was linearized around a steady state
operating point. At points far from the steady state condi-
tions the linear equation may result in significant error,
hence, in order not to exceed engine limitations, the control
must program fuel on the conservative side. This procedure
results in an acceleration less than the optimum possible.

The study contained herein consists of an analogue
computer solution of the non-linear equation for accelera-
tion. The computer solution furnishes continuous informa-

tion on the response of all engine variables. A control can then sense the change during acceleration of various engine parameters and generate a fuel flow which will maintain the maximum allowable turbine inlet temperature throughout the acceleration. This procedure should provide the maximum acceleration.

Engine design calculations are shown in Appendix B. Equilibrium running calculations are shown in Appendix C.

This work was performed at the University of Michigan by Lt. Charles H. Rockcastle, USN and Lt. Andrew J. Yates, USN under the supervision and guidance of Dr. James E. Broadwell.

ANALYSIS AND PROCEDURE

Analysis

A typical configuration of a direct coupled turbojet engine is shown in Fig. 1.

When turbine torque equals compressor torque a turbojet engine will run at some equilibrium speed. A change in fuel flow causes a change in turbine inlet temperature which results in a change in turbine torque. The difference between turbine torque and the torque absorbed by the compressor then accelerates or decelerates the engine according to the following equation:

$$Q = I\propto \qquad\qquad (1)^{[1]}$$

Equation (1) may be written:

$$\frac{\dfrac{P_t}{w\,T_2} - \dfrac{P_c}{w\,T_2}}{N/\sqrt{T_2}} = I\;\frac{d\;(N/\sqrt{T_2})}{dt} \qquad (2)$$
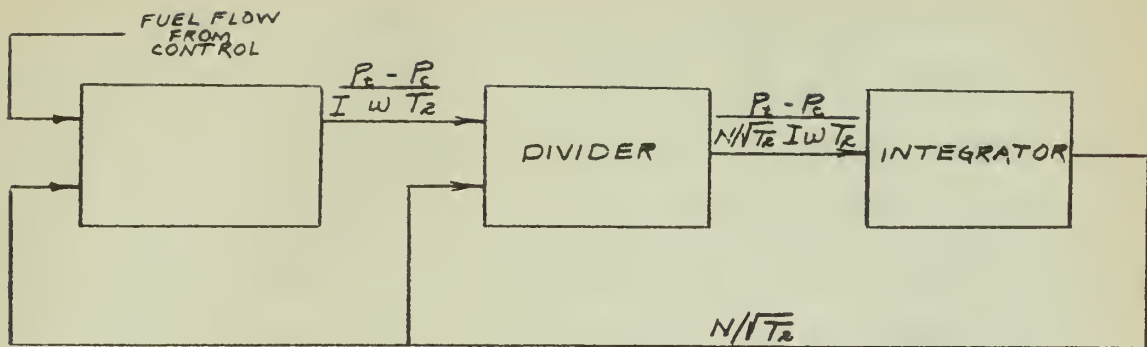
where $\dfrac{P_t}{w\,T_2}$ and $\dfrac{P_c}{w\,T_2}$ are turbine and compressor power

parameters as developed in Appendix C and $N/\sqrt{T_2}$ is the

speed parameter.[2] The use of these parameters keeps the

analysis quite general and permits altitude corrections to

the computer circuit by simply changing the effective

moment of inertia.

If engine speed and fuel flow are used as independent variables then compressor power and turbine power can be computed as will be shown in detail in the next section. Hence, equation (2) may be solved on an analogue computer as is indicated schematically on following page:

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[1] All symbols are defined in Appendix A.

[2] Henceforth such parameters as these will be referred to as simply "turbine power", etc. Definitions will be found in Appendix A.

FUEL FLOW FROM CONTROL

$\dfrac{P_t - P_c}{I\,\omega\,T_2}$

DIVIDER

$\dfrac{P_t - P_c}{N/\sqrt{T_2}\;\;I\,\omega\,T_2}$

INTEGRATOR

$N/\sqrt{T_2}$

A standard electronic analogue computer, Reeves
Computer C-101 Mod 5, was used for the engine simulation.
No special equipment was necessary.

Engine Simulation on the Computer

As previously described, the basic equation used
for the operation of the engine is:

$$\frac{P_t}{w\,T_2} - \frac{P_c}{w\,T_2} = I\;(N/\sqrt{T_2})\;\frac{d(N/\sqrt{T_2})}{dt} \qquad (3)$$

In the simulation of the components of this equa-
tion the two independent variables, fuel flow and engine
speed, were used for the following reasons: (1) fuel flow,
as it represents a quantity supplied by the control circuit
and, therefore, isolates the simulated engine from the con-
trol, (2) engine speed, as it is almost universally used as
a thrust control parameter and may be easily and accurately
measured in an actual engine. In this study fuel flow was
supplied by the control circuit (to be described later) and

engine speed was determined by the engine circuit and fed back as an input.

For the engine simulation, circuits were set up to simulate the compressor, the burner, and the turbine (including exhaust nozzle). The operation of these components then determined the inputs to the basic engine equation (Equation 3) which was solved for the engine running speed. Thus, an actual engine was simulated and the same engine parameters were made continuously available in the computer solution that would be available in an actual engine.

Compressor Simulation. For derivation of an equation to be used for the generation of compressor pressure ratio, Fig. 3 was plotted from Fig. 2. From this plot the equation for compressor pressure ratio (for design turbine inlet temperature) was approximated by:

$$(P_3/P_2)_{design} = .01419 \ (N/\sqrt{T_2}) - 0.65 \qquad (4)$$

An empirical relationship was derived for compressor pressure ratio for an arbitrary value of turbine inlet temperature:

$$P_3/P_2 = (P_3/P_2)_{design} + (T_4/T_2 - 1) \ (.01035 \ N/\sqrt{T_2} - 1.45) \qquad (5)$$

This value was then used in the circuits for generation of turbine inlet temperature and compressor power.

The equation for compressor power (given in exact form in equation (C7)) was plotted versus compressor pressure ratio in Fig. 5. This was approximated by:

$$\frac{P_c}{w \ T_2} = .0152 + .0305 \ (P_3/P_2) \qquad (6)$$

Burner Simulation. For burner simulation it was necessary to convert fuel flow to turbine inlet temperature. The equation used for this purpose was:

$$T_4/T_2 = (P_3/P_2)^{\frac{k-1}{\gamma_c}} + \frac{k \, \eta_b \, (w_f/P_2 \sqrt{T_2})}{(w_a \sqrt{T_2}/P_2)} \qquad (7)$$

At this point it was decided to include burner efficiency with fuel flow for further calculations. From Fig. 2, a cross plot of air flow versus engine speed for different turbine inlet temperatures (Fig. 4) was made. The approximate equation below was found:

$$\frac{w_a \, T_2}{P_2} = .710 + .002825 \, (N/\sqrt{T_2} - 310) \qquad (8)$$

The term including the compressor pressure ratio was approximated as follows:

$$(P_3/P_2)^{\frac{\gamma_c - 1}{\gamma_c}} = 1.442 + .11 \, (P_3/P_2 - 3.6) \qquad (9)$$

An approximate value of k was assumed to be 80,000. Thus, if the input of fuel flow from the control circuit is multiplied by k, divided by airflow, and added to the function of the compressor pressure ratio, the resulting value is the turbine inlet temperature. This output was then divided by the design value of the turbine inlet temperature in order to obtain a value in percent which was used in all subsequent circuits.

Turbine Simulation. An expression for turbine power was known as a function of P_5/P_4 and T_4/T_2. A graphical determination was made of turbine power as a function of com-

pressor pressure ratio for T_4/T_2 of 100% (Fig. 5). However, in determining an approximate relationship to be used in the computer circuit in the area of interest where the turbine nozzles are choked, the turbine power was found to be a function of turbine inlet temperature only. This relationship was given by:

$$\frac{P_t}{w\,T_2} = .1372 \ (T_4/T_2)_{\% \ design} \qquad (10)$$

It will be noted that several complicated functions have been approximated by straight line equations in order to simplify their use on the computer. If greater accuracy is desired, function generators may be used for these expressions. Observation of this circuit in operation, however, indicated sufficient accuracy for this study.

This completed the determination of equations for the components of the basic engine equation. The difference between turbine and compressor powers was divided by engine speed. The resultant quantity, when divided by the moment of inertia of the engine, was equal to the derivitive of the engine speed which was integrated to determine engine speed. The speed was then fed back to the circuit components where needed. Suitable scale factors were introduced where necessary throughout the circuit. Fig. 6 shows the schematic diagram of the computer circuit.

It will be noted that within this circuit are explicit outputs of compressor pressure ratio, turbine and compressor powers, turbine inlet temperature, and air flow.

All may be plotted, observed while the engine is running, or made available for control parameters if desired.

Description of an Illustrative Control

For an illustrative control it was decided to use the difference between the operating engine speed and the desired or set engine speed, as determined by the throttle position, as the controlling parameter. The control delivers fuel flow proportional to the difference between these speeds and proportional to the integral of the difference between these speeds. The proportional control is used for speed of response and the integral control for elimination of steady state errors. The equation for the control is:

$$\frac{w_f \, \eta_b}{P_2 \sqrt{T_2}} = \beta \int\!\!\left(\frac{N_s - N}{T_2}\right) dt + \epsilon \left|\frac{N_s - N}{T_2}\right| \tag{11}$$

A preliminary theoretical analysis indicated that the control coefficients β and ϵ should each be approximately 0.50. The control was then set up on a second computer and applied to the previously determined engine circuit. Runs were made with β set at one of several values (of the order of magnitude indicated by the preliminary analysis) with a range of values for ϵ for each value of β. In this manner an optimum combination was determined to be 0.65 for β and 0.50 for ϵ.

An upper limit of turbine inlet temperature is critical within the engine. Therefore, some method of limiting fuel flow is necessary so that this predetermined maximum level of temperature is reached and maintained, but not exceeded, during acceleration. A continuously generated limit was obtained by rearranging equation (7) to give:

$$\frac{w_f \eta_b}{P_2 \sqrt{T_2}} = (w_a \sqrt{T_2}/P_2) \left[\frac{T_4/T_2 - (P_3/P_2)^{\frac{\gamma_c - 1}{\gamma_c}}}{k} \right] \quad (12)$$

In this equation, when the turbine inlet temperature is made a constant at the desired limiting value and is combined in a circuit with the function of compressor pressure ratio and air flow generated in the engine circuit, a limiting value of the fuel flow parameter is continuously generated.

This value is then introduced into the control circuit as an electronic limiter (equivalent to controlling a by-pass valve in an engine). Thus, when the control circuit generates a value of fuel flow that exceeds the limiting value, this limiting value takes over and controls the engine. Fig. 6 shows the control circuit with the limiter in schematic form.

This circuit does not take into consideration physical limitations of a specific burner. Therefore, in a given engine the maximum rate of introduction of fuel as indicated by this control circuit might result in burner blowout. Such a condition could be avoided by changing the values of β and ϵ . An extremely low value of fuel flow

might also cause burner blow-out. A control limiter based
on compressor pressure ratio could be incorporated to insure
against this condition. No such electronic limiting has been
incorporated in this study.

Engine Operation

To adequately show the results of this study three
types of response of the engine were obtained. The first
was the response of the engine to an accelerating step input
of set engine speed. The second was the response of the
engine to an accelerating ramp input of set engine speed.
Then, to show the response of the engine to a decelerating
command, a decelerating step input of set engine speed was
made. In each of these three types of response the engine
was accelerated or decelerated to several equilibrium run-
ning speeds.

In order to obtain the behavior of the engine at
maximum limits of acceleration and deceleration one of the
equilibrium engine speeds used was that determined by allow-
ing the turbine inlet temperature to remain at 110% of the
design operation. A limiting turbine inlet temperature of
115% of the design operation was used to allow for reason-
able time to accelerate to the maximum speed. This was
thought compatible with the concept of "military power",
where engine running time would be limited but would be
available in case of emergency. With this concept it will
be noted that minimum time to accelerate to the high speed

condition would be vital, so the control coefficients were set to obtain a minimum accelerating time to this condition at some sacrifice in acceleration to lower speeds. Such limits on an actual engine would, of course, be set with regard to the specific burner and turbine in use on that engine.

RESULTS AND DISCUSSION

Engine Acceleration - Step Input. The engine was accelerated from an initial equilibrium speed to several final speeds by using step inputs to the control set speed. The initial speed of $N_o/\sqrt{T_2}$ of 280 represents equilibrium for a turbine inlet temperature of approximately 80% of design operation. The control set speeds, $N_s/\sqrt{T_2}$, were selected to obtain final turbine inlet temperatures of 90%, 100%, and 110% of design operation. Plots of the following engine variables versus time were made:

1. Speed
2. Fuel Flow
3. Turbine Inlet Temperature
4. Air Flow
5. Compressor Pressure Ratio
6. Accelerating Torque

The results are shown in Figs. 7 through 12.

Fig. 7 shows the speed response of the engine. A slight overshoot for $N_s/\sqrt{T_2}$ of 345 may be observed. This overshoot was tolerated to reduce the time to accelerate to the equilibrium speed. It may be controlled by varying the

control coefficients β and ϵ. It will be noted that there is no overshoot when accelerating to the lower equilibrium speeds. If the control coefficients were made to vary with engine speed, however, a slight overshoot could result for all values of $N_s/\sqrt{T_2}$ and acceleration time would be improved throughout the speed range.

The time to accelerate from $N_o/\sqrt{T_2}$ to $N_s/\sqrt{T_2}$ of 345 is approximately eight seconds. This represents an acceleration of about 1500 RPM under standard conditions. Comparison with acceleration data given in Ref. 2 shows that this time is typical of an actual engine.

Fig. 8 shows the variation of fuel flow. As would be expected, the fuel flow rapidly increases to some maximum value and then decreases to the new equilibrium value. For the set speed of 345 the fuel flow is limited (by-passed in an actual engine) so as not to exceed the upper limit of turbine inlet temperature. A run without the limiter was made for comparison purposes. It is noted that the limited fuel flow is not constant but varies with time.

