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# EVALUATION OF JITTER IN MULTICHANNEL TAPE RECORDERS USED IN SHAFT MOTION REPRODUCERS

Milton Gussow S. Russell Hawe and Robert C. Thyberg



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### EVALUATION OF JITTER IN MULTICHANNEL TAPE RECORDERS USED IN SHAFT MOTION REPRODUCERS

by

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## SUBMITTED IN PARTIAL FULFILLMENT OF THE

### REQUIREMENTS FOR THE DEGREE OF

### MASTER OF SCIENCE

at the

### MASSACHUSETTS INSTITUTE OF TECHNOLOGY

May, 1957



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### EVALUATION OF JITTER IN MULTICHANNEL TAPE RECORDERS USED IN SHAFT MOTION REPRODUCERS

by

Milton Gussow S. Russell Hawe Robert C. Thyberg

Submitted to the Department of Aeronautical Engineering on May 20, 1957, in partial fulfillment of the requirements for the degree of Master of Science.

### ABSTRACT

An investigation was undertaken on a highly accurate system based on phase information for the recording of shaft motion in an airborne test vehicle or in other severe and confined environment for later reproduction.

The investigation showed that the major contribution to the error of the system was due to interchannel time jitter in a multichannel magnetic tape recorder. Measurement and calibration techniques were devised to obtain experimental data on interchannel jitter to determine the characteristics of this interference. Results from the experimental data indicated that jitter could introduce large errors in the shaft recording and reproduction system. Analysis of the data formed a basis for the design of a compensation circuit to reduce the effects of jitter on the system error in order to meet the high accuracy requirements.

Experimental data of a jitter compensating circuit showed a significant decrease in interchannel jitter. Improvement of circuit design will result in further reduction of jitter error.

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The investigation showed that jitter can be reduced so that a high degree of accuracy can be attained for instrumentation utilizing multichannel tape recorders.

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The graduate work for which this thesis is a partial requirement was performed while the authors were assigned to the U.S. Naval Administrative Unit, Massachusetts Institute of Technology, Cambridge 39, Massachusetts.

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### **OBJECT**

The object of the work described in this thesis was to investigate a highly accurate system for the recording and reproduction of shaft motion. Emphasis was placed on a study of interchannel time jitter since it is the major source of error to the proposed system. Both experimental and analytical techniques were used.



### CHAPTER 1

### INTRODUCTION

### 1.1 Applications of Shaft Motion Recording and Reproducing Systems

Many commercial and military programs now require recording and reproducing of shaft motion with predictable fidelity. The inherent limitations of the components and type of modulation determine the over-all fidelity of a system. Many systems of shaft motion recording and reproduction have been designed and operated.

Several corporations are now controlling machine tools with prerecorded signals on magnetic tape to position several shafts which in turn position the cutting head, grinder or the object being worked on itself. Fig. 1-1 is a block diagram of the basic units in a typical system  ${}^{(1)*}$ .

In the military field, dynamic testing of complete systems may be desirable as a regular maintenance check. This is especially true where static tests do not give a true picture of the condition of a system, as for example, a fire control system. Another military application is the accurate recording of shaft motion in a system undergoing tests under extreme environmental conditions, and later reproducing these motions for detailed study. Flight testing of components in aircraft or missiles are examples of this.

A typical dynamic tester,<sup>(2)</sup> is shown in Fig. 1-2. This system is used for dynamic test of a fire control system.

<sup>\*</sup>Superscript numerals refer to similarly numbered references in the Bibliography, Appendix D.

Prerecorded signals are fed to the various parts of the system. The correct solution is also prerecorded and is compared with the actual output of the system under test. The errors can then be recorded and analyzed.

Naval Research Laboratory developed a system known as  $Digitar^{(3)}$  for the purpose of translating shaft rotation into a digital code with a resolution of one-hundredth of a degree and for storing this digital data on magnetic tape. It is expected that the data converter and storage units of the digitar will be used at each instrumentation station for coding and storing the radar data.

The foregoing applications of shaft motion systems all involve the use of multichannel tape recorders. These tape recorders are not necessarily restricted for use in shaft motion systems. Other uses to which they are put are numerous. Some are:

- 1. Computer programming (digital and analog)
- 2. Binaural audio recording
- Multichannel stereophonic sound tracks in motion pictures (sound comes from different parts of the theater)
- 4. Recording of telemetered signals,

### 1.2 Requirements of the Skipper Test Program

The Skipper test program at the M.I.T. Instrumentation Laboratory was faced with the problem of accurately recording shaft motions generated by integrating accelerometers while undergoing tests in a varying environment, and later reproducing these shaft motions for analysis. The system is applicable to many environments.

The shaft motions to be recorded are the outputs of three pendulous integrating gyros (PIG) which are used as integrating accelerometers. The requirements of the program state that the



Fig. 1-1 A Typical Machine Tool Control System



# Fig. 1-2 A Typical Fire Control System Dynamic Tester

accuracy be better than 0.1 ft/sec which corresponds to an angle of 10 minutes of arc at the shaft. This means that the playback shaft can not vary more than 10 minutes from the position of the PIG shaft except for transients which should integrate to zero. The system should be sensitive to lost revolutions in case the servo driven playback shaft loses: synchronism. In other words, the error signal supplied to the servo should increase proportional to the angular displacement from the PIG shaft. This can be shown graphically in Fig. 1-3.





The requirement to avoid lost motion can not be met by all systems even though the system might otherwise be adequate. If, by the proper design of the servo, it can be assured that the total error will never exceed a specified amount, then this requirement can be relaxed to permit unstable nulls. The dynamic requirements of the servo and also the expected dynamics of the PIG units will be discussed in Chapter 2. Another requirement to be incorporated in the system is the use of standard-stock readily available components. All electronic subassemblies are to use printed circuit and sub-miniaturization techniques.

In the following sections, a comparison is made of several systems of shaft to shaft reproducing techniques utilizing continuous modulation which include amplitude, frequency, and phase.

### 1.3 Amplitude Modulation System

The first of several systems to be considered is an amplitude modulation system of the synchro transmitter - control transformer repeater servo type. The information is transmitted on three wires plus a reference which may be common to several loops. Fig. 1-4 shows the basic configuration with a record-playback device incorporated in the transmission line. From the diagram, we can see that 3n + 1 channels are necessary for n shafts. This could be reduced to 2n + 1 channels if connected in the grounded delta configuration.

This one-speed system has unstable nulls 180 degrees away from the correct null which is not necessarily so in some multiple speed systems. The system has stable nulls every 360 degrees and is insensitive to lost revolutions if the servo loses synchronism. However, as mentioned before this is not too serious a drawback.

Three-phase one-speed synchros can now be obtained which have accuracies on the order of several minutes of arc in a nulling





loop. To this must be added the dynamic requirements and possible servo noise giving a best possible accuracy of about five to ten minutes of arc. These accuracy figures do not include the errors inherent in the recording-playback medium. The amplitude distortion must be less than one part in 350, (the sine of 10 minutes compared to the sine of 90 degrees) to recover 10 minutes and much greater to bring the total error down to 10 minutes or less. Amplitude distortion however is quite prominent in all magnetic tape recorders and distortion inaccuracies on the order of  $\frac{1}{2}$  1 db are common. Use of tape recorders in an amplitude modulation system does not appear feasible with the above accuracy requirements.

### 1.4 Frequency Modulation System

A relatively simple frequency modulation system based on a multivibrator whose period is constant but whose duty cycle or on-off time is controlled by two capacitors in the plate circuit is described next. The capacitors are mechanically coupled and driven by the shaft motion to be reproduced. The plates of the capacitors are shaped so that the capacity is a known and simple function of the angular displacement. An advantage of this FM system is that only one tape recorder channel per shaft is required. The reference is contained in the constant basic frequency of the multivibrator while the information is available in the on-off times of each cycle.

Two methods for recovering the shaft motion from this information are possible. First, the on-off time can be used to key an electronic switch which in turn connects in a precision signal source such as a  $\pm$  d.c. source. The average d.c. level is then a measure of the shaft angle. A precision d.c. servo could convert this signal to an angle. The second way involves driving a multivibrator circuit similar to that used to record the information by a servo. The waveforms from the tape recorder and servo may then be compared and drive the servo on the

difference in the average level of the two waves.

To be able to recover a minimum 10 minutes of arc both recovery systems require detection processes accurate to better than one part in 2000. Further, the recording and playback electronics must not distort the waveform or introduce time leads or lags since the waveform contains the information. The capacitors must be capable of being graduated to considerably better than one part in 2000, since the duty cycle can not go to zero either way (all on or all off) or else the time reference mark will be lost. The nulling loop of the second recovery system requires capacitors which will track exactly with those of the recording multivibrator and these capacitors are not standard stock items. However, this problem could possibly be avoided by using the same multivibrator in the reproduction as was used in recording.

Both recovery systems as described have no inherent unstable null. Depending on the construction of the capacitor the loop can be made to work through less than or more than one revolution. Both systems have finite inherent synchronism beyond which cycles can be lost without the servo distinguishing this loss and catching up. Because of the special components required this system was not considered for development.

### 1.5 Phase Modulation System

A third scheme for reproducing shaft angles is a phase modulation system in which the information is contained in the difference in phase between two equal frequency carriers. This system is best visualized by using the following block diagram.





A two-phase resolver is used as a mechanically controlled electrical phase shifting device by exciting its windings with in phase and quadrature signals of equal amplitude and frequency. The reference and phase shifted signals are recorded on magnetic tape. Later as the tape is played back the reference signal reconstructs proper excitation for a comparison two-phase resolver while the phase shifted signal is fed into a phase sensitive detector where it is compared with the signal output of the comparison resolver. The output of the phase sensitive detector drives a servo which is mechanically connected to the comparison resolver to null the relative phase shift. This system as described can have an unstable null 180<sup>°</sup> from the proper null and it can not recover lost revolutions once synchronism is lost.

Commercial resolvers and resolver drive amplifiers are available with an accuracy of one to five minutes of arc. Null detectors (phase sensitive) are manufactured which are sensitive to considerably less than 10 minutes of arc. This system requires n + 1 tape channels for each n shafts.

The system appears feasible if the tape recorder phase errors are small or can be compensated. Chapter 3 is devoted to disturbances in the tape recorder. The phase modulation system is the one which was chosen for use. Chapter 2 gives a detailed description of the proposed system and individual components.

### 1.6 Statement of Problem

Preliminary investigation indicated that the major source of error in the phase system is in the data handling medium, a multichannel magnetic tape recorder. Of all the errors present in a tape recorder interchannel time jitter represents the largest effect. The basic problem of this thesis is an attempt at an analysis of jitter and a method of reducing jitter effects in order to meet the accuracy requirements of the over-all system.



### CHAPTER 2

### DESCRIPTION OF THE MAJOR COMPONENTS IN A SHAFT RECORDING AND REPRODUCING SYSTEM

### 2.1 Introduction

This chapter describes the major components of the proposed system and the flow of information data between them. The description includes the precision power supply, phase sensitive detector, resolver drive amplifier, phase shift amplifier, playback servo, magnetic tape recorder and reproducing equipment, and the digitalizer, The proposed system is shown in Fig. 2-1. The source of signals is a carrier oscillator at one kilocycle. One kilocycle was chosen as the carrier frequency because the frequency response and phase shift characteristics of the direct or AM record and playback amplifiers in the tape recording and playback equipment were not good enough in the vicinity of 400 cps, the most obvious choice of carrier frequency in an aircraft installation. Also one kilocycle is available from other sources in a modern jet aircraft. It is also possible to use FM record and playback amplifiers in the tape recorder instead of AM or direct amplifiers. FM amplifiers should reduce tape drop out (see Chapter 3) while retaining all the other characteristics of the system using AM amplifiers. No FM amplifiers were available for testing. Further investigation using FM amplifiers is recommended.

