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Relation between acoustic phenomena and dynamic thrust.

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RELATION BETWEEN ACOUSTIC PHENOMENA AND DYNAMIC THRUST

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SUMMARY

An investigation of the relation of acoustic phenomena to dynamic thrust was made at the University of Minnesota Aeronautical Engineering Department by a Naval Officer Postgraduate Student. A preliminary survey of the scientific literature concerning the acoustic jet indicated that, with the exception of the work done recently by the Cornell Aeronautical Laboratory, no analysis of this problem had been made.

These phenomena, however, were known to Dvorak who performed a number of descriptive experiments in Vienna in 1876, but made only quantitative measurements. Lord Rayleigh completed a mathematical analysis of the circulation of air in pipes sounding in resonance to a given source in 1883. Others have done work on the allied problems of acoustic circulation and streaming, but have made no mention of acoustic repulsion or thrust.

A series of experiments were conducted in the Aeronautical Engineering Laboratory at the University of Minnesota under the guidance of Prof. N. A. Hall. First, small models of resonating chambers of paper and glass were constructed to perform qualitative experiments to determine the nature of thrust produced by the chamber in resonance with sound made by an electrical oscillator and radio speaker. Then, larger resonating chambers of plexiglas tubing about three inches in diameter and of various lengths were used in conjunction with the electrically produced

sound to accurately measure dynamic thrust. The circulation and streaming of the air was also investigated with the aid of smoke.

For a given energy level, it was found that the thrust varies approximately inversely as the acoustic frequency and therefor that the more significant values of thrust are associated with the frequencies between 10 and 150 cycles per second.

In proportion to the small amount of energy supplied to the acoustic jet, a relatively large magnitude of thrust is produced.

RELATION BETWEEN ACOUSTIC PHENOMENA AND DYNAMIC THRUST

OBJECTIVES OF THIS INVESTIGATION

Introduction. During the years spanned by the dates 1870 to 1885 there was a considerable amount of interest displayed in scientific circles over acoustic phenomena apparent to the musician and to the physicist in his laboratory experiments. Vibrating strings, vibrating reeds, vibrating membranes, and vibrating columns of air in organ pipes were known to the artist, of course, from an early age; but in the period between 1870 and 1885 scientific experimenters in Europe and America began to investigate the well-known phenomena systematically in an effort to correlate the various types of effects produced by vibrating bodies in fluid mediums.

The production of sound in tubes and pipes was of considerable interest because of the pipe organ and the various wind instruments. While experimenting with various apparatus to permit observations of sound in air in tubes and pipes, Dvorak /1/* hit upon the effects of acoustic repulsion and attraction noticed when sound was resonated in a vessel. He published his findings in 1878 in the "Philosophical Magazine". His experiments were all descriptive and accurate measurements were not made. However, a fairly wide variety of configurations of vessels in the shapes

*See reference number 1.

of spheres, cylinders, and cones was experimented with and it was found that a reaction from the ambient atmosphere acting inward through the open mouth of the resonating chamber was experienced when the vessel was actually in resonance with a given sound. Not only did there appear to be a force of repulsion between the sound source and the resonator, but under certain conditions there seemed to be a force of attraction present. In addition to these unbalanced forces, which produced an acceleration of the resonating chamber, the air (or fluid medium filling the vessel) was observed to be in movement. Definite currents of air interested Rayleigh when the results of the experiments conducted by Dvorak and Mayer came to his attention.

In 1883 Lord Rayleigh published a paper in the "Philosophical Transactions" in which he presented a mathematical explanation of the circulation of fluid in a tube resonating to a continuous sound. Other allied acoustical problems were considered in the same paper. The results were arrived at by beginning with the fundamental flow equations and treating the fluid as compressible and viscous. Flow involving two directions only was considered, and the analysis was not extended beyond the confines of the tube walls and stopped ends. Furthermore, no effort was made to develop expressions for the unbalanced thrust produced at the open end of a resonating tube, which Dvorak and

Mayer had discovered. It is this effect which is of special interest in this particular research problem.

Current Interest in the Acoustic Jet. During more recent years a great deal of attention has been centered on jet propulsion and the aerodynamics of compressible fluids. Much experimental and mathematical work has been done on the flow of viscous and compressible gases through tubes, channels, and chambers of many different shapes and configurations. Very little of this work to date has taken into consideration the secondary effects caused by sound waves propagated by various disturbances in the fluid flow. Discrepancies have been noted in the expected flow of gases in the combustion chambers of turbojet engines and ramjet engines as well as in the flow in and around the tail pipes and inlet ducts. These discrepancies have not been explained by the examination of the flow phenomena by merely considering the boundary conditions imposed by channel contours, pressure conditions, mechanical work, addition or removal of heat energy, and velocities. It has been suspected that the propagation of sound (which is merely the production of small pressure disturbances in the fluid) and allied acoustical problems might have an important bearing upon the questions arising in this field.

Recently the Cornell Aeronautical Laboratory set up some experiments in which the unbalanced thrust produced by a cylindrical tube, sounding in resonance with a continuous source, was measured. Although this thrust is of very small magnitude it might be arranged to help and not hinder the thrust developed by the mass flow of the fluid in a jet engine produced by the various other mechanical and thermodynamic characteristics of the given engine. There is also the possibility that the related acoustical phenomena of circulation and streaming will have an interesting effect on the efficiency of the jet engine. The reports of CAL embodying the above are not available.

Others, in the past few years, have considered the interesting phenomena of circulation and streaming due to resonant sound waves. Ingaard and Eckart, particularly, have made mathematical studies of these effects and have endeavored to check their results experimentally. Neither of these have given any attention to dynamic thrust produced by a resonating chamber.

Areas Encompassed by This Investigation. It is proposed to set up various configurations of air-filled chambers and tubes so that the unbalanced thrust, produced by the vessel when resonating to a controlled sound source, can be measured to a high degree of accuracy. Measurements will be made of the in-

tensity and frequency of the sound in conjunction with the corresponding thrust.

Secondarily, the phenomena of circulation and streaming produced in the air within the resonator and in the ambient atmosphere will be examined.

FUNDAMENTAL PRINCIPLES

Vibrating Air Columns. It is assumed that the diameter of the pipe in which the vibration of the air column takes place is small compared with the length of the pipe and the wave length of sound. Moreover, it can be assumed that the pressure waves are propagated in a direction parallel to the axis of the pipe or tube, and that the sound waves are very close to being plane surfaces. The walls of the pipe are rigid; and the diameter of the pipe in which the vibrations take place is sufficiently great to justify neglect of viscosity effects when dealing with the longitudinal motion of the sound waves.

There are two types of pipe which are of practical importance: the pipe open at both ends, known as the "open pipe", and the pipe closed at one end, known as the "closed pipe". The elementary facts about the possible modes of vibration in these cases have been arrived at.

In the open pipe there must be a displacement antinode at each end and therefor, in the fundamental mode of vibration, a node in the middle. The wave length of the corresponding sound, which is four times the distance between a node and the adjacent antinode, must be 2L, where L is the length of the pipe. The frequency of the fundamental, a/λ, is therefor a/2L. In the next possible mode of vibration there must be two nodes in the pipe and an antinode at the center. The wave length is now L

and the frequency a/L. In the next possible mode there are three nodes in the pipe; the wave length is 2L/3 and the frequency 3a/2L. The first three possible modes of vibration are shown in the figure; note the arrows show the direction of the movement of the air. Figure 17 shows the distribution for each mode at the two instants of maximum velocity in each vibration. It will be seen that the fundamental mode of vibration has a frequency a/2L and the other modes frequencies which bear to this the ratios 2:1 and 3:1.

The closed pipe must always have a node at the closed end and an antinode at the open end. In the fundamental mode of vibration the wave length is therefor 4L and the frequency a/4L. When there is a second node in the pipe the wave length is 4L/3 and the frequency 3a/4L. When there is a third node in the pipe the wave length is 4L/5 and the frequency 5a/4L. The possible modes are shown in Figure 17.

Another point of view from which the frequency of a vibrating air column may be deduced is that of a travelling pulse of air. The time of vibration of the column of air is the same as the time in which a pulse of air within the pipe completes its cycle of changes. Considering first of all a pipe open at both ends, we start a compression from one end and follow its course. From the farther end it is reflected as a rarefaction, and re-

turning to its point of initiation it is reflected as a compression, and its cycle is complete. The time occupied is 2L/a, so that the frequency is a/2L. In the case of a pipe closed at one end we start a compression from the open end. It is reflected as a compression from the closed end, as a rarefaction from the open end, as a rarefaction again from the closed end and as a compression from the open end, completing its cycle. Thus the time occupied is 4L/a and the frequency a/4L.

See Alexander Wood: Acoustics, 1943, Interscience Publishers, Inc., N.Y., pages 396 to 401 for mathematical expressions for vibration of air in pipes, open and closed, as well as cones.

End Correction. Even in the case of a cylindrical pipe with a clean-cut end the antinode does not coincide with the end of the pipe, and the effective length of the pipe is always greater than its geometrical length. For a cylindrical pipe the "end correction" is about 0.6 R, where R is the radius of the cross section of the pipe, so that of two pipes of the same geometrical length but different diameters the wider pipe gives the lower note.

The correction for a tube ending in an infinite flange, however, has been calculated by Rayleigh, who found it to be 0.824 R. The effect of removing the flange is found by experimant

to be 0.22 R (Rayleigh obtained 0.25 and Bosanquet 0.20) so that for an ordinary open end it is usual to take the correction as 0.6 R.

The magnitude of the correction for other than cylindrical ends depends on the degree of openness of "conductivity" of the end. Thus an uncovered hole in a flute is an open end, but the correction will be greater than for the case previously discussed.

The correction is approximately independent of the wave length of the sound.

Kundt' Tube. A convenient device was developed by Kundt /20/ in about 1866 to measure the velocity of sound in gases. The movement of the fluid in the tube as it is sounding in resonance to a sound of a given frequency is of interest because of its possible bearing on the problem of thrust produced by the resonator. The Kundt's tube, or dust tube, however, produces no unbalanced thrust because both ends are stopped.

The principle of the device is very simple as far as velocity of sound measurement is concerned. A wide tube is closed at one end by an adjustable piston and at the other end by a diaphram fixed to a rod clamped at the center. The diaphram nearly fills the cross section of the tube. The tube between

The diaphram and the adjustable piston contains a dry powder dusted along the inner walls. When the rod is stroked with a wet cloth or resined rubber, or struck by a hammer, longitudinal vibrations are set up in it, which are communicated by the diaphram to the gas in the tube. If the piston is now adjusted so that the length of tho gas column gives an exact number of stationary waves, the dust will be violently disturbed at the antinodes and will form an unmistakable series of striations marking these positions. The frequency of the note given by the rod is determined and the wave length of the sound in the tube is calculated by measuring the distance from the diaphram to the piston and dividing by the number of vibrating segments. This gives the half wave length of the sound in the gas, and therefor the velocity.

However, the phenomena occurring in Kundt's tube are much more complex than early observers supposed. Between the nodes the powder arranges itself in striae, of which no accurate measurements were made and about the formation of which no theory was suggested. Experimenters were handicapped by the fact that the stroked rod is an intermittent source of sound and measurements on the striae and the nodal heaps can only be made after the sound has ceased.

A detailed study of the phenomena has been made by Irons /21/, Cook /22/, Henry /23/, Andrade /24/, and Hutchisson and Morgan /25/. Using a valve-maintained diaphram as the source of sound and fine smoke particles observed in scattered light as tracing points, Andrade was able to measure the amplitude of the vibrations in the tube and establish the existence of the circulation that was predicted by Rayleigh. This circulation takes place from antinode to node in the neighborhood of the walls and from node to antinode along the center, as shown in Figure 18. The form of the circulation was given by Rayleigh in the formula $\psi = C(r^4 - r^2R^2)\sin 2\pi x/\lambda$, where ψ is the velocity potential, r the radial distance, x the axial distance, and R the radius of tube, and the observed circulation gave good agreement with Rayleigh's formula.

Very light objects like fine smoke particles follow almost exactly the motion of the air; around larger particles vortex systems are formed, and from observations of these considerable light is thrown on hydrodynamical problems. All the phenomena of Kundt's tube are explicable in terms of the vortex motion and the circulation. When two particles come close enough together to coalesce they arrange themselves side by side across the tube. When a number do this they range themselves in striae whose longitudinal spacing varies from node to antinode, the

edges of the corresponding vortex systems being contiguous. The spacing depends on sound intensity, size of particles, gas pressure and density of particles. When the sound is very intense the center stria coincides in position with a disc of particles extending right across the tube. These discs are very convenient for measurement. The particles rise up the walls of the tube and fall back across its whole section.

Rayleigh on the Circulation of Air Observed in Kundt's Tubes. In 1883 Lord Rayleigh /3/ published a paper on this subject in "Philosophical Transactions". Quoting from parts of this paper:

"In the present paper three problems of this kind are considered, two of which are illustrative of phenomena observed by Faraday /26/. In these problems the fluid may be treated as incompressible. The more important of them relates to the currents generated over a vibrating plate, arranged as in Chladni's experiments. It was discovered by Savart that very fine powder does not collect itself at the nodal lines, as does sand in the production of Chladni's figures, but gathers itself into a cloud which, after hovering for a time, settles itself over the places of maximum vibration. This was traced by Faraday to the action of currents of air, rising from the plate at the places of maximum vibration, and falling back to it at the nodes. In a vacuum the

phenomena observed by Savart do not take place, all kinds of powder collecting at the nodes. In the investigation of this, as of the other problems, the motion is supposed to take place in two dimensions.