Fig. 9 shows the variation of turbine inlet temperature. It is seen that the variable limited fuel flow shown in Fig. 8 produces a constant maximum turbine inlet temperature of 115%.

The effect of limiting on the compressor pressure ratio and torque may be observed in Figs. 11 and 12.

Engine Acceleration - Ramp Input. Acceleration runs between the same initial and final conditions listed above were made for a slow ramp input to the control. The set

speed was made to vary at the rate of approximately 100 RPM per second. Variation of engine speed, fuel flow and turbine inlet temperature versus time were plotted. These results are shown in Figs. 13 through 15.

Fig. 13 shows that the engine speed parallels the control set speed with a time lag of about one-half second. Figs. 14 and 15 show that very little limiting is encountered for a slow ramp input.

Engine Deceleration - Step Input. Deceleration runs were made using a step input of $N_s/\sqrt{T_2}$ of 280. Initial speeds for 110%, 100% and 90% of design turbine inlet temperatures were used. Variation of engine speed, fuel flow, and turbine inlet temperature versus time were plotted. The results are shown in Figs. 16 through 18. Deceleration time is about seven seconds. Comparison with data given in Ref. 2 indicates that this time is typical of an actual engine. Again, if the control coefficients were made to vary with engine speed, time to decelerate from the lower speeds would be reduced.

Fig. 17 shows the variation in fuel flow for deceleration. As is pointed out in Ref. 3, deceleration is accompanied by the possibility of unstable burner operation or blow-out. Ref. 3 further indicates that the compressor discharge pressure is a suitable parameter for controlling minimum fuel flow. Although compressor pressure ratio is available for such a lower limit in the control, no effort was made in this study to provide for such a limit.

It will be noted that the equilibrium running speeds as determined by the computer are 345, 328, and 307 for operation at 110%, 100%, and 90% of design turbine inlet temperature. The speeds calculated in Appendix C for these conditions are 349, 328, and 311. The percentage errors are 1.14%, zero, and 1.28% respectively.

The foregoing results show that a reasonably accurate dynamic and static simulation of a turbojet engine in the region of greatest interest is possible on a standard electronic analogue computer without resort to special equipment. The non-linear differential equation governing engine operation has been solved without reducing the equation to a linear form. As a result, operation far from equilibrium is made possible. The accuracy is satisfactory and the range of operation of the simulated engine covers one of the important ranges for control studies on an actual engine. The accuracy and range of operation could be improved, however, by use of function generators in place of the straight line relationships used to simplify the computer circuit.

With this simulation it is possible to make control studies on any engine whose characteristics (design or actual) are known, using any of several continuously generated engine parameters for control. As has been shown in this study by the illustrative control, the control may be applied to the simulated engine and the time history of all important engine parameters plotted. Then theoretical or empirical changes may be made to the control to optimize it or to stay

within limits of engine operation. In this way it was shown
that the control coefficients used on the illustrative con-
trol should be made variable with engine speed for optimiza-
tion. The need for the upper and, possibly, the lower limi-
ters on fuel flow also became apparent as the result of such
an analysis.

CONCLUSIONS

A reasonably accurate dynamic and static simulation
of a turbojet engine is possible on a standard electronic
analogue computer. Such a simulation has the following
characteristics:

1. No special equipment is necessary.

2. No linearizing assumptions are necessary in the
equation for engine operation.

3. Operation far from equilibrium is possible.

4. Compressor pressure ratio, air flow, turbine in-
let temperature, turbine and compressor powers, and engine
speed are available as computer outputs for use as control
parameters, for observation, and for plotting.

5. Parameters above are continuously generated.

6. The simulation is equally applicable to engines
in the "hardware" or design stages of development.

7. Accuracy and range of operation may be improved,
if necessary, by use of function generators in the circuit.

Improvement of the illustrative control may be accomplished by:

1. Making the control coefficients vary with engine speed.

2. Incorporating a lower limiter to fuel flow to prevent unstable burner operation or blow-out.

REFERENCES

1. Otto, Edward W., and Taylor, Burt L., III: Dynamics of a Turbojet Engine Considered as a Quasi-static System. NACA TR No. 1011, 1951.

2. Bell, Arthur H., and Farmer, J. Elmo: Transient operating Characteristics of a Turbojet Engine When Subjected to Step Changes in Fuel Flow. NACA RM No. E9K25A, 1950.

3. Novik, David, and Otto, Edward W.: An Analysis of Control Requirements and Control Parameters for Direct-Coupled Turbojet Engines. NACA RM No. E7I25a, 1948.
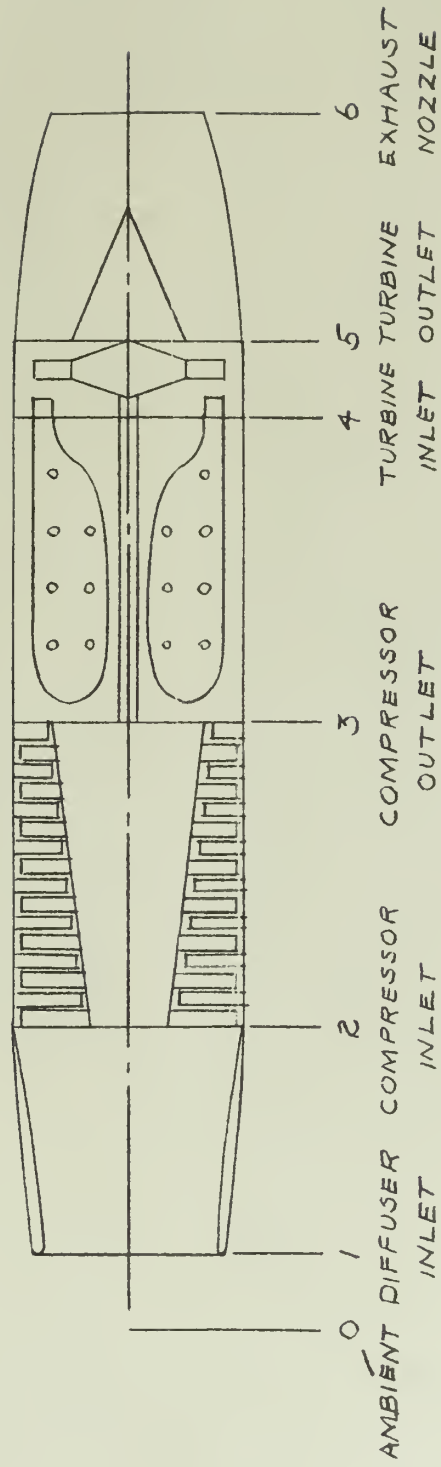
FIG. 1. TYPICAL CONFIGURATION OF A DIRECT COUPLED TURBOJET ENGINE
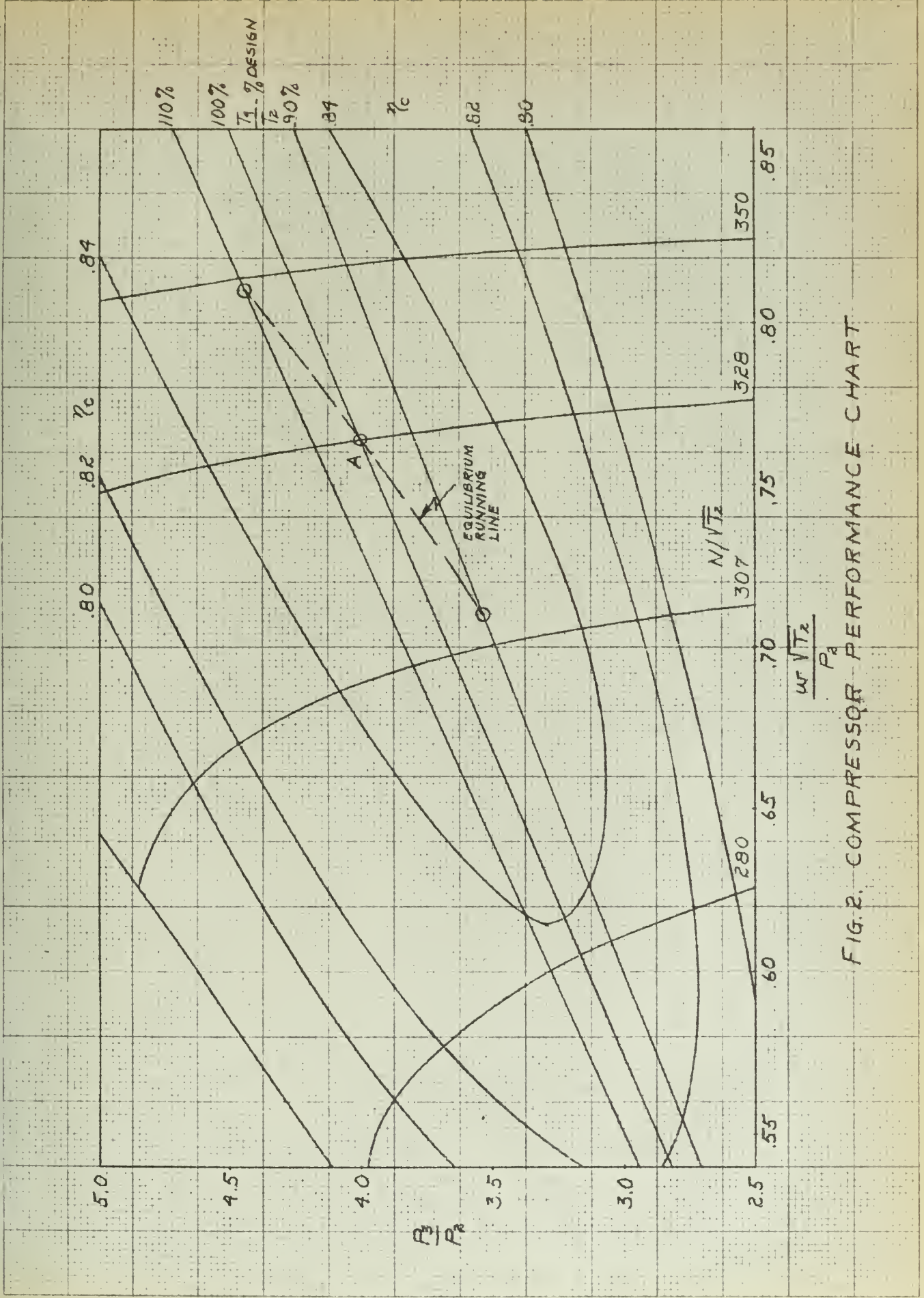
FIG. 2. COMPRESSOR PERFORMANCE CHART.
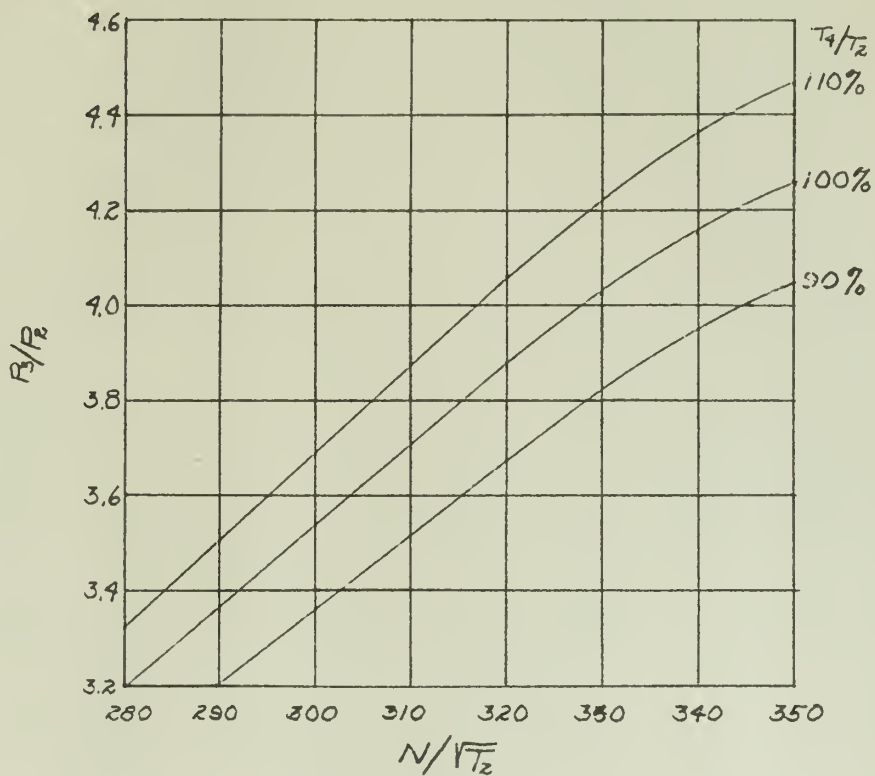
FIG. 3. P_3/P_2 VS. $N/\sqrt{T_2}$ FOR CONSTANT T_4/T_2
(FROM COMPRESSOR CHARACTERISTICS)
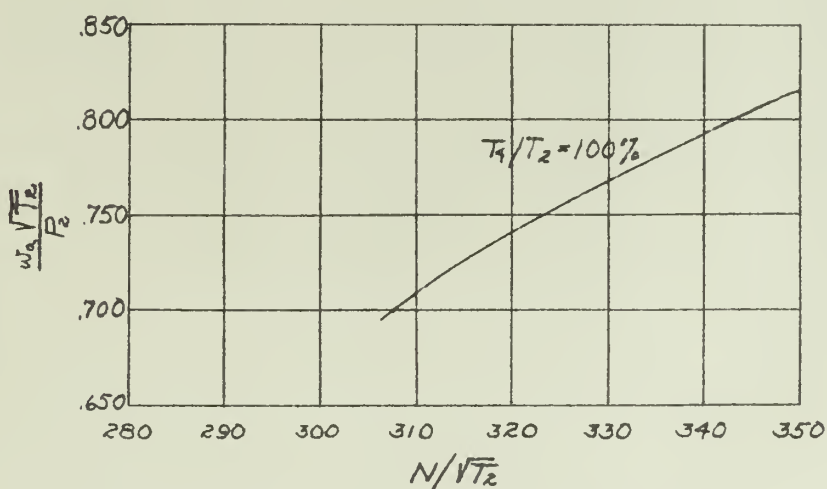
FIG. 4. AIR FLOW VS. ENGINE SPEED
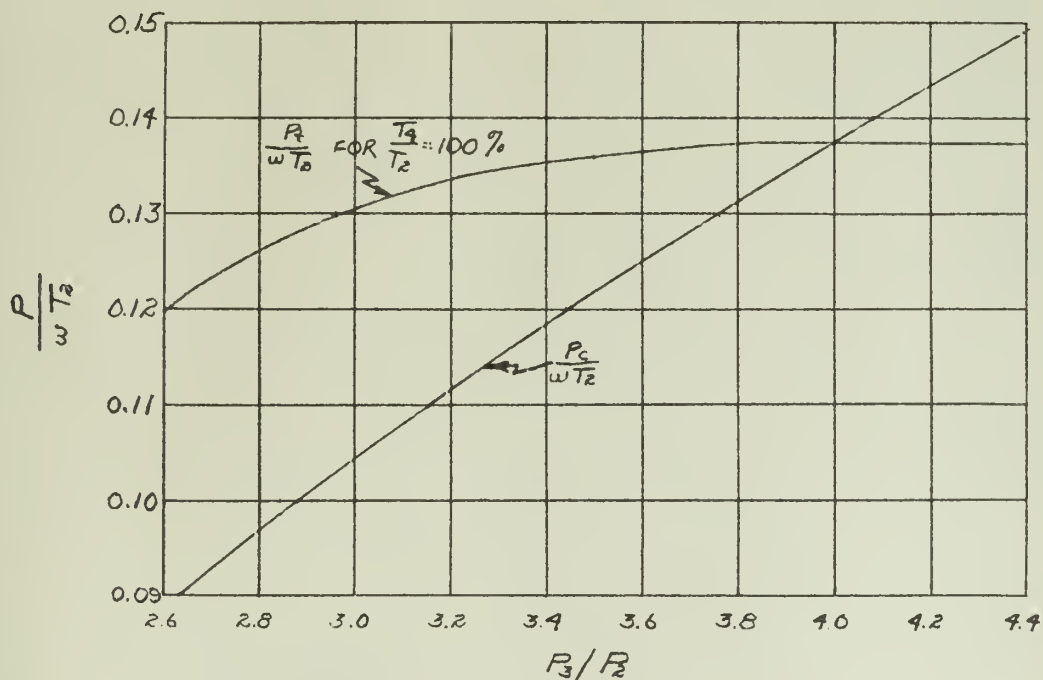
FIG. 5. TURBINE AND COMPRESSOR POWERS
VS.
COMPRESSOR PRESSURE RATIO
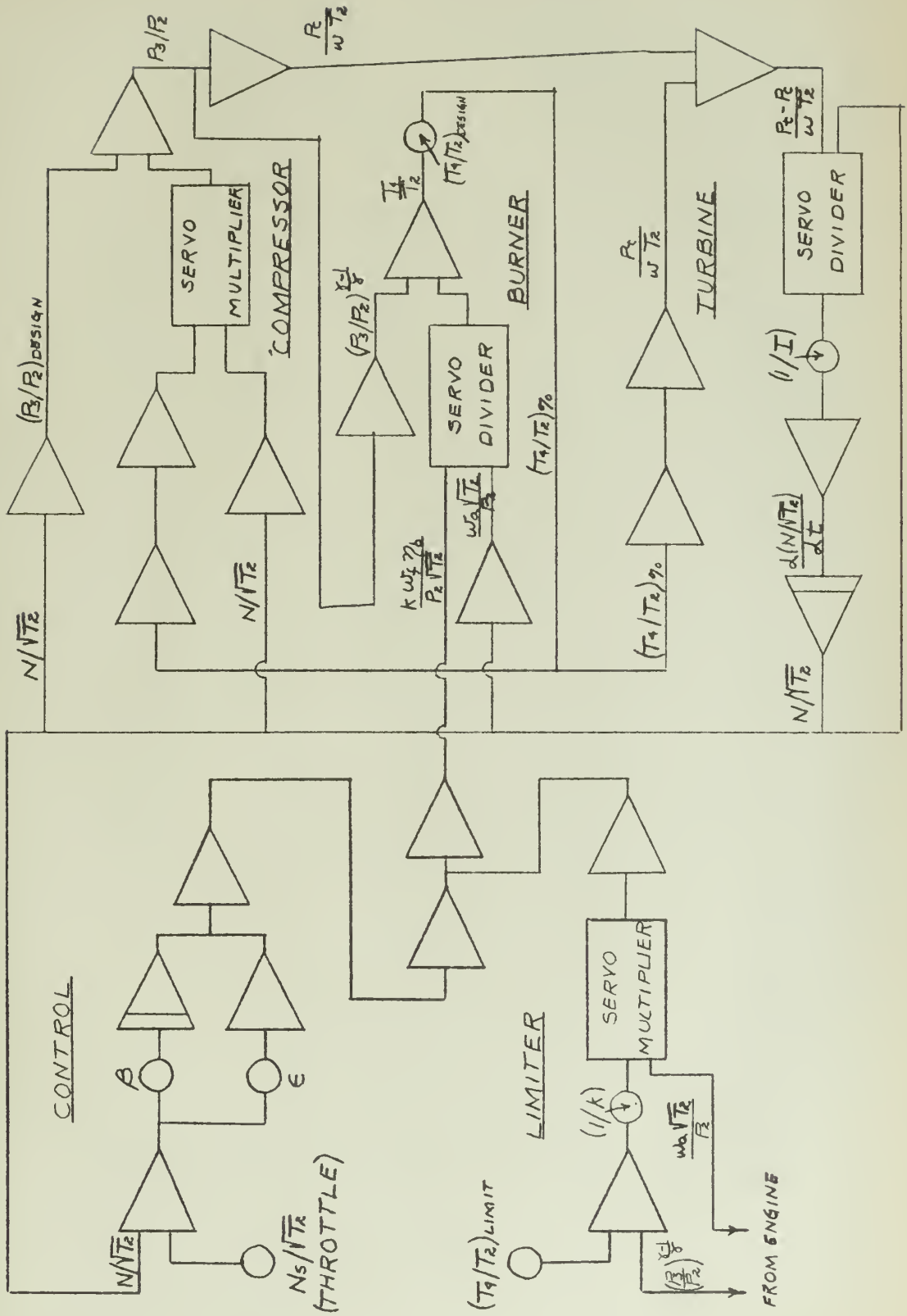
FIG. 6. COMPUTER CIRCUIT

