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THE EFFECT OF BOBWEIGHT AND DOWNSPRING ON THE LONGITUDINAL DYNAMIC STABILITY OF AN AIRPLANE

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THE EFFECT OF BOBWEIGHT AND DOWNSPRING

ON THE LONGITUDINAL DYNAMIC STABILITY

OF AN AIRPLANE

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Summary

A theoretical and experimental investigation of a modified Navion type airplane was undertaken to determine the effect on the phugoid mode of obtaining artificial static stick-free stability with bobweights and/or downsprings while the center of gracity was well aft of the basic airplane's neutral points.

The theoretical study was very rewarding and indicated that even though adequate static stability could be introduced with gadgetry, if the effect of the gadgetry were to move the static stick-free neutral point aft of the meneuvering neutral point, the phugoid mode will become rapidly divergent although remaining oscillatory. This change in the relative positions of the neutral points can be accomplished only with downspring.

The experimental flight research program confirmed the qualitative results of the theoretical study and even agreed surprisingly well with the quantitative results.

An interesting sidelight of the experimental program concerned stick-fixed stability. It was concluded, after many hours in the air with the center of gravity aft of the stick-fixed neutral point, that the pilot is insensitive to negative static stick-fixed, or elevator position, stability. It appears that if the pilot has satisfactory force stability, satisfactory dynamic stability, and adequate elevator for the flight requirements, the relative positions of the stick-fixed neutral point and the center of gravity is unimportant.

There appears to be only two minor qualifications to the acceptability of static and dynamic stability when artificially acquired with

gadgetry. The gadgetry may introduce such static moments into the system that it is unsatisfactory from ground-handling considerations, or it may increase the system inertia so that the pilot will find it unsatisfactory because of high forces necessary to overcome the inertia even in level flight.

THE EFFECT OF BOBWEIGHT AND DOWNSPRING OF THE
LONGITUDINAL DYNAMIC STABILITY OF AN AIRPLANE

INTRODUCTION

Since the beginning of World War II, the airplane designer has
had to produce airplanes of greater and greater capabilities, load
carrying capacities and versatility, so that many design limitations
such as size, cost, complexity, runway lengths, etc., have forced on him
the necessity of making many more major compromises in his design. Re-
gardless of compromises and means necessary to achieve an end, however,
there have always been certain minimum requirements for airplane stability
and control which had to be satisfied. These requirements have been
established by the various customers of the aircraft industry, with the
assistance of the National Advisory Committee for Aeronautics, and estab-
lish a criteria for all major measures of handling qualities, with one
notable exception. In making his compromises and in using his imaginative
powers in turning out a final design, the airplane manufacturer has never
had to trouble himself with one mode of the airplane's longitudinal dynamic
stability, the long period oscillatory phugoid mode. This mode of oscil-
lation in past airplanes has always been of such long period that it was
relatively unimportant whether it was damped or undamped, as long as any
possible divergence was not too rapid. Consequently scant attention has
been paid to this mode of dynamic response, and comparatively little work
has been done in studying the phugoid mode.

In recent years, the necessary compromises in design have resulted
in many airplanes being built with their center of gravity too far aft,

so that static and maneuvering longitudinal stability of the finished airplane has been unsatisfactory. Designers have met this problem with the introduction of the relatively familiar downspring and/or bobweight into the longitudinal control system. These devices are satisfactory in improving static stability but their effect on dynamic stability was only lightly considered, since the short period mode is generally very heavily damped, and the phugoid mode historically was of such little consequence.

Within the military services and some educational institutions, however, there has always been a group of aerodynamicists who have felt that the phugoid mode was not getting deserved attention whenever control systems were modified with mechanical devices producing artificial static stability. Their feelings on the subject have received support from airplane pilots, particularly those flying all-weather, who greatly desire that the transient response of an airplane to any disturbance such as a gust be stable, and any resulting oscillations be either damped out in a few cycles or of such long period that they are barely noticeable. As a result, the U. S. Air Force awarded Princeton University a contract for applied research to study the phugoid mode as it is affected by gadgetry in the longitudinal control system.

The author of this thesis asked to be allowed to participate in this research and was greatly flattered when he was allowed to take a considerable part in the program. This thesis is a presentation of the study, flight research, and results of that part of the program dealing with bobweights and downsprings. Although it is complete as to this phase of the program, this thesis is not to be construed in any way as a report

on the results of the Air Force program which is the peculiar responsibility of Princeton University.

The thesis will cover the subject matter in the following general fashion. First will be presented a brief discussion on the effects of bobweights and downsprings on static stability in order that the reader may more easily understand their effect on dynamic stability. This will be followed by a brief report on the analytical study of these effects and a presentation of the results. The flight research program will then be introduced. In order that the very interesting conclusions in regard to the effect of gadgetry on the phugoid mode be not obscured by the many relatively trivial but time-consuming ancillary problems, those matters of merely incidental interest will receive only brief mention.

THE EFFECT OF DOWNSPRINGS AND BOBWEIGHTS ON STATIC STABILITY

Both the downspring and the bobweight affect stick-free, or control force, stability by introducing a moment into the longitudinal control system which changes the stick-free floating angle of the elevator in the downward direction. In the case of the downspring this moment is independent of normal accelerations, i.e., is not affected by maneuvering. Increasing normal acceleration with a bobweight installed, however, increases the inertia force of the bobweight so that the hinge moment produced by the bobweight is directly proportional to normal acceleration.

The floating angle of the elevator resulting from aerodynamic and mechanical effects may be expressed in coefficient form as follows:

$$\delta_{e_{float}} = -\frac{C_{h_{\alpha}}}{C_{h_{\delta}}} \alpha_s - \frac{HM_g}{q S_e c_e} \frac{1}{C_{h_{\delta}}}$$

where $\alpha_s = \alpha_w - i_w + i_s - \epsilon$

Terms are defined in the Appendix, page 24. The contribution of the free elevator to the airplane pitching moment equation is simply:

$$\Delta C_{m_{free}} = C_{m_{\delta}} \delta_{e_{float}} = -C_{m_{\delta}} \frac{C_{h_{\alpha}}}{C_{h_{\delta}}} \alpha_s - \frac{C_{m_{\delta}}}{C_{h_{\delta}}} \left(\frac{HM_g}{q S_e c_e} \right)$$

Neutral points occur wherever the change in pitching moment with change in lift coefficient is zero. These points are found by taking the derivative of the pitching moment equation with respect to C_L. If the static stick-free neutral point is desired this derivative must be taken holding $n = 1$, or the product $C_L V^2$ equal a constant. If the maneuvering neutral point is desired, velocity must be held constant and n may vary in direct proportion to angle of attack or lift coefficient.

The shift in neutral point due to the free elevator is obtained by taking the derivative of its contribution to the moment equation. Holding $C_L V^2$ equal to a constant, then $\frac{L}{q} = \frac{c_e}{W/S}$, and the derivative is:

$$\frac{d(\Delta C_{m_{free}})}{dC_L} = -\frac{C_{m_{\delta}}}{C_{L_{\alpha_w}}} \frac{C_{h_{\alpha}}}{C_{h_{\delta}}} \left(1 - \frac{d\epsilon}{d\alpha} \right) - \frac{HM_g}{S_e c_e} \frac{1}{W/S} \frac{C_{m_{\delta}}}{C_{h_{\delta}}}$$

This shows that a gadget producing a hinge moment coefficient equivalent to $\frac{HM_g}{S_e c_e q}$ increases the static stick-free stability margin by a factor

$$\Delta (N_o' - x_{cg}) = \frac{HM_g}{S_e c_e} \frac{1}{W/S} \frac{C_{m_{\delta}}}{C_{h_{\delta}}}$$

If the derivative is taken holding V a constant which is the con-
dition obtaining in maneuvering flight, then $\frac{1}{q}$ is not equal to $\frac{C_L}{W/S}$,
but also is a constant, and the entire second term vanishes. This indicates
that a mechanical hinge moment which is not a function of normal accelera-
tion such as that introduced by a downspring, has no effect on the stick
free maneuver margin.

However, if the mechanical hinge moment is a direct function of
normal acceleration, such as that introduced by a bobweight, then the
addition to the moment equation produced by freeing the elevator is:

$$\Delta C_{m_{free}} = -C_{m_\delta} \frac{C_{h\alpha}}{C_{h\delta}} \alpha_s - n \left(\frac{C_{m\delta}}{C_{h\delta}}\right) \frac{HM_g}{q \, S_e \, c_e}$$

Where n = 1, as in rectilinear flight where $C_L V^2$ equals a constant,
it is plain that the bobweight has the same effect as the downspring in
increasing the static stability margin, since the equations are identical.
However, where V is held constant, then n is not equal to 1, but equals
$\frac{q \, C_L}{W/S}$, so that the resulting contribution to the moment equation, velocity
held constant, is:

$$\Delta C_{m_{free}} = -C_{m_\delta} \frac{C_{h\alpha}}{C_{h\delta}} \alpha_s - \frac{q \, C_L}{W/S} \left(\frac{C_{m\delta}}{C_{h\delta}}\right) \frac{HM}{S_e c_e \, q}$$

Taking the derivative with respect to C_L to find the effect of the
mechanical hinge moment in changing the maneuvering neutral point:

$$\Delta \left(\frac{dC_m}{dC_L}\right)_{free} = -\frac{C_{m\delta}}{C_{L_\alpha}} \frac{C_{h\alpha}}{C_{h\delta}} \left(1 - \frac{d\epsilon}{d\alpha}\right) - \frac{HM_g}{S_e c_e} \frac{1}{W/S} \frac{C_{m\delta}}{C_{h\delta}}$$

From this equation is seen the fact that the bobweight shifts the maneuvering neutral point aft by a factor

$$\Delta N_m' = \frac{H M_g}{S_e C_e} \frac{1}{W/S} \frac{C_{m\delta}}{C_{n\delta}}$$

which is exactly equivalent to the shift in static neutral point produced by the bobweight.

In summary it has been shown that the downspring changes the static neutral point but has no effect on the maneuvering neutral point, whereas the bobweight affects each by an equivalent amount. To express this con-conclusion in the form in which it will be referred to in the remainder of this thesis, the downspring increases the static margin ($N_o' - x_{cq}$) but not the maneuver margin ($N_m' - x_{cq}$) whereas the bobweight increases both an equivalent amount. The downspring reduces the margin ($N_m - N_o'$) whereas the bobweight keeps this margin, the difference between the two neutral points, a constant.

It is essential that this distinction be understood and accepted in order that the remainder of this thesis be fully appreciated.

ANALYTICAL STUDY OF THE EFFECT OF DOWNSPRINGS AND BOBWEIGHTS ON DYNAMIC STABILITY

The generally accepted equations of motion of an airplane in the longitudinal plane and with elevator free to rotate are, in operator form, as follows:

$$\text{DRAG EQ:} \quad (C_D + d)u + \frac{1}{2}(C_{D_\alpha} - C_L)x + \frac{C_L}{2}\theta = 0$$

$$LIFT\ EQ: \quad C_L u + \left(\frac{C_{L\alpha}}{2} + d\right)\alpha - d\theta = 0$$

$$MOMENT\ EQ: \quad C_{m_u}u + \left(C_{m_\alpha} + C_{m_{d\alpha}}\right)\alpha + \left(C_{m_{d\theta}}d - hd^2\right)\theta + \left(C_{m_\delta} + C_{m_{d\delta}}d\right)\delta_e = 0$$

$$HINGE\ MO.\ EQ: \quad 2C_{h_o}u + \left(C_{h_\alpha} - h_1 d\right)\alpha + \left(C_{h_{d\theta}}d - h_2 d^2 + h_1 d - \ell_1 d^2\right)\theta$$

$$+ \left(C_{h_\delta} + C_{h_{d\delta}} - h_2 d^2\right)\delta_e = 0$$

where all variables are incremental values, $u = \frac{\Delta V}{V}$, and the time
parameter is $\frac{t}{\tau} = \frac{t}{m/\rho S V}$

C_{h_o} is the initial aerodynamic hinge moment from any cause, includ-
ing the aerodynamic hinge moment necessary to balance the downspring and/or
bobweight.

h_1 is the term taking into account the mass unbalance of the
elevator, including the effect of a bobweight in the system.

ℓ_1 is the term accounting for the effect of pitching accelerations
on the mass unbalance of the elevator.

h is the inertia term for the airplane.

h_2 is the term accounting for the elevator's moment of inertia
about its own axis.

All coefficients which are stability derivatives are expressed in
a short-hand notation so that, eg., C_{h_δ} is equivalent to $\frac{\partial C_h}{\partial \delta}$

All terms are defined in detail in the appendix but for the purposes
of this discussion it is only necessary to know the origin of the terms.

Before proceeding with the solution of these equations, it was assumed that $h_1 = C_{n_{\delta\delta}} = 0$, since in most airplanes the elevator mode of oscillation is of such short period and so heavily damped that an assumption that the elevator instantaneously assumes its trim position does not affect other airplane motions appreciably.

Since it is not the purpose of this thesis to demonstrate how such differential equations are solved in order to determine the transient motions following a disturbance, only a brief description of the process followed by the solution for damping and period of the transient oscillations resulting from a disturbance will be presented. For details of this process the reader is referred to Ref. 1.

The solution of the four equations for the transient motion is assumed to be of the form $y = y_1 e^{\lambda \frac{t}{\tau}}$, $\alpha = \alpha_1 e^{\lambda \frac{t}{\tau}}$, etc. These assumed solutions are substituted into the four differential equations and the result is four homogeneous algebraic equations. Since they are consistent, the determinant of the coefficients of the variables must equal zero. This determinant is expanded and can be presented as a quartic in . known as the characteristic equation, of the form:

$$\lambda^4 + A\lambda^3 + B\lambda^2 + C\lambda + D = 0$$

The roots of this quartic determine the character of the motion of the airplane. If any roots are real numbers, the motion is aperiodic, convergent if negative, divergent if positive. If there is a complex pair of roots, there is oscillatory motion, damped if the real part is negative, undamped if the real part is positive.

After the determinant was expanded, the coefficients of the characteristic equation were found, each of which consists of many of the coefficients of the equations of motion grouped in an algebraic relationship.

These coefficients were simplified by making appropriate substitutions, which will be defined later, and the result was as follows:

$$A = \frac{C_{L_0}}{2} - \frac{1}{h'}\left(C_{m_{d\alpha}} + C_{m_{d\theta}}\right)'$$

$$B = \frac{C_L}{2}\left(C_L - C_D\right) + \frac{C_D\,C_{L_\alpha}}{2} + \frac{C_{L_\alpha}}{h'}\left(N_m' - x_{cg}\right)$$

$$C = \frac{1}{h'}\left[\left(C_D\,C_{L_\alpha} - C_L\,C_{D_\alpha}\right)\left(N_m' - x_{cg}\right) + C_L\,C_{D_\alpha}\left(N_0' - x_{cg}\right) - \frac{C_L^2}{2}\left(C_{m_{d\alpha}} + C_{m_{d\theta}}\right)'\right]$$

$$D = \frac{1}{h'}\,\frac{C_L^2\,C_{L_\alpha}}{2}\left(N_0' - x_{cg}\right)$$

where h' is the effective airplane inertia, defined as follows:

$$h' \equiv h - \frac{C_{m\delta}}{C_{n\delta}}\,\ell_t$$

$\left(C_{m_{d\alpha}} + C_{m_{d\theta}}\right)'$ is the effective aerodynamic damping, defined as follows:

$$\left(C_{m_\alpha} + C_{m_{d\theta}}\right)' \equiv \left(C_{m_{d\alpha}} + C_{m_{d\theta}}\right)\left(1 - \tau_e\,\frac{C_{n_{\tau e}}}{C_{n\delta}}\right)$$

$-C_{L_\alpha}(N_m' - x_{cg})$ is the stick-free maneuvering stability and is
defined as follows:

$$-C_{L_\alpha}(N_m' - x_{cg}) \equiv C_{m_\alpha} + \frac{C_{L_\alpha} C_{m_{b_\theta}}}{2} b_\theta - \frac{C_{m_\delta}}{C_{n_\delta}}\left[C_{h_\alpha} + \frac{C_{L_\alpha}}{2}(h_1 + C_{h_{\dot{\delta_\theta}}})\right]$$

$-C_{L_\alpha}(N_0' - x_{cg})$ is the stick-free static stability, and is defined
as follows:

$$-C_{L_\alpha}(N_0' - x_{cg}) \equiv C_{m_\alpha} - \frac{C_{L_\alpha}}{2C_\circ} C_{m_u} - \frac{C_{m_\delta}}{C_{n_\delta}}\left(C_{h_\alpha} - \frac{C_{L_\alpha}}{C_{n_\delta}} C_{h_0}\right)$$

$N_m' - x_{cg}$ and $N_0' - x_{cg}$ are the maneuvering and static stick-free
stability margins, positive if stable. For a detailed discussion of these
equivalences, see Ref. 1.

