THEORY AND SENSITIVITY OF WAVE-DIGITAL FILTER

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THESIS

THEORY AND SENSITIVITY OF WAVE-DIGITAL FILTER

by

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Theory and Sensitivity of Wave-Digital Filter

by

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It has been conjectured by Fettweis that wave-digital filters designed after doubly terminated ladder networks by means of a transmission line transformation have a low sensitivity against coefficient variation. The theory of wave-digital filters is summarized together with the design procedure for deriving wave-digital ladder structures based upon an analog design. To investigate this conjecture nineteen Chebyshev low-pass filters are realized and experimentally evaluated using the IBM-360 general purpose digital computer. The realization was done in two forms: as wave-digital ladder structures and as recursive digital filter. The sinusoidal steady state response is determined for both filters in floating-point arithmetic with quantized coefficients. The analysis shows the validity of the conjecture for all nineteen cases over a wordlength range of the mantissa of four to twenty two bits.

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I. INTRODUCTION

Transfer properties of analog LC-filters in a ladder configuration are known to have exceptionally low sensitivity characteristics against element variations. Digital ladder structures derived from analog L-C filters retain this low coefficient sensitivity (Fettweis conjecture [Ref. 1]).

This thesis summarizes wave-digital ladder filter theory and presents a design procedure for deriving wave-digital ladder filter based upon an analog design, and investigates the coefficient sensitivity properties of the digital filter experimentally. This is accomplished by realizing a Chebyshev low-pass filter in two forms: as a wave-digital ladder filter and as a recursive digital filter. The theoretical sinusoidal steady state response for both filter types is calculated in floating-point arithmetic with the filter coefficients being quantized. The experimental response is determined using a computer program on the IBM 360 by carrying out the filter algorithm in floating-point representation under truncation of the mantissa for wordlength of four to twenty two bits. A comparison of the results demonstrates the validity of the conjecture.

II. WAVE DIGITAL FILTER THEORY

A. PRINCIPLES

In conventional digital filter theory filters are designed directly from the required transfer function in the s-domain so that lengthy analog realization procedures can be avoided. Two problems that arise in all digital filter design procedures are that of the high sensitivity of the transfer function versus coefficients, and roundoff noise effects due to finite precision arithmetic. Since analog LC-ladder structures exhibit low sensitivity for element variations Fettweis [Ref. 1] has suggested modeling digital filters directly after these analog structures. Most of the following introduction is based on that article.

The basic principle involved is to carry out a frequency transformation to convert the system of differential equations of the analog structure to a system of difference equations that allows implementation by a digital computer. The first step in doing this is to perform a Richard's transformation [Ref. 2] which converts the analog LC-ladder structure to a transmission line circuit by replacing each inductor with lengths of short-circuited transmission lines and each capacitor with lengths of open-circuited transmission lines. The length, ℓ, of the transmission line is related to the inherent delay, τ_1, by

$$\tau_1 = \frac{\ell}{v} \qquad \text{II-1}$$

where v is the propagation velocity. If one assumes a traveling sinusoidal signal of wavelength λ moving along the transmission line then the period τ_2 between arrival of two successive peaks is given by

$$\tau_2 = \frac{\lambda}{v} \qquad \text{II-2}$$

By choosing the length of the transmission line to be equal to λ one has

$$\tau_1 = \tau_2 = \tau \qquad \text{II-3}$$

Modeling the transmission line by two parallel pieces of delay lines each with an inherent delay τ one arrives at the unit-element [Fig. 1] which is the basic building block for transmission line or also called microwave filters.

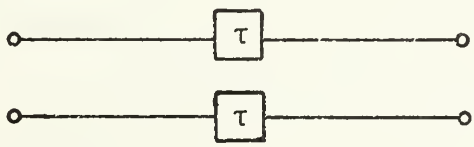


FIGURE 1. Unit-element of length $\ell = \lambda$ and delay τ

The input impedance to a transmission line of length ℓ and velocity v is given by

$$Z(s) = R_o \tanh(s\tau) \qquad \text{II-4}$$

where R_o is the characteristic resistance of the transmission line and $\tau = \ell/v$.

Defining a new frequency variable as

$$\psi = \tanh\,(s\tau) = \tanh\,(\frac{sT}{2}) \quad \text{with } T = 2\tau \qquad \text{II-5}$$

one arrives at the Richard's transformation. II-5 can also be written as

$$\psi = \frac{e^{\frac{sT}{2}} - e^{-\frac{sT}{2}}}{e^{\frac{sT}{2}} + e^{-\frac{sT}{2}}} = \frac{z-1}{z+1} \qquad \text{II-6}$$

for

$$z = e^{s\tau} \qquad \text{II-7}$$

$Z = e^{s\tau}$

Now the input impedance of a short-circuited transmission line can be written as

$$Z(\psi) = \psi R_o \,. \qquad \text{II-8a}$$

and of an open circuited

$$Z(\psi) = \frac{R_o}{\psi} \qquad \text{II-8b}$$

In the standard bilinear z-transform, s in T(s) is replaced by

$$s = \frac{2}{T}\left(\frac{z-1}{z+1}\right) \qquad \text{II-9}$$

This is equivalent to the following process starting from any analog LC-circuit which has T(s) as its transfer function:

(a) Replace each sL by $R_o \tanh\left(\frac{s\ell_1}{v}\right) = R_o\left(\frac{z-1}{z+1}\right)$ where $R_o = L\,\frac{2}{T}$ and $z = e^{sT}$; $T = \frac{\ell_1}{2v}$.

(b) Replace each $\frac{1}{sC}$ by $R_o \text{ctnh}\left(\frac{s\ell_2}{v}\right)$ where $R_o = \frac{T}{2C}$ and $z = e^{sT}$; $T = \frac{\ell_2}{2v}$.

Since resistors are frequency independent elements they remain unchanged by this frequency transformation. Thus all analog LC-elements are replaced by pieces of open- or short-circuited transmission line.

Fettweis proves very elegantly in Ref. 1 that the realization of a digital filter algorithm directly based upon the circuit by interconnecting transformed elements is physically unrealizable. Also there is no direct way of writing an algorithm which takes care of the interconnection of elements.

Take as an example an inductance. One has to realize the voltage-current relation

$$v = \psi R_o I \qquad \text{II-10}$$

with $R_o = \frac{2}{T} L$.

Using II-6 to carry out a z-transformation leads to

$$v = \frac{z-1}{z+1} R_o I \qquad \text{II-11}$$

which results after inverse z-transformation in the following difference equation

$$v(nT) + v[(n-1)T] = R_o \cdot \{i(nT) - i[(n-1)T]\} \ . \qquad \text{II-12}$$

Small letters are used to denote instantaneous values. It is easily seen that the computation of v(t) at t = nT not only requires the knowledge of v(t) at the previous sampling instant but also the current at the present one. Representing the above difference equation in a signal-flow graph in the ψ-domain, one has generated a feedback loop without delay which is known to be physically unrealizable. This will be true for the v-i relationships of all other digitized circuit elements.

This can be avoided by using wave quantities in signal-flow graphs and getting from there the algorithm for the digital filter. For any transmission line the voltage and

current at any point can be defined by a wave traveling to the right, A, and a wave, B, traveling to the left after reflection. At any instant of time the effective voltage is a sum of both waves at any point along the line

$$V = A + B \qquad \text{II-13}$$

and the current is given by

$$I = \frac{1}{R_o} (A - B) \ . \qquad \text{II-14}$$

Adding both equations gives

$$A = \frac{1}{2} (V + R_o I) \qquad \text{II-15a}$$

and

$$B = \frac{1}{2} (V - R_o I) \ . \qquad \text{II-15b}$$

Leaving off the scaling factor 1/2 gives as the general relations between the analog circuit quantities current and voltage and the wave quantities A and B

$$A = V + R_o I \qquad \text{II-16a}$$

$$B = V - R_o I \qquad \text{II-16b}$$

The next step in the procedure has to be to convert a signal-flow graph to a wave-flow diagram. By using II-16 which holds at any point along the line one calculates the v-i relationship which holds only at the input port to the transmission line.

Taking as an example an inductor one has

$$V = \psi R_o I \qquad \text{II-17}$$

with $R_o = \frac{2}{T} L$. Substituting II-17 into II-16 and dividing II-16a and b gives

$$B = \frac{\psi - 1}{\psi + 1} A = - \frac{1 - \psi}{1 + \psi} A \qquad \text{II-18}$$

The final step in the process is now to carry out a z-transformation of the standard z-transform as given by II-7. By substituting

$$\psi = \frac{z - 1}{z + 1}$$

one finally arrives at

$$B = -z^{-1} A \qquad \text{II-19}$$

In carrying out an inverse z-transform one finally arrives at

$$b(nT) = -a[(n-1)T] \qquad \text{II-20}$$

using small letters to denote instantaneous values. The sampling interval is twice the basic delay τ. Since a signal has to return to the input port after reflection it has gone twice through the delay τ of the transmission line on a round trip to arrive at the next sampling instance. The overall process is summarized in Figure 2.

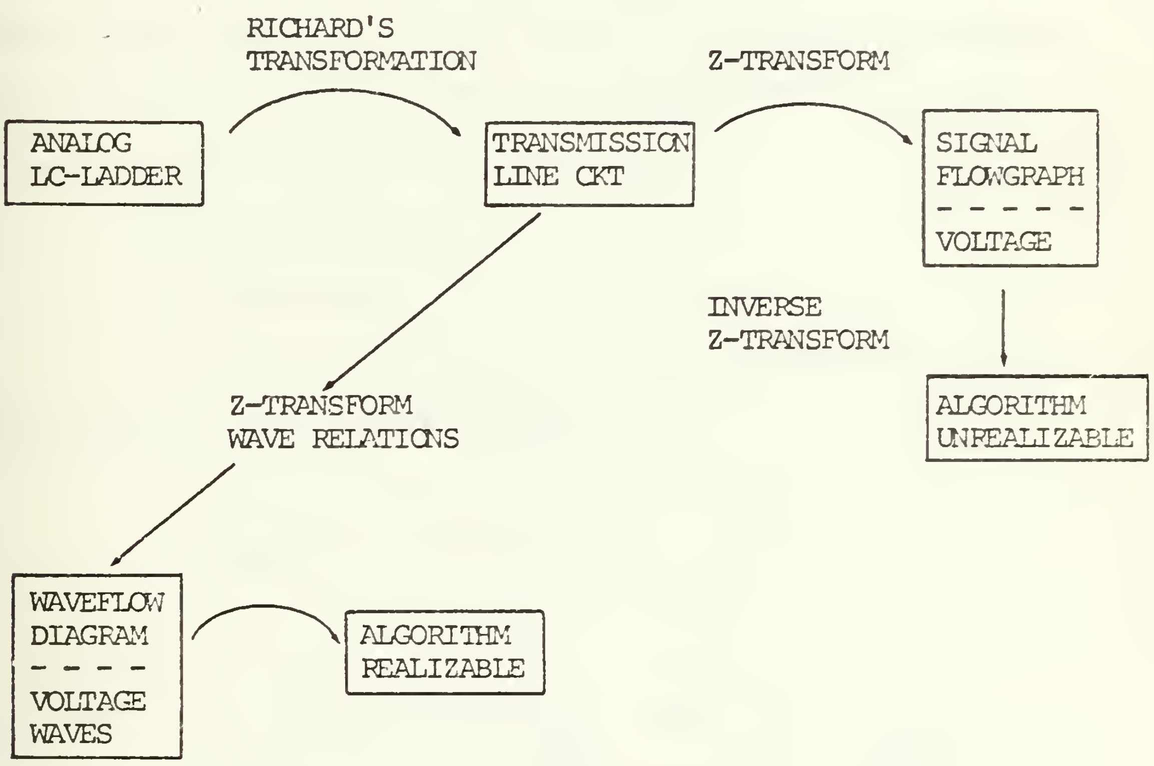


FIGURE 2. Summary of the steps involved

Because of the use of waves these digital filters are called wave-digital filters.

B. REALIZATION

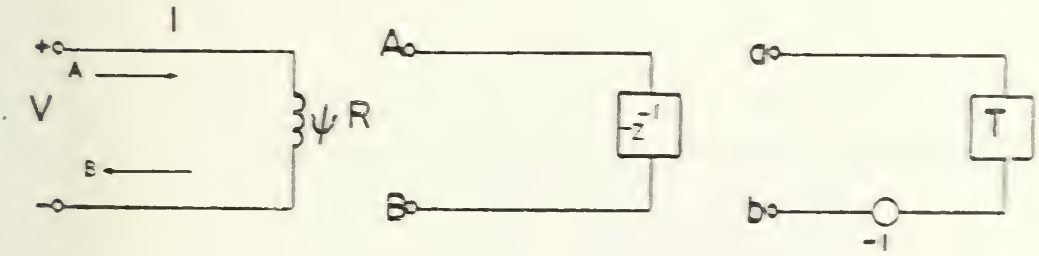
The characteristic resistance of a piece of transmission line will be called from here on R which is the port impedance for the wave quantities to the one-port transmission line circuit element. To ensure propagation of the wave without reflection from the port the port resistances of interconnections have to be matched. This matching has to be done by an algorithm in the digital realization. The general approach will be to look at the current-voltage relationship to be satisfied and use II-16 to get the related relation for voltage waves.

1. Passive Elements

a. Inductors

The realization of inductors was outlined in the previous section and results in

$$b(nT) = -a[(n-1)T]$$



a. b. c.

FIGURE 3. Realization of an inductor
a. circuit diagram
b. wave-flow diagram
c. algorithm

b. Capacitors

The V-I relation for a capacitor is defined as

$$V = \frac{R}{\psi} I \qquad \text{II-21}$$

Substituting II-21 into II-16 and dividing II-16a and b results in

$$B = \frac{1 - \psi}{1 + \psi} A = z^{-1} A \qquad \text{II-22}$$

The final difference equation is

$$b(nT) = a[(n-1)T] \qquad \text{II-23}$$

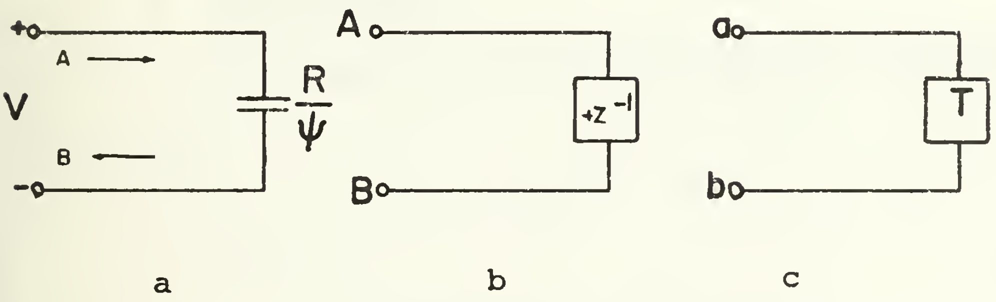


FIGURE 4. Realization of a capacitor

a. circuit diagram
b. wave-flow diagram
c. algorithm

c. Resistors

The V-I relation for a resistor is defined as

$$V = RI \qquad \text{II-24}$$

which leads immediately by II-16 to

$$B = 0 \qquad \text{II-25}$$

and for the difference equation

$$b(nT) = 0 \qquad \text{II-26}$$

There is no reflected wave from a resistor if $R_o = R$, and hence what has to be required, and hence it is also called a wave sink.

d. Open Circuit

The relation to be realized is $I = 0$ which leads by II-16 to

$$A = B \qquad \text{II-27}$$

and

$$a(nT) = b(nT) \qquad \text{II-28}$$

e. Short Circuit

A short circuit has $V = 0$ giving by II-16

$$A = -B \qquad \text{II-29}$$

and

$$a(nT) = -b(nT) \qquad \text{II-30}$$

f. Unit-Element

A unit-element is only defined in transmission line filters by the relations

$$B_1 = A_2 e^{-\frac{sT}{2}}$$
$$B_2 = A_1 e^{-\frac{sT}{2}} \qquad \text{II-31a,b}$$

Note from Figure 1 that the unit-element is used as a two-port network.

The difference equations are

$$b_1(nT) = a_2[(n-\tfrac{1}{2})T]$$
$$b_2(nT) = a_1[(n-\tfrac{1}{2})T] \qquad \text{II-32a,b}$$

The delay of T/2 has to be taken into account in determining the calculation sequence for the filter algorithm.

2. Active Elements

a. Resistive Source

A source of voltage V_s with a series resistance R_s has a V-I relation

$$V_s = V + R_s I \qquad \text{II-33}$$

where V is the voltage at the output port of this oneport. Using II-16 yields

$$V_s = A \qquad \text{II-32}$$

and

$$a(nT) = v_s(t) \quad \text{at } t = nT \ . \qquad \text{II-33}$$

b. Ideal Voltage Source

An ideal voltage source has the source voltage appearing at the output port and hence

$$V_s = V \qquad \text{II-34}$$

which leads upon substitution into II-16 to

$$A = V_s + RI$$

II-35a,b

$$B = V_s - RI$$

taking R to be an arbitrary constant which will drop out. Adding II-35a and b gives

$$B = 2V_s - A \qquad \text{II-36}$$

and

$$b(nT) = 2v_s(nT) - a(nT) \qquad \text{II-38}$$

c. Ideal Current Source

Taking the ideal source in combination with a series arbitrary R one gets by use of II-16

$$B = 2V_s - A \qquad \text{II-38}$$

and

$$b(nT) = 2v_s(nT) - a(nT) \qquad \text{II-39}$$

The wave-flow diagrams for the elements of section a. and b. are summarized in Table I.

3. Structural Elements

So far the different circuit elements have been realized. Two problems arise now in interconnecting these elements: Reflections from interconnections that are not matched in port impedance and realization of the nodes of the analog structure. In the transmission line filter these nodes collect and redistribute the forward and reverse traveling voltage waves.

