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DISSERTATION

OPTIMUM SIGNAL PROCESSING

IN DISTRIBUTED SENSOR SYSTEMS

by

Abdel-Aziz M. Al-Bassiouni

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Examples are given to illustrate the trade off between performance and communications between the sensors. Our results match that of centralized processing at one extreme and that of decentralized processing at the other. The way is graded between extreme ends. Finally a faster algorithm is given to solve the system of nonlinear equations for the optimum system parameters.

Optimum Signal Processing in Distributed Sensor Systems

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LIST OF SYMBOLS

CD	Centralized Detection
C_{ij}	Cost of detecting H_i while H_j is true
CSP	Centralized Signal Processing
DD	Distributed or Decentralized Detection
DDN	Distributed Decision Network
DSN	Distributed Sensor Network
DSP	Distributed Signal Processing
E{.}	Expectation Operator
erf(x)	The error function given by $$\mathrm{erf}(x) = \frac{2}{\sqrt{2\pi}} \int_0^x \exp(-u^2/2)\, du$$
erfc(x)	The complement of the error function = 1-erf(x)
f(y)	Probability density function of x
H	The phenomena to be detected
H_0	The null hypothesis
H_1	The alternative hypothesis
LMMS	Linear Minimum Mean Square
MMS	Minimum Mean Square
P_{di}	Probability of detection of the i_{th} detector
P_{fi}	Probability of false alarm of the i_{th} detector
P_d	Probability of detection of the fusion center
P_f	Probability of false alarm of the fusion center
PDM	Primary Decision Maker
QD	Quantized Detection
Q_i	i_{th} quantizer
R	The average cost of detection, also risk of decision
SNR	Signal-to-noise ratio
T_{ilo}	Locally optimum threshold of the i_{th} detector
U_i	Decision of detector i
U_o	Decision of the fusion center
Var(.)	Variance

LIST OF SYMBOLS (CONT'D)

Y_i	i_{th} sensor observation
Y_{iq}	Quantized version of y_i
Z_{i0}	Null decision subspace of Detector i
Z_{i1}	Alternative decision subspace of Detector i
α_i	Quantization rule of the i_{th} quantizer
η	Ratio of the powers at the input and output of the quantizer
Φ	The empty set
$\int_{\Psi} dy$	Integeral over the set Ψ
$\Lambda_i(x)$	Likelihood ratio of $x = f_i(x/H_1)/f_i(x/H_0)$
ρ	Correlation coefficient
ρ_{cr}	Critical value of ρ
$\mathcal{R}$	The set of real numbers
$>$	Greater than
$\leq$	Less than or equal to

DEDICATION

To My Parents

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I. INTRODUCTION

This thesis deals with detection and estimation using spatially separated sensors. A typical practical situation is a surveillance system [1] in which a large number of sensors monitor some region of space, earth or sea and report their findings to a global processor. The sensors themselves may use thermal, acoustic or infrared effects to form their observations. The global processor performs some processing on the data to come with a decision or for taking actions. Because of many considerations such as bandwidth communication limitations, time delay or because the amount of information is too massive to be processed by a single processor, the processing is carried out on many levels. As an example consider the case of distributed detection. Detection is performed at the sensor level and at the fusion center.

Due to the loss of information in the local processing, the overall performance degrades. However a great communication bandwidth reduction results. If the communication channels can support more information flow, then it is wise to perform "softer" processing at the local level, to send more information to the fusion center, and to use the information available there effectively.

The purpose of this chapter is to define the Distributed Signal Processing (DSP) problem in general and to show some reasons and situations in which it replaces Centralized Signal Processing (CSP) techniques. We then will review the status of the research on Decentralized Detection (DD) problem, one of the basic problems of DSP. Finally the contributions and organization of this thesis are described.

A. OVERVIEW

Classical (Centralized) Signal Processing (CSP) assumes complete availability of all information (signals) at one central processor for processing (decision making, computing, detection, estimation, etc...). While this situation is realistic in some cases, many real world systems are too large for the classical processing to be practically applied. Power systems, detection networks, large manufacturing systems and military organizations are among those systems in which total centralized signal processing is hard to apply. Some of the reasons and considerations for the limations of CSP are [2,3]:

1. In large systems, each processor has partial information of some credibility. While total information is distributed in the whole system, total centeralization of the information at one processor is impractical, inconvenient or expensive due to limitations in the system's communication channels, memory or computation and information capabilities.
2. In some cases, processing speed is a bottleneck. Increasing local processing of the data at each processor and sending processed data to the next level of processors will help relieve the problem.
3. When reliability of the system is of major concern, distributed processing may better tolerate various kinds of equipment failures. Less complex centralized processing is more easily shifted to a new location.
4. In cases when security is a major problem, increasing local processing will decrease the information handled between the processors, so limit any other system's access to the process.
5. As the cost of computation has decreased dramatically relative to the cost of communication, it is advantageous to trade off increased computation for reduced communication. So in Distributed Sensor Networks (DSN) involving geographically distributed sensors that collect data, it may be more economical to locally process the data and send condensed summaries to other processors.

Distributed Signal Processing (DSP), in contrast to CSP, has several processors that cooperate together to best achieve a global task according to some criterion. A basic problem in DSP, which has attracted much attention recently, is the Decentralized Detection (DD) problem (hypothesis testing). The DD problem will be a major concern in this thesis. A summary of its status is given in the following section.

B. MOTIVATION

There has been an increased interest in the DD problem since Tenney and Sandell introduced it in 1981 [5]. They extended the classical Bayesian formulation of the detection problem to distributed environments. Because their work was the pioneering one in DD and because we will refer to it often in this thesis , let us consider it now in some detail together with the Centralized Detection (CD) problem. Also, because detection is dealt with throughout a large portion of this thesis, we will make some remarks about the phenomena to be detected and about detection criterion.

The Phenomena

Consider observing a phenomena H of M possible states in order to determine which of them is true. For $M = 2$, the state H_0 is called the null hypothesis and H_1 the alternative hypothesis. Their probabilities of occurrence

$$P(H_0) = P_0, \quad P(H_1) = P_1 \tag{1.1}$$

are assumed to be known.

The Sensor Observations

The phenomena H is observed by N sensors $S_1, S_2, ..., S_N$. The sensor observations are $y_1, y_2, ..., y_N$. The sensor observations have known conditional distributions

$$p(y_1, y_2, ..., y_N / H_0), \quad p(y_1, y_2, ..., y_N / H_1). \tag{1.2}$$

Detection Criterion

The function of the detection process is to make a decision, U_o, about which state of the phenomena is true. The optimality criterion is a function

$$J: U_o \, xH \rightarrow \mathscr{R}, \tag{1.3}$$

that assigns to the event of deciding u_i when H_j is true a real number C_{ij}, i,j=0,1, called the detection cost, so

$$J(U_o = u_i, H = H_j) = C_{ij}. \tag{1.4}$$

The objective of the decision rule will be to minimize the expected decision cost

$$\min E\{J(u,H)\}. \tag{1.5}$$

An important ratio in our analysis is the constant given by

$$C = \frac{P_0 (C_{10} - C_{00})}{P_1 (C_{01} - C_{11})}. \tag{1.6}$$

Van Trees [6] showed that the average decision cost is given by,

$$R = C_{00} P_0 + C_{01} P_1 + P_0 (C_{01} - C_{11}) P_f - P_1 (C_{01} - C_{11}) P_d \tag{1.7}$$

where P_f and P_d are the probability of false alarm[1] and probability of detection[2] respectively. At this point we will make the assumptions that

$$C_{01} > C_{11}, \tag{1.8}$$

and

$$C_{10} > C_{00}, \tag{1.9}$$

These assumptions implies that it is more costly to err than to make a correct decision. Equation (1.7) can then be written in the form:

$$R = \left[\frac{C_{00} P_0 + C_{01} P_1}{P_1 (C_{01} - C_{11})} + \frac{P_0 (C_{10} - C_{00})}{P_1 (C_{01} - C_{11})} P_f - P_d \right] P_1 (C_{01} - C_{11}) . \tag{1.10}$$

Ignoring positive constants that will not affect our analysis, the average decision cost R is given by

$$R = 1 + C P_f - P_d . \tag{1.11}$$

1. The Centralized Detection (CD) Problem

The problem of centralized binary hypothesis testing can be posed in its most general form as follows. For the structure of Figure 1.1 it is assumed that all sensor observations can be sent to one (central) location for processing. The function of the processor is to map the vector $\underline{Y} = [\, y_1\ y_2 \ldots y_N\,]^t$ into the decision space U&subo(0,1)

$$U_o : \underline{Y} \rightarrow (0,1) \tag{1.12}$$

as follows;

$$U_o = \begin{cases} 0, & H_0 \text{ is declared to have been detected} \\ 1, & H_1 \text{ is declared to have been detected.} \end{cases} \tag{1.13}$$

---

[1]Probability of deciding $U_o = 0$ while H_1 is true

[2]Probability of deciding $U_o = 1$ while H_1 is true

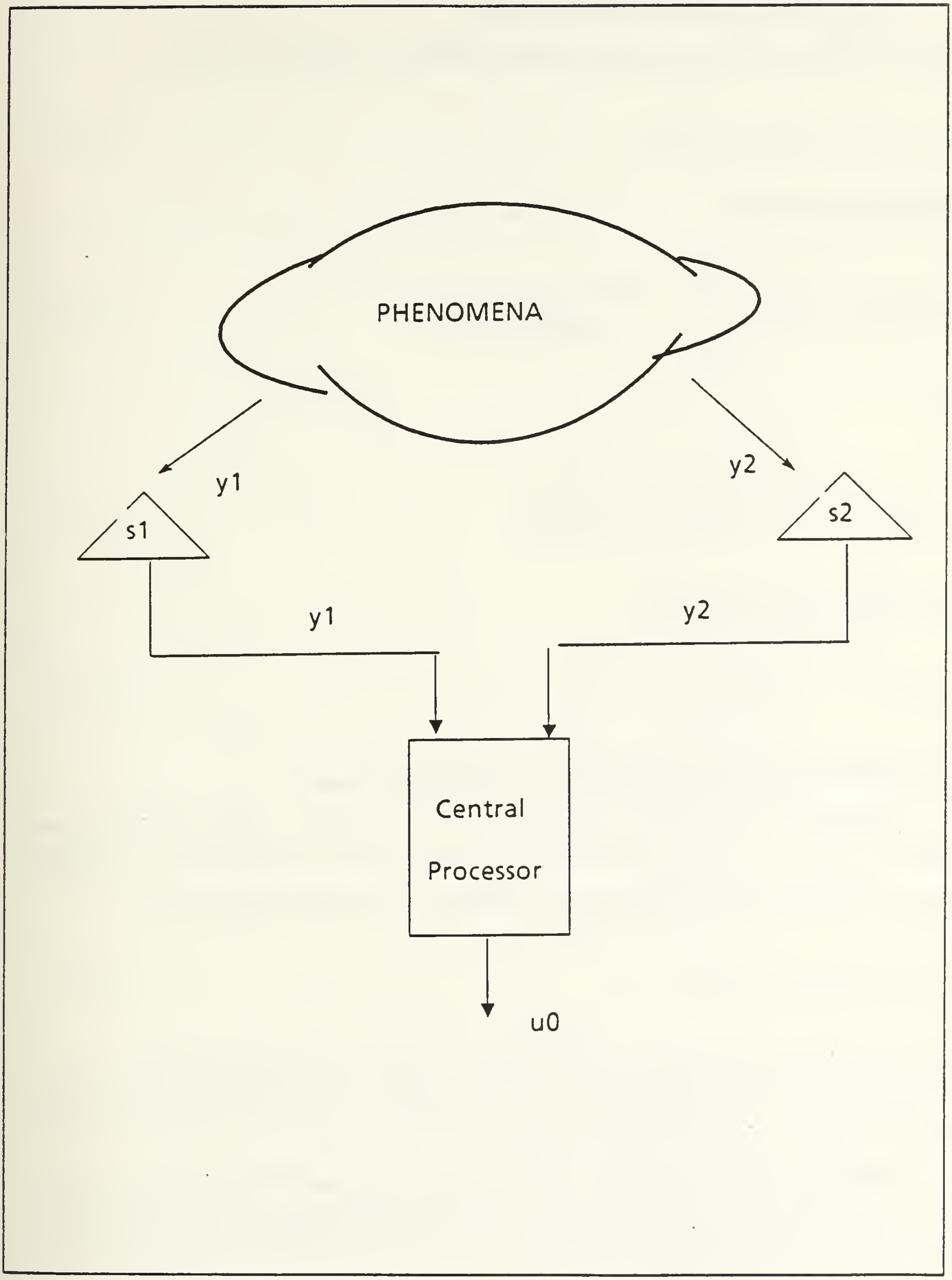

Figure 1.1 Centralized Detection.

Solution of the CD Problem

The solution to the CD problem is [6]

a) deterministic, so that the decision rule is a function of the observations

$$\gamma : Y_1 \, x Y_2 \, x \ldots x Y_N \rightarrow (0,1), \tag{1.14}$$

b) a likelihood ratio test,

$$U_o(y_1, y_2, \; y_N) = \begin{cases} 0, & \text{if } \Lambda\,(y_1, y_2 .., y_N) \geq t \\ 1, & \text{if } \Lambda\,(y_1, y_2, \ldots, y_N) < t \end{cases} \tag{1.15}$$

where

$$\Lambda\,(y_1, y_2, \ldots, y_N) = \frac{f(y_1, y_2, \ldots, y_N / H_1)}{f(y_1, y_2, \ldots, y_N / H_0)}\,, \tag{1.16}$$

c) and the threshold t is given by

$$t = C\,. \tag{1.17}$$

2. **The Decentralized Detection (DD) Problem with Fusion**

Consider the structure of Figure 1.2 with H and Y being as before; the decisions U_1 ,U_2 ,... and U_N are sent to a fusion center. The activity of the fusion center is to make the global decision U_o according to some preset fusion rule.

$$U_o : U_1 \, x U_2 .. x U_N \rightarrow (0,1). \tag{1.18}$$

In the DD problem with fusion it is required to design local decision rules U_1,U_2,... and U_N and a global fusion rule (1.18) so as to minimize the expected cost $E\{J(U_o\,, H)\}$ incurred by deciding $U_o = i$ when H_j is true.

Choosing an AND fusion rule apriori, Tenney and Sandell solved this problem for N=2. They set the decision rule as $U_o = U_1 \; U_2$ and optimized the local decision rules.

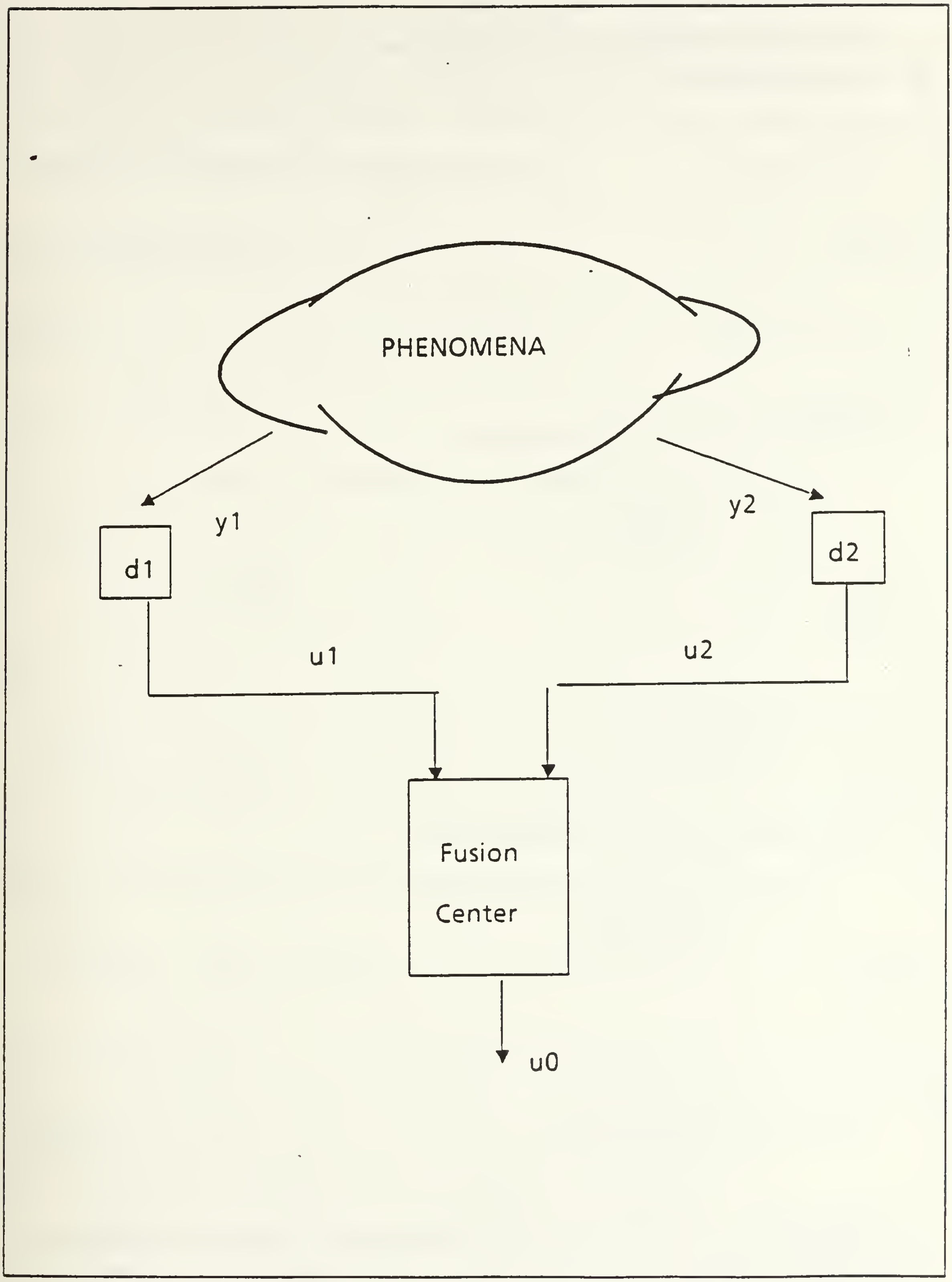

Figure 1.2 Decentralized Detection with Fusion.

Solution of DD Problem with Fusion

The solution to the DD problem with fusion is

1. deterministic

$$\gamma_1 : Y_1 \rightarrow (0,1) \tag{1.19}$$

and

$$\gamma_2 : Y_2 \rightarrow (0,1) \tag{1.20}$$

2. a likelihood ratio test for each detector

$$U_i = \begin{cases} 0, & \text{if } \Lambda_i (y_i) \geq t_i \\ 1, & \text{if } \Lambda_i (y_i) < t_i \end{cases} \tag{1.21}$$

where

$$\Lambda_i (y_i) = \frac{f(y_i / H_1)}{f(y_i / H_0)} . \tag{1.22}$$

3. with coupled thresholds t_1 and t_2 given by

$$t_1 = C \frac{Pr(F_2 / y_1)}{Pr(D_2 / y_1)} \tag{1.23}$$

and

$$t_2 = C \frac{Pr(F_1 / y_2)}{Pr(D_1 / y_2)} \tag{1.24}$$

where $Pr(F_i / y_j)$ and $Pr(D_i / y_j)$ are respectively the conditional probability of false alarm and the conditional probability of detection of the i_{th} detector given the j_{th} detector's observation.

Equations (1.23) and (1.24) are two coupled functional equations in t_1 and t_2 . For general distributions, a functional expression for each of them in terms of its own observation and the other detector's decision is impossible. We shall consider the complexity of these decision rules later. A special case of the DD problem is the case of conditionally independent sensor observations, i.e.

$$f(y_1 / y_2, H) = f(y_1 / H) \tag{1.25}$$

and

$$f(y_2 / y_1, H) = f(y_2 / H). \tag{1.26}$$

In this case, the conditional probabilities in (1.23) and (1.24) reduce to

$$t_1 = C \frac{P_{f2}}{P_{d2}} \tag{1.27}$$

and

$$t_2 = C \frac{P_{f1}}{P_{d1}} \tag{1.28}$$

Equations (1.27) and (1.28) are two coupled algebraic equations in the form of

$$t_1 = g_1 (t_2) \quad \text{and} \quad t_2 = g_2 (t_1) \tag{1.29}$$

since P_{fi} and P_{di} depend on t_i . This coupling represents cooperation between the two sensors. The threshold equations are necessary conditions for optimality. There may be several local minima; each must be checked to assure the global minima. The threshold equations are strongly coupled for general cost assumptions.

Tenney and Sandell came to the following conclusions:

1. Increasing the signal-to-noise ratio improves the performance of the system. However a centralized system makes more efficient use of the increased information.

2. As the imbalance between the two detectors increases the performance improves. If the signal-to-noise ratio of one of the detectors goes to zero then the system decision is that of the other detector. This is equivalent to the performance of a CD system of the same signal-to-noise ratio.

The case of conditionally independent observations has been considered by many authors. Sarma and Rao [7] extended Tenney and Sandell's results to the case of three sensors. They assumed a majority logic fusion rule and evaluated the threshold settings for some specific cases. Chair and Varshney [8] considered the problem of optimal fusion of N local decisions from prespecified local decision rules. Their optimum fusion structure is a weighted sum of local decisions according to their reliabilities. Reibman and Nolte [9] optimize both local decision rules and the fusion rule under the assumption of identical local decision rules. The global decision is then k out of N. They optimize the local decision rule for each k ,k = 1,2,...,N, then pick the value of k corresponding to the minimum decision cost.

A sub-class of the DD problem with fusion, that will be referrd to as the "Second Opinion" problem, is the fusion of one's observation with another's decisions. An example of this is the second opinion in a medical examination, or even asking for legal advise. Ekchian [10] and Ekchian and Tenney [11] consider some specific topologies of this problem. Each decision maker has to make his decision based on his own observation and a predecessor's decisions. All the decision rules are likelihood ratio tests using the actual data. The thresholds are determined by incoming communication messages. The number of thresholds at each decision maker grows exponentially with the number of message inputs. Their results suggest putting the noisy sensor "up stream" in the detection network.

Papastavrou and Athans [12] also consider the second opinion problem.They examine the structure of a primary decision maker, PDM, and a secondary decision maker, SDM (a consultant). The PDM makes his decision based on his own observation if it is of good quality. If his observation is noisy, the PDM asks, at a communication cost, the opinion of the SDM. Being activated by the request of the PDM, the SDM sends his decision to the PDM or ignores the request if his observation is noisy. In either case the PDM has to make a final decision. Again the thresholds are coupled. The threshold of the PDM is determined by the message of the SDM.

This thesis is motivated mainly by three of the above works namely;

1. Bayesian formulation of the DD problem by Tenney and Sandell [5].

2. Extension of the DD problem to the Distributed Detection Networks by Ekchian. [10]
3. Extention of the DD problem to the case of correlated sensor observations by Lauer and Sandell [4].

C. THE COMPLEXITY OF THE DD PROBLEM

We saw that the DD problem can be solved optimally for conditionally independent sensor observations. If this condition does not hold local decisions are not likelihood ratio tests with constant thresholds. Tenney and Sandell show that for conditionally dependent observations, local decision rules are likelihood ratio tests but with data dependent thresholds (see e.g. (1.23) and (1.24)). These two equations are coupled. This means that the observation of one sensor is necessary for the other sensor's decision, which contradicts the principle of decentralization. In terms of the terminology of the Theory of Combinatorial Complexity [13], Tsitisiklis and Athans [14] show that

1. The DD problem with independent observations is a polynomial time problem.
2. The DD problem with dependent sensor observations in its simplest form is a nondeterministic polynomial NP-complete. This means that exhaustive enumeration is necessary to find the optimum local decision rules. Optimality may be an illusive goal. So, suboptimal solutions must be sought.

A suboptimal solution to the problem for the case of AND fusion was considered by Lauer and Sandell [4]. They considered the case of known signals in correlated noise. They took as a suboptimal solution local decision rules which are likelihood tests but having constant, not data dependent, thresholds satisfying the necessary condition of optimality. These thresholds are given by the implicit equations:

$$\Lambda_1 (T_1) = C \frac{\Pr(F_2 / T_1)}{\Pr(D_2 / T_1)} \tag{1.30}$$

and

$$\Lambda_2 (T_2) = C \frac{\Pr(F_1 / T_2)}{\Pr(D_1 / T_2)} . \tag{1.31}$$

D. CONTRIBUTIONS OF THIS THESIS

We have reviewed the complexity of the DD problem and its current status. The research reported here has significantly advanced this status in several important ways. Specifically the contributions of this thesis have been to :

1. Answer the question of the optimum fusion rule at the fusion center for the case of two sensors.
2. Specify the exact relation between the performance of the optimum fusion rule and the correlation coefficient between sensor observations.
3. Solve the the second opinion decision problem.
4. Solve the multi-level DD problem with fusion; i.e. detection with quantized sensor data for the known signal in noise case.
5. Introduce the minimum risk quantizer.
6. Grade the road between DD detection and CD detection.
7. Optimally design quantizers for minimum mean square estimation.
8. Present an efficient procedure to calculate parameters of a large variety of quantizers.

E. ORGANIZATION OF THE THESIS

The thesis is organized as follows. In Chapter II we consider the problem of fusion in DD. Optimum detection with quantized sensor data is considered in Chapter III, where the Quantized Detection algorithm, QD, is introduced. Numerical examples to illustrate the algorithm are given in Chapter IV. Generalization to the case of vector observations is presented in Chapter V. Optimum regeneration of sensor observations from their quantized versions and another sensor observation is considered in Chapter VI. A summary of the thesis, conclusions and suggestions for future research are given in Chapter VII. Proofs to some equations and FORTRAN programs to calculate parameters of the minimum risk and the minimum distortion quantizers are given in the appendices.

II. OPTIMUM FUSION OF LOCAL DECISIONS

In this chapter the important question of the optimum fusion rule will be answered. The relationship of the optimum fusion policy to the ratio of costs and the correlation coefficient between observations is determined.

A. INTRODUCTION

Distributed Detection with fusion is a two level optimization problem. The problem can be formulated in the following three ways:

1. Local Decision Optimization

The first way is to select the fusion rule apriori and optimize the local decision rules accordingly. Setting the activity of the fusion center as AND fusion, Tenney and Sandell [5] derived optimum local decision rules for a pair of spatially separated detectors with conditionally independent observations. They prove that local decision rules are simple likelihood ratio tests with constant thresholds. The thresholds are the solution of a pair of coupled algebraic equations that correspond to the global minimum of the detection cost function. They also show that for the case of correlated observations local decision rules are likelihood ratio tests but with data dependent thresholds. Functional solution of the threshold equations in the later case violates the principle of decentralization. Realizing the difficulty of the problem in the case of correlated observations, Lauer and Sandell [4] designed suboptimal local decisions for AND fusion. Their local decision rules are likelihood ratio tests with constant thresholds satisfying the necessary conditions of optimality. Kovatana [15] considered AND fusion for two detectors. Fefjar [16] compared AND to OR fusion for two detectors. He claimed that OR is better than AND. Stearns [17] contradicts Fefjar's results. He showed by an example that OR combining is better for higher cost of missing the target while AND combining is better for higher cost of false alarms.

2. Fusion Rule Optimization

In the second formulation of the problem, local decision rules are set apriori. Optimization is carried out with respect to the fusion rules. An example of this situation could be factory built sensors that cannot be adjusted. Assuming local threshold settings Chair and Varshney [8] prove that for the case of conditionally independent sensor observations, the optimum fusion rule is a likelihood ratio test that sums local decisions weighted according to their reliability.

3. Global optimization of the Local decisions and the Fusion Rule

The third formulation involves optimization at both levels. Here local decisions are optimized for every possible fusion rule. The optimum fusion rule is the one that minimizes cost.

The main issue of this chapter is the global optimization of the DD system for general correlated observations. First we will state the main results for the case of N conditionally independent and identically distributed sensor observations. Then, the problem of fusing two local decisions of sensors with correlated observations is considered.

B. GLOBAL OPTIMIZATION OF DISTRIBUTED DETECTION

In CD all sensor observations are available at one central processor for detection. The decision rule in CD is a likelihood ratio test in the observations $y_1,y_2,...,y_N$. It declares H_1 is true if the likelihood ratio

$$\Lambda\,(y_1,y_2,...,y_N\,) \geq C\,, \tag{2.1}$$

otherwise it will declare H_0 to be true.

In DD only local decisions are sent to the central processor (fusion center). The objective of the fusion center is to mix (fuse) the local decisions into a single global decision with minimum decision cost. So given the local decisions the observation space of the fusion center consists of 2^N discrete points. The activity of the fusion center is to divide this space into two decision regions Z_0 and Z_1. The decision rule of the fusion center is a likelihood ratio test [8.] The fusion center declares H_1 is true if

$$\Lambda\,(u_1,u_2,...,u_N\,) \geq C. \tag{2.2}$$

otherwise it will declare that H_0 is true. In the special case of conditionally independent and identically distributed observations, the fusion rule is a k out of N rule. Reibman and Nolte [9] considered this problem. Assuming the same decision rule for every detector they optimize local decisions for every k, k = 1,2,...,N then pick the k with the minimum decision cost.

If sensor observations are not conditionally independent, there is no guarantee that local decisions are simple likelihood ratio tests. The problem turns out to be NP-complete which needs exhaustive enumerations to find the optimum decision rules [14.] Moreover if sensor observations are not identically distributed, there are as many as 2^{2^N} possible fusion rules for the N sensor decisions. Any algorithm that goes through the entire fusion list optimizing local decisions will be impractical[3] for $N \geq 6$.

Our approach to avoid this exhaustive enumeration is the following:

1. We assume that local decisions are likelihood ratio tests with constant thresholds. Again we emphasize that this assumption is valid only for conditionally independent observations, there is no guarantee that it is correct for correlated observations [5]. So the constant threshold likelihood ratio test is optimum for conditionally independent observations and perhaps suboptimum for correlated observations. However the solution tends to the optimum solution as the correlation coefficient tends to zero [4].
2. Those fusion rules which agree with the CD solution will be tested. The rest of the fusion rules will be disregarded. The meaning of this will be made clear in the following example.

Let us consider the case of two sensors ($N = 2$) in detail. To be explicit, consider detection of known signals in gaussian noise. The sensor observations are given by:

$$H_0 : \ y_i = n_i , \qquad i = 1,2 \tag{2.3}$$

and

$$H_1 : \ y_i = a_i + n_i , \qquad i = 1,2. \tag{2.4}$$

The a_i's are positive constants and $\underline{N} = [n_1 \ n_2]^t$ is vector of zero mean with covariance

$$K = \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix} \tag{2.5}$$

where ρ is given by

$$\rho = E\{ n_1 n_2 \}. \tag{2.6}$$

[3]A computer that spends 1 μ second in every optimization process, will spend 40000 years to determine the optimum fusion rule, for $N = 6$.

The threshold equation of the CD problem is given by [6],

$$(a_1 - \rho a_2)y_1 + (a_2 - \rho a_1)y_2 = (a_1^2 + a_2^2 - 2\rho a_1 a_2)/2 + (1-\rho^2)\log(C) \tag{2.7}$$

which is a straight line in the y_1 y_2 plane. Figure 2.1 shows decision rules based only on D_1, only on D_2, both decision rules together, and the decision rule of CD.

The global optimization requires optimizing local decision rules for every fusion rule then picking the fusion rule with minimum average cost. The observation space of the fusion center consists of four discrete points (0,0), (0,1), (1,0), (1,1). Any fusion rule divides this space into two decision regions Z_1 and Z_0. There are $2^4 = 16$ methods to subdivide four points into two groups. Table 1 contains a list of those fusion rules. Some special cases for the detection problem are as follows.

1. If $C_{10} \rightarrow \infty$, i.e. the cost of missing the target is extremely high, the CD solution assigns all the observation space to Z_0. The fusion center can perform the same. This is fusion rule one.
2. Similarly if $C_{01} \rightarrow \infty$, the fusion center will always decide H_1, this is fusion rule two.
3. If $a_2 = \rho a_1$, the CD will decide based only on y_1. So will the fusion center. This is fusion rule three. This can only happen when $a_1 \geq a_2$.
4. If $a_1 = \rho a_2$, the CD will decide based on y_2. This is fusion rule four. This can only happen when $a_2 \geq a_1$.

The first two situations represent extreme conditions of C. The next two conditions deal with specific values of ρ. We also distinguish the following two cases.

Case a

$-1 \leq \rho \leq \min(a_1, a_2)/\max(a_1, a_2)$.

In this case the y_1 and y_2 intersections of the threshold equation (2.7) are of the same sign.

Case b

$\min(a_1, a_2)/\max(a_1, a_2) < \rho \leq 1$.

In this case the y_1 and y_2 intersections of the threshold equation are of different signs. We shall consider these intervals of ρ when we study the effect of correlation between sensor observations.

The CD threshold in the y_1 y_2 plane suggests assigning the decision point (0,0) to Z_0 and (1,1) to Z_1. The fusion rules from 5 to 14 do not do this. They either assign (0,0) to Z_1 or assign (1,1) to Z_0 or assign (0,0) and (1,1) to the same decision region.

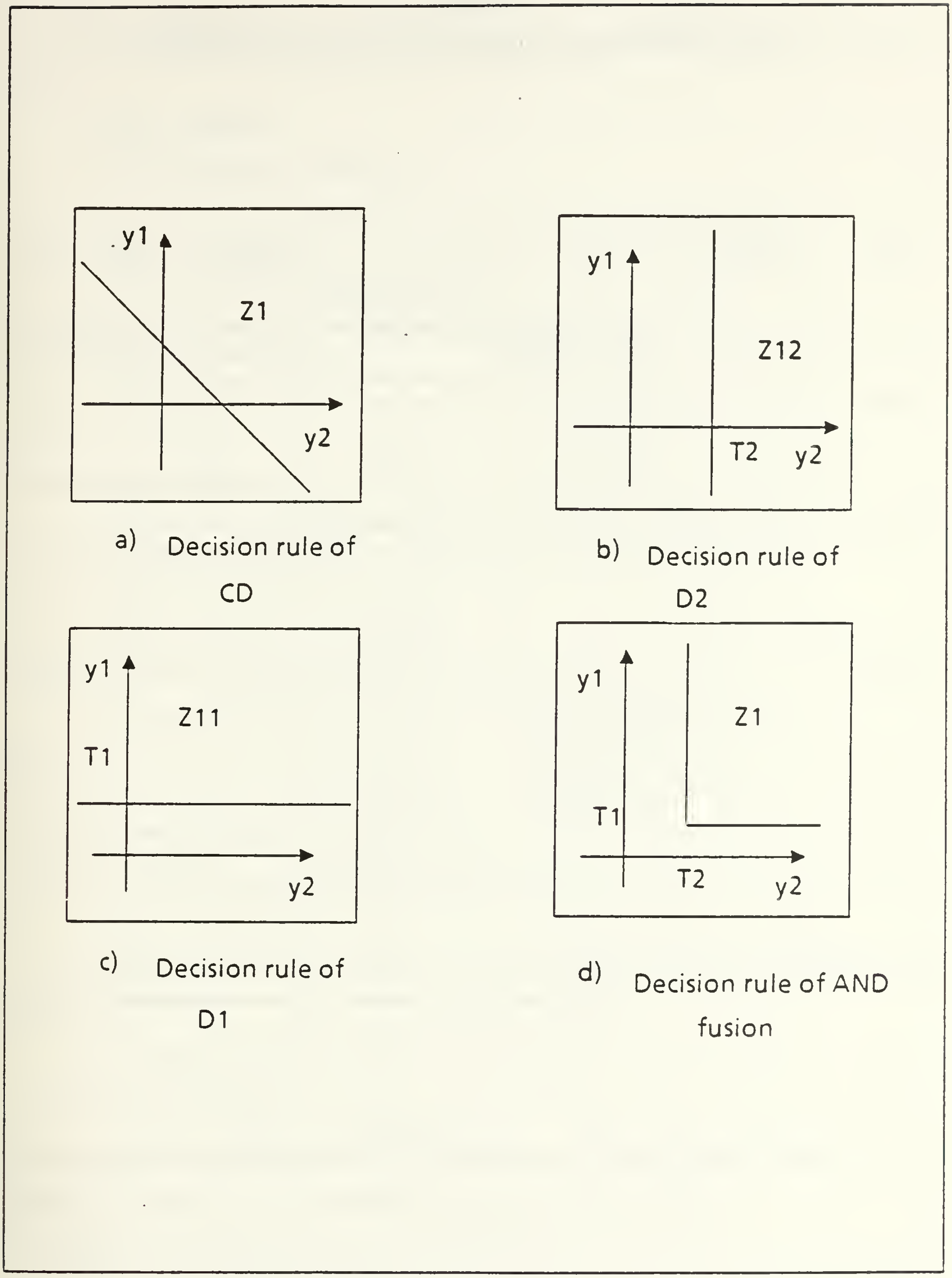

Figure 2.1 Decision Rules.

