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DONALD E. CAVERLY

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By

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Submitted in partial fulfillment of
the requirements for the degree of

MASTER OF SCIENCE
IN
METEOROLOGY

United States Naval Postgraduate School
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ABSTRACT

Beginning with linearized forms of the vorticity equation and the first law of thermodynamics applied to a simple baroclinic model including friction, an analytic solution is obtained which describes the time variation of the relative phase angle between the temperature and pressure waves in the atmosphere. It is found that for unstable waves the atmosphere tends toward a state where the temperature wave lags the pressure wave. The time variation of the amplitude and relative phase depends on the initial value of the phase difference, but the ultimate angle by which the temperature field lags the pressure field depends only on the drag coefficient, certain atmospheric parameters, and the wavelength of the waves. Figures are included to show the variation with time and initial phase difference of the amplitude and the relative phase. Also shown are the effects on the amplitude and terminal value of the phase difference due to variations in the drag coefficient and other parameters.

The writer wishes to express his appreciation to Professor George J. Haltiner of the U. S. Naval Postgraduate School for his guidance, contributions and encouragement in this work.

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

TABLE OF SYMBOLS

ω -- the total derivative of pressure with respect to time (dynes cm^{-2} sec^{-1})

P -- the atmospheric pressure (dynes cm^{-2})

ζ -- the relative vorticity: $\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$

$\mathbb{V}$ -- the wind velocity vector: $\mathbb{V} = u\mathbf{i} + v\mathbf{j}$

$\mathbb{V}_T$ -- the thermal wind vector

f -- the coriolis parameter (sec^{-1})

ΔX -- the finite difference of the parameter X

f_m -- the average value of the coriolis parameter and is equal to the value at latitude 45N (sec^{-1})

∇ -- the gradient operator: $\nabla = \mathbf{i}\frac{\partial}{\partial x} + \mathbf{j}\frac{\partial}{\partial y}$

σ -- the static stability parameter (cm^5 $dynes^{-2}$)

h -- the thickness of an atmospheric layer (cm)

θ -- the potential temperature (°K)

Z -- the vertical height coordinate; Z=0 at the surface (cm)

V -- the magnitude of the vector $\mathbb{V}$ (cm sec^{-1})

g -- the vertical component of the acceleration due to gravity (cm sec^{-2})

C_D -- the surface drag coefficient (dyne sec cm^{-3})

ψ_T -- the stream function: $\psi_T = \frac{g}{f}h$

μ -- the wave number: $\mu = 2\pi/L$

L -- the wavelength (cm)

C -- the phase speed of the waves

β -- the Rossby parameter: $\beta = \frac{\partial f}{\partial y}$

$\mathbf{i}$ and $\mathbf{j}$ -- the x and y unit vector respectively

i -- the $\sqrt{-1}$

C_R -- the Rossby wave speed

A' -- the amplitude of the pressure wave at t=0

A_T -- the amplitude of the temperature wave at t=0

α -- the relative phase between the temperature and pressure wave at t=0

B -- the time varying amplitude of the pressure wave

B_T -- the time varying amplitude of the temperature wave

ϕ -- the time varying phase angle of the pressure wave

ϕ_T -- the time varying phase angle of the temperature wave

U_T -- the basic zonal thermal wind (cm sec^{-1})

U -- the basic zonal wind (cm sec^{-1})

t -- time

1. Introduction

In 1960 Wiin-Nielsen [5] published the results of an investigation of the mechanisms in the atmosphere whereby the conversion of potential to kinetic energy is possible. He stated that for this conversion to occur it was necessary that the temperature wave lag behind the pressure wave. Using a simple baroclinic model with no friction he obtained an analytic solution for the relative phase between the temperature and pressure fields. Based on this solution he indicated how the relative phase angle changed with time for various initial values and how the early tendency of the pressure wave to amplify was affected by the initial value of the relative phase angle.

The purpose of this work is to include friction in the simple baroclinic model and compare the results to the non-friction case. Also the values of the initial relative phase angle, wavelength, drag coefficient, Coriolis parameter, thermal wind, and static stability were varied to investigate their effects on the amplification of the pressure wave, and the time variations including the limiting value of the relative phase between the temperature and pressure waves.

2. Development.

The atmosphere is represented by a simple baroclinic model. The 750- and 250-mb levels are used as data levels, and the 500-mb level is taken as the level of interest. This gives a 4-layer model where $\omega=0$ at the upper boundary, P=0, (level zero); and $\omega=\omega_f$, the frictionally induced value at level four, the lower boundary of the atmosphere. It is also assumed that the atmospheric motions are adiabatic. A zonal basic wind field which is independent of latitude and a linear function of pressure is utilized.

The equations used are the vorticity equation in the form

$$\frac{\partial \zeta}{\partial t} + \mathbb{V}\cdot\nabla(\zeta+f) - f_m \frac{\Delta\omega}{\Delta P} = 0, \tag{1}$$

and an expanded form of the first law of thermodynamics for adiabatic motion given by Haltiner [2]

$$\frac{\partial h}{\partial t} + \mathbb{V}\cdot\nabla h - \Delta P \sigma \omega = 0. \tag{2}$$

Here σ is a measure of the static stability,

$$\sigma = \frac{1}{\theta'}\frac{\partial Z}{\partial P}\frac{\partial \theta'}{\partial P},$$

and $\frac{\partial Z}{\partial P}$ has been replaced by $-h/\Delta P$. It will be assumed that the basic flow pattern remains constant and that the perturbations grow, but no attempt will be made to account for the form of the necessary energy input for this to occur notwithstanding the dissipating effect of the friction. The

vorticity equation will be applied at 25C mb, level one, and at 750 mb, level three. The thermal equation is applied at 500 mb, level two. The wind at level i is denoted $\mathbb{V}_i$, except that at level two the wind is taken to be $\overline{\mathbb{V}} = \frac{1}{2}(\mathbb{V}_1 + \mathbb{V}_3)$. The thermal wind for the layer of thickness ΔP centered about level two is given by $\mathbb{V}_T = \frac{1}{2}(\mathbb{V}_1 - \mathbb{V}_3)$.