"The third problem relates to the air currents observed by Dvorak /1/ in a Kundt's tube, to which is apparently due the formation of the dust figures. In this case we are obliged to take into account the compressibility of the fluid.

"In the third problem, relating to Kundt's tubes, the fluid must be treated as compressible, as the motion is supposed to be approximately in one dimension, parallel (say) to x. The solution to a first approximation is merely an adaptation to two dimensions of the corresponding solution for a tube of revolution by Kirchhoff /27/, simplified by the neglect of the terms relating to the development and conduction of heat. It is probable that the solution to the second order would be practicable also for a tube of revolution, but for the sake of simplicity I have adheredto the case of two dimensions. The most important point in which the two problems are likely to differ can be investigated very simply, without a complete solution.

"If we suppose $p = a^2\rho$, and write σ for $\log\rho - \log\rho_0$, the fundamental equations are,

$$a^2 \frac{d\sigma}{dx} = -\frac{du}{dt} - u\frac{du}{dx} - v\frac{du}{dy} + \nu\nabla^2 u + \nu' \frac{d}{dx}\left(\frac{du}{dx} + \frac{dv}{dy}\right)$$

with a corresponding equation for v, and the equation of continuity,

$$\frac{du}{dx} + \frac{dv}{dy} + \frac{d\sigma}{dt} + u\frac{d\sigma}{dx} + v\frac{d\sigma}{dy} = 0$$

also $$u = \frac{d\phi}{dx} + \frac{d\psi}{dy} \qquad v = \frac{d\phi}{dy} - \frac{d\psi}{dx}$$

and $$\nabla^2\phi = \frac{du}{dx} + \frac{dv}{dy} \qquad \nabla^2\psi = \frac{du}{dy} - \frac{dv}{dx}$$

Then,

$$\left(a^2 + \nu'\frac{d}{dt}\right)\frac{d\sigma}{dx} = -\frac{du}{dt} + \nu\nabla^2 u - u\frac{du}{dx} - v\frac{du}{dy} - \nu'\frac{d}{dx}\left(u\frac{d\sigma}{dx} + v\frac{d\sigma}{dy}\right)$$

$$\left(a^2 + \nu'\frac{d}{dt}\right)\frac{d\sigma}{dy} = -\frac{dv}{dt} + \nu\nabla^2 v - u\frac{dv}{dx} - v\frac{dv}{dy} - \nu'\frac{d}{dy}\left(u\frac{d\sigma}{dx} + v\frac{d\sigma}{dy}\right)$$

continuing, ...

after a good deal of manipulation, we have

$$u = -\frac{3u_0^2 \sin 2kx}{16a}\left\{1 - \frac{3(y_1 - y)^2}{y_1^2}\right\}$$

$$v = -\frac{3u_0^2\, 2k \cos 2kx}{16a}\left\{y_1 - y - \frac{(y_1 - y)^3}{y_1^2}\right\}$$

From the equation above we see that u changes sign as we pass from the boundary $y = 0$ to the plane of symmetry $y = y_1$, the critical value of y being $y_1(1 - \sqrt{1/3})$ or $0.423\, y_1$. See Figure 19.

The principal motion being $u = u_0 \cos kx \cos nt$, the loops correspond to $kx = 0, \pi, 2\pi \ldots$, and the nodes correspond to $\frac{\pi}{2}, \frac{3\pi}{2},$

$\frac{5\pi}{2}$, ... Thus v is positive at the nodes and negative at the loops (or anti-nodes), vanishing of course in either case both at the wall, $y = 0$, and at the plan of symmetry $y = y_1$.

To obtain the mean velocities of the particles parallel to x, we must make an addition to u,

$$u' = - \frac{u_0^2 \sin 2kx}{8a} \left\{ - 2\beta y + \dots \right\}$$

"We have seen that the width of the direct current along the wall is 0.423 y_1, and that of the return current (measured up to the plane of symmetry) is 0.577 y_1, so that the direct current is distinctly narrower than the return current. This will be still more the case in a tube of circular section. The point under consideration depends only upon a complementary function analogous to one previously considered. The equation for ψ is,

$$\left(\frac{d^2}{dr^2} - \frac{1}{r} \frac{d}{dr} - 4k^2 \right)^2 \psi = 0$$

but if we suppose the radius of the tube is small in comparison with λ , k^2 may be omitted. The general solution is

$$\psi = \left\{ A + Br^2 + B'r^2 \log r + Cr^4 \right\} \sin 2kx$$

so that,

$$u = \frac{1}{r} \frac{d\psi}{dr} = \left\{ 2B + B'(2 \log r + 1) + 4Cr^2 \right\} \sin 2kx$$

whence

$B' = 0$, by the condition at $r = 0$

Again $v = - \frac{1}{r} \frac{d\psi}{dx} = -2k \left\{ Ar^{-1} + Br + Cr^3 \right\} \cos 2kx$

whence $A = 0$

and $u = \left\{ 2B + 4Cr^2 \right\} \sin 2Kx$

$$v = -2K \left\{ Br + Cr^3 \right\} \cos 2Kx$$

If $v = 0$, when $r = R$, $B + CR^2 = 0$ and

$$u = 2C(2r^2 - R^2) \sin 2Kx$$

Thus u vanishes when

$$r = \frac{R}{\sqrt{2}} = 0.707 R \qquad R - r = 0.293 R$$

"The direct current is thus limited to an annulus of thickness 0.293 R, the return current occupying the whole interior, and having therefor a diameter of 2 x .707 R = 1.414 R."

No consideration was given in this paper to the flüid flow outside the tube (in the case of an open-ended tube) or to the unbalanced thrust which might be produced by any of the various characteristics of pressure or motion of the fluid when compared with the properties of the ambient atmosphere, or a suitable frameof reference.

Acoustic Repulsion. The phenomenon of acoustic repulsion, accompanied by circulation of the air in a chamber resonating to a given sound source, together with evidence of the air streaming from the open end of the chamber, was demonstrated by Dvorak /1/ as early as 1878. He carried out several interesting experiments in which a glass resonator tuned to the note of a tuning

fork is fastened by sealing wax to the end of a light wooden rod, the other end of which carries a small lead counterpoise. To the center of the rod is fixed a glass cap which rests on a vertical needle point. When the fork is strongly bowed and the open end of the resonance box (upon which the fork is mounted) is presented to the open end of the resonator the wooden arm rotates on the needle point.

When the resonator is filled with tobacco smoke, and the fork is sounded, currents of air are seen to enter and leave the chamber, as the resonator is accelerated by the unbalanced thrust acting inward on the open mouth of the resonating chamber. A current of air which is nearly as large in diameter as the cross section of the tube or chamber is noted to proceed out of the tube, while a very narrow ring of air is noticed to flow into the tube adjacent to the walls of the resonator or tube.

There is also an unbalanced pressure on the interior wall of the resonator opposite to its open end. Dvorak noted by careful measurements with a manometer that the pressure at the node at the interior wall of the resonator* opposite to the open end is greater than the ambient pressure outside the tube. Also the pressure at the open end of the tube is greater than the outside

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*See also, Morse /4/, p. 258, "Vibration and Sound" 1948, for cavity resonance and formula for ratio of pressure at closed end to pressure at open end of tube.

pressure, although the pressure at the open end of the tube (at the antinode) is less than the pressure at the closed end.

DESCRIPTION OF APPARATUS

The principal apparatus in this investigation consisted of resonators of various sizes and shapes, a controlled sound source which could be adjusted in frequency and amplitude, and a thrust measuring device.

The resonators were constructed of glass, stiff paper, and plexiglas to the dimensions shown in Figure 1.

The sound source was a Stromberg-Carlson eight inch cone type radio loudspeaker, Model RC-25, with good reproduction qualities from 60 to 7000 cycles per second. This speaker was driven by an amplifier generally used in public address systems, RCA Model MI-12222; and an audio oscillator, Hewlett Packard, Model 200C, frequency range 18 cps to 200,000 cps. The shape of the wave form produced was sinusoidal. This was checked on a cathode ray oscilloscope screen. See Figure 2 for diagrammatic sketch of loudspeaker arrangement.

Thrust measurement was made by supporting the resonator by means of a cantilever strip of metal (a strip of Alcoa 24ST one inch wide, 0.125 inches thick, and 30 inches in length) mounted vertically. Standard Baldwin and Southwark wire strain gages, with a factor of 2.04, were cemented on either side of the strip in order to measure the bending moment in the strip produced by the unbalanced thrust acting on the resonator. The wire strain

gages and bridge measurement equipment, SR4 type L box, were calibrated by means of a force triangle arrangement shown in Figure 3. It was found that the relationship between thrust and strain was linear and in the proportions: 10 micro-inches strain equal 0.00570 pounds thrust.

EXPERIMENTAL PROCEDURE

Descriptive Tests and Experiments. Work was begun by constructing several small models of resonators shown in Figure 1 as numbers five, six, and seven. These were taken from the designs by Dvorak /1/. The controlled sound source was the radio loudspeaker driven by the oscillator and amplifier circuit. Variation of frequency as well as amplitude of the sound source was easily obtainable. When the resonators were mounted so that rotational freedom was achieved and the resonant frequency was produced by the loudspeaker, the vessels revolved and a jet of air issued from the openings. The sense of the rotation was such that the resonators moved away from the direction in which their open ends were pointing. When the resonators were filled with smoke, and the resonant frequency sounded, the smoke was seen to be expelled from the openings.

Although these experiments were interesting and showed that the phenomena of thrust could be produced by the secondary effects of acoustic energy, the amount of thrust was too small to measure, and the resonators were extremely critical to frequency adjustment.

Quantitative and Qualitative Experiments. The investigation was continued on a larger scale to permit the measurement of thrust precisely as well as to study the flow of fluid during resonance. Figure 4 shows the experimental setup used. The first

step was to mount the cantilever strip and install and calibrate the strain gages. The strain gages were cemented to the strip of 24ST aluminum alloy approximately one inch from the clamped end of the strip. A small weight, weighing 0.1102 pounds (or 49.988 grams) was suspended by a silk thread as shown in Figure 3. Another thread was attached to the end of the resonator tube by a drop of cement and the other end of the thread was secured by a knot to the thread of the weight. The thrust, R, on the resonator was then determined by measuring x and y and using the relation: $x/y = R/w$, where w equals 0.1102 pounds.

See Table 1 and Figure 5 for the results of the strain gage calibration. It should be noted that, although the variation in the weight of the resonator displaces the calibration curve upward or downward, the curve remains a straight line and the slope is identical within the accuracy of the setup. Measurements could be made to an accuracy of 0.0001 pounds.

Suitable instruments calibrated to measure the absolute intensity of the sound produced by the loudspeaker were not available. However, a Westinghouse sound level meter and condenser microphone were used to compare the intensities of sounds at various frequencies. This device had a constant response to frequency. The sound level meter microphone was placed 0.40 inches in front of the loudspeaker aperature. The loudspeaker was then

made to sound at the lowest frequency of resonance for the resonating tubes used in the experiment. The amplitude of the oscillator and amplifier combination was adjusted to produce a sound of arbitrary intensity. Then, all the other frequencies were set up, successively, and the amplitude of the oscillator was adjusted to produce the same arbitrary sound intensity as indicated by the sound level meter. The amplitude calibration for the various frequencies at constant sound intensity 0.40 inches in front of the speaker aperature is shown in Figure 6.

The three-voltmeter method was used to determine the power input to the loudspeaker. A cathode ray oscilloscope was employed to determine the phase angel le of the sinusoidal signal. A diagrammatic sketch of the electrical circuit for this purpose is shown in Figure 7. It was desired to obtain the power input to the speaker at various frequencies with constant energy output from the speaker. Hence, the sound level meter was used to adjust the amplitude of the oscillator and amplifier units. The power input to the speaker was then measured.

The power input to the speaker, P, is given by

$$P = E_s \, I \cos \phi$$

where ϕ is the phase angel between the speaker voltage, E_s, and the current, I. The current, I, is obtained from the voltage drop, E_r, across the known resistance, r. The voltage, E_L, supplied by

the power amplifier exceeds E_s and the vector diagram is indicated in Figure 7. From this diagram

$$E_s = \sqrt{E_L^2 + E_r^2 - 2E_LE_r\cos\theta}$$

$$\sin\phi = (E_L/E_s)\sin\theta$$

θ was determined by the oscilloscope, see Figure 7.

$$X = k_1 E_L \sin\omega t$$

$$Y = k_2 I \sin(\omega t - \theta)$$

where

ω = circular frequency

t = time

k_1 and k_2 = constants for oscilloscope.

Since $\sin\theta = X_A/X_B$, the phase angle may be determined, and the power, P, is then computed.

See Figure 8 for the plot of power input at various frequencies for constant power output from the speaker.

The measurement of the thrust produced by the vessel under investigation when resonating to the loudspeaker sound source was made by obtaining the deflection in micro-inches of the strain gage attached to the cantilever strip supporting the resonator. The setup is shown in Figure 4. The eight inch cone of the loudspeaker was masked with a sound-absorbent insulation board to a diameter of approximately 3.5 inches to prevent undue dissipation of the sound energy to the atmosphere. The speaker was then

placed so that the aperature was 0.40 inches from the open end of the resonating tube. This was an arbitrary arrangement found by experimentation to provide just about a maximum amount of sound concentration without interference with the flow of air in and out of the tube. To prevent disturbance from stray air currents, the setup was made in a small compartment with an open side. The walls of the compartment were covered with quilting to decrease echoing and chamber resonance.