TABLE 1

EXHAUSTIVE FUSION LIST OF TWO DECISIONS

Rule #	Z_0	Z_1	Comments
1	Φ	(0,0),(0,1),(1,0),(1,1)	$C_{01} \rightarrow \infty$
2	(0,0),(1,0),(0,1),(1,1)	Φ	$C_{10} \rightarrow \infty$
3	(0,0),(1,0)	(0,1),(1,1)	$U_o = U_2$
4	(0,0),(0,1)	(1,0),(1,1)	$U_o = U_1$
5	(0,1)	(0,0),(1,0),(1,1)	
6	(1,0)	(0,0),(0,1),(1,1)	
7	(1,1)	(0,0),(1,0),(0,1)	
8	(0,1),(1,0)	(0,0),(1,1)	
9	(0,1),(1,1)	(0,0),(1,0)	
10	(1,0),(1,1)	(0,0),(0,1)	
11	(0,0),(1,1)	(0,1),(1,0)	
12	(0,1),(1,0),(1,1)	(0,0)	
13	(0,0),(0,1),(1,1)	(1,0)	
14	(0,0),(1,0),(1,1)	(0,1)	
15	(0,0),(0,1),(1,0)	(1,1)	AND
16	(0,0)	(0,1),(1,0),(1,1)	OR

We will not consider these ten fusion rules further. The remaining two decision rules are the AND fusion and the OR fusion. Let us now consider optimizing each of them.

1. ' AND ' Fusion

In AND fusion u_o is given by:

$$u_o = u_1 u_2. \tag{2.8}$$

The individual rules are given by assigning y_i to Z_1 if

$$y_i \geq T_i \quad , i=1,2. \tag{2.9}$$

Otherwise they assign it to Z_0.

The probability of detection P_d(AND) and probability of false alarm P_f(AND) of the fusion center are given by:

$$P_d\,(\text{AND}) = \int_{T_1}^{\infty} \int_{T_2}^{\infty} f(y_1, y_2 / H_1)\, dy_1\, dy_2 \tag{2.10}$$

and

$$P_f\,(\text{AND}) = \int_{T_1}^{\infty} \int_{T_2}^{\infty} f(y_1, y_2 / H_0)\, dy_1\, dy_2 \tag{2.11}$$

It has been shown in Chapter I that, to within positive multiplicative and additive constants, the average decision cost is given by

$$R = 1 + C\, P_f - P_d. \tag{2.12}$$

Substituting for P_d and P_f in (2.12) from (2.10) and (2.11) expresses R(AND) as a function of T_1 and T_2. The necessary conditions for optimality are

$$\partial R / \partial T_1 = 0 \text{ and } \partial R / \partial T_2 = 0 , \tag{2.13}$$

which can be written in the forms:

$$C \int_{T_2}^{\infty} f(T_1,y_2/H_0)\,dy_2 = \int_{T_2}^{\infty} f(T_1,y_2/H_1)\,dy_2 \qquad (2.14)$$

and

$$C \int_{T_1}^{\infty} f(y_1,T_2/H_0)\,dy_1 = \int_{T_1}^{\infty} f(y_1,T_2/H_1)\,dy_1. \qquad (2.15)$$

Applying Bayes rule and rearranging terms, one can write (2.14) and (2.15) as follows:

$$\Lambda_1(T_1) = C\,\frac{\int_{T_2}^{\infty} f(y_2/T_1,H_0)\,dy_2}{\int_{T_2}^{\infty} f(y_2/T_1,H_1)\,dy_2} \qquad (2.16)$$

and

$$\Lambda_2(T_2) = C\,\frac{\int_{T_1}^{\infty} f(y_1/T_2,H_0)\,dy_1}{\int_{T_1}^{\infty} f(y_1/T_2,H_1)\,dy_1}. \qquad (2.17)$$

To insure minima the Hessian matrix of R with respect to T_1 and T_2

(2.18)

$$H = \begin{bmatrix} \partial^2 R/\partial T_1^2 & \partial^2 R/\partial T_1\,\partial T_2 \\ \partial^2 R/\partial T_2\,\partial T_1 & \partial^2 R/\partial T_2^2 \end{bmatrix} \qquad (2.19)$$

must be positive definite. Optimum threshold settings T_1 and T_2 are the solution of (2.16) and (2.17) that corresponds to the global minima, so all possible solutions of (2.16) and (2.17) must be tried. The coupling between (2.16) and (2.17) to determine the thresholds represents the cooperation that can occur between the two local detectors to mininize the overall decision cost.

2. 'OR' Fusion

The decision of the OR fusion is given by

$$u_o = u_1 + u_2 - u_1 u_2. \tag{2.20}$$

The probability of detection P_d (OR) and probability of false alarm P_f (OR) are given by

$$P_d\,(OR) = 1 - \int_{-\infty}^{T_1} \int_{-\infty}^{T_2} f(y_1, y_2 / H_1)\, dy_1\, dy_2 \tag{2.21}$$

and

$$P_f\,(OR) = 1 - \int_{-\infty}^{T_1} \int_{-\infty}^{T_2} f(y_1, y_2 / H_0)\, dy_1\, dy_2 \tag{2.22}$$

while the necessary conditions for optimality are

$$\Lambda\,(T_1) = C \frac{\int_{-\infty}^{T_2} f(y_2 / T_1, H_0)\, dy_2}{\int_{-\infty}^{T_2} f(y_2 / T_1, H_1)\, dy_2} \tag{2.23}$$

and

$$\Lambda\,(T_2) = C \frac{\int_{-\infty}^{T_1} f(y_1 / T_2, H_0)\, dy_1}{\int_{-\infty}^{T_1} f(y_1 / T_2, H_1)\, dy_1}\,. \tag{2.24}$$

Again the Hessian matrix must be positive definite.

3. **Solution of the Nonlinear Threshold Equations**

The pair of coupled equations (2.16), (2.17) for the AND fusion and (2.23) and (2.24) for the OR fusion can be solved using Max's technique [18]. The technique is summarized as follow: pick a value of T_1 and calculate T_2 from (2.16) or (2.23). If the calculated value of T_2 does not agree with that value calculated from (2.17) or (2.24) then T_1 must be chosen again. This approach is time consuming. Another approach is the method of successive substitution [19]. We first put the two equations in the form

$$T_{1_{k+1}} = G(T_{1_k}, T_{2_k}) \ , \ T_{2_{k+1}} = F(T_{1_{k+1}}, T_{2_k}) \tag{2.25}$$

then start with a reasonable guess for $(T_1)_0$ and $(T_2)_0$. A suitable initial guess is the locally optimum solutions, i.e. the thresholds that would optimize the detection if each sensor works alone. These will be denoted by T_{1lo} and T_{2lo}. For known signals in gaussian noise these are

$$(T_i)_0 = a_i/2 + \log(C)/a_i \ . \tag{2.26}$$

4. **Numerical Results**

We have solved the threshold equations for both fusion rules for $a_1 = 1.7$ and $a_2 = 2.3$ for several values of ρ and C.

To compare AND and OR fusion, define **K** as the ratio of the AND cost to the OR cost.

$$K = \frac{1 + C\,P_f(AND) - P_d(AND)}{1 + C\,P_f(OR) - P_d(OR)} \tag{2.27}$$

We have also computed the Receiver Operating Characteristic[4] (ROC) curves of classical communication theory [20] for each fusion rule.

Figure 2.2 shows the ratio **K** as a function of C for $\rho = 0, 0.2, 0.4$. The figure shows that AND fusion is optimum for $C \geq 1$ and OR fusion is optimum for lower values of C. The same is clear from Figure 2.3; ROC curves of AND fusion are above those of OR fusion for $C \geq 1$ and lower otherwise. The performance difference becomes smaller as the correlation coefficient increases. Also the figures show that the performance degrades for both fusion rules as ρ tends to one. This is in sharp contrast to CD which has perfect detection for $\rho = 1$.

[4] P_d as a function of P_f

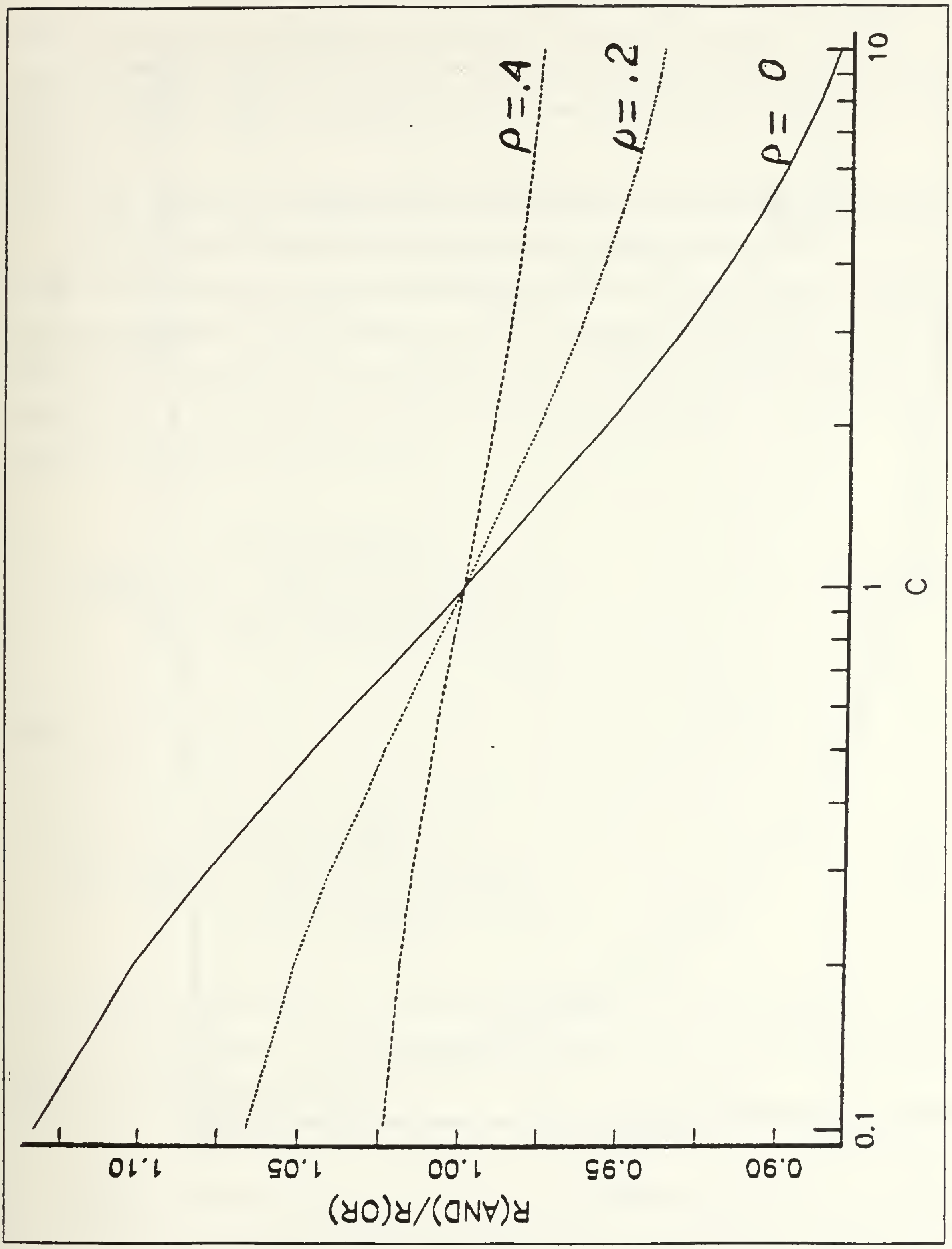

Figure 2.2 Ratio of Costs of AND and OR Fusion Rules
$a_1 = 1.7$, $a_2 = 2.3$.

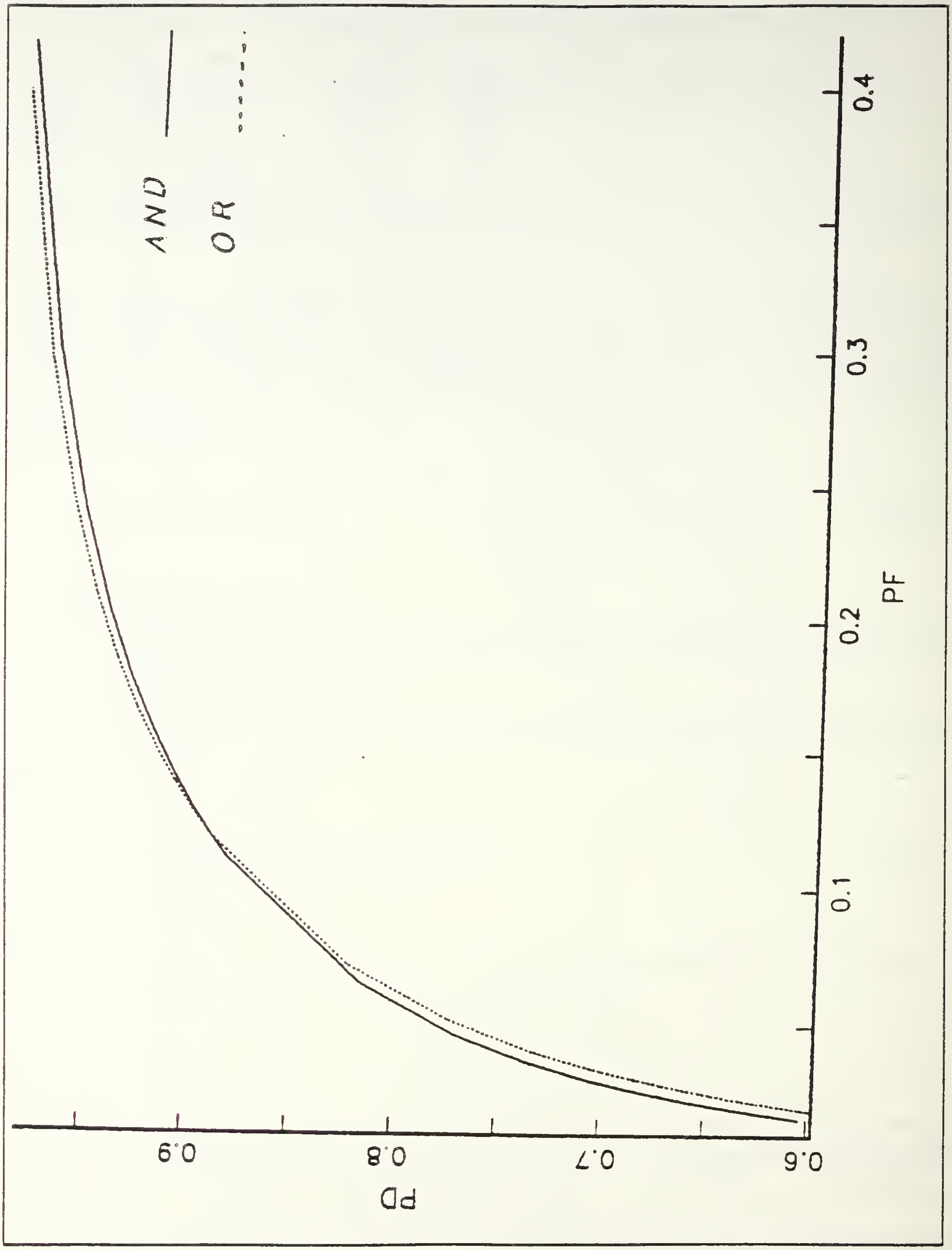

Figure 2.3 ROC Curves of AND and OR Fusion Rules
$a_1 = 1.7, a_2 = 2.3$.

The same effects can be concluded from Figure 2.4 and Figure 2.5. Figure 2.4 shows the ratio K as a function of C for $a_1 = 1$ and $a_2 = 2$ and for $\rho = 0, 0.25, 0.5$. Figure 2.5 shows the ROC curves for both fusion rules for the same case. The figure shows that AND fusion is optimum for $C \geq 1$ and OR fusion is optimum for lower values of C.

C. THE EFFECT OF CORRELATION BETWEEN SENSOR OBSERVATIONS

So far we have answered the question of the optimum fusion rule. For $C \geq 1$ AND fusion is optimum. Let us now examine the effect of the correlation coefficient ρ on the performance of AND fusion for $C \geq 1$ (its range of superiority). We assume without loss of generality that a_2 is greater than a_1. The two necessary conditions for optimality of AND fusion are (2.16) and (2.17). For the problem of known signal in gaussian noise these can be written as:

$$\Lambda_1 (T_1) = C \frac{\operatorname{erfc}\left\{ \frac{T_2 - \rho T_1}{\sqrt{(1-\rho^2)}} \right\}}{\operatorname{erfc}\left\{ \frac{T_2 - a_2 - \rho (T_1 - a_1)}{\sqrt{(1-\rho^2)}} \right\}} \tag{2.28}$$

and

$$\Lambda_2 (T_2) = C \frac{\operatorname{erfc}\left\{ \frac{T_1 - \rho T_2}{\sqrt{(1-\rho^2)}} \right\}}{\operatorname{erfc}\left\{ \frac{T_1 - a_1 - \rho (T_2 - a_2)}{\sqrt{(1-\rho^2)}} \right\}} . \tag{2.29}$$

Notice that C appears only as a multiplicative constant in the two equations. The role of ρ is not that obvious. Examining the two equations leads to the following insights about the role of ρ:

1. $T_1 = -\infty$ and $T_2 = T_{21o}$ is a solution. This corresponds to the decision rule of D_2.
2. $T_2 = -\infty$ and $T_1 = T_{11o}$ is a solution. This corresponds to the decision rule of D_1.
3. If a_2 is greater than a_1 we expect the performance of D_2 alone to be better than that of D_1 alone and that of the selfish decision rule in which each detector tries to minimize its own detection cost, not the system decision cost, by using T_{11o}, T_{21o}.

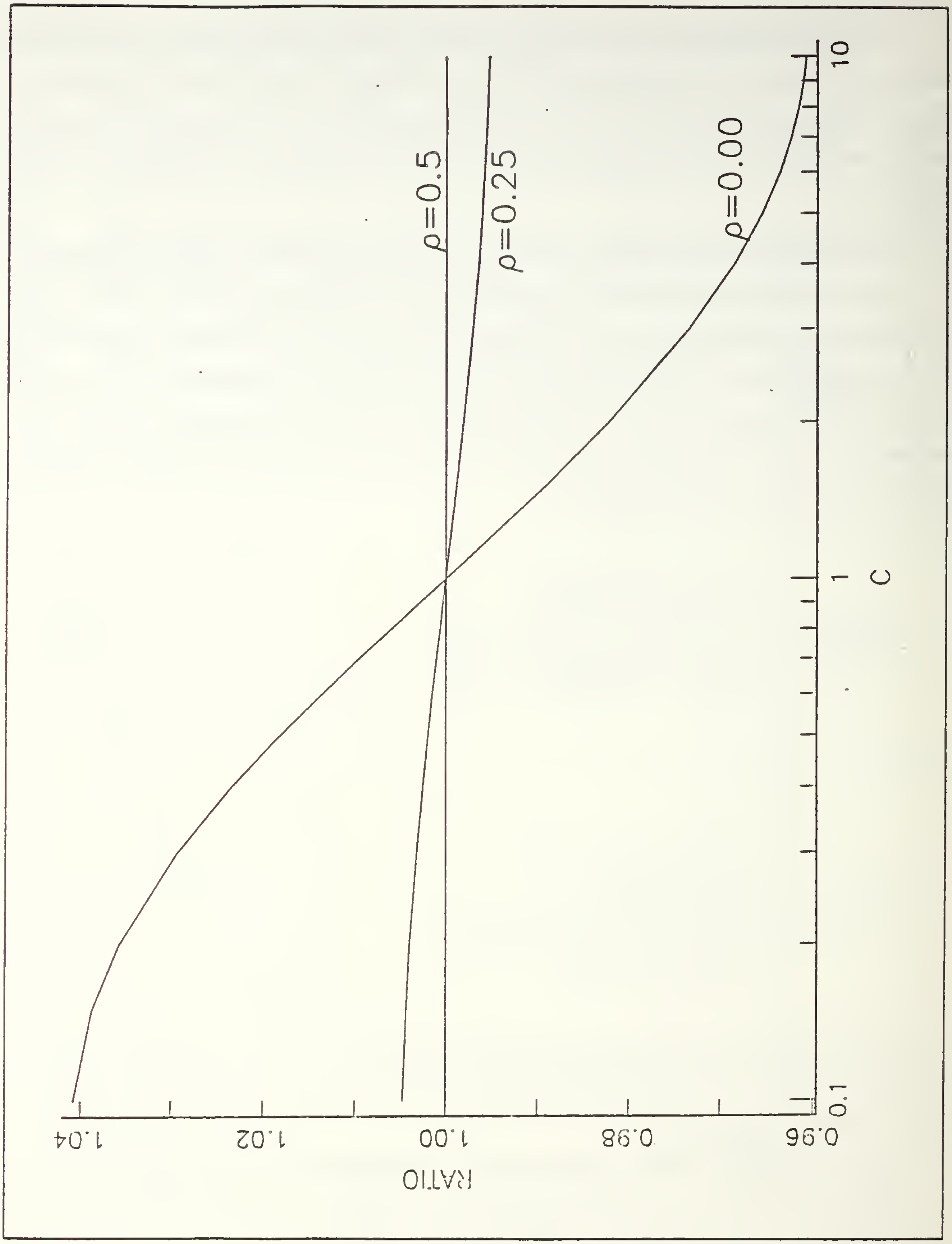

Figure 2.4 Ratio of Costs of AND and OR Fusion Rules
$a_1 = 1$, $a_2 = 2$.

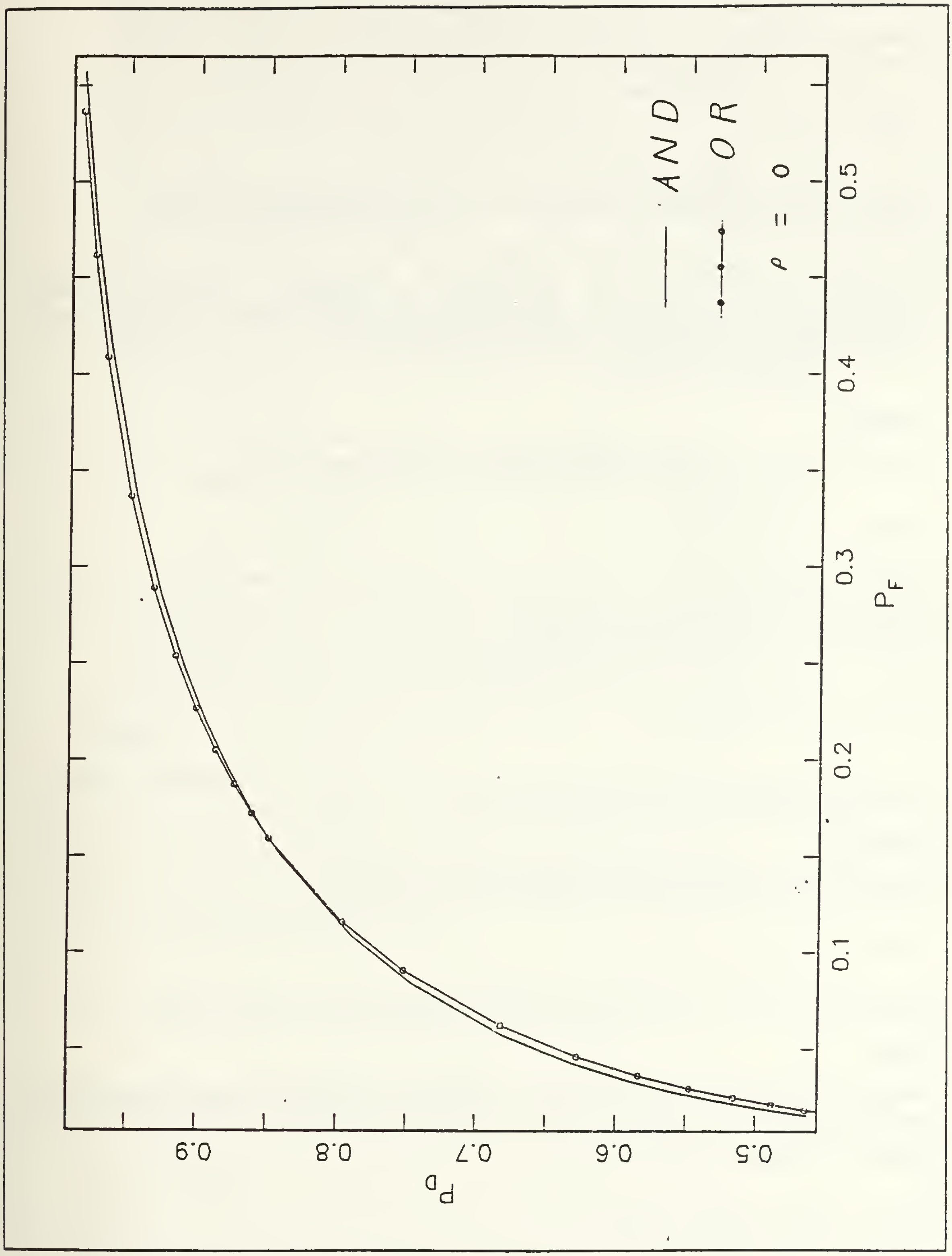

Figure 2.5 ROC for AND and OR Fusion Rules
$a_1 = 1, a_2 = 2$.

We now prove three lemmas concerning these equations.

1. **Lemma 1.**

For $\rho \leq a_1/a_2$,

$$T_1 \leq T_{11o}$$

and

$$T_2 \leq T_{21o}$$

where T_{i1o} is the optimum threshold of the ith detector operating alone.

Proof:

Since the argument of the complement of the error function in each denominator is less than the argument in the corresponding numerator, the fraction is always less than one. This implies that

$$\Lambda_i(T_i) \leq C, i=1,2.$$

2. **Lemma 2**

For $\rho = a_1/a_2$, the only possible solution of (2.28) and (2.29) is:

$$T_2 = T_{21o}$$

and

$$T_1 = -\infty.$$

Proof:

For $\rho = a_1/a_2$ equation (2.29) becomes

$$\Lambda_2(T_2) = C = \Lambda_2(T_{21o}). \tag{2.30}$$

The corresponding value of T_1 is $T_1 = -\infty$.

3. **Lemma 3**

For $\rho \geq a_1/a_2$ the optimum solution for T_1 and T_2 is:

$$T_1 = -\infty$$

and

$$T_2 = T_{21o}.$$

This means that the decision of the optimum AND fusion is that of D_2.

Proof:

Recall that the CD threshold line divides the observation space into two decision regions. For positive signals the following inequality is satisfied in the region to the right of the CD line:

$$C f(y_1, y_2/H_0) < f(y_1, y_2/H_1). \tag{2.31}$$

The converse of this inequality is true in the left region. The decision region Z_1 of any other decision rule contains areas from the right and from the left of the CD line. Areas to the right will have a negative contribution to the decision cost while areas to the left will have positive contributions. Now assume that T_1 and T_2, where both are finite, satisfy the necessary condition (2.28) and (2.29). We shall prove that they cannot correspond to the optimum solution. The finite point (T_1 ,T_2) either lies to the left or to the right of the CD threshold line as shown in Figure 2.6 a and b respectively. In Figure 2.6 a the intersection of the CD line with the line $y_1 = T_1$ is a better solution since it excludes an area in which C $f(y_1, y_2/H_0)$ is greater than $f(y_1, y_2/H_1)$. A better solution than this has the same T_2 but with $T_1 = -\infty$ since the added area has negative contribution to the cost. In Figure 2.6 b, $T_1 = -\infty$ and T'_2 is a better solution than T_1 and T_2, since the added area has a negative contribution to the cost. In both cases $T_1 = -\infty$ is the optimum solution and the corresponding optimum value of T_2 is T_{21o}.

As a result of the above three lemmas it is clear that

1. Any solution of the necessary conditions must satisfy

   $T_1 \leq T_{11o}$ and

   $T_2 \leq T_{21o}$

2. The performance of the AND fusion saturates to that of D_2 alone for $\rho \geq a_1/a_2$. We might recall that the threshold line of the CD system changes slope at that value of ρ. We will refer to this value of ρ by ρ_{cr}. This result is in contradiction with Lauer and Sandell's results [4] which shows performance continuing to degrade with increasing ρ for

   $\rho \geq \rho_{cr}$.

Limiting behavior for = ρ-1.

For $\rho = -1$ the joint probability density function $f(y_1, y_2/H_0)$ has values only on the line $y_1 = -y_2$. So any threshold values T_1 and T_2 such that $T_1 = -T_2$ will produce AND fusion with zero probability of false alarm. This can be visualized from Figure 2.7 . Consequently, P_d will be given by

$$P_d = 0.5 \operatorname{erfc}\left\{ T_2 - a_2 \right\} - 0.5 \operatorname{erfc}\left\{ T_2 + a_1 \right\}. \qquad (2.32)$$

Maximizing P_d with respect to T_2 yields

$$T_2 = (a_2 - a_1)/2 . \qquad (2.33)$$

For the special case of equal SNR sensors, $T_2 = 0$.

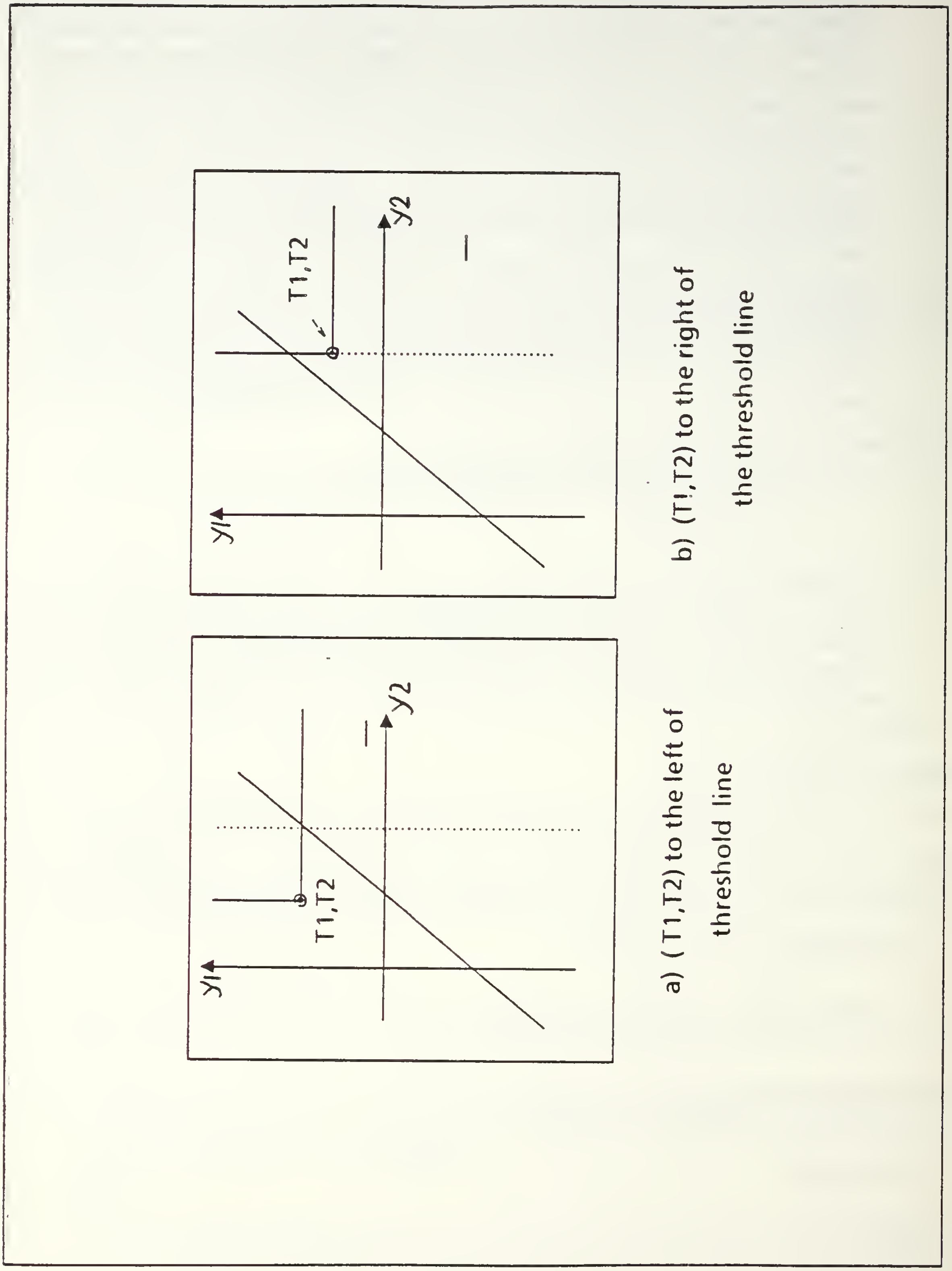

Figure 2.6 Optimum T_1 and T_2 for $\rho \geq a_1/a_2$.

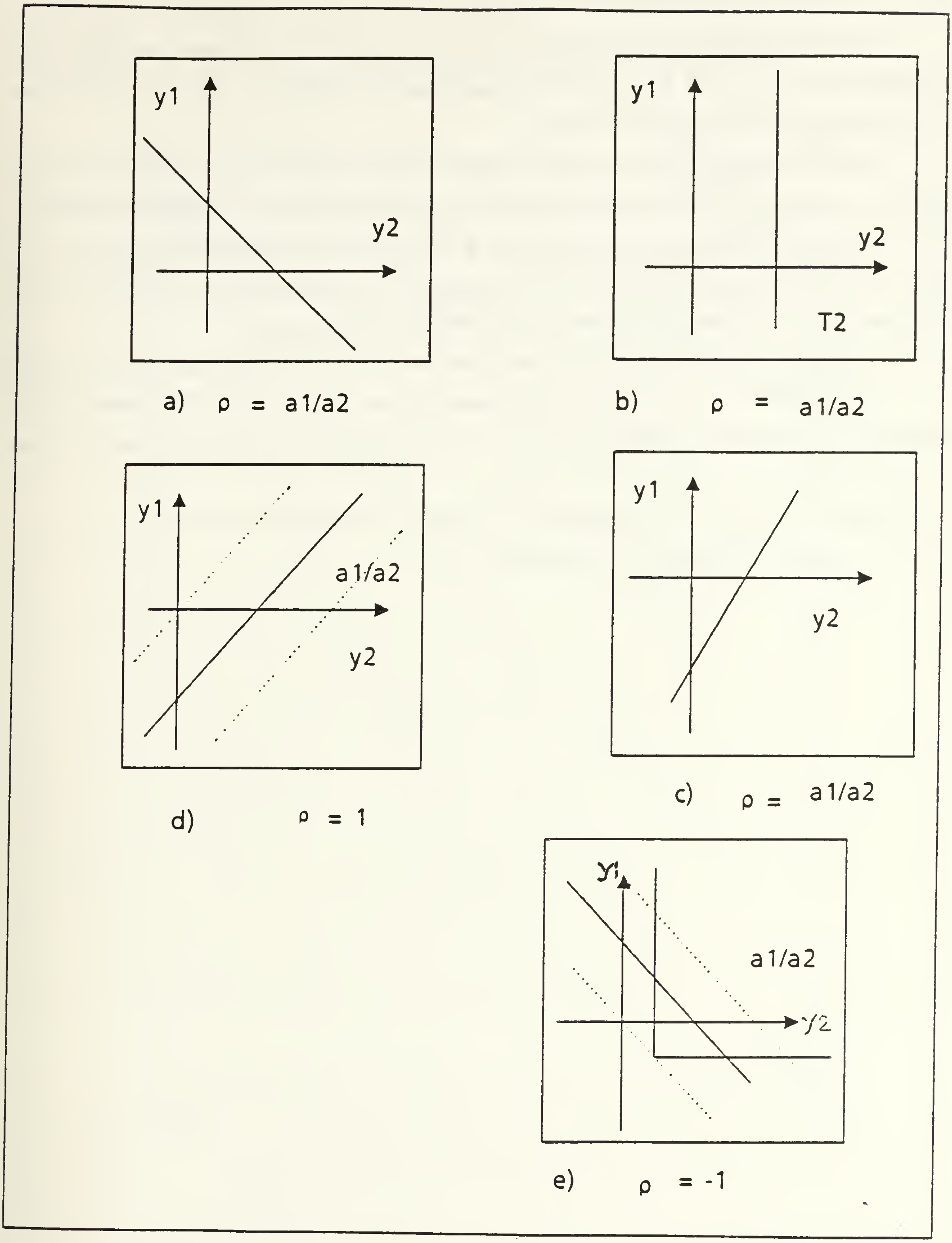

Figure 2.7 Centralized Threshold Line For Different Values of ρ.