For simplicity in the equations it was assumed that the frictional stress is directly proportional to the surface wind. The geostrophic wind approximation leads to the result $\omega_f = -\frac{g}{f} C_D \zeta_g$. Here C_D is a drag coefficient, ζ_g is geostrophic relative vorticity, g is gravity, and f is the Coriolis parameter.

Applying the vorticity equation at levels one and three yields:

$$\frac{\partial \zeta_1}{\partial t} + \mathbb{V}_1 \cdot \nabla(\zeta_1 + f) = f_m \frac{\omega_2}{2\Delta P};$$

$$\frac{\partial \zeta_3}{\partial t} + \mathbb{V}_3 \cdot \nabla(\zeta_3 + f) = f_m \frac{\omega_4 - \omega_2}{2\Delta P};$$

and expressing $\mathbb{V}_1$ and $\mathbb{V}_3$ in terms of $\overline{\mathbb{V}}$ and $\mathbb{V}_T$ yields for level one the result

$$\frac{\partial}{\partial t}(\overline{\zeta} + \zeta_T) + (\overline{\mathbb{V}} + \mathbb{V}_T) \cdot \nabla(\overline{\zeta} + \zeta_T + f) = f_m \frac{\omega_2}{2\Delta P}, \qquad (3)$$

and for level three

$$\frac{\partial}{\partial t}(\overline{\zeta} - \zeta_T) + (\overline{\mathbb{V}} - \mathbb{V}_T) \cdot \nabla(\overline{\zeta} - \zeta_T + f) = f_m \frac{\omega_4 - \omega_2}{2\Delta P}. \qquad (4)$$

Here, the thermal equation is applied at level two to obtain

$$\omega_2 = \left(\frac{\partial h}{\partial t} + \mathbb{V}_2 \cdot \nabla h\right) / (\sigma \Delta P)$$

and the linear extrapolation $\mathbb{V}_4 = \mathbb{V}_3 + \left(\frac{\Delta \mathbb{V}}{\Delta P}\right)_3 \Delta P$ is used to obtain ω_4. Adding equations (3) and (4) gives

$$\frac{\partial}{\partial t}\bar{\zeta} + \bar{\mathbb{V}}\cdot\nabla(\bar{\zeta}+f) + \mathbb{V}_T\cdot\nabla\zeta_T = -\frac{f_m g C_D}{4f\Delta P}(\bar{\zeta} - 2\zeta_T). \qquad (5)$$

Subtracting equation (4) from equation (3) produces

$$\frac{\partial}{\partial t}\zeta_T + \mathbb{V}_T\cdot\nabla(\bar{\zeta}+f) + \bar{\mathbb{V}}\cdot\nabla\zeta_T = \frac{f_m}{2\sigma(\Delta P)^2}\left(\frac{\partial h}{\partial t} + \bar{\mathbb{V}}\cdot\nabla h\right) +$$

$$+ \frac{f_m g C_D}{4f\Delta P}(\bar{\zeta} - 2\zeta_T). \qquad (6)$$

To linearize these last two equations assume that $\mathbb{V}_T$ and $\bar{\mathbb{V}}$ are given by

$$\bar{\mathbb{V}} = (U+u)\hat{i} + v\hat{j} \quad and \quad \mathbb{V}_T = (U_T+u_T)\hat{i} + v_T\hat{j},$$

where the U and U_t represent the basic zonal flow which is a function of the pressure only, and the small u's and v's represent the perturbation quantities which are independent of latitude, the y-axis. Also take h to be a basic part plus a perturbation, i.e., $h = H(y) + h'(x,t)$. Now by neglecting terms involving products of perturbation quantities and replacing v by $v = \frac{\partial \psi}{\partial x}$, where ψ is a stream function given by $\psi = \frac{g}{f} z_2$; and v_T by $v_T = \frac{\partial \psi_T}{\partial x}$, where ψ_T is a stream function given by $\psi_T = \frac{g}{f} h$, we obtain two third order linear partial differential equations in ψ and ψ_T:

$$\frac{\partial}{\partial t}\left(\frac{\partial^2 \psi_T}{\partial x^2}\right) + U\frac{\partial^3 \psi_T}{\partial x^3} + U_T\frac{\partial^3 \psi}{\partial x^3} + \beta\frac{\partial \psi_T}{\partial x} - \delta\frac{\partial \psi_T}{\partial x} + \delta U_T\frac{\partial \psi}{\partial x} -$$

$$-\delta U\frac{\partial \psi_T}{\partial x} + \lambda\left(\frac{\partial^2 \psi}{\partial x^2} - 2\frac{\partial^2 \psi_T}{\partial x^2}\right) = 0 \qquad (7)$$

$$\frac{\partial}{\partial t}\left(\frac{\partial^2 \psi}{\partial x^2}\right) + U\frac{\partial^3 \psi}{\partial x^3} + U_T\frac{\partial^3 \psi_T}{\partial x^3} + \beta\frac{\partial \psi}{\partial x} - \lambda\left(\frac{\partial^2 \psi}{\partial x^2} - 2\frac{\partial^2 \psi_T}{\partial x^2}\right) = 0 \qquad (8)$$

where $\beta = \frac{\partial f}{\partial y}$, $\lambda = -f_m g C_D/4f\Delta P$, and $\delta = f f_m/2g\sigma(\Delta P)^2$.