The system as shown is practically self-descriptive, however it should be noted that there will be a constant 45 degree difference between the input and reproduced shaft angles. The way the components operate individually and are integrated into a system is described in the following sections.



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-1

2.2 Precision Power Supply



### Fig. 2-2 Basic precision power supply

The precision power supply above is an amplifier incorporating an automatic gain control (AGC) stage to minimize the effect of input and load changes. The output signal is compared to a reference signal in a detector. Any difference is amplified by the d. c. amplifier and applied to the AGC stage, thereby keeping the output constant. The precision power supply is adjusted to give less than 10 minutes of arc phase shift for  $a \pm 20$  cps change in frequency about 1000 cps. The amplifier output amplitude is flat at 20 volts from 250 cps to over 3000 cps. This characteristic makes the system insensitive to flutter. The frequency response curve is shown in Fig. 2-3. With a constant load of 1,500 ohms, Fig. 2-4 shows the wide range of input levels possible with










Fig. 2-5 Basic Phase Sensitive Detector

negligible change in output amplitude. This range is approximately 0.35 to 3 volts. Normal operating input is 1 volt rms.

This amplifier has greater power capability than needed in this application because it is used as a general purpose reference for test instrumentation. It may also be used at 400 cps simply by changing four capacitors. The circuit diagram is shown in Fig. C-1.

# 2.3 Phase Sensitive Detector

The phase sensitive detector, Fig. C-2,uses two high loop gain feedback amplifiers driving a double bridge of silicon diodes which is the phase sensing device. A simplified block diagram of the phase sensitive detector is shown below. The phase difference between the two inputs is demodulated to a d. c. level at the output. When the phase shift is  $90^{\circ}$  the d. c. level is zero. The phase difference - d. c. level characteristic is illustrated in Fig. 2-6. Phase shift in the exciting transformer has been eliminated by local feedback and varying of the a. c. signal in one of the pushpull output tubes. The designed input is 20 volts rms at 1000 cps. Balance adjustments in the demodulator permit exact null output with a  $90^{\circ}$  phase shift. There will be a constant  $90^{\circ}$  mechanical difference in position between the original and reproduced resolver shafts.

# 2.4 Precision Resolver

The precision resolver used is a two phase synchro excited by two equal amplitude 90<sup>°</sup> out of phase signals and used as a mechanically controlled phase shifting device. The phase shift mechanism and associated mathematics are shown below.



Note: Stable and unstable nulls can be interchanged depending on the internal connections within the phase sensitive detector.

Fig. 2-6 Phase difference - d.c. level characteristic of phase sensitive detector



 $e_{out} = e \cos \omega t \cos \theta + e \sin \omega t \sin \theta$   $e_{out} = e \cos (\omega t - \theta)$  $e_{out} = e \cos (\omega t - \omega_{shaft} t)$ 

Fig. 2-7 Basic two-phase resolver phase shift mechanism

The resolvers are designed to operate with both input and output voltages equal 20 volts rms. Fig. 2-8 shows the power supply and amplifier arrangements for driving the resolver. The precision power supply (2.2) is adjusted for 20 volt output independent of input voltage or frequency. It drives resolver drive amplifers (2.5) through a phase shift amplifer (2.6) adjusted for  $-45^{\circ}$  and  $+45^{\circ}$ , to give two signals  $90^{\circ}$  apart to apply to the stator windings. To reduce amplitude variations two precision power supplies are used between the phase shift amplifer and stator coils of the resolver. The use of two phase shift networks at plus and minus 45<sup>°</sup> instead of a single 90<sup>°</sup> phase shift network results in a frequency independent phase network insuring better accuracy for a varying reference signal. Each resolver drive amplifier can be adjusted to take account of the individual gains of the resolver coils to give an effective gain of unity. Adjustments can also be made with networks on the resolver rotor coils to correct minor phase errors since only one half of the rotor winding is used. The resolver schematic diagram is shown in Fig. C-3. Errors on the order of three minutes of arc maximum can be expected.

# 2.5 Resolver Drive Amplifier

The resolver drive amplifier, Fig. C-4, is a conventional three-stage high loop gain feedback amplifier used to excite the resolver at a frequency of one kilocycle per second. Feedback is accomplished through use of a compensating winding on the stator of the resolver (see Fig. C-3) in order to reduce nonlinearities due to the iron in the resolver. The null output of the compensating winding through the amplifier produces a quadrature plus harmonic content voltage equivalent to less than three minutes of arc. Broadband (low-Q) phase correctors are used on both the input and output of the amplifier which results in an essentially flat phase shift characteristic over a substantial band either side of 1000 cycles. The amplifier input is 20 volts rms with unity overall gain.





#### 2.6 Phase Shift Amplifier

The phase shift amplifier, Fig. C-5, is used to supply two signals 90<sup>°</sup> apart in phase to drive the resolver stator coils. The phase shift network is a simple resistance-capacitance bridge driven by a cathode follower output amplifier. The over-all gain, input to output, is less than unity. Fig. 2-8, the resolver excitation system, shows how the phase shift amplifier is incorporated. In order to ensure constant amplitude signals input to the resolvers, precision power supplies are used as buffers between the phase shift amplifier and resolver drive amplifer.

# 2.7 Playback Servo\*

The playback servo is required to reproduce accurately the shaft motion impressed on the magnetic tape. The maximum expected input corresponds to shaft motion of only two revolutions per second. However no velocity error can can be tolerated. The digitalizer system, for which the servo provides the input by driving the digitalizing disk, has a sampling rate of 20 cps. This rules out a servo with an undamped natural frequency in the vicinity of 20 cps. An undamped natural frequency of about 10 cps is proposed which is removed from the sampling frequency. Another reason for choosing an undamped natural frequency as low as possible is that the servo would then act as a low-pass filter eliminating high frequency phase shift effects due to jitter, flutter, and also high frequency noise in the system. The peak angular acceleration of the accelerometer shaft is expected to be less than 25 radians/sec $^2$  for a duration of 0.1 sec or less. The transients incurred during these peak accelerations integrate to zero in the servo by use of a feedback integrator. The effects on

<sup>\*</sup>The servo is at the time of this writing being developed and was not available for testing.





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the digitalizer will be small because of the ratio of sampling time to the undamped natural frequency of the servo. Two other requirements of the proposed servo to ensure proper functioning are tied together. Full motor torque is desired for an input corresponding to 50 minutes of arc error at the shaft. Also it is desired to incorporate jitter compensation which means an additional input to the servo. Any additional input to a shunt feedback type d. c. amplifier lowers the loop gain which would change the over-all gain of the servo. A specially designed chopper stabilized d. c. amplifier is now being developed to ensure adequate gain characteristics for multiple inputs. Full motor torque is developed with 115 volts a. c. applied to the control winding by a magnetic amplifier. Then for a phase sensitive detector sensitivity of approximately 0. 1 volt/degree, the overall loop gain of the servo must be

$$\frac{115}{\frac{5}{6}(0.1)} = 1400 \frac{\text{volts a.c.}}{\text{volt d.c.}}$$

The proposed servo system is shown in Fig. 2-9. The chopper stabilized d.c. amplifier has integral feedback compensation to remove velocity error. Tachometer feedback is to be used to stabilize the servo if needed. The loop is closed by comparing the resolver shaft position signal with the signal from the magnetic tape in a phase sensitive detector. The simplicity and straightforward design of the servo is apparent from Fig. 2-9.

# 2.8 Airborne Magnetic Tape Recorder

This system uses an Ampex Series 800 magnetic tape recorder<sup>(4)</sup>, which is a record-only equipment, specifically designed to operate in environments usually encounted in airborne applications. This equipment consists of an assembly of separate units to give a flexibility in installation and use. These units are shown in Fig. 2-10 and a typical arrangement in an aircraft is shown in Fig. 2-11.



Fig. 2-10 Multichannel Airborne Recorder Components

The tape transport mechanism provides the means of transporting the tape past the record heads. It consists of the capstan drive motor, take-up reel torque motor, 10-1/2 inch supply and take-up reels, 14-track record head assembly, and associated braking controls.

The electronics are packaged separately to eliminate all possible sources of heat to avoid excessive tape expansion at high ambient temperatures.

While there are six possible tape speeds available, this system will normally operate at a fixed tape speed of 15 inches per second (ips) which will give about 48 minutes of recording time.

The record amplifier assembly provides the housing for seven record amplifiers in any combination of amplifer types. Three amplifier types are available to permit AM, FM, or pulse width modulation recording. An amplifier is required for each tape channel.

The plate and filament power for the record amplifiers is furnished by the Power Supply for Electronics. Both the highvoltage d.c. and filament voltages are regulated.

The Capstan Drive Power Supply supplies a precision 60 cycle power source from a precision R-C oscillator and a power amplifier to drive the capstan motor in the tape transport mechanism.

A remote control box which may be mounted in an aircraft instrument panel performs the functions of power switching, start-stop control of the tape, and indication of tape quantity remaining.

Outputs of the pick-offs and transducers in the aircraft are fed to the record amplifier inputs. For operation as a directrecord amplifier, the information signals are recorded directly through a feedback amplifier.



Fig. 2-11 Typical Arrangement of the Airborne Equipment

Shaft rotation to be recorded is transmitted mechanically to the resolver rotors. The output of the resolver rotor has a phase shift proportional to the shaft angle. This signal is then fed to a record amplifier and thence on a channel of the magnetic tape.

#### 2.9 Tape Playback Equipment

The tape playback unit used in this system is an Ampex FR 114<sup>(5)</sup> and is shown in Fig. 2-12. In this assembly the tape transport mechanism, the head assemblies, the reproduce amplifiers, and the power supplies are all housed in a single cabinet. These units make up part of the system ground equipment.

The tape transport mechanism has a mechanical servocontrolled brake system on the supply turntable to ensure constant tension in the tape passing over the head assemblies. The tight loop tape drive used produces accurate tape tracking and holds flutter to a minimum. Six possible tapes speed are available and in addition a fast rewind is provided to reduce rewind time.

A staggered head arrangement is used, with seven oddnumbered tracks on one stack, and seven even-numbered heads on the other stack as shown in Fig. 2-13.

The reproduce amplifiers are plug-in type units of modular construction and are mounted in an interconnecting chassis below the tape transport mechanism.

All power supplies are mounted on the interconnecting chassis.

Outputs from the reproduce amplifiers are accessible from the rear of the assembly and are connected to the system ground system by shielded cables to the gain of 20 amplifiers.

Speedlock equipment to reduce flutter and wow is also contained in the assembly and is described in Chapter 7.

