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An energy analysis of the pseudo Wigner-Ville distribution in support of machinery monitoring and diagnostics

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An Energy Analysis Of The Pseudo
Wigner-Ville Distribution In Support Of
Machinery Monitoring And Diagnostics

by

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Lieutenant, United States Navy
B.S., United States Naval Academy, 1984

Submitted in partial fulfillment
of the requirements for the degree of

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ABSTRACT

The Pseudo Wigner-Ville Distribution is a signal transformation of an input time signal into a joint time-frequency domain, that provides an excellent characterization of an input signal as well as it's respective energy content. A smoothing window and energy sensitivity analysis of the discretized Pseudo Wigner-Ville Distribution is presented along with a machinery monitoring application using the resulting Wigner-Ville energy of the sampled signal. The ability to apply the Pseudo Wigner-Ville Distribution to both steady-state and transient machinery may enable the monitoring and diagnostics of virtually any piece of equipment.

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I would like to express my sincere appreciation to my thesis advisor, Dr. Y.S. Shin, for his indispensible assistance and guidance throughout the course of my studies at the Naval Postgraduate School. I would also like to thank Alan Pride and Kathy Medlin of NAVSEA for their support and funding of this project. Additionaly, I would like to thank the Mechanical Engineering Department technicians for their support on a multitude of topics. Finially, I would like to thank my family for their understanding, and specifically my wife Karen for her extreme patience.

I. INTRODUCTION

The recent world events have caused drastic changes in many areas and made the future that much more unpredictable. With this high degree of uncertainty and drastic financial cutbacks as evidenced in the Defense budget, the emphasis of technological advance and dominance is all the more heightened. With the requirement to "do more with less", it is imperative to find ways to save money. It is with this mind-set that better ways to prolong the life of operating machinery and defray unneeded maintenance costs be investigated. For many years, most transient machinery has gone unmonitored and removed at periodic time intervals for refurbishment regardless of the current machinery condition. Developing a method to monitor and diagnose these machines of a transient nature will provide a way to determine when overhaul or replacement is needed.

Vibrational data has long been used as the tool to measure a machine's health. Early methods of monitoring machinery had been for the *experienced* personnel to listen or touch the machine to detect if a fault exists, however, this method is dependent on human senses and the same individual conducting the monitoring. It was for this reason that vibration levels as monitored by a dynamic signal-analyzer are now used for machinery health monitoring. The vibration levels of a *good* condition establish a baseline to allow a means of comparison when subsequent measurements are taken. These dynamic signal analyzers are time independent and do not show the time dependency required by transient machinery. It is because of this limitation, that a three-dimensional time, frequency and energy representation such as the Pseudo Wigner-Ville Distribution is being investigated as a method to monitor transient machinery.

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A. PREVIOUS WORK AND ACCOMPLISHMENTS

This thesis is a follow-on work which has been preceded by LT G. Rossano and LCDR J. Calahorrano, both in conjunction with Prof. Y.S. Shin. Rossano established the initial form of the computer program that calculates the discretized Pseudo Wigner-Ville Distribution. He evaluated the effect that different signals and procedures had on the Distribution along with characterizing an actual pump signal. Calahorrano made some further improvements to the computer program and then applied this improved version of the computer program to characterize some classified signals. Both of these works have been a great benefit in dealing with characterizing signals, but have not investigated the smoothing method and energy or amplitude of the discretized Pseudo Wigner-Ville Distribution.

B. OBJECTIVES OF CURRENT RESEARCH

The Pseudo Wigner-Ville Distribution provides an outstanding time-frequency domain spectra representation of an input signal as shown through the works among others of Rossano and Calahorrano here at the Naval Postgraduate School. The purpose of this research is:

- To review and modify the computer program developed here at the Naval Postgraduate School to support the smoothing window and energy analysis work.

- To conduct a smoothing window sensitivity analysis.

- To investigate the energy content of the discretized Pseudo Wigner-Ville Distribution.

- To apply the developed smoothing window and energy principles to an actual machinery monitoring application.

- To investigate the feasibility of using the Pseudo Wigner-Ville Distribution energy as a pre-processed input to a Neural Network.

II. THE PSEUDO WIGNER-VILLE DISTRIBUTION

A. THEORETICAL BACKGROUND

1. Evolution

The current day Pseudo Wigner-Ville Distribution is a three dimensional (time, frequency, and energy) representation of a signal that is particularly well suited for analysis of non-stationary signals. Eugene Wigner [Ref. 1] first introduced the Wigner distribution in 1932 to study the problem of statistical equilibrium in the area of quantum mechanics. His work was furthered in 1948 by J. Ville [Ref. 2], who used the Wigner distribution in the area of signal analysis. A major contribution of Ville's, is the use of analytic signals as an input to the distribution vice the customary real signal. The advantages of using an analytic signal in the distribution are two-fold [Ref. 3]. First, the distribution of a real signal results in a symmetrical spectrum with only half of the result containing useful information, while the use of an analytic signal avoids this negative frequency redundancy. Secondly, by accounting for only the positive frequencies it satisfies both the practical and the mathematical completeness of the problem. The third part of the Pseudo Wigner-Ville Distribution (ie. Pseudo) , arises from the need to apply a smoothing window to the resulting distribution. This is a result of the presence of cross terms that arise from multiple component signals and will be covered in depth later.

The works of T.A.C.M. Claasen and W.F.G. Mecklenbrauker [Refs. 4,5, and 6] in 1980 have paved the way for many recent applications of the Pseudo Wigner-Ville Distribution. Their three part paper is an all encompassing work that has served to highlight the capabilities of the Pseudo Wigner-Ville Distribution and allow for greater knowledge of the subject with an emphasis on signal processing and analysis.

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A considerable amount of recent work using the Pseudo Wigner-Ville Distribution have come in the areas of optics [Refs. 7,8, and 9] and speech [Ref. 10 and 11]. Additionally, T.J. Wahl and J.S. Bolton of Purdue University [Ref. 12] have used the Pseudo Wigner-Ville Distribution to analyze structural impulse responses. Two recent papers have investigated the use of the Wigner-Ville Distribution in the area of machinery monitoring. Flandrin et. al. [Ref. 13] proposed the Wigner-Ville as a means to confirm a machinery diagnosis in the specific application of a Pressurized Water Reactor power plant and lastly, Forrester [Ref. 14] has investigated the Wigner-Ville Distribution as a method for fault detection in gears. As evidenced by the cited examples, the capability and adaptability of the Wigner-Ville Distribution is enormous with applications of it's use rapidly expanding.

2.   Function Definition

The Wigner-Ville Distribution is a transformation of a signal into the time-frequency domain. By definition, the cross Wigner-Ville Distribution is defined as:

$$WDF_{r,s} = \int_{-\infty}^{\infty} e^{-j\omega\tau} \; r(t+\frac{\tau}{2}) \; s^*(t-\frac{\tau}{2}) \; d\tau$$

where: $r(t)$ = a complex time history

$s(t)$ = a complex time history

t = time

w = frequency

$*$ = complex conjugate

Similarly, the auto Wigner-Ville Distribution is defined as:

$$WDF_{r,r} = \int_{-\infty}^{\infty} e^{-j\omega\tau} \; r(t+\frac{\tau}{2}) \; r^*(t-\frac{\tau}{2}) \; d\tau$$

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Since the concentration of this research is with the representation of a single input signal, the auto Wigner-Ville Distribution will be used. This expression is essentially a Fourier transform of the auto-correlation of a signal, which may be thought of as a three dimensional spectral density function.

3. Distribution Properties

There are several properties of the Distribution that are worth noting. The properties listed below have been shown in many works concerning the Wigner-Ville Distribution. These properties will be shown in the form of the auto Wigner-Ville Distribution [Ref. 15].

Time and frequency shift properties follow:

- A time shift in the signal corresponds to a time shift of the Distribution:

$$WDF_{s(t-\tau)s(t-\tau)}(t, \omega) = WDF_{s,s}(t-\tau, \omega)$$

- A frequency shift in the signal corresponds to a frequency shift in the Distribution:

$$WDF_{e^{j\Omega t}s, e^{j\Omega t}s}(t, \omega) = WDF_{s,s}(t, \omega-\Omega)$$

- Both a frequency and time shift in the signal corresponds to the same in the Distribution:

$$WDF_{e^{j\Omega t}s(t-\tau), e^{j\Omega t}s(t-\tau)}(t, \omega) = WDF_{s,s}(t-\tau, \omega-\Omega)$$

Since our concern is in the area of machinery monitoring and diagnostics, the aforementioned properties are a nessesity in the development of a suitable monitoring program.

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The next three properties provide information concerning the *energy* contained within the Wigner-Ville Distribution.

- The integration of the Distribution over the frequency domain provides the instantaneous signal power:

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} WDF_{s,s}(t,\omega)\,d\omega = |s(t)|^2$$

- The integration of the Distribution over the time domain provides the energy density spectrum:

$$\int_{-\infty}^{\infty} WDF_{s,s}(t,\omega)\,dt = |S(\omega)|^2$$

- The integration over both the time and frequency domain provides the total energy of the input signal:

$$\frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} WDF_{s,s}(t,\omega)\,dt\,d\omega = \|s\|^2$$

Again, as with the time and frequency properties, the energy contained in the Wigner-Ville Distribution must be definable in order to use this in a machinery monitoring program. Additional properties may be found in Reference 4.

4.   Cross Terms Within The Distribution

A particularly troublesome characteristic of the Wigner-Ville Distribution is the presence of cross terms. These cross terms result in the Distribution taking on negative values as shown later in Figure 2. Since the properties above have shown that the Wigner-Ville Distribution represents an energy value, these negative values raise an area of concern. Jones

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and Parks [Ref. 16] show that these cross terms are essentially the interference of multiple component signals that occur when there is a non-zero cross-correlation within the signal. The equation listed below shows the cross-term that results from a signal that consists of two components.

$$WDF_{x+y}(t, \omega) = WDF_x(t, \omega) + 2RE[WDF_{x,y}(t, \omega)] + WDF_y(t, \omega)$$

As shown, the cross-term is represented by the center term while the other tems represent their respective signal components. There are several methods for eliminating these terms and providing a more presentable Distribution and there has been a considerable amount of research in the are of an optimum smoothing method for the Wigner-Ville Distribution as indicated by References 3,4,17, and 18. The method chosen for smoothing the Wigner-Ville Distribution in this work is a post processing, sliding Gaussian window function. What is meant by post processing, is that the distribution is smoothed after calculation of the Wigner-Ville Distribution.

5. Gaussian Smoothing Function

The Gaussian window function was first proposed mathematically as a means to smooth the Wigner-Ville Distribution by N.D. Cartwright [Ref. 19]. The continuous form of the Gaussian window function follows:

$$G(t, \omega) = \frac{1}{2\pi\sigma_t\sigma_\omega} e^{-(\frac{t^2}{2\sigma_t^2} + \frac{\omega^2}{2\sigma_\omega^2})}$$

Wahl and Bolton in Reference 12 have furtherd this work and developed the sampled form of the window function that is used in our discretized form of the Pseudo Wigner-Ville

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Distribution for smoothing. The effect of various size smoothing windows will be presented in greater depth in Chapter III.

B.   WIGNER-VILLE DISTRIBUTION COMPUTER PROGRAM

The computer program listed in the Appendix is a method that was developed here at the Naval Postgraduate School for calculating the discretized Pseudo Wigner-Ville Distribution. As mentioned earlier, the program that computes the discretized Pseudo Wigner-Ville Distribution is largely the work of G. Rossano [Ref. 15], who initially investigated the use of the Wigner-Ville Distribution as a tool for machinery monitoring. For the purposes of this work, the program has been thoroughly reviewed and rewritten to allow for ease of readibility and modification. The current version enables the user to look at not only the resulting smoothed version of the Pseudo Distribution, but also the unsmoothed Wigner-Ville Distribution. The second major modification was to program the smoothing window subroutine to allow for the use of different size windows in smoothing the Distribution. These changes have proven invaluable for evaluating the effect of various sized smoothing windows and in conducting the energy sensitivity analysis. Presented below are the two major equations that comprise the calculation of the discretized Pseudo Wigner-Ville Distribution as performed in the program listed in the Appendix.

1.   Discrete Time Wigner-Ville Distribution

As mentioned earlier, the Wigner-Ville Distribution is essentially a Fourier transform of an auto-correlation function. There have been a couple of equations developed to represent the discrete Wigner-Ville Distribution. The version that has been used in the development of the program listed in the Appendix, is one that was developed in Reference 4 by Classen and Mecklenbrauker and and later used in the work of Wahl and Bolton [Ref. 12] and Rossano [Ref. 15]. The only difference of note between the various discrete versions is essentially a difference in constants. All of the versions are basically a Fourier transform

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of an auto-correlation function. Shown below is the discretized equation that has been used for programming purposes in this work:

$$WDF_{s,s}(i\Delta\omega, j\Delta t) - \sum_{j-1}^{N} Re\{FFT[2\Delta t\ CORR(i)]\}$$

where: Re = Real part

Corr(i) = complex auto-correlation of the signal s(t)

FFT = Fast Fourier Transform

Δt = sample time

2.   Discretized Gaussian Window Function

Due to the need to deemphasis the resulting cross-terms within the Wigner-Ville Distribution, a suitable method for smoothing has been obtained. The sampled equation of the selected smoothing window is shown below [Ref. 12]:

$$\hat{G}(t,\omega) - \frac{1}{2\pi\sigma_t\sigma_\omega}e^{-[\frac{(m\Delta t)^2}{2\sigma_t^2} + \frac{(n\Delta\omega)^2}{2\sigma_\omega^2}]}$$

where: $-2M\leq m\leq 2M$ & $-2N\leq n\leq 2N$

$\sigma_t = M\Delta t$ & $\sigma_\omega = N\Delta\omega$

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III. SMOOTHING WINDOW ANALYSIS

As shown above, the smoothing function or window that is used in this work is a Gaussian window that is applied to the *unsmoothed* Wigner-Ville Distribution. Figure 1 is the raw time input signal of a 100 & 400 Hz sine wave that will be processed through the Wigner-Ville Distribution. Figure 2 is the unsmoothed Distribution that results from this combined 100 and 400 Hz sine wave. As can easily be seen in Figure 2, the resulting cross terms are very dominant and need to be deemphasized. This Gaussian smoothing function is applied after the Wigner-Ville Distribution has been calculated (ie. post processing), which allows for visual inspection of both the unsmoothed Wigner-Ville Distribution and the effect that different sized windows may produce on the smoothed Distribution (ie. Pseudo). For example, Figure 3 is the smoothed result of Figure 2 after applying a 13 by 13 sized window. The next few sections will discuss some of the characteristics of this post-processing smoothing window and what may be gained or lost through varying the size of the applied window.
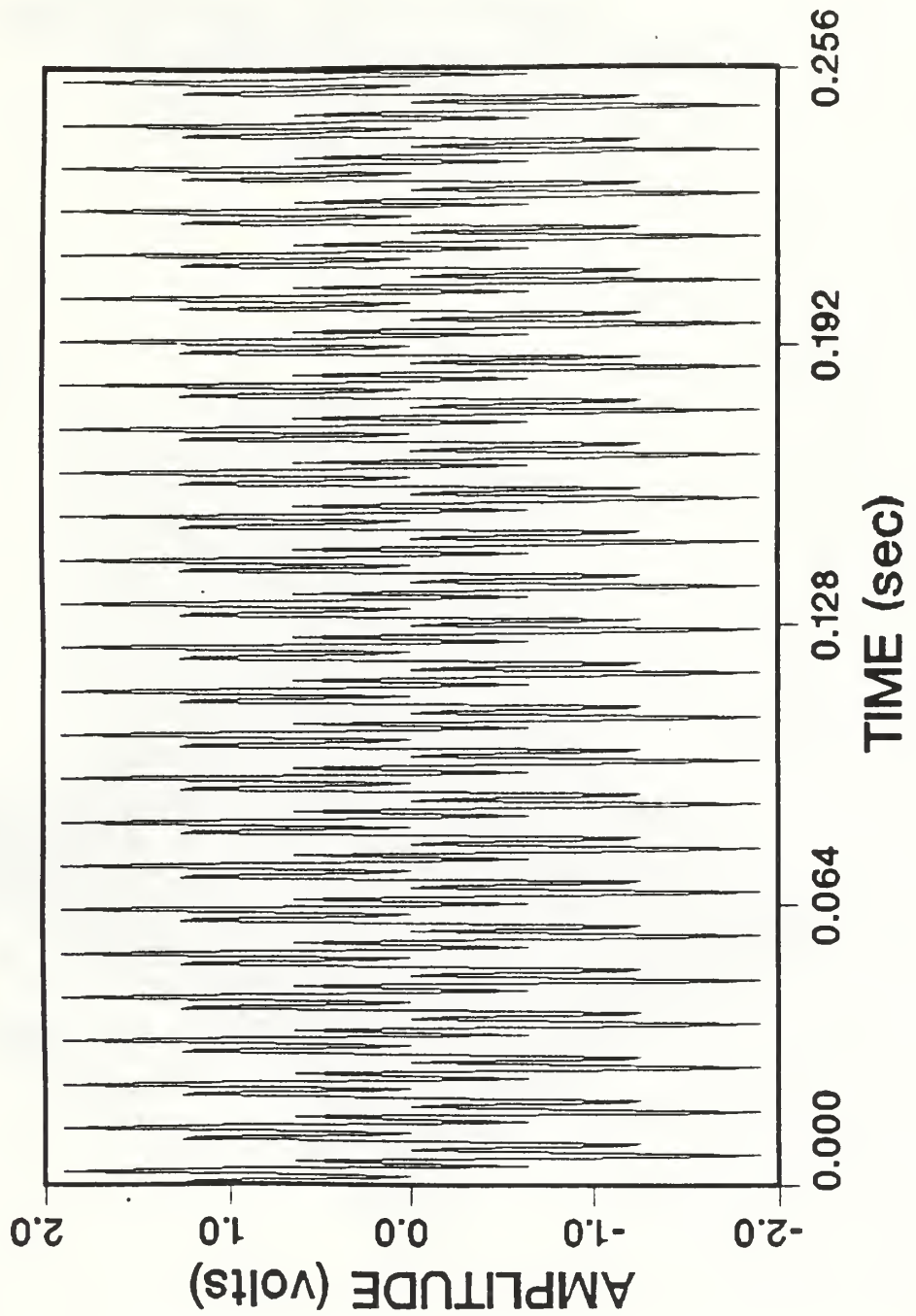
Figure 1. 100 & 400 Hz Sine Wave Input

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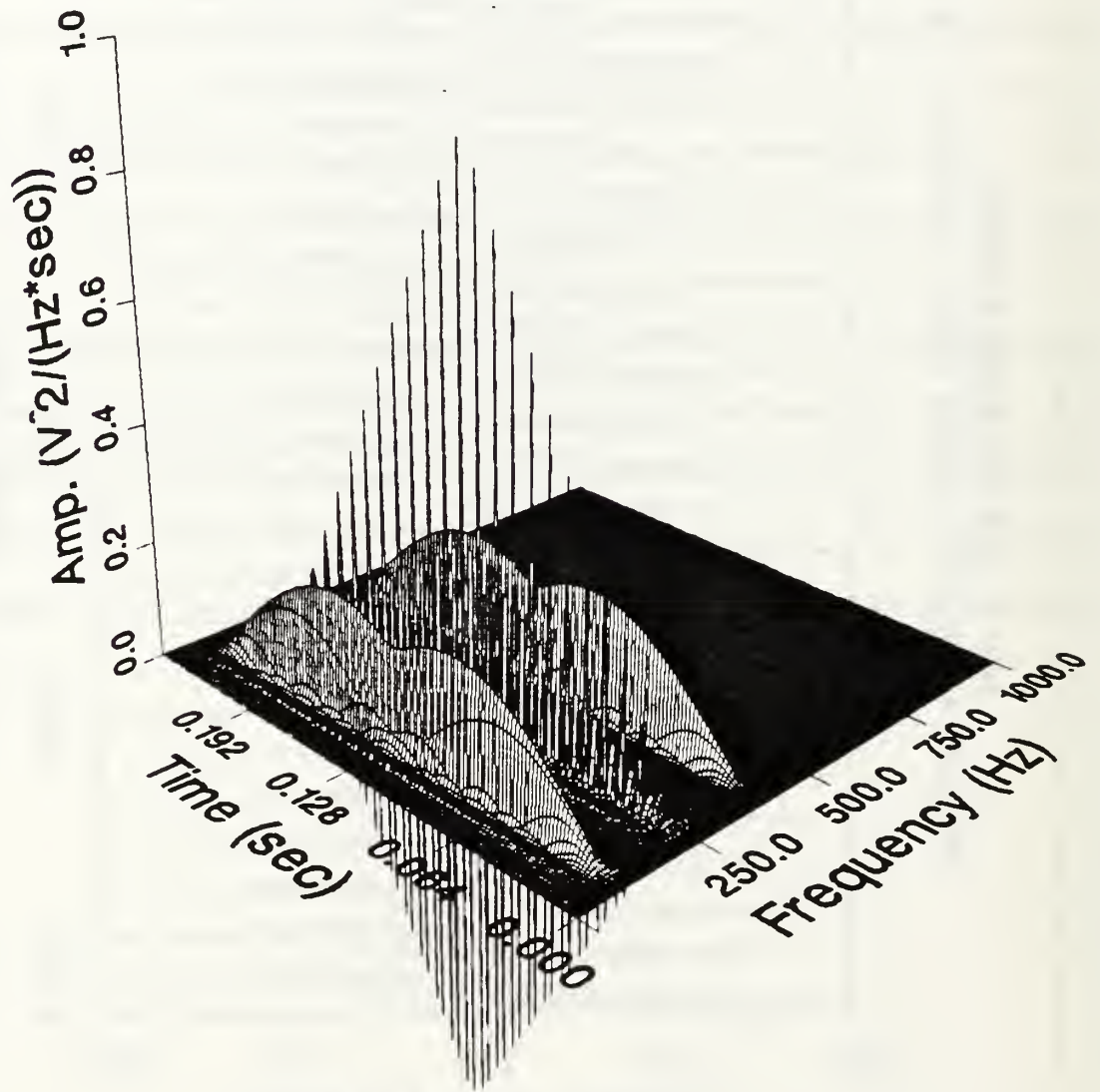
Figure 2. Unsmoothed Wigner-Ville Distribution
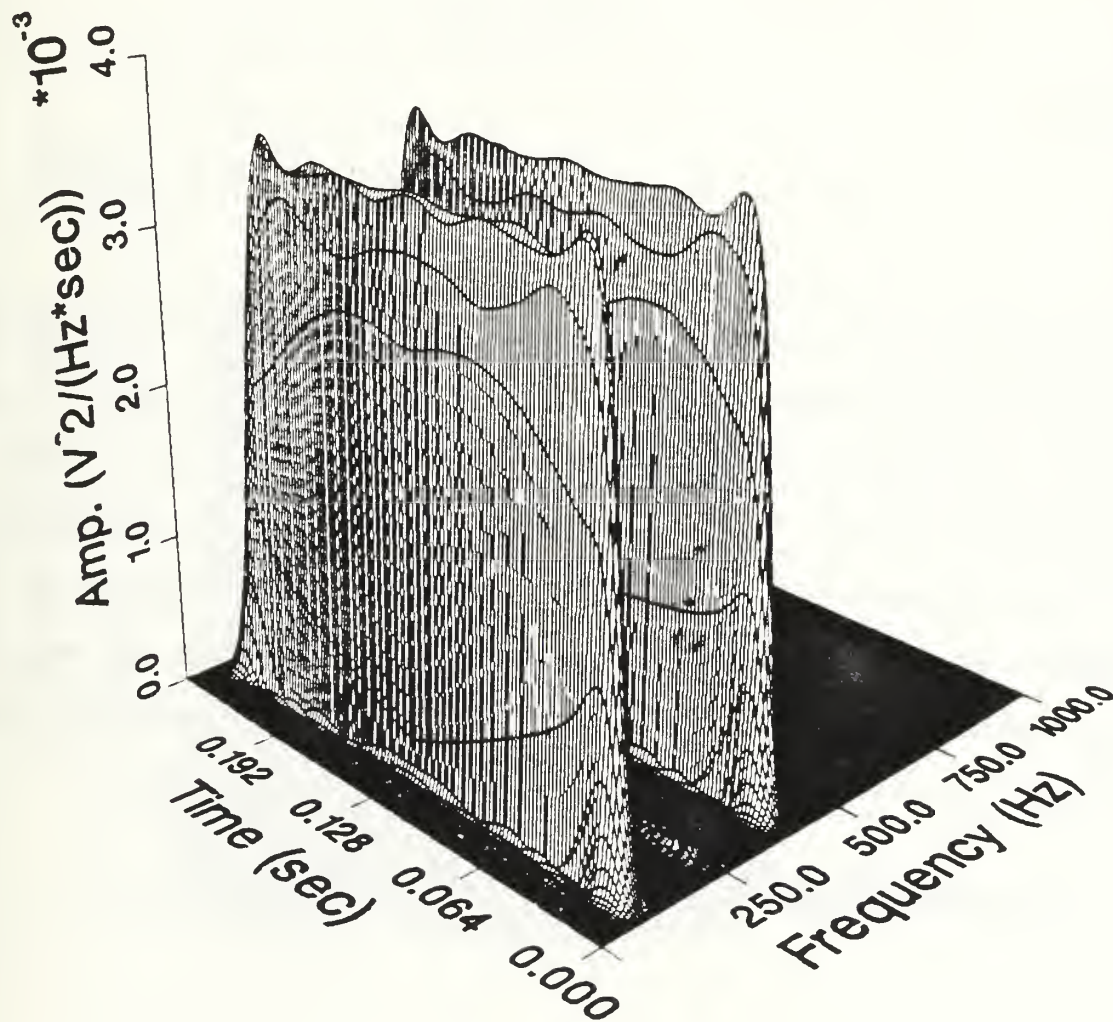
Figure 3.   Pseudo Wigner-Ville Distribution (13 X 13 Window)