The coefficients of the characteristic equation were arranged in
this fashion to facilitate study of the stick-free static and maneuvering
margins on the characteristic transient motions of the airplane. With the
equation as set up, values for $N_0' - x_{cg}$ and $N_m' - x_{cg}$ were varied and solu-
tions for the equation were made for each variation. The values of the
other components of the equations, including the stability derivatives,
were computed from theory and wind tunnel tests, using the Navion as the
subject airplane. Approximately eighty variations were solved by M. J. H.
Goldberg. so that the root loci for the equation as $N_0' - x_{cg}$ and $N_m' - x_{cg}$
were varied could be quantitatively defined. The real roots, and the real
part of the complex roots were converted to the inverse of the time to damp
to 1/2 amplitude $\left(\frac{1}{T_{1/2}}\right)$ and the imaginary parts of the complex roots were

converted to period (P) by the following relationships.

If a complex root is as follows:

$$\lambda = m \pm in$$

$$\text{Then } \frac{1}{T_{1_{2}}} = \frac{1}{\frac{\log_e 2}{m}} \tau = \frac{1}{seconds} \quad \text{and} \quad P = \frac{2\pi}{n} \tau \text{ seconds}$$

The root loci of the characteristic equation are plotted in Fig. 1 of the Appendix for various values of $N_m' - N_o'$. The real roots and the real part of the complex root are plotted on the upper family of curves, with the inverse of time to damp to half amplitude as the argument. Negative values of the inverse time parameter indicate negative damping, and are equivalent to time to double amplitude. Dotted lines indicate oscillatory motion and solid lines indicate a pure, or real root, divergence. The imaginary part of the complex roots are plotted in the lower family of curves with period as the argument.

Discussion of Results of Theoretical Study

The most prominent fact revealed by the curves of Fig. 1 is that the short period mode, the upper family of dotted lines, is independent of the maneuver margin and of the margin $N_m' - N_o'$. In any event, as far as the Navion is concerned the short period mode, even where it is aperiodic, is so heavily damped that it is of no particular interest.

Almost equally prominent is the fact that the phugoid mode represented

by the lower family of dotted lines is very greatly affected by the margin $N_m' - N_o'$. For some values of this parameter it is apparent that the phugoid mode is very divergent, going to double amplitude in a very few seconds.

Of small interest is the fact that for certain negative values of $N_m' - x_{cg}$ two real roots combine to become complex and produce an oscillatory motion, since at the same time there is a real root so rapidly divergent that any such oscillatory motion is completely obscured.

Accepting the fact that the short period mode is of scent interest while the phugoid mode is of great interest, a cross plot of this family of curves was made indicating the root loci of the phugoid roots only. This cross plot is presented on Fig. 2 and more clearly illustrates the effect of stability margins on the phugoid. This plot was made with $N_x - x_{cg}$ and $N_m' - N_o'$ as the arguments. Varying downspring moves the roots along the horizontal lines of constant $N_m' - x_{cg}$ while varying bobweight moves the roots along the vertical lines of constant $N_m' - N_o'$. Of course, lines of constant $N_o' - x_{cg}$ are diagonal, as indicated.

It is readily apparent that there are definite stability boundaries setting off distinct regions. In one region motion is aperiodic and divergent. In another the phugoid motion is oscillatory but damped, while in the third the motion is oscillatory but undamped. Lines of constant time (in seconds) to double or half amplitude, depending on whether the motion is undamped or damped, are included in the upper plot.

The bottom half of the cross plot include lines of constant period. It is seen that within the regime presented the period of the phugoid varies

from 12 to 50 seconds. The rectangle superscribed on the plot indicates the region in which flight tests were made.

In order to point out a region of particular interest, and to illustrate the use of the plot of phugoid characteristics, consider a maneuver margin, $N_m' - x_{cg}$ of $+.02c$. By adding downspring to the system, we move horizontally, with $N_m' - x_{cg}$ remaining unchanged. Moving from right to left, it is seen that for a margin $N_m' - N_o'$ equal to $+.04c$ the motion is aperiodic and undamped. Increasing downspring moves the airplane into the region of damped oscillatory motion, which shows an improvement. Increasing downspring further moves the airplane into undamped oscillatory motion with a fairly short period which very rapidly becomes so divergent that amplitude is doubled in less than 20 seconds. Notice that this occurs even though both the static and maneuver margins are stable.

The effect of changing bobweight is to move the airplane along the vertical lines of constant $N_m' - N_o'$. As long as both static and maneuver margins are positive, it is apparent that there is no serious change in the oscillatory mode due to varying bobweight.

These curves are deserving of more extensive discussion, but they are also susceptible to easy analysis. It is very apparent from this analytical study that an indiscriminate use of downspring can have a serious adverse effect on the phugoid mode whereas the effect of bobweight is not so pronounced. A flight research program was undertaken to see whether the airplane appreciated this distinction. The results of this program will be presented forthwith.

THE FLIGHT RESEARCH PROGRAM

Description of the Test Airplane

The flight research was conducted on a Navion airplane which was modified as follows. The area of the horizontal tail surfaces was reduced _18_ percent by reducing the span 36 inches. The chord of the elevator trim tabs was increased one inch, or _20_ percent, by adding a flat plate which was then bent upwards 20 degrees to provide a fixed trimming moment in addition to the adjustable trim.

The elevator control system was modified as indicated in the sketch in Fig. _5_ of the Appendix. This system permitted unrestricted adjustment of bobweights and downsprings while airborne. Throughout the range of stick positions, the moment produced by the device was essentially constant.

For the static stability tests, elevator position and stick force was measured with autosyn and strain gages so that deflections of .1 degree and forces of 1 pound could be measured. An accelerometer constructed with spring and mass and enclosed in a freely suspended glass tube, accurate to .01 g, was used in the maneuvering tests.

For the dynamic stability tests, where only the phugoid response was required, a photo panel was used which contained, among other things, an airspeed indicator with the pickup from a boom on the starboard wing tip.

Fixed ballast of 80 pounds was anchored in the tail and movable ballast of 150 pounds was carried within the cabin so that the center of gravity could be varied from .32c to .40c while airborne. This required operating the airplane 8% over design maximum gross weight.

Static Stability Phase

For the sake of standardization, bobweights and downsprings were varied in increments of 8 pounds equivalent stick force per increment of either mechanical device. Innumerable static and maneuvering tests were performed to determine the neutral points with various combinations of downspring and bobweight. Enough data was taken to support a comprehensive report on static stability. Due to extreme elevator deflections, tab deflections, and center of gravity positions, various nonlinearities were encountered. However, since the purpose of these tests, and the need for them, was merely to locate neutral points, any discussion of the static data would be superfluous to the topic of this thesis. Suffice it to say that standard pilot technique was employed, the neutral points were determined carefully and with reasonable accuracy, downspring was found to have no effect on maneuvering stability whatever, and that static stability is in fact as indicated by the summary in Fig. 4 of the Appendix.

Dynamic Stability Phase, General

The dynamic test program was designed to determine the phugoid responses at a constant static stability margin as bobweights and downsprings were varied. In order that as many combinations of downsprings and bobweights as possible could be used without introducing too much static stability, it was necessary to test the airplane at center of gravity positions well aft of the neutral point.

No phugoid responses were taken with the center of gravity aft of .363c, due to a critical shortage of down elevator deflection. Full down elevator

was required at 90 mph with the c.g. at .39c. It was not felt that the
added information to be obtained at center of gravity positions aft of .363c
justified the very real possibility of losing the airplane due to inability
to recover from the nose-up swing of an oscillatory or divergent response.
As the flights progressed, it soon became apparent that oscillations obtained
at 36% chord could be as extreme as the Navion could safely withstand.

A trim speed of 110 mph was used in taking all of the phugoid
responses. Very great attention was devoted to getting trim as closely as
possible. Although control friction was reduced to 1 pound, trimming to
exactly 110 mph when the force gradients were very low was difficult but the
results show that any discrepancy was minor.

Responses were recorded by carefully trimming the airplane at 110
mph, then applying the necessary force to cause absolutely rectilinear flight
5 mph either above or below trim speed, and then releasing the stick. The
resulting motion was determined by a plot of airspeed versus time taken from
the photo panel.

Dynamic Stability Phase, Presentation of Results

Upwards of 80 different responses were recorded during this phase
of the flight research program. Most of them are included in this report.
It was necessary to discard some which indicated a gust input during the
transient response, or unsatisfactory initial conditions. For the sake of
emphasizing certain important conclusions, and pictorializing the literal
statements, various groups of these responses are presented separately for
the following purposes.

In order to illustrate the fact that phugoid response can be unsat-
isfactory even though static stick-free stability is positive, and to show
that the response at a given center of gravity with the same static margin
worsens rapidly as downspring is employed in lieu of bobweight in order to
obtain the static margin, responses are plotted in Fig. 5 , page 30 of the
Appendix all of which were taken at a static margin $N_o' - x_{cg} = .064_c$
and center of gravity at $.342_c$. The tendency of the phugoid to become
undamped and diverge rapidly with increase in the proportion of downspring
in the system is quite obvious.

In order to illustrate the fact that if the static margin be kept
constant and the margin $N_m' - N_o'$ be kept constant the phugoid response
will remain unchanged even though the center of gravity is varied, responses
meeting these conditions are presented in Fig. 6 , page 31 of the Appendix.
These conditions can be met only by correcting center of gravity shift with
bobweight.

In order to illustrate the fact that there is a great change in the
phugoid response as the center of gravity is varied from .32c to .36c with-
out correcting with bobweights or other devices, responses are presented in
Fig. 7 , page 32 of the Appendix and require no further comment.

The general mass of the responses, including those already presented,
is presented on page 35 et subs. On the page preceding is tabulated the
margins and c.g. positions obtaining for each response. All responses are
grouped in major subdivisions of approximately equal static margin,
with the margin $N_m' - N_o'$ in descending order within the subdivisions.
An interesting fact, consistent with simple dynamics, is illustrated in this

type of presentation. As long as the static margin, which is analogous
to the spring in a spring-mass-damper system, is positive, motions are
oscillatory. As long as $N_m' - N_o'$, which is analogous to the damping in
the simple dynamic system, is positive, the oscillatory motion is damped,
or very nearly so. If $N_m' - N_o'$ becomes negative, indicating negative
damping, the oscillatory motion always increases in amplitude. Recall that
the difference between N_m' and N_o in a simple aerodynamic airframe is
due to aerodynamic damping. Therefore it is safe to conclude that even
though the static stability is positive, if the static force neutral point
is aft of the maneuvering neutral point, the phugoid will be undamped.

These points just discussed and illustrated definitely confirm at
least the trend disclosed by the theoretical study. In particular, the
flight research confirms the analytical conclusion that the downspring will
have a serious adverse effect on the phugoid mode whenever it moves the
static stick-free neutral point. A quantitative comparison of the experi-
mental and theoretical results will be made later in this paper.

Pilot Observations

In the long period or phugoid oscillations there was a very large
time lag in airplane response between attitude and airspeed, with attitude
leading airspeed by a rather extreme amount in some cases. For example it
often occurred that the airplane was pitching quite rapidly, on the order
of 12 degrees per second, and had obtained an extremely high nose attitude
while the airspeed, although falling rapidly, was of the order of 140 mph.
Naturally a recovery was necessary, and this often had to be accomplished

before the airspeed had fallen considerably from its maximum value. Consequently the velocity traces are not truly indicative of the severity of the oscillations. Another example of an extreme condition which occurred several times during the tests, and indicative of the lag between attitude and velocity, occurred at the high speed peaks of the oscillations. The airplane would be pitching downward at a great rate, at least 12 degrees per second since it would go from a conservatively estimated nose-up attitude of 30 degrees to a nose down attitude of 50 degrees in 7 seconds. It would reach its maximum nose-down attitude and rapidly reverse its direction of pitch while the airspeed was still building up. Consequently before the airspeed ever reached its maximum value, the accelerations at the bottom of the dive would build up so rapidly that a recovery was necessary before the airspeed ever reversed to show another peak in the oscillation on the airspeed plot of the maneuver. For these reasons the airspeed traces of the oscillations in many cases were cut short nearly a half cycle earlier than a trace of airplane attitude versus time would show a cut-off. In other words, were the phugoid response to be recorded by an attitude versus time plot, these very divergent curves would show an extra half cycle.

COMPARISON OF RESULTS FROM FLIGHT TESTS AND THEORETICAL INVESTIGATIONS

Since the Navion was used in both the theoretical and experimental phases of the investigation, there appears to be a good opportunity to compare results from each type of investigation and possibly to confirm quantitatively the theoretical results. It would seem rash, at first blush, to hope for

any close quantitative comparison, since the theoretical study required the use of engineering estimates of stability derivatives in many cases, and the solution of the equations of motion required certain approximations in order to linearize them. In particular, the theoretical study was necessarily limited by linear approximations to small perturbations, whereas the actual motions, while very perturbing, were not at all small. In detail power effects undoubtedly varied considerably during the oscillations since the Navion does not have a constant speed propeller, but the theoretical study could not take this very nonlinear variation into account.

However, since a quantitative comparison is inevitable, the author presents on page 45 of the Appendix the plot of phugoid characteristics with constant damping lines, and with various flight test results susceptible to reasonably accurate measurement plotted where they fall. Suitable captions identifying the points are presented beneath the curves. On page 46, the plot of phugoid characteristics with constant period lines is presented with a similar treatment of actual flight test results. All things being considered, the comparisons are very good and far better than expected.

PILOT'S QUALITATIVE OPINION ON HANDLING QUALITIES AS INFLUENCED BY DOWNSPRINGS AND BOBWEIGHTS

This topic normally would be of extreme interest to an aircraft designer with the pilot's interest at heart, if there are any, but unfortunately all of these opinions are not strictly applicable to the general airplane as the quantitative trends disclosed in this investigation appear to be.

Accept the fact that a bobweight will provide stable force gradients in an unstable airplane. Furthermore, this study showed the bobweight to be most ineffectual in causing an unsatisfactory phugoid. In spite of these obvious advantages, the pilot may well object to having very much bobweight in the elevator control system. The elevator generally has a fairly low moment of inertia. When masses of lead on a moment arm are introduced into the system, the elevator moment of inertia may increase considerably. In the case of the Navion, the elevator moment of inertia increased 32 times while moving the neutral points 5.1 percent chord with bobweight. This tremendous increase was due to the very small moment of inertia of this small airplane, and a larger airplane would not show such an increase percentage-wise while getting the same effect on stability. Yet it is true that a large increase in inertia in the system will be very objectionable to the pilot, both on the ground and in rough air, and it may well be that the amount of bobweight which may be introduced into the system will be limited by adverse pilot opinions.