# 2.10 Digitalizer

This unit as proposed accepts information from two sources;



Fig. 2-12 Ampex FR 100 Equipment



Fig. 2-13 Arrangement of Magnetic Head Stacks

first, the reproduced shaft position of a transducer, and second, the time signals from a magnetic tape channel. The unit, functionally, consists of a encoder, which accepts the shaft angle of the system servo and converts this into pulses. The encoder emits a pulse for every 0.1 ft/sec change of velocity as indicated by the reproduced shaft.

These pulses are fed to a pulse counter which has a capacity of 18 binary bits and which accumulates the pulses from the encoder. The pulses are sampled every  $\frac{1}{20}$  of a second and these are accumulated in a partial summer. Every 2 seconds the 16 most significant bits in the pulse counter and in the partial summer are punched on a paper tape.

Timing signals from the magnetic tape control the functions of (1) no operation, (2).add pulse to partial summer and (3) perform step 2 and read the results from a buffer storage and punch the paper tape.

The encoder is an incremental type using a coded glass disc with a lamp and photocell pick-off. A disc with 512 opaque sectors and 512 transparent sectors is used. The associated logical circuits accept information from the outputs of two phased photocells, and produce four counts for each opaque sector on the glass discs, giving 2048 increments per rotation of the disc. The logic also senses direction and adds or subtracts increments depending on direction of rotation. For a one to one ratio with respect to the shaft position, each increment is equivalent to a 0.1 ft/sec change in velocity.



Fig. 2-14 Precision Power Supply



Fig. 2-15 Phase Sensitive Detector



Fig. 2-16 Dual-Channel Resolver Drive Amplifier



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Fig. 2-17 Phase Shift Amplifier

#### CHAPTER 3

# DISTORTIONS INTRODUCED BY MAGNETIC TAPE RECORDERS

# 3.1 Introduction

The system data handling medium is the Ampex Series 800 airborne multichannel magnetic tape recorder and the Ampex FR 100 Series reproduce equipment. The airborne recorder with associated electronic equipment was developed by the Ampex Corporation specifically for Project DATUM, <sup>(7)</sup> which is a data acquistion and processing system currently under development for the Air Force Flight Test Center by the Electronic Engineering Company of California. This is the latest magnetic tape recording and reproducing equipment of instrument caliber available at this writing. An investigation of this equipment was undertaken to determine its usefulness and limitations from an instrumentation standpoint in a system using phase information.

# 3.2 Classification of Distortions

The distortions introduced by magnetic tape recorders may be divided into five general classifications:

- 1. Spectral distortion including frequency response limitations and phase response characteristics
- 2. Non-linear distortions
- 3. Noise
- 4. Time fluctuations which variously are called flutter and wow.

5. Interchannel time displacements referred to as interchannel jitter or simply jitter.

Specific disturbances introduced by the magnetic tape multichannel recorder and reproduce equipment are listed below:

- 1. Flutter and Wow
- 2. Drop-out
- 3. Gap Scatter
- 4. Noise
- 5. Interchannel time displacement or jitter
- 6. Equalization matching
- 7. Crosstalk between AM tracks
- 8. Variations in tape dimensions
- 9. Print out
- 10. Magnetostriction

# 3.3 Descriptions of Disturbances

1. Flutter and Wow - is the term used to designate any variation from uniform tape motion along the longitudinal axis of the tape. Flutter usually refers to cyclic deviations occurring at a relatively high rate, in the order of 10 cycles per second.  $^{(8)}$ A sine wave signal of constant amplitude recorded on the tape would experience a change in frequency due to flutter. The effect is a frequency modulation of the signal being recorded or reproduced. In an FM recording system, flutter will cause a noise signal on demodulation of the carrier. Wow is the term sometimes used to describe low frequency flutter such as the once-perrevolution speed variation of phonograph turntables. Flutter and wow are disturbances common to all magnetic tape recorders, however, compensation methods are available which reduce their effects to a minimum. The importance of holding the effects of flutter and wow to a minimum in data recording machines is that they can appear in the useful signal as noise or as an error in many types of data signals, such as in Pulse Width Modulation.

A treatment of flutter and wow is given in Chapter 7.

2. <u>Drop-out</u> - is the term used to designate a highly attenuated or complete loss of signal due to the motion of the tape away from the magnetic heads. Occasional imperfections in the tape also are a source of drop-outs. Drop-outs due to tape imperfections give rise to greater than 50 percent loss of signal approximately 100 cycles out of 2-1/2 million for a sine wave of 2 mil wavelength. An amplitude level variation occurs during reproduction which is common to all magnetic recording instruments. This variation is greater for the higher frequency carriers which have short recorded wavelengths. Any bouncing of the tape or any non-uniform contact with the head gives rise to a loss of signal for a greater portion of the cycle for the short wavelength signals. Amplitude variations are minimized by the precision power supplies used in the system.

3. <u>Gap Scatter</u> - is the term used to describe the tolerances of the magnetic head geometry and is a measure of the alignment of the magnetic heads with respect ot each other in a stack and with respect to each other between stacks.



Fig. 3-1 Definition of gap scatter and azimuth alignment

The head geometry of the Ampex Series recorder are shown in Fig. 3-1. All trailing edges will be within two parallel lines that are 0.0001" apart. Reproduce equipment Gap Scatter is that the center line of reproduce gaps are within a band 100 microinches wide.

Interstack Tolerance: Gaps within any pair of stacks, record or reproduce, are separated by 1.5 plus or minus 0.005 inches.

There is a criteria for determining whether to use a single stack head or interleaved heads which depends on the type of information to be recorded and on the number of tape tracks that must be used. If time and phase coincidence among tracks are at all important, use single stack heads since precisely aligned heads are essential for this type information. This is particularly true in digital recording where crosstalk is not much of a problem but where time and phase coincidence is a necessity for pulse control.

When it is essential that a large number of tracks must be recorded without crosstalk and considerable time and phase displacement between channels can be tolerated then use interleaved heads.

The effect of gap scatter on the data in the shaft rotation measuring system is to introduce a small constant phase angle between data channels. A small constant phase angle is initially accepted by the servo as a constant error at its shaft. The servo is given a few seconds to reach steady state. Then the digital computer is turned on by a timing signal on the tape. Since it is an incremental type computer, the initial false zero does not affect it.

4. Noise - The noise level with the tape stationary was measured and found to be greater than 36 db down from the nominal output level of 1 volt rms. In the thesis of R. H. Prager<sup>(9)</sup> it is stated that the output noise level goes up approximately 8 db when the tape was in motion than when it was stationary. With

higher tape speeds the noise induced by tape motion increases. This noise is due principally to the magnetic coating used in the manufacture of magnetic tape.

Magnetic tapes are produced by coating the base film with a layer of lacquer in which the magnetic materials are dispersed. Highly volatile solvents are used to promote quick drying and in drying much of the solvent must escape through the surface. As the escaping vapors boil through the surface of the lacquer, microscopic craters are formed. Small craters are a source of noise and occasional large craters can cause drop-outs and amplitude variations. Other main sources of noise are:

- a. Inhomogeneity in the magnetic coating
- b. Variations in the cross section of the tape channel
- c. Variations in the contact between the tape and the recording heads.

A tape with the same mechanical properties but with a nonmagnetic coating will show no increase in the noise level except for sporadic noise spikes which may be attributed to the tape physically slapping against the head and producing a magnetostriction effect.

5. <u>Interchannel Jitter</u> - is defined as a varying time displacement between tape channels caused by a change in the orientation of the tape relative to the record or playback heads as it moves across the heads. Skewing motion, defined as motion of the tape about some axis perpendicular to the longitudinal axis of the magnetic tape as it passes between the heads, is the principal source of jitter. Jitter is peculiar to multichannel tape recorders and is a phenomenon not present in single track recorders or in audio transcription.

The relative phase of signals recorded in different channels will be affected by jitter. This is illustrated in Fig. 3-2. As the servo output shaft position is an accurate reflection of the phase difference between the signals on the data and the reference channels, the presence of jitter will introduce spurious signals into the



(c) ORIENTATION OF TAPE UNDER MAGNETIC HEAD WHEN NO SKEWING MOTION IS TAKING PLACE. NO PHASE ERROR IS INTRODUCED.



(b) skewed tape motion under the recorder head introduces a phase error due to  $\phi_l.$ 



- (c) skewed tape motion under the reproduce head introduces a phase error due to  $\varphi_2,$
- (d) RESULTANT EFFECTIVE PHASE ERROR DUE TO  $\phi_1$  and  $\phi_2$  showing the cumulative effect of the skewing motion of the tape under the record and reproduce heads.

# Fig. 3-2 Development of Phase Error by Skewing Motion of Tape Under the Magnetic Heads

servo loop. Jitter has been observed to be the source of the largest error in the phase information data. Fortunately, the servo of the system has a bandwidth of approximately 10 cycles per second so that the high frequency components of jitter are effectively filtered out.

A survey of the literature in this field indicates that this disturbance has been recognized but that no thorough investigation has been made into this type of distortion. The results of a limited investigation into the properties of jitter are reported in the next three chapters.

6. Equalization Matching - Equalization is the corrective means employed in the recording and reproducing process to obtain a frequency response that is flat over the frequency range desired. If the equalization response curves of the record and reproduce amplifiers for the different tape channels are not identical some signals will undergo more phase displacement than others. This will produce constant errors in phase angle betwen data channels. As explained before, the system digitalizer will not respond to this type of error.

7. Crosstalk - By crosstalk is meant a situation where a portion of a signal on an adjacent channel is picked up on a data channel. For the recorders used in the system, crosstalk between AM channels is below the specified system noise, except when an AM track is next to an FM track. Crosstalk pickup in the AM track then deteriorates it signal-to-noise ratio to approximately 30 db below the normal operating level. This interference may be minimized by using interleaved heads where this is possible or by using different types of modulation on adjacent channels. In the Ampex recorder crosstalk is over 55 db below tape saturation level (16 ma. head current), except when an AM track is adjacent to an FM track as mentioned before.

8. Tape dimension variations - Thickness variations will produce variations in the signal amplitude and noise. Variations

in the width of the tape will cause skewing and lateral motion between the transport guides. Precision slit tapes are available which minimize these distortions. Tapes are also available with polished surfaces to give a higher signal-to-noise ratio. Metal backed tape stretches less under varying tension thus reducing flutter and is less noisy but is thicker and takes up more reel space which shortens the recording time available.

<u>9. Print out</u> - is the result of magnetic variations on layers of tape in the tape reel being in close proximity to the magnetic variations of the layer of tape above and below it. This interaction of the small magnetic fields on one another results in the impression of small residual magnetic variations on adjacent tape layers. This effect is negligible if the recording operating level is well below the tape saturation level.

# 3.4 Accuracy and Jitter Error

The accuracy requirements of the system is the reproduction of shaft motion to within 10 minutes of arc. This high accuracy requirement corresponds to 0.5 microsecond of time at a carrier frequency of 1000 cycles per second. This requires that the error due to jitter must be compensated to less than 10 minutes of arc.

With this requirement in mind an investigation of jitter was undertaken for the purpose of determining the effects of jitter on the accuracy of the system and to discover if possible some of its sources. This involved the determination of the best measuring technique, the instrumentation, and the collection of sufficient data to permit an adequate evaluation of the characteristics of jitter. The measurement and instrumentation is discussed in the next two chapters and leads to a method of jitter compensation described in Chapter 6.