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A. SIZE CRITERION

As shown by N.D. Cartwright [Ref. 19] for the continuous form of the Wigner-Ville Distribution, the size of the smoothing window may be selected such that a positive Distribution will result. Following, are these criterion that have been mathematically proven for the continuous case to assure a positive distribution:

$$\sigma^2_t \; \sigma^2_\omega \; \geq \; \frac{1}{4}$$

where:

$$\sigma_t \; - \; M\Delta t \qquad\qquad \sigma_\omega \; - \; N\Delta\omega$$

By chosing M and N such that this criterion is met, the resulting smoothed discretized Wigner-Ville Distribution does not necessarily result in all of the values being positive. This may be seen in Figure 4, where the window that was used was again a 13 by 13 (ie. N = 13 & M = 13) sized window. However, it does deemphasize the large cross-terms to the point that they are nearly zero and yields a very presentable Pseudo Wigner-Ville Distribution. The ability to deemphasize these cross terms is a necessity if the desire is to evalute the energy change within a signal. For the remainder of this work, the window size selected (ie. N & M, N X M, or N by M) will be such that this criterion is met as closely as possible.
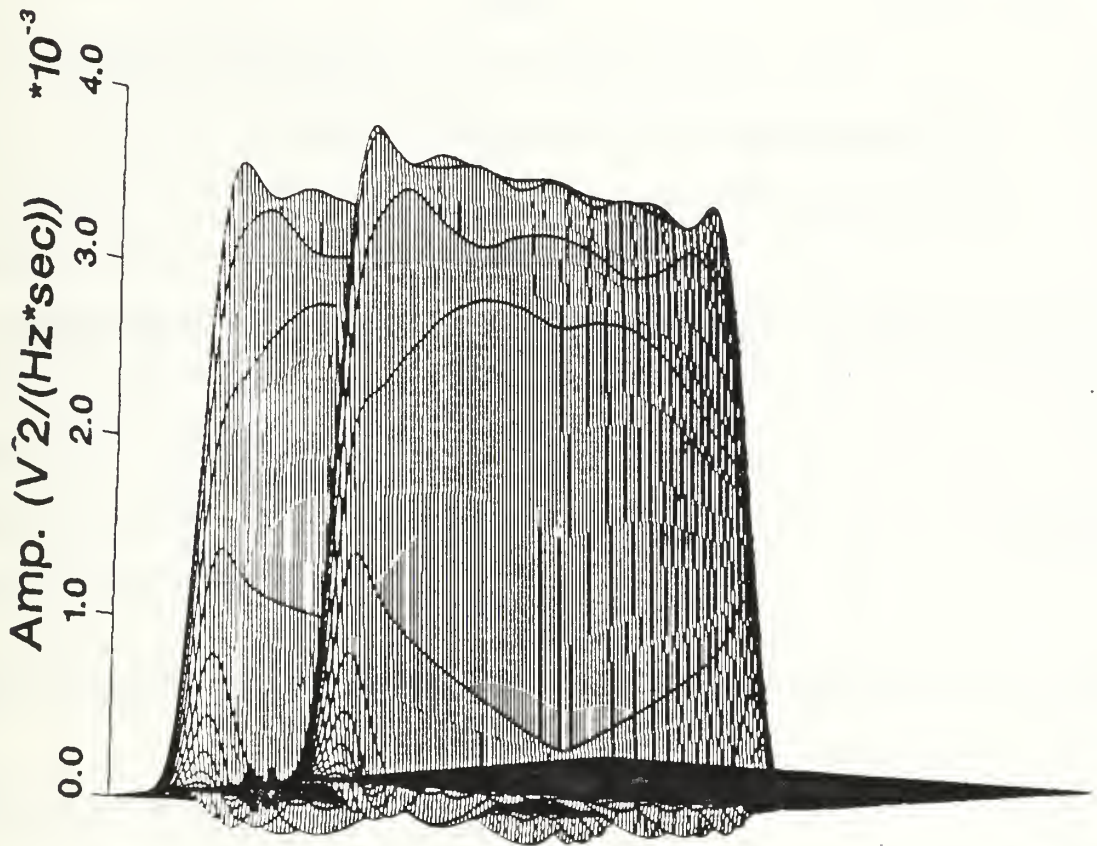
Figure 4. Smoothed Distribution with Negative Values

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B. TIME AND FREQUENCY DEPENDANCE

Since the criterion shown previously are time and frequency dependent, there must be a relation between the two that can be reduced to one parameter. The customary time and frequency relations of signal analysis are slightly changed in the context of the Wigner-Ville Distribution. Shown below are the time and frequency relations when working with the Wigner-Ville Distribution.

$$t_{max} = DP \times \Delta t \qquad ; \qquad \Delta f = \frac{1}{4 \times t_{max}} \qquad ; \qquad f_{max} = 2 \times DP \times \Delta f$$

where: t_{max} = maximum signal length   & f_{max} = maximum frequency

Δt = sampling time   & Δf = frequency resolution

DP = number of data points

The definitions of the size criterion discussed earlier for the smoothing window follow:

$$\sigma^2_t \sigma^2_\omega \geq \frac{1}{4}$$

and since:

$$\sigma^2_t = (M \Delta t)^2 \qquad \sigma^2_\omega = (N \Delta \omega)^2$$

and the time and frequency are related in the Wigner-Ville Distribution through the number of data points as shown:

$$\Delta \omega = \frac{\pi}{2 \ DP \ \Delta t}$$

Making this substitution into the size criterion, yields a very simple result for determining the size of the smoothing window required.

$$M \times N \geq \frac{DP}{\pi}$$

In looking at this result it seems as if the window is completely independent of time and frequency, but it must be rememberd that M and N are parameters that specify the lengths of the smoothing window in the time and frequency directions respectively.

C.   RELATION TO NUMBER OF DATA POINTS

Another area of interest is the relation of the window size to the number of data points in the sampled signal. The best way to show this relation is through an example. For instance, a signal of a length 4.0 seconds that is sampled using 512 data points, will result in a $\Delta t = 7.8125$ msec and $\Delta f = 0.0625$ Hz as calculated using the constraints of time and frequency as defined by the Wigner-Ville Distribution. Since the example has used 512 data points, the previous section has shown that chosing N = 10 and M = 16 will approximately satisfy the size criterion limitations. Additionaly, by definition of the smoothing window, the area of coverage will be:

$$-2M \leq m \leq 2M \qquad -2N \leq n \leq 2N$$

This means that for the example shown, the coverage of the window will be 40 steps in the frequency domain and 64 steps in the time domain.

$$40 \times \Delta f = 2.5 \qquad 64 \times \Delta t = 0.5 \qquad AREA = 1.25$$

where: $\Delta f = 0.0625$ Hz & $\Delta t = 7.8125$ msec

The AREA = 1.25 is a unitless quantity that defines the area that the smoothing window encompasses while smoothing one point in the Distribution. The unitless dimension is the result of the time being in seconds and the frequency in hertz. For comparison purposes, the total area of the distribution is the product of f_{max} and t_{max} which is equal to 256.0, resulting in the window smoothing over 0.488% of the total Distribution, while smoothing one point.

Now consider another example that uses 1024 data points. As shown earlier, the product of M and N must be greater than or equal to the number of data points divided by pi in order to meet the window size criterion as established by Reference 12. With respect to the earlier example, the size of the window (ie. N times M) must increase two times. Therefore, a selection of N = 10 and M = 32 will be used for this example. These values are very close to the required criterion and will suffice for this illustrative example. The signal length (t_{max}) will be 2.0 seconds resulting in $f_{max} = 256.25$ Hz, $\Delta t = 1.953$ msec, and $\Delta f = 0.125$ Hz. In relation to the area of coverage of the window, the following results apply:

$$40 \times \Delta f = 5.0 \qquad 128 \times \Delta t = 0.249984 \qquad AREA = 1.25$$

where: $\Delta f = 0.125$ Hz & $\Delta t = 1.953$ msec

This is the same result as the 512 data point case, with the following exception. The total area of the Distribution ($t_{max} \times f_{max} = 512.0$) has doubled, while the area of the smoothing window coverage has remained the same. This results in the window only covering 0.244% of the entire Distribution when smoothing a single point within the Distribution. In conclusion, the greater the number of sampled data points within the processed raw time signal, the smaller the area the smoothing window will encompass while smoothing the Distribution. An

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important trade-off to consider, however, is the increase in computer time required to run the additional number of data points.

D.   TIME/FREQUENCY RESOLUTION

Since the smoothing window used in this work gives the ability to vary the time and frequency size of the window, the following examples show the results that can be obtained. Figure 5, is the input signal that is sampled using 512 data points. It is a 400 Hz sine wave that has been computer generated with an amplitude of 2.0. In order to be consistent with the previous work, the size criterion established earlier will be used as a guide for the selection of M and N in the smoothing window. Figure 6, is the result of a window in which M and N are both equal to 13. Since this smoothing window does not require M and N to be equal, varying these parameters yielded some intersting results. Figure 7, shows the result of the input signal being smoothed with a 35 by 5 window. This means that N = 35 and M = 5, or in other words, the window is longer in the frequency domain than it is in the time domain. The resulting distribution has an amplitude that is much more constant throughout the entire time length, but the amplitude itself has been reduced considerably as compared to the 13 by 13 example. Figure 8 is just a different view of this example, showing the loss of frequency resolution with a 35 by 5 window. Just the opposite case to this example is where the window used for smoothing is a 5 by 35 window. In this case, the window is longer in the time domain than the frequency domain and just the opposite results are obtained. Figure 9, shows the result of this window, while Figure 10 gives a view showing the degree of frequency resolution that can be obtained. This can be very beneficial in the monitoring of machinery, since in many cases the ability to differentiate between signals that are very close to one another is paramount.
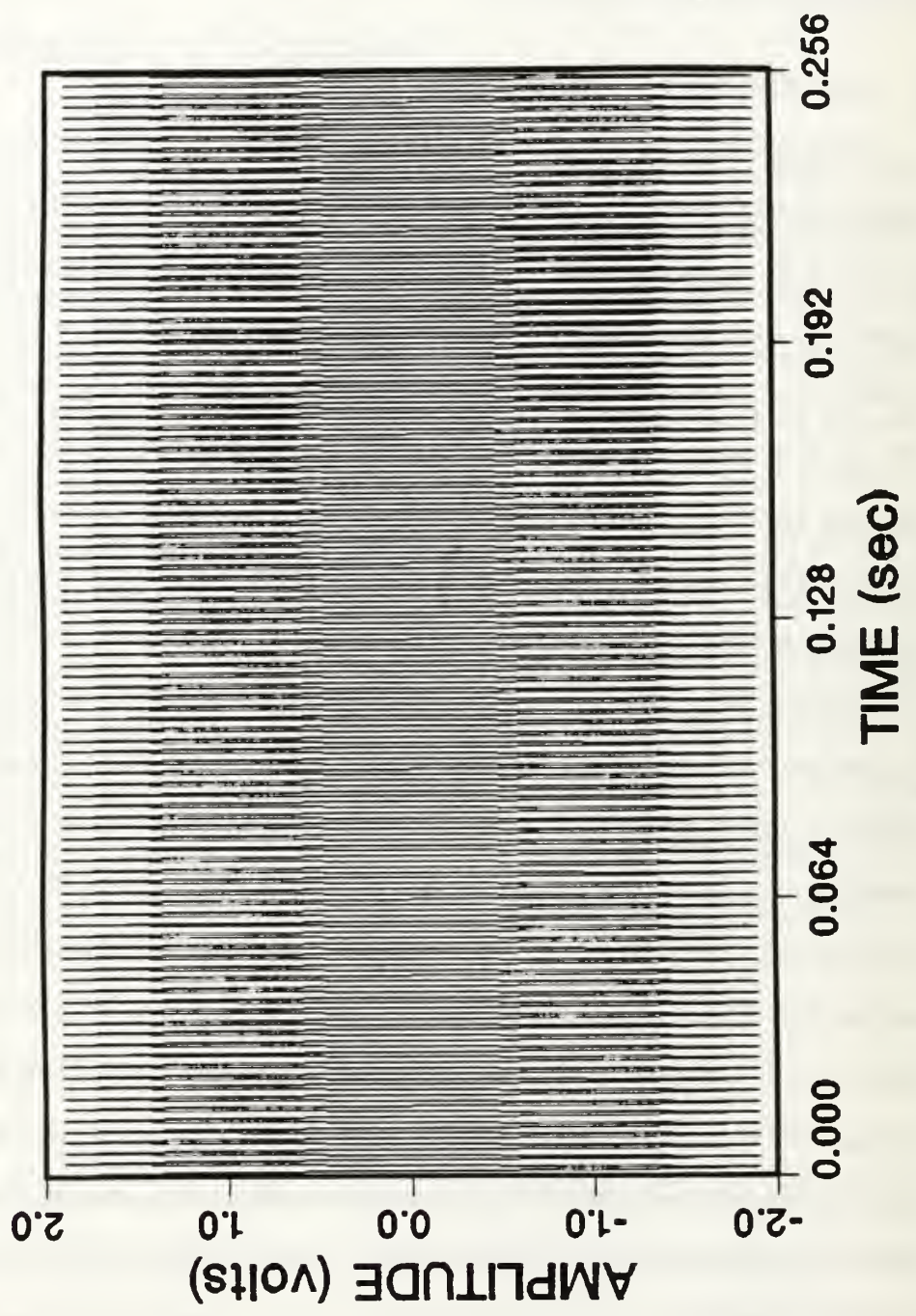
Figure 5. 400 Hz Sine Wave Input

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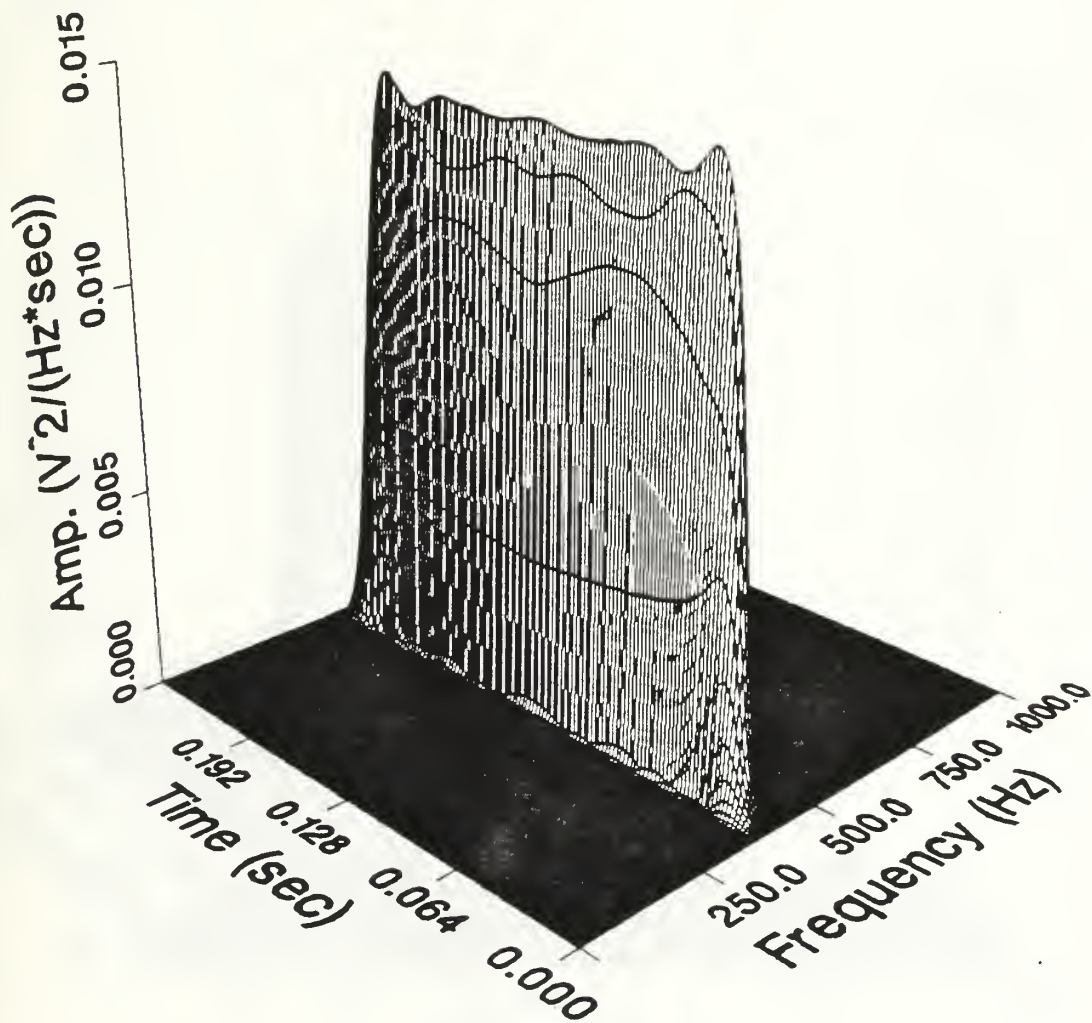
Figure 6. Pseudo Wigner-Ville Distribution (13 X 13 Window)
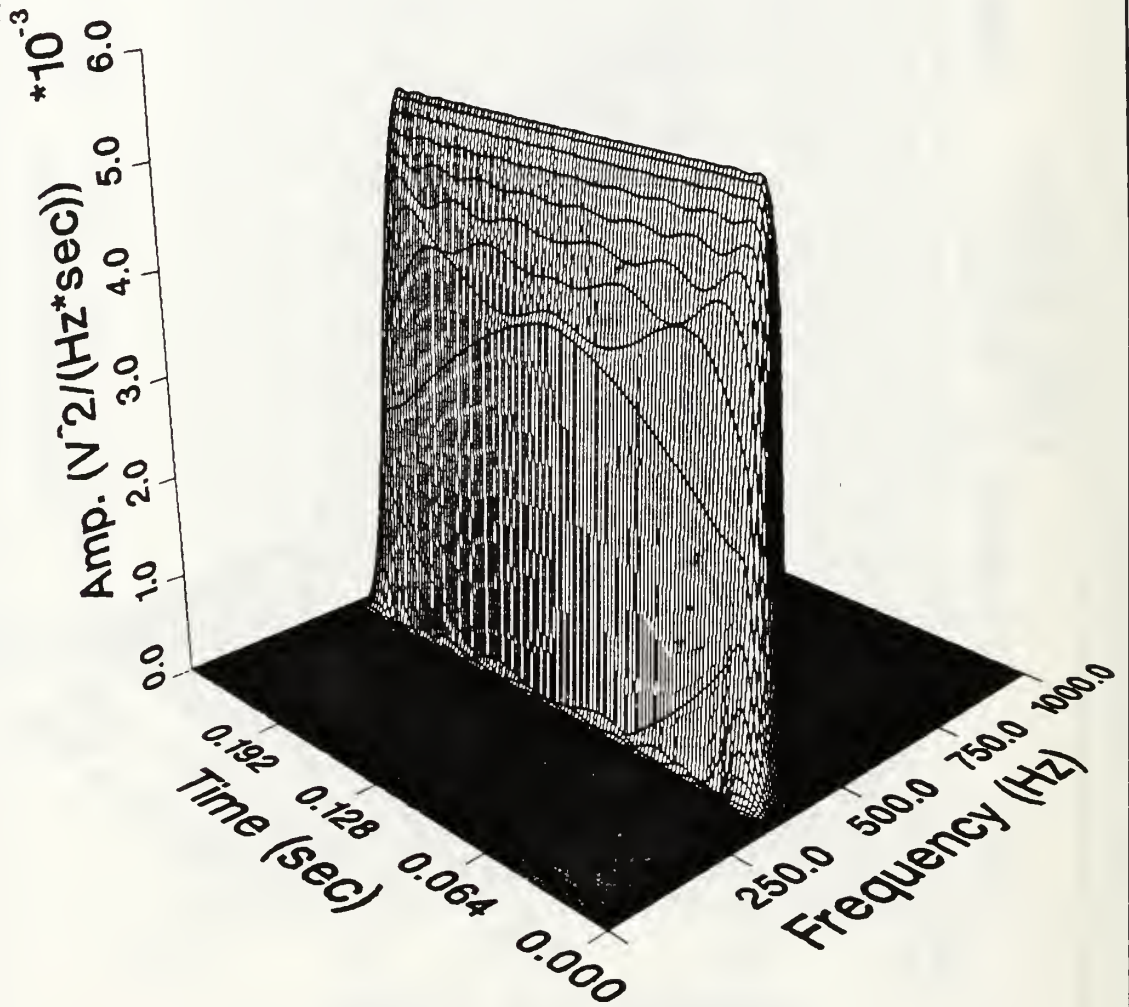
Figure 7. Pseudo Wigner-Ville Distribution (35 X 5 Window)

Figure 8. Frequency Resolution (35 X 5 Window)

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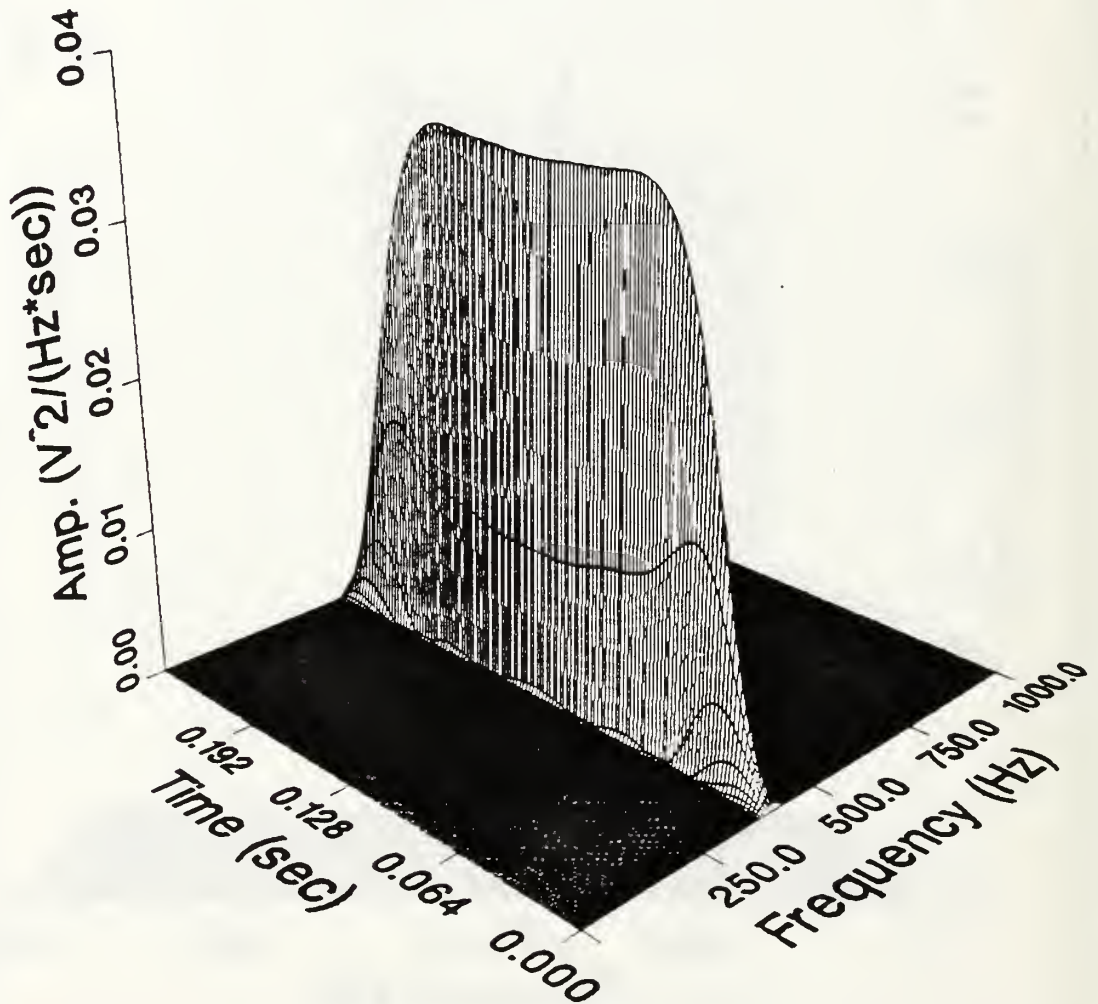
Figure 9. Pseudo Wigner-Ville Distribution (5 X 35 Window)

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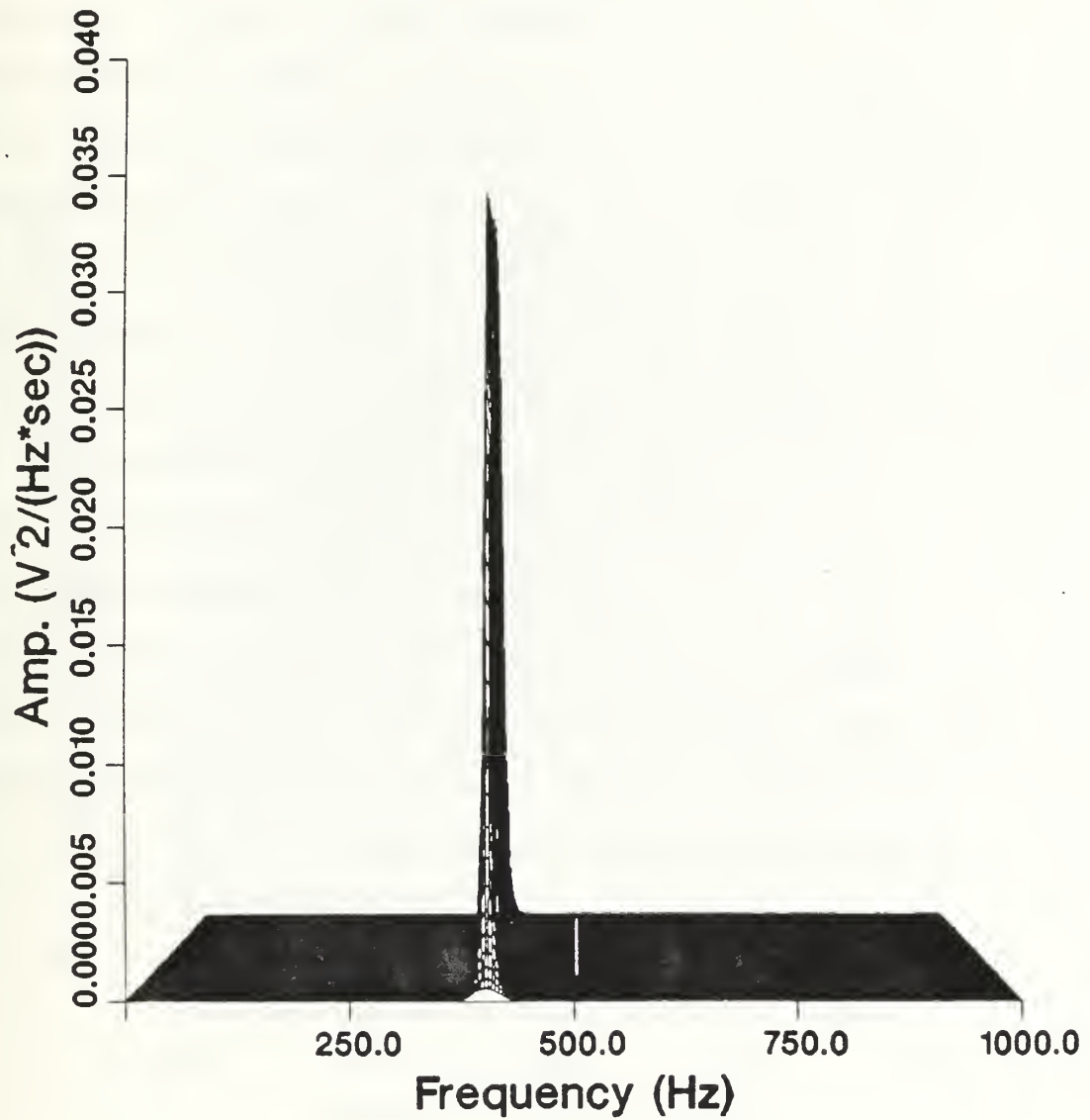
Figure 10. Frequency Resolution (5 X 35 Window)

E. TRANSITIONAL PEAK PHENOMENON

In works that have given examples of the Wigner-Ville Distribution for transient signals, the point at which the signal's frequency is changing from increasing to decreasing or vice versa has a large amplitude increase in the Wigner-Ville Distribution. For example, Rossano [Ref. 15] shows an artificially generated linear chirp and also an actual pump signature that has what appears to be a large concentration of energy at the transition point. In order to use the Wigner-Ville Distribution for transient machinery monitoring, there must be an explanation or way to quantify this "ghost peak" that is occuring at the transition point. To date, there has not been an explanation to this phenomenon, however, by looking at the unsmoothed Wigner-Ville Distribution, the reason for this is easily seen. Figure 11 is a linear chirp sine wave that is used as the input to the Wigner-Ville Distribution. Figure 12, shows the resulting *unsmoothed* Wigner-Ville Distribution of this linear chirp and as discussed and shown earlier, the cross-terms can be seen occuring exactly at the mid-point of the conflicting frequencies and times. These cross-terms build until they reach a maximum concentration at the transition or frequency change point. Since these are nothing more than cross-terms, there must be a way to deemphasis these through the judicial use of the smoothing window. Figures 13 through 16 show the result of varying the smoothing window from a point of essentially no effect to entire elimination of the ghost peak.
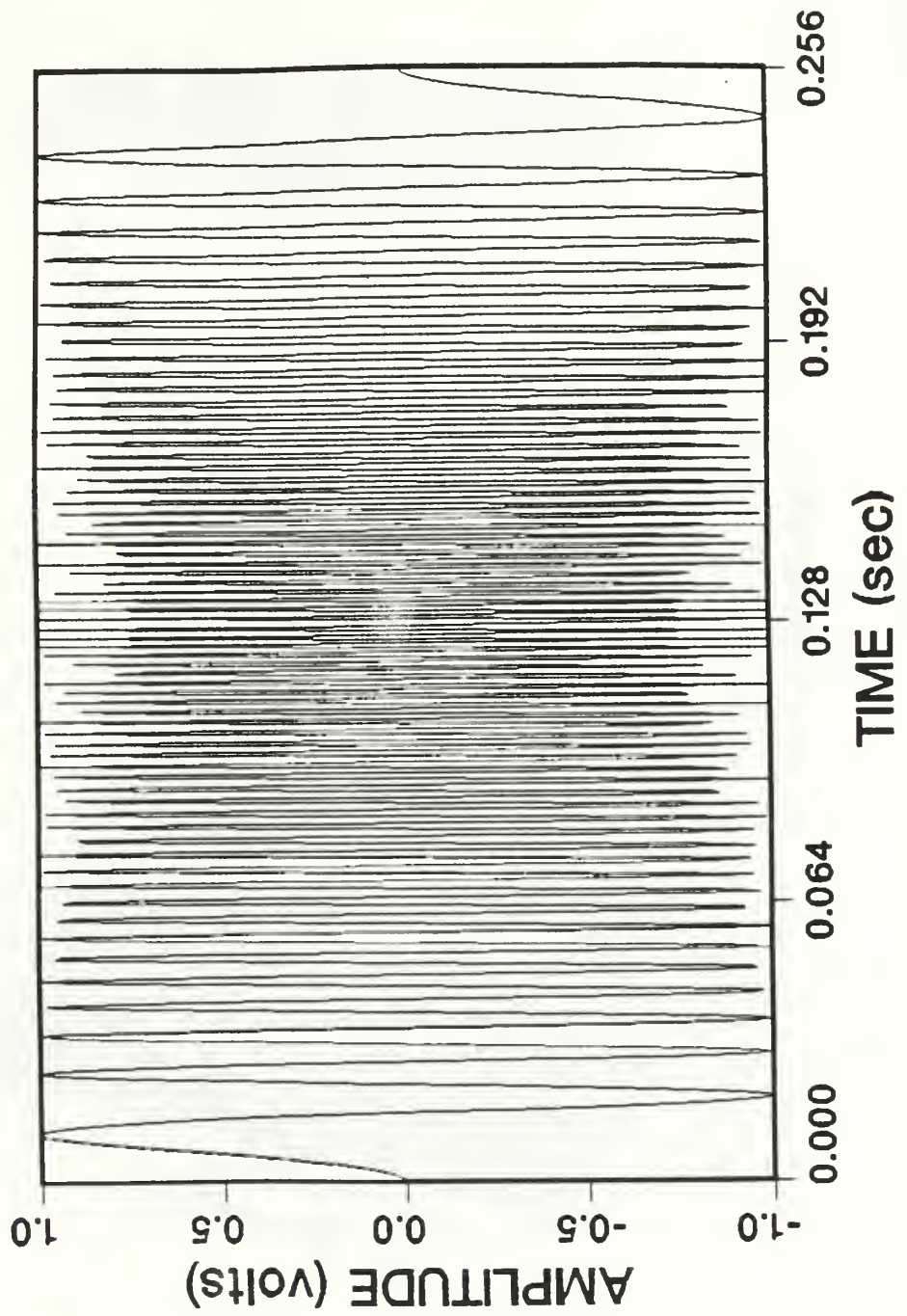
Figure 11. Linear Chirp Sine Wave Input

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LINEAR CHIRP SINE WAVE
UNSMOOTHED DISTRIBUTION
AMPLITUDE = 1.0
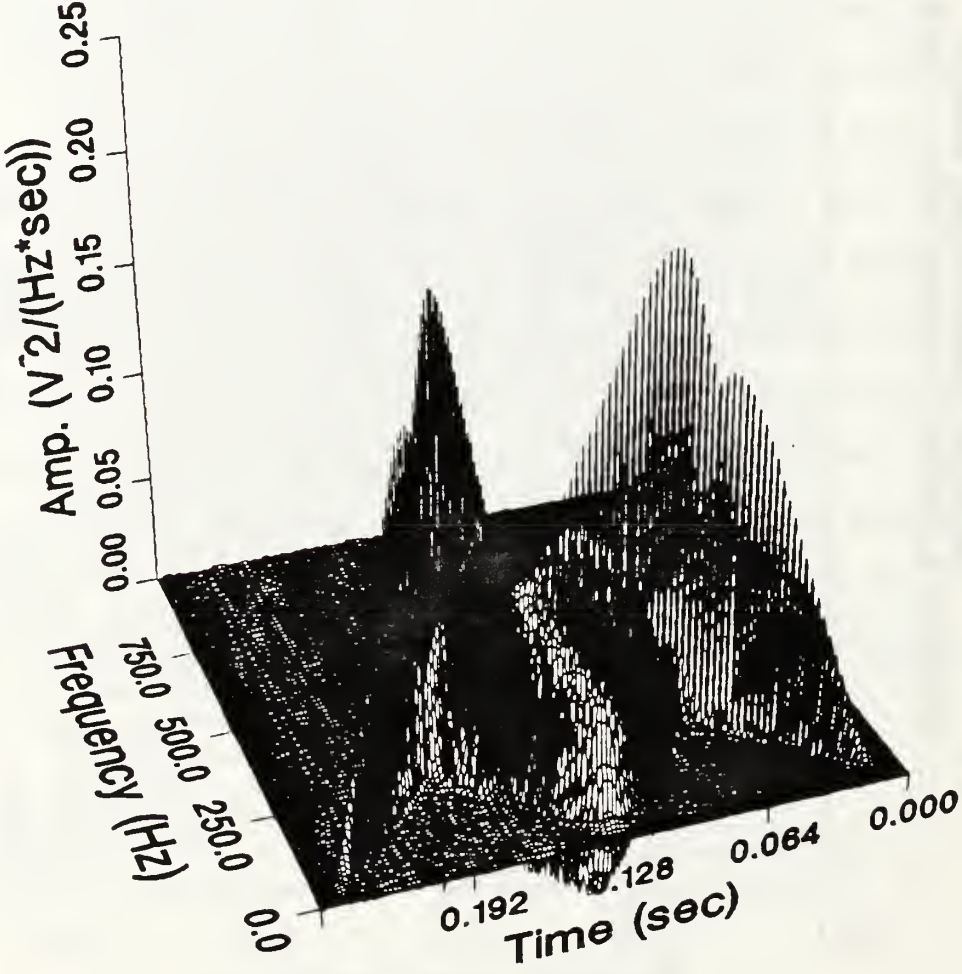
Figure 12. Unsmoothed Wigner-Ville Distribution

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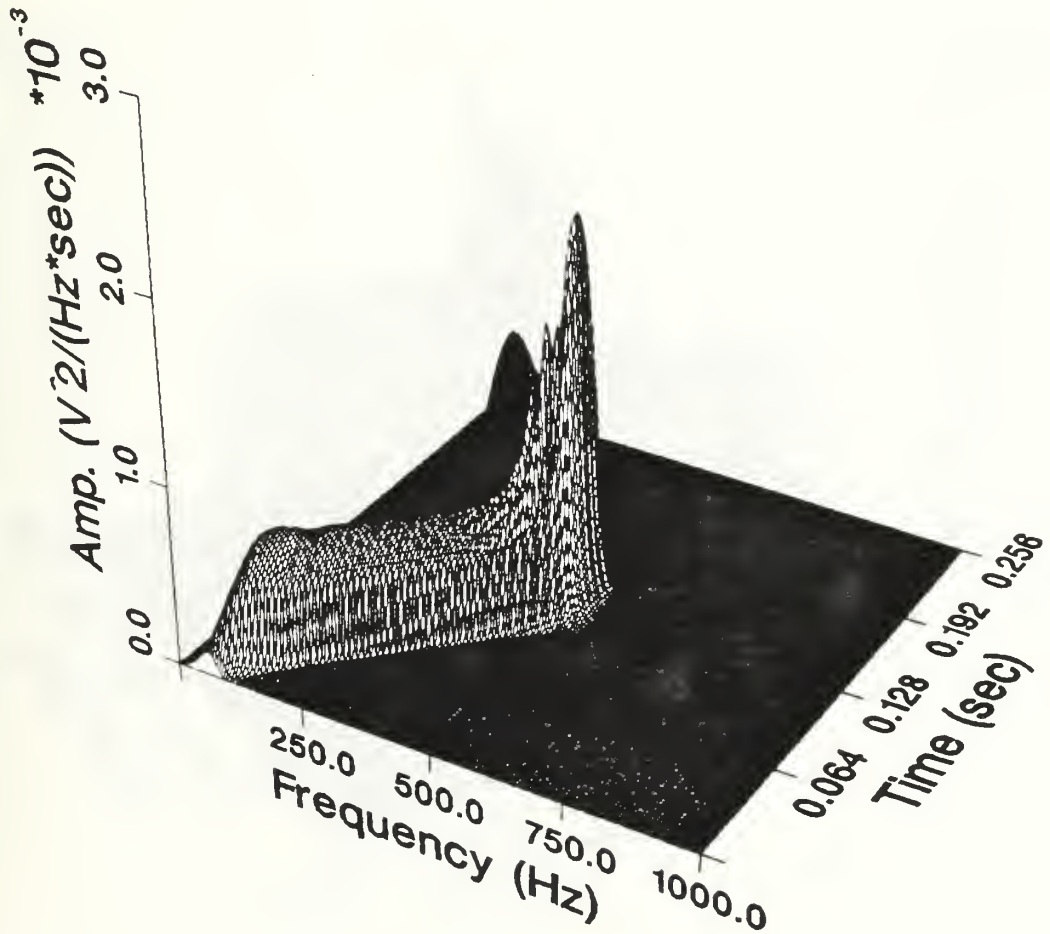
Figure 13. Pseudo Wigner-Ville Distribution (5 X 35 Window)
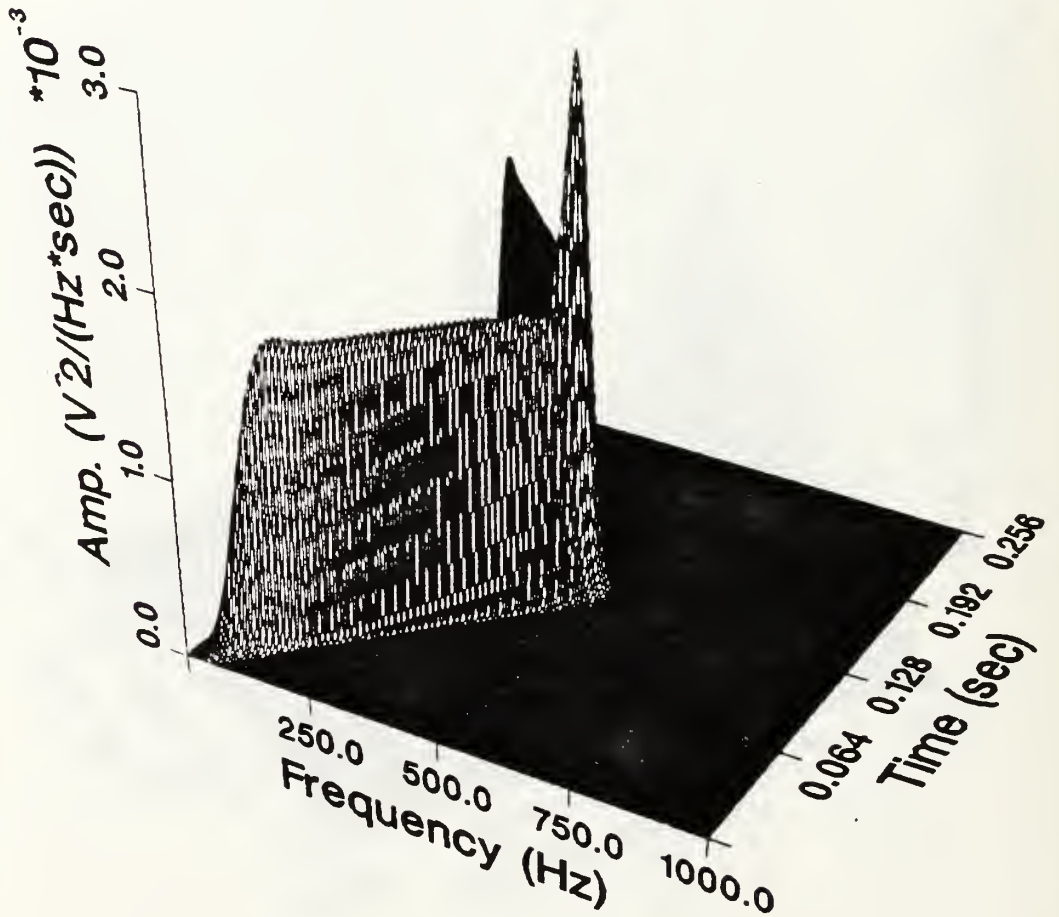
Figure 14. Pseudo Wigner-Ville Distribution (13 X 13 Window)

30
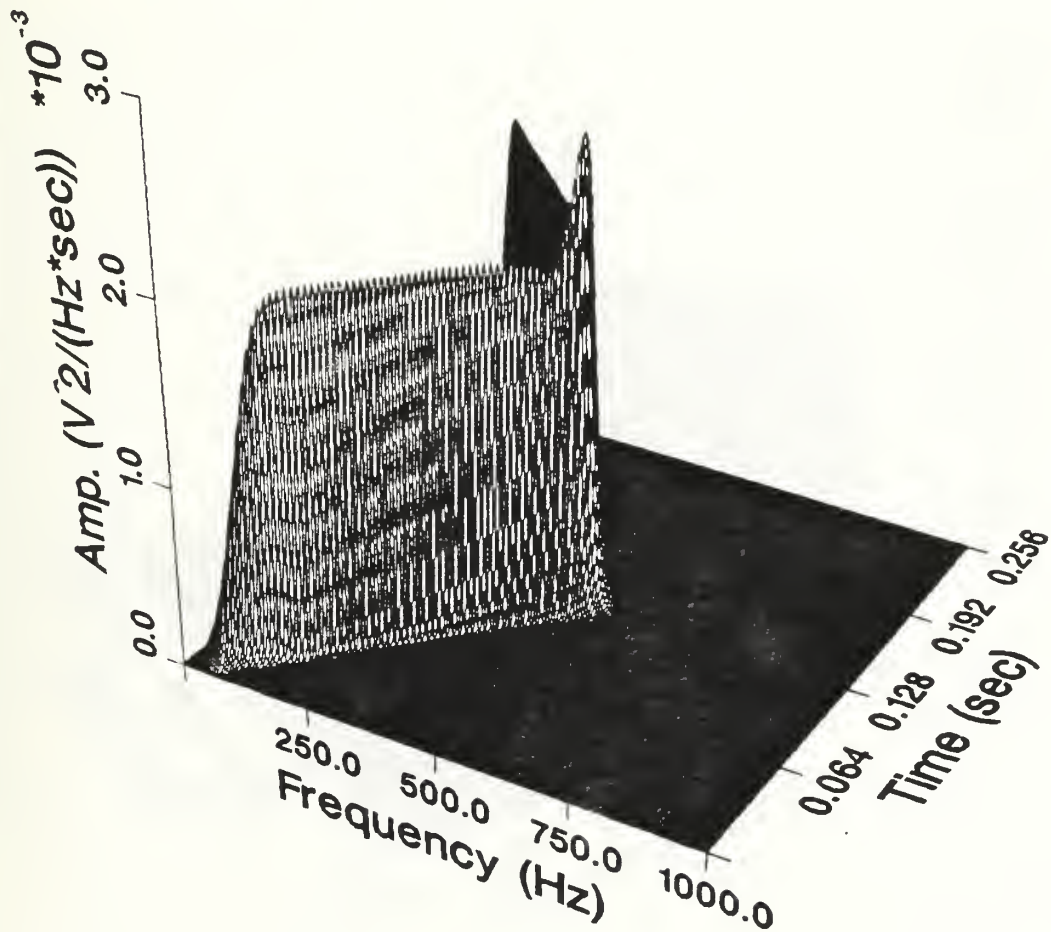
Figure 15. Pseudo Wigner-Ville Distribution (18 X 10 Window)

31
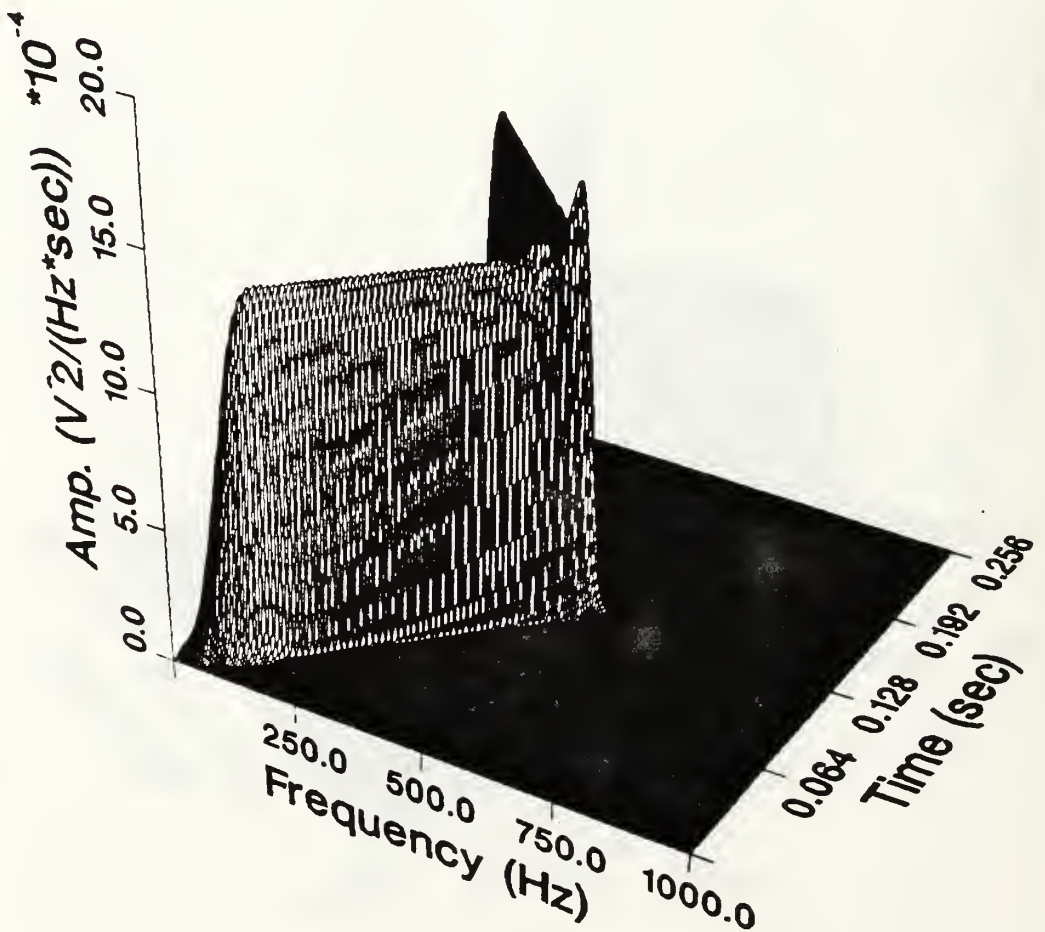
Figure 16. Pseudo Wigner-Ville Distribution (35 X 5 Window)

32

This chapter has shown the benefits that may be gained through the use of the smoothing window. The frequency resolution and amplitude are the two major areas affected by the window changes. The last area presented, showed the reason for the large amplitude increase at the frequency transition point and how these "ghost peaks" may be smoothed and to what extent deemphasized by adjusting the smoothing window size. Within the next chapter, a thorough energy analysis of the discretized Pseudo Wigner-Ville Distribution will be conducted.

IV. WIGNER-VILLE DISTRIBUTION ENERGY ANALYSIS

A. ENERGY DEFINITION

As mentioned earlier, the Wigner-Ville Distribution is essentially the Fourier transform of the auto-correlation of an input signal. This may be thought of as the spectral density function in a three dimensional sense. The total energy of the Wigner-Ville Distribution was listed earlier as:

$$\frac{1}{2\pi} \int\limits_{-\infty}^{\infty} \int\limits_{-\infty}^{\infty} WDF(\omega, t) \, d\omega \, dt - \psi^2$$

where:

$$\psi^2 - mean \ square \ value \ of \ the \ signal$$

There are a few points that must be made concerning this equation. The first is that the integration is from minus infinity to plus infinty and another is that it is the integration of the resulting Distribution of a real signal, not of an analytic signal as used in most discretized forms of the Wigner-Ville Distribution. Lastly is that this equation has been proven mathematically, which unfortunately does not take into account the fact that there are resulting cross-terms and that the Distribution is subsequently smoothed because of this. Since the last point is indicative of what is seen in actual real world problems, the energy must be defined in a discretized sense that accounts for the above mentioned points and also these unavoidable cross-terms.

B. EFFECT OF WINDOW SIZE ON ENERGY

Since smoothing of the Distribution is a necessary requirement, the size of the applied window is something that must be decided upon. Another important point is knowing what effect this will have on the energy of the Distribution. In the previous chapter, the size criterion that was established by Cartwright [Ref. 19] and later used by Wahl and Bolton in Reference 12 was discussed. Within this discussion, the visual benefits of varying the window size in the time and frequency domain were presented. In order to define the energy, the effect on the energy of the Pseudo Wigner-Ville Distribution as caused by varying these windows will be presented.

Figure 17 is a 400 Hz sine wave of length 0.256 seconds and amplitude 2.0 that will be sampled and have the Wigner-Ville Distribution calculated. Figure 18 is the resulting smoothed Wigner-Ville Distribution that has been processed using a 13 by 13 sized smoothing window. Figure 18 also shows that the volume is equal to 0.0709118 V^2. This is representative of the energy contained within the Wigner-Ville Distribution of the signal in Figure 17 and is nothing more than the volume under the resulting Distribution. By definition, this volume should be equal to the mean square value of the 400 Hz sine wave, however, due to the points mentioned above, this is not going to hold true and will be discussed in depth later. Figures 19 and 20 are also smoothed Distributions of this same 400 Hz signal only using a 35 by 5 window in the case of Figure 19, and a 5 by 35 window in Figure 20. Since N times M (the size criterion) are very close in these three examples, the resulting energy values of the three cases are essentially the same. The point to be emphasized from this is that as long as the area of the window does not vary significantly, the energy will be represented in a constant manner and independent of the time and frequency window size selected. The last example that will be presented in this section is that this result is independent of the frequency of the signal as shown by the combined 100 and 400 Hz sine wave example of Figure 21, in which the energy is equal to 0.1418314 V^2. This is essentially

twice the value of the single sine waves of Figures 18 through 20. Of special importance for transient machinery applications is Figure 22, which is a transient or ramped sine wave that shows the same resulting energy as the previous stationary signals. This shows that the Wigner-Ville Distribution energy is independent of the nature of the signal allowing for the possibility of monitoring a piece of machinery during both it's steady state and transient phases of operation.
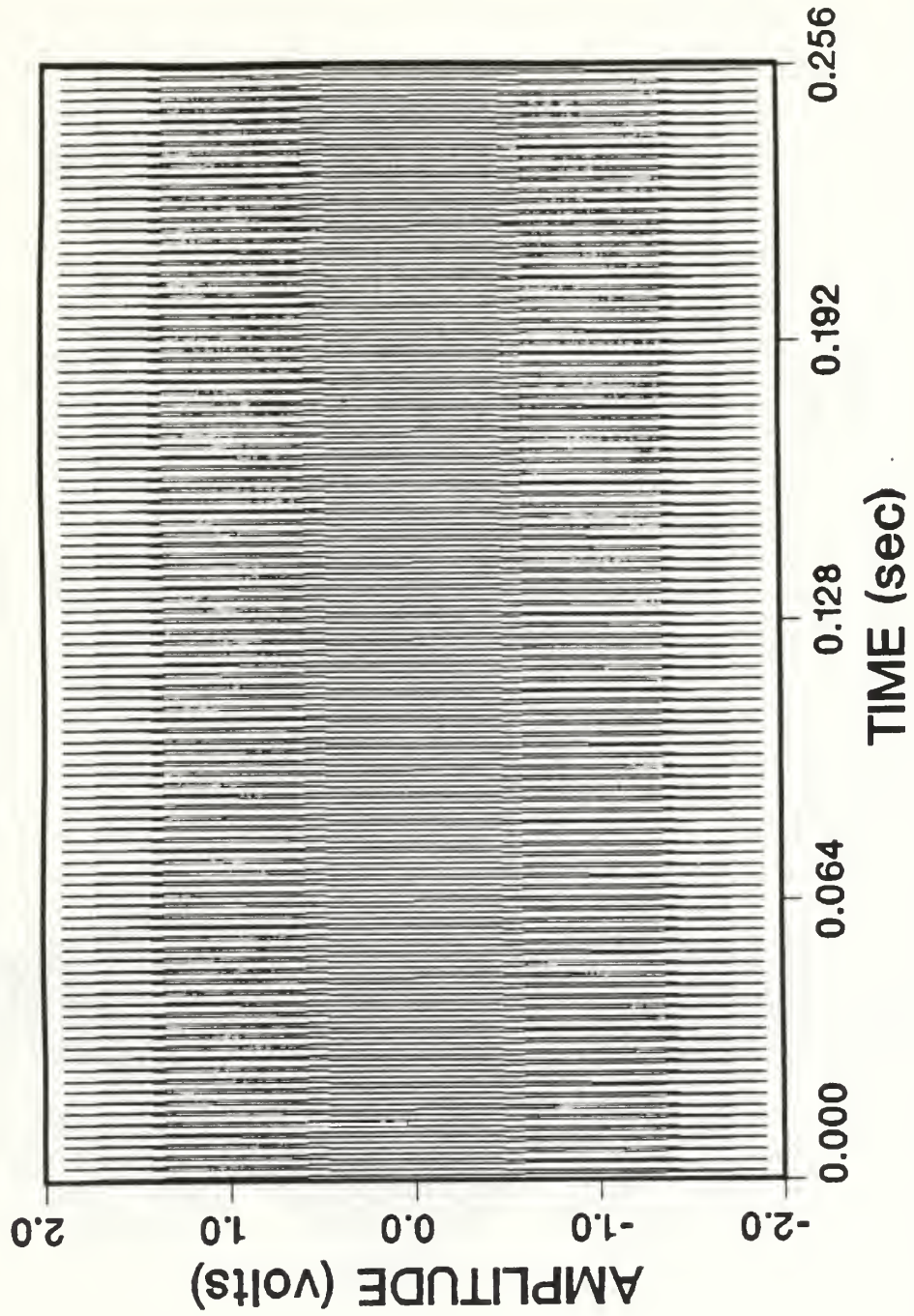
Figure 17. 400 Hz Sine Wave Input

37
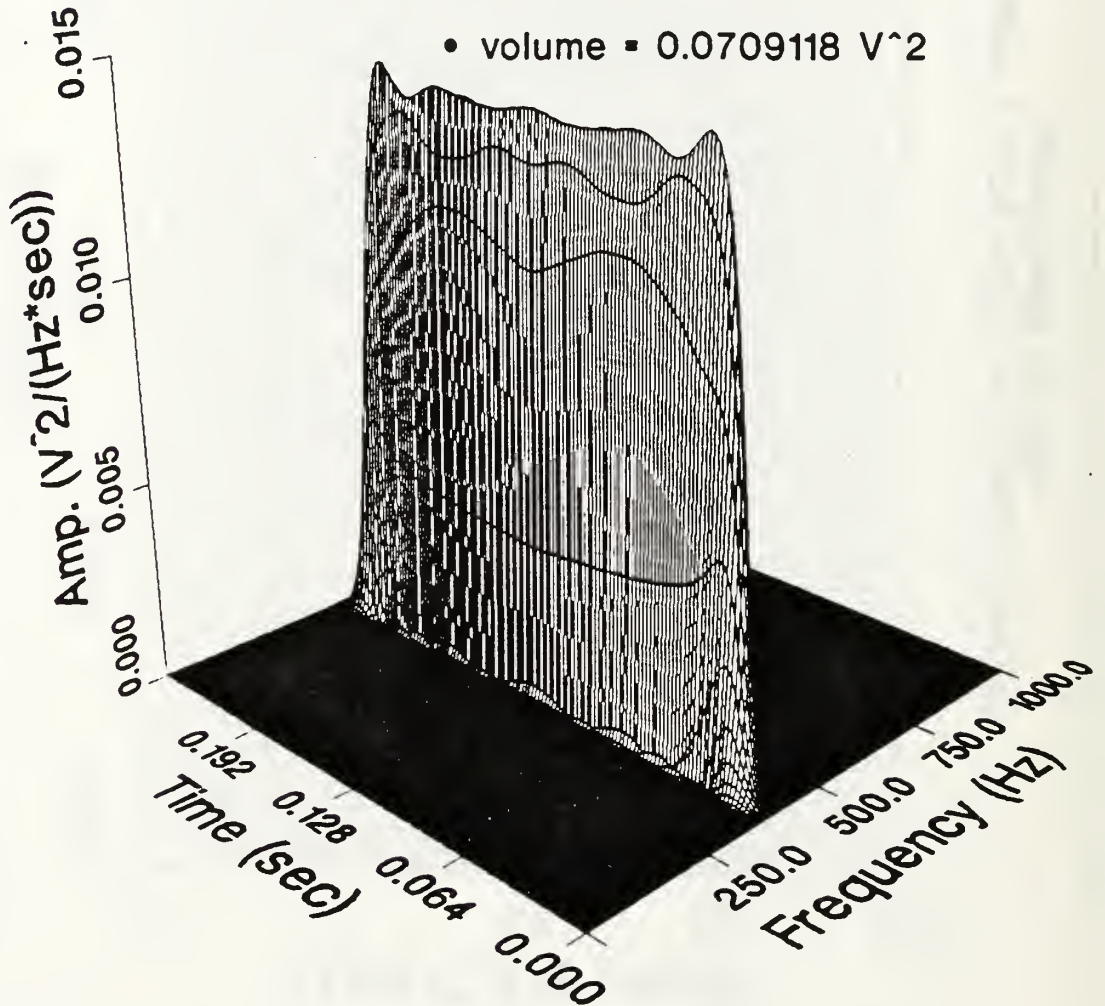
Figure 18. Pseudo Wigner-Ville Distribution Energy (13 X 13 Window)

38
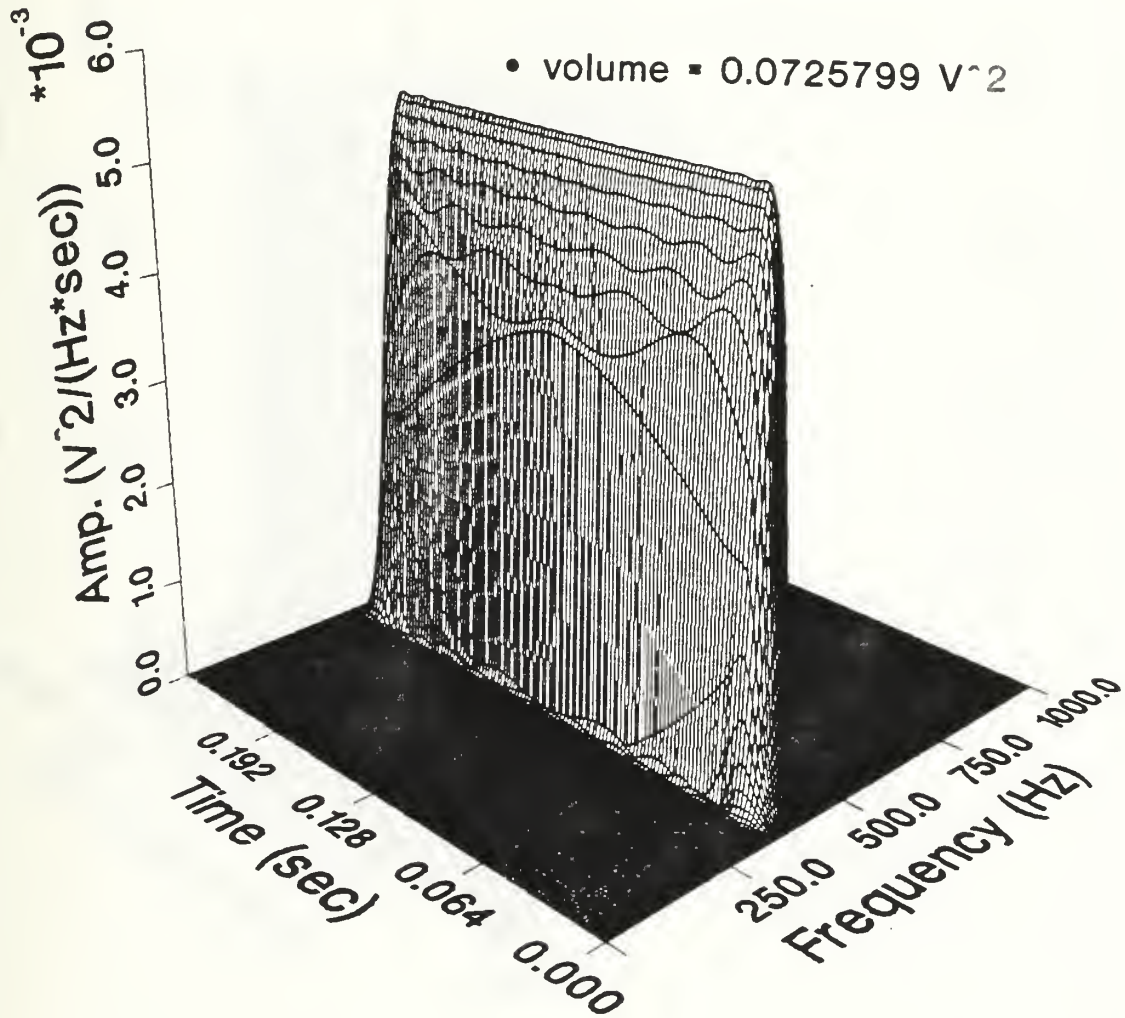
Figure 19. Pseudo Wigner-Ville Distribution Energy (35 X 5 Window)

39
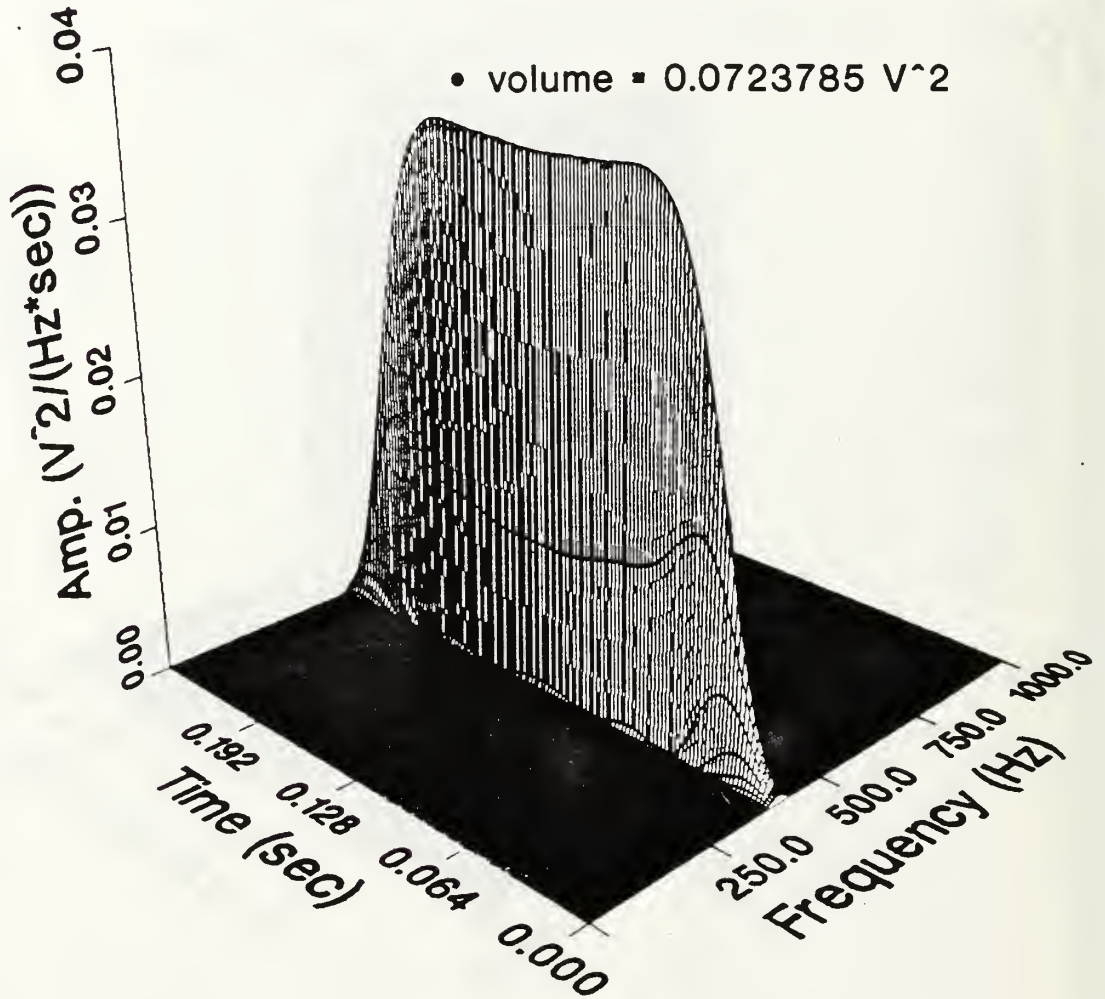
Figure 20. Pseudo Wigner-Ville Distribution Energy (5 X 35 Window)

40
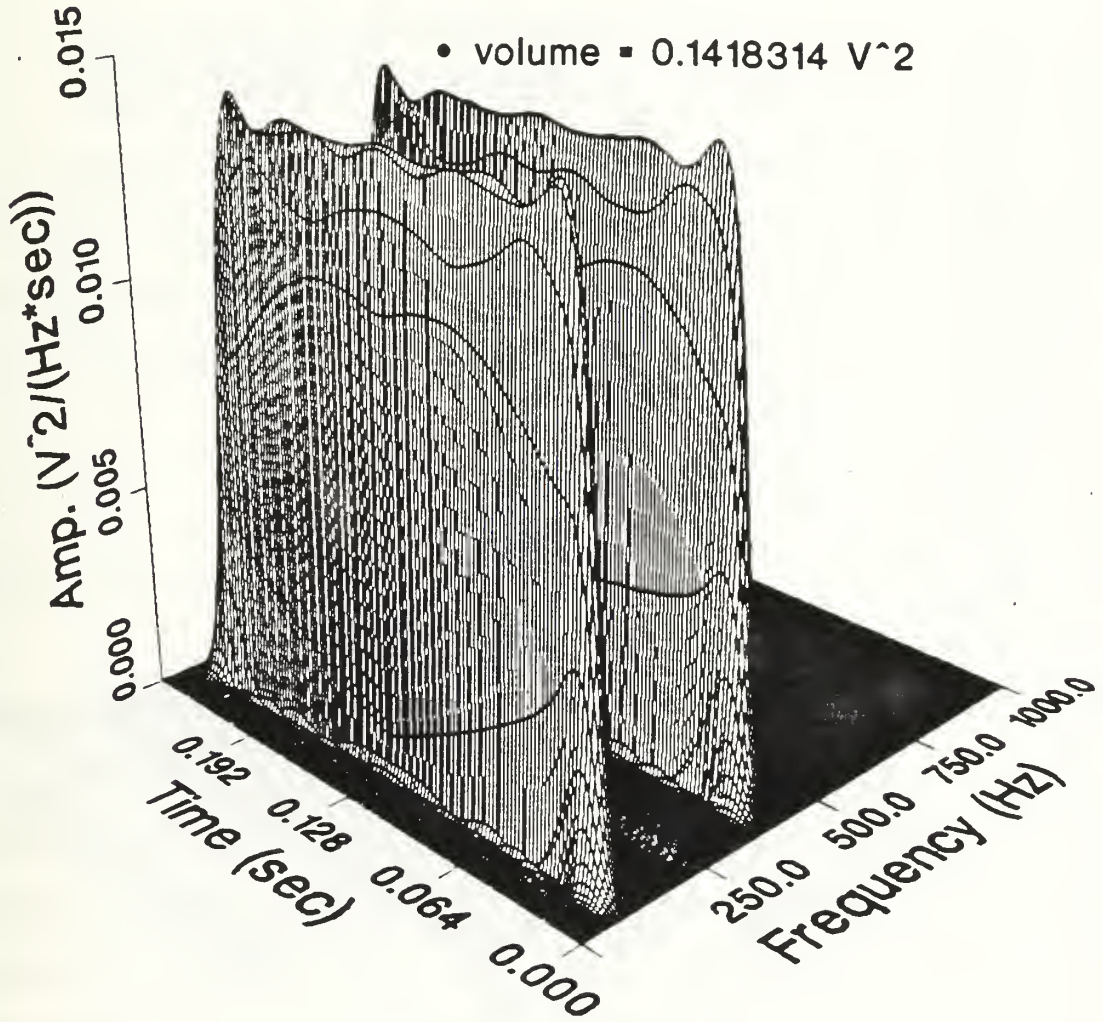
Figure 21. Combined Sine Wave Distribution Energy (13 X 13 Window)

41
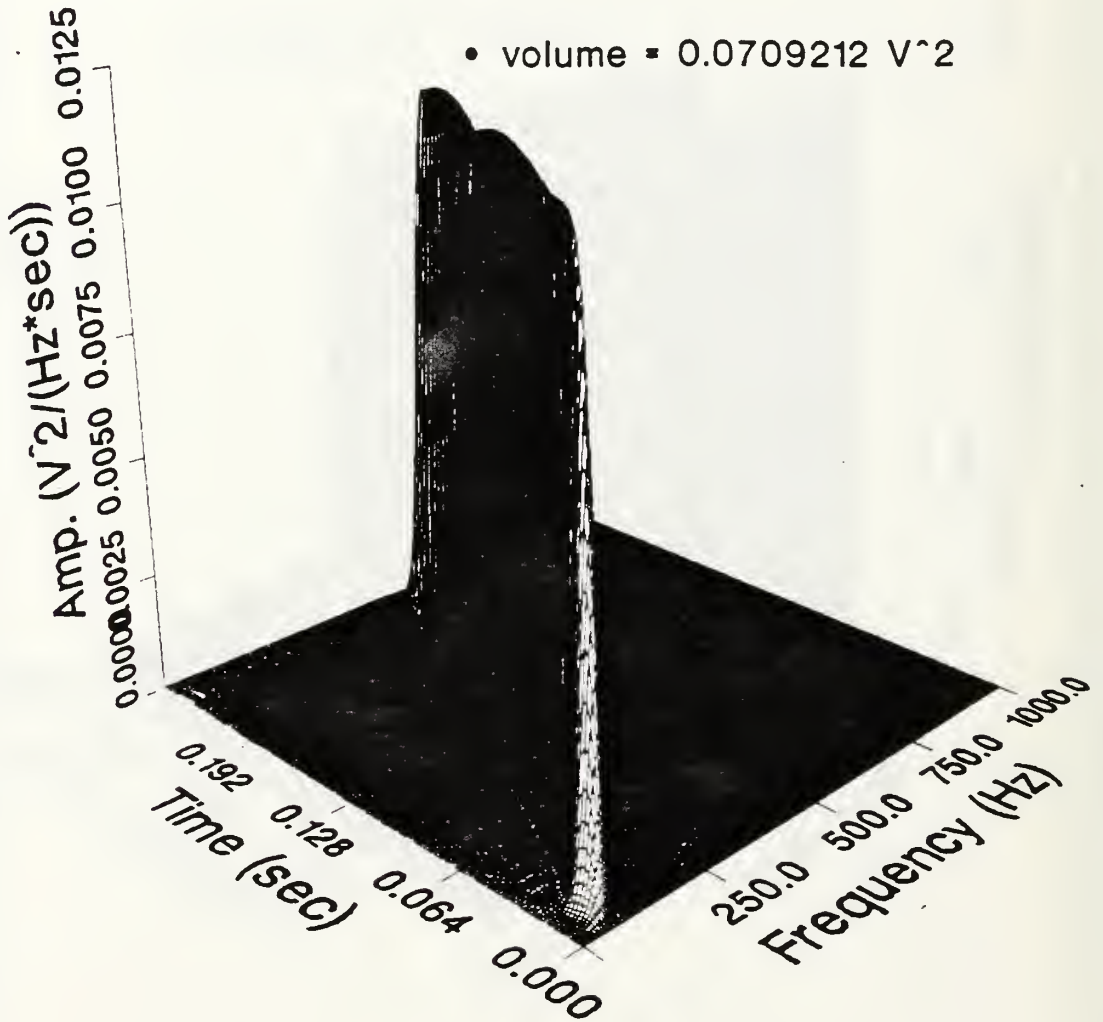
Figure 22. Ramped Sine Wave Distribution Energy (13 X 13 Window)

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The previous figures show that if the Wigner-Ville Distribution is going to be used to accurately portray or maintain signal information, then the window size must at least remain constant and better yet to keep the identical window size for each measurement of comparison. This requirement to maintain a constant sized smoothing window is not the only constraint that must be meet for energy analysis purposes. The next section will discuss another of these elements.

C.   TIME DEPENDENCE OF WIGNER-VILLE ENERGY

Not only is the window size going to influence the energy result, but the time length of the signal will also play a large part. This varies considerably from the conventional knowledge of the mean square value of a signal being independent of time. The standard mean square value will be the same for a signal of time length 2.0 seconds as it will for a signal of 4.0 seconds. For instance, the mean square value of a sine wave x(t) is:

$$\psi^2 = \int_0^\infty (auto\ spectrum)\, df = \frac{X^2}{2}$$

where: $x(t) = X \sin (\omega t)$

To compute the mean square value of this sine wave, it is independent of time and will equal 0.5 for $X = 1.0$. This is not the case for the mean square value or total energy of the signal as found using the discretized form of the Wigner-Ville Distribution. The following figures will show this second important point that must be realized when comparing Wigner-Ville energies of various signals. Figure 23 is the 400 Hz input signal of length 0.128 seconds. This time length is half of that of the 400 Hz signal used in the previous section. The resulting Wigner-Ville Distribution is shown in Figure 24 with an energy value of 0.0354558 V^2. As can easily be seen, this resulting energy value is not the same and in fact, the energy of Figure

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24, is one half that of the earlier 400 Hz signals of length 0.256 seconds. Had the signal been of a length 0.512 seconds or twice as long as the earlier examples, the resulting Wigner-Ville energy would have doubled. The window size used for this Pseudo Wigner-Ville Distribution was a 13 by 13 sized window.
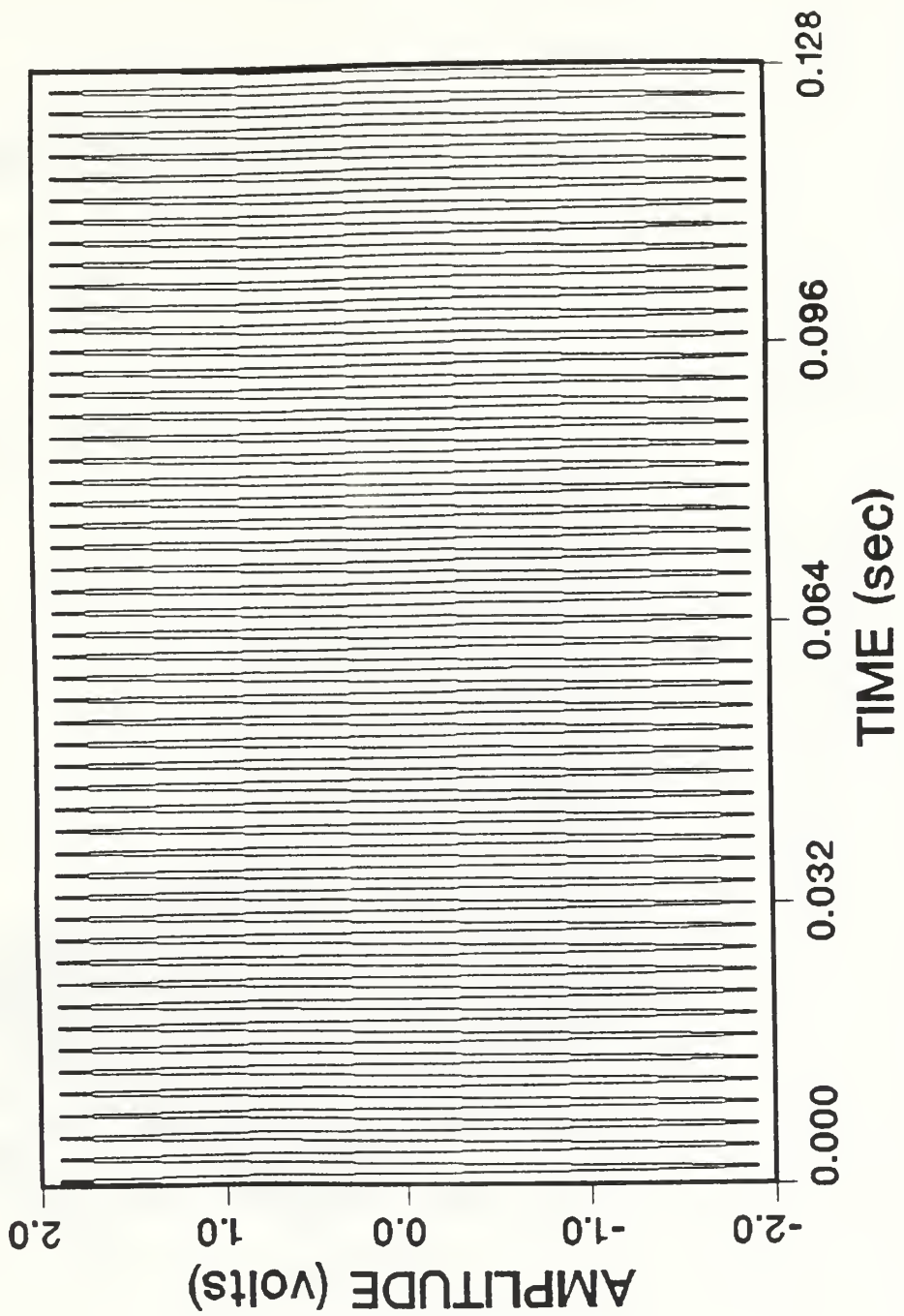
Figure 23. 400 Hz Sine Wave (T_L = 0.128 sec)

45
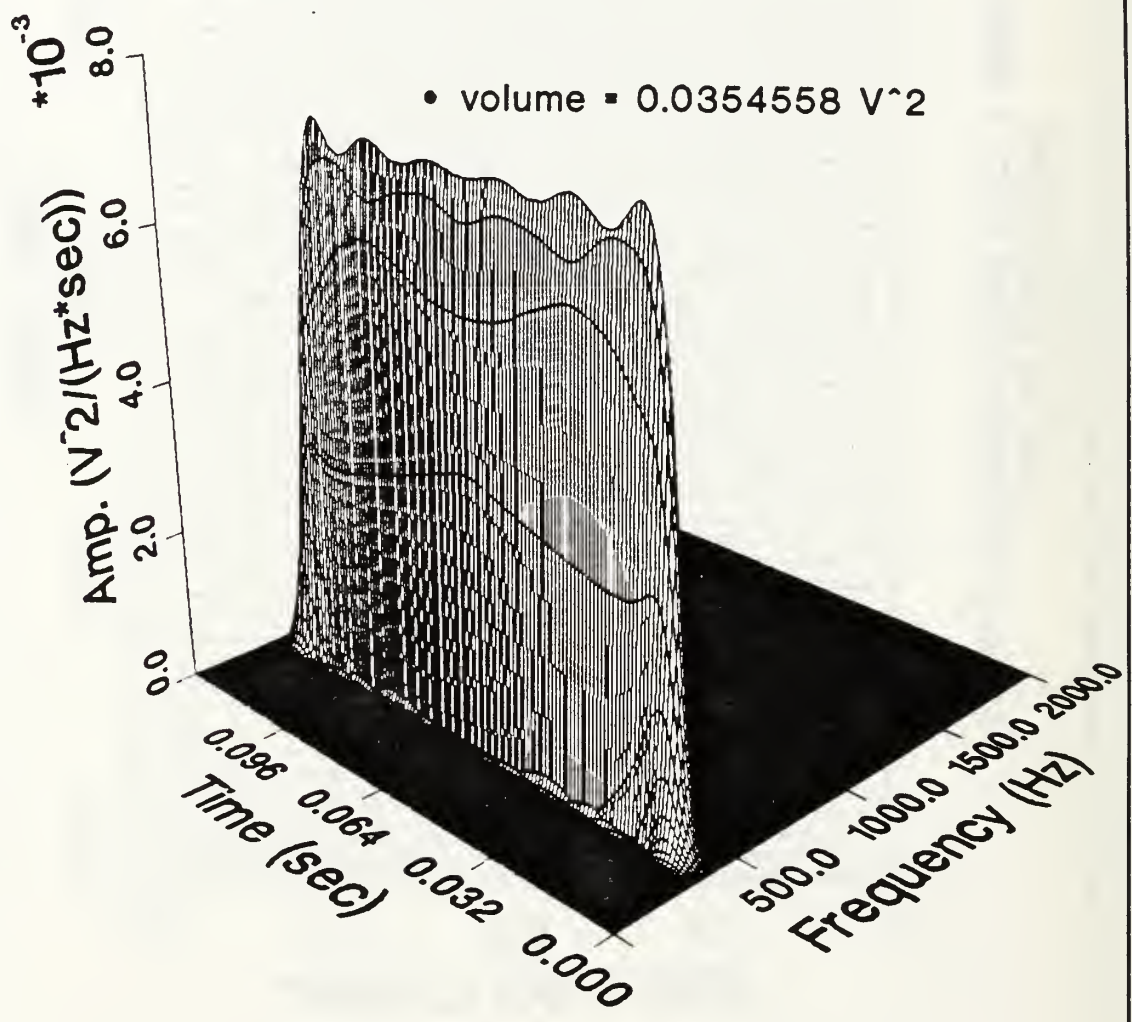
Figure 24. Pseudo Wigner-Ville Distribution Energy (T_L = 0.128 sec)

This has yielded a second concern that must be kept in mind when using the energy of the discretized Pseudo Wigner-Ville Distribution. Since the energy is proportional to the signal length, an easy conversion can be made by using the lengths of the individual raw input signals. This time dependence holds true for not only steady state signals as shown, but also for signals of a transient nature such as previously shown with Figure 22. Throughout these last two sections, the comparison of *energy* values of signals has been mentioned a great deal. The thought process behind this is that if there is an amplitude change in the input signal, this will be reflected in the resulting Pseudo Wigner-Ville Distribution. By ensuring that these previous two conditions are met, an investigation into the resulting energy changes from an amplitude increase or decrease of an input signal can be made.

D.   ENERGY PROPORTIONALITY OF THE DISTRIBUTION

The mean square value of a simple sine wave is the amplitude squared divided by two. This was shown in an earlier equation and now will be used to show the proportionality of the Wigner-Ville Distribution. For instance, the following applies for signals of differing amplitudes as shown:

$$amplitude = 1.0 \qquad \psi^2 = 0.5$$

$$amplitude = 2.0 \qquad \psi^2 = 2.0$$

This amplitude increase from 1.0 to 2.0 represents an increase of four in the mean square value of the signal. Figures 25 and 26 are the raw input sine waves of amplitude 1.0 and 2.0 respectively. The resulting Pseudo Wigner-Ville Distributions are shown in Figures 27 and 28, which acurately portray the four-fold energy increase. These Distributions have been smoothed using a window of 13 by 13.

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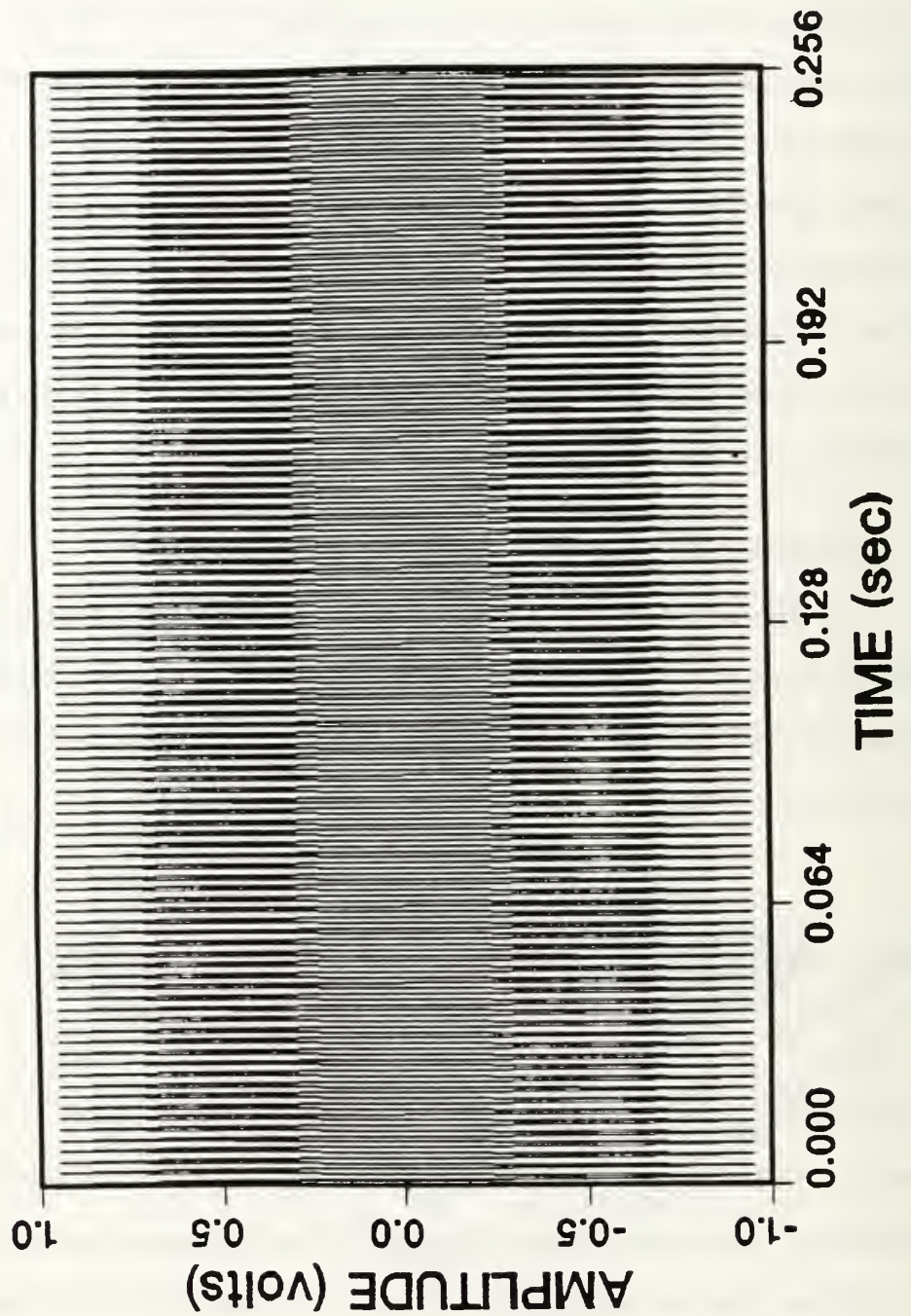
Figure 25. 400 Hz Sine Wave (Amplitude = 1.0)

48
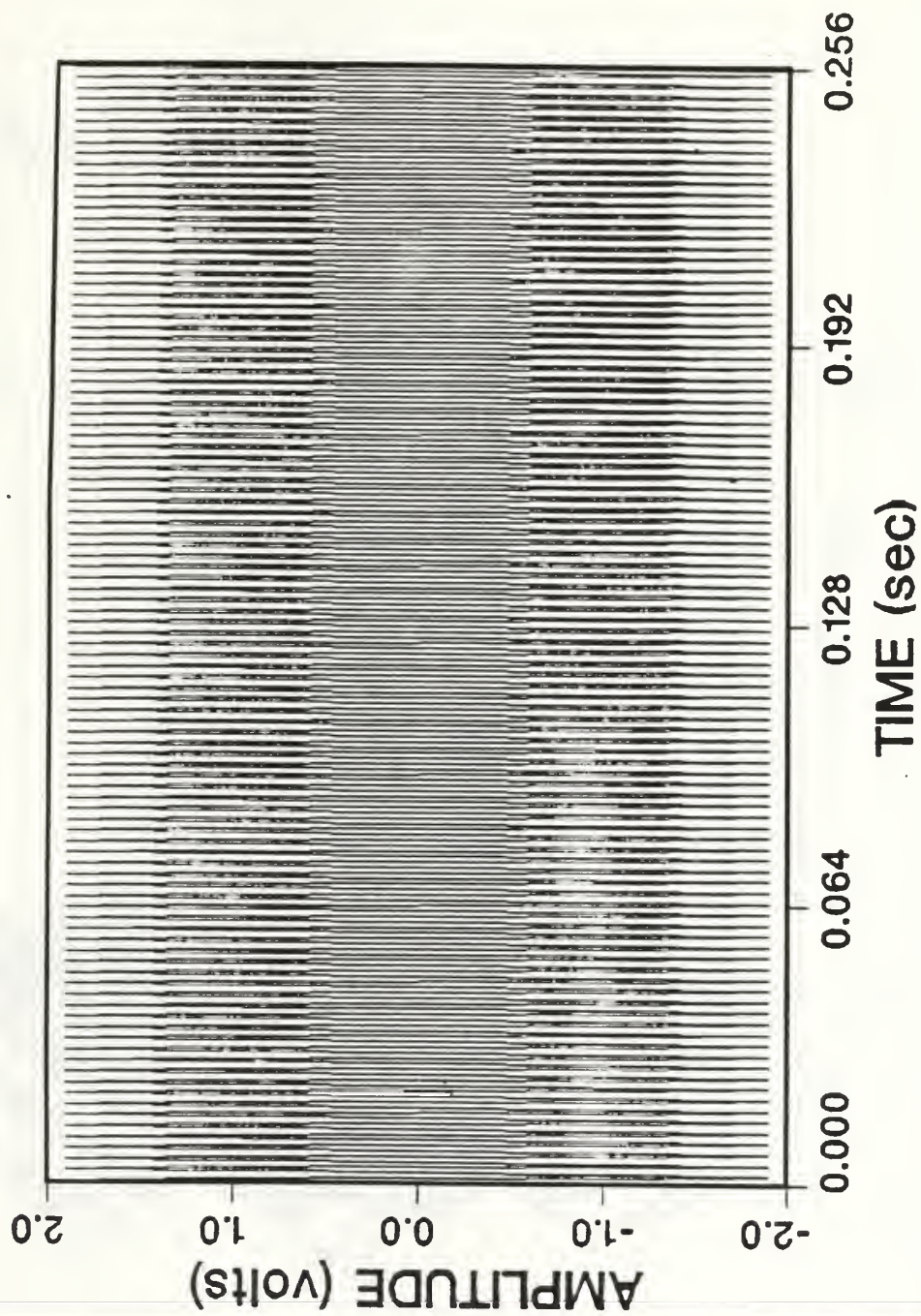
Figure 26. 400 Hz Sine Wave (Amplitude = 2.0)
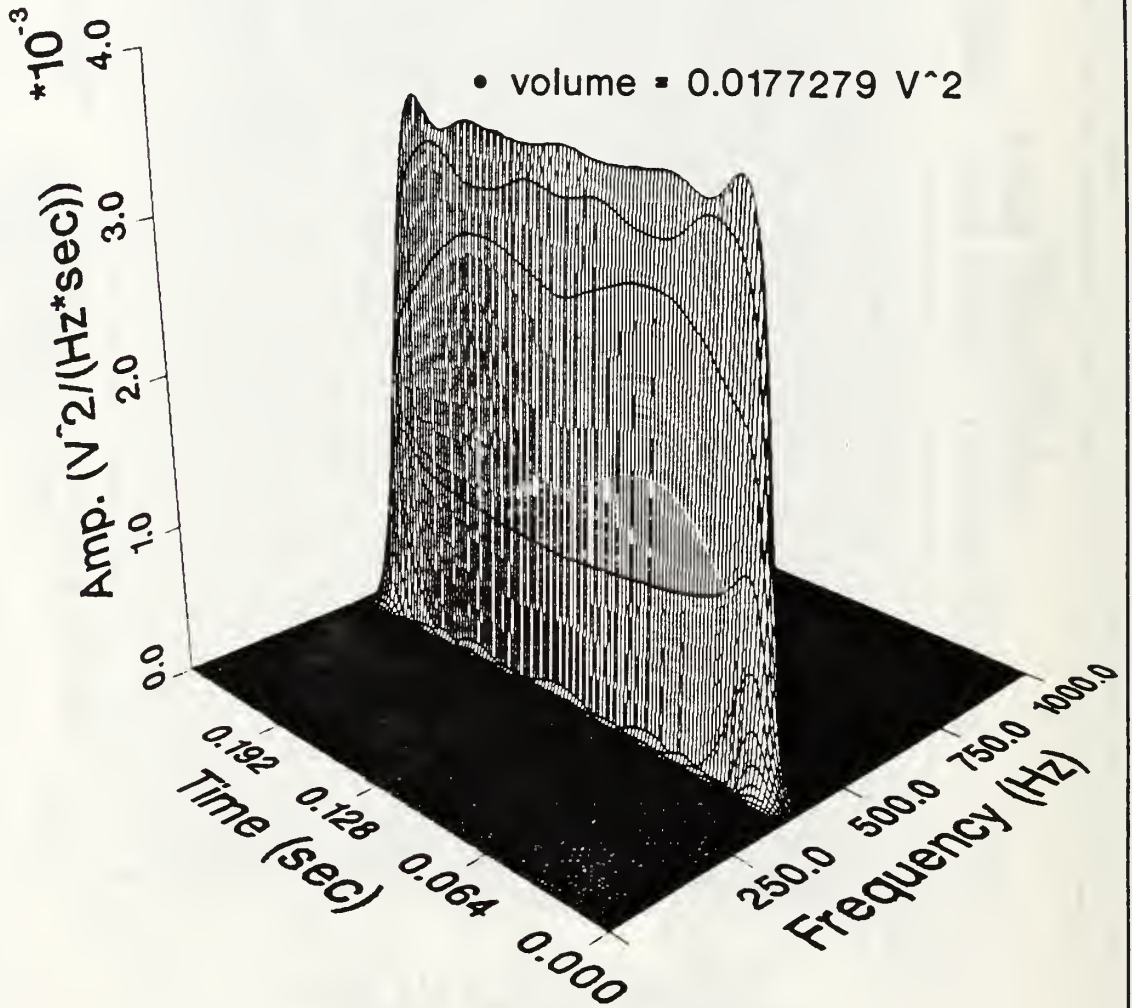
Figure 27. Pseudo Wigner-Ville Distribution (Amplitude = 1.0)
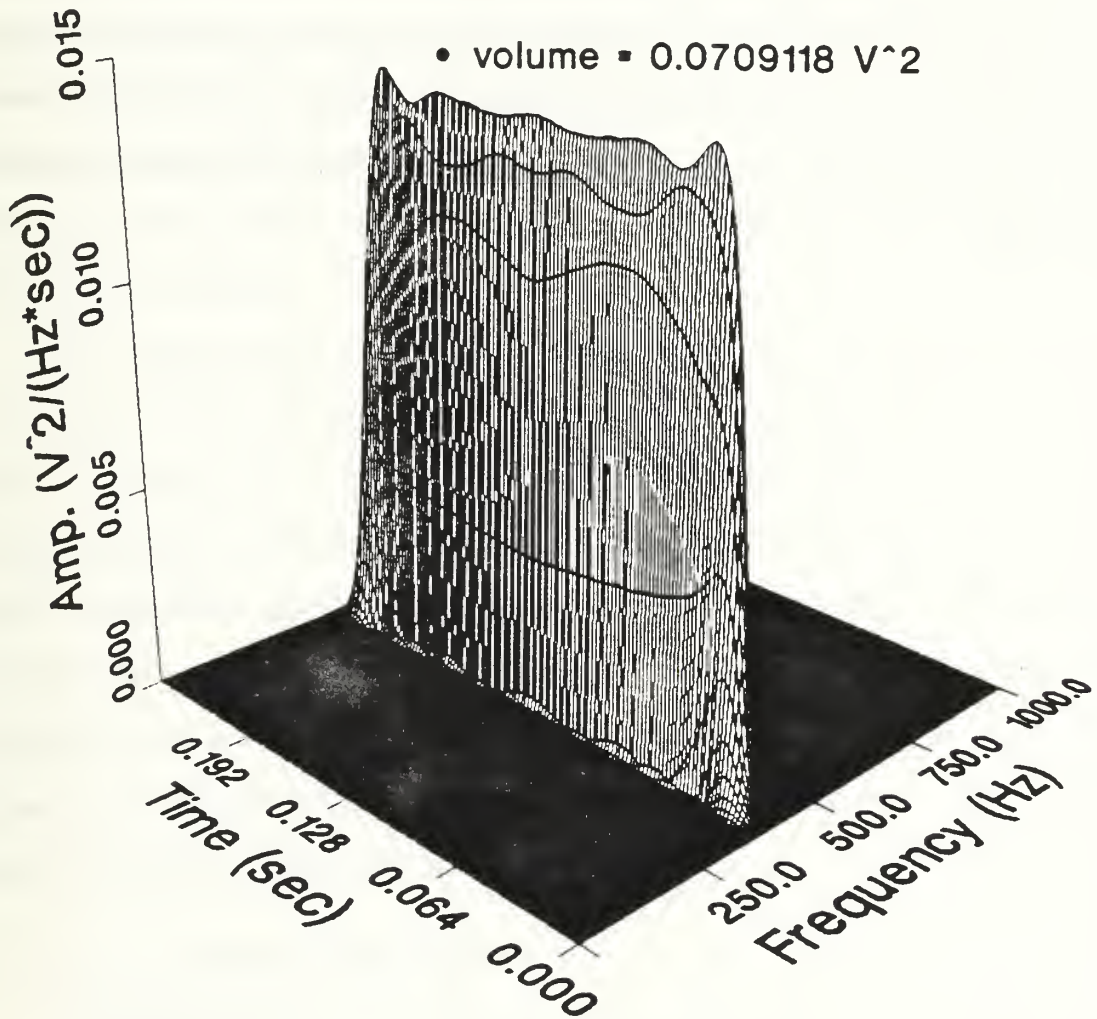
Figure 28. Pseudo Wigner-Ville Distribution (Amplitude = 2.0)

These previous four figures show the proportionality of the energy within the Wigner-Ville Distribution. These two cases are just an example to show the proportionality of the energy. Runs were made for many more cases (ie. X = 4.0, X = 8.0, etc) and the proportionality relation held true as shown with the previous Figures. This was another required element in order to use the energy as the basis for a method of machinery monitoring. Had the cross-terms increased in such a fashion as to not allow the Distribution to show the proportional energy change, the energy content of the Pseudo Wigner-Ville Distribution would have been rendered useless for the purposes of comparison and evaluation. The next chapter will use these requirements and basics that have been developed to show an actual application of how the Wigner-Ville energy may be used to conduct machinery monitoring.

V. GEAR MODEL APPLICATION

A. DESCRIPTION OF GEAR MODEL

The simple complexity gear model used in this work was built based upon a model used by D.K. Carlson [Ref. 20] for Artificial Neural Network applications for machinery monitoring. It may be of a simple nature, however, it provides all of the necessary components that may be found in more complex systems. A schematic of the gear model can be seen in Figure 29. The two major changes of this model over the prototype used by Carlson is that the components are much larger enabling a better signal to noise ratio and secondly, the motor used to drive the gear model is mounted on a separate base to reduce motor enduced vibrations.

The model consists of a single reduction gear train, with the pinion gear being a 90 tooth spur gear and the driven gear a 120 tooth spur gear. The gears were manufactured by Martin Corporation with a 1/2 inch bore and a 14.5 degree pressure angle. Aluminum blocks housed the NICE 1/2 inch bore radial ball bearings that were used to support the shaft and gears. These aluminum blocks were bolted to the support base which was a 1.0 inch thick plexiglass slab. Driving the single shaft is a General Electric 1/6th HP motor, that is controlled by the use of a Bodine Electric Company combination rectifier and variable potentiometer speed controller. The shaft speed was monitored through use of a Power Instruments Model 1720 RPM Optical Proximeter. [Ref 20]

B. SIGNAL SAMPLING EQUIPMENT

This section will describe the components shown in Figure 30, that were used to sample the vibrational signal of the gear model. The signal was monitored using an ENDEVCO model 303A03 accelerometer that was amplified and powered with a PCB model 483B07

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power unit. The signal was then processed through a Krohn-Hite model 3342 analog filter configured in a band pass mode, prior to being sent to the HP 3565S system for sampling and storage. Lastly, the signal was transfered to the VAX 3520 workstation for the calculation and plotting of the Pseudo Wigner-Ville Distribution.

1.   ENDEVCO Model 303A03 Accelerometer

The ENDEVCO Isotron PE accelerometer is a minature accelerometer that has a medium range high frequency capability. The operation of the accelerometer is based upon a piezoelectric quartz transducer sensing element. The sensitivity of the accelerometer is 10 mV/g with a resonant frequency of 70 kHz. The resolution available is 0.02 g, with a maximum range of $\pm$ 500 g.

The accelerometer was mounted on top of the aluminum bearing housing that supports the shaft and specifically at the position shown in Figure 30. The accelerometer wass attached by means of a mounting wax which yields a maximum operating range out to 15k - 20k Hz. This accelerometer signal was then sent through the power unit described below.

2.   PCB Model 483307 Power Unit

This power unit is used to power the low impedance quartz transducers and amplify the signal if desired. The power unit provides an adjustable 2 to 20 mA current for purposes of transducer excitation. The gain adjustment is available from 0 to 100 and is set through the use of a ten turn vernier gain pot and a three position gain multiplier switch. For the purposes of these tests, the gain was set to 20 before sending the signal to the analog filter.

3.   Krohn-Hite Model 3342 Anolog Filter

The model 3342 variable filter is a digitally tunned filter that will function as a High-Pass or Low-Pass filter. When the two channels of the filter are connected together, the filter will function as a band pass filter, which is the configuration for the gear model work. The range of the filter is from 0.001 Hz to 99.9 kHz as adjusted by three rotary decade

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switches and a rotary six positiohn multiplier switch. In addition, the filter unit has a gain setting of unity (0 dB) and 10 (20 dB). The signal was not further amplified using this capability of the filter prior to sampkling by the HP 3565S system.

4.   HP 3565S Measurement Hardware System

The HP 3565S measurement hardware with the HP-Vista software uses a HP-9000 series computer system for controlling purposes. The HP 3565S system is a modular multichannel system that can process data in both the time and frequency domain. The system is capable of handling 64 source and input modules, but is presently configured for eight. The HP-Vista software allowed for the sampling and storage of the preprocessed accelerometer signal that was later processed through the Pseudo Wigner-Ville distribution.
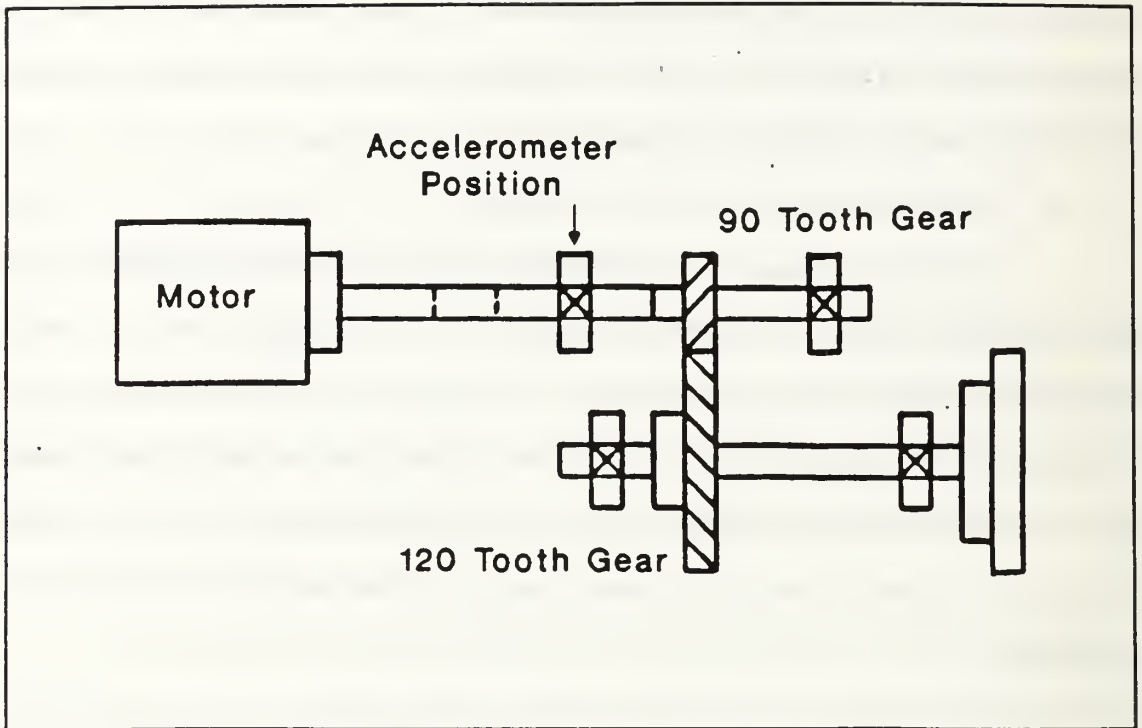
Figure 29. Gear Model Schematic
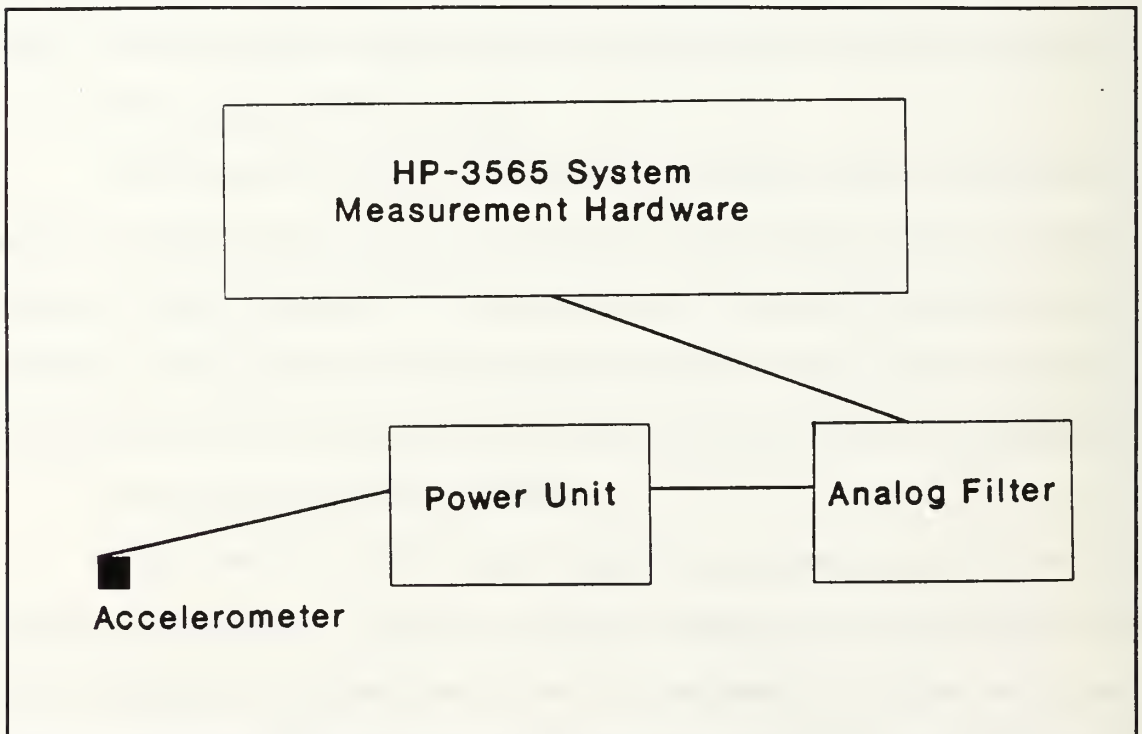
Figure 30. Signal Sampling Equipment Schematic

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C. ESTABLISHING MONITORING FREQUENCIES

This section will describe how the monitoring frequencies have been determined and how they will be used to monitor the gear model and in future applications a piece of transient machinery. As shown earlier in Chapter III, the energy contained in a signal is accurately portrayed within the resulting time-frequency domain of the Wigner-Ville Distribution. Through the Wigner-Ville Distribution energy that is contained in the established monitoring frequencies, faults may be detected. The shaft frequency is just the speed of the shaft, while the gear mesh frequency is the number of teeth on the rotating gear multiplied by the shaft speed or frequency. Both of these frequencies have associated harmonics that can be of equal importance as the frequencies themselves in the monitoring of machinery, It is for these reasons that the frequencies listed below have been chosen based upon a shaft frequency of 15 Hz.

1. Monitoring Frequencies

For the purposes of this work, just the shaft and gear mesh frequencies of the model will be included. Just as in standard established machinery monitoring methods, the shaft frequency and associated harmonics can provide a great deal of information for fault detection and diagnosis. Along with the shaft frequencies, the gear meshing frequencies provide valuable information of faults and will also be monitored. The purpose of the two examples will be to show that the Wigner-Ville Distribution energy (ie. volume) changes as a result of damage and will prove to be a very valuable tool in future machinery monitoring methods. The established monitoring frequencies have been filtered as follows:

- Shaft Input 1: (5 - 100 Hz) This is the spectrum of signals in the lower frequency range selected to determine the total energy contained within the shaft frequency and it's harmonics.

- Shaft Input 2: (10 - 20 Hz) This frequency range captures just the shaft frequency.

- Shaft Input 3: (25 - 35 Hz) This frequency range captures the 1st shaft harmonic.

- Shaft Input 4: (40 - 50 Hz) This frequency range captures the 2nd shaft harmonic.

- Shaft Input 5: (55 - 100 Hz) This frequency range captures the upper harmonics of the shaft frequency.

- Gear Mesh Input 1: (1250 - 1450 Hz) This frequency range captures the gear meshing frequency.

- Gear Mesh Input 2: (2600 - 2800 Hz) This frequency range captures the 1st gear mesh harmonic.

- Gear Mesh Input 3: (3950 - 4150 Hz) This frequency range captures the 2nd gear mesh harmonic.

2. Combined Monitoring Technique

Using the Wigner-Ville energy as an input to an Artificial Neural Network is a follow-on goal of this research. It is for this reason, that the above established frequencies have been identified as *inputs*. Within this scheme, the Wigner-Ville is used to characterize the signal and give a single energy value at the particular frequency of interest as an input to an Artificial Neural Network. By filtering these signals prior to processing, just the frequency of interest will be captured along with it's respective Wigner-Ville energy content. The Wigner-Ville energy data can now be used to produce a comprehensive training set for the Artificial Neural Network. By pre-processing the inputs to a Neural Network with the filtering and the Wigner-Ville Distribution, the number of inputs is greatly reduced. Since these associated energy levels are fairly stable for a given machinery condition, the Neural Network will be capable of detecting patterns in energy level shifts that are associated with particular machinery faults. To support this last statement, the following section will show two examples of machinery faults imposed on the gear model and the response of the Wigner-Ville energy to these faults.

D. MACHINERY FAULT EXAMPLES

1. Mechanical Looseness

This fault was imposed on the gear model by loosening the aluminum support blocks that house the bearings and support the shaft. Specifically, the blocks that support the driver shaft were loosened. The degree of looseness imposed was difficult to judge and made this particular fault a difficult one to diagnose. The blocks were not completely undone, but loosened just a small amount from their original firmly bolted posture. Shown in the table below are the results of averaging two *good* runs and two post-damage runs and comparing the respective energy values at the different frequencies. These energy values are nothing more than the volume under the Pseudo Wigner-Ville Distribution as discussed earlier. Only the shaft frequencies are shown for purposes of clarity and since it is in these monitored frequencies that a fault of mechanical looseness should present itself.

Table I. MECHANICAL LOOSENESS

MECHANICAL LOOSENESS ENERGY VALUES (V^2)

Frequency	Good	Damaged
Shaft Spectrum	41.0	28.0
Shaft Frequency	1.07	0.83
1st Harmonic	2.31	1.22
2nd Harmonic	1.23	2.40
Upper Spectrum	31.4	26.31

As can be seen in the table above, the energy content approximately doubled at the second shaft harmonic. This is a very encouraging result for the purposes of using the Pseudo Wigner-Ville Distribution energy as a fault detection tool.

2. Gear Tooth Damage

This particular fault was imposed on the gear model by shaving a gear tooth on the driver gear. The degree of damage was much easier to predict in this case and made this particular fault an easier one to control. The gear tooth that was damaged had approximately, $1/8^{th}$ of the leading edge removed. Shown in the table below are again the results of averaging two *good* runs and two post-damage runs and comparing the respective energy values at the different frequencies. For this case, just the frequencies associated with the gear

mesh are presented, since it is in these monitored frequencies that a fault involving the damage of a gear train would present itself.

Table II. GEAR TOOTH DAMAGE

GEAR TOOTH DAMAGE ENERGY VALUES (V^2)

MESH FREQUENCY	GOOD	DAMAGED
Fundamental	17.14	154.4
1st Harmonic	0.53	3.44
2nd Harmonic	0.16	0.48

As shown in the above table, the energy content greatly increased at the fundamental gear mesh frequency. Additionaly, there was also increases at the harmonics and shown in Figures 31 and 32 is a graphical presentation of the energy increase that occured due to the damaged tooth.
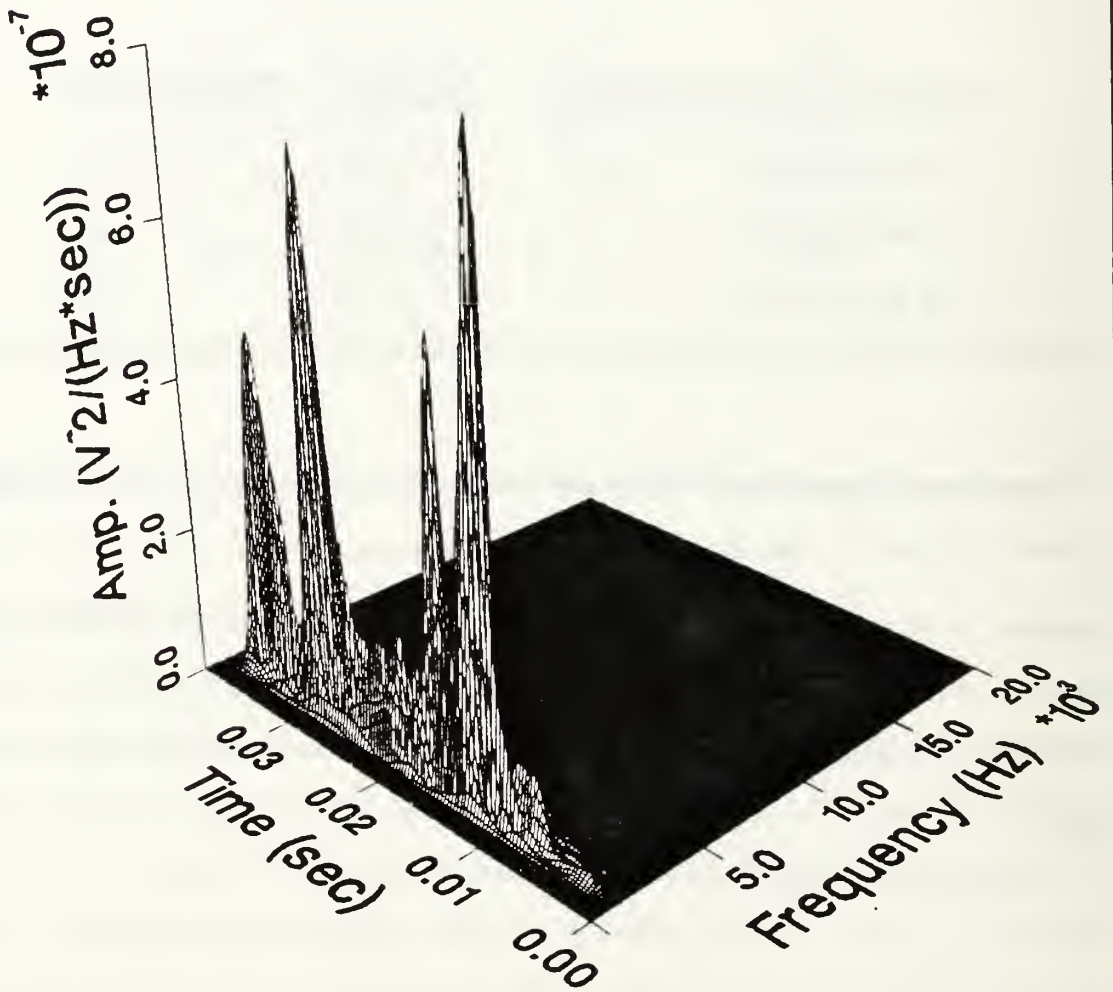
Figure 31. Gear Mesh Frequency (Pre-Damage)

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Figure 32. Gear Mesh Frequency (Post-Damage)

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This chapter has shown two examples of machinery damage and the resulting changes of the Pseudo Wigner-Ville Distribution energy. By applying the window and energy requirements discussed in Chapters III and IV, the Wigner-Ville energy has yielded a definite change to allow a means for machinery monitoring. The Wigner-Ville energy will make a stable input to a Neural Network and provide a quick efficient method by pre-processing the machinery signal with the Wigner-Ville Distribution as shown. This pre-processing will allow the number of inputs to the Neural Network to remain at a reasonable level and provide for a manageable, quicker hardware system.

VI. CONCLUSIONS

The Pseudo Wigner-Ville Distribution has been shown to provide an excellent characterization of an input signal. Based upon the research conducted, it will also provide a means for machinery condition monitoring and diagnostics. The research has also shown that in order to use the Wigner-Ville Distribution in a machinery monitoring scheme, the following conditions or conclusions must be met and understood.

- The smoothing window size applied must remain constant for purposes of comparing various signals.

- The individual time and frequency lengths of the smoothing window may be adjusted to provide a better frequency resolution or constant amplitude.

- The energy contained within the Pseudo Wigner-Ville Distribution of a signal is proportionaly time dependent.

- The energy of the Distribution is proportional to the mean square value of the input signal and is not adversely affected by the cross-terms.

- Lastly, the damaged gear model conditions showed a definite change in the energy of the Pseudo Wigner-Ville Distribution at the respective monitoring frequencies.

VII. RECOMMENDATIONS FOR FUTURE RESEARCH

The Pseudo Wigner-Ville Distribution computer program that was established here at the Naval Postgraduate School is very stable. The pronounced peaks at the transition points are now definable and additionally, with the energy of the Pseudo Wigner-Ville Distribution now being a definable and understood quantity, the next additional research in this area may include:

- Establish a baseline or data bank of "good" Wigner-Ville Distribution data for the present gear model.

- Conduct additional runs of various levels of damage of gear model components for comparison with good data.

- Establish a Torpedo Ejection Pump baseline of good data as with the gear model.

- Continue with the research of linking the Pseudo Wigner-Ville Distribution as an input to a Neural Network Application.

APPENDIX   COMPUTER PROGRAMS

The following programs have all been developed here at the Naval Postgraduate School

and used in the conduct of the research presented. The language used is FORTRAN 77 with

the plotting programs using calls to CA-DISSPLA software.

A.   WVD3 COMPUTER PROGRAM

```
      PROGRAM WVD3
*
****************************************************************
*                                                              *
* THE FOLLOWING INDIVIDUALS HAVE BEEN INSTRUMENTAL IN THE      *
* DEVELOPMENT OF THIS PROGRAM AND PREVIOUS VERSIONS:           *
*                                                              *
*                                                              *
*      Graham Rossano                                          *
*                                                              *
*      Jose G. Calahorrano                                     *
*                                                              *
*      Scott G. Spooner                                        *
*                                                              *
*      Prof. Y.S. Shin                                         *
*                                                              *
*                                                              *
****************************************************************
*
*          This program calculates the Wigner Distribution
*   Function of a signal.  It includes the option of applying a
*   smoothing function and reducing the distribution to different
*   sizes while writing the results to files for subsequent
*   plotting.
*
****************************************************************
*                                                              *
*                     VERSION 3                                *
*                                                              *
****************************************************************
*                                                              *
*                VARIABLE DECLARATION                          *
*                                                              *
****************************************************************
*
**********--- setting number of data points ---*************
*
*
*
      integer dp,dp2,mvopt,redopt,dimt,dimf,mm,nn,rf,rdp,rdp2,
     :        nf,mt,nf2,mt2
      PARAMETER  (DP = 512)
      real pi,tin(dp),ain(dp),rawa(dp),rwdf(dp*2,dp),rswdf(dp*2,dp),
     :    rawt(dp),dtsum,delt,dt,asum,meanv,mtime,del1,del2,
     :    const,t,sum,sumb,val,sval,wdf(dp*2,dp),coef,df,AREA
      complex s(dp*2),dum,c(dp*5),dum3,dum2
      character*25  inname
*
      common/in/ mvopt,redopt,dimt,dimf,mm,nn,rf,rdp,rdp2,
```

```
      :         nf,mt,nf2,mt2
      common/rl/ pi,tin,ain,rawa,rwdf,rswdf,
      :    rawt,dtsum,delt,dt,asum,meanv,mtime,del1,del2,
      :    const,t,sum,sumb,val,sval,wdf,coef,df,area
      common/comp/ s,dum,c,dum3,dum2
      dp2 = dp*2
*
***************************************************************
*                 INITIALIZING VARIABLES                      *
***************************************************************

*      --- Print description of program ---
      print*
      print*,' Program to calculate the'
      print*,' Pseudo Wigner-Ville Distribution'
      print*

*      --- Set initial values ---

      print*,' Enter name of signal input file'
      read(5,904) inname
      print*
*
      print*,' Do you wish to remove the mean value?'
      print*,' Enter 1 for yes - 0 for no'
      read(5,910)  mvopt
      print*
*
      print*,' Input the reduction size desired'
      print*,' input 1 for    64 by 32       '
      print*,' input 2 for    128 by 64      '
      print*,' input 3 for    128 by 128     '
      print*,' input 4 for    256 by 128     '
      read(5,910) redopt
      print*
*
      print*,' Input the smoothing window size desired'
      print*,' input the size for the frequency axis'
      read(5,910) nf
      print*,' input the size for the time axis'
      read(5,910) mt
*
*
*
***************************************************************
*                                                             *
*      calculation of some common used constants              *
*                                                             *
***************************************************************

      pi = 4.0 * atan(1.0)


***************************************************************
*      THIS IS THE CALCULATION PART OF THE PROGRAM            *
***************************************************************

      open(4,file=inname,status='old')
       rewind 4


*   PRELIMINARY CALCULATIONS:

      CALL DATAIN3

       write(6,*) , ' done with datain3'

      CALL DTCALC3
```

```
          write(6,*) , ' done with dtcalc3'

      CALL MEAN3
       write(6,*) , ' done with mean3'
       write(6,*) , ' mean value removed =', meanv

*   SIGNAL MODIFICATIONS:

*       Window application (modified hamming window)

          CALL HAMMG3
       write(6,*) , ' done with hammg3'

*       Conversion of real signal to an analytical one

          CALL ANAL3
       write(6,*) , ' done with anal3'



*   CALCULATION OF THE WIGNER VILLE DISTRIBUTION:

      CALL WIGH3

       write(6,*) , ' done with wigh3'


*   REDUCTION & SMOOTHING OF THE WIGNER VILLE DISTRIBUTION:

      CALL REDUCE3

          write(6,*) , ' done with reduce3'

      CALL SMOOTH3

          write(6,*) , ' done with smooth3'
*****************************************************************
*                                                               *
*       WRITING OF THE DIFFERENT ARRAYS GENERATED               *
*            TO FILES FOR PLOTTING PURPOSES.                    *
*                                                               *
*****************************************************************
*
*   OPENING OF THE OUTPUT FILES
*
      OPEN(UNIT=7,FILE='RAWTIME.OUT',STATUS='NEW')
      OPEN(UNIT=8,FILE='WNDWTIM.OUT',STATUS='NEW')
      OPEN(UNIT=9,FILE='RWDF.OUT',STATUS='NEW')
      OPEN(UNIT=10,FILE='RSWDF.OUT',STATUS='NEW')
      OPEN(UNIT=11,FILE='WDF.OUT',STATUS='NEW')


*
*   WRITING OF RAW AND WINDOWED TIME SIGNAL TO FILES
*
      DO 300 I = 1,DP
        WRITE(7,930)  TIN(I),RAWA(I)
        WRITE(8,930)  TIN(I),AIN(I)
300     CONTINUE
*
*   WRITING OF REDUCED/UNSMOOTHED WVD TO FILE
*
      DO 400 I = 1,DP/mm
        DO 400 J = 1,DP2/nn
          WRITE(9,906)  RWDF(J,I)
400     CONTINUE
*
*   WRITING OF REDUCED & SMOOTHED WVD TO FILE
```

```
*
        DO 500 I = 1,DP/mm
          DO 500 J = 1,DP2/nn
            WRITE(10,906)  RSWDF(J,I)
500       CONTINUE
*
*  WRITING OF ENTIRE WDF TO A FILE
*
        DO 600 I = 1,DP
          DO 600 J = 1,DP2
            WRITE(11,906)  WDF(J,I)
600       CONTINUE

* Format statements

904     format(a25)
906     format(2X,g16.8)
910     format(i6)
930     format(2x,g16.8,5x,g16.8)

        END

* SUBROUTINES

        include 'ANAL3.INC'
        include 'CORR3.INC'
        include 'DATAIN3.INC'
        include 'DTCALC3.INC'
        include 'FFT3.INC'
        include 'HAMMG3.INC'
        include 'MEAN3.INC'
        include 'SMOOTH3.INC'
        include 'REDUCE3.INC'
        include 'WIGH3.INC'
```

B.   SUBROUTINES OF WVD3

Listed below in alphabetic order are the subroutines used in the main program WVD3.

```
      SUBROUTINE ANAL3
*
************************************************************
*                                                          *
*  This subroutine converts a real signal to an            *
*  analytic one                                            *
*                                                          *
************************************************************
*
*
*
      integer dp,dp2,mvopt,redopt,dimt,dimf,mm,nn,rf,rdp,rdp2,
     :        nf,mt,nf2,mt2
      PARAMETER  (DP = 512)
      real pi,tin(dp),ain(dp),rawa(dp),rwdf(dp*2,dp),rswdf(dp*2,dp),
     :     rawt(dp),dtsum,delt,dt,asum,meanv,mtime,del1,del2,
     :     const,t,sum,sumb,val,sval,wdf(dp*2,dp),coef,df,area
      complex s(dp*2),dum,c(dp*5),dum3,dum2
      character*25  inname
*
      common/in/ mvopt,redopt,dimt,dimf,mm,nn,rf,rdp,rdp2,
     :        nf,mt,nf2,mt2
      common/rl/ pi,tin,ain,rawa,rwdf,rswdf,
     :     rawt,dtsum,delt,dt,asum,meanv,mtime,del1,del2,
     :     const,t,sum,sumb,val,sval,wdf,coef,df,area
      common/comp/ s,dum,c,dum3,dum2
      dp2 = dp*2
*
*
        do 100 i=1,dp
           sum=0.0

        do 200 j=1,dp
           sumb=0.0

           if(i-j.eq.0) go to 200

              n=i-j
              val=pi*n/2.
              sval=sin(val)
              sumb=ain(j)*sval*sval/val

200        sum=sum+sumb

           s(i)=cmplx(ain(i),sum)

100        continue

        END
```

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```
          SUBROUTINE DATAIN3
**************************************************************
*                                                            *
* This subroutine assumes the input file is in the following *
* format: time  amplitude (2x,e16.8,5x,e16.8) with an end of *
* file indicator of  a last entry  9999.,9999.               *
*                                                            *
**************************************************************
*
*
*
      integer dp,dp2,mvopt,redopt,dimt,dimf,mm,nn,rf,rdp,rdp2,
     :          nf,mt,nf2,mt2
      PARAMETER  (DP = 512)
      real pi,tin(dp),ain(dp),rawa(dp),rwdf(dp*2,dp),rswdf(dp*2,dp),
     :     rawt(dp),dtsum,delt,dt,asum,meanv,mtime,del1,del2,
     :     const,t,sum,sumb,val,sval,wdf(dp*2,dp),coef,df,AREA
      complex s(dp*2),dum,c(dp*5),dum3,dum2
      character*25  inname
*
      common/in/ mvopt,redopt,dimt,dimf,mm,nn,rf,rdp,rdp2,
     :          nf,mt,nf2,mt2
      common/rl/ pi,tin,ain,rawa,rwdf,rswdf,
     :     rawt,dtsum,delt,dt,asum,meanv,mtime,del1,del2,
     :     const,t,sum,sumb,val,sval,wdf,coef,df,area
      common/comp/ s,dum,c,dum3,dum2
      dp2 = dp*2
*

********simple loop to read in time & amplitude*************

        do 100 j=1,dp

         read(4,902)  rawt(j) ,  rawa(j)
            tin(j) = rawt(j)
            ain(j) = rawa(j)


100       continue

*     FORMAT STATEMENT
902       FORMAT(2X,E16.8,5X,E16.8)

200       END
```

```fortran
      SUBROUTINE DTCALC3
*
*************************************************************
*                                                           *
*   This subroutine calculates the delta t of the signal    *
*                                                           *
*************************************************************
*
*
*
      integer dp,dp2,mvopt,redopt,dimt,dimf,mm,nn,rf,rdp,rdp2,
     :          nf,mt,nf2,mt2
      PARAMETER  (DP = 512)
      real pi,tin(dp),ain(dp),rawa(dp),rwdf(dp*2,dp),rswdf(dp*2,dp),
     :      rawt(dp),dtsum,delt,dt,asum,meanv,mtime,del1,del2,
     :      const,t,sum,sumb,val,sval,wdf(dp*2,dp),coef,df,AREA
      complex s(dp*2),dum,c(dp*5),dum3,dum2
      character*25  inname
*
      common/in/ mvopt,redopt,dimt,dimf,mm,nn,rf,rdp,rdp2,
     :          nf,mt,nf2,mt2
      common/rl/ pi,tin,ain,rawa,rwdf,rswdf,
     :      rawt,dtsum,delt,dt,asum,meanv,mtime,del1,del2,
     :      const,t,sum,sumb,val,sval,wdf,coef,df,area
      common/comp/ s,dum,c,dum3,dum2
      dp2 = dp*2
*
*
*
*************************************************************

      dtsum = 0.0

       do 100 i = 1 , dp-1

         delt = tin(i+1) - tin(i)
         dtsum = dtsum + delt

100     continue

      dt = dtsum / (dp-1.)


      END
```

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```
      SUBROUTINE FFT3
*
*************************************************************
*                                                           *
* This subroutine calculates the Fast Fourier Transform     *
*                                                           *
*************************************************************
*
*
*
*
      integer dp,dp2,mvopt,redopt,dimt,dimf,mm,nn,rf,rdp,rdp2,
     :          nf,mt,nf2,mt2
      PARAMETER  (DP = 512)
      real pi,tin(dp),ain(dp),rawa(dp),rwdf(dp*2,dp),rswdf(dp*2,dp),
     :    rawt(dp),dtsum,delt,dt,asum,meanv,mtime,del1,del2,
     :    const,t,sum,sumb,val,sval,wdf(dp*2,dp),coef,df
      complex s(dp*2),dum,c(dp*5),dum3,dum2
      character*25  inname
*
      common/in/ mvopt,redopt,dimt,dimf,mm,nn,rf,rdp,rdp2,
     :          nf,mt,nf2,mt2
      common/rl/ pi,tin,ain,rawa,rwdf,rswdf,
     :    rawt,dtsum,delt,dt,asum,meanv,mtime,del1,del2,
     :    const,t,sum,sumb,val,sval,wdf,coef,df,area
      common/comp/ s,dum,c,dum3,dum2
      dp2 = dp*2.0
*
*
*
      const=dp2

      val=alog(const)/alog(2.)+.1

      j=1
      do 40 i=1,dp2-1
        if (i.ge.j) go to 10
        dum3=c(j)
        c(j)=c(i)
        c(i)=dum3
 10     k=dp
 20     if (k.ge.j) go to 30
        j=j-k
        k=k/2
        go to 20
 30     j=j+k
 40   continue
      do 70 n=1,val
        coef=2**n
        coef=coef/2
        dum2=cmplx(1.,0.)
        dum=cmplx(cos(pi/coef),-sin(pi/coef))

        do 60 j=1,coef
          do 50 i=j,dp2,2*coef
            ii=i+coef
            dum3=c(ii)*dum2
            c(ii)=c(i)-dum3
            c(i)=c(i)+dum3
 50       continue
          dum2=dum2*dum
 60     continue
 70   continue

      END
```

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```
        SUBROUTINE HAMMG3
*
**********************************************************
*                                                        *
*   This subroutine applies a modified hamming window    *
*   to the signal ain(t)                                 *
*                                                        *
**********************************************************
*
*
      integer dp,dp2,mvopt,redopt,dimt,dimf,mm,nn,rf,rdp,rdp2,
     :        nf,mt,nf2,mt2
      PARAMETER  (DP = 512)
      real pi,tin(dp),ain(dp),rawa(dp),rwdf(dp*2,dp),rswdf(dp*2,dp),
     :     rawt(dp),dtsum,delt,dt,asum,meanv,mtime,del1,del2,
     :     const,t,sum,sumb,val,sval,wdf(dp*2,dp),coef,df,area
      complex s(dp*2),dum,c(dp*5),dum3,dum2
      character*25  inname
*
      common/in/ mvopt,redopt,dimt,dimf,mm,nn,rf,rdp,rdp2,
     :        nf,mt,nf2,mt2
      common/rl/ pi,tin,ain,rawa,rwdf,rswdf,
     :     rawt,dtsum,delt,dt,asum,meanv,mtime,del1,del2,
     :     const,t,sum,sumb,val,sval,wdf,coef,df,area
      common/comp/ s,dum,c,dum3,dum2
      dp2 = dp*2
*

      mtime=dp*dt
      del1=0.1*mtime
      del2=0.9*mtime
      const=pi/del1

      do 100 j = 1 , dp

        t = (j-1) * dt

        if (t.le.del1) then

          ain(j) = ain(j) * (.5*(1.-cos(const*t)))

        elseif ((t.ge.del2).and.(t.le.mtime)) then

          ain(j)=ain(j)*(.5*(1.-cos(const*(mtime-t))))

        else

        endif

100     continue

      END
```

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```fortran
        SUBROUTINE MEAN3
*
*************************************************************
*                                                           *
* This subroutine calculates and removes the mean value     *
* of the signal.                                            *
*                                                           *
*************************************************************

*
*
      integer dp,dp2,mvopt,redopt,dimt,dimf,mm,nn,rf,rdp,rdp2,
     :        nf,mt,nf2,mt2
      PARAMETER  (DP = 512)
      real pi,tin(dp),ain(dp),rawa(dp),rwdf(dp*2,dp),rswdf(dp*2,dp),
     :     rawt(dp),dtsum,delt,dt,asum,meanv,mtime,del1,del2,
     :     const,t,sum,sumb,val,sval,wdf(dp*2,dp),coef,df,AREA
      complex s(dp*2),dum,c(dp*5),dum3,dum2
      character*25  inname
*
      common/in/ mvopt,redopt,dimt,dimf,mm,nn,rf,rdp,rdp2,
     :        nf,mt,nf2,mt2
      common/rl/ pi,tin,ain,rawa,rwdf,rswdf,
     :     rawt,dtsum,delt,dt,asum,meanv,mtime,del1,del2,
     :     const,t,sum,sumb,val,sval,wdf,coef,df,area
      common/comp/ s,dum,c,dum3,dum2
      dp2 = dp*2
*
*
*

        asum = 0.0

        do 100 i = 1 , dp

           asum = asum + ain(i)

100     continue

        meanv = asum / dp

         if (mvopt.eq.1) then

          do 200 i = 1 , dp
           ain(i) = ain(i) - meanv
200        continue

         endif

        END
```

```
      SUBROUTINE REDUCE3
*
***********************************************************
*                                                         *
*    This subroutine reduces the wigner ville             *
*    to a workable size.                                  *
*                                                         *
***********************************************************

*
*
      integer dp,dp2,mvopt,redopt,dimt,dimf,mm,nn,rf,rdp,rdp2,
     :        nf,mt,nf2,mt2
      PARAMETER  (DP = 512)
      real pi,tin(dp),ain(dp),rawa(dp),rwdf(dp*2,dp),rswdf(dp*2,dp),
     :     rawt(dp),dtsum,delt,dt,asum,meanv,mtime,del1,del2,
     :     const,t,sum,sumb,val,sval,wdf(dp*2,dp),coef,df,area
      complex s(dp*2),dum,c(dp*5),dum3,dum2
      character*25  inname
*
      common/in/ mvopt,redopt,dimt,dimf,mm,nn,rf,rdp,rdp2,
     :        nf,mt,nf2,mt2
      common/rl/ pi,tin,ain,rawa,rwdf,rswdf,
     :     rawt,dtsum,delt,dt,asum,meanv,mtime,del1,del2,
     :     const,t,sum,sumb,val,sval,wdf,coef,df,area
      common/comp/ s,dum,c,dum3,dum2
      dp2 = dp*2
*
*
*

      IF (REDOPT.EQ.1) THEN
           RF = 64
           RT = 32
        ELSEIF (REDOPT.EQ.2) THEN
           RF = 128
           RT = 64
        ELSEIF (REDOPT.EQ.3) THEN
           RF = 128
           RT = 128
        ELSE
           RF = 256
           RT = 128
      ENDIF

      nn = dp2 / RF
      mm = dp / RT

        l = 0
      do 100 j = 1 , dp2 , nn
        l = l + 1
        k = 0
      do 100 i = 1 , dp , mm
        k = k + 1
           rwdf(l,k) = wdf(j,i)
100   continue

      END
```

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```fortran
      SUBROUTINE SMOOTH3
*
****************************************************************
*                                                          *
* This subroutine reduces and smooths the WDF along both    *
* the time and frequency axes for purposes of plotting      *
* clarity.                                                  *
*                                                          *
****************************************************************
*
      real hg(-100:100,-100:100)



*

      integer dp,dp2,mvopt,redopt,dimt,dimf,mm,nn,rf,rdp,rdp2,
     :         nf,mt,nf2,mt2
      PARAMETER  (DP = 512)
      real pi,tin(dp),ain(dp),rawa(dp),rwdf(dp*2,dp),rswdf(dp*2,dp),
     :     rawt(dp),dtsum,delt,dt,asum,meanv,mtime,del1,del2,
     :     const,t,sum,sumb,val,sval,wdf(dp*2,dp),coef,df,area
      complex s(dp*2),dum,c(dp*5),dum3,dum2
      character*25  inname
*
      common/in/ mvopt,redopt,dimt,dimf,mm,nn,rf,rdp,rdp2,
     :         nf,mt,nf2,mt2
      common/rl/ pi,tin,ain,rawa,rwdf,rswdf,
     :     rawt,dtsum,delt,dt,asum,meanv,mtime,del1,del2,
     :     const,t,sum,sumb,val,sval,wdf,coef,df,area
      common/comp/ s,dum,c,dum3,dum2
      dp2 = dp*2

*
*
*
      df=1./(4.*dp*dt)

      rdp=dp/mm
      rdp2=dp2/nn

      nf2=nf*2
      mt2=mt*2

      val=1./((2.*pi)**2*nf*mt)

      do 20 j=-mt2,mt2
        do 10 i=-nf2,nf2
          coef=-((j*j)/(2.*mt*mt)+(i*i)/(2.*nf*nf))
          hg(i,j)=val*exp(coef)
 10     continue
 20   continue
      do 100 j=1,rdp
      do 100 i=1,rdp2
 100  rswdf(i,j)=0.0
*
*
            ii = 0
        DO 500 N=1,DP2,nn
            jj = 0
            ii = ii+1
          DO 450 M = 1,DP,mm
              jj = jj+1
            DO 400 K=m-MT,m+MT
              DO 350 L = n-nf,n+nf

                IF (L.LT.1) THEN
                   RSWDF(ii,jj) = RSWDF(ii,jj) + 0
                 ELSEIF (K.LT.1) THEN
```

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```fortran
                    RSWDF(ii,jj) = RSWDF(ii,jj) + 0
                ELSEIF (L.GT.DP2) THEN
                    RSWDF(ii,jj) = RSWDF(ii,jj) + 0
                ELSEIF (K.GT.DP) THEN
                    RSWDF(ii,jj) = RSWDF(ii,jj) + 0
                ELSE
                    RSWDF(ii,jj) = RSWDF(ii,jj)+WDF(L,K)*HG(L-N,K-M)
                ENDIF

350             CONTINUE
400          CONTINUE
          RSWDF(ii,jj)=RSWDF(ii,jj)
450       CONTINUE
500     CONTINUE

      END
```

```
          SUBROUTINE WIGH3
*
**********************************************************
*                                                        *
*    This subroutine calculates the WDF of the signal    *
*                                                        *
**********************************************************
*
*
*
      integer dp,dp2,mvopt,redopt,dimt,dimf,mm,nn,rf,rdp,rdp2,
     :        nf,mt,nf2,mt2
      PARAMETER  (DP = 512)
      real pi,tin(dp),ain(dp),rawa(dp),rwdf(dp*2,dp),rswdf(dp*2,dp),
     :     rawt(dp),dtsum,delt,dt,asum,meanv,mtime,del1,del2,
     :     const,t,sum,sumb,val,sval,wdf(dp*2,dp),coef,df,area
      complex s(dp*2),dum,c(dp*5),dum3,dum2
      character*25  inname
*
      common/in/ mvopt,redopt,dimt,dimf,mm,nn,rf,rdp,rdp2,
     :        nf,mt,nf2,mt2
      common/rl/ pi,tin,ain,rawa,rwdf,rswdf,
     :     rawt,dtsum,delt,dt,asum,meanv,mtime,del1,del2,
     :     const,t,sum,sumb,val,sval,wdf,coef,df,area
      common/comp/ s,dum,c,dum3,dum2
      dp2 = dp*2
*

      do 100  j = 1 , dp

*******auto correlation of the signal corr3
         coef = 2.0 * dt
         do 300 i = 1 , dp+1

           if(j.ge.i) then
             dum = s(j-i+1)
            else
             dum = cmplx(0.,0.)
           endif

          c(i) = coef * (s(j+i-1)*conjg(dum))

           if(i.ne.j.and.i.ne.dp+1) then
             c(dp2-i+2) = conjg(c(i))
           endif

300      continue

         call fft3

         do 200 i=1,dp2
             wdf(i,j)=real(c(i))
200      continue

100   continue

      END
```

C. PLOTTING PROGRAMS

Listed below are the two computer programs that were used to produce the Figures in this work.

1. 2-D Plotting Routine

```
      PROGRAM SIGNALPLT
*
************************************************************
*
*    THIS PROGRAM USES THE GRAPHIC PACKAGE DISSPLA TO
*    PLOT THE A 2-D FUNCTION.
*
************************************************************
*
*    ***---DECLARING VARIABLES---***
*
      REAL XARAY(1000),YARAY(1000)
      REAL X,Y,F,XSTART,XEND,YSTART,YEND,XSTEP
      INTEGER IPROPT,NPNTS,IX,IY
*
*    ***---INITIALIZING VARIABLES AND SETTING GRAPH OPTIONS---***
*
*          ---set option for x starting & ending value---
      WRITE(*,*),'DECIDE THE STARTING AND ENDING VALUES OF X'
      WRITE(*,*),'INPUT THE STARTING VALUE OF X'
      READ(*,*),  XSTART
      WRITE(*,*),'INPUT THE ENDING VALUE OF X'
      READ(*,*),  XEND
*          ---set desired x step size---
      WRITE(*,*),'DECIDE THE X STEP SIZE'
      WRITE(*,*),'INPUT THE X STEP SIZE'
      READ(*,*),  XSTEP
*          ---set option for y starting & ending value---
      WRITE(*,*),'DECIDE THE STARTING AND ENDING VALUES OF Y'
      WRITE(*,*),'INPUT THE STARTING VALUE OF Y'
      READ(*,*),  YSTART
      WRITE(*,*),'INPUT THE ENDING VALUE OF Y'
      READ(*,*),  YEND
*          ---set option for number of points to plot---
      WRITE(*,*),'DECIDE THE NUMBER OF POINTS TO PLOT'
      WRITE(*,*),'INPUT THE NUMBER OF POINTS'
      READ(*,*),  NPNTS
*          ---set option for line style (IMARK)---
      WRITE(*,*),'DECIDE THE IMARK STYLE DESIRED'
      WRITE(*,*),'INPUT THE IMARK STYLE'
      READ(*,*),  IMARK
*          ---set option for grid style---
      WRITE(*,*),'DECIDE THE GRID STYLE DESIRED'
      WRITE(*,*),'INPUT THE X GRID STYLE'
      READ(*,*),  IX
      WRITE(*,*),'INPUT THE Y GRID STYLE'
      READ(*,*),  IY
*
*          ---set option for output device---
      WRITE(*,*),'YOU WILL NOW DECIDE WHERE TO SEND THE OUTPUT'
      WRITE(*,*),'ENTER 0 FOR OUTPUT TO THE SCREEN'
      WRITE(*,*),'ENTER 1 FOR OUTPUT TO THE LASER'
      READ(*,*),  IPROPT
```

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```
*
C               CALL PDEV('LNO3',IEER)
            IF (IPROPT.EQ.0) THEN
                CALL PGPX
              ELSE
                CALL LNO3I
            ENDIF
*
*
*     **FUNCTION EXECUTION TO DETERMINE PLOTTING POINTS**
*
      OPEN (20,FILE='SIN400256A2.DAT',STATUS='OLD')
      CALL F(XARAY,YARAY,NPNTS,XSTART,XEND)
*
*
*     ***---LEVEL ONE COMMANDS---***
        CALL HWROT('COMIC')
        CALL PAGE(11.0,8.5)
        CALL NOBRDR
        CALL AREA2D(9.0,6.0)
*
*     ***---LEVEL TWO COMMANDS---***
*
        CALL SWISSB
        CALL HEADIN('400 HZ SINE WAVES$',100,1.5,2)
        CALL HEADIN('SIGNAL LENGTH = 0.128 SEC  / AMP = 2.0$',100,1.5,2)
        CALL SWISSM
        CALL XNAME('TIME (sec)$',100)
        CALL YNAME('AMPLITUDE$',100)
        CALL GRAF(XSTART,XSTEP,XEND,YSTART,'SCALE',YEND)
*
*     ***---LEVEL THREE COMMANDS---***
*
        CALL CURVE(XARAY,YARAY,NPNTS,IMARK)
        CALL FRAME
        CALL GRID(IX,IY)
*
*     ***---CLOSING COMMANDS---***
*
        CALL ENDPL(0)
        CALL DONEPL
        END
*
*
*      ---FUNCTION SUBROUTINE TO DETERMINE PLOTTING POINTS---
*
        SUBROUTINE F(XARAY,YARAY,NPNTS,XSTART,XEND)
*
        REAL XARAY(NPNTS),YARAY(NPNTS),XSTART,XEND,STEP,XTEMP
        INTEGER NPNTS
*         ---READ IN THE XARAY AND YARAY VALUES---
        DO 100 I = 1,NPNTS
            READ(20,*)  XARAY(I), YARAY(I)
100     CONTINUE
        END
```

2.   3-D Plotting Routine

```
      PROGRAM WDFPLTVER3
*
***********************************************************
*
*    THIS PROGRAM USES THE GRAPHIC PACKAGE DISSPLA TO
*    PLOT A 3-D FUNCTION.
*
***********************************************************
*
*    ***---DECLARING VARIABLES---***
*
      REAL RWDF(32768)
      INTEGER IPROPT,IXPTS,IYPTS
*
*    ***---INITIALIZING VARIABLES AND SETTING GRAPH OPTIONS---***
         OPEN(15,FILE='RSWDF.OUT',STATUS='OLD')
*
      DO 100 I = 1,32768
        READ(15,*) RWDF(I)
100    CONTINUE
*
*
*
              call pdev('ln03', ieer)
*
*    **FUNCTION EXECUTION TO DETERMINE PLOT**
*
*    ***---LEVEL ONE COMMANDS---***
*
      CALL PAGE(8.5,11.0)
      CALL AREA2D(8.0,7.0)
*
*    ***---LEVEL TWO COMMANDS---***
*
      CALL VOLM3D(6.,6.,4.)
      CALL HEADIN('200 & 600 Hz SINE WAVE$',100,-1.5,2)
      CALL HEADIN('FREQUENCY RESOLUTION ; 5 X 35$',100,-1.5,2)
      CALL X3NAME('FREQUENCY$',100)
      CALL Y3NAME(' ',1)
      CALL Z3NAME('AMPLITUDE$',100)
      CALL VUANGL(-90.,0.,30.)
      CALL GRAF3D(0.,1250./5.,1250.0,
     :            0.,.2048/2.,0.2048,
     :            0.,'SCALE',.01)
*
*    ***---LEVEL THREE COMMANDS---***
*
      CALL SURMAT(RWDF,1,256,1,128,0)
      CALL FRAME
*
*    ***---CLOSING COMMANDS---***
*
      CALL ENDPL(0)
      CALL DONEPL
*
      END
```

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