Assume solutions of equations (7) and (8) of the form

$$\psi = \psi_0 e^{i\mu(x-ct)} \quad \text{and} \quad \psi_T = \psi_{T0} e^{i\mu(x-ct)}$$

where ψ_0 and ψ_{T0} are complex amplitudes, $\mu = \frac{2\pi}{L}$ is the wave number for wavelength equal to L , and C is the phase speed. Substituting these two solutions into the differential equations yields two homegeneous linear equations in ψ_0 and ψ_{T0}

$$\left(C - U + \frac{\beta}{\mu^2} - i\frac{\lambda}{\mu}\right)\psi_0 - \left(U_T - i\frac{2\lambda}{\mu}\right)\psi_{T0} = 0 \tag{9}$$

$$\left[U_T\left(\frac{\delta}{\mu^2} - 1\right) + i\frac{\lambda}{\mu}\right]\psi_0 + \left(C - U + \frac{\beta}{\mu^2} + \frac{\delta C}{\mu^2} - \frac{\delta U}{\mu^2} - i\frac{2\lambda}{\mu}\right)\psi_{T0} = 0 \tag{10}$$

In order to have non-trivial solutions to this system of equations it is required that the determinant of the coefficients of ψ_0 and ψ_{T0} be equal to zero. Setting the determinant equal to zero and defining $C_R = U - \frac{\beta}{\mu^2}$, the Rossby wave speed, yields a quadratic equation in $(C - C_R)$ Solving this equation for $(C - C_R)$ gives

$$C - C_R = F + iE \pm \frac{1}{2A}\sqrt{H + iG} \tag{11}$$

$$F = \beta\delta / 2\mu^2(\mu^2 + \delta) \qquad E = (\lambda/2\mu)(3\mu^2 + \delta)/(\mu^2 + \delta)$$

$$G = \left[4U_T\left(\frac{2\delta^2}{\mu^4} - \frac{\delta}{\mu^2} - 3\right) + \frac{2\delta\beta}{\mu^4}\left(1 - \frac{\delta}{\mu^2}\right)\right](\lambda/\mu)$$

$$H = 4U_T^2\left(1 - \frac{\delta^2}{\mu^4}\right) - \frac{\lambda^2}{\mu^2}\left(9 + \frac{\delta^2}{\mu^4} + \frac{6\delta}{\mu^2}\right) + \frac{\delta^2\beta^2}{\mu^8} \qquad A = \left(1 + \frac{\delta}{\mu^2}\right)$$

By comparing equation (11) to the result given by Wiin-Nielsen [5] it can be seen that the effect of including

friction in the model has been the addition of those terms involving λ as a factor to the frequency equation. If the drag coefficient, C_D , is set equal to zero, λ vanishes in equation (11) and the result is identical to Wiin-Nielsen's equation.

Equation (11) gives C as a complex number where the real part determines the speed of propogation of the perturbations, and the imaginary part when multiplied by the wave number gives the amplification for the wave. Taking first the plus sign and then the minus sign with the radical gives two solutions which are designated as C_+ and C_- respectively. The complete solutions for ψ and ψ_T are then given by:

$$\psi = \psi_{0+} e^{i\mu(x - C_+ t)} + \psi_{0-} e^{i\mu(x - C_- t)} \tag{12}$$

$$\psi_T = \psi_{T0+} e^{i\mu(x - C_+ t)} + \psi_{T0-} e^{i\mu(x - C_- t)} \tag{13}$$

Next take as initial conditions at time t=0

$$\psi = A' e^{i\mu x} \quad \text{and} \quad \psi_T = A_T e^{i(\mu x + \alpha)}$$

where A' and A_T are real numbers and α is a relative phase angle which is positive when the temperature field lags the pressure field. Then by considering equations (12) and (13) at time zero it follows immediately that

$$\psi_{0+} + \psi_{0-} = A' \quad \text{and} \quad \psi_{T0+} + \psi_{T0-} = A_T e^{i\alpha}$$

By utilizing equation (9) for C_+ and C_- we obtain two more equations for $\psi_{0+}, \psi_{0-}, \psi_{T0+},$ and ψ_{T0-} giving a system of four equations and four unknowns. Solving the system gives

$$\psi_{0+} = (KA_T e^{i\alpha} - A' X_-)/(X_+ - X_-) \qquad \psi_{T0+} = \frac{X_+}{K}\psi_{0+} \quad (14)$$

$$\psi_{0-} = (KA_T e^{i\alpha} - A' X_+)/(X_- - X_+) \qquad \psi_{T0-} = \frac{X_-}{K}\psi_{0-} \quad (15)$$

where $K = U_T - i\frac{2\lambda}{\mu}$,

$X_+ = C_+ - C_R - i\frac{\lambda}{\mu}$ and $X_- = C_- - C_R - i\frac{\lambda}{\mu}$.

Then by substituting the expressions for ψ_{0+}, ψ_{0-}, ψ_{T0+}, and ψ_{T0-} into equations (12) and (13) the complete solutions for ψ and ψ_T are obtained.