# CHAPTER 4

### MEASUREMENT OF JITTER

# 4.1 Introduction

The initial concept of jitter by intuition was that it would be composed of some nearly cyclic time varying components and other components of a random nature with frequencies in the lower portion of the frequency spectrum. For the purposes of the shaft reproducing system, jitter components in the range of from 1 to 10 cycles per second were of major interest as this is the signal bandwidth of the servo in this system. However, to gain a full understanding of the nature of jitter it was decided to investigate as wide a bandwidth as possible with the equipment available. Once the frequency range of the major energy components of jitter were known the bandwidth of the measuring system could be adjusted to accommodate these frequencies. It was further decided to use a phase measuring technique as the basis for the measuring system since interchannel jitter is actually a time or phase displacement between the data channel and the reference channel

# 4.2 Measurement Technique

The purpose of the measuring system was to obtain a photographic recording of the jitter components of several tape channels simultaneously. Such a record would yield the peak-topeak jitter that could be expected from each channel and in addition would show if the maximum peaks occurred on all channels at the same time. A photographic record with time would also give the frequency of any components that proved to be cyclic and of more importance would indicate the degree of proportionality of jitter

between several channels.

Since interchannel jitter represents a phase displacement between the data channel and the reference channel, a phase sensitive detector was used to give a varying d.c. voltage output that is proportional to the phase difference between the signals on two tape channels. To examine the jitter between a data channel and its reference channel requires two precision power supplies to take the 1000 cps signals from the reproduce amplifiers and amplify these to 20 volts rms. The two amplified signals then become the inputs to the phase sensitive detector. Circuit diagrams of the precision power supply and of the phase sensitive detector are given in Appendix C-2. The phase sensitive detector gives a null output for two signals exactly 90 degrees out of phase and a d.c. voltage output proportional to the magnitude of the jitter induced phase shifts that occurred during the recording and playback of the magnetic tape. The phase sensitive detector output also contained a 2000 cps ripple of sufficient magnitude to mask small amplitudes of jitter. This interference was removed by the insertion of a low-pass filter designed with a cutoff frequency of 160 cps.

It was initially planned to measure the jitter between all 14 tape channels simultaneously. To do this would require 14 precision power supplies and 13 phase sensitive detectors. The equipment available for test purposes permitted the recording of jitter data from only 2 tape channels with respect to a reference channel at any one time.

To provide the photographic record of the jitter data a Midwestern recording oscillograph was used as this recorder may be used with interchangeable galvanometers that have a range of bandwidths from 0 to 60 cps to as high as 0 to 3000 cps. The sensitivity of the galvanometers was sufficiently high so that no amplification of the jitter signals was necessary.

#### 4.3 Jitter Measuring System

A block diagram of the Jitter Measuring system used is shown in Fig. 4-1. Descriptions of the operation of the major components of the measuring system were given in Chapter 2.

The first method of measurement was done with a 1000 cps signal at 1 volt rms recorded on all 14 tape channels with a tape speed of 15 inches per second. A block diagram of the method used to record the test tape is shown in Fig. 4-1. A bandwidth of jitter components of 0 to 120 cps could be examined practically by using type 102-200 galvanometers in the Midwestern recording oscillograph. This would yield data on the components of jitter that would most affect the system servo and at the same time have a broad enough band to give an indication of the higher frequency components. This recording oscillograph has a considerable range of paper speeds and in addition provides trace identification and timing markers. DuPont LINO-WRIT Extra Thin type photo recording paper was used and run at a speed of 2-5/16 inches per second with timing markers every 0.1 seconds.

The galvanometers in the recording oscillograph require a shunting resistor across the galvanometer coil to give a critically damped system, which is the normal mode of operation of the galvanometers. The proper value of damping resistance to give critical damping depends on the output impedance of the source driving the galvanometers. The nominal d. c. resistance of the galvanometer coil was 28 ohms which loaded the output of the phase sensitive detectors considerably. A 10 K current limiting resistor in series with the low-pass filter on the output of the phase sensitive detector gave a better impedance match but reduced the sensitivity of the measuring system a small amount. Step inputs of voltage were applied to the galvanometer and the damping resistance was varied to give a critically damped response from the galvanometers.

The system was calibrated by a method described later in





this chapter and its sensitivity was found to be linear. Several hundred runs were recorded using various combinations of tape channels and with each channel as the reference channel. Runs were also taken over various portions of the tape supply reel to determine whether or not there was any increased jitter near the beginning or near the end of a reel. All runs were made with the equipment at nearly the same temperature to minimize the effects of tape expansion or shrinkage between the time the test tape was recorded and played back.

#### 4.4 Test Data Observations

Fig. 4-2 shows samples of actual test data obtained by this measurement scheme. This figure illustrates the comparision of the jitter between channels 1 and 4, 1 and 7, and 1 and 14. It is immediately evident that the peak-to-peak jitter is greater for those channels that are at a greater distance from the reference channel. Further examination of the data also shows the major component of jitter to be nearly cyclic at a frequency of approximately 0.5 cps. It was also observed that this 0.5 cps variation occurred on all tape channels in time phase. Other components are in the frequency range of 4 to 10 cycle component. These, however, seldom occur on all tape channels simultaneously and may be observed to occur on one channel and not on another. These smaller components of jitter which occur on one channel and not another are called random jitter. Still higher frequency components are present but these are near the noise level and can be almost completely eliminated by a filter with a cutoff frequency of about 18 cps.

# 4.5 Modified Jitter Measuring System

Since the phase sensitive detector requires two input signals 90 degrees out of phase, this first measuring system accomplished this phase shift by inserting two 45 degree phase shift networks ahead of the two precision power supplies. With the 45 degree







Fig. 4-3 Actual Test Record of Jitter Data
phase shift networks in this position the signals into them contained the carrier of 1000 cps frequency modulated by any flutter that took place during the recording and playback of the test tape. In addition the 1000 cps signals were also phase modulated by the jitter induced during the record and playback process. The two 45 degree phase shift networks were designed to be frequency insensitive provided signals of the same frequency were fed to the networks at the same time. It was conceivable that the dynamics of the two 45 degree networks on the 1000 cps carrier with flutter and jitter present could exaggerate the jitter output signal of the phase sensitive detector. To prevent this from occurring a second method of measurement that required only a few changes was used.

The second method of measurement differed from the previous method in that the tape channels 2 to 14 were recorded on the magnetic tape 90 degrees out of phase with channel 1, the reference channel. Recording the test tape in this way permitted the 90 degree phase shift to be done on the carrier without any flutter or jitter present. The only changes required were the removal of the two 45 degree phase shifters from the precision power supply inputs and placement of these between the audio oscillator and the record amplifiers.

Fig. 4-3 shows actual test data of jitter from this method and a comparison of the test data of the two methods shows that the second measuring scheme which avoids the dynamics of the two phase shift networks is superior. (Test data is tabulated in Appendix B-1 and B-2 and compared mathematically in Table 5-1). All subsequent test runs on jitter were recorded by this latter method.

# 4.6 Calibration of the MeasurementSystem

A calibration of the entire measuring system was necessary to establish the system sensitivity for jitter phase angle input recorder scale deflection output. The system calibration was

performed by using a variable phase shift network. The actual phase shift was determined with a Hewlett Packard Electronic Counter, model 524B with an associated Time Interval Unit, model 526 B. This arrangement was used to shift the phase of the 1000 cps signal over a range of phase angles sufficient to give a calibration curve. A graph of this calibration data is shown in Fig. 4-4. A block diagram of the calibration arrangement is shown in Fig. 4-5. The resulting system sensitivity for jitter phase angle input - recorder scale deflection output is 10.3 degrees per inch of record deflection. The calibration sensitivities in this chapter are given in the notation developed in "Instrument Engineering", Volume 1 (10).

Sensitivity is defined as

S (oc) 
$$\begin{bmatrix} q_{in} & q_{out} \end{bmatrix} = \frac{q_{out}}{q_{in}}$$
 where  
 $q_{in}$  - input quantity  
 $q_{out}$  - output quantity  
oc. - operating component

Calibration sensitivity of the Jitter Measuring System for Phase Angle in - Record Deflection out,

S(jms) 
$$\left[\phi;d\right] = 10.3 \frac{\text{degrees phase shift}}{\text{in ch record deflection}}$$

The phase sensitive detector and the Midwestern recording oscillograph were calibrated independently and yielded the following sensitivities:

Calibration sensitivity of Recording Oscillograph for voltage in inches record deflection out.

S (ro) 
$$[e;d] = 0.592 \frac{\text{inch deflection}}{\text{volt input}}$$

Calibration sensitivity of Phase Sensitive Detector for phase angle in-voltage out.

S (psd) 
$$[\phi;e] = 0.165$$
 volts output  
degree phase angle input









Sensitivity of the Jitter Measuring System for phase angle in inches record deflection out.

$$S_{(jms)} \phi; d = \frac{1}{S_{(psd)} [\phi; e]} x \frac{1}{S_{(ro)} [e; d]}$$
$$= \frac{1}{(0.592)(0.165)} = 10.2 \frac{\text{degrees}}{\text{inch}}$$

This served to verify the calibration of the overall measuring system.

# 4.7 General Observations

It was noted earlier in this chapter that the major component of jitter was a near cyclic component of 0.5 cps. Careful observation of the tape motion the tape transport mechanism during the test runs showed a skewing motion of the tape between the tape guides. This skewing motion of the tape was excited by the motion of the constant tension arms that maintain a constant tension on the magnetic tape as it is fed past the magnetic heads. Careful timing of this skewing motion showed that the motion of the tension arms was cyclic at the same frequency as that of the major component of jitter. This physically explained one source of jitter. The other components of jitter which are random in nature do not lend themselves to easy correlation to the data. They are, fortunately, much smaller in magnitude that the 0.5 cps component but they do give a limitation to the accuracy of any system using phase or pulse time information on a multichannel tape recorder as the data handling medium. Furthermore, the higher frequency components that occur at random between the tape channels do not appear to depend on the distance of the tape channel. The frequencies of some of these components are within the bandwidth of the servo of the shaft reproducing system and these as well as the 0.5 cps component will have to be compensated for to meet the accuracy requirements specified.







System for measurement and recording of interchannel jitter

System for Measuring and Recording of Interchannel Jitter

Fig. 4-7

# CHAPTER 5

# TABULATION AND CORRELATION OF JITTER DATA

# 5.1 Introduction

It was pointed out in Chapter 3 that jitter or phase angle difference, caused by the relative motion of the magnetic tape with respect to the record and playback heads, is the largest source of error to the servo loop. The object of this chapter is to tabulate jitter data obtained by methods described in Chapter 4, evaluate this data mathematically, and correlate it to other parameters of the system so that a logical basis may be established for compensating the jitter error.

#### 5.2 Determination of Model for Jitter



Fig. 5-1 Formulation of jitter equation

Let m and n be any two tape channels separated by a distance  $S_{m-n}$  and whose sinusoidal signals are to be compared in phase. In space domain, skew angle  $\theta$  introduces a phase difference  $J_{m-n}$ 

In space domain,  $J_{m-n} = S_{m-n} \tan \theta$ .

For a given time interval, phase difference  $J_{m-n}$  is proportional to carrier frequency W and inversely proportional to the tape speed V.