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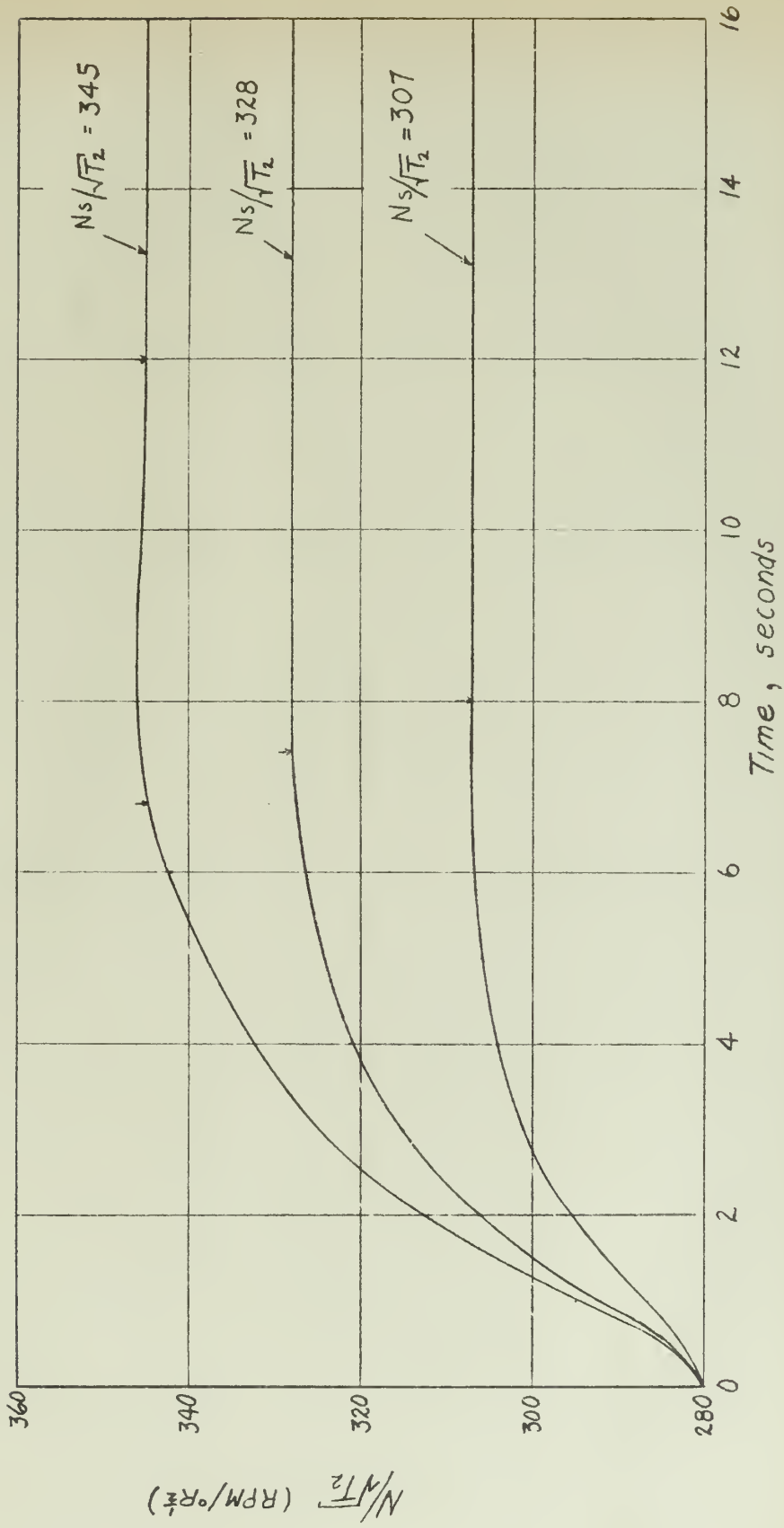
Fig. 7. Engine speed response for step inputs of $N_s/\sqrt{T_2}$
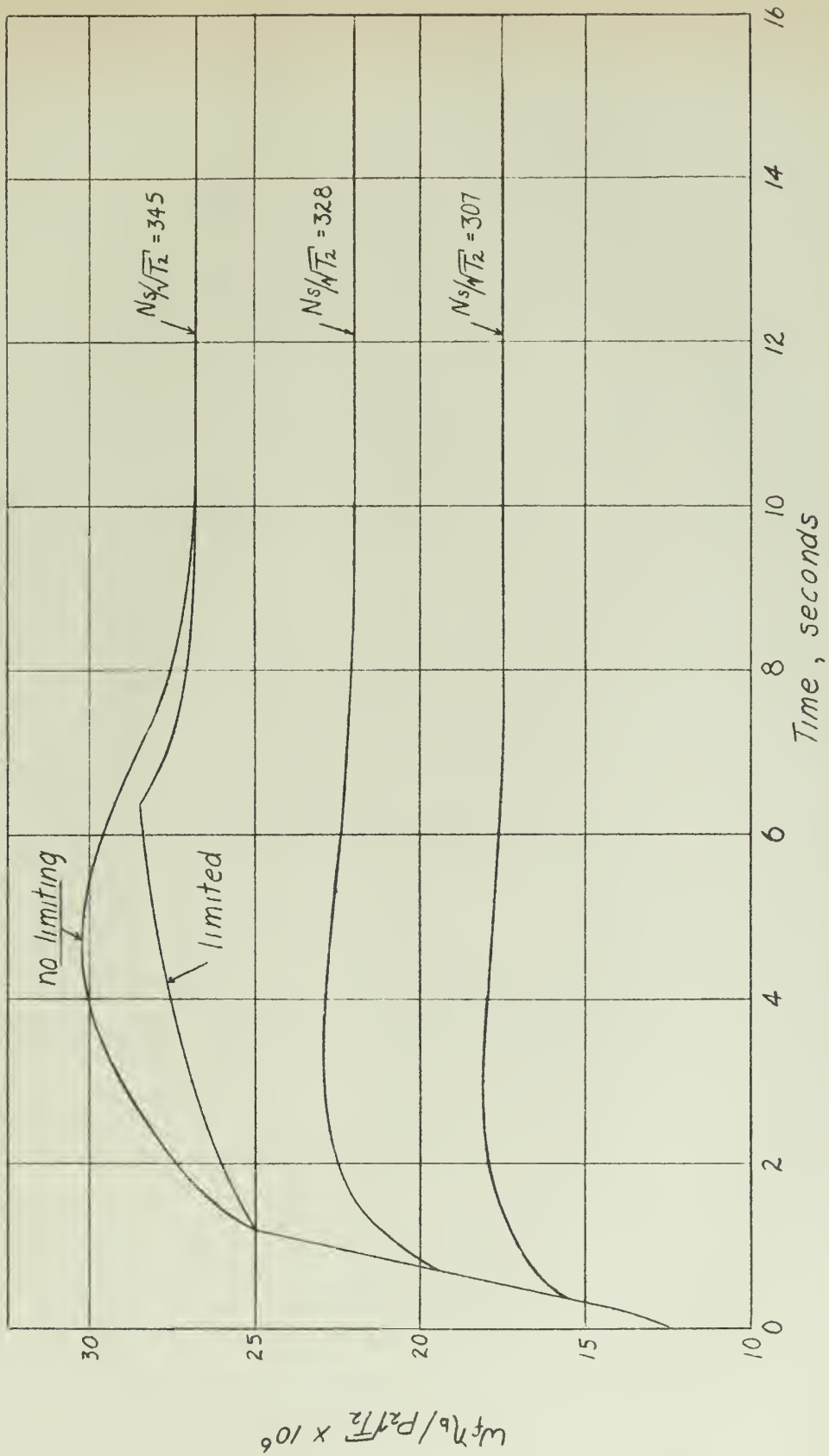
FIG. 8. Fuel flow parameter response for step inputs of $N_s/\sqrt{T_2}$. ($N_o/\sqrt{T_2} = 280$)
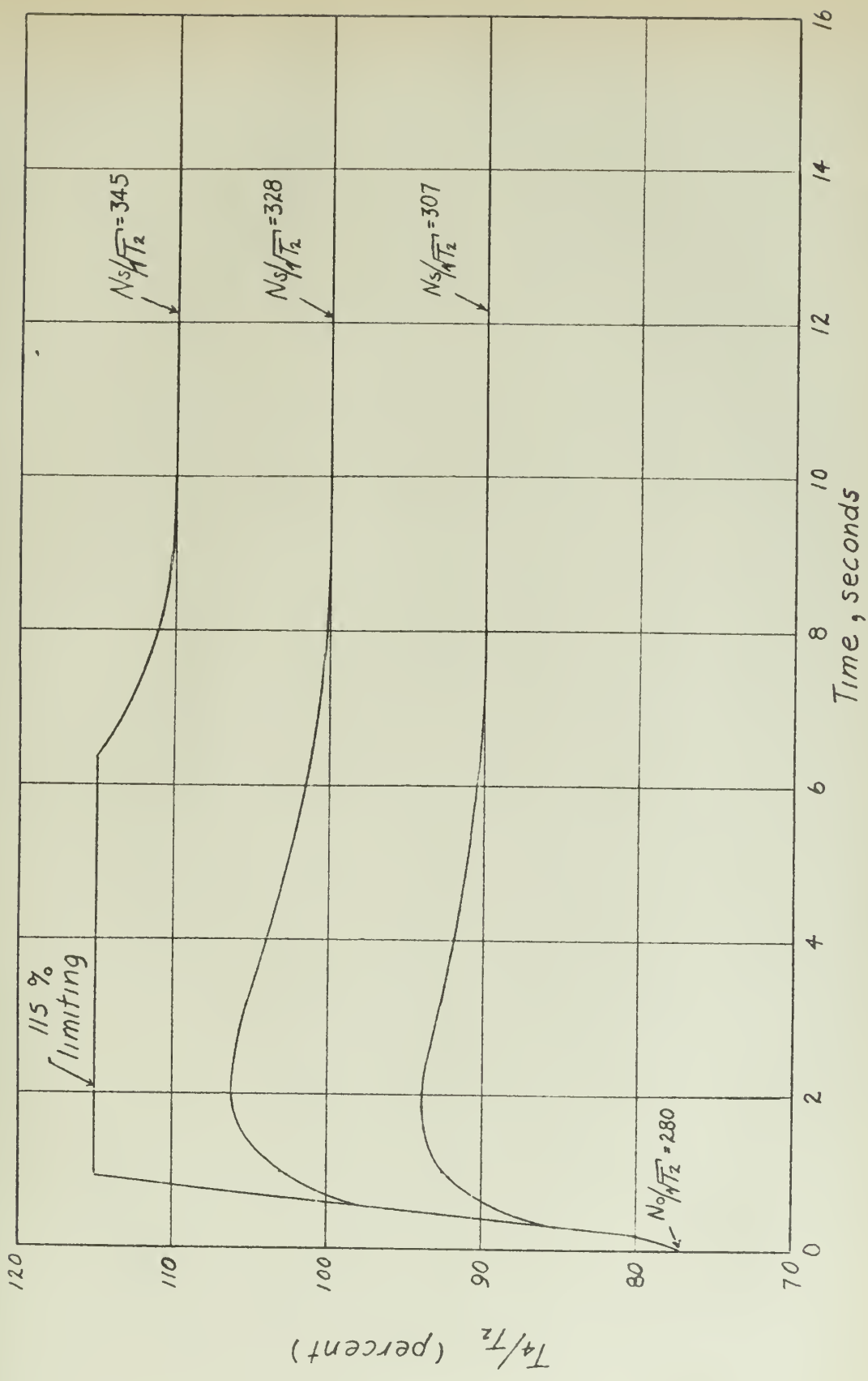
FIG. 9. Turbine inlet temperature ratio response for step inputs of $N_s/\sqrt{T_2}$.
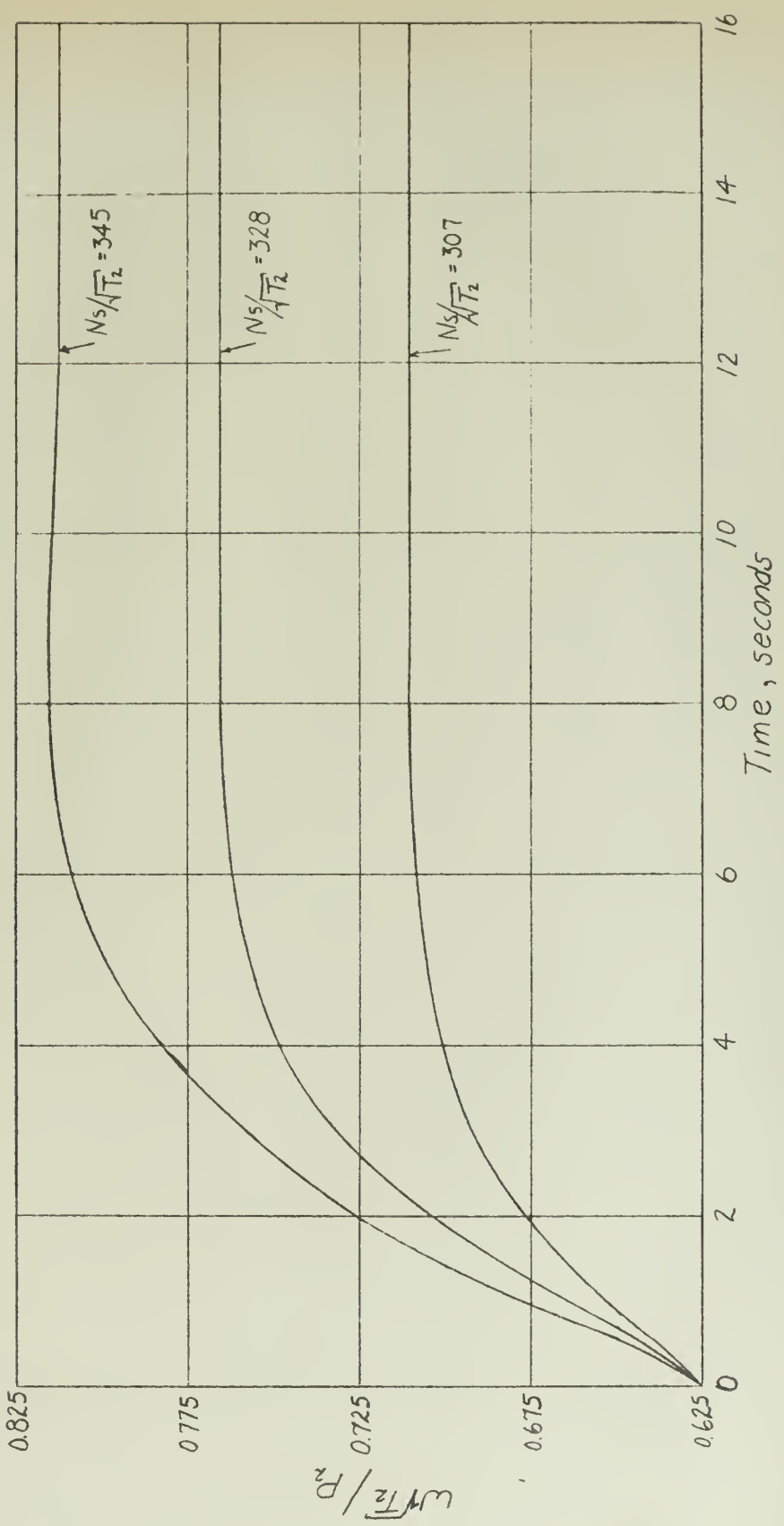
FIG. 10. Air Flow parameter response for step inputs of $N_s/\sqrt{T_2}$. ($N_0/\sqrt{T_2}=280$)
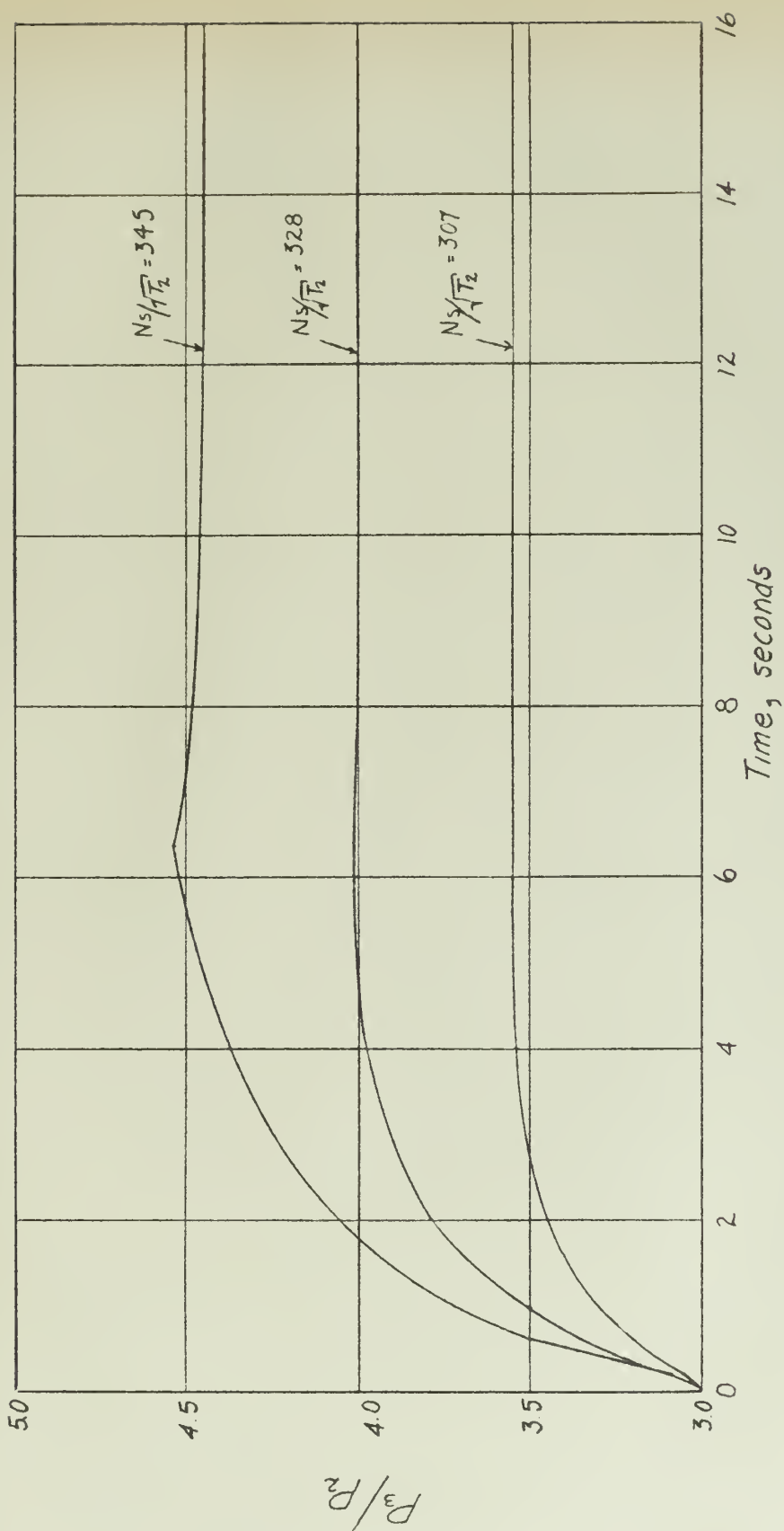
Fig. 11. Compressor pressure ratio response for step inputs of $N_s/\sqrt{T_2}$. ($N_0/\sqrt{T_2} = 280$)
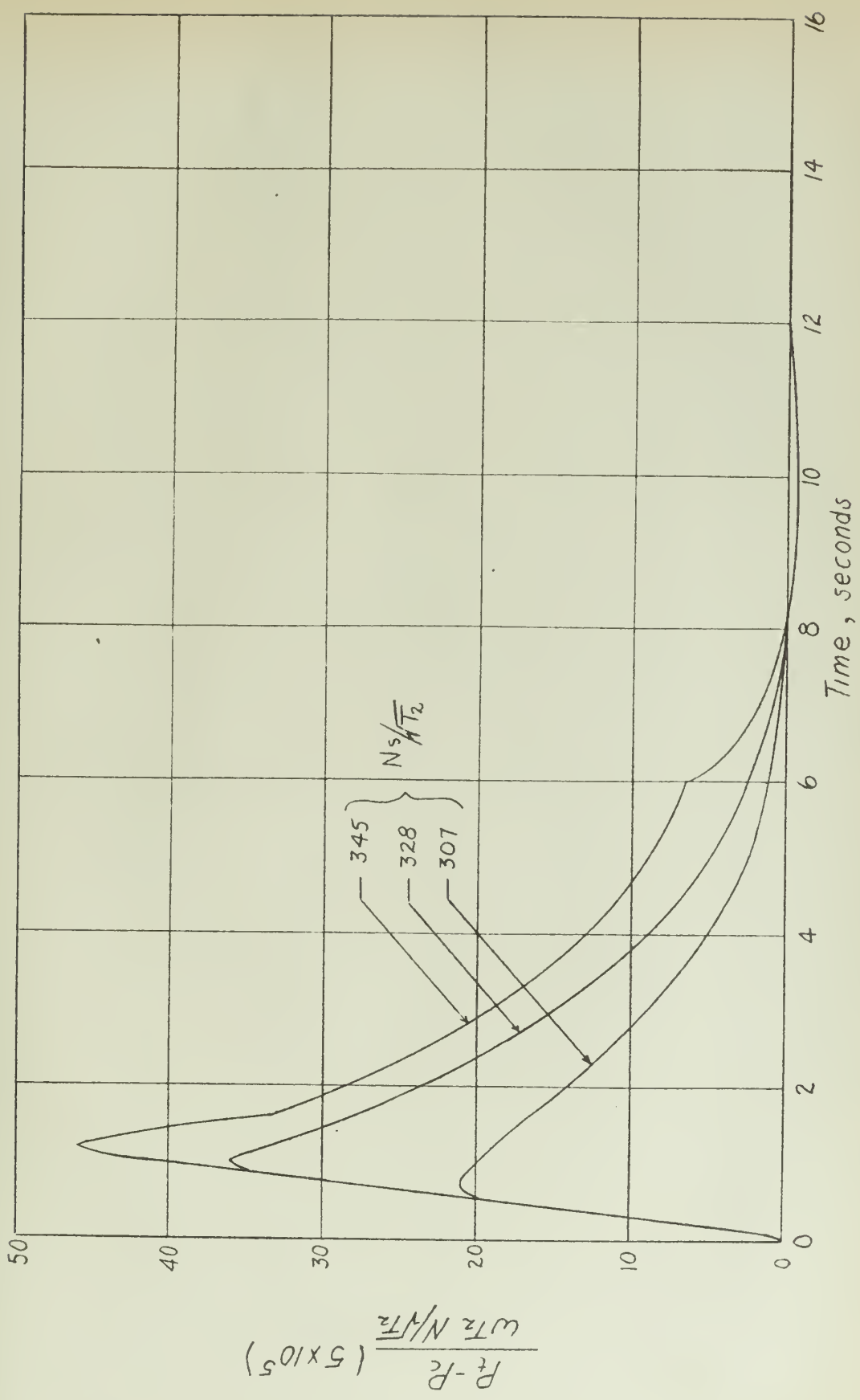
FIG. 12. Torque response for step inputs of $N_s/\sqrt{T_2}$. ($N_0/\sqrt{T_2} = 280$)
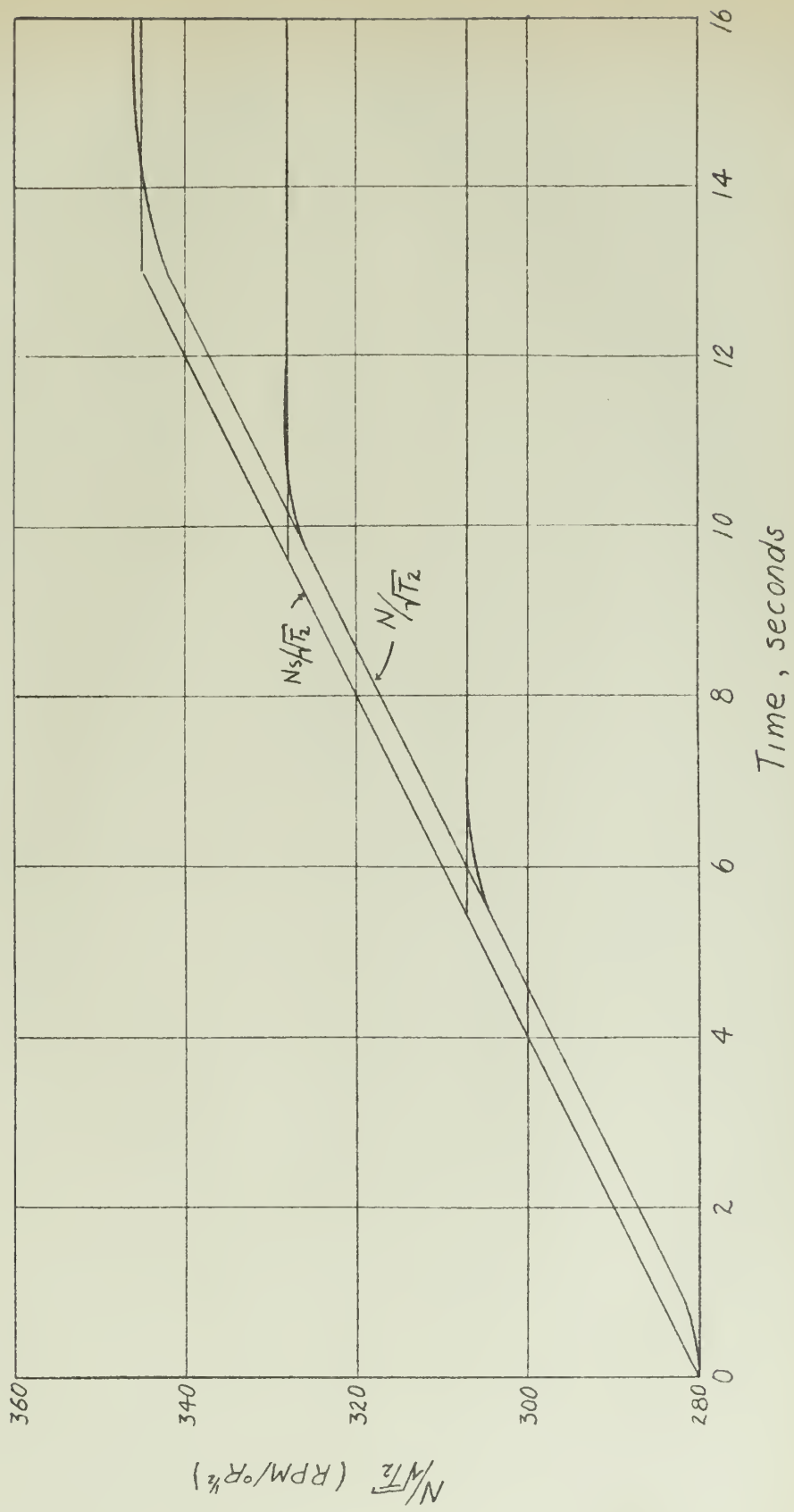
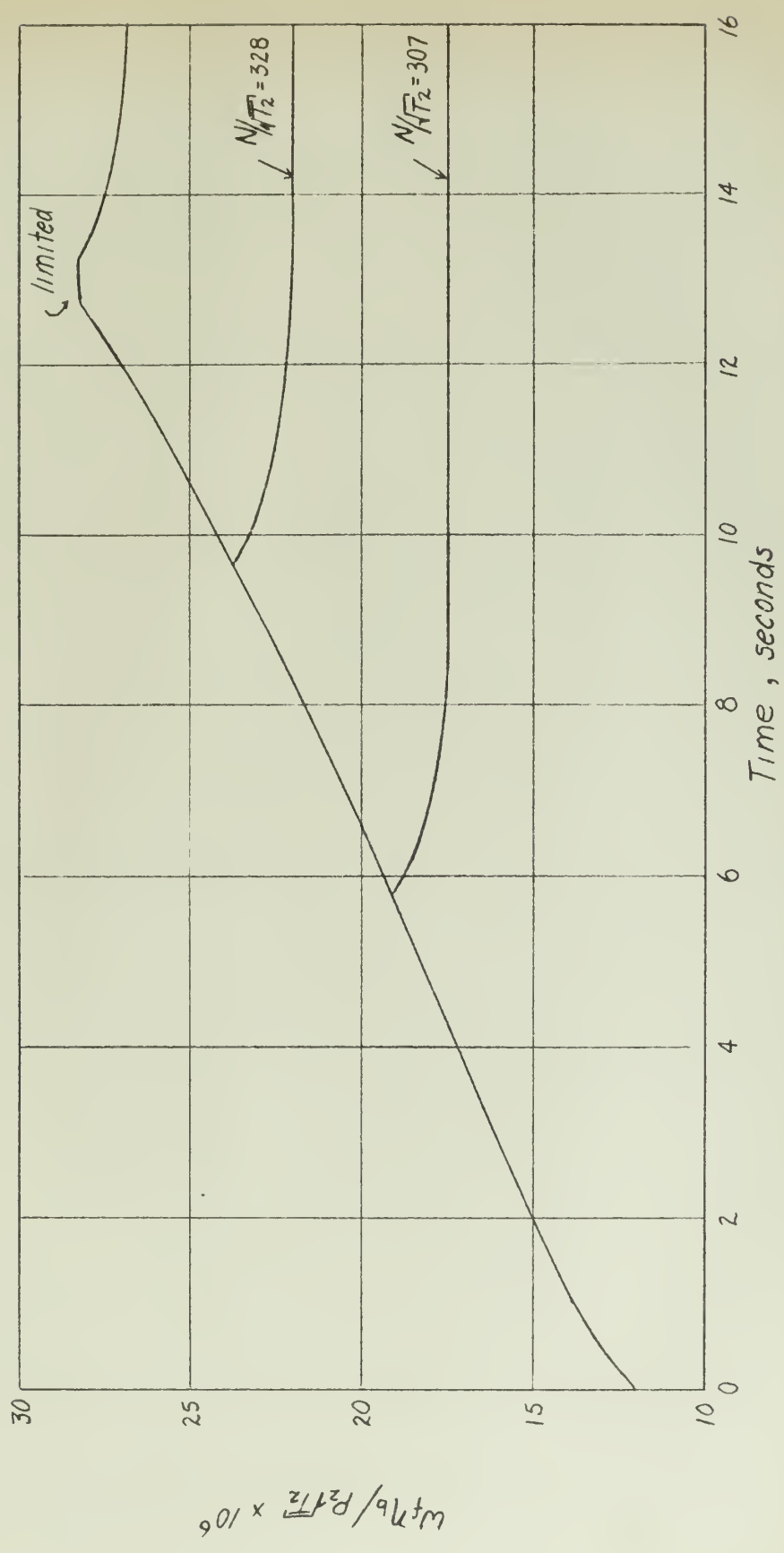FIG. 13. Engine speed response for ramp input of $N_S/\sqrt{T_2}$. ($N_S/\sqrt{T_2} = 5/\text{sec.}$)

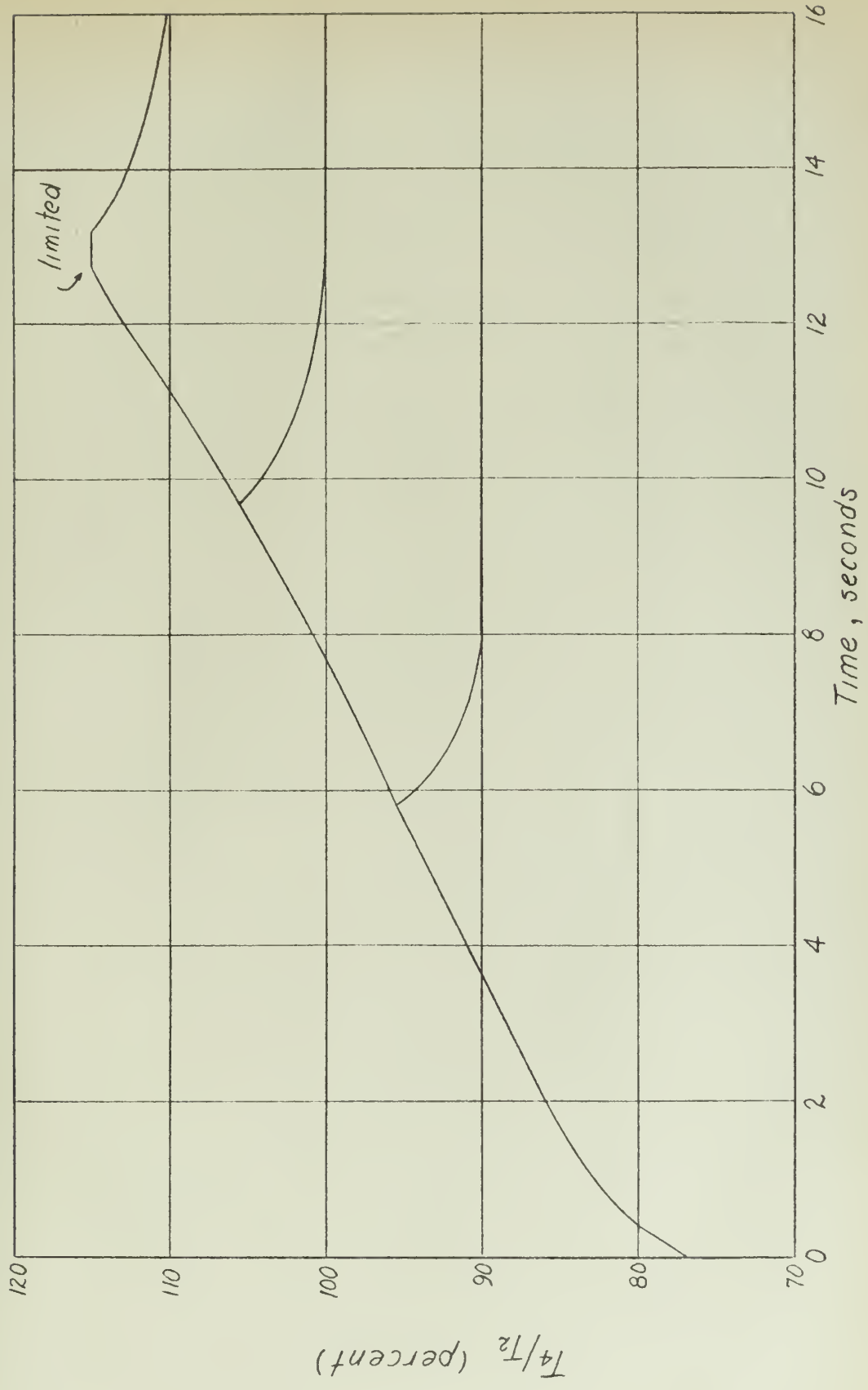FIG. 14. Fuel flow parameter response for ramp inputs of $N_s/\sqrt{T_2} = 5/sec.$