D. NUMERICAL RESULTS

The average decision costs vs ρ for $a_1 = a_2 = 2$, and $C = 1$ are shown in Figure 2.8. Threshold values T_1 and T_2 vs ρ for the same case are shown in Figure 2.9. Figures 2.10 and 2.11 show the same for $C = 10$.

These four figures for the case of equal signal-to-noise ratio show that the two detectors cooperate with each other using the same decision rule (equal thresholds). Their threshold is an increasing function of ρ. The limit of this threshold as $\rho \rightarrow -1$ is zero. This behavior agrees with (2.33). The limit of the threshold as $\rho \rightarrow 1$ is T_{1o}. This is because for $\rho \rightarrow 1$ the two systems have identical observations.

The detection cost curves show that the cost is an increasing function of ρ. The curve of the AND fusion has the same shape as the curve of the CD system. Both systems attain their best performance at $\rho = -1$. They have the same worst performance for $\rho = 1$.

Figures 2.12 and 2.13 represent the case of unequal SNR sensors for $C = 1$. Figures 2.14 and 2.15 show the same for $C = 10$.

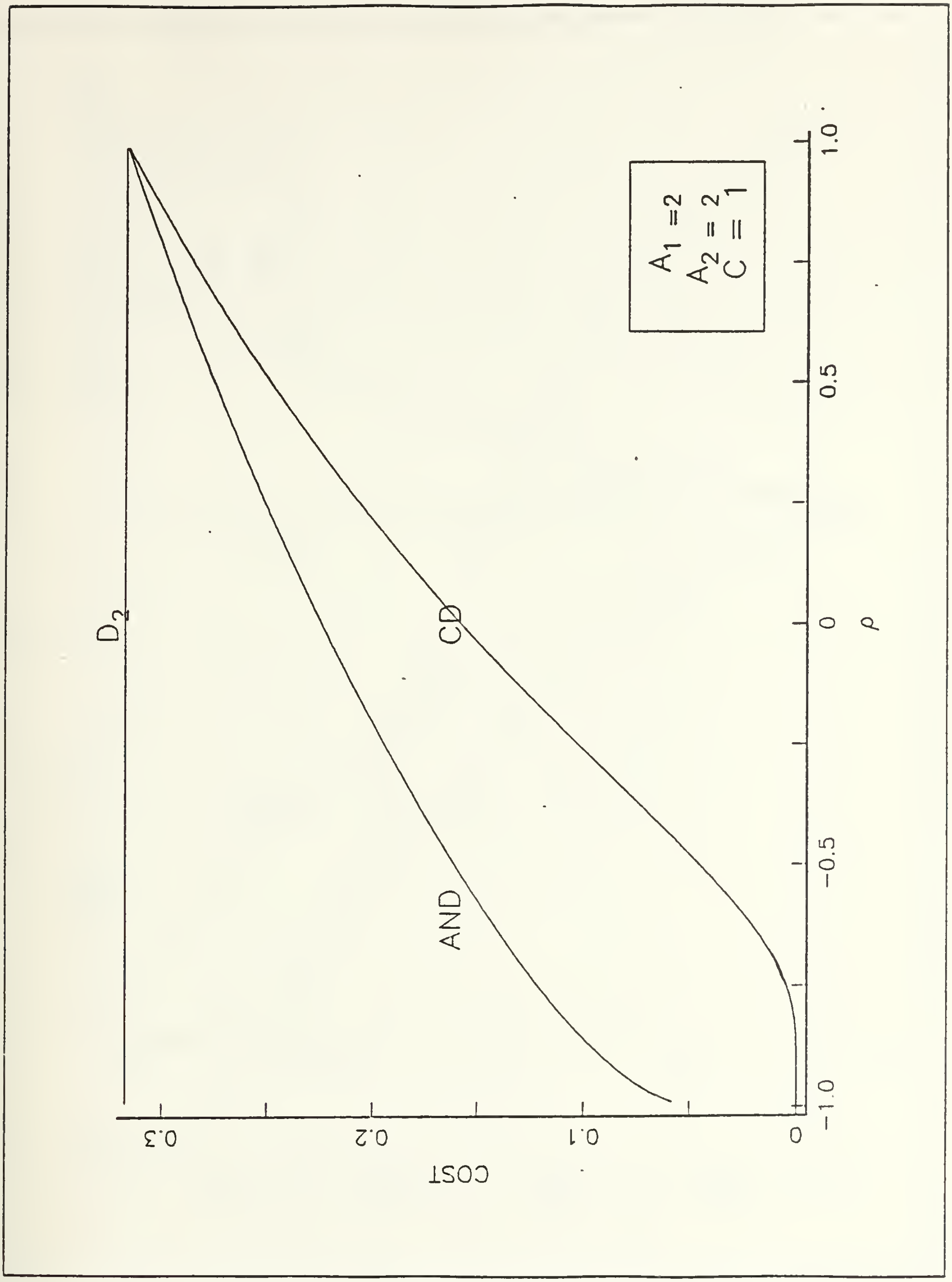

Figure 2.8 Average Decision Costs for Equal SNRs, C = 1.

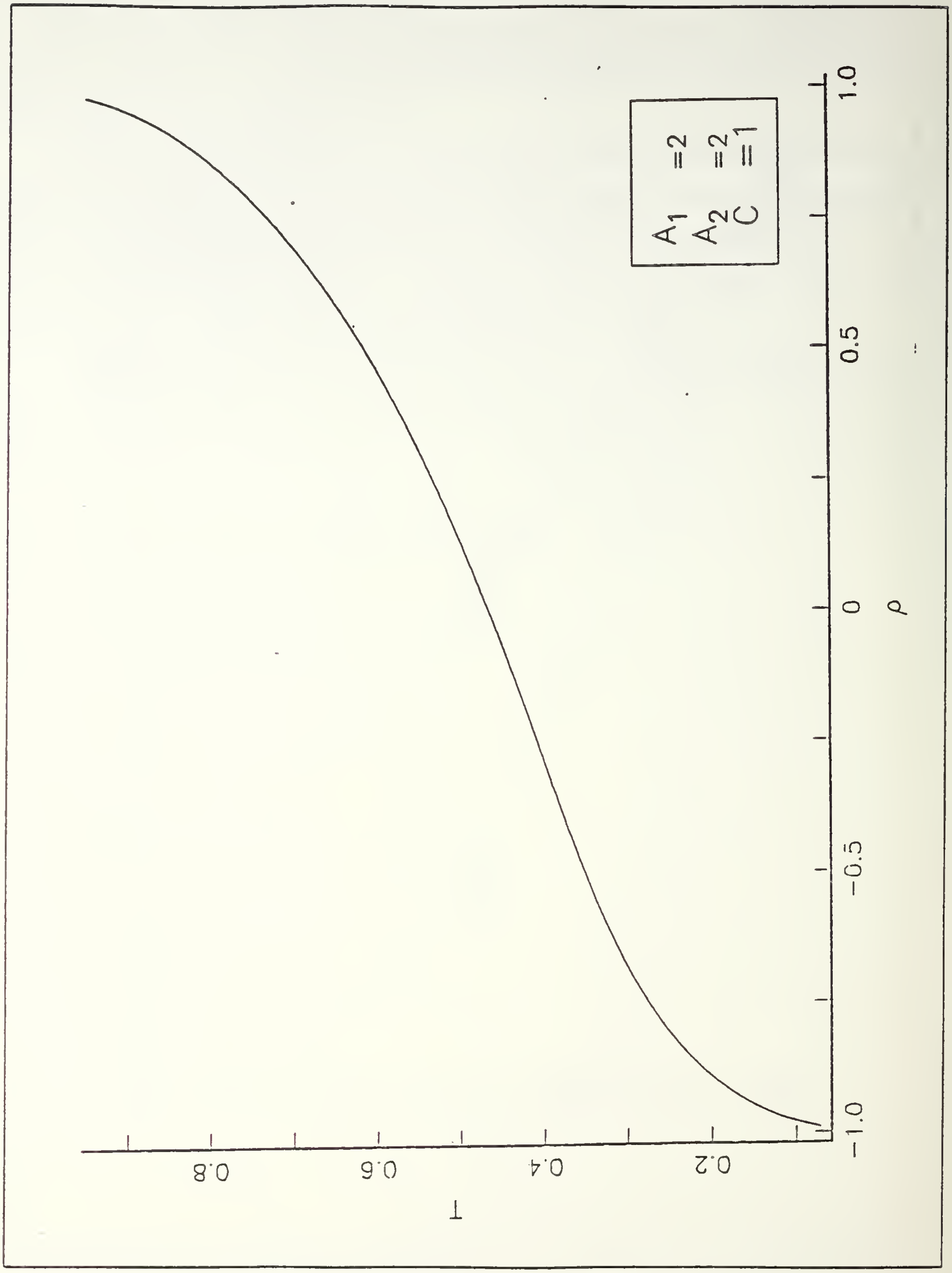

Figure 2.9 Threshold Value for Equal SNRs, C = 1.

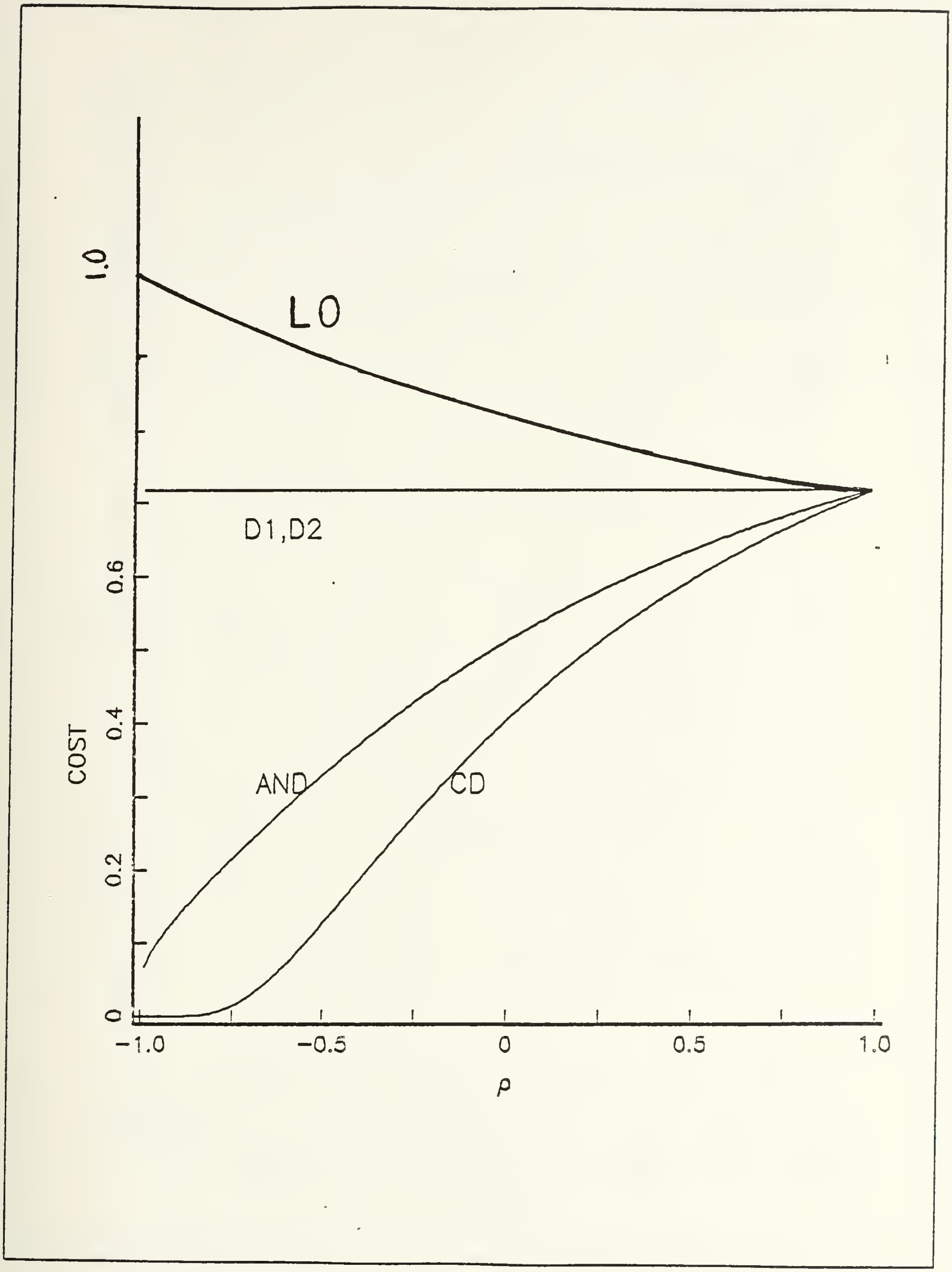

Figure 2.10 Average Decision Costs for Equal SNRs,C = 10.

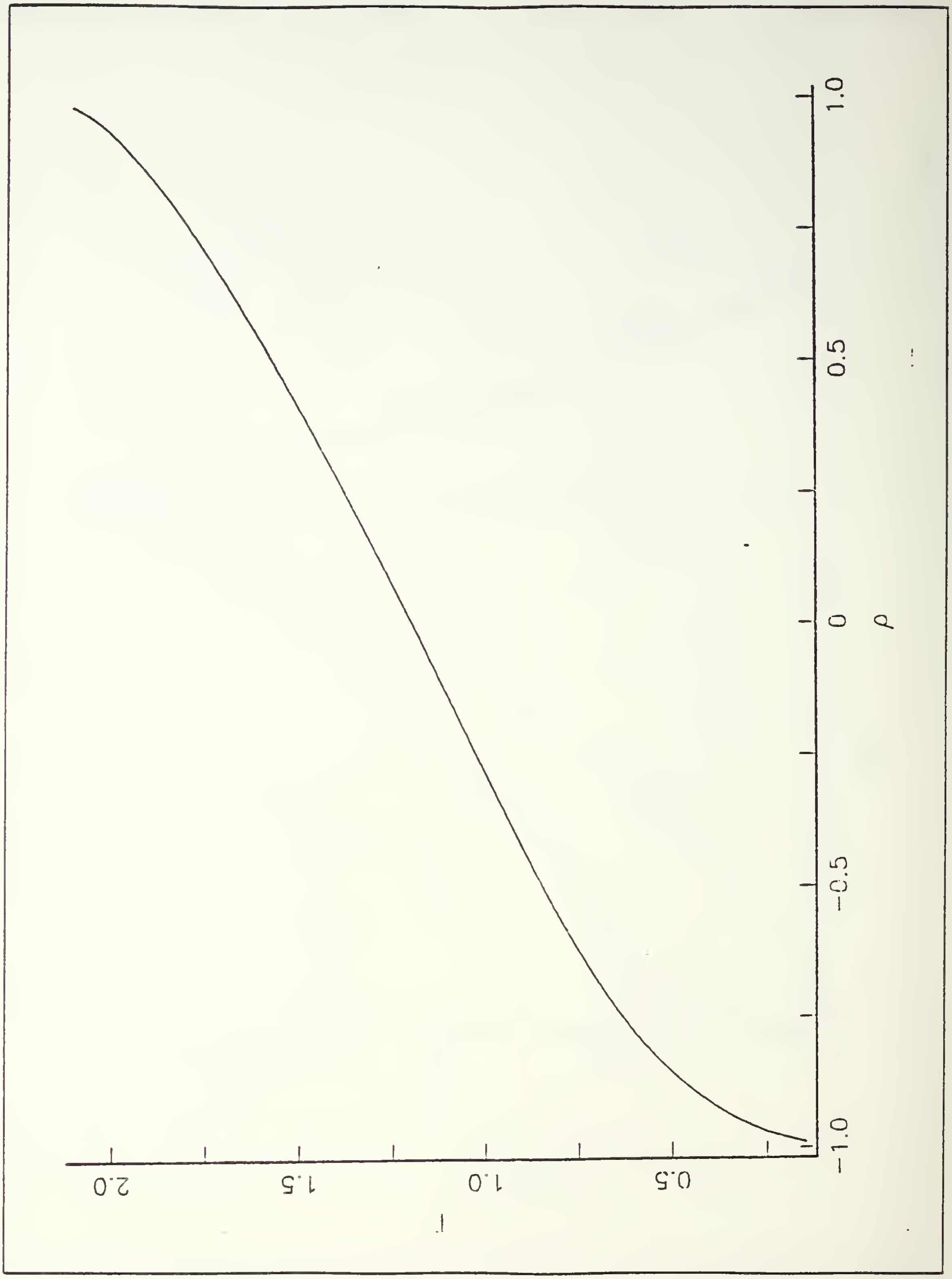

Figure 2.11 Threshold Value for Equal SNRs, C = 10.

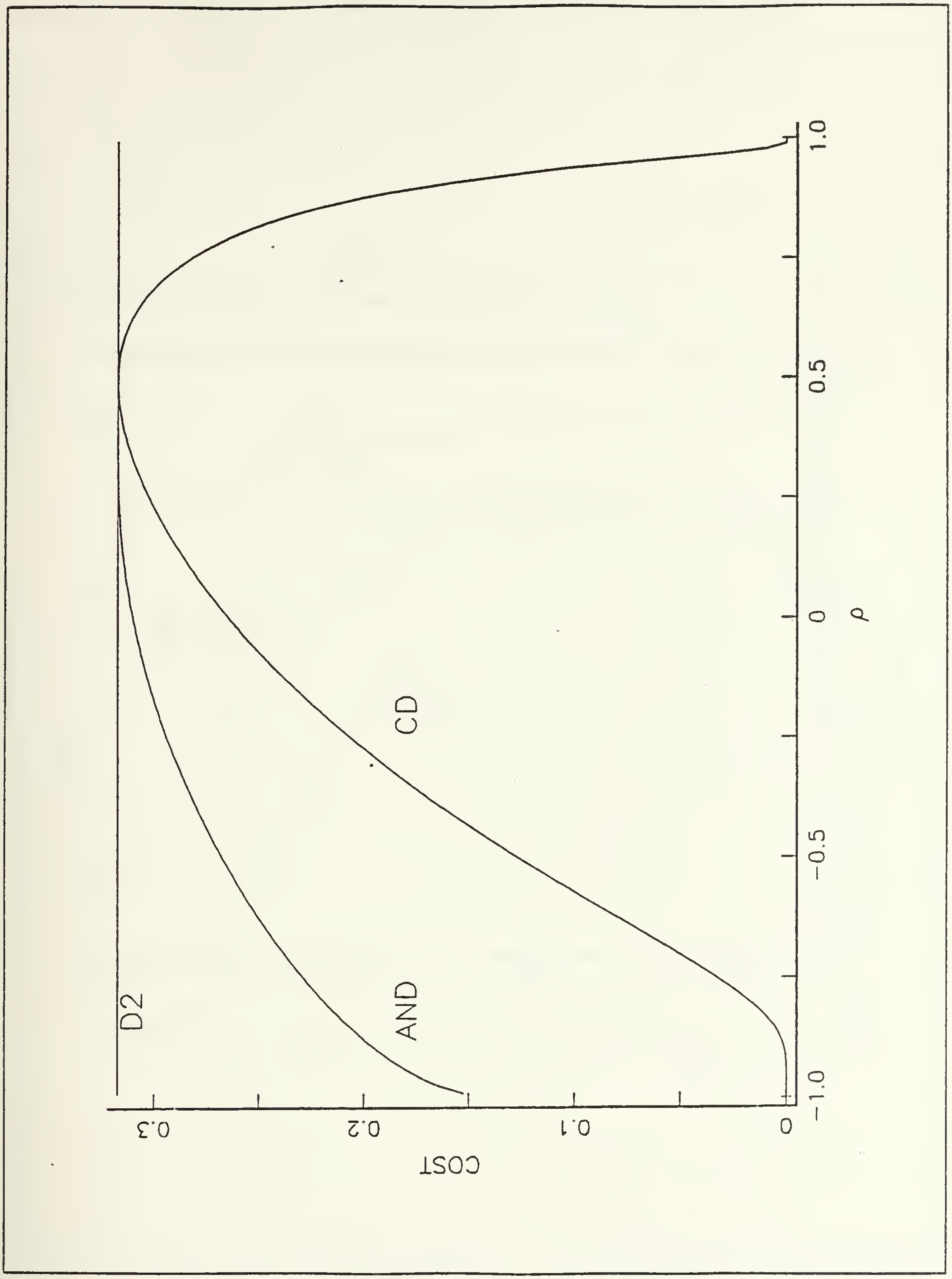

Figure 2.12 Average Decision Costs for Unequal SNRs, C = 1.

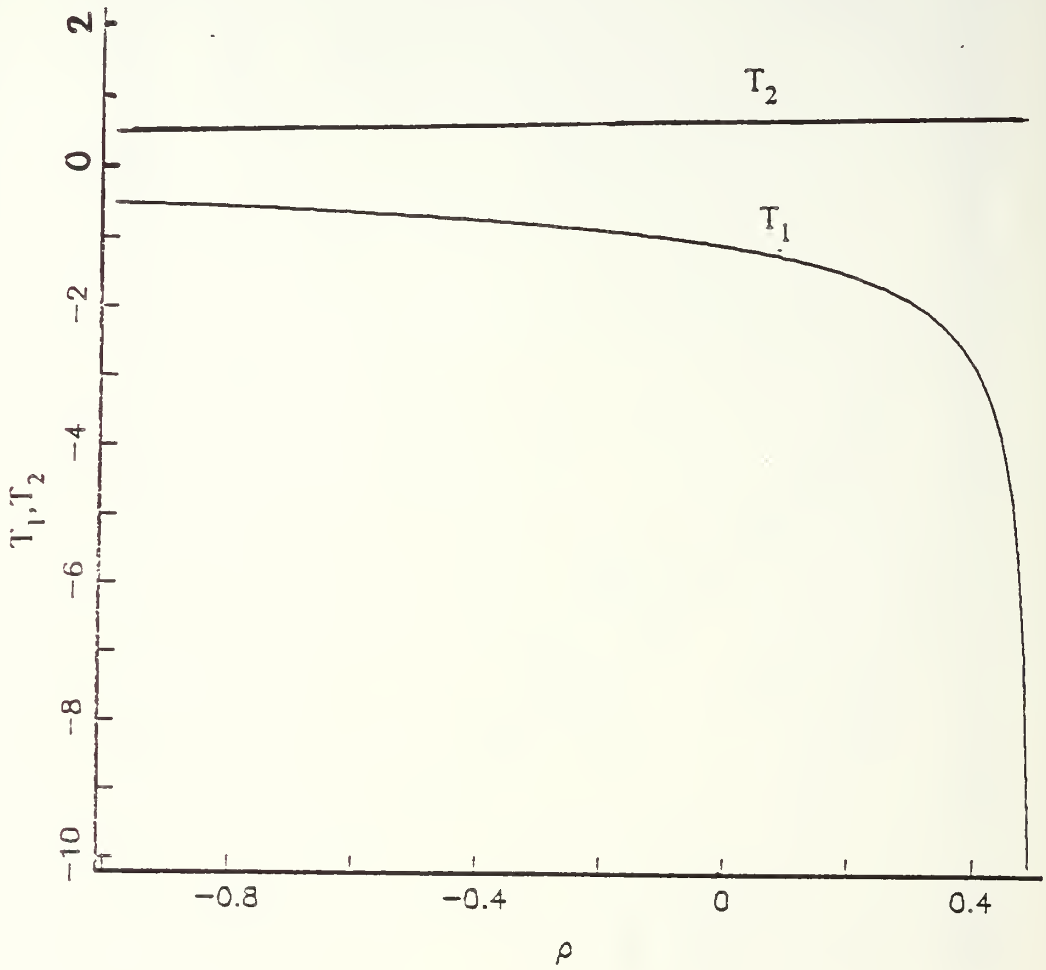

Figure 2.13 Threshold Value for Unequal SNRs, C = 1.

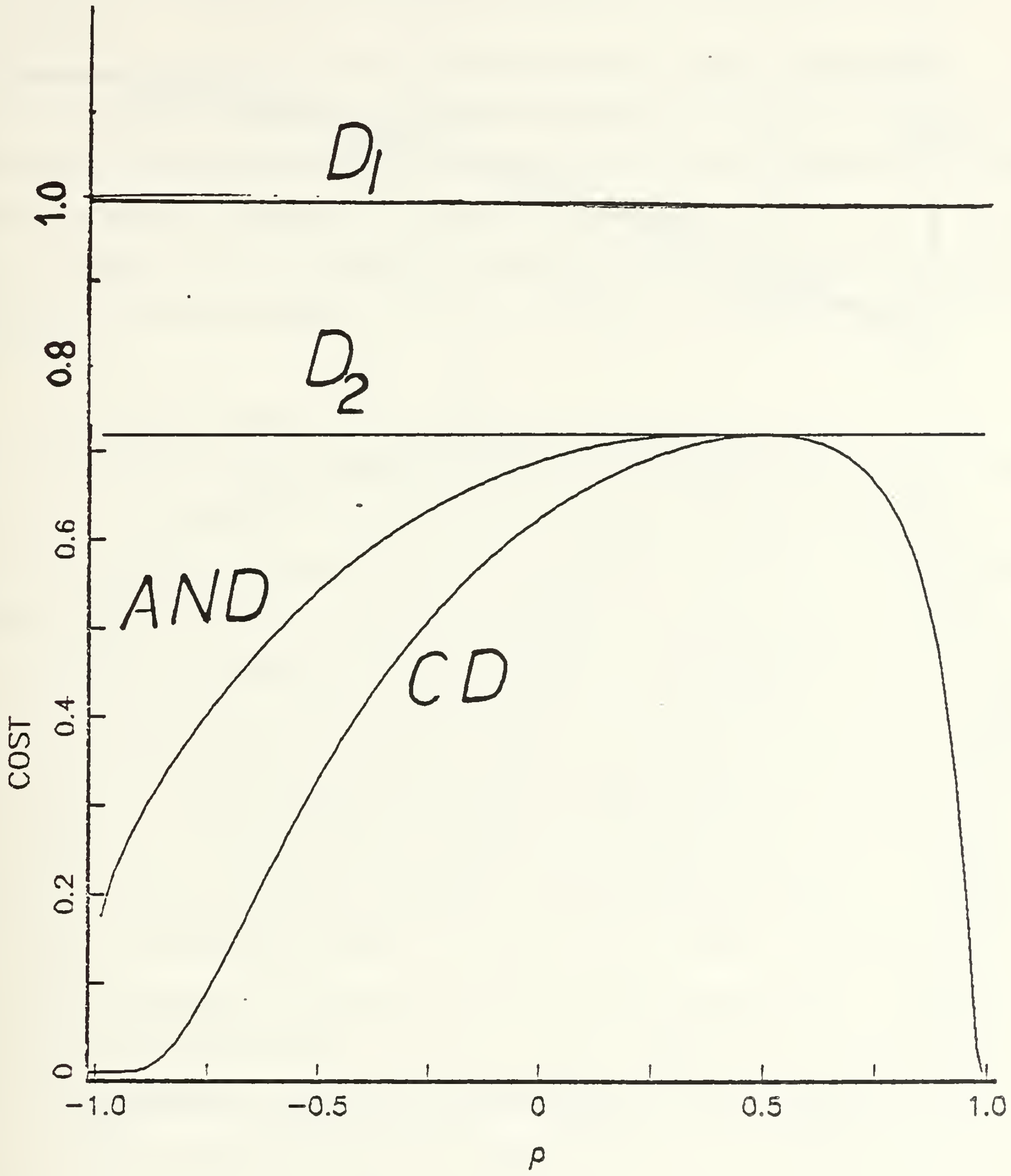

Figure 2.14 Average Decision Costs for Unequal SNRs, C = 10.

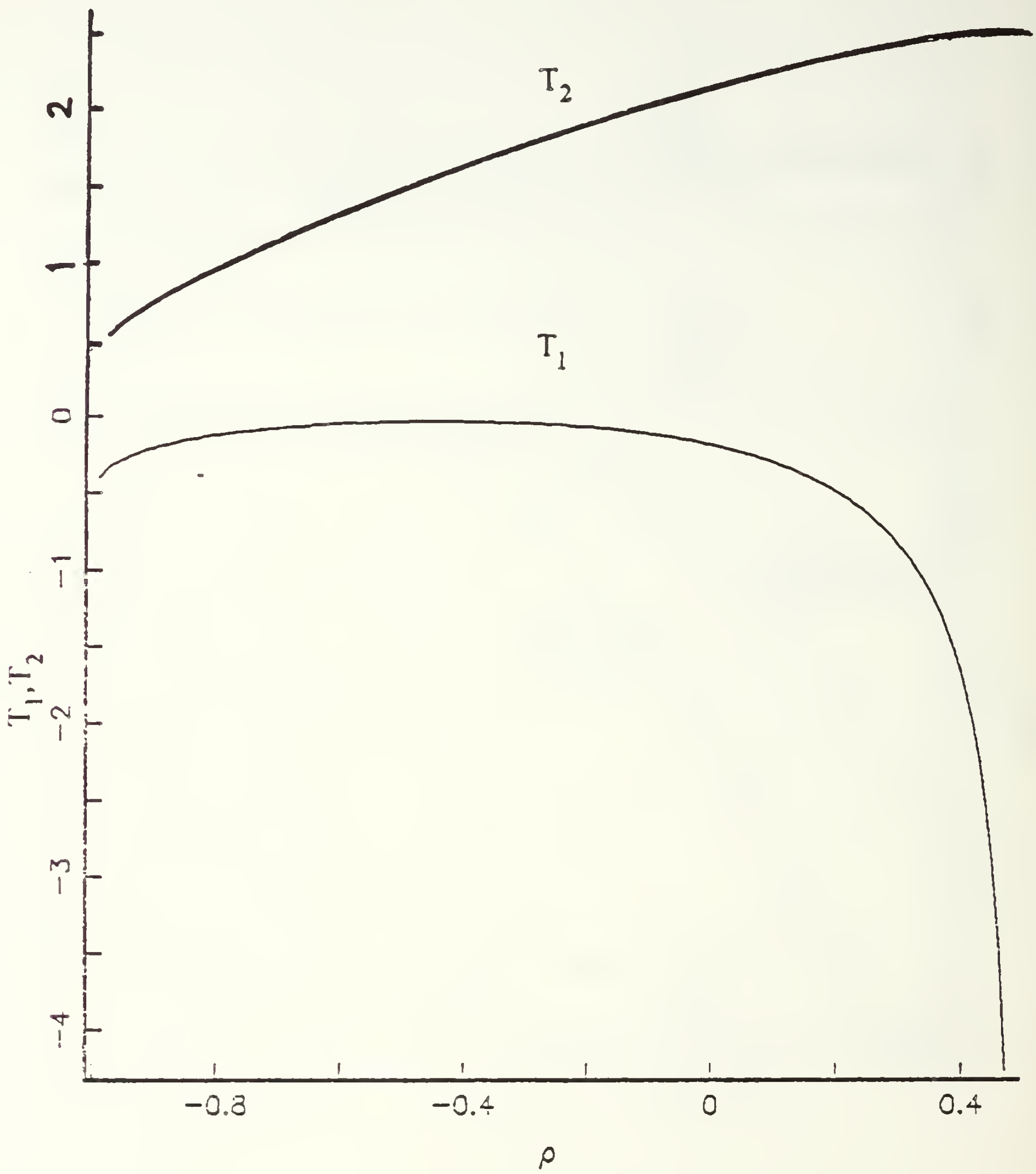

Figure 2.15 Threshold Value for Unequal SNRs, C = 10.

These four figures for the case of unbalanced sensors show that, the two detectors cooperate using different thresholds. The threshold of the higher signal-to-noise detector is an increasing function of ρ while the other threshold is a decreasing function of ρ.

The cost curves show that the fusion rule has its best performance at $\rho = -1$. Both DD and CD have their worst performance at $\rho = \rho_{cr}$. For $\rho \geq \rho_{cr}$ the performance of the optimum fusion rule is the same as the detector of the higher signal-to-noise ratio. Recall that CD system has perfect detection for $\rho = 1$ when the SNR's are unequal. As C increases the average cost of each system increases. This can be explained from the expression for R in which the probability of false alarm is weighted by C.

E. DISCUSSION AND CONCLUSIONS

We have shown that the optimum fusion rule is determined by the ratio of costs and the apriori probabilities. For equal error costs AND and OR fusion rules are equivalent. This is not surprising since each system turns out to be the minimum probability of error detector; thresholds are adjusted such that $1 - P_d = P_f$. It might also be noted that the optimality of the fusion rule is independent of the correlation coefficient and the signal-to-noise ratio in this case. We also note that the detection cost of the optimum fusion rule has its minimum value at $\rho = -1$. It has its maximum value at $\rho = a_1/a_2$. The performance saturates at the cost of decision of the detector of higher SNR. In the interval ($\rho \in [a_1/a_2, 1]$), the optimum fusion rule ignores the decision of the detector of lower SNR. As a good dynamical example that agrees with this result is the switched diversity combiner [21] in fading environments and its centralized counterpart, the maximum ratio diversity combiner [22]. Recall that for unequal SNRs the performance of the CD system improves in this interval and has perfect detection for $\rho = 1$. Also it is important to note that the optimum thresholds of the individual observers are not the same as if they were operating independently, but must be determined by simultaneous solution of two coupled nonlinear equations. This represents the cooperation between the two detectors to work as a team. Lastly the performance difference between CD and DD is due to the information loss in local data processing. However DD has fewer requirements on the communication channel in contrast to CD which requires infinite bandwidth. A compromise between these two extremes is to allow more information than just decisions to be sent to the fusion center. This is the concept behind the Quantized Detection algorithm considered in the following two chapters.

III. DETECTION USING QUANTIZED SENSOR OBSERVATIONS

A. INTRODUCTION

So far detection with sensor observations has been described using two methods. In the first method all sensor observations are sent to some central processor which makes a decision based on a likelihood ratio test. In the second method only local decisions are sent to the central processor which fuses these decisions into a global decision. While the first method is very easy to design it requires in principle infinite bandwidth communication channels. The second method requires only one information bit per detection. Detection with quantized sensor observations will be introduced in this chapter. The main goal of the chapter is to grade the road from the DD problem to the CD problem. It will be referred to by Quantized Detection, QD. The performance improvement of the DD problem will be traced as the amount of information delivered to the fusion center increases.

First let us consider the problem of the Primary Decision Maker (PDM) and its quantized second opinion (consultant). We will prove three theorems concerning the decision rule of the PDM. Then fusion of two quantized observations of an arbitrary number of levels will be considered. As a special case, fusion of two sensor observations, one quantized to N levels and the other to N+1 levels, will be proven equivalent to the PDM and an N-level quantizer. Comparison between different configurations will follow.