In space and time domain,

$$J_{m-n} = K \frac{W S_{m-n} \tan \theta}{V}$$
(5-1)

And where  $\theta^*$  is a small angle, tan  $\theta \cong \theta$  so that

$$J_{m-n} = K \frac{W S_{m-n} \theta}{V}$$
(5-2)

where

 $J_{m-n}$  = Jitter of channel n referred to channel m

K = proportionality constant W = carrier frequency  $S_{m-n}$  = distance between channels m and n  $\theta = \theta_1 + \theta_2$  = algebraic sum of the instantaneous skew angles of the magnetic tape under record playback heads

- 5.3 Physical Interpretation of Equation (5-2)
  - (1) Assume  $S_{m-n} = constant$

\*  $\theta < .05^{\circ}$  by calculation

If it is assumed that W and V\* are constant, then for a given channel n ( $S_{m-n}$  = constant)

$$J_{m-n} = K_{l} \theta$$
where  $K_{l} = K \frac{W S_{m-n}}{V}$ 
(5-3)

 $\theta$  = constant is physically impossible for it implies that the tape must run off the roller of the constant tension arm guide. What happens is that contact between the tape and the spindle of this guide forces the tape to reverse its direction of skewing and thus accounts for the maximum and minimum peaks of the jitter curves as seen in Fig. 4-2. It is the instantaneous value of skew angle that primarily causes jitter which introduces spurious phase error signals into the servo system.

(2) Assume  $\theta$  = constant at a given instant of time.

$$J_{m-n} = K_2 S_{m-n}$$
(5-4)  
where  $K_2 = K \frac{W \theta}{V}$ 

In the tabulation of jitter data the reference channel m was conveniently chosen as channel 1. Equation (5-4) indicates that the jitter between channels compared to a common reference channel is proportional to their spacing to the same reference channel. The assumption that V, the tape speed, is constant is not necessary since a change in tape speed would insert into all channels simultaneously the same variation in  $K_2$  and, therefore, does not change the proportionality.

Increased jitter for channels farther separated from the reference channel is readily observable from the appearance of the jitter curves of Fig. 4-2. But this raw data must be evaluated and correlated to verify the linearity of this simple but

<sup>\*</sup>In practice W is made constant by using stable oscillators and V by a speedlock mechanism.

very important relationship. A linear relationship will be assumed and then a measure of this linearity will be made by mathematical methods explained in Sections 5.4 and 5.5.

# 5.4 Curve Fitting by Method of Least Squares

Since jitter is a varying quantity, averaging statistical methods were employed to measure linear correlation between jitter and channel spacing and the expected values of jitter between any channel referred to channel 1.

The method used to fit a straight line objectively and measure the amount of deviation of the points about the line is known as the method of least squares. This straight line or line of regression is fitted to a pair of observation points  $(X_i, Y_i)$  such that the sum of the squares of the deviation of the observation points from the fitted line is a minimum.



Fig. 5-2 Representation of line of regression

Let the line of regression be represented by the equation

$$Y_{rgr} = a + b X_{i}$$

where

$$Y_{rgr} = value of Y on the regression line
X_i = value of X on the regression line
a = intercept of regression line on Y axis
b = slope of regression line
N = number of observations
$$\sum_{i=1}^{N} \sum_{i=1}^{N} \sum_{i=1}$$$$

Then according to the method of least squares, a and b are determined from the condition that

$$\sum (Y_i - Y_{rgr})^2 = \sum (Y_i - a - b X_i)^2$$

over all the observation points be a minimum.

It is shown in Appendix A.1 that

$$b = \frac{\sum x_i Y_i - \sum x_i \sum Y_i}{\sum x_i^2 - \frac{\left(\sum x_i\right)^2}{N}}$$
$$a = \frac{\sum Y_i}{N} - b \frac{\sum x_i}{N}$$

The reasons for using the method of least squares are that it provides a mathematical and impersonal means of comparing regression linesand determines the reliability of estimates of Y made from the regression line if the variations around it are random.

### 5.5 Correlation Coefficient

With each linear regression there is associated a correlation coefficient r which measures the degree of association between two variables. With reference to the regression line of Y on X the correlation coefficient is defined by the equation

$$r^2 \equiv 1 - \frac{\sigma^2_{xy}}{\sigma_y^2}$$

where

r = correlation coefficient  $\sigma^2 xy = \frac{1}{N} \sum (Y_i - Y_{rgr})^2$  $\sigma^2_y = \frac{1}{N} \sum (Y_i - \overline{Y})^2$ 

Appendix A.2 develops the correlation coefficient into a working computational form

$$r = \frac{N \sum X_{i} Y_{i} - \sum X_{i} Y_{i}}{\sqrt{N \sum X_{i}^{2} - (\sum X_{i})^{2}} \sqrt{N \sum Y_{i}^{2} - (\sum Y_{i})^{2}}}$$
(5-5)

The correlation coefficient measures the usefulness of a regression line for estimating purposes. It will have a value close to zero for a line incapable of prediction and close to  $\pm 1$  for a line capable of nearly perfect prediction. r = (+) 1 if the regression line slopes upward, and r = (-) 1 if it slopes downward. It is the magnitude of r that is important. But the value of r by itself is qualitative, i.e., no scale of measurement is provided to enable one to compare two sets of data for their relative strength of association. However, the correlation coefficient may be interpreted quantitatively by stating that the square of a correlation coefficient is equal to the percentage of the value of Y that has been accounted for by the relationships with X. For example, if the correlation coefficient between jitter and channel spacing is 0.95, then 90.25% of the variation in jitter

could be explained by the relationships of jitter to channel spacing. The remaining 9.75% would be due to other factors.

5.6 Limits for Individual Values





By definition  $(SE)^2 \equiv \frac{1}{N-2} \sum (Y_i - Y_{rgr})^2$ (5-6)

where

(SE) = standard error of estimate

 $Y_i - Y_{rgr}$  = difference between estimated value of Y and the actual value of Y

N - 2 = number of observations less two because the degrees of freedom have been reduced by two.

It can be shown that

$$(SE)^2 = \frac{N-1}{N-2} \sigma_y^2 (1-r^2)$$

where

$$\frac{1}{N-1} \left[ \sum Y_i^2 - \left( \sum Y_i \right)^2 \right]$$

so that

$$(SE)^2 = \frac{1}{N-2} \left[ \sum Y_i^2 - \left( \frac{\sum Y_i}{N} \right) \right] (1 - r^2)$$
 (5-7)

2

By equation (5-6),  $(SE)^2$  can be regarded as an index of the amount of variation around the regression line. The smaller the value of  $(SE)^2$ , the smaller the variation. If it is assumed that the individual values of Y for a given value of X are normally distributed about the regression line, then parallel lines placed at a distance of 2(SE) on each side of the regression line will include about 95% of the values of Y. Normal distribution is a reasonable assumption since Fig. 5-4 and 5-5 indicate that a high percentage of jitter values for a given channel number are included within the 95% limit lines.

#### 5.7 Regression Line of Jitter on Channel Spacing

From the jitter curves like those of Fig. 4-2, peak-topeak deflection in inches (Y) of the low frequency component were measured and plotted against channel number (X) to form a scatter diagram, Fig. 5-4 and 5-5. A straight line was fitted through these points by the method of least squares since equation (5-2) indicated that jitter is a linear function of channel spacing. It is assumed that for a given channel number, the range of  $\theta$ ,  $\theta_{range} = \theta_{max} - \theta_{min}$  is a constant over a period of time. The data taken established this assumption to be reasonable.

To evaluate two methods for measuring jitter, their regression lines were compared. The first method included the dynamic effect of jitter due to a phase network input to the precision power supply and the second method excluded this effect.



Fig. 5-4 Values of Jitter for Various Tape Channels Referred to Channel 1. Measurement Includes Dynamics of Jitter Due to Phase Networks



Fig. 5-5 Values of Jitter for Various Tape Channels Referred to Channel 1. Measurement Excludes Dynamics of Jitter Due to Phase Networks

Refer to Section 4.2 on the instrumentation of these two methods, to Fig. 4-2 and 4-3 for a comparison of their jitter curves, to Tables B-1 and B-2 in the appendix for computation of data, and to Fig. 5-4 and 5-5 for their plotted results. Table 5-1 compares the statistical data of these two methods.

Quantity	(lst method) Dynamic Effect	(2nd method) No Dynamic Effect
Regression line	$Y_{rgr} = .04 + .19X$	$Y_{rgr} =01 + .12X$
Correlation coefficient, r	. 987	.996
Correlation coefficient squared, r <sup>2</sup>	. 974	. 992
Standard Error of Estimate, (SE)	. 14	. 05

# Table 5-1Summary of statistical data for comparing twomethods for measuring jitter

It is evident that the second method is superior for measuring jitter. For a given channel, its jitter is less. Since its correlation coefficient is higher, jitter data of the second method is fitted more linearily to its regression line and its individual values show about one third less deviations from it. This method of measurement was then followed for the remainder of jitter tests.

The range of expected values of jitter for 95% probability were taken from Fig. 5-5 and tabulated in Table 5-2.

The high value of the correlation coefficient squared of both

methods indicate a nearly perfect linear relationship between jitter and channel spacing. Immediately this suggests a method of jitter compensation where jitter between channels 1 and 14,  $J_{1-14}$ , can be linearly reduced by a potentiometer arrangement and then subtracted from  $J_{1-n}$  so that jitter from n channel is reduced within tolerable limits before the data signal of n channel is fed into the servo input, Jitter compensation using this scheme is discussed in Chapter 6.

# 5.8 Deviations of Jitter from the Line of Regression

Equipment available limited the examination to only two jitter curves with the same time base. Therefore the largest source of error in presenting jitter data against channel spacing is that jitter of only two channels referred to channel 1 could be recorded, measured, and compared simultaneously. Thus for a given time interval, different ranges of skew angles between photographic recordings account for most of the deviation of jitter from the line of regression.

How much the change in skew angle affects the jitter measurement can be shown simply. Recalling equation (5-2),  $J_{m-n} = K \frac{W S_{m-n}}{V}^{\theta}$ It can be shown that  $\frac{d J_{m-n}}{J_{m-n}} = \frac{d S_{m-n}}{S_{m-n}} + \frac{d \theta}{\theta}$ where W, V are assumed constant.

For a given channel number,  $dS_{m-n} = 0$  so that  $\frac{dS_{m-n}}{S_{m-n}} = 0$ .

and

$$\frac{d J_{m-n}}{J_{m-n}} = \frac{d\theta}{\theta}$$
(5-8)

Thus a relative change of skew angle,  $\frac{d\theta}{\theta}$ , causes a relative change in jitter  $\frac{dJ_{m-n}}{J_{m-n}}$  in a one to one ratio.

Another source of jitter deviation is the fact that the magnetic

Channels	Range of Jitter		
	Peak-to-peak deflection (inches)	Phase difference (degrees)	
1-2	0.14-0.34	1.4-3.5	
1-3	0.26-0.46	2,7-4.7	
1-4	0.38-0.58	3.9-5.9	
1-5	0.50-0.70	5.1-7.1	
1-6	0.62-0.82	6.3-8.4	
1-7	0.74-0.94	7.6-9.6	
1-8	0,86-1.06	8.8-10.8	
1-9	0.98-1.18	10.0-12.0	
1-10	1.08-1.29	11, 1-13, 1	
1-11	1.22-1.42	12.4-14.5	
1-12	1.34-1.54	13.7 <b>-</b> 15.7	
1-13	1.46-1.66	14.9-16.9	
1-14	1.58-1.78	16.1-18.1	

# Table 5-2 Range of expected values of jitter for 95% probability

heads are interlaced, i.e., there are two record heads one which records on a given section of tape the odd number channels first, 1,3,...,13, and the other head the even number channels second, 2,4,..., 14. The reproduce magnetic heads are interlaced in the same order so that what is recorded first is played back first.