Although a downspring will have a bad effect on the phugoid, as far as moving the elevator and controlling the airplane is concerned, it has no discernible effect. Except for improving the static force gradient, the pilot would not even know a downspring was in the system, except while taxiing. However, when the airplane's phugoid mode, although oscillatory, is rapidly divergent, it is extremely difficult to trim the airplane. And of course, even if a trim were obtained, the first gust that hit the airplane would send it off trim speed never to return except to pass through on its way to other extreme speeds and attitudes. In other words, if the

downspring were responsible for producing an undamped phugoid, the pilot would have to fly the plane at all times, keeping a positive control of the airspeed.

Many hours of flying time was spent with the center of gravity aft of the static stick-fixed neutral point. The unstable slope of the elevator position versus lift coefficient curve was completely unnoticeable, with the exception that in extreme cases a shortage of down elevator as speed is decreased is very noticeable and can be embarrassing. With this exception, which can not apply to most aircraft, and which can easily be corrected by adjusting stabilizer incidence angle, the author can see no objection to unstable stick-fixed stability as a routine situation.

Pilots' opinion at best is not susceptible to quantitative definition. These opinions are offered as a qualitative guide to possible objections to extreme amounts of gadgetry in the system. This author, as a pilot, has flown many military airplanes equipped with the bobweight none of which had quite as pronounced an effect on the elevator system as did the devices in the Navion. Consequently it is not felt that the adverse comments on handling qualities as influenced by the bobweight is necessarily applicable to a larger airplane. The downspring is a different story. No pilot will like any airplane which he can not trim longitudinally.

CONCLUSIONS

Static and maneuvering stability may be improved considerably with the use of bobweights and downsprings. Even though an airplane is basically very unstable, judicious use of these devices will provide stable force

gradients. No attention need be paid to providing stable stick-fixed stability, providing adequate elevator power and range of deflections is provided.

Correcting an unsatisfactory static force stability with bobweight can only have a favorable effect on the phugoid mode. Correcting the force stability with downspring in some conditions will change the mode from a pure divergence to a damped oscillatory motion, but in all conditions it is possible to produce very divergent oscillations by an indiscreet use of downsprings.

The results of the study clearly indicate that an indifference to the phugoid mode is unjustified where gadgetry is used in the elevator control system, and prove a need for a rational specification as to minimum requirements in regard to this mode.

RECOMMENDATIONS

Since this investigation has been limited to a study of the airplane in the cruising configuration, it is recommended that there be some further study of the phugoid response of an airplane in a wider range of conditions.

REFERENCES

1. Perkins, C. D. and Hage, R., "Airplane Performance Stability and Control" John Wiley & Sons, Inc. 1949.

DESCRIPTION OF SYMBOLS

C_h Elevator hinge moment coefficient

C_m Pitching moment coefficient

C_L & C_D Lift and Drag coefficients

All stability derivatives are expressed in shorthand notation. For example:

$$C_{m_\alpha} = \frac{\partial C_{m_s}}{\partial \alpha} \quad \text{and} \quad C_{m_{d\theta}} = \frac{\partial C_m}{\partial (d\theta)}$$

d The differential operator, using $\frac{t}{\zeta}$ as the time variable rather than t. $\quad Eq. \quad d\theta = \frac{d\theta}{d(\frac{t}{\zeta})} = \dot{\theta}\,\zeta$

δ_e, δ Elevator deflection

S_e Elevator area

C_e Elevator chord

τ_e Elevator Effectiveness

α Angle of attack, s for stabilizer, w for wing

ι Angle of incidence, s for stabilizer, w for wing

ε Downwash angle

θ Pitch angle

u $\frac{\Delta V}{V}$, incremental change from trim speed

n Number of normal accelerations, in g.

C Mean Aero. Chord of wing

X_{cg} Location of center of gravity with respect to c (from leading edge)

N_o' Static stick-free neutral point with respect to c.

N_m' Maneuvering stick-free neutral point with respect to c.

Description of Symbols, Cont'd

$$h = \frac{2 k_v^2}{\mu c^2}$$

$$n_1 = \frac{2 \mu_e x_e}{c \mu}$$

$$\mu = \frac{m}{\rho s c}$$

$$h_{12} = \frac{x_1 \mu_e k_e^2}{\mu^2 c^2}$$

$$l_1 = \frac{2 \mu_e x_e l_c}{\mu^2 c^2}$$

Normal Short Period Mode

Oscillatory Motion — — —
Aperiodic Motion ————

Normal Phugoid Mode

$Nm' - N\delta' = 0$
$= .04$

$Nm' - N\delta'$ $-.20$
$-.16$
$-.12$
$-.08$
$-.04$

DAMPING $\frac{1}{T}$ $(\frac{1}{sec})$

.10
.08
.06
.04
.02
0
-.02
-.04
-.06
-.08
-.10
-.12

.16   .12   .08   .04   0   -.04   -.08   -.12   -.16

$Nm' - Xcg$

Fig. 1
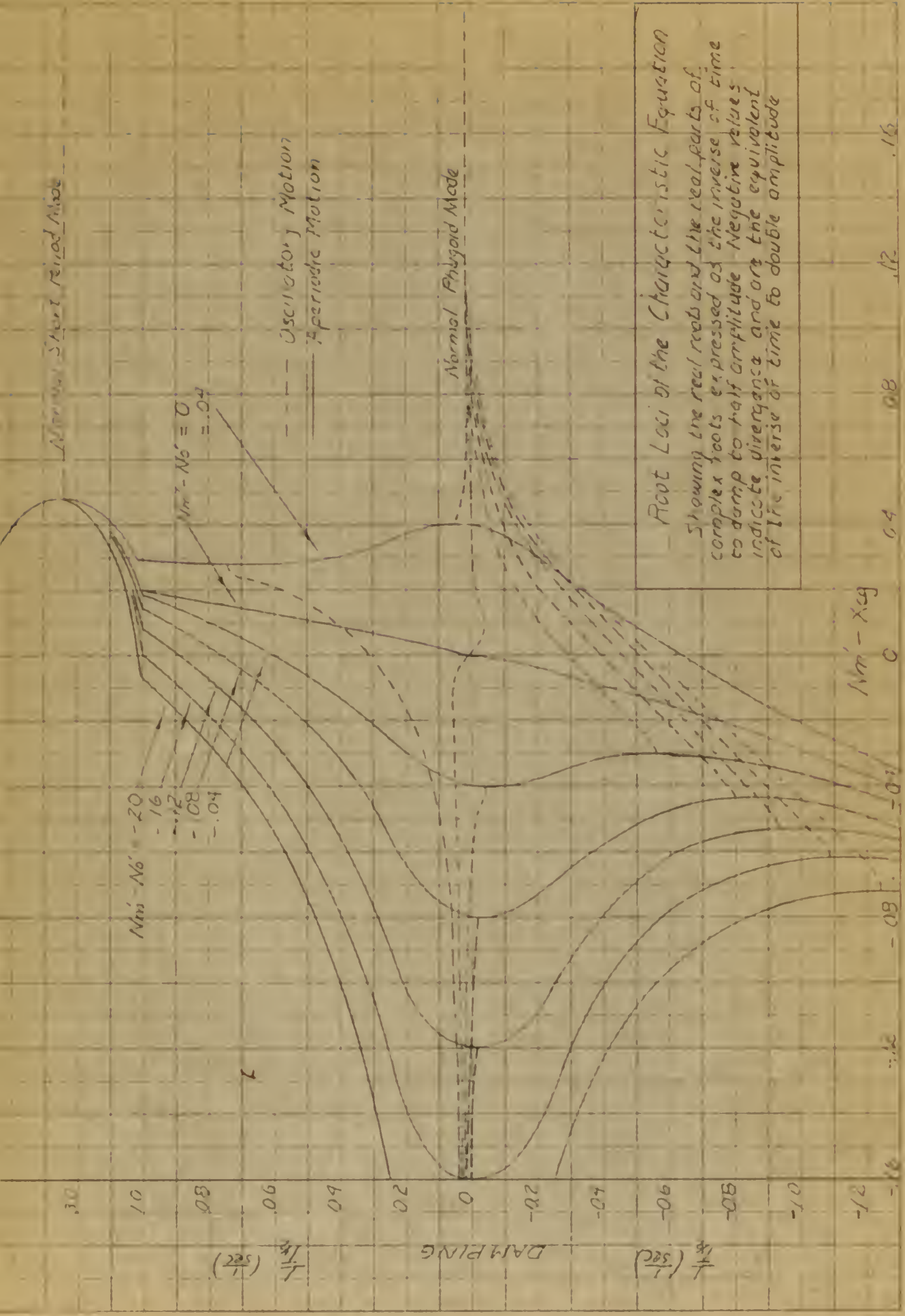
Root Loci of the Characteristic Equation

Showing the real roots and the real parts of
complex roots expressed as the inverse of time
to damp to half amplitude. Negative values
indicate divergence and are the equivalent
of the inverse of time to double amplitude

Root loci of the Characteristic Equation

Showing imaginary parts of complex z roots expressed as Period, in seconds.

Short Period Mode

$Nm' - N\delta' = 0, \; G \, 4$

$N\alpha' - N\delta' = -.04 \; to \; -.20$

$Nm' - N\delta'$

Those lines curving down and to right are for normal Phugoid Mode

$Nm - Xcg$

$F_{IG} \; 16$

Period — secs.
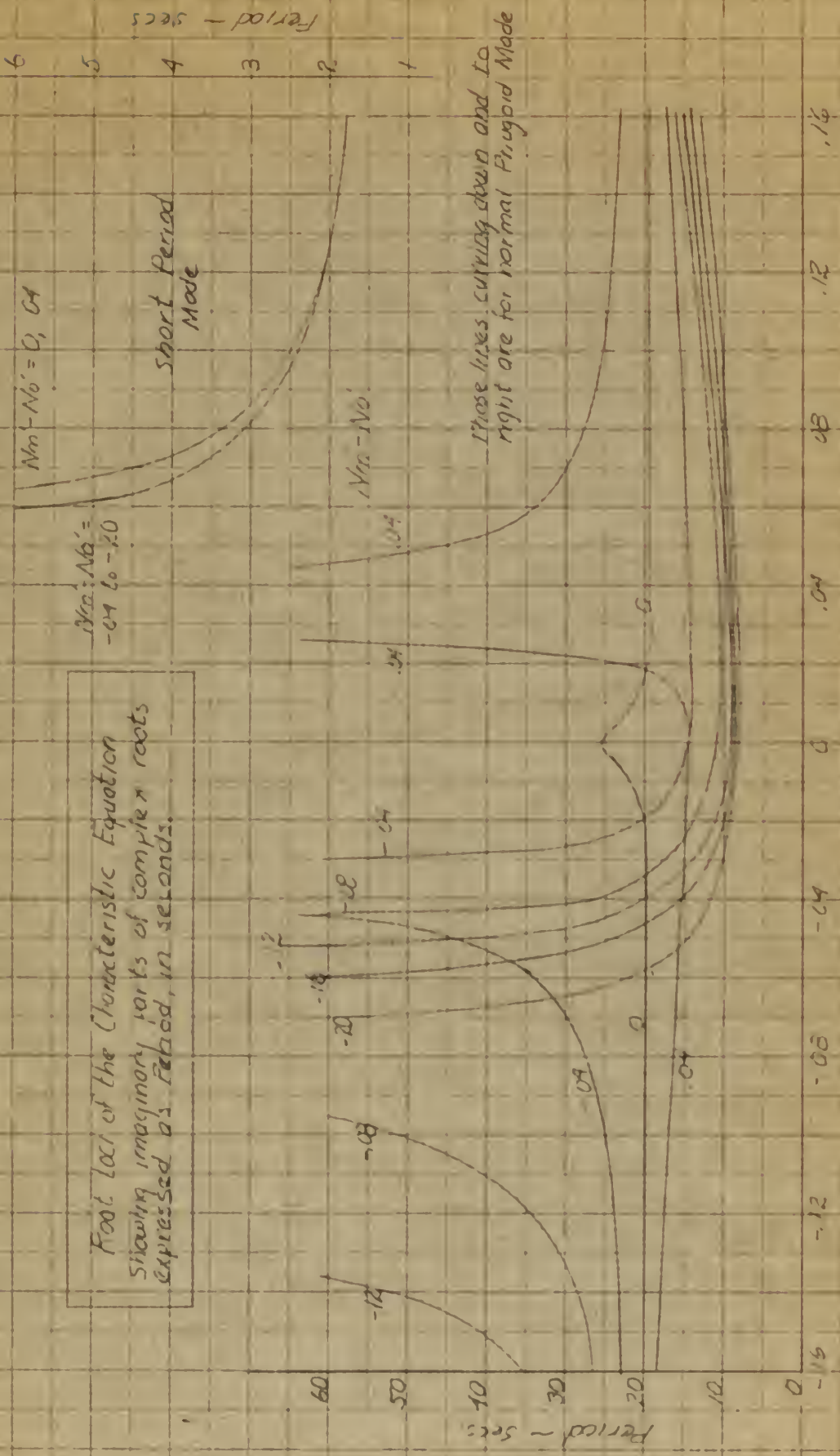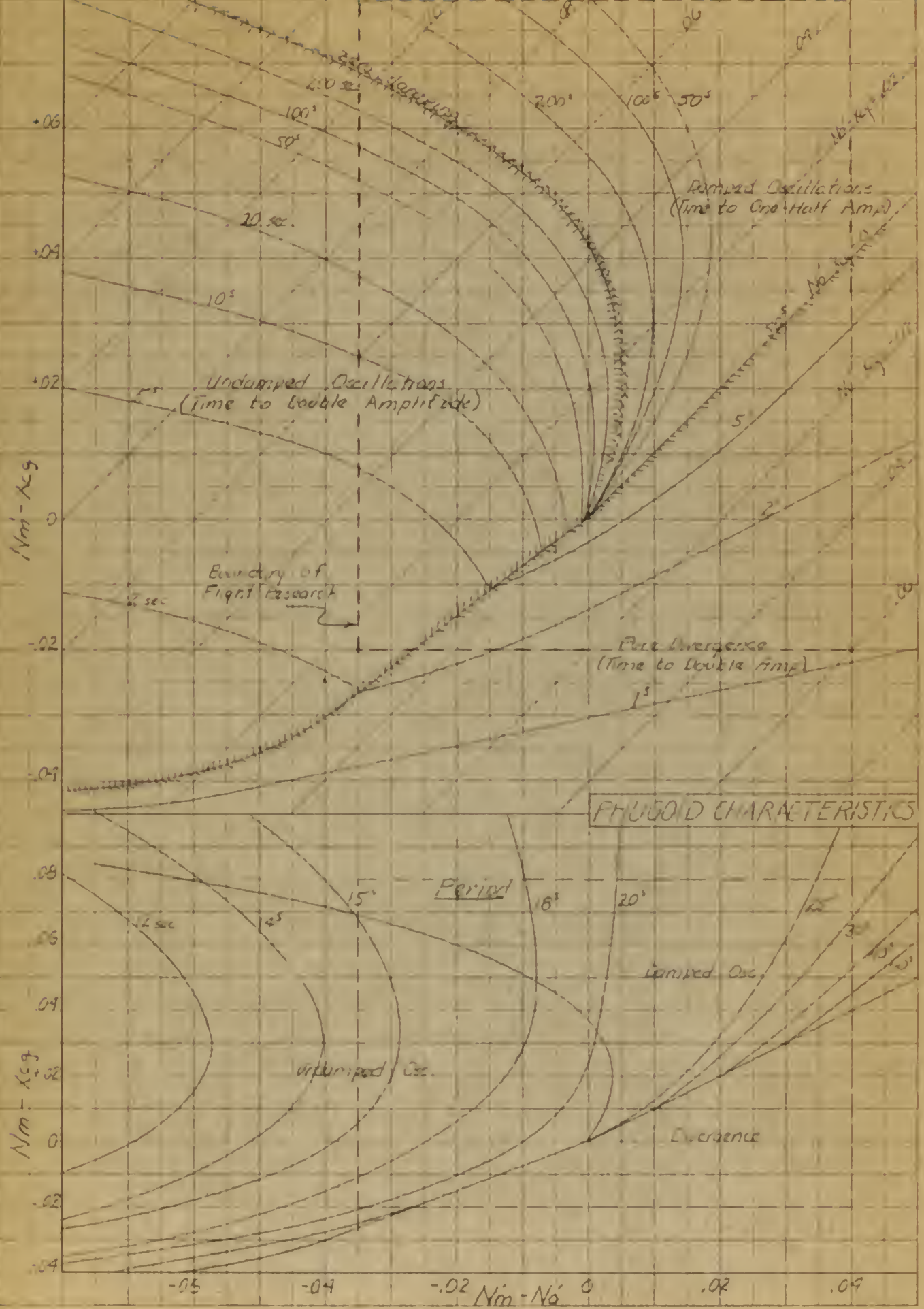
FIG 2