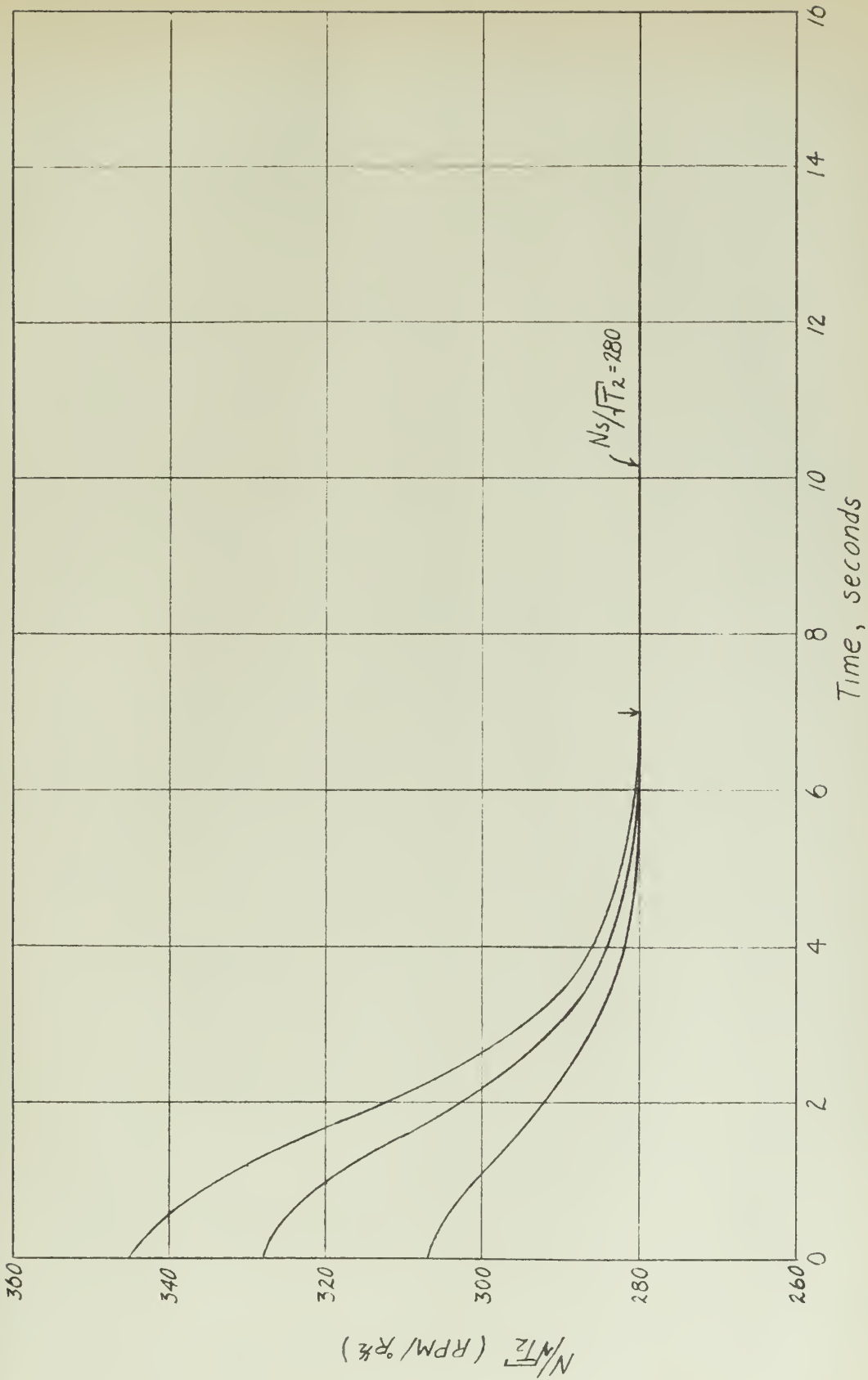FIG. 15. Turbine inlet temperature ratio response for ramp inputs of $N_s/\sqrt{T_2}$.