The process of impedance matching is shown best on a two-port interconnection

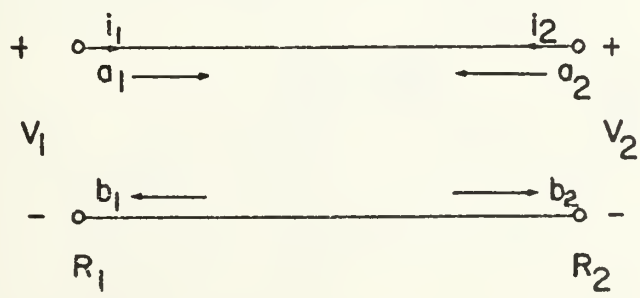
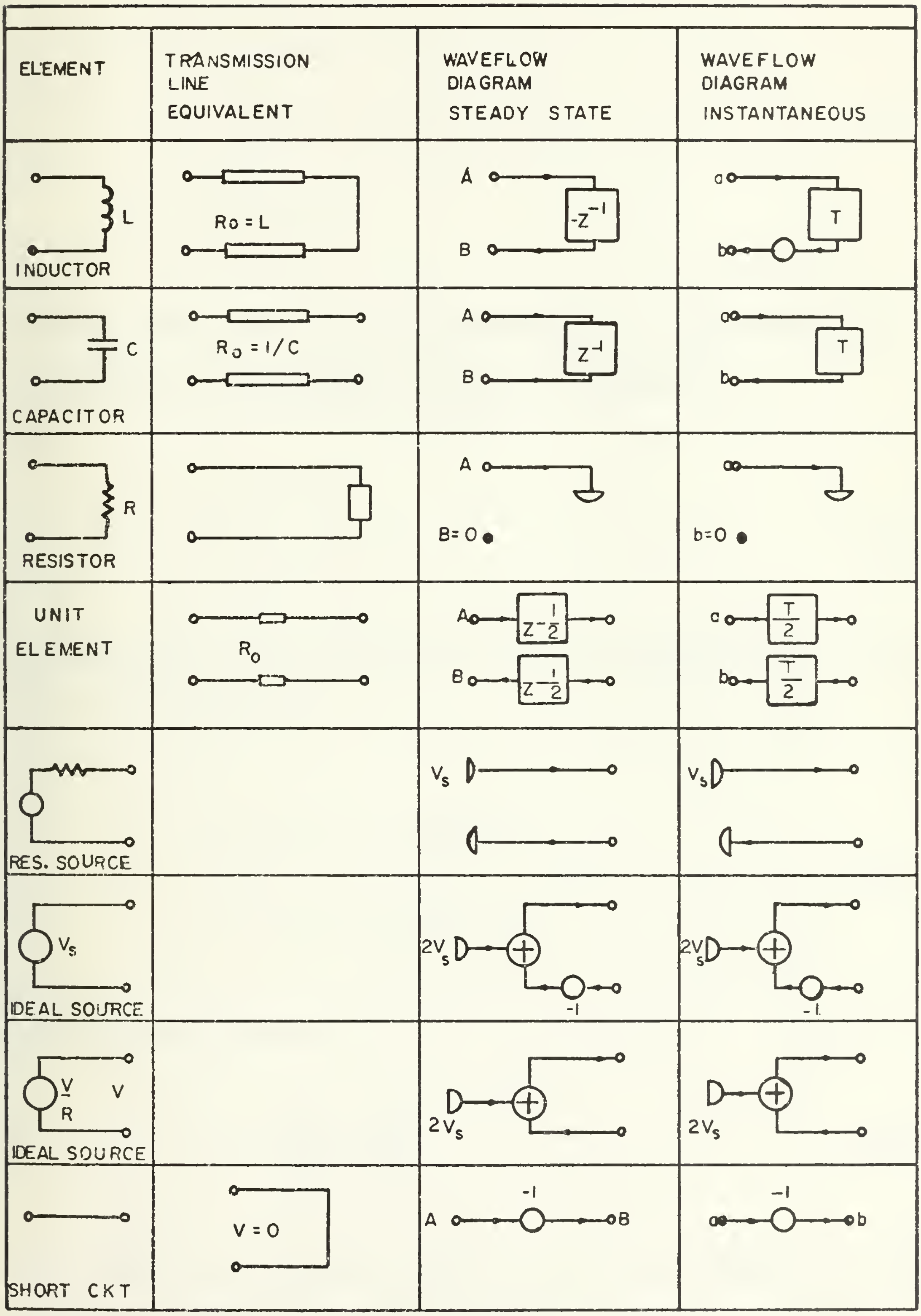


FIGURE 5. Interconnection of two ports with port resistances R_1 and R_2

TABLE I


ELEMENT
TRANSMISSION LINE EQUIVALENT
WAVEFLOW DIAGRAM STEADY STATE
WAVEFLOW DIAGRAM INSTANTANEOUS
INDUCTOR
L
Ro = L
A
B
$-Z^{-1}$
a
b
T
CAPACITOR
C
$R_0 = 1/C$
Z^{-1}
RESISTOR
R
B= 0
b=0
UNIT ELEMENT
R_0
$Z^{-\frac{1}{2}}$
$\frac{T}{2}$
RES. SOURCE
V_s
IDEAL SOURCE
$2V_s$
-1
$\frac{V}{R}$
V
SHORT CKT
V = 0

At each port II-16 is valid which can be written in closed form as

$$a_k = v_k + R_k i_k$$

$$b_k = v_k - R_k i_k \qquad \text{II-40a,b}$$

$$k = 1,2.$$

Since above shown interconnections are ideal without attenuation and delay we have by inspection

$$v_1 = v_2$$

$$i_1 = -i_2 \quad . \qquad \text{II-41a,b}$$

Substituting II-41 into II-40 and eliminating the v's and the i's results in

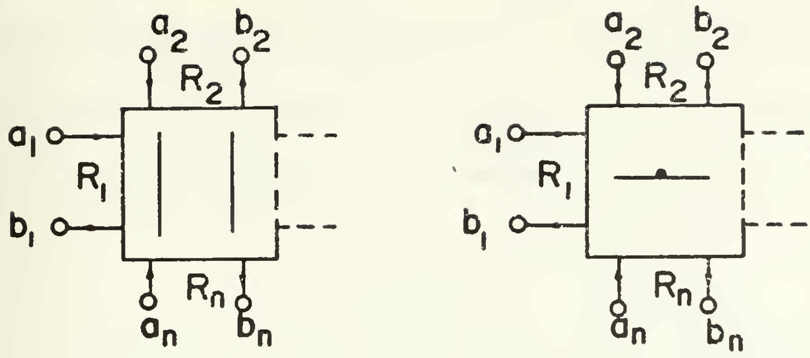
$$b_1 = a_2 + \beta(a_2 - a_1)$$

$$b_2 = a_1 + \beta(a_2 - a_1) \qquad \text{II-42a,b}$$

where β is given by

$$\beta = \frac{R_1 - R_2}{R_1 + R_2} \qquad \text{II-43}$$

The wave-flow diagram representation for equation II-42 is called an adaptor and the symbol is shown in Figure 6.



a b

FIGURE 6. Wave-flow diagram symbol of a
a. parallel adaptor
b. series adaptor

Note that due to the fact of $R_1 > 0$ and $R_2 > 0$ we have $|\beta| < 1$ and one can see clearly that β is the reflection coefficient of transmission theory. As shown in Figure 6 this matching can be extended bo any number of ports. The second purpose of an adaptor is to serve as node for the wave-flow diagram.

To arrive at a digital ladder structure of the combination of elements one has to generate an algorithm that is equivalent to Kirchoff's current or voltage law. A series connection of any number of elements has to satisfy the relation

$$v_1 + v_2 + v_3 + \ldots + v_n = 0$$

II-44a,b

$$i_1 = i_2 = i_3 = \ldots = i_n$$

Using II-40 for k = 1,n one arrives after use of II-44 and elimination of v's and i's at a system of linear algebraic equations that can be written in matrix form. For a three-port series adaptor this matrix is

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} 1-\beta_1 & -\beta_1 & -\beta_1 \\ -\beta_2 & 1-\beta_2 & -\beta_2 \\ -\beta_3 & -\beta_3 & 1-\beta_3 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \qquad \text{II-45}$$

The constant matrix relating output waves to input waves is often referred to as scattering matrix. The β's are given by

$$\beta_k = \frac{2R_k}{R_1 + R_2 + \ldots + R_n} \qquad \text{II-46}$$

and equation in II-45 is of the form

$$b_k = a_k - \beta_k \sum_{m=1}^{n} a_m \qquad \text{II-47}$$

The sum of the coefficients β can easily be shown to be constant

$$\beta_1 + \beta_2 + \ldots + \beta_n = 2 \qquad \text{II-48}$$

This signifies the presence of only n-1 independent coefficients with the nth coefficient as a linear combination of all the other ones.

The corresponding relations for a parallel connection of elements are

$$v_1 = v_2 = v_3 = \ldots = v_n \qquad \text{II-49}$$

$$i_1 + i_2 + \ldots + i_n = 0 \qquad \text{II-50}$$

which upon substitution into II-40 can be shown to lead to after elimination of the v's and the i's

$$b_k = \sum_{m=1}^{n} \alpha_m a_m - a_k \qquad \text{II-51}$$

where the α's are given by

$$\alpha_k = \frac{2G_k}{G_1 + G_2 + \ldots + G_n} \qquad \text{II-52}$$

and G = 1/R. The sum of the coefficients is again constant

$$\alpha_1 + \alpha_2 + \ldots + \alpha_n = 2 \qquad \text{II-53}$$

giving again n-1 independent ones. The matrix equation for a three-port parallel adaptor is given as

$$
\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} \alpha_1 - 1 & \alpha_2 & \alpha_3 \\ \alpha_1 & \alpha_2 - 1 & \alpha_3 \\ \alpha_1 & \alpha_2 & \alpha_3 - 1 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \qquad \text{II-54}
$$

4. Ground Rules

Any realization of a wave-digital ladder filter diagram from analog structures has to obey the following ground rules:

1. Connections of ports only pairwise to ensure proper wave-flow.
2. The wave-flow must be compatible, i.e. the outflow of one port must flow into the connected port.
3. The wave flow diagram must not contain delay free loops.

III. REALIZATION

There exist several methods in design of digital ladder structures. A direct design was proposed by Bruton [Ref. 3]. Another possibility is to synthesize transmission line filters by using the theory of unit-element filters [Ref. 4]. As a third way one starts from the analog ladder structure consisting of reactive elements with resistive termination and converts this structure to a digital ladder structure by the digitization methods shown earlier. This way was chosen in this thesis.

A. ANALOG DESIGN

The design of an analog ladder structure was found to be quite difficult because driving-point impedance synthesis procedures cannot be used to guarantee a maximum power transfer over most of the passband range. The synthesis procedure to be used is the insertion loss design procedure a good introduction to which can be found in Skwirzynski [Ref. 5] where also a great amount of design data is supplied. Numerous evaluated filters can be found by Saal [Ref. 6] from where the values for the analog ladder filters for the experimental realization of the wave-digital ladder filter had been obtained.

B. DIGITIZATION

The first step is to replace each analog circuit element in the circuit diagram by its digital counterpart and each node by the appropriate adaptor. This procedure leads to direct connections of series - and parallel adaptors which generates at the interconnection inner loops without delays since any output wave is dependent on any input wave without delay. A violation of one of the ground rules can be avoided if one changes the structure of the analog network by use of Kuroda's identities [Ref. 4].

With these identities it is possible to convert reactive elements from one type to another by introducing unit-elements: Unit-elements if matched to the load can be introduced between network and load without changing the insertion loss of the network (a phase delay will be introduced). By applying Kuroda's identities two of which are shown in Figure 8 it is possible in a step by step procedure to move these unit-elements through the network one by one until all the shunt capacitors are converted to series inductors and all inductors are separated by unit-elements as shown in Figures 9, 10 and 11.

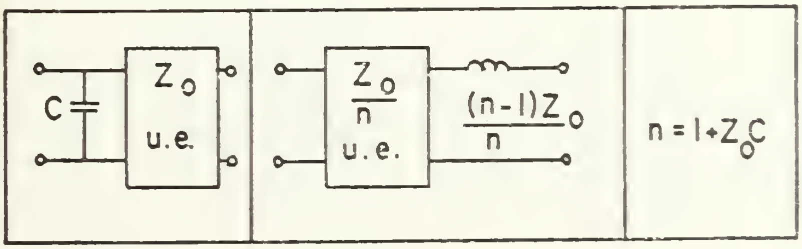


Figure 8a. One of Kuroda's Identities

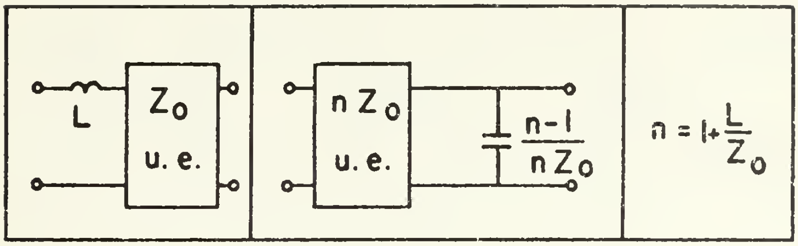


Figure 8b. One of Kuroda's Identities.

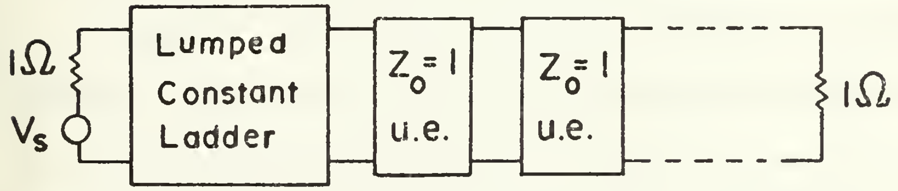


Figure 9. Introduction of Arbitrary Number of Unit-Elements Between Ladder and Load.

The resistors at both ends in Figure 10 don't have to be equal depending on the particular design.

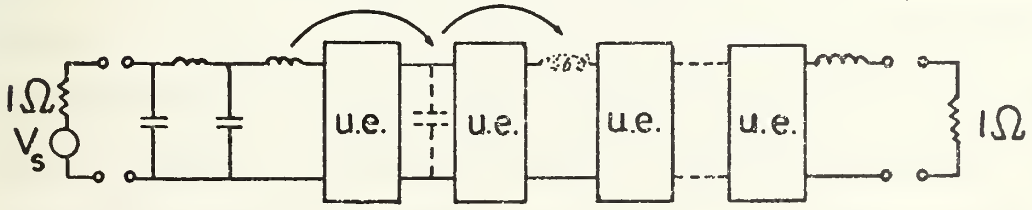


Figure 10. Application of Kuroda's Identites.

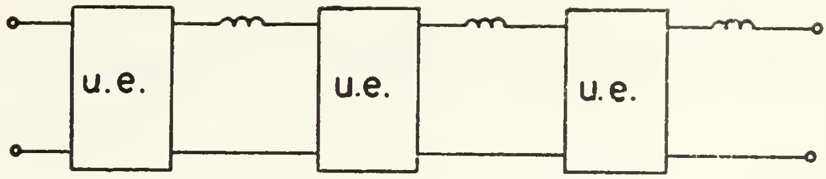


Figure 11. Final network with Terminations not Shown since they Remain Unchanged.

This new structure can now be converted to a signal-flow diagram without violation of any of the ground rules since each adaptor will be separated by a delay represented by the unit-element. This realization approach introduces unit-elements not contained in the original filter. Since they do not generally contribute to the filter action of the network their use is not desired. Crochiere [Ref. 7] in his work on sensitivity of wave-digital ladder filters uses this approach. The main purpose of this section is to introduce Kuroda's identities and the use of unit-elements.

C. TRUE DIGITAL LADDER STRUCTURES

Another way of solving the problem with the delay free loop at the interconnection of adaptors was proposed by Fettweis [Ref. 8]. The delay free loop was created due to the dependence of any of the output signals on any of the input signals. If one of the two multipliers of the interconnected ports is chosen to be equal to 1 then the dependence of the output signal on the input signal at that port

is broken and a direct interconnection of adaptors becomes possible. No unit-elements are required. This is shown on an interconnection of a series - and a parallel 3-port adaptor [Fig. 12].

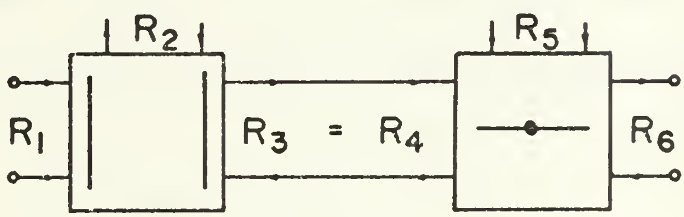


Figure 12. Interconnection of a Series 3-port and a Parallel 3-port Adaptor.

The output equation for port R_3 is given by

$$b_3 = \alpha_1 a_1 + \alpha_2 a_2 + \alpha_3 a_3 - a_2 \qquad \text{III-1}$$

where the coefficients are given by

$$\alpha_k = \frac{2\ G_k}{G_1 + G_2 + G_3} \qquad \text{III-2}$$

and for port R_4 by

$$b_4 = a_4 - \alpha_4(a_4 + a_5 + a_6) \qquad \text{III-3}$$

where one has

$$\alpha_4 = \frac{2\ R_4}{R_4 + R_5 + R_6} \qquad \text{III-4}$$

The delay free loop is easily recognized. Making an arbitrary choice between α_3 and α_4, say α_3, one wants

$$\alpha_3 = 1\ . \qquad \text{III-5}$$

Since R_3 is the port of the parallel adaptor this equation is arrived at from III-2 only if

$$G_3 = G_1 + G_2\ . \qquad \text{III-6}$$

Since the port impedances of ports at interconnection of adaptors are not determined by analog circuit element values this choice of G_3 can be made. The resultant output equation for R_3 is

$$b_3 = \alpha_1 a_1 + \alpha_2 \alpha_2 \qquad \text{III-7}$$

Applying the condition that the sum of all multipliers for each adaptor equals two, one can replace α_2 by $(1 - \alpha_1)$ and arrives finally at

$$b_3 = a_2 + \alpha_1(a_1 - a_2) \quad . \qquad \text{III-8}$$

The equations for the second adaptor remain unchanged unless R_5 or R_6 are again connected to an adaptor at which interconnection the same principle can be applied. Figures 13 and 14 show the wave-flow diagram for the modified series and parallel three-port adaptors using only one multiplier.

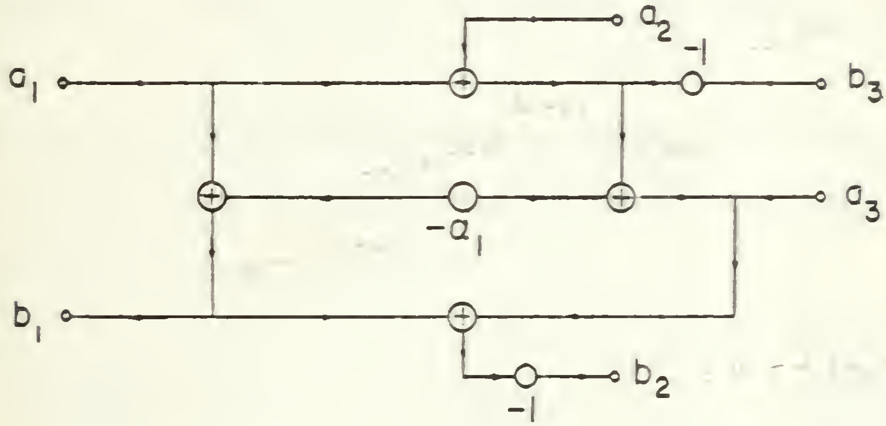


Figure 13. Series Three-port adaptor with α_3=1 and α_2=1-α_1

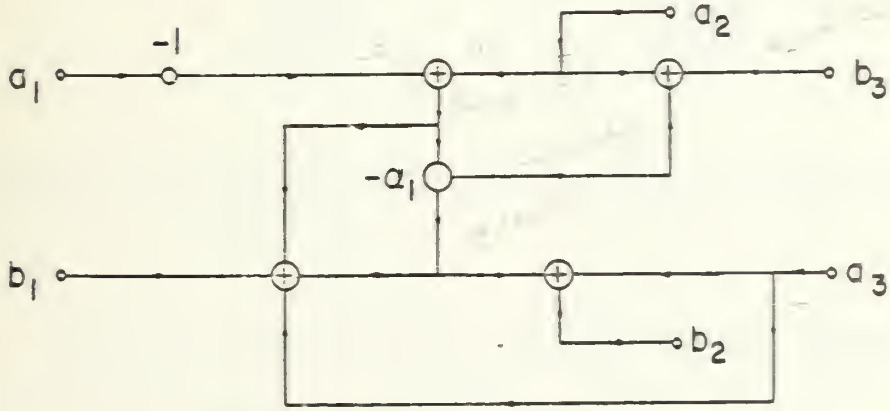


Figure 14. Parallel three-port adaptor with α_3=1 and α_2=1-α_1

D. SUMMARY OF STEPS

(a) By insertion loss design one gets a resistivity terminated analog LC-ladder structure [Fig. 15].

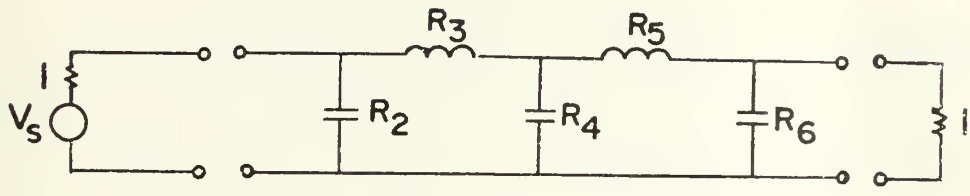


Figure 15. Resistivity terminated LC-ladder network

(b) Apply digitization methods element by element one arrives at the wave-digital wave-flow diagram [Fig. 16].