Time interval between pair of heads =  $\frac{\text{head separation}}{\text{tape speed}}$ =  $\frac{1.5''}{15''/\text{sec}}$  = 0.1 sec. Therefore if there is a different skew angle between the pair of record heads or playback heads and this is likely since the heads are displaced 0.1 sec in time, a phase difference is introduced between the even and odd numbered channels. Thus it is conceivable and was corroborated by data taken simultaneously that occasionally  $J_{1-4} \stackrel{>}{=} J_{1-5}$ .

A third error was that in estimating the maximum and minimum points of the jitter curves. The presence of high frequency noise and high frequency jitter components on the envelope of the low frequency component of jitter and the extension of the flatness of the peaks by the occasional curling of the tape on the roller of the constant tension arm guide made peak-to-peak estimating difficult at times, particularly for channels 1-2.

To make the measurements independent of time, two schemes are proposed;

(1) Weight jitter measurements with respect to one measurement, say  $J_{1-7}$ . Using  $J_{1-7}$  as a weighting factor, modify other  $J_{1-n}$  measurements. An example may make this procedure clear. If our reference measurement  $J_{1-7} = 0.70^{"}$ , then on measuring  $J_{1-7} = 0.50^{"}$  and  $J_{1-n} = 1.00^{"}$  on the same time base, modified  $J_{1-n} = \frac{0.70}{0.50} \times 1.0 = 1.40^{"}$ . However since  $J_{1-7}$  measurement varied only by 0.2", the accuracy with which jitter could be measured did not warrant any modification of the data.

(2) Normalize equation (5-2). Then at any given instant of time, ideally

$$\frac{\frac{J_{1-n}}{J_{1-n_{1}}}}{\frac{S_{1-n}}{S_{1-n_{1}}}} = 1$$
(5-9)

for perfect linearity between jitter and channel spacing. Measure jitter of two curves at the same instant of time, calculate the jitter ratio over the channel separation ratio as in equation (5-9) and compare it to 1.

 $J_{1-6}$  and  $J_{1-14}$  were compared since their curves were relatively smooth, thus permitting less error in estimating instantaneous values of jitter. However the presence of frequency components of noise and jitter above 0.5 cps limited the accuracy of the jitter measured to  $\pm$  0.05". Below is a table that compares

of the jitter measured to  $\frac{J_{1-n}}{J_{1-n_1}}$ the experimental ratio of  $\frac{\frac{S_{1-n}}{J_{1-n_1}}}{\frac{S_{1-n}}{S_{1-n_1}}}$  to the ideal value of 1.

$\frac{J_{1-14}}{J_{1-6}}$	$\frac{S_{1-14}}{S_{1-6}}$	$\frac{J_{1-14}}{J_{1-6}} / \frac{S_{1-14}}{S_{1-6}}$	
		Experimental	Ideal
2.80	2.60	1.08	1.00
2.50	2.60	. 96	1.00
2.61	2.60	1.00	1.00
2.70	2.60	1.04	1.00
2.75	2.60	1.05	1.00

Table 5-3 Comparison of jitter ratio-channel separation ratio ratio to ideal value as a measure of linearity The data shows a trend of linearity but is limited for further mathematical analysis.

#### CHAPTER 6

#### JITTER COMPENSATION

#### 6 1 Introduction

In Chapter 5 jitter was shown to be proportional to the channel spacing, all measurements referred to channel 1 the reference channel. In order to compensate for jitter a signal must be applied to the input of the servo instantaneously proportional but opposite in sign to the jitter component of phase shift present in an information channel. A separate channel of the tape recorder must be used which can be compared with the reference channel in order to get the required jitter compensation voltage. This separate channel is called the jitter compensating channel. Because of the linearity of jitter to channel spacing a simple resistive network suggests itself.

#### 6.2 Approach to Compensation

The input stage of the servo is a d.c. amplifier and requires a summing network to combine the d.c. voltage from the phase sensitive detector and the required d.c. jitter compensating voltage.

Fig. 6-1 illustrates a simple additive high gain feedback amplifier similar to that used in the first stage of the servo, except that no velocity compensation is shown.



Fig. 6-1 High-gain d.c. summing amplifier

If

$$A = \frac{e_0}{e_g} >> 1,$$

then

e<sub>g</sub> = 0

and

$$\mathbf{e}_{o} = - \begin{bmatrix} \mathbf{R}_{\underline{f}} & \mathbf{R}_{\underline{f}} \\ \mathbf{e}_{1} & \mathbf{R}_{1}^{T} + \mathbf{e}_{2} & \mathbf{R}_{2}^{T} \end{bmatrix}$$

where

A = voltage amplification e<sub>1</sub> = signal plus jitter voltage e<sub>2</sub> = negative jitter or compensating voltage e<sub>g</sub> = grid voltage e<sub>o</sub> = output voltage R<sub>1</sub>, R<sub>2</sub> = summing resistors R<sub>f</sub> = feedback resistor

Since  $e_2$  is instantaneously opposite in sign to  $e_1$ , and  $e_0$  is to be zero for complete cancellation, it is then apparent that by varying  $R_2$  complete compensation is theoretically possible.

#### 6.3 Method of Compensation

The Skipper test program requires the output of three shafts to be recorded and reproduced. A block diagram of the proposed system incorporating jitter compensation is given in Fig. 6-2. The only additional requirement for airborne recording is the use of an extra tape channel. For ground playback, the additional equipment required is a gain of twenty amplifier, a phase sensitive detector, and some resistors for the summing network.

As explained in Chapter 2 the phase sensitive detector has null output when the inputs are 90 degrees out of phase. Therefore





ANGULAR DISPLACEMENT OF SHAFT

SUMMING NETWORK.

SERVO MOTOR

ΣW

SUMMING NETWORK RESISTOR

TWO-PHASE RESOLVER

ຂຶ

PRECISION POWER SUPPLY 90° PHASE SHIFT NETWORK

PP PSN PSD RDA

æ

SERVO

s o

PHASE SENSITIVE DETECTOR RESOLVER DRIVE AMPLIFIER

AMPLIFIER (GAIN = 20)

۹ ۵

DIGITALIZER



Test Setup for Recording Compensated and Uncompensated Jitter Signals Fig. 6-3



Fig. 6-4 Jitter Compensation Summer

a signal which is 90 degrees out of phase with the reference signal is recorded on the jitter compensating channel during the test.

Jitter between the reference channel and jitter compensating channel will thus be impressed on the tape. On the ground the tape is played back and the jitter correction signal is amplified and fed to a phase sensitive detector where it is compared with the amplified reference signal. At this point the sign of the d. c. output signal must be changed so that the two are not added. Examination of Fig. 2-6 indicates that this can be accomplished by changing the sign of the phase angle in-voltage out sensitivity of the phase sensitive detector from negative to positive. This means interchanging stable and unstable nulls. Changing the sign can easily be done by switching the leads which are tied between the 10K, 1% resistors in the bridge circuit in Fig. C-2. The compensating signal is then fed to summing networks on the input of each servo.

At the time of this writing the servo had not been completed and verifying results could not be obtained with the servo. However, the Midwestern Recording Galvonometer affords an excellent means of checking the proposed scheme of compensation. Because the input impedance of the galvonometer is very low compared with that of the d. c. amplifier in the servo, all that need be done is to change the values of the resistances in the summing network. Fig. 6-3 illustrates the general setup for test of the jitter compensation scheme. Fig. 6-4 is a detailed schematic diagram of the jitter compensation summing network constructed for this test.

#### 6.4 Jitter Compensating Network (Fig. 6-4)

A simple test showed that the output impedance of the phase sensitive detector when operating within 80 to 100 degrees phase shift to be approximately 4000 ohms. This value must be added to each summing leg. The RC filter on each input is used to cut down the 2000 cycle a.c. ripple present at the phase sensitive

detector output. This filter has a cutoff frequency of approximately 160 cps. The galvonometer cutoff frequency is 120 cps. The values for the signal channel resistors are the same as used previously in recording jitter. Using the same value of filter resistor in the compensating input, the correct settings of the potentiometers were calculated by use of the general expression

$$R_{comp} = \frac{N}{n-1} R_{sig}$$

where

$$R_{comp} = r_{n-pot} + R_{psd} + R_{F}$$

and

$$R_{sig} = R_{psd} + R_F + R_l$$

N = number of spaces between reference and jitter compensating channels

R<sub>comp</sub> = total resistance in compensating leg

R<sub>sig</sub> = total resistance in information signal leg

Rn-pot = potentiometer setting for channel n

R<sub>psd</sub> = output resistance of the phase detector

R<sub>F</sub> = resistance of filter resistor (arbitrary)

Substituting and solving for R<sub>n-pot</sub>

$$R_{n-pot} = \frac{N}{n-1} (R_{psd} + R_F + R_1) - (R_{psd} + R_F)$$

It must be noted here that channel 1 was used as reference and channel 14 as jitter compensating channel. There are 13 spaces on the tape between the reference and the jitter compensating channels.



# Fig. 6-5 Actual Test Record of Recorded Uncompensated and Compensated Jitter

Any channel may be used for the reference and for jitter compensation. However it is desirable to straddle the information signal channels to avoid the possibility of making a summing leg too low in resistance thereby loading the phase sensitive detectors excessively. In the test setup N=13,  $R_F = R_1$  were picked as 10,000 ohms,  $R_{psd} = 4000$  ohms. With these values the last equation becomes

$$R_{n-pot} = \frac{13}{n-1} (24,000) - (14,000)$$

The calculated values of  $R_{n-pot}$  for n = 2, 3, ..., 13 are shown in the diagram of the summer, Fig. 6-4.

#### 6.5 Results and Conclusions

The jitter compensation summer as shown in Fig. 6-4 was used to make actual measurements of the effectiveness of the compensation. Samples of actual test recordings of compensated and uncompensated signals channels are shown in Fig. 6-5. It was necessary to adjust the potentiometers of the compensation summer from that value of resistance calculated to give best compensation. The method of adjusting the compensation for each channel was to place simutaneously an uncompensated signal on one channel of the recording oscillograph and a compensated signal on another oscillograph channel, and then by observation of the compensated signal, adjust the potentiometer for that channel until the least jitter was observed.

The results show that this method of compensation is most effective for the channels most remote from the reference channel 1. For channels 11, 12, and 13 the reduction of jitter in the compensated signal is in the order of 90%. The reduction of jitter on channel 7 is approximately 80%. The percentage reduction is considerably less for tape channels closer to the reference channel and this is due in part to the difficulty of determining the proper potentiometer setting in the presence of noise on the compensated signal. A refinement in the method of adjusting the

summer should lead to even higher percentage reduction of jitter then that obtained.