Fig. 16. Engine speed response for deceleration step input of $N_s/\sqrt{T_2}$.
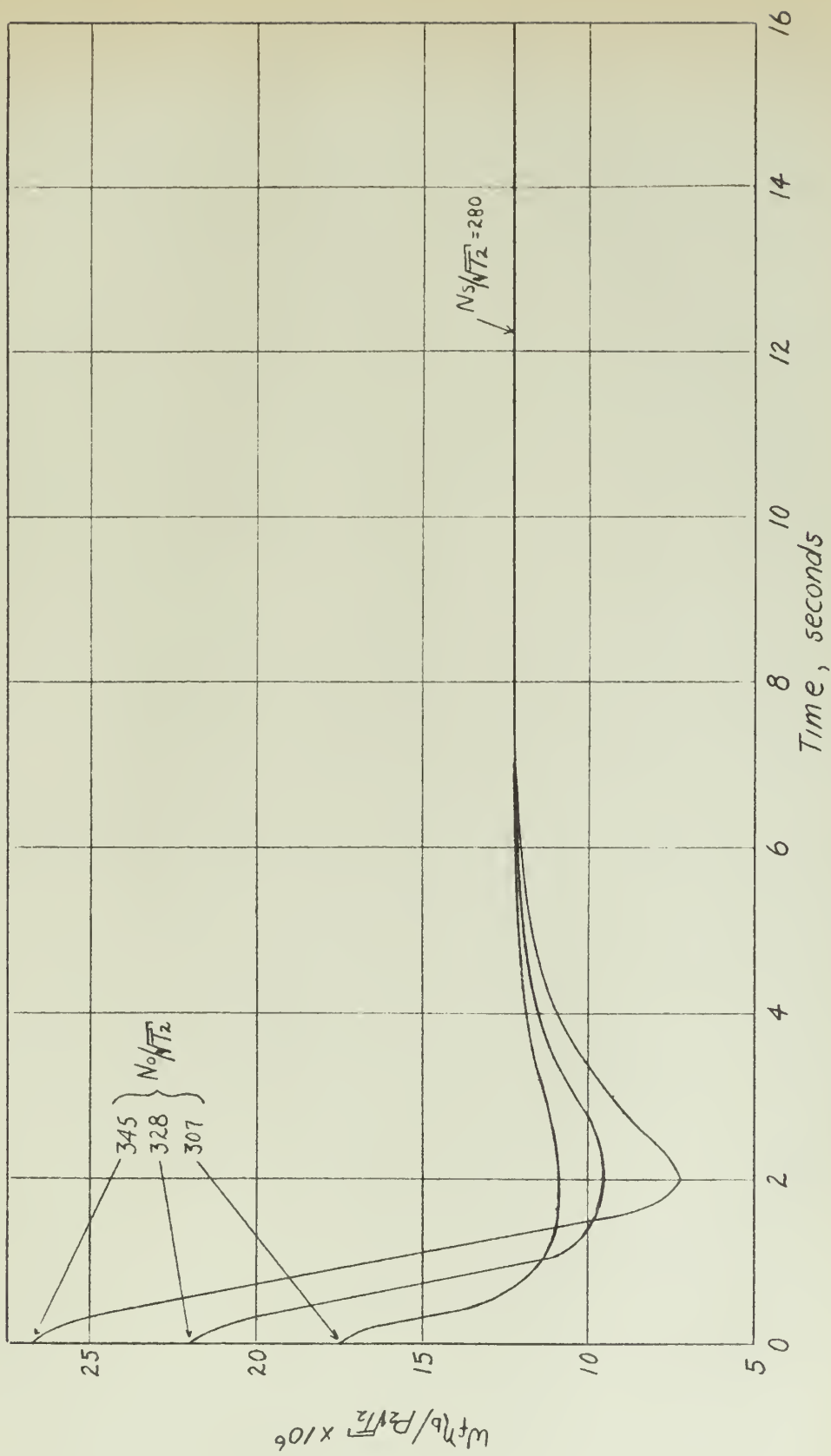
Fig. 17. Fuel flow parameter response for deceleration step inputs of $N_s/\sqrt{T_2}$.
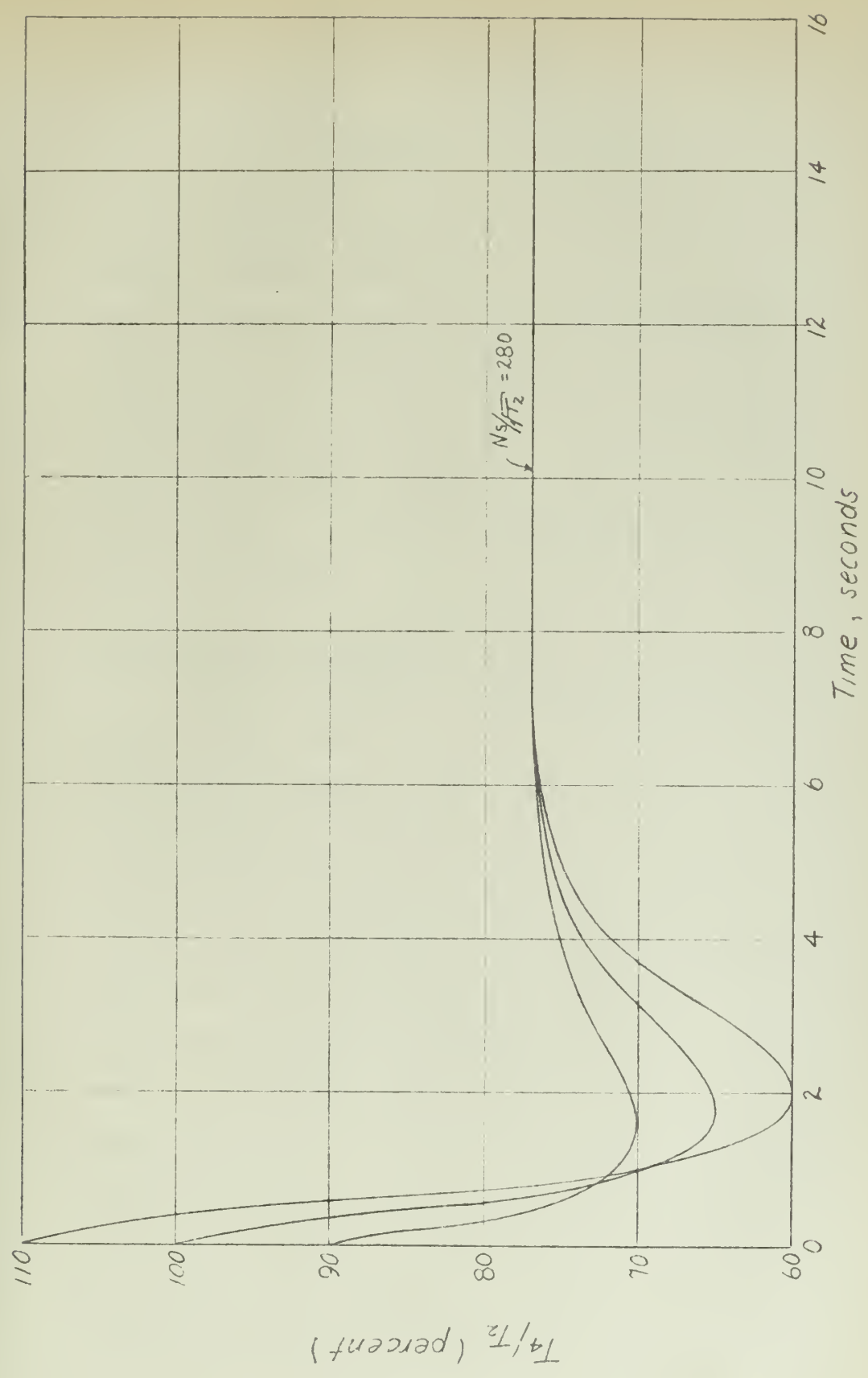
Fig. 18 Turbine inlet temperature response for deceleration step input of $N_s/\sqrt{T_2}$.

Symbols

A_6 Exhaust nozzle area, sq. ft.

A_t Turbine nozzle area, sq. ft.

c_{pc} Average specific heat of gas passing through compressor (assumed .240 BTU/lb. ° R)

c_{pt} Average specific heat of gas passing through turbine (assumed .270 BTU/lb. °R)

I Polar moment of inertia of rotating parts (20 lb-ft sec^2)

K Constant

k $\dfrac{\text{Heating value of fuel}}{\text{specific heat}}$, (assumed 80,000 °R)

N Engine speed, rpm

N_s Set value of engine speed, rpm

P Total pressure, lb/sq. ft.

p Static pressure, lb/sq. ft.

P_t Turbine power, BTU/sec

P_c Compressor power, BTU/sec

Q Accelerating torque, lb-ft.

T Total temperature, °R

t Time, sec

w_a Air flow, lb/sec

w_f Fuel flow, lb/hr

w_g Total gas flow, lb/sec

w Equals w_a and w_g by assumption

α	Acceleration of engine speed, rad/sec^2
β	Coefficient of integral control component
γ_c	Ratio of specific heats in compressor (1.40)
γ_t	Ratio of specific heats in turbine (1.34)
ϵ	Coefficient of proportional control component
η_b	Burner efficiency, percent
η_c	Compressor efficiency, percent
η_t	Turbine efficiency, percent (85%)

Subscripts

0	Ambient
1	Diffuser inlet
2	Compressor inlet
3	Compressor outlet
4	Turbine inlet
5	Turbine outlet
6	Exhaust

Parameters

$N/\sqrt{T_2}$	Engine speed
$P_c/\,w\,T_2$	Compressor power
$P_t/\,w\,T_2$	Turbine power
P_3/P_2	Compressor pressure ratio
T_4/T_2	Turbine inlet temperature
$\dfrac{w_a\sqrt{T_2}}{P_2}$	Air flow
$\dfrac{w_f\,\eta_b}{P_2\sqrt{T_2}}$	Fuel flow

APPENDIX B

Engine Design Calculations

Fig. 1 shows the configuration of a typical direct-coupled turbojet for which typical compressor characteristics (Fig. 2) are assumed to apply. It is necessary to choose a point of design operation for the compressor and then to determine a reasonably sized turbine and tailpipe area to complete the design of the engine. The method used to determine the turbine nozzle throat area consists of determining the choked nozzle area necessary to pass the mass flow. Equating compressor and turbine powers gives temperature and pressure at the turbine outlet from which the tail pipe area is calculated.