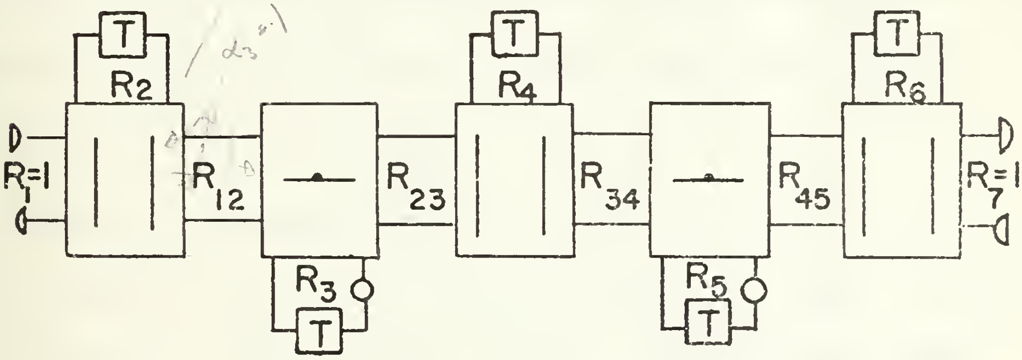


Figure 16. Wave-flow diagram for analog filter of Figure 15.

The port impedances R_1 through R_7 are predetermined by the analog element values.

(c) Choose the port-impedance for interconnections.

Carrying out step three on Figure 16, choosing in each case the right port of each adaptor to have $\alpha = 1$, (except for the last adaptor), leads to the following port impedance

$$G_{12} = G_1 + G_2 \qquad \text{III-9a}$$

$$R_{23} = R_{12} + R_2 \qquad \text{III-9b}$$

$$G_{34} = G_{23} + G_3 \qquad \text{III-9c}$$

$$R_{45} = R_{34} + R_4 \qquad \text{III-9d}$$

The total number of multipliers is six, one for each adaptor and two for the last one which corresponds directly to the degree of freedom of the analog network (number of resistive passive elements minus one).

E. NUMERICAL IMPLEMENTATION

To actually implement the wave-digital ladder filter one must carefully evaluate the correct computational order to ensure the knowledge of all input quantities for the output at hand. Figure 17 shows a conventional signal flow graph for the fifth order digital filter of Figure 16 with the input and output waves as state variables. For each adaptor port two is connected to a delay. From the signal flow graph of Figure 17 the computational order can be obtained by inspection. Calculations are assumed to take place

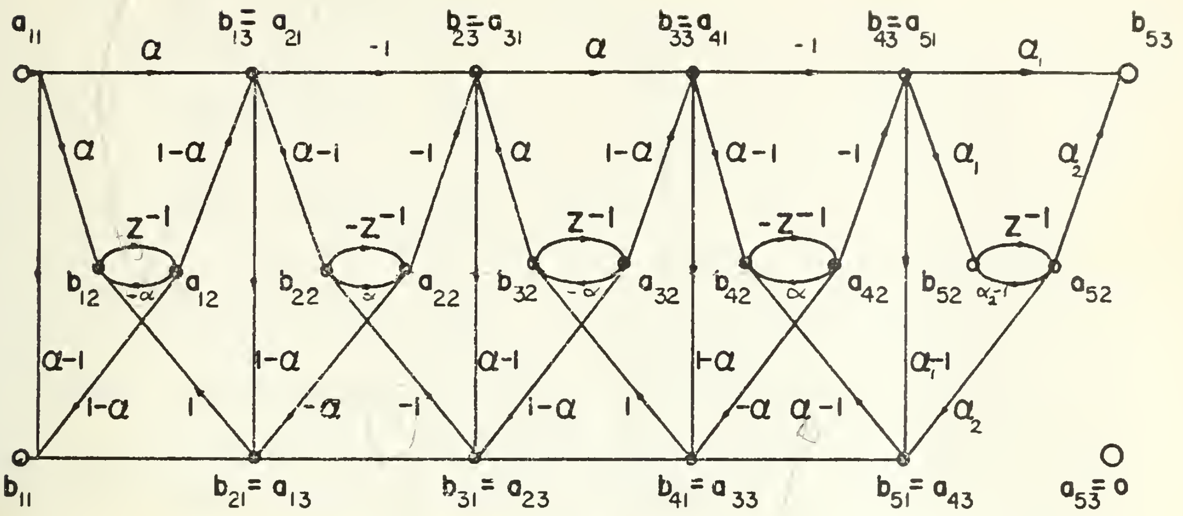


Figure 17. Signal flow graph for filter of Figure 16. a_{31} denotes input to port 1 of adaptor number 3.

simultaneously and to take now time. The values at the nodes are assumed to be zero between sampling instants except for the wave values stored in the delay line. Since an actual computer algorithm cannot provide this ideal situation one has to follow the signal flow graph. Initial conditions are assumed to be zero. First the output values for node three can be evaluated since it is independent from the present input to that port. Going through the signal flow graph to the end one is able to calculate all output values for the last adaptor since the resistive termination of the last port supplies zero input. Going back through the signal flow graph the other outputs can be evaluated. The equations for the fifth order filter example in correct computational order are:

(1) $b_{13} = \alpha_1(a_{11}-a_{12}) + a_{12}$

(2) $b_{23} = -a_{21} - a_{22}$

(3) $b_{33} = \alpha_3(\alpha_{31}-a_{32}) + a_{32}$

(4) $b_{43} = -a_{41} - a_{42}$

(5) $b_{53} = \alpha_{51}a_{51} + \alpha_{52}a_{52}$

(6) $b_{52} = \alpha_{51}a_{51} + \alpha_{52}a_{52} - a_{52}$

(7) $b_{51} = \alpha_{51}a_{51} + \alpha_{52}a_{52} - a_{51}$

(8) $b_{42} = -a_{41} - a_{43} + \alpha_4(a_{41}+a_{42}+a_{43})$

III-10

(9) $b_{41} = a_{41} - \alpha_4(a_{41}+a_{42}+a_{43})$

(10) $b_{32} = \alpha_3(a_{31}-a_{32}) + a_{33}$

(11) $b_{31} = \alpha_3(a_{31}-a_{32}) + a_{32} + a_{33} - a_{31}$

(12) $b_{22} = -a_{21} - a_{23} + \alpha_2(a_{21}+a_{22}+a_{23})$

(13) $b_{21} = a_{21} - \alpha_2(a_{21}+a_{22}+a_{23})$

(14) $b_{12} = \alpha_1(a_{11}-a_{12}) + a_{13}$

(15) $b_{11} = \alpha_1(a_{11}-a_{12}) + a_{12} + a_{13} - a_{11}$

In addition to these equations a storage of output to the delays is required. One should also keep in mind the equality of waves at directly connected ports, i.e. at any instant of time for example b_{31} is identical to a_{21} and a_{31} to b_{21}.

F. THEORETICAL RESPONSE

To get the theoretical response it is possible to evaluate the transfer function from Figure 17 and let $z = e^{j\omega T}$. A considerably shorter way via matrix theory is shown on the fifth order filter. At the interconnections of ports in Figure 16 the right hand values of a left adaptor are identical to the lefthand values of a right adaptor. So all one has to do is to eliminate port two by algebraic procedures which means for a 3-port adaptor:

(a) Get $b_2 = \pm z a_2$

(b) Substitute (a) into the corresponding output equation of the adaptor to eliminate b_2

(c) Substitute the resultant $a_2 = fn(a_1, a_3)$ into the remaining output equations.

By changing the dependent quantities in the resultant equations of the form

$$b_1 = fn(a_1, a_3)$$

$$b_3 = fn(a_1, a_3) \qquad \text{III-11a,b}$$

one finally arrives at

$$a_1 = fn(b_3, a_3)$$

$$b_1 = fn(b_3, a_3) \qquad \text{III-12a,b}$$

In matrix form, II-12 can be written as

$$\begin{bmatrix} a_1 \\ b_1 \end{bmatrix} = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \begin{bmatrix} b_3 \\ a_3 \end{bmatrix} \qquad \text{III-13}$$

The entries in the P-matrix are generally functions of α and of z. Since b_3 and a_3 are identical to the leftside values of the next adaptor one immediately generates a chain matrix relating a_1 and b_1 of the first adaptor to b_3 and a_3 of the last one. The last a_3 will always be zero (wave sink at port 3 of adaptor) so that the chain matrix finally is of the form for the example. The matrices for the usual

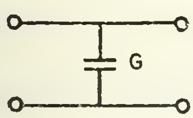
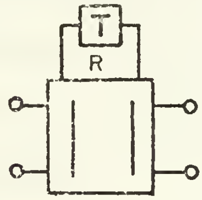
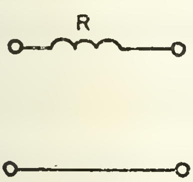
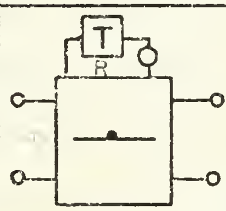
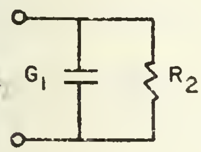
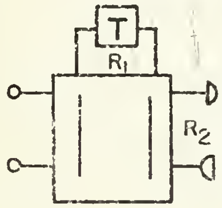
$$\begin{bmatrix} a_{11} \\ b_{11} \end{bmatrix} = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{21} & Q_{22} \end{bmatrix} \begin{bmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{bmatrix} \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \end{bmatrix} b_{53}$$

circuit arrangements are shown in Table II. Solution of the matrix equation will directly lead to the transfer function $H_1(z) = \frac{b_{53}}{a_{11}}$. Since the complementary output can be taken from b_{11} one also is able to evaluate the transfer function for the high pass complementary filter

$$H_2(z) = \frac{b_{11}}{a_{11}} .$$

Using $z = e^{j\omega T}$ one generates the frequency response.

TABLE II

CIRCUIT ELEMENTS	TRANSMISSION LINE EQUIVALENT	WAVE MATRIX	MATRIX ELEMENTS
 	 	$\begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}$	$\Delta = \alpha(z+1)$ $P_{11} = +(z+\alpha)/\Delta$ $P_{12} = (\alpha-1)/\Delta$ $P_{21} = z(\alpha-1)/\Delta$ $P_{22} = (z\cdot\alpha+1)/\Delta$
 	 	$\begin{bmatrix} Q_{11} & Q_{12} \\ Q_{21} & Q_{22} \end{bmatrix}$	$\Delta = z+1$ $Q_{11} = -(z+\alpha)/\Delta$ $Q_{12} = (\alpha-1)/\Delta$ $Q_{21} = z(\alpha-1)/\Delta$ $Q_{22} = -(\alpha z+1)/\Delta$
 	 	$\begin{bmatrix} U_1 \\ U_2 \end{bmatrix}$	$\Delta = \alpha_1\cdot(z+1)$ $U_1 = (z-\alpha_2+1)/\Delta$ $U_2 = \dfrac{\Delta - z + \alpha_2 - 1}{\Delta}$

IV. THEORETICAL SENSITIVITY

In analog ladder structures resonant frequencies corresponding to attenuation poles are determined by the product of two element values. The pole and zero positions are shifted about their true values with variation of circuit element values causing a change in magnitude and phase characteristic of the response including bandwidth and cutoff frequencies. This variation is either caused by inaccurate circuit element values or drifting due to various physical phenomena.

In digital filters drifting of values does not exist since the filter coefficients are stored in the computer and can be considered constant as long as the filter is operational. But since the design generates decimal coefficients their representation in a binary number of finite wordlength generates what is called quantization error. Assuming a coefficient obtained with infinite accuracy to be $\alpha = 0.82$ its representation in the binary system is 0.11010001 ... . A wordlength of 4 binary digits after the decimal point gives 0.1101 which converted to decimal leads to .8125, close to .82 but not exact.

The inaccuracy which is dependent on the number of binary digits causes a change in the response. Hence the effect of finite wordlength for the digital filter is the same as that of inaccurate analog element values for the

analog filter. Analog LC-ladder filters are very insensitive in their transfer characteristics against element variations. It has to be determined whether these sensitivity properties are preserved and carried over by transformations. Gupta and Renner [Ref. 9] prove this as follows: The voltage-wave into the load in Figure 17 is equal to the effective voltage V_2 over R_2 since $a_2 = 0$ for all times.

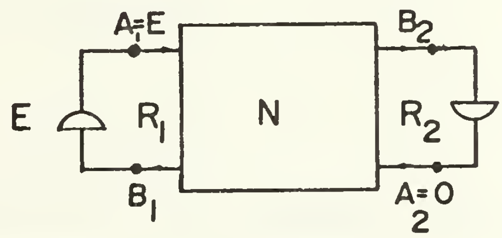


Figure 17. Wave-Digital Two Port

Since $a_1 = e(t)$ the voltage transfer functions H(s) is given by

$$|H(s)| = \frac{V_2}{E} \qquad \text{IV-1}$$

and the voltage-wave transfer function is given by

$$|H(z)| = \frac{b_2}{a_1}\sqrt{\frac{R_1}{R_2}} \qquad \text{IV-2}$$

They show the sensitivity of H(s) to be zero against analog element variation at particular points, the maximum available

power points, only.

$$\left.\frac{\delta|H(s)|}{\delta\beta}\right|_{f_o} = 0 \qquad \text{IV-3}$$

where f_o is the MAP transfer point and $\delta\beta$ is given by IV-10. Consider the wave-digital filter multiplier α as a function of the analog network elemaents:

$$\alpha = fn(L,C,R) \qquad \text{IV-4}$$

where L,C,R are row vectors.

A small variation of the nominal value of $\overline{\alpha}$ will cause a corresponding change in L, C and R:

$$\overline{\alpha} + \delta\alpha = f(\overline{L} + \delta L, \overline{C} + \delta C, \overline{R} + \delta R). \qquad \text{IV-5}$$

Taking only the first order terms of a Taylor series expansion leads to

$$\delta\alpha = \frac{\delta f}{\delta \overline{L}} (\delta L)^T + \frac{\delta f}{\delta \overline{C}} (\delta C)^T + \frac{\delta f}{\delta \overline{R}} (\delta R)^T \qquad \text{IV-7}$$

or

$$\delta\alpha = K\ \delta\beta^T \qquad \text{IV-8}$$

where

$$K = \left(\frac{\delta f}{\delta \overline{L}} \cdot \frac{\delta f}{\delta \overline{C}} \cdot \frac{\delta f}{\delta \overline{R}}\right) \qquad \text{IV-9}$$

$$\delta\beta = (\delta L \delta C \delta R). \qquad \text{IV-10}$$

From IV-1 and IV-2 $|H(z)|$ and $|H(s)|$ are equal:

$$|H(z)| = |H(s)|.$$

Hence

$$\frac{\delta |H(z)|}{\delta \alpha} = \frac{\delta |H(s)|}{\delta \alpha} \qquad \text{IV-11}$$

which upon substitution of IV-8 can be written as

$$\frac{\delta |H(z)|}{\delta \alpha} = \frac{\delta |H(s)|}{K \; \delta\beta^T} \qquad \text{IV-12}$$

so the zero sensitivity of $|H(s)|$ with respect to variations of the analog element values is carried over to the sensitivity of $|H(z)|$ with respect to variations of provided only small variations in the digital-filter coefficients are made such that the approximation of the Tayler series expansion by the first order term is valid. The zero sensitivity of $|H(z)|$ can also be shown in a more qualitative picture

directly on the wave-digital filter shown in Figure 17. Introducing the concept of theoretical power (no power is consumed by an algorithm) absorbed by the wave-twoport one can write the relation

$$P = \frac{|E|^2 - |B_1|^2}{R_1} - \frac{|B_2|^2}{R_2} . \qquad \text{IV-13}$$

Solving IV-13 for $|B_2|^2$ and substituting into IV-2 leads to

$$|H(z)|^2 = 1 - \frac{|B_1|^2}{|E|^2} - \frac{PR_1}{|E|^2} \qquad \text{IV-14}$$

This gives the obvious condition

$$|H(z)|^2 \leq 1 \qquad \text{IV-15}$$

At points where the full voltage wave is transfered to the output and no reflections back to the source occur, equivalent to the MAP points, one will get the limiting case

$$|H(z)|^2 = 1 . \qquad \text{IV-16.}$$

The attenuation A of the wave-digital filter may be defined as:

$$A = - \ln |H(z)| . \qquad \text{IV-17}$$

The power P absorbed in the two-port will always be zero. So as long as IV-16 is valid at MAP points ω_o we have

$$A = 0 \qquad \text{IV-18}$$

This in turn implies

$$\frac{\delta A}{\delta \alpha} = 0 \qquad \text{IV-19}$$

One can conclude with the statement the smaller the attenuation in the passband the smaller is the sensitivity of the wave-digital filter transfer properties with respect to coefficient variations.

V. RESULTS

To investigate the sensitivity properties of wave-digital filters experimentally 19 different Chebyshev low-pass filters were realized. On the basis of analog ladder structures taken from Saal [Ref. 7], five filters of order N = 5 with ripple (in dB) in the range from R = 1.25 to R = 0.1 were realized, in addition six filters of order N = 7 with ripple in the range from R = 0.28 to R = 0.1. The cutoff frequency was normalized to $\omega_c = 1$. The sampling time was taken to be $T_s = 2.0$ for all filters which corresponds to $f_c/f_s = 0.32$. The filter with ripple R = .177 for order N = 5 and order N = 7 was arbitrarily chosen for an investigation of dependence of sensitivity properties versus sampling time. The sampling times taken were $T_s = 2.0$, 1.0, 0.5, 0.1, and 0.05 corresponding to $f_c/f_s = 0.32$, .16, .08, .016 and .008.

Corresponding to the realization of the wave-digital filter a recursive series-cascade digital filter with first and second order sections [Fig. 17] was realized directly from the transfer function.

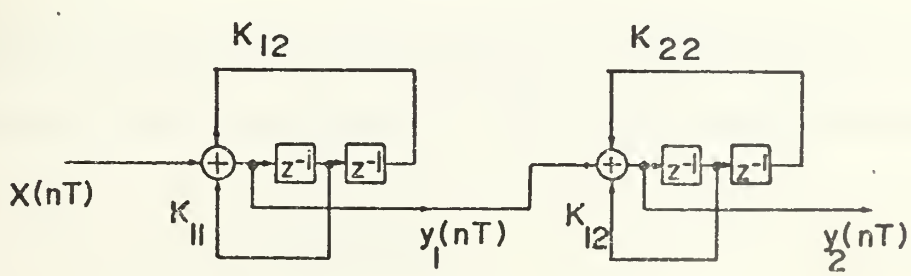


Figure 17. Series-Cascade Second Order Recursive Digital Filter Sections.

The choice was made because both filters have the same number of multipliers and in general cascade structures are known to be less noisy and less sensitive against multiplier variations than direct realizations. The design of the recursive filter is based upon the transfer function H(s) for analog Chebyshev low-pass filters

$$H(s) = \frac{K}{1 + \varepsilon^2 T_n^2(s)} \qquad \text{V-1}$$

where T_n is the n-th Chevyshev polynomial and ε is directly related to the ripple by

$$\varepsilon = (10^{.1R} - 1.0)^{1/2} \qquad \text{V-2}$$

with R denoting the desired ripple in dB and K being a constant. Factorization of the denominator polynomial gives the pole locations. Collection of complex conjugate poles results in the transfer function

$$H(s) = \frac{K}{(s+a_o)(s^2+b_1s+c_1)\ldots(s^2+b_ms+c_m)} \qquad \text{V-3}$$

the relation between pole locations and coefficients in V-3 is obvious. After application of the bi-linear z-transform

$$z = \frac{2}{T}\left(\frac{z-1}{z+1}\right) \qquad \text{V-4}$$

one gets H(z) as

$$H(z) = \frac{K_1 (z+1)^n}{(z^2+d_1 z+e_1)(z^2+d_2 z+e_2)\cdots(z+d_o)}$$

which can be written as

$$H(z) = K_1 \cdot \frac{z+1}{z+d_o} \cdot \frac{(z+1)^2}{(z^2+d_1 z+e_1)} \cdot \frac{(z+1)^2}{(z^2+d_2 z+e_2)} \qquad \text{IV-5a}$$

$$= K_1 \cdot H_1(z) \cdots H_m(z) \qquad \text{IV-5b}$$

Application of the inverse z-transformation gives the system of difference equations that constitute the algorithm of the digital filter.