One limitation to the degree of compensation achieved is concerned with the bandwidth of the measuring system. The compensation measurements were made with a system that has a bandwidth of approximately 120 cps. This permits jitter components of frequency greater than 10 cps to enter the summer These components appear to be random in occurrence between tape channels and can not be compensated. These higher frequency jitter components that occur on one channel and not the others represent the lower limit of jitter reduction. They also obscure the compensation adjustments especially near the reference channel. Actual jitter compensation can best be effected while driving the servo. The servo would filter out components of jitter and noise above 10 cps, its undamped natural frequency.

The compensating circuit used a portion of the interchannel jitter between channels 1 and 14 depending on the distance that the channel to be compensated 15 from the reference channel.

Any random jitter that occurs on other tape channels but which do not occur on the jitter picked off between channels 1 to 14 cannot be compensated by this scheme.

A compensation method to compensate jitter transients peculiar to a particular channel will have to provide some means to separate the information signal from the jitter signal. Then the jitter signal can be fed back out of phase to eliminate the jitter entirely. This refinement in compensation will require more equipment than the linear scheme described.

Jitter compensation would be more effective in the channels on one record and reproduce head, either even or odd numbered channels, (see Fig. 2-13) since there would be no time displacement in recorded signals due to the separation of heads.

#### CHAPTER 7

#### FLUTTER AND WOW MEASUREMENTS

#### 7.1 Introduction

As defined in Chapter 3, flutter and wow represents any variation from uniform tape motion along the longitudinal axis of the tape. Wow is commonly referred to as low frequency flutter in the trade and in the literature. Hereafter flutter will be used to designate both flutter and wow.

Changes in frequency between the recorded and the reproduced data result from variations in tape speed. Flutter then must be kept to a minimum as it can appear as noise in the useful signal or as an error in many types of data signal. The high frequency flutter components appear to be of litte importance for normal audio application. On the other hand, if the recorder is used as a frequency modulation medium or is used for data recording, these flutter components become of considerable importance. Therefore the accuracy of recorded data depends on the consistency of tape motion during the recording and reproducing procedure. The tape transport mechanism as well as factors external to it may cause both real and apparent variations in tape speed. The tape transport mechanism must be designed for maximum precision in tape speed.

Real variation in tape speed can be caused by a change in power line frequency. The rotational speed of the capstan drive motor is directly proportional to the line frequency, and thus the tape speed is accuarate only when the drive motor is powered by an absolute frequency. Precision frequency power is necessary so that the drive motor speed will not fluctuate as the line

frequency varies.

Apparent variations in tape speed can be introduced by dimensional changes in the tape between the recording and reproduction process. Thus, if the tape is reproduced at a temperature different from that in the recording process, the longitudinal dimensions of the tape will change. Even with the most precise frequency sources, an apparent error in tape speed will be introduced. For example, the tape tension may not be precisely the same during the two processes, and the tape length will not be identical, introducing an apparent error in tape speed. This effect may be particularly pronounced when tapes are recorded on one machine and reproduced on another.

If the tape speed is increased for a given carrier frequency, peak-to-peak percent flutter (also called peak-to-peak frequency drift from the carrier frequency) is decreased. For a given tape speed, a higher carrier frequency results in increased peak-topeak percent flutter. Therefore in any system using tape recorders for storage, the effects of flutter may be reduced by using as low a carrier frequency and as high a tape speed that the system will permit.

#### 7.2 Instrumentation and Test Results

Fig. 7-1 is a block diagram of the units used to measure and calibrate flutter of the Ampex 800 series record and Ampex FR 100 series reproduction equipment. A recorded tape signal of 1000 cps and lv rms was played back at 15 ips and fed as an input to the flutter-wow meter.

The following principle was used in the measurement of flutter. If a signal of constant frequency is recorded on the tape, the instantaneous flutter will be proportional to the instantaneous change in the frequency of the playback signal. If the reproduced signal is applied to an F-M discriminator tuned to the carrier frequency, the output will be a d.c. voltage which is proportional






to the difference between the playback signal frequency and the carrier frequency and is positive or negative in sign depending whether signal frequency is greater or less than the carrier frequency.

Fig. 7-2 is a schematic of the flutter-wow circuit. A 1 KC bandpass constant K filter attenuates the tape noise and harmonic content of the recorded signal. A limiter prevents erroneous readings in the output due to variations in the input amplitude. These variations are due mainly to defects in the tape rather than the transport mechanism. The discriminator is in essence a dual tank in which each tank is tuned 100 cps off the center frequency of 1000 cps.

A smoothing filter was added to reduce the 2000 cps ripple frequency at the output of the flutter-wow meter. A d.c. amplifier and a coupling network consisting of a current-limiting and a damping resistor were necessary to adjust signal strength within the sensitivity of the Midwestern oscillograph recorder. Fig. 7-3 (a) shows the appearance of flutter curves obtained by the measurement scheme described above.

The flutter calibration setup is the same as the measurement one except that the source of signal is a pure sine wave generated from an audio oscillator and a frequency counter is added to monitor the oscillator's frequency. Deflections of the Midwestern recorder were recorded for each change of frequency. From this data system sensitivity was calculated,

System Sensitivity =  $\frac{1.2 \text{ inches record deflection}}{1 \text{ cps frequency change}}$ 

Fig. 7-3 (a) shows that the amplitude of peak-to-peak flutter is 1.3 cps and occurs mainly at a rate of .5 cps and 3 cps. 1.3 cps represents a variation of playback signal frequency between 998.7 and 1001.3 cps and also a 0.1 percent peak-to-peak flutter that is well within the 0.3 percent peak-to-peak flutter value listed in the specifications for this equipment. The very small value of flutter

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reflects the vast improvement upon the performance of tape transports that has been made recently. Flutter can be further compensated by adding a speedlock amplifier to the recording system and a speedlock servo loop ito the reproducing system.

This flutter study was made at a tape speed of 15 ips only. A more detailed treatment of flutter at various tape speeds and for various carrier frequencies is given in the thesis by R. H. Prager<sup>(9)</sup> and in January-March 1954, Quarterly Progress Report, M.I. T. Acoustics Laboratory.<sup>(11)</sup>

## 7.3 Speedlock Mechanism and Test Results

The speedlock mechanism provides a means for compensating the real and apparent variations of tape speed. Basically the system works as follows: $^{(5)}$ 

- A 60 cps precision frequency source in the model 800 drives a power amplifier which, in turn, powers the capstan drive motor.
- This same 60 cps, called the reference signal, amplitude modulates an 18.24 kc oscillator. This amplitude modulated signal is recorded on one track of the tape, called the control track.
- 3. During reproduction (see Fig. 7-4) the control track signal is fed to a demodulator that strips the carrier from the reproduced signal leaving the 60 cps signal and the flutter components.
- 4. This recovered 60 cps signal plus flutter components is fed to a phase comparator in which it is compared in phase with the output of the 60 cps precision frequency source in the FR 100.
- 5. Any difference in phase causes an error signal to be generated.



Scale sensitivity =  $\frac{1.2 \text{ cycles per second}}{\text{inch record deflection}}$ 

Fig. 7-3 Actual Test Record of Recorded Flutter with and without Speedlock Flutter Compensation





- 6. This error signal, proportional to the flutter components, is fed to an oscillator controller which controls the frequency of a R-C oscillator operating at a nominal frequency of 60 cps.
- 7. The output of the oscillator is amplified and fed to the capstan drive motor.

Hence if the tape has been stretched between recording and reproduction, the apparent decrease in tape speed produces a phase difference between the control track and reference signals. An error signal is generated which increases the frequency of the oscillator so that the tape speed is increased until equilibrium is reached.

The use of the speedlock mechanism reduced the observed change in frequency from 1.3 cps to 0.8 cps, a reduction of about 38%. Fig. 7-3 compares the observed flutter with and without the use of the speedlock mechanism.

7.4 Effect of Flutter on Jitter

From equation (5-4)  $J_{m-n} = K \frac{W S_{m-n} \theta}{V}$  it is seen that tape speed is inversely proportional to jitter. Since variation in tape speed is well within  $\frac{1}{2}$ . 2% of the tape speed, it should cause no significant change in the vaue of jitter. Jitter data with an without the use of the speed lock mechanism were compared and indicated no significant difference.

## 7.5 Effect of Flutter on System Performance

- The precision power supply is insensitive to flutter. (see Fig. 2-3).
- 2. The major flutter components are small so that the effect of different phase slopes with frequency of several channels in the vicinity of 1000 cps is likely to be small compared to 10 minutes of arc, the system specification.

3. Also the frequency components are slow enough so that flutter (of order 3 cps) and the peak signal change (of order 2 cps) are not likely to run into problems described in (2).

## CHAPTER 8

## CONCLUSIONS AND RECOMMENDATIONS

## 8.1 Experimental Data

Three major conclusions can be drawn from the experimental data on interchannel jitter.

- 1. Interchannel jitter in a multichannel magnetic tape recorder is the largest source of error to the accuracy of the shaft motion recording and reproducing system.
- 2. The value of jitter with respect to the tape channel distance from the reference channel was mathematically found to be approximately linear.
- This linear relationship provided a basis in which a simple method of jitter compensation could be instrumented.

#### 8.2 Jitter Compensation

Typical values of jitter ranged from 2 degrees phase difference between channels 1 and 2 to 20 degrees between channels 1 and 14. Peak-to-peak jitter errors of 8 degrees or more would have to be reduced by at least 98 percent to meet the system specification of 10 minutes of arc at the shaft.

A simple linear summing network reduced the peak-to-peak jitter of about 15 degrees between channels 1 and 12 about 90 percent. Jitter compensation on the order of 95 percent can be obtained using only the linear resistive network with additional filtering. More sophisticated methods of compensation will make possible even greater reductions in interchannel jitter.

#### 8.3 Jitter Reduction

1. One physical source of jitter excitation was found to be in the movement of the constant tension arm in the playback constant tension servo. Motion of this arm in correcting the tape tension excites the magnetic tape to skewing between the guide spindle clearances. This skewing motion of the tape as it passes the magnetic heads was found to be the principal source of jitter error. It is believed that refinements to the constant tension servo would eliminate that part of the jitter excited by motion of the constant tension arms.

2. Physical separation of record heads introduced an additional jitter error. This error is eliminated if the reference channel and data channels are located on the same head.

3. For instrumentation work it is recommended that test data recording and playback be recorded at the same temperature so that there is no resultant expansion or contraction of the magnetic tape which will lead to tape speed variation.

4. Data handling and data reduction systems based on phase information or on pulse time information may reduce the effects of jitter by resorting to lower carrier frequencies and higher tape speeds. Again the use of single stacked heads is recommended where time coincidence of data channels is important and where data channels may be spaced so as to avoid crosstalk.

#### 8.4 General

1. Flutter and wow measurements verified that variation in tape speed is very small and is of no consequence on the error of the system. Flutter and wow effects were so small that it was not possible to establish an empirical relationship between jitter and flutter.

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2. All jitter data was obtained at a tape speed of 15 ips. Since the jitter equation given in Chapter 5 indicates that higher tape speed will produce less jitter, a study of jitter at different tape speeds would serve to verify the relationship between jitter and tape speed.

3. Sufficient equipment was not available to make simultaneous recordings of the jitter between all 14 tape channels. It is recommended that future investigations should attempt to obtain a simultaneous record of jitter between all tape channels. This would permit the determination of the relationship of jitter with respect to channel spacing independent of time.

4. The effect of jitter was studied using phase modulated signals recorded through direct record (AM) amplifiers. Phase information recorded through (FM) amplifiers has the advantage of being much less susceptible to the effects of dropout. The effects of jitter on the FM recording process could be investigated by utilizing the same techniques of jitter measurement as described in Chapter 4.

5. The basic system appears to have a high inherent accuracy and to be applicable to a variety of test programs where little "on board" equipment is allowed and to the general problem of repeater servos for the control of several simultaneous operations on machine tools from programmed information.

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#### APPENDIX A

#### DERIVATION OF FORMULAS

# A.1 Determination of the Regression Line of Y on X by the Method of Least Squares

Given N observations of two variables  $(X_1 Y_2, X_2 Y_2, \ldots X_i Y_i \ldots X_N Y_N)$ , fit a straight line known as the regression line of Y on X through these observed points by the method of least squares, i.e., fit a straight line in the X, Y plane so chosen that the sum of the Y deviations from it is a minimum.



$$Y_{rgr} = a + b X_i$$
 (A-1)

where

 $Y_{rgr}$  = value of Y on the regression line

- $X_{i}$  = value of X on the regression line
- a = intercept of regression line on Y axis
- b = slope of regression line

$$\sum_{i=1}^{N} = \sum_{i=1}^{N}$$

$$S = \sum_{i=1}^{N} (Y_{i} - Y_{rgr})^{2} \text{ is to be a minimum} (A-2)$$

$$= \sum_{i=1}^{N} (Y_{i} - a - b X_{i})^{2}$$

We now choose a and b so as to minimize S.

$$\frac{\partial S}{\partial a} = 0$$
 ,  $\sum (Y_i - a - bX_i) = 0$  (A-3)

$$\frac{\partial S}{\partial b} = 0$$
 ,  $\sum (Y_i - a - b X_i) X_i = 0$  (A-4)

Expanding (A-3) and (A-4),

$$\sum Y_i - Na - b \sum X_i = 0$$
 (A-5)

$$\sum X_{i} Y_{i} - a \sum X_{i} - b \sum X_{i}^{2} = 0$$
 (A-6)

Solving (A-5) and (A-6) simultaneously,

$$a = \frac{\sum Y_i}{N} - b \frac{\sum X_i}{N} = \overline{Y} - b \overline{X}$$
(A-7)

$$b = \frac{\sum x_i Y_i}{\sum x_i^2 - \frac{\sum x_i \sum Y_i}{N}}$$
(A -8a)

$$b = \frac{\frac{1}{N} \sum X_i Y_i - \overline{X} \overline{Y}}{\frac{1}{N} \sum X_i^2 - \overline{X}^2}$$
(A-8b)

$$b = \frac{\sum (Y_i - \overline{Y}) (X_i - \overline{X})}{\sum (X_i - \overline{X})^2}$$
(A-8c)

$$\overline{Y} = \frac{\sum Y_i}{N} = \text{mean value of } Y$$
$$\overline{X} = \frac{\sum X_i}{N} = \text{mean value of } X$$

Regression line is now uniquely determined.

where

By substituting (A-7) in (A-1), another form of the regression line is

$$Y_{rgr} - \overline{Y} = b(X_i - \overline{X})$$
 (A-9)

Hence  $\overline{X}, \overline{Y}$  lie on the regression line.

#### A.2 Determination of Correlation Coefficient

With each linear regression there is associated a correlation coefficient which measures the degree of association between the two variables. With reference to the <u>regression</u> <u>line of Y on X</u>, the correlation coefficient is defined by the formula

$$r^{2} \equiv 1 - \frac{\sigma_{XY}^{2}}{\sigma_{Y}^{2}}$$
 (A-10)

where r = correlation coefficient

$$\sigma_{XY}^{2} \equiv \frac{1}{N} \sum (Y_{i} - Y_{rgr})^{2}$$
 (A-11)

$$\sigma_{\rm Y}^2 \equiv \frac{1}{N} \sum \left( {\rm Y}_{\rm i} - \overline{{\rm Y}} \right)^2 \qquad (A-12)$$

$$\sigma_{rgr}^{2} \equiv \frac{1}{N} \sum (Y_{rgr} - \overline{Y})^{2}$$
 (A-13)

$$\sigma_{\rm X}^{\ 2} \equiv \frac{1}{N} \sum_{i} (X_i - X)^2 \equiv \frac{1}{N} \sum_{i} X_i^2 - \overline{X}^2$$
 (A-14)

By using relationships (A-8c) thru (A-13), it can be shown that

$$\sigma_{XY}^2 = \sigma_Y^2 - \sigma_{rgr}^2 \qquad (A-15)$$

Substituting (A-15) in (A-10),

$$r = \frac{\sigma_{rgr}}{\sigma_{Y}}$$
(A-16)  
$$\frac{\sqrt{\sum (Y_{rgr} - Y)^{2}}}{\sqrt{\sum (Y_{i} - Y)^{2}}}$$

Substituting (A-9), (A-12), and (A-14) in (A-16),

$$r = b \frac{\sigma_X}{\sigma_Y}$$
 (A-17)

A more convenient form must be derived for computation. Substituting (A-14) in (A-8b),

$$b = \frac{\frac{1}{N} \sum X_i Y_i - X Y}{\sigma_X^2}$$
(A-18)

Then

$$r = \frac{\frac{1}{N} \sum x_i y_i - x y}{\sigma_X \sigma_Y}$$

$$\mathbf{r} = \frac{N \sum \mathbf{x}_{i} \mathbf{y}_{i} - \sum \mathbf{x}_{i} \sum \mathbf{y}_{i}}{\sqrt{N \sum \mathbf{x}_{i}^{2} - \left(\sum \mathbf{x}_{i}\right)^{2} \sqrt{N \sum \mathbf{y}_{i}^{2} - \left(\sum \mathbf{y}_{i}\right)^{2}}}$$

# APPENDIX B

Tables for computing statistical parameters of jitter by methods outlined in Chaper 5 and Appendix A.

#### TABLE B-1

#### Table for Computing Statistical Parameters of Jitter. Measurement Includes Dynamics of Jitter Due to Phase Network

x	х ү		x <sup>2</sup>	Y <sup>2</sup>	
2	2 .40		4	. 16	
2	. 40	. 80	4	. 16	
2	. 50	1.00	4	. 25	
2	. 50	1.00	4	. 25	
2	. 50	1.00	4	. 25	
4	. 78	3.12	16	. 61	
4	.83	3.32	16	. 69	
4	. 83	3.32	16	. 69	
4	. 83	3.32	16	. 69	
4	1.00	4.00	16	1.00	
5	.85	4.25	25	. 72	
5	. 92	4.60	25	. 85	
5	. 92	4.60	25	. 85	
5	. 92	4.60 25		.85	
6	1.18	7.08	36	1.39	
6	1.18	7.08	36	1.39	
6	1.30	7.80	36	1,69	
7	1.30	9,10	49	1, 69	
7	1.30	9.10	49	1.69	
7	1.30	9.10	49	1.69	
7	7 1.45		49	2.10	
7	7 1.45		49	2.10	
14 2.50		35.00	196	6.25	
14 2.50		35.00	196	6.25	
14	14 2.70		196	7.29	
14 2.70		37.80 196		7.29	
14	3.00	42.00	196	9.00	
14	3.10	43.40	196	9.61	
Σ 187	37.14	340.29	1729	67.45	

X = Channel No. referred to channel l

$$N = 28$$
  
b =  $\frac{\Sigma X Y - \frac{\Sigma X \Sigma Y}{N}}{\Sigma X^2 - (\Sigma X)^2} = .15$ 

N

$$a = \frac{\Sigma Y}{N} - b \frac{\Sigma X}{N} = .04$$
$$(SE)^{2} = \frac{1}{N-2} \left[ \Sigma Y^{2} - \frac{(\Sigma Y)^{2}}{N} \right] (1 - r^{2})$$
$$= .0181$$

$$Y_{rgr} = a + bX = .04 + .19X$$

$$r = \frac{N\Sigma X Y - \Sigma X \Sigma Y}{\sqrt{N\Sigma X^2 - (\Sigma X)^2} \sqrt{N\Sigma Y^2 - (\Sigma Y)^2}} = .987$$

#### TABLE B-2

#### Table for Computing Statistical Parameters of Jitter.

Measurement Excludes Dynamics of Jitter Due to Phase Network

х	Y	ХҮ	x <sup>2</sup>	ү <sup>2</sup>	x	Y	ХҮ	x <sup>2</sup>	Y <sup>2</sup>
2	. 30	. 60	4	. 09	7	. 70	4.90	49	. 49
2	. 35	. 70	4	. 12	7	. 80	5.60	49	. 64
2	. 35	. 70	4	. 12	7	. 80	5.60	49	. 64
2	. 40	. 80	4	. 16	7	. 85	5,95	49	. 72
2	. 40	.80	4	. 16	7	. 90	6.30	49	. 81
4	. 50	2.00	16	. 25	12	1, 50	18.00	144	2.25
4	. 50	2.00	16	. 25	12	1.50	18.00	144	2.25
4	. 50	2.00	16	. 25	12	1.60	19.20	144	2.56
4	. 55	2.20	16	. 30	12	1.60	19.20	144	2.56
4	. 60	2.40	16	.36	12	1,80	21.60	144	3.24
5	. 50	2.50	25	. 25	14	1.60	22.40	196	2.56
5	.55	2.75	25	. 30	14	1.70	23.80	196	2.89
5	. 55	2.75	25	. 30	14	1.70	23.80	196	2.89
5	. 60	3.00	25	. 36	14	1.80	25.20	196	3.24
5	.60	3.00	25	.36	14	1,80	25.20	196	3.24
6	. 60	3.60	36	.36	Σ 7, 12, 14 165	20.65	244.75	1945	30.98
6	. 60	3.60	36	. 36	Σ 2, 4, 5, 6 85	10.35	46.80	405	5.25
6	. 60	3.60	36	. 36	STotal 250	21.00	201 55	2250	36.22
6	. 65	3.90	36	. 42	210tal 250	31.00	201.00	2350	30.23
6'	. 65	3.90	36	. 42					
Σ 85	10.35	46.80	405	5.25					

$$X = \text{Channel No. referred to}$$

$$x = \text{Channel l}$$

$$Y = \text{Peak-to-peak deflection}$$
in inches
$$N = 35$$

$$x = \frac{\Sigma Y}{N} - b \frac{\Sigma X}{N} = -.01$$

$$Y_{rgr} = a + b X = -.01 + .12X$$

$$r = \frac{N \Sigma X Y - \Sigma X \Sigma Y}{\sqrt{N \Sigma X^2 - (\Sigma X)^2} \sqrt{N \Sigma Y^2 - (\Sigma Y)^2}} = .00532$$

# APPENDIX C

A compilation of the circuit diagrams of the major components of the shaft measuring system.



Fig. C-1 Precision Power Supply











Fig. C-4 Dual-Channel Resolver Drive Amplifier



Fig. C-5 Phase Shift Amplifier

ALL CAPACITORS IN  $\mu f$ 

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#### APPENDIX D

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# Thesis 100411 G87 Gussow Evaluation of jitter in multichannel tape recordters used in shaft motion pe reproducers. JA 17 58 23 JUL 93 BINDERY 13044 Thesis G87 Gussow Evaluation of jitter in multichannel tape recorders used in shaft motion reproducers.