The following assumptions were made:

1. Negligible velocity at stations 2, 3, 4, and 5.
2. No ram ($p_0 = P_2$).
3. Burner pressure drop equals 4% ($P_4/P_3 = .96$).
4. Turbine efficiency constant at 0.85.
5. Weight of fuel neglected ($w_a = w_g = w$).
6. Standard atmospheric temperature and pressure at station 2.
7. $T_4 = 1960$ °R at design operation.
8. Neglect nozzle and tailpipe losses, i.e., isentropic.
9. Neglect friction.
10. Turbine nozzles are choked.

Nozzle Throat Area (A_t):

For choked nozzles the following equation may be used:

$$\frac{w_g \sqrt{T_4}}{A_t \, P_4} = \text{constant} \tag{B1}$$

which may be written:

$$\frac{w_g \sqrt{T_4}}{A_t \, P_4} = C = \frac{w_a \sqrt{T_2}}{P_2} \left| \frac{P_2}{P_3} \right| \left| \frac{P_3}{P_4} \right| \sqrt{\frac{T_4}{T_2}} \cdot \frac{w_g}{w_a} \cdot \frac{1}{A_t} \tag{B2}$$

where $C = 0.525 \; ^\circ R^{\frac{1}{2}} \, / \, \sec$ for $\gamma_t = 1.34$.

At design point "A" on the compressor chart the following values are obtained:

$$w = 71.2 \text{ lbs/sec} \qquad \frac{w \sqrt{T_2}}{P_2} = .766$$

$$P_3/P_2 = 4.0$$

Solving equation (B2) gives a turbine nozzle throat area of:

$$A_t = 0.737 \text{ sq. ft.}$$

Tail Pipe Area (A_6):

Compressor power is given by the equation:

$$P_c = w_a \, c_{pc} \, (T_3 - T_2) \tag{B3}$$

which may be written:

$$P_c = w_a \, c_{pc} \, \frac{1}{\eta_c} \, T_2 \left[\left| \frac{P_3}{P_2} \right|^{\frac{\gamma_c - 1}{\gamma_c}} - 1 \right] \tag{B4}$$

Turbine power is given by the equation:

$$P_t = w_g \, c_{pt} \, (T_4 - T_5) \tag{B5}$$

At equilibrium, compressor and turbine powers are equal which permits solving for T_5 by equating equations (B4) and (B5) giving a T_5 of 1696 $^\circ R$. P_5 is determined by turbine efficiency as expressed by the following equation:

$$\eta_t = 0.85 = \frac{1 - T_5/T_4}{1 - \left(\dfrac{P_5}{P_4}\right)^{\frac{\gamma_t - 1}{\gamma_t}}} \tag{B6}$$

Solution of equation (B6) gives a P_5 of 4110 lbs/sq. ft.
$p_0/P_5 = 0.516$ but $p_6/P_5 = 0.536$ for a choked nozzle at
$\gamma_t = 1.34$. Therefore, $p_6 > p_0$ and the tailpipe is choked.
Then the equation

$$\frac{w_g \sqrt{T_5}}{A_6 \; P_5} = C = 0.525 \; {}^\circ R^{\frac{1}{2}} / \sec \tag{B7}$$

may be solved for A_6 giving a tailpipe area of

$$A_6 = 1.36 \text{ sq. ft.}$$

APPENDIX C

Equilibrium Running Calculations

An equilibrium operating point for a direct coupled turbojet is determined by satisfying two conditions:

(1) The mass flow through the turbine and compressor must be equal.

(2) The powers of the turbine and compressor must be equal.

For choked turbine nozzles:

$$\frac{w\sqrt{T_4}}{A_t\,P_4} = \text{constant} \tag{C1}$$

where the constant is $0.525\,^{\circ}R^{\frac{1}{2}}/sec$ for $\gamma_t = 1.34$. Equation (C1) may be written:

$$\frac{w\sqrt{T_2}}{P_2}\sqrt{\frac{T_4}{T_2}} = K\frac{P_3}{P_2} \tag{C2}$$

where $K = 0.525\;A_t\left|\frac{P_4}{P_3}\right| = (0.525)\,(.737)\,(.96) = .3715$.

Equation (C2) is used to draw lines of constant T_4/T_2 on the compressor performance chart (Fig.2). Lines for T_4/T_2 of 90%, 100%, and 110% of design operation are shown on Fig. 2. On a given line of $T_4/T_2 = $ constant, the mass flow through the turbine and the compressor is equal. An equilibrium running point is determined by finding the point on the line where compressor and turbine powers are equal.

A relation between turbine pressure ratio (P_4/P_5) and turbine temperature ratio $(\sqrt{T_4/T_5})$ is determined by turbine efficiency as expressed by the following equation:

$$\eta_t = \frac{1 - T_5/T_4}{1 - \left|\dfrac{P_5}{P_4}\right|^{\frac{\gamma_t - 1}{\gamma_t}}} \tag{C3}$$

For this analysis η_t was assumed to be 85% and γ_t was assumed to be 1.34. The resulting relation between turbine temperature ratio and turbine pressure ratio is plotted in Fig. C1.

Equation (C1) may be expanded as follows:

$$\frac{w\sqrt{T_5}}{P_5} = K \sqrt{\frac{T_5}{T_4}} \; \frac{P_4}{P_5} \tag{C4}$$

also:

$$\frac{w\sqrt{T_5}}{P_5} = f\left|\frac{P_0}{P_5}\right| = \sqrt{\frac{2g\gamma}{R(\gamma-1)}} \left|\frac{P_0}{P_5}\right|^{\frac{1}{\gamma}} \sqrt{1 - \left|\frac{P_0}{P_5}\right|^{\frac{\gamma-1}{\gamma}}} \left(A_6\right) \tag{C5}$$

then:

$$\sqrt{\frac{T_5}{T_4}} \; \frac{P_4}{P_5} = f\left|\frac{P_0}{P_5}\right| \tag{C6}$$

From equation (C6) values may be obtained for a plot of P_5/p_0 as a function of $\dfrac{P_4/P_5}{\sqrt{T_4/T_5}}$ for an exhaust nozzle area of 1.36 sq. ft. This relation is shown in Fig. C2. Figs. C1 and C2 may then be used to obtain P_4/p_0 as a function of P_4/P_5. This relation is shown in Fig. C3.

Compressor power is given by the equation:

$$P_c = w \, c_{pc} \, (T_3 - T_2) \tag{C7}$$

which may be written:

$$\frac{P_c}{w \, T_2} = \frac{c_{pc}}{\eta_c} \left[\left| \frac{P_3}{P_2} \right|^{\frac{k-1}{k}} - 1 \right] \tag{C8}$$

Turbine power is given by the equation:

$$P_t = w \, c_{pt} \, (T_4 - T_5) \tag{C9}$$

which may be written:

$$\frac{P_t}{w \, T_2} = c_{pt} \eta_t \left| \frac{T_4}{T_2} \right|_{100\%} \left| \% \, \frac{T_4}{T_2} \right| \left[1 - \left| \frac{P_5}{P_4} \right|^{\frac{k-1}{k}} \right] \tag{C10}$$

Entering the compressor performance chart (Fig. 2) along a line of constant T_4/T_2, P_3/P_2 may be obtained for various $N/\sqrt{T_2}$. Equation (C8) is then used to determine P_c/wT_2 vs. $N/\sqrt{T_2}$ for a given $T4/T_2$. P_4/p_0 is obtained from P_3/P_2 from the relation:

$$\frac{P_4}{p_0} = \frac{P_3}{P_2} \cdot \frac{P_2}{p_0} \cdot \frac{P_4}{P_3} \tag{C11}$$

where P_2/p_0 is 1.0 and P_4/P_3 is 96% in this analysis. Knowing P_4/p_0 permits the determination of P_5/P_4 from Fig. C3. Equation (C10) is then used to determine the turbine power parameter vs. the engine speed parameter for a given T_4/T_2. A plot of both the turbine and compressor power parameters vs. the engine speed parameter for T_4/T_2 of 90% and 110% of design operation is shown in Fig. C4. The intersection of the powers determines the equilibrium operating point. Equilibrium points obtained are $N/\sqrt{T_2} = 311$ for T_4/T_4 of 90% and $N/\sqrt{T_2} = 349$ for T_4/T_2 of 110%. These points are plotted on Fig. 2.
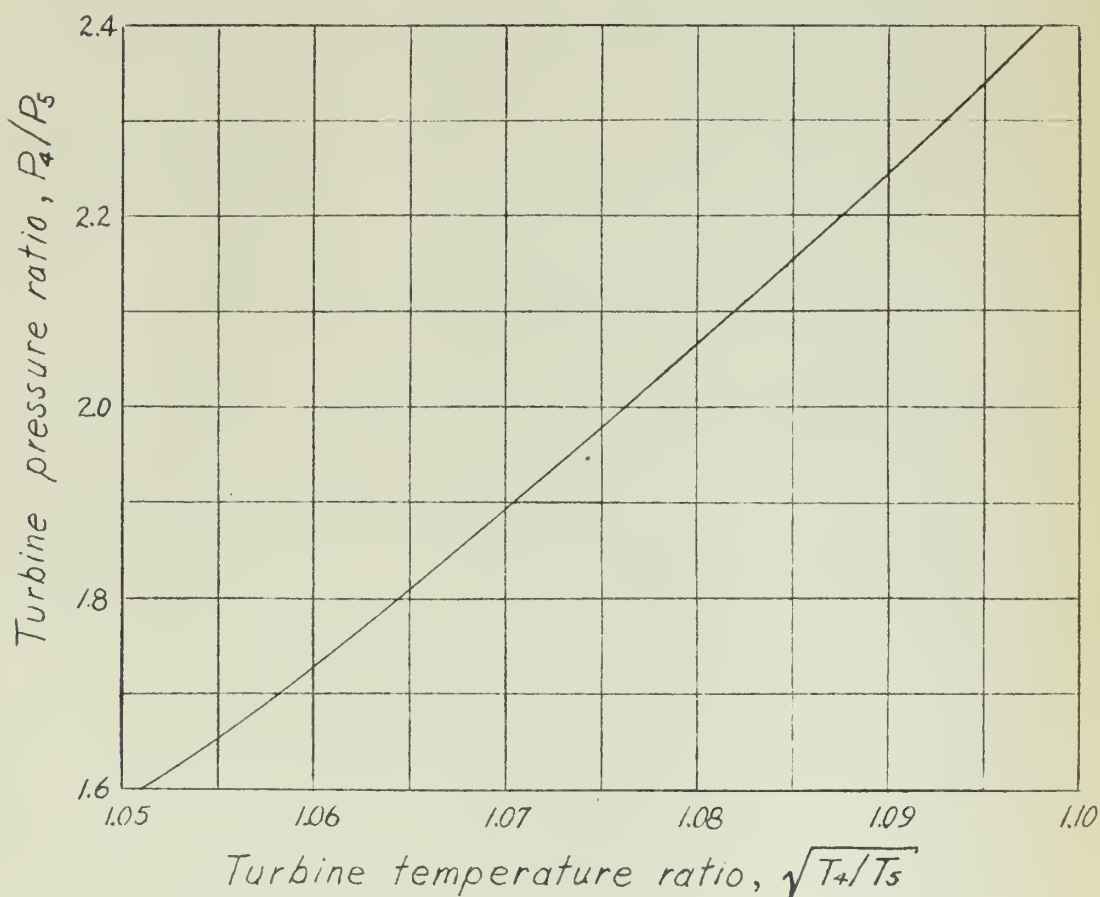
FIG. C1. Relation between turbine total-pressure ratio and total-temperature ratio at constant turbine efficiency of 0.85.
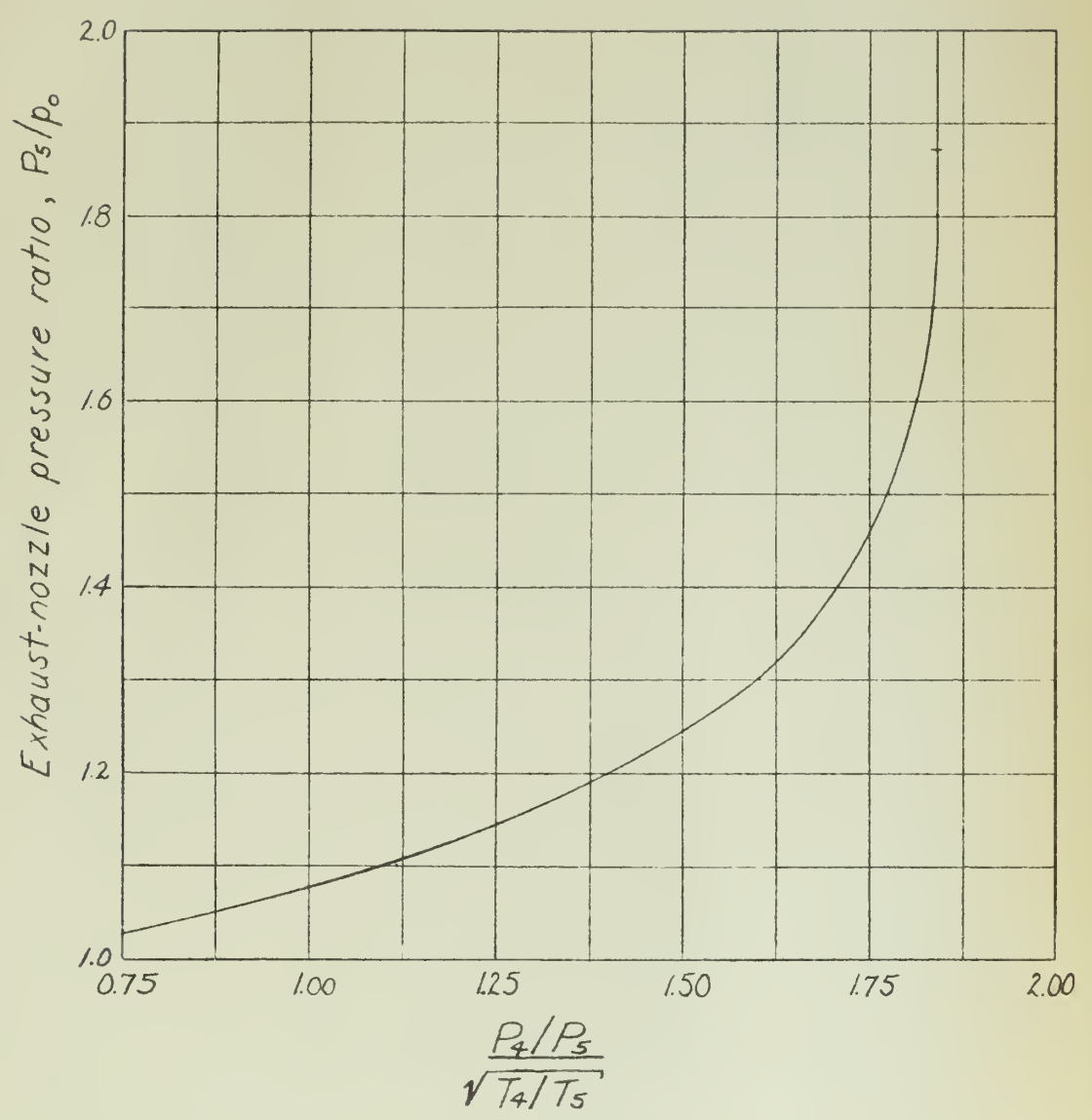
FIG. C2. Relation of exhaust-nozzle pressure ratio
and turbine total pressure-temperature ratio
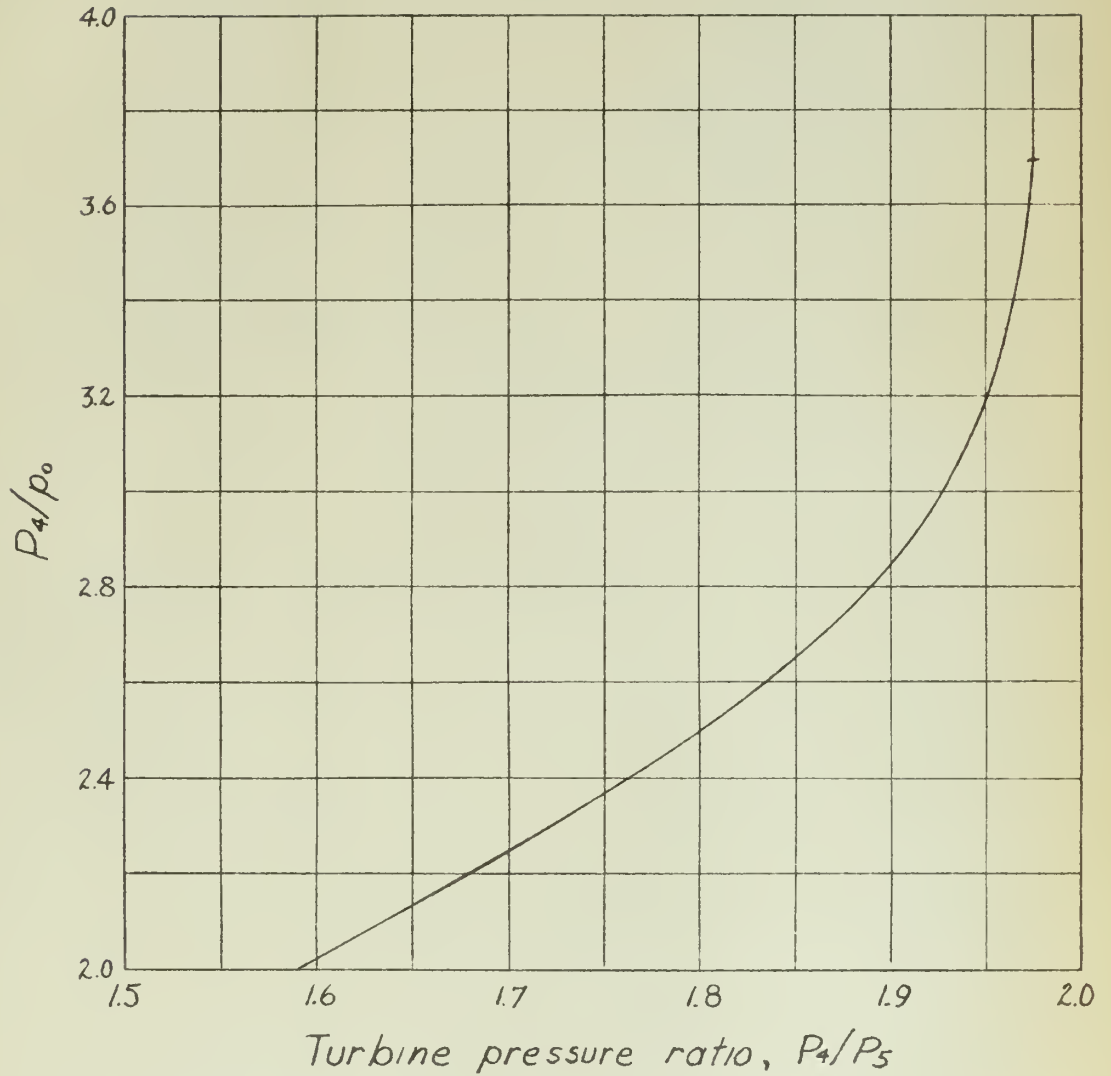for exhaust-nozzle area of 1.36 Sq. FT.