SKETCH OF COCKPIT
(left seat, canopy removed)
showing mechanism with which bobweight
and downspring were introduced into the
longitudinal control system.

instrumentation control board

downspring (variable)

photopanel

bobweight (variable)

PRINCETON UNIVERSITY
EXPERIMENTAL
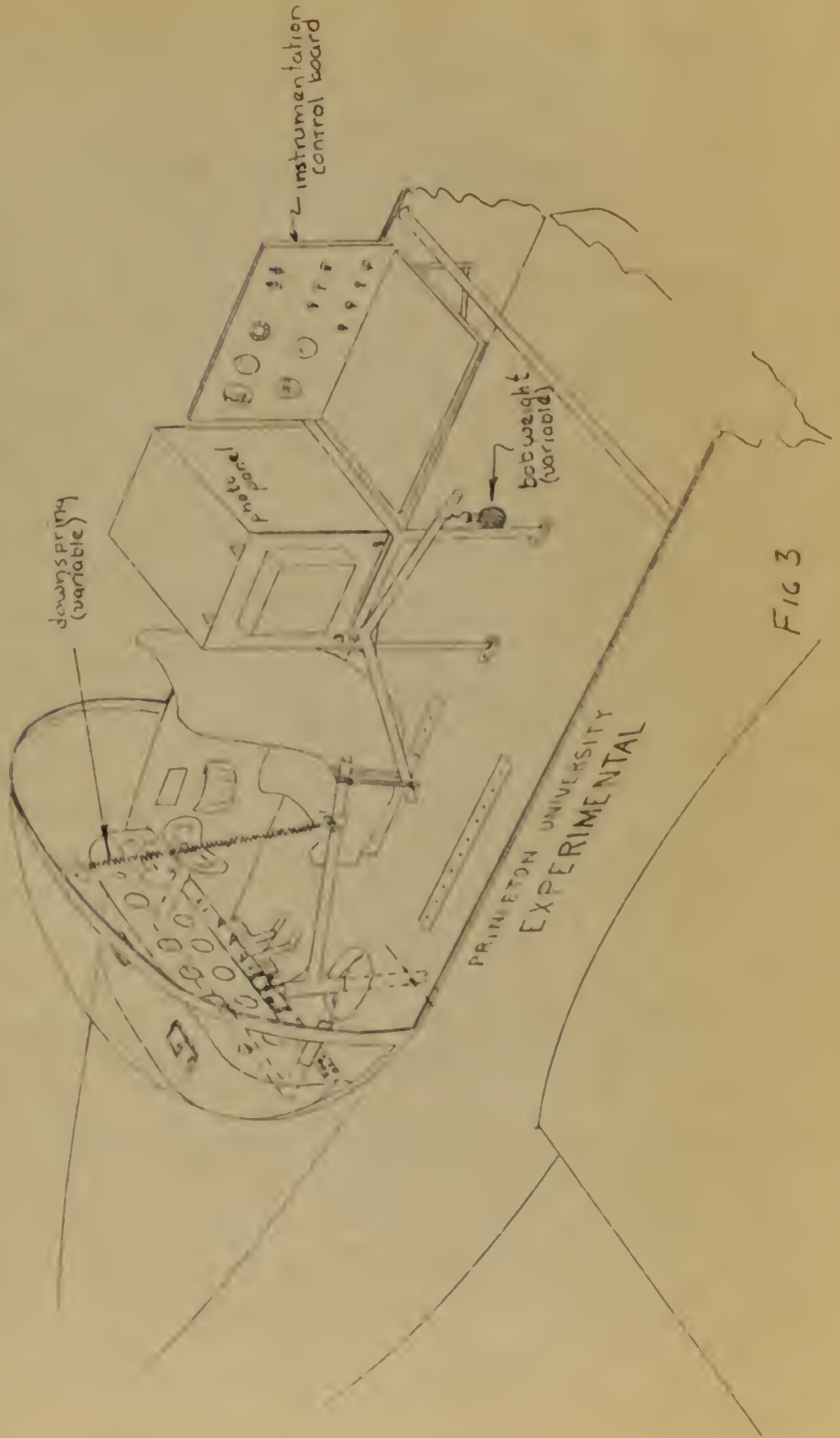
FIG 3

SUMMARY OF THE STATIC STABILITY
of modified Navion No 1566
as determined by flight tests

(A) No Bobweight or Downspring
(B) 8 lbs. Equiv. F_S of either device
(C) 16 lbs. " " " " "
(D) 24 lbs " " " " " "
(E) 32 lbs. " " " " " "

$\dfrac{dF_S}{dC_L}$

Neutral Points No's

STATIC STICK-FREE STABILITY

Xcg

.26   .30   .34   .38   .42

(A) No Bobweight
(B) 8 lbs Bobweight
(C) 16 lbs "
(D) 24 lbs "
(E) 32 lbs. "

$\dfrac{dF_S}{dn}$

Neutral Points N'm

MANEUVERING STICK FREE STABILITY
(independent of Downspring)

Xcg

.26   .30   .34   .38   .42
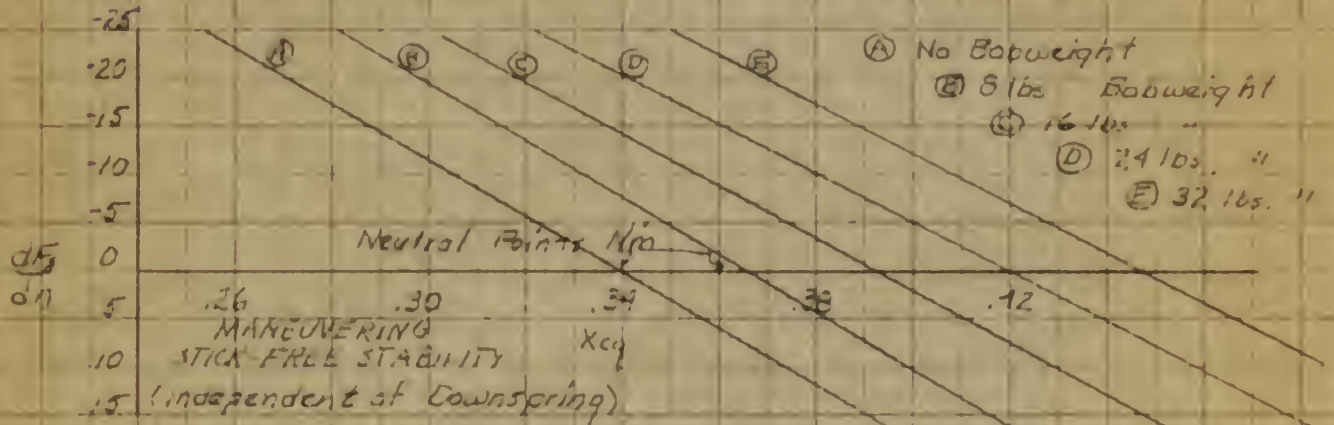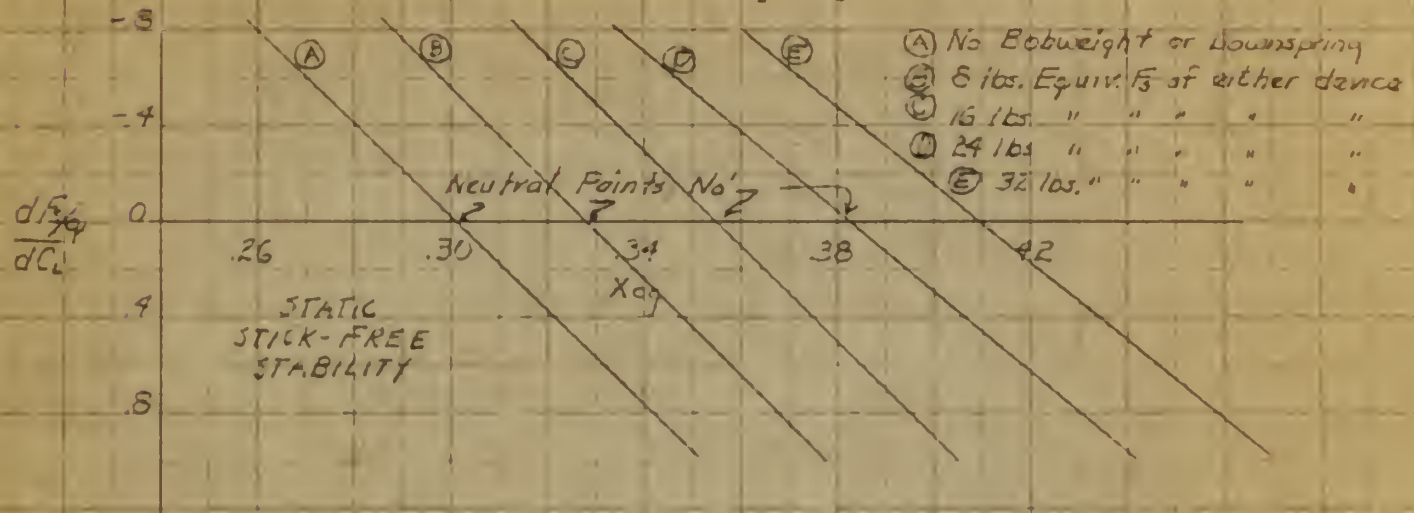
Note: Pull Forces are (−) Forces

$\dfrac{d\delta e}{dn}$ or $\dfrac{d\delta e}{dC_L}$

Maneuvering

STICK FIXED STABILITY
(independent of gadgetry)

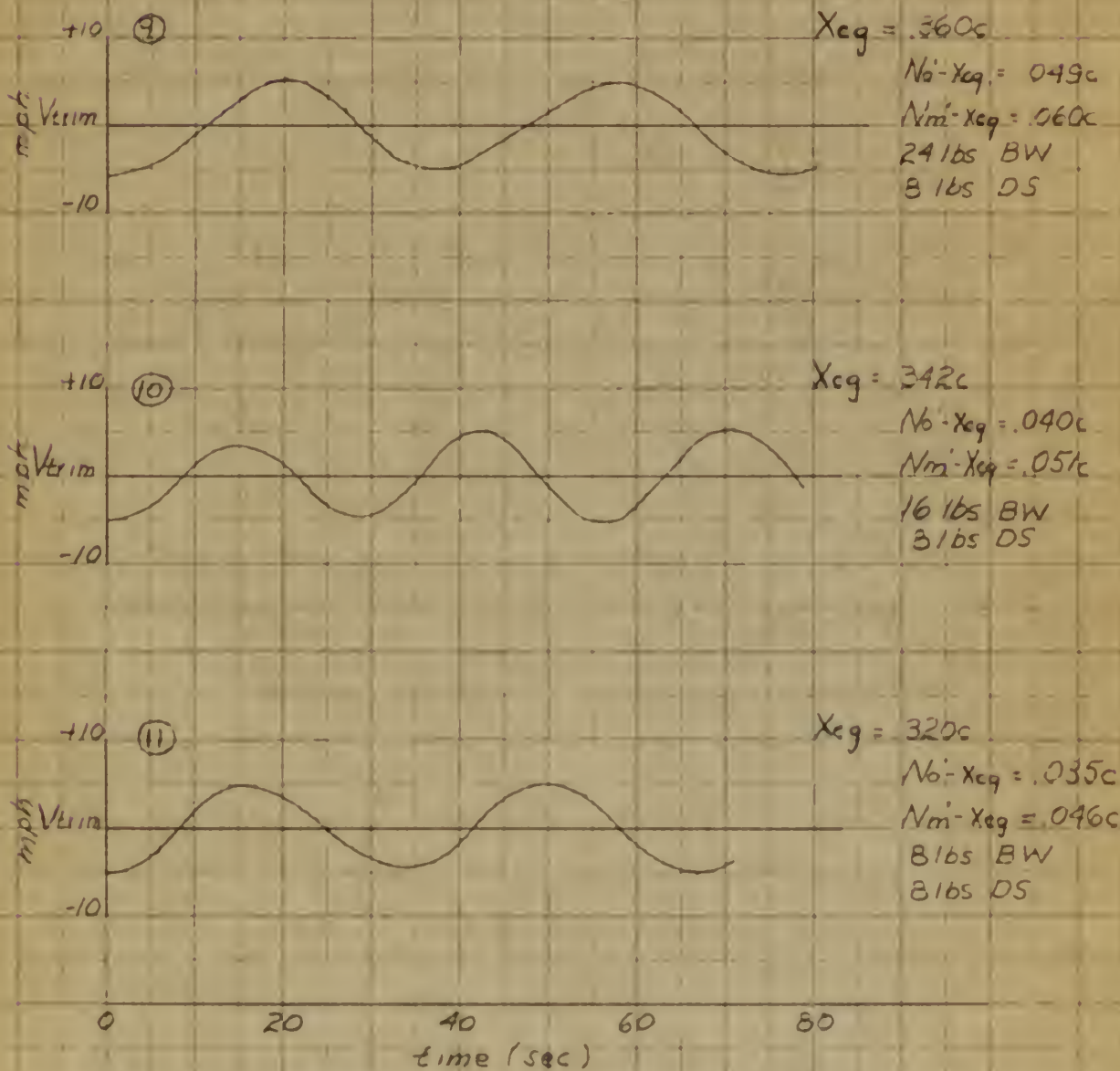Static   No   Nm

.26   .30   .34   .38   .42

FIG. 4

FIG. 5
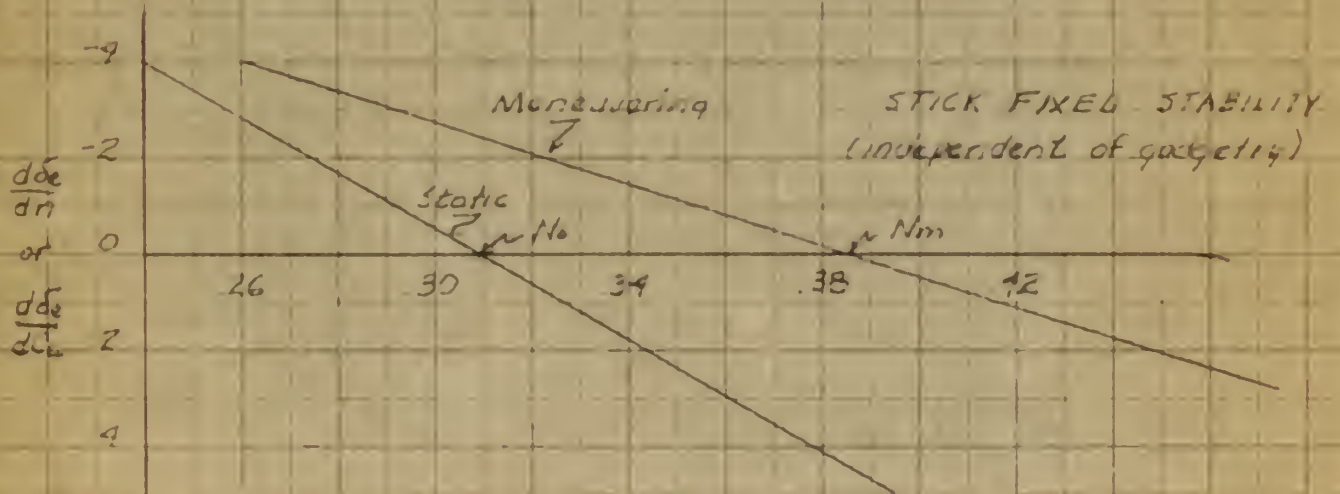
PHUGOID RESPONSES
Flight Test Results