The response was generated in two ways. After conversion of the ideal multipliers to binary floating-point representation the wordlength of the mantissa was reduced in steps of two bits from 22 bits to four bits. The truncated multipliers were substituted back into the filter and by use of $z = e^{j\omega T}$ the theoretical response was calculated for each step of wordlength for both filters, wave-digital and conventional. The calculation was carried out in double precision on the IBM 360 general purpose digital computer.

The experimental response was obtained by going through the digital filter algorithm up to ten thousand times to arrive at the steady-state value. It was assumed that the steady state value was arrived at once the normalized output

peak had settled down to within 0.001 of the previous peak for a sinusoidal input. This was done in steps of $\Delta\omega = .02$ within the passband. Multipliers were quantized according to the present wordlength of the mantissa and after each algebraic step the mantissa of the result was truncated to the present wordlength to simulate operation of the digital-filter with shorter wordlength for both, memory locations and arithmetic element.

The quantities measured in either case were the maximum and the minimum output in the passband, $|H(j\omega)|_{max}$ and $|H(j\omega)|_{min}$, respectively. The experimentally determined ripple is given by

$$R_{max} = 20 \log_{10} \frac{|H(j\omega)|_{min}}{|H(j\omega)|_{max}} . \qquad \text{V-6}$$

The measure of the error in the response is the relative error

$$\text{relative error} = \frac{R_{max} - R_{spec}}{R_{zpec}} \quad \text{for } R_{max} > R_{spec}$$

$$\text{V-7}$$

$$\text{relative error} = 0 \quad \text{for } R_{max} \leq R_{spec}$$

where R_{spec} is the specified ripple for the design and denotes the maximum variation of the response in the passband.

The experimental results confirm Fettweis conjecture if one looks just at the theoretical response errors. The response after simulation shows no remarkable difference in general except for the cases with higher sampling frequency. The simulated response is very much affected by the truncation noise in the calculation process which is larger for the wave-digital filter due to the higher number of additions than for the recursive filter, thirty-nine versus fourteen.

VI. CONCLUSIONS

From the observed results it can be concluded that the wave-digital filter is a valuable alternative to conventional digital filters. In most cases wave-digital filter can be realized with a considerably shorter wordlength than conventional digital filters thus allowing an overall reduction in size and cost. The larger number of adders is insignificant due to the steadily decreasing hardware cost and the particular cheapness of adders. The difficult access to wave-digital filter due to the unfamiliar insertion loss design procedures for the analog LC-ladder filter could be eased by generation and publicizing of an insertion loss design computer program. Since the sensitivity of wave-digital filter transfer function properties to coefficient variation does not change noticeably as it is the case with the recursive digital filter an application in systems with varying or high sampling rates seems very desirable.

EXPERIMENTAL RESULTS

In the following figures the relative error, as specified earlier, is plotted versus the number of bits in the wordlength of filter coefficients and, for the simulation of each filter, also in the wordlength of the arithmetic unit. Since the range of three decades is not sufficient to display all data the results are also shown in tables. An "X" was used in all cases where either the filter had become unstable or the cutoff frequency was outside the range of observation, 1.1 radians for a design continuous cutoff frequency of one radian.