In order to study unstable baroclinic waves in some detail, it is desirable to have the expressions for ψ and ψ_T in slightly different forms. Since C_+ and C_- are complex they can be expressed as $C_+=Y+iW$ and $C_-=M+iN$. Then the expression for ψ becomes

$$\psi = \psi_{0+} e^{\mu W t} e^{i\mu(x-Yt)} + \psi_{0-} e^{\mu N t} e^{i\mu(x-Mt)}.$$

Now define $B+$, $B-$, $\Phi+$, and $\Phi-$ by

$$\psi_{0+} e^{\mu W t} = B_+ e^{i\Phi_+} \quad \text{and} \quad \psi_{0-} e^{\mu N t} = B_- e^{i\Phi_-}. \quad (16)$$

Then

$$\psi = B_+ e^{i\Phi_+} e^{i\mu(x-Yt)} + B_- e^{i\Phi_-} e^{i\mu(x-Mt)},$$

which after some manipulation can be written as

$$\psi = B e^{i[\mu x - \mu(C_R+F)t + \Phi]}.$$

Here B is given by

$$B^2 = B_+^2 + B_-^2 + 2B_+B_- \cos[(\Phi_+ - \Phi_-) - \gamma], \quad (17)$$

ϕ is given by

$$\tan\phi = \frac{B_+ \sin(\phi_+ - \frac{\gamma}{2}) + B_- \sin(\phi_- + \frac{\gamma}{2})}{B_+ \cos(\phi_+ - \frac{\gamma}{2}) + B_- \cos(\phi_- + \frac{\gamma}{2})}; \qquad (18)$$

γ is defined as $\gamma = \frac{\mu}{A}\sqrt{r}\cos\frac{\theta}{2}t$ where $r = \sqrt{G^2+H^2}$ and θ=arctan (G/H); F, G, H, and A are defined immediately following equation (11). The treatment for ψ_T is similar to the above procedure for ψ. Define B_{T+}, B_{T-}, ϕ_{T+}, and ϕ_{T-} by

$$\psi_{TO+}e^{\mu wt} = B_{T+}e^{i\phi_+} \text{ and } \psi_{TO-}e^{\mu wt} = B_{t-}e^{i\phi_-}, \qquad (19)$$

and then obtain

$$\psi_T = B_T e^{i[\mu x - \mu(C_R+F)t + \phi_T]}$$

with B_T given by

$$B_T^2 = B_{T+}^2 + B_{T-}^2 + 2B_{T+}B_{T-}\cos[(\phi_{T+} - \phi_{T-}) - \gamma], \qquad (20)$$

and ϕ_T given by

$$\tan\phi_T = \frac{B_{T+}\sin(\phi_{T+} - \frac{\gamma}{2}) + B_{T-}\sin(\phi_{T-} + \frac{\gamma}{2})}{B_{T+}\cos(\phi_{T+} - \frac{\gamma}{2}) + B_{T-}\cos(\phi_{T-} + \frac{\gamma}{2})}. \qquad (21)$$

We now have obtained expressions which give the variations of ψ and ψ_T as simple sinusoidal waves having amplitudes and phase angles which are functions of time.

We now want to determine how the relative phase angle between the thermal and pressure waves, $(\phi_T - \phi)$ changes with time. Considering the exponential nature of B_+, B_-, B_{T+}, and B_{T-}, it can be shown from equations (18) and (21) that as time approaches infinity, ϕ approaches $(\phi_+ - \frac{\gamma}{2})$ and ϕ_T

approaches $(\phi_{T+} - \frac{\pi}{2})$. Therefore, in the limiting case $(\phi_T - \phi) = (\phi_{T+} - \phi_+)$. Now by suitable combining equations (14), (15), (16), and (19), it can be shown that

$$\tan(\phi_{T+} - \phi_+) =$$

$$= \frac{U_T(\mu^5\sqrt{r}\sin\frac{\theta}{2} + \lambda\mu^4 - \lambda\delta\mu^2) + 2\lambda(\beta\delta + \mu^4\sqrt{r}\cos\frac{\theta}{2})}{U_T(\beta\delta\mu + \mu^5\sqrt{r}\cos\frac{\theta}{2}) - 2\lambda(\lambda\mu^3 - \lambda\delta\mu + \mu^4\sqrt{r}\sin\frac{\theta}{2})} \quad (22)$$

Equation (22) was obtained by considering only the amplifying part of the wave. Unstable baroclinic waves tend toward a state such that the limiting phase angle between the thermal and pressure waves, $(\phi_{T+} - \phi_+)$, is given by $\lim_{t\to\infty}[\tan(\phi_+ - \phi)] = FUNCTION(U_T, L, \sigma, f, \beta, C_D, g)$ where the function is given by the right side of equation (22). It is to be noted that the initial value of the relative phase difference, α, does not appear in the expression for the limiting value of $(\phi_T - \phi)$.

3. Discussion of Results.

Equation (22) was solved numerically and the results are plotted in figure 1, which gives $(\phi_{T+} - \phi_{+})$ as a function of wavelength and drag coefficient. Using a surface geostrophic wind of 10 m/sec, the values of C_D in figure 1 correspond to frictional stresses of from zero to 20 dynes/cm^2. C_D=0.004 dyne sec/cm^3 corresponds to the value given by Sutton [4] for grass about 15 centimeters in height. The value of the static stability parameter, σ, of 2.90×10^{-4} cm^5/dyne^2 corresponds to the value given by Gates [1] as the mean value over the United States in January. Those curves in figure 1 for which $(\phi_{T+} - \phi_{+})$ is zero when the drag coefficient is zero, are the curves for which term H in equation (11) is positive, i.e. these wavelengths are the stable waves since the requirement for instability in the frictionless case is that H be negative. The apparent amplification depicted in figure 2 for what should be stable waves with C_D=0 can be explained by reference to equations (16) and (17) from which it can be seen that when C_+ and C_- are real numbers, the maximum possible value of B is given by $B^2 = B_+^2 + B_-^2 + 2B_+B_-$. The time variation of B for two stable waves, 3,000 km and 12,000 km, with C_D=0 is shown in figure 3. As is apparent from equation (17), B is a sinusoidal function of time, oscillating about the initial value. Figure 2 shows the effects of variations in the drag coefficient on the value

of the amplitude for the wavelength band from 2,000 km to 14,000 km. The values of B used to construct the figures are the values attained at the end of the first 36 hours for those wavelengths having $(\Phi_{T+} - \Phi_{+}) > 0$, for $C_D=0$, in figure 1. For the stable waves, due to the oscillatory nature of B mentioned above, the maximum value of B attained during the 36-hour period was used. For each wavelength the value of the initial phase difference between the temperature and pressure fields, α, was such as to maximize the amplification in 36 hours (α given to the nearest 45 degrees). As is obvious from figure 2, the effect of increasing the drag coefficient is to reduce the amplification for all wavelengths considered. This effect of friction is in agreement with the results presented by Holopainen [3]. In comparing the results it should be noted that the ordinate in figure 2 is wavelength, whereas the ordinate in Holopainen's figure is a function of wavelength, static stability, and latitude. In figure 2 it can be seen that the wavelength for which the amplification is maximum decreases as the drag coefficient increases.

Figures 4, 5, and 6 depict the effects of variations in thermal wind, static stability, and latitude on the maximum value of the amplitude for the same wavelengths as were used in figures 1 and 2, with $C_D=0.004$ dyne sec/cm^3. The central curves in figures 4, 5, and 6 are identical and

are the same as the middle curve in figure 2. From figure 4 it is apparent that as the thermal wind increases the amplitude increases for all wavelengths, and that the wavelength for which the amplitude is a maximum also increases. Figure 5 shows that the effect of increasing the static stability is to decrease the amount of amplification for all wavelengths, except that for wavelengths greater than about 13,000 km the amplitude increases slightly. Also as σ increases the wavelength of maximum amplification increases. The effects of moving the latitude northward are, as depicted in figure 6, to increase the amplification of the entire waveband, and to shift the wavelength of maximum amplitude toward lower values. It appears from the figures that increasing the static stability and decreasing the latitude have essentially the same effects on the amplification, and that decreasing the thermal wind corresponds to increasing the drag coefficient in the resulting effect on the wave amplitude. One might conclude from the information presented in the figures that at low latitudes with a high value of the static stability parameter and a low thermal wind the pressure waves would not tend to develop, and that at high latitudes with low static stability and high thermal wind the tendency would be toward amplification of the pressure waves.

Wiin-Nielsen [5] discusses the validity of applying the results of a linearized treatment of the dynamical equations

to obtain a limiting case for very long times. He justified the application on the basis that the initial trends were toward the situation given as the limiting case. With similar logic in mind, equations (17), (18), (20), and (21) were used to numerically investigate the values of the amplitude factors B and B_T, and the relative phase difference $(\phi_T - \phi)$ for periods of time of less than 72 hours. It should be noted that while the limiting value of $(\phi_T - \phi)$ is independent of the initial phase difference, α, the short period variations of B, B_T, and $(\phi_T - \phi)$ are very much dependent on the value of α. Figures 7 and 8 were constructed for a 5,000 km wavelength wave to depict the time variation of $(\phi_T - \phi)$ and B, to show the dependence on α of the time variation, and to show the fact that the terminal value of $(\phi_T - \phi)$ is independent of the value of α. Because of the very similar behavior in the variations of B and B_T with time, only B is shown in the figures. Figures 7 and 8 may be compared with similar figures presented by Wiin-Nielsen [5], using the same wavelength, for the frictionless case. In comparing the two sets of curves consideration should be given to the differences in the values of the atmospheric parameters used in the calculations performed to construct the curves in figures 7 and 8, and the values used by Wiin-Nielsen. One conclusion that can be drawn from the figures is that increasing the drag coefficient decreases the time required for $(\phi_T - \phi)$ to approach its limiting value.

The number at the top of each curve is the wavelength x10^{-3} km.

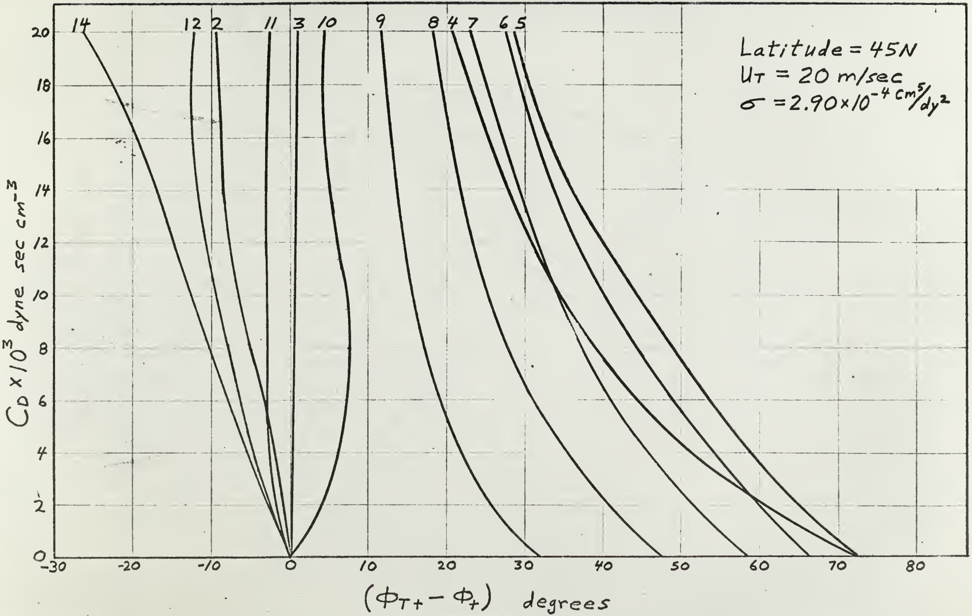


Figure 1

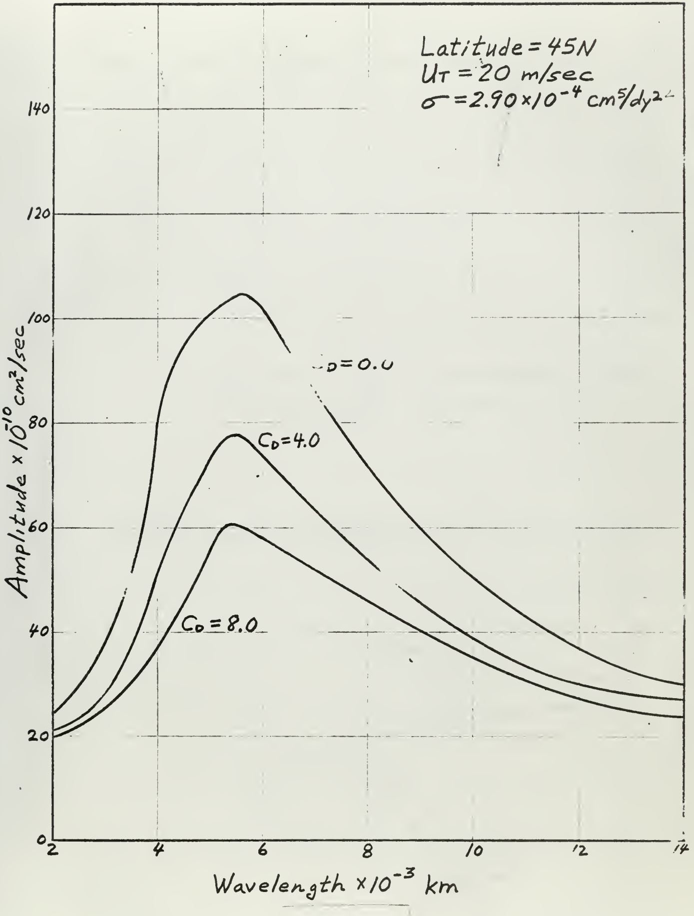
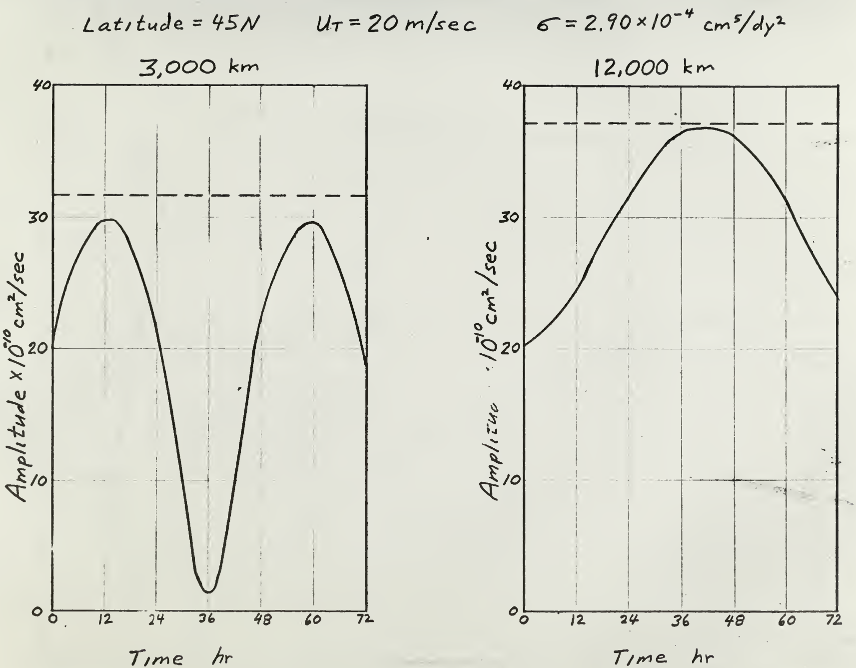

Latitude = 45N
U_T = 20 m/sec
$\sigma = 2.90 \times 10^{-4}$ cm^5/dy^2
C_D = 0.0
C_D = 4.0
C_D = 8.0
Amplitude $\times 10^{-10}$ cm^2/sec
Wavelength $\times 10^{-3}$ km
0
20
40
60
80
100
120
140
2
4
6
8
10
12
14

Figure 2



Latitude = 45N
U_T = 20 m/sec
σ = 2.90×10^-4 cm^5/dy^2
3,000 km
12,000 km
Amplitude x10^10 cm^2/sec
Time hr
0
10
20
30
40
0
12
24
36
48
60
72

Figure 3

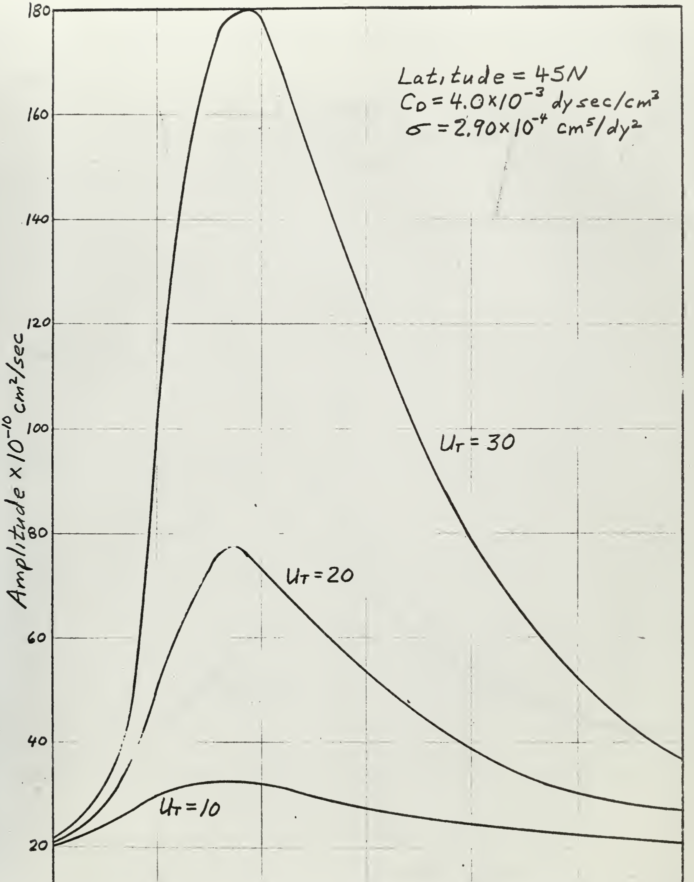
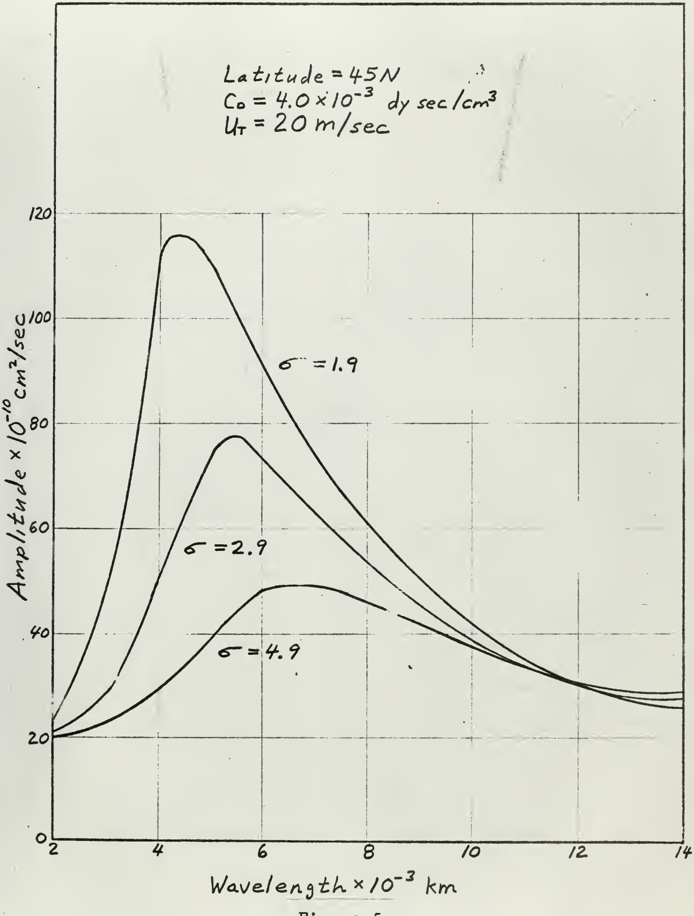
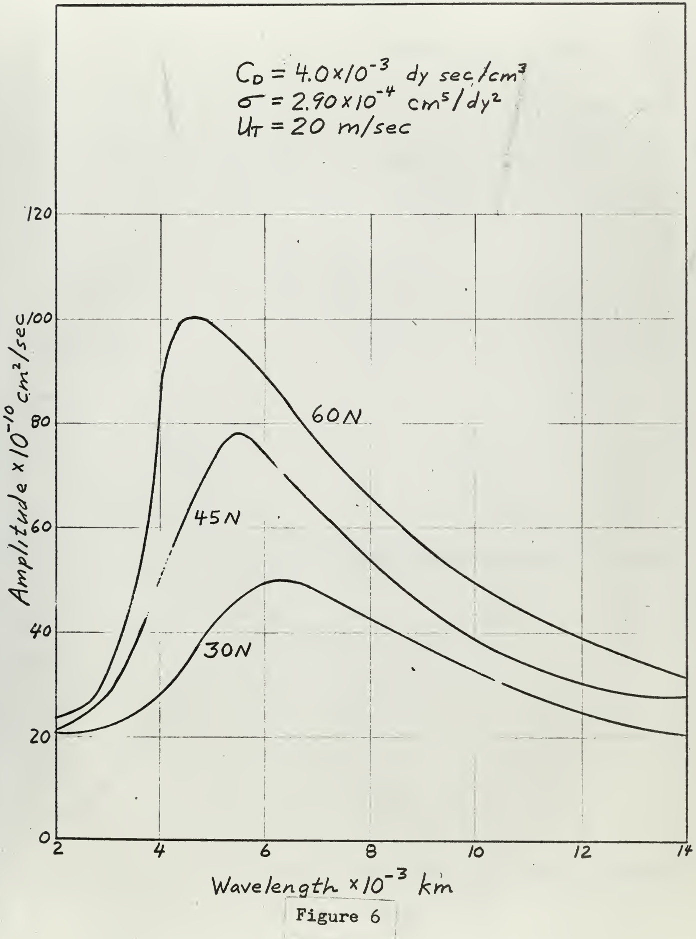

Latitude = 45N
$C_D = 4.0 \times 10^{-3}$ dy sec/cm³
$\sigma = 2.90 \times 10^{-4}$ cm⁵/dy²
$U_T = 30$
$U_T = 20$
$U_T = 10$
Amplitude × 10⁻¹⁰ cm²/sec
Wavelength × 10⁻³ km
180
160
140
120
100
80
60
40
20
10
2
4
6
8
10
12
14

Figure 4



Latitude = 45N
Co = 4.0 × 10⁻³ dy sec/cm³
UT = 20 m/sec
σ = 1.9
σ = 2.9
σ = 4.9
0
20
40
60
80
100
120
2
4
6
8
10
12
14
Amplitude × 10⁻¹⁰ cm²/sec
Wavelength × 10⁻³ km