Static and Maneuvering Stability approximately equal
in all three tests
Center of Gravity varied from 32c to 36c
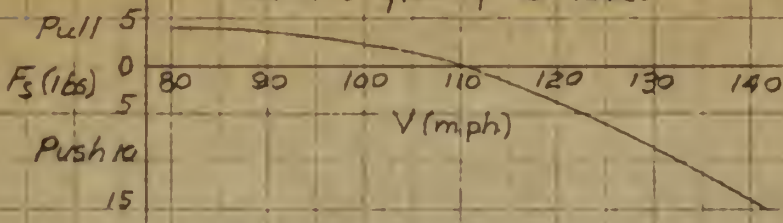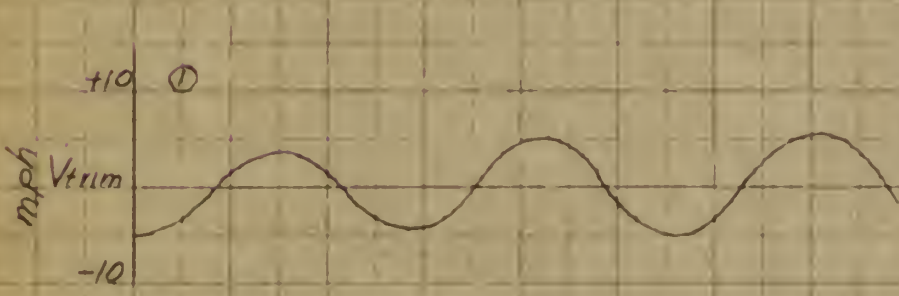
$X_{cg} = .360c$

$N'_o - X_{cg} = .049c$

$N'_m - X_{cg} = .060c$

24 lbs BW

8 lbs DS

$X_{cg} = 342c$

$N'_o - X_{cg} = .040c$

$N'_m - X_{cg} = .051c$

16 lbs BW

3 lbs DS

$X_{cg} = 320c$

$N'_o - X_{cg} = .035c$

$N'_m - X_{cg} = .046c$
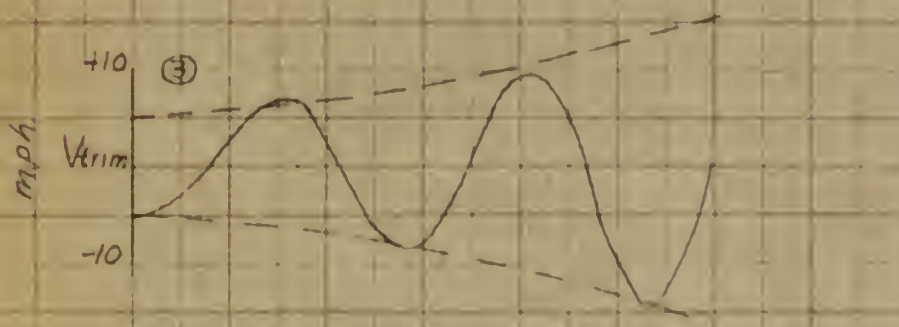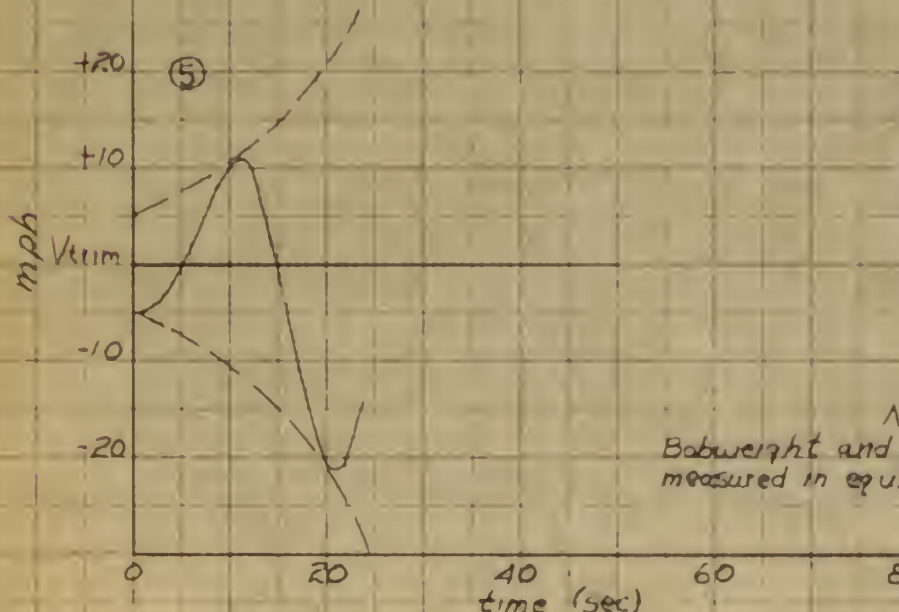
8 lbs BW

8 lbs DS

Note
Bobweight and Downspring (BW and DS)
measured in equivalent stick force

FIG 6

PHUGOID RESPONSES
Flight Test Results

Series of test results, indicating difference in response
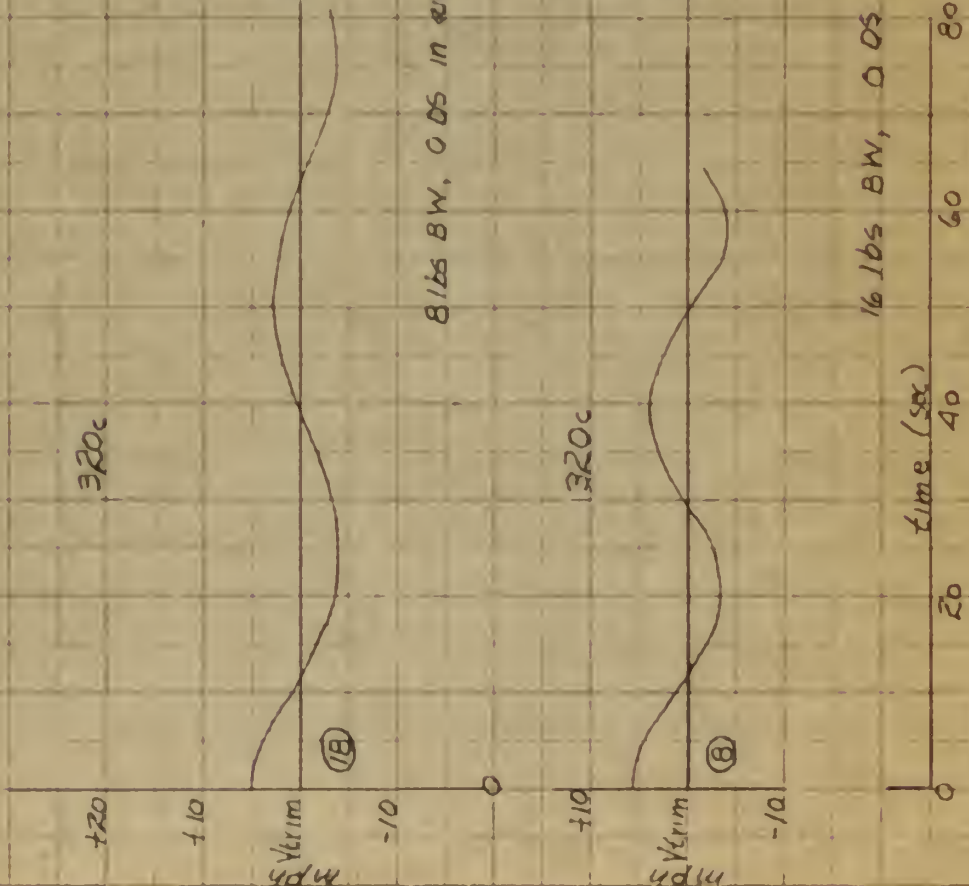due to center of Gravity variation from .32c to .36c
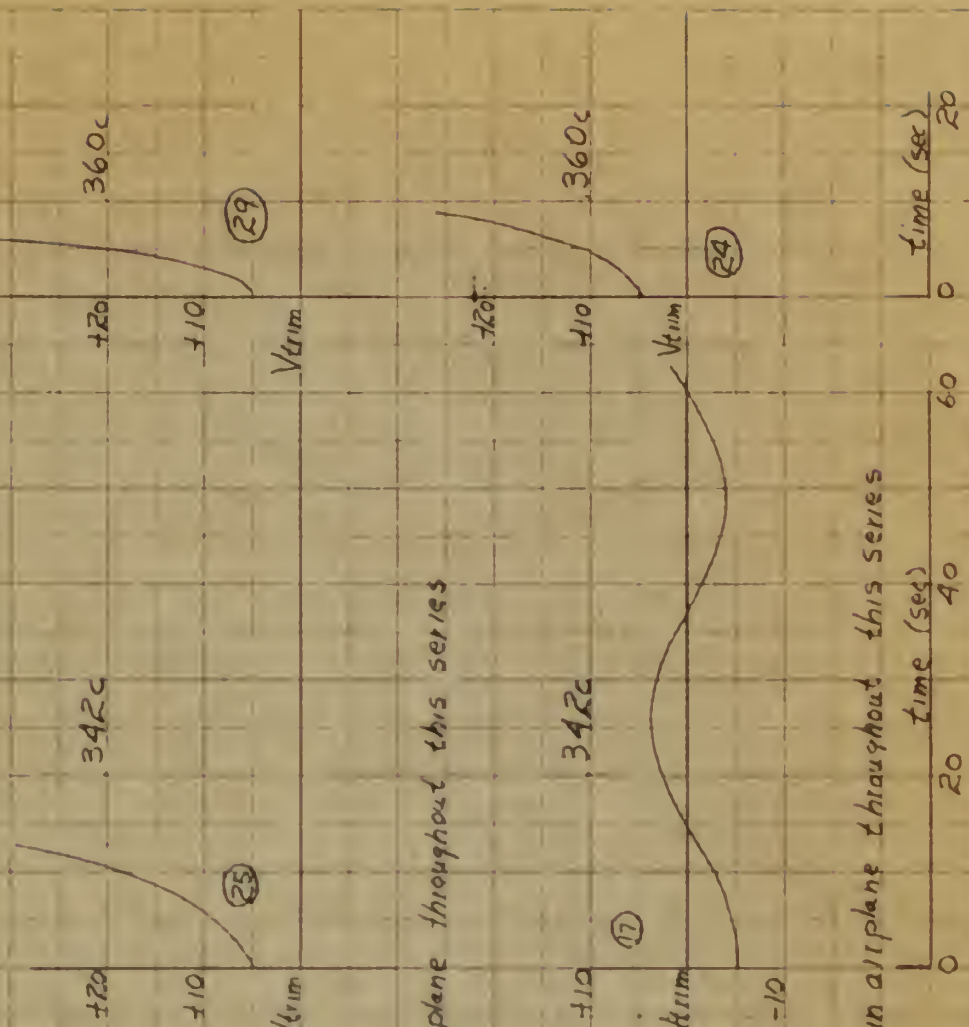
FIG 7

TABULATION
for the
General Presentation of Flight Test Results
starting pf

Each Response numbered according to this table.
BW & DS columns list amount of bobweight and
downspring in pounds of equivalent stick force

No	$N_o' - x_{cg}$	$N_{rr} - x_{cg}$	$N_{rr} - N_o'$	x_{cg}	BW (lbs)	DS (lbs)	$T_{1/2}$ (sec.)	P (kts)
GROUP A			$N_o' - x_{cg} \approx .064c$					
1	.067	.078	.011	.342	24	8	∞	27
2	.062	.073	.011	.320	16	8	48	30
3	.067	.051	-.016	.342	16	16	-37	26
4	.062	.046	-.016	.320	8	16	-20	22
5	.067	.024	-.043	.342	8	24	-10	21
6	.062	.019	-.043	.320	0	24	-7	22
GROUP B			$N_o' - x_{cg} \approx .091c$					
7	.090	.078	.038	.342	24	0	∞	40
8	.035	.073	.038	.320	16	0	40	38
9	.049	.061	.011	.360	24	8	∞	38
10	.090	.051	.011	.342	16	8	∞	32
11	.035	.046	.011	.320	8	8	∞	34
12	.049	.033	-.016	.360	16	16	-37	26
13	.040	.024	-.016	.342	8	16	-16	25
14	.035	.019	-.016	.320	0	16	-15	25
15	.049	.006	-.043	.360	8	24	-10	23

TABULATION (conʈd)

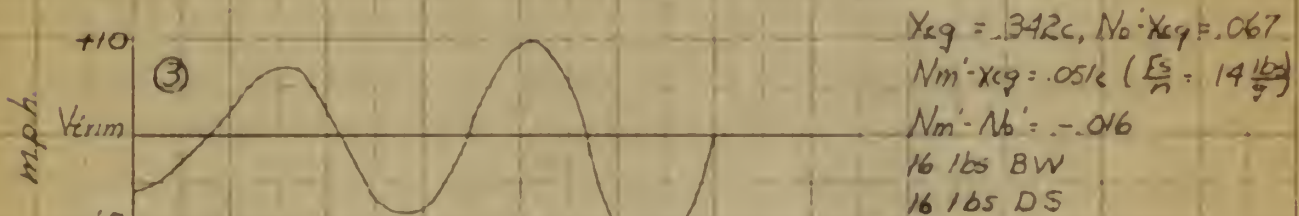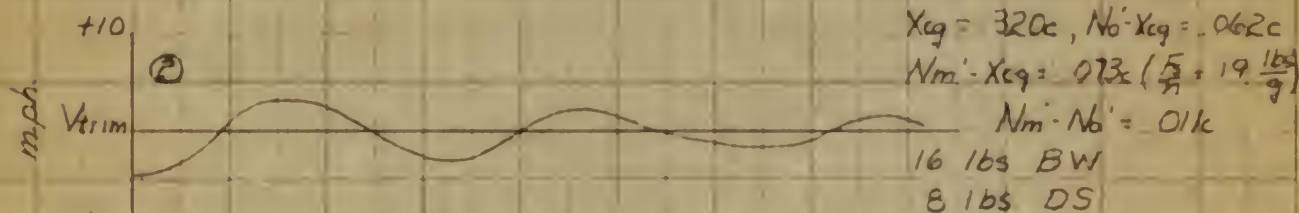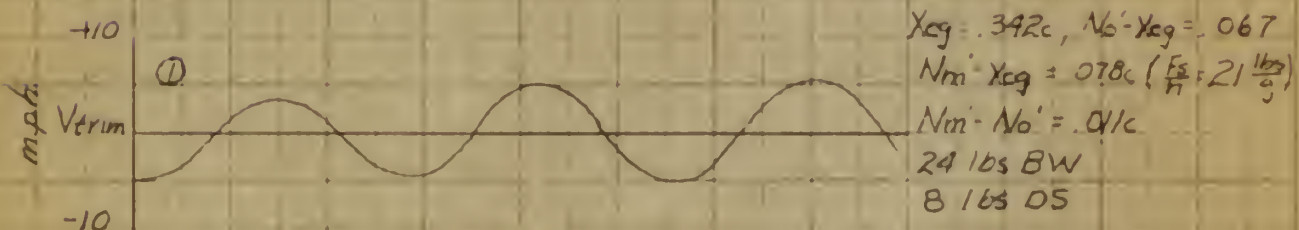| No	$N_o'-X_{cg}$	$N_{mi}-X_{cg}$	$N_{mi}-N_o$	X_{cg}	BW (lbs)	DS (lbs)	$T_{1/2}$ (sec)	P (sec)
GROUP C $N_o'-X_{cg} \approx .014c$								
16	.022	.060	.038	.360	24	0	∞	65
17	.013	.051	.038	.342	16	0	220	48
18	.008	.046	.038	.320	8	0	95	51
19	.022	.033	.011	.360	16	8	−45	45
20	.013	.024	.011	.342	8	8	−85	43
21	.008	.019	.011	.320	0	8	−55	43
22	.022	.006	−.016	.360	8	16	−16	32
23	.013	−.003	−.016	.342	0	16	−7	27
GROUP D $N_o'-X_{cg} \approx -.013c$								
24	−.005	.033	.038	.360	16	0	−5	∞
25	−.014	.024	.038	.342	8	0	−5	∞
26	−.019	.019	.038	.320	0	0	−5	∞
27	−.014	−.003	.011	.342	0	8	−35	∞
GROUP E $N_o'-X_{cg} \approx -.045c$								
28	−.041	−.003	.038	.342	0	0	−3	∞
29	−.032	+.006	.038	.360	1	0	−3	∞
30	−.059	−.021	.038	.360	0	0	−2	∞

PHUGOID RESPONSES
General Presentation of Flight Test Results

GROUP A

For all tests in this group, the airplane was rigged to have approximately the same static stick-free stability
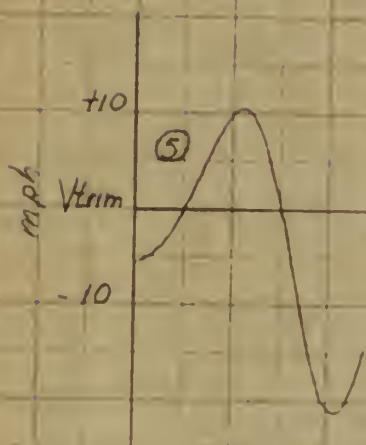

static force stability curve
applying to this group

① $X_{cg} = .342c$, $N_o' - X_{cg} = .067$
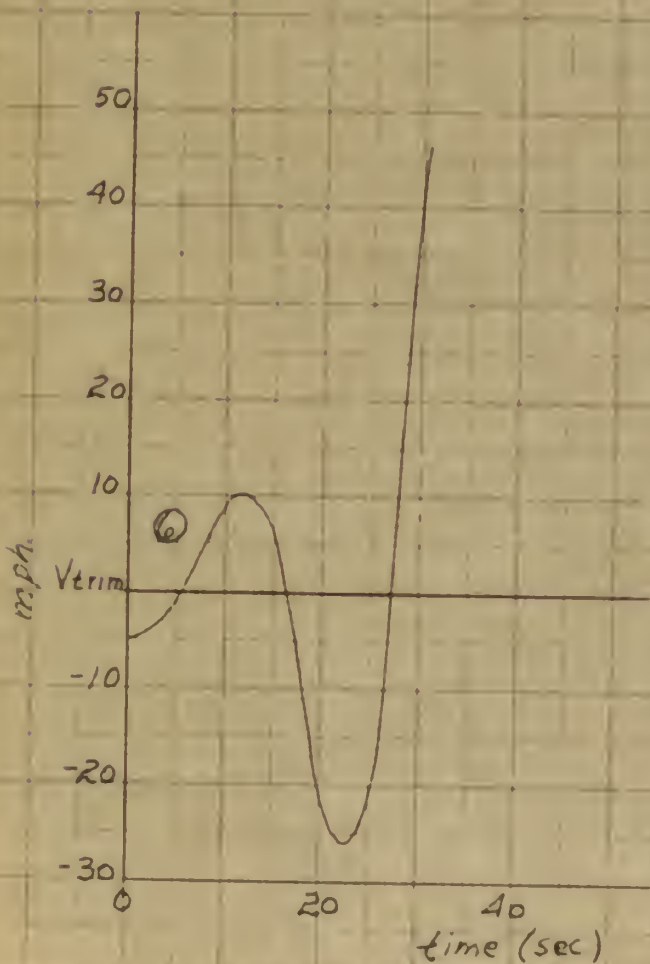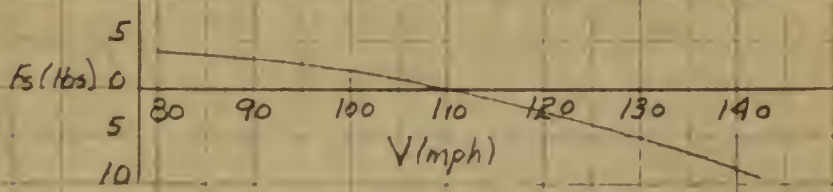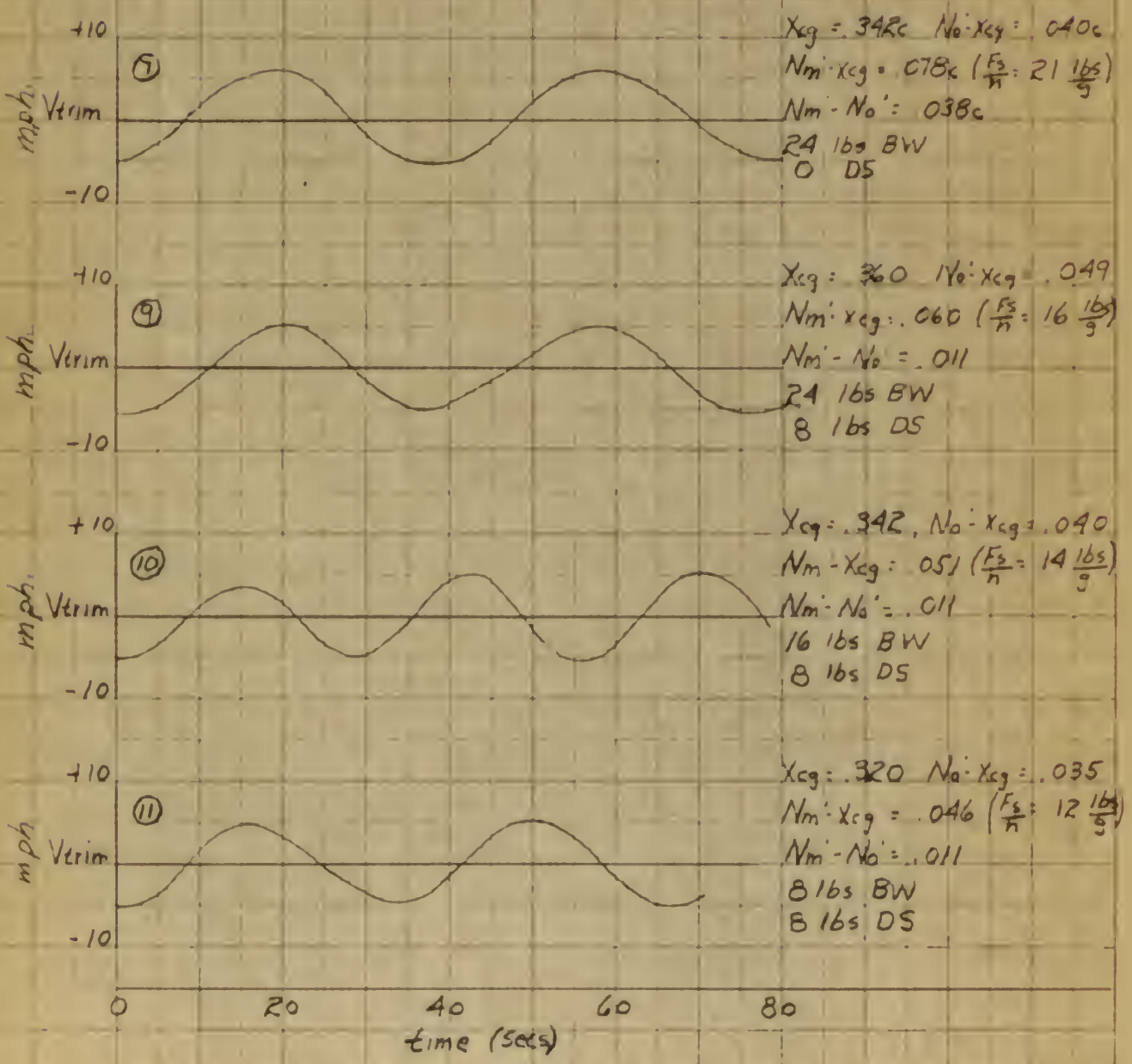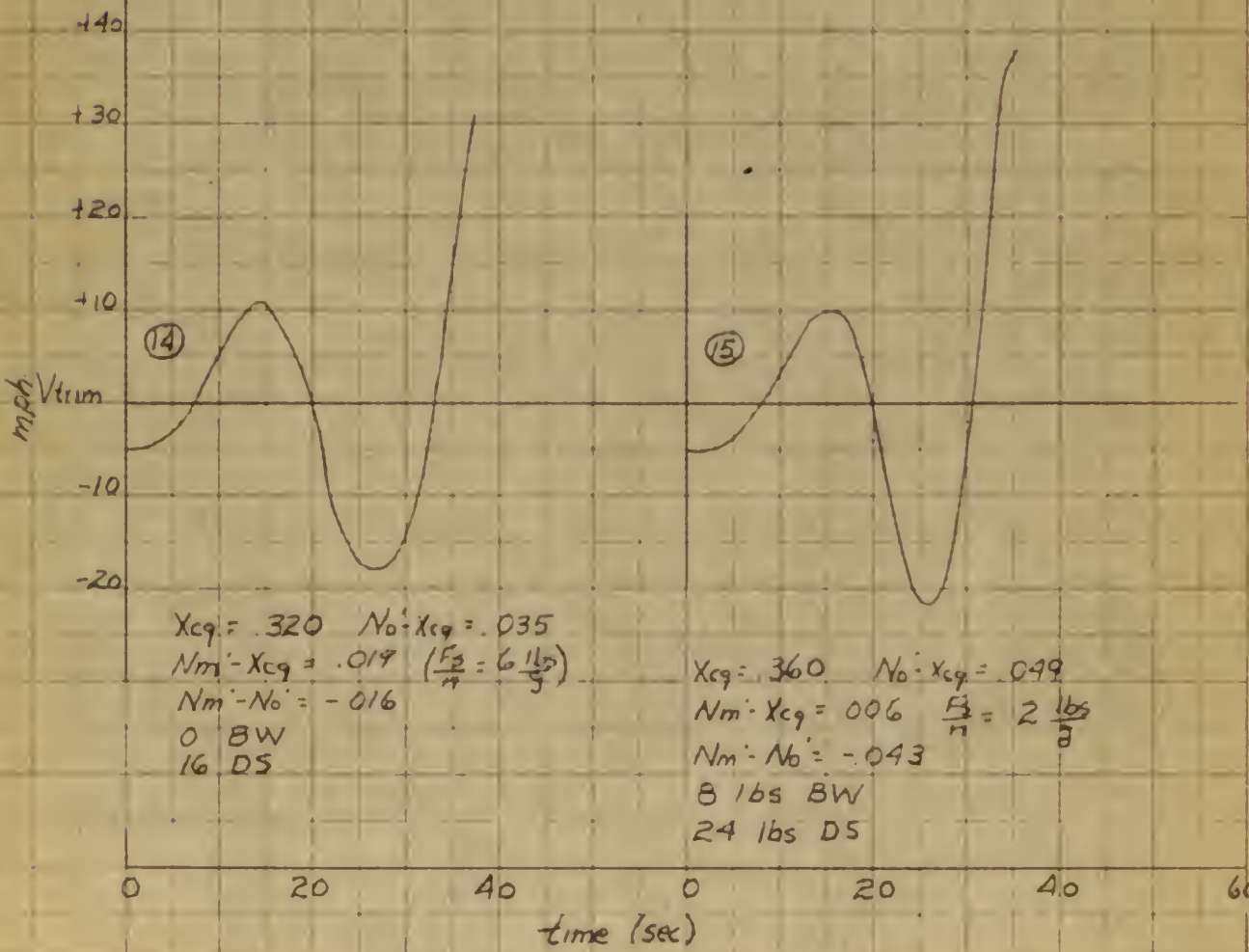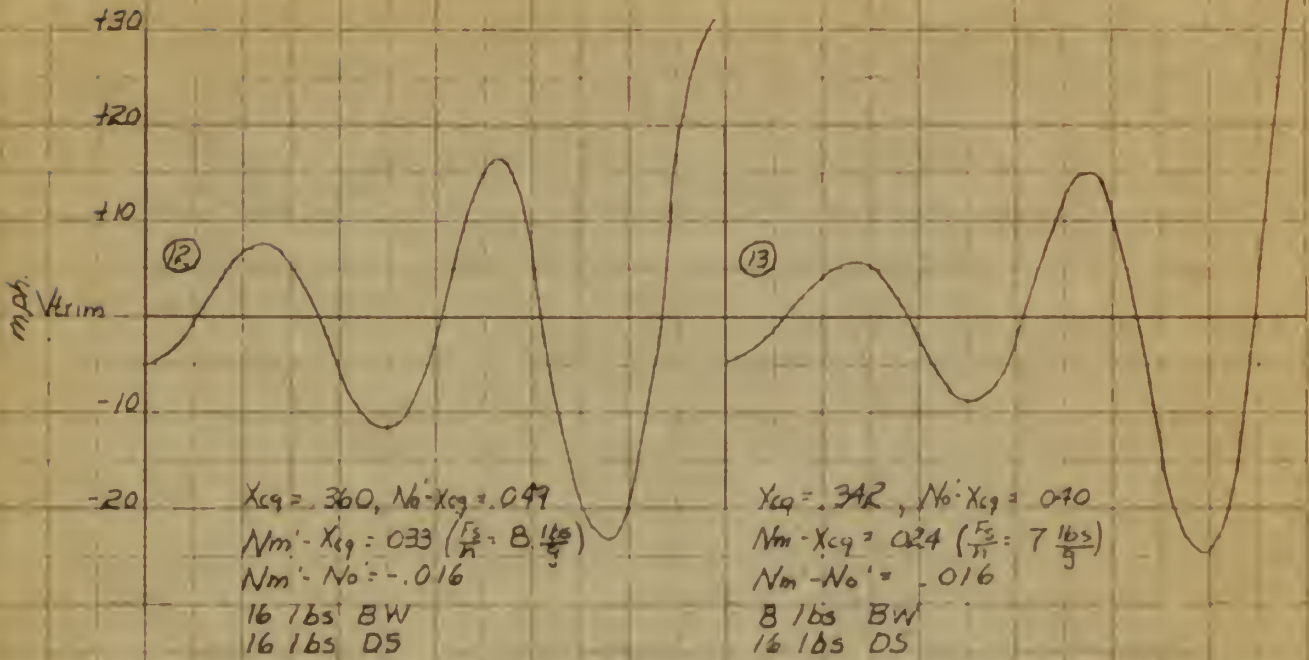$N_m' - X_{cg} = 0.78c$ $\left(\frac{F_s}{n} = 21 \frac{lbs}{g}\right)$
$N_m' - N_o' = .01/c$
24 lbs BW
8 lbs DS

② $X_{cg} = .320c$, $N_o' - X_{cg} = .062c$
$N_m' - X_{cg} = 0.73c$ $\left(\frac{F_s}{n} = 19 \frac{lbs}{g}\right)$
$N_m' - N_o' = .01/c$
16 lbs BW
8 lbs DS

