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# NAVAL POSTGRADUATE SCHOOL

## Monterey, California



# THESIS

CASCADE COMPENSATION FOR SYSTEMS  
WITH MECHANICAL RESONANCES

by

Sozon A. Constandoulakis  
December 1982

Thesis Advisor

G. Thaler

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Cascade Compensation for Systems with Mechanical Resonances

by

Sozon A. Constandoulakis  
Lieutenant, Hellenic Navy  
B.S., Naval Postgraduate School, 1982

Submitted in partial fulfillment of the  
requirements for the degree of

MASTER OF SCIENCE IN ENGINEERING SCIENCE

from the

NAVAL POSTGRADUATE SCHOOL  
December 1982



## ABSTRACT

This thesis presents two methods of compensation for control systems including mechanical resonances. The first method is the use of a filter including pure imaginary zeros and complex poles and the second method is a filter using only complex poles. One basic model has been used to develop the methods.



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I wish to dedicate this thesis to my wife, Nantia for without her constant love, support, and devotion, this work would not have been possible.



## I. INTRODUCTION

### A. PROBLEM DESCRIPTION

It is common in the real world to use a motor to drive a mechanical load. So, when the structure is subjected to periodic forcing, it may well exhibit a resonance or a number of resonances (harmonics) which can lead the system to instability. Two typical examples of the above problem are the head servo in a disk memory and the electric typing machine used with word processors.

In order to be familiar with the problem and see what causes our system to instability we have to study the time and frequency response of a system which becomes unstable due to mechanical resonances.

In the following paragraph we consider the problem in the frequency domain where it is easier to define and understand.

### B. FREQUENCY DOMAIN

Considering a typical control system we know that in order for this system to be stable we need a positive phase margin. The positive phase margin happens if the gain cross-over of the system is at a lower frequency than the phase



crossover on the Bode diagram as shown in Figure 1.1. So the resulting phase margin vector ( $\gamma$ ) is positive.

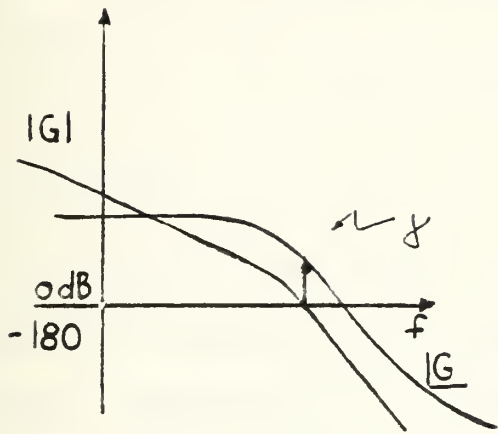
If now we attempt to drive one or more mechanical loads with our control system then it is possible that the system may become unstable due to the mechanical loads. The reason for this instability is the resonance peak or peaks created by the mechanical loads. In general we can summarise the problem by the following two cases.

- (1) At the resonances the peak of the gain curve does not rise above the zero dB axis.
- (2) At the resonances the peak of the gain rises above the zero dB axis

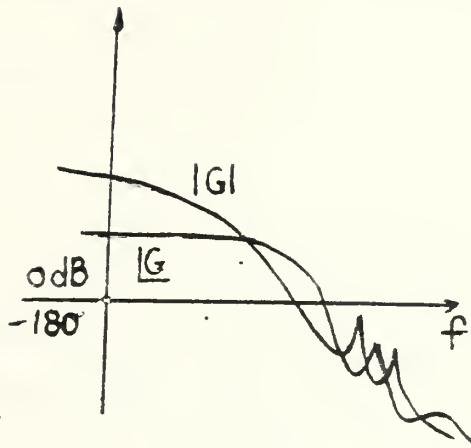
Analyzing each case separately, we can easily understand that for the first case there is no stability problem since the resonances do not exceed the zero dB axis, which means that the phase margin we had remains the same and if our system was stable it remains stable.

Considering now the second case we see that if one of the resonance peaks exceeds the zero dB axis it will create a new gain crossover which can make the system unstable if the resulting phase crossover is at a lower frequency than the gain crossover i.e. the resulting phase margin will be negative.

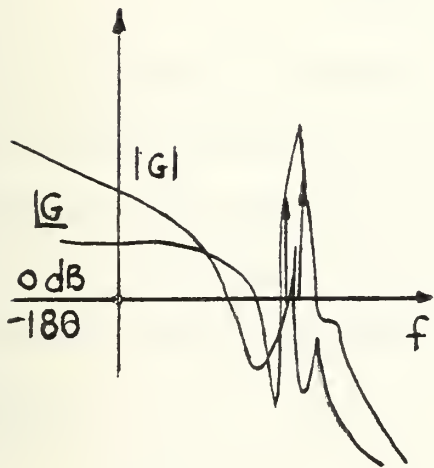




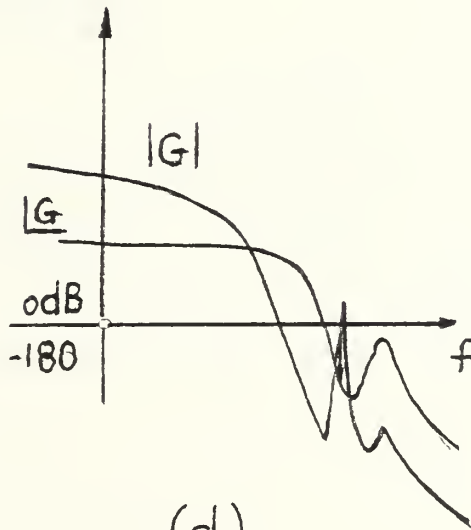
(a)



(b)



(c)



(d)

Figure 1.1 Possible Phase Margins of the 'Model'.





Assuming that the reader now is familiar with the mechanical resonances problem we can proceed and try to solve the problem i.e. to compensate a system which has been unstable due to these resonances, using two different methods.

### C. THE MODEL

The model which we will use in the major part of the study has to be initially stable. So we choose conveniently a third degree system with one zero and three poles with locations that might appear in a physical system whose Laplace transform has the following form:

$$G(s) = \frac{2300(0.1s+1)}{s(0.2s+1)(0.0033s+1)} \quad (\text{Eqn 1.1})$$

As we see in Figures 1.2 , 1.3 the frequency response of the system proves that the system is stable since we have a positive phase margin of 42 degrees. In Figure 1.4 we can see the time response of the system with a small oscillation for 0.04 seconds, which also proves the stability of the system.



GAIN CURVE  
SOZON CONSTANDOULAKIS

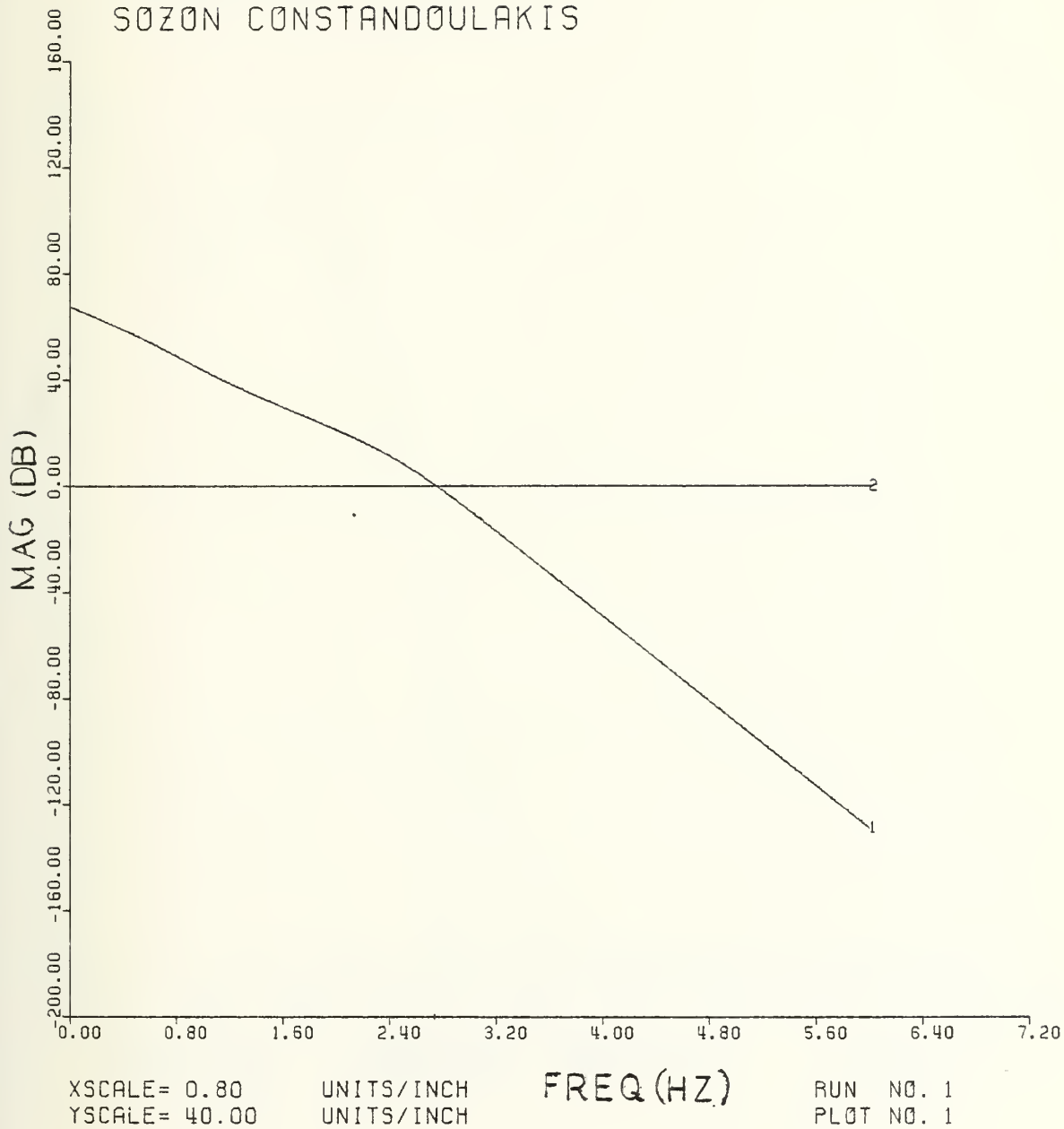
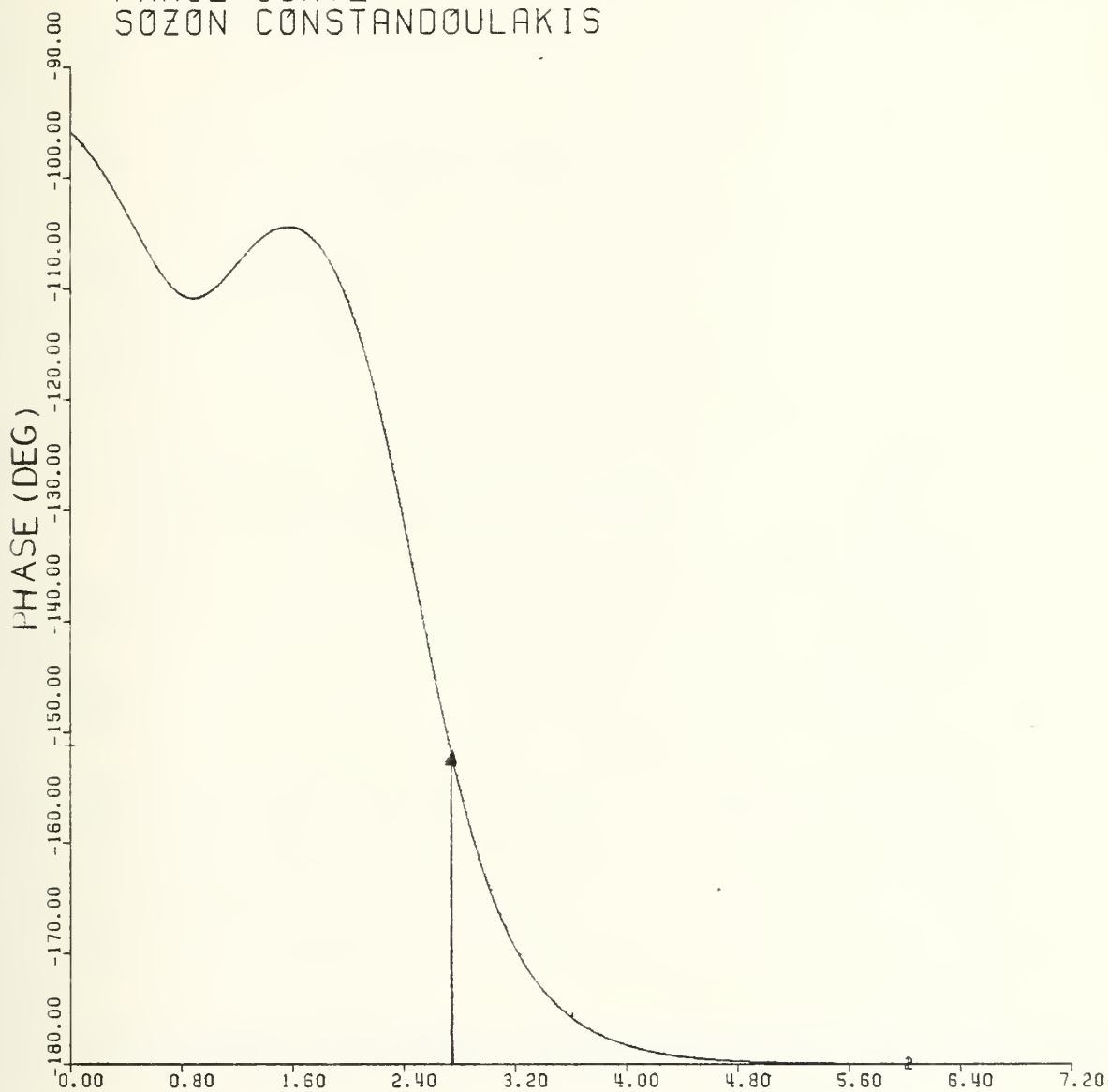


Figure 1.2 Gain Curve of the Stable System.



PHASE CURVE  
SOZON CONSTANDOULAKIS



XSCALE= 0.80  
YSCALE= 10.00

UNITS/INCH  
UNITS/INCH

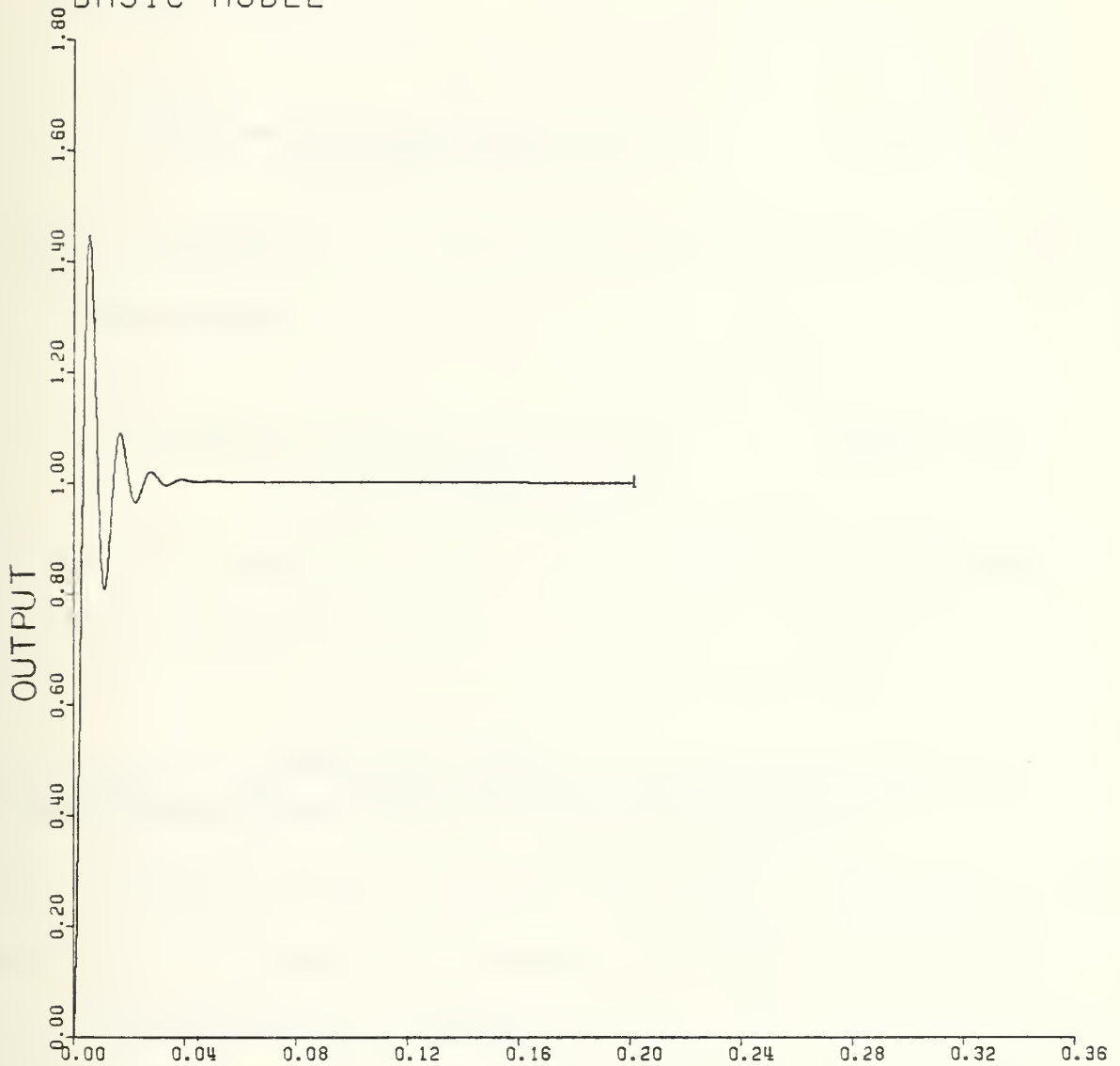
FREQ (HZ)

RUN NO. 1  
PLOT NO. 2

Figure 1.3 Phase Curve of the Stable Model.



TRANSIENT RESPONSE OF THE COMPENSATED SYSTEM  
BASIC MODEL



XSCALE= 0.04  
YSCALE= 0.20

UNITS/INCH  
UNITS/INCH

TIME (SEC)

RUN NO. 1  
PLOT NO. 1

Figure 1.4 Time Response of the Stable System.





Now we have to add two typical resonances in order to see how the system will become unstable.

The two selected mechanical resonances are as follows:

FIRST HARMONIC at frequency  $f_1=2000$  HZ with damping ratio  $\zeta=0.005$

$$G(s) = \frac{1}{0.25 \cdot 10^6 s^2 + 0.5 \cdot 10^5 s + 1} \quad (\text{Eqn 1.2})$$

SECOND HARMONIC at frequency  $f_2=5000$  HZ with damping ratio  $\zeta=0.0000158$

$$G(s) = \frac{1}{0.4 \cdot 10^7 s^2 + 0.632 \cdot 10^8 s + 1} \quad (\text{Eqn 1.3})$$

After the introduction of the two mechanical resonances the transfer function becomes:

$$G(s) = \frac{2300 (0.1s + 1)}{s(0.2s + 1)(0.033s + 1)(0.25 \cdot 10^6 s^2 + 0.5 \cdot 10^5 s + 1)(0.4 \cdot 10^7 s^2 + 0.632 \cdot 10^8 s + 1)} \quad (\text{Eqn 1.4})$$

The following pages present frequency responses on Figures 1.5, 1.6 and time response on Figure 1.7 of this system, including the two resonances.

From this point in this Thesis we will call  $G_u(s)$  'Model' wherever it is needed. The 'u' means uncompensated.

In Appendix the programs mostly used in this Thesis are given. Program 1 is suitable for calculating the frequency



response of the system with suitable modification for each case. Program 2 is suitable for calculating the time response of the system with suitable modification for each case also. Program 3 is suitable for calculating the time response of the system, when we use as a compensation method imaginary zeros and complex poles. The graphs have been obtained using the Versatec plotter of the Naval Postgraduate School computer IBM-3033 using the Digital Simulation Package (DSL) and/or CSMP III. In the phase curves when the phase exceeds the  $-360$  degrees the plotter jumps to its next value and the curve looks discontinued but in reality it is not.



GAIN CURVE  
SOZON CONSTANDOULAKIS

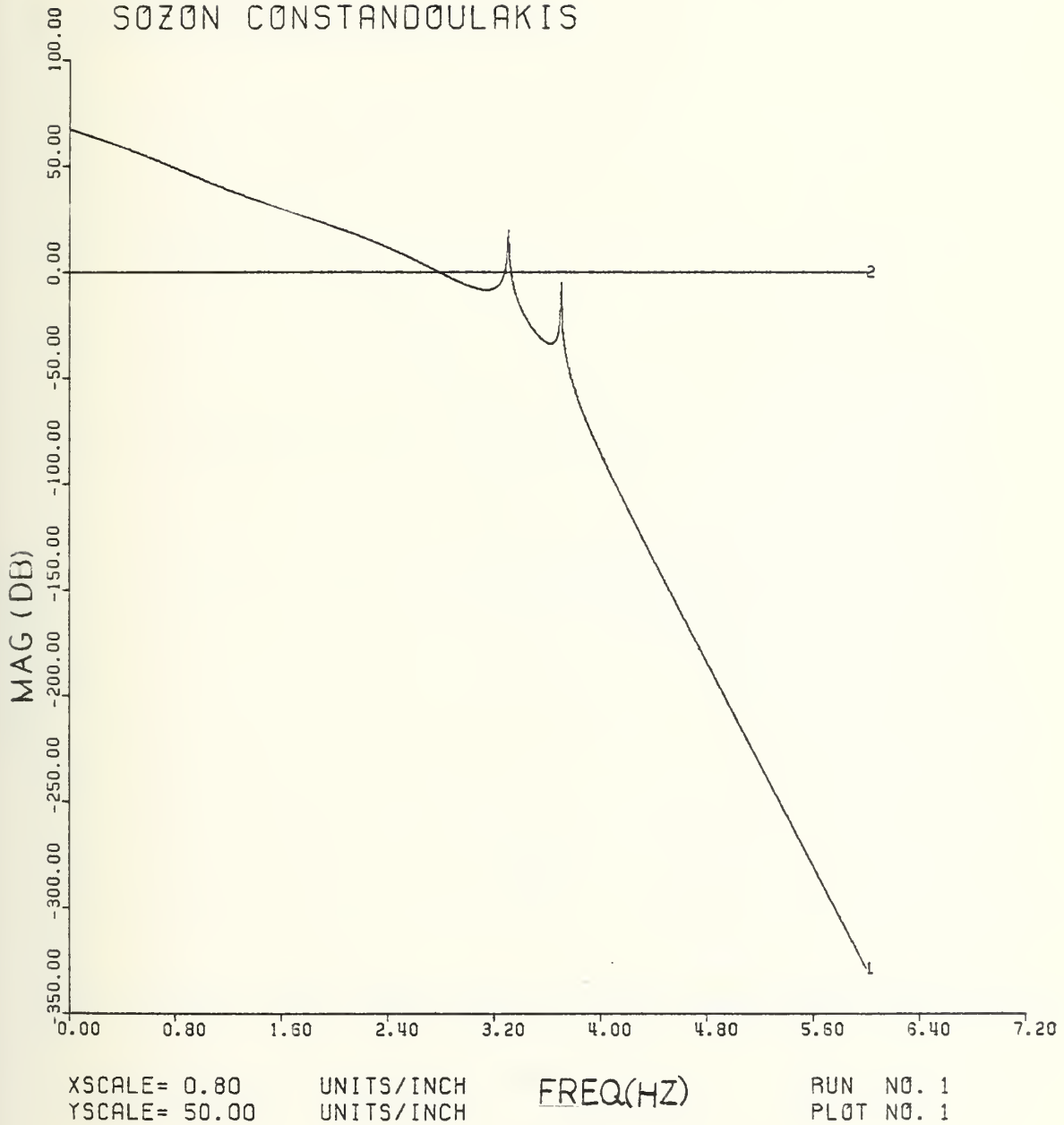


Figure 1.5 Gain Curve of the System with the Resonances.