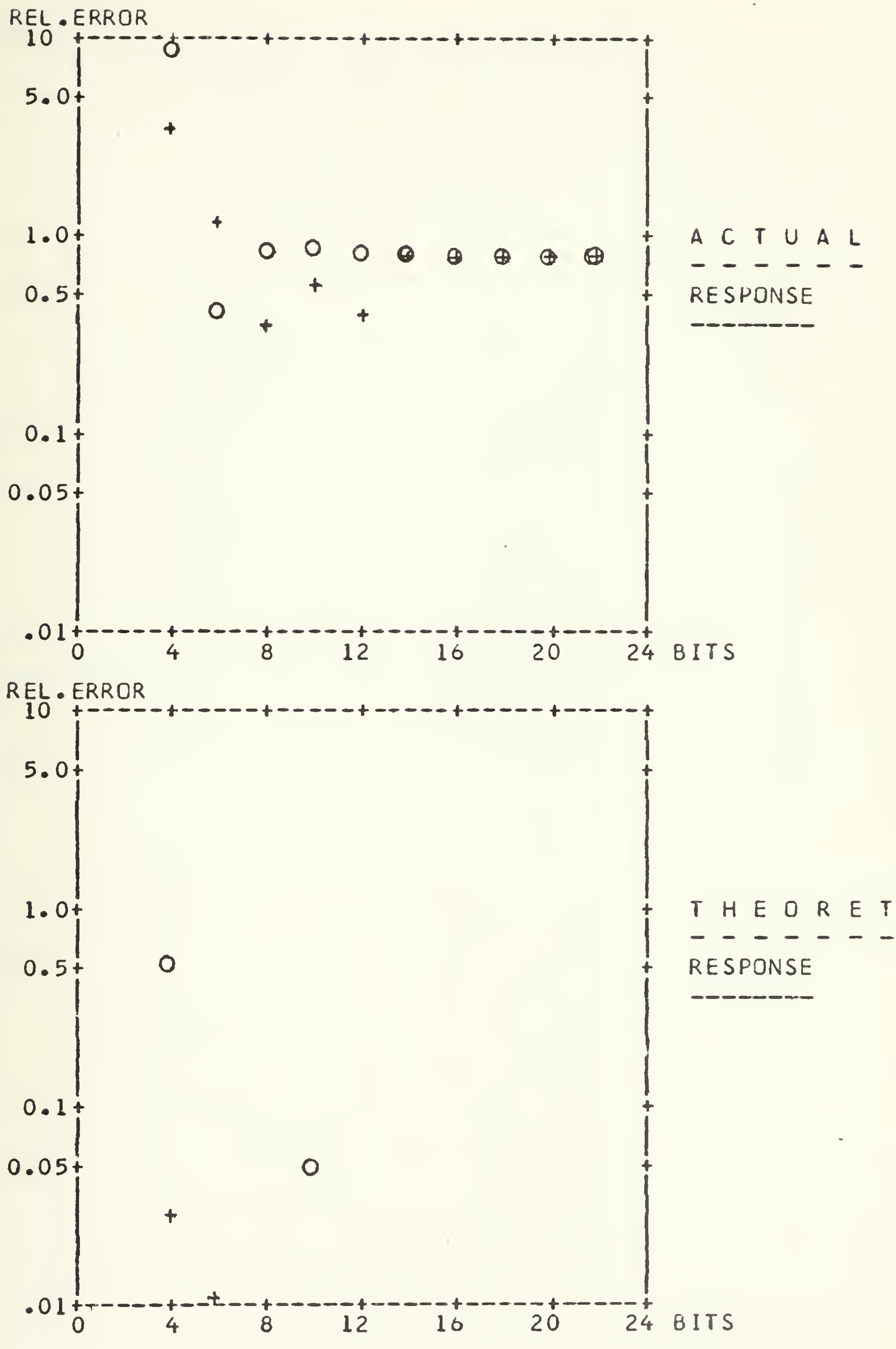


FIGURE A1. N=5, R=1.2498, TS=2.0

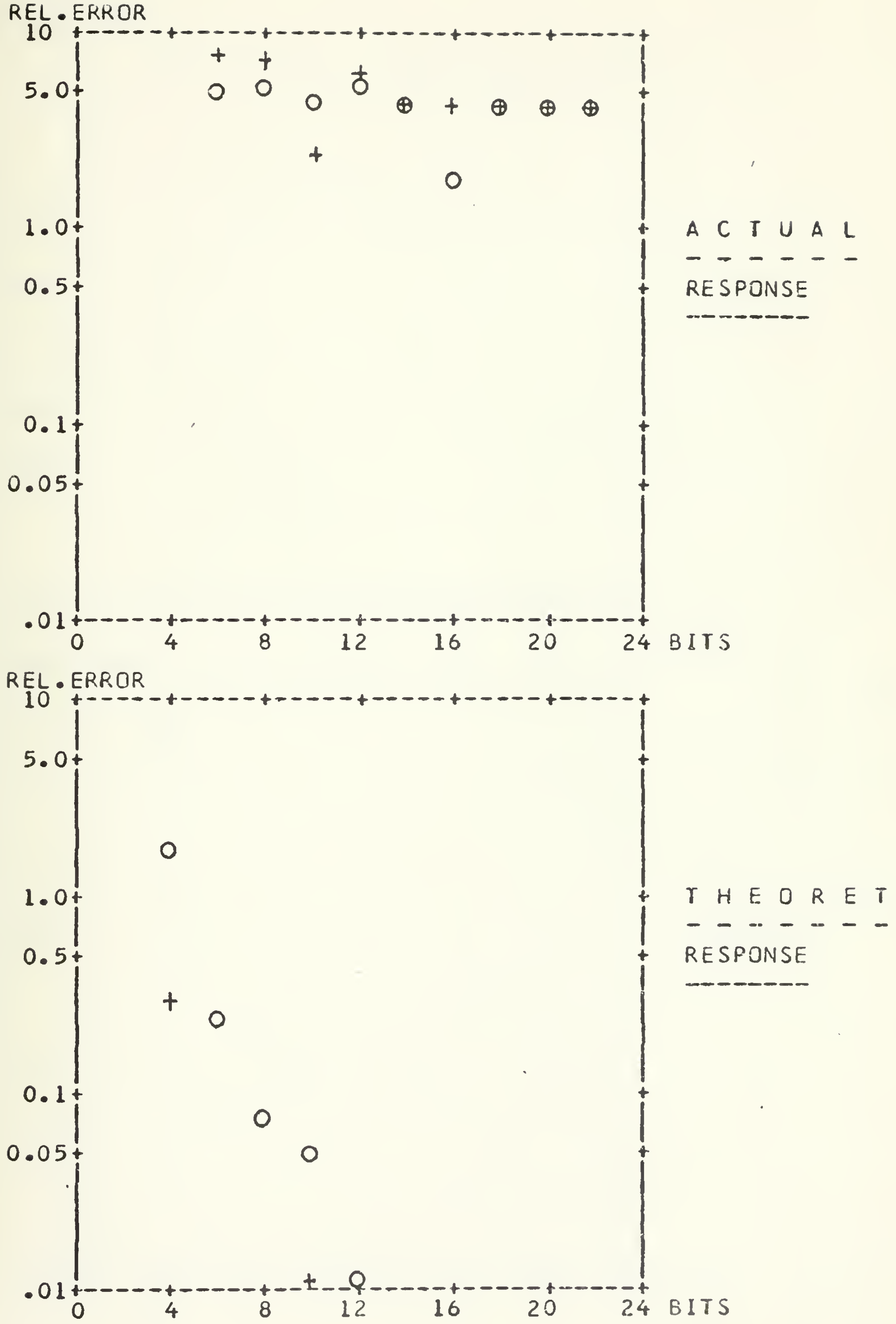


FIGURE A2. N=5, R=0.28037, TS=2.0

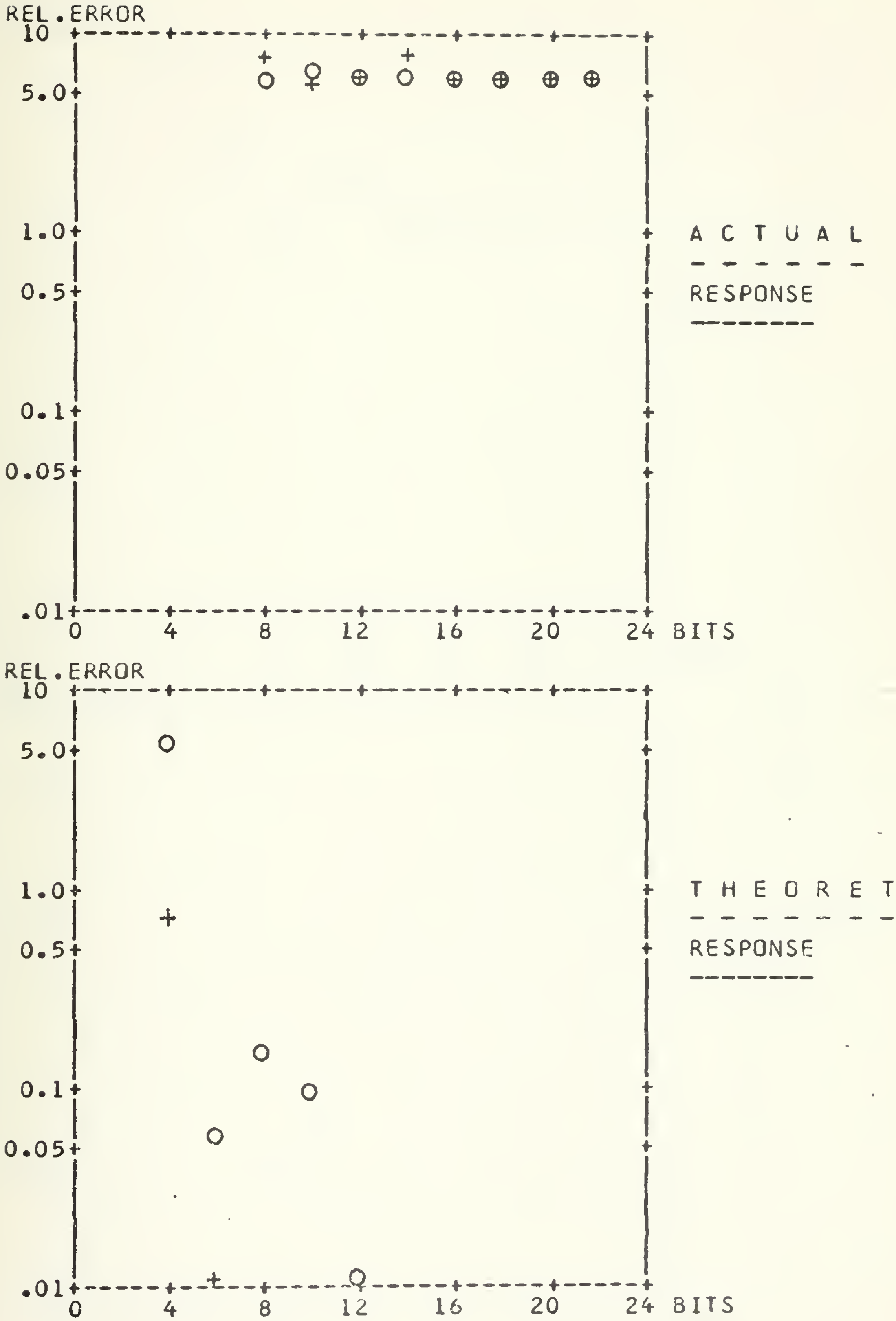


FIGURE A3. N=5, R=0.17734 , TS=2.0

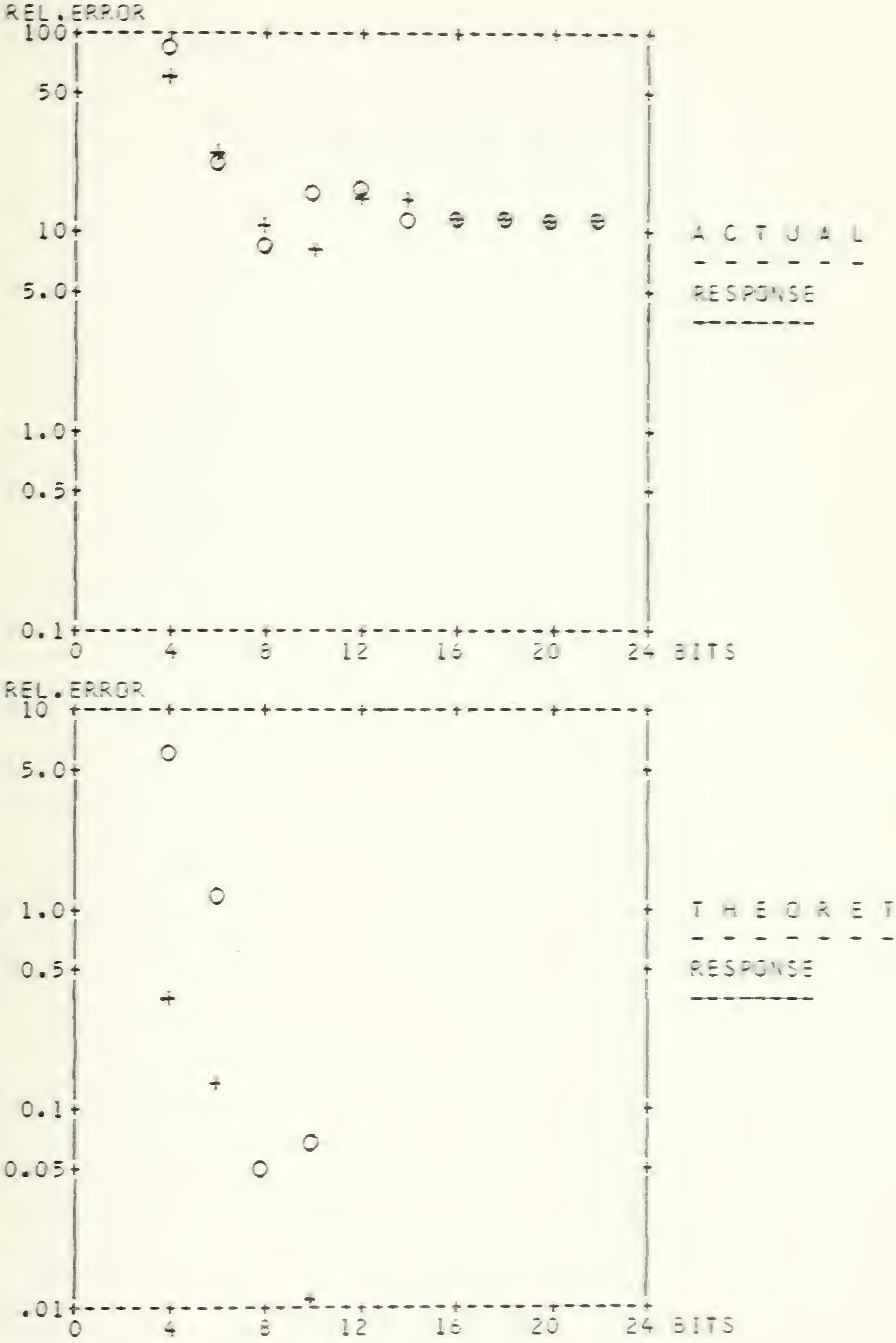


FIGURE A4. N=5, R=0.09866 , TS=2.0

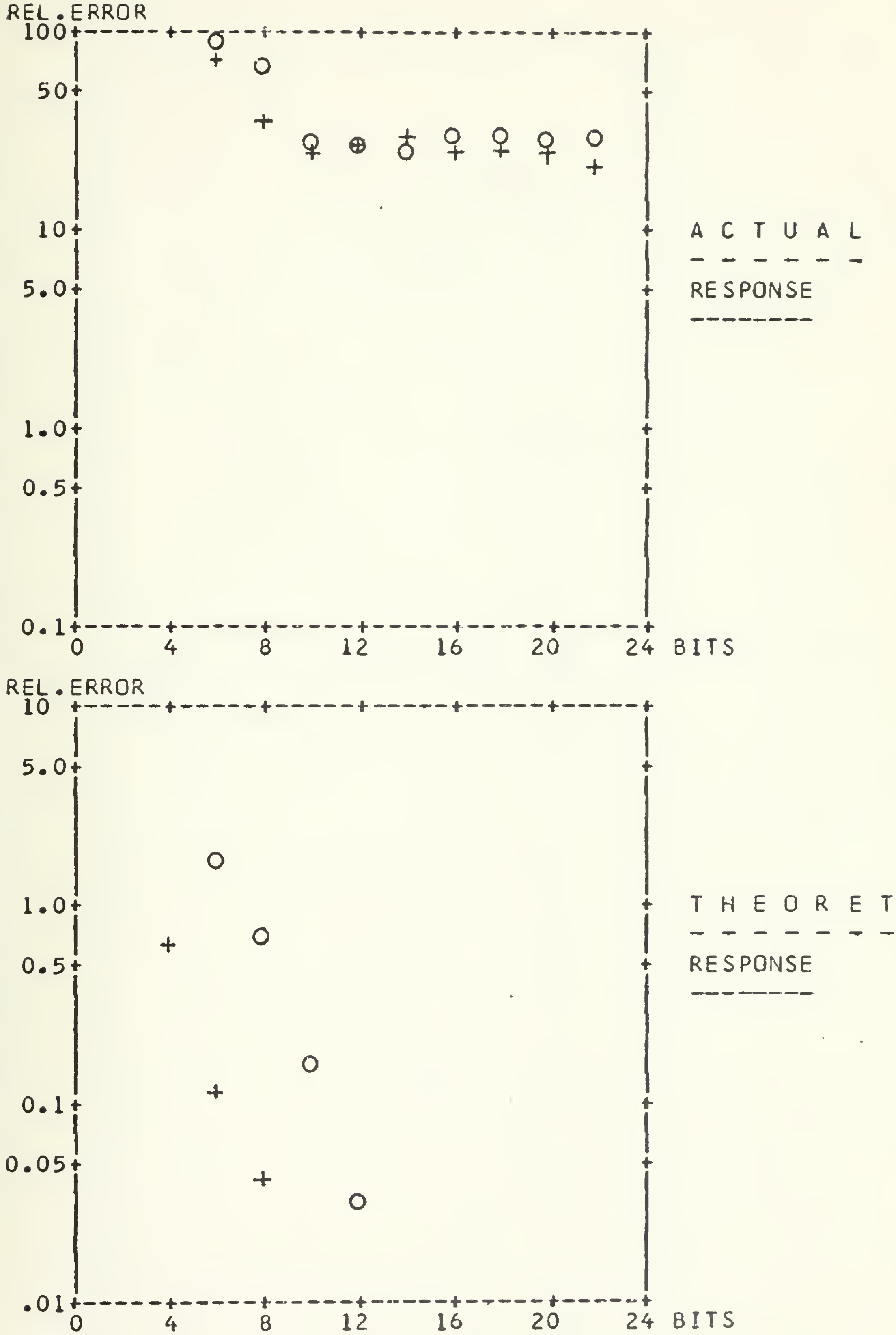
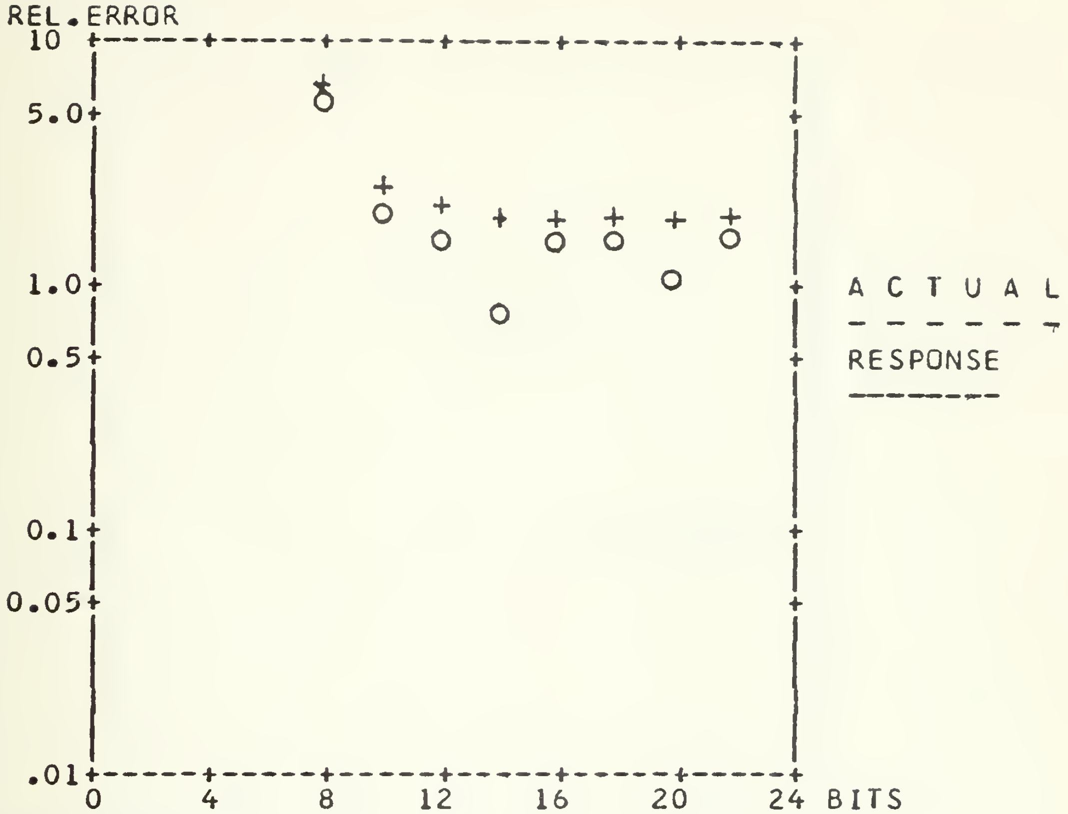


FIGURE A5. N=5, R=0.04366 , TS=2.0



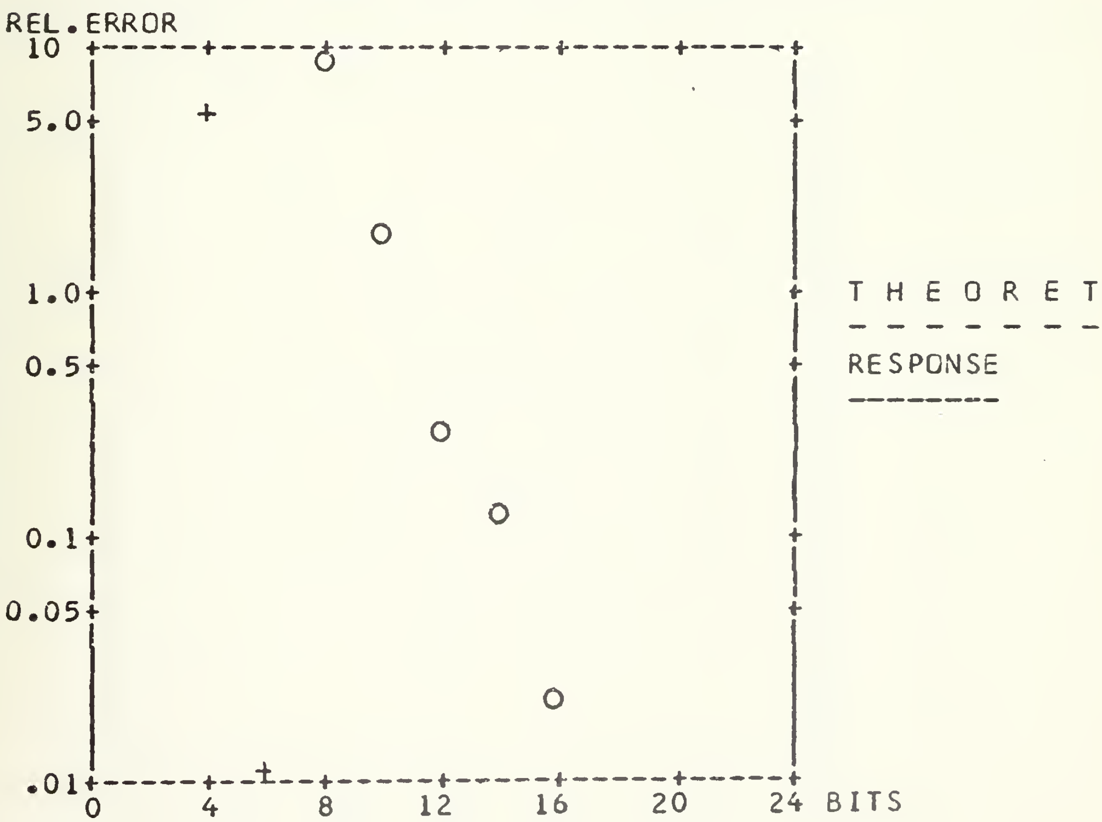


FIGURE A6. N=5, R=0.17734 , TS=1.0

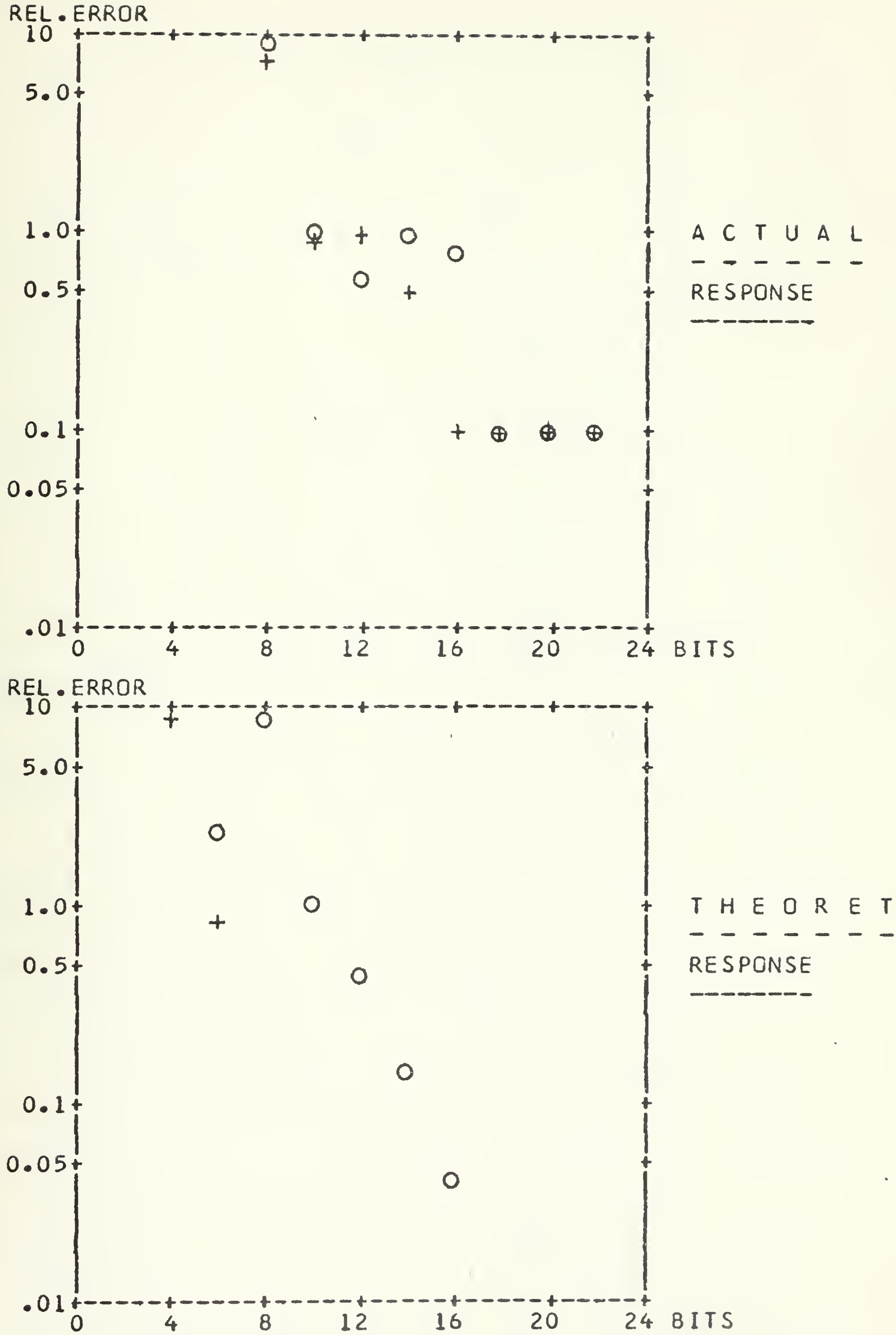


FIGURE A7. N=5, R=0.17734 , TS=0.5

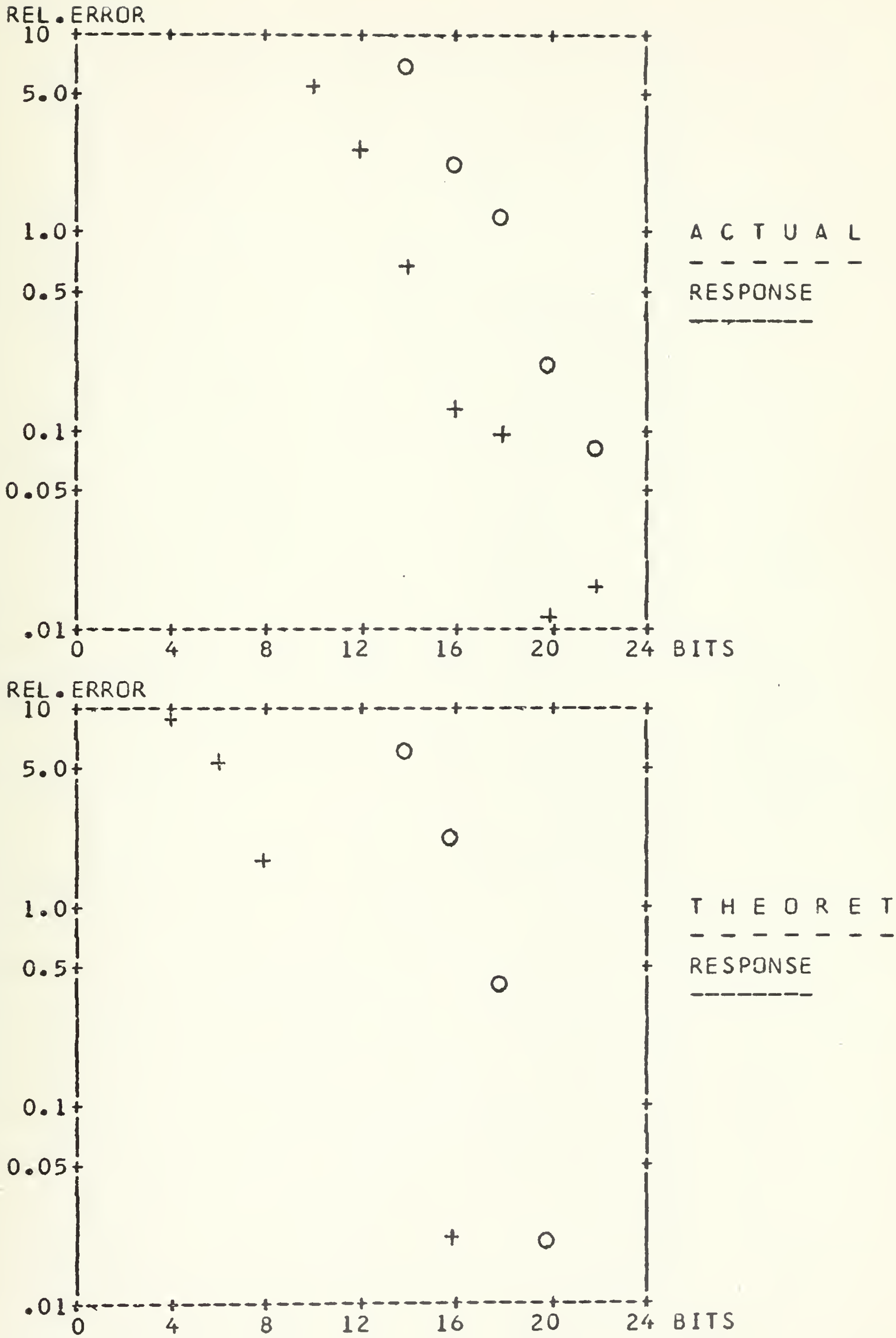
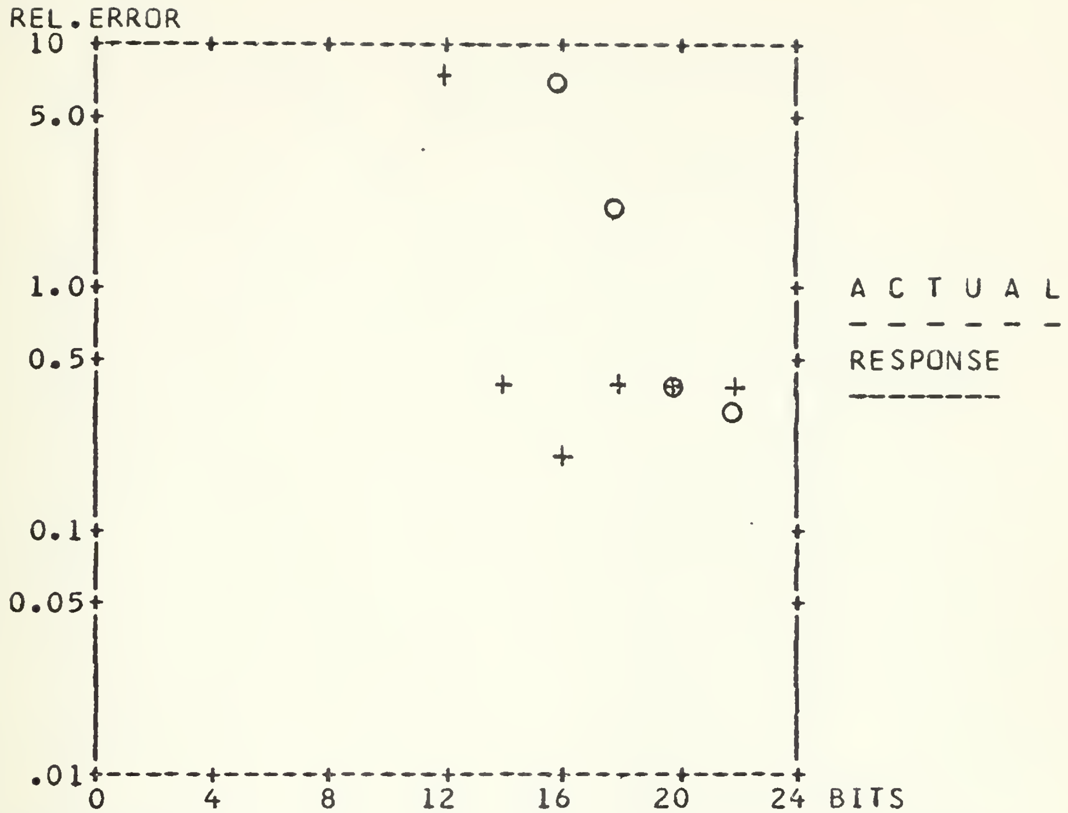