③ $X_{cg} = .342c$, $N_o' - X_{cg} = .067$
$N_m' - X_{cg} = .05/c$ $\left(\frac{F_s}{n} = 14 \frac{lbs}{g}\right)$
$N_m' - N_o' = -.016$
16 lbs BW
16 lbs DS
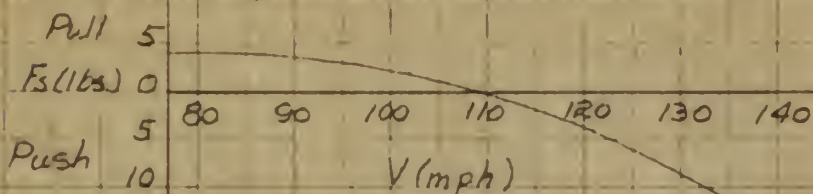
④ $X_{cg} = .320c$, $N_o' - X_{cg} = .062c$
$N_m' - X_{cg} = .046$, $\left(\frac{F_s}{n} = 12 \frac{lbs}{g}\right)$
$N_m' - N_o' = -.016$
8 lbs BW
16 lbs DS.
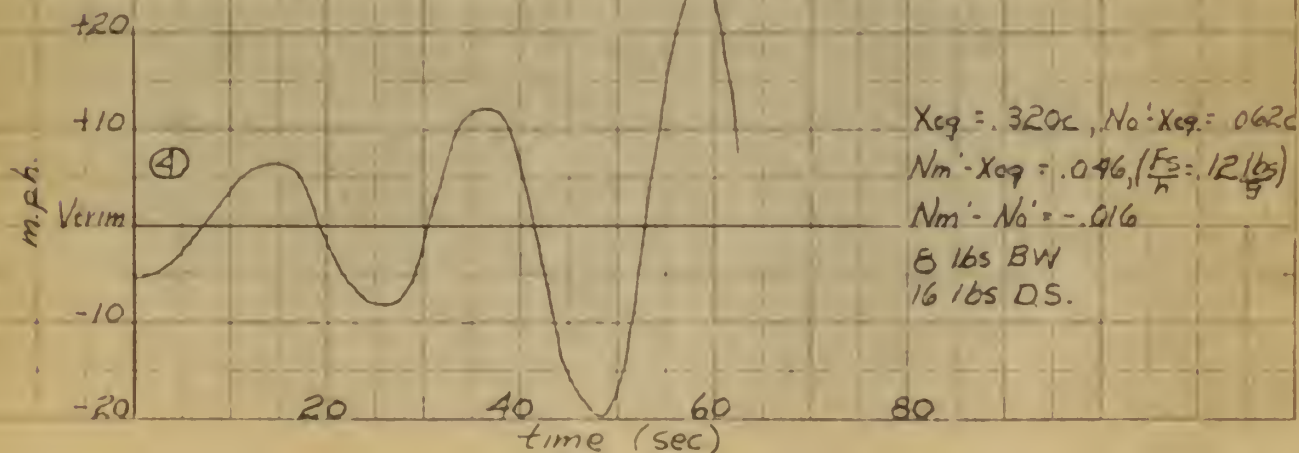
PHUGOID RESPONSES

GROUP A (cont'd)

+10

V_{trim}

-10

m.p.h.

⑤

$x_{cg} = .342c$ $N_o' - x_{cg} - .067c$
$N_m' - x_{cg} = .024c$ ($\frac{F_s}{n} = 7 \frac{lbs}{g}$)
$N_m' - N_o' = -.043$
8 lbs BW
24 lbs DS

50

40

30

20

10

V_{trim}

-10

-20

-30

m.p.h.

⑥

$x_{cg} = .320c$ $N_o' - x_{cg} = .062c$
$N_m' - x_{cg} = .019$ ($\frac{F_s}{n} = 6 \frac{lbs}{g}$)
$N_m' - N_o' = -.043$
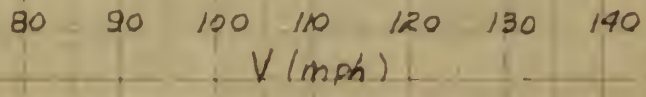O BW
24 lbs DS

0   20   40   60   80

time (sec)

PHUGOID RESPONSES
General Presentation of Flight Test Results

GROUP B

For all tests in this group the airplane was rigged
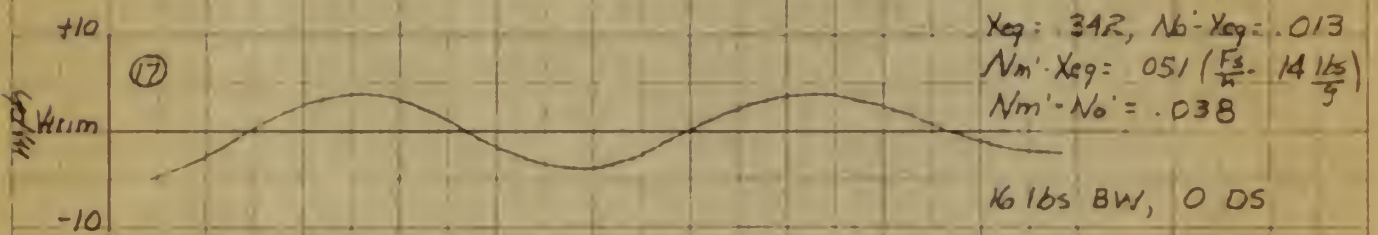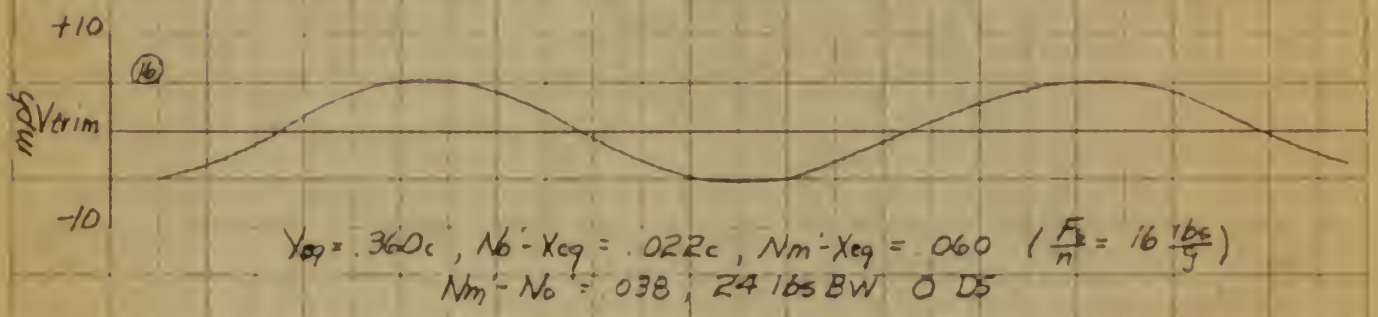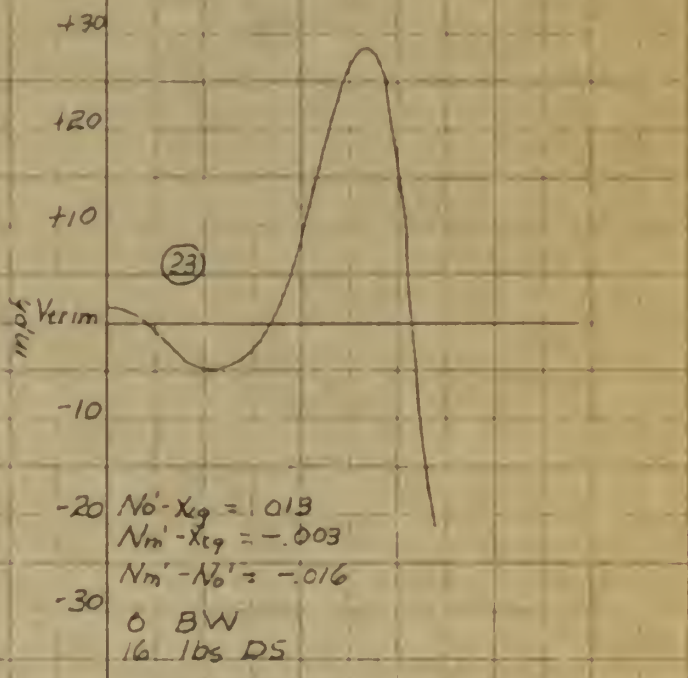to have approximately the same static stick free stability

Static force stability curve
applying to this group

Xcg = .342c   No - Xcg = .040c
Nm - Xcg = .078c ($\frac{Fs}{n}$ = 21 $\frac{lbs}{g}$)
Nm' - No' = .038c
24 lbs BW
0 DS

Xcg = .360   No - Xcg = .049
Nm - Xcg = .060 ($\frac{Fs}{n}$ = 16 $\frac{lbs}{g}$)
Nm' - No' = .011
24 lbs BW
8 lbs DS

Xcg = .342 , No - Xcg = .040
Nm - Xcg = .051 ($\frac{Fs}{n}$ = 14 $\frac{lbs}{g}$)
Nm' - No' = .011
16 lbs BW
8 lbs DS

Xcg = .320   No - Xcg = .035
Nm - Xcg = .046 ($\frac{Fs}{n}$ = 12 $\frac{lbs}{g}$)
Nm' - No' = .011
8 lbs BW
8 lbs DS

PHUGOID RESPONSES
GROUP. B. (cont'd)

⑫ $Xcg = .360, No - Xcg = .049$
$Nm' - Xcg = .033 \left(\frac{F_s}{n} = 8 \frac{lbs}{g}\right)$
$Nm' - No' = -.016$
16 7bs BW
16 lbs DS

⑬ $Xcg = .342, No - Xcg = .070$
$Nm' - Xcg = .024 \left(\frac{F_s}{n} = 7 \frac{lbs}{g}\right)$
$Nm - No' = .016$
8 lbs BW
16 lbs DS

⑭ $Xcg = .320 \quad No - Xcg = .035$
$Nm' - Xcg = .019 \left(\frac{F_s}{n} = 6 \frac{lbs}{g}\right)$
$Nm' - No' = -.016$
0 BW
16 DS

⑮ $Xcg = .360 \quad No - Xcg = .099$
$Nm' - Xcg = .006 \quad \frac{F_s}{n} = 2 \frac{lbs}{g}$
$Nm' - No' = -.043$
8 lbs BW
24 lbs DS

time (sec)

PHUGOID RESPONSES
General Presentation of Flight Test Results

GROUP C

For all tests in this group the same static stability obtains

Static force stability curve
applying to this group

$X_{cg} = .360c$, $N_o - X_{cg} = .022c$, $N_m' - X_{cg} = .060$ ($\frac{F_s}{n} = 16 \frac{lbs}{g}$)

$N_m' - N_o' = .038$, 24 lbs BW 0 DS

$X_{cg} = .342$, $N_o' - X_{cg} = .013$
$N_m' - X_{cg} = .051$ ($\frac{F_s}{n} = 14 \frac{lbs}{g}$)
$N_m' - N_o' = .038$

16 lbs BW, 0 DS

$X_{cg} = .320$, $N_o' - X_{cg} = .008$
$N_m' - X_{cg} = .046$ ($\frac{F_s}{n} = 12 \frac{lbs}{g}$)
$N_m' - N_o' = .038$
8 lbs BW, 0 DS

$X_{cg} = .360$, $N_o' - X_{cg} = .022$
$N_m' - X_{cg} = .083$ ($\frac{F_s}{n} = 8 \frac{lbs}{g}$)
$N_m' - N_o' = .011$
16 lbs BW, 8 lbs DS
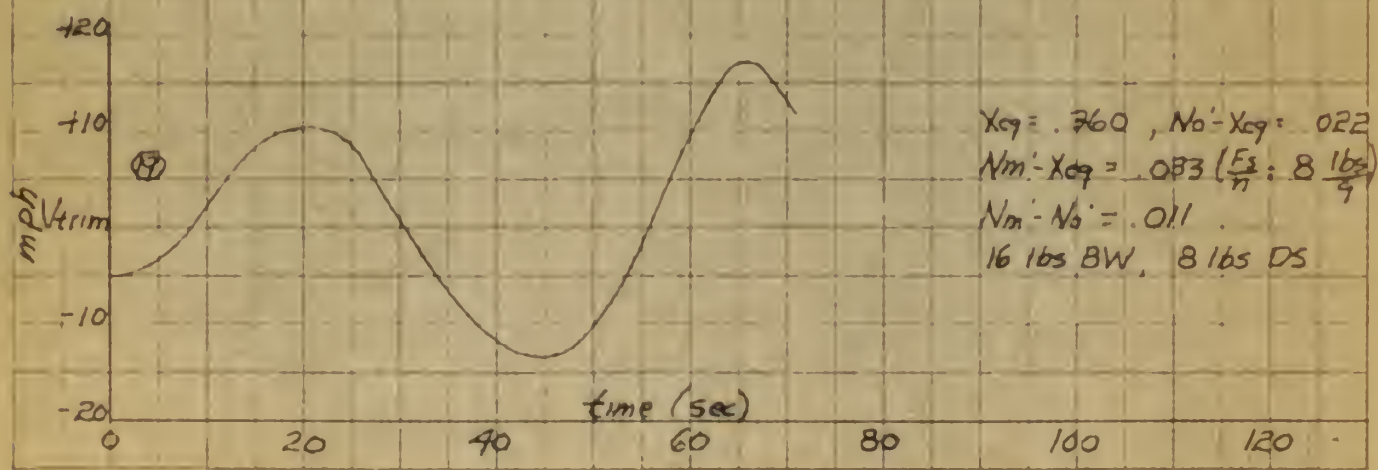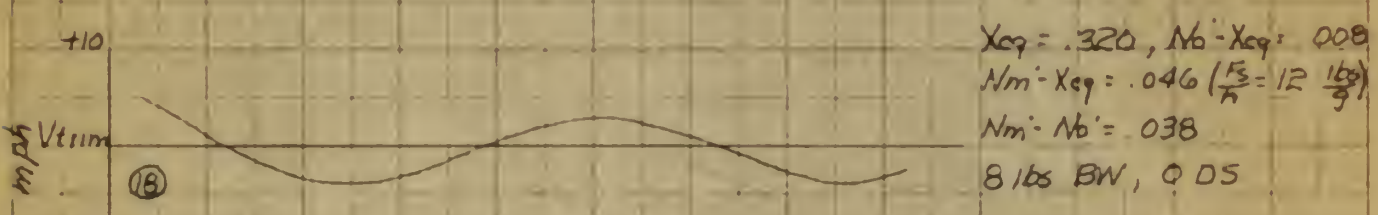
time (sec)

PHUGOID RESPONSES

GROUP C (cont'd)

$X_{cg} = .392$ $N_b' - X_{cg} = .013$
$N_m' - X_{cg} = .024$ $\frac{F_s}{n} = 7 \frac{lbs}{c}$
$N_m' - N_b' = .011$

8 lbs BW., 8 lbs DS

$X_{cg} = .320$ $N_b' - X_{cg} = .008$
$N_m' - X_{cg} = .019$ $\frac{F_s}{n} = 6 \frac{lbs}{g}$
$N_m' - N_b' = .011$

0 BW
16 lbs DS

$X_{cg} = .360$, $N_b' - X_{cg} = .022$
$N_m' - X_{cg} = .046$ $\frac{F_s}{n} = 2 \frac{lbs}{g}$
$N_m' - N_b' = .016$