B. TEAM DECISION OF A PRIMARY DECISION MAKER AND A SECOND OPINION QUANTIZER.

1. Formulation of the PDM Problem

Consider the structure of Figure 3.1 in which y_1 is quantized into y_{1q} by the quantization rule α of N levels.

$$\alpha : Y_1 \rightarrow Y_{1q} . \qquad (3.1)$$

The primary decision maker will make his decision u_0, about the phenomena H based on its own observation y_2 and the quantized observation y_{1q}.

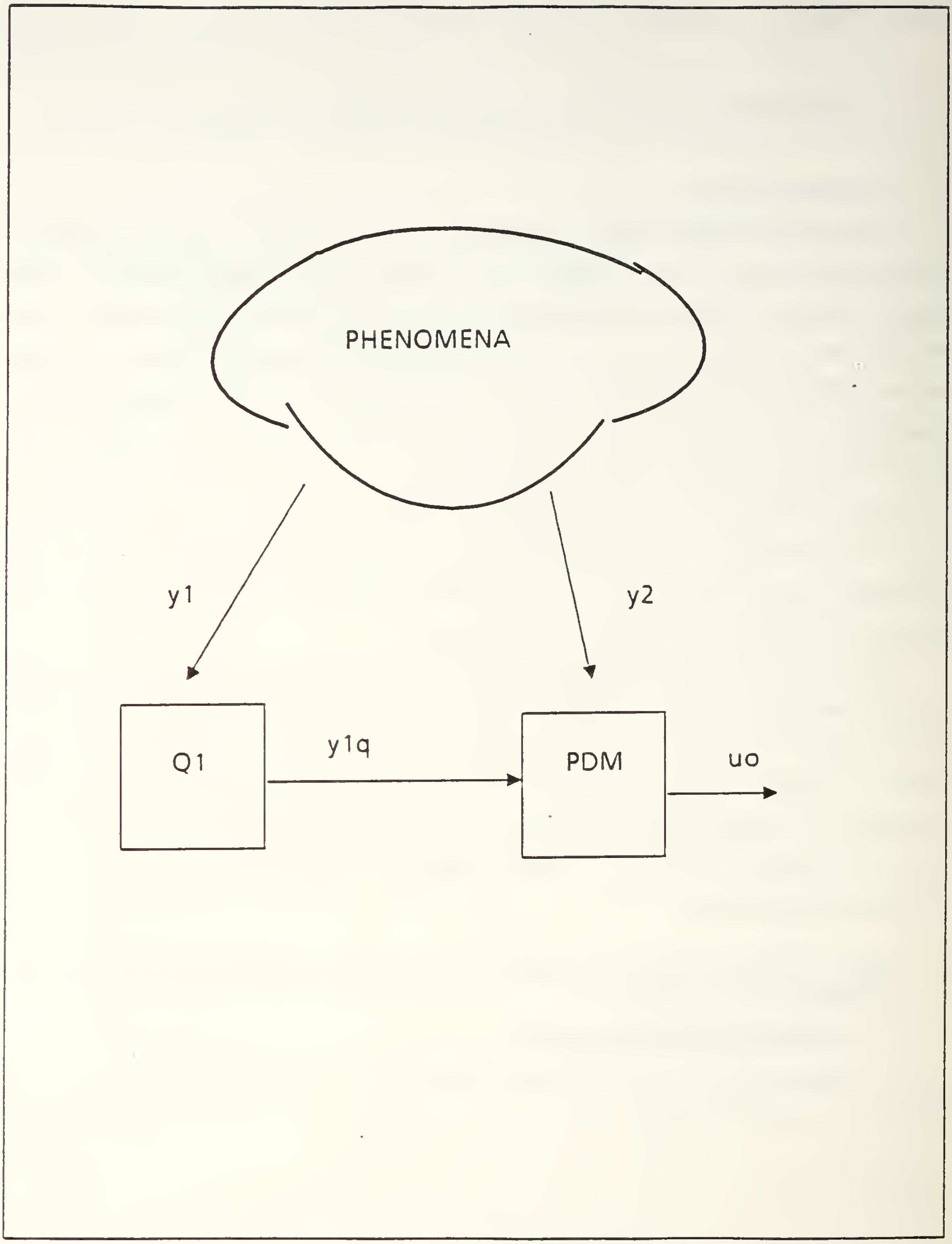

Figure 3.1 Configuration A, The Primary Decision Maker and its Quantized Consultant.

The problem of the PDM is

1. to design the quantization rule α i.e. to specify the set of N points

   $-\infty = X_1 \leq X_2 \leq \ldots \leq X_N < \infty$

   that defines the quantizer intervals, and

2. to design the decision rule γ_2

$$\gamma_2 : Y_{1q} \times Y_2 \rightarrow (0, 1) \qquad (3.2)$$

in order to minimize the decision cost.

2. **Problem Analysis**

Our approach is as follows. We first design the optimum Bayes decision rule given a set of quantizer parameters. Next, the average cost is expressed as a function of these parameters. We then minimize the average cost with respect to them.

a. The Optimum PDM Given Some Quantization Rule α

We have shown in Chapter I that, to within an additive and a multiplicative positive constant the average cost is given by [6]

$$R = C\, P_f - P_d \qquad (3.3)$$

where C is the ratio of error costs and P_f and P_d are the probability of false alarm and probability of detection respectively. The PDM receives a quantized level $y_{1q} = Q_j$. He will make his decision on the basis of his own observation y_2 and y_{1q}. The performance of the the primary decision maker, given some quantization rule α, is given by the following lemma.

Lemma 3.1

The probability of detection and probability of false alarm of the Primary Decision Maker are given by:

$$P_d = \sum_{i=1}^{N} \int_{X_j}^{X_{j+1}} \int_{y_2 \in Z_{1j}} f(y_1, y_2 / H_1)\, dy_1\, dy_2 \qquad (3.4)$$

and

$$P_f = \sum_{i=1}^{N} \int_{X_i}^{X_{i+1}} \int_{y_2 \in Z_{1j}} f(y_1, y_2 / H_0)\, dy_1\, dy_2 \tag{3.5}$$

where Z_{1j} is the decision region Z_1 given that $y_1 \in [X_j, X_{j+1}]$.

Proof:

The proof is given in Appendix (A).

The decision rule of the Primary Decision Maker is given by Theorem 3.1.

Theorem 3.1

Given y_{1q} and y_2 the decision rule of the Primary Decision Maker of Figure 3.1 is

1. deterministic

$$\gamma_2 : Y_{1q} \times Y_2 \rightarrow (0, 1) \tag{3.6}$$

2. a likelihood ratio test

$$u_o = \begin{cases} 1 & \text{if } \Lambda(y_2) \geq \Theta_j(y_2) \\ 0 & \text{if } \Lambda(y_2) < \Theta_j(y_2) \end{cases} \quad ,j = 1,2,...,N \tag{3.7}$$

where $\Lambda(y_2) = f(y_2/H_1)/f(y_2/H_0)$

3. the threshold function $\Theta_j(y_2)$ is given by

$$\Theta_j(y_2) = C \frac{\int_{X_j}^{X_{j+1}} f(y_1/y_2, H_0)\, dy_1}{\int_{X_j}^{X_{j+1}} f(y_1/y_2, H_1)\, dy_1} \quad ,j = 1,2,...N. \tag{3.8}$$

Proof

We first insert (3.4) and (3.5) into (3.3). Each term of the detection cost (3.3) is then given by

$$R_j = \int_{y_2 \in Z_{1j}} \int_{X_j}^{X_{j+1}} [\, C f(y_1, y_2 / H_0) - f(y_1, y_2 / H_1)\,]\, dy_1\, dy_2 \tag{3.9}$$

To make R_j in (3.9) negative an optimum decision rule assigns y_2 to Z_1 if

$$C \int_{X_j}^{X_{j+1}} f(y_1, y_2 / H_0)\, dy_1 - \int_{X_j}^{X_{j+1}} f(y_1, y_2 / H_1)\, dy_1 \geq 0 \quad ,j=1,2,...N \tag{3.10}$$

otherwise it will assign y_2 to Z_0 .

Applying Bayes rule and rearranging terms, decision rule (3.10) can be written as

$$\Lambda(y_2) \geq C \frac{\int_{X_j}^{X_{j+1}} f(y_1/y_2, H_0)\, dy_1}{\int_{X_j}^{X_{j+1}} f(y_1/y_2, H_1)\, dy_1} \quad , j=1,2,...N \tag{3.11}$$

which completes the proof.

b. Optimum Quantization of $\underline{Y}_1$

According to Theorem 3.1, the decision rule of the PDM is a likelihood ratio test with data dependent threshold. The threshold depends on the choice of X_j's. To find an optimum solution for the X_j 's is not any easier than that of the DD problem. Recall that for the DD problem optimum solutions are possible only for the case of conditionally independent observations. Only suboptimal solutions are possible for the case of correlated observations. We will not expect more for the QD problem. Let us consider each case separately.

3. **Conditionally Independent Observations**

Under the assumption of conditionally independent observations, i.e.

$$f(y_1/y_2, H) = f(y_1/ H) \tag{3.12}$$

the decision rule of the Primary Decision Maker can be simplified. This decision rule is given by the following corollary of Theorem 3.1.

Corollary 1

Assuming conditionally independent sensor observations, and given y_{1q} and y_2, the decision rule of the Primary Decision Maker of Figure 3.1 is

1. deterministic

$$\gamma_2 : Y_{1q} \times Y_2 \rightarrow (0, 1) \tag{3.13}$$

2. a likelihood ratio test

$$u_o = \begin{cases} 1 & \text{if } \Lambda(y_2) \geq \Theta_j \\ 0 & \text{if } \Lambda(y_2) < \Theta_j \end{cases} \quad ,j=1,2,\ldots,N \tag{3.14}$$

where $\Lambda(y_2) = f(y_2/H_1)/f(y_2/H_0)$

3. the threshold Θ_j is given by

$$\Theta_j = C \frac{\int_{X_j}^{X_{j+1}} f(y_1/H_0)\, dy_1}{\int_{X_j}^{X_{j+1}} f(y_1/H_1)\, dy_1} \quad ,j=1,2,\ldots N. \tag{3.15}$$

Proof

By applying condition (3.12) in the threshold equation (3.8) one obtains (3.15) which completes the proof.

Let us denote the conditional probability of detection and the conditional probability of false alarm of the PDM given that the j_{th} quantization level of y_1 is received by P_{dj} and P_{fj}. Let Ψ_j be the set of all points y_2 for which

$$\Lambda_2(y_2) \geq \Theta_j \tag{3.16}$$

Then P_{dj} and P_{fj} can be written as

$$P_{dj} = \int_{\Psi_j} f(y_2/H_1)\, dy_2 \tag{3.17}$$

and

$$P_{fj} = \int_{\Psi_j} f(y_2/H_0)\, dy_2 \tag{3.18}$$

Equations (3.4) and (3.5) are now given by

$$P_d = \sum_{i=1}^{N} \int_{X_i}^{X_{i+1}} f(y_1/H_1)\, dy_1 \int_{\Psi_j} f(y_2/H_1)\, dy_2 \qquad (3.19)$$

and

$$P_f = \sum_{i=1}^{N} \int_{X_i}^{X_{i+1}} f(y_1/H_0)\, dy_1 \int_{\Psi_j} f(y_2/H_0)\, dy_2 \,. \qquad (3.20)$$

Substituting (3.19) and (3.20) in (3.3), then differentiating R with respect to X_j ,$\{j=2,3,...N\}$ will yield necessary conditions for optimality of the set of N equations.

$$C\left[\int_{\Psi_k} f(X_k, y_2/H_0)dy_2 - \int_{\Psi_{k-1}} f(X_k, y_2/H_0)dy_2\right] - \left[\int_{\Psi_k} f(X_k, y_2/H_1)dy_2 - \int_{\Psi_{k-1}} f(X_k, y_2/H_1)dy_2\right] = 0 \quad ,k=2,3,...,N. \qquad (3.21)$$

Applying Bayes rule and rearranging terms, (3.21) can be written in the following way.

$$\Lambda(X_k) = C\,\frac{\int_{\Psi_k} f(y_2/H_0)\, dy_2 - \int_{\Psi_{k-1}} f(y_2/H_0)\, dy_2}{\int_{\Psi_k} f(y_2/H_1)\, dy_2 - \int_{\Psi_{k-1}} f(y_2/H_1)\, dy_2} \quad ,k=2,3,...,N \qquad (3.22)$$

The set of N-1 necessary conditions (3.22) are general for any statistics of y_1 . For the special case when $\Lambda(y_2)$ is monotonic in y_2 , let T_j be the value of y_2 for which

$$\Theta_j = \Lambda_2(T_j) \quad ,j=1,2,...,N. \qquad (3.23)$$

So T_j is given by

$$\Lambda(T_j) = C\,\frac{\int_{X_j}^{X_{j+1}} f(y_1/H_0)\, dy_1}{\int_{X_j}^{X_{j+1}} f(y_1/H_1)\, dy_1} \quad ,j=1,2,...N \qquad (3.24)$$

For this case of monotonic $\Lambda(y_2)$ the set of necessary conditions for optimality (3.22) can be written as

$$\Lambda(X_k) = C\,\frac{\int_{T_k}^{\infty} f(y_2/H_0)\,dy_2 - \int_{T_{k-1}}^{\infty} f(y_2/H_0)\,dy_2}{\int_{T_k}^{\infty} f(y_2/H_1)\,dy_2 - \int_{T_{k-1}}^{\infty} f(y_2/H_1)\,dy_2} \quad ,k=2,3,\ldots,N \qquad (3.25)$$

Equivalently we can write (3.25) in the form

$$\Lambda(X_k) = C\,\frac{\int_{T_k}^{T_{k-1}} f(y_2/H_0)\,dy_2}{\int_{T_k}^{T_{k-1}} f(y_2/H_1)\,dy_2} \quad ,k=2,3,\ldots,N\,. \qquad (3.26)$$

P_d and P_f in this case are given by

$$P_d = \sum_{i=1}^{N} \int_{X_i}^{X_{i+1}} f(y_1/H_1)\,dy_1 \int_{T_i}^{\infty} f(y_2/H_1)\,dy_2 \qquad (3.27)$$

and

$$P_f = \sum_{i=1}^{N} \int_{X_i}^{X_{i+1}} f(y_1/H_0)\,dy_1 \int_{T_i}^{\infty} f(y_2/H_0)\,dy_2 \qquad (3.28)$$

Equations (3.24) and (3.26) are only necessary conditions for optimality for monotonic likelihood ratio. They correspond to minima if the Hessian matrix $[\partial^2 R/\partial X_i \partial X_j]$ is positive definite. All solutions must be checked for the global minima.

4. **Solution of the Primary Decision Maker Problem with Independent Sensor Observations and Monotonic Likelihood Ratio**

The following theorem summarizes the above solution of the PDM with independent sensor observations and monotonic likelihood ratio.

Theorem 3.2

The decision rule of the Primary Decision Maker with a Quantized Consultant (for independent sensor observations and monotonic likelihood ratio) is;

1. deterministic

$$\gamma_2 : Y_{1q} \times Y_2 \rightarrow (0, 1) \tag{3.29}$$

2. a likelihood ratio test

$$u_o = \begin{cases} 1 & \text{if } \Lambda(y_2) \geq \Theta_j \\ 0 & \text{if } \Lambda(y_2) < \Theta_j \end{cases} \quad ,j = 1,2,\ldots,N \tag{3.30}$$

where $\Lambda(y_2) = f(y_2/H_1)/f(y_2/H_0)$

3. the threshold function $\Theta(y_2)$ is given by

$$\Theta_j = C \frac{\int_{X_j}^{X_{j+1}} f(y_1/H_0)\, dy_1}{\int_{X_j}^{X_{j+1}} f(y_1/H_1)\, dy_1} \quad ,j = 1,2,\ldots N . \tag{3.31}$$

The optimum set of quantizer interval end points must satisfy the set (3.26), where T_k's are given by (3.24). All possible solutions must be checked for the global minimum cost.

5. **The Case of Correlated Observations**

We now move to a more realistic situation by removing the condition of independent sensor observations. In many radar and sonar problems noise in nearby sensors is likely to be correlated. As we mentioned before the decision rules (3.11) are likelihood ratio tests with data dependent thresholds. It is impossible to come with their optimum functional expressions [4.] **A suboptimal solution** for the case of correlated observations is to use likelihood ratio tests with constant thresholds as local decision rules. These constant thresholds for y_2 are the values of y_2 for which the inequality (3.11) is an equality. i.e.;

$$\Lambda(T_k) = C \frac{\int_{X_k}^{X_{k+1}} f(y_1/T_k,H_0)\,dy_1}{\int_{X_k}^{X_{k+1}} f(y_1/T_k,H_1)\,dy_1} \quad ,k=1,2,\ldots,N \ . \tag{3.32}$$

In terms of these thresholds T_k 's and the quantizer points X_k 's one can write expressions for the probability of detection and the probability of false alarm in the form of (3.4) and (3.5). Substituting for P_d and P_f in (3.3) and differentiating R with respect to X_k for k=2,3,...N yields the following set of necessary conditions for the case of monotonic $\Lambda_2(y_2)$:

$$\Lambda(X_k) = C \frac{\int_{T_k}^{T_{k-1}} f(y_2/X_k,H_0)\,dy_2}{\int_{T_k}^{T_{k-1}} f(y_2/X_k,H_1)\,dy_2} \quad ,k=2,3,\ldots,N. \tag{3.33}$$

The set of equations in (3.32) and (3.33) constitute 2N-1 equations that specify the quantizer interval end points $\{X_k\}$ for y_1 and the thresholds $\{T_k\}$ for y_2 .

C. TEAM DECISION OF TWO QUANTIZERS AND A FUSION CENTER

In this section we will consider the problem of making a global decision based on two quantized observations.

1. Formulation of the QD problem

For the structure of Figure 3.2, y_1 is quantized into N levels by the quantization rule α_1

$$\alpha_1 : Y_1 \rightarrow Y_{1q} \tag{3.34}$$

and y_2 is quantized into M levels by the quantization rule α_2

$$\alpha_2 : Y_2 \rightarrow Y_{2q} \ . \tag{3.35}$$

The quantized values y_{1q} and y_{2q} are sent to the fusion center which must decide which state of the phenomena is true. It is required to design the quantization rules α_1 and α_2 and the decision rule γ

$$\gamma : Y_{1q} \times Y_{2q} \rightarrow (0,1) \qquad (3.36)$$

to minimize the global cost.

2. Problem Analysis and the QD Algorithm

The observation space of the fusion center contains NM points to be divided into two decision regions. Since there are as many as 2^{NM} fusion methods, checking all of them will consume a very long time even for small values of N and M. A suboptimal solution is to approximate the threshold equation of the corresponding CD problem by a piecewise curve in the y_1 y_2 plane. This is illustrated in Figure 3.3.

The figure shows a schematic diagram of a CD threshold curve and its staircase approximation. The approximate curve consists of segments of straight lines connected together. The coordinates of the connecting points will play the role of the interval end points of the quantizers. Let us first write an expression for P_d and P_f in terms of these point coordinates. If this expression of the cost is minimized with respect to each coordinate there will be as many equations as the number of coordinates. Solving these equations simultaneously yields the quantizer parameters. This is the core of the QD algorithm which is summarized as follows:

1. Derive the threshold equation of the CD system.

$$\Lambda (y_1, y_2) = C \qquad (3.37)$$

2. Approximate the threshold equation by a stepwise curve satisfying the N and M constraints.
3. Write an expression for the cost in terms of the curve parameters.
4. Minimize the average cost with respect to the curve parameters.

Let us illustrate how the algorithm works for the case of detection of a known signal in gaussian noise.

3. An Example: The Known Signal in Gaussian Noise

Consider Figure 3.2 when y_1 and y_2 are given by

$$H_0 : y_i = n_i$$
$$H_1 : y_i = a_i + n_i \quad ,i = 1,2 \qquad (3.38)$$

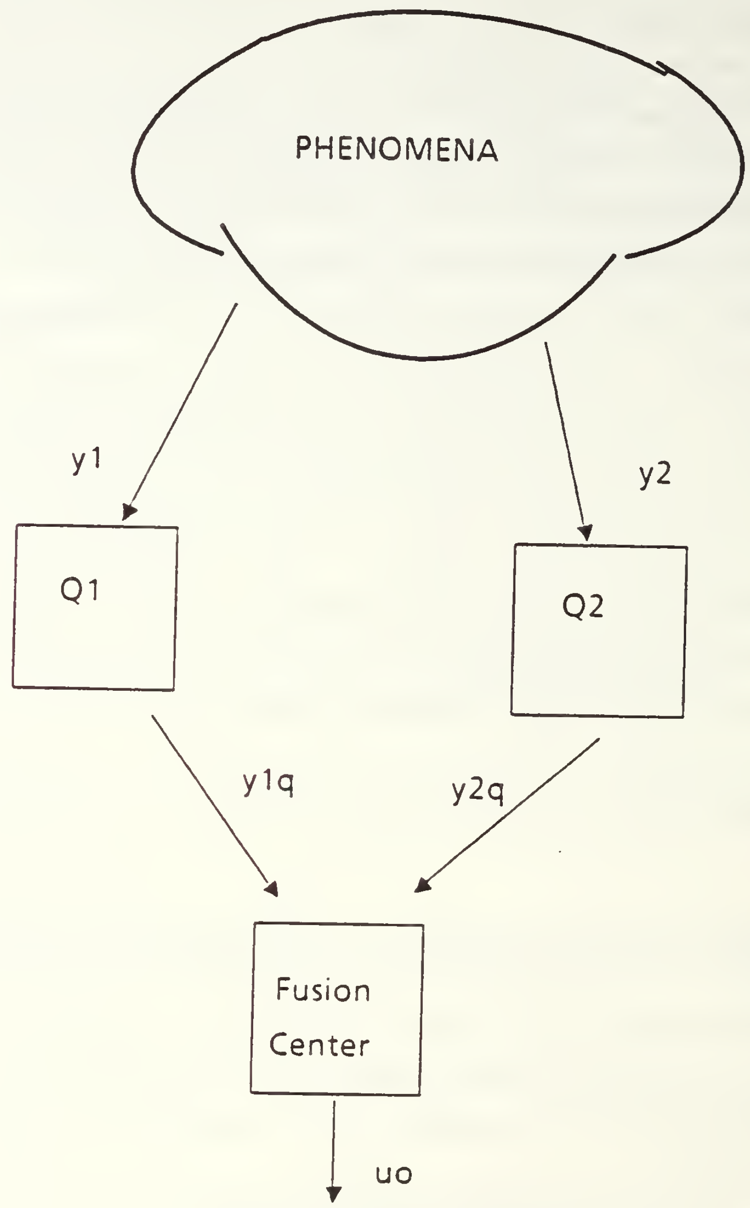

Figure 3.2 Configuration B. The Team of Two Quantizers and a Fusion Center.

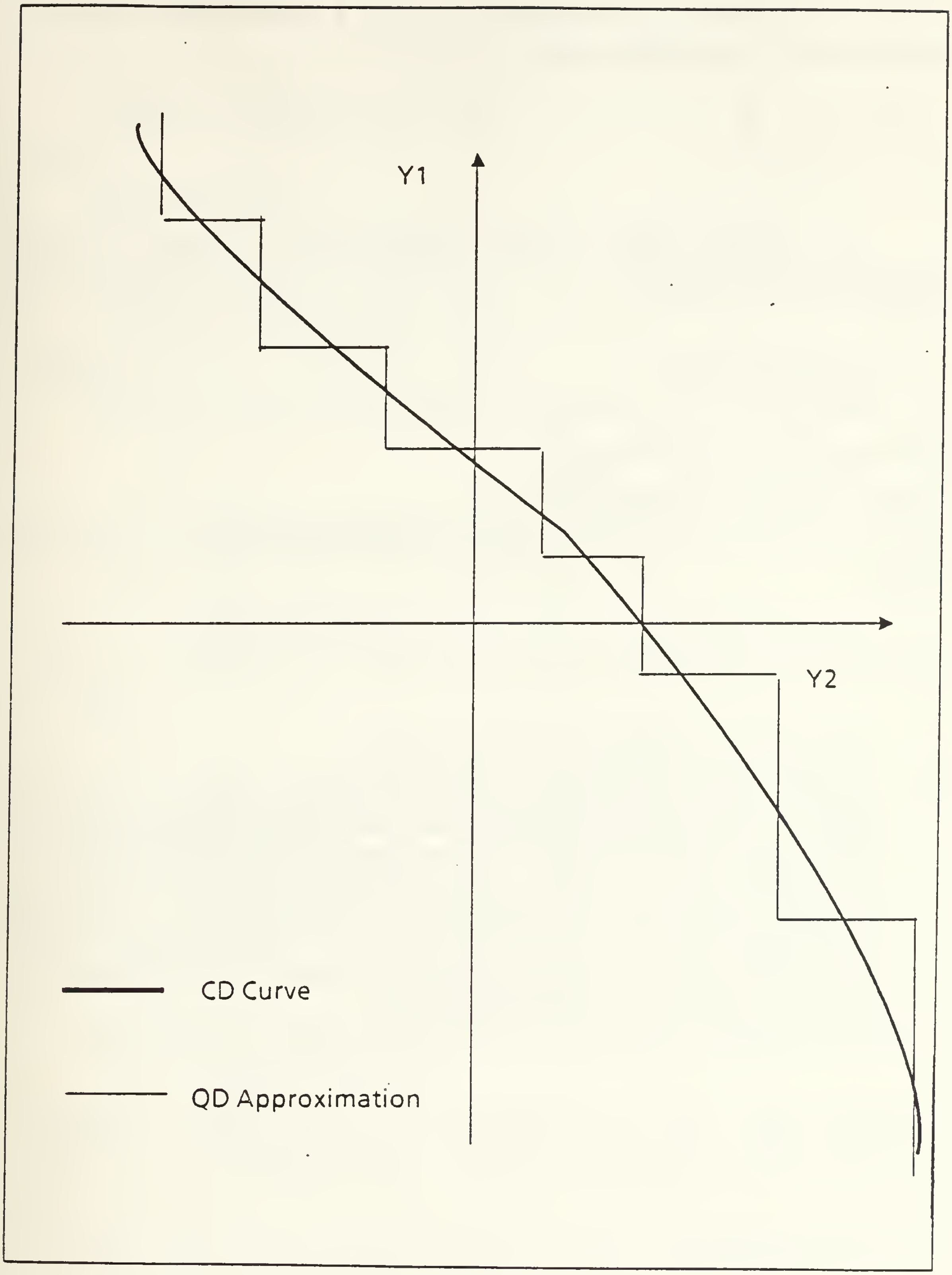

Figure 3.3 Quantized Threshold Curve.

where the a_i's are positive constants and $\underline{N} = [n_1\ n_2\ ]^t$ is a gaussian random vector of zero vector mean with covariance matrix:

$$K = \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix} \tag{3.39}$$

It is required to design the N-level and the M-level quantizers Q_1 and Q_2 and the decision rule γ where

$$\gamma : Y_{1q} \times Y_{2q} \rightarrow (0,1) \tag{3.40}$$

to minimize the average decision cost.

Procedure following the QD algorithm

The threshold equation of the CD problem has been shown in Chapter II to have the form:

$$(a_1 - \rho a_2)y_1 + (a_2 + \rho a_1)y_2 = (a_1^2 + a_2^2 - \rho a_1 a_2)/2 + (1-\rho^2)\log(C). \tag{3.41}$$

1. The CD curve is a straight line in the $y_1\ y_2$ plane.
2. Possible stepwise approximations for the threshold equation are shown in Figure 3.4 . We notice that in Figure 3.4 a and c the two quantizers have the same number of quantizer levels. While in Figure 3.4 b and d one quantizer has one more level than the other. From Chapter II, we can expect that the constant C will decide the superiority of a or c and of b or d. We shall consider optimum parameters of Figure 3.4 a and b. Similar treatment can be considered for Figure 3.4 c and d. In Figure 3.4 a the point $X_1 = -\infty$ while T_1 is finite. In Figure 3.4 b $X_1 = -\infty$ and $T_1 = \infty$.
3. The probability of detection of the decision rule of Figure 3.4 a is given by

$$P_d = \sum_{i=1}^{N} \int_{X_i}^{X_{i+1}} \int_{T_i}^{\infty} f(y_1,y_2 /H_1)\, dy_1\, dy_2 \tag{3.42}$$

and P_f is given by

$$P_f = \sum_{i=1}^{N} \int_{X_i}^{X_{i+1}} \int_{T_i}^{\infty} f(y_1,y_2 /H_0)\, dy_1\, dy_2 \,. \tag{3.43}$$

For the detection rule of Figure 3.4 b P_d is given by :

$$P_d = \sum_{i=2}^{N} \int_{X_i}^{X_{i+1}} \int_{T_i}^{\infty} f(y_1, y_2 / H_1)\, dy_1\, dy_2 \tag{3.44}$$

and P_f is given by

$$P_f = \sum_{i=2}^{N} \int_{X_i}^{X_{i+1}} \int_{T_i}^{\infty} f(y_1, y_2 / H_0)\, dy_1\, dy_2 \tag{3.45}$$

4. Necessary conditions for optimality of parameters of the curve in Figure 3.4 a are:

$$\Lambda(T_i) = C \frac{\int_{X_i}^{X_{i+1}} f(y_1/T_i, H_0)\, dy_1}{\int_{X_i}^{X_{i+1}} f(y_1/T_i, H_1)\, dy_1} \quad ,i = 1,2,...,N \tag{3.46}$$

and

$$\Lambda(X_i) = C \frac{\int_{T_i}^{T_{i-1}} f(y_2/X_i, H_0)\, dy_2}{\int_{T_i}^{T_{i-1}} f(y_2/X_i, H_1)\, dy_2} \quad ,i = 2,3,...,N\ . \tag{3.47}$$

For Figure 3.4 b ,the optimality conditions are

$$\Lambda(T_i) = C \frac{\int_{X_i}^{X_{i+1}} f(y_1/T_i, H_0)\, dy_1}{\int_{X_i}^{X_{i+1}} f(y_1/T_i, H_1)\, dy_1} \quad ,i = 2,3,...,N \tag{3.48}$$

and

$$\Lambda(X_i) = C \frac{\int_{T_i}^{T_{i-1}} f(y_2/X_i,H_0)\, dy_2}{\int_{T_i}^{T_{i-1}} f(y_2/X_i,H_1)\, dy_2} \quad ,i=2,3,\ldots,N . \tag{3.49}$$

The last two equations are exactly the same as the necessary conditions for optimizing detection using a Primary Decision Maker and its quantized second opinion for the same signals in gaussian noise. Recall that the information available at the PDM is more complete than that available at the fusion center of two quantized observations. Yet the two problems have the same solution. This is a proof of the following lemma.

Lemma 3.2

Optimum detection of known signal in gaussian noise using two quantized observations of N and N+1 levels is equivalent to optimum detection using the first quantized observation and the second continuous observation.

Lemma 3.2 is applicable to any case with a monotonic likelihood ratio. This can be easily proved by writing the necessary conditions of optimality for the two configurations. A special case of **Lemma 3.2** is that of N = 2. It corresponds to the tandem configuration of two detectors in a Distributed Detection Network (DDN) [10]. The "downstream" detector (decision maker) makes its decision based on its own observation and the "upstream" detector's decision.

D. NUMERICAL SOLUTION FOR THE SYSTEM PARAMETERS

It is of interest to compare the four sets of equations {(3.24),(3.26)}, {(3.32),(3.33)}, {(3.46),(3.47)} and {(3.48),(3.49)} with that of Lloyd and Max [18,23] for minimum distortion quantizer parameters.

Max's trial and error algorithm to solve this set of nonlinear equations can be used. However Max's algorithm is very time consuming [24]. We have used instead the method of successive substitutions with an initial guess satisfying

$$X_2 \leq X_3 \leq \ldots \leq X_N \tag{3.50}$$

and put the equations in the form

$$Z = G(Z) \tag{3.51}$$

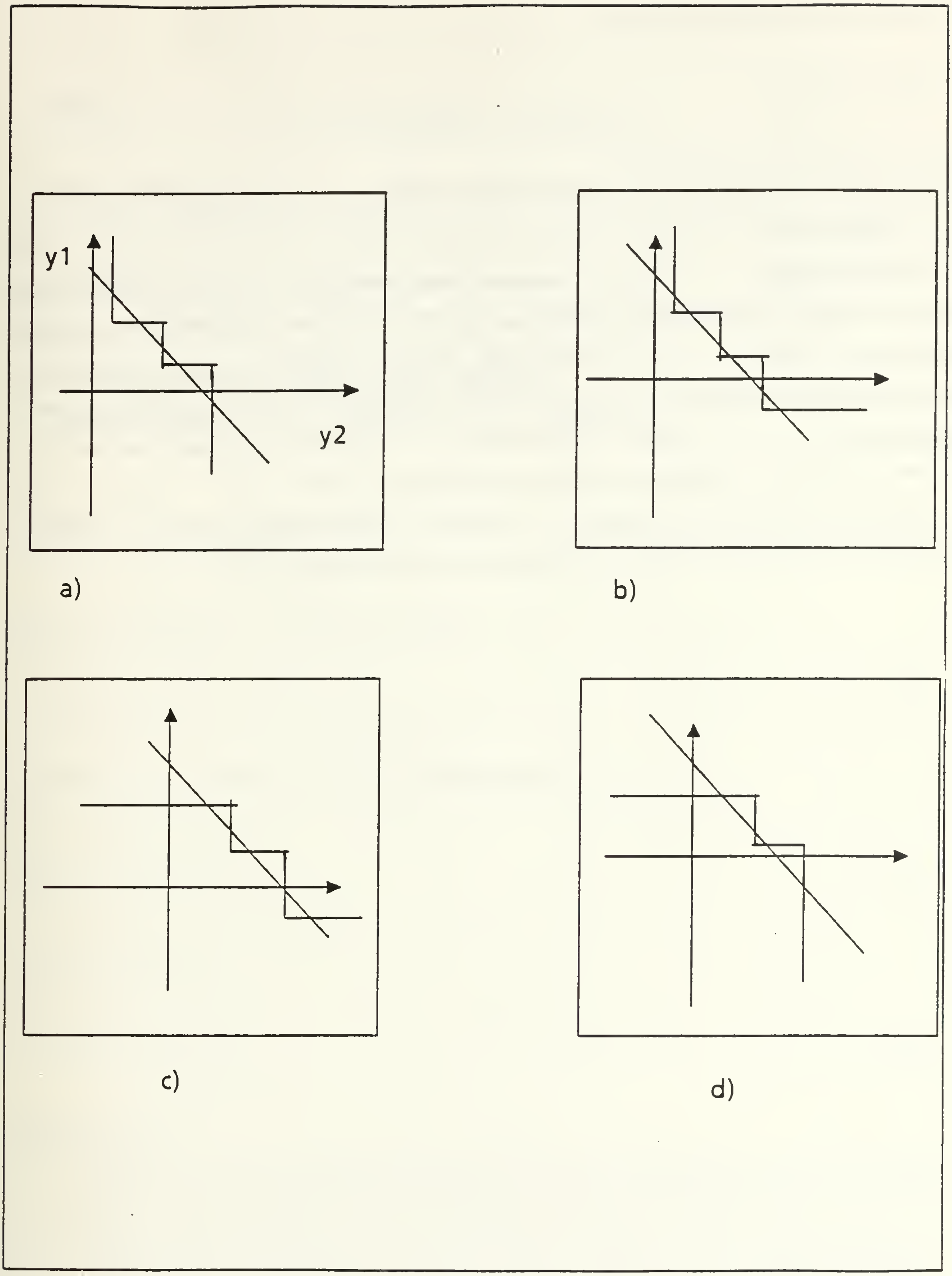

Figure 3.4 Possible Approximations of the Threshold Equation.

The kth iteration is then given by

$$Z_k = G (Z_{k-1}) \tag{3.52}$$

We will devote the next chapter to solving some numerical examples using this method.

E. SUMMARY

In this chapter the method of detection using quantized sensor observations has been introduced. This method, referred to by QD, can have significant performance improvement compared to the distributed detection algorithm (DD) with only marginally more demand on the communication channels. The QD algorithm involves approximating the CD threshold hyperplane by a stepwise hyperplane that can be spanned with the quantized data and that minimizes the detection cost.