FIGURE A8. N=5, R=0.17734 , TS=0.1



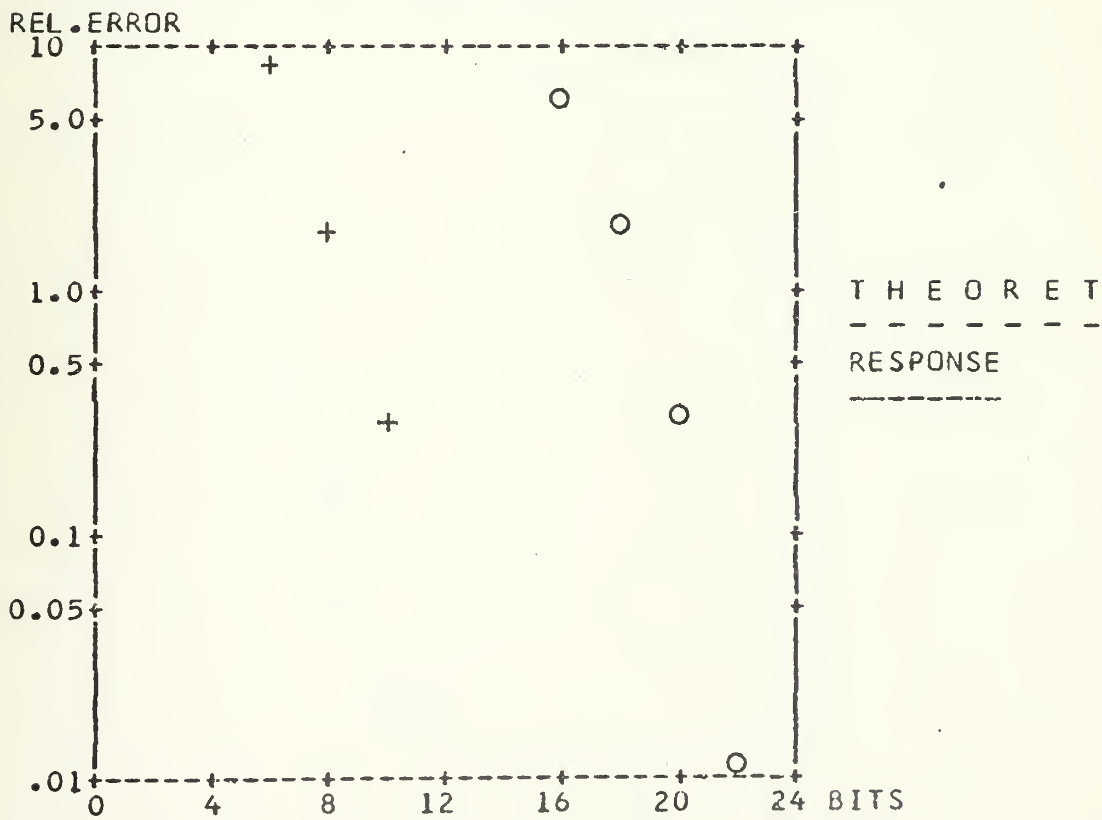
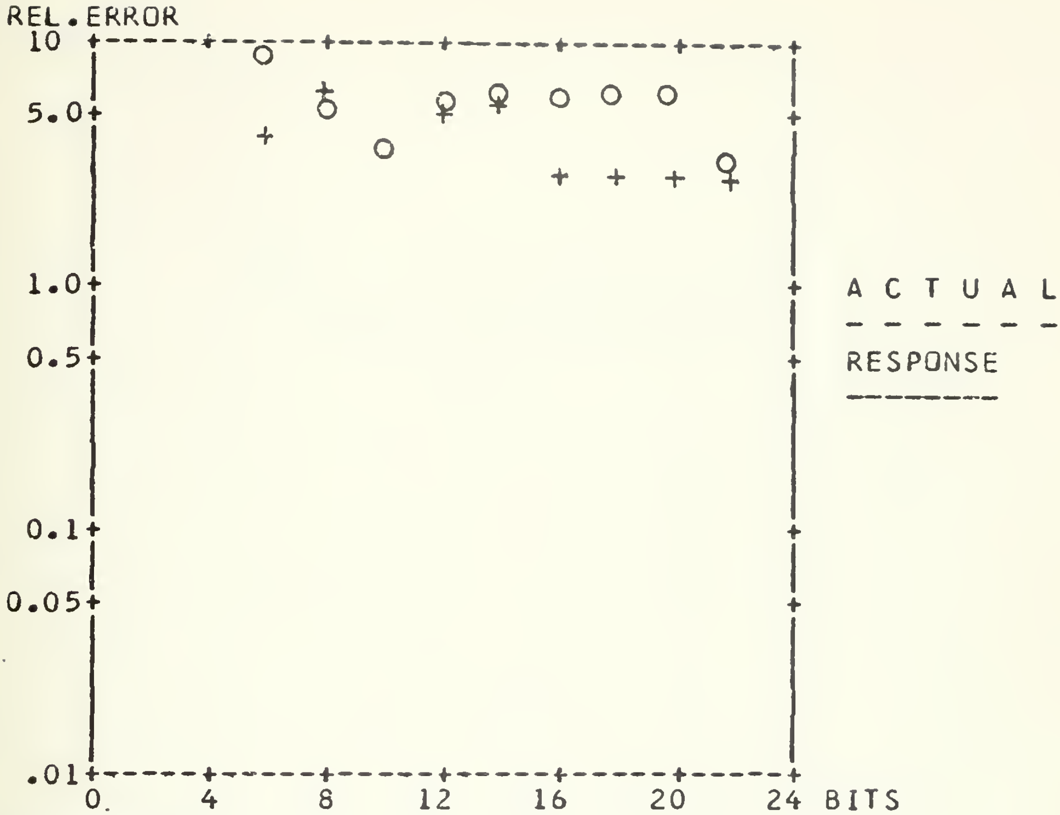
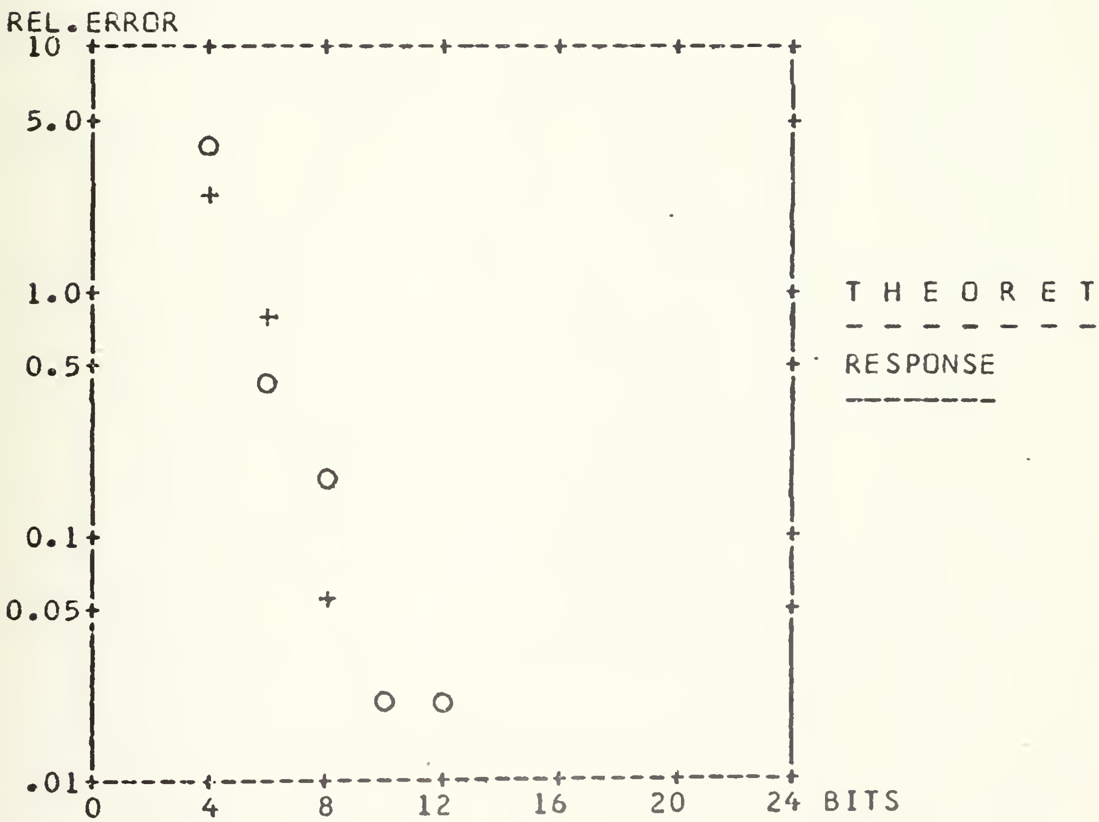
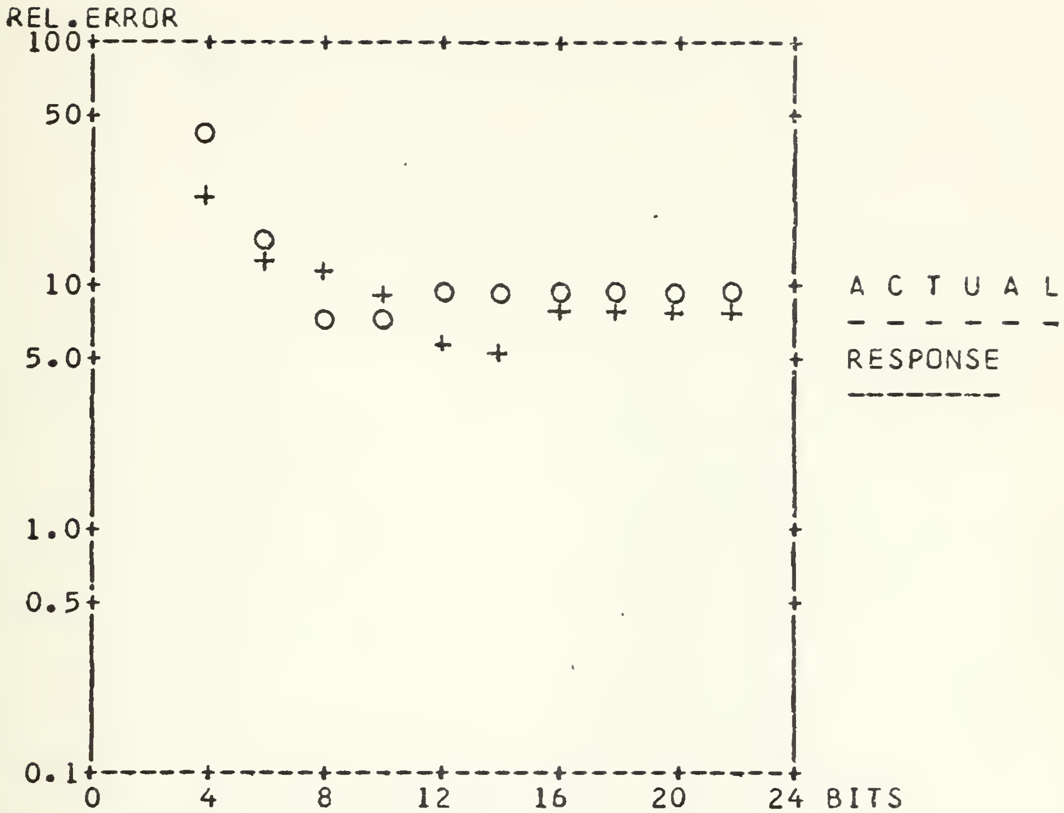