8 lbs BW - 16 lbs DS

$N_b' - X_{cg} = .013$
$N_m' - X_{cg} = -.003$
$N_m' - N_b' = -.016$

0 BW
16 lbs DS
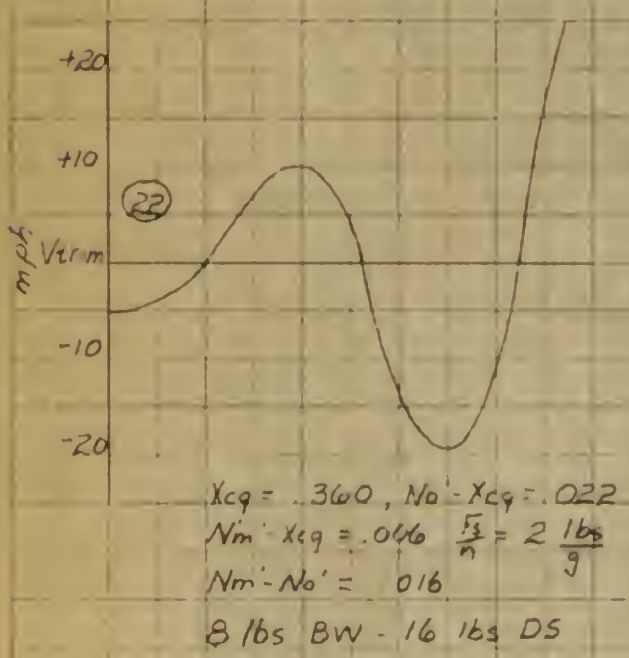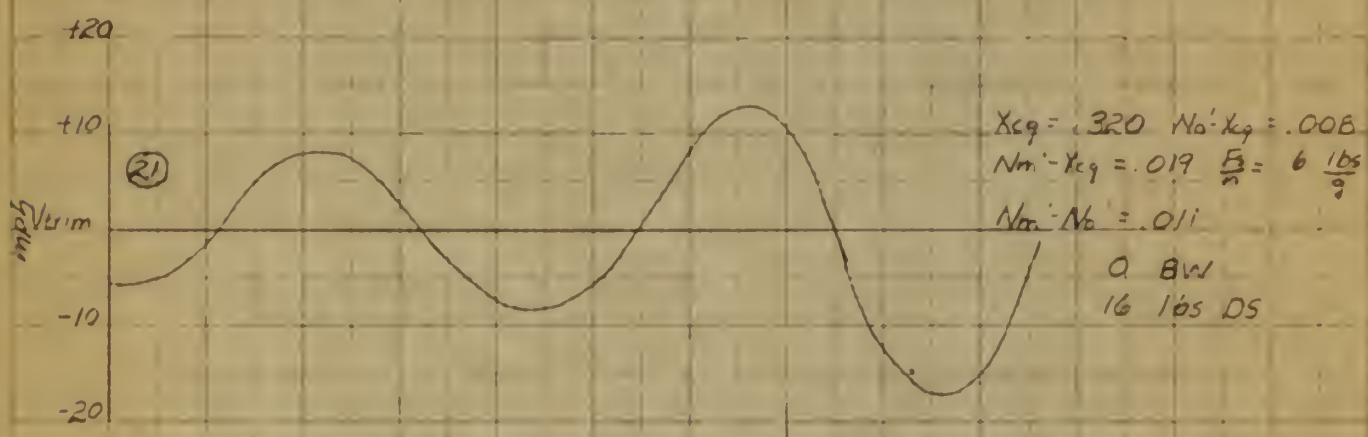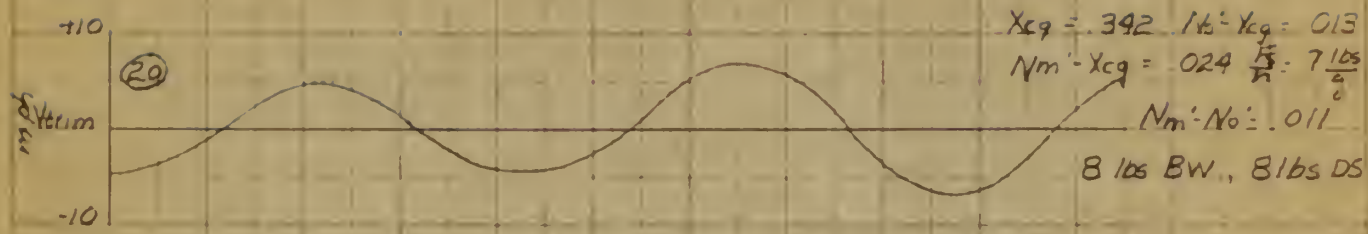
time (secs)

PHUGOID RESPONSES
General Presentation of Flight test Results

GROUPS D & E
Center of Gravity aft of Static Stick-free Neutral Point
See Tabulation, pp   for details

GROUP D
No.- Xcg = -.013

GROUP E
No.- Xcg = -.045

time (sec)

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Correlation of Experimental and Theoretical Results
Damping Only

Correlation of Experimental and Theoretical Results
Period Only
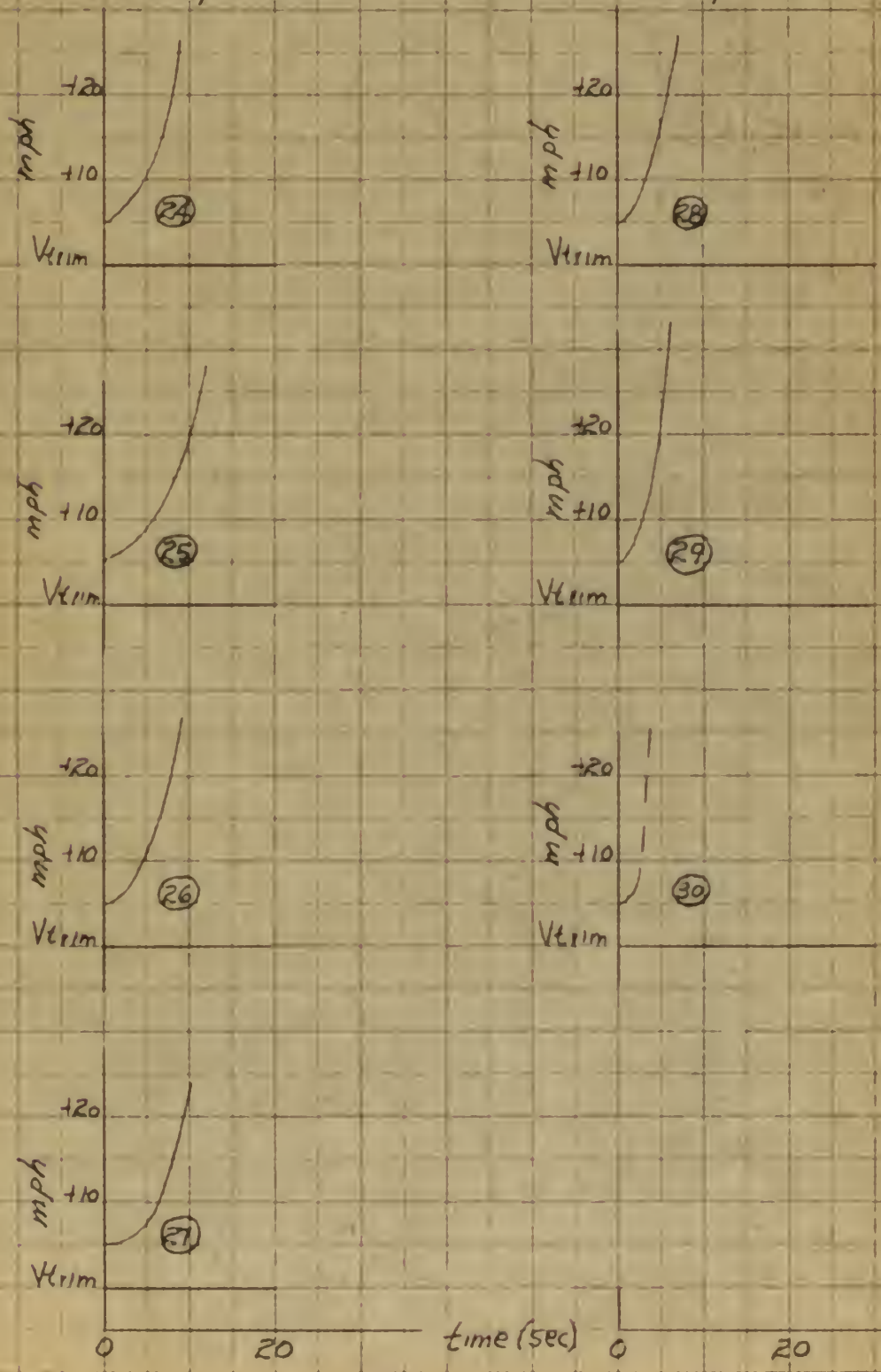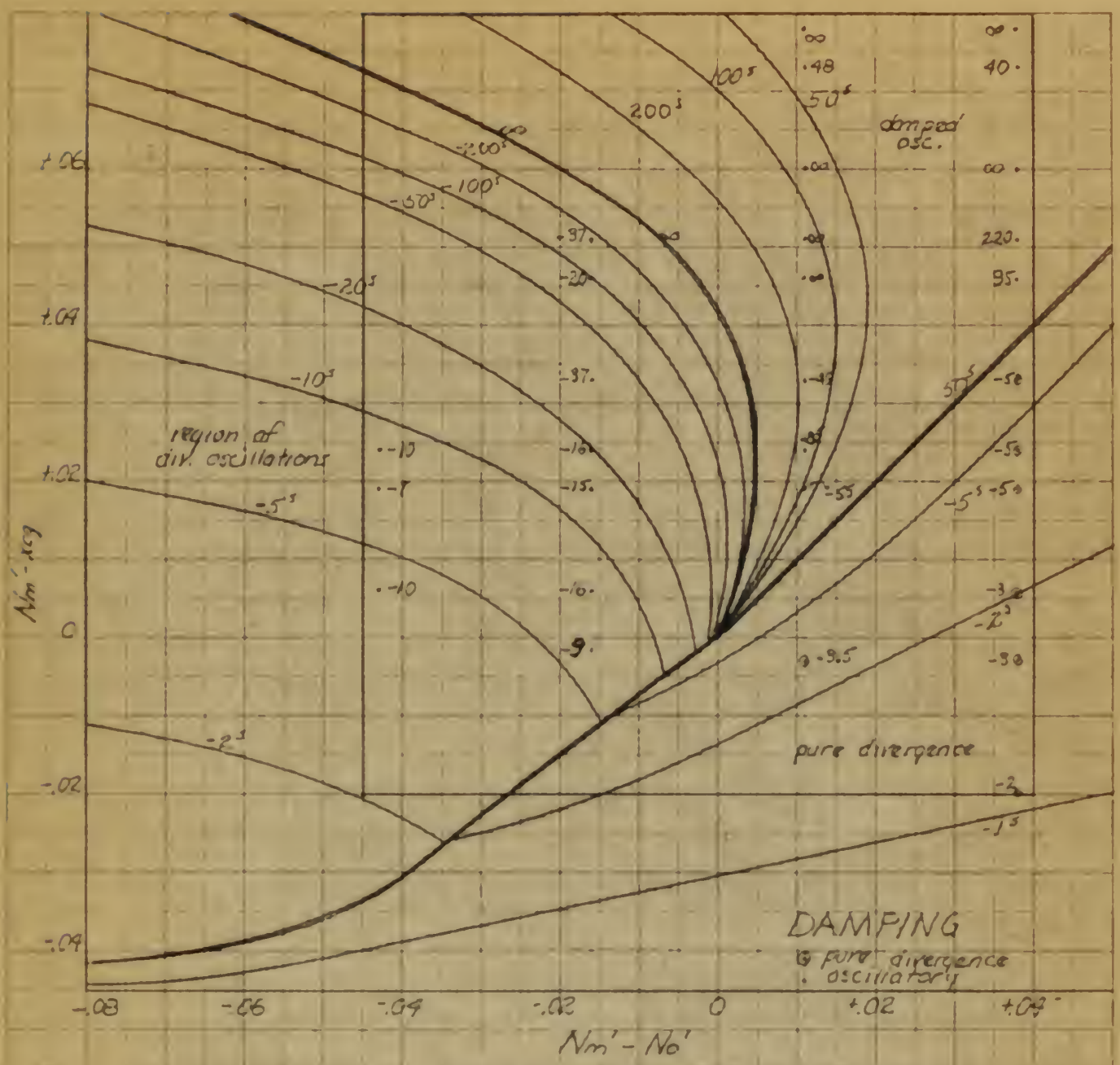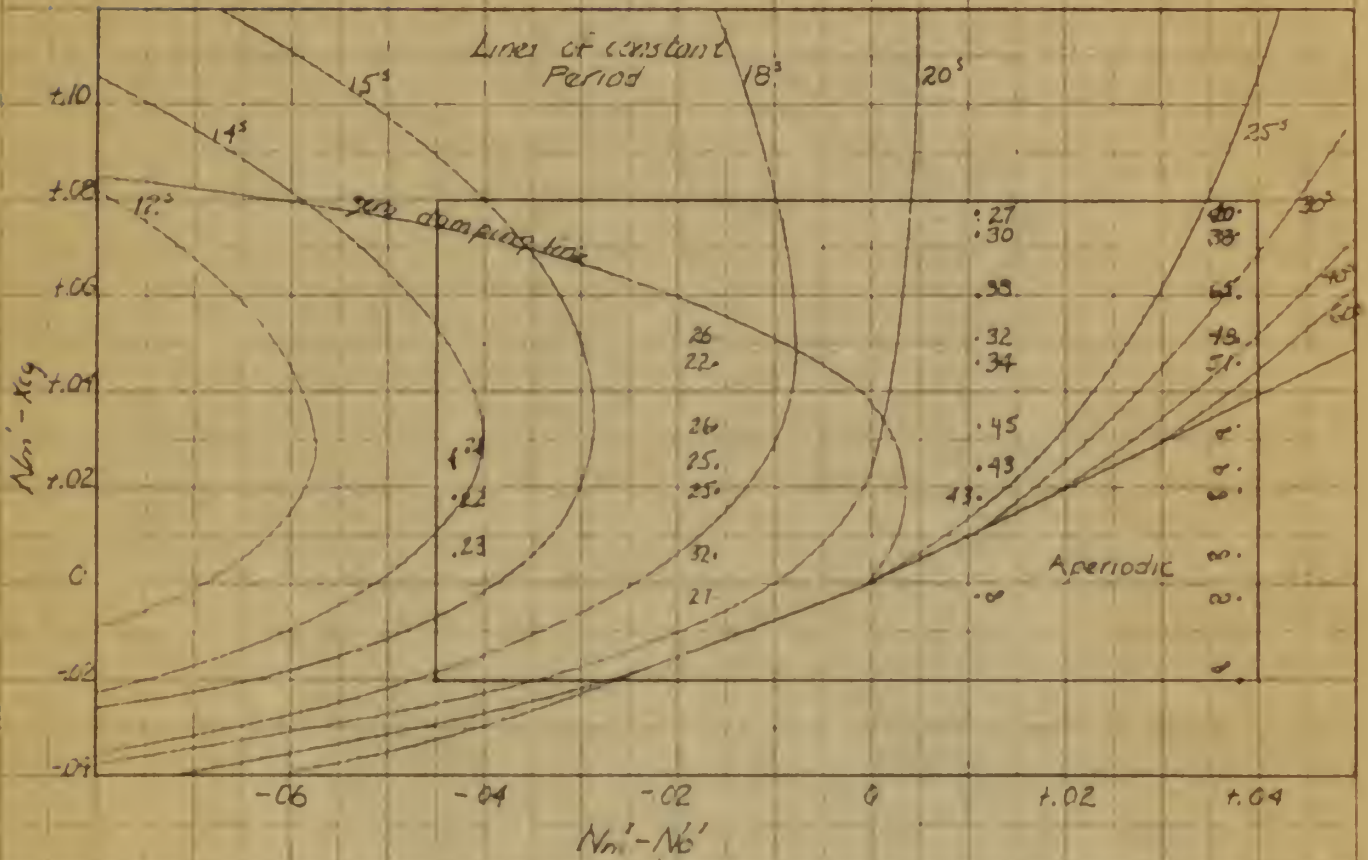
Key to test results, giving run number
in same relative position as test point above.
See Tabulation.