FIG. C 3. Relation between engine-pressure ratio
and turbine-pressure ratio for exhaust
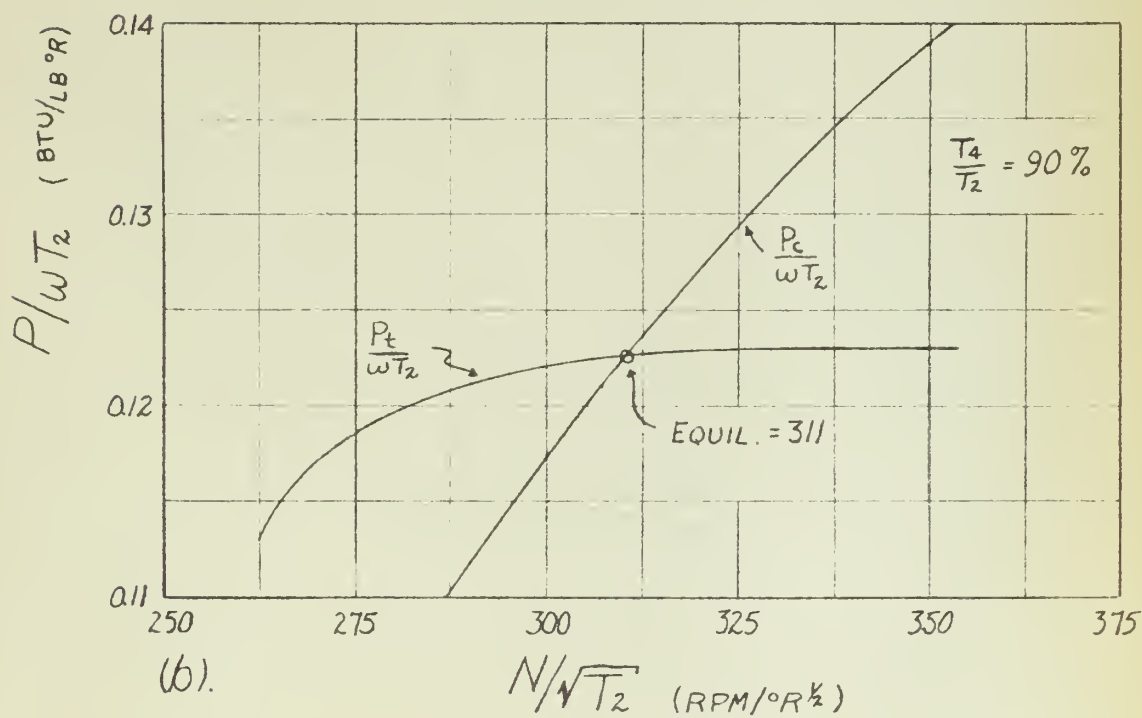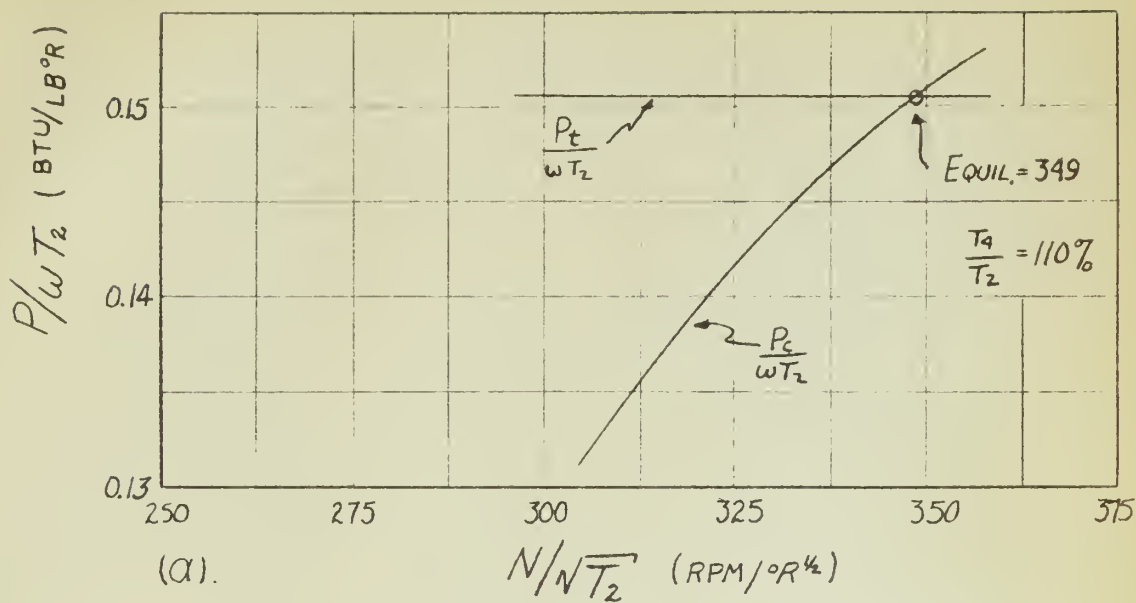nozzle area of 1.36 SQ. FT.

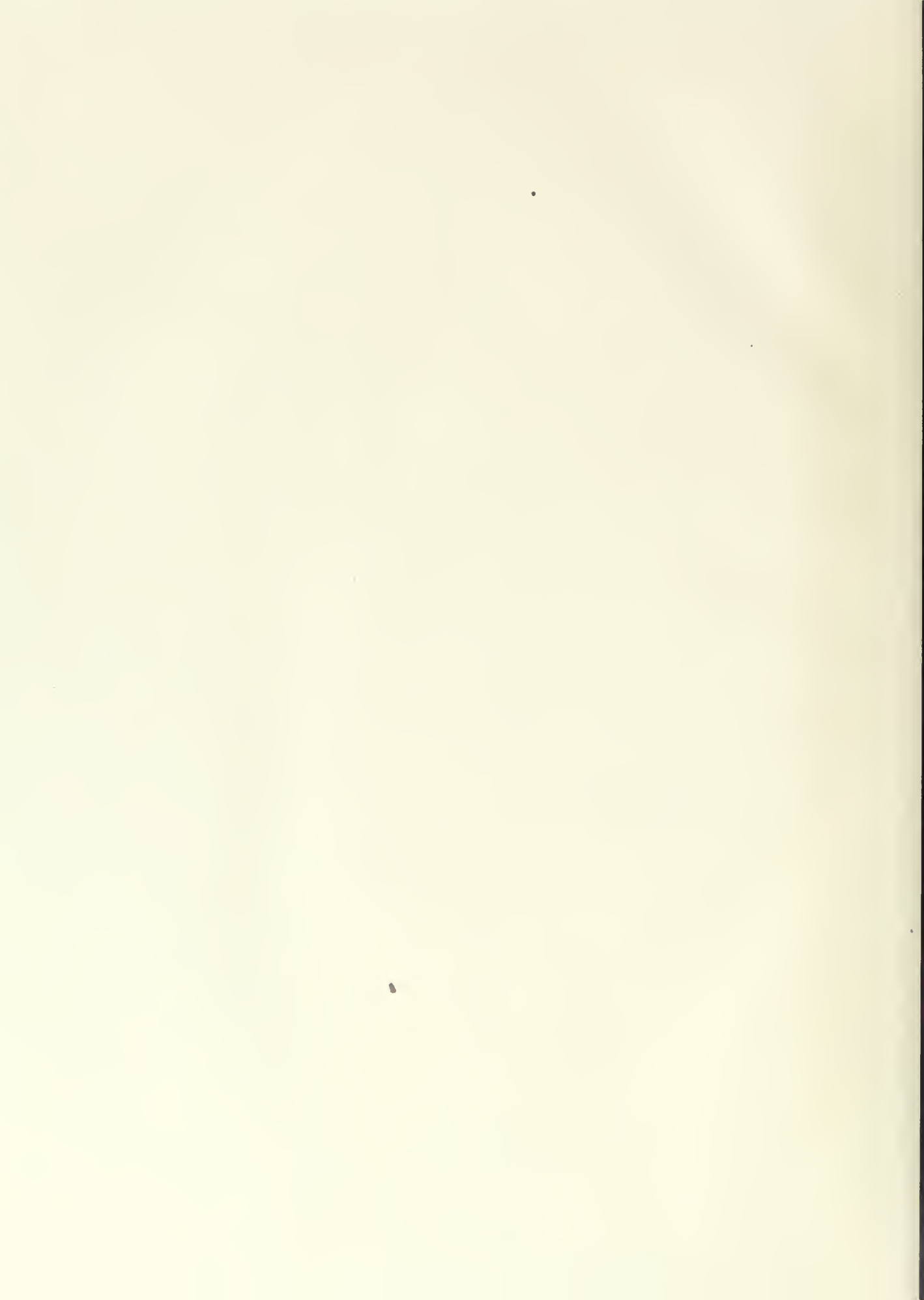FIG. C 4. Turbine and compressor power parameters versus speed parameter.