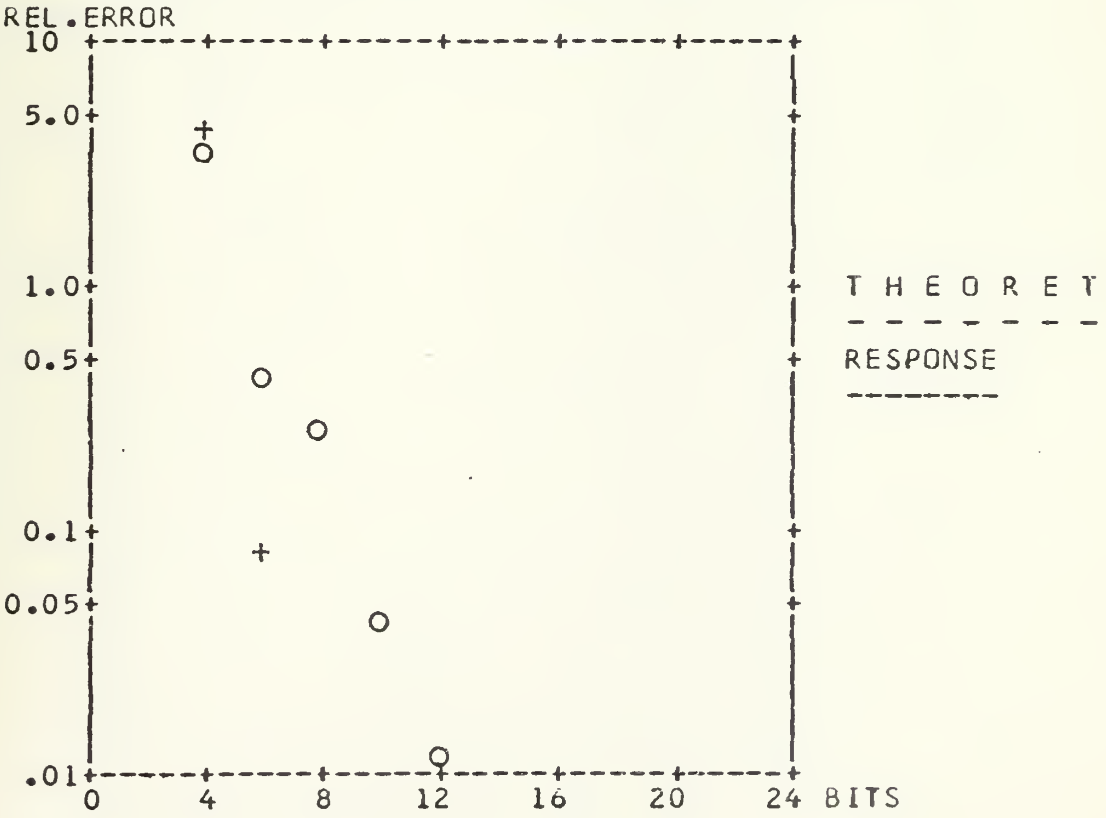
FIGURE A9. N=5, R=0.17734 , TS=0.05



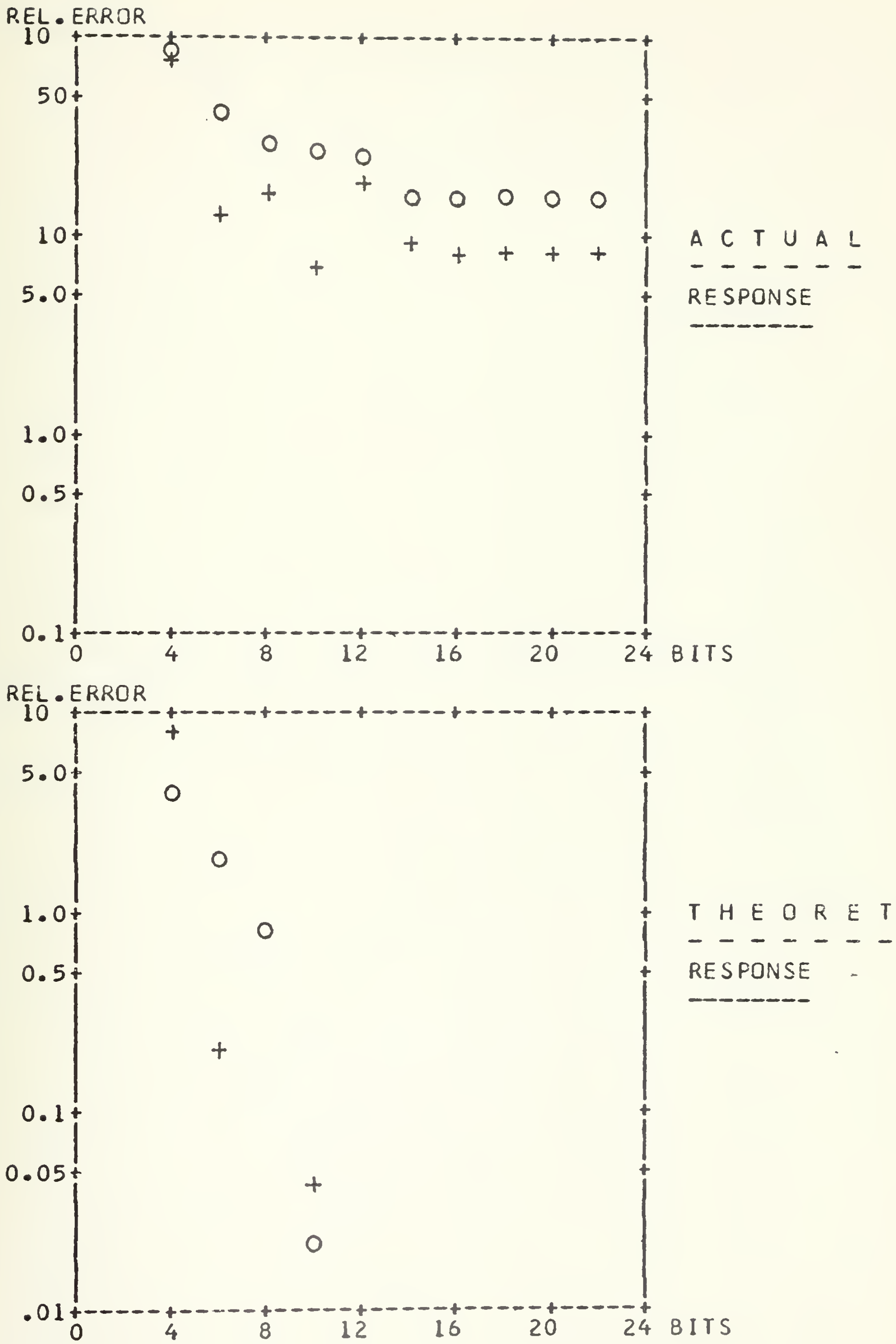


FIGUREA10. N=7, R=0.28037 , TS=2.0





FIGUREA11 N=7, R=0.17734 , TS=2.0



FIGUREA12 N=7, R=0.09886 , TS=2.0

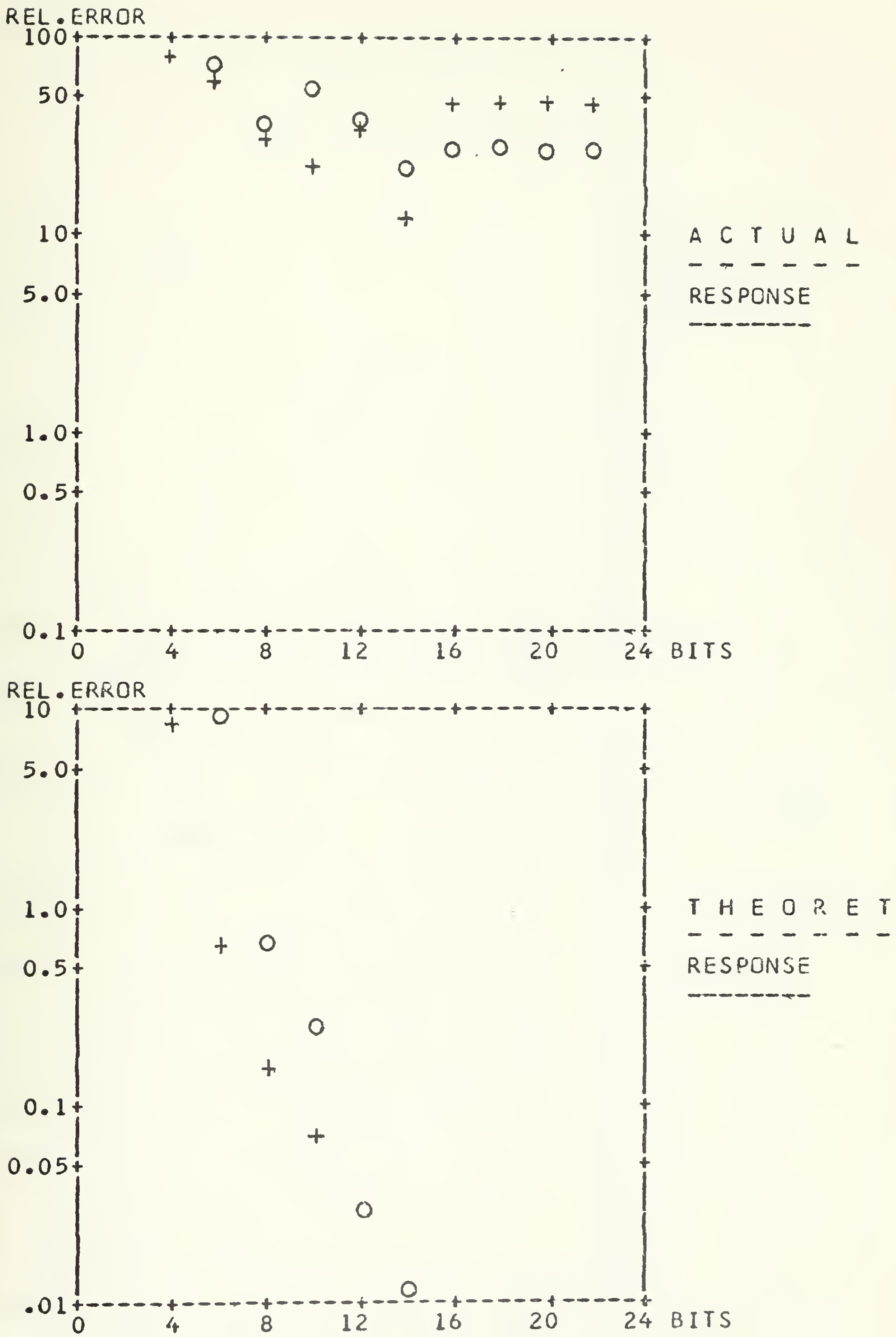
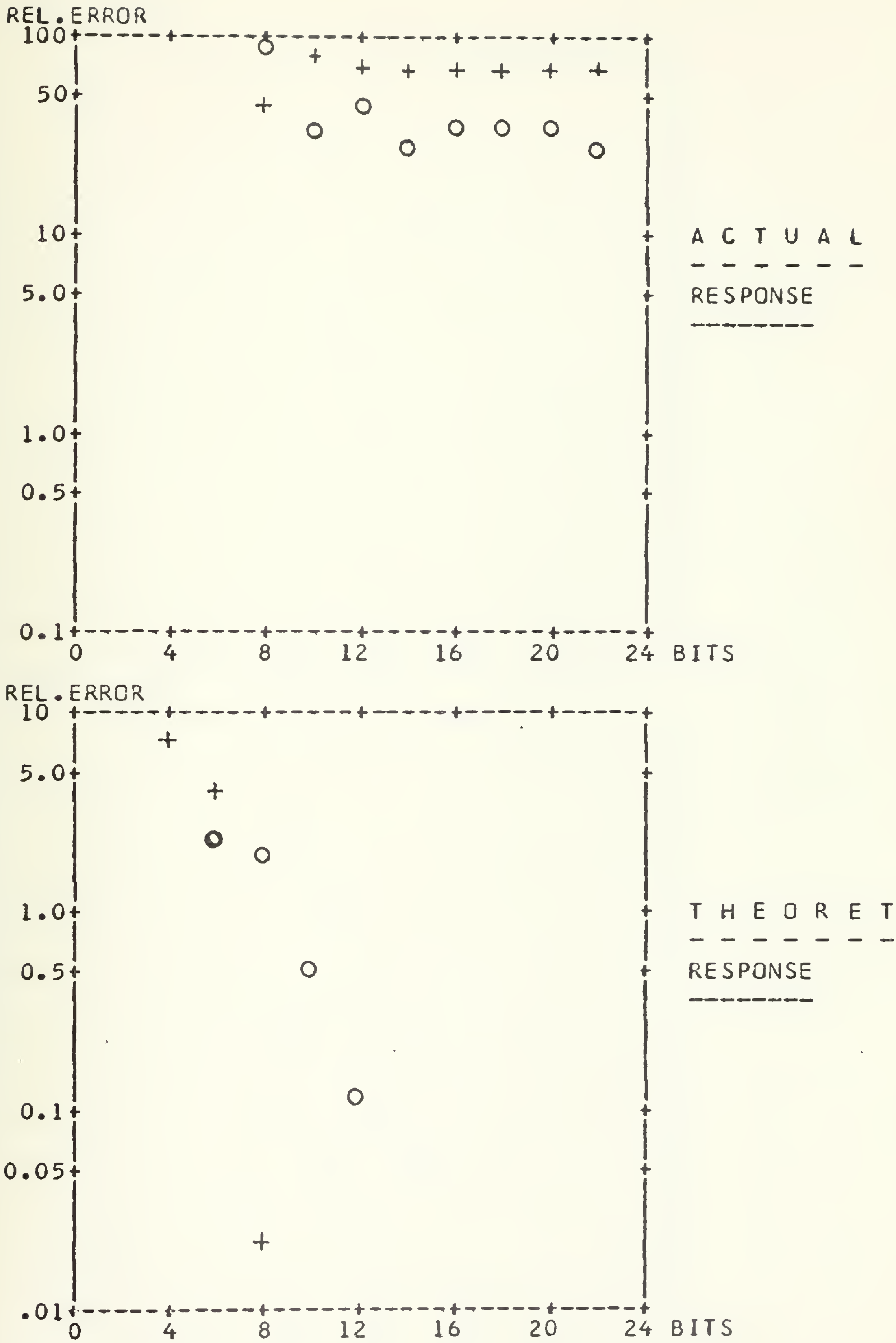


FIGURE A13 N=7, R=0.04366 , TS=2.0



FIGUREA14 N=7, R=0.02790 , TS=2.0

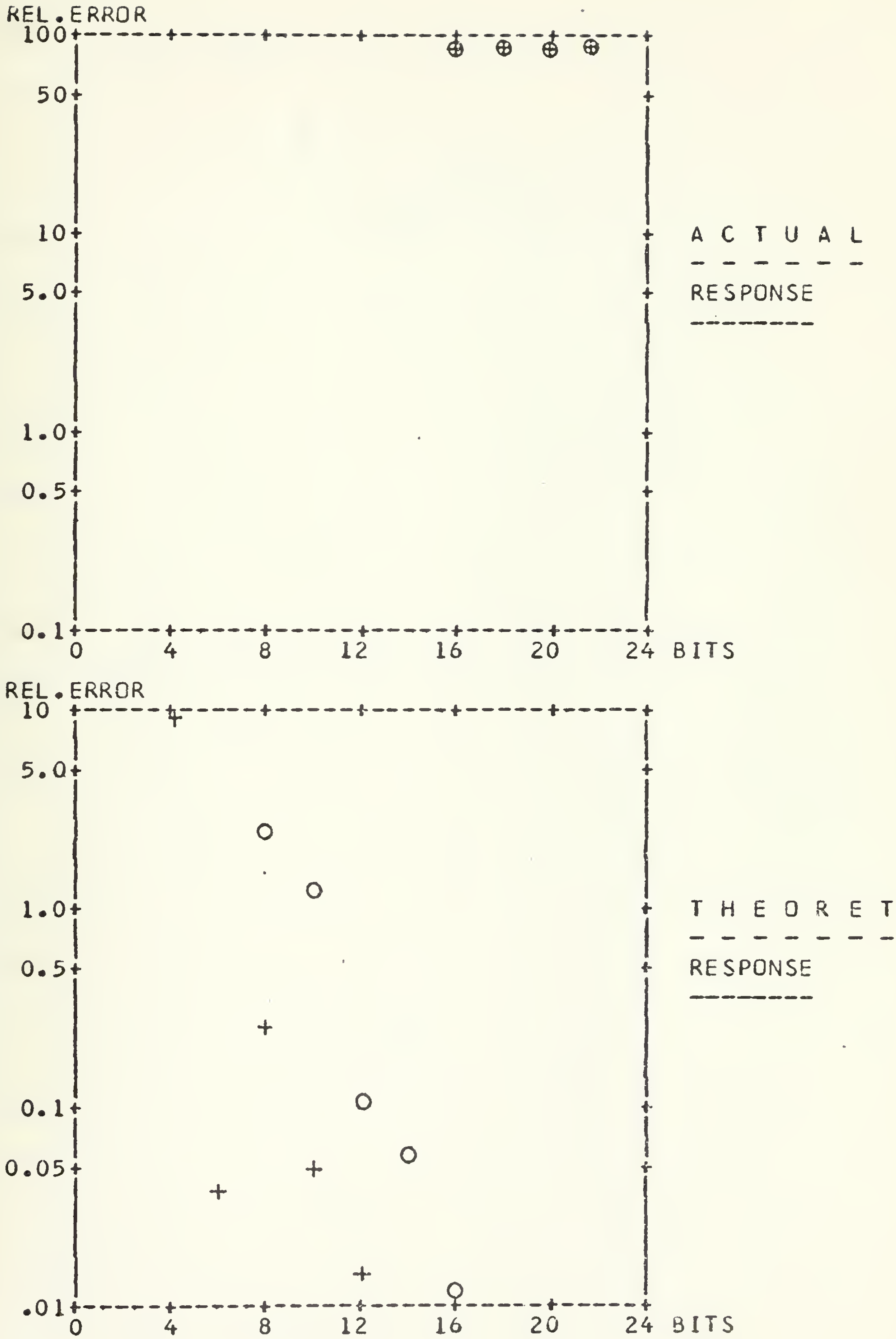


FIGURE A15 N=7, R=0.01090 , TS=2.0

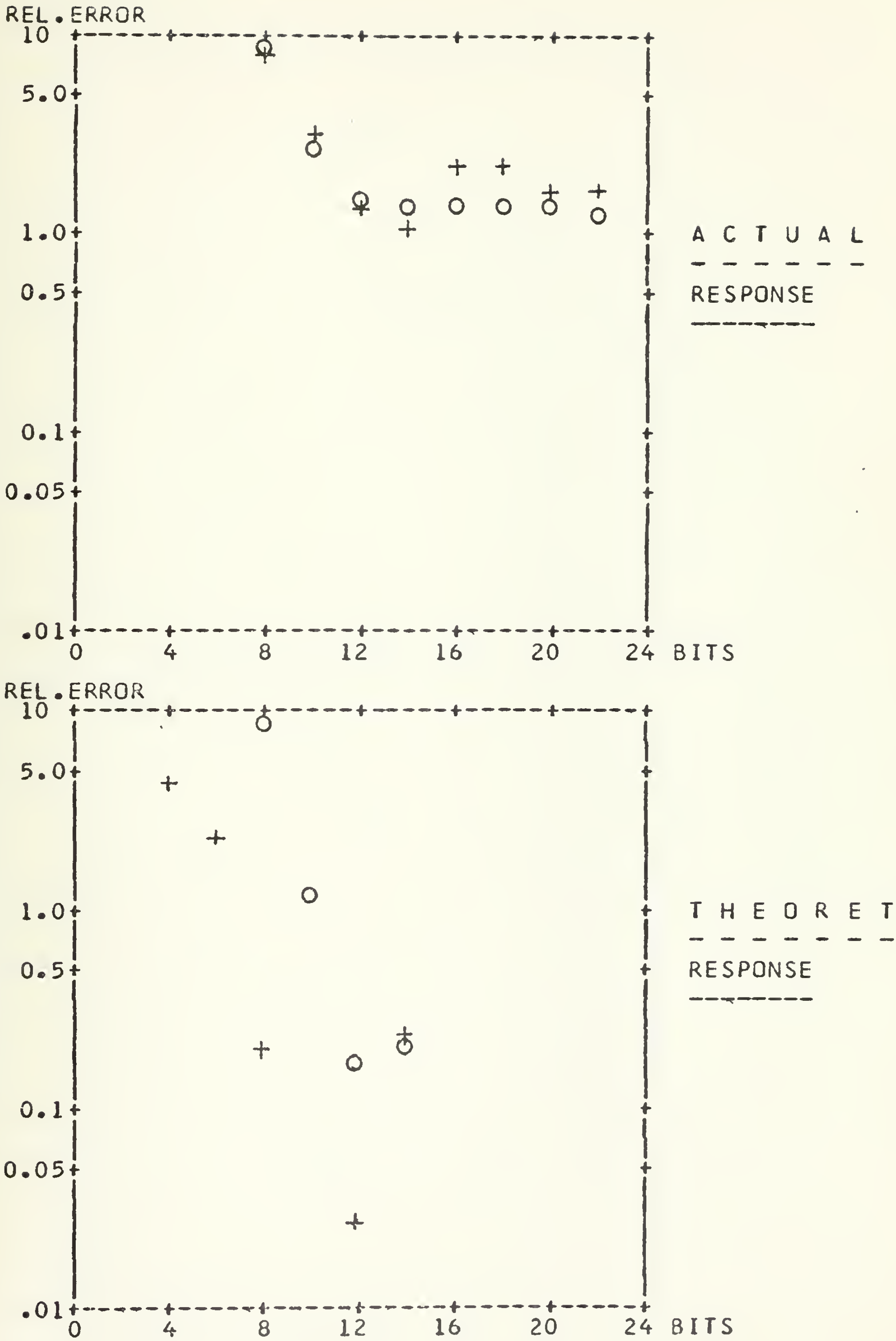
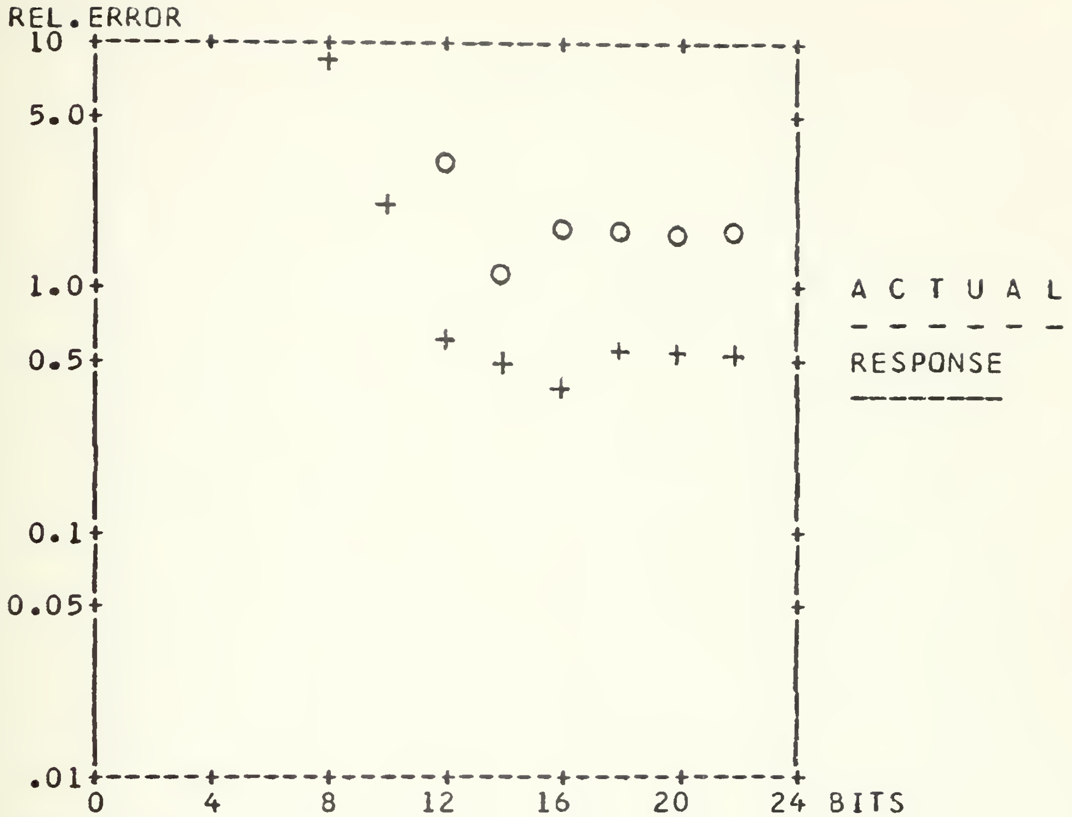
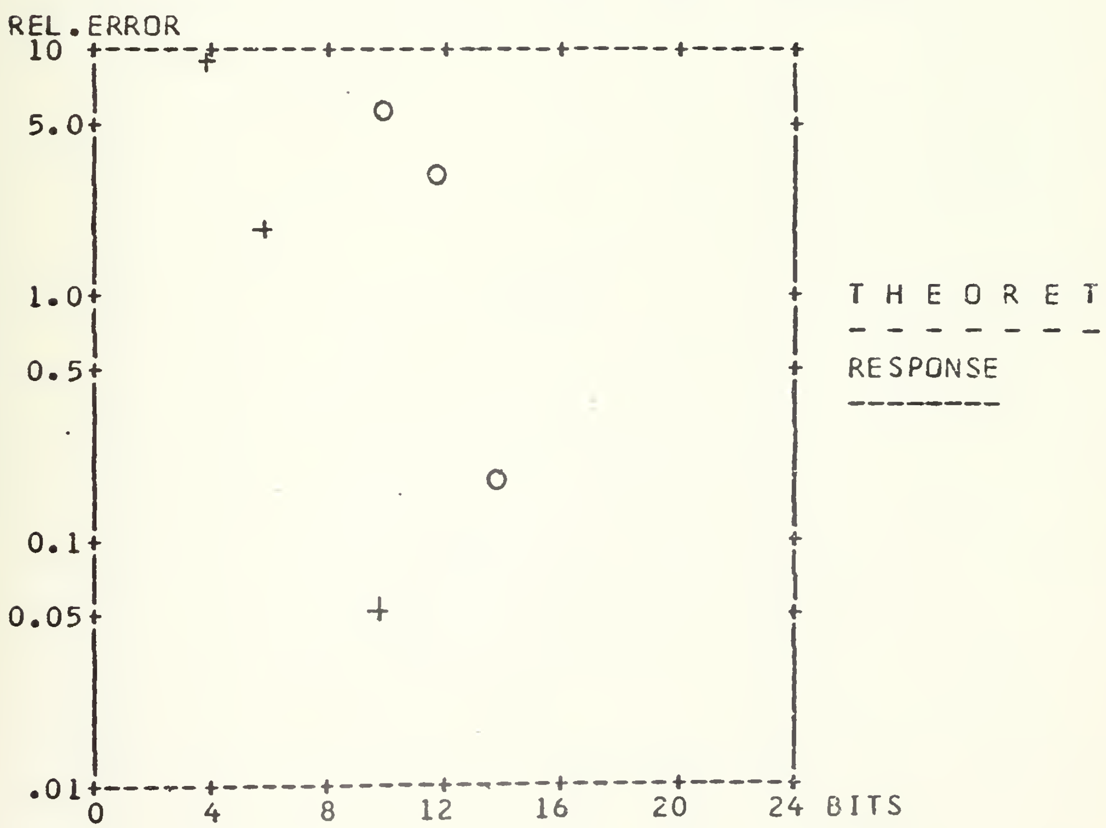
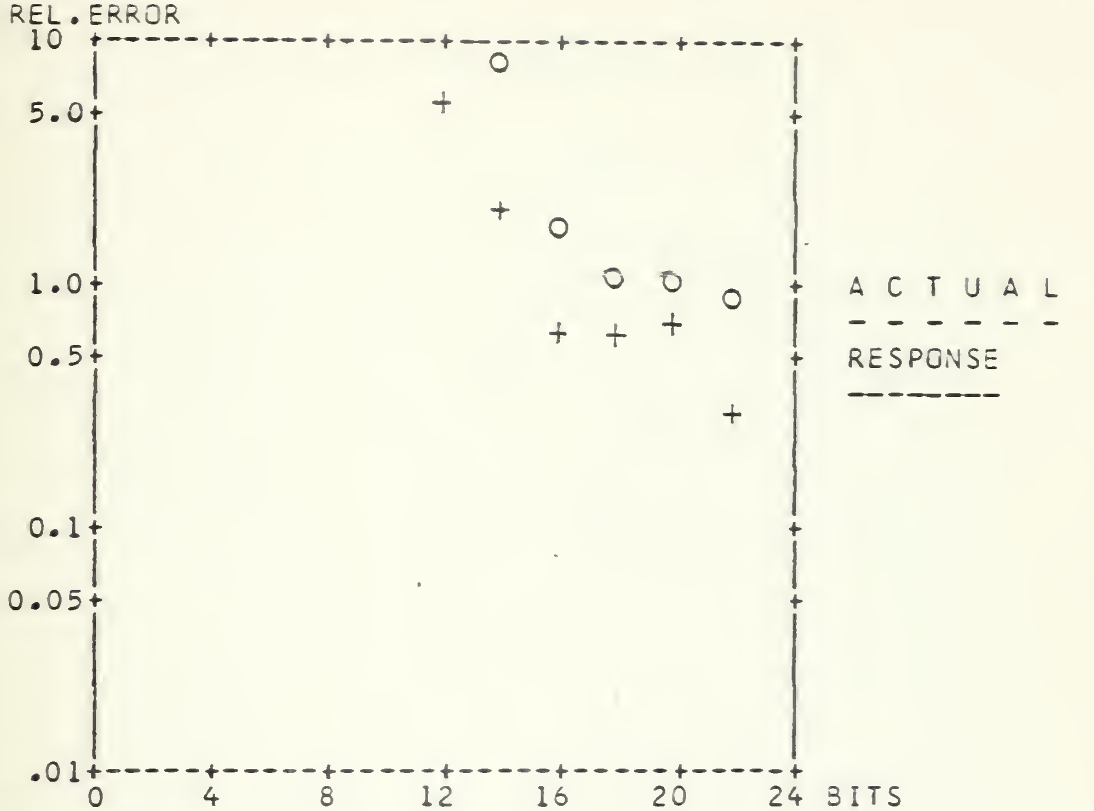