From the thrust calibration it was determined that the deflection of the strain gage in the amount of 10 micro-inches was equivalent to 0.00570 pounds of thrust acting on the resonator. It was also found that this relationship was linear, so that it was only necessary to measure the deflection for a given test, multiply the deflection by 0.00570, and thus obtain the value of the thrust in pounds.

Resonators of various dimensions and configurations were constructed and tested for thrust in the manner mentioned above. The peculiarities of the air flow in and out of the resonators was determined by soaking a small wick placed on a probe with titanium tetrachloride. This is a yellowish liquid which produces dense smoke when it comes in contact with air. Tobacco smoke was found to be entirely inadequate because of the rapid movement of air and the immediate dispersal of the smoke.

RESULTS AND DISCUSSION

One of the interesting factors noted in the experimentation was the preciso reproducability of all the data taken. Each point on the thrust-frequency curves was reproduced on different days with varying ambient temperatures and humidity conditions. Furthermore, all thrust values were very steady conditions which would be found to obtain as long as the sound frequency and intensity conditions were maintained. None of the thrust or jet flow was found to be unstable when the sound source was constant.

Thrust versus Frequency. The variation of dynamic thrust with constant sound energy but varying frequency is plotted in Figure 9. This thrust was produced by the unbalanced forces acting on the resonating vessel sounding in response to the loudspeaker sound source placed 0.40 inches in front of the open end of the resonator and directed toward it. From the plot it may be seen that the thrust drops off very rapidly as the frequency is increased. It appears that the thrust is a function of frequency, therefor, as well as of the energy output of the sound source. Furthermore, the thrust seems to be dependent on frequency and not the length of the tube, because the first harmonic of the longer tubes produced a thrust which agreed with its proper place on the thrust-frequency curve.

A curve of similar shape to that exhibited in Figure 9 was

obtained when the thrust was plotted versus frequency with constant volume setting (not exactly constant energy output, however, due to the response of the speaker) of the loudspeaker-amplifier-oscillator system. This curve is plotted in Figure 10.

Thrust versus Power. The amount of dynamic thrust developed by the acoustic jet is directly dependent on the sound energy supplied to the resonating system. This could be readily observed by the rapid decrease in thrust when the volume control of the loud speaker was turned down as well as when the loudspeaker was moved away from the resonator. Moving the source of sound about 12 inches away from the open end of the resonator caused the thrust to drop off to a very small quantity, even at the low frequencies.

In Figure 13, the thrust/input power was plotted versus frequency. It was found that the former dropped off rapidly as the frequency was increased. From the standpoint of both actual thrust, measured in pounds, and thrust/power input, measured in pounds per watt, the most promising region is the acoustic frequency band between about 10 and 150 cycles per second.

Ventilated Resonators. One resonator was selected for tests to determine the effect of openings of various sizes and

positions on the thrust produced by the resonator sounding at its fundamental frequency. In all cases the energy output of the loudspeaker was maintained at the same constant value used in previous experiments.

Resonator No. 10, 12.08 inches long and 2.73 inches inside diameter, was used for this experiment. A series of holes, each 9/64 inches in diameter were drilled through the sides of the tube in radial directions. Pairs of holes were drilled diametrically opposite each other and spaced one inch apart from the open end to the closed end of the tube. Thrust measurements were then made with the tube sounding at it fundamental frequency and with one pair of holes open at a time. The other holes were covered with scotch tape to prevent the passage of air. The variation of thrust with axial position of these pairs of holes along the length of the tube may be noted in the plot in Figure 11. The total area of two holes open at one time was 0.031 square inches. From the curve it can be seen that the effect of openings in the tube is much greater at the closed end of the tube than at the open end. This follows from the fact that the static pressure in a closed tube of this kind is greater at the closed end than at the open end; and an opening at the closed end will have greater effect in reducing the pressure acting on the end plate (which is largely responsible for the thrust produced by the resonator). See, also, the diagrams in Figure 14.

The departure from a smooth curve, indicated by the variation in thrust found when the pairs of holes from three to six inches from the open end of the tube were uncovered, is indicative of the rather turbulent flow conditions noted in this section of the tube by smoke. Apparently the inflow of air, which enters the tube at the open end in a thin annulus, moves along the inner wall of the tube until about one-third to one-half the length of the tube, at which point the flow becomes turbulent and the outflow of air begins to develop there. The outflow of air picks up velocity and becomes more smooth and laminar, as it proceeds toward the mouth of the tube, until at the open end of the resonator it issues forth in a rapidly flowing jet much larger in diameter than the thickness of the inflowing annulus.

All the holes had air flowing out of them when the tube was sounding at the resonant frequency. The intensity of the jets of air issuing from the holes varied as the position of the holes from the open end of the tube. Holes close to the open end showed air coming out in the form of a jet, but the intensity of the jets increased very markedly as the location of the hole moved toward the closed end. This, of course, is in agreement with the fact that the static pressure is greater at the closed end of the tube.

A diagram showing the pressure and velocity distribution in a closed tube is given in Figure 14.

The general circulation of the air in the tube and the inflowing and outflowing streams were unchanged by the presence of the holes mentioned above. The fundamental frequency showed no appreciable change.

The resonator listed in Figure 1 as No. 9 was used to determine the effect an orifice in the center of the end plate of the closed tube resonator has on the thrust. The orifices were drilled through the 1/16 inch plexiglas plate at right angles to the surface of the plate. Holes were not bevelled. The variation of thrust with the ratio of orifice area to cross-sectional area of the tube can be seen in the plot in Figure 12. As the orifice area ratio increased the thrust decreased very rapidly. Evidently, as the size of the orifice is enlarged, the resonant properties of the tube begin to approach those of an open tube; i.e., open at both ends. However, in this case the resonant frequency of the open tube of length 12.08 inches was so high that no appreciable thrust was developed.

An experiment was also made with the 2.73 inch diameter resonator 12.08 inches long in which the end plate was closed and two 9/64 inch diameter holes were drilled in opposite sides of the

wall about 0.08 inches from the closed end. The momentum of the jets of air issuing from the two holes thus balances each other and did not disturb the equilibrium of the tube. The thrust measured on the tube in this case was found to be 0.00513 pounds, which was slightly higher than that observed when an orifice of equal area was drilled in the end plate so that its jet of air was along the axis of the tube. See Figure 12. The difference in the two thrust values would be the measure of the momentum developed by the jet from the orifice opposing the momentum of the larger jet of air flowing from the open end of the resonator.

Thrust With Resonator Reversed. In order to compare the thrust developed by a resonator when sounding in sympathy to the sound source with the apparent thrust produced by a single flat plate placed in front of the same sound source, the closed end of a tube was presented to the aperature of the loudspeaker. In this case the thrust on the closed end of the tube was only 1/20 of that developed by the resonator with open end toward the sound source. Also, the thrust in the first case was toward the sound source, while the thrust produced by the resonator was always away from the sound source. This negative thrust was caused by the low static pressure produced over the face of the end plate by the rapid flow of air across it. Smoke tests showed no movement

of the air within the tube and no jet flow at its open end away from the speaker.

Thrust on Open Tube. When a straight tube, shown as resonator No. 8 in Figure 1, with both ends open, was made to sound in resonance to its fundamental frequency, a thrust was produced. This thrust was very small -- only 0.00228 pounds at a frequency of 231 cycles per second -- but it was in the same direction as that of the closed tube (away from the sound source). This compares with a thrust of about 0.0190 pounds to be expected of a closed tube resonator at 231 cycles per second.

When the flow was examined with the aid of titantium tetrachloride smoke, it was found that there was no outflow whatsoever from the open end of the resonator adjacent to the sound source. The air flowed in at this end and continued straight through the tube in what appeared to be smooth, laminar flow. It issued from the far end of the tube in a jet, which was of the same diameter as the tube. Hence, the thrust of the tube was probably due to the drag associated with the friction along the tube walls.

Air Circulation and Streaming. When the experiments were first begun, a great deal of difficulty was encountered in obtaining air flow in the manner to be expected from the observations

of Dvorak /1/ and Lord Rayleigh /3/. At first, the closed tube resonator, constructed of plexiglas tubing 2.88 inches in diameter with a 1/8 inch wall thickness and closed at one end with a plate of 1/16 inch plexiglas, was placed about 0.40 inches in front of the aperature (which measured about 3.50 inches in diameter) of the loudspeaker. The tube was found to resonate strongly when the proper frequency was set up, but the thrust obtained was such that the tube was drawn toward the sound source. And when the flow of air was investigated by means of smoke, it was discovered that there was a good deal of turbulent flow in and around the mouth of the resonator; none of which could be identified as having any properties of a jet.

The edge of the open end of the tube had been left square and smooth, and although it was only 1/8 inch in thickness, it was found to present enough of a surface to ruin the desired circulation and streaming of the air in and out of the resonator. Instead, the rapid movement of turbulent air produced a low static pressure over the edge of the lip such that the atmospheric pressure of the air, acting at the other end of the tube, pushed the tube toward the sound source.

When the edge of the tube was sanded off to a knife edge (without changing the inside dimensions of the tube) the flow was smoothed out so that a thin annulus of air flowed rapidly around

the end of the tube and along the inner wall to a point about one third to one half the length of the tube and then turned around and flowed out through the center portion of the tube in the form of a jet of circular cross-section.

A sketch of the air circulation and streaming during resonance may be seen in Figure 15. Photographs of smoke plumes to trace the flow are presented in Figure 16.

It was found in all cases that the phenomena of dynamic thrust, air circulation and streaming, together with jet formation were only associated with resonance. When frequencies other than resonant ones (either fundamental or harmonic) were produced by the sound source, there were none of the phenomena mentioned above. Moreover, tuning was quite critical. A few cycles per second variation from the resonant frequency one way or the other were enough to eliminate the effects. See Figure 9 for an illustration of this.

The thickness of the ring of air moving along the inner wall of the resonator was found to be approximately 1/4 of the diameter of the resonator, while the diameter of the jet of air flowing out through the mouth of the tube was 1/2 of the diameter of the tube. This is roughly in agreement with the mathematical analysis presented by Lord Rayleigh /3/ for the circulation of air

in a Kundt's tube. See page 16 of this thesis. At the thinnest point, the thickness of the annulus of air moving inward along the wall of the tube was given by Rayleigh as about 1/6 of the diameter of the tube. The diameter of the outflowing jet then, of course, occupies 2/3 of the diameter of the tube. The difference may be attributed to conditions imposed by the open end of the tube.

CONCLUSIONS

Acoustic resonance of a vessel with a single opening, filled with air, sounding in response to an external sound source, is accompanied by the following interesting phenomena:

(1) An unbalanced thrust is produced on the vessel acting in a direction inward through the opening of the vessel.

(2) Two distinct streams of air are recognizable flowing in and out of the open mouth of the vessel. The inflowing air current is a thin ring moving rapidly along the inner wall of the resonator. The outflowing current, which is circular in cross-section is approximately 1/2 to 2/3 the diameter of the tube.

(3) The static pressure in the vessel is everywhere greater than atmospheric pressure. It is largest at the nodal positions in the tube and lowest at the anti-nodal points.

The jet action (with air as the fluid medium), and hence the thrust, is many times greater at lower frequencies between 10 to 150 cycles per second than at higher frequencies with the same power input.

The formation of the acoustic jet is extremely sensitive to lip conditions at the opening of the resonator.

The effect of small openings in the wall of the resonator, while not making an appreciable difference in the resonant frequency of the vessel, is to permit a loss in pressure within the

tube and therefor a reduction in jet action and unbalanced thrust. The loss in pressure and accompanying reduction in thrust is more significant at the high pressure regions in the vicinity of the nodal points of the resonator than in the anti-nodal regions. Openings large enough to greatly change the resonant frequency of the vessel reduce the thrust to a negligible quantity.

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to 19 mm and diameters of 3.5 mm to 20 mm. Frequency between 150 to 1000 cps. Velocities in orifice cover range of 0 to 700 cm/sec. Eckart, in his theory of streaming caused by sound waves, shows that time independent streams necessarily follow as part of the solution of the complete wave equation, taking into account viscosity and second order terms. He proves driving force of streams is proportional to frequency squared. Suggests that slow streams might also be carried in air at audio frequencies.

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verify the theoretical results. The resonant frequency of resonators is also calculated and agreement with measured values obtained.

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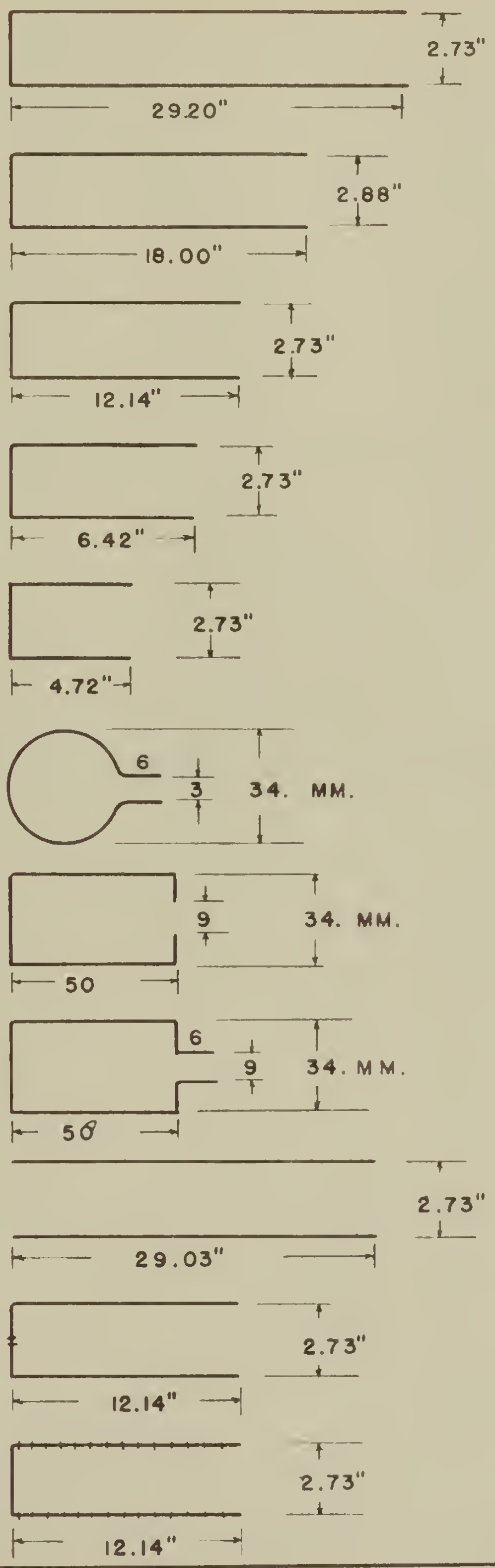
FIG. 1. DIMENSIONS OF RESONATORS.