Figure 5



$C_D = 4.0 \times 10^{-3}$ dy sec/cm^3
$\sigma = 2.90 \times 10^{-4}$ cm^5/dy^2
$U_T = 20$ m/sec
120
100
80
60
40
20
0
Amplitude $\times 10^{-10}$ cm^2/sec
60N
45N
30N
2
4
6
8
10
12
14
Wavelength $\times 10^{-3}$ km

Figure 6

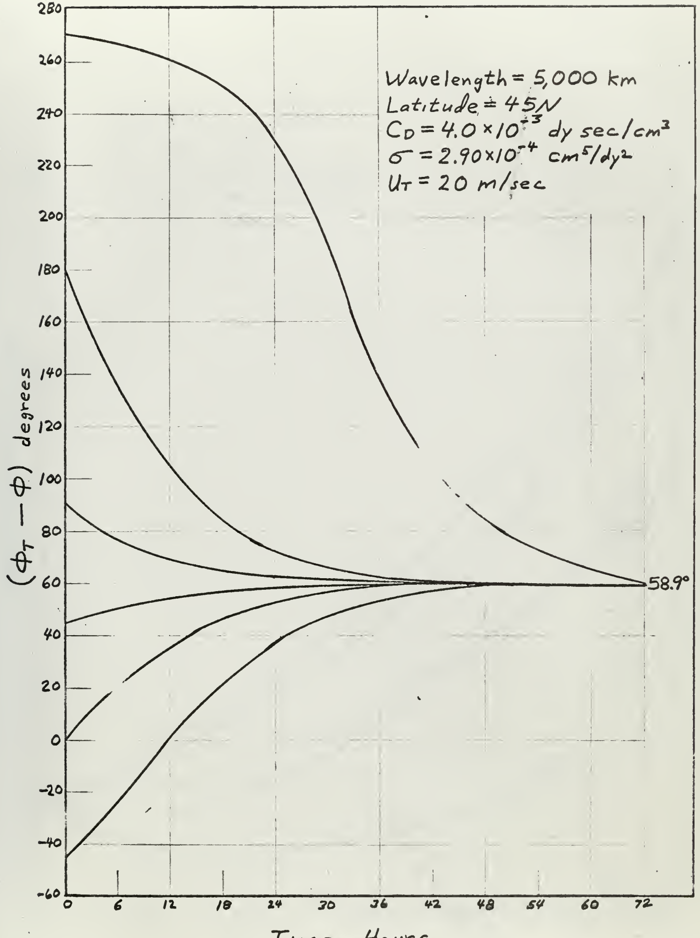
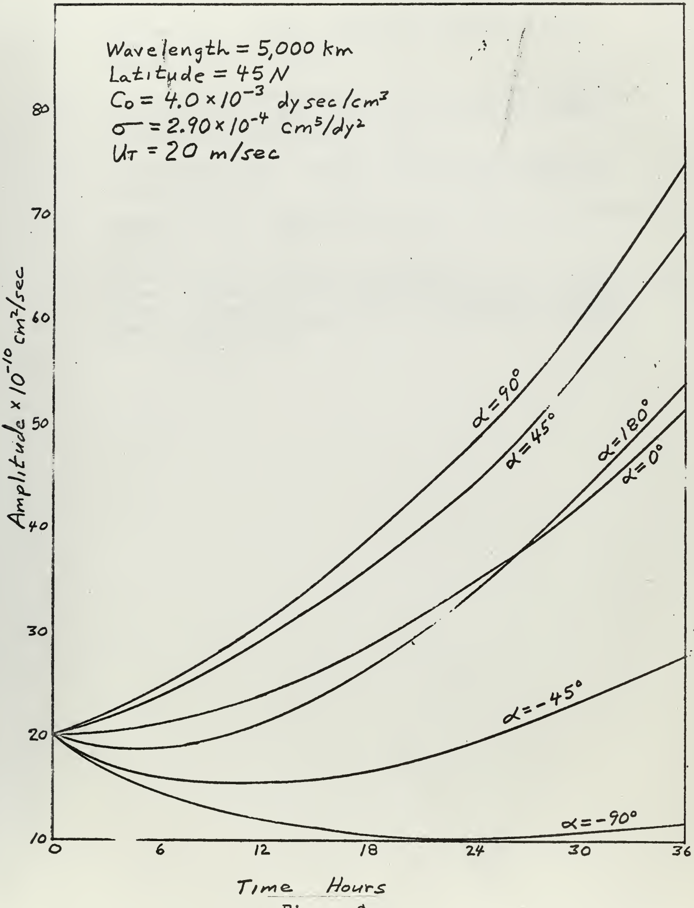

Wavelength = 5,000 km
Latitude = 45N
$C_D = 4.0 \times 10^{-3}$ dy sec/cm^3
$\sigma = 2.90 \times 10^{-4}$ cm^5/dy^2
$U_T = 20$ m/sec
58.9°
$(\Phi_T - \Phi)$ degrees
Time Hours

Figure 7



Wavelength = 5,000 km
Latitude = 45 N
$C_0 = 4.0 \times 10^{-3}$ dy sec/cm^3
$\sigma = 2.90 \times 10^{-4}$ cm^5/dy^2
$U_T = 20$ m/sec
Amplitude $\times 10^{-10}$ cm^2/sec
80
70
60
50
40
30
20
10
$\alpha = 90°$
$\alpha = 45°$
$\alpha = 180°$
$\alpha = 0°$
$\alpha = -45°$
$\alpha = -90°$
0
6
12
18
24
30
36
Time Hours

Figure 8

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