Also the equivalence between two detection configurations, one with tandem connection and the other with hierarchical structure, has been shown.

IV. NUMERICAL RESULTS

In this Chapter some examples are solved numerically using the QD algorithm. First the detection of known signals in gaussian noise is considered. Next detection of signals with exponential distribution is considered. Finally, the algorithm will be applied to differentiating between gaussian signals with different variances.

A. KNOWN SIGNAL IN GAUSSIAN NOISE

Again consider Figure 3.2 when y_1 and y_2 are given by

$$H_0 : y_i = n_i$$
$$H_1 : y_i = a_i + n_i, i = 1,2 \tag{4.1}$$

with $a_1 = 4$ and $a_2 = 2$. The noise vector

$$\underline{N} = [\, n_1 \; n_2 \,]^t \tag{4.2}$$

is of zero vector mean and with covariance matrix given by:

$$K = \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix} \tag{4.3}$$

where ρ is given by

$$\rho = E\{ n_1 n_2 \} . \tag{4.4}$$

It is required to:

1. Design the primary decision maker PDM and its N-level Quantizer to minimize the average decision cost. We have designated this structure configuration A.
2. Design the N-level quantizers Q_1 and Q_2 and the decision rule u_o to minimize the average decision cost. We have designated this structure configuration B.
3. Compare the performance of the two configurations and that of the completely centralized system.

Following the algorithm we have:

1. The threshold equation for the CD problem given by,

$$(a_1 - \rho a_2) y_1 + (a_2 - \rho a_1) y_2 = (a_1^2 + a_2^2 - 2\rho a_1 a_2)/2 + (1-\rho^2)\log(C) \quad (4.5)$$

a straight line in the y_1 y_2 plane.

2. Figures 3.4 a and 3.4 b show the decision regions for configuration A and configuration B respectively.
3. Probability of detection P_{d1} and probability of false alarm P_{f1} of PDM are given by:

$$P_{d1} = \sum_{i=1}^{N} \int_{X_i}^{X_{i+1}} \frac{1}{\sqrt{2\pi}} \exp(-y_1^2/2) \ \mathrm{erfc}\left\{\frac{T_k - a_2 - \rho(y_1 - a_1)}{\sqrt{(1-\rho^2)}}\right\} dy_1 \quad (4.6)$$

and

$$P_{f1} = \sum_{i=1}^{N} \int_{X_i}^{X_{i+1}} \frac{1}{\sqrt{2\pi}} \exp(-y_1^2/2) \ \mathrm{erfc}\left\{\frac{T_k - \rho y_1}{\sqrt{(1-\rho^2)}}\right\} dy_1 . \quad (4.7)$$

Also P_{d2} and P_{f2} of configuration B are given by:

$$P_{d2} = \sum_{i=2}^{N} \int_{X_i}^{X_{i+1}} \frac{1}{\sqrt{2\pi}} \exp(-y_1^2/2) \ \mathrm{erfc}\left\{\frac{T_k - a_2 - \rho(y_1 - a_1)}{\sqrt{(1-\rho^2)}}\right\} dy_1 \quad (4.8)$$

and

$$P_{f2} = \sum_{i=2}^{N} \int_{X_i}^{X_{i+1}} \frac{1}{\sqrt{2\pi}} \exp(-y_1^2/2) \ \mathrm{erfc}\left\{\frac{T_k - \rho y_1}{\sqrt{(1-\rho^2)}}\right\} dy_1 . \quad (4.9)$$

4. For configuration A equations of the quantizer interval end points and corresponding PDM's thresholds for the gaussian case are given by

$$\Lambda(X_k) = C \frac{\mathrm{erfc}\left\{\frac{T_{k-1} - \rho X_k}{\sqrt{(1-\rho^2)}}\right\} - \mathrm{erfc}\left\{\frac{T_k - \rho X_k}{\sqrt{(1-\rho^2)}}\right\}}{\mathrm{erfc}\left\{\frac{T_{k-1} - a_2 - \rho(X_k - a_1)}{\sqrt{(1-\rho^2)}}\right\} - \mathrm{erfc}\left\{\frac{T_k - a_2 - \rho(X_k - a_1)}{\sqrt{(1-\rho^2)}}\right\}} \quad ,k=2,3, \ N \quad ($$

and

$$A(T_k) = C \frac{\operatorname{erfc}\left\{\frac{X_{k+1}-\rho T_k}{\sqrt{(1-\rho^2)}}\right\} - \operatorname{erfc}\left\{\frac{X_k-\rho T_k}{\sqrt{(1-\rho^2)}}\right\}}{\operatorname{erfc}\left\{\frac{X_{k+1}-a_1-\rho(T_k-a_2)}{\sqrt{(1-\rho^2)}}\right\} - \operatorname{erfc}\left\{\frac{X_k-a_1-\rho(T_k-a_2)}{\sqrt{(1-\rho^2)}}\right\}} \quad ,k=1,2,\ N. \quad (4.11)$$

For configuration B the quantizer end point intervals X's and T's are given by:

$$A(X_k) = C \frac{\operatorname{erfc}\left\{\frac{T_{k-1}-\rho X_k}{\sqrt{(1-\rho^2)}}\right\} - \operatorname{erfc}\left\{\frac{T_k-\rho X_k}{\sqrt{(1-\rho^2)}}\right\}}{\operatorname{erfc}\left\{\frac{T_{k-1}-a_2-\rho(X_k-a_1)}{\sqrt{(1-\rho^2)}}\right\} - \operatorname{erfc}\left\{\frac{T_k-a_2-\rho(X_k-a_1)}{\sqrt{(1-\rho^2)}}\right\}} \quad ,k=2,3,\ N \quad (4.12)$$

and

$$A(T_k) = C \frac{\operatorname{erfc}\left\{\frac{X_{k+1}-\rho T_k}{\sqrt{(1-\rho^2)}}\right\} - \operatorname{erfc}\left\{\frac{X_k-\rho T_k}{\sqrt{(1-\rho^2)}}\right\}}{\operatorname{erfc}\left\{\frac{X_{k+1}-a_1-\rho(T_k-a_2)}{\sqrt{(1-\rho^2)}}\right\} - \operatorname{erfc}\left\{\frac{X_k-a_1-\rho(T_k-a_2)}{\sqrt{(1-\rho^2)}}\right\}} \quad ,k=2,3,\ N. \quad (4.13)$$

We have solved the system of equations of the two configurations using the method of successive substitution for $N = 2, 3, 4, 5$ and 6 . Figure 4.1 shows the receiver operating characteristics ROC for the two configurations for $\rho=0$, for different values of N. The ROC for the CD system is also shown. The effect of ρ is illustrated in Figure 4.2. The figure shows ROC curves for Configuration A for different values of N and for $\rho=0$ and 0.25. Figure 4.3 shows the average cost of Configuration B and CD vs. C, for different values of N. The relation between the cost of detection for Configuration B vs. the number of quantization levels is shown in Figure 4.4 . The figure shows the exponential decay of the detection cost as the amount of information available at the fusion center increases.

The following results are noted from the curves.

1. Configuration A has better ROC curves than Configuration B. The performance difference is large for $N = 2$ but gets smaller as N increases.
2. Both performances converge to that of the CD in a uniform manner.
3. As the correlation coefficient increases the performance difference decreases.

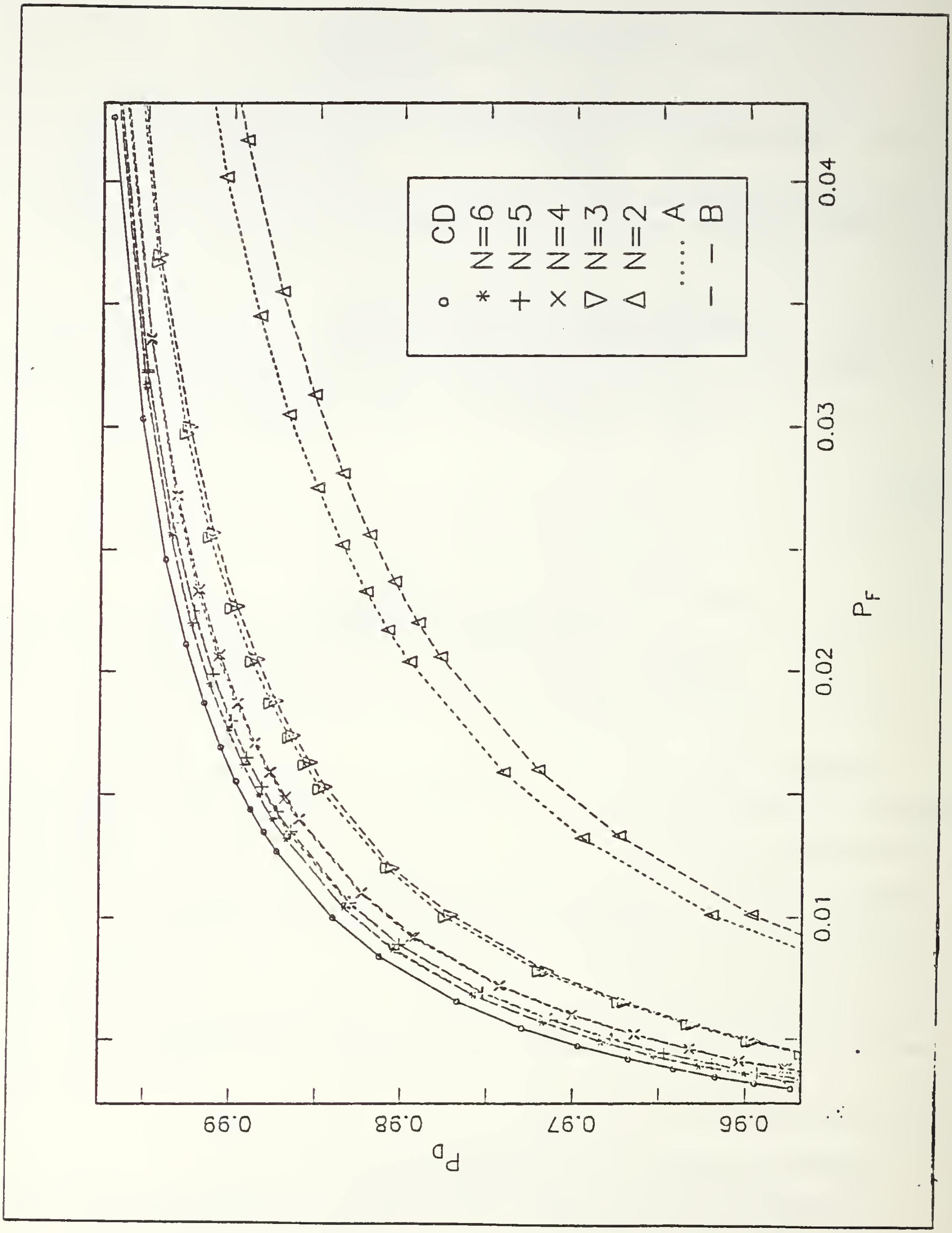

Figure 4.1 ROC Curves for Configuration A , Configuration B and CD.

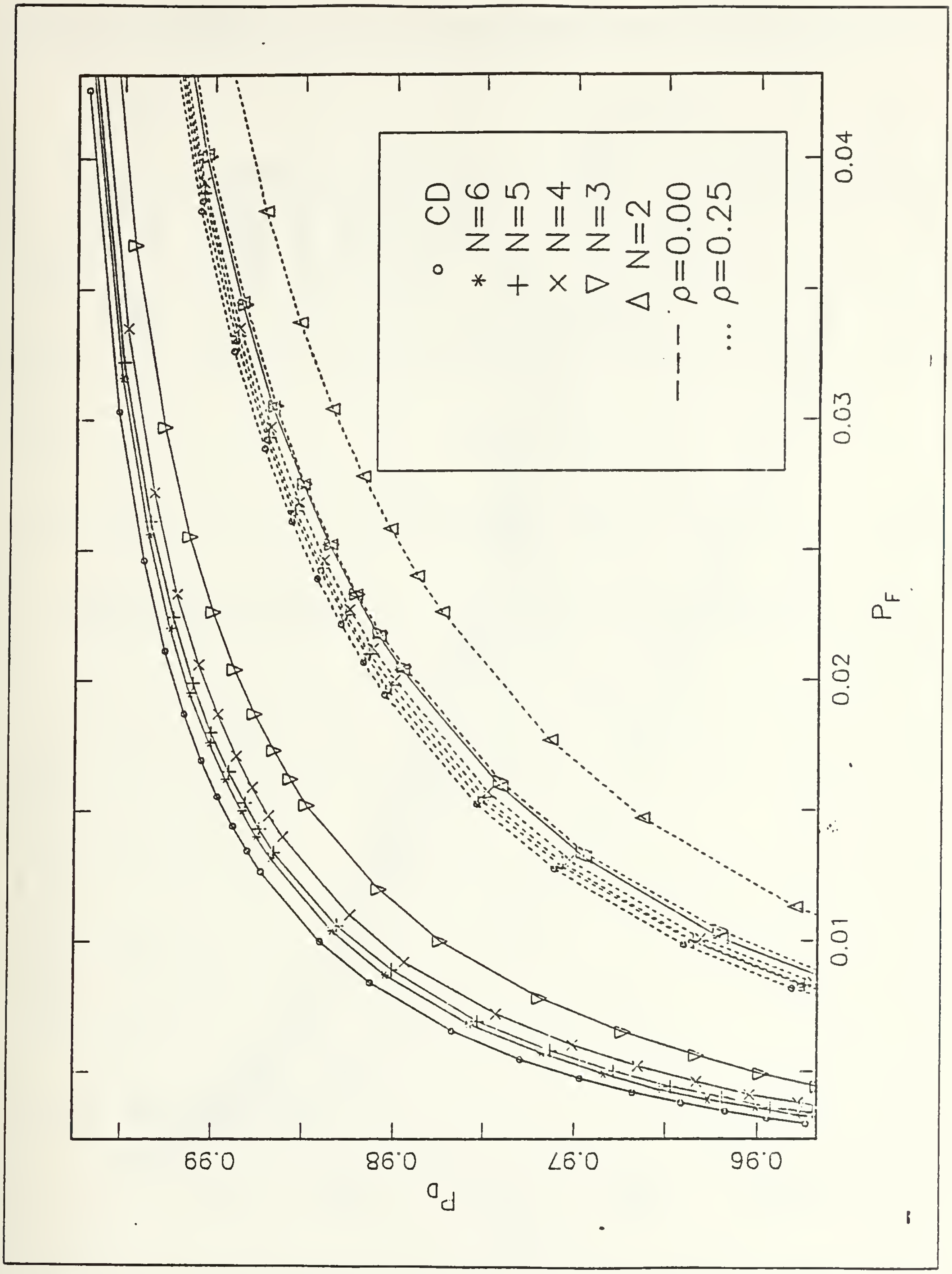

Figure 4.2 ROC Curves for Configuration A for ρ = 0 and 0.25.

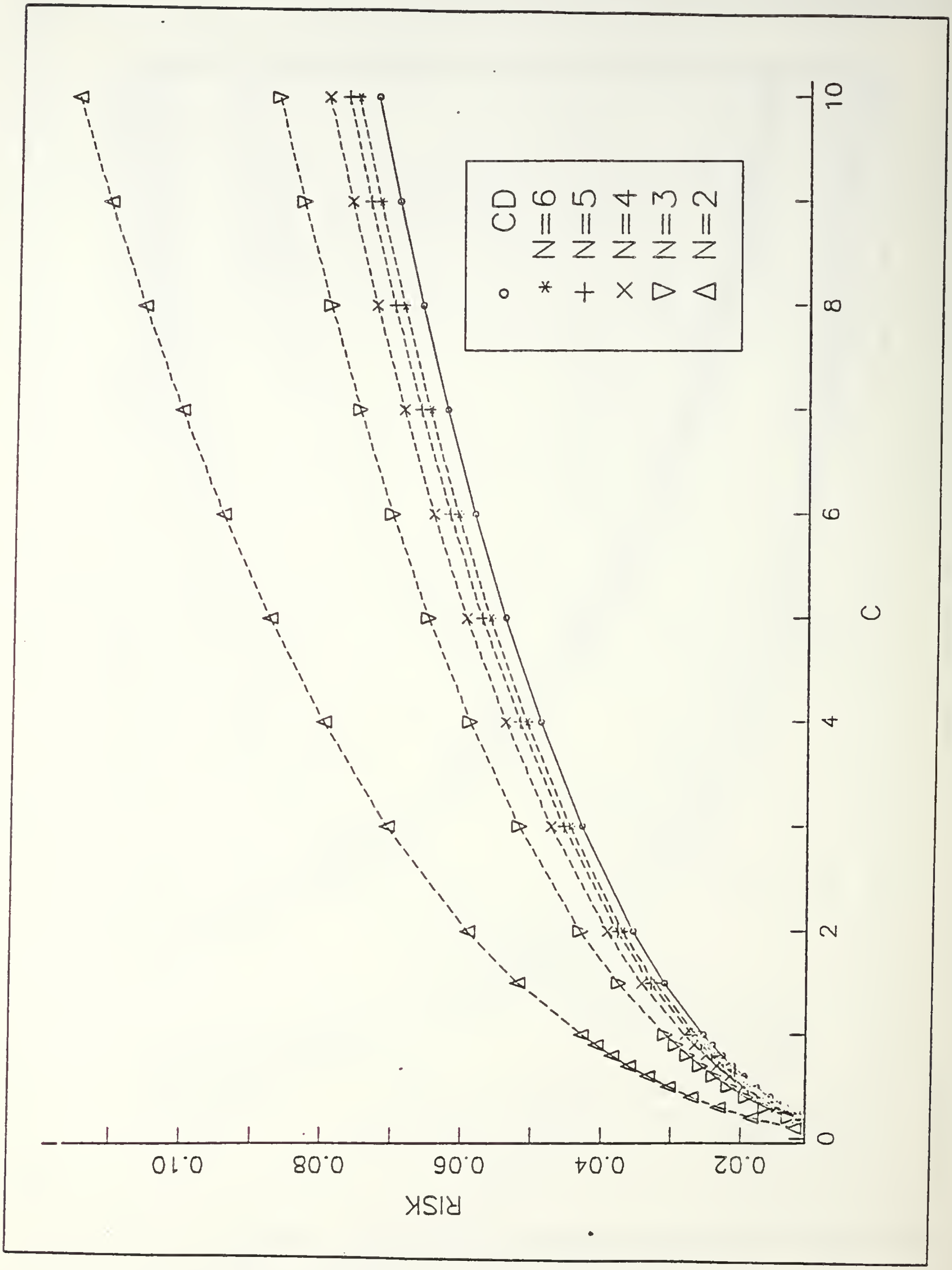

Figure 4.3 Average Detection Cost for Configuration B and CD.

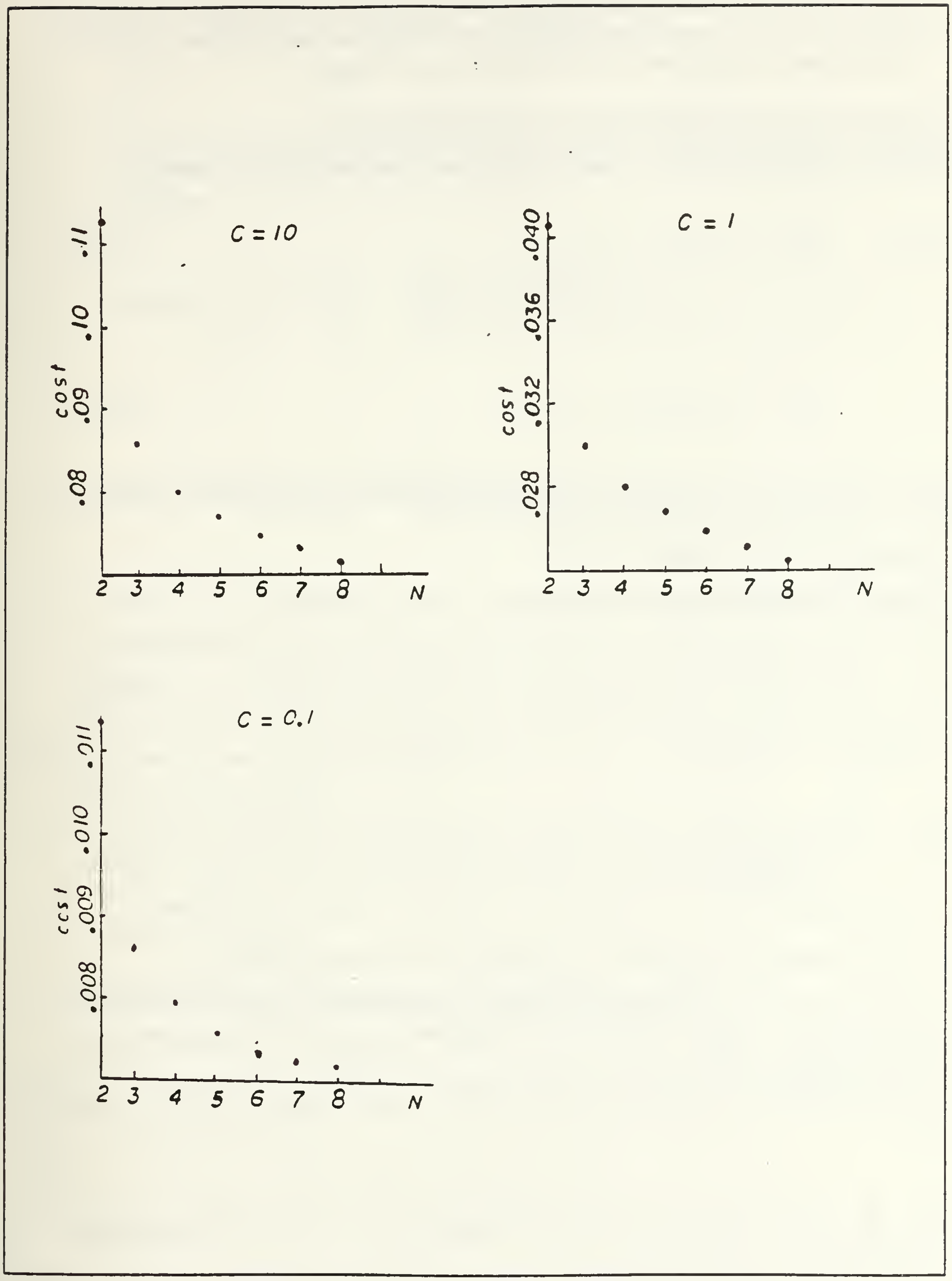

Figure 4.4 Average detection cost for Configuration B vs. N.

4. As N increases the average detection cost gets smaller and tends to that of the CD. Since the number N reflects the mutual information between the input and the output of the quantizers, the relation between the performance degradation and information delivered to the fusion center is strong.

B. SIGNALS WITH EXPONENTIAL DISTRIBUTIONS

Consider again Figure 3.2 . Let y_1 and y_2 have the following distributions:

$$H_0 : \qquad f(y_i) = \lambda_0 \exp(-\lambda_0 y_i) \tag{4.14}$$

and

$$H_1 : \qquad f(y_i) = \lambda_1 \exp(-\lambda_1 y_i) \ , i=1,2 \tag{4.15}$$

and assume that λ_1 is less than λ_0 . It is required to design the quantizers and fusion rule that minimize the average decision cost.

Following the QD algorithm we have:

1. The CD threshold equation is given by

$$y_1 + y_2 = C_1 \tag{4.16}$$

where C_1 is given by

$$C_1 = \frac{(\lambda_0 / \lambda_1)^2}{\lambda_0 - \lambda_1} C . \tag{4.17}$$

The CD threshold equation is a straight line in the first quadrant.

2. Figure 4.5 shows possible approximations of the threshold equation. For $N=2$, the symmetry suggests equal detector thresholds. For $N \geq 3$ let us fix X_1 and T_N to zero.

3. The probability of detection and probability of false alarm P_d and P_f are given by

$$P_d = \sum_{i=1}^{N} [\exp(-\lambda_1 X_i) - \exp(-\lambda_1 X_{i+1})] \exp(-\lambda_1 T_i) \tag{4.18}$$

and

$$P_f = \sum_{i=1}^{N} [\exp(-\lambda_0 X_i) - \exp(-\lambda_0 X_{i+1})] \exp(-\lambda_0 T_i) \quad (4.19)$$

4. Writing an expression of the average cost in P_d and P_f as before and minimizing with respect to X_k, $k=2,3,..N$ and T_k, $k=1,2,..$ $N-1$ one obtains the set of equations

$$\exp\{(\lambda_0 - \lambda_1) T_k\} = \frac{\lambda_0}{\lambda_1} C \frac{\exp(-\lambda_0 X_k) - \exp(-\lambda_0 X_{+1})}{\exp(-\lambda_1 X_k) - \exp(-\lambda_1 X_{k+1})}, k=1,2,\ldots N-1 \quad (4.20)$$

and

$$\exp\{(\lambda_0 - \lambda_1) X_k\} = \frac{\lambda_0}{\lambda_1} C \frac{\exp(-\lambda_0 T_k) - \exp(-\lambda_0 T_{k+1})}{\exp(-\lambda_1 T_k) - \exp(-\lambda_0 T_{k+1})}, k=2,3,\ldots N. \quad (4.21)$$

This set of equations Have been solved by the method of successive substitutions for $\lambda_0 = 2$, $\lambda_1 = 1$, and for $N=2,3,4,5$ and 6. A FORTRAN program to calculate the quantizer parameters is given in Appendix D.

Figure 4.6 shows ROC curves for the quantized as well as the CD systems. The average detection cost is shown in Figure 4.7.

We note the following:

1. The largest performance improvement occurs when we switch from $N = 2$ to $N = 3$ (i.e. only less than one more information bit per detection).
2. The performance curves { ROC(N) } and { R(N) } converge uniformally to the performance of CD

C. GAUSSIAN SIGNALS WITH DIFFERENT VARIANCE

Consider again the structure of Figure 3.2 . Let sensor observations y_1 and y_2 be independent, identically distributed gaussian random variables of zero mean. However,

under H_0, $Var(y_i) = \sigma_0^2$,

and

under H_1, $Var(y_i) = \sigma_1^2$, $i=1,2$. For specificity, let

$\sigma_0 = 1$ and $\sigma_1 = \sqrt{2}$.

Quantized sensor observations are sent to the fusion center to decide which of the hypothesis is true. It is required to design the quantizers Q_1 and Q_2 as well as the fusion rule to minimize the average decision cost.

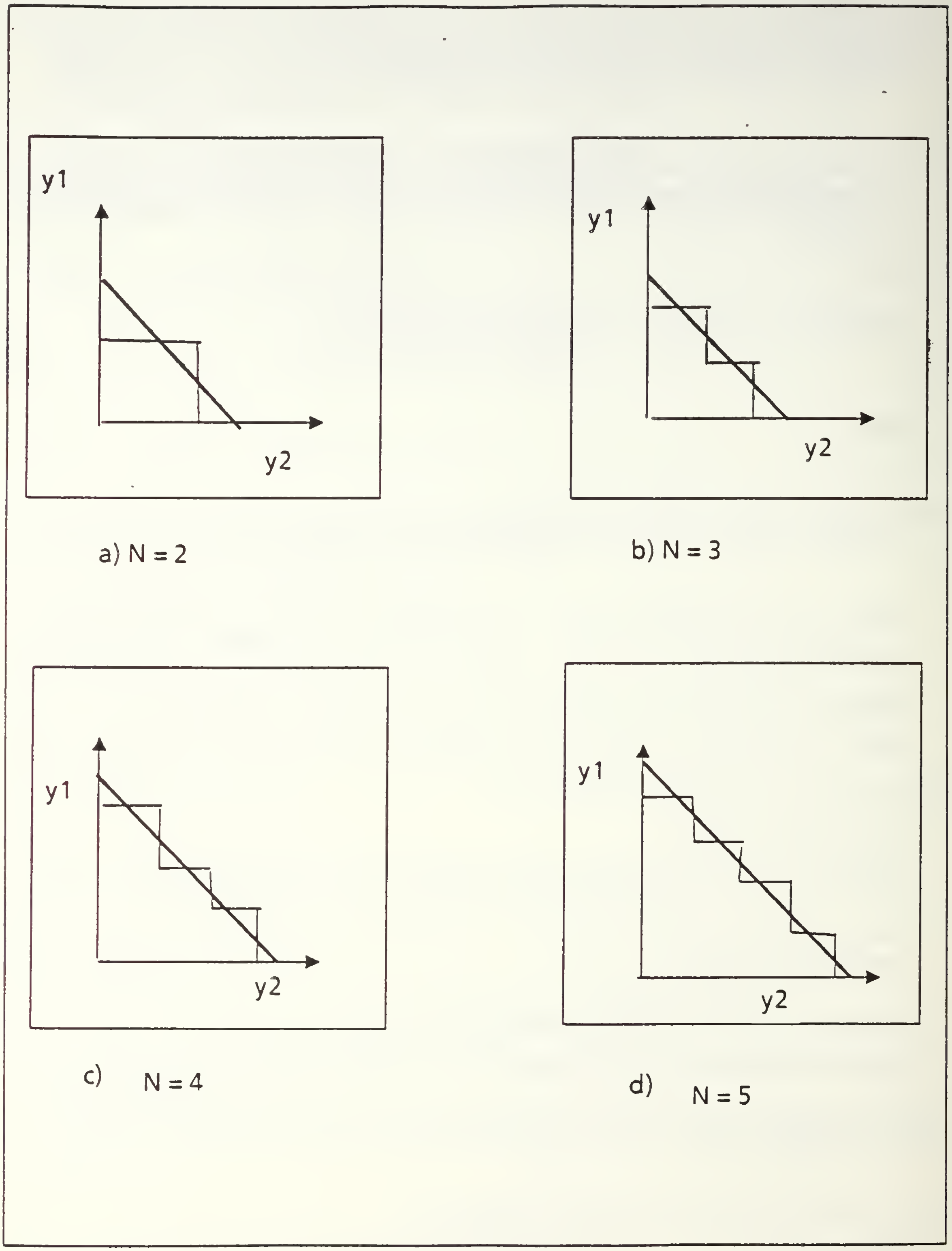

Figure 4.5 Approximation of the Threshold equation for Different Values of N, for Exponential Signals.

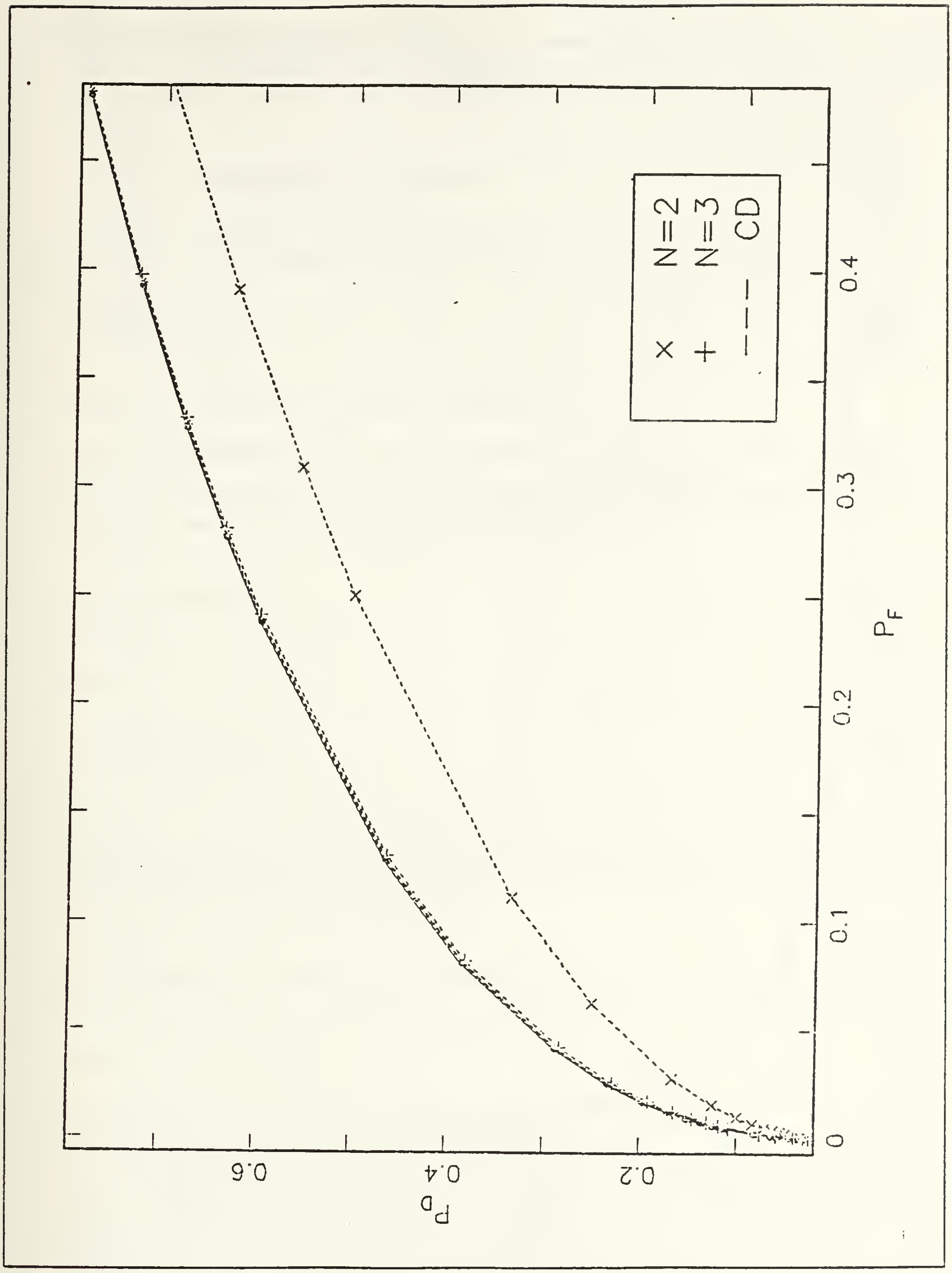

Figure 4.6 ROC curves for Exponential Signals.