NO. 0	PLEXIGLAS TUBING 0.14" WALL 1/16" PLATE ON END OPEN EDGE SHARPENED.
NO. 1	SAME AS ABOVE BUT 0.12" WALL
NO. 2	SAME AS RES. NO. 0
NO. 3	SAME
NO. 4	SAME
NO. 5	GLASS SPHERE 0.02" WALL
NO. 6	STIFF PAPER CYLINDER WITH ORIFICE
NO. 7	STIFF PAPER CYLINDER WITH NECK
NO. 8	SAME AS NO. 0 BUT OPEN AT BOTH ENDS
NO. 9	SAME AS NO. 2 WITH 0.0625" ORIFICE
NO. 10	SAME AS NO. 2 WITH 12 9/64" HOLES IN SIDE TOTAL OF 24

FIG. 2. DIAGRAM OF LOUDSPEAKER SETUP.

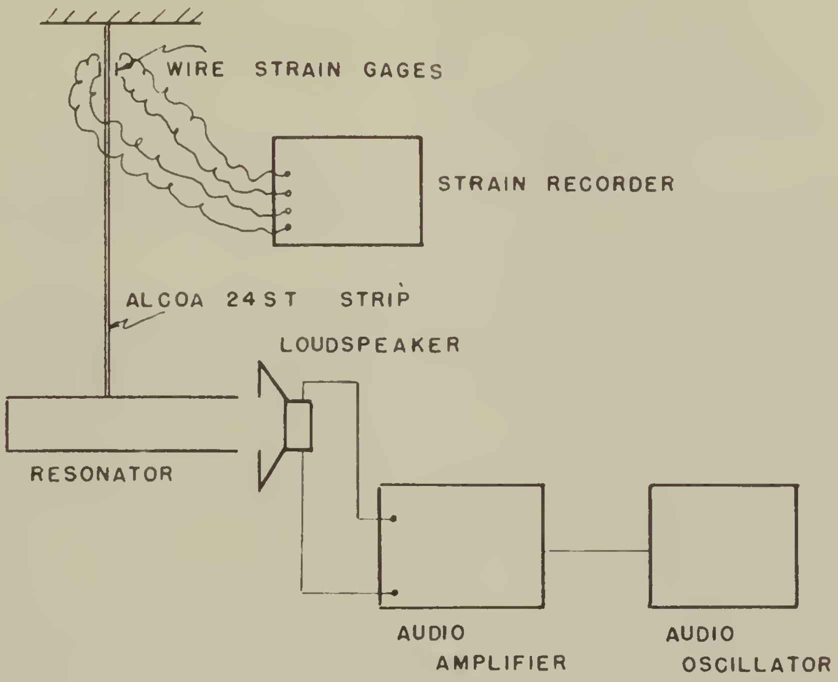
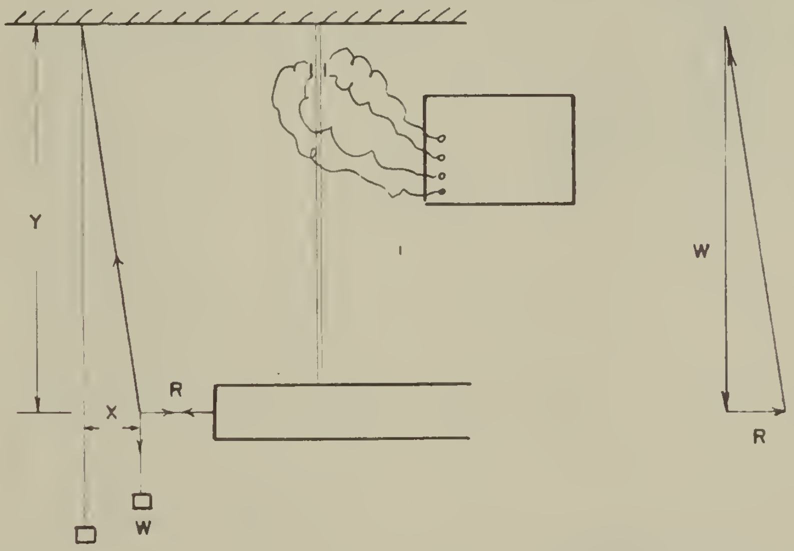


FIG. 3. FORCE TRIANGLE FOR STRAIN GAGE CALIBRATION.



R/W = X/Y

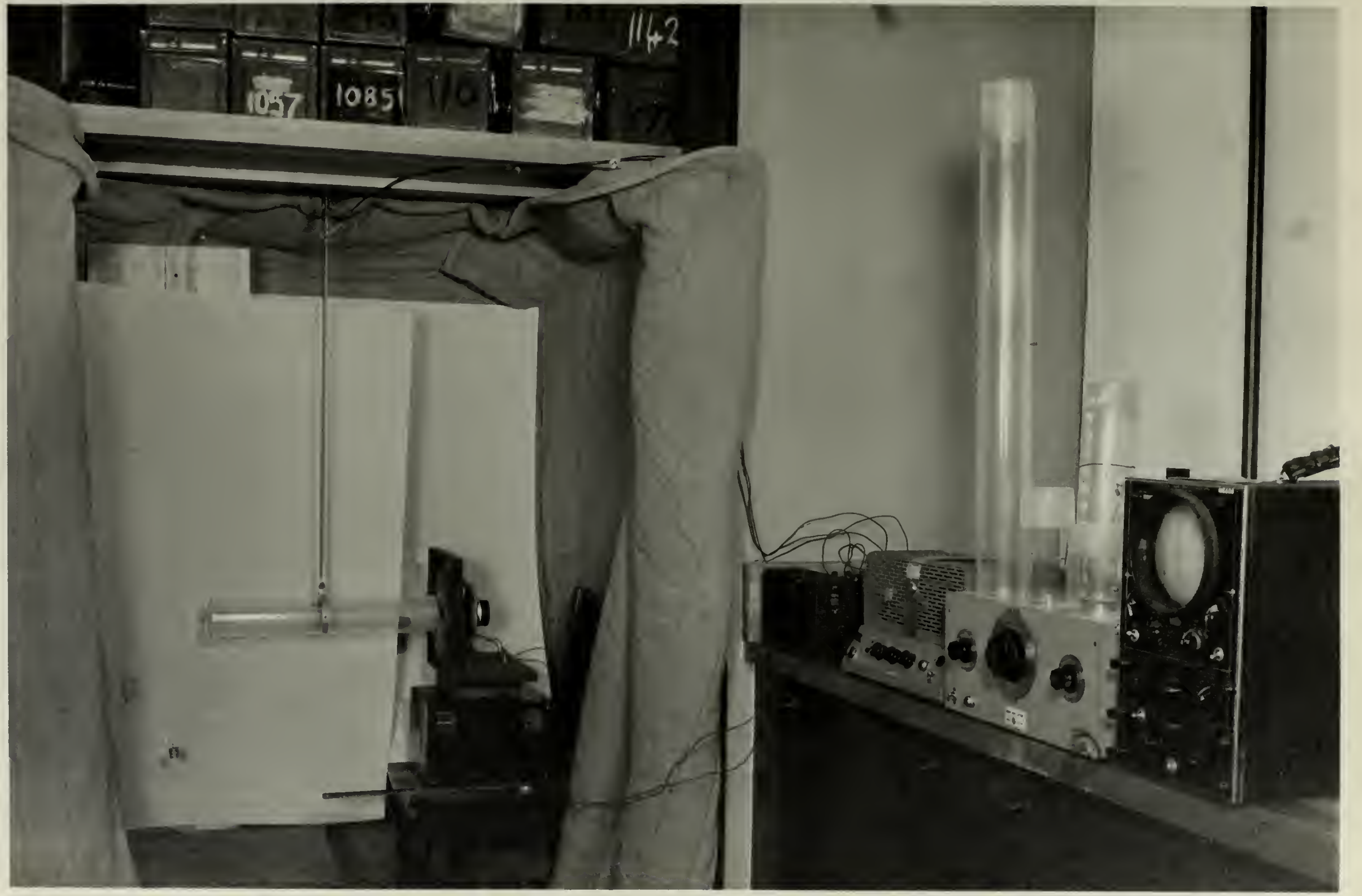


FIG. 4 PRINCIPAL APPARATUS.

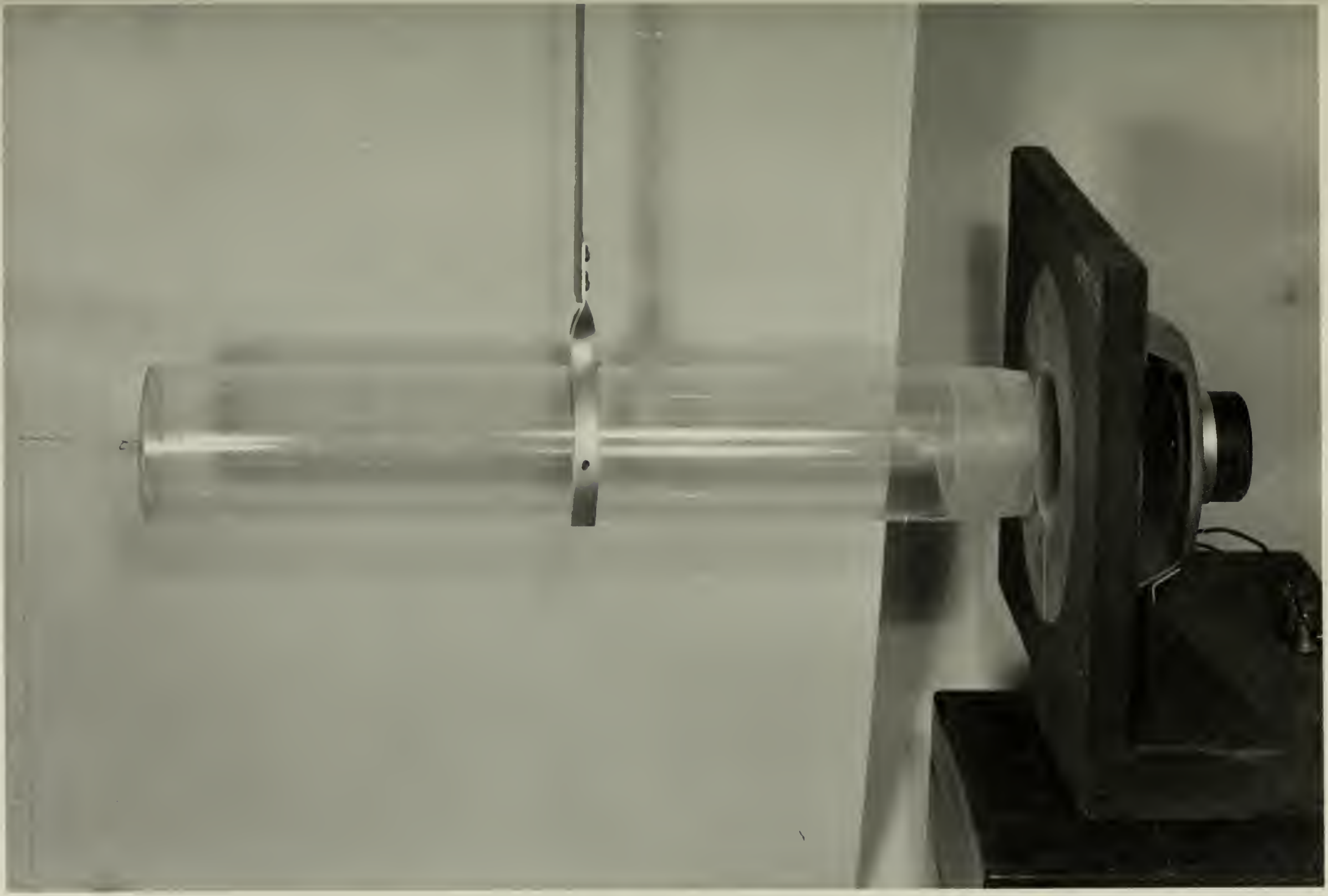
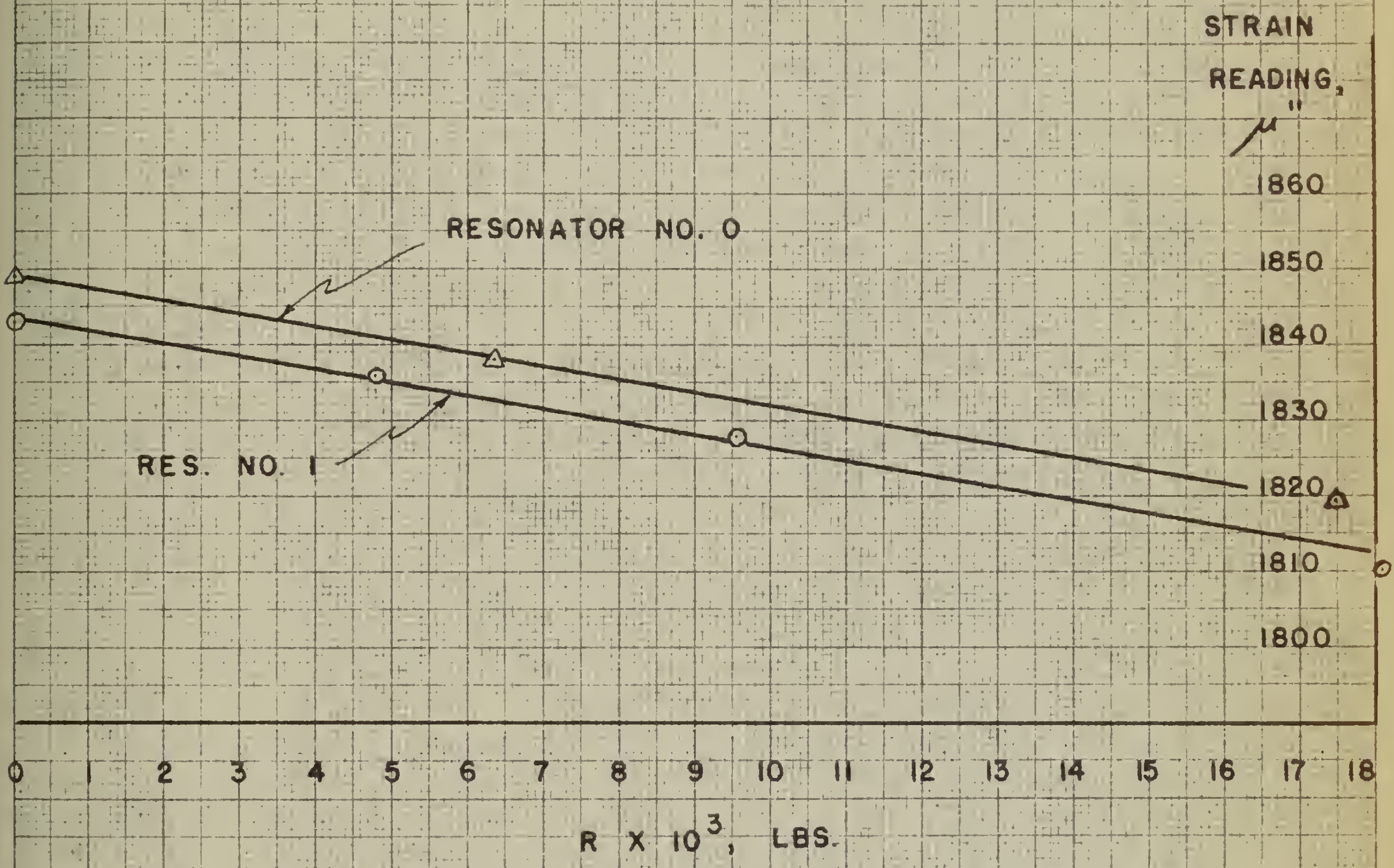


FIG. 4.1 RESONATOR MOUNTED FOR STRAIN GAGE CALIBRATION.

FIG. 5. STRAIN GAGE CALIBRATION.



BALDWIN SOUTHWARK SR4 STRAIN INDICATOR TYPE L

NO. H-59223 FACTOR 2.04 REF. 8

RES. NO. 0 $10 \mu'' = 0.00570$ LBS.

RES. NO. 1 $10 \mu'' = 0.00570$ LBS.

FIG. 6. CALIBRATION OF OSCILLATOR AMPLITUDE CONTROL WITH CONSTANT ENERGY OUTPUT FROM LOUD SPEAKER.

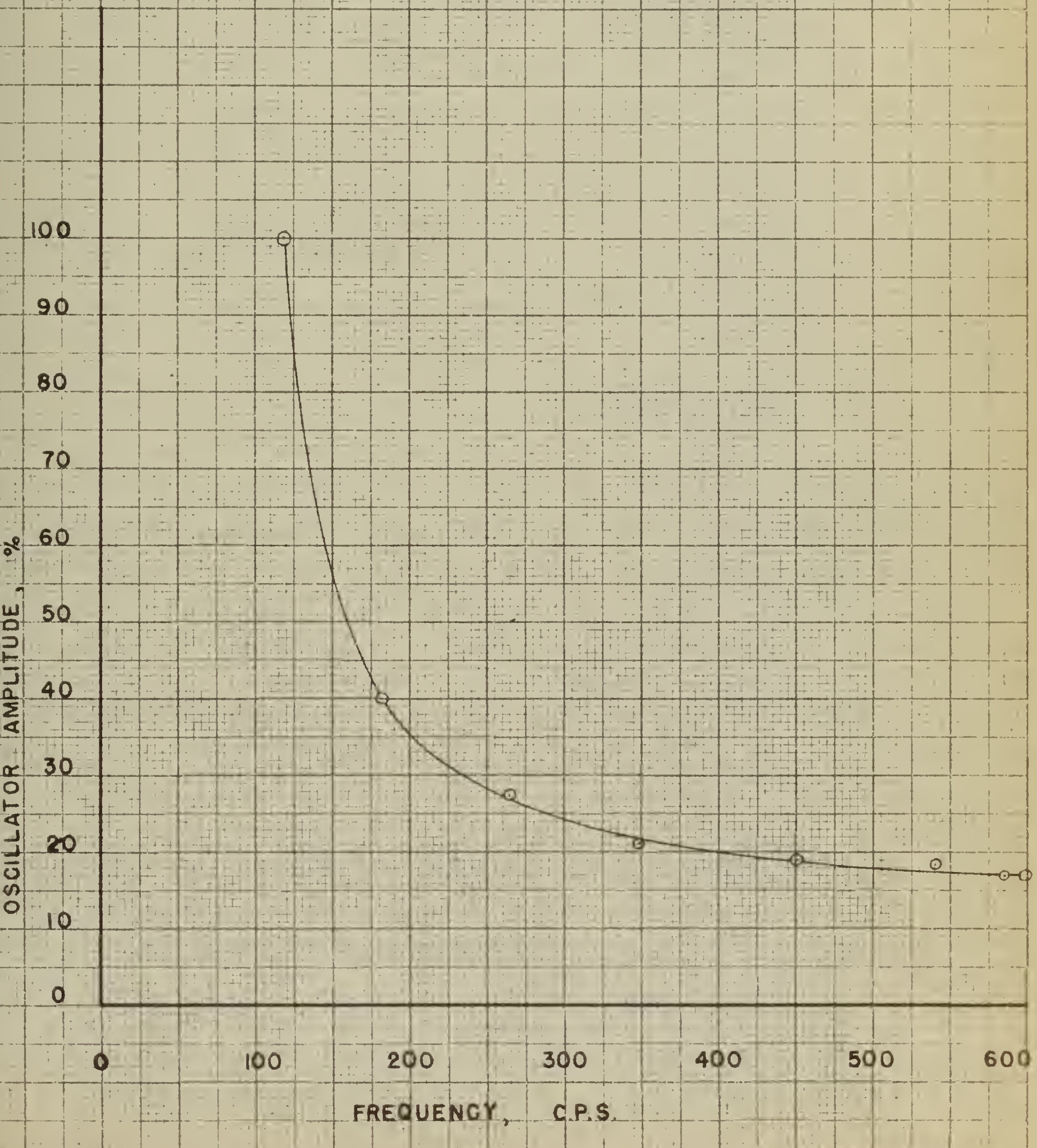
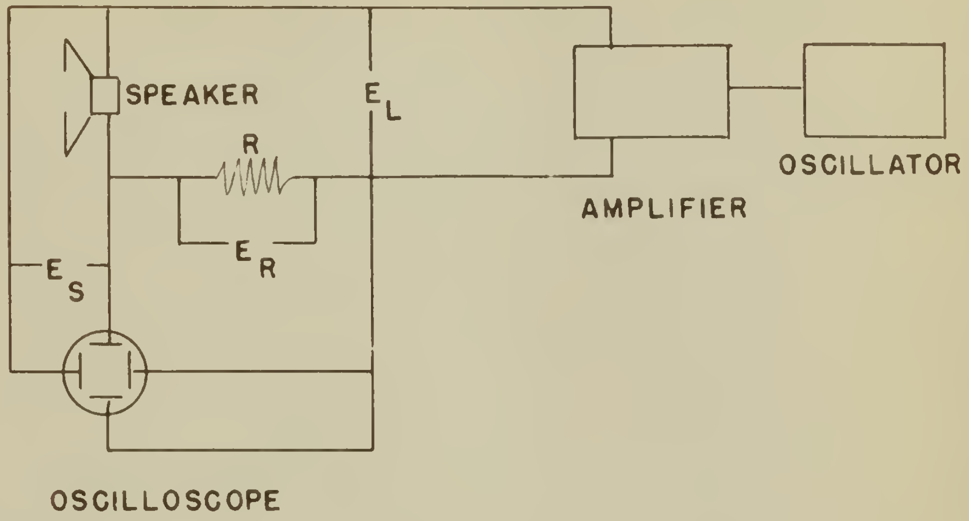
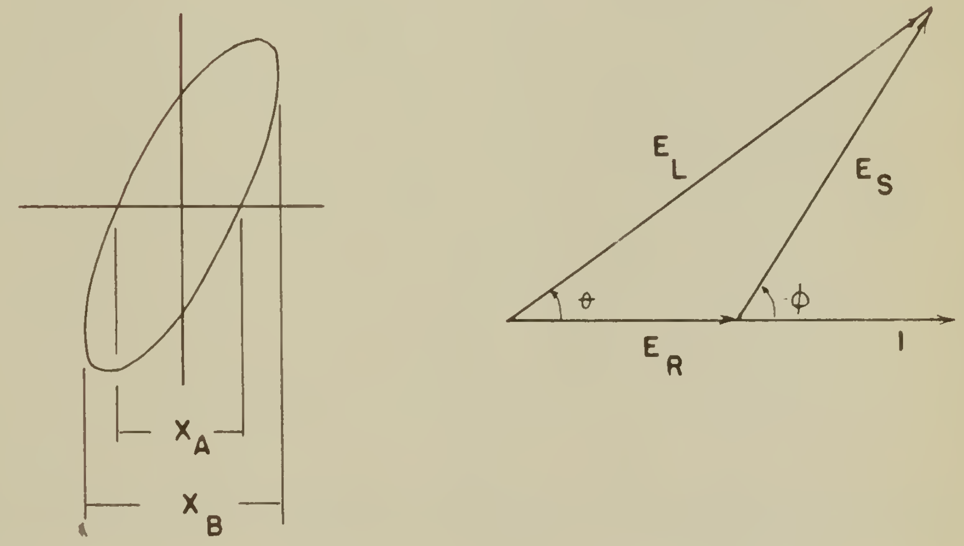


FIG. 7. MEASUREMENT OF POWER INPUT TO SPEAKER.



VECTOR DIAGRAM



ELLIPSE ON OSCILLOSCOPE

FIG. 8. SPEAKER POWER INPUT AT VARIOUS FREQUENCIES WITH CONSTANT POWER OUTPUT OF SAME VALUE USED TO CALIBRATE OSCILLATOR IN FIG. 6.

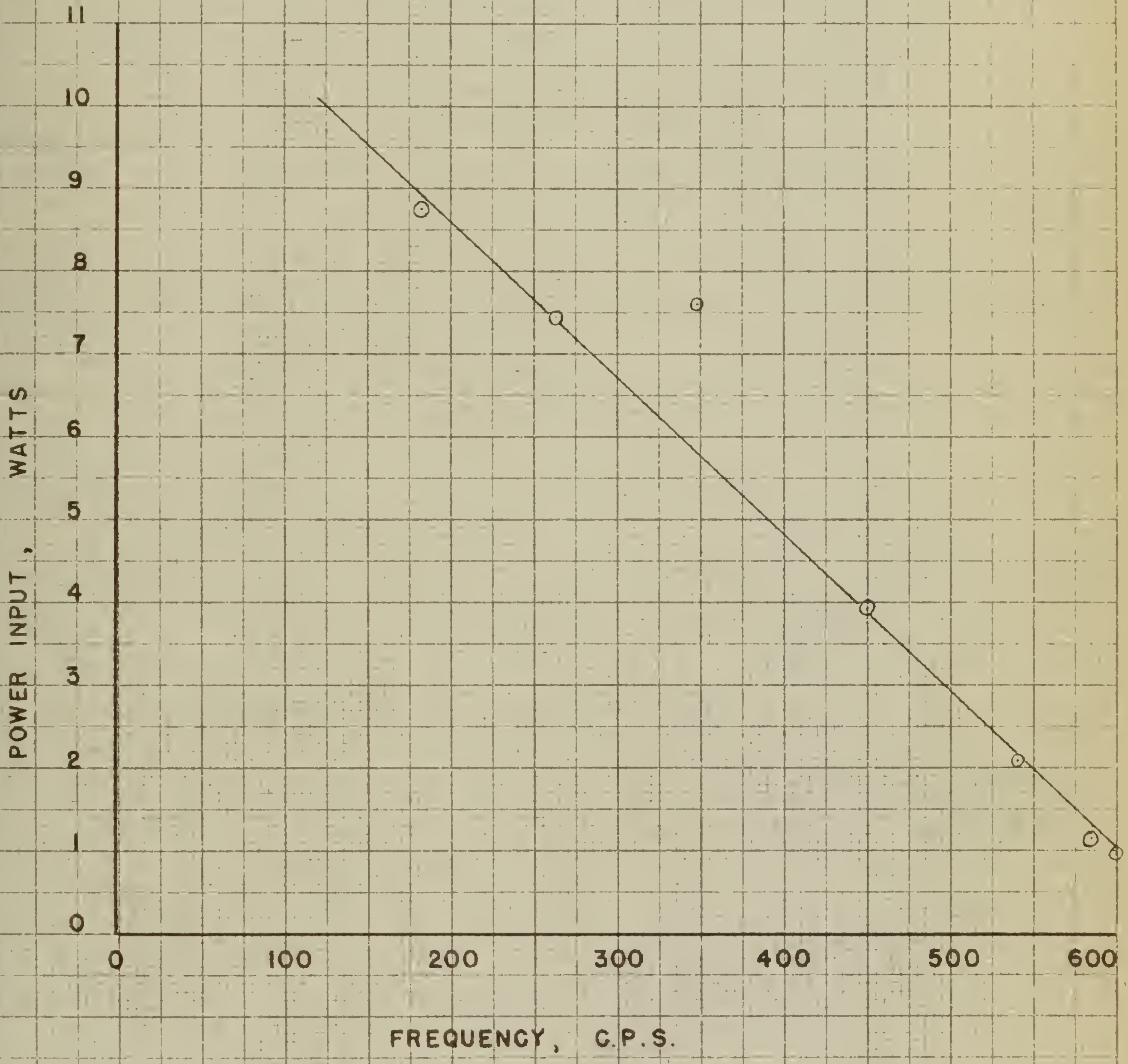
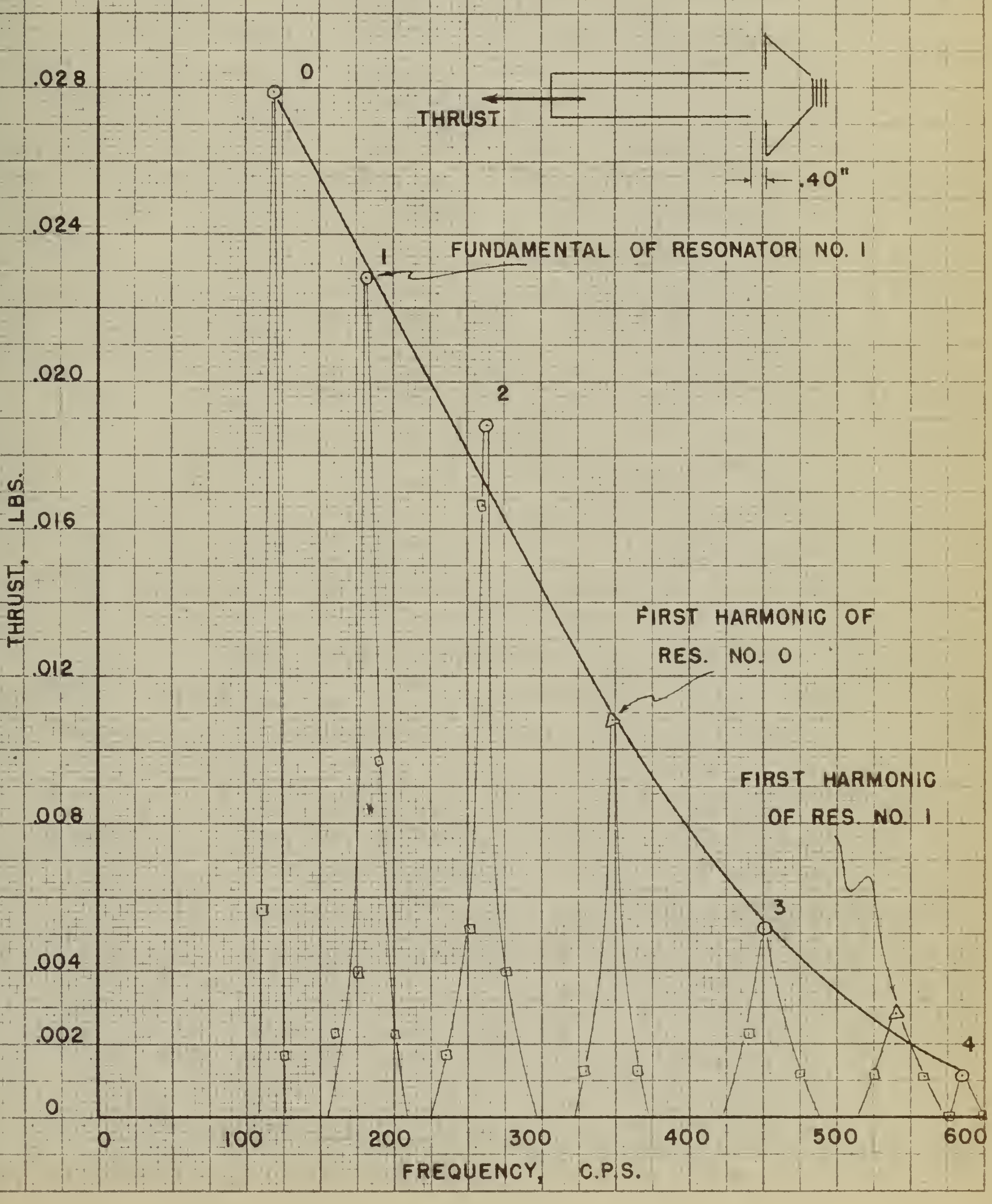


FIG. 9. VARIATION OF THRUST PRODUCED BY CLOSED TUBE RESONATORS OF SAME CROSS-SECTIONAL AREA BUT DIFFERENT LENGTHS, SOUNDING IN RESPONSE TO EXTERNAL SOUND SOURCE OF CONSTANT INTENSITY.



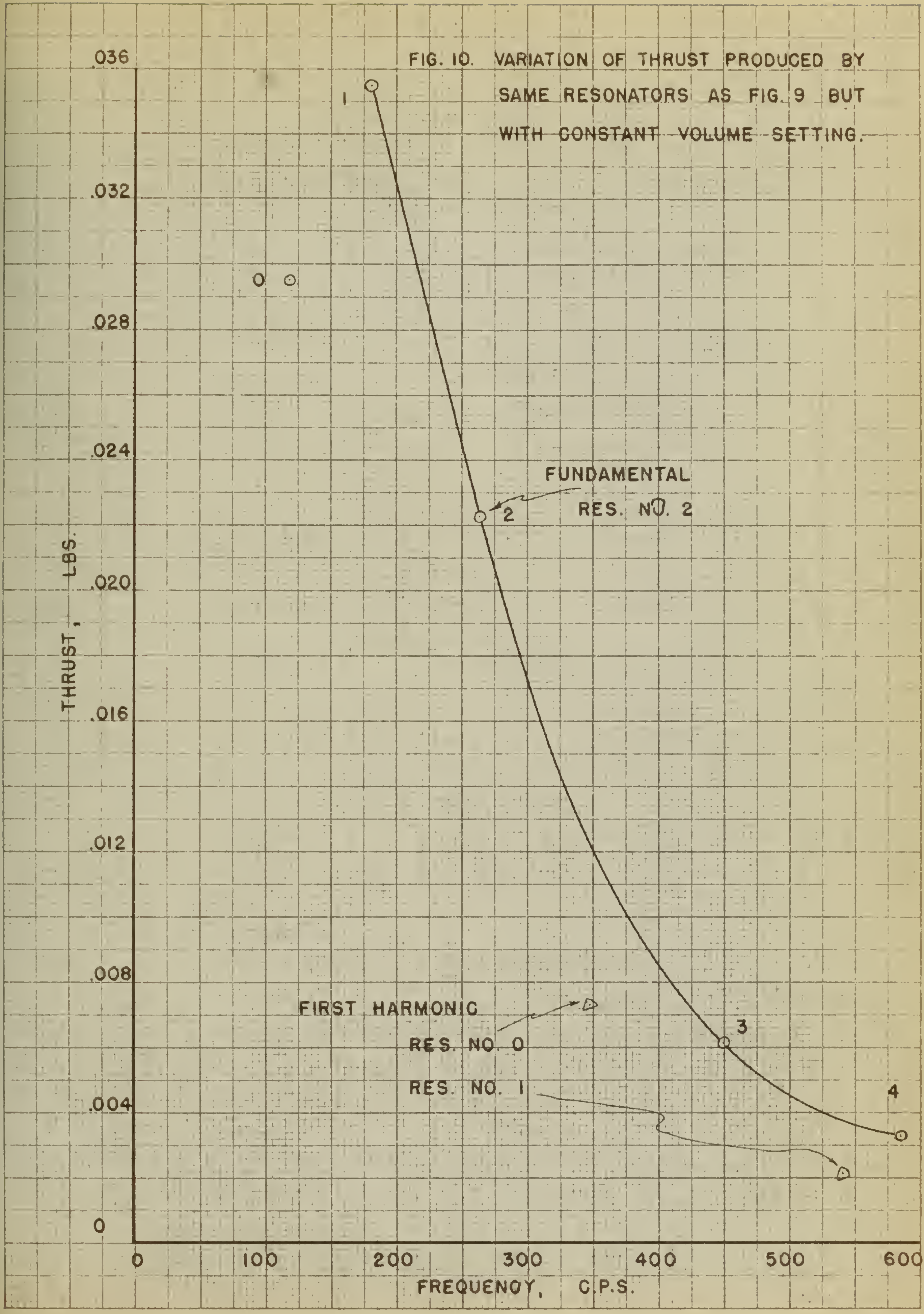
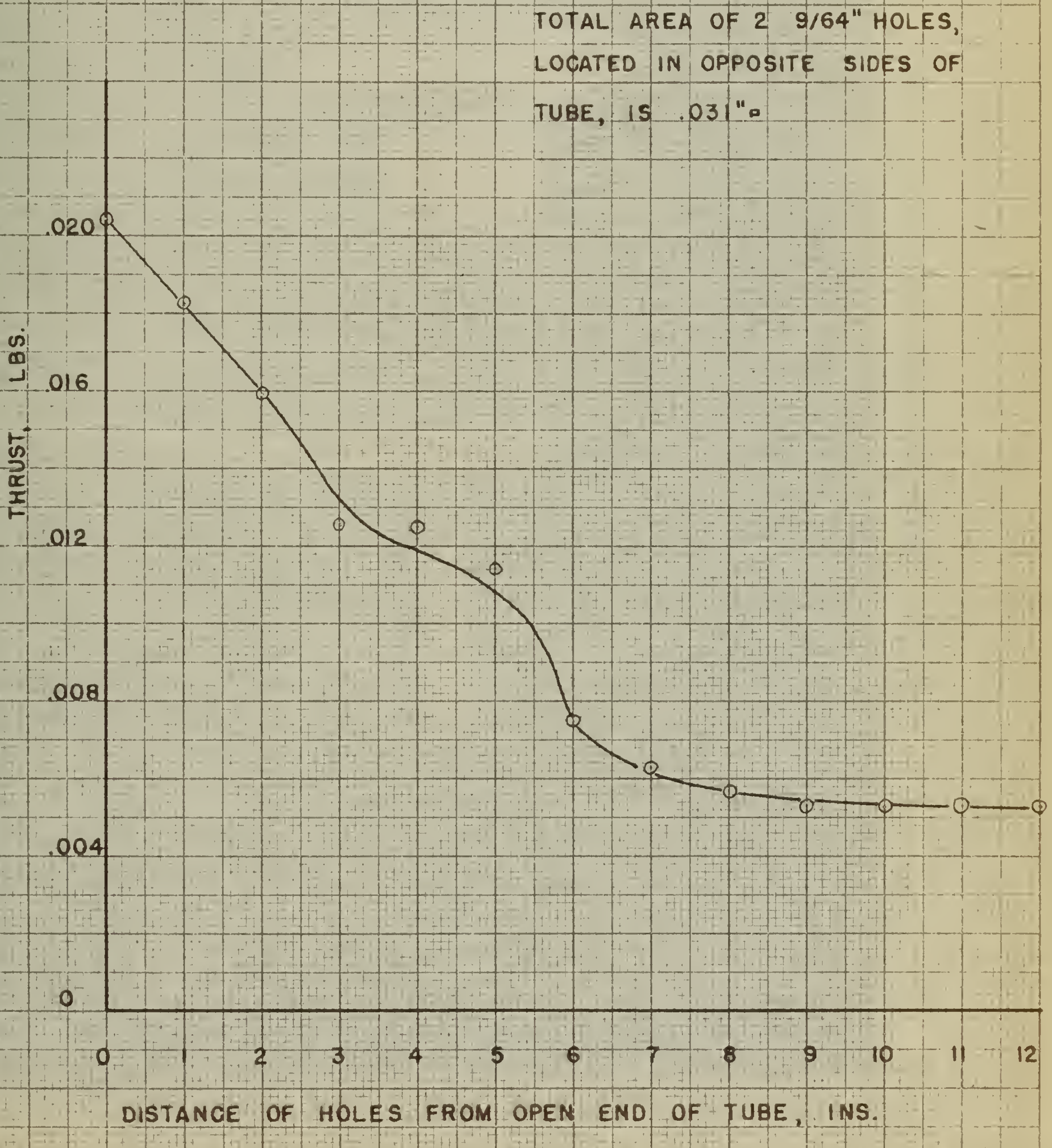


FIG. 10. VARIATION OF THRUST PRODUCED BY SAME RESONATORS AS FIG. 9 BUT WITH CONSTANT VOLUME SETTING.

FIG. 11. RELATION BETWEEN THRUST AND AXIAL POSITION OF HOLES IN SIDE OF CLOSED TUBE RESONATOR.



TOTAL AREA OF 2 9/64" HOLES, LOCATED IN OPPOSITE SIDES OF TUBE, IS .031"▫

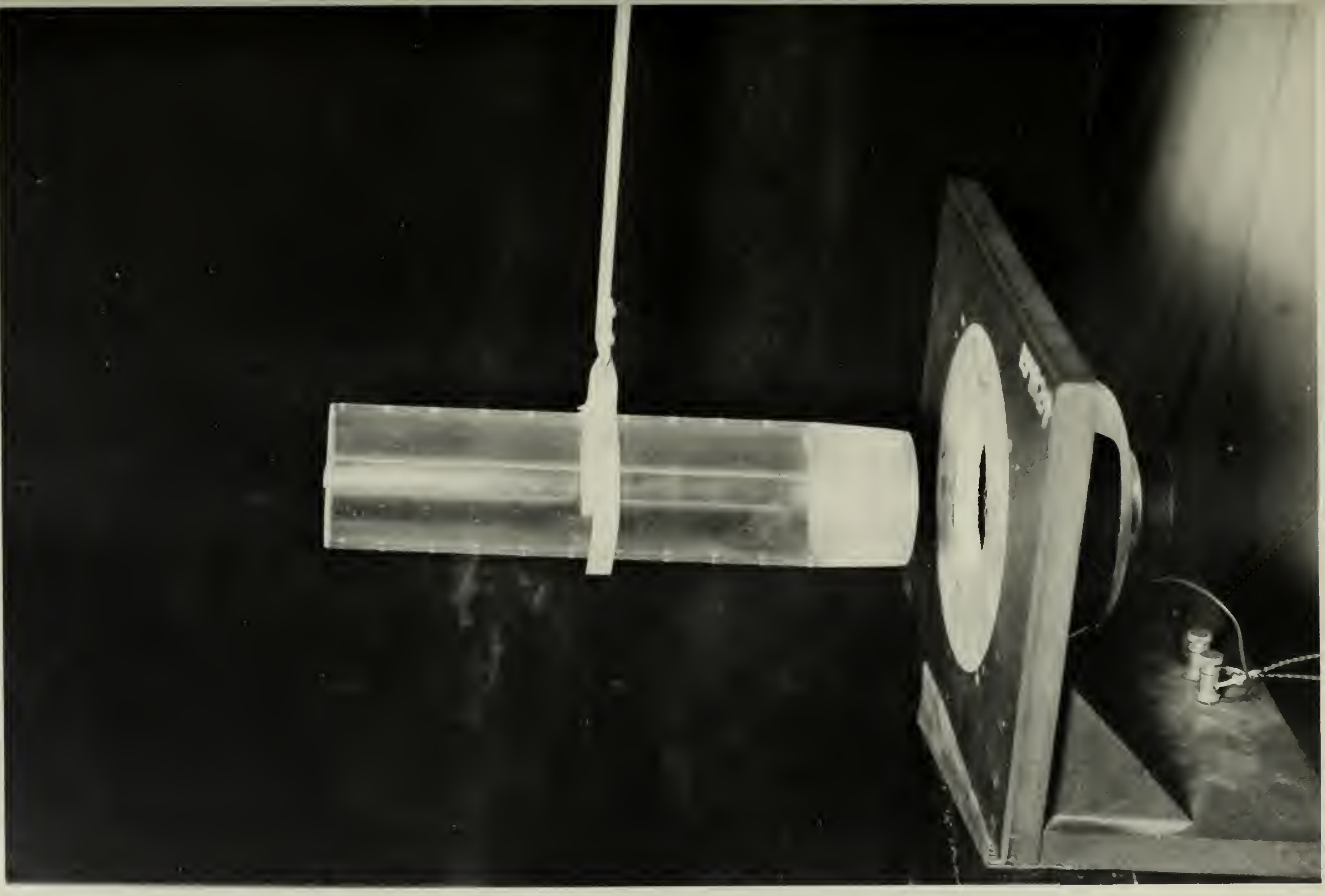


FIG. 11.1 CLOSED TUBE RESONATOR WITH HOLES IN WALL.

FIG. 12. VARIATION OF THRUST WITH AREA OF ORIFICE IN CENTER OF END PLATE OF CLOSED TUBE RESONATOR.

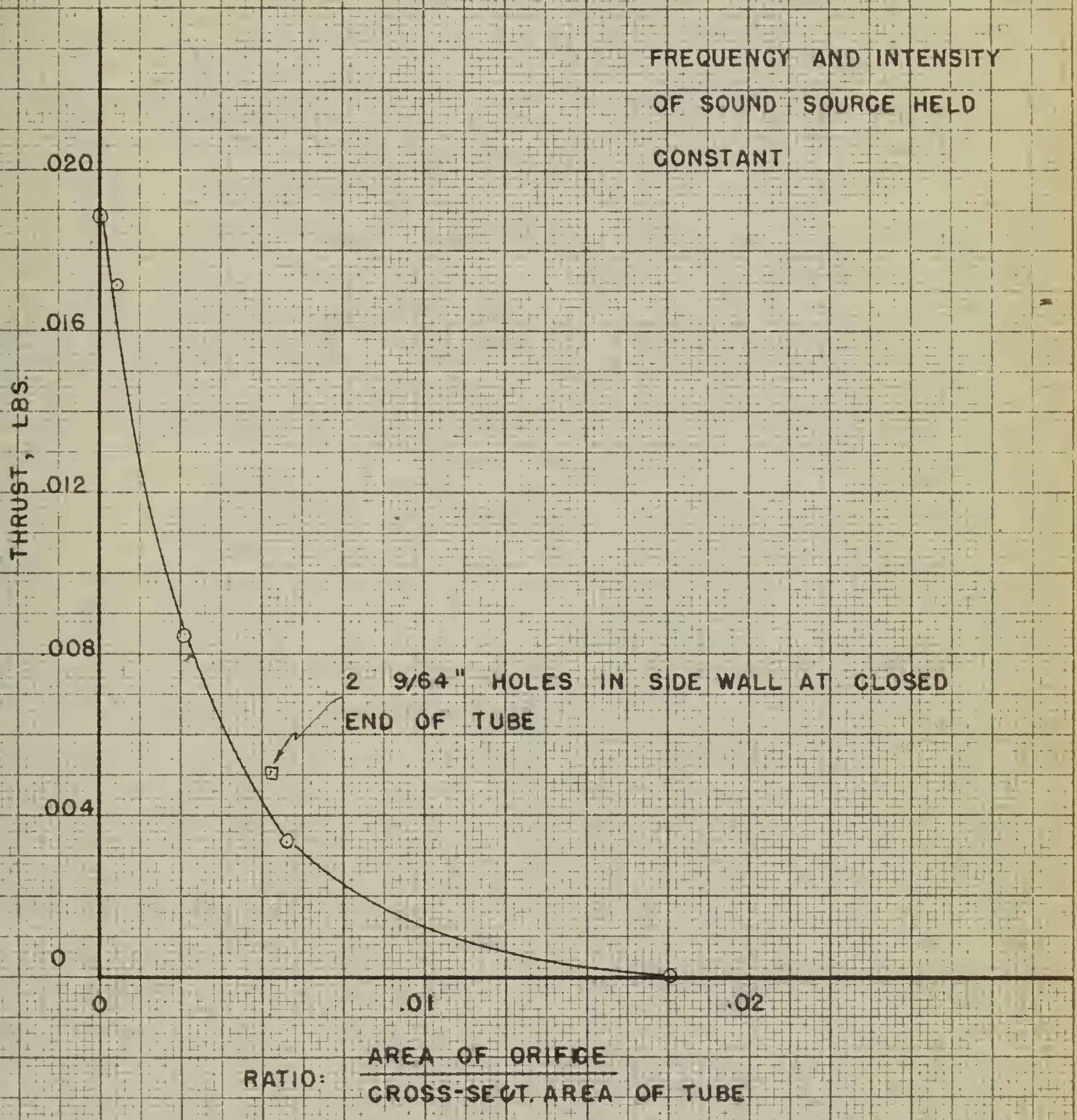


FIG. 13. VARIATION OF $\frac{\text{THRUST}}{\text{POWER INPUT}}$ WITH FREQUENCY, OUTPUT ENERGY REMAING CONSTANT AT SOUND SOURCE.

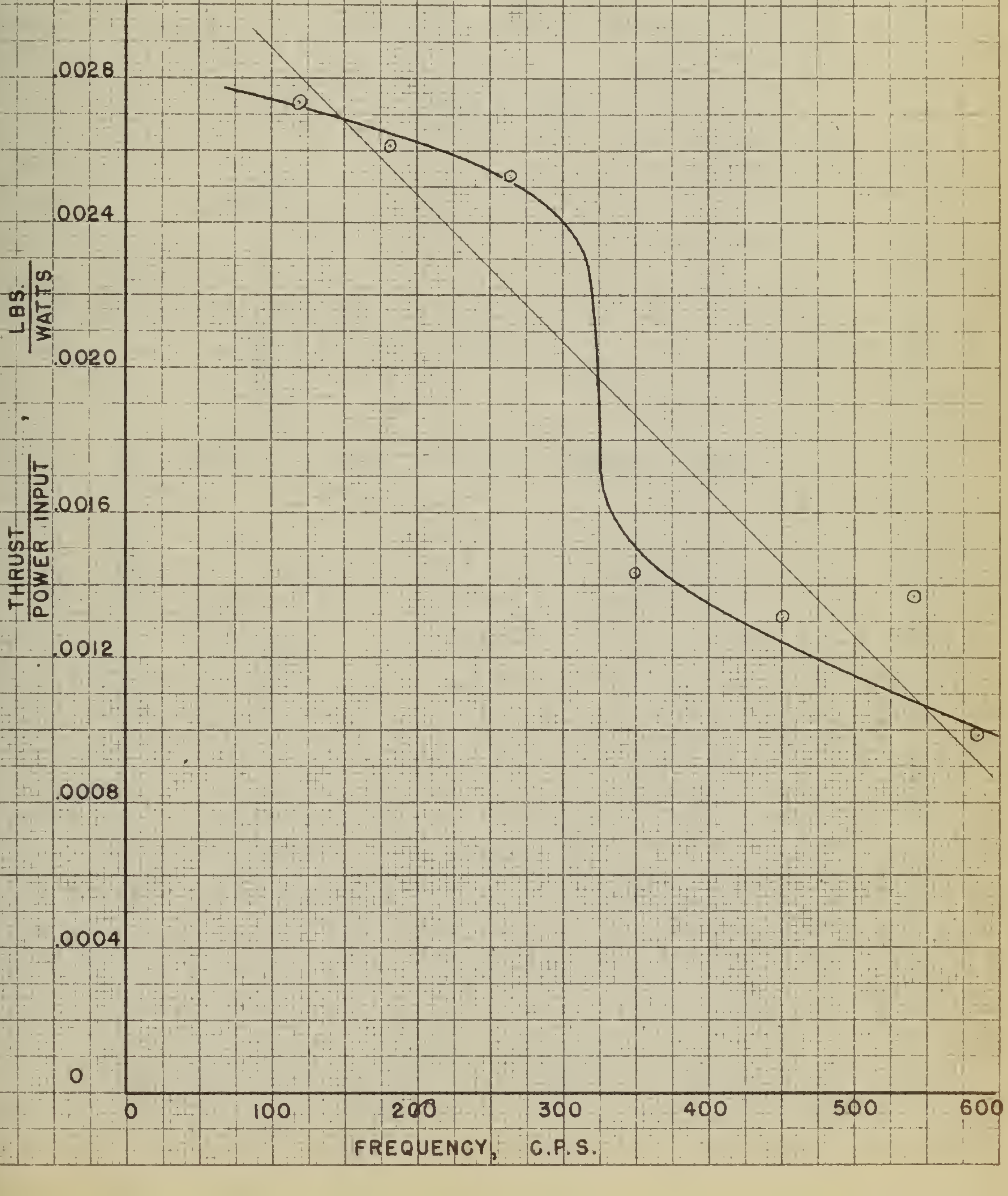
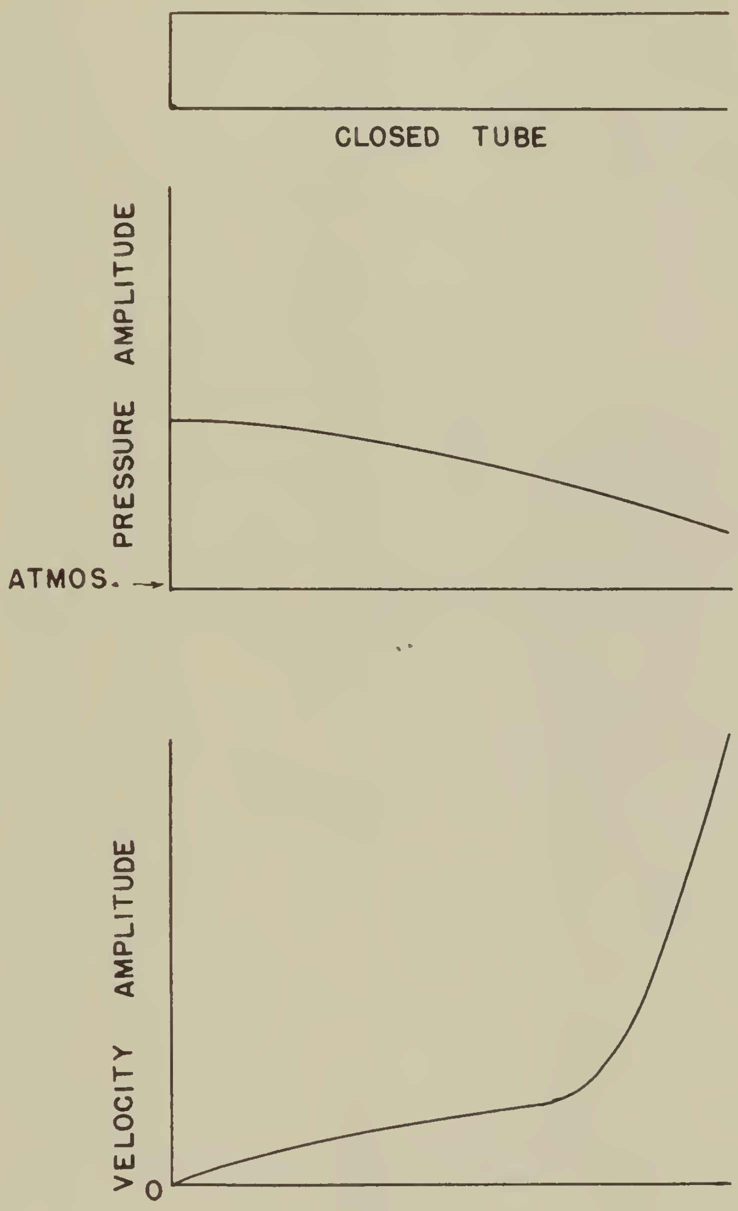


FIG. 14. DIAGRAM OF VELOCITY AND PRESSURE AMPLITUDES IN CLOSED TUBE RESONATOR.



SEE A.B.WOOD, "TEXTBOOK OF SOUND"

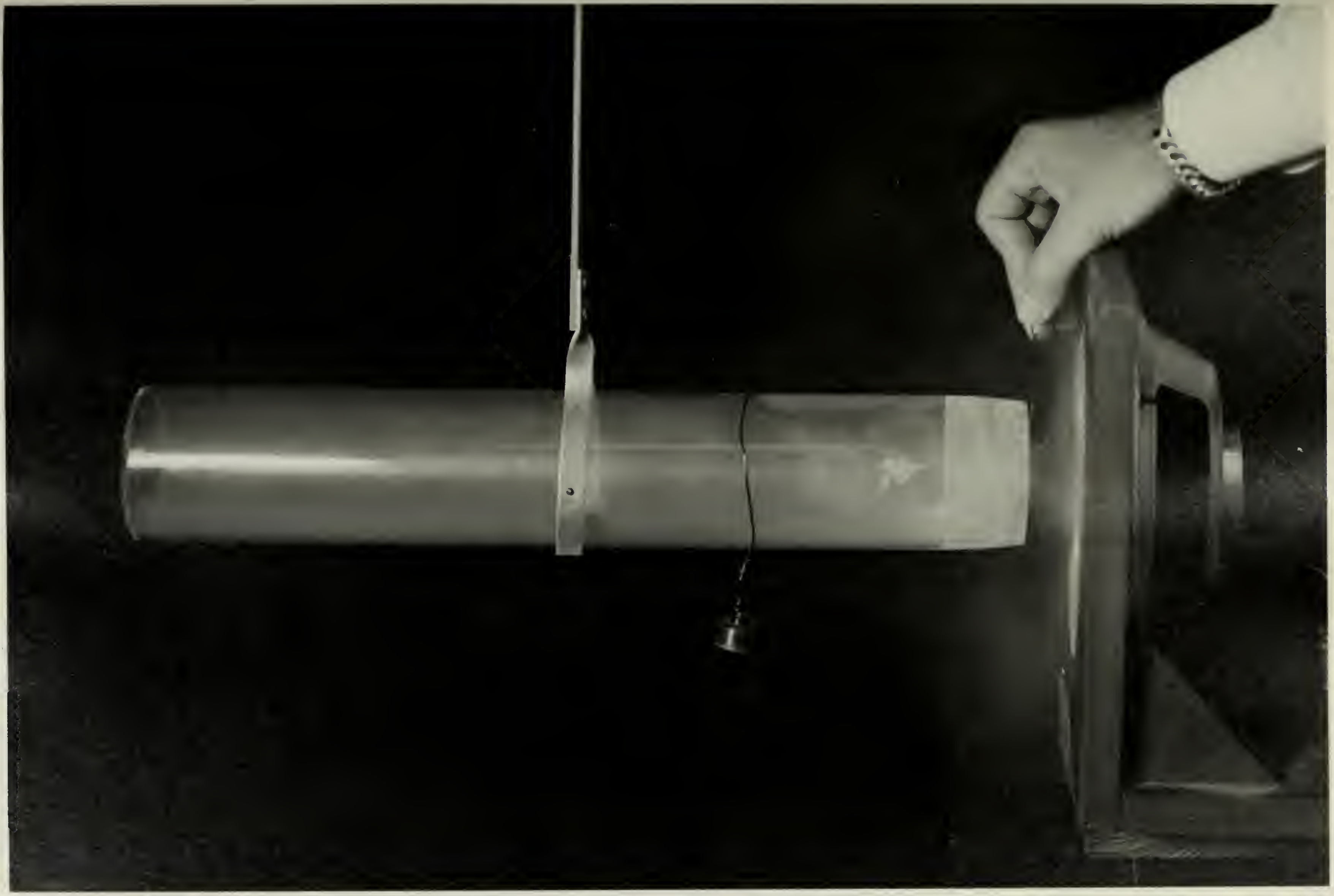


FIG. 16 CLOSED TUBE, AT RESONANCE, SHOWING OUTFLOWING JET OF AIR INDICATED BY SMOKE.

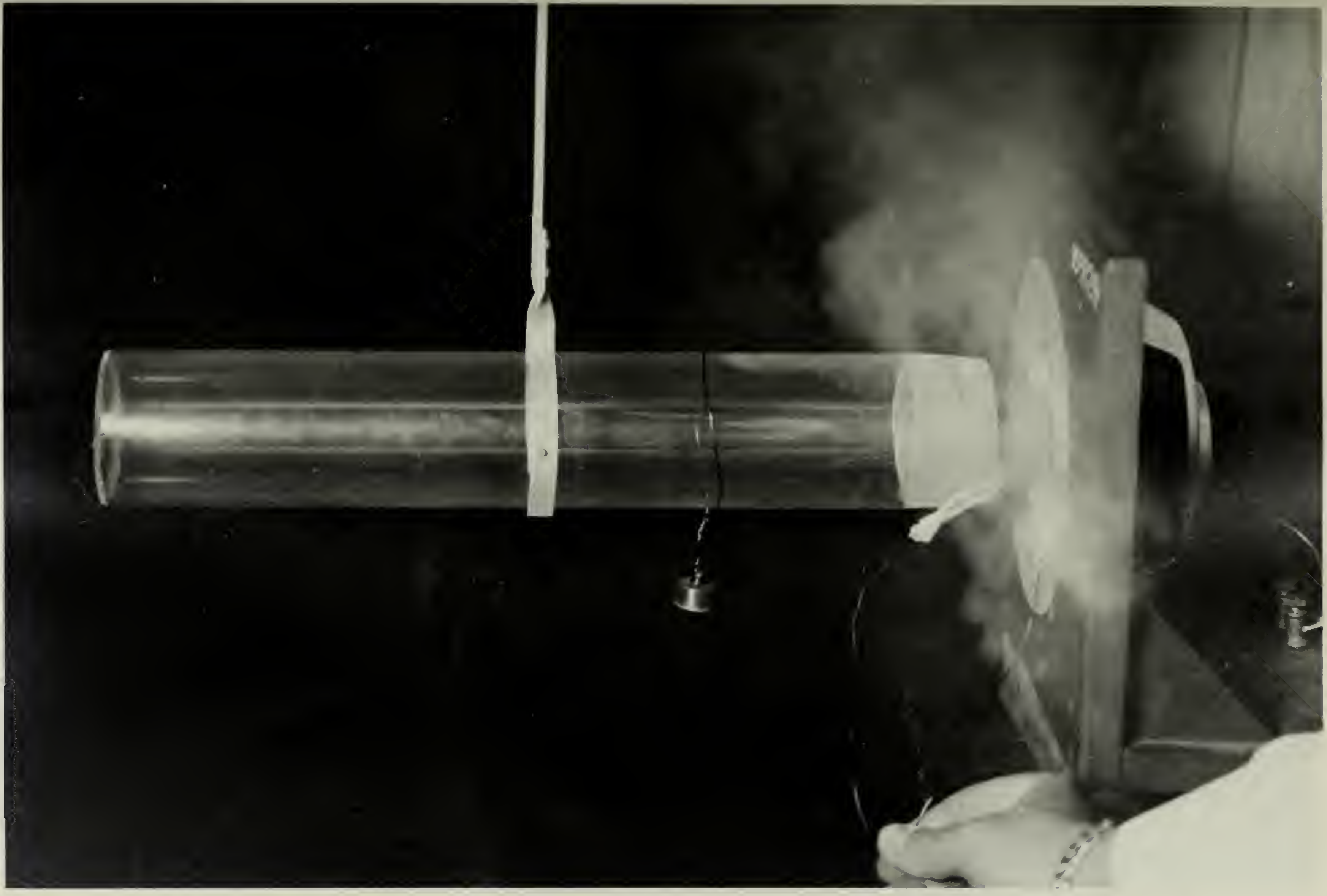
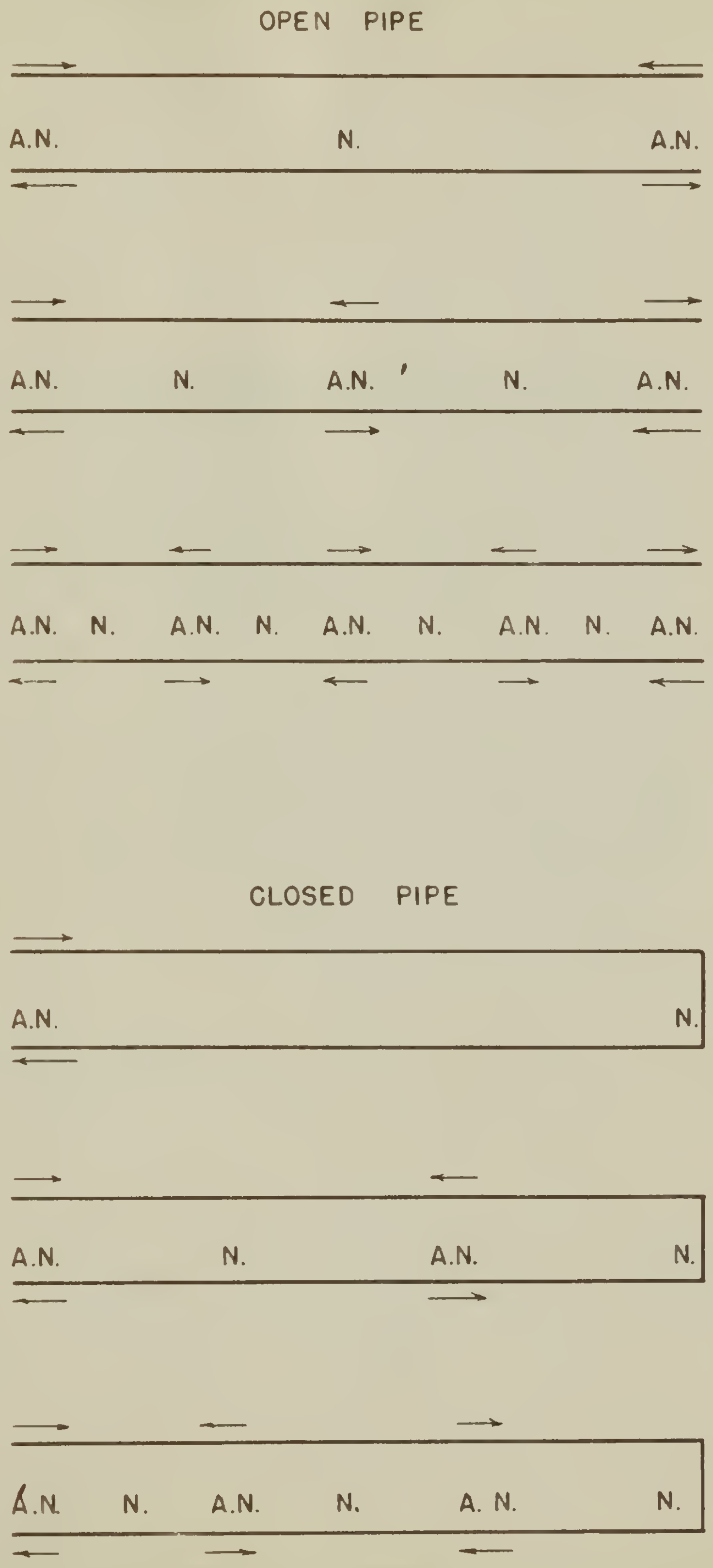


FIG. 16.1 CLOSED TUBE, AT RESONANCE, SHOWING AIR INFLOW AND OUTFLOW INDICATED BY SMOKE.

FIG. 17. AIR VIBRATIONS IN PIPES.



Thesis 13189
S85 Strieter
Relation between acoustic phenomena and dynamic thrust.