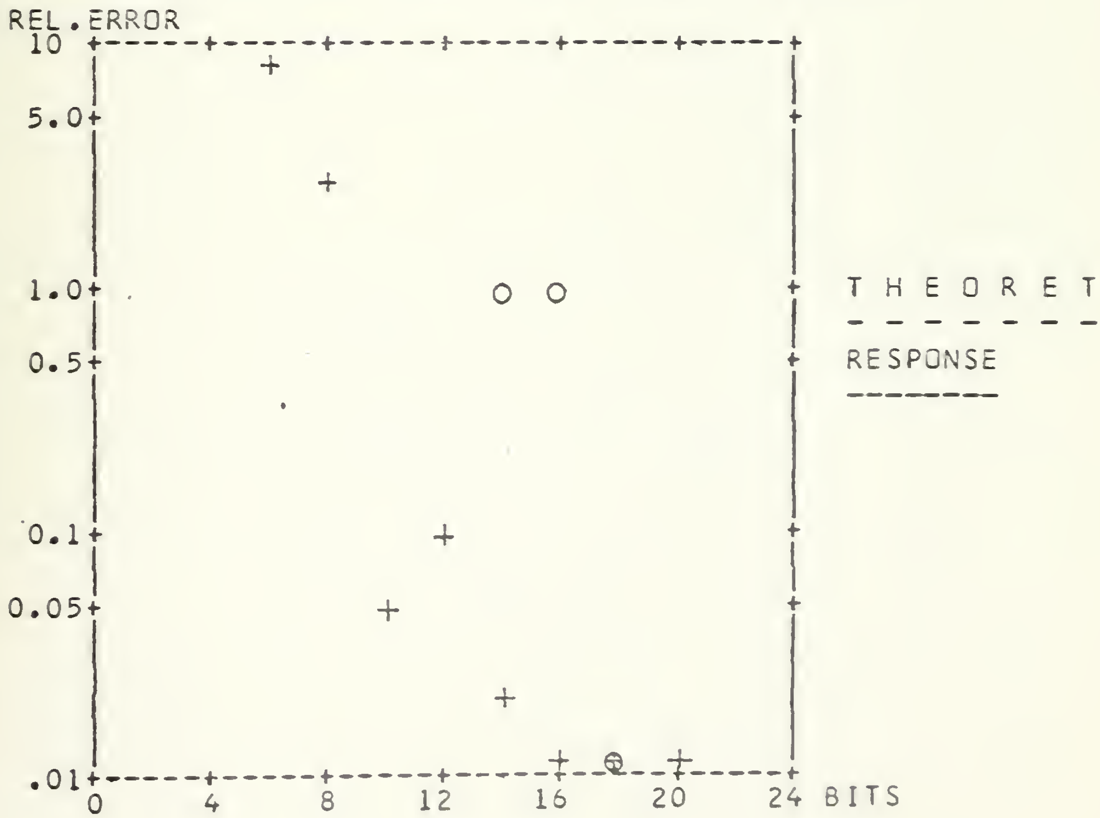
FIGURE A16 N=7, R=0.17734 , TS=1.0



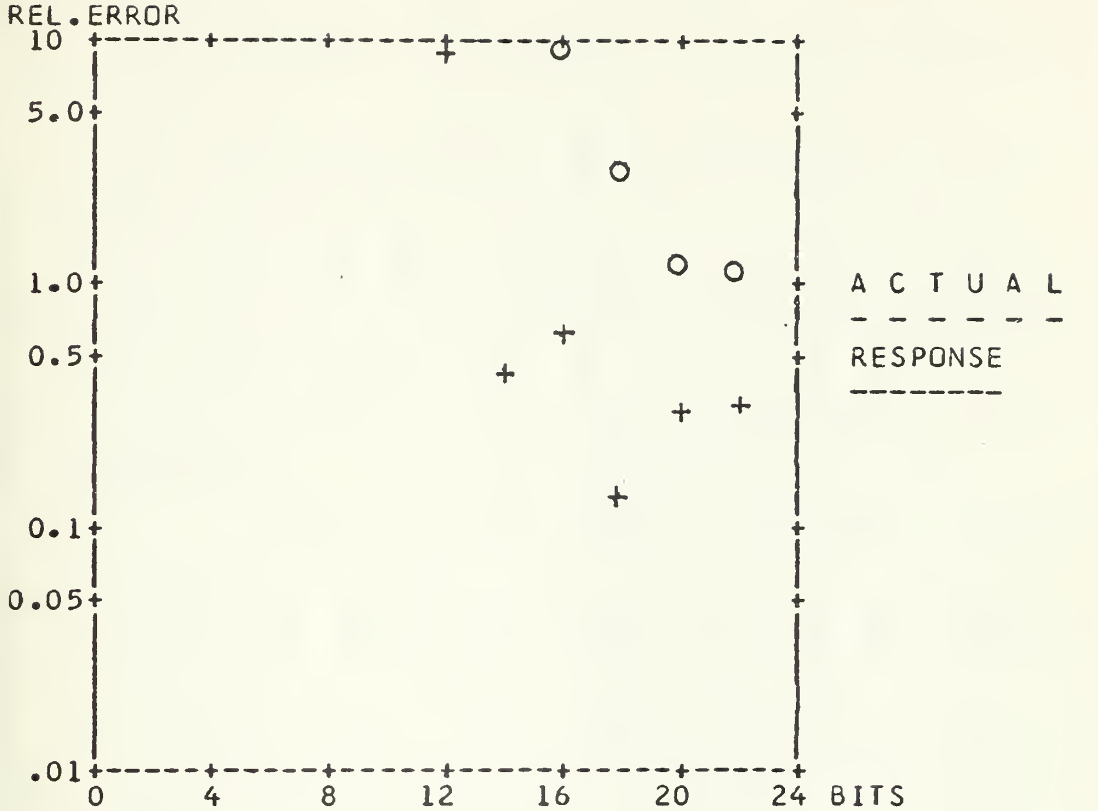


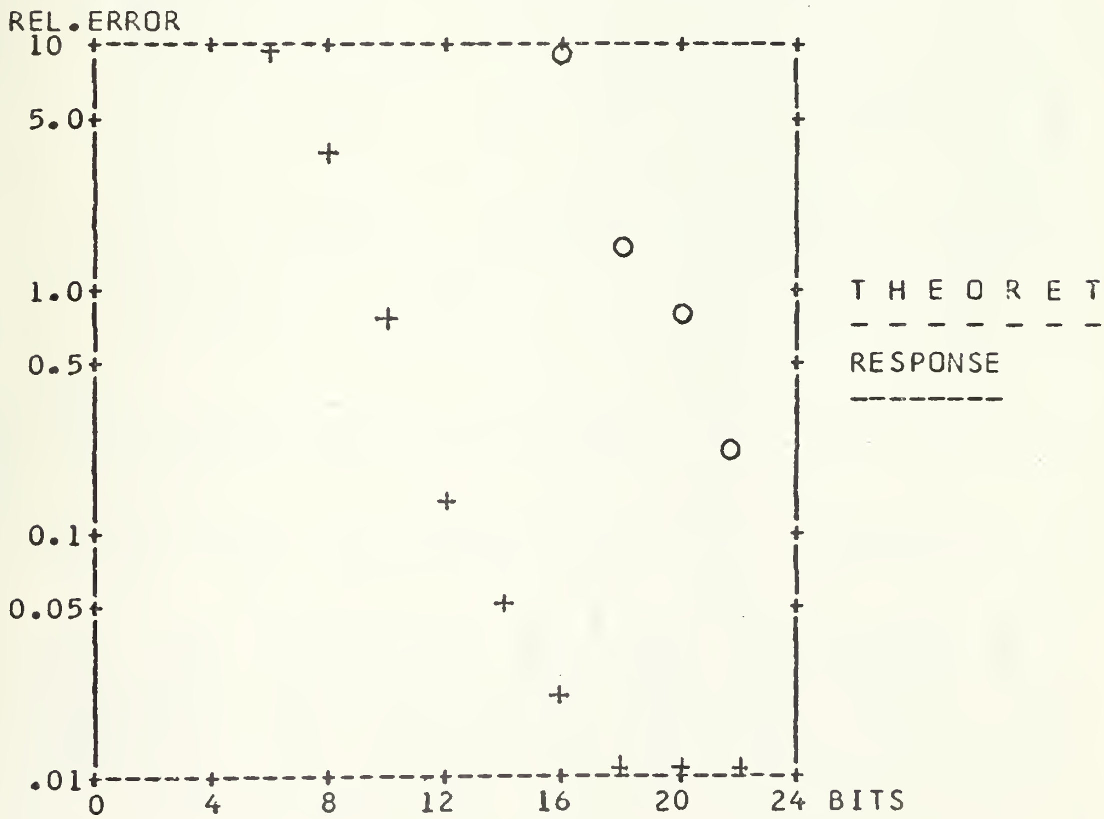
FIGUREA17 N=7, R=0.17734 , TS=0.5





FIGUREA18 N=7, R=0.17734 , TS=0.1





FIGUREA19 N=7, R=0.17734 , TS=0.05

TABLE AI. SAMPLING TIME VARYING

	R(dB)=.17734	FILTER TYPE	QUANTIZATION, No. of Bits									
ORDER N / Ts	Sinusoidal Steady-State Response	A: Cascade B: W.D.F.	22	20	18	16	14	12	10	8	6	4
7 / 1.0	THEORETICAL	A	0	0	0	0	.21	.17	1.2	11.2	28	74
		B	0	0	0	0.001	.25	.025	0	.2	2.5	4.5
	EXPERIMENTAL	A	1.2	1.4	1.4	1.4	1.4	1.5	3.1	9.6	36.1	135
		B	1.7	1.7	2.2	2.2	1.1	1.4	3.4	8.4	26	134
7 / 0.5	THEORETICAL	A	0	0	0	0	.16	3.4	5.8	53.6	X	X
		B	0	0	0	0	0	0	.05	0	1.8	10.0
	EXPERIMENTAL	A	1.6	1.6	1.6	1.6	1.3	3.1	17.7	73	X	X
		B	.6	.6	.6	.4	.5	.7	2.4	11.5	40.5	134
7 / 0.1	THEORETICAL	A	0	0	0	1.0	1.0	26.5	21	48	X	X
		B	.01	.01	.01	.01	.02	.1	.05	3.0	8.0	17.8
	EXPERIMENTAL	A	.878	1.1	1.1	1.7	8.7	130	112	X	X	X
		B	.33	.7	.66	.66	2.2	6.9	25	116	X	X
7 / 0.05	THEORETICAL	A	.2	.86	1.4	11.3	40	25	X	X	X	X
		B	.01	.01	.01	.02	.05	.12	.68	3.3	13	22.4
	EXPERIMENTAL	A	1.1	1.3	3.0	14.9	36.5	X	X	X	X	X
		B	.36	.35	.16	.6	4.5	25	38.3	134	X	X

TABLE AII. SAMPLING TIME VARYING

	R(dB)=.17739	FILTER TYPE	QUANTIZATION, No. of Bits									
ORDER N / T_s	Sinusoidal Steady State Response	A: Cascade B: W.D.F.	22	20	18	16	14	12	10	8	6	4
7 / 1.0	THEORETICAL	A	0	0	0	.02	.12	.28	1.75	9.5	14.5	6.0
		B	0	0	0	0	0	0	0	0	0	5.6
	EXPERIMENTAL	A	1.5	1.1	1.5	1.5	.82	1.5	2.0	6.6	20.8	112
		B	1.9	1.9	1.9	1.9	1.9	2.1	2.4	7.3	27	14
7 / 0.5	THEORETICAL	A	0	0	0	.04	.14	.46	1.05	9.9	2.0	11.6
		B	0	0	0	0	0	0	0	0	.8	9.3
	EXPERIMENTAL	A	.1	.1	.1	.8	1.0	.6	1.0	10.7	76.4	100
		B	.1	.1	.1	.1	.5	1.0	.9	7.9	28.7	35
7 / 0.1	THEORETICAL	A	.015	.01	.1	.13	.8	3.0	6.0	130	129	X
		B	0	.02	.43	2.2	6.9	12.5	22.6	15	X	X
	EXPERIMENTAL	A	.08	.21	1.2	2.4	8.4	14.4	49	156	X	X
		B	.015	.01	.1	.13	.8	3.0	6.0	130	124	X
7 / 0.05	THEORETICAL	A	.01	.37	1.9	6.4	28.4	18.3	X	X	X	X
		B	0	0	0	0	0	0	.3	1.8	8.6	17
	EXPERIMENTAL	A	.3	.4	2.1	7.8	29.2	31.6	165	X	X	X
		B	.4	.4	.4	.2	.4	7.5	20	25	X	X

TABLE AIII. SAMPLING TIME FIXED

	R = RIPPLE(dB)	FILTER TYPE	QUANTIZATION, No. of bits									
ORDER N / Ts	Sinusoidal Steady State Response	A: Cascade B: W.D.F.	22	20	18	16	14	12	10	8	6	4
5	THEORETICAL R=1.2498	A	0	0	0	0	0	0	.05	0	0	.54
		B	0	0	0	0	0	0	0	0	0	.03
2.0	EXPERIMENTAL R=1.2498	A	.83	.83	.83	.83	.83	.84	.85	.9	.46	9.2
		B	.83	.83	.83	.83	.83	.42	.58	.39	1.25	3.8
5	THEORETICAL R=.28037	A	0	0	0	0	.001	.01	.05	.07	.26	1.78
		B	0	0	0	0	0	0	.004	0	0	.34
2.0	EXPERIMENTAL R=.28037	A	4.3	4.3	4.3	1.9	4.4	5.1	4.4	5.1	5.1	20.5
		B	4.4	4.4	4.4	4.4	4.4	6.5	2.5	7.5	7.7	70
5	THEORETICAL R=.17734	A	0	0	0	0	0	.01	.1	.14	.06	5.6
		B	0	0	0	0	0	0	0	0	0	.74
2.0	EXPERIMENTAL R=.17734	A	6.4	6.4	6.4	6.4	6.4	6.4	6.4	5.9	14.4	60
		B	6.4	6.4	6.4	6.4	8.5	6.3	5.7	8.1	11.9	16.5
5	THEORETICAL R=.09886	A	0	0	0	0	.003	0	.07	.05	1.18	6.5
		B	0	0	0	0	.002	0	.01	.05	.12	.35
2.0	EXPERIMENTAL R=.09886	A	11.5	11.5	11.5	11.5	11.5	16.9	16.0	9.0	24.3	89.5
		B	11.5	11.5	11.5	11.5	15.4	15.7	8.5	11.5	26.1	60

TABLE AIV. SAMPLING TIME FIXED

	R=RIPPLE (dB)	FILTER TYPE	QUANTIZATION, No. of bits									
ORDER N / Ts	Sinusoidal Steady State Response	A: Cascade B: W.D.F.	22	20	18	16	14	12	10	8	6	4
5	THEORETICAL R=.04366	A	0	0	0	0	0	.03	.15	.7	1.3	16.6
		B	0	0	0	0	0	0	0	.04	.11	.67
2.0	EXPERIMENTAL R=.04366	A	32	32	32	32	26.2	26.3	26.7	63.7	83	167
		B	21.2	26.2	26.2	26.2	32.2	26.1	23.4	34.1	70	120
7	THEORETICAL R=.28037	A	0	0	0	0	0	.02	.02	.15	.44	4.4
		B	0	0	0	0	0	0	0	.06	.81	2.7
2.0	EXPERIMENTAL R=.28037	A	3.2	6.6	6.6	6.6	6.6	5.6	4.3	5.5	9.5	70
		B	3.1	3.1	3.1	3.1	6.7	5.2	24.5	6.7	4.2	14.8
7	THEORETICAL R=.17739	A	0	0	0	0	0	.01	.04	.26	.4	3.6
		B	0	0	0	0	0	0	0	0	.08	4.8
2.0	EXPERIMENTAL R=.17739	A	9.5	9.5	9.5	9.5	9.5	9.4	7.0	6.9	15.6	112
		B	7.9	7.9	7.9	7.9	5.4	5.8	9.0	11.4	12.7	24
7	THEORETICAL R=.09886	A	0	0	0	0	0	0	.02	.85	1.9	3.7
		B	0	0	0	0	0	0	.04	0	.2	8.1
2.0	EXPERIMENTAL R=.09866	A	15	15	15	15	15	23.2	24.3	27.5	44.9	87.9
		B	8.8	8.8	8.8	8.8	9.6	19.8	7.8	16.5	13.8	79.5

TABLE AV. SAMPLING TIME FIXED

	R=RIPPLE(dB)	FILTER TYPE	QUANTIZATION, No. of Bits									
ORDER N / Ts	Sinusoidal Steady State Response	A: Cascade B: W.D.F.	22	20	18	16	14	12	10	8	6	4
7	THEORETICAL R=.04366	A	0	0	0	0	.01	.03	.24	.71	9.3	12.7
		B	0	0	0	0	0	0	.07	.17	.7	9.1
2.0	EXPERIMENTAL R=.04366	A	33.2	33.2	33.2	33.2	22.4	38.5	52.1	37.3	71	X
		B	46	46	46	46	12.6	35.5	22.4	27.3	61.8	80
7	THEORETICAL R=.0279	A	0	0	0	0	0	.11	.51	1.9	2.3	25.1
		B	0	0	0	0	0	0	0	.2	4.2	7.4
2.0	EXPERIMENTAL R=.0279	A	34.1	39	39	38.9	33	47.8	38.1	78	107	430
		B	72.4	72.4	72.4	72.4	72	71.6	80.5	48.5	108	182
7	THEORETICAL R=.0109	A	0	0	0	0	.06	.11	1.2	2.6	25	73
		B	0	0	0	0	0	.015	.05	.27	.03	19
2.0	EXPERIMENTAL R=.0109	A	142	141	142	157	144	144	151	219	302	X
		B	132	132	132	132	133	143	142	151	177	X

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