PHASE CURVE  
SOZON CONSTANDOULAKIS

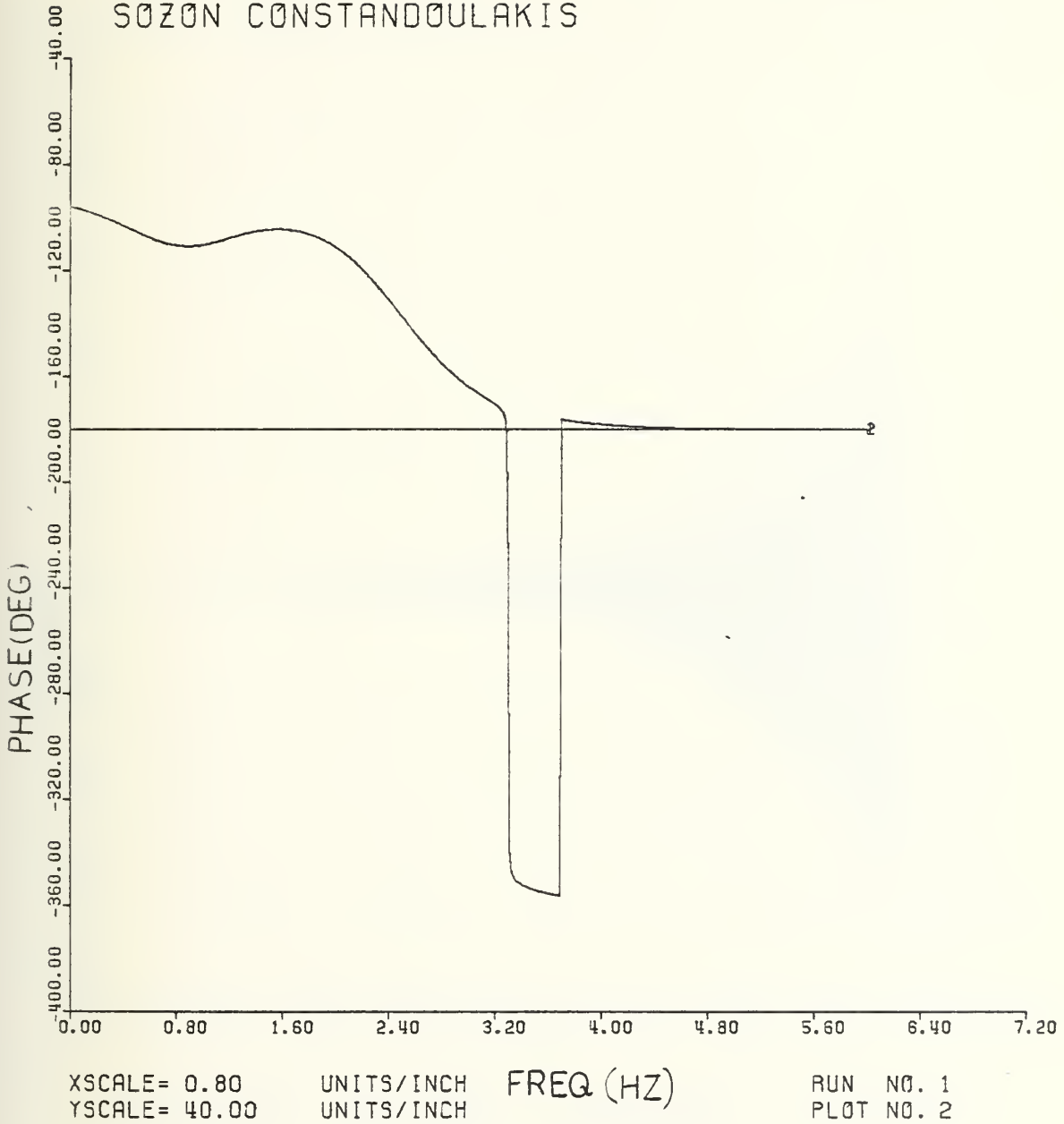


Figure 1.6 Phase Curve of the System with the Resonances.





TRANSIENT RESPONSE OF THE UNCOMPENSATED SYSTEM  
BASIC MODEL

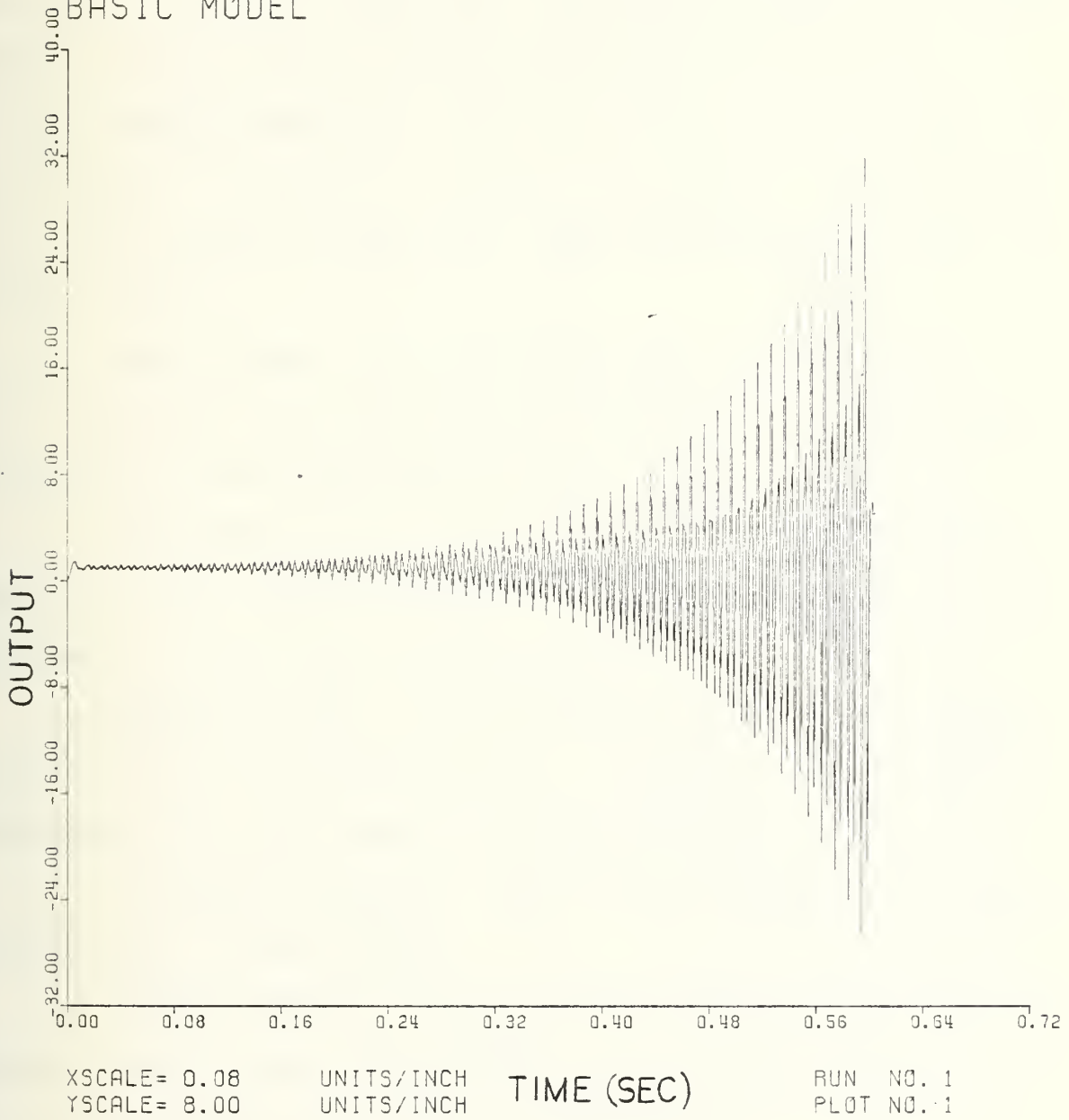


Figure 1.7 Time Response of the System with the Resonances.



#### D. DEFINITIONS

In order to follow the thesis easily, it is a good idea to refresh our memory with a few definitions.

1. GAIN CROSSOVER : This is the point on the plot of the transfer function at which the magnitude is unity i.e. the point at which the gain curve crosses the zero dB axis.

2. PHASE CROSSOVER : This is the point on the plot of the transfer function at which the phase is  $-180$  degrees i.e. the point at which the phase curve crosses the  $-180$  degrees axis.

3. PHASE MARGIN : This is defined by the equation:

$\gamma = 180 + |G(s)|$  and determines the amount of phase shift required to place the system at the stability limit.

4. ROOT LOCUS : Is a plot of the roots of the characteristic equation of the closed loop system as a function of the gain or some other variable parameter.

5. RESONANCE PEAK : Is the maximum amplitude of the gain curve. Of the frequency response. It occurs at the resonant frequency,  $\omega_r$ , and is due to complex poles (roots).

6. RESONANT FREQUENCY ( $\omega_r$ ): The frequency at which  $|G(j\omega)|$  has its peak value

7. DAMPING RATIO : Is the factor  $\zeta = F/F_c$  where  $F$  is the actual damping and  $F_c = 2\zeta\sqrt{Jxk}$  the critical factor of a quadratic having form:  $G(s) = k/(Js^2 + Fs + k)$



## II. COMPENSATION USING IMAGINARY ZEROS AND COMPLEX POLES

### A. METHOD DESCRIPTION

In this method we will use a compensator which includes imaginary zeros and complex poles. This compensator has the following general form:

$$G(s) = \frac{s^2 + a^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \quad (\text{Eqn 2.1})$$

Where  $a$  is the frequency of the pure zero on the imaginary axis and  $\omega_n$  is the frequency of the complex poles.

The philosophy of this compensator is that we try to raise the phase curve at a frequency slightly lower than the resonant frequency in order to avoid the negative phase margin. Then we lower the phase curve after the resonance has occurred so that the gain curve lies under this phase peak and the resulting new phase margins are positive at both gain crossovers, in other words we try to reshape the phase curve. For this reason we put the zero in the vicinity of the resonant frequency (2000 Hz) and specifically at a slightly lower frequency (1950 Hz). In this way we try to raise the phase curve before the resonance happens. Then in order to return the system to its original condition we introduce the complex poles at a frequency of 2050 Hz.



In the following pages we see the frequency response and the time response of the 'Model' when we apply this method.

## B. METHOD APPLICATION

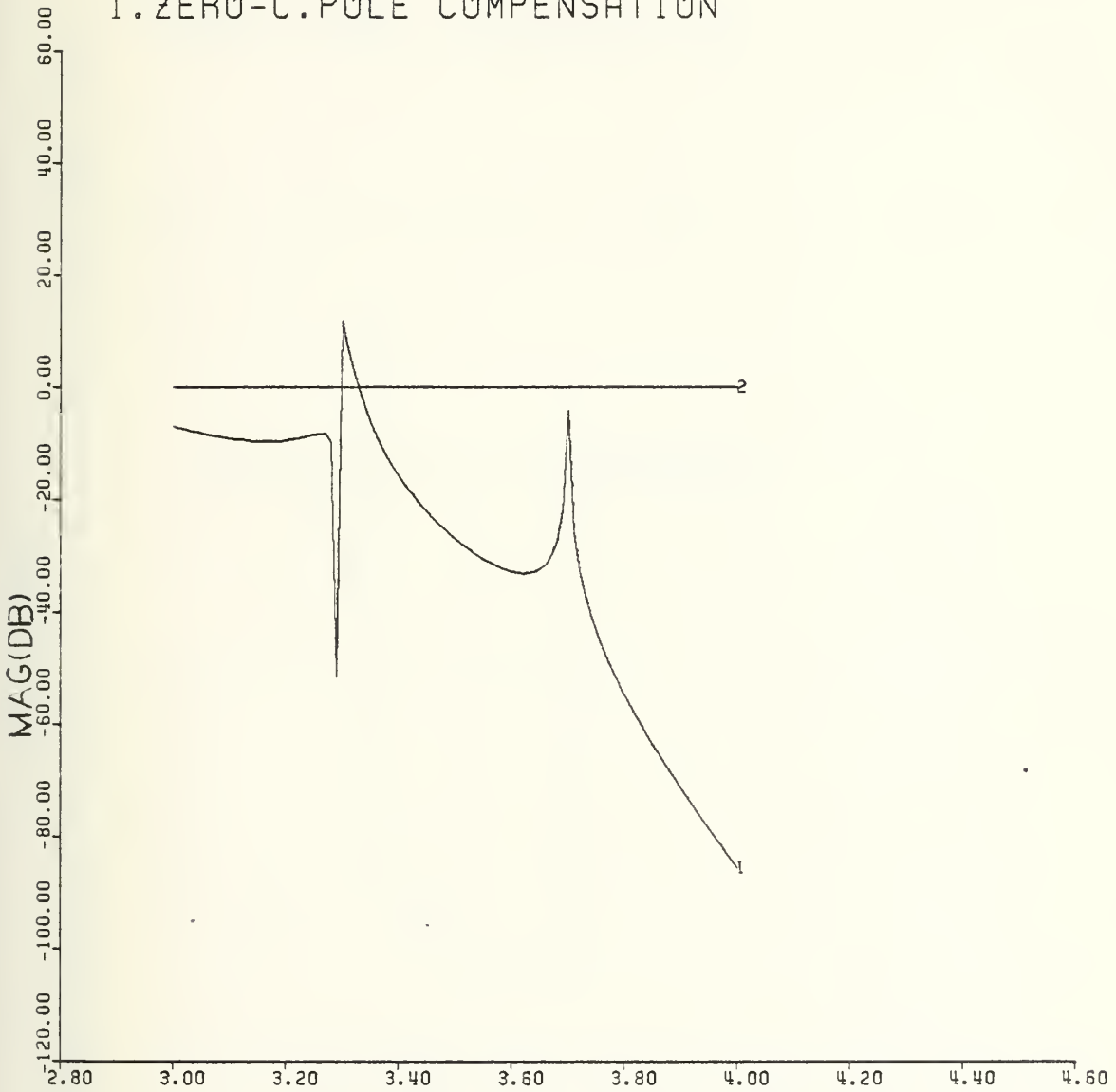
The first data we use for our compensator is a frequency 1950 Hz for the imaginary zero and a slightly greater frequency for the complex poles, 2050 Hz. The resonant frequency is 2000 Hz. Keeping the above data constant we vary the damping ratio of the compensator in order to see how this variation affects the filter .

In Figures 2.1, 2.2, 2.3 we have the gain and phase curves and the Time response of the system with damping ratio  $\zeta=0.05$ . In Figures 2.4, 2.5, 2.6 we have the gain, phase and time response of the system with damping ratio  $\zeta=0.1$ .





GAIN CURVE  
I. ZERO-C. POLE COMPENSATION



XSCALE= 0.20  
YSCALE= 20.00

UNITS/INCH  
UNITS/INCH

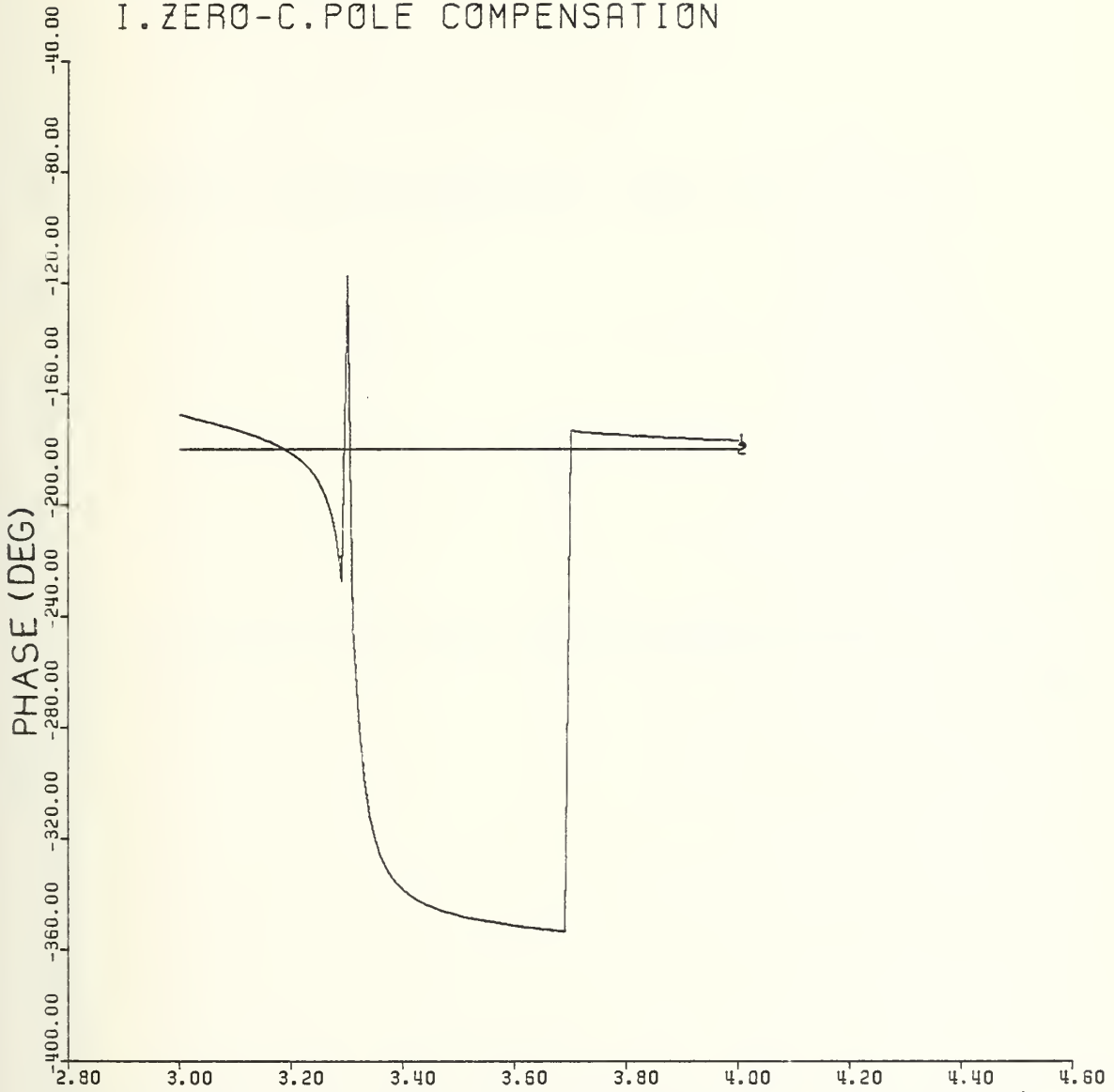
FREQ (HZ)

RUN NO. 1  
PLOT NO. 1

Figure 2.1 Gain Curve with  $\zeta=0.05$ .



PHASE CURVE  
I. ZERO-C. POLE COMPENSATION



XSCALE= 0.20  
YSCALE= 40.00

UNITS/INCH  
UNITS/INCH

FREQ (HZ)

RUN NO. 1  
PLOT NO. 2

Figure 2.2 Phase Curve with  $\zeta=0.05$ .



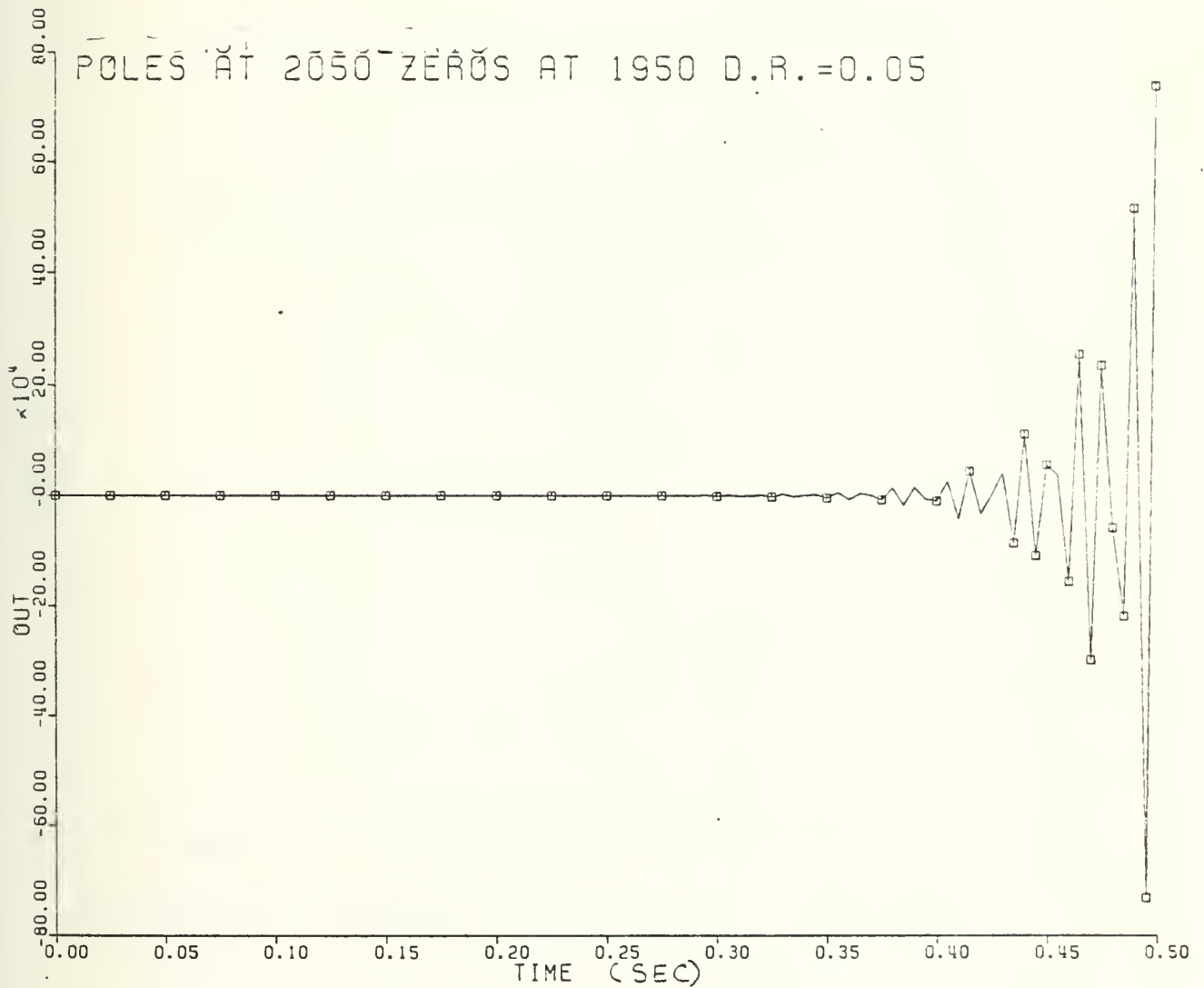
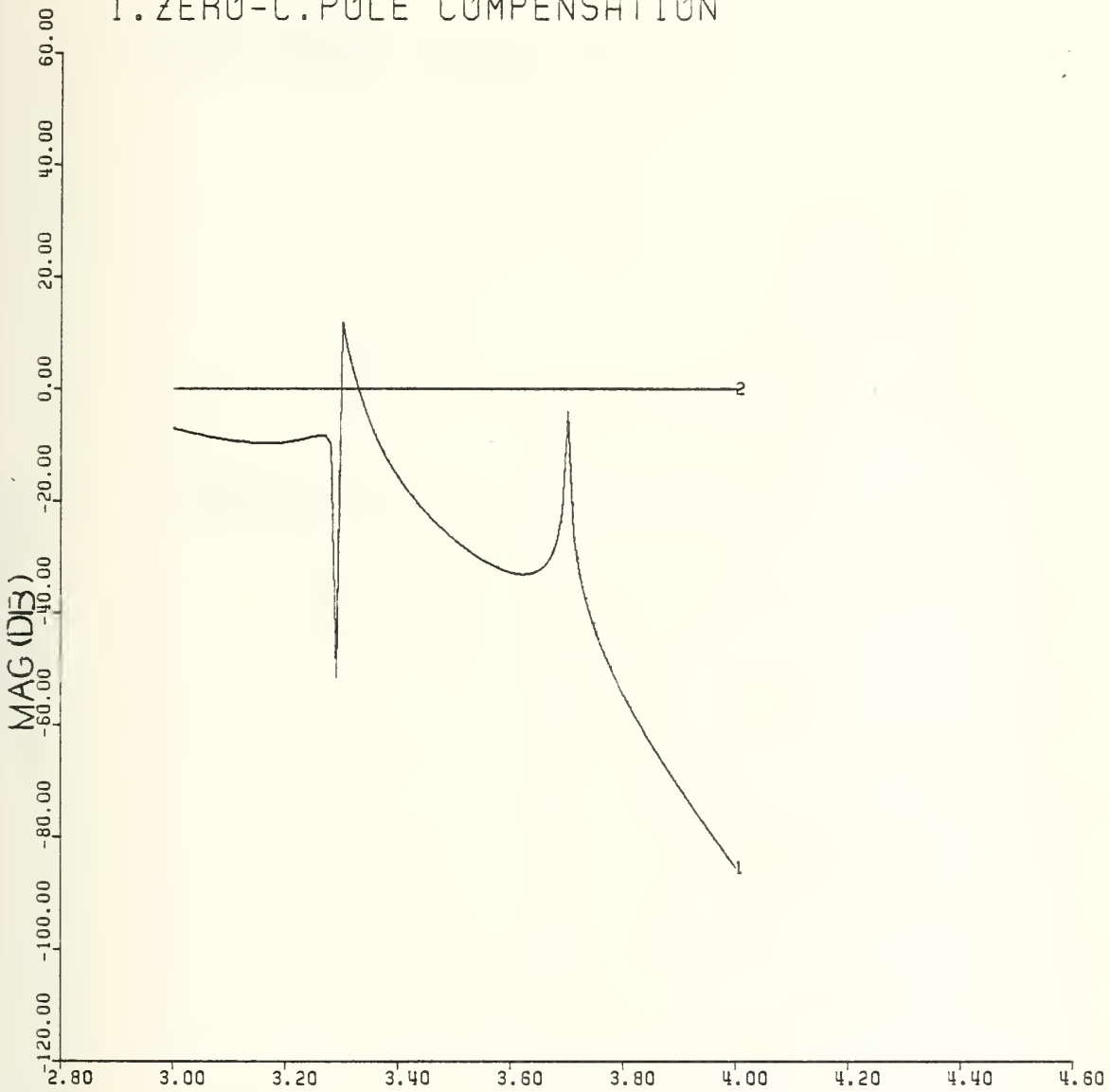


Figure 2.3 Time Response of the System with  $\zeta=0.05$ .



GAIN CURVE  
I. ZERO-C. POLE COMPENSATION



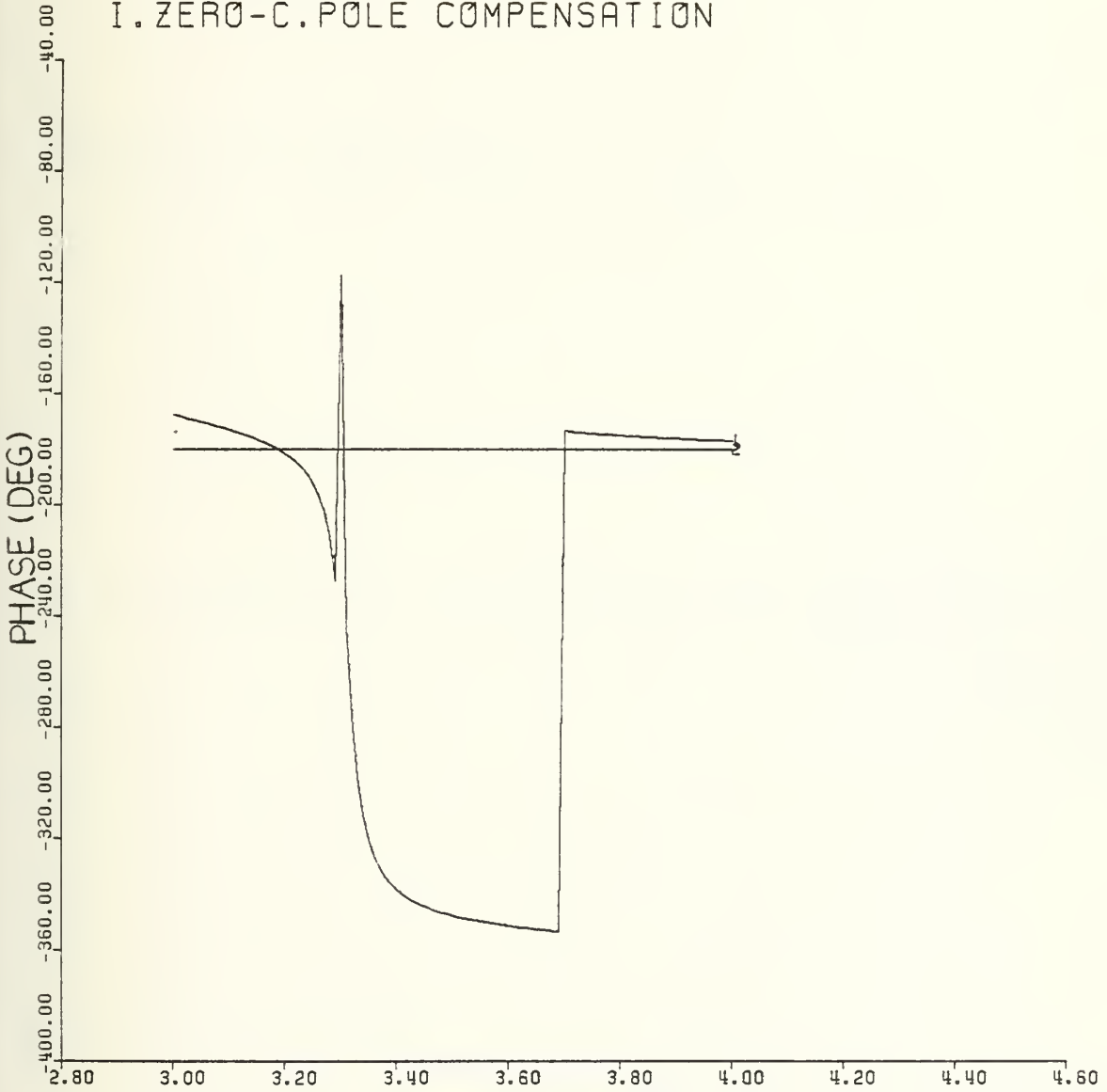
XSCALE= 0.20      UNITS/INCH      FREQ (HZ)      RUN NO. 1  
YSCALE= 20.00    UNITS/INCH                                      PLOT NO. 1

Figure 2.4 Gain Curve with  $\zeta=0.1$ .





PHASE CURVE  
I. ZERO-C. POLE COMPENSATION



XSCALE= 0.20  
YSCALE= 40.00

UNITS/INCH  
UNITS/INCH

FREQ (HZ)

RUN NO. 1  
PLOT NO. 2

Figure 2.5 Phase Curve with  $\zeta=0.1$ .



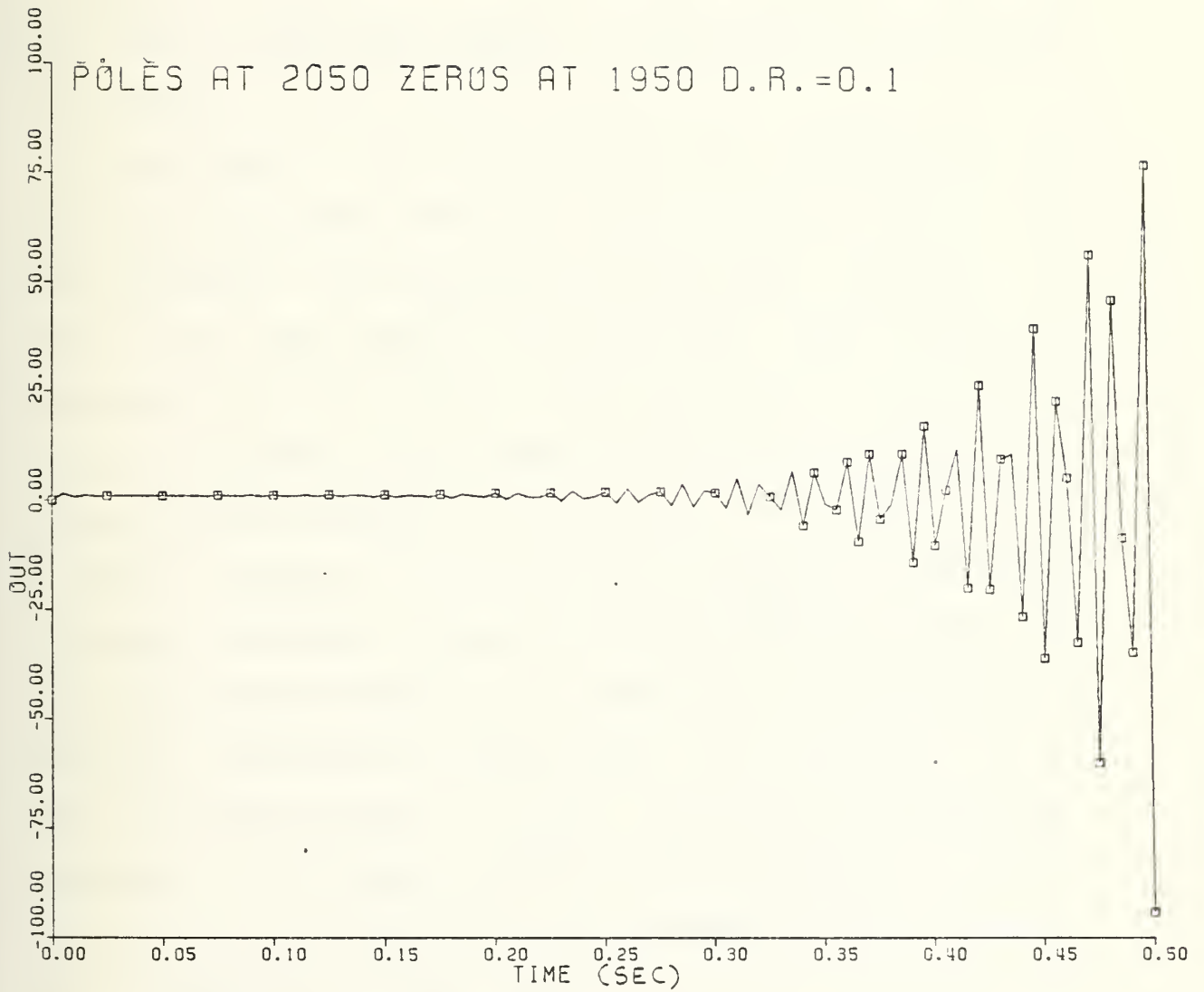


Figure 2.6 Time Response of the System with  $\zeta=0.1$ .



Observing Figures 2.1 through 2.6 it is obvious that the method has not worked since the system remains unstable. The two different damping ratios used i.e. 0.05 and 0.1 are the two representatives for this point since we do not want to increase the size of the thesis with unimportant graphs, but in reality we tried many more, concluding that the damping ratio does not have much effect on the system at this point of the study.

If we observe the graphs closer or we overlap the gain curves over the phase curves we will see that we have three gain crossovers and consequently three phase margins. The first is the phase margin of the original model before the resonances and it is stable. The other two are created from the gain crossover of the first peak of the 'Model' which crosses the zero dB axis at two points. The first of these two phase margins is positive and the other is negative. That means something happened because before we used the filter both phase margins were negative. So the filter has worked but partially and specifically the frequency of the zeros was correct since the phase curve has been raised and the second of the phase margins became positive. What we have to do now is to adjust the frequency of the poles in order to reshape the high frequency part of the phase curve and have the third phase margin positive too, which means that the system will be stable. In Figures 2.7 through 2.15 we can see the gain, phase and time response of the system

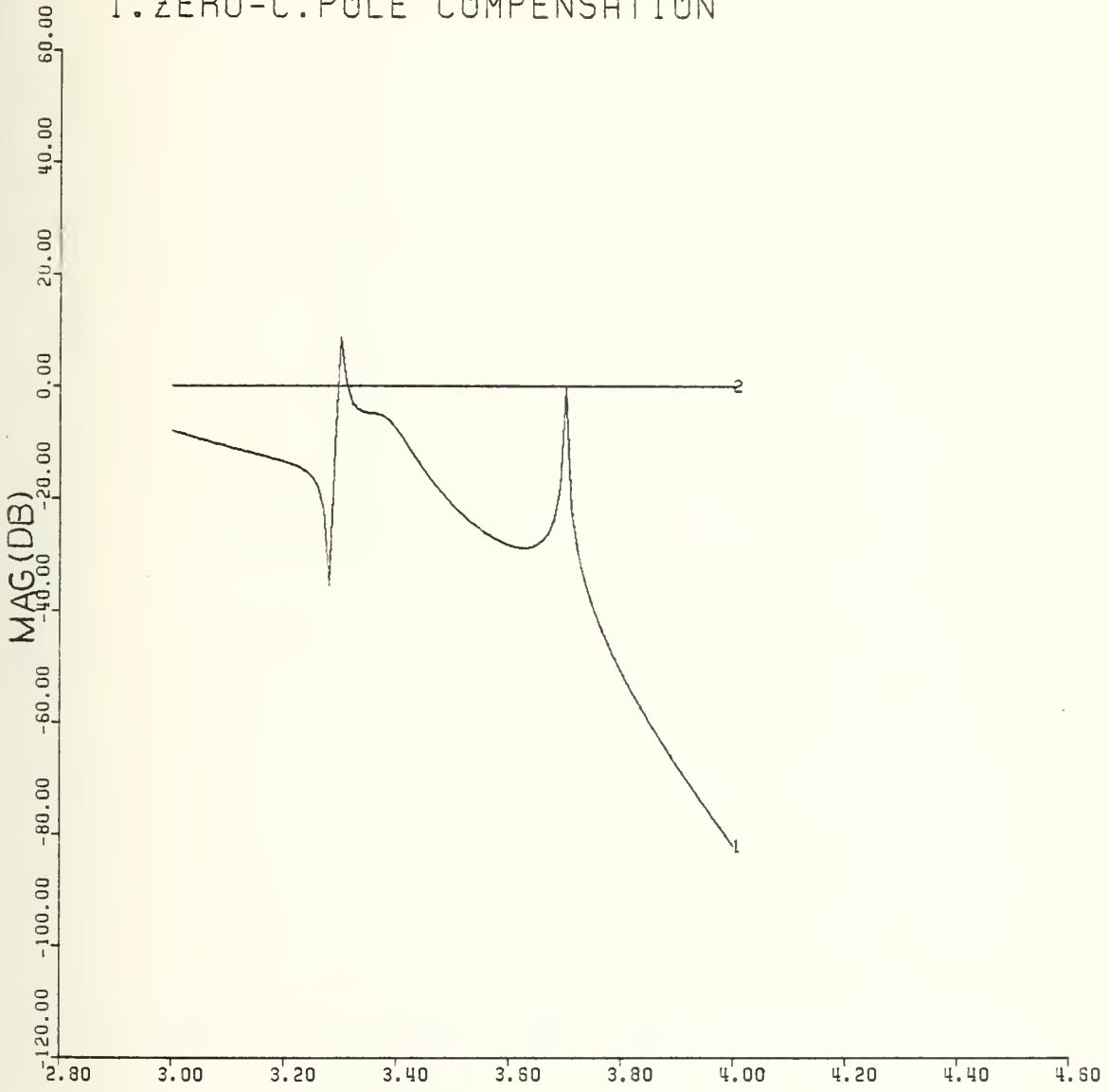


at pole frequencies 2400, 2600, 2800 Hz respectively. The frequency of the zeros is also reduced at 1900 Hz i.e. 50 Hz less in order to see if we can have a small variation of the zeros for application purposes.





GAIN CURVE  
I. ZERO-C. POLE COMPENSATION

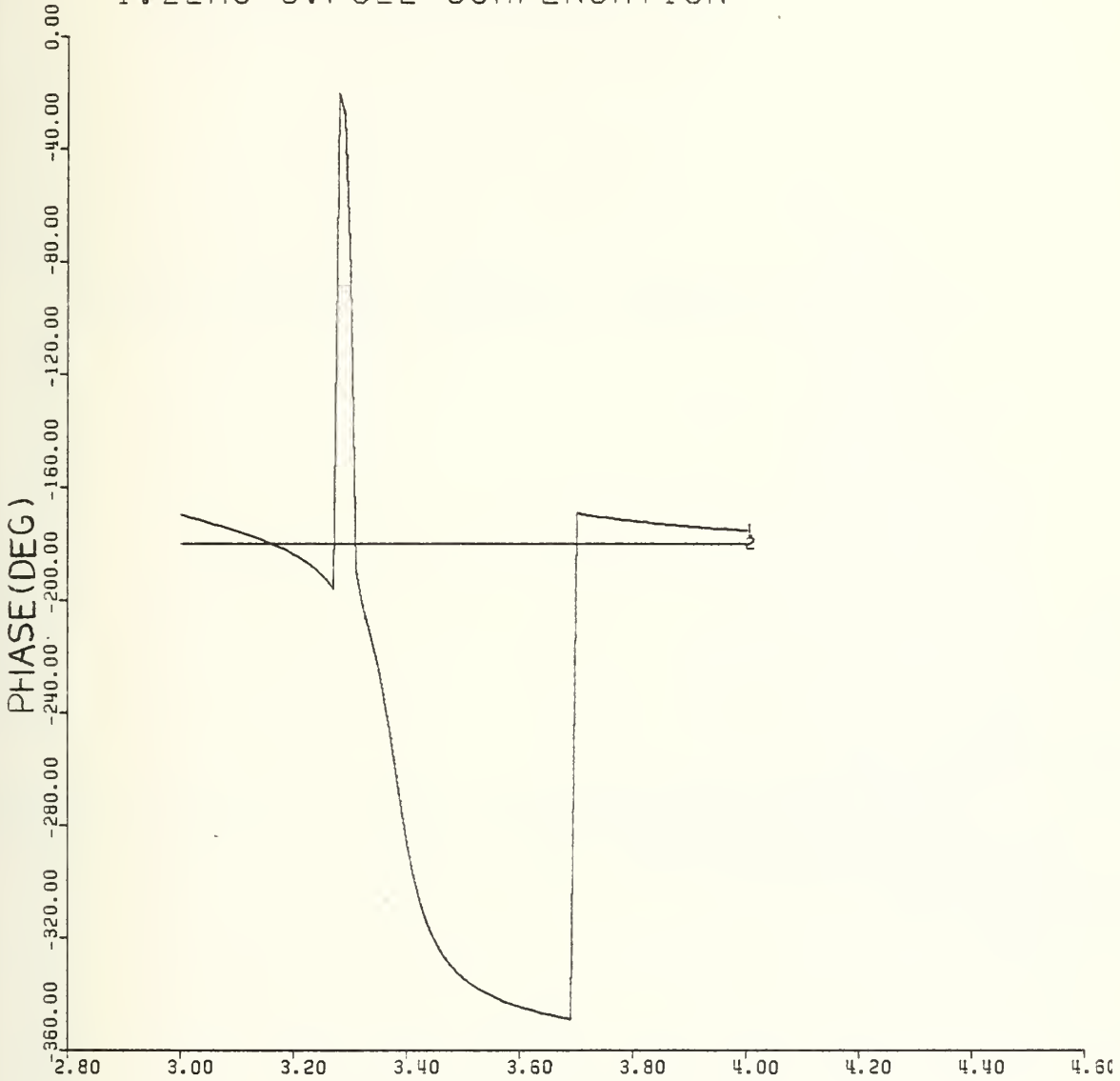


XSCALE= 0.20      UNITS/INCH      FREQ (HZ)      RUN NO. 1  
YSCALE= 20.00      UNITS/INCH                              PLOT NO. 1

Figure 2.7      Gain Curve with Pole at Frequency 2400 Hz.



PHASE CURVE  
I. ZERO-C. POLE COMPENSATION



XSCALE= 0.20  
YSCALE= 40.00

UNITS/INCH  
UNITS/INCH

FREQ (HZ)

RUN NO. 1  
PLOT NO. 2

Figure 2.8 Phase Curve with Pole at Frequency 2400 Hz.



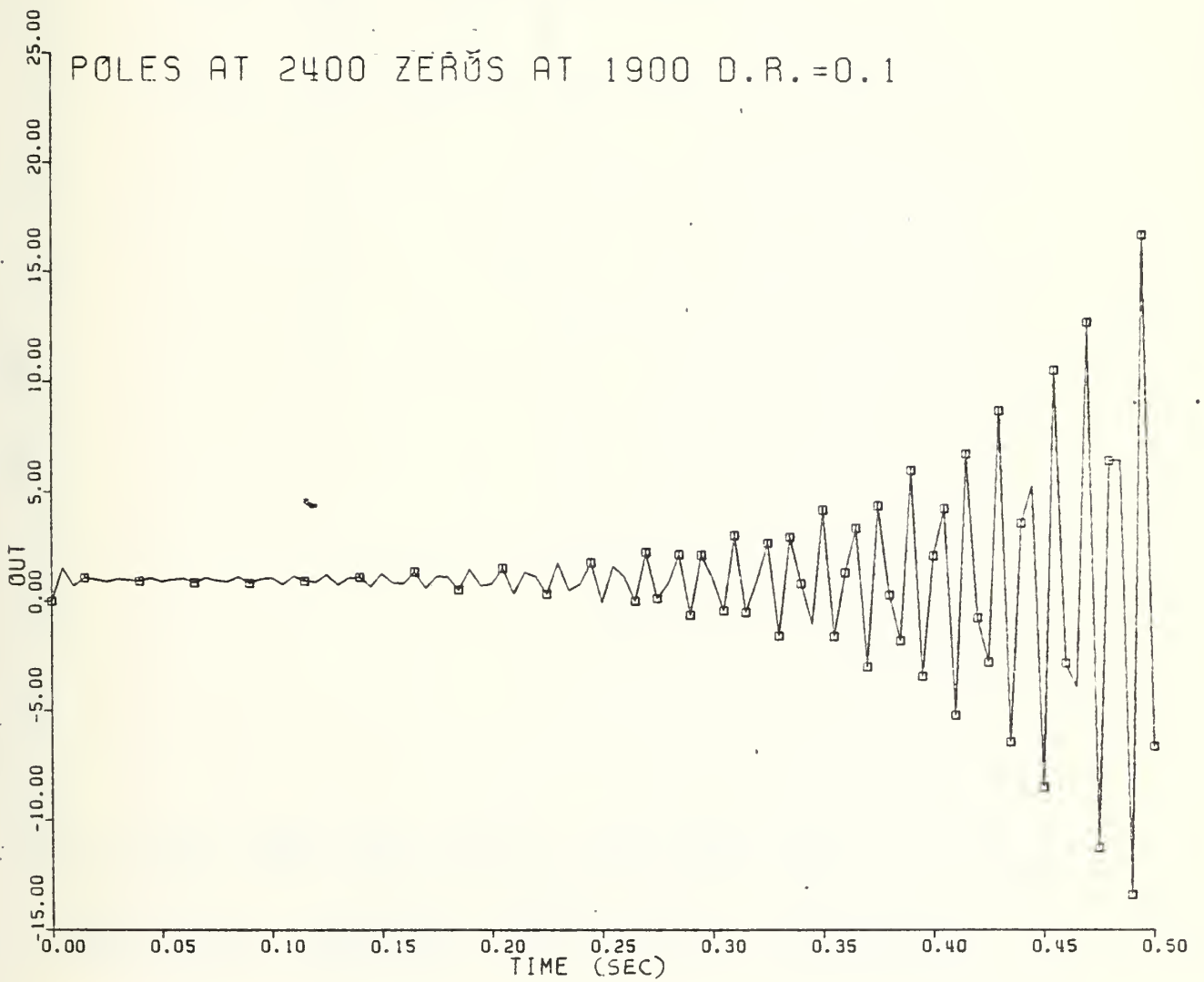
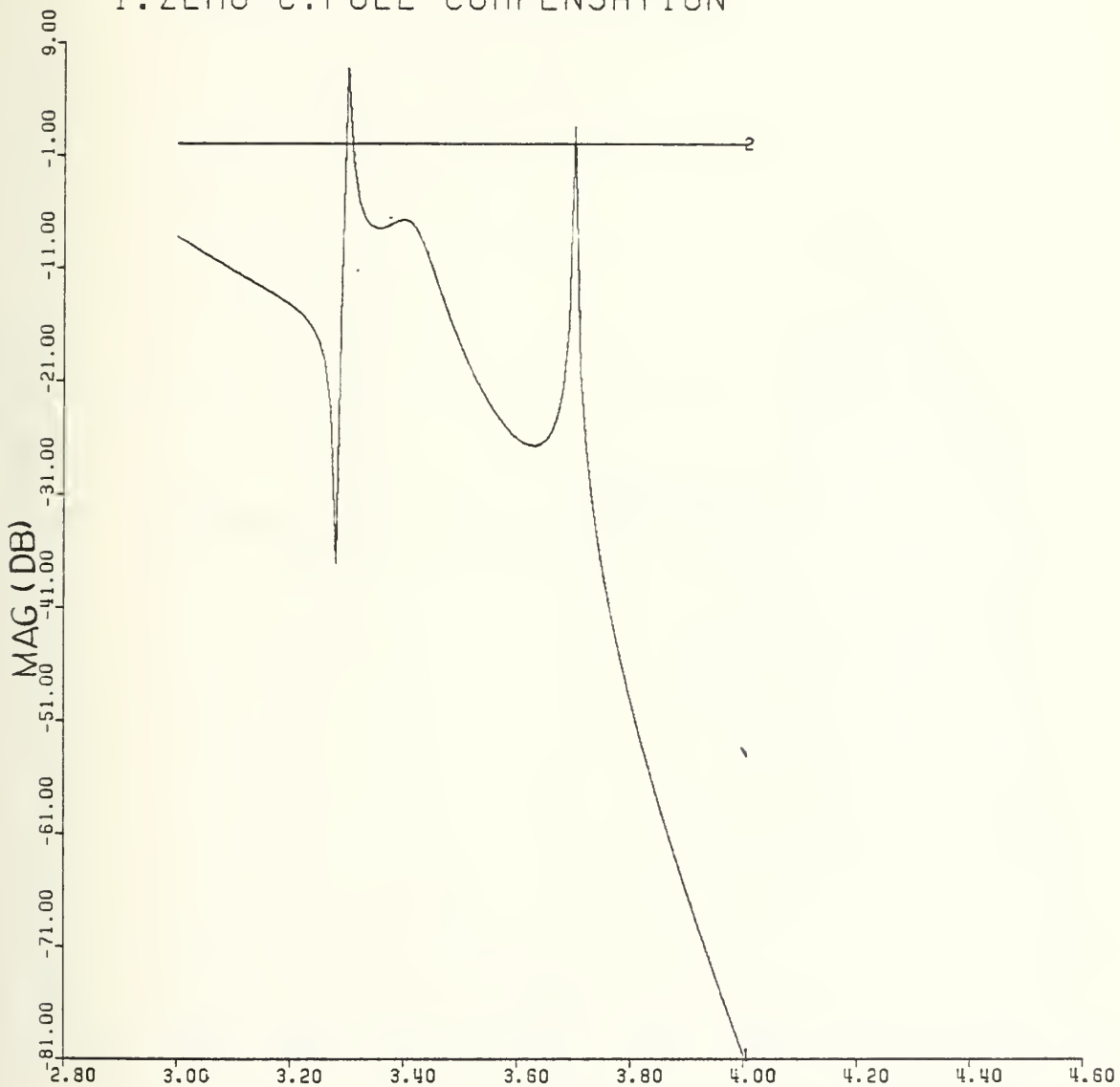


Figure 2.9 Time Response with Poles at 2400 Hz.



GAIN CURVE  
I. ZERO-C. POLE COMPENSATION



XSCALE= 0.20  
YSCALE= 10.00

UNITS/INCH      FREQ (HZ)  
UNITS/INCH

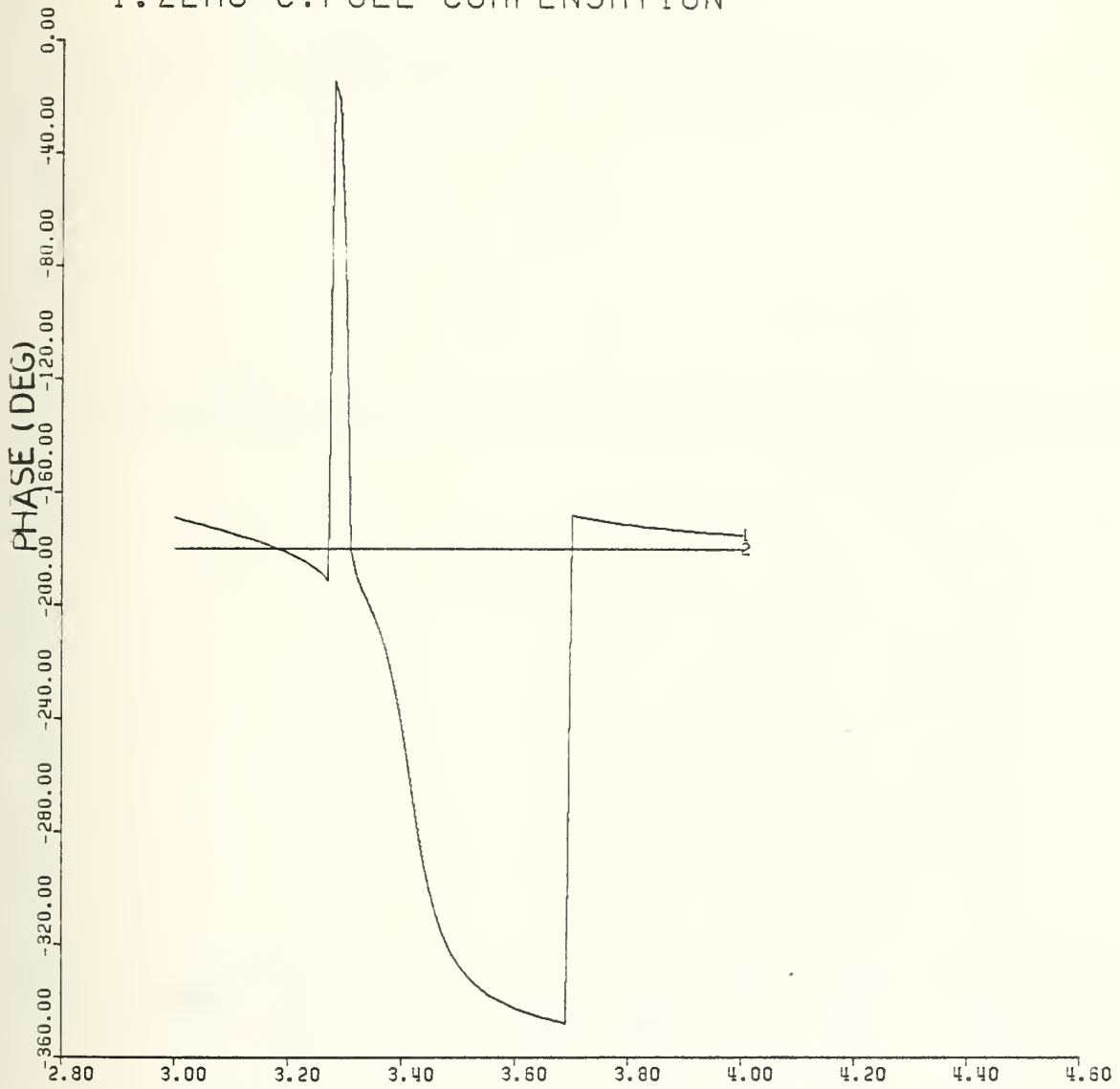
RUN NO. 1  
PLOT NO. 1

Figure 2.10      Gain Curve with Poles at 2600 Hz.





PHASE CURVE  
I. ZERO-C. POLE COMPENSATION



XSCALE= 0.20  
YSCALE= 40.00

UNITS/INCH      FREQ (HZ)  
UNITS/INCH

RUN NO. 1  
PLOT NO. 2

Figure 2.11      Phase Curve with Poles at 2600 Hz.



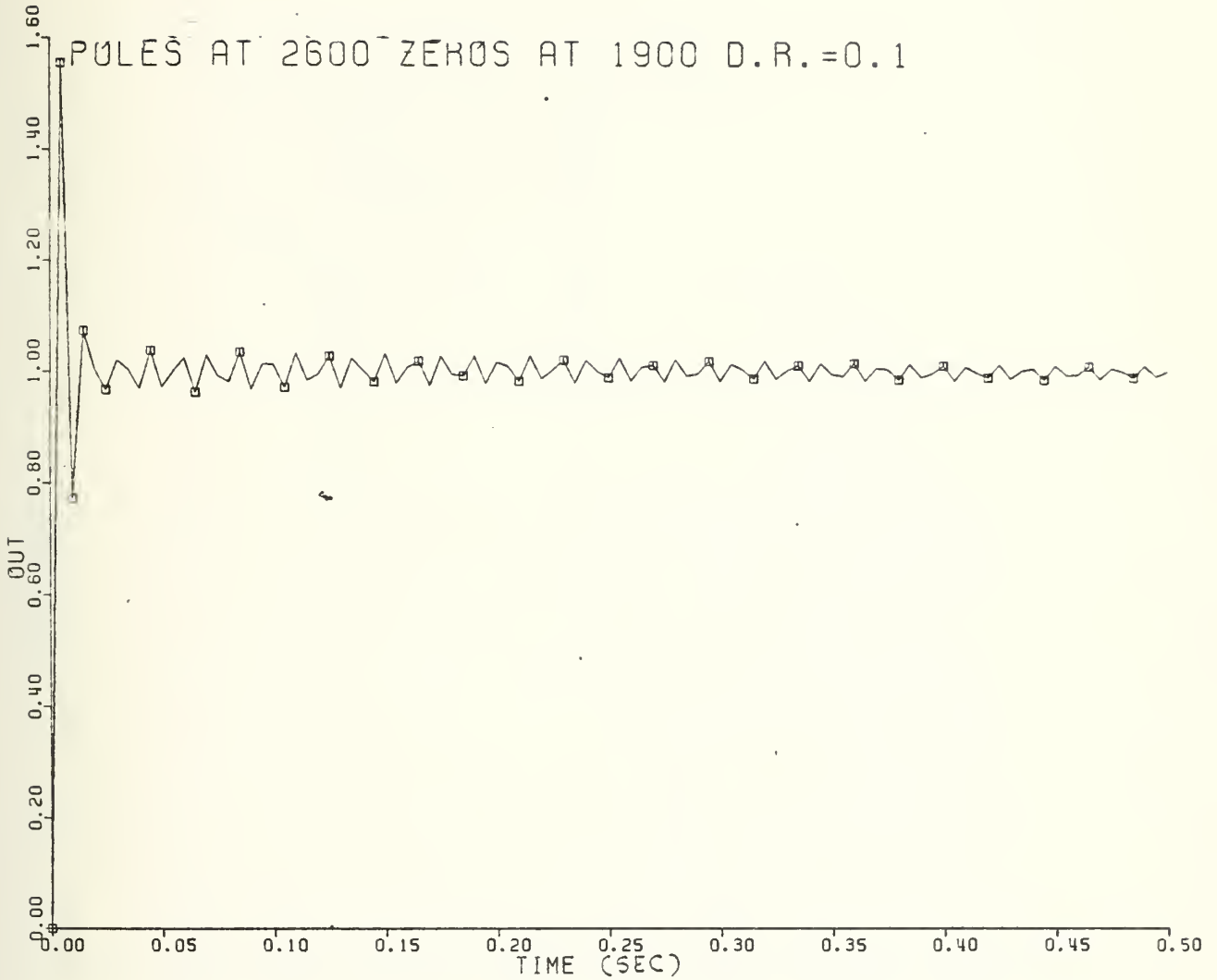
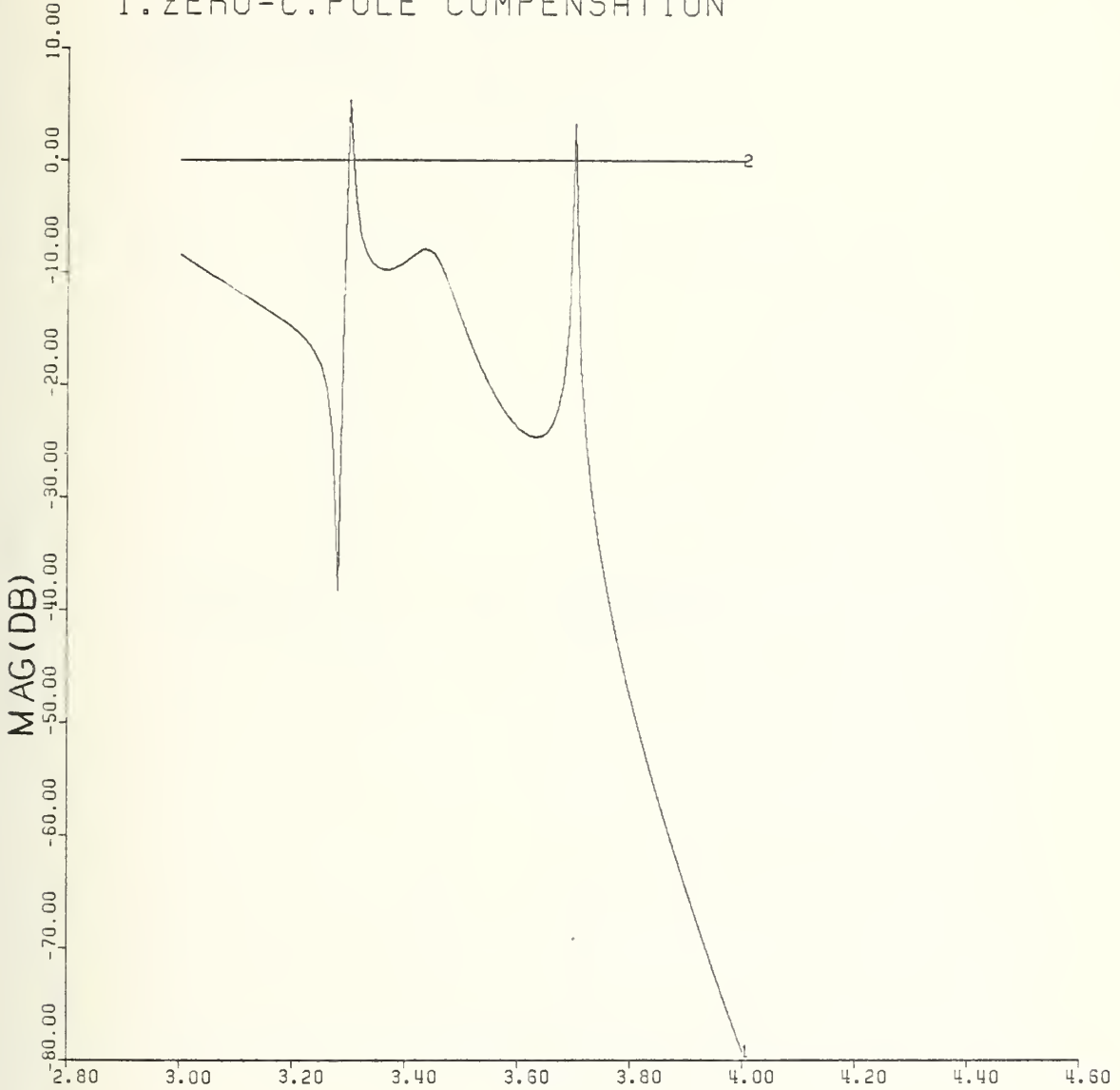


Figure 2.12 Time Response with Poles at 2600 Hz.



GAIN CURVE  
I. ZERO-C. POLE COMPENSATION



XSCALE= 0.20  
YSCALE= 10.00

UNITS/INCH  
UNITS/INCH

FREQ (HZ)

RUN NO. 1  
PLOT NO. 1

Figure 2.13 Gain Curve with Poles at 2800 Hz .



PHASE CURVE  
I. ZERO-C. POLE COMPENSATION

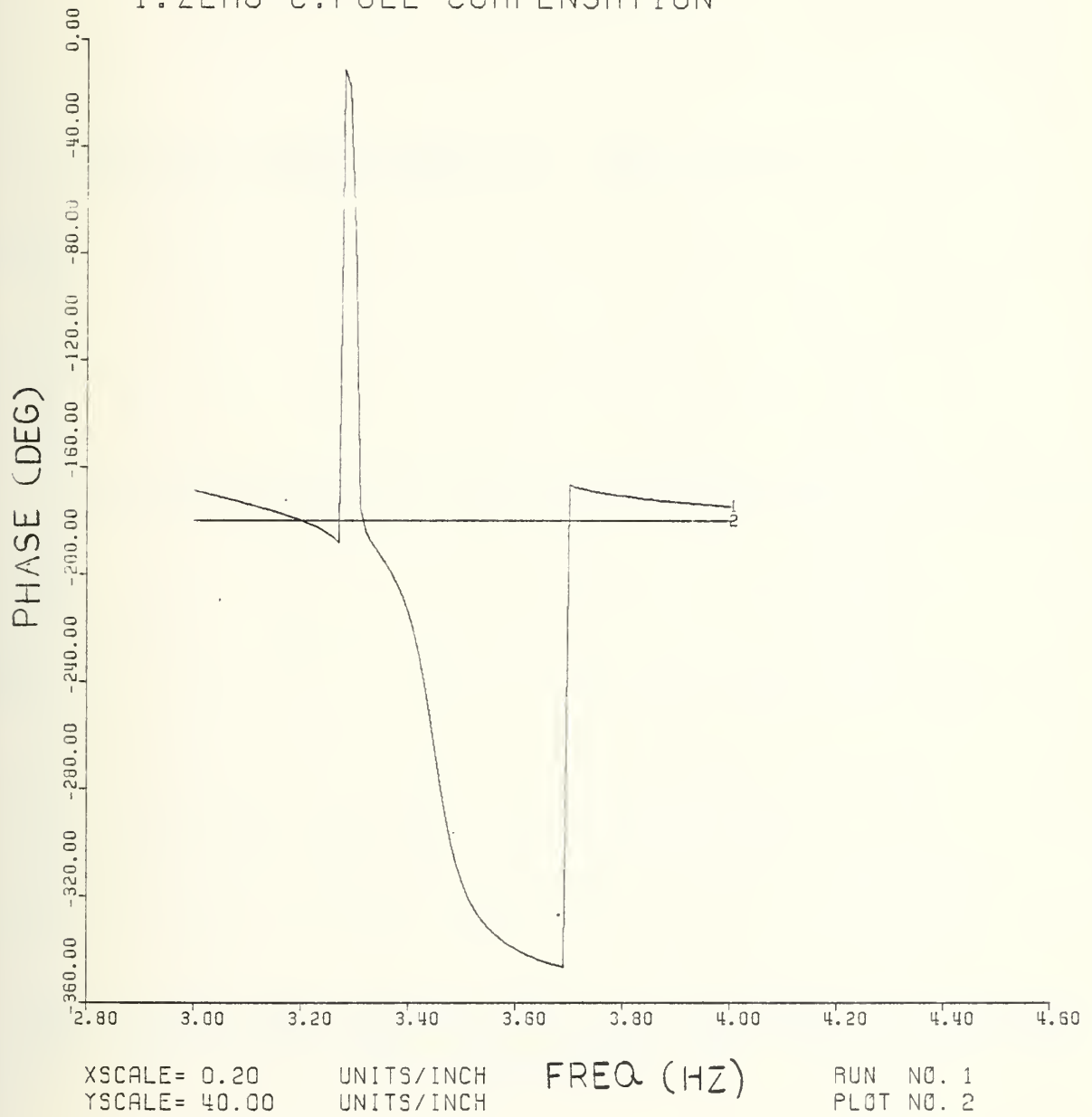


Figure 2.14 Phase Curve with Poles at 2800 Hz .





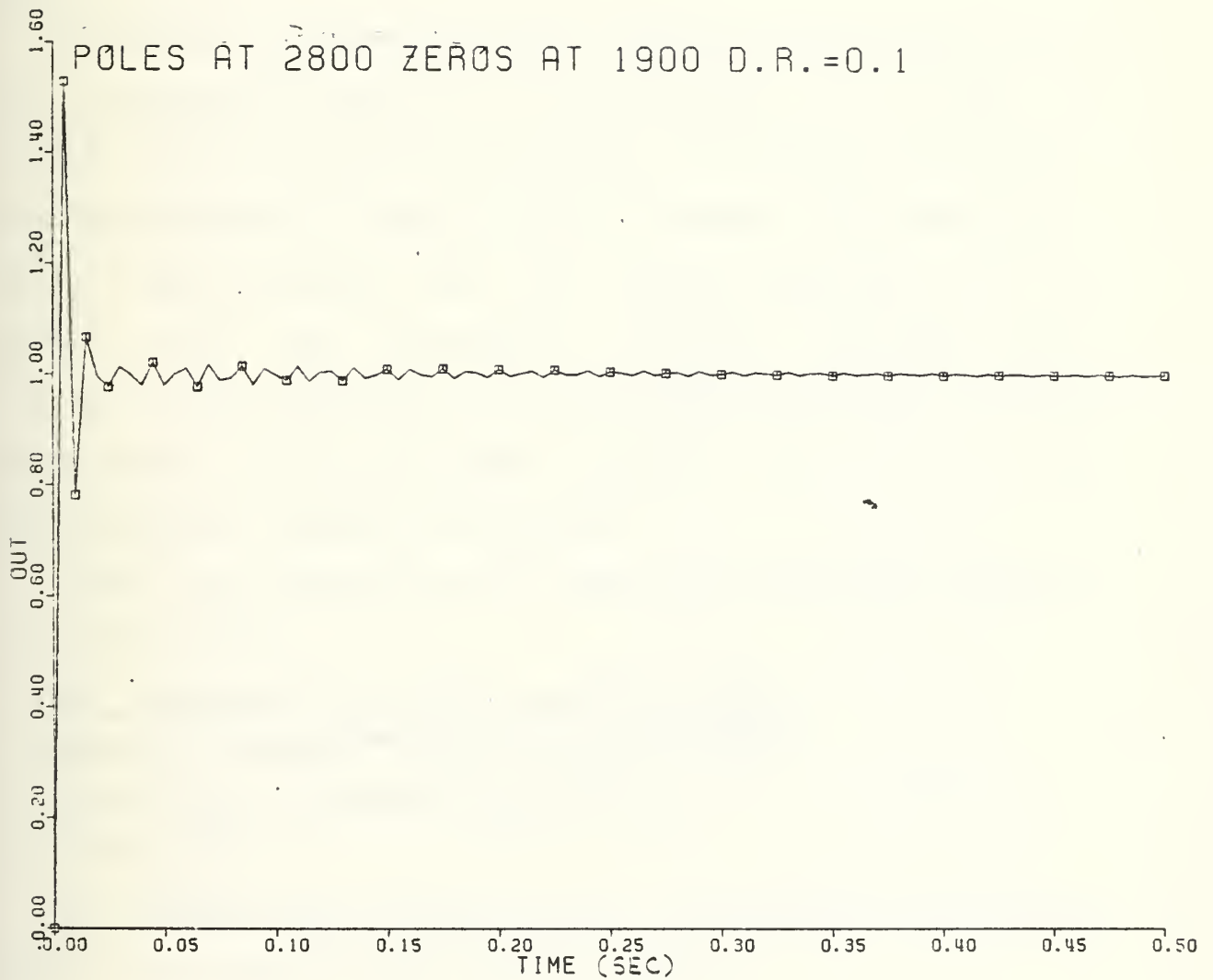


Figure 2.15 Time Response with Poles at 2800 Hz .



After examination of Figures 2.7 through 2.15 we observe that the system becomes stable for pole frequencies 2600 and 2800 Hz respectively with the only difference being that the system has less transient oscillation when we put the poles at frequency 2800 Hz (system oscillate oscillates for 0.25 seconds) than at frequency 2600 Hz (system oscillates for 0.5 seconds)

In order to understand more completely the behavior of the compensated system for various gains it is wise to obtain the root locus of the system. So we have to calculate the characteristic equation of the system. The characteristic equation comes from the transfer function of the system with the following formula  $G(s)+1=0$  but  $G(s)=N(s)/D(s)$  where  $N(s)$  is the numerator and  $D(s)$  is the denominator of the transfer function and finally the CHARACTERISTIC EQUATION  $=N(s)+D(s)$ .

After the calculations we obtain the following equation of ninth degree:

$$\begin{aligned}
 \text{C.E} = & s^9 + 885.158s^8 + 37.02 \cdot 10^6 s^7 + 2.813 \cdot 10^{10} s^6 + 3.32 \cdot 10^{14} s^5 + \\
 & + 1.59 \cdot 10^{17} s^4 + s^3(8.028 \cdot 10^{20} + 3.257 \cdot 10^{16} K) + s^2(2.39 \cdot 10^{23} + \\
 & + 3.257 \cdot 10^{17} K) + s(1.176 \cdot 10^{24} + 1.176 \cdot 10^{23} K) + \\
 & + 1.176 \cdot 10^{24} K
 \end{aligned}
 \tag{Eqn 2.2}$$

In Figures 2.16 we have plotted the roots of the system varying the gain (K) from 0.01 to 933444 .



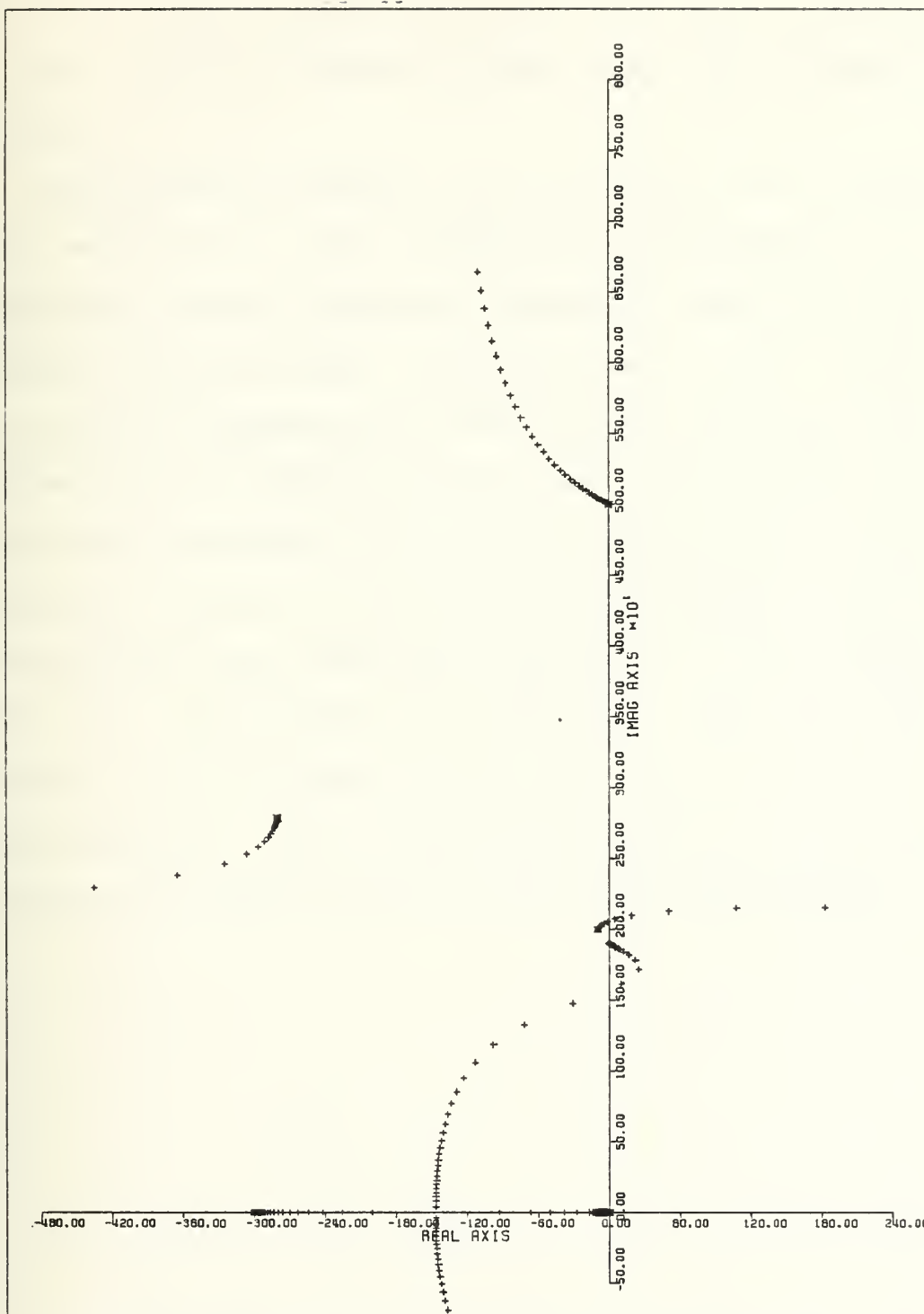


Figure 2.16 Root Locus of the System.



From the root locus of the system we observe that the system can have roots in the right half plane, meaning that for some specific values of gain ( $K$ ) the system becomes unstable.

Summarizing our results up to this point we can say that this method will work successfully and generally we can stabilize a system including mechanical resonances with the proposed filter putting imaginary zeros in the vicinity of the resonant frequency and specifically 100-150 Hz below the frequency of the system complex poles. The complex poles of the filter are placed a few hundred Hertz above the resonant frequency. Choice of the filter poles depends also on the frequencies of any additional resonances in the system, and trial and error methods are needed to find the best pole location. It is wise also to focus our attention on the system gain ( $K$ ) when we apply this method in order to avoid transition of other roots into the right half plane.





### III. COMPENSATION USING COMPLEX POLES

#### A. METHOD DESCRIPTION

When a system is unstable because of a resonance, the bode diagram shows that the peaking of the magnitude curve generates two additional gain crossover points, while the rapid decrease in phase due to the complex poles causes the phase curve to pass through an odd multiple of  $\pi$  (-180 or -540 degrees) at a frequency between these gain crossover points. Thus, at the higher frequency gain crossover there is a negative phase margin indicating that the system is unstable. In using an additional pair of complex poles to stabilize the system, we reshape the phase curve so that the phase crossover occurs at a frequency lower than that of either gain crossover due to the resonance. This eliminates the possibility of the negative phase margin (i.e. eliminates the encirclement of the Nyquist point). Note that in designing the compensator the reshaping of the gain curve (due to the compensator complex poles) must not generate any additional gain crossover points. This means that the damping ratio,  $\zeta$ , of the compensator poles must not be too small. Some compromise may therefore be expected, since we would prefer a very sharp decrease in phase which in turn requires small  $\zeta$ .

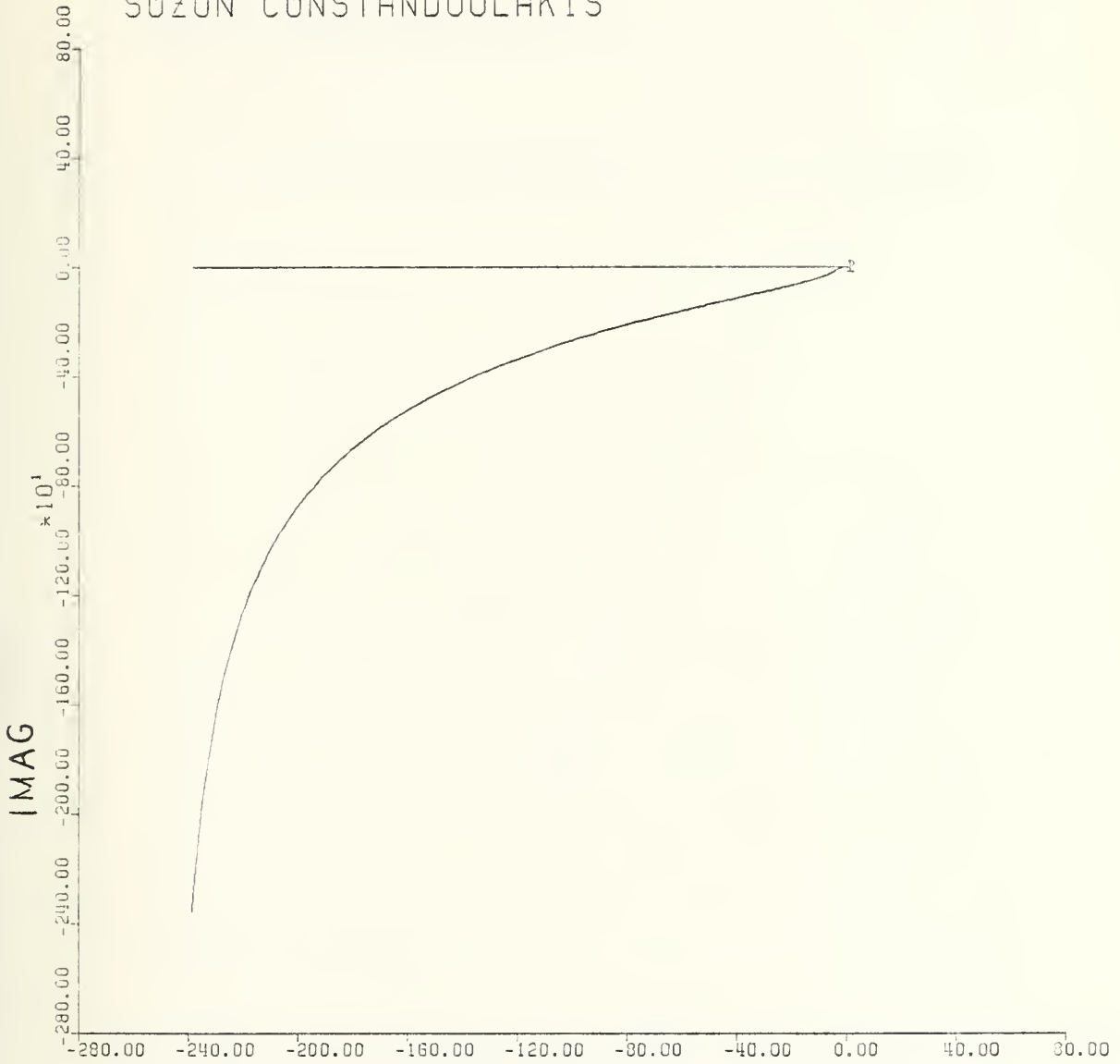


## B. METHOD APPLICATION

It is desirable to have the plots for the system in order to realise how the filter works. In the following Figures we have the Nyquist plots of the stable system without the resonances and with the resonances. Figure 3.1 shows the Nyquist plot of the system before the introduction of the resonances. This plot is in large scale in order to show all the values of the imaginary versus real coordinates, but it is not clear whether the  $-1$  point is inside or outside the curve. In Figure 3.2 we can see that the  $-1$  point is not included in the curve. In Figure 3.3 we can see the system after the introduction of resonances. It is easy to observe that the  $-1$  point is inside the curve which makes the system unstable.



NQUIST PLOT  
SOZON CONSTANDOULAKIS



XSCALE= 40.00 UNITS/INCH REAL RUN NO. 1  
YSCALE= 400.00 UNITS/INCH PLOT NO. 3

Figure 3.1 Stable System Nyquist plot (large scale).



NQUIST PLOT  
SOZON CONSTANDOULAKIS

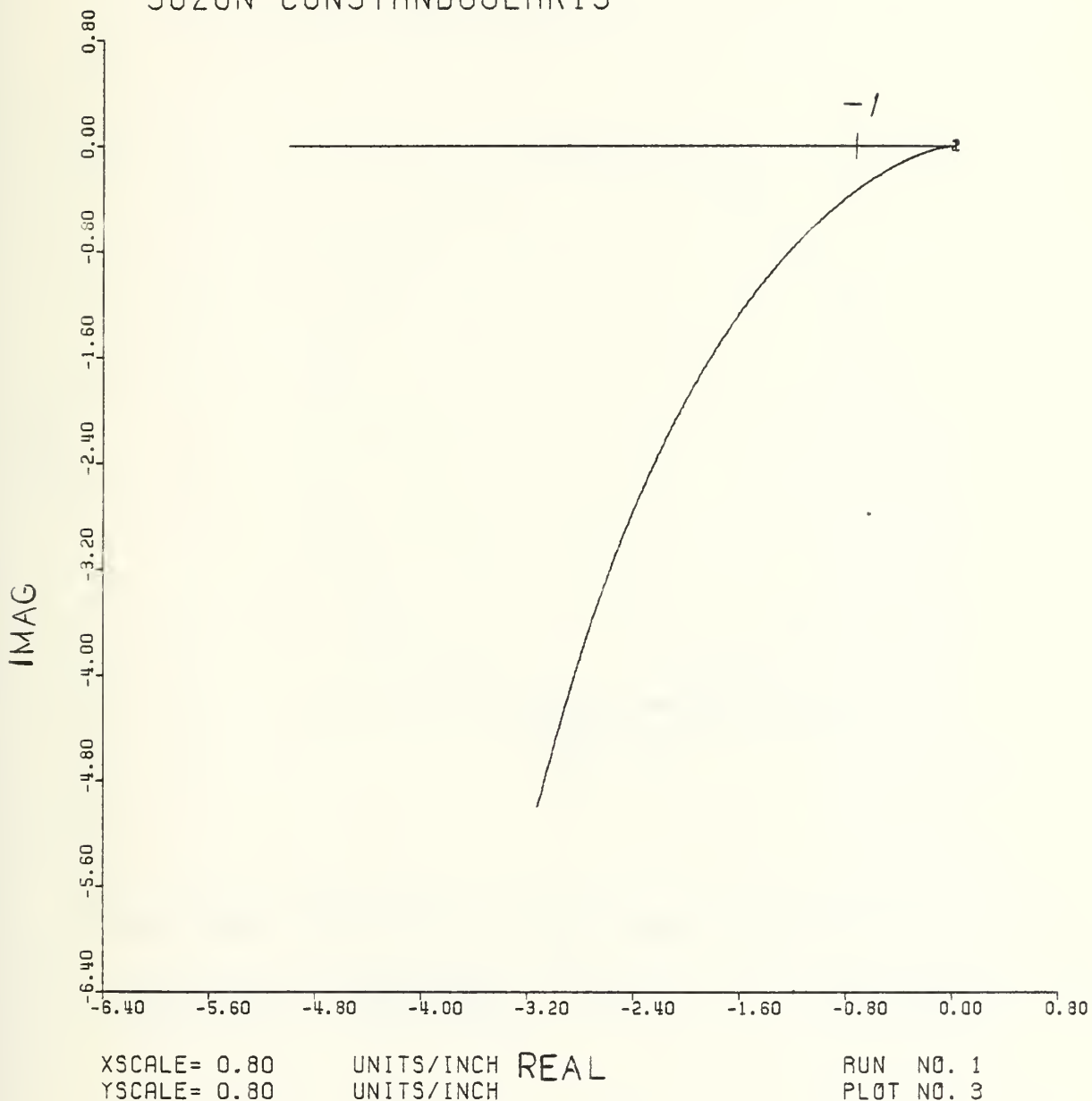


Figure 3.2 Stable System Nyquist plot (small scale).





NYQUIST PLOT  
UNSTABLE SYSTEM WITH RESONANCES

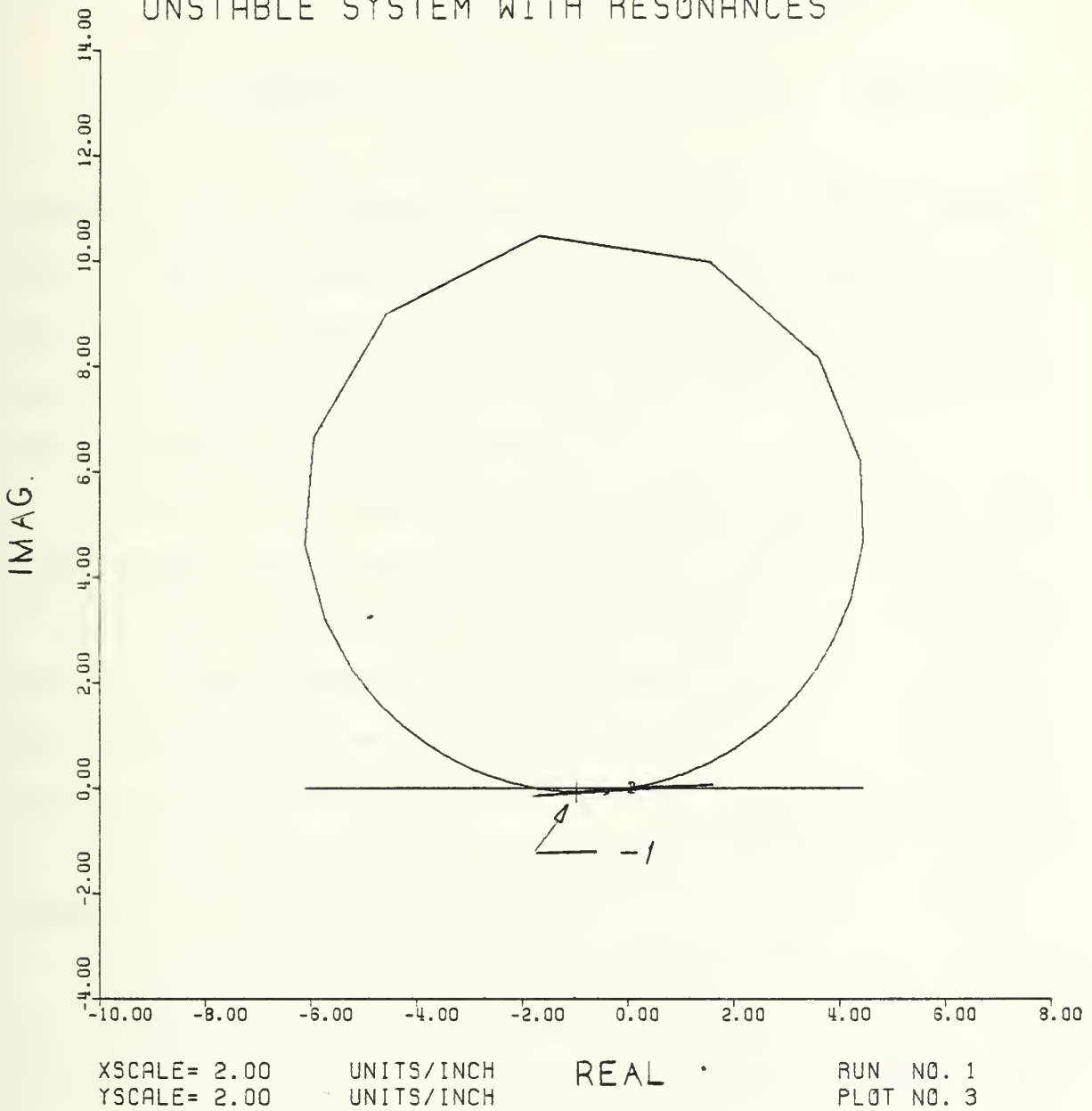


Figure 3.3 The System with the Resonance Peaks.



Now we will try to stabilise the system using the complex pole filter. The filter will have the following transfer function:

$$G(s) = \frac{1}{s^2 + 2\zeta\omega_n s + \omega_n^2} \quad (\text{Eqn 3.1})$$

Where we have to specify the pole location and the damping ratio. From the experience of the previous chapter we know that  $\zeta=0.1$  is a reasonable value which does not create additional peaks. So we will use this value and we will vary the pole location by trial and error.

Since we want the phase lag to be increased to more than  $-360$  degrees what we think first is to locate the poles in the vicinity of the first resonance and see if this choice stabilizes the system. If that happens we will change the pole location to lower and higher frequencies from the resonance peak in order to see how flexible the method is, since in the next chapter we are planning to compare the two methods.

As a first approach we will use pole frequency  $1950$  Hz i.e.  $50$  Hz lower than the resonance peak with  $\zeta=0.1$ . In the following Figures we can see the gain, the phase and the time response of the system using this compensator.



GAIN CURVE  
COMPENSATION WITH COMPLEX POLES

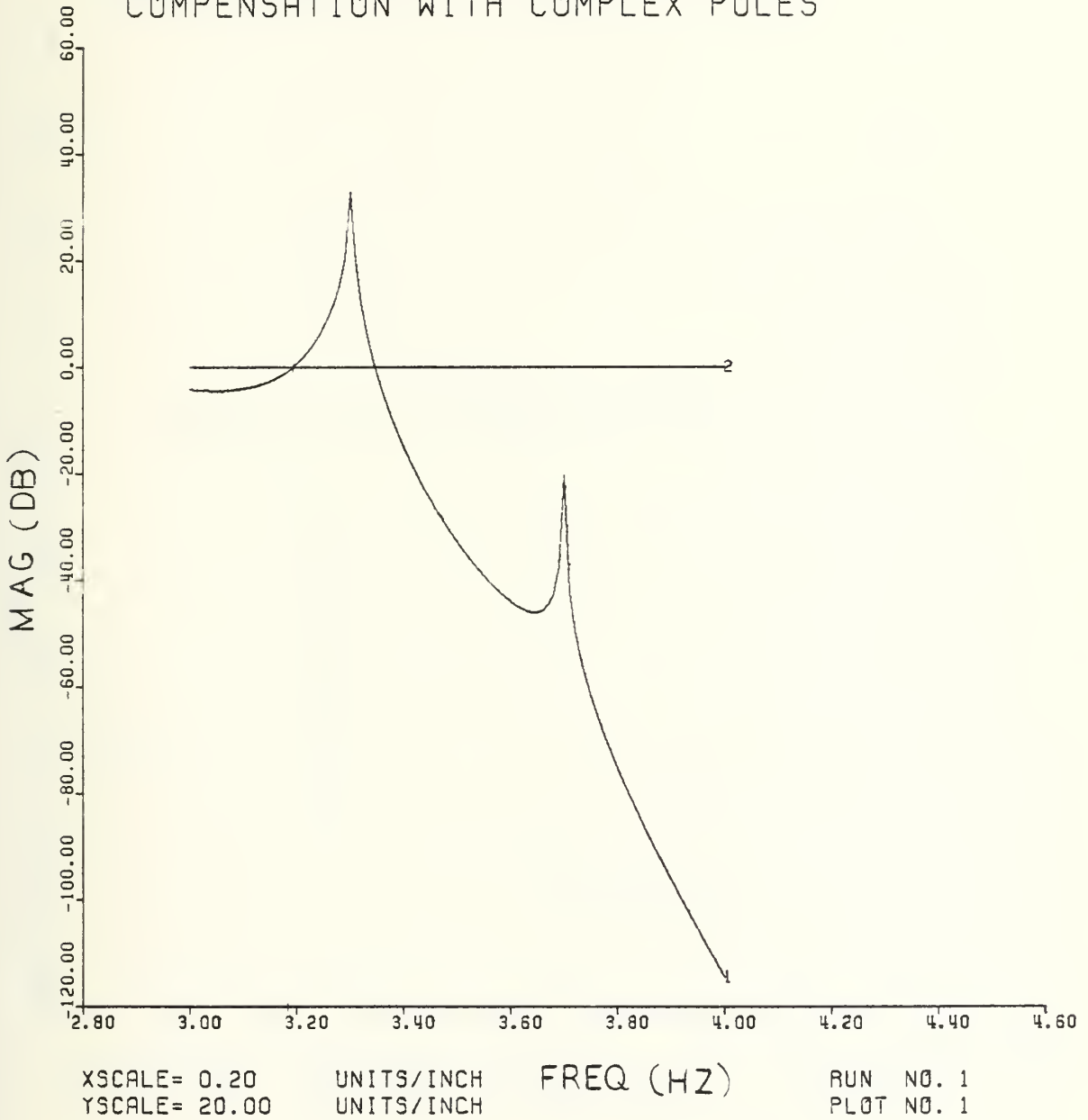


Figure 3.4 Gain Curve of the System with Poles at 1950 Hz.



PHASE CURVE  
COMPENSATION WITH COMPLEX POLES

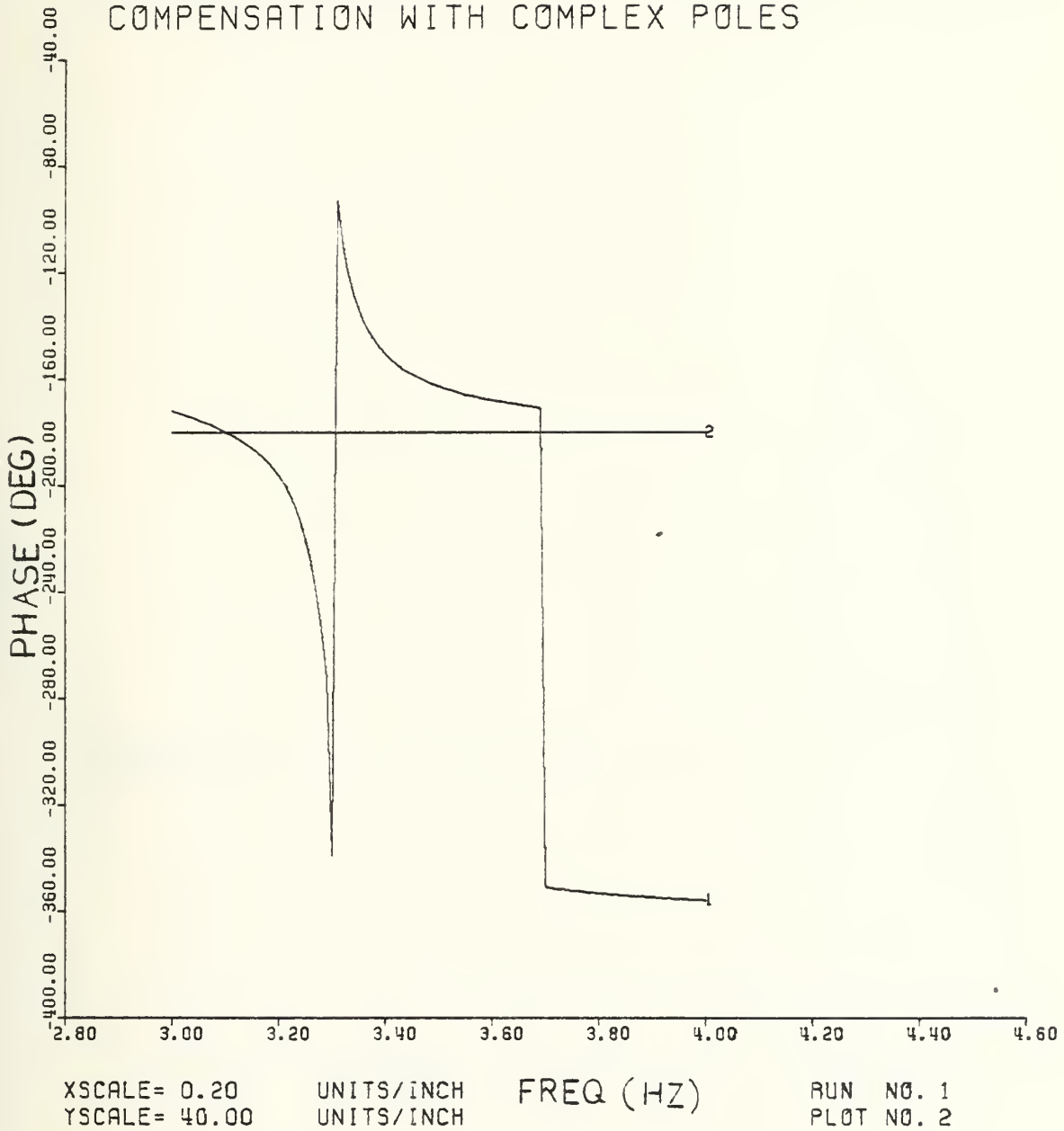
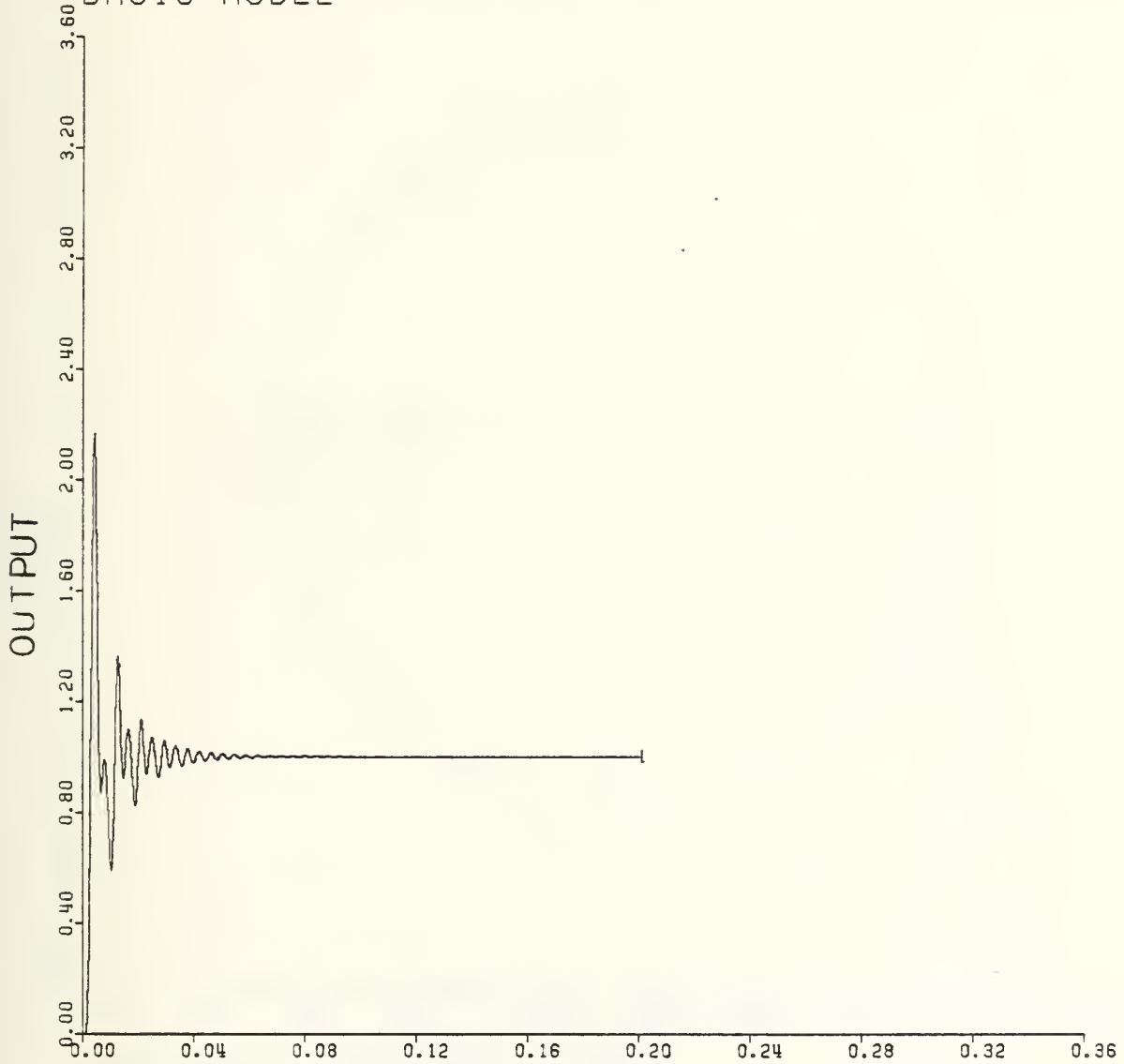


Figure 3.5 Phase Curve of the System with Poles at 1950 Hz.





TRANSIENT RESPONSE OF THE COMPENSATED SYSTEM  
BASIC MODEL

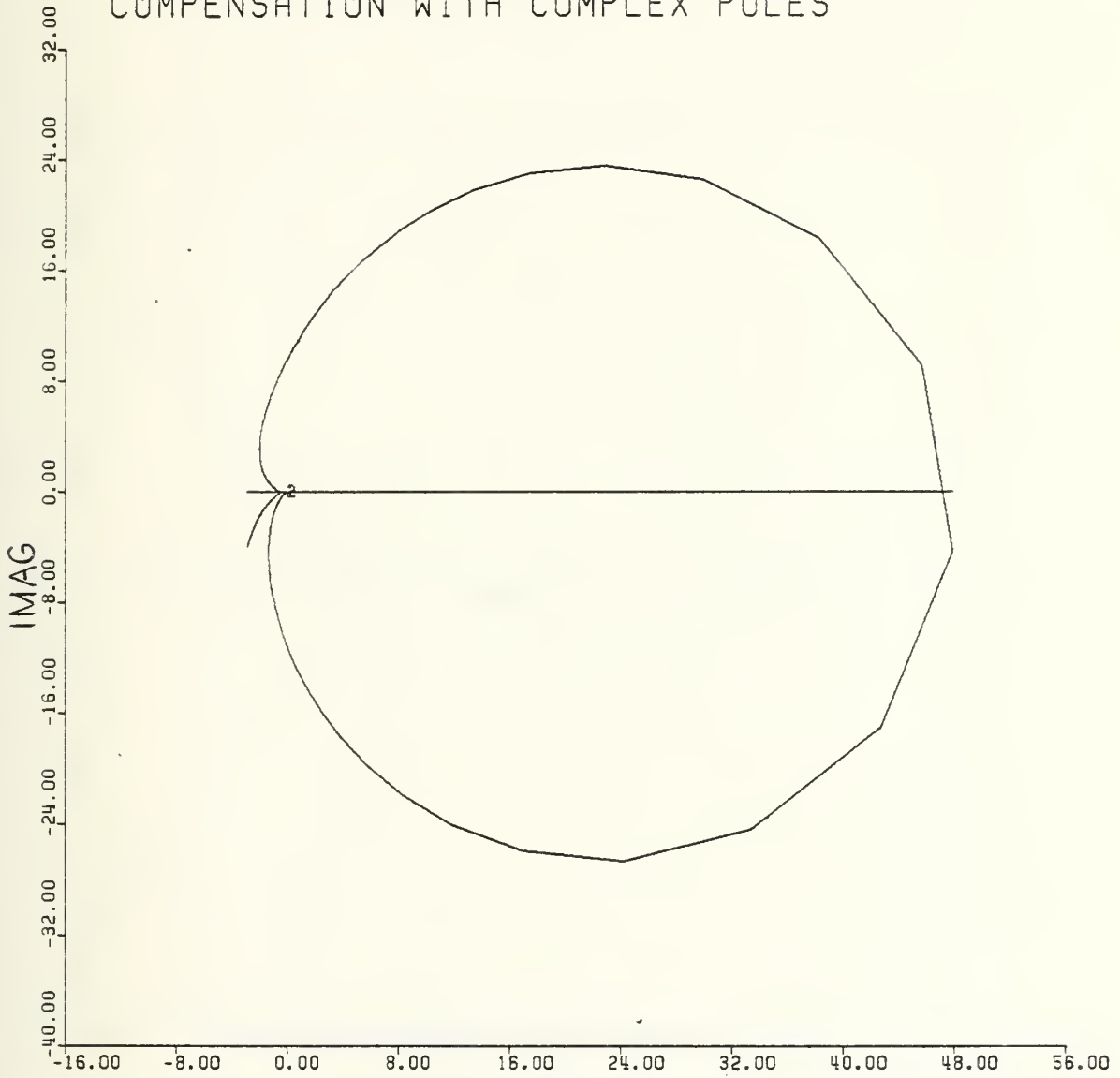


XSCALE= 0.04      UNITS/INCH      TIME (SEC)      RUN NO. 1  
YSCALE= 0.40      UNITS/INCH      PLOT NO. 1

Figure 3.6 Time Response of the System with Poles at 1950 Hz.



NYQUIST PLOT  
COMPENSATION WITH COMPLEX POLES



XSCALE= 8.00  
YSCALE= 8.00

UNITS/INCH  
UNITS/INCH

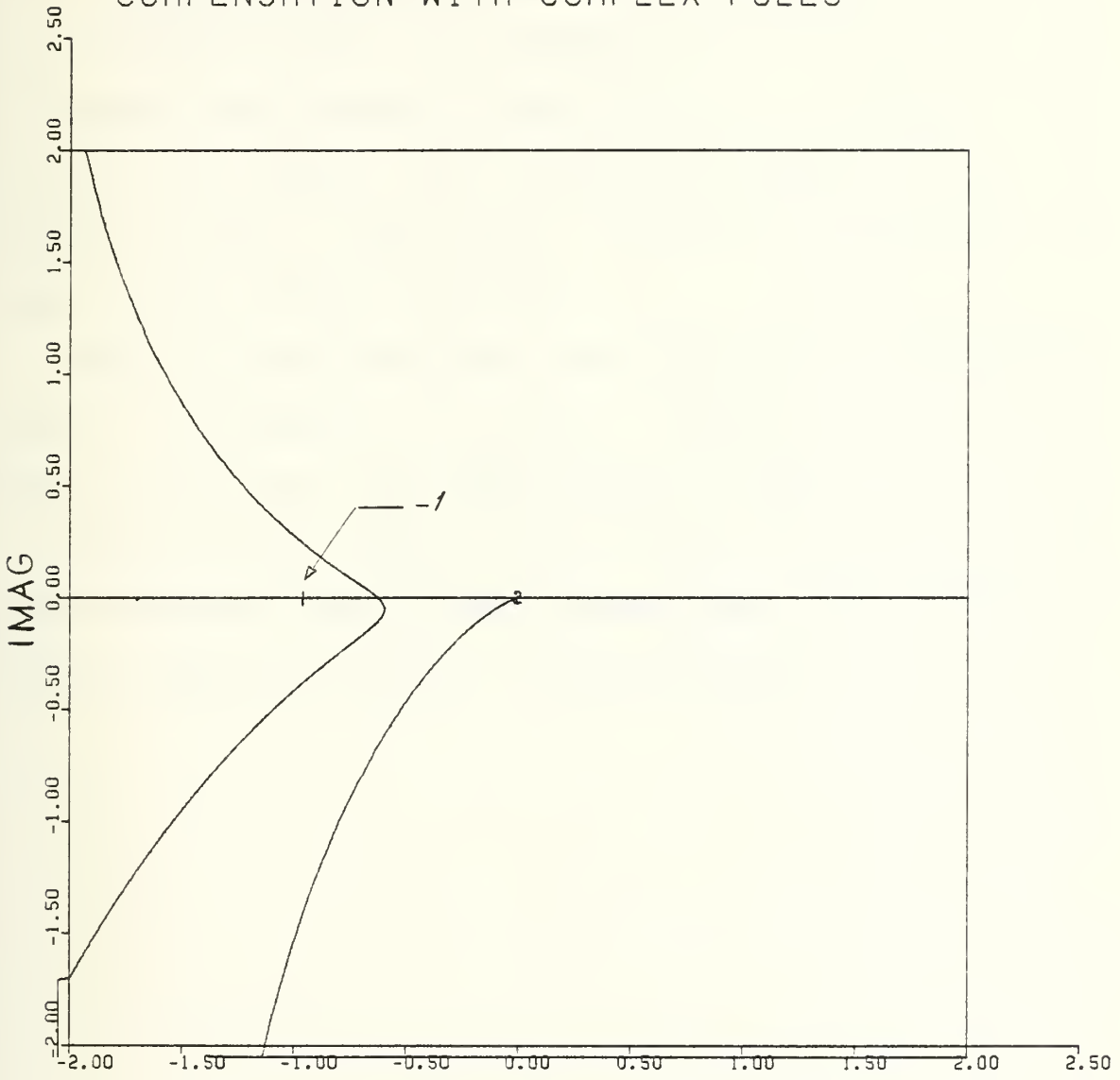
REAL

RUN NO. 1  
PLOT NO. 3

Figure 3.7 Nyquist Plot of the System with Poles at 1950 Hz.



NYQUIST PLOT  
COMPENSATION WITH COMPLEX POLES



XSCALE= 0.50  
YSCALE= 0.50

UNITS/INCH  
UNITS/INCH

REAL

RUN NO. 1  
PLOT NO. 3

Figure 3.8 Nyquist Plot Magnified at the -1 point.



Observing Figures 3.4 through 3.3 we see that the filter has worked as expected. The phase lag has exceeded  $-360$  degrees. From the time response curve of Figure 3.6 we can see a small oscillation for about  $0.06$  seconds and then the system becomes stable. The Nyquist plots of Figures 3.7 and 3.8 verify the stability of the system too. In Figure 3.8 we can see the Nyquist plot magnified specifically at the vicinity of the  $-1$  point and it is easy to observe that the point is lied outside the curve.

After we have proved that the method has worked sucessfully it is necessary to find the limits at the frequency axis we can locate the complex poles in order for the system to become stable. First we will remove the poles to higher frequencies and then to lower frequencies. In the following Figures appear the results of that research.





GAIN CURVE  
COMPENSATION WITH COMPLEX POLES

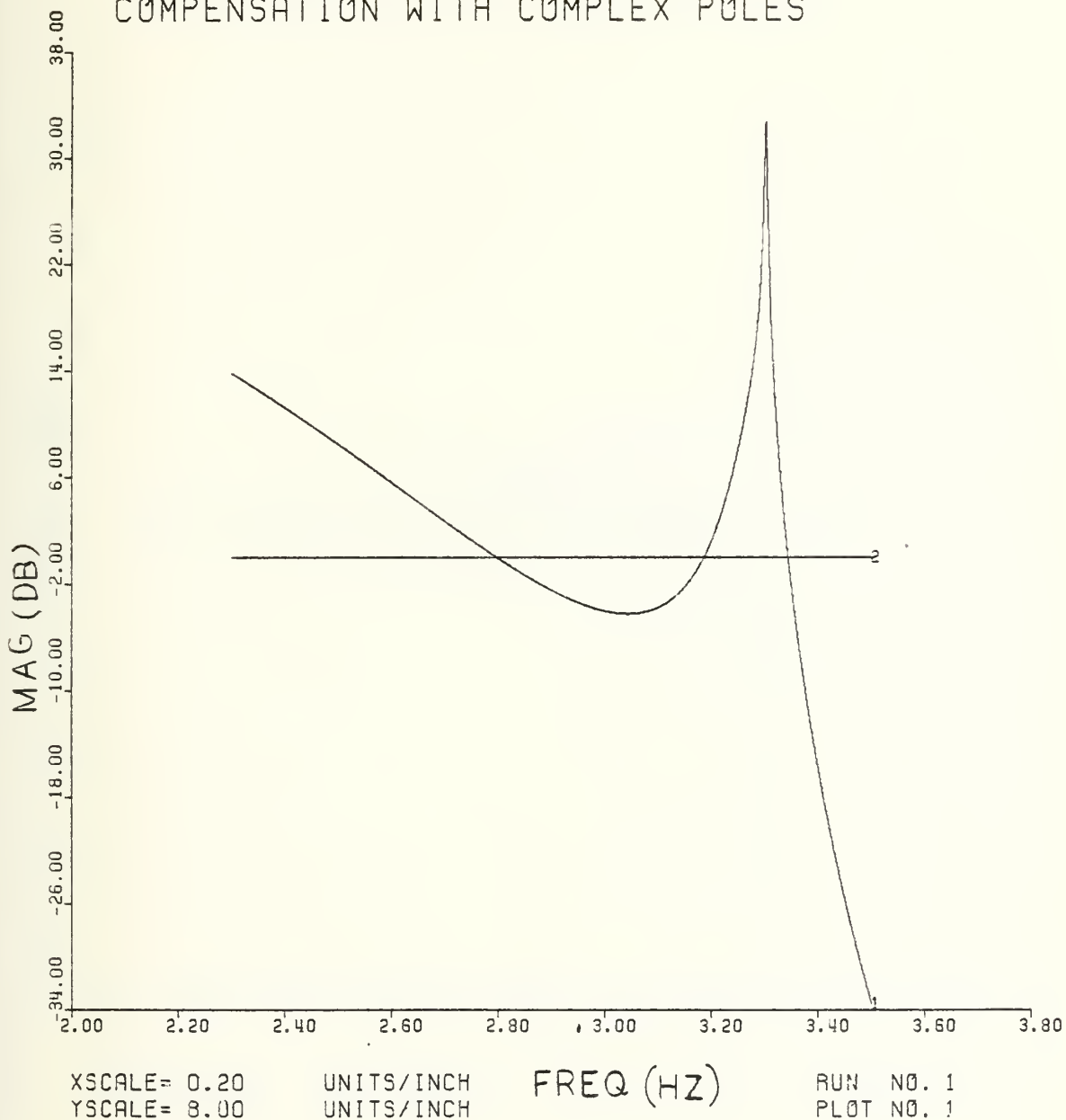


Figure 3.9 Gain Curve of the System with Poles at 1900 Hz.



PHASE CURVE  
COMPENSATION WITH COMPLEX POLES

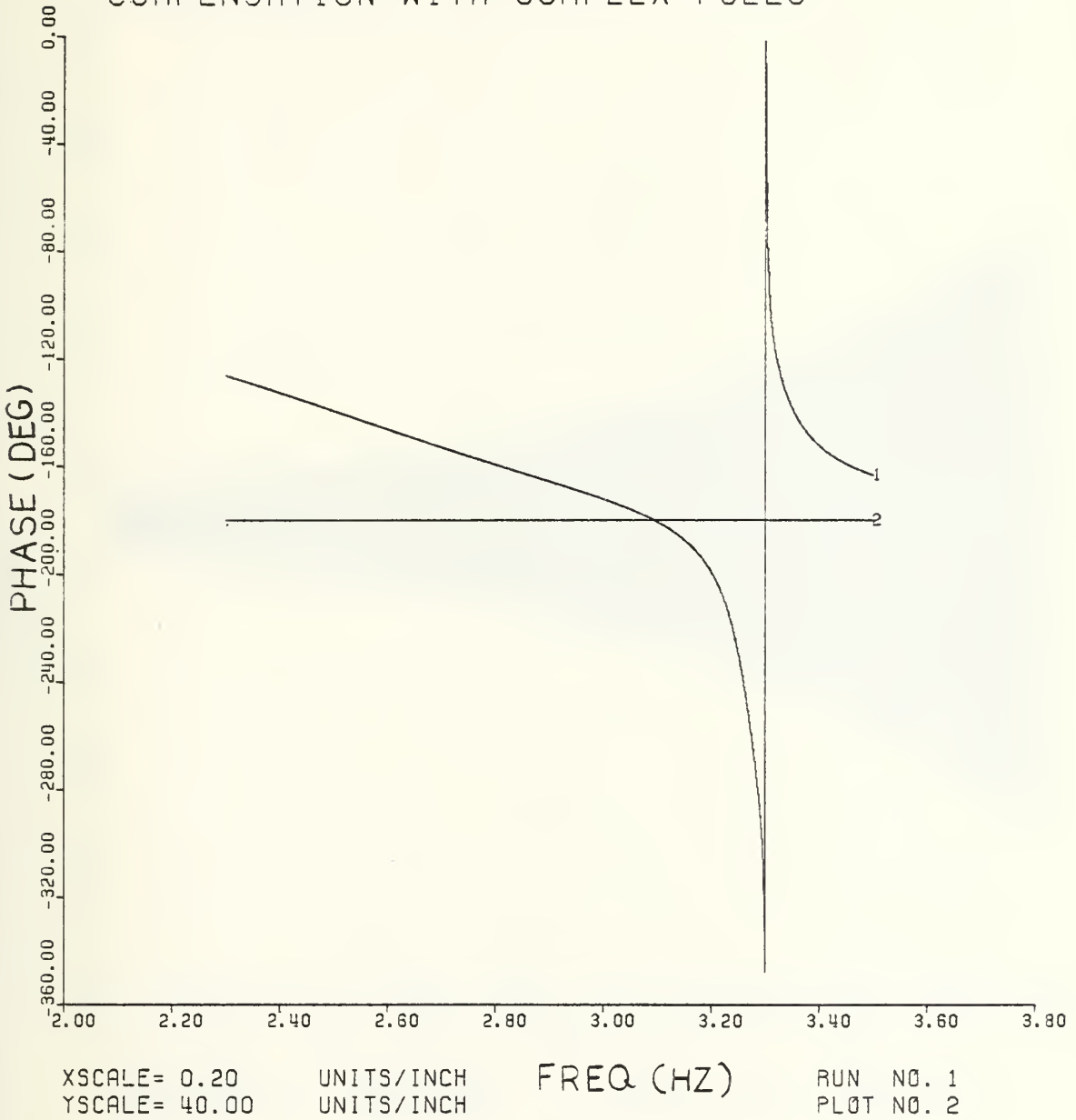


Figure 3.10 Phase Curve of the System with Poles at 1900 Hz.



TRANSIENT RESPONSE OF THE COMPENSATED SYSTEM  
BASIC MODEL

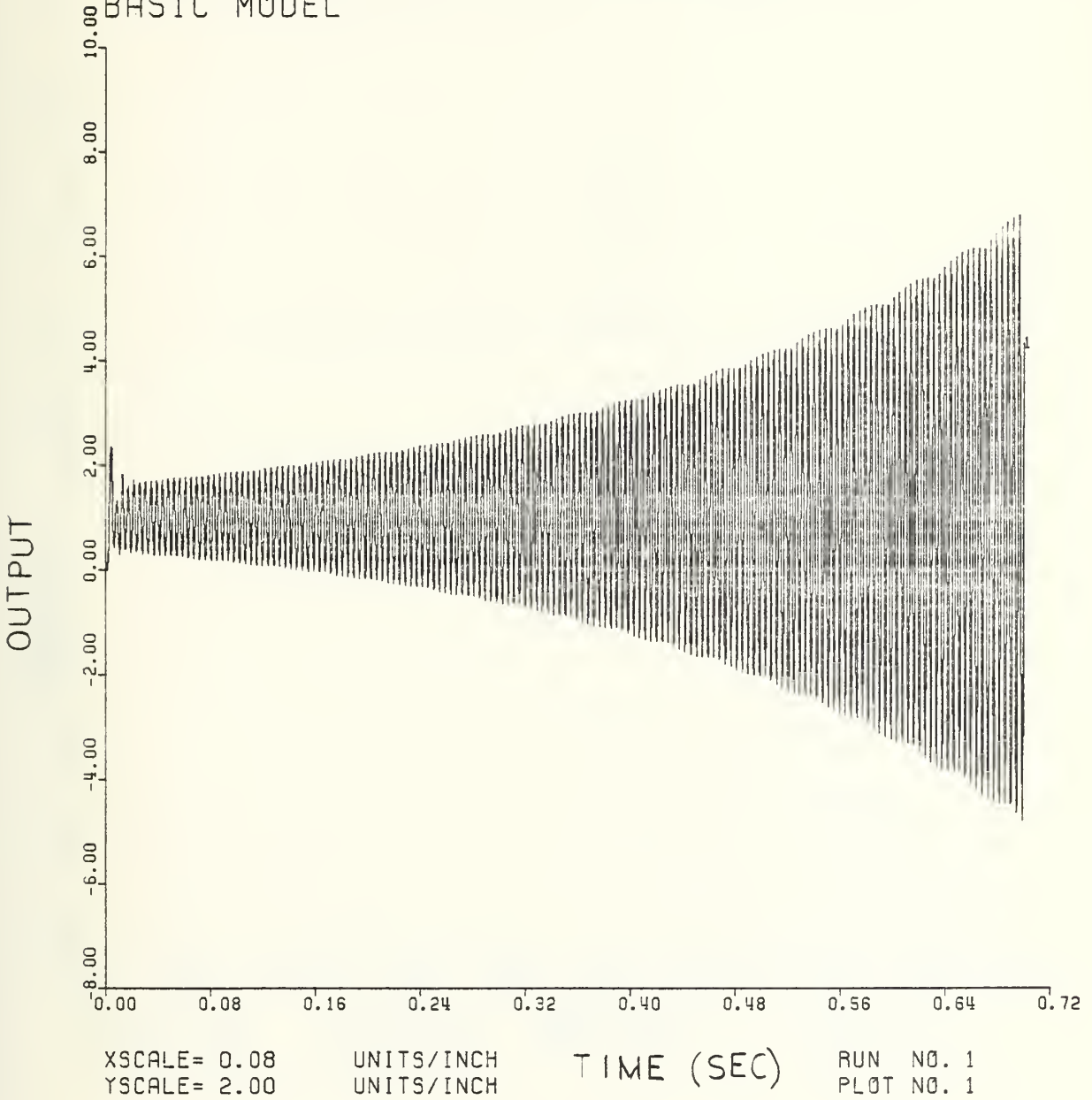


Figure 3.11 Time Response of the System with Poles at 1900 Hz.



GAIN CURVE  
COMPENSATION WITH COMPLEX POLES

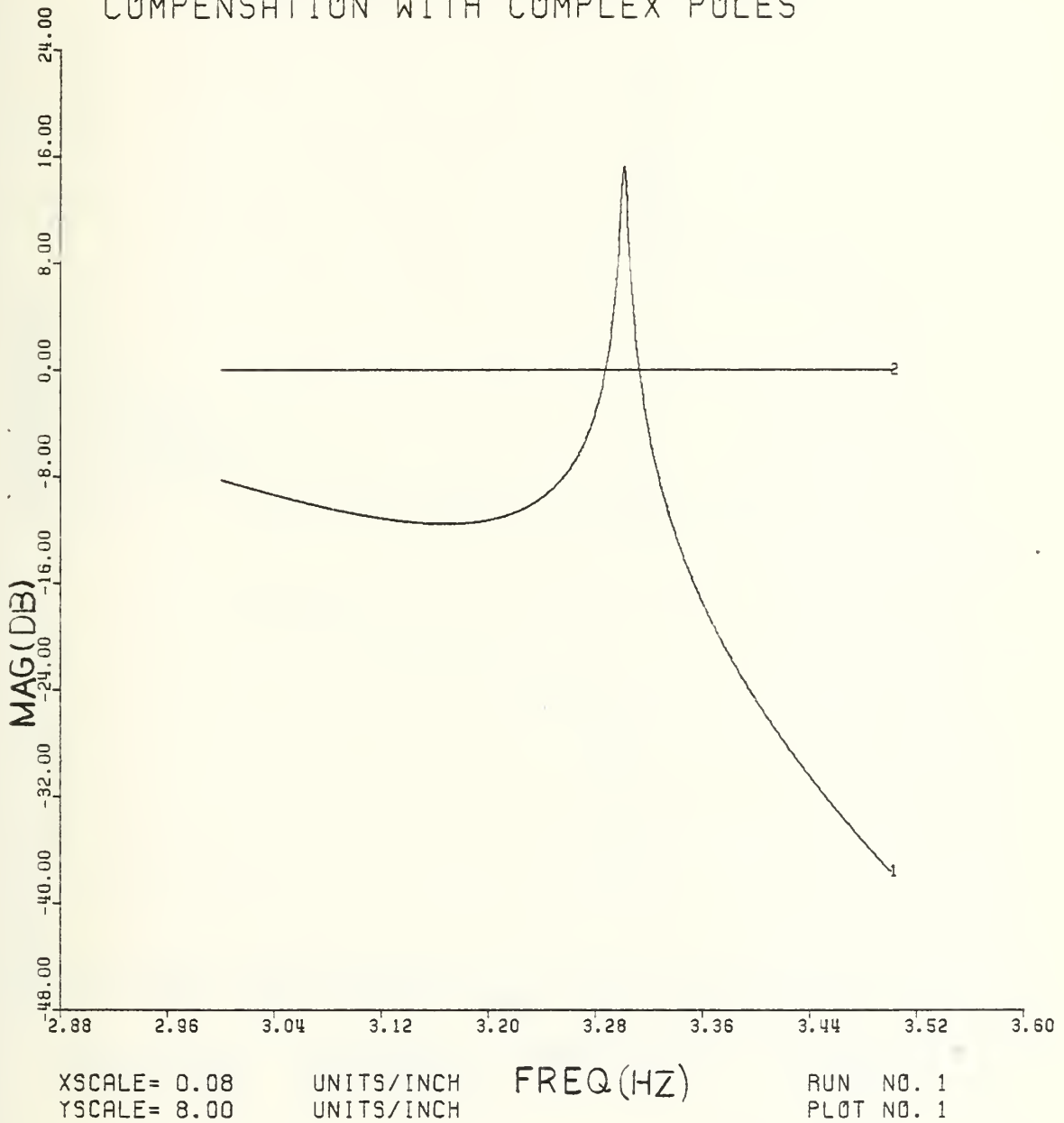
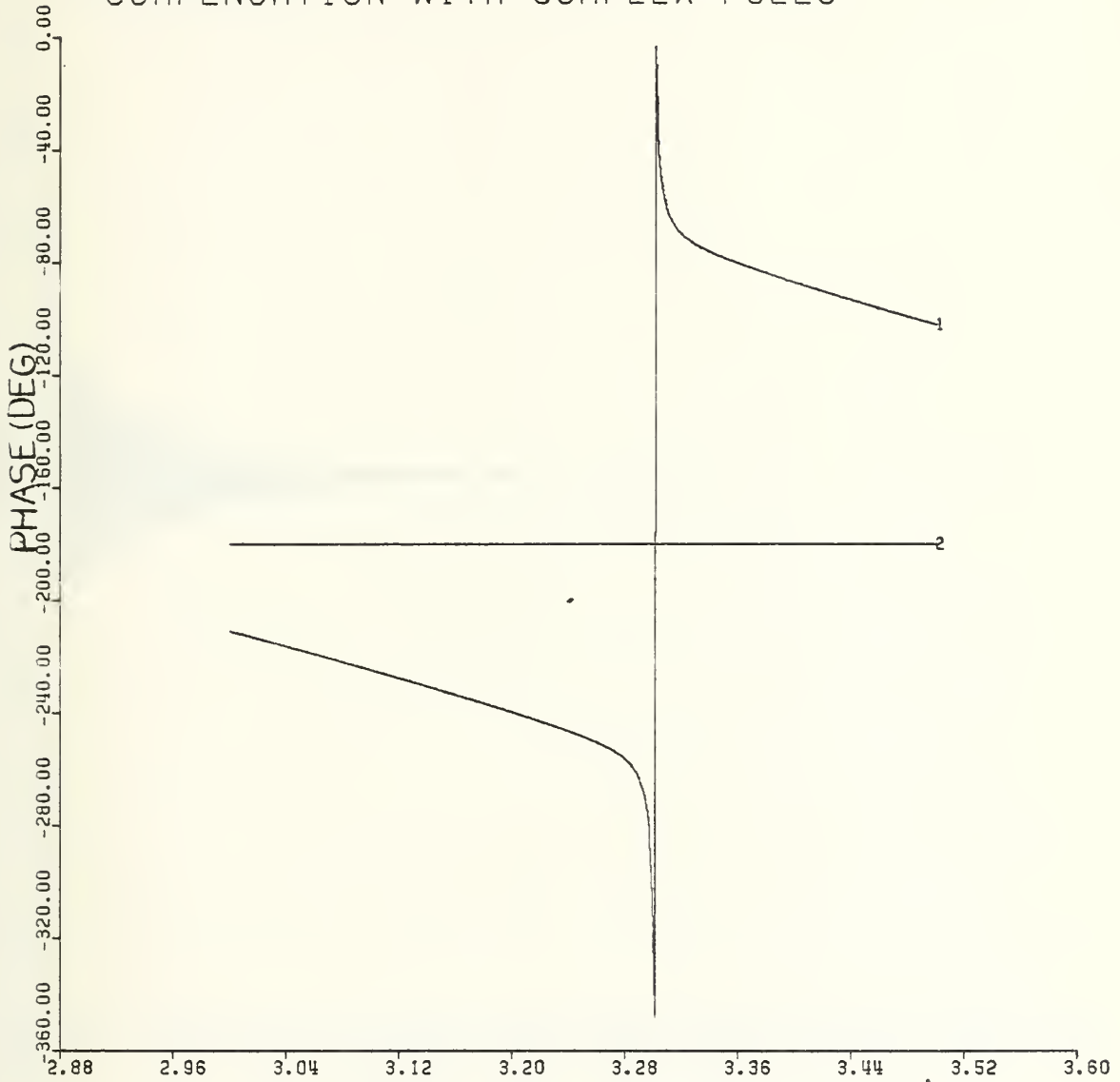


Figure 3.12 Gain Curve of the System with Poles at 2300 Hz.





PHASE CURVE  
COMPENSATION WITH COMPLEX POLES

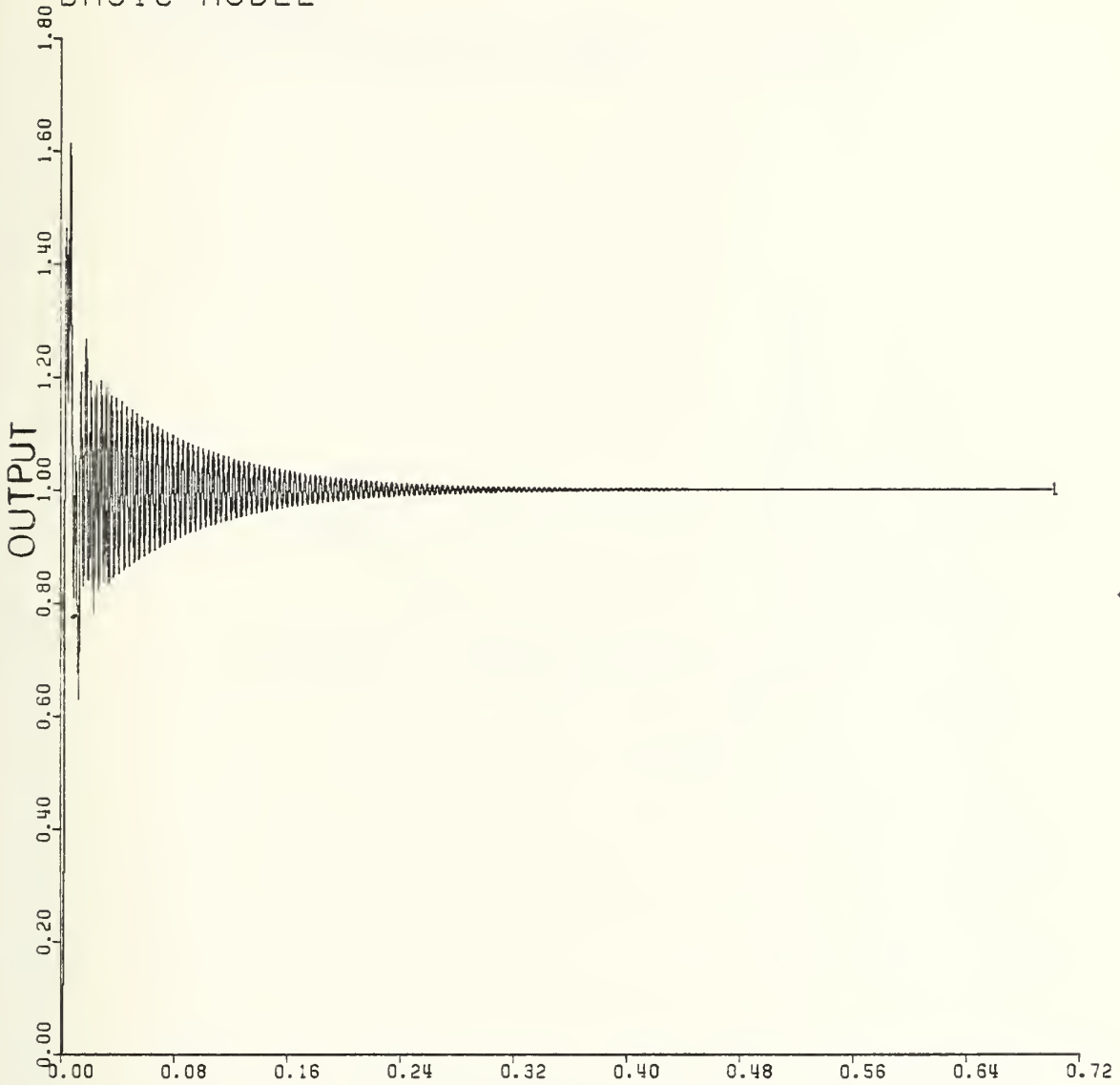


XSCALE= 0.08      UNITS/INCH      FREQ (HZ)      RUN NO. 1  
YSCALE= 40.00    UNITS/INCH                              PLOT NO. 2

Figure 3.13 Phase Curve of the System with Poles at 2300 Hz.



TRANSIENT RESPONSE OF THE COMPENSATED SYSTEM  
BASIC MODEL



XSCALE= 0.08  
YSCALE= 0.20

UNITS/INCH    TIME (SEC)  
UNITS/INCH

RUN NO. 1  
PLOT NO. 1

Figure 3.14    Time Response of the System with Poles at 2300 Hz.



GAIN CURVE  
COMPENSATION WITH COMPLEX POLES

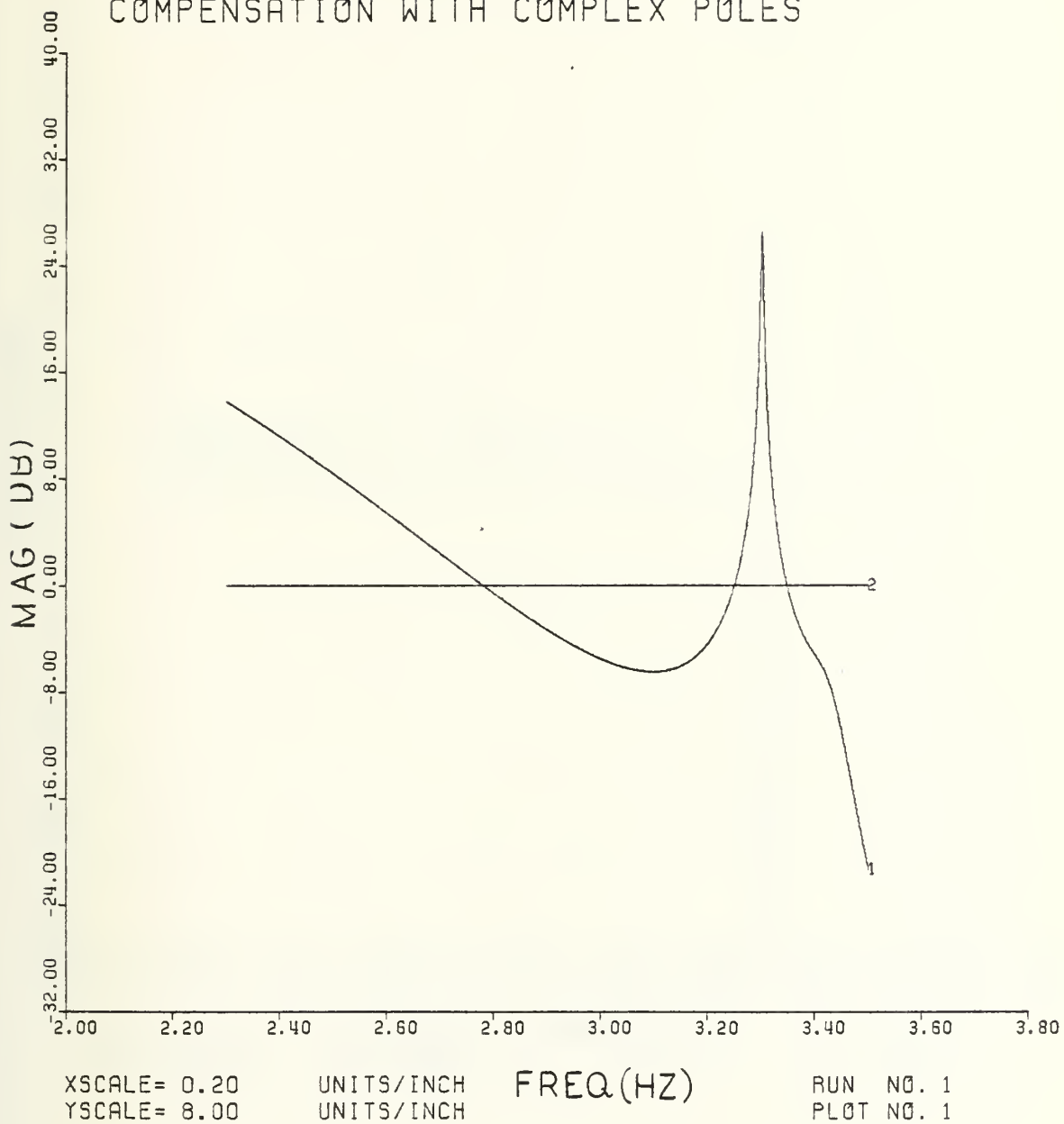


Figure 3.15 Gain Curve of the System with Poles at 2700 Hz.



PHASE CURVE  
COMPENSATION WITH COMPLEX POLES

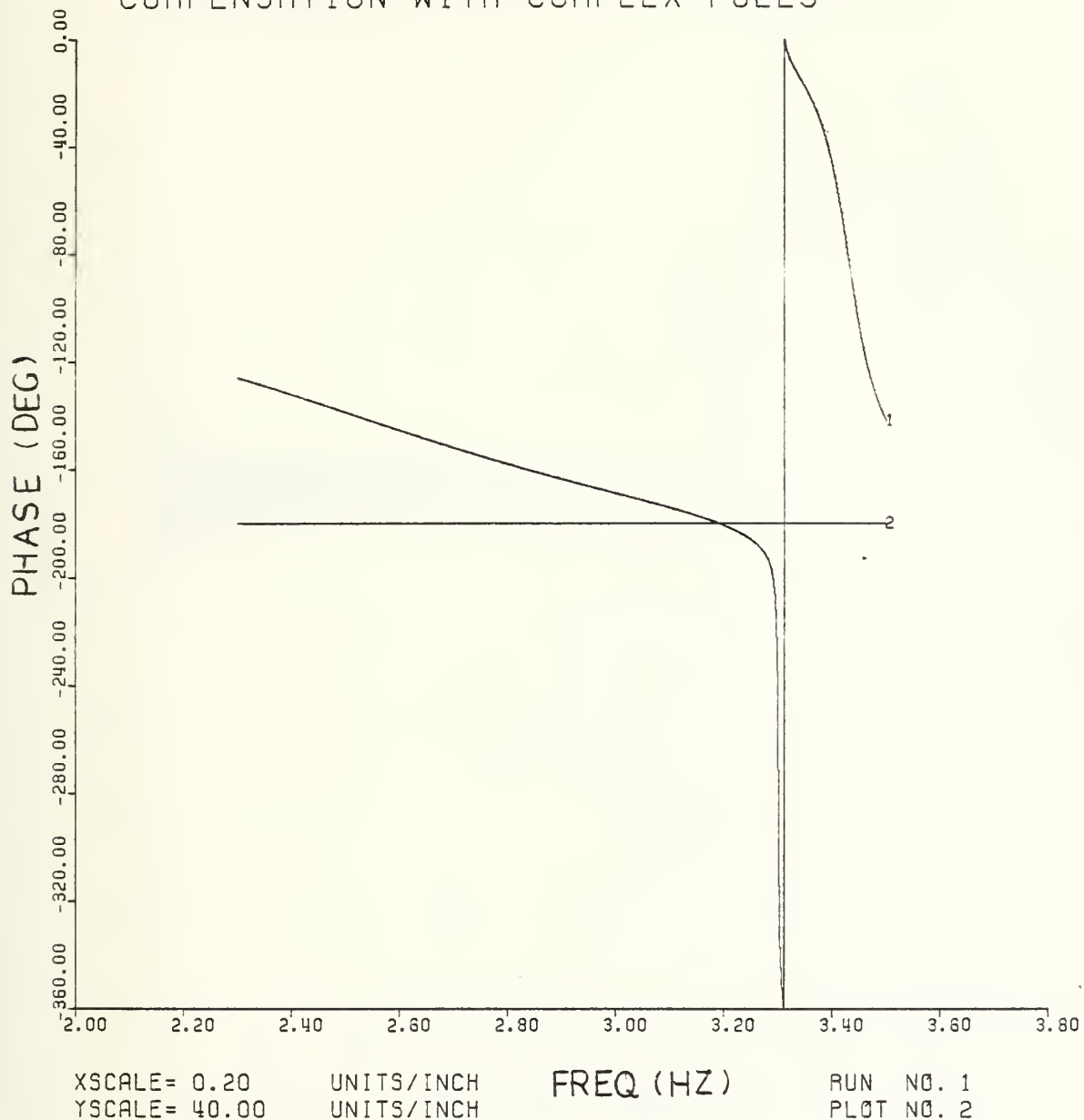


Figure 3.16 Phase Curve of the System with Poles at 2700 Hz.





TRANSIENT RESPONSE OF THE COMPENSATED SYSTEM  
BASIC MODEL

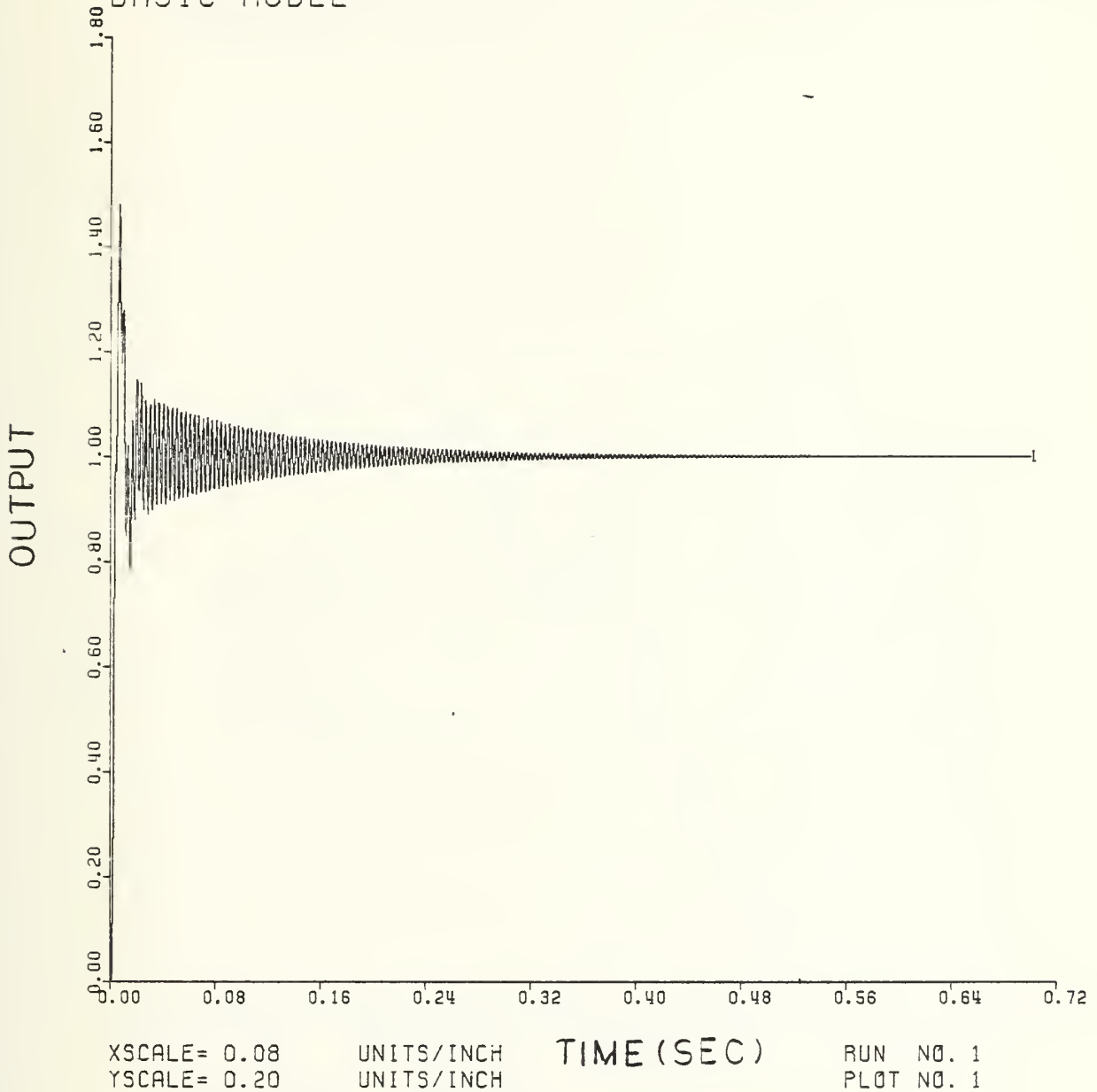


Figure 3.17 Time Response of the System with Poles at 2700 Hz.







PHASE CURVE  
COMPENSATION WITH COMPLEX POLES

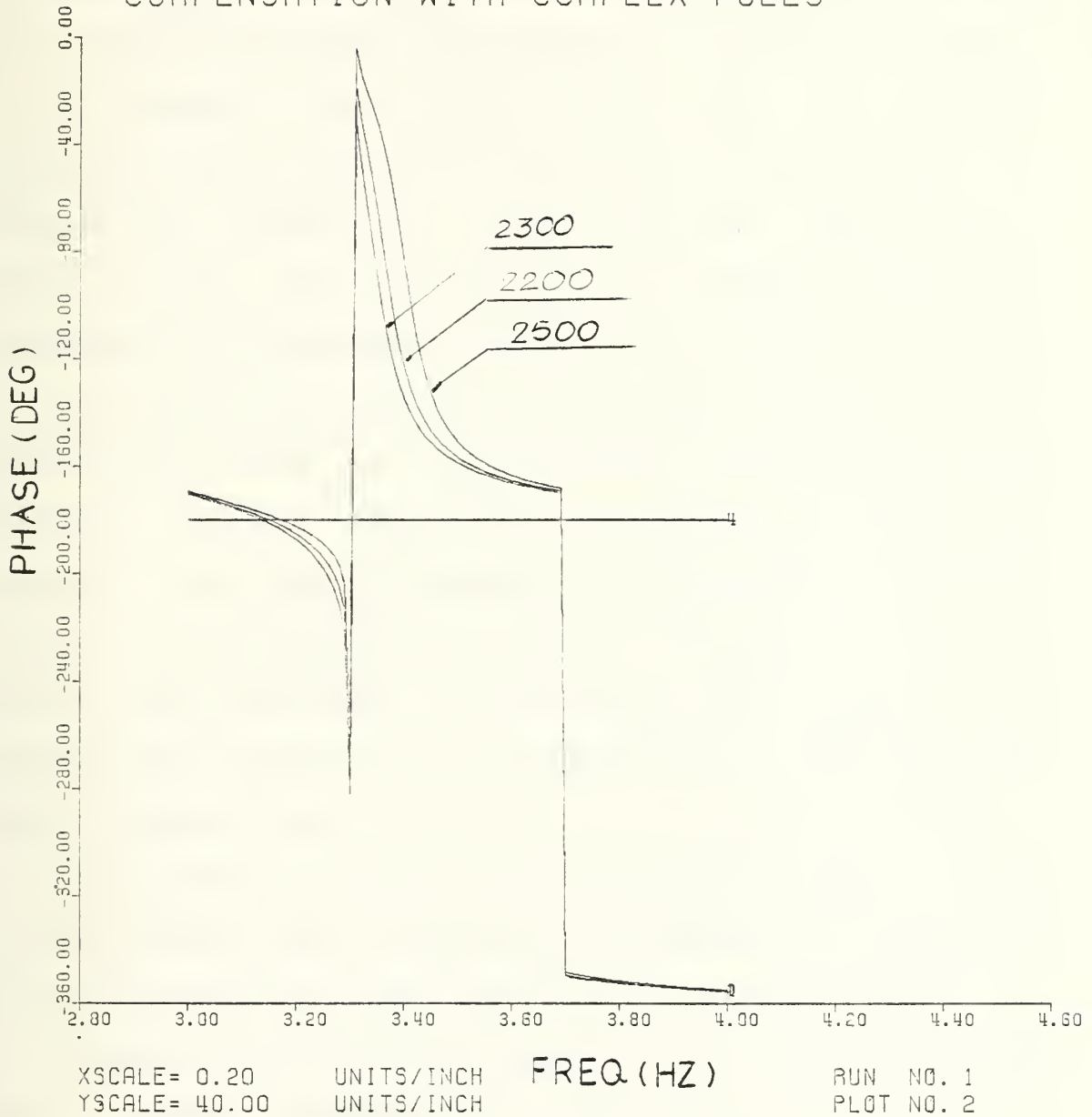


Figure 3.19 Phase Curves for Various pole Frequencies.



In Figures 3.9 through 3.11 we can see what happens if we reduce the pole location 50 Hz. So we put the poles at 1900 Hz and what we see is that the system becomes unstable as shown in Figure 3.11, the time response of the system. That indicates that the lower limit of the pole location is at a frequency about 100 Hz lower than the resonant frequency. Now we try to find the upper frequency limit. In Figures 3.12 through 3.14 we can see that the system is stable as we move the poles to higher frequencies. Specifically if we observe Figure 3.13 we see that there is a longer oscillation period (0.24 seconds) compared with the oscillation period of the stabilised system with poles located at 1950 Hz which was 0.09 seconds. This is one indication that when we increase the frequency of the poles we have a larger oscillation time. In Figures 3.15 through 3.17 we place the poles at 2700 Hz and the oscillation time has further increased to 0.53 seconds which certifies again what we claimed above.

In the last two Figures we can see the change in shape of the gain and phase graphs as we increase the frequency of the pole location to 2200, 2300, 2500 Hz, respectively.

Closing this chapter we conclude that this method worked for our 'Model'. Specifically we can say that when we locate the poles of the filter we must take into account the oscillation period which is very important for a system. The range of frequency with which we can locate the poles is





large enough. The lower limit is about 50 to 100 Hz lower than the resonant frequency. The upper limit is larger depending always on the location of the next peak and the permissible oscillation period for the system.



## IV. CONCLUSIONS AND RECOMMENDATIONS

### A. CONCLUSIONS

Two methods of compensation for instability due to resonances have been studied. We can conclude that both were successful and both can be used to compensate systems including mechanical resonances. In the first method, using pure imaginary zeros and complex poles we need to locate the zeros at a frequency 100-150 Hz lower than the resonance frequency of the system, and the complex poles a few hundred Hz higher. This method has the disadvantage that it gives an oscillation period that is undesirably long and additionally there are some gain values that which make the system unstable. The second method using only complex poles appears more successful because we have a larger range of permissible pole location. The oscillation period can be significantly reduced if we build the filter poles with frequency near the resonant frequency.

### B. RECOMMENDATIONS

In this study we used only a 'Model' built for this problem. Before we conclude that the methods are useful in applications it is wise to try to apply both methods to more



examples and specifically to examples with data collected from the real world. Also it will be wise to study the effect of tolerances.



## APPENDIX A

### COMPUTER PROGRAMS

This appendix is composed of listings of the various DSL/360 and CSMP III programs used in the computer simulations throughout this study.

Specifically on page 73 is the program used to obtain the frequency response of the 'Model' without the resonances. On page 74 is the program used to obtain the frequency response of the 'Model' with the resonances. On page 75 is the program used to obtain the frequency response of the compensated system using complex poles and imaginary zeros. On page 76 is the program used to obtain the time response of the compensated system using complex poles and imaginary zeros. On page 77 is the program used to obtain the frequency response of the compensated system using only complex poles. On page 78 is the program used to obtain the time response of the compensated system using only complex poles. To obtain the root locus of page 43 we used the subroutine ROOTLO on the IBM 3033 of the Naval Postgraduate School. On page 79 is the program used to obtain Figures 3.18 and 3.19.





```

TITLE FREQUENCY RESPONSE
TITLE MODEL WITHOUT RESONANCES
O COMPLEX S,N,D,G
D COMMON/CAREA/S,N,C,G
INTEGER NPLCT
CONST NPLCT=1
CONTRL FINTIM=4.0, CELT=C.COC6,DELS=0.0025
PRINT 4.0F-3,W,PHI,RE,IM,MDB,MAG
DYNAMIC
RW=RAMP(0.0)
LCGW=RW
W=10.**LCGW
S=CMPLX(0.,W)
D=S*(0.2*S+1.C)*(0.003333*S+1.0)
N=2400*(C.1*S+1.0)
G=N/D
RE=REAL(G)
IM=AIMAG(G)
PHI=57.3*ATAN2(IM,RE)
IF(PHI.GT.0.) PHI=PHI-360
MAG=CABS(G)
MDB=20.*ALOG10(MAG)
GREAL=LIMIT(-5., 5., RE)
GIMAG=LIMIT(-5., 5., IM)
PHI180=PHI+180
SAMPLE
CALL DRWG(1,1,LOGW,MDB)
CALL DRWG(2,1,LOGW,PHI)
CALL DRWG(1,2,LOGW,0.0)
CALL DRWG(2,2,LOGW,-180.)
CALL DRWG(3,1,GREAL,GIMAG)
CALL DRWG(4,1,PHI,MDB)
CALL DRWG(3,2,GREAL,C.0)
CALL DRWG(4,2,LOGW,0.0)
TERMINAL
CALL ENDRW(NPLCT)
END
STOP

```



```

TITLE FREQUENCY RESPONSE
TITLE BODE
D COMPLEX S,N,D,G,C1,D2
D COMMON/CAREA/S,N,C,G,D1,D2
INTGER NPLOT
CONST NPLOT=1
CONTRL FINITIM=1.C,DELT=C.0005,DELS=0.0005
PRINT 4.0E-3,W,PHI,RE,IM,MDB,MAG
DYNAMIC

```

```

*-----*
* THE SYSTEM WITH THE RESONANCE PEAKS. *
*-----*

```

```

RW=RAMP(C.C)
LOGW=3.C+RW
W=10.**LOGW
S=CMPLX(0.,W)
D1=S*(C.2*S+1.C)*(0.003333*S+1.C)
D2=(S*(0.25E-6*S+C.5E-5)+1.0)*(S*(0.4E-7*S+0.632E,....
-8)+1.0)
N=2400*(C.1*S+1.0)
D=D1*D2
G=N/D
RE=REAL(G)
IM=AIMAG(G)
PHI=57.3*ATAN2(IM,RE)
IF(PHI.GT.C.) PHI=PHI-36C
MAG=CABS(G)
MDB=20.*ALCG1C(MAG)
GREAL=LIMIT(-5., 5., RE)
GIMAG=LIMIT(-5., 5., IM)
PHI18C=PHI+18C

```

```

SAMPLE
CALL DRWG(1,1,LOGW,MDB)
CALL DRWG(2,1,LOGW,PHI)
CALL DRWG(1,2,LOGW,0.C)
CALL DRWG(2,2,LOGW,-180.)
CALL DRWG(3,1,RE,IM)
CALL DRWG(4,1,PHI,MDB)
CALL DRWG(3,2,RE,0.C)
CALL DRWG(4,2,LOGW,0.0)

```

```

TERMINAL
CALL ENDRW(NPLOT)

```

```

END
STOP

```



```

TