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A computerized investigation using the method of images to predict the sound field in a fluid wedge overlying a slow fluid hal-space

Kaswandi, Carolus

Monterey, California: U.S. Naval Postgraduate School

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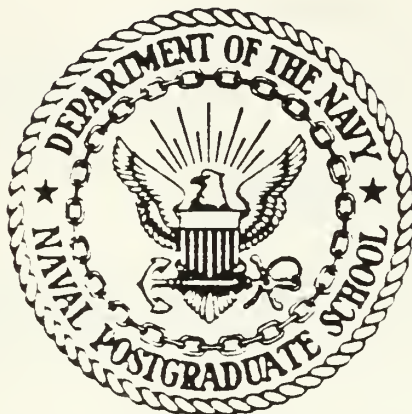






# NAVAL POSTGRADUATE SCHOOL

Monterey, California



## THESIS

K1451

A Computerized Investigation Using the Method  
of Images to Predict the Sound Field in a  
Fluid Wedge Overlying a Slow Fluid Half-Space

by

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December 1987

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A Computerized Investigation Using the Method  
of Images to Predict the Sound Field in a  
Fluid Wedge Overlying a Slow Fluid Half-Space

by

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### ABSTRACT

Sound fields in wedge-shaped ocean layers, modeling conditions on the continental shelf, have been studied at the Naval Postgraduate School in the last few years using the method of images. These studies are carried further in the present work. The method is implemented in different environmental conditions. This thesis examines the influence of several parameters on the sound field for downslope propagation in a wedge-shaped fluid of speed of sound  $c_2$  overlying a slow bottom of speed of sound  $c_1$ . On the basis of qualitative and semi-quantitative analysis of the behavior of the pressure-depth profile for various geometrical and physical parameters, we can conclude that:

1. A defined distance, the "characteristic distance"  $X_0 = \pi / (2k_2 \sin \theta_0 \tan \beta)$ , where  $\cos \theta_0 = c_1/c_2$ ,  $k_2 = \omega/c_2$ , and  $\beta$  is the vertex angle of the wedge, has physical meaning as a useful scaling distance.
2. The distance of the source from the apex, in terms of the  $X_0$ , plays a major role in determining the downslope sound field.

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Finally, I hope that in the future this work will be useful to others by expanding their knowledge in this field.

## I. INTRODUCTION

### A. SOUND PROPAGATION IN SHALLOW WATER CHANNEL

Experimental investigations of sound propagation in shallow water channels have been done by several investigators. Shallow water propagation is of interest because of the applications to coastal defense. These investigations are expensive and time consuming. The use of a computer model should provide a relatively inexpensive alternative to observation.

One of the techniques uses normal mode theory. The normal mode theory, which was introduced and developed by C. L. Pekeris [Ref. 1], gave the exact solution in water of constant depth. Further development of normal mode theory was made by L. Brekovskikh [Ref. 2], who initiated pressure as an integral involving Bessel functions and solution of the normal mode equation. Another theoretical approach to sound propagation in a horizontally stratified ocean of constant depth is given by the method of multiple scattering [Ref. 1]. With this method, all the previous theories can be simplified by conversion into an asymptotic form which is valid when the acoustic wavelength is small compared to the distance over which the sound speed varies appreciably. These theories agree with the laboratory experiments.

For a water channel with a small bottom slope, the sound field may be expressed approximately in terms of adiabatic normal modes. To facilitate prediction, R.D. Groves, Anton Nagl, H. Uberall, and G. L. Zauer [Ref. 3] modeled a wedge-shaped isovelocity ocean with a linearly-sloping, perfectly-rigid ocean floor using adiabatic normal modes. For a penetrable bottom the normal mode description fails when modes propagating upslope encounter the "critical depth" ( $H_C$ ), defined as the depth where the associate mode changes from fully trapped within the water channel to radiating energy into the bottom (cut off) [Ref. 4-6]. The parabolic equation can be used to explain the mechanism of sound energy radiation into the bottom [Ref. 7,8]. Such an equation was studied by F. B. Jensen and W. A. Kuperman [Ref. 9], with predictions that satisfactorily agreed with the experimental results for small ray angles. With some restrictions, normal mode theory is applicable for sound propagation in the wedge-shaped fluid with a fast bottom. The parabolic equation is good for fast and slow bottom, but with the restriction that horizontal ray angles must be less than  $20^\circ$ .

Another technique introduced to predict the propagation of sound in the wedge is the method of images. This method was derived from the simplest case; a monofrequency point source in a homogeneous ocean with parallel boundaries. The total pressure is the sum of an infinite number of spherical

waves from an infinite set of images. The restriction of this method is that it does not generalize to the case of inhomogeneous media or non-planar boundaries. In this work, this method will be studied.

## B. THE METHOD OF IMAGES

In 1978, Coppens, Sanders, Ioannou, and Kawamura [Ref. 10], predicted the pressure amplitude and phase of the sound field along the bottom of a wedge-shaped fluid layer of density  $\rho_1$ , and speed of sound  $c_1$ , overlying a fast fluid bottom of density  $\rho_2$ , and speed of sound  $c_2 > c_1$  by applying the method of images in a computer program implementation. In 1984, Baek [Ref. 11], and LeSesne [Ref. 12], implemented further improvements. Baek's computer program, WEDGE, and LeSesne's computer program XSLOPE were validated for several cases. WEDGE was developed for two-dimensional upslope propagation (the source and receiver are in the same vertical plane perpendicular to the shore line, and the receiver is closer to the apex than the source (Figure 1.1a)) and downslope propagation (the source is closer to the apex than the receiver (Figure 1.1b)). XSLOPE was developed for upslope, downslope, or cross-slope propagation (the source, receiver, and apex, are not necessarily in the same plane perpendicular to the shore line (Figure 1.1c)). In both programs, Baek and LeSesne assume that the fluid in the wedge and fluid in the bottom have constant densities, that

the speed of sound is constant, and that the interface between the fluids and the surface is smooth.

In both WEDGE and XSLOPE, all distances are scaled in units of the "dump distance." A dump distance  $X$ , as stated in Reference 10, is the distance from the apex measured along the interface at which the lowest mode attains cutoff. If the wedge angle is  $\beta$  (Figure 1.1c), then

$$X = \frac{\pi/2}{k_1 \sin\theta_c \tan\beta} \quad (1.1)$$

$$\theta_c = \arccos(c_1/c_2) \quad (1.2)$$

where  $k_1$  is the wave number in the wedge and  $\theta_c$  is the critical grazing angle for reflection of sound from the bottom. For  $\beta \ll 1$

$$X = H \tan \beta \quad (1.3)$$

This scaling distance negates the necessity of specifying frequency.

### C. COMPUTER PROGRAM DSLOW

At the start of the work reported in this thesis, a computer program was obtained [Ref. 13], which is an extension of the WEDGE and XSLOPE for downslope configuration with a slow bottom. The computer model, DSLOW, developed to run on a desktop computer (Wang 2000), uses the method of images to predict the pressure amplitude and phase anywhere

within the wedge fluid overlying a slow bottom in a cross-slope configuration. A geometrical picture of this configuration is shown in Figure 1.1c.

Mathematically, the model used in WEDGE and XSLOPE is applicable in any condition. But consideration must be given for making it work for a slow bottom. In the case of a fast bottom, the dump distance has a physical meaning. The dump distance is expressed as a function of the critical angle. The critical angle is equal to  $\arccos(c_1/c_2)$ . In the case of slow bottom,  $c_1/c_2$  is greater than 1, thus  $\arccos(c_1/c_2)$  is invalid; therefore, so is the dump distance. To facilitate the scaling factor, a "characteristic distance" or "scaling distance" is introduced. We need the scaling distance because, with this distance, our model will be independent of frequency as in the fast bottom case. There is also the hope that the use of a scaling distance will allow systematic observation of the pressure field. This scale distance  $X_0$  is the distance measured along the interface from the apex to the point where the lowest mode would attain cutoff if the fluids in the wedge and in the bottom were to be interchanged. The characteristic distance is defined by the following equation:

$$X_0 = \frac{\pi/2}{k_2 \sin\theta_0 \tan\beta} \quad (1.4)$$

where  $\theta_0 = \arccos(c_2/c_1)$  and  $K_2 = \omega/c_2$  is the wave number in the bottom.

The following terms will be used throughout (see Figure 1.1):

$\beta$  = wedge angle

$R_1$  = distance of the source from the apex in units of  $X_0$

$R_2$  = distance of the receiver from the apex in units of  $X_0$

$\gamma$  = angle of elevation of the source above the bottom

$\delta$  = angle of elevation of the receiver above the bottom

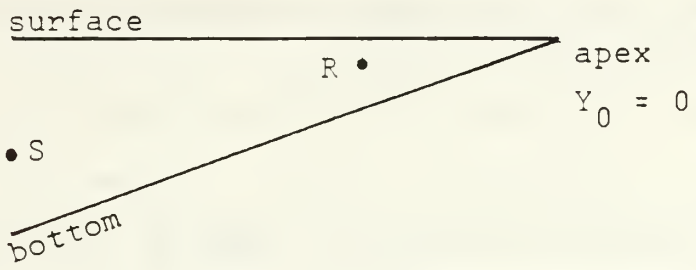
$Y_0$  = distance between the projection of the source and receiver on the shore line, scaled by  $X_0$

$\rho_1/\rho_2$  is the ratio between the density of the fluid in the wedge ( $\rho_1$ ) and the density of the fluid in the bottom ( $\rho_2$ )

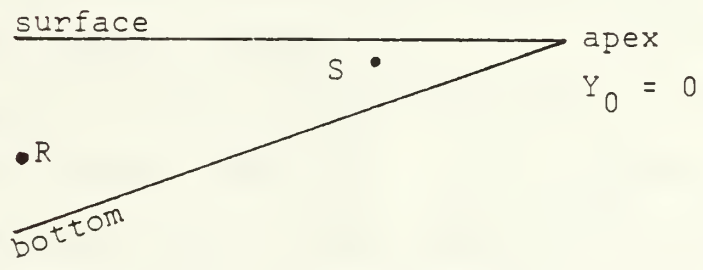
$c_1/c_2$  is the ratio between the speed of sound in the wedge ( $c_1$ ) and the speed of sound in the bottom ( $c_2$ ). A fast bottom occurs when  $c_2 > c_1$ ; a slow bottom occurs when  $c_2 < c_1$

The purpose of this research is the following:

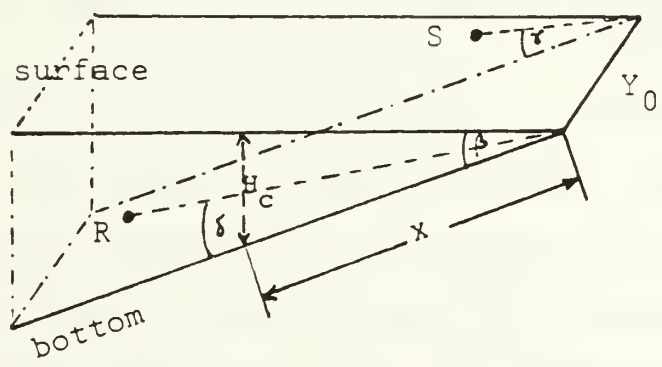
1. To transfer, test, and evaluate DSLOW program on the IBM 3300;
2. To obtain numerical and graphical output for a number of cases; and
3. To attempt to develop plausible explanations for any significant features observed.



a. upslope configuration



b. downslope configuration



c. cross-slope configuration

Figure 1.1 Geometry of the wedge



## II. THEORY

### A. GENERAL VIEW OF A WEDGE PRESSURE DISTRIBUTION IN THE DOWNSLOPE CONFIGURATION

A general picture of the sound energy propagation within the wedge in downslope direction is given in Figure 2.1. If a sound source is placed at point S, ray 1 will reach the surface at point P with an incident angle  $\alpha$  with respect to the normal to the surface at this point. This ray is reflected by the surface at the same angle but with the phase  $180^\circ$  different. (On the surface, sound pressure is zero everywhere.) The reflected ray reaches the bottom with an incident angle  $\beta + \alpha$ . At great enough distance, ray 1 never reaches the bottom again. This ray does not contribute to a sound pressure field at the bottom. The pressure at the bottom should be very small according to the ray theory argument.

Using these ray-tracing methods, an estimated profile of the pressure amplitude versus the receiver depth can be made. When the source and the receiver are placed near the apex, the pressure amplitude is zero at the surface, a maximum somewhere within the wedge, and greater than zero at the bottom. In the case where the source is at a far distance, the pressure amplitude is equal to zero at the surface, a maximum somewhere within the wedge, and zero at the bottom.

Ray tracing will only give a rough approximation, not an exact solution, but ray tracing may be used as a guide. The method of images calculates the exact pressure amplitude at each point within the wedge subject only to the assumption inherent in using the plane-wave Rayleigh reflection coefficients.

B. SOUND PRESSURE AT A POINT IN THE WEDGE DOWNSLOPE PREDICTED BY THE METHOD OF IMAGES

Let the source be a scaled distance  $R_1$  from the apex and at an angle of  $\gamma$  measured from the bottom of the wedge. Let the receiver be a scaled distance  $R_2$  from the apex and at an angle  $\delta$  measured from the bottom.

Using Figure 2.2, let the upper half family of images be  $n = 1, 2, 3, 4, \dots$  and the lower half family be  $n' = 1, 2, 3, 4, \dots$ . Calculating the field resulting from source and images proceeds along the lines developed in [Ref. 14]. If  $\phi_n$  is the angle formed at the apex between the  $n_{th}$  image of the source and the receiver, then

$$\phi_1 = 2\beta - \delta - \gamma$$

$$\phi_2 = 2\beta - \delta + \gamma$$

$$\phi_3 = 4\beta - \delta - \gamma$$

$$\phi_4 = 4\beta - \delta + \gamma$$

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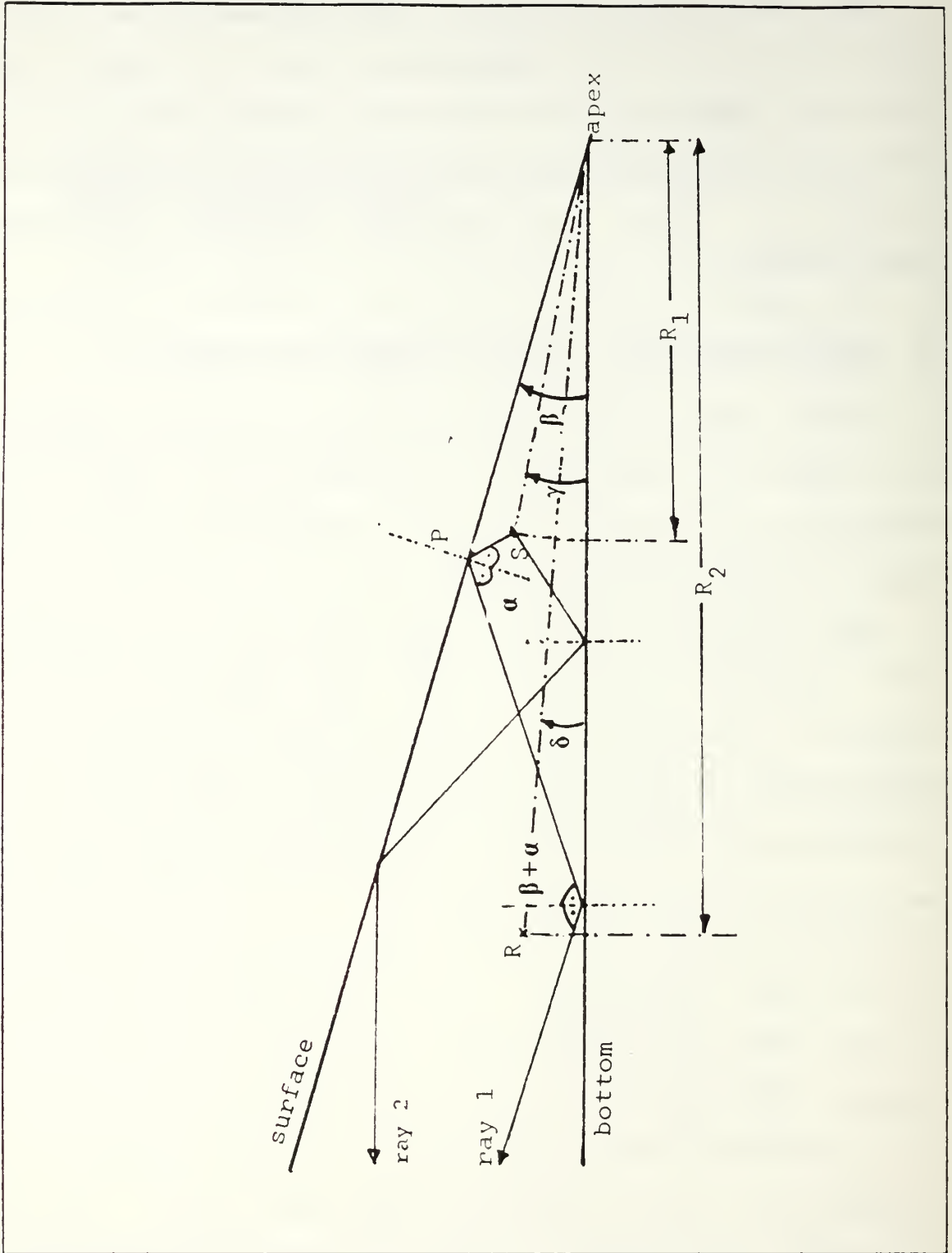


Figure 2.1 Ray tracing in the wedge downslope

or more generally

$$\phi_n = (n + 1)\beta - \delta - \gamma \quad \text{for } n \text{ odd}$$

$$\phi_n = n\beta - \delta + \gamma \quad \text{for } n \text{ even}$$

Which can be reduced to:

$$\phi_n = \{n + (1/2)[1 - (-1)^n]\}\beta + (-1)^n\gamma - \delta \quad (2.1)$$

or

$$\phi_n = 2 \text{ INT} \left[ \frac{n + 1}{2} \right] \beta + (-1)^n\gamma - \delta \quad (2.2)$$

where  $\text{INT}[\ ]$  denotes the largest integer which is equal to, or smaller than the argument. Using the same method for the member  $n'$  of the lower family of images we obtain:

$$\phi_{n'} = \{n + (1/2)[1 - (-1)^n]\}\beta + (-1)^n\gamma + \delta \quad (2.3)$$

or

$$\phi_{n'} = 2 \text{ INT} \left[ \frac{n + 1}{2} \right] \beta + (-1)^n\gamma + \delta \quad (2.4)$$

Using the geometry of Figures 2.2 and 2.4, the distance between the  $n^{\text{th}}$  and  $n'^{\text{th}}$  images to the receiver is respectively

$$r_n = \sqrt{R_1^2 + R_2^2 - 2R_1R_2\cos\phi_n} \quad (2.5)$$

and

$$r_{n'} = \sqrt{R_1^2 + R_2^2 - 2R_1R_2\cos\phi_{n'}} \quad (2.6)$$

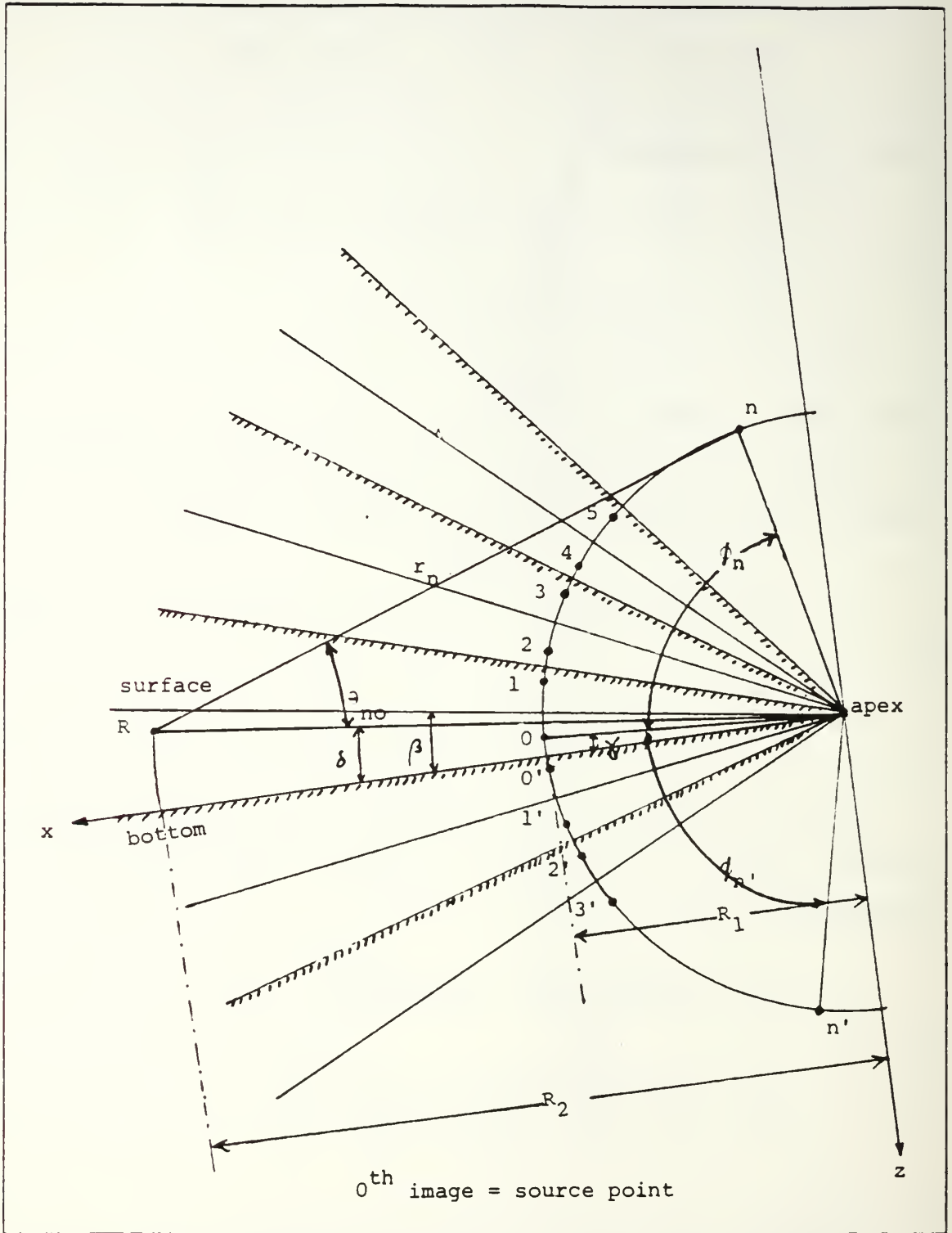


Figure 2.2 Geometry of a wedge by the method of images

The angles  $\theta_{n0}$  and  $\theta_{n'0}$  for the  $n^{\text{th}}$  and  $n'^{\text{th}}$  images respectively are

$$\theta_{n0} = \arctan \left[ \frac{\sin \phi_n}{R_2/R_1 - \cos \phi_n} \right] \quad (2.7)$$

and

$$\theta_{n'0} = \arctan \left[ \frac{\sin \phi_{n'}}{R_2/R_1 - \cos \phi_{n'}} \right] \quad (2.8)$$

Define  $\theta_{nm}$  and  $\theta_{n'm}$  as the angles of incidence for the  $m^{\text{th}}$  bounces from the bottom for the  $n$  and  $n'$  image respectively;  $m = 1, 2, 3, \dots$ . (The  $0^{\text{th}}$  bounce is the last one before reaching the receiver.) The geometry of Figures 2.2 and 2.3 give  $\theta_{nm}$  as follows:

$$\theta_{21} = \theta_{20} - 2\beta - \delta$$

$$\theta_{32} = \theta_{30} - 4\beta - \delta$$

$$\theta_{41} = \theta_{40} - 2\beta - \delta$$

$$\theta_{52} = \theta_{50} - 4\beta - \delta$$

....

....

....

The general expression is

$$\theta_{nm} = \theta_{n0} - 2m\beta - \delta$$

Using the same method

$$\theta_{n'm} = \theta_{n'0} - 2m\beta + \delta$$

The maximum number of bottom bounces of the  $n^{\text{th}}$  and  $n'^{\text{th}}$  image is

$$m_{\text{max}} = M = \text{INT} [\phi_n/2\beta] = \text{INT} [\phi_{n'}/2\beta]$$

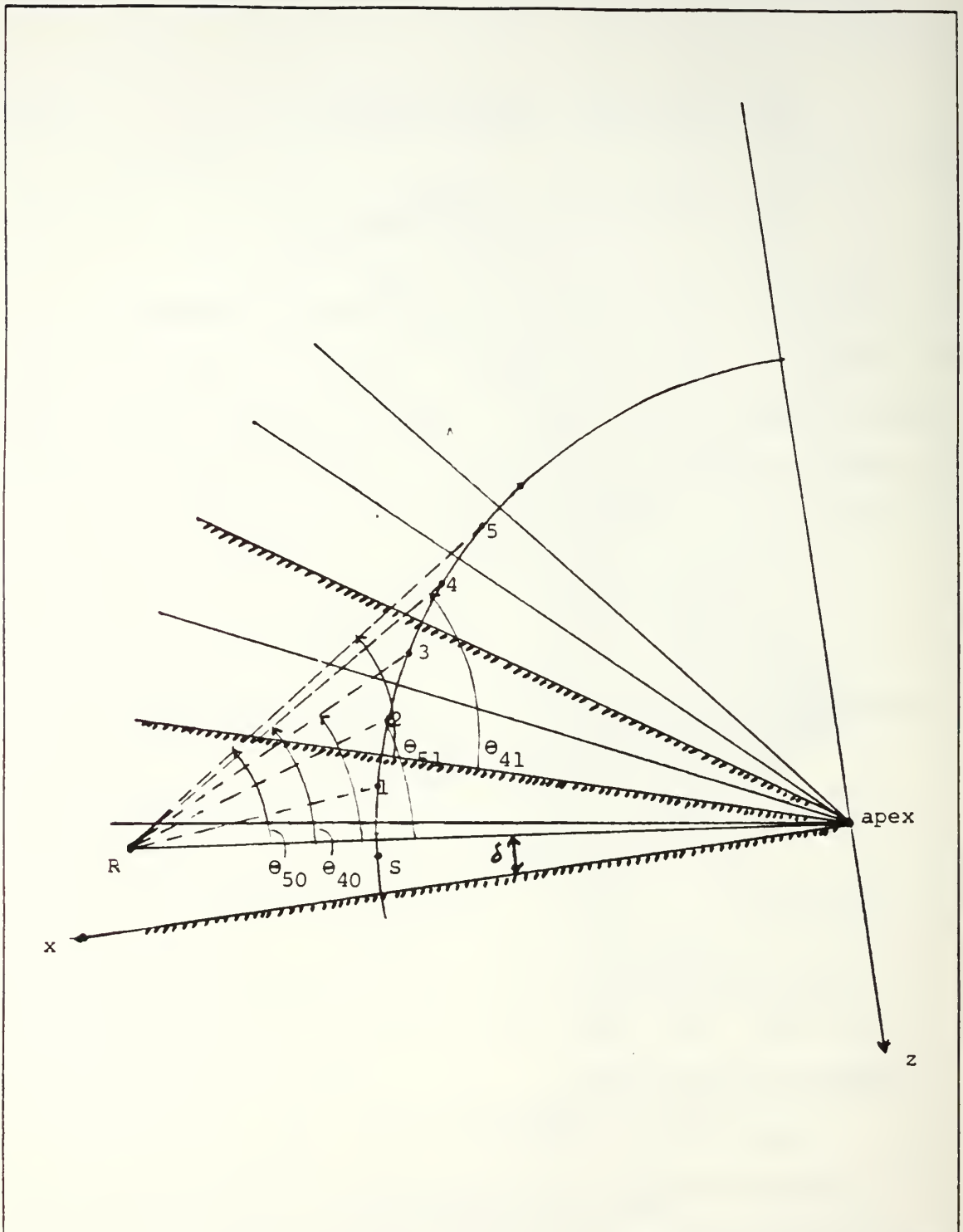


Figure 2.3 Geometric development of reflection angles

The maximum number of images is

$$n_{\max} = N = \text{INT} [\pi/\beta]$$

The reflection coefficients for the  $n^{\text{th}}$  and  $n'^{\text{th}}$  images for a plane wave are:

$$R(\theta_{nm}) = \frac{\frac{\rho_1 c_1}{\rho_2 c_2} - \psi_{nm}}{\frac{\rho_1 c_1}{\rho_2 c_2} + \psi_{nm}} \quad (2.9)$$

and

$$R(\theta_{n'm}) = \frac{\frac{\rho_1 c_1}{\rho_2 c_2} - \psi_{n'm}}{\frac{\rho_1 c_1}{\rho_2 c_2} + \psi_{n'm}} \quad (2.10)$$

where

$$\psi_{nm} = \frac{\sqrt{1 - (c_1/c_2)^2 \cos^2 \theta_{nm}}}{\sin \theta_{nm}} \quad (2.11)$$

and

$$\psi_{n'm} = \frac{\sqrt{1 - (c_1/c_2)^2 \cos^2 \theta_{n'm}}}{\sin \theta_{n'm}} \quad (2.12)$$

The contribution from the upper family of images is

$$P_u = \sum_{n=1}^N \frac{1}{r_n} \exp(-jkr_n) (-1)^{\text{INT}[n+1]/2} \prod_{m=0}^M R_{nm} \quad (2.13)$$



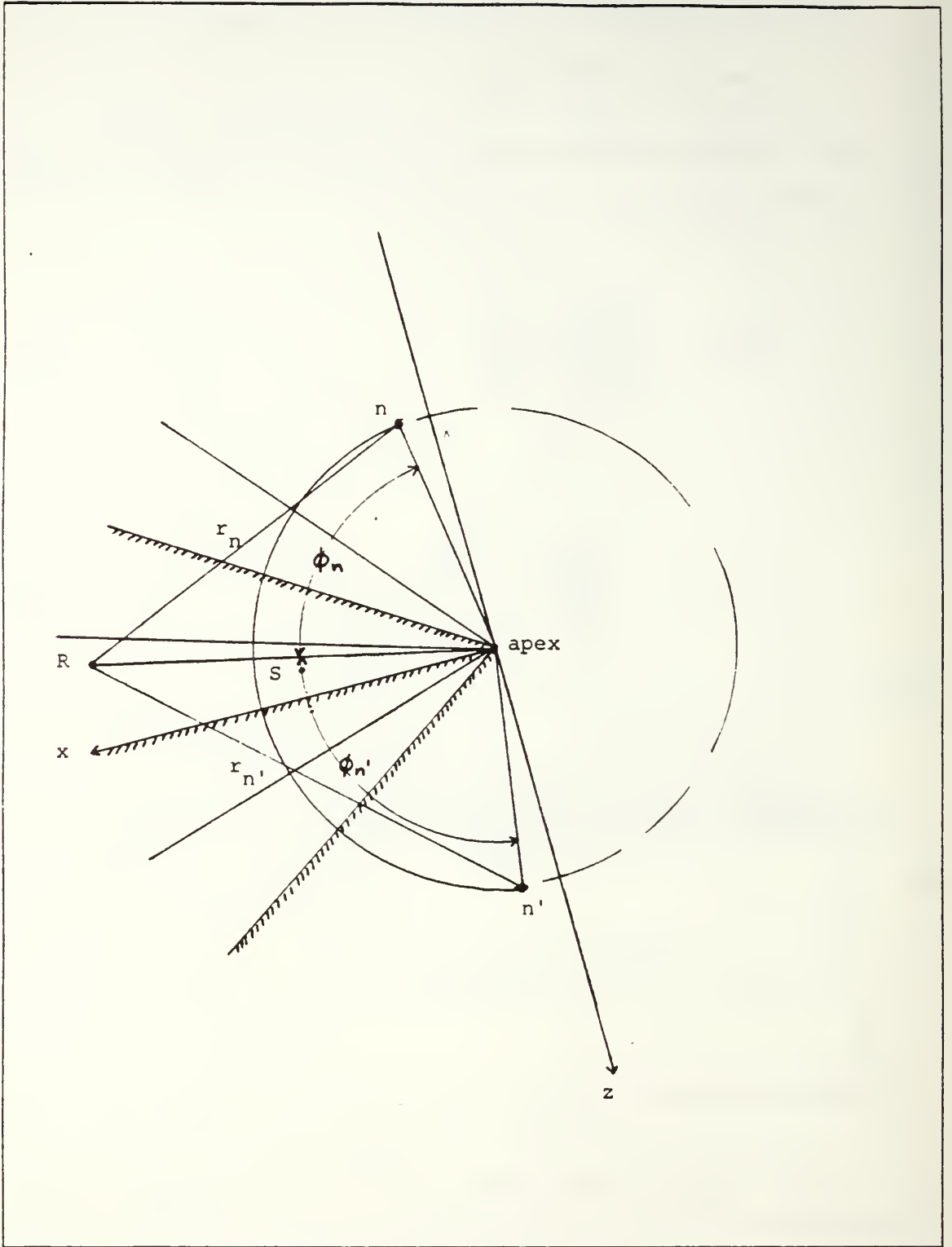


Figure 2.4 Geometry of symmetric images

and for lower family of images is

$$P_u = \sum_{n=1}^N \frac{1}{r_n'} \exp(-jkr_n') (-1)^{\text{INT}[(n+1)/2]} \prod_{m=0}^M R_{n',m} \quad (2.14)$$

The total complex pressure is

$$P(x) = P_u + P_l \quad (2.15)$$

### III. DSLOW PROGRAM IMPLEMENTATION

#### A. PROGRAMS FEATURES

Since the mainframe graphics computer was available, the DISSPLA graphical program was used. The only programming language compatible with DISSPLA is FORTRAN. The numerical and graphical output is provided by this program. To give the pressure amplitude versus received angle graphs, two-dimensional plotting is used.

The program DSLOW is run by placing the point source anywhere in the wedge and then placing the receiver at a distance downslope from the source. The receiver position was varied from zero degrees at the bottom to  $\beta$  at the surface. High resolution plotting was achieved by dividing the y-axis (received angle) into two regions. The first region covers the receiver angles from zero to  $1/5$  of the wedge angle. In this region  $\Delta\delta$  is equal to  $\beta/100$ . The second region covers the remaining wedge angle with  $\Delta\delta$  equal to  $\beta/10$ . This method provides 29 predictions of the pressure amplitude. Another method of plotting carried out was in the region of  $\delta > \beta/2$ ,  $\Delta\delta = \beta/10$ , and in the region of  $\delta < \beta/2$ ,  $\Delta\delta = \beta/100$ . This method provides 54 points to be plotted.

## B. NORMALIZATION

The main goal of this research was to investigate the profile of the pressure amplitude as a function of a number of variables. An example of the numerical values of the pressure amplitude, the normalized pressure amplitude, and the phase at each receiver position is displayed in Appendix C. The sound pressure becomes smaller as the receiver is moved away from the source. If the pressure amplitude were plotted directly, it would be difficult to compare the curves at near distances to the curves at far distances since at the near distances the pressure amplitude is much greater than the pressure amplitude in far distance. Thus, a normalized pressure amplitude is needed. The normalized pressure is obtained as follows: (see Figure 3.1)

We know that the sound pressure at the surface is zero and that the sound pressure is a small number greater than zero at a point near the surface. The first non-zero value of pressure  $P_1$  is at the receiver angle,  $\delta_1 = 9\beta/10$ . We use this first calculated non-zero pressure amplitude as the normalization unit. The normalized pressure is

$$P_N = P(\delta)/(P_1) \quad (3.1)$$

where  $P(\delta)$  is the pressure at any point within the wedge.

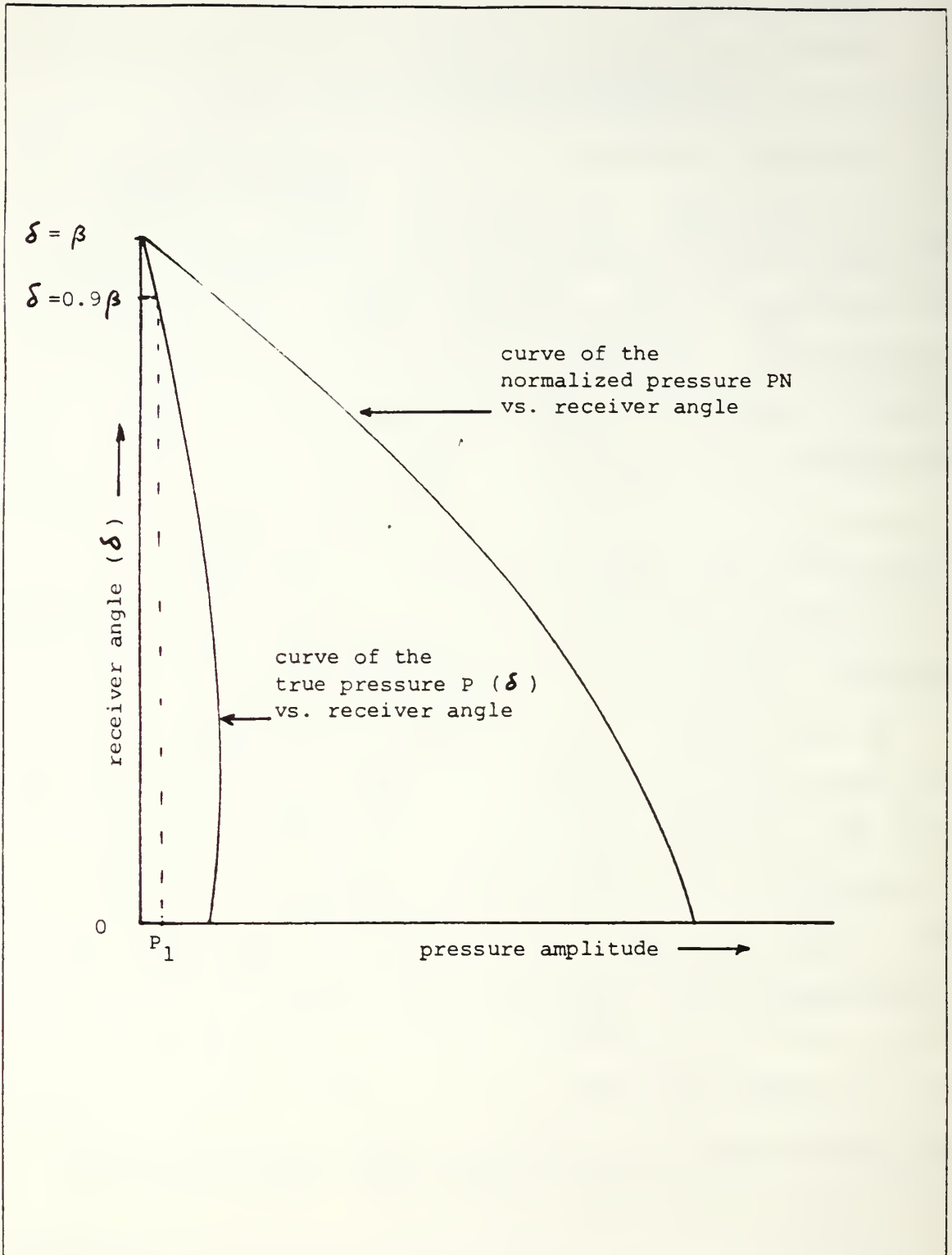


Figure 3.1 Pressure amplitude normalization

### C. PROCEDURE

Figures 3.2 through 3.7 represent the results when the receiver distance and source angle are fixed and the source distance and receiver angle are varied. Figures 3.8 through 3.13 represent the results when the source distance and angle are fixed and the received distance and angle are varied. These cases will be the foundation of our subsequent discussions.

The solid lines indicate the fitted curve and the dots indicate some values of the normalized pressure amplitude. In DSLOW, the dot appears at each third datum.

### D. PROGRAM IMPROVEMENT

DSLOW was designed to provide three-dimensional graphs. For example, the x-axis represents the scaled source distance, the y-axis represents the scaled received distance, and z-axis represents the normalized pressure amplitude. To simplify the presentation, only two-dimensional graphs were presented with the x-axis the normalized pressure amplitude and the y-axis the receiver angle  $\delta$ . All curves are presented with the data fitted with a cubic spline.

The DSLOW program was executed to obtain numerical results of the phase angle, the pressure amplitude, and the normalized pressure amplitude at each receiver position. The first run used double precision for accuracy. Difficulties were encountered when the DISSPLA subprogram

was attached for making the graphical output. When double precision and DISSPLA were not successful, the single precision was used, resulting in round-off error. (See Figure 3.8 at  $R_2 = 10.0$ .)

# RECEIVER ANGLE VS. PRESSURE

WEDGE ANGLE : 6.0  
 RH01/RH02 : 0.80  
 C1/C2 : 1.50  
 SOURCE ANGLE : 3.00  
 SOURCE.DIST.INIT : 0.25  
 SOURCE.DIST.INCR : TWICE  
 REC.DIST : 3.00  
 SHORE DIST : 0.00  
 KIX : 13.37

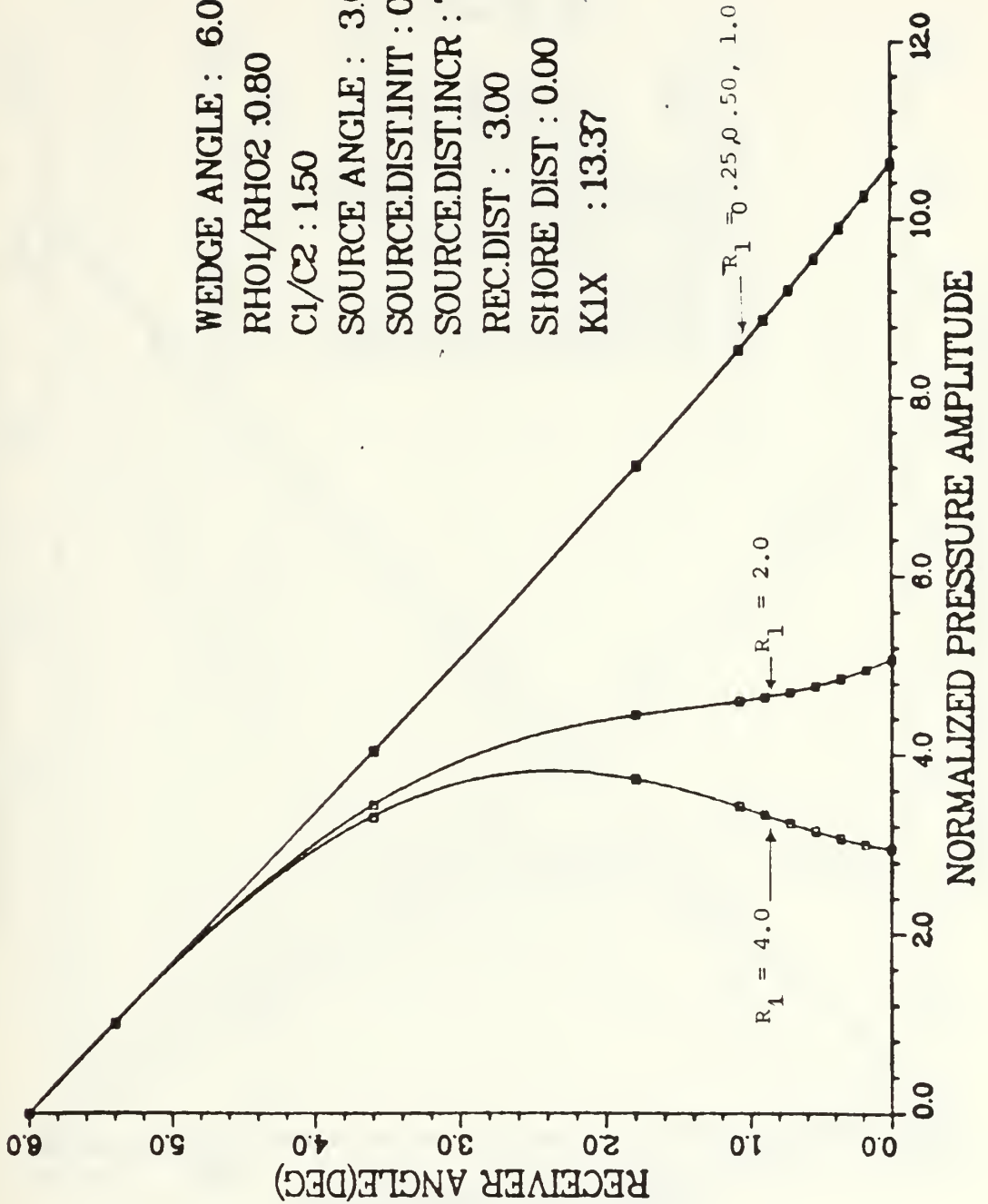


Figure 3.2 Graphs of receiver angle  $\delta$  versus pressure amplitude with  $R_2$  fixed,  $R_1$  varied



# RECEIVER ANGLE VS. PRESSURE

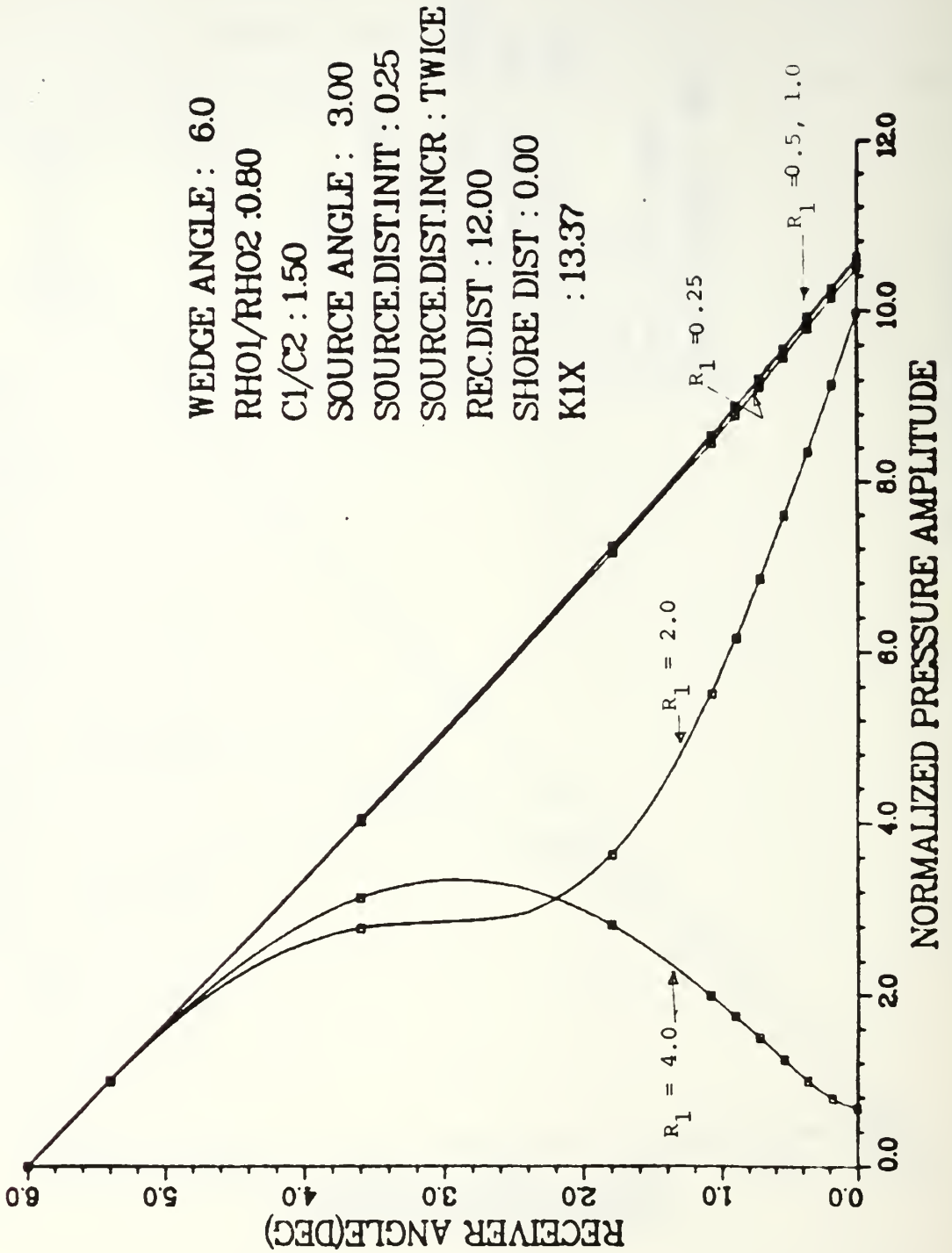


Figure 3.3 Graphs of receiver angle  $\delta$  versus pressure amplitude with  $R_2$  fixed,  $R_1$  varied

# RECEIVER ANGLE VS. PRESSURE

WEDGE ANGLE : 10.0  
 RH01/RH02 : 0.80  
 C1/C2 : 1.10  
 SOURCE ANGLE : 5.00  
 SOURCE.DIST.INIT : 0.25  
 SOURCE.DIST.INCR : TWICE  
 REC.DIST : 3.00  
 SHORE DIST : 0.00  
 KIX : 19.44

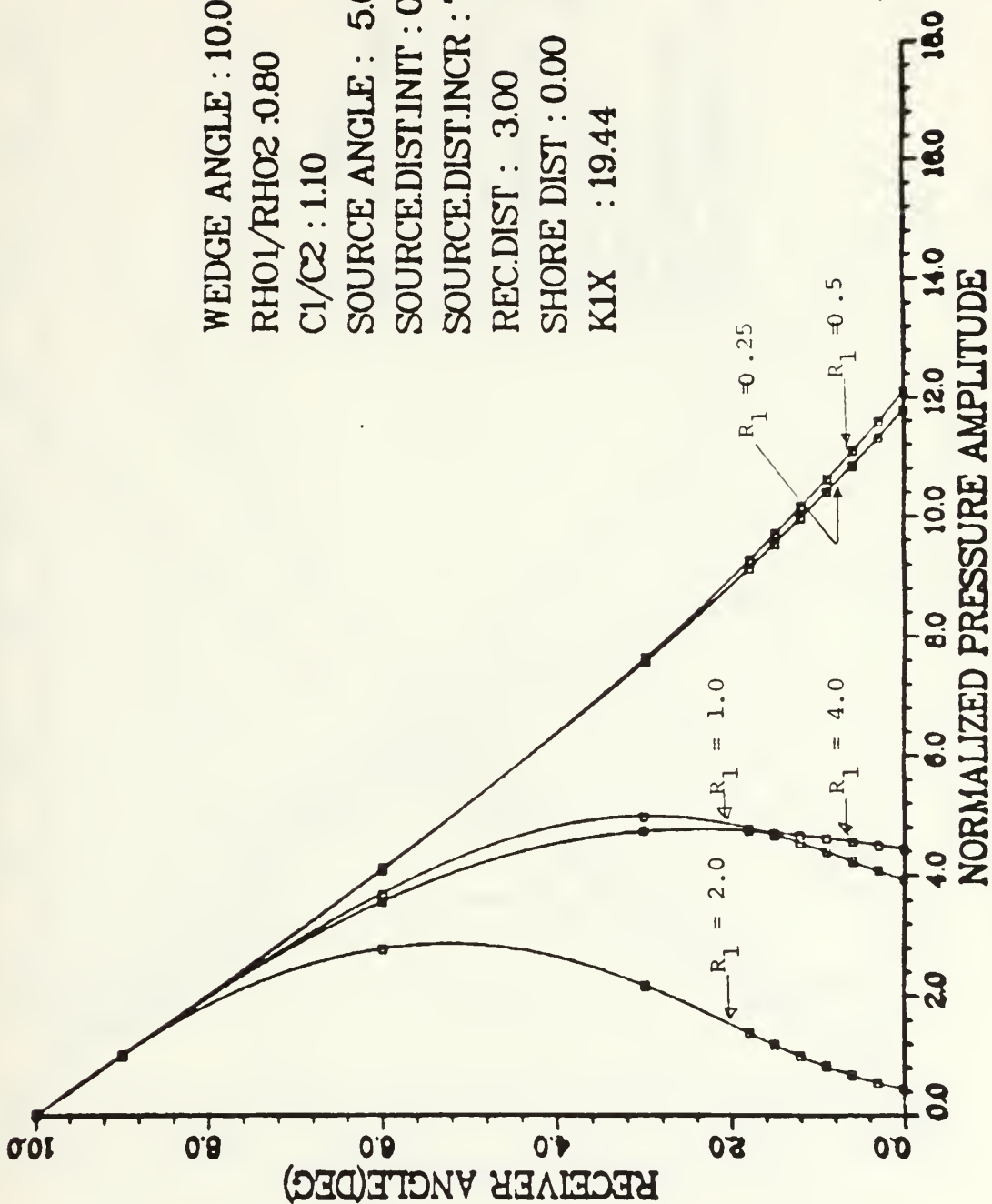


Figure 3.4 Graphs of receiver angle  $\delta$  versus pressure amplitude with  $R_2$  fixed,  $R_1$  varied

# RECEIVER ANGLE VS. PRESSURE

WEDGE ANGLE : 10.0  
 RH01/RH02 : 0.80  
 C1/C2 : 1.10  
 SOURCE ANGLE : 5.00  
 SOURCE.DIST.INIT : 0.25  
 SOURCE.DIST.INCR : TWICE  
 REC.DIST : 12.00  
 SHORE DIST : 0.00  
 KIX : 19.44

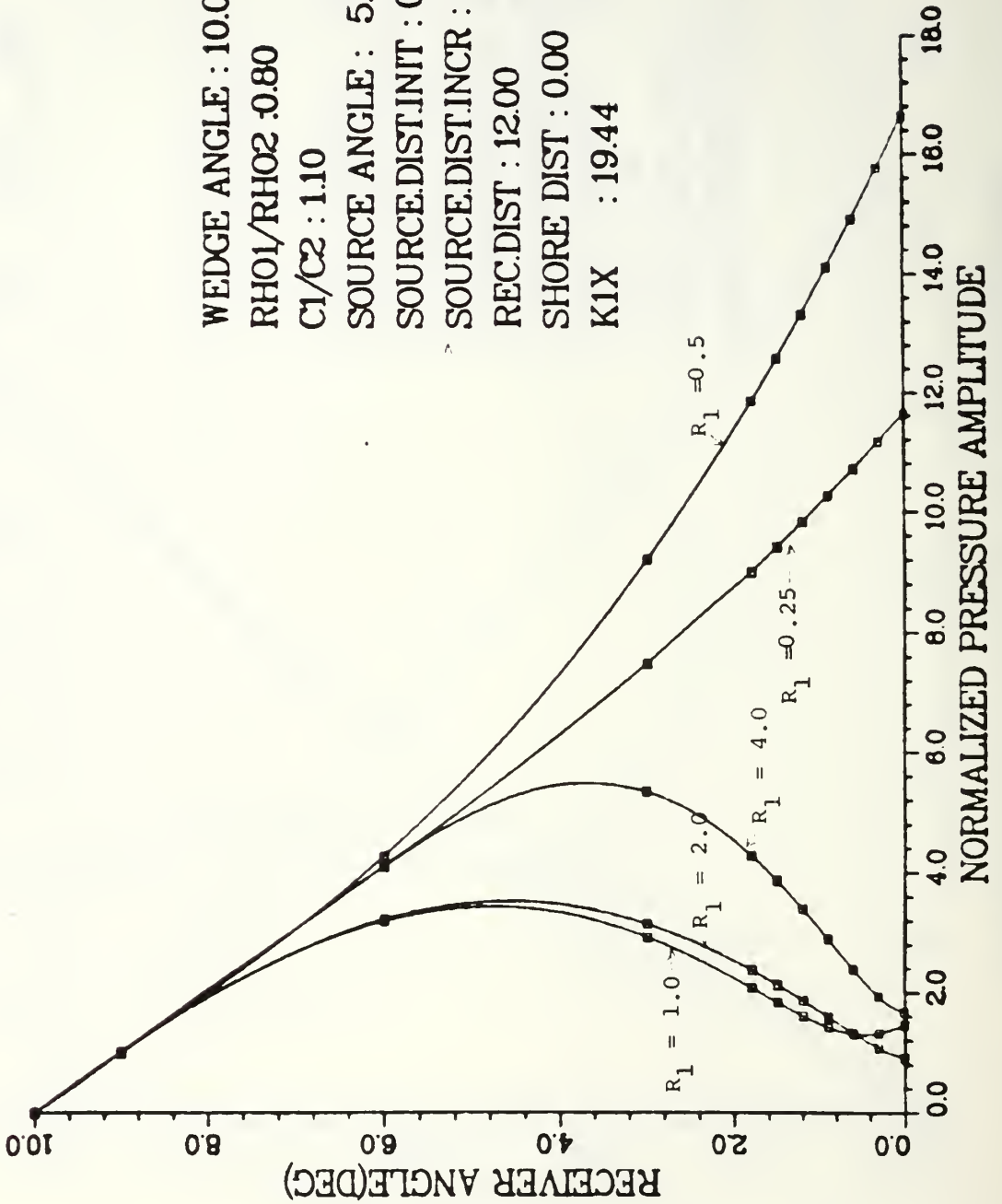


Figure 3.5 Graphs of receiver angle  $\delta$  versus pressure amplitude with  $R_2$  fixed,  $R_1$  varied

# RECEIVER ANGLE VS. PRESSURE

WEDGE ANGLE : 15.0  
 RH01/RH02 : 0.80  
 C1/C2 : 1.10  
 SOURCE ANGLE : 7.50  
 SOURCE.DIST.INIT : 0.25  
 SOURCE.DIST.INCR : TWICE  
 REC.DIST : 6.00  
 SHORE DIST : 0.00  
 K1X : 12.79

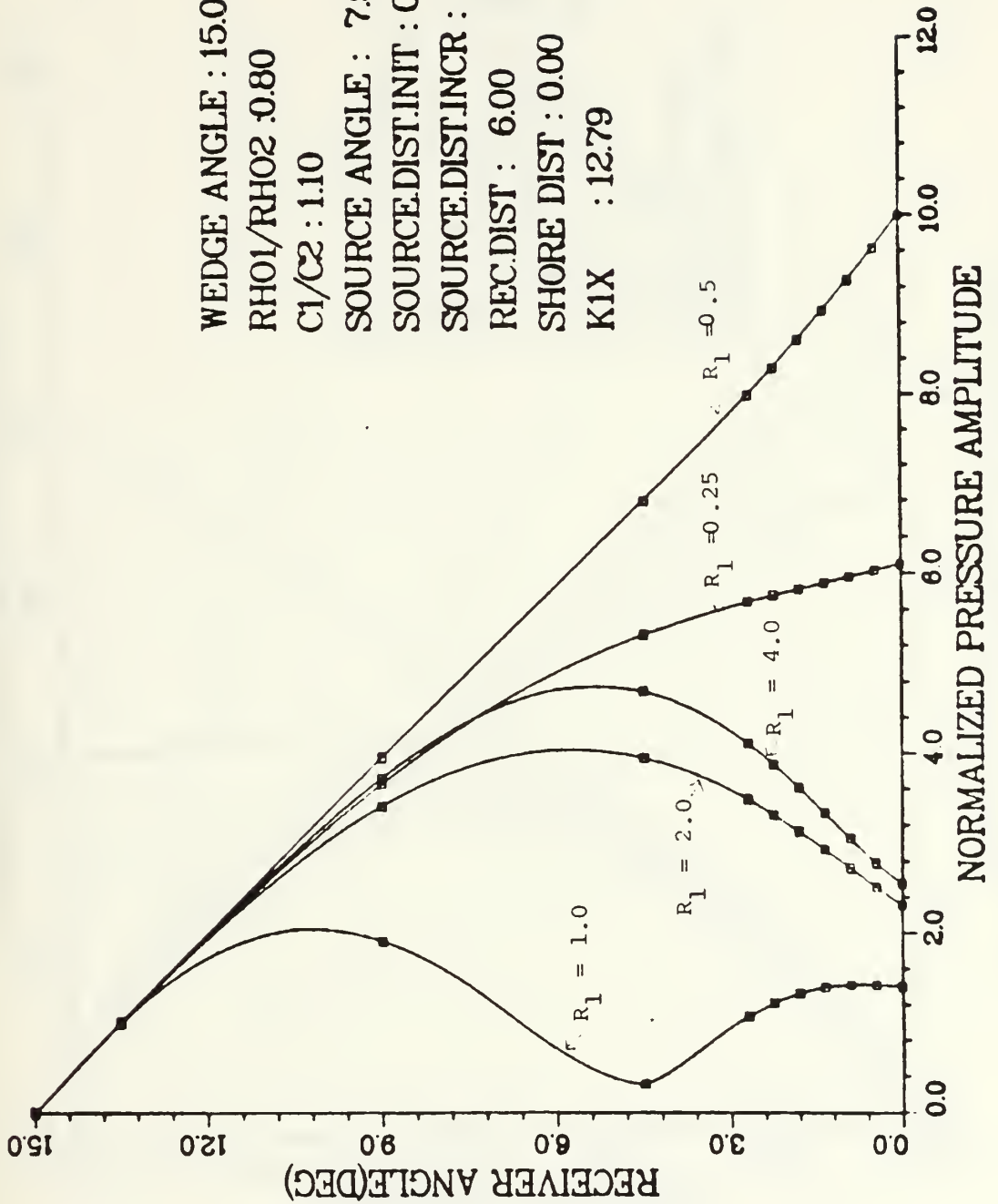


Figure 3.6 Graphs of receiver angle  $\delta$  versus pressure amplitude with  $R_2$  fixed,  $R_1$  varied

# RECEIVER ANGLE VS. PRESSURE

WEDGE ANGLE : 15.0  
 RH01/RH02 : 0.80  
 C1/C2 : 1.10  
 SOURCE ANGLE : 7.50  
 SOURCE.DIST.INIT : 0.25  
 SOURCE.DIST.INCR : TWICE  
 REC.DIST : 12.00  
 SHORE DIST : 0.00  
 KIX : 19.44

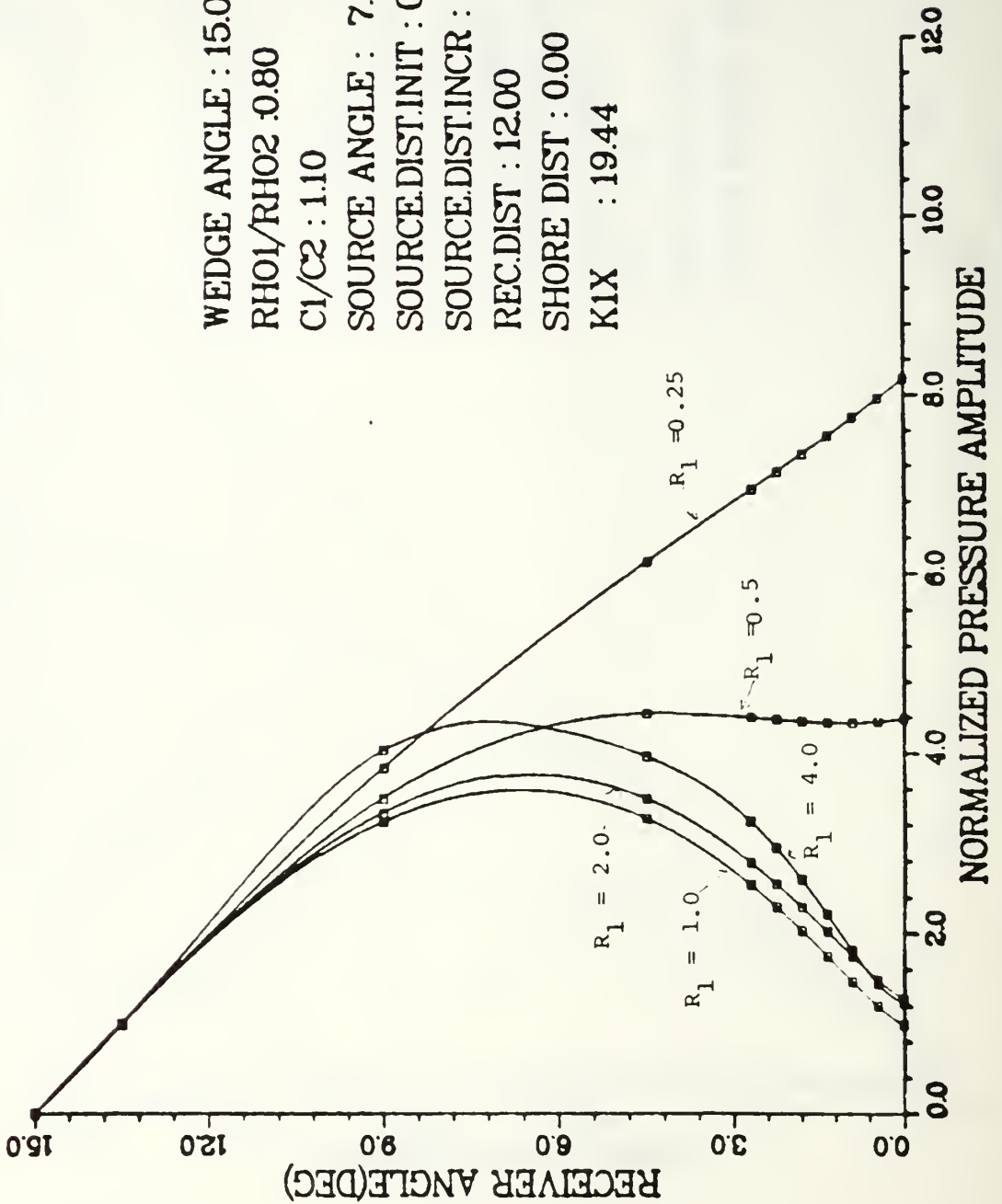


Figure 3.7 Graphs of receiver angle  $\delta$  versus pressure amplitude with  $R_2$  fixed,  $R_1$  varied

# RECEIVER ANGLE VS. PRESSURE

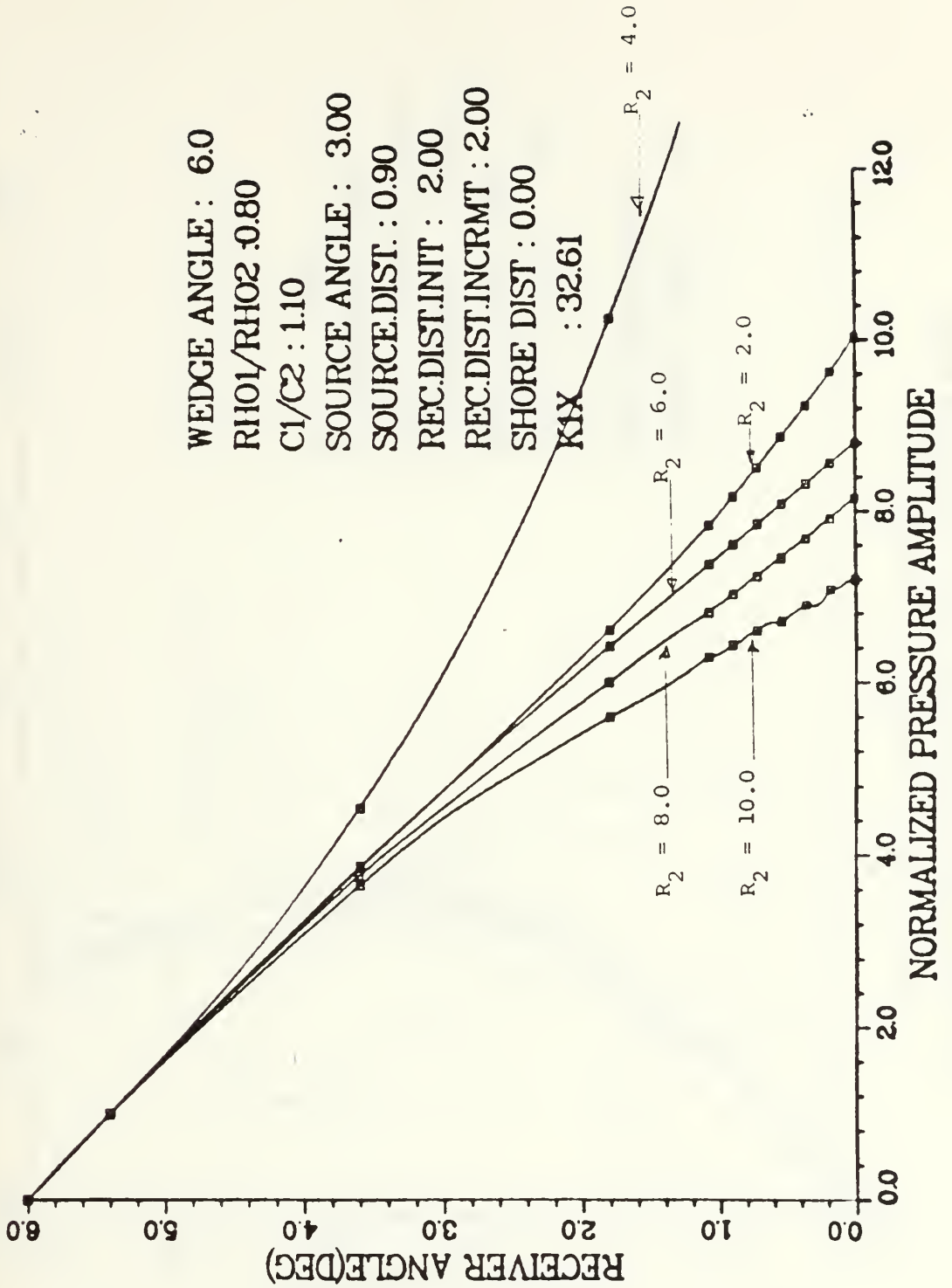


Figure 3.8 Graphs of receiver angle  $\delta$  versus pressure amplitude with  $R_2$  fixed,  $R_1$  varied

# RECEIVER ANGLE VS. PRESSURE

WEDGE ANGLE : 6.0  
 RH01/RHO2 : 0.80  
 C1/C2 : 1.10  
 SOURCE ANGLE : 3.00  
 SOURCE.DIST. : 1.10  
 REC.DIST.INIT : 5.00  
 REC.DIST.INCRMT : 2.00  
 SHORE DIST : 0.00  
 KIX : 32.61

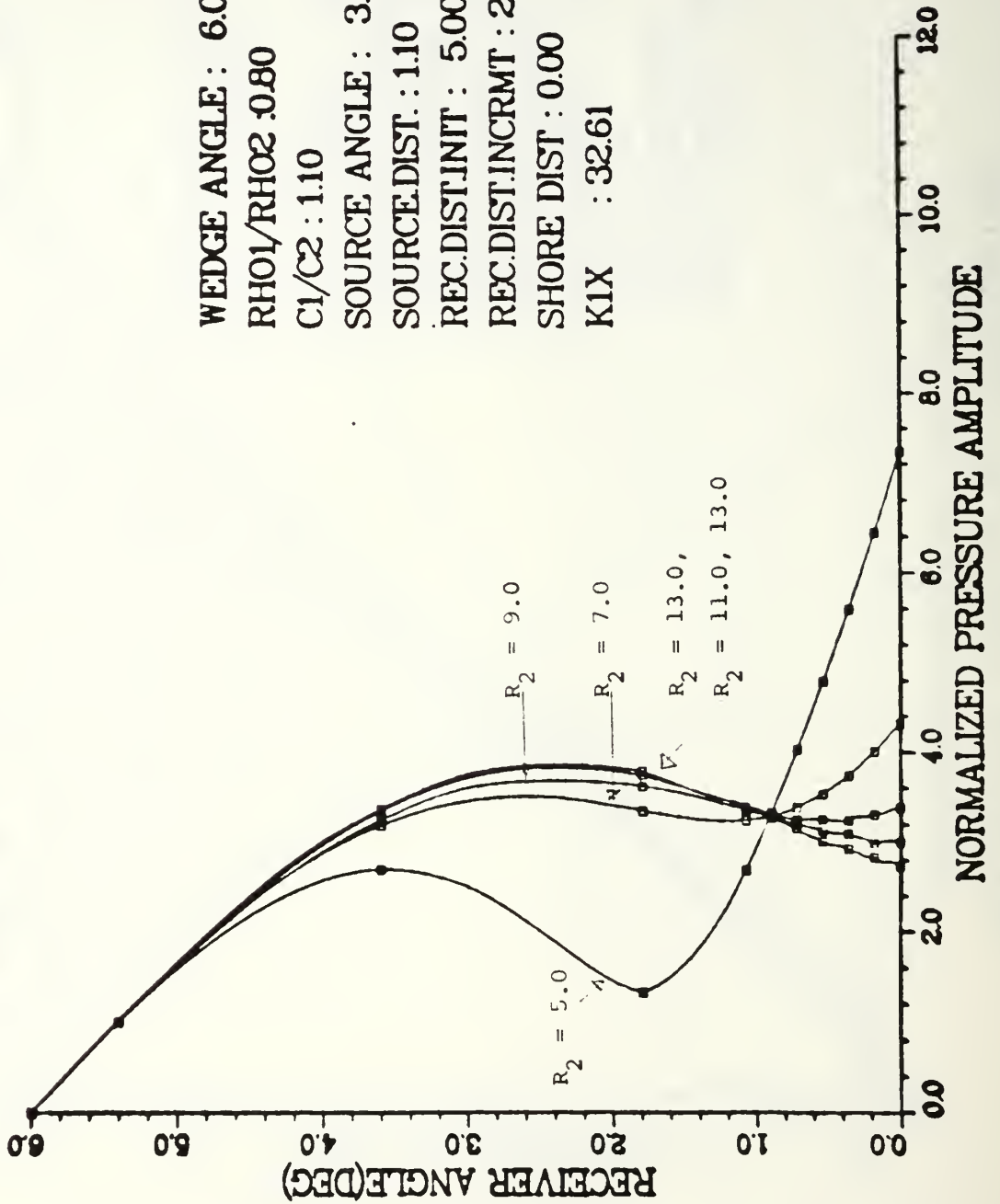


Figure 3.9 Graphs of receiver angle  $\delta$  versus pressure amplitude with  $R_2$  fixed,  $R_1$  varied

# RECEIVER ANGLE VS. PRESSURE

WEDGE ANGLE : 10.0  
 RH01/RH02 : 0.80  
 C1/C2 : 1.10  
 SOURCE ANGLE : 5.00  
 SOURCE.DIST. : 1.10  
 REC.DIST.INIT : 2.00  
 REC.DIST.INCRMT : TWICE  
 SHORE DIST : 0.00  
 KIX : 19.44

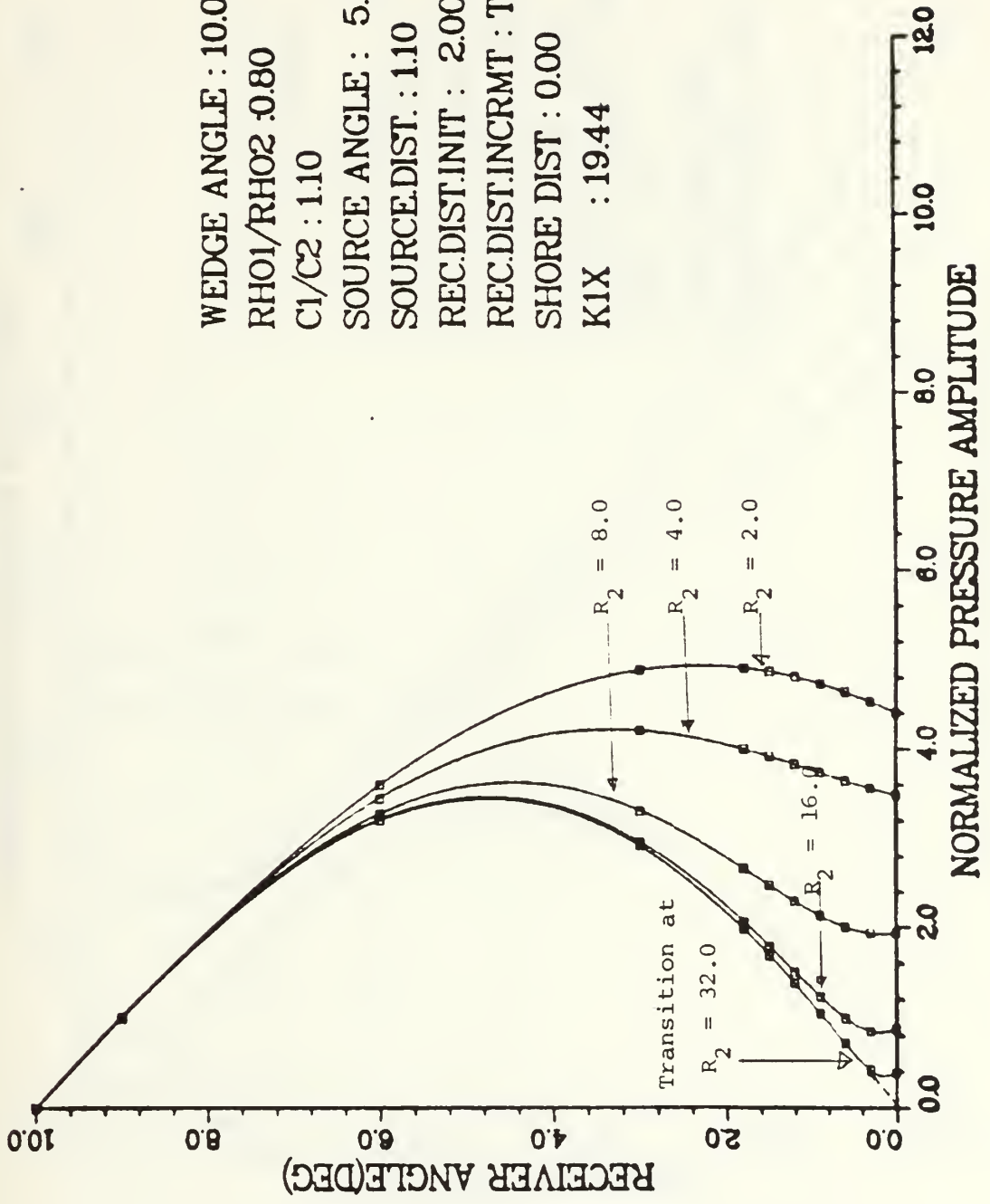


Figure 3.10 Graphs of receiver angle  $\delta$  versus pressure amplitude with  $R_2$  fixed,  $R_1$  varied



# RECEIVER ANGLE VS. PRESSURE

WEDGE ANGLE : 10.0  
 RH01/RH02 : 0.80  
 C1/C2 : 1.10  
 SOURCE ANGLE : 5.00  
 SOURCE.DIST. : 0.90  
 REC.DIST.INIT : 2.00  
 REC.DIST.INCRMT : TWICE  
 SHORE DIST : 0.00  
 K1X : 19.44

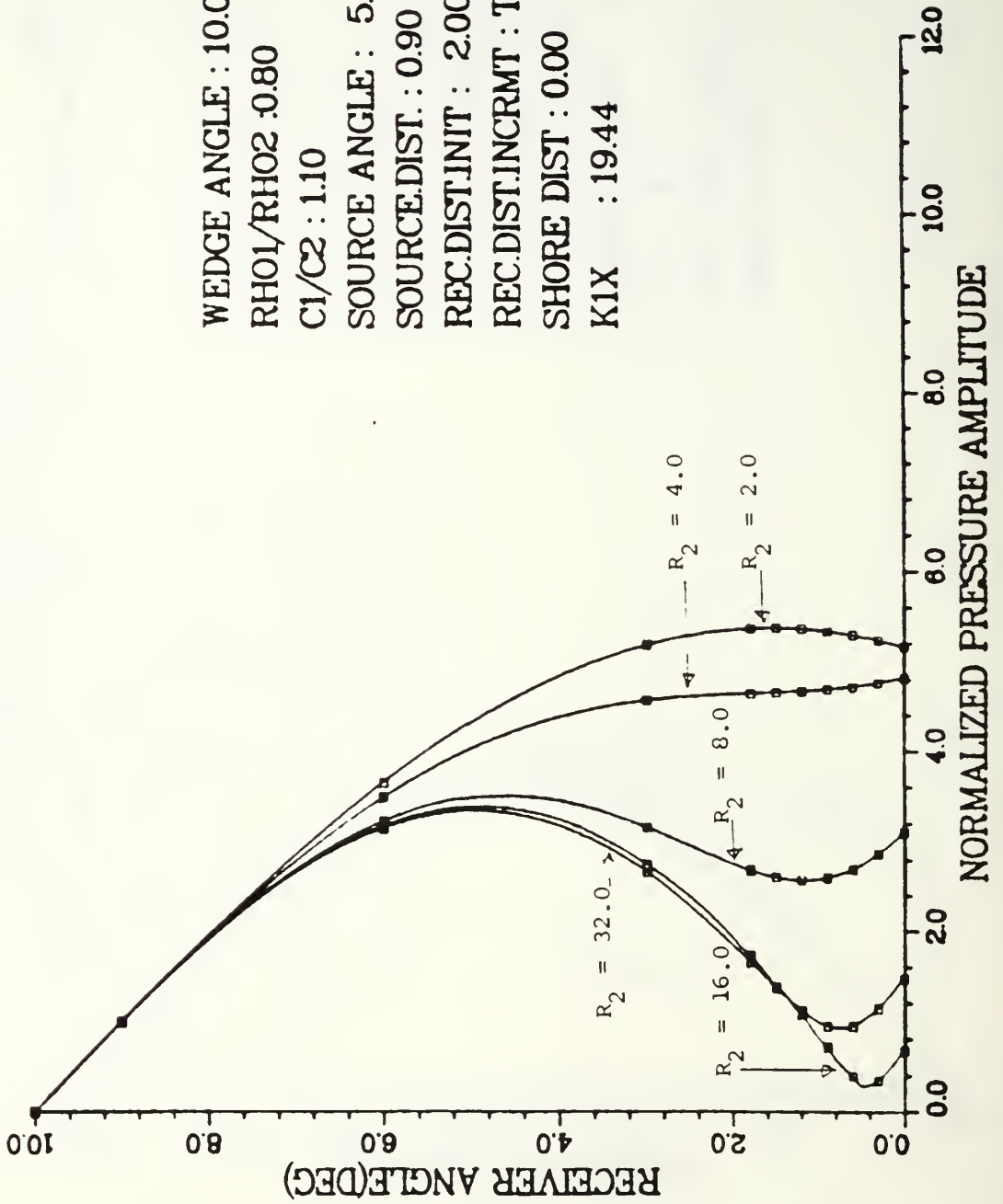


Figure 3.11 Graphs of receiver angle  $\delta$  versus pressure amplitude with  $R_2$  fixed,  $R_1$  varied

# RECEIVER ANGLE VS. PRESSURE

WEDGE ANGLE : 15.0  
 RH01/RH02 : 0.80  
 C1/C2 : 1.10  
 SOURCE ANGLE : 7.50  
 SOURCE.DIST. : 0.25  
 REC.DIST.INIT : 0.50  
 REC.DIST.INCRMT : TWICE  
 SHORE DIST : 0.00  
 K1X : 12.79

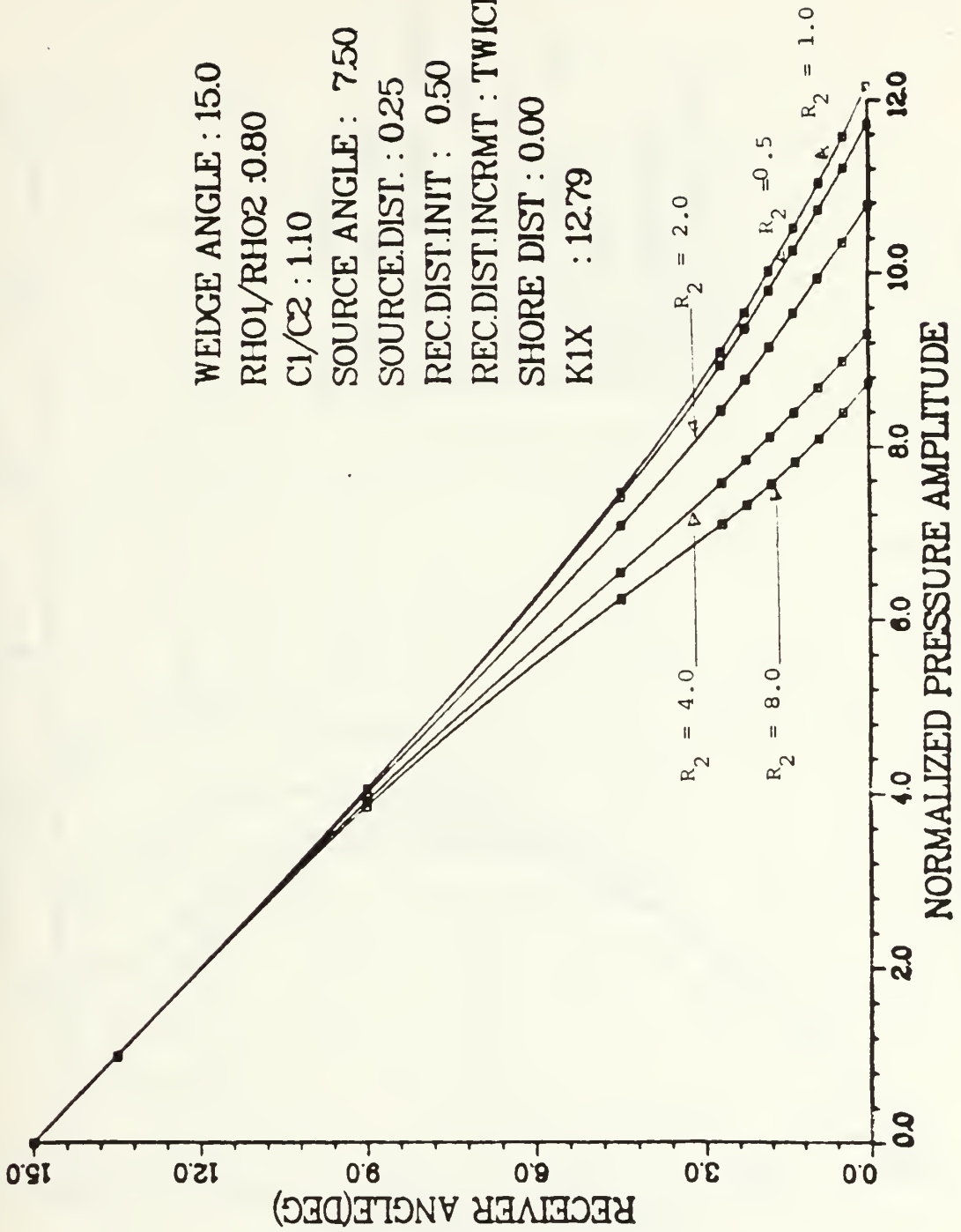


Figure 3.12 Graphs of receiver angle  $\delta$  versus pressure amplitude with  $R_2$  fixed,  $R_1$  varied

# RECEIVER ANGLE VS. PRESSURE

WEDGE ANGLE : 15.0  
 RH01/RH02 : 0.80  
 C1/C2 : 1.10  
 SOURCE ANGLE : 7.50  
 SOURCE.DIST. : 1.10  
 REC.DIST.INIT : 2.00  
 REC.DIST.INCRMT : TWICE  
 SHORE DIST : 0.00  
 KIX : 12.79

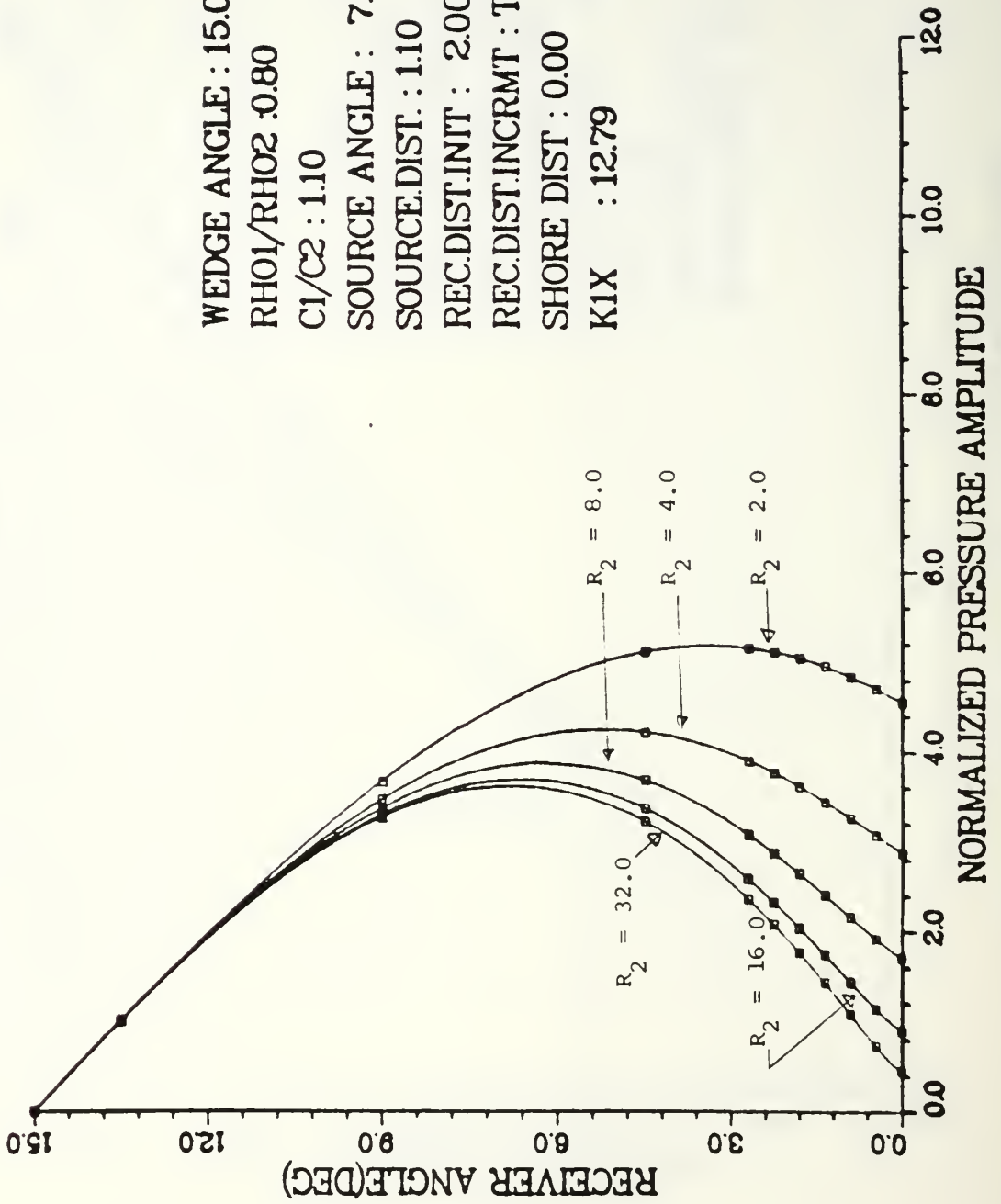


Figure 3.13 Graphs of receiver angle  $\delta$  versus pressure amplitude with  $R_2$  fixed,  $R_1$  varied

## IV. DISCUSSIONS

### A. GRAPHICAL OUTPUT

The graphs of normalized pressure amplitude as a function of receiver angle  $\delta$  were investigated for various source distances  $R_1$  and receiver distances  $R_2$  (Fig. 3.2 to 3.13) while the other parameters are held constant. For a given wedge angle  $\beta$  and sufficiently small source distance (Fig. 3.8 and 3.12), at all receiver distances, the pressure increases uniformly towards the bottom. For greater source distances, (Fig. 3.9 -3.11) the pressure attains a maximum within the wedge for all receiver distance.

As the receiver distance is increased (Fig. 3.9), a pressure minimum develops between the maximum and the bottom. An important property of the curves of pressure versus receiver angle when there is a maximum and minimum is that, at a specific receiver distance, the pressure above the minimum can be extrapolated to zero pressure on the bottom. (See Fig. 3.10 with  $R_2 = 32$ .) This receiver distance is called the "transition point." So far, we do not know the properties of the transition point. We use the transition point for indicating the behavior of the curves when the parameter involved is varied. The transition point appeared twice in some cases, but in the following discussions the first transition point is the only point we

will be concerned with. (See Fig. 4.1 for transitions correspond to  $R_2 = 4.6$  and  $6.4$ .)

## B. GRAPHS CLASSIFICATION

The development of curves with the source distance ( $R_1$ ) and the receiver distance ( $R_2$ ) as variables was observed. As  $R_1$  or  $R_2$  are varied the curve changes from a linear curve to a curve with an observable minimum (Fig. 3.9,  $R_2 = 5.0$ ) and finally to a curve without a minimum (Fig. 3.9,  $R_2 = 9.0$ ). Three different types of curves resulted from the series of two-dimensional plotting. They are described below:

### 1. Type 1 Curves

Type 1 curves (Fig. 4.2) are those where the sound pressure is equal to zero at the surface and maximum at the bottom and is almost linearly dependent on depth. These curves are most pronounced when the source distance is much smaller than the characteristic distance. The closer the source is to the characteristic distance, the more nonlinear the curves (Figs. 3.3 and 3.4).

### 2. Type 2 Curves

Type 2 curves (Fig. 4.3) are those where the sound pressure is zero at the surface, maximum somewhere between the surface and the bottom with no minimum. These types of curves are generated when the source is placed at a point much greater than the characteristic distance. Type 2

# RECEIVER ANGLE VS. PRESSURE

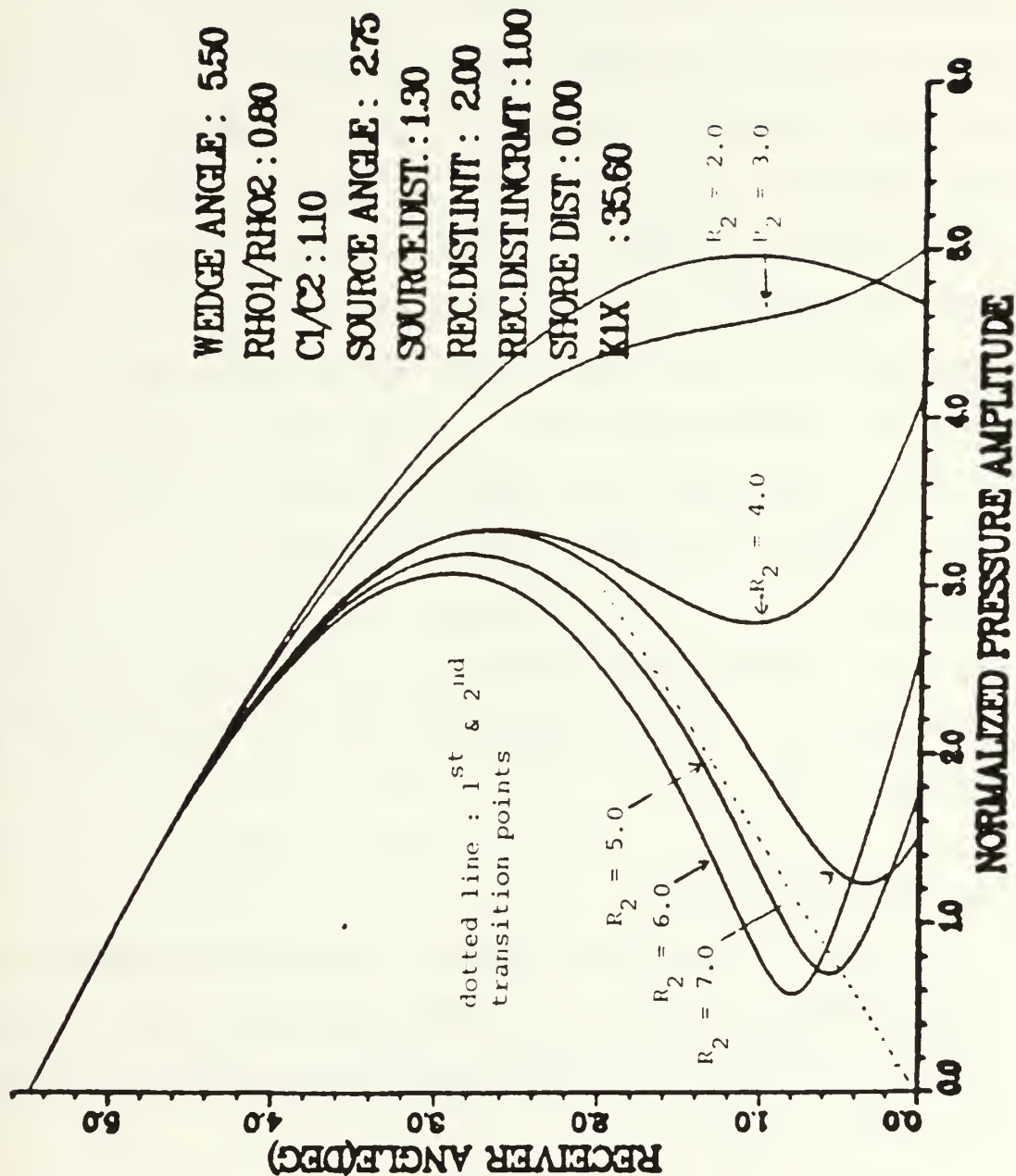


Figure 4.1 The plots where there are two transition points,  $R_2=4.6$  is the first and  $R_2=6.4$  is the second

curves indicate that the sound energy in the wedge is well collimated and that reflection is negligible.

### 3. Type 3 Curves

Type 3 curves (Fig. 4.4) are those that have a minimum pressure. These curves occur when the source is a distance slightly greater than, or less than, the characteristic distance. Tables 1, 2, and 3 of Appendix D show the receiver positions at the first transition points. Three different values of  $\beta$ , two different values of  $\rho_1/\rho_2$ , and two different values of  $c_1/c_2$  were used in making these tables. The transition point did not occur when  $\beta = 15^\circ$ ,  $\rho_1/\rho_2 = 0.90$ ,  $c_1/c_2 = 1.10$ . An explanation can be offered using the fact that for these particular sound-speed and density ratios an angle of intromission exists [Ref. 15]. Since the angle of intromission is the grazing angle at which the sound energy is completely transmitted into the slow bottom, it is plausible that no transition point occurs.

#### C. TRANSITION POINT

By varying the wedge angle  $\beta$  in small increments  $\Delta\beta = 0.5^\circ$  starting with  $\beta = 5^\circ$ , and ending at  $\beta = 7^\circ$ , it was found that transition occurs for source distances within the range from 1.0 to 1.5.

For  $R_1 < 1.0$ , no transition point was observed; the curves are the Type 1. For  $1.0 < R_1 < 1.5$ , the evolution of

curves as the receiver distance varied can be explained as follows: first, the receiver is placed near the source and gradually it is shifted further from the source. The minimum in the pressure decreases reaching the point where the curves extrapolate to zero (the first transition point). Further detailed observations were made on this particular facet by varying the source distance and the receiver distances. The results of these observations are tabulated and graphed in Appendices D and E. When the receiver is moved away from the source, the minimum will reach a minimum pressure then the pressure increases until it reaches the point where the curves again can be extrapolated to zero, this is the second transition point (See Fig. 4.1).

For  $R_1 > 1.5$ , there will be no transition point. The curves are the Type 2.

The transition point as a function of source angle can be observed using the tables in Appendix D. In most cases the greater the source angle  $\gamma$ , the closer the transition point is to the apex. Graphs of transition point as a function of  $R_1$  (Appendix E) indicate that the smaller  $\beta$  the more regular the curves. This is easy to understand because the smaller  $\beta$ , the more accurate the observation of transition point; the greater  $\beta$  the less accurate the data.



# RECEIVER ANGLE VS. PRESSURE

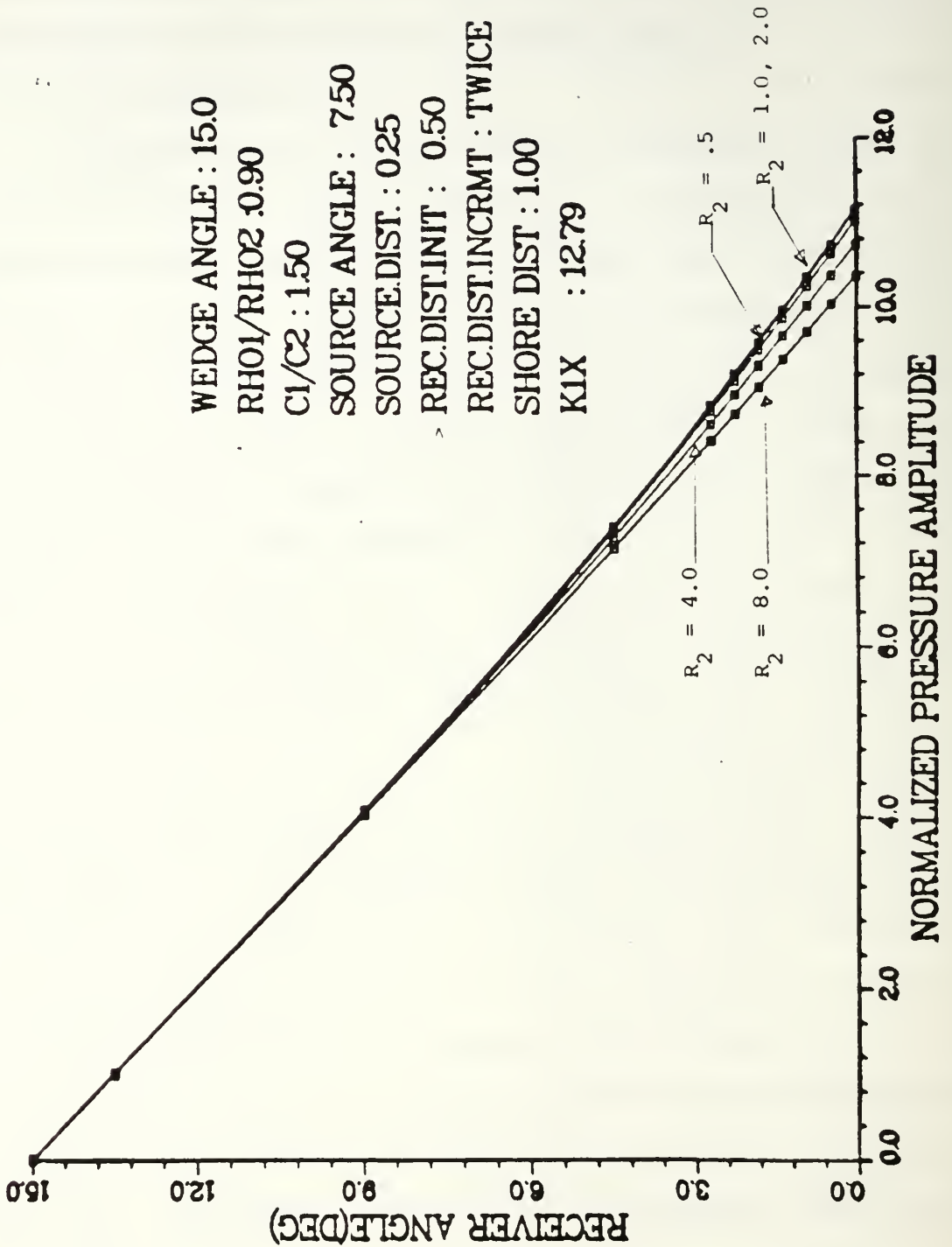


Figure 4.2 Type 2 curves, indicating a pressure amplitude nearly linear with depth

# RECEIVER ANGLE VS. PRESSURE

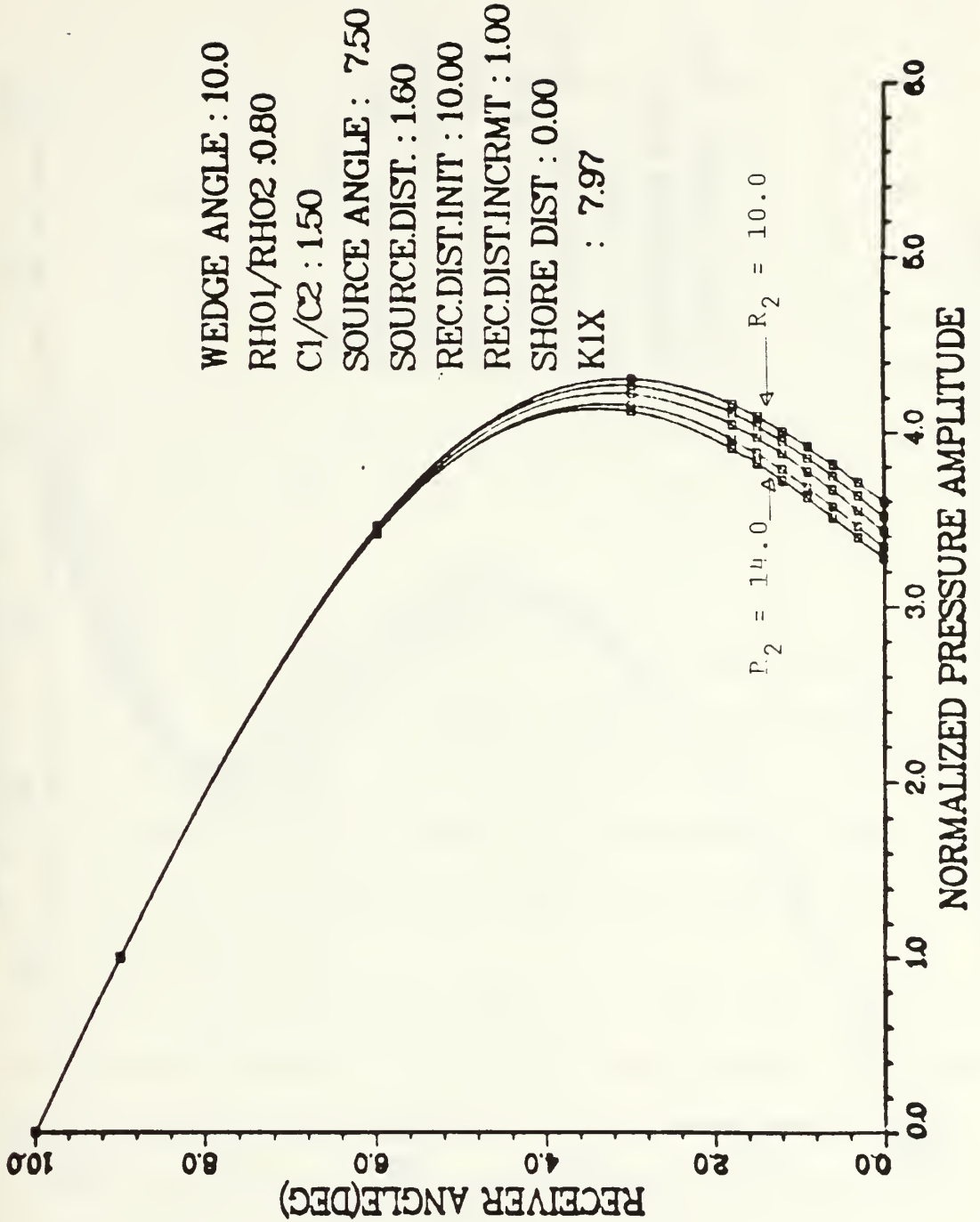


Figure 4.3 Type 2 curves, indicating a well-collimated sound field as the source away from the apex

# RECEIVER ANGLE VS. PRESSURE

WEDGE ANGLE : 15.0  
 RH01/RH02 : 0.80  
 C1/C2 : 1.50  
 SOURCE ANGLE : 11.25  
 SOURCE.DIST. : 1.30  
 REC.DIST.INIT : 10.00  
 REC.DIST.INCRMT : 0.50  
 SHORE DIST : 0.00  
 KIX : 524

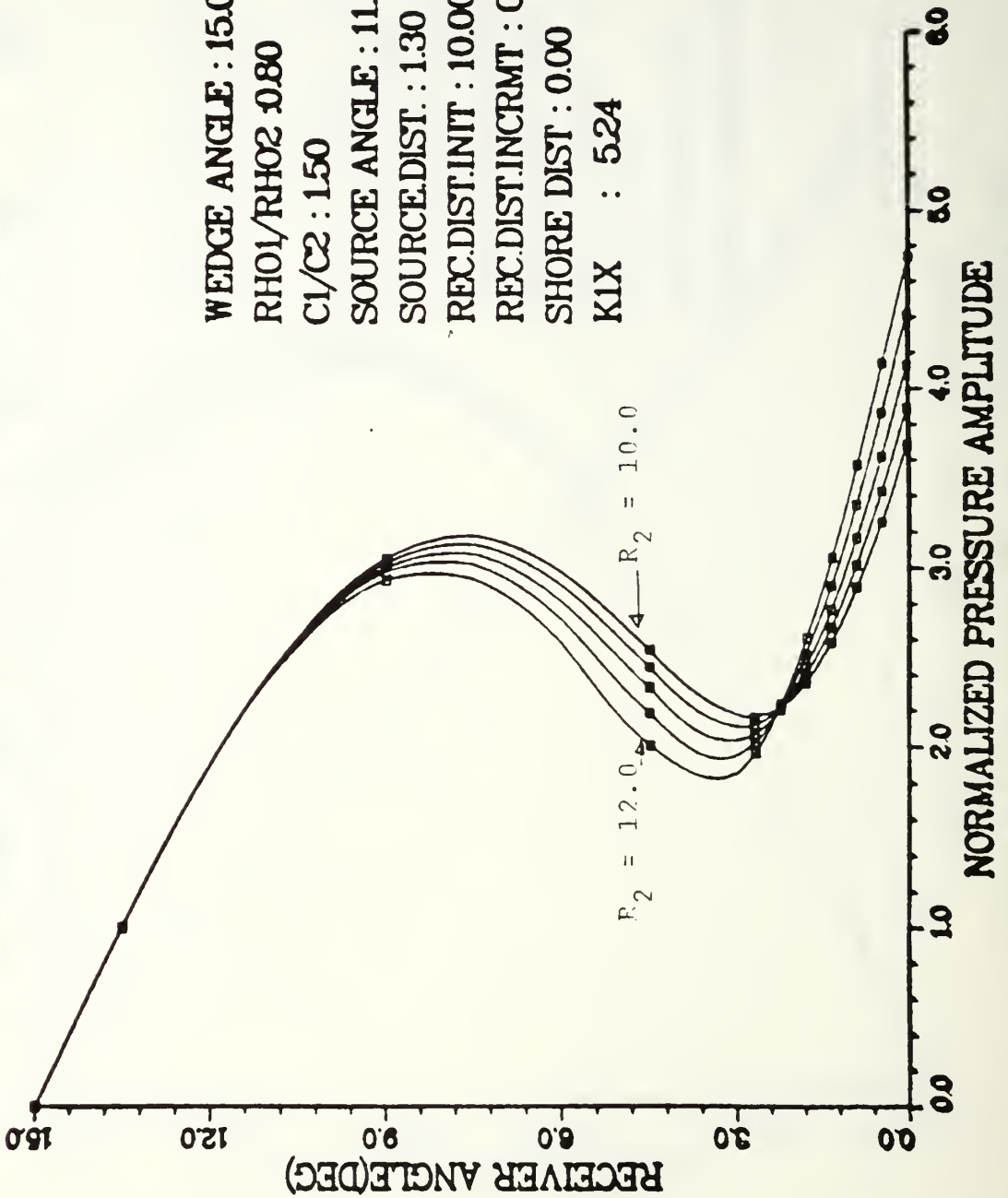


Figure 4.4 Type 3 curves, indicates the presence of reflection and refraction near the bottom

#### D. PARAMETER VARIATIONS

Variation of parameters was done by changing one parameter of interest while all others were held constant, for fixed source and receiver distances, and plotting the receiver angle versus normalized pressure amplitude.

The parameters  $\beta$ ,  $\rho_1/\rho_2$ , and  $c_1/c_2$ , were held constant and the pressure amplitude was plotted for various  $\delta$ ,  $R_1$ , and  $R_2$ . The  $\gamma$ 's are set at  $\beta/4$ ,  $\beta/2$ , and  $3\beta/4$ . Variations in the shore distance ( $Y_0$ ) can be made because the program is available, but to simplify the investigation,  $Y_0$  was set equal to zero for all plots (Fig 4.2 is included as an example for  $Y_0 \neq 0$ ).

##### 1. Variations of $\beta$

Initially, the values of  $\beta$  investigated were:  $6^\circ$ ,  $10^\circ$ , and  $15^\circ$ . The major effect created by altering the value of  $\beta$  is that, for the same values of  $R_1$ ,  $\rho_1/\rho_2$ , and  $c_1/c_2$ , the smaller  $\beta$ , the shorter the transition point (see Figs. 3.9 and 3.10, Tables 1, 2, and 3 of Appendix D).

##### 2. Variations of $\gamma$

The variations of  $\gamma$  from  $\gamma = \beta/4$  (the source is placed near the bottom) to  $\gamma = 3\beta/4$  (the source is placed near the surface) are presented in Tables 1, 2, and 3 indicated that the greater  $\gamma$  the shorter the transition point. It is not always true, for instance in Table 1 at  $\beta = 6^\circ$ ,  $R_1 = 1.50$ , the greater  $\gamma$  the longer transition point, for the

rises. (See Appendix D.)

3. Variations of  $c_1/c_2$  and  $\rho_1/\rho_2$

Variations of the acoustical parameters  $c_1/c_2$  and  $\rho_1/\rho_2$  were done, but did not give a significant variation of the sound pressure profile.

## V. CONCLUSIONS AND RECOMMENDATIONS

### A. CONCLUSIONS

The program DSLOW gives good plots representative of the sound energy distribution within the wedge. The sound energy can be well collimated by the wedge. This phenomenon is strongly affected by the source position. At a source position close enough to the apex, sound energy is distributed linearly with respect to the depth. As the source moved away from the apex, the distribution of sound energy becomes more complex. Sometimes a minimum is found; this minimum may be caused by the presence of sound energy reflected by the bottom.

The source position plays a major role in forming the pressure distribution profile. The pressure distribution is also very sensitive to the parameter variation at small source distances, but it becomes insensitive at large source distance. The characteristic distance must have physical meanings rather than just an arbitrary number, because when the source distance in proximity to the characteristic distance, the model is most sensitive.

The model is restricted when the single precision mode generates round-off error and rough curves which do not allow for accurate analysis.

## B. RECOMMENDATIONS

1. Single precision produces good results, but failed in some cases. Double precision would improve the program, but increase the execution time. This must be done by running the program in double precision, and accumulating the result in single precision before plotting the data by DISSPLA.
2. It is suggested that the program be run using more realistic parameters and observing the effects on the characteristic distance and transition point.
3. Further study validating DSLOW in comparison with experimental results is suggested.

## APPENDIX A DSLOW ALGORITHM

The pressure amplitude calculation

$$N_1 = \text{INT}[180/\beta] \quad (\text{eqn A.1})$$

$$K_1 X = \frac{\pi}{2 \tan \beta \tan[\arccos(c_2/c_1)]} \quad (\text{eqn A.2})$$

$$AL = \alpha \cdot K_2 = 0.0001 \quad (\text{constant}) \quad (\text{eqn A.3})$$

$$D_2 = Y_0^2 + R_1^2 + R_2^2 \quad (\text{eqn A.4})$$

$$R_3 = 2R_1 R_2 \quad (\text{eqn A.5})$$

$$R_8 = \sqrt{D_2 - R_3 \cos[(N-1)\beta + \gamma - \delta]} \quad (\text{eqn A.6})$$

$$R_9 = \sqrt{D_2 - R_3 \cos[(N-1)\beta + \gamma + \delta]} \quad (\text{eqn A.7})$$

$$S_2 = (-1)^{\text{INT}(N_1/2)} \quad (\text{eqn A.8})$$

$$W_1 = 2AL(c_1/c_2)^2 \quad (\text{eqn A.9})$$

$$SI = \{ [R_1 \sin[(N-1)\beta + \gamma] - 2[\text{INT}((N-1)/2)\beta + R_2 \sin[2\text{INT}((N-1)/2)\beta - \delta]] / R_8 \} \quad (\text{eqn A.10})$$

$$CI = \sqrt{(1-SI^2)} \quad (\text{eqn A.11})$$

$$T = SI/D_1 \quad (\text{eqn A.12})$$

$$W_0 = (-c_2 + c_1/c_2) \quad (\text{eqn A.13})$$



$$Y = \sqrt{W_0^2 + W_1^2} \quad (\text{eqn A.14})$$

$$Z = |W_0| \quad (\text{eqn A.15})$$

$$Y_1 = \sqrt{(Y + W_0)/2} \quad (\text{eqn A.16})$$

$$Y_2 = \sqrt{(Y - W_0)/2} \quad (\text{eqn A.17})$$

$$Z_1 = \frac{T_1 - Y_2}{(T - Y_2)^2 + Y_1^2} \quad (\text{eqn A.18})$$

$$Z_2 = \frac{Y_1}{(T - Y_2)^2 + Y_1^2} \quad (\text{eqn A.19})$$

$$Z_5 = \frac{(T^2 - Y_2)^2 - Y_1^2}{(T^2 - Y_2)^2 + Y_1^2} \quad (\text{eqn A.20})$$

$$Z_6 = \frac{2Y_1T}{(T^2 - Y_2)^2 + Y_1^2} \quad (\text{eqn A.21})$$

$$P_1 = \sum_{n=1}^{N_1} (-1)^{\text{INT}(N/2)} \{Z_5 \cos(R_{8n}K_1X) + Z_6 \sin(R_{8n}K_1X)\} / R_{8n} \quad (\text{eqn A.22})$$

$$P_2 = \sum_{n=1}^{N_1} (-1)^{\text{INT}(N/2)} \{-Z_5 \sin(R_{8n}K_1X) + Z_6 \cos(R_{8n}K_1X)\} / R_{8n} \quad (\text{eqn A.23})$$

$$P_3 = \sum_{n=1}^{N_1} (-1)^{\text{INT}(N/2)} \{Z_5 \cos(R_{9n}K_1X) + Z_6 \sin(R_{9n}K_1X)\} / R_{9n} \quad (\text{eqn A.24})$$

$$P_4 = \sum_{n=1}^N (-1)^{\text{INT}(N/2)} \{-Z_5 \sin(R_{9n} K_1 X) + Z_6 \cos(R_{9n} K_1 X)\} / R_{9n} \quad (\text{eqn A.25})$$

$$P_5 = P_1 + P_2 \quad (\text{eqn A.26})$$

$$P_6 = P_3 + P_4 \quad (\text{eqn A.27})$$

$$P_{\text{tot}} = R_1 \sqrt{P_5^2 + P_6^2} \quad (\text{eqn A.28})$$

# APPENDIX B

## DSLOW PROGRAM

```

*****
** THIS PROGRAM CALLED DSLOW IS CALCULATING THE SOUND PRESSURE **
** WITHIN THE WEDGE OVERLYING SLOW BOTTOM FLUID AND DOWN-SLOPE **
** DIRECTION **
*****
-
- INTEGER A, I, I1, M, N, S1, S2, N1, J, K, P
- REAL*4 B, CC, C2, D, D1, D2, G, PI, P1, P2, Q1, R1, R2, T,
* T4, T6, W0, W1, Y0, Y1, Y2, Z1, Z2, Z3, Z4, Z5, Z6,
* T1(80), R8(80), R9(80), S(30), C(30), E(30), AT(30),
* F(30), V, Z, R3, AL, PZ(30), DZ(30), V, DX, XP, DD, PN(30)
-
- REAL*4 TOO, TOO1, TQQ2, TQQ3
- PI = ACOS(-1.0D00)
*****
** INPUT PARAMETERS **
*****
C B = WEDGE ANGLE (DEG)
C G = SOURCE ANGLE (DEG)
C D = RECEIVER ANGLE (DEG)
C N1 = # OF IMAGE POINTS
C R1 = SOURCE DISTANCE (IN CHARACTERISTIC DISTANCES)
C R2 = RECEIVER DISTANCE (IN CHARACTERISTIC DISTANCES)
C Y0 = APEX DISTANCE (IN CHARACTERISTIC DISTANCES)
C D1 = RHO 1/RHO 2
C CC = C-1/C-2
C AL = ALPHA/K2
C A = # OF RECEIVER POSITIONS
*****
** INITIAL INPUT & RELATIONS **
*****
C Y0 = 0.00
C Z0 = 9.00
C ON = 111 P=1,6
C W = 10.0
C W1 = 10.0
C W2 = 10.0
C W3 = 10.0
C W4 = 10.0
C W5 = 10.0
C W6 = 10.0
C W7 = 10.0
C W8 = 10.0
C W9 = 10.0
C W10 = 10.0
C W11 = 10.0
C W12 = 10.0
C W13 = 10.0
C W14 = 10.0
C W15 = 10.0
C W16 = 10.0
C W17 = 10.0
C W18 = 10.0
C W19 = 10.0
C W20 = 10.0
C W21 = 10.0
C W22 = 10.0
C W23 = 10.0
C W24 = 10.0
C W25 = 10.0
C W26 = 10.0
C W27 = 10.0
C W28 = 10.0
C W29 = 10.0
C W30 = 10.0
C W31 = 10.0
C W32 = 10.0
C W33 = 10.0
C W34 = 10.0
C W35 = 10.0
C W36 = 10.0
C W37 = 10.0
C W38 = 10.0
C W39 = 10.0
C W40 = 10.0
C W41 = 10.0
C W42 = 10.0
C W43 = 10.0
C W44 = 10.0
C W45 = 10.0
C W46 = 10.0
C W47 = 10.0
C W48 = 10.0
C W49 = 10.0
C W50 = 10.0
C W51 = 10.0
C W52 = 10.0
C W53 = 10.0
C W54 = 10.0
C W55 = 10.0
C W56 = 10.0
C W57 = 10.0
C W58 = 10.0
C W59 = 10.0
C W60 = 10.0
C W61 = 10.0
C W62 = 10.0
C W63 = 10.0
C W64 = 10.0
C W65 = 10.0
C W66 = 10.0
C W67 = 10.0
C W68 = 10.0
C W69 = 10.0
C W70 = 10.0
C W71 = 10.0
C W72 = 10.0
C W73 = 10.0
C W74 = 10.0
C W75 = 10.0
C W76 = 10.0
C W77 = 10.0
C W78 = 10.0
C W79 = 10.0
C W80 = 10.0
C W81 = 10.0
C W82 = 10.0
C W83 = 10.0
C W84 = 10.0
C W85 = 10.0
C W86 = 10.0
C W87 = 10.0
C W88 = 10.0
C W89 = 10.0
C W90 = 10.0
C W91 = 10.0
C W92 = 10.0
C W93 = 10.0
C W94 = 10.0
C W95 = 10.0
C W96 = 10.0
C W97 = 10.0
C W98 = 10.0
C W99 = 10.0
C W100 = 10.0
*****
** THE MAIN PROGRAM CALCULATES UPPER AND LOWER PATH OF RAYS **
*****
- K = 0
- N1 = INT(180./B)
- T6 = 180./PI
- W = B/T6
- G = G/T6
- C2 = CC**2
- D2 = (Y0*Y0)+(R1*R1)+(R2*R2)
- R3 = 2.*R1*R2
- T4 = PI/(2*TAN(ACOS(1/CC))*TAN(B))
- TOO = TAN(B)
- TOO1 = ACOS(1/CC)
- TQQ2 = TAN(TOO1)
- TQQ3 = 2.*TQQ2*TQQ
- T4 = PI/TQQ3

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```

C      K1X= PI/(2*TOO*TOO2)
-      D2 = Y0*Y0+R1*R1+R2*R2
-      O1 = 1/DSORT(2.0D00)
C240  *FORMAT('NOP',7X,'THETA(N)',7X,'IMGE SR R8',7X,
C      *IMGE SR R9')
C      WRITE(6,240)
-241  FORMAT(6,241)
-      WRITE(6,241)
-250  FORMAT('WEDGE ANGLE = ',F5.2,1X,' SOURCE ANGLE = ',F5.2)
-      WRITE(6,250) B*T6,G*T6
-251  *FORMAT('SOURCE DISTANCE=',F4.2,1X,'RECEIVER DISTANCE=',F6.2,1X,
-      *SHORE DISTANCE=',F4.2)
-      WRITE(6,251) R1,R2,Y0
-270  FORMAT('RHO1/RHO2=',F5.2,5X,' C1/C2=',F5.2,5X,' ALPHA/K2=',F8.4)
-      WRITE(6,270) D1,CC,AL
-271  FORMAT('K1X = ',F5.2)
-      WRITE(6,271) T4
-272  FORMAT(6,272)
-      WRITE(6,272)
-800  *FORMAT(1X,'REC.POS',2X,' REC.ANGLE ',2X,'PRES.AMPLITUDE ',3X,
-      *PHASE ANGLE ',4X,'NORM.PRESS')
-      WRITE(6,800)
-801  FORMAT(6,801)
-      WRITE(6,801)
-      DO 10 M = 0,A
-      D = M*B/A
-      DD=B/A
-      V = 2*B/10
-      IF(D.LT.V) GOTO 110
-      IF(D.GE.V) GOTO 120
-110  DX = D
-      DO 15 J = 1,10
-      D = DX+(J-1)*B/(10*A)
-      DD = B/(10*A)
-120  S1 = 1.0
-      DO 20 N = 1,N1
-      IF(S1.GT.0) T1(N)=(N-1)*B+G
-      IF(S1.LT.0) T1(N)=N*B-G
-      S1 = -S1
-      R8(N) = SORT(D2-R3*COS(T1(N)-D))
-      R9(N) = SORT(D2-R3*COS(T1(N)+D))
C310  *FORMAT(3X,I2,5X,F5.2,4X,F6.4,6X,F6.4)
-20  *WRITE(6,310) N,T1(N),R8(N),R9(N)
-      CONTINUE
-      P1 = 0.0
-      P2 = 0.0
-      DO 30 N = 1,N1
-      S2 = (-1)**(INT(N/2))
-      W1 = 2*C2*AL
-      I1 = INT((N-1)/2)
-      DO 40 I = 1,I1
-      *S(I) = ABS(R1*SIN(T1(N)-2*I*B)
-      +R2*SIN(2*I*B-D))/R8(N)
-      C(I) = SORT(I-(S(I)*S(I)))
-      T = S(I)/D1
-      W0 = (-C2+(C(I)*C(I)))
-      Y = SORT((W0*W0)+(W1*W1))
-      Z = ABS(W0)
-      IF(Y.LE.Z) Y = Z
-      Y1 = O1*SORT(Y+W0)
-      Y2 = -O1*SORT(Y-W0)
-      Z1 = T-Y2
-      Z2 = -Y1
-      Z12= CMPLX(Z1R,Z2I)
-      Z3 = Z1/(Z1*Z1+Z2*Z2)
-      Z4 = -Z2/(Z1*Z1+Z2*Z2)
-      Z1 = T+Y2
-      Z2 = Y1
-      Z13= CMPLX(Z3R,Z4I)
-      Z5 = Z1*Z3-Z2*Z4
-      Z6 = Z1*Z4+Z2*Z3
-      Z14= CMPLX(Z5R,Z6I)
-      F(I) = Z5
-      E(I) = Z6
C400  *FORMAT('NOP',5X,'I',5X,'ARSIN(S(I)*T6)',5X,'E(I)',
C      *5X,'F(I)',5X
C      *DSORT(E(I)*E(I)+F(I)*F(I))',5X,'T6*ATAN(F(I)/E(I))')
C      WRITE(6,400)

```

```

C420  FORMAT(3X,I2,6X,I2,5X,F6.4,10X,F6.4,3X,F6.4,3X,F6.4,10X,F7.4)
C      *WRITE (6,420) N,I,ASIN(S(I))*T6,E(I),F(I),
C      *SQRT(E(I)*E(I)+F(I)*F(I)),ATAN(F(I)/E(I))
40  CONTINUE
      Z1 = 0
      Z2 = 0
      Z3 = 0
      Z4 = 0
      Z5 = 1
      Z6 = 0
      IF(N,LE,2.00) GOTO 50
      DO 45 I = 1, I1
      Z1 = E(I)
      Z2 = F(I)
      Z3 = Z5
      Z4 = Z6
      Z5 = Z1*Z3-Z2*Z4
      Z6 = Z1*Z4+Z2*Z3
45  CONTINUE
50  Z1 = Z5
      Z2 = Z6
      T = T4*R8(N)
      Z3 = COS(T)
      Z4 = -SIN(T)
      Z5 = Z1*Z3-Z2*Z4
      Z6 = Z1*Z4+Z2*Z3
      P1 = S2*Z5/R8(N)
      P2 = P2+S2*Z6/R8(N)
C500  FORMAT(6,NO,OF,I.P',3X,'RE(REFL)= ',3X,'IM(REFL)= ')
C      *WRITE (6,500)
C510  FORMAT(3X,I2,6X,F6.4,6X,F6.4)
C      *WRITE (6,510) N,S2*Z1,S2*Z2
C600  *FORMAT(5X,I',5X,'ARCSIN(S(I))*T6=',3X,'E(I)',5X,'F(I)',5X,'EF',
C      *WRITE (6,600)
      I1 = I1+1
      DO 60 I = 1, I1
      *S(I) = ABS(R1*SIN(T1(N)-2*(I-1)*B)
      *      + R2*SIN(2*(I-1)*B+D))/R9(N)
      C(I) = SORT(I-S(I))*S(I)
      T = S(I)/D1
      W0 = -C2+C(I)*C(I)
      Y = SORT((W0*W0)+(W1*W1))
      Z = ABS(W0)
      IF(Y.LE,Z) Y = Z
      Y1 = O1*SORT(Y+W0)
      Y2 = O1*SORT(Y-W0)
      Z1 = T-Y2
      Z2 = -Y1
      Z3 = Z1/(Z1*Z1+Z2*Z2)
      Z4 = -Z2/(Z1*Z1+Z2*Z2)
      Z1 = T+Y2
      Z2 = Y1
      Z5 = Z1*Z3-Z2*Z4
      Z6 = Z1*Z4+Z2*Z3
      E(I) = Z5
      F(I) = Z6
      AN1 = ASIN(S(I))*T6
      EF = SORT(E(I)*E(I)+F(I)*F(I))
      AN2 = ATAN(F(I)/E(I))*T6
C610  FORMAT(2X,I2,5X,F6.4,5X,F6.4,5X,F6.4,5X,F7.4)
C      *WRITE (6,610) I,AN1,E(I),F(I),EF,AN2
60  CONTINUE
      Z1 = 0
      Z2 = 0
      Z3 = 0
      Z4 = 0
      Z5 = 1
      Z6 = 0
      DO 80 I = 1, I1
      Z1 = E(I)
      Z2 = F(I)
      Z3 = Z5
      Z4 = Z6
      Z5 = Z1*Z3-Z2*Z4
      Z6 = Z1*Z4+Z2*Z3
80  CONTINUE
      Z1 = Z5

```

```

-          Z2 = Z6
-          T  = T4*R9(N)
-          Z3 = COS(T)
-          Z4 = -SIN(T)
-          Z5 = Z1*Z3 - Z2*Z4
-          Z6 = Z1*Z4 + Z2*Z3
-          P1 = P1+S2*Z5/R9(N)
-          P2 = P2+S2*Z6/R9(N)
C700  FORMAT(' LOWER PATH NO=',3X,'RE(REFL)= ',3X,'IM(REFL)= ')
C      WRITE(6,700)
C710  FORMAT(6X,I2,12X,F6.4,12X,F6.4)
C      WRITE(6,710) N,S2*Z1,S2*Z2
- 30  CONTINUE
-      K = K+1
-      DZ(K)=D*T6
-      PZ(K)=SORT(P1*P1+P2*P2)*R1
-      AT(K)=ATAN(P2/P1)
C      WRITE(6,810) K,DZ(K),PZ(K),ATAN(P2/P1),PN(K)
-810  FORMAT(6X,I3,11X,F5.2,12X,F7.4,12X,F7.4,10X,F7.4)
-15  CONTINUE
-10  CONTINUE
-      DO 31 L=1,29
-      XP=PZ(28)
-      PN(L)=PZ(L)/XP
-      WRITE(6,811) L,DZ(L),PZ(L),AT(L),PN(L)
-811  FORMAT(2X,I3,7X,F5.2,8X,F9.6,8X,F9.6,6X,F9.6)
-31  CONTINUE
C      Y0=Y0+0.5
-      R2=R2*0.50
-111 CONTINUE
-      STOP
-      END

```

```

*****
** A SUBPROGRAM FOR PLOTTING BY TEK618OR SHERPA **
*****

```

```

C      CALL MEDBUF
C      CALL TEK618
C      CALL COMPRS
C      CALL NOBRDR
C      CALL PAGE(15,12)
C      CALL AREA2D(11,9)
C      CALL HEIGHT(.2)
C      CALL XNAME('NORMALIZED PRESSURE AMPLITUDES',29)
C      CALL YNAME('RECEIVER ANGLE(DEG)$',19)
C      CALL YTICKS(5)
C      CALL XTICKS(5)
C      CALL GRAF(0.,0.10,0.50,0.,5.0,15.0)
C      CALL GRAF(0.,10.,50.0,0.,5.0,10.0)
C      CALL GRAF(0.,3.0,15.0,0.,3.0,15.0)
C      CALL DOT
C      CALL GRID(2,2)
C      CALL HEADIN('REC.ANGLE VS. PRESSURES',-100,1.8,1)
C      CALL MESSAG('WEDGE ANGLE: $',100,8,7)
C      CALL MESSAG('15.0S',100,'ABUT','ABUT')
C      CALL MESSAG('RHO1/RHO2: $',100,8,6.5)
C      CALL MESSAG('0.80 $',100,'ABUT','ABUT')
C      CALL MESSAG('C1/C2: $',100,8,6.0)
C      CALL MESSAG('1.50S',100,'ABUT','ABUT')
C      CALL MESSAG('SOURCE ANGLE: $',100,8,5)
C      CALL MESSAG('11.25S',100,'ABUT','ABUT')
C      CALL MESSAG('7.50S',100,'ABUT','ABUT')
C      CALL MESSAG('3.75S',100,'ABUT','ABUT')
C      CALL MESSAG('7.50S',100,'ABUT','ABUT')
C      CALL MESSAG('SOURCE DIST: $',100,8,5)
C      CALL MESSAG('0.25 $',100,'ABUT','ABUT')
C      CALL MESSAG('1.10 $',100,'ABUT','ABUT')
C      CALL MESSAG('1.60$ ',100,'ABUT','ABUT')
C      CALL MESSAG('0.95 $',100,'ABUT','ABUT')
C      CALL MESSAG('REC.DIST.INIT: $',100,8,4.5)
C      CALL MESSAG('0.50 $',100,'ABUT','ABUT')
C      CALL MESSAG('1.00 $',100,'ABUT','ABUT')
C      CALL MESSAG('3.00 $',100,'ABUT','ABUT')
C      CALL MESSAG('3.00 $',100,'ABUT','ABUT')
C      CALL MESSAG('REC.DIST.INCRMT: $',100,8,4.0)
C      CALL MESSAG('TWICES',100,'ABUT','ABUT')
C      CALL MESSAG('2.00S',100,'ABUT','ABUT')
C      CALL MESSAG('SHO.DIST.INIT: $',100,8,3.5)
C      CALL MESSAG('1.00 $',100,'ABUT','ABUT')

```

```

C      CALL MESSAG('SHO,DIST: $' 100.8. 3.5)
C      CALL MESSAG('0.00 $',100,'ABUT','ABUT')
C      CALL RESEI('ALL')
C      CALL PARA3
C      CALL POLY3
C      CALL NOCHK
C      CALL CURVE(PN,DZ,29,0)
111   CONTINUE
C      CALL ENDPL(0)
C      CALL DONEPL
C      STOP
C      END

```

```

*****
** For plotting the calculations result on TEK618 or Sherpa, erase **
** the proper C's in DISSPLA subprogram above. Put C's in front of **
** 111 CONTINUE STOP and END in Main Program. **
*****

```

# APPENDIX C

## NUMERICAL RESULTS OF DSLOW

WEDGE ANGLE = 10.00      SOURCE ANGLE = 2.50  
 SOURCE DISTANCE = 0.75      RECEIVER DISTANCE = 9.00      SHORE DISTANCE = 0.00  
 RHO1/RHO2 = 0.80      C1/C2 = 1.10      ALPHA/K2 = 0.0001  
 K1X = 19.44

REC.POS	REC.ANGLE	PRES.AMPLITUDE	PHASE ANGLE	NORM.PRESS
1	0.00	0.007611	-1.129606	6.792701
2	0.10	0.007390	-1.110871	6.595129
3	0.20	0.007179	-1.108508	6.403362
4	0.30	0.006995	-1.085083	6.215911
5	0.40	0.006837	-1.061422	6.033560
6	0.50	0.006699	-1.037255	5.85664
7	0.60	0.006573	-1.012591	5.68555
8	0.70	0.006458	-0.987420	5.520390
9	0.80	0.006354	-0.961727	5.361308
10	0.90	0.006260	-0.935597	5.207738
11	1.00	0.006176	-0.909028	5.059000
12	1.10	0.006101	-0.882027	4.914440
13	1.20	0.006034	-0.854699	4.77441
14	1.30	0.005974	-0.827047	4.638044
15	1.40	0.005920	-0.799075	4.505240
16	1.50	0.005871	-0.770794	4.376038
17	1.60	0.005827	-0.742208	4.250238
18	1.70	0.005788	-0.713321	4.127756
19	1.80	0.005753	-0.684137	4.008556
20	1.90	0.005721	-0.654659	3.892477
21	2.00	0.005692	-0.624890	3.779377
22	3.00	0.004400	-0.500000	3.572277
23	4.00	0.003391	-0.411111	3.497443
24	5.00	0.002541	-0.341111	3.427758
25	6.00	0.001890	-0.283333	3.366851
26	7.00	0.001414	-0.233333	3.311988
27	8.00	0.001081	-0.187500	3.260608
28	9.00	0.000829	-0.142857	3.211787
29	10.00	0.000620	-0.100000	3.165000
30	11.00	0.000440	-0.060000	3.120229



WEDGE ANGLE = 10.00 SOURCE ANGLE = 2.50  
 SOURCE DISTANCE = 0.75 RECEIVER DISTANCE = 4.50 SHORE DISTANCE = 0.00  
 RHO1/RHO2 = 0.80 C1/C2 = 1.10 ALPHA/K2 = 0.0001  
 K1X = 19.44

REC.POS	REC.ANGLE	PRES.AMPLITUDE	PHASE ANGLE	NORM.PRESS
1	0.00	0.037832	1.537560	7.554441
2	0.10	0.0337361	-1.5553782	7.4660249
3	0.20	0.036906	-1.55551594	7.3669024
4	0.30	0.036460	-1.5551594	7.2800349
5	0.40	0.0366019	-1.5385668	7.1922361
6	0.50	0.035594	-1.5219799	7.1077576
7	0.60	0.035178	-1.5053866	7.0244406
8	0.70	0.034766	-1.4888472	6.9422334
9	0.80	0.034371	-1.471694	6.8633304
10	0.90	0.0339981	-1.4544889	6.7865799
11	1.00	0.0336662	-1.4379888	6.7099667
12	1.10	0.0333229	-1.4220986	6.6352274
13	1.20	0.0329867	-1.4070230	6.5622953
14	1.30	0.0326511	-1.3928717	6.4911957
15	1.40	0.0323263	-1.3796529	6.4224443
16	1.50	0.0320149	-1.3673691	6.3554453
17	1.60	0.0317166	-1.3560005	6.2909080
18	1.70	0.0314324	-1.3455534	6.2286366
19	1.80	0.0311622	-1.3360270	6.1684499
20	1.90	0.0309053	-1.3274225	6.1102689
21	2.00	0.0306626	-1.3197393	6.0540270
22	2.10	0.0304341	-1.3129721	6.0005668
23	2.20	0.0302206	-1.3071255	5.9498333
24	2.30	0.0300222	-1.3022034	5.9017683
25	2.40	0.0298388	-1.2982001	5.8563099
26	2.50	0.0296706	-1.2951186	5.8134099
27	2.60	0.0295176	-1.2929601	5.7730274
28	2.70	0.0293798	-1.2917271	5.7351223
29	2.80	0.0292572	-1.2914211	5.7006499
30	2.90	0.0291500	-1.2919556	5.6695734
31	3.00	0.0290588	-1.2933411	5.6418222
32	3.10	0.0290828	-1.2955899	5.6174499
33	3.20	0.0292208	-1.2987069	5.5964276
34	3.30	0.0294769	-1.3027001	5.5786999
35	3.40	0.0298466	-1.3075721	5.5642499
36	3.50	0.0303333	-1.3133271	5.5530276
37	3.60	0.0309400	-1.3199721	5.5449764
38	3.70	0.0316663	-1.3275171	5.5396222
39	3.80	0.0325166	-1.3359721	5.5364499
40	3.90	0.0334866	-1.3453471	5.5350276
41	4.00	0.0345766	-1.3556421	5.5359764
42	4.10	0.0357866	-1.3668671	5.5389222
43	4.20	0.0371166	-1.3790321	5.5436499
44	4.30	0.0385666	-1.3921471	5.5498764
45	4.40	0.0401366	-1.4062221	5.5574222
46	4.50	0.0418266	-1.4212671	5.5662499
47	4.60	0.0436366	-1.4372821	5.5762276
48	4.70	0.0455666	-1.4542671	5.5872499
49	4.80	0.0476166	-1.4722221	5.5992276
50	4.90	0.0497866	-1.4911471	5.6121499
51	5.00	0.0520666	-1.5110321	5.6259764
52	5.10	0.0544566	-1.5318771	5.6406222
53	5.20	0.0569466	-1.5536821	5.6560499
54	5.30	0.0595366	-1.5764471	5.6721276
55	5.40	0.0622266	-1.6001721	5.6888499
56	5.50	0.0650166	-1.6248571	5.7061276
57	5.60	0.0679066	-1.6505021	5.7239764
58	5.70	0.0708966	-1.6771171	5.7423222
59	5.80	0.0739866	-1.7047021	5.7611499
60	5.90	0.0771766	-1.7332571	5.7804276
61	6.00	0.0804666	-1.7627821	5.8001499
62	6.10	0.0838566	-1.7932771	5.8203276
63	6.20	0.0873466	-1.8247421	5.8409499
64	6.30	0.0909366	-1.8571771	5.8620276
65	6.40	0.0946266	-1.8905821	5.8835499
66	6.50	0.0984166	-1.9249471	5.9055276
67	6.60	0.1023066	-1.9602721	5.9279499
68	6.70	0.1062966	-1.9965571	5.9508276
69	6.80	0.1103866	-2.0338021	5.9741499
70	6.90	0.1145766	-2.0720171	5.9979276
71	7.00	0.1188666	-2.1112021	6.0221499
72	7.10	0.1232566	-2.1513671	6.0468276
73	7.20	0.1277466	-2.1925121	6.0719499
74	7.30	0.1323366	-2.2346371	6.0975276
75	7.40	0.1370266	-2.2777421	6.1235499
76	7.50	0.1418166	-2.3218271	6.1499276
77	7.60	0.1467066	-2.3668921	6.1766499
78	7.70	0.1516966	-2.4129371	6.2037276
79	7.80	0.1567866	-2.4600621	6.2311499
80	7.90	0.1619766	-2.5082671	6.2589276
81	8.00	0.1672666	-2.5575421	6.2870499
82	8.10	0.1726566	-2.6078871	6.3155276
83	8.20	0.1781466	-2.6593021	6.3443499
84	8.30	0.1837366	-2.7117871	6.3735276
85	8.40	0.1894266	-2.7653421	6.4030499
86	8.50	0.1952166	-2.8199671	6.4329276
87	8.60	0.2011066	-2.8756621	6.4631499
88	8.70	0.2070966	-2.9324271	6.4937276
89	8.80	0.2131866	-2.9902621	6.5246499
90	8.90	0.2193766	-3.0491671	6.5559276
91	9.00	0.2256666	-3.1091421	6.5875499
92	9.10	0.2320566	-3.1701871	6.6195276
93	9.20	0.2385466	-3.2323021	6.6518499
94	9.30	0.2451366	-3.2954871	6.6845276
95	9.40	0.2518266	-3.3597421	6.7175499
96	9.50	0.2586166	-3.4250671	6.7509276
97	9.60	0.2655066	-3.4914621	6.7846499
98	9.70	0.2724966	-3.5589271	6.8187276
99	9.80	0.2795866	-3.6274621	6.8531499
100	9.90	0.2867766	-3.6970671	6.8879276
101	10.00	0.2940666	-3.7677421	6.9229499
102	10.10	0.3014566	-3.8394871	6.9582276
103	10.20	0.3089466	-3.9123021	6.9937499
104	10.30	0.3165366	-3.9861871	7.0295276
105	10.40	0.3242266	-4.0611421	7.0655499
106	10.50	0.3320166	-4.1371671	7.1018276
107	10.60	0.3399066	-4.2142621	7.1383499
108	10.70	0.3478966	-4.2924271	7.1751276
109	10.80	0.3559866	-4.3716621	7.2121499
110	10.90	0.3641766	-4.4519671	7.2494276
111	11.00	0.3724666	-4.5333421	7.2869499
112	11.10	0.3808566	-4.6157871	7.3247276
113	11.20	0.3893466	-4.6993021	7.3627499
114	11.30	0.3979366	-4.7838871	7.4010276
115	11.40	0.4066266	-4.8695421	7.4395499
116	11.50	0.4154166	-4.9562671	7.4783276
117	11.60	0.4243066	-5.0440621	7.5173499
118	11.70	0.4332966	-5.1329271	7.5566276
119	11.80	0.4423866	-5.2228621	7.5961499
120	11.90	0.4515766	-5.3138671	7.6359276
121	12.00	0.4608666	-5.4059421	7.6759499
122	12.10	0.4702566	-5.4990871	7.7162276
123	12.20	0.4797466	-5.5933021	7.7567499
124	12.30	0.4893366	-5.6885871	7.7975276
125	12.40	0.4990266	-5.7849421	7.8385499
126	12.50	0.5088166	-5.8823671	7.8798276
127	12.60	0.5187066	-5.9808621	7.9213499
128	12.70	0.5286966	-6.0804271	7.9631276
129	12.80	0.5387866	-6.1810621	7.9951499
130	12.90	0.5489766	-6.2827671	8.0274276
131	13.00	0.5592666	-6.3855421	8.0609499
132	13.10	0.5696566	-6.4893871	8.0947276
133	13.20	0.5801466	-6.5943021	8.1287499
134	13.30	0.5907366	-6.7002871	8.1630276
135	13.40	0.6014266	-6.8073421	8.1975499
136	13.50	0.6122166	-6.9154671	8.2323276
137	13.60	0.6231066	-7.0246621	8.2673499
138	13.70	0.6340966	-7.1349271	8.3026276
139	13.80	0.6451866	-7.2462621	8.3381499
140	13.90	0.6563766	-7.3586671	8.3739276
141	14.00	0.6676666	-7.4721421	8.4099499
142	14.10	0.6790566	-7.5866871	8.4462276
143	14.20	0.6905466	-7.7023021	8.4827499
144	14.30	0.7021366	-7.8189871	8.5195276
145	14.40	0.7138266	-7.9367421	8.5565499
146	14.50	0.7256166	-8.0555671	8.5938276
147	14.60	0.7375066	-8.1754621	8.6313499
148	14.70	0.7494966	-8.2964271	8.6691276
149	14.80	0.7615866	-8.4184621	8.7071499
150	14.90	0.7737766	-8.5415671	8.7454276
151	15.00	0.7860666	-8.6657421	8.7839499
152	15.10	0.7984566	-8.7909871	8.8227276
153	15.20	0.8109466	-8.9173021	8.8617499
154	15.30	0.8235366	-9.0446871	8.9010276
155	15.40	0.8362266	-9.1731421	8.9405499
156	15.50	0.8490166	-9.3026671	8.9803276
157	15.60	0.8619066	-9.4332621	9.0203499
158	15.70	0.8748966	-9.5649271	9.0606276
159	15.80	0.8879866	-9.6976621	9.1011499
160	15.90	0.9011766	-9.8314671	9.1419276
161	16.00	0.9144666	-9.9673421	9.1829499
162	16.10	0.9278566	-10.1042871	9.2242276
163	16.20	0.9413466	-10.2423021	9.2657499
164	16.30	0.9549366	-10.3813871	9.3075276
165	16.40	0.9686266	-10.5215421	9.3495499
166	16.50	0.9824166	-10.6627671	9.3918276
167	16.60	0.9963066	-10.8050621	9.4343499
168	16.70	1.0102966	-10.9484271	9.4771276
169	16.80	1.0243866	-11.0928621	9.5201499
170	16.90	1.0385766	-11.2383671	9.5634276
171	17.00	1.0528666	-11.3849421	9.6069499
172	17.10	1.0672566	-11.5325871	9.6507276
173	17.20	1.0817466	-11.6813021	9.6947499
174	17.30	1.0963366	-11.8310871	9.7390276
175	17.40	1.1109266	-11.9819421	9.7835499
176	17.50	1.1256166	-12.1338671	9.8283276
177	17.60	1.1404066	-12.2868621	9.8733499
178	17.70	1.1552966	-12.4409271	9.9186276
179	17.80	1.1702866	-12.5960621	9.9641499
180	17.90	1.1853766	-12.7522671	10.0099276
181	18.00	1.2005666	-12.9095421	10.0559499
182	18.10	1.2158566	-13.0678871	10.1022276
183	18.20	1.2312466	-13.2273021	10.1487499
184	18.30	1.2467366	-13.3877871	10.1955276
185	18.40	1.2623266	-13.5493421	10.2425499
186	18.50	1.2780166	-13.7119671	10.2898276
187	18.60	1.2938066	-13.8756621	10.3373499
188	18.70	1.3096966	-14.0404271	10.3851276
189	18.80	1.3256866	-14.2062621	10.4331499
190	18.90	1.3417766	-	

WEDGE ANGLE = 10.00 SOURCE ANGLE = 2.50  
 SOURCE DISTANCE=0.75 RECEIVER DISTANCE= 2.25 SHORE DISTANCE= 0.00  
 RHO1/RHO2= 0.80 C1/C2= 1.10 ALPHA/K2= 0.0001  
 K1X = 19.44

REC.POS	REC.ANGLE	PRES.AMPLITUDE	PHASE ANGLE	NORM.PRESS
1	0.00	0.224816	1.084841	6.519929
2	0.10	0.225248	1.099217	6.505968
3	0.20	0.225700	1.109943	6.491550
4	0.30	0.226170	1.106665	6.476150
5	0.40	0.226650	1.111389	6.459500
6	0.50	0.227140	1.114113	6.441500
7	0.60	0.227640	1.115827	6.422500
8	0.70	0.228150	1.116538	6.403000
9	0.80	0.228670	1.116246	6.383500
10	0.90	0.229200	1.114943	6.363500
11	1.00	0.229740	1.112637	6.342500
12	1.10	0.230290	1.109338	6.320500
13	1.20	0.230850	1.105044	6.297500
14	1.30	0.231420	1.100757	6.273500
15	1.40	0.232000	1.096479	6.248500
16	1.50	0.232590	1.092210	6.222500
17	1.60	0.233190	1.087950	6.195500
18	1.70	0.233800	1.083700	6.167500
19	1.80	0.234420	1.079460	6.138500
20	1.90	0.235050	1.075230	6.108500
21	2.00	0.235690	1.071010	6.077500
22	3.00	0.236340	1.066800	6.045500
23	4.00	0.237000	1.062600	6.012500
24	5.00	0.237670	1.058410	5.978500
25	6.00	0.238350	1.054230	5.943500
26	7.00	0.239040	1.050060	5.907500
27	8.00	0.239740	1.045910	5.870500
28	9.00	0.240450	1.041770	5.832500
29	10.00	0.241170	1.037640	5.793500
-	1.00	0.243458	1.033510	5.753500

WEDGE ANGLE = 10.00  
SOURCE DISTANCE=0.75  
RHO1/RHO2= 0.80  
K1X = 19.44

SOURCE ANGLE = 2.50  
RECEIVER DISTANCE= 1.13  
C1/C2= 1.10

SHORE DISTANCE= 0.00  
ALPHA/K2= 0.0001

REC.POS	REC.ANGLE	PRES.AMPLITUDE	PHASE ANGLE	NORM.PRESS
1	0.00	1.9002229	-0.125102	4.791066
1	0.10	1.9232264	-0.111954	4.849142
1	0.20	1.9442552	-0.111474	4.903825
1	0.30	1.9652253	-0.110949	4.955010
1	0.40	1.9842233	-0.104495	5.002865
1	0.50	2.0011944	-0.099689	5.047551
1	0.60	2.0183499	-0.096651	5.088881
1	0.70	2.033438	-0.098221	5.126922
1	0.80	2.047371	-0.098825	5.162555
1	0.90	2.060022	-0.097748	5.195908
1	1.00	2.071478	-0.077214	5.222233
1	1.10	2.081736	-0.066788	5.242899
1	1.20	2.090866	-0.061446	5.259177
1	1.30	2.099001	-0.056122	5.271768
1	1.40	2.105888	-0.050809	5.281558
1	1.50	2.111934	-0.045508	5.289555
1	1.60	2.117599	-0.040234	5.295800
1	1.70	2.122351	-0.034997	5.300400
1	1.80	2.126518	-0.029773	5.303548
1	1.90	2.129900	-0.024576	5.305405
1	2.00	2.132631	-0.019431	5.306099
1	2.10	2.134863	0.002980	5.305500
1	2.20	2.136317	0.007410	5.303785
1	2.30	2.136991	0.007410	5.300929
1	2.40	2.136804	0.011247	5.296708
1	2.50	2.135811	0.014435	5.291378
1	2.60	2.134082	0.016694	5.284300
1	2.70	2.131646	0.018733	5.275866
1	2.80	2.128519	0.019825	5.265500
1	2.90	2.124801	-0.153182	5.252500
1	3.00	2.120501	-0.153182	5.236500

WEDGE ANGLE = 10.00 SOURCE ANGLE = 2.50 SHORE DISTANCE = 0.00  
 SOURCE DISTANCE = 0.75 RECEIVER DISTANCE = 0.56 ALPHA/K2 = 0.0001  
 RHO1/RHO2 = 0.80 C1/C2 = 1.10  
 K1X = 19.44

REC. POS	REC. ANGLE	PRES. AMPLITUDE	PHASE ANGLE	NORM. PRESS
1	0.00	3.99	0.30	5.47
1	0.10	3.92	0.36	5.12
1	0.20	3.85	0.41	4.79
1	0.30	3.78	0.46	4.48
1	0.40	3.71	0.51	4.18
1	0.50	3.64	0.56	3.89
1	0.60	3.57	0.61	3.61
1	0.70	3.50	0.66	3.34
1	0.80	3.43	0.71	3.08
1	0.90	3.36	0.76	2.83
1	1.00	3.29	0.81	2.59
1	1.10	3.22	0.86	2.35
1	1.20	3.15	0.91	2.12
1	1.30	3.08	0.96	1.90
1	1.40	3.01	1.01	1.69
1	1.50	2.94	1.06	1.49
1	1.60	2.87	1.11	1.30
1	1.70	2.80	1.16	1.12
1	1.80	2.73	1.21	0.95
1	1.90	2.66	1.26	0.79
1	2.00	2.59	1.31	0.64
1	2.10	2.52	1.36	0.50
1	2.20	2.45	1.41	0.37
1	2.30	2.38	1.46	0.25
1	2.40	2.31	1.51	0.14
1	2.50	2.24	1.56	0.04
1	2.60	2.17	1.61	0.00
1	2.70	2.10	1.66	0.00
1	2.80	2.03	1.71	0.00
1	2.90	1.96	1.76	0.00
1	3.00	1.89	1.81	0.00
1	3.10	1.82	1.86	0.00
1	3.20	1.75	1.91	0.00
1	3.30	1.68	1.96	0.00
1	3.40	1.61	2.01	0.00
1	3.50	1.54	2.06	0.00
1	3.60	1.47	2.11	0.00
1	3.70	1.40	2.16	0.00
1	3.80	1.33	2.21	0.00
1	3.90	1.26	2.26	0.00
1	4.00	1.19	2.31	0.00
1	4.10	1.12	2.36	0.00
1	4.20	1.05	2.41	0.00
1	4.30	0.98	2.46	0.00
1	4.40	0.91	2.51	0.00
1	4.50	0.84	2.56	0.00
1	4.60	0.77	2.61	0.00
1	4.70	0.70	2.66	0.00
1	4.80	0.63	2.71	0.00
1	4.90	0.56	2.76	0.00
1	5.00	0.49	2.81	0.00
1	5.10	0.42	2.86	0.00
1	5.20	0.35	2.91	0.00
1	5.30	0.28	2.96	0.00
1	5.40	0.21	3.01	0.00
1	5.50	0.14	3.06	0.00
1	5.60	0.07	3.11	0.00
1	5.70	0.00	3.16	0.00
1	5.80	0.00	3.21	0.00
1	5.90	0.00	3.26	0.00
1	6.00	0.00	3.31	0.00
1	6.10	0.00	3.36	0.00
1	6.20	0.00	3.41	0.00
1	6.30	0.00	3.46	0.00
1	6.40	0.00	3.51	0.00
1	6.50	0.00	3.56	0.00
1	6.60	0.00	3.61	0.00
1	6.70	0.00	3.66	0.00
1	6.80	0.00	3.71	0.00
1	6.90	0.00	3.76	0.00
1	7.00	0.00	3.81	0.00
1	7.10	0.00	3.86	0.00
1	7.20	0.00	3.91	0.00
1	7.30	0.00	3.96	0.00
1	7.40	0.00	4.01	0.00
1	7.50	0.00	4.06	0.00
1	7.60	0.00	4.11	0.00
1	7.70	0.00	4.16	0.00
1	7.80	0.00	4.21	0.00
1	7.90	0.00	4.26	0.00
1	8.00	0.00	4.31	0.00
1	8.10	0.00	4.36	0.00
1	8.20	0.00	4.41	0.00
1	8.30	0.00	4.46	0.00
1	8.40	0.00	4.51	0.00
1	8.50	0.00	4.56	0.00
1	8.60	0.00	4.61	0.00
1	8.70	0.00	4.66	0.00
1	8.80	0.00	4.71	0.00
1	8.90	0.00	4.76	0.00
1	9.00	0.00	4.81	0.00
1	9.10	0.00	4.86	0.00
1	9.20	0.00	4.91	0.00
1	9.30	0.00	4.96	0.00
1	9.40	0.00	5.01	0.00
1	9.50	0.00	5.06	0.00
1	9.60	0.00	5.11	0.00
1	9.70	0.00	5.16	0.00
1	9.80	0.00	5.21	0.00
1	9.90	0.00	5.26	0.00
1	10.00	0.00	5.31	0.00

WEDGE ANGLE = 10.00      SOURCE ANGLE = 2.50  
 SOURCE DISTANCE = 0.75      RECEIVER DISTANCE = 0.28      SHORE DISTANCE = 0.00  
 RHO1/RHO2 = 0.80      C1/C2 = 1.10      ALPHA/K2 = 0.0001  
 K1X = 19.44

REC.POS	REC.ANGLE	PRES.AMPLITUDE	PHASE ANGLE	NORM.PRESS
1	0.00	0.6423356	-0.271448	9.791314
00	0.10	0.6033351	-0.270972	9.692724
00	0.20	0.6229941	-0.270501	9.602075
00	0.30	0.623727	-0.270035	9.507351
00	0.40	0.617507	-0.269567	9.412549
00	0.50	0.600112	-0.269099	9.317747
00	0.60	0.592558	-0.268631	9.222945
00	0.70	0.588825	-0.268163	9.128143
00	0.80	0.589258	-0.267695	9.033341
00	0.90	0.590000	-0.267227	8.938539
10	0.00	0.590000	-0.266759	8.843737
11	1.00	0.590000	-0.266291	8.748935
11	1.10	0.590000	-0.265823	8.654133
11	1.20	0.590000	-0.265355	8.559331
11	1.30	0.590000	-0.264887	8.464529
11	1.40	0.590000	-0.264419	8.369727
11	1.50	0.590000	-0.263951	8.274925
11	1.60	0.590000	-0.263483	8.180123
11	1.70	0.590000	-0.263015	8.085321
11	1.80	0.590000	-0.262547	7.990519
11	1.90	0.590000	-0.262079	7.895717
20	0.00	0.590000	-0.261611	7.800915
22	0.00	0.590000	-0.261143	7.706113
22	0.10	0.590000	-0.260675	7.611311
22	0.20	0.590000	-0.260207	7.516509
22	0.30	0.590000	-0.259739	7.421707
22	0.40	0.590000	-0.259271	7.326905
22	0.50	0.590000	-0.258803	7.232103
22	0.60	0.590000	-0.258335	7.137301
22	0.70	0.590000	-0.257867	7.042499
22	0.80	0.590000	-0.257399	6.947697
22	0.90	0.590000	-0.256931	6.852895
22	1.00	0.590000	-0.256463	6.758093
22	1.10	0.590000	-0.255995	6.663291
22	1.20	0.590000	-0.255527	6.568489
22	1.30	0.590000	-0.255059	6.473687
22	1.40	0.590000	-0.254591	6.378885
22	1.50	0.590000	-0.254123	6.284083
22	1.60	0.590000	-0.253655	6.189281
22	1.70	0.590000	-0.253187	6.094479
22	1.80	0.590000	-0.252719	6.000000
22	1.90	0.590000	-0.252251	5.905521
22	2.00	0.590000	-0.251783	5.811042
22	2.10	0.590000	-0.251315	5.716563
22	2.20	0.590000	-0.250847	5.622084
22	2.30	0.590000	-0.250379	5.527605
22	2.40	0.590000	-0.249911	5.433126
22	2.50	0.590000	-0.249443	5.338647
22	2.60	0.590000	-0.248975	5.244168
22	2.70	0.590000	-0.248507	5.149689
22	2.80	0.590000	-0.248039	5.055210
22	2.90	0.590000	-0.247571	4.960731
22	3.00	0.590000	-0.247103	4.866252
22	3.10	0.590000	-0.246635	4.771773
22	3.20	0.590000	-0.246167	4.677294
22	3.30	0.590000	-0.245699	4.582815
22	3.40	0.590000	-0.245231	4.488336
22	3.50	0.590000	-0.244763	4.393857
22	3.60	0.590000	-0.244295	4.299378
22	3.70	0.590000	-0.243827	4.204899
22	3.80	0.590000	-0.243359	4.110420
22	3.90	0.590000	-0.242891	4.015941
22	4.00	0.590000	-0.242423	3.921462
22	4.10	0.590000	-0.241955	3.826983
22	4.20	0.590000	-0.241487	3.732504
22	4.30	0.590000	-0.241019	3.638025
22	4.40	0.590000	-0.240551	3.543546
22	4.50	0.590000	-0.240083	3.449067
22	4.60	0.590000	-0.239615	3.354588
22	4.70	0.590000	-0.239147	3.260109
22	4.80	0.590000	-0.238679	3.165630
22	4.90	0.590000	-0.238211	3.071151
22	5.00	0.590000	-0.237743	2.976672
22	5.10	0.590000	-0.237275	2.882193
22	5.20	0.590000	-0.236807	2.787714
22	5.30	0.590000	-0.236339	2.693235
22	5.40	0.590000	-0.235871	2.598756
22	5.50	0.590000	-0.235403	2.504277
22	5.60	0.590000	-0.234935	2.409798
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22	5.80	0.590000	-0.234000	2.220840
22	5.90	0.590000	-0.233532	2.126361
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22	6.10	0.590000	-0.232596	1.937403
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22	6.40	0.590000	-0.231192	1.653966
22	6.50	0.590000	-0.230724	1.559487
22	6.60	0.590000	-0.230256	1.465008
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22	6.80	0.590000	-0.229320	1.276050
22	6.90	0.590000	-0.228852	1.181571
22	7.00	0.590000	-0.228384	1.087092
22	7.10	0.590000	-0.227916	0.992613
22	7.20	0.590000	-0.227448	0.898134
22	7.30	0.590000	-0.226980	0.803655
22	7.40	0.590000	-0.226512	0.709176
22	7.50	0.590000	-0.226044	0.614697
22	7.60	0.590000	-0.225576	0.520218
22	7.70	0.590000	-0.225108	0.425739
22	7.80	0.590000	-0.224640	0.331260
22	7.90	0.590000	-0.224172	0.236781
22	8.00	0.590000	-0.223704	0.142302
22	8.10	0.590000	-0.223236	0.047823
22	8.20	0.590000	-0.222768	-0.046656
22	8.30	0.590000	-0.222300	-0.141137
22	8.40	0.590000	-0.221832	-0.235618
22	8.50	0.590000	-0.221364	-0.330099
22	8.60	0.590000	-0.220896	-0.424580
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22	8.80	0.590000	-0.219960	-0.613542
22	8.90	0.590000	-0.219492	-0.708023
22	9.00	0.590000	-0.219024	-0.802504
22	9.10	0.590000	-0.218556	-0.896985
22	9.20	0.590000	-0.218088	-0.991466
22	9.30	0.590000	-0.217620	-1.085947
22	9.40	0.590000	-0.217152	-1.180428
22	9.50	0.590000	-0.216684	-1.274909
22	9.60	0.590000	-0.216216	-1.369390
22	9.70	0.590000	-0.215748	-1.463871
22	9.80	0.590000	-0.215280	-1.558352
22	9.90	0.590000	-0.214812	-1.652833
22	10.00	0.590000	-0.214344	-1.747314

## APPENDIX D

### TABLES

TABLE 1

RECEIVER DISTANCE AT THE FIRST TRANSITION POINT,  
FOR CONSTANT  $\rho_1 \rho_2 = 0.80, c_1 c_2 = 1.10$

$\beta = 6^\circ, K_1 X = 32.61$

	$R_1 = 1.10$	$R_1 = 1.20$	$R_1 = 1.30$	$R_1 = 1.40$	$R_1 = 1.50$
$\gamma = \beta/4$	5.2	5.0	5.9	7.3	9.8
$\gamma = \beta/2$	5.2	4.7	5.7	6.8	9.9
$\gamma = 3\beta/4$	4.0	4.5	5.4	7.0	10.5

$\beta = 10^\circ, K_1 X = 19.44$

	$R_1 = 0.80$	$R_1 = 0.90$	$R_1 = 1.00$	$R_1 = 1.10$	$R_1 = 1.20$
$\gamma = \beta/4$	17.5	24.0	33.0	52.0	72.0
$\gamma = \beta/2$	12.5	17.0	24.0	42.0	60.0
$\gamma = 3\beta/4$	10.6	17.0	22.0	40.0	58.0

$\beta = 15^\circ, K_1 X = 12.79$

	$R_1 = 1.30$	$R_1 = 1.40$	$R_1 = 1.50$	$R_1 = 1.60$	$R_1 = 1.70$
$\gamma = \beta/4$	8.1	8.2	8.7	9.8	11.4
$\gamma = \beta/2$	6.9	7.0	7.8	8.9	10.9
$\gamma = 3\beta/4$	9.8	12.0	15.0	19.0	29.0

TABLE 2

RECEIVER DISTANCE AT THE FIRST TRANSITION POINT,  
FOR CONSTANT  $\rho_1/\rho_2 = 0.80$ ,  $c_1/c_2 = 1.20$

$\beta = 6^\circ$ ,  $K_1 X = 22.53$

	$R_1 = 1.30$	$R_1 = 1.40$	$R_1 = 1.50$	$R_1 = 1.60$	$R_1 = 1.70$
$\gamma = \beta - 4$	3.37	3.6	3.92	4.4	4.96
$\gamma = \beta - 2$	3.155	3.42	3.78	4.36	4.96
$\gamma = 3\beta - 4$	3.07	3.34	3.74	4.2	5.1

$\beta = 10^\circ$ ,  $K_1 X = 13.43$

	$R_1 = 0.80$	$R_1 = 0.90$	$R_1 = 1.00$	$R_1 = 1.10$	$R_1 = 1.20$
$\gamma = \beta - 4$	6.1	11.3	18.0	40.0	60.0
$\gamma = \beta - 2$	6.5	8.2	14.5	25.0	40.0
$\gamma = 3\beta - 4$	5.05	5.45	12.0	18.0	30.0

$\beta = 15^\circ$ ,  $K_1 X = 8.84$

	$R_1 = 0.80$	$R_1 = 0.90$	$R_1 = 1.00$	$R_1 = 1.10$	$R_1 = 1.20$
$\gamma = \beta - 4$	36.0	40.0	52.0	60.0	64.0
$\gamma = \beta - 2$	24.0	26.0	32.0	46.0	54.0
$\gamma = 3\beta - 4$	19.50	23.0	28.0	38.0	58.0

TABLE 3  
 RECEIVER DISTANCE AT THE FIRST TRANSITION POINT,  
 FOR CONSTANT  $\rho_1 \rho_2 = 0.90$ ,  $c_1 c_2 = 1.10$

$\beta = 6^\circ$ ,  $K_1 X = 32.61$

	$R_1 = 1.10$	$R_1 = 1.20$	$R_1 = 1.30$	$R_1 = 1.40$	$R_1 = 1.50$
$\gamma = \beta \cdot 4$	3.66	4.1	4.9	6.2	11.0
$\gamma = \beta \cdot 2$	3.35	3.9	4.7	7.0	no
$\gamma = 3\beta \cdot 4$	3.24	3.76	4.75	7.3	9.8

$\beta = 10^\circ$ ,  $K_1 X = 19.44$

	$R_1 = 1.30$	$R_1 = 1.40$	$R_1 = 1.50$	$R_1 = 1.60$	$R_1 = 1.70$
$\gamma = \beta \cdot 4$	9.8	11.0	14.0	40.0	60.0
$\gamma = \beta \cdot 2$	7.6	8.6	11.0	19.0	40.0
$\gamma = 3\beta \cdot 4$	6.7	7.6	10.0	21.0	45.0



APPENDIX E

GRAPHS OF  $R_1$  VERSUS  $R_2$  AT THE FIRST TRANSITION POINT

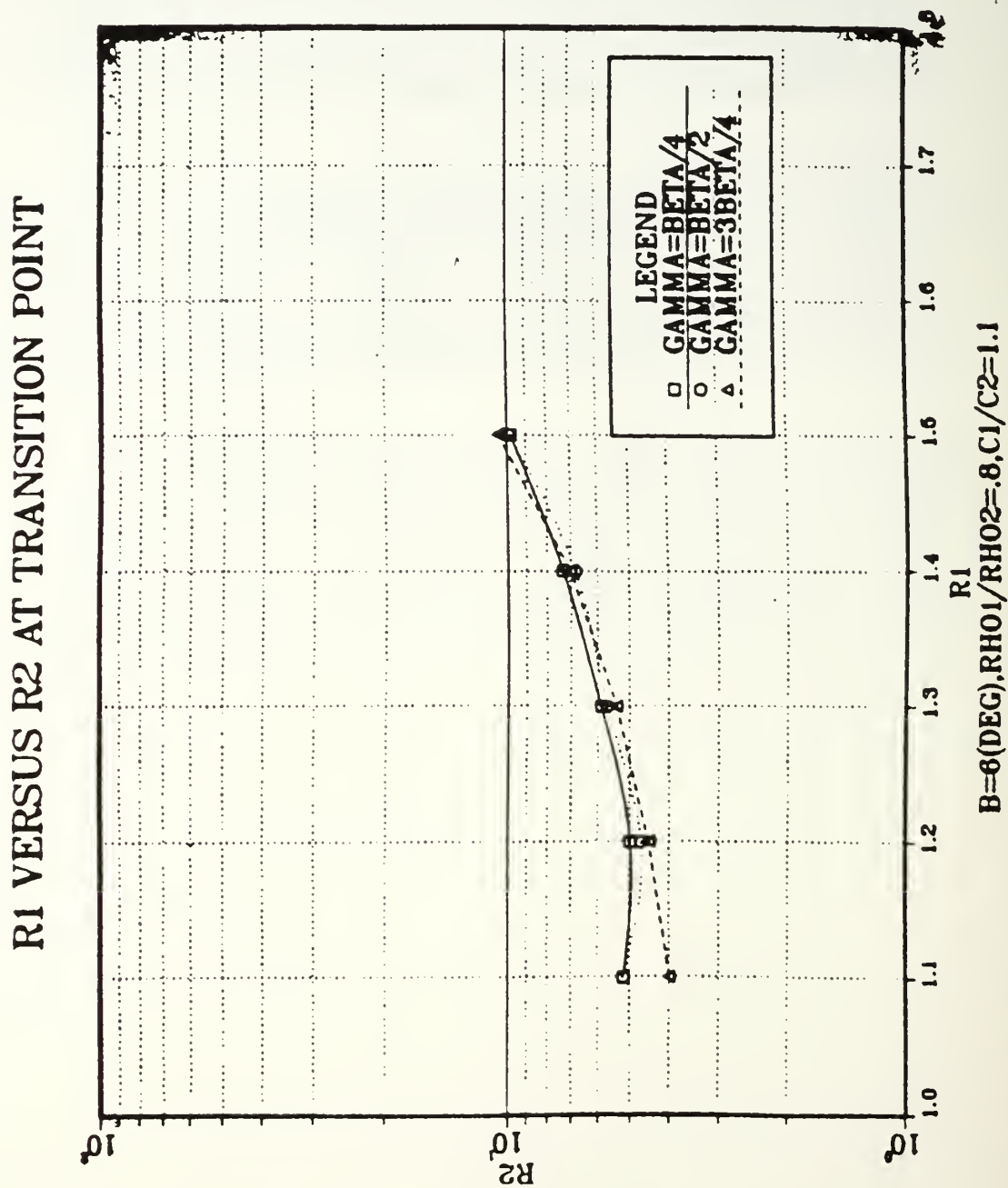


Figure E.1  $R_1$  vs  $R_2$  at the first trans.points, for  $\beta = 6^\circ$ ,  $\rho_1/\rho_2 = 0.80$ ,  $c_1/c_2 = 1.10$ .

# R1 VERSUS R2 AT TRANSITION POINT

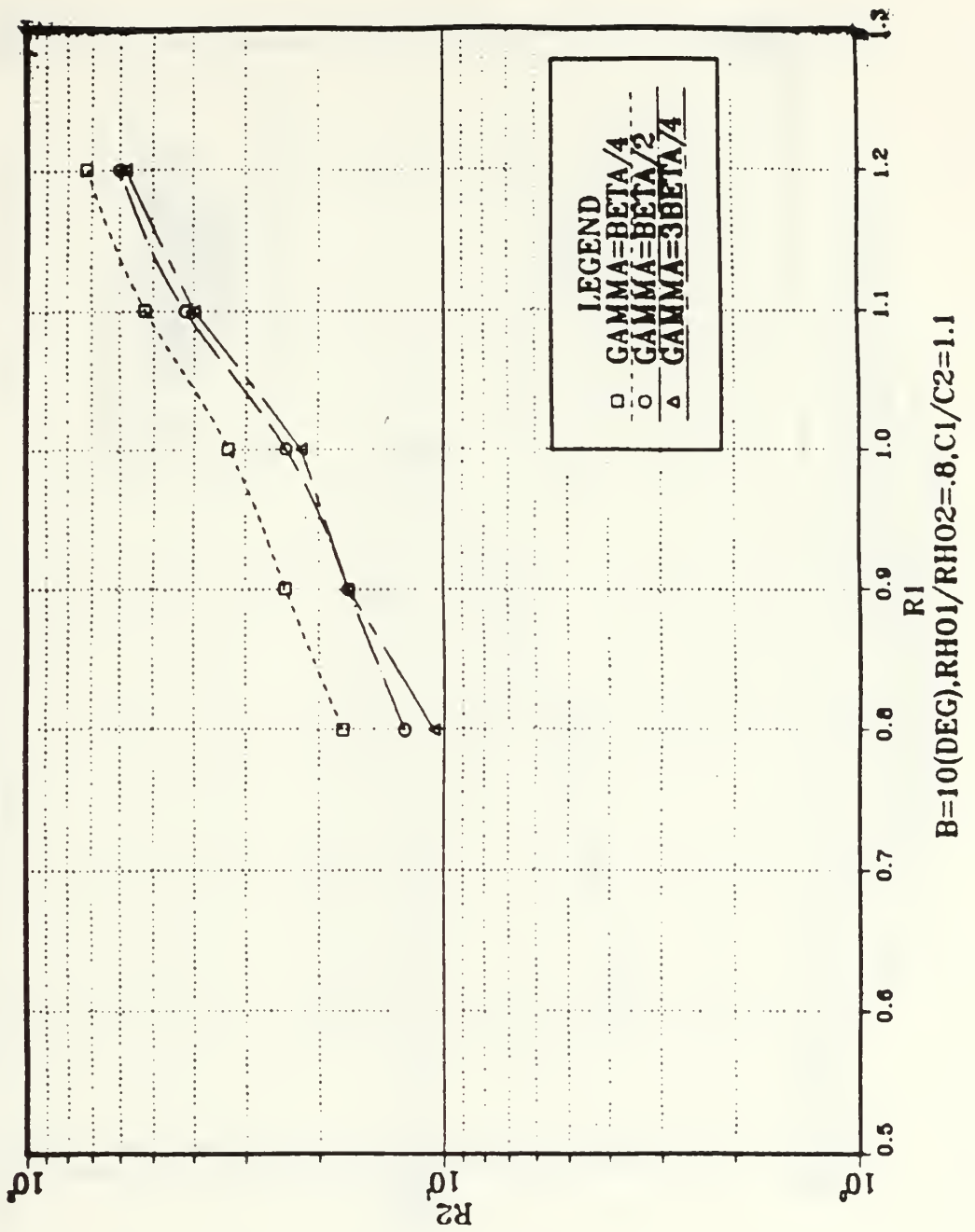


Figure E.2  $R_1$  vs  $R_2$  at the first trans. points, for  $\beta = 10^\circ$ ,  $\rho_1/\rho_2 = 0.80$ ,  $c_1/c_2 = 1.10$ .

# R1 VERSUS R2 AT TRANSITION POINT

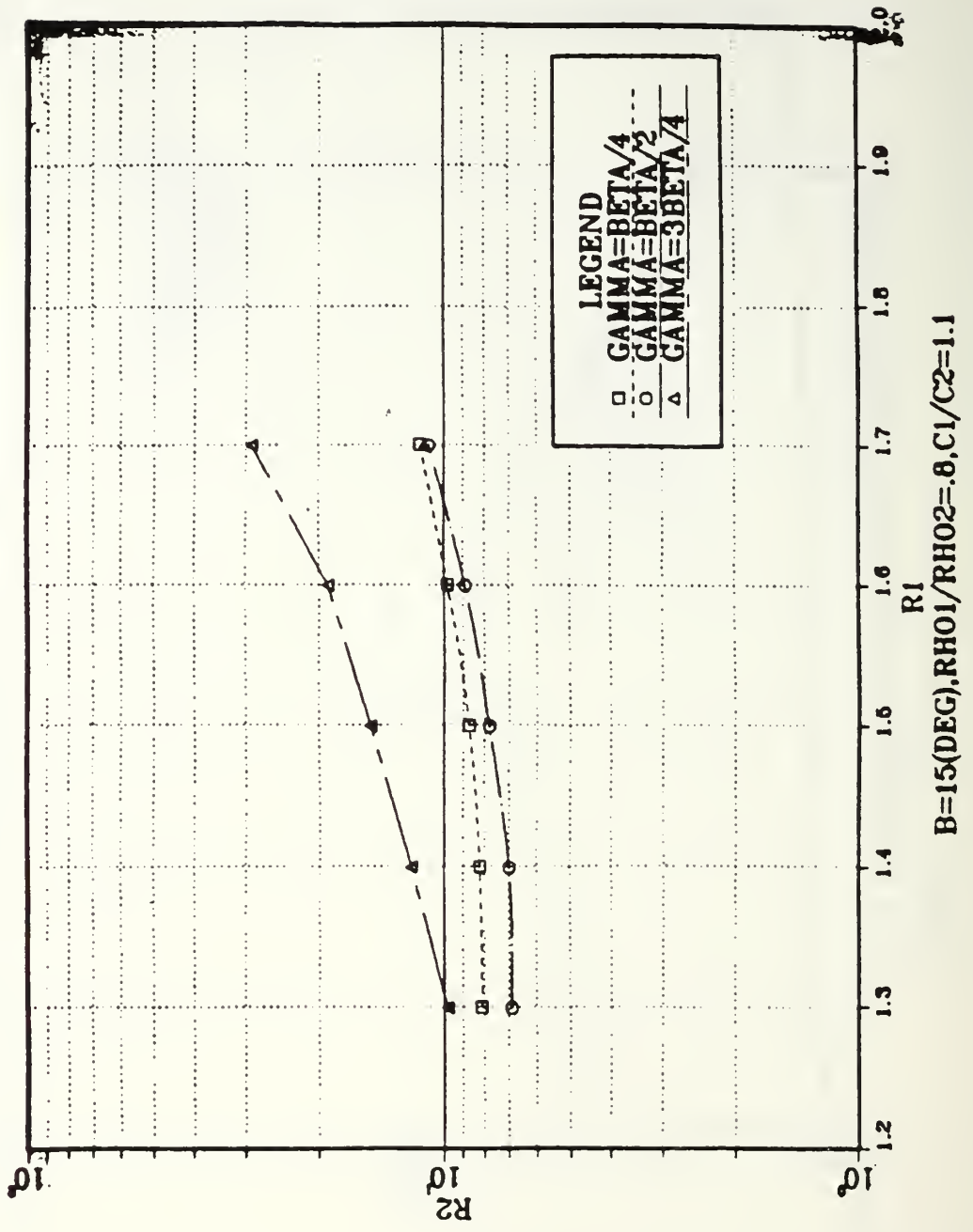


Figure E.3 R<sub>1</sub> vs R<sub>2</sub> at the first trans. points, for β = 15°, ρ<sub>1</sub>/ρ<sub>2</sub> = 0.80, c<sub>1</sub>/c<sub>2</sub> = 1.10.

# R1 VERSUS R2 AT TRANSITION POINT

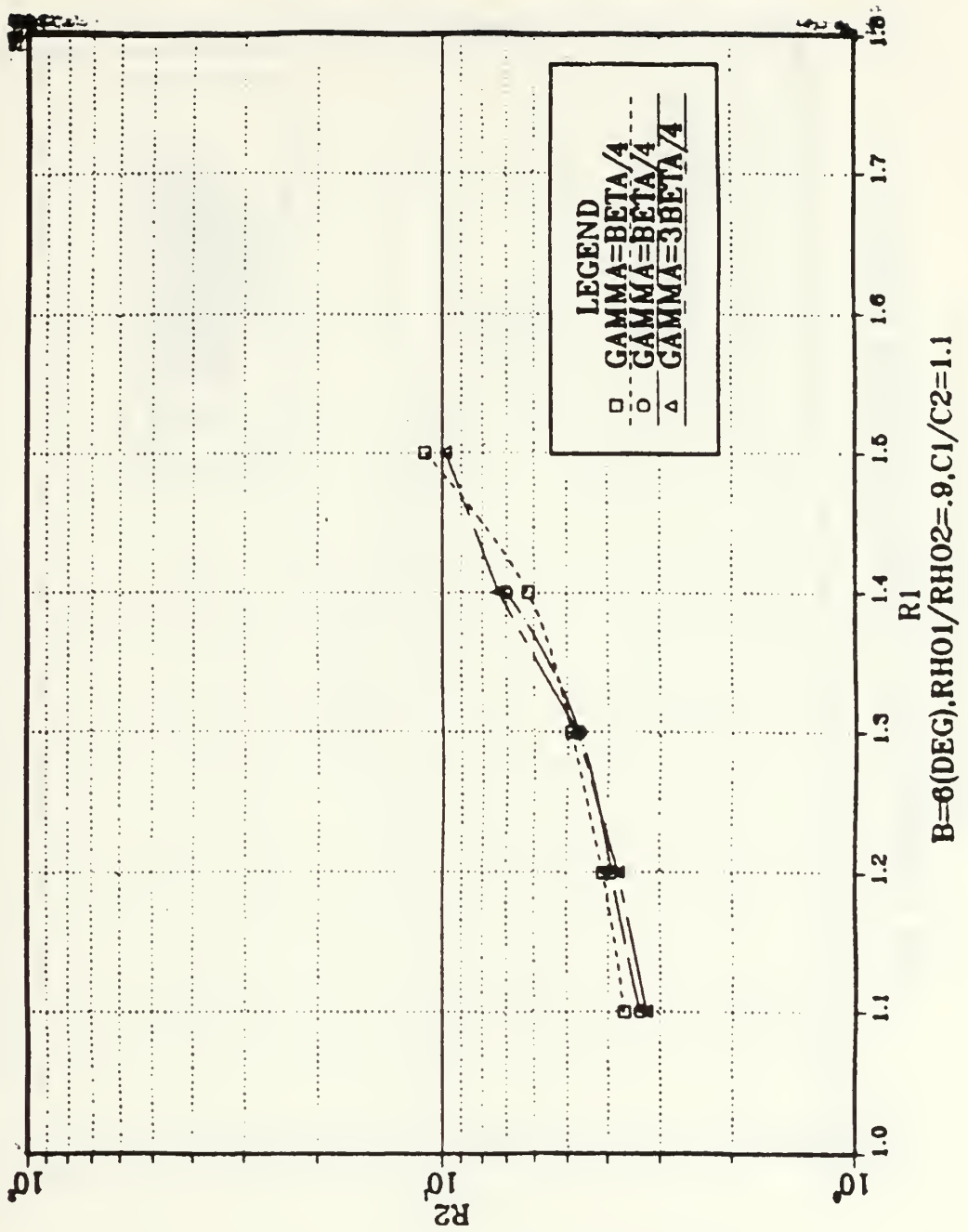


Figure E.4  $R_1$  vs  $R_2$  at the first trans. points, for  $\beta = 6^\circ$ ,  $\rho_1/\rho_2 = 0.80$ ,  $c_1/c_2 = 1.20$ .

# R1 VERSUS R2 AT TRANSITION POINT

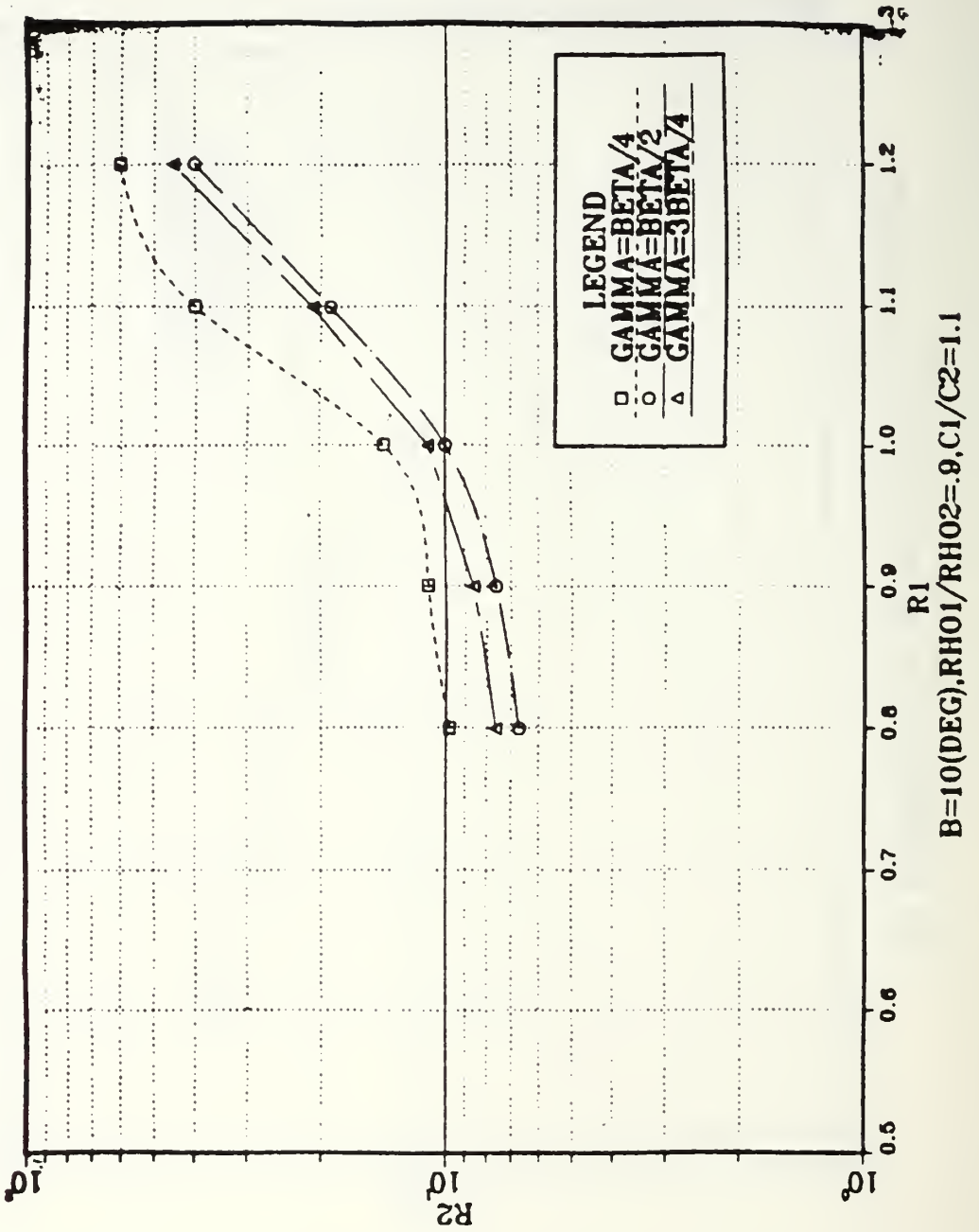


Figure E.5  $R_1$  vs  $R_2$  at the first trans. points, for  $\beta = 10^\circ$ ,  $\rho_1/\rho_2 = 0.80$ ,  $c_1/c_2 = 1.20$ .

# R1 VERSUS R2 AT TRANSITION POINT

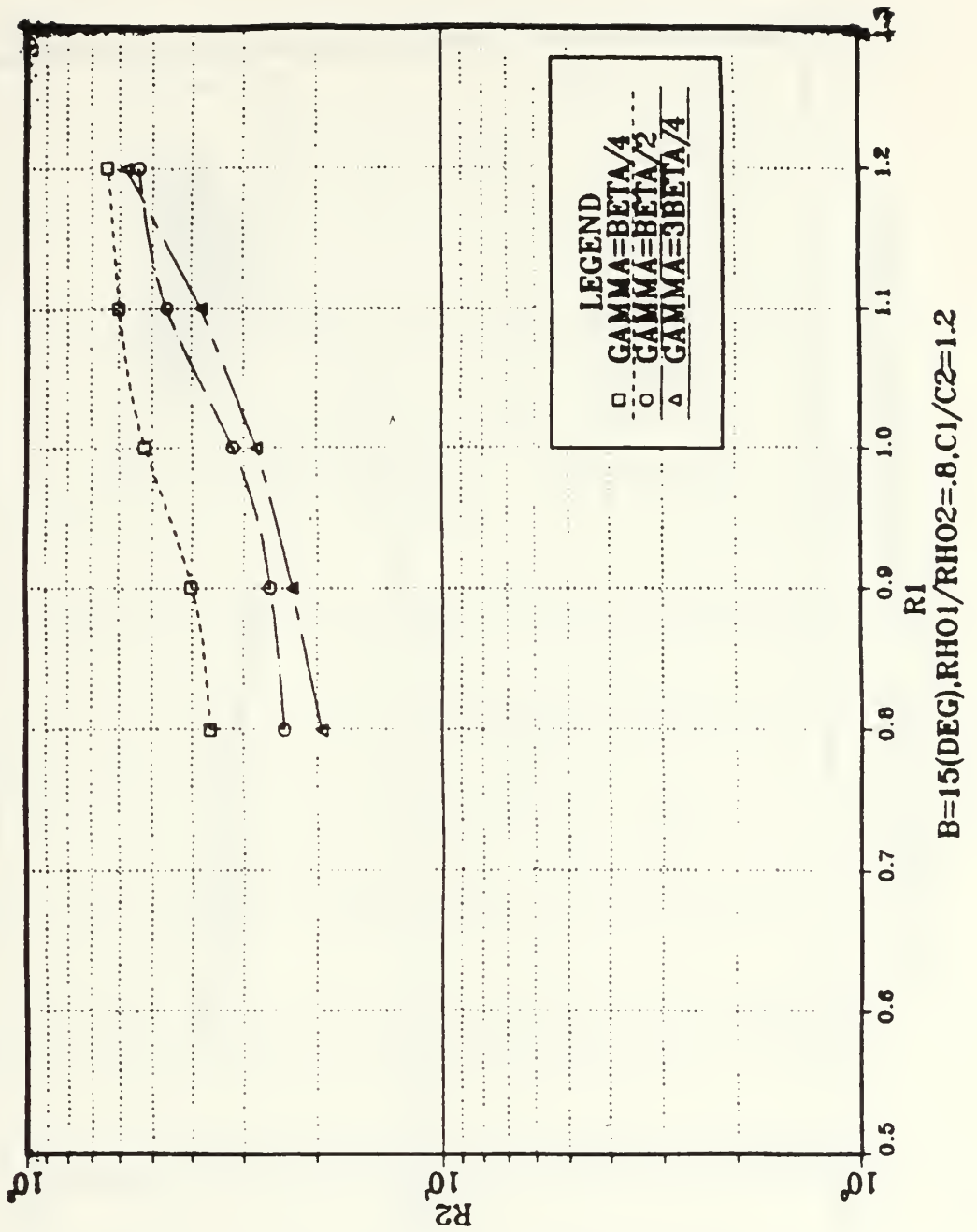


Figure E.6  $R_1$  vs  $R_2$  at the first trans. points, for  $\beta = 15^\circ$ ,  $\rho_1/\rho_2 = 0.80$ ,  $c_1/c_2 = 1.20$ .

# R1 VERSUS R2 AT TRANSITION POINT

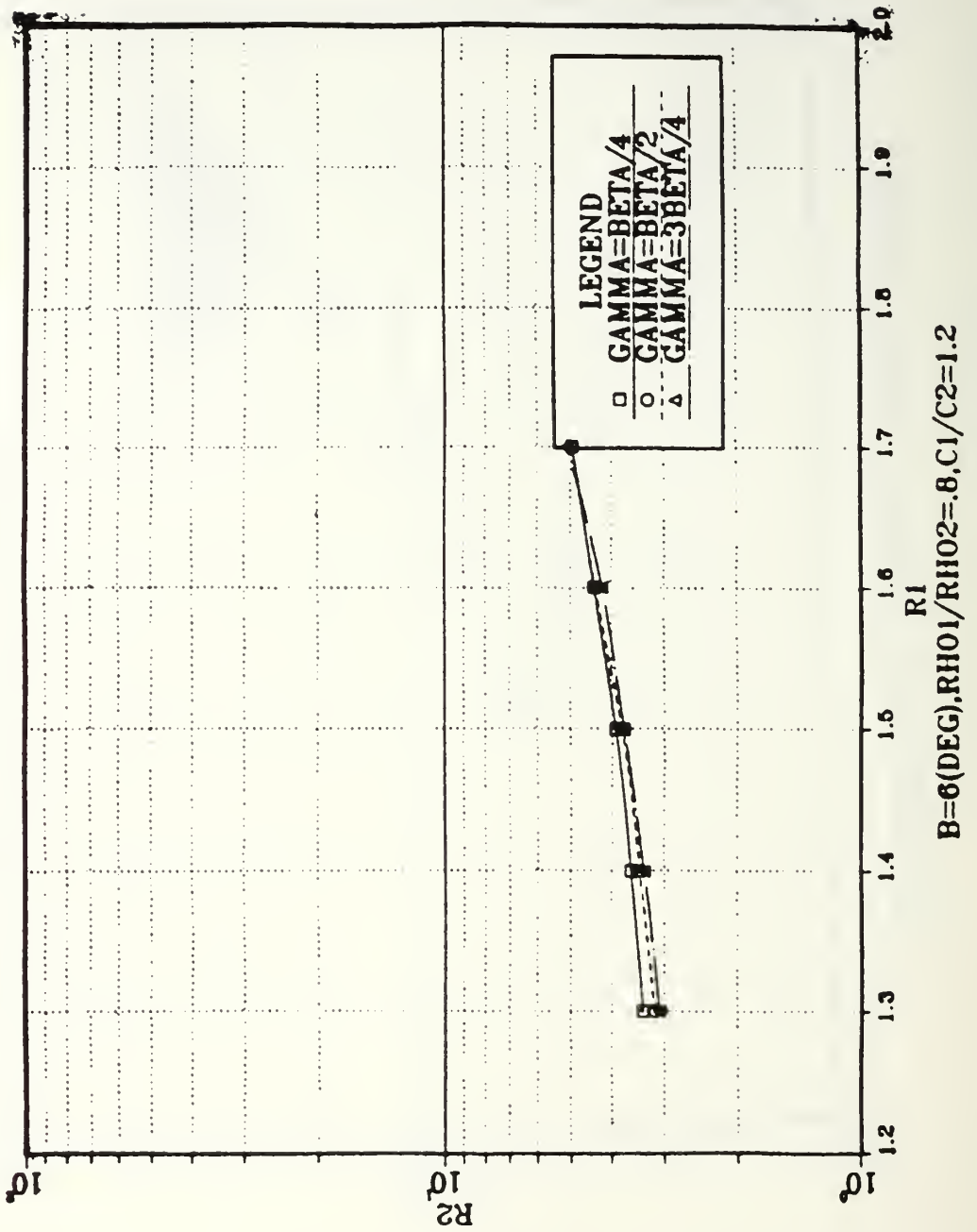


Figure E.7  $R_1$  vs  $R_2$  at the first trans. points, for  $\beta = 6^\circ$ ,  $\rho_1/\rho_2 = 0.90$ ,  $c_1/c_2 = 1.10$ .

# R1 VERSUS R2 AT TRANSITION POINT

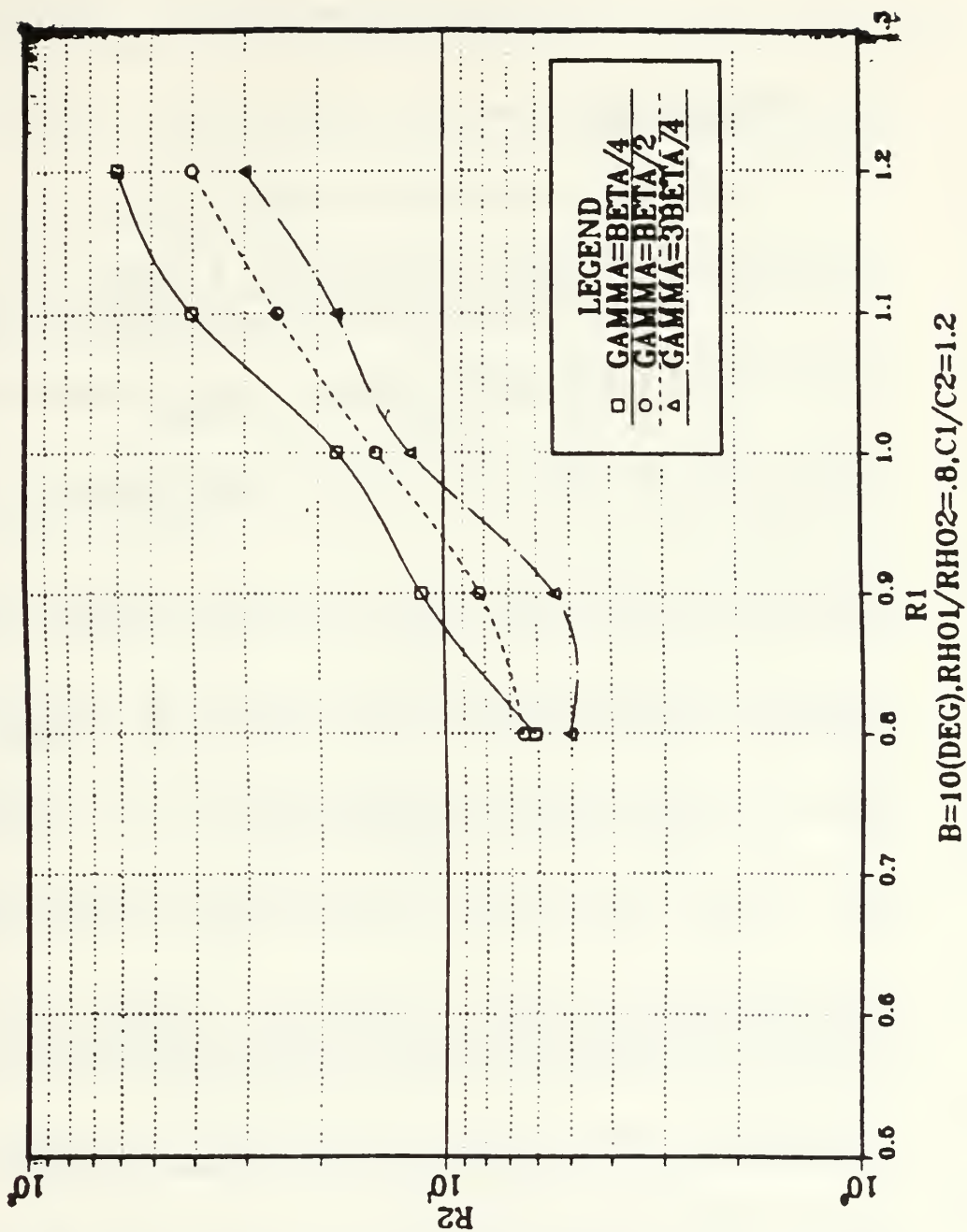


Figure E.8  $R_1$  vs  $R_2$  at the first trans. points, for  $\beta = 10^\circ$ ,  $\rho_1/\rho_2 = 0.90$ ,  $c_1/c_2 = 1.10$ .



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