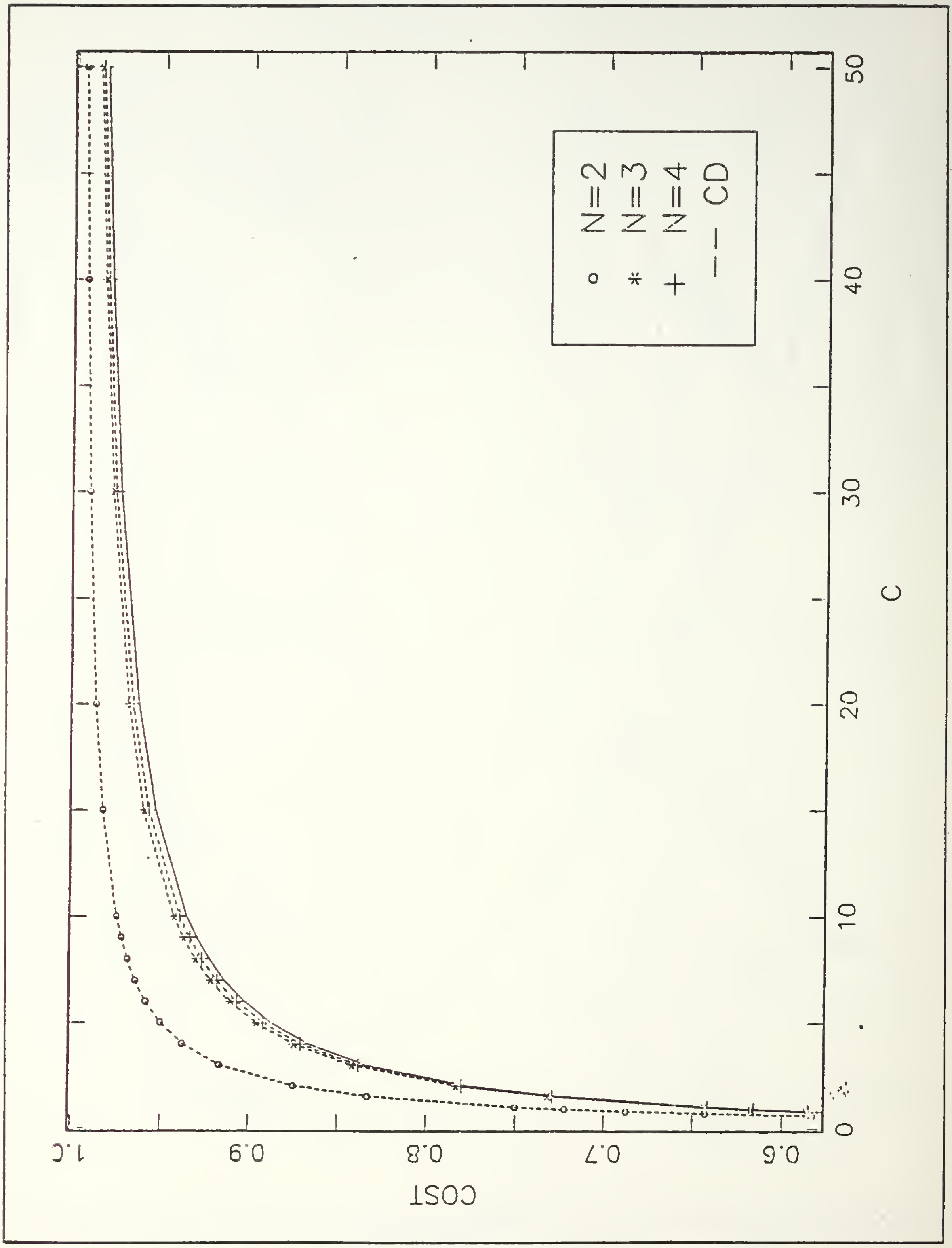

Figure 4.7 Average Detection Cost for Different Values of N for Exponential Signals.

Following the QD algorithm we have:

1. The CD system decision rule is a likelihood ratio test. The CD detector declares H_1 is true if

$$y_1^2 + y_2^2 < (1/2)\log[\sigma_1^2 /(\sigma_0^2 C)]/(\sigma_0^{-2} - \sigma_1^{-2})^{-1} \tag{4.22}$$

otherwise it will declare H_0 is true. The threshold equation is the circle

$$y_1^2 + y_2^2 = R_0^2 \tag{4.23}$$

where R_0^2 is the right hand side of inequality (4.22).

2. Possible approximations of the CD threshold equation are shown in Figure 4.8 .
3. Figure 4.8 a corresponds to 3-level quantizers. The corresponding probability of detection and probability of false alarm are given by;

$$P_d(3) = [\mathrm{erf}(-X/\sigma_1)]^2 \tag{4.24}$$

and

$$P_f(3) = [\mathrm{erf}(-X/\sigma_0)]^2 \tag{4.25}$$

where y_1 and y_2 are subdivided by the points X and -X. For the 5-level quantization approximation of Figure 4.8 b, the probability of detection and probability of false alarm are given by

$$P_d(5) = \mathrm{erf}(X_3/\sigma_1)\{2\mathrm{erf}(X_2/\sigma_1) - \mathrm{erf}(X_3/\sigma_1)\} \tag{4.26}$$

and

$$P_f(5) = \mathrm{erf}(X_3/\sigma_0)\{2\mathrm{erf}(X_2/\sigma_0) - \mathrm{erf}(X_3/\sigma_0)\} \tag{4.27}$$

where X_2 , X_3 ,$-X_3$ and $-X_2$ define the the quantization intervals of both y_1 and y_2 .

4. Inserting $P_f(3)$ and $P_d(3)$ into R in (3.3) and minimizing R with respect to X gives

$$\Lambda(X) = C\frac{\sigma_1}{\sigma_0}\frac{\mathrm{erf}(X/\sigma_0)}{\mathrm{erf}(X/\sigma_1)} \tag{4.28}$$

Also inserting $P_d(5)$ and $P_f(5)$ into (3.3) and minimizing R with respect to X_2 and X_3 gives;

$$\Lambda(X_2) = C\frac{\sigma_1}{\sigma_0}\frac{\mathrm{erf}(X_3/\sigma_0)}{\mathrm{erf}(X_3/\sigma_1)}\,. \tag{4.29}$$

$$\Lambda(X_3) = C\frac{\sigma_1}{\sigma_0}\frac{\mathrm{erf}(X_2/\sigma_0) - \mathrm{erf}(X_3/\sigma_0)}{\mathrm{erf}(X_2/\sigma_1) - \mathrm{erf}(X_3/\sigma_1)}\,. \tag{4.30}$$

Solution of these implicit equations in the quantizer parameters can be carried out by the method of successive substitution. The FORTRAN program to calculate them for any value of σ_0 and σ_1 is given in Appendix F.

Figure 4.9 shows the average detection cost vs. C for 3-level and 5-level quantizer systems. Detection cost of CD is also shown. The figure shows that the detection cost decreases dramatically using 5-level quantizers in comparison to 3-level quantizers. The cost of the CD system is only slightly lower than that of the 5-level quantizers.

Similar procedures can be carried out for the case of correlated observations. The CD curve in this case is an ellipse with principle axes passing through the origin. It can be approximated in a similar way as the circle.

D. CONCLUSION

The above examples show the uniform convergence of the Quantized Detection Algorithm to the Centralized Detection Algorithm. The Distributed Detection Algorithm is a special case of QD. It follows that Quantized Detection is an efficient utilization of bandlimited communication channels.

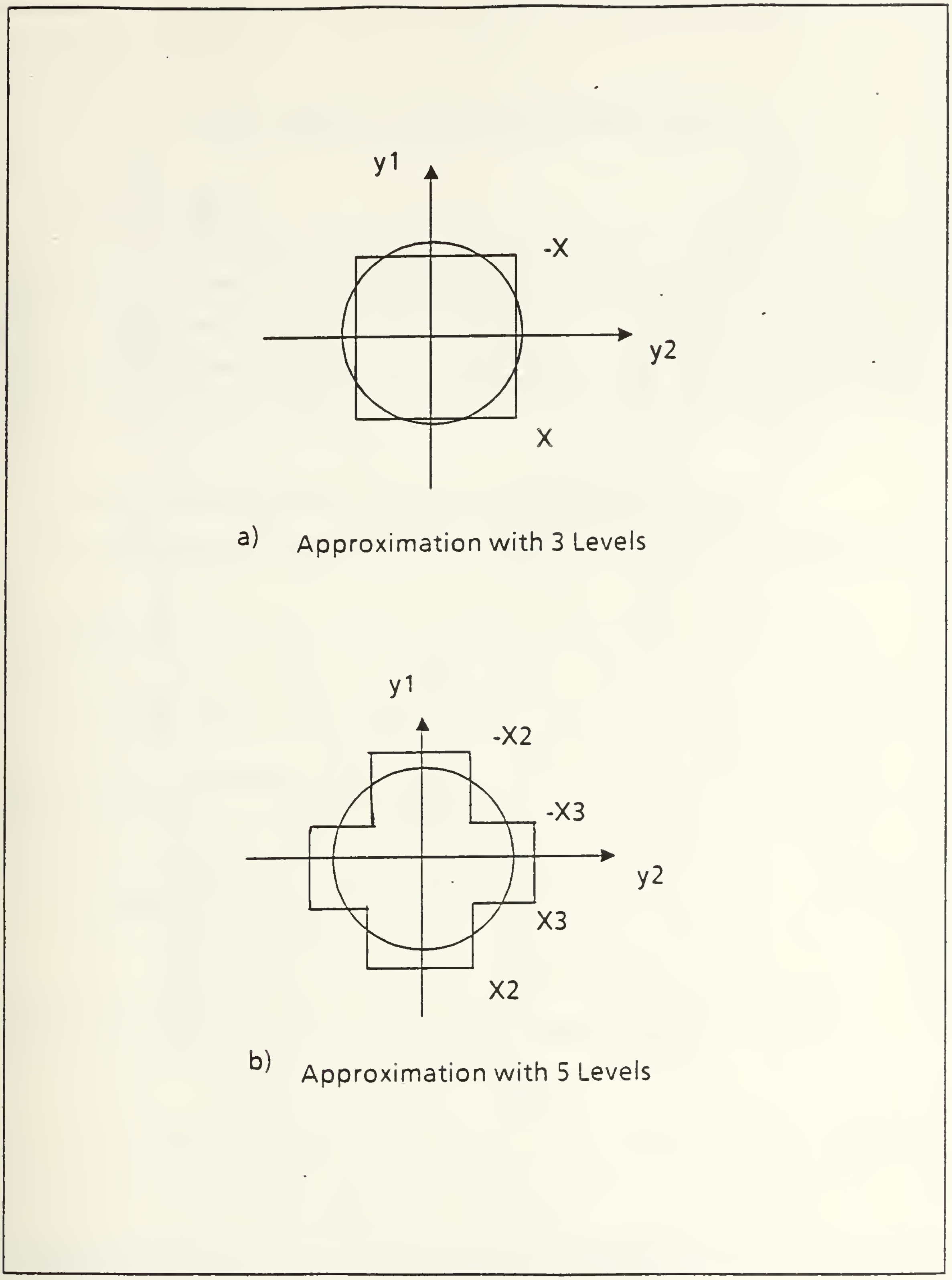

Figure 4.8 Possible Approximation Of the Threshold Equation.

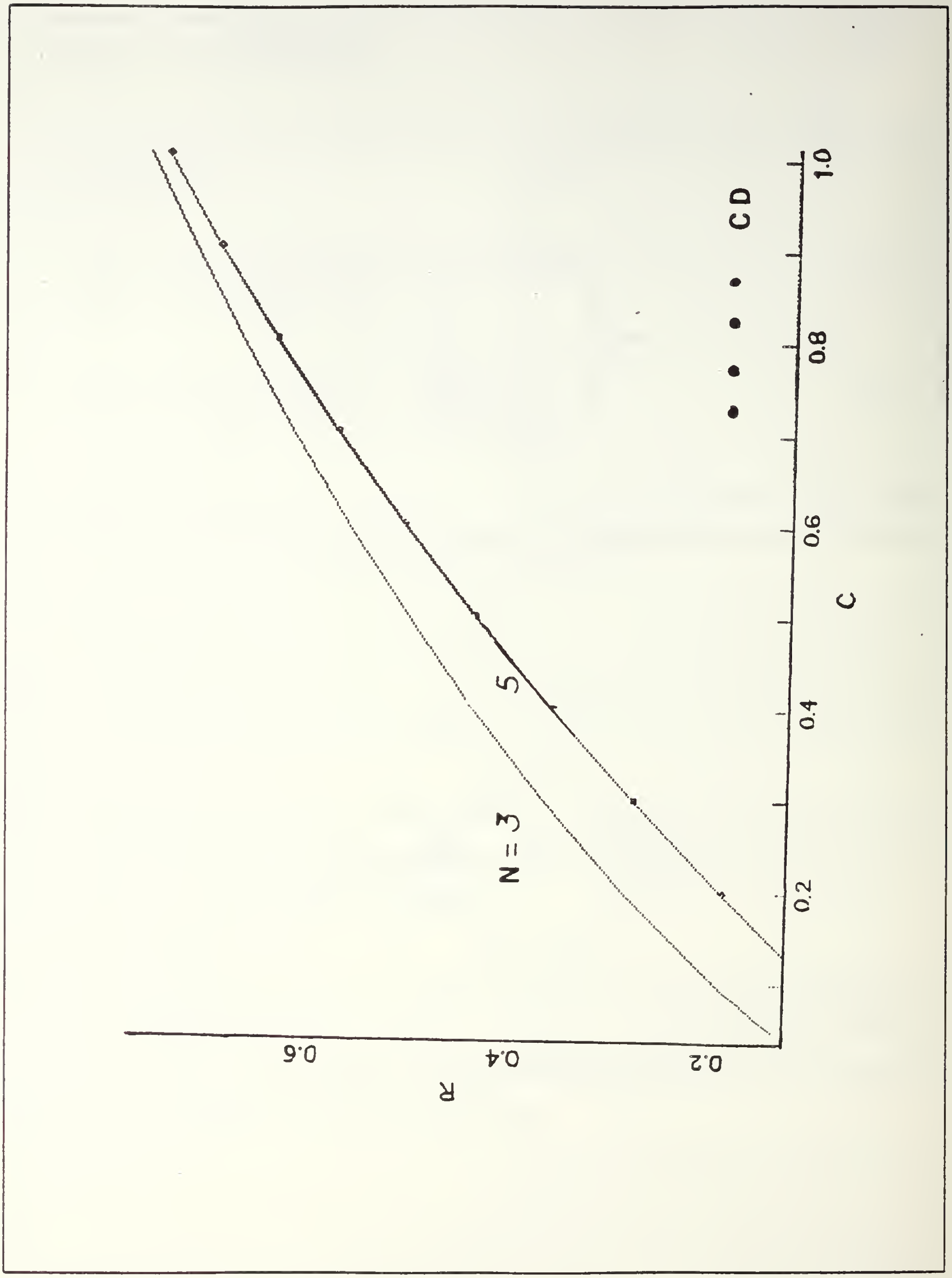

Figure 4.9 Average Detection Cost.

V. THE CASE OF VECTOR OBSERVATIONS

A. INTRODUCTION

In the previous two chapters the QD problem for the case of scalar sensor observations was solved. It is now time to extend the QD algorithm to the case where each local observation is a vector Y_i. The QD algorithm can be applied as long as the corresponding sufficient statistic for the centralized detection problem can be divided into local statistics to be quantized. Let us consider the gaussian case and put it in the previous framework.

B. QUANTIZED DETECTION WITH VECTOR OBSERVATIONS

For the structure of Figure 5.1 the observations at locations 1 and 2 are given by

$$H_0 : \underline{Y}_i = \underline{N}_i$$

and

$$H_1 : \underline{Y}_i = \underline{A}_i + \underline{N}_i \quad ,i=1,2. \tag{5.1}$$

Let us denote the observation vector by $\underline{Y}$

$$\underline{Y} = \begin{bmatrix} \underline{Y}_1 \\ \\ \underline{Y}_2 \end{bmatrix}. \tag{5.2}$$

The noise vector $\underline{N}$, given by

$$\underline{N} = \begin{bmatrix} \underline{N}_1 \\ \\ \underline{N}_2 \end{bmatrix}, \tag{5.3}$$

is multivariate gaussian with zero vector mean and covariance

$$\underline{R} = \begin{bmatrix} \underline{R}_1 & \underline{R}_{12} \\ \\ \underline{R}_{21} & \underline{R}_2 \end{bmatrix} \tag{5.4}$$

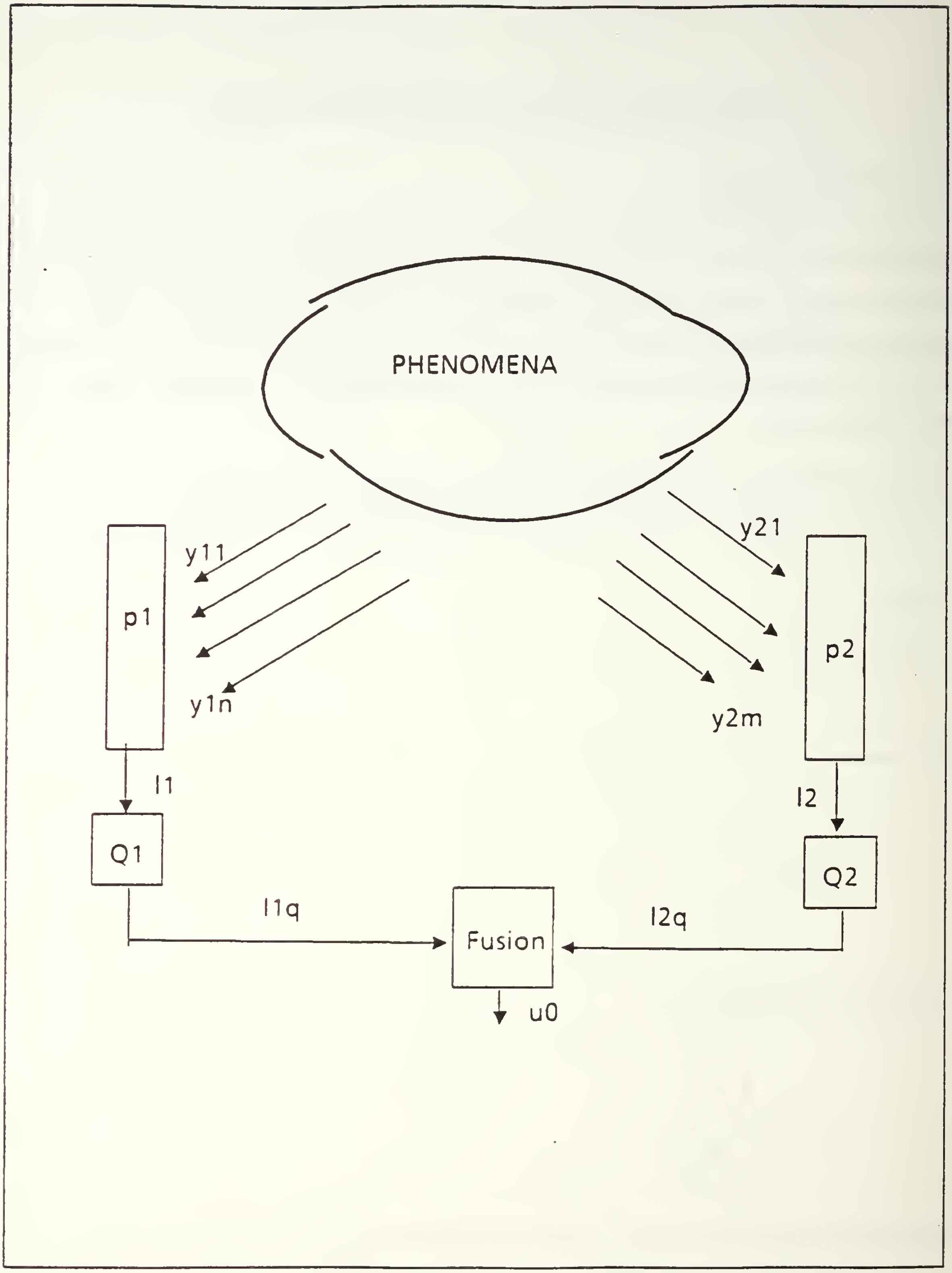

Figure 5.1 Vector Observations.

$\underline{R}_1$, $\underline{R}_2$ and $\underline{R}_{12}$ are the covariance matrices of the noises at locations 1 and 2 and their common covariance matrix. The signal vector $\underline{A}$ is given by

$$\underline{A} = \begin{bmatrix} \underline{A}_1 \\ \\ \underline{A}_2 \end{bmatrix}. \tag{5.5}$$

The CD system decides that $\underline{Y}$ belongs to Z_1 if [6]

$$\exp \{ (-1/2) [(\underline{Y} - \underline{A})' \underline{R}^{-1} (\underline{Y} - \underline{A}) - \underline{Y}' \underline{R}^{-1} \underline{Y}] \} \geq C. \tag{5.6}$$

The CD threshold equation can be written in the form

$$\underline{A}' \underline{R}^{-1} \underline{Y} = \log(C) - (1/2) \underline{A}' \underline{R}^{-1} \underline{A}. \tag{5.7}$$

Using the block matrix inversion lemma [25], (5.7) can be written in the form

$$\alpha \ \underline{Y}_1 + \beta \ \underline{Y}_2 = \log(C) - (1/2) \underline{A}' \underline{R}^{-1} \underline{A}. \tag{5.8}$$

In (5.8) α and β are given by

$$\alpha = \underline{A}'_1 (\underline{R}_1 - \underline{R}_{12} \underline{R}_2^{-1} \underline{R}_{21})^{-1} - \underline{A}'_2 (\underline{R}_2 - \underline{R}_{21} \underline{R}_1^{-1} \underline{R}_{21})^{-1} \underline{R}_{21} \underline{R}_1^{-1} \tag{5.9}$$

and

$$\beta = -\underline{A}'_1 (\underline{R}_1 - \underline{R}_{12} \underline{R}_2^{-1} \underline{R}_{21})^{-1} \underline{R}_{12} \underline{R}_2^{-1} + \underline{A}'_2 (\underline{R}_2 - \underline{R}_{21} \underline{R}_1^{-1} \underline{R}_{12})^{-1}. \tag{5.10}$$

Denoting

$$l_1 = \alpha \ \underline{Y}_1 \tag{5.11}$$

and

$$l_2 = \beta \underline{Y}_2 \tag{5.12}$$

(5.8) becomes

$$l_1 + l_2 = \log(C) - (1/2)\underline{A}'\underline{R}^{-1}\underline{A} \tag{5.13}$$

where l_1 and l_2 are bivariate gaussian with zero vector mean under hypothesis H_0. Under hypothesis H_1 their vector mean is

$$E\left\{\begin{bmatrix} l_1 \\ l_2 \end{bmatrix}\right\} = \begin{bmatrix} \alpha \underline{A}_1 \\ \beta \underline{A}_2 \end{bmatrix}. \tag{5.14}$$

Covariance of l_1 and l_2 is given by

$$Cov(l_1, l_2) = \begin{bmatrix} \alpha' \underline{R}_1 \alpha & \rho \\ \rho & \beta' \underline{R}_2 \beta \end{bmatrix} \tag{5.15}$$

In (5.15) ρ is given by

$$\rho = \alpha \underline{R}_{12} \beta \sqrt{\left[(\alpha \underline{R}_1 \alpha)^{-1} (\beta \underline{R}_2 \beta)^{-1}\right]}. \tag{5.16}$$

The distributed signal processing is to form local linear combinations l_1 and l_2, then quantize them as before. This processing is also shown in Figure 5.1.

C. SUMMARY

In this Chapter it is shown that the QD algorithm can be extended to the case of sensor vector observations. An application is the case of high quality local area communication and lower quality long distance communications. In this case sensor observations in local areas are gathered at a local processor to form the local sufficient statistics. Quantized local statistics are then sent to the global far away processor for fusion.

VI. OPTIMUM ESTIMATION USING QUANTIZED SENSOR OBSERVATIONS

A. INTRODUCTION

In the previous part of this thesis there are situations in which a group of observers make local decisions that, taken in combination determine the overall performance of a system. The observers may or may not be interconnected. However, even when they are, for a variety of considerations such as limitations on communications bandwidth, transmitter power,security, or perhaps the very nature of the observers themselves, only decisions may be interchanged between them and not all the observations upon which their decisions are based [1,5,26-33].

Another case of interest concerns the encoding of high resolution measurements for transmission between observers using a small number of bits. Here a remote observer must decide which of N possible discrete values best represents his observation. A second observer is to combine his local observations with the discrete data from the first in an optimum manner. In this chapter we consider the problem of regeneration of a remote sensor observation using its quantized representation and a local observation. The design of the quantizer at the remote sensor location and the optimum linear estimator to combine the quantized data with the local observation to minimize the expected mean square estimation error will be considered. Generalization of the results to the vector case is also shown.

B. THE LINEAR MINIMUM MEAN-SQUARE ESTIMATE OF Y_1

Consider the structure of Figure 6.1 in which the observation y_1 is quantized into y_{1q} by a quantization rule γ

$$\gamma : y_1 \rightarrow y_{1q}. \tag{6.1}$$

The quantized data y_{1q} is sent to sensor S_2 site.
The linear minimum mean square estimate of the observation y_1 from y_{1q} and y_2 is shown in (Appendix F) to be

$$\hat{y}_1 = \left[(1-\rho^2)\mu\, y_{1q} + \rho\,\frac{\sigma_1}{\sigma_2}(\eta-\rho^2)y_2\right]/(\eta-\mu^2\rho^2) \tag{6.2}$$

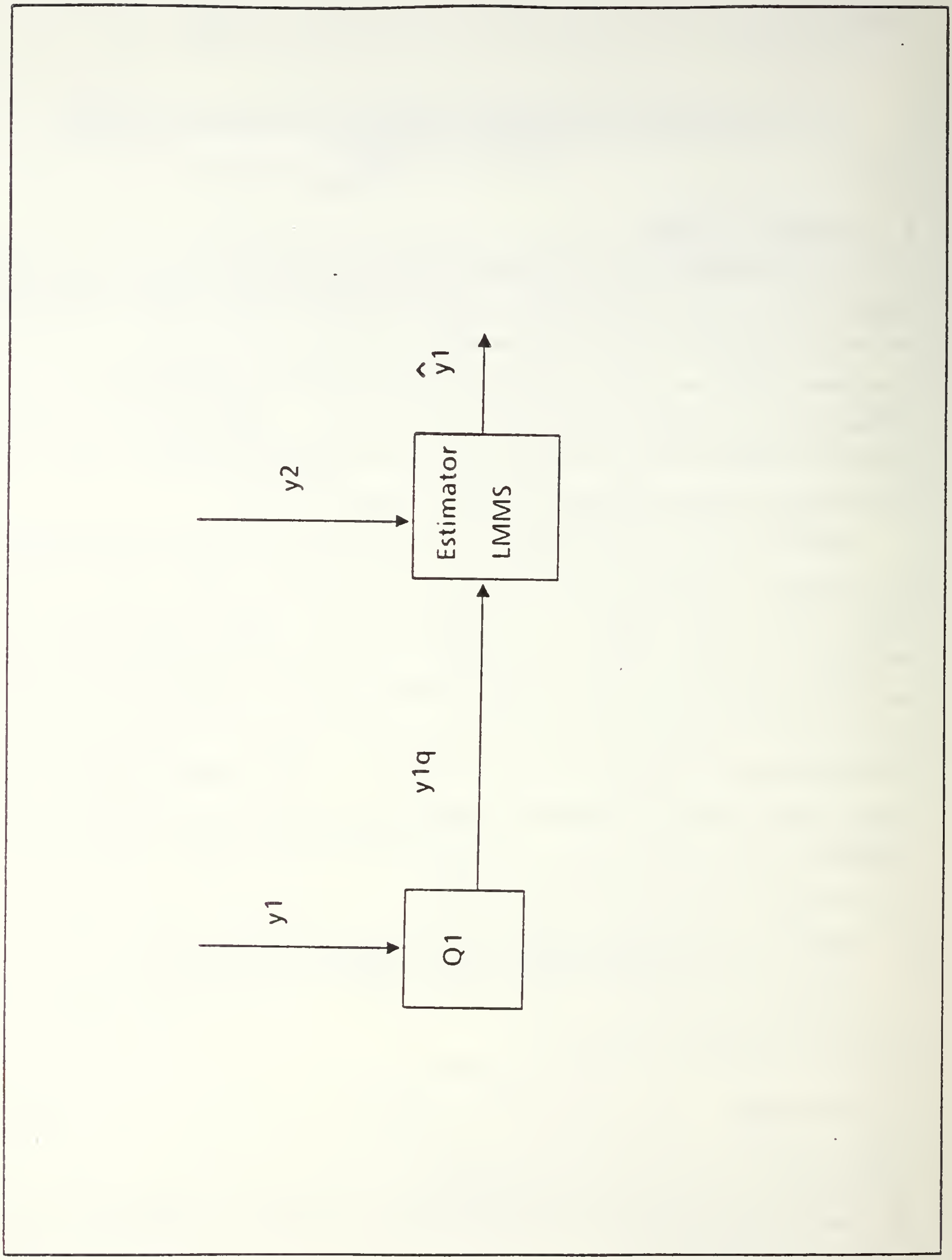

Figure 6.1 Estimation Using Quantized Sensor Observations.

where we have assumed that y_1 and y_2 are random variables with zero mean and variances

$$E\{y_i^2\} = \sigma_i^2 \quad ,i=1,2 \tag{6.3}$$

and correlation

$$\rho = E\{y_1 y_2\}/\sigma_1 \sigma_2 \tag{6.4}$$

The scalar quantities η and μ are parameters of the quantizer and are given by

$$\eta = (\sum_{k=1}^{N} P_k Q_k^2)/\sigma_1^2 \tag{6.5}$$

and

$$\mu = (\sum_{i=1}^{N} P_k Q_k C_k)/\sigma_1^2 . \tag{6.6}$$

N is the number of quantization levels and C_k and P_k are given by

$$C_k = (\int_{X_k}^{X_{k+1}} y_1 \; f(y_1)\, dy_1)/P_k \tag{6.7}$$

and

$$P_k = \int_{X_k}^{X_{k+1}} f(y_1)\, dy_1 \; . \tag{6.8}$$

Q_k is the k_{th} quantization value and $Q_k \in [.X_k, X_{k+1}]$. In (6.7) and (6.8), X_k ,k=1,2,....N, are the quantization interval end points, with

$X_1 = -\infty$ and $X_{N+1} = \infty$.

It is required to design a quantizer that minimizes the mean square estimation error.

The expected mean square error is given by [34]

$$E\{(y_1 - \hat{y}_{1q})^2\} = \sigma_1^2 - E\{y_1 \underline{Y}^t\} E\{\underline{Y}\,\underline{Y}^t\}^{-1} E\{\underline{Y}\, y_1\} \tag{6.9}$$

where $\underline{Y}$ isthe vector

$$\underline{Y} = [\, y_{1q} \;\; y_2 \,]^t . \tag{6.10}$$

Equation (6.9) can be written in the form (Appendix D)

$$E\{e^2\} = \sigma_1^2 (1-\rho^2)(1-\omega)/(1-\rho^2 \omega) \tag{6.11}$$

where ω is given by:

$$\omega = \mu^2 / \eta . \tag{6.12}$$

A plot of $E\{e^2\}/\sigma_1^2$ vs. ω is shown in Figure 6.2 for $\rho^2 = 0$, 0.25, 0.5, 0.75. The figure shows that the mean square error is decreasing with ω. Recall that the criterion is to minimize the mean square error.

Equivalently the problem now is to maximize ω over all quantization rules where

$$\omega = \frac{1}{\sigma_1^2} \frac{\left(\sum_{K=1}^{N} P_k Q_k C_k\right)^2}{\sum_{K=1}^{N} P_k Q_k^2} \tag{6.13}$$

Appling the Cauchy Inequality [35] to the numerator yields

$$\left(\sum_{K=1}^{N} P_k Q_k C_k\right)^2 \le \left(\sum_{k=1}^{N} P_k Q_k^2\right)\left(\sum_{k=1}^{N} P_k C_k^2\right) \tag{6.14}$$

with equality if and only if

$$Q_k = C_k . \tag{6.15}$$

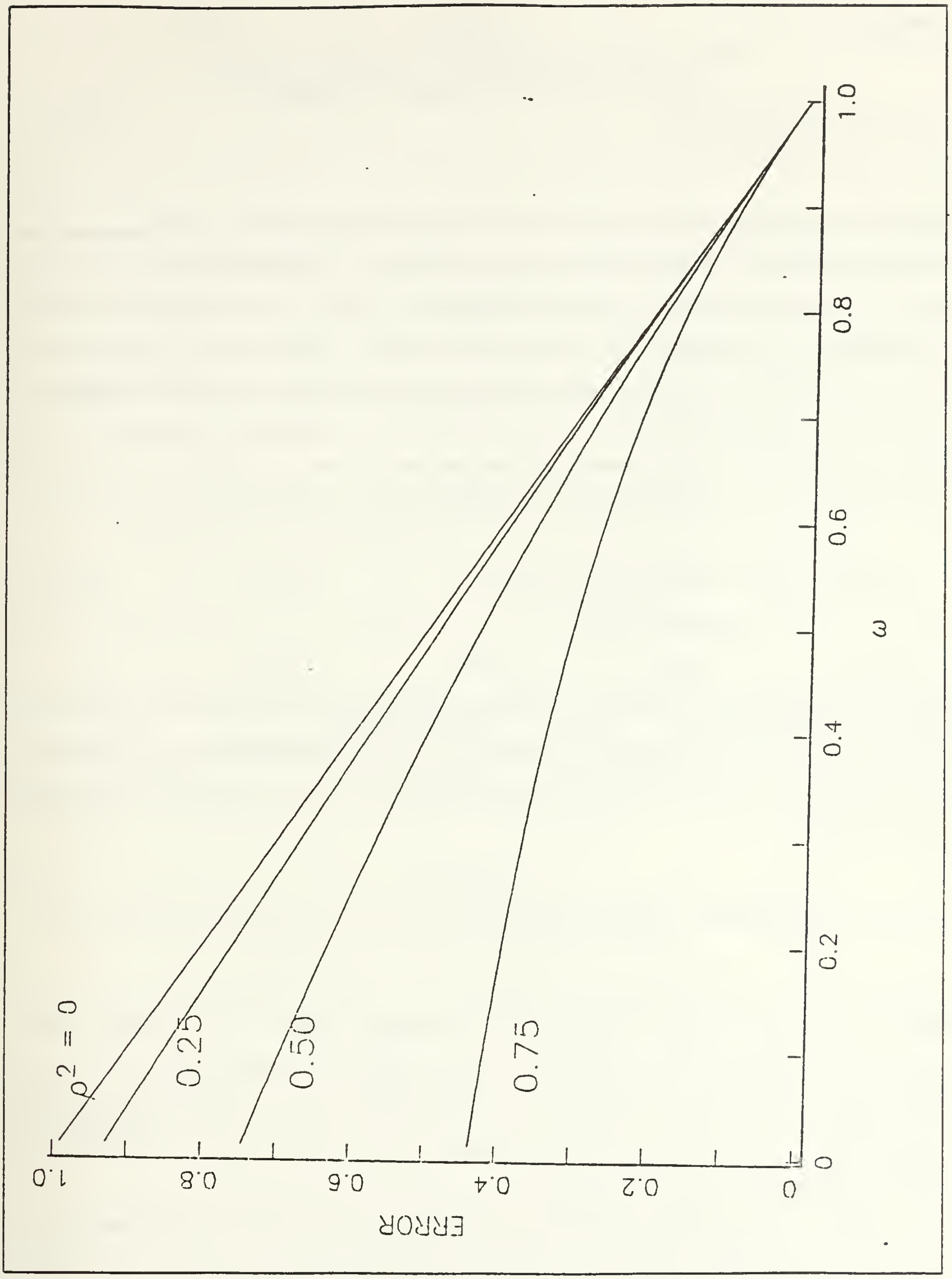

Figure 6.2 Relative Mean Square Estimation Error vs. ω.

Therefore

$$\omega \le \frac{1}{\sigma_1^2} \sum_{k=1}^{N} P_k Q_k^2 \tag{6.16}$$

gives an upper bound of ω. Equation (6.15) says we maximize ω , and thus minimize $E\{e^2\}$ by making the quantization level Q_k equal to the conditional mean of y_1 given that y_1 lies in the kth quantization interval. This is one of the conditions characterizing the classical Lloyd-Max quantizer [18,23.] There remains the problem of how to pick X_k ,k=1,2,...N, so that the upper bound of ω in (6.16) is maximum. Notice that the upper bound of ω is η. Therefore, the optimum quantizer will be a Lloyd-Max quantizer if we prove that maximizing η over all choices of the set of points $\{X_k\}$, k=1,2,...N, is equivalent to minimizing the distortion $E\{(y_1-y_{1q})^2\}$. Since

$$\begin{aligned} E\{(y_1-y_{1q})^2\} &= \sigma_1^2-2\sigma_1^2\eta+\sigma_1^2\eta \\ &= \sigma_1^2(1-\eta) \end{aligned} \tag{6.17}$$

then maximizing η will minimize the distortion $E\{(y_1-y_{1q})^2\}$ and vice versa. Since the Lloyd-Max quantizer is the optimum quantizer for minimum distortion it follows that it is also optimum for our problem. Accordingly choose X_k 's such that [23,18], (see also Appendix G)

$$X_k = \frac{Q_k + Q_{k-1}}{2} \qquad ,k=1,2,...N. \tag{6.18}$$

Equations (6.15) and (6.18) along with (6.7) completely design the quantizer [23,18]. Parameters of the Lloyd-Max quantizer can be calculated efficiently using the method of successive substitution (Appendix G). Values of $E\{e^2\}/\sigma_1^2$ vs. N are listed in Table 2 for ρ =0, 0.25, 0.5, 0.75. The table shows the exponential decay of the MMSE as the number of quantization levels increases.

Table 3 shows a comparison between the average number of bits per sample used in this system and another method in which the Maximum Output Entropy (MOE) Quantizer [36] is used. Huffman coding [37] is assumed for both quantizers.

TABLE 2

MINIMUM MEAN SQUARE ERROR VS. THE NUMBER OF QUANTIZATION LEVELS

N ρ	0	0.25	0.5	0.75
2	0.3634	0.3548	0.3241	0.2477
4	0.1175	0.1166	0.1131	0.1021
8	0.0345	0.0345	0.0342	0.0330
16	0.0095	0.0095	0.0095	0.0094
32	0.0025	0.0025	0.0025	0.0025
64	0.0006	0.0006	0.0006	0.0006
128	0.00016	0.00016	0.00016	0.00016

TABLE 3

COMPARISON OF THE AVERAGE NUMBER OF BITS IN THE MMSE AND THE MOE SYSTEMS

N	2	4	6	8
Optimum System	1	1.989	2.4768	2.8842
MOE	1	2	2.667	3

C. CONCLUSION

The trade off between performance and communication is clear from Table 2. For $\rho = 0.5$ the relative MMSE is 0.75 without communication. This corresponds to substituting $\omega = 0$ in (6.11). The relative MMSE decreases to 0.32 using one information bit per sample. The relative MMSE is 0.11 using two bits/sample. It is only 0.03 using 3 information bits/sample (N= 8) and is 0.00016 using 7 bits/sample. We also notice that for high number of quantization levels the estimation error is approximately equal to the the quantization error. This means that the estimator depends mainly on y_{1q} for fine quantization. For coarser quantization the estimator depends heavily on y_2 to reduce the MMSE. Table 3 shows that the designed system has considerable reduction in the number of bits per sample compared to the MOE quantizer system.

D. GENERALIZATION TO THE VECTOR CASE

In this section we will consider regeneration of a random vector $\underline{Y}_1$ from its quantized version $\underline{Y}_{1q}$ and a correlated continuous scalar y_2 . As an application consider a sensor S_2 monitoring the activities of N stations. Due to some considerations, perhaps of safety nature, only simple sensors can be placed near the stations. Because of other considerations, such as limited bandwidth communication channels, only quantized sensor measurements can be sent to the monitor. Specific examples can be the case of monitoring the states of a target in a far field or the positions of N targets in a multitracking problem [38,39]. Another example is to monitor the radiation levels outside of N nuclear reactors. A third example is monitoring the activities of N enemy transmitters.

Let us design the quantizers at the N sensor sites and the estimation rule at the monitor site so as to minimize the mean square error of each component of $\underline{Y}$. Let $\underline{Y}_1$,the sensor observation vector be given by;

$$\underline{Y}_1 = [\, y_{11}\; y_{12} \cdots y_{1N}\,]^t , \tag{6.19}$$

where y_{1j} is the j_{th} sensor observation ,j= 1,2,...,N. We will assume that components of $\underline{Y}$ are independent. i.e.

$$f(y_{1i} / y_{1j} , y_2) = f(y_i / y_2) \quad ,i \neq j,\ i,j = 1,2,...,N. \tag{6.20}$$

Under the above conditions, also y_{1i} and y_{1jq} are conditionally independent for $i \neq j$, so

$$f(y_{1i}/\underline{Y}_{1q},y_2) = f(y_{1i}/y_{1iq},y_2),\ i=1,2,\ldots,N. \tag{6.21}$$

The MMS estimate of $\underline{Y}_1$ given $\underline{Y}_{1q}$ is given by

$$\hat{\underline{Y}}_1 = E\{\underline{Y}_1/\underline{Y}_{1q},y_2\}. \tag{6.22}$$

Or

$$\hat{\underline{Y}}_1 = \begin{bmatrix} E\{y_{11}/y_{11q},y_2\} \\ E\{y_{12}/y_{12q},y_2\} \\ \cdot \\ \cdot \\ \cdot \\ E\{y_{1N}/y_{1Nq},y_2\} \end{bmatrix}. \tag{6.23}$$

Let us denote the error vector by $\underline{E}$, so

$$\underline{E} = [\,e_1\ e_2 \ldots e_N\,]^t \tag{6.24}$$

where e_i is the error in estimating y_i, $i=1,2,\ldots,N$. The MMS error covariance matrix is

$$E\{\underline{E}\,\underline{E}^t\} = E\{(\underline{Y}_1-\hat{\underline{Y}}_1)(\underline{Y}_1-\hat{\underline{Y}}_1)^t\}. \tag{6.25}$$

The trace of the error covariance matrix is given by

$$\mathrm{trace}(E\{\underline{E}\,\underline{E}^t\}) = \sum_{i=1}^{N} E\{e_i^2\} \tag{6.26}$$

where

$$e_i = y_i - E\{y_{1i}/y_{1iq},y_2\}. \tag{6.27}$$

Minimizing the trace of the covariance matrix in (6.26) is accomplished by minimizing each summand alone since every summand is nonnegative. Now assuming Linear Minimum Mean Square Estimation, the problem of minimizing $E\{e_i^2\}$ implies using the Lloyd-Max Quantizer to quantize y_{1i} as was shown previously.

In conclusion the Linear Minimum Mean Square Estimate of the observation vector $\underline{Y}$ implies using the Lloyd-Max quantizers at the sensor sites and the same linear combining considered in the scaler case at the central processor.

VII. SUMMARY, RESULTS, CONCLUSIONS AND SUGGESTIONS FOR FUTURE RESEARCHS

A. SUMMARY

This Thesis begins by listing some reasons why Distributed Signal Processing is more practical than Centralized Signal Processing. The status of Distributed Detection, an important case of Distributed Signal Processing, and its complexity are reviewed in Chapter I.

Chapter II deals with the problem of optimum fusion of local decisions into a global decision. The relationship between the optimum fusion rule and the ratio of error cost is shown. The dependance of the performance of the optimum fusion rule on the correlation coefficients between sensor observations is throughly analyzed. For higher values of the correlation coefficients the Distributed Detection system is shown to reduce to the detector of the highest signal-to-noise ratio.

A compromise technique between Centralized Detection and Distributed Detection, Quantized Detection, is suggested in Chapter III. The main issue of that chapter is to control the degree of centralization according to the communications channel constraints. The Quantized Detection technique replaces local detectors by quantizers and sets a global fusion rule that approximates the centralized decision rule. The algorithm matches the other techniques at extreme limits.

Chapter IV contains some specific applications of the Quantized Detection Algorithm for detection problems. A significant performance improvement is attained by replacing Distributed Detection with Quantized Detection with three quantization levels (one and half information bits per sample vice one information bit per sample).

Chapter V considers applicability of the Quantized Detection Algorithm to the case of vector observations. In this case local sufficient statistics are quantized in the same way as before.

Chapter VI deals with the regeneration of sensor observations from their quantized versions and another correlated observation. The local quantizers and the optimum linear data fusion are designed. We arrive at the following results and conclusions.

B. RESULTS

1. Detection with Distributed Sensors

a. Optimum Fusion Rules in Distributed Detection

The optimum fusion rule depends on the ratio of costs of different types of detection errors. For high cost of false alarm relative to the cost of missing the target the AND fusion rule is better than the OR fusion rule, and vice versa.

The performance of the optimum fusion rule depends on the degree of correlation between sensors. The performance degrades as the correlation coefficient increases. The worst performance of the optimum fusion rule is at and above a critical value of the correlation coefficient ρ_{cr} . In that region of correlation the best system employs only the detector of higher signal-to-noise ratio, ignoring the lower signal-to-noise ratio sensor entirely. The performance of the Distributed Detection system improves as the signal-to-noise ratio imbalance between sensors increases. However there is a large performance difference between the Centralized Detection and the Distributed Detection for values of the correlation coefficient above ρ_{cr}.

Below ρ_{cr} the performance of the Distributed Detection system improves as the correlation coefficient gets smaller. The best performance (lowest detection cost) of the Distributed Detection system is achieved at $\rho=-1$. Recall that the Centralized Detection system has perfect detection at $\rho=-1$. This is due to the efficient use of the information contained in two observations of positive signals and anticorrelated noise samples.

The large performance difference between Centralized Detection and Distributed Detection systems is due to the loss of information in the local detection processes. As a remedy to the performance degradation in Distributed Detection we have introduced the Quantized Detection Algorithm.

b. Quantized Detection

There is a great improvement in the system performance using Quantized Detection with three quantization levels in comparison to Distributed Detection. This performance difference between Quantized Detection and Distributed Detection decreases as the correlation between sensors increases.

The Quantized Detection algorithm is applicable to the case of vector observations and waveform observations. In those cases, the local sufficient statistics are to be quantized at the local processor and transmitted to the central site for fusion.

The Quantized Detection algorithm is implemented by the quantizers as local processors and a fusion rule, suggested by the Centralized Detection decision rule, at the central site. The quantizers used in the Quantized Detection algorithm are designed to minimize the detection cost.

2. Minimum Mean Square Estimation in Distributed Sensor Systems

Minimum mean square estimation in Distributed Sensor Systems involves the classical Lloyd-Max minimum distortion quantizers at the local levels and linear processing at the global central level. A faster iterative algorithm to calculate the Lloyd-Max quantizer parameters is the method of successive substitution. It also has more accurate results than previously reported techniques.

C. CONCLUSIONS

We conclude the following:

1. Global optimization of the Distributed Detection implies picking the fusion rule and corresponding local decisions that minimizes the detection cost.
2. The optimum fusion rule in Decentralized Detection depends on the correlation coefficient , the a priori probabilities and the ratio of costs.
3. For optimum fusion of two local unbalanced decisions there is a particular value of ρ that decides the optimum fusion rule.
4. For $\rho \leq \rho_{cr}$ OR fusion is better for higher cost of missing target while AND fusion is better for higher costs of false alarm.
5. For $\rho > \rho_{cr}$ the optimum fusion rule is to ignore the sensor of lower signal-to-noise ratio and optimize the decision of the higher signal-to-noise ratio sensor.
6. The poor performance of Distributed Detection compared to Centralized Detection is due to the loss of information at the local levels.
7. The Quantized Detection system matches the Distributed Detection system and the Centralized Detection system for the two extreme conditions of quantization. As the number of quantization levels increases the Quantized Detection converges to Centralized Detection.
8. The Quantized Detection algorithm has a tremendous improvement in performance over Distributed Detection even with only 3 quantization levels.
9. The performance difference between Quantized Detection and Distributed Detection increases as the correlation of the observations gets smaller.
10. In case of linear Centralized Detection threshold equations in the observation space, Distributed Detection and Centralized Detection are special cases of Quantized Detection.
11. The Quantized Detection algorithm can get the maximum allowable performance in the presence of communication constraints.

12. The Quantized Detection algorithm can be applied to arbitrary distributions for the observations.

13. The method of successive substitution is applicable to the design of many types of quantizers. It has a simple programming procedure and very accurate results.

D. SUGGESTIONS FOR FUTURE RESEARCH

The following are some areas the Quantized Detection algorithm can extend to:

1. Optimum detection using quantized sensor observations for the case of unknown signal in noise.
2. Detection of M-ary phenomena using quantized sensor data.
3. Utilizing the Quantized Detection algorithm over noisy channels.
4. Illustration of the relation between the complexity in some suitable units and the amount of information delivered to the fusion center.
5. Utilizing the Quantized Detection algorithm to meet the Neyman-Pearson criterion.
6. Extension of Distributed Detection and Quantized Detection to more than two sensors with correlated observations and unequal SNR's.
7. Development of general principles for parsing fusion rules given a Centralized Detection surface in N-dimensional space.
8. Application of the Quantized Detection method to target detection,classification and tracking using distributed sensors.

APPENDIX A

PROBABILITY OF DETECTION AND PROBABILITY OF FALSE ALARM OF THE PRIMARY DECISION MAKER

Given that the Primary Decision Maker (PDM) receives Q_j (the j_{th} quantization level of the Consultant observation y_1), and that its own observation is y_2, its observation space is divided into two decision subspaces Z_{1j} and Z_{0j}. Let us denote the conditional probability of detection and probability of false alarm given Q_j by P_{dj} and P_{fj} respectively. P_{dj} and P_{fj} are given by:

$$P_{dj} = \Pr(\text{Declare } H_1 / y_{1q} = Q_j, H_1 \text{ is true}) \tag{A.1}$$

and

$$P_{fj} = \Pr(\text{Declare } H_1 / y_{1q} = Q_j, H_0 \text{ is true}). \tag{A.2}$$

These can be expressed as:

$$P_{dj} = \int_{y_2 \in Z_{1j}} f(y_2 / y_{1q} = Q_j, H_1)\, dy_2 \tag{A.3}$$

and

$$P_{fj} = \int_{y_2 \in Z_{1j}} f(y_2 / y_{1q} = Q_j, H_0)\, dy_2 \tag{A.4}$$

or equivalently as ,

$$P_{dj} = \int_{y_2 \in Z_{1j}} \int_{y_1} f(y_1, y_2 / y_{1q} = Q_j, H_1)\, dy_1\, dy_2 \tag{A.5}$$

and

$$P_{fj} = \int_{y_2 \in Z_{1j}} \int_{y_1} f(y_1, y_2 / y_{1q} = Q_j, H_0)\, dy_1\, dy_2 \tag{A.6}$$

But $f(y_1, y_2 / Q_j, H_i)$ is given by [40]

$$f(y_1, y_2 / Q_j, H_i) = \begin{cases} f(y_1, y_2 / H_i)/Pr(Q_j / H_i) & , y_1 \in [X_j, X_{j+1}] \\ 0 & \text{otherwise} \end{cases} \tag{A.7}$$

where $Pr(Q_j / H_i)$ is the probability of the j_{th} quantization level of y_1 under hypothesis H_i. It is given by:

$$Pr(Q_j/H_i) = \int_{X_j}^{X_{j+1}} f(y_1/H_i)\, dy_1 \quad , j = 1,2,...N \ , i = 0,1 \tag{A.8}$$

The probability of detection and probability of false alarm are now

$$P_d = \sum_{j=1}^{N} Pr(Q_j / H_1) P_{dj} \tag{A.9}$$

and

$$P_f = \sum_{j=1}^{N} Pr(Q_j / H_0) P_{fj} \tag{A.10}$$

Inserting (A.5 and (A.6) into (A.9) and (A.10) yields

$$P_d = \sum_{i=1}^{N} \int_{X_i}^{X_{i+1}} \int_{y_2 \in Z_{1j}}^{\infty} f(y_1, y_2 / H_1)\, dy_1\, dy_2 \tag{3.4}$$

and

$$P_f = \sum_{i=1}^{N} \int_{X_i}^{X_{i+1}} \int_{y_2 \in Z_{1j}}^{\infty} f(y_1, y_2 / H_0)\, dy_1\, dy_2 \tag{3.5}$$

where Z_{1j} is the decision region Z_1 given that $y_1 \in [X_j, X_{j+1}]$.

APPENDIX B

PROGRAM LISTING TO CALCULATE PARAMETERS OF THE N AND THE (N + 1)-LEVEL QUANTIZERS

In this appendix we give a program listing to calculate the parameters of the N and the (N + 1)-level quantizers in configuration A

```
C*******************************************************************
C   THIS PROGRAM CALCULATES THE OPTIMUM N-LEVEL AND (N+1)-level
c QUANTIZER PARAMETERS
C FOR A SYSTEM OF TWO QUANTIZERS AND THEIR FUSION CENTER.
C THE RATIO OF COSTS C RANGES FROM 0.1 AND 10 .
C THE VALUE OF N =2,3,4,5 AND 6.
C THE PROGRAM USES THE MODIFIED METHOD OF SUCCESSIVE SUBSTITUTIONS .
C*******************************************************************
C               ******************************
        REAL*8 X(9),T(9),XX(9),TT(9),C,S1,S2,A,B,A12,A21,R,C1(20),R1(2)
       1,AERR,RERR,ERROR,PD,PF,PD1(20,9,6),PF1(20,9,6),PD2
       1,PF2,PDC,PFC,X2,R3(20,2,6),X11,DCADER,F1,F2,APD,APF,A1,A2,AP
        INTEGER K,N,I,P,N1,N2,M,J,MX
        EXTERNAL F1,F2
        COMMON X11,R
        DATA C1/10.0D0,9.0D0,8.0D0,7.0D0,6.0D0,5.0D0,4.0D0,3.0D0,2.0D0,
       11.50D0,
       11.0D0,.90D0,.80D0,.70D0,.60D0,.50D0,.40D0,.30D0,.20D0,0.10D0/
        A1=1.0D0
        A2=2.0D0
        S1=1.0D0
        S2=1.0D0
        R1(1)=0.600D0
        R1(2)=0.8000D0
        AERR=0.0D0
        RERR=0.00010D0
c*****************************************************
c      INITIAL GUESS
        XX(1)=-9.50000D0
        XX(2)=-1.9000D0
        XX(3)=-1.50000D0
        XX(4)=-0.89000D0
        XX(5)=00.50000D0
        XX(6)=0.09000D0
        XX(7)=00.99000D0
        XX(8)=01.50000D0
        XX(9)=2.09000D0
C      TT(1)=4.67000D0
C      TT(2)=3.89000D0
C      TT(3)=2.67000D0
C      TT(4)=2.29000D0
C      TT(5)=1.90000D0
C      TT(6)=1.49000D0
C      TT(7)=1.09000D0
C      TT(8)=00.50000D0
C      TT(9)=0.19000D0
C*******************************************************
C   THE FOLLOWING INITIAL VALUES OF T'S CORRESPOND TO THE CASE
C   OF CORRELATION COEFFICIENT GREATER THAN A1/A2
        TT(9)=4.67000D0
        TT(8)=3.89000D0
        TT(7)=2.67000D0
        TT(6)=2.29000D0
        TT(5)=1.90000D0
        TT(4)=1.49000D0
        TT(3)=1.09000D0
```

```
      TT(2)=00.50000D0
      TT(1)=0.19000D0
      DO 500 N=2,2
         WRITE(9,60)N
         DO 600  I=6,9
c****************************************
c        INPUT VALUE(S) OF THR CORRELATION COEFFICIENTS
         R=0.10D0*DFLOAT(I)
         WRITE(9,61)R
C        DO 200 M=1,20
            M=1
            C=C1(M)
            WRITE(9,61)C
c****************************************
c        INPUT MAXIMUM NUMBER OF ITERATIONS HERE
            MX=250
            K=0
 5          K=K+1
            IF (K .GT. MX) GO TO 10
            IF (K .GT.1) X(N)=XX(N)
            IF (K .GT.1) T(N)=TT(N)
            IF (K .GT. 1)X(N)=XX(N)
            IF (K .GT. 1)T(N)=TT(N)
            A=(S1*XX(2)-R*S2*TT(1))/(S1*S2*DSQRT((1.0D0-R**2)*2.0D0))
               A12=(S2*A1-R*S1*A2)/(S1*S2*DSQRT((1.0D0-R**2)*2.0D0))
               A21=(S1*A2-R*S2*A1)/(S1*S2*DSQRT((1.0D0-R**2)*2.0D0))
            TT(1)=(A2*A2/2.0D0+DLOG(C)+DLOG((DERFC(A)-
     1         2.0D0)/(DERFC(A-A12)-2.0D0)))/A2
            T(1)=TT(1)
            A=(S1*TT(2)-R*S2*XX(2))/(S1*S2*DSQRT((1.0D0-R**2)*2.0D0))
            B=(S2*T(1)-R*S1*XX(2))/(S2*S1*DSQRT((1.0D0-R**2)*2.0D0))
            XX(2)=(A1*A1/2.0D0+DLOG(C)+DLOG((DERFC(A)-
     1      DERFC(B))/(DERFC(A-A21)-DERFC(B-A21))))/A1
            X(2)=XX(2)
            IF (N .EQ. 2) GO TO 17
            DO 15  P=2,N-1
               X(P)=XX(P)
               A=(S1*X(P)-R*S2*T(P))/(S1*S2*DSQRT((1.0D0-R**2)*2.0D0))
               B=(S2*X(P+1)-R*S1*T(P))/(S2*S1*DSQRT((1.0D0-R**2)*2.0D0))
               TT(P)=(A2*A2/2.0D0+DLOG(C)+DLOG((DERFC(A)-
     1         DERFC(B))/(DERFC(A-A12)-DERFC(B-A12))))/A2
               T(P)=TT(P)
             A=(S1*T(P+1)-R*S2*X(P+1))/(S1*S2*DSQRT((1.0D0-R**2)*2.0D0))
               B=(S2*T(P)-R*S1*X(P+1))/(S2*S1*DSQRT((1.0D0-R**2)*2.0D0))
               XX(P+1)=(A1*A1/2.0D0+DLOG(C)+DLOG((DERFC(A)-
     1         DERFC(B))/(DERFC(A-A21)-DERFC(B-A21))))/A1
 15   CONTINUE
 17  CONTINUE
      TT(N)=(A2*A2/2.0D0+DLOG(C)+DLOG(DERFC((XX(N)-R*TT(N))/
     1DSQRT((1.0D0-R**2)*2.0D0))
     1/DERFC((XX(N)-A1-R*(TT(N)-A2))/
     1DSQRT((1.0D0-R**2)*2.0D0))
     1))/A2
C*****************************************************************
C CHECKING THE ACCURACY
C INPUT REQUIRED PRECISION HERE
      AP=0.10d-07
      IF((DABS(T(N)-TT(N)) .GT. AP).OR.(DABS(X(N)-XX(N)) .GT.
     1           AP))GO TO 5

 10    CONTINUE
      X(1)=-10.0D0
      X(N+1)=10.0D0
      APD=0.0D0
      APF=0.0D0
      DO 81 Q=1,N
         A=X(Q)
         B=X(Q+1)
         X11=T(Q)
         APD=APD+0.50D0*DCADRE(F1,A,B,AERR,RERR,ERROR,IRE1)
```

```
           IF (IRE1 .NE. 0) WRITE(6,60)IRE1
           APF=APF+0.50D0*DCADRE(F2,A,B,AERR,RERR,ERROR,IRE1)
           IF (IRE1 .NE. 0) WRITE(6,60)IRE1
  81  CONTINUE
            PD1(M,I,N)=APD
            PF1(M,I,N)=APF
            R3(M,I,N)=1.0D0+C1(M)*APF-APD
C*****************************************************
C     QUANTIZER PARAMETERS
        DO 120 J=1,N
            WRITE(9,62) X(J),T(J)
 120   CONTINUE
C      WRITE(9,90)
C200  CONTINUE
C      WRITE(9,90)
C      WRITE(9,90)
 600  CONTINUE
C      WRITE(9,90)
C      WRITE(9,90)
C      WRITE(9,90)
 500  CONTINUE
          DO 501 J=1,2
            DO 502 I=1,20
C***********************************************************
C              OUTPUT DETECTION COST
                 WRITE(10,61)C1(I),( R3(I,J,N) ,N=2,6)
C***********************************************************
C -            OUTPUT PROBABILITY OF DETECTION AND PROBABILITY OF
C              FALSE ALARM FOR DIFFERENT VALUES OF N
                 WRITE(8,61)( PD1(I,J,N),PF1(I,J,N) ,N=2,6)
 502      CONTINUE
 501  CONTINUE
 60    FORMAT(1X,I4,4(1X,F15.8))
C90    FORMAT(2X,'CON A ************************************')
 61    FORMAT(1X,10(1X,F6.4))
 62    FORMAT(1X,2(1X,F15.8))
        STOP
        END
       FUNCTION F1(X)
       REAL*8 X,F1,A1,A2,R,X11,F11,F12
       COMMON X11,R
       A1=1.0D0
       A2=2.0D0
       F11=DEXP(-(X-A1)**2/2.0D0)/
      1DSQRT(8.0D0*DATAN(1.0D0))
       F12=DERFC((X11-A2-(X-A1)*R)/
      1(DSQRT(2.0D0*(1.0D0-R**2))))
       F1=F11*F12
       RETURN
       END
       FUNCTION F2(X)
       REAL*8 X,F2,R,X11,F11,F12
       COMMON X11,R
       F11=DEXP(-X**2/2.0D0)/
      1DSQRT(8.0D0*DATAN(1.0D0))
       F12=DERFC((X11-X*R)/
      1DSQRT(2.0D0*(1.0D0-R**2)))
       F2=F11*F12
       RETURN
       END
```

APPENDIX C

PROGRAM LISTING TO CALCULATE PARAMETERS OF THE TWO QUANTIZERS

In this appendix we give a program listing to calculate the parameters of the two N-level quantizers in Configuration B.

```
C*****************************************************************
C   THIS PROGRAM CALCULATES THE OPTIMUM N-LEVEL QUANTIZER PARAMETERS
C FOR A SYSTEM OF TWO QUANTIZERS AND THEIR FUSION CENTER .
C THE CORRELATION COEFFICIENT IS ASSUMED TO BE LESS THAN A1/A2.
C FOR THE CORRELATION COEFFICIENT IS GREATER THAN A1/A2 THE
C THE PROGRAM NEEDS SLITE MODIFICATIONS ACCORDING TO THE QD ALGORITHM.
C THE RATIO OF COSTS C RANGES FROM 0.1 AND 10.
C THE VALUE OF N =2,3,4,5 AND 6.
C THE PROGRAM USES THE MODIFIED METHOD OF SUCCESSIVE SUBSTITUTIONS .
C*****************************************************************
       REAL*8 X(8),T(8),XX(8),TT(8),A1,A2,S1,S2,R,T12,T21,AA2,A,B
      1,AERR,RERR,ERROR,PD,PF,PD1(20,9,6),PF1(20,9,6),PD2
      1,PF2,PDC,PFC,C1(20),A21,A12,C,X2
      1,X11,DCADER,F1,F2,APD,APF,R3(20,2,6)
       INTEGER K,N,I,IER1,IER2,M,P,Q,L
       EXTERNAL F1,F2
       DATA C1/10.0D0,9.0D0,8.0D0,7.0D0,6.0D0,5.0D0,4.0D0,3.0D0,2.0D0,
      11.50D0,
      11.0D0,.90D0,.80D0,.70D0,.60D0,.50D0,.40D0,.30D0,.20D0,0.10D0/
       COMMON X11,R
C*************************************************************
C INPIT SIGNALS HERE
       A1=4.0D0
       A2=2.0D0
C INPIT VARIANCE HERE
       S1=1.0D0
       S2=1.0D0
C************************************************************
C INITIAL GUESS OF THE PARAMETERS
       XX(1)=-0.50000D0
       XX(2)=0.89000D0
       XX(3)=01.50000D0
       XX(4)=2.89000D0
       XX(5)=03.50000D0
       XX(6)=4.09000D0
C************************************************************
C  INITIAL VALUES OF T'S FOR CORRELATION COEFFICIENT GREATER THAN A1/A2
       TT(1)=-0.67000D0
       TT(2)=0.89000D0
       TT(3)=01.67000D0
       TT(4)=2.89000D0
       TT(5)=03.50000D0
       TT(6)=4.89000D0
C************************************************************
C  INITIAL VALUES OF T'S FOR CORRELATION COEFFICIENT LESS THAN A1/A2
C  PUT TT(1) > TT(2)>.....>TT(N)
C*************************************************************
       AERR=0.0D0
       RERR=0.00010D0
       DO 500 N=1,5
       DO 11 I=1,2
       R=DFLOAT(I-1)*0.250D0
       DO 20 M=1,20
       C=C1(M)
       K=0
 5     K=K+1
```

```
C          WRITE(6,60)K,T1,T11,T2,T22
       IF (K .GT.100) GO TO 10
       DO 25 L=1,N
       T(L)=TT(L)
       X(L)=XX(L)
 25    CONTINUE
       XX(1)=(A1*A1/2.0D0+DLOG(C)+DLOG(DERFC((T(1)-R*X(1))/
      1DSQRT((1.0D0-R**2)*2.0D0))
      1/DERFC((T(1)-A2-R*(X(1)-A1))/
      1DSQRT((1.0D0-R**2)*2.0D0))
      1))/A1
         IF (N .EQ. 1) GO TO 16
         A21=(S1*A2-R*S2*A1)/(S1*S2*DSQRT((1.0D0-R**2)*2.0D0))
         A12=(S2*A1-R*S1*A2)/(S1*S2*DSQRT((1.0D0-R**2)*2.0D0))
         DO 15  P=1,N-1
            A=(S1*X(P)-R*S2*T(P))/(S1*S2*DSQRT((1.0D0-R**2)*2.0D0))
           B=(S2*X(P+1)-R*S1*T(P))/(S2*S1*DSQRT((1.0D0-R**2)*2.0D0))
        TT(P)=(A2*A2/2.0D0+DLOG(C)+DLOG((DERFC(A)-
      1          DERFC(B))/(DERFC(A-A12)-DERFC(B-A12))))/A2
            A=(S1*T(P+1)-R*S2*X(P+1))/(S1*S2*DSQRT((1.0D0-R**2)*2.0D0))
           B=(S2*T(P)-R*S1*X(P+1))/(S2*S1*DSQRT((1.0D0-R**2)*2.0D0))
        XX(P+1)=(A1*A1/2.0D0+DLOG(C)+DLOG((DERFC(A)-
      1 DERFC(B))/(DERFC(A-A21)-DERFC(B-A21))))/A1
 15             CONTINUE
 16    CONTINUE
       TT(N)=(A2*A2/2.0D0+DLOG(C)+DLOG(DERFC((X(N)-R*T(N))/
      1DSQRT((1.0D0-R**2)*2.0D0))
      1/DERFC((X(N)-A1-R*(T(N)-A2))/
      1DSQRT((1.0D0-R**2)*2.0D0))
      1))/A2
       IF((DABS(T(N)-TT(N)) .GT. 0.10D-05).OR.(DABS(X(N)-XX(N)) .GT.
      1            0.10D-05))GO TO 5
 10     CONTINUE
       X(N+1)=10.0D0
       APD=0.0D0
       APF=0.0D0
       DO 81 Q=1,N
           A=X(Q)
           B=X(Q+1)
           X11=T(Q)
           APD=APD+0.50D0*DCADRE(F1,A,B,AERR,RERR,ERROR,IRE1)
           IF (IRE1 .NE. 0) WRITE(6,60)IRE1
           APF=APF+0.50D0*DCADRE(F2,A,B,AERR,RERR,ERROR,IRE1)
           IF (IRE1 .NE. 0) WRITE(6,60)IRE1
  81   CONTINUE
            PD1(M,I,N)=APD
            PF1(M,I,N)=APF
          R3(M,I,N)=1.0D0+C1(M)*PF1(M,I,N)-PD1(M,I,N)
C*************************************************************
C OUTPUT SYSTEM PARAMETERS
        DO 120 J=1,N
            WRITE(9,62) X(J),T(J)
 120    CONTINUE
C          WRITE(9,61)C,T(1),T(2),APD,APF
C          WRITE(8,61)APD,APF
C          WRITE(6,61)C,X(1),X(2),APD,APF,PDC,PFC
 20     CONTINUE
 11        CONTINUE
C       WRITE(9,90)
        WRITE(8,90)
  500  CONTINUE
C*************************************************************
C OUTPUT AVERAGE COST
           DO 501 J=1,2
           DO 502 I=1,20
           WRITE(10,61)C1(I),( R3(I,J,N) ,N=1,5)
C*************************************************************
C OUTPUT  PROB. OF DETECTION AND PROB. OF FALSE ALARM
           WRITE(8,61)( PD1(I,J,N),PF1(I,J,N) ,N=1,5)
 502   CONTINUE
```

```
        WRITE(8,90)
501   CONTINUE
60      FORMAT(1X,I2,9(1X,F6.4))
90      FORMAT(2X,'CON B  ************************************')
61      FORMAT(1X,10(1X,F6.4))
62      FORMAT(1X,2(1X,F15.8))
        STOP
        END
      FUNCTION F1(X)
      REAL*8 X,F1,A1,A2,R,X11,F11,F12
      COMMON X11,R
      A1=4.0D0
      A2=2.0D0
      F11=DEXP(-(X-A1)**2/2.0D0)/
     1DSQRT(8.0D0*DATAN(1.0D0))
      F12=DERFC((X11-A2-(X-A1)*R)/
     1(DSQRT(2.0D0*(1.0D0-R**2))))
      F1=F11*F12
      RETURN
      END
      FUNCTION F2(X)
      REAL*8 X,F2,R,X11,F11,F12
      COMMON X11,R
      F11=DEXP(-X**2/2.0D0)/
     1DSQRT(8.0D0*DATAN(1.0D0))
      F12=DERFC((X11-X*R)/
     1DSQRT(2.0D0*(1.0D0-R**2)))
      F2=F11*F12
      RETURN
      END
```

APPENDIX D

PROGRAM LISTING TO CALCULATE PARAMETERS OF THE TWO QUANTIZERS FOR THE CASE OF EXPONENTIAL DISTRIBUTIONS

```
C                *************************
C   THIS PROGRAM CALCULATES THE OPTIMUM N-LEVEL QUANTIZERS OF TWO SEN-
C SOR OBSERVATIONS OF EXPONENTIAL DISTRIBUTIONS
C TO MINIMIZE A GLOBAL SYSTEM RISK FOR  FUSION
C SEE CHAPTER IV
C                *************************
      REAL*8 X(8),T(8),XX(8),TT(8),PD,PF,PD1(20,9,6),PF1(20,9,6),PD2
     1,PF2,PDC,PFC,C1(20),C,X2,X11,APD,APF,R3(20,2,6),AL,BL,Y
     1,APC(20,2),AFC(20,2),R4(20,2),AD2,AF2,RR2,AP
      INTEGER K,N,I,IER1,IER2,M,P,Q,L,MX
      DATA C1/50.0D0,40.0D0,30.0D0,20.0D0,15.0D0,10.0D0,9.0D0,
     18.0D0,7.0D0,6.0D0,5.0D0,4.0D0,3.0D0,2.0D0,1.50D0,
     11.0D0,.90D0,.80D0,.70D0,.60D0/
C                *************************
C INITIAL VALUES OF QUANTIZER PARAMETERS
C FIRST QUANTIZER
      XX(1)=00.00000D0
      XX(2)=0.89000D0
      XX(3)=01.50000D0
      XX(4)=1.89000D0
      XX(5)=02.50000D0
      XX(6)=3.09000D0
C SECOND  QUANTIZER
      TT(1)=3.090000D0
      TT(2)=2.50000D0
      TT(3)=01.89000D0
      TT(4)=1.50000D0
      TT(5)=00.89000D0
      TT(6)=0.00000D0
      DO 500 N=3,6
      DO 11 I=1,2
C PARAMETERS OF THE EXP. DISTRIBUTIONS
      AL=DFLOAT(I)*0.50D0
      DO 20 M=1,20
      C=C1(M)
      DO 55 P=1,N
          TT(P)=DLOG(2.0D0*C)*DFLOAT(N-P+1)/DFLOAT(N)/AL
          XX(N-P+1)=DLOG(2.0D0*C)*DFLOAT(N-P+1)/DFLOAT(N)/AL
 55   CONTINUE
      K=0
 5    K=K+1
      DO 89 PP=1,N
          T(PP)=TT(PP)
          X(PP)=XX(PP)
 89   CONTINUE
C********************
C   INPUT MAXIMUM NUMBER OF ITERATIONS
      MX=200
      IF (K .GT.MX) GO TO 10
C     XX(1)=DLOG(2.0D0*C)/AL-TT(1)
      XX(1)=0.0D0
      TT(1)=(DLOG(DEXP(-AL*XX(1))+DEXP(-AL*XX(2)))+DLOG(2.0D0*C))/AL
       DO 15  P=2,N-1
      XX(P)=(DLOG(DEXP(-AL*TT(P))+DEXP(-AL*TT(P-1)))+DLOG(2.0D0*C))/AL
      TT(P)=(DLOG(DEXP(-AL*XX(P))+DEXP(-AL*XX(P+1)))+DLOG(2.0D0*C))/AL
 15             CONTINUE
      XX(N)=(DLOG(2.0D0*C)+DLOG(DEXP(TT(N))+DEXP(-AL*TT(N-1))))/AL
C     TT(N)=DLOG(2.0D0*C)/AL-XX(N)
      TT(N)=0.0D0
C ACCURACY CHECKING
```

```
C INPUT PRECESSION HERE
      AP=0.10d-05
      IF((DABS(XX(N)-X(N)) .GT. AP).OR.(DABS(TT(1)-T(1)) .GT.
     1           AP))GO TO 5
 10     CONTINUE
        WRITE(8,60) K
      BL=2.0D0*AL
      APD=0.0D0
      APF=0.0D0
      DO 81 Q=1,N-1
        APD=APD+DEXP(-AL*TT(Q))*(DEXP(-AL*XX(Q))-DEXP(-AL*XX(Q+1)))
        APF=APF+DEXP(-BL*TT(Q))*(DEXP(-BL*XX(Q))-DEXP(-BL*XX(Q+1)))
  81  CONTINUE
          APD=APD+DEXP(-AL*X(N))
          APF=APF+DEXP(-BL*X(N))
          APC(M,I)=(1.0D0+DLOG(4.0D0*C))/(4.0D0*C)
          AFC(M,I)=(1.0D0+DLOG((4.0D0*C)**2))/(4.0D0*C)**2
           PD1(M,I,N)=APD
           PF1(M,I,N)=APF
          R3(M,I,N)=1.0D0+C1(M)*PF1(M,I,N)-PD1(M,I,N)
          R4(M,I)=1.0D0+C1(M)*AFC(M,I)-APC(M,I)
        DO 120 J=1,N
C******************************
C  OUTPUT QUANTIZER PARAMETERS
        WRITE(9,62) XX(J),TT(J)
 120  CONTINUE
        WRITE(9,90)
 20     CONTINUE
 11        CONTINUE
        WRITE(9,90)
C      WRITE(9,90)
        WRITE(8,90)
 500  CONTINUE
          DO 501 J=1,2
          DO 502 I=1,20
          AP2=1.0D0/(2.0D0*C1(I))
          AF2=AP2**2
          RR2=1.0D0+C1(I)*AF2-AP2
C******************************
C  OUTPUT DETECTION COST
          WRITE(10,61)C1(I),RR2,( R3(I,J,N) ,N=3,6),R4(I,J)
C******************************
C  OUTPUT PROB. OF DETECTION AND PROB. OF FALSE ALARM
       WRITE(8,61)AP2,AF2,( PD1(I,J,N),PF1(I,J,N) ,N=3,5),APC(I,J)
     1,AFC(I,J)
 502  CONTINUE
        WRITE(8,90)
 501  CONTINUE
 60    FORMAT(1X,I6,9(1X,F6.4))
 90    FORMAT(2X,'CON B  ***********************************')
 61    FORMAT(1X,10(1X,F6.4))
 62    FORMAT(1X,3(1X,F15.8))
        STOP
        END
```

APPENDIX E

PROGRAM LISTING TO CALCULATE PARAMETERS OF THE TWO QUANTIZERS FOR EXAMPLE 3, CHAPTER IV

```
C   THIS PROGRAM CALCULATES THE OPTIMUM QUANTIZER PARAMETERS OF TWO
C  N-LEVEL QUANTIZERS IN ORDER TO MINIMIZE A GLOBAL SYSTEM RISK FOR
C  DETECTION OF SIGNALS WITH DIFFERENT VARIANCE. N=3 , N=5.
c  S1 = SIGNAL VARIANCE UNDER H1
c  S0 = SIGNAL VARIANCE UNDER H0
c  T1 = QUANTIZATION POINT FOR N=3 ( T1,-T1 )
c  X1,X2 QUANTIZATION POINTS FOR N=5 (X1,X2,-X1,-X2)
c  C|(20) ARRAY OF RATIO OF COSTS
c  K  = Number of iterations.
       REAL*8 T1,T11,S1,S2,R,C,X2,PD,PF,C1(15),Z31,Z21,X3,X33,X22
     1,TS,SSS,TTT(10),X1,PD3,PF3,PDC,PFC,R2,R3,RC
       INTEGER K,N,K1
       DATA  C1/01.0D0,.900D0,.800D0,.700D0,0.60D0,0.50D0,0.40D0,
     10.30D0,0.20D0,0.10D0,.090D0,.080D0,.070D0,0.060D0,0.050D0/
       S1=1.0D0
       S0=DSQRT(2.0D0)
c INITIAL VALUES OF THE QUANTIZER PARAMETERS
       T1=01.15800D0
       X1=01.15800D0
       X2=00.15800D0
       WRITE(6,60)K,S1,S0,C
       WRITE(9,60)K,S1,S0,C
       WRITE(8,60)K,S1,S0,C
       SSS=(1.0D0/S0**2-1.0D0/S1**2)/2.0D0
       DO 101 N=2,2
       DO 100 I=1,15
       C=C1(I)
       K=0
 5    K=K+1
       IF (K .GT.100) GO TO 10
       T11=T1
       TS=(DLOG(S1/S0)+
     1DLOG(C)+DLOG((DERF(T1/DSQRT(2.0D0)
     1/S0))/(DERF(T1/DSQRT(2.0D0)/S1)))*DFLOAT(N-1))/SSS
C     IF (TS .GT. 0.0D0) T1=DSQRT(TS)
        T1=DSQRT(DSQRT(TS))
             IF((DABS(T1-T11) .GT. 0.10D-05)) GO TO 5
             CONTINUE
 10     TTT(N)=T11
       K1=0
55    K1=K1+1
       IF (K1 .GT.100) GO TO 15
       X33=X3
       X22=X2
       X2=(DLOG(S1/S0)+
     1DLOG(C)+DLOG((DERF(X3/DSQRT(2.0D0)
     1/S0))/(DERF(X3/DSQRT(2.0D0)/S1))))/SSS
C     IF (TS .GT. 0.0D0) T1=DSQRT(TS)
        X2=DSORT(X2)
       Z30=DERF(X3/DSQRT(2.0D0)/S0)
       Z20=DERF(X2/DSQRT(2.0D0)/S0)
       Z31=DERF(X3/DSQRT(2.0D0)/S1)
       Z21=DERF(X2/DSQRT(2.0D0)/S1)
       X3=(DLOG(S1/S0)+
     1DLOG(C)+DLOG((Z30-Z20)/(Z31-Z21)))/SSS
       X3=DSQRT(DABS(X3))
       IF(((DABS(X2-X22) .GT. 0.10D-05)) .OR.(( DABS(X3-X33) .GT.
     10.10D-05))) GO TO 55
             CONTINUE
 15     TTT(N)=T11
```

```
            TS=-DLOG(S0**2/S1**2/C)/SSS
            PDC=1.0D0-DEXP(-TS/(2.0D0*S1*S1))
            PFC=1.0D0-DEXP(-TS/(2.0D0*S0*S0))
            RC=1.0D0+C*PFC-PDC
            WRITE(9,60)K,T1,T11,X2,X22,X3,X33
            WRITE(6,60)K,T1,T11
        PD=DERF(T11/DSQRT(2.0D0)/S1)**N
        PF=DERF(T11/DSQRT(2.0D0)/S0)**N
            R2=1.0D0+C*PF-PD
        PD3=(DERF(X22/DSQRT(2.0D0)/S1)*2.0D0-DERF(X33/DSQRT(2.0D0)/S1))
     1 *DERF(X33/DSQRT(2.0D0)/S1)
        PF3=(DERF(X22/DSQRT(2.0D0)/S0)*2.0D0-DERF(X33/DSQRT(2.0D0)/S0))
     1 *DERF(X33/DSQRT(2.0D0)/S0)
            R3=1.0D0+C*PF3-PD3
            WRITE(8,60)N,PDC,PFC,PD,PF,PD3,PF3
            WRITE(10,60)N,C,R2,R3,RC
C         IF ((I .EQ. 1) .OR. (I .EQ. 10 )) WRITE(10,60)N,C,R2,R3,RC
            WRITE(6,60)N,C,TTT(N),PD,PF,PD3,PF3
 100      CONTINUE
 101      CONTINUE
 60    FORMAT(1X,I3,6(1X,F10.7))
        STOP
        END
```

APPENDIX F
LINEAR MINIMUM MEAN SQUARE ESTIMATE OF Y_1

Having $\underline{Y} = [\, y_{1q} \; y_2 \,]^t$ the LMMS estimate of y_1 and the corresponding mean square error are given by [34]:

$$\hat{y}_1 = E\{y_1 \underline{Y}^t\} E\{\underline{Y}\,\underline{Y}^t\}^{-1} \underline{Y} \tag{F.1}$$

and

$$E\{e^2\} = E\{y_1^2\} - E\{y_1 \underline{Y}^t\} E\{\underline{Y}\,\underline{Y}^t\}^{-1} E\{\underline{Y} y_1\} \tag{F.2}$$

where

$$E\{y_1 \underline{Y}^t\} = [\, E\{y_1 y_{1q}\} \quad E\{y_1 y_2\} \,] \tag{F.3}$$

and

$$E\{\underline{Y}\,\underline{Y}^t\} = \begin{bmatrix} E\{y^2_{1q}\} & E\{y_{1q} y_2\} \\ E\{y_2 y_{1q}\} & E\{y^2_2\} \end{bmatrix} \tag{F.4}$$

The entries of these matrices are:

$$\begin{aligned} E\{y_1 y_{1q}\} &= \sum_{j=1}^{N} P_j Q_j E\{y_1 / y_{1q} = Q_j\} \\ &= \sum_{j=1}^{N} P_j Q_j C_j \\ &= \mu \sigma_1^2 \end{aligned} \tag{F.5}$$

$$E\{y_2 y_{1q}\} = \sum_{j=1}^{N} P_j Q_j E\{y_2 / y_{1q} = Q_j\} \tag{F.6}$$

but

$$E\{y_2/y_{1q} = Q_j\} = E\{\{y_2/y_1\}/y_{1q} = Q_j\} \quad \text{(F.7)}$$

For the case where y_1 and y_2 are jointly gaussian, we can write

$$E\{y_2\, y_{1q}\} = \rho\, \sigma_1\, \sigma_2 \sum_{j=1}^{N} P_j\, Q_j\, C_j/\sigma_1^2 \quad \text{(F.8)}$$

$$E\{y_2\, y_{1q}\} = \rho\, \sigma_1\, \sigma_2\, \mu \quad \text{(F.9)}$$

$$E\{y_{1q}^2\} = \sum_{j=1}^{N} P_j\, Q_{j2} \quad \text{(F.10)}$$
$$= \sigma_1^2\, \eta\,.$$

Inserting these in (F.1) and (F.2) and performing matrix multiplications yields

$$\hat{y}_1 = \left[(1-\rho^2)\mu\, y_{1q} + \rho\, \frac{\sigma_1}{\sigma_2}(\eta - \rho^2)y_2\right]/(\eta - \mu^2\rho^2) \quad \text{(6.2)}$$

and

$$E\{e^2\} = \sigma_1^2\,(1-\rho^2)(1-\omega)/(1-\rho^2\omega) \quad \text{(6.11)}$$

where ω is given by:

$$\omega = \mu^2/\eta. \quad \text{(6.12)}$$

APPENDIX G
SOLUTION OF THE LLOYD-MAX QUANTIZER PARAMETERS BY THE METHOD OF SUCCESSIVE SUBSTITUTION

1. INTRODUCTION

The minimum distortion quantizer parameters [18,23], as well as parameters based on other criterion such as quantizers for signal detection [41], minimum risk quantizers and quantizers for LMMS estimation error dealt with in this thesis, can be solved by Max's trial and error technique [18]. There are also many other approximation methods to calculate the quantizer parameters [42], [43] and [44].

In this Appendix we apply the method of successive substitution and its modifications [19] to solve for the Lloyd-Max quantizer parameters. It is more accurate and computationally more efficient than the previously reported methods. It is shown to easily generate 7 bit (128 level) optimum quantization.

2. STATEMENT OF THE PROBLEM

The Lloyd-Max minimum mean square distortion quantizer problem deals with transforming a random variable X of differentiable probability density function f(x) into the N-level discrete random variable Y.

$$Y(X) = Y_i \text{ for } X \in [x_i, x_{i+1}] \tag{G.1}$$

The optimum parameters minimize the distortion D

$$D = \sum_{i=1}^{N} \int_{x_i}^{x_{i+1}} (x-y_i)^2 f(x)\, dx \tag{G.2}$$

with

$$-\infty = x_1 \leq x_2 \leq \ldots \leq x_N \leq x_{N+1} = \infty$$

Differentiating D with respect to x_i and y_i yields the following necessary conditions of optimality :

$$x_i = (y_i + y_{i+1})/2 \quad , i = 2,3,\ldots N \tag{G.3}$$

$$y_i = \left(\int_{x_i}^{x_{i+1}} x f(x)dx\right) / \left(\int_{x_i}^{x_{i+1}} f(x)dx\right), i = 1,2,..N \qquad (G.4)$$

a set of simultaneous equations of propagating character. That is, if y_1 is chosen correctly then x_2 can be calculated from (G.4), y_2 from (G.3), x_3 from (G.4) and so forth [18]. In this case the value of y_N calculated from (G.3) must agree with its value calculated from (G.4) with $x_{N+1} = \infty$. This was the core of Max's trial and error algorithm; to pick a value for y_1 and calculate the parameters up to and including y_N, which must agree with the value of y_N calculated from (G.4), otherwise, to pick another value of y_1. Let us put the system of equations in the form

$$\underline{Z} = \underline{G}(\underline{Z}) \qquad (G.5)$$

where $\underline{Z}$ is a 2N-1 vector given by:

$$\underline{Z} = [\, y_1\ x_2\ y_2 \ldots\ldots y_N \,]^t \qquad (G.6)$$

and apply the iterative substitution

$$\underline{Z}_{new} = \underline{G}(\underline{Z}_{old}) \qquad (G.7)$$

with a suitable initial guess. The convergence is guaranteed if $\partial G_k / \partial Z_j$ is sufficiently small for every k,j = 1,2,...,2N-1 [19]. From (G.4)

$$\partial G_j / \partial y_j = [(x_{j+1} - y_j) f(x_{j+1}) + (y_j - x_j) f(x_j)] / (2P_j) \qquad (G.8)$$

where P_j is the probability the input of the quantizer is in the j_{th} interval.

$$P_j = \int_{x_j}^{x_{j+1}} f(x)dx. \qquad (G.9)$$

The numerator in (G.8) is an approximation of the integral in (G.9) by the trapezoidal rule with the subdivision $[x_j, y_j, x_{j+1}]$, so the value of the derivative is very likely less than one. Also, substituting for y_j and y_{j+1} in (G.3) from (G.4) and differentiating with respect to x_j it is easily to show that

$$\partial G_j / \partial x_j = (y_j - x_j) f(x_j)/(2P_j) + (x_j - y_{j-1}) f(x_j)/2P_{j-1} \qquad \text{(G.10)}$$

which is less than ($\partial G_j / \partial y_j$). The method can be more efficient if we use the updated values in the same iteration. In this modification of the method the best current values of the parameters are used. This choice may also enhance convergence. The method also avoids the tedious calculation of the upper limit of the integral to solve for the next x_j in (G.4).

3. NUMERICAL RESULTS

We have solved for the quantizer parameters for a gaussian random variable of zero mean and unit variance for several values of N up to 128. Also the mean square error D and the output entropy ($-\sum_k P_k \log_2 (P_k)$) have been calculated. The results presented in Table 4 show that in several cases Max's results, which were only available up to N=36, are not accurate in the last digit.

Key to Table 4

The numbering in the table is as follows.

1. For N even, each table begins with the $(N/2+1)_{th}$ parameters. In this case the $(N/2+1)_{th}$ value of x is zero.
2. For N odd, Each table begins with the $(N/2+2)_{th}$ parameters. In this case the $(N/2+2)_{th}$ value of y is zero.

Negative parameters can be calculated from the symmetry relation

$$X_j = X_{n-j+2} \qquad \text{(G.11)}$$

and

$$y_j = y_{n-j+1} \, . \qquad \text{(G.12)}$$

A FORTRAN program to calculate the parameters ,distortion and entropy follows Table 4. The only input to the program is N, the number of quantization levels.

TABLE 4

MAX'S QUANTIZER PARAMETERS FOR THE NORMAL DISTRIBUTION

N = 2

J	X	Y
2	0.000000	0.797885

ERROR = 0.363380
ENTROPY = 1.000000

N = 3

J	X	Y
2	-0.612003	0.000000
3	0.612003	1.224006

ERROR = 0.190174
ENTROPY = 1.535789

N = 4

J	X	Y
3	0.000000	0.452780
4	0.981599	1.510418

ERROR = 0.117482
ENTROPY = 1.911099

N = 5

J	X	Y
3	-0.382284	0.000000
4	0.382284	0.764567
5	1.244357	1.724147

ERROR = 0.079941
ENTROPY = 2.202916

N = 6

J	X	Y
4	0.000000	0.317716
5	0.658911	1.000106
6	1.446850	1.893595

ERROR = 0.057978
ENTROPY = 2.442789

N = 7

J	X	Y
4	-0.280289	0.000000
5	0.280288	0.560577
6	0.874362	1.188147
7	1.610758	2.033369

ERROR = 0.044000

ENTROPY = 2.646931

N = 8

J	X	Y
5	0.000000	0.245094
6	0.500550	0.756005
7	1.049957	1.343909
8	1.747927	2.151946

ERROR = 0.034548
ENTROPY = 2.824865

N = 9

J	X	Y
5	-0.221819	0.000000
6	0.221819	0.443639
7	0.681217	0.918796
8	1.197594	1.476392
9	1.865528	2.254664

ERROR = 0.027853
ENTROPY = 2.982695

N = 10

J	X	Y
6	0.000000	0.199623
7	0.404740	0.609857
8	0.833841	1.057825
9	1.324583	1.591340
10	1.968218	2.345096

ERROR = 0.022937
ENTROPY = 3.124584

TABLE 4

MAX'S QUANTIZER PARAMETERS FOR THE NORMAL DISTRIBUTION (CONT'D.)

N = 11

J	X	Y
6	-0.183729	0.000000
7	0.183729	0.367458
8	0.559913	0.752367
9	0.965597	1.178826
10	1.435733	1.692639
11	2.059193	2.425746

ERROR = 0.019220
ENTROPY = 3.253506

N = 12

J	X	Y
7	0.000000	0.168438
8	0.340142	0.511846
9	0.694313	0.876779
10	1.081245	1.285711
11	1.534371	1.783030
12	2.140733	2.498435

ERROR = 0.016340
ENTROPY = 3.371666

N = 13

J	X	Y
7	-0.156887	0.000000
8	0.156887	0.313773
9	0.476012	0.638251
10	0.812600	0.986949
11	1.184106	1.381263
12	1.622890	1.864518
13	2.214522	2.564525

ERROR = 0.014063
ENTROPY = 3.480744

N = 14

J	X	Y
8	0.000000	0.145706
9	0.293513	0.441321
10	0.595882	0.750443
11	0.918039	1.085635
12	1.276582	1.467528
13	1.703070	1.938612
14	2.281837	2.625062

ERROR = 0.012232
ENTROPY = 3.582050

N = 15

J	X	Y
8	-0.136929	0.000000
9	0.136928	0.273857
10	0.414310	0.554764
11	0.702949	0.851134
12	1.013007	1.174879
13	1.360468	1.546057
14	1.776266	2.006474
15	2.343670	2.680866

ERROR = 0.010737
ENTROPY = 3.676630

N = 16

J	X	Y
9	0.000000	0.128395
10	0.258222	0.388048
11	0.522404	0.656759
12	0.799550	0.942340
13	1.099286	1.256231
14	1.437139	1.618046
15	1.843532	2.069017
16	2.400803	2.732590

ERROR = 0.009501

ENTROPY = 3.765328

TABLE 4

MAX'S QUANTIZER PARAMETERS FOR THE NORMAL DISTRIBUTION (CONT'D.)

N = 17

J	X	Y
9	-0.121497	0.000000
10	0.121497	0.242994
11	0.366938	0.490882
12	0.620085	0.749287
13	0.887442	1.025597
14	1.178246	1.330896
15	1.507669	1.684442
16	1.905707	2.126971
17	2.453866	2.780762

ERROR = 0.008467

ENTROPY = 3.848840

N = 18

J	X	Y
10	0.000000	0.114769
11	0.230557	0.346345
12	0.465324	0.584302
13	0.709082	0.833862
14	0.967981	1.102100
15	1.250963	1.399827
16	1.572915	1.746003
17	1.963465	2.180927
18	2.503372	2.825817

ERROR = 0.007593
ENTROPY = 3.927741

N = 19

J	X	Y
10	-0.109205	0.000000
11	0.109204	0.218409
12	0.329382	0.440355
13	0.555076	0.669797
14	0.790731	0.911666
15	1.042234	1.172801
16	1.318296	1.463791
17	1.633568	1.803345
18	2.017359	2.231373
19	2.549745	2.868116

ERROR = 0.006848
ENTROPY = 4.002518

N = 20

J	X	Y
11	0.000000	0.103762
12	0.208277	0.312791
13	0.419640	0.526488
14	0.637511	0.748533
15	0.866088	0.983642
16	1.111055	1.238467
17	1.380941	1.523414
18	1.690196	1.856977
19	2.067846	2.278714
20	2.593337	2.907961

ERROR = 0.006208
ENTROPY = 4.073583

TABLE 4

MAX'S QUANTIZER PARAMETERS FOR THE NORMAL DISTRIBUTION (CONT'D.)

N = 21

J	X	Y
11	-0.099179	0.000000
12	0.099178	0.198357
13	0.298856	0.399355
14	0.502624	0.605892
15	0.713667	0.821442
16	0.935997	1.050552
17	1.175138	1.299725
18	1.439469	1.579214
19	1.743269	1.907323
20	2.115306	2.323289
21	2.634448	2.945607

ERROR = 0.005653
ENTROPY = 4.141290

N = 22

J	X	Y
12	0.000000	0.094686
13	0.189942	0.285198
14	0.382215	0.479232
15	0.579359	0.679485
16	0.784380	0.889276
17	1.001147	1.113019
18	1.235056	1.357093
19	1.494358	1.631622
20	1.793180	1.954739
21	2.160062	2.365386
22	2.673330	2.981274

ERROR = 0.005170
ENTROPY = 4.205942

N = 23

J	X	Y
12	-0.090844	0.000000
13	0.090844	0.181688
14	0.273544	0.365400
15	0.459363	0.553325
16	0.650668	0.748012
17	0.850333	0.952654
18	1.062107	1.171561
19	1.291284	1.411007
20	1.546005	1.681003
21	1.840265	1.999527
22	2.202390	2.405252
23	2.710201	3.015150

ERROR = 0.004746
ENTROPY = 4.267806

N = 24

J	X	Y
13	0.000000	0.087072
14	0.174587	0.262101
15	0.350977	0.439853
16	0.531112	0.622370
17	0.717227	0.812084
18	0.912088	1.012091
19	1.119352	1.226612
20	1.344223	1.461834
21	1.594750	1.727666
22	1.884807	2.041948
23	2.242523	2.443098
24	2.745248	3.047398

ERROR = 0.004372
ENTROPY = 4.327112

TABLE 4

MAX'S QUANTIZER PARAMETERS FOR THE NORMAL DISTRIBUTION (CONT'D.)

N = 25

J	X	Y
13	-0.083805	0.000000
14	0.083805	0.167610
15	0.252208	0.336806
16	0.423045	0.509283
17	0.598128	0.686972
18	0.779592	0.872212
19	0.970115	1.068019
20	1.173279	1.278540
21	1.394213	1.509886
22	1.640881	1.771876
23	1.927050	2.082224
24	2.280667	2.479110
25	2.778634	3.078159

ERROR = 0.004041
ENTROPY = 4.384064

N = 26

J	X	Y
14	0.000000	0.080593
15	0.161536	0.242480
16	0.324498	0.406516
17	0.490402	0.574288
18	0.660961	0.747635
19	0.838229	0.928823
20	1.024813	1.120803
21	1.224230	1.327657
22	1.441544	1.555432
23	1.684648	1.813865
24	1.967207	2.120549
25	2.316997	2.513445
26	2.810502	3.107559

ERROR = 0.003746
ENTROPY = 4.438843

N = 27

J	X	Y
14	-0.077781	0.000000
15	0.077780	0.155561
16	0.233975	0.312389
17	0.392106	0.471823
18	0.553594	0.635364
19	0.720073	0.804782
20	0.893532	0.982281
21	1.076518	1.170756
22	1.272495	1.374235
23	1.486469	1.598704
24	1.726267	1.853829
25	2.005461	2.157093
26	2.351670	2.546247
27	2.840977	3.135707

ERROR = 0.003483
ENTROPY = 4.491610

N = 28

J	X	Y
15	0.000000	0.075012
16	0.150307	0.225602
17	0.301760	0.377919
18	0.455569	0.533219
19	0.613076	0.692934
20	0.775854	0.858775
21	0.945836	1.032897
22	1.125522	1.218147
23	1.318326	1.418505
24	1.529205	1.639905
25	1.765925	1.891945
26	2.041975	2.192005
27	2.384821	2.577637
28	2.870169	3.162701

ERROR = 0.003246
ENTROPY = 4.542507

TABLE 4

MAX'S QUANTIZER PARAMETERS FOR THE NORMAL DISTRIBUTION (CONT'D.)

N = 29

J	X	Y
15	-0.072566	0.000000
16	0.072566	0.145132
17	0.218211	0.291291
18	0.365424	0.439557
19	0.515338	0.591119
20	0.669236	0.747352
21	0.828638	0.909923
22	0.995431	1.080939
23	1.172074	1.263209
24	1.361940	1.460671
25	1.569941	1.679211
26	1.803788	1.928364
27	2.076890	2.225415
28	2.416571	2.607727
29	2.898177	3.188627

ERROR = 0.003032
ENTROPY = 4.591663

N = 30

J	X	Y
16	0.000000	0.070155
17	0.140542	0.210928
18	0.282019	0.353110
19	0.425412	0.497714
20	0.571795	0.645876
21	0.722402	0.798927
22	0.878709	0.958490
23	1.042565	1.126640
24	1.216393	1.306147
25	1.403530	1.500912
26	1.608846	1.716779
27	1.840001	1.963224
28	2.110332	2.257440
29	2.447027	2.636614
30	2.925088	3.213562

ERROR = 0.002839
ENTROPY = 4.639193

N = 31

J	X	Y
16	-0.068008	0.000000
17	0.068008	0.136016
18	0.204446	0.272876
19	0.342170	0.411464
20	0.482101	0.552739
21	0.625267	0.697794
22	0.772858	0.847921
23	0.926315	1.004710
24	1.087456	1.170202
25	1.258670	1.347138
26	1.443261	1.539385
27	1.646065	1.752745
28	1.874694	1.996643
29	2.142413	2.288184
30	2.476285	2.664385
31	2.950981	3.237577

ERROR = 0.002664
ENTROPY = 4.685201

N = 32

J	X	Y
17	0.000000	0.065890
18	0.131971	0.198052
19	0.264715	0.331378
20	0.399039	0.466699
21	0.535816	0.604934
22	0.676035	0.747136
23	0.820850	0.894565
24	0.971674	1.048783
25	1.130294	1.211804
26	1.299072	1.386340
27	1.481284	1.576228
28	1.681731	1.787233
29	1.907981	2.028728
30	2.173234	2.317739
31	2.504429	2.691120
32	2.975926	3.260732

ERROR = 0.002505
ENTROPY = 4.729784

TABLE 4

MAX'S QUANTIZER PARAMETERS FOR THE NORMAL DISTRIBUTION (CONT'D.)

N = 64

J	X	Y
33	0.000000	0.033409
34	0.066844	0.100278
35	0.133787	0.167297
36	0.200932	0.234567
37	0.268380	0.302193
38	0.336238	0.370283
39	0.404616	0.438950
40	0.473632	0.508314
41	0.543408	0.578503
42	0.614079	0.649655
43	0.685789	0.721922
44	0.758695	0.795468
45	0.832972	0.870476
46	0.908816	0.947155
47	0.986446	1.025736
48	1.066112	1.106488
49	1.148104	1.189720
50	1.232757	1.275794
51	1.320468	1.365141
52	1.411709	1.458276
53	1.507054	1.555831
54	1.607210	1.658589
55	1.713065	1.767542
56	1.825759	1.883977
57	1.946794	2.009611
58	2.078211	2.146810
59	2.222896	2.298981
60	2.385143	2.471305
61	2.571789	2.672274
62	2.794840	2.917407
63	3.078922	3.240437
64	3.492269	3.744101

ERROR = 0.000644
ENTROPY = 5.710078

TABLE 4

MAX'S QUANTIZER PARAMETERS FOR THE NORMAL DISTRIBUTION (CONT'D.)

N = 128

J	X	Y
65	-0.000001	0.016828
66	0.033659	0.050490
67	0.067331	0.084172
68	0.101029	0.117886
69	0.134765	0.151644
70	0.168552	0.185460
71	0.202404	0.219347
72	0.236333	0.253318
73	0.270353	0.287387
74	0.304477	0.321567
75	0.338720	0.355874
76	0.373097	0.390320
77	0.407620	0.424921
78	0.442307	0.459693
79	0.477172	0.494651
80	0.512231	0.529812
81	0.547502	0.565192
82	0.583001	0.600810
83	0.618747	0.636684
84	0.654758	0.672833
85	0.691055	0.709277
86	0.727658	0.746039
87	0.764589	0.783139
88	0.801871	0.820602
89	0.839528	0.858454
90	0.877587	0.896719
91	0.916074	0.935428
92	0.955019	0.974609
93	0.994453	1.014296
94	1.034409	1.054523
95	1.074924	1.095326
96	1.116037	1.136747
97	1.157788	1.178828
98	1.200223	1.221617
99	1.243391	1.265165
100	1.287346	1.309527
101	1.332146	1.354766
102	1.377857	1.400948
103	1.424548	1.448148
104	1.472299	1.496450
105	1.521196	1.545943
106	1.571339	1.596733
107	1.622833	1.648933
108	1.675805	1.702677
109	1.730394	1.758111
110	1.786759	1.815407
111	1.845085	1.874762
112	1.905583	1.936404
113	1.966503	2.000601
114	2.034136	2.067671
115	2.102332	2.137993
116	2.175010	2.212027
117	2.251193	2.290339
118	2.331996	2.373634
119	2.418222	2.462811
120	2.510926	2.559040
121	2.611463	2.663886
122	2.721700	2.779511
123	2.844280	2.909047
124	2.983146	3.057246
125	3.144589	3.231933
126	3.339681	3.447430
127	3.591183	3.734936
128	3.962315	4.189694

ERROR = 0.000163
ENTROPY = 6.699583

4. PROGRAM LISTING TO CALCULATE THE LLOYD-MAX QUANTIZER PARAMETERS

```
C THIS PROGRAM CALCULATES LLOYD-MAX QUANTIZER PARAMETERS BY THE METHOD
C OF SUCCESSIVE SUBSTITUTION FOR THE NORMAL DISTRIBUTION OF ZERO MEAN
C AND UNIT VARIANCE
C The INPUT TO THE PROGRAM IS
C   (1)    THE NUMBER OF QUANTIZATION LEVELS   N
C   (2)    THE MAXIMUM NUMBER OF ITERATIONS    M
C   (2)    THE ACCURACY                        AP
C               ****************************
      REAL*8 X(199),T(199),XX(199),TT(199),C ,DELTA,AP(199),AP
     1,ERROR,ENTROP
      INTEGER K,N,I,P,N1,N2,N3,M
      C=DSQRT(00.50D0/DATAN(1.0D0))
C     DO 99 N=110,110
C************************************************************
C  INPUT THE NUMBER OF QUANTIZATION LEVELS
      N=100
      WRITE(9,65)N
      WRITE(9,66)
      WRITE(9,67)
      WRITE(9,66)
C*************************************************************
C    INITIALIZATION OF THE QUANTIZER PARAMETERS
C
      DELTA=0.0150D0*DFLOAT(N)
      XX(1)=-10.50000D0
      TT(1)=-5.50000D0
      X(1)=XX(1)
      T(1)=TT(1)
      DO 50  L=2,N
            TT(L)=TT(L-1)-DELTA
            XX(L)=(TT(I)+TT(I-1))/2.0D0
            X(L)=XX(L)
            T(L)=TT(L)
 50   CONTINUE
C*****************************************************
C    BEGINING OF THE ITERATIONS
c     M = MAXIMUM NUMBER OF ITERATIONS
      M = 1050
      K=0
 5    K=K+1
      IF (K .GT. M) GO TO 10
       IF (K .GT. 1)X(N)=XX(N)
       IF (K .GT. 1)T(N)=TT(N)
      TT(1)=-C*DEXP(-XX(2)*XX(2)/2.0D0)/(DERFC(-10.0D0)-
     1DERFC(XX(2)/DSQRT(2.0D0)))
      T(1)=TT(1)
      IF ( N .EQ. 2 ) GO TO 17
       DO 15  P=2,N-1
            XX(P)=(T(P)+T(P-1))/2.0D0
            X(P)=XX(P)
            TT(P)=DEXP(-X(P)*X(P)/2.0D0)-DEXP(-X(P+1)*X(P+1)/2.0D0)
            TT(P)=TT(P)*C/(DERFC(X(P)/DSQRT(2.0D0))-DERFC(X(P+1)/DSQRT(2.0
     1      0D0)))
            T(P)=TT(P)
 15             CONTINUE
 17             CONTINUE
         XX(N)=(TT(N)+T(N-1))/2.0D0
      TT(N)=DEXP(-XX(N)*XX(N)/2.0D0)*C/DERFC(XX(N)/DSQRT(2.0D0))
      X(N)=XX(N)
      T(N)=TT(N)
      N2=IDINT(DFLOAT((N+2)/2))
      N1=IDINT(DFLOAT((N+1)/2))
C*****************************************************
C    CHECKING THE PRECISION OF THE SOLUTION
c    AP = REQUIRED ACCURACY
     AP=0.10D-6
```

```
      IF((MOD(N,2) .EQ. 0).AND.(DABS(X(N2)).GT. AP))GO TO 5
      IF((MOD(N+1,2) .EQ. 0).AND. (DABS(T(N1)).GT. AP))GO TO 5
        CONTINUE
 10     CONTINUE
C*******************************************************
C     OUTPUT RESULTS
      IF (MOD(N,2) .EQ. 0)N3=N2
      IF (MOD(N+1,2) .EQ. 0)N3=N1
       WRITE(6,60) K
       DO 120 J=1,N3
           IF (J .EQ. 1)
     1     WRITE(9,71)J, T(J)
           IF (J .GT. 1)
     1     WRITE(9,61)J, X(J),T(J)
 120  CONTINUE
       X(N+1)=10.0D0
       X(1)=-10.0D0
       ERROR=0.0D0
       ENTROP=0.0D0
       DO 222 I=1,N
              AP(I)=DERFC(X(I)/DSQRT(2.0D0))-DERFC(X(I+1)/DSQRT(2.0D0))
              AP(I)=AP(I)/2.0D0
              ERROR=ERROR+AP(I)*T(I)**2
              ENTROP=ENTROP-AP(I)*DLOG(AP(I))/DLOG(2.0D0)
 222  CONTINUE
       ERROR=1.0D0-ERROR
       WRITE(9,66)
       WRITE(9,62) ERROR
       WRITE(9,66)
       WRITE(9,63) ENTROP
       WRITE(9,66)
       WRITE(6,72) K
       WRITE(9,90)
       WRITE(9,66)
C99  CONTINUE
 65   FORMAT(3X,'  N = ',I7)
 66   FORMAT(3X,'--------------------------')
 67   FORMAT(3X,' J          X           Y      ')
 60   FORMAT(1X,I7,8(1X,F6.4))
 90   FORMAT(2X,'==========================================')
 61   FORMAT(1X,I4, 2(2X,F9.6))
 71   FORMAT(1X,I4,11X, 2(2X,F9.6))
 62   FORMAT(7X,'ERROR   =', 2(1X,F9.6))
 72   FORMAT(3X,'# ITERATIONS = ',I7 )
 63   FORMAT(7X,'ENTROPY =', 2(1X,F9.6))
       STOP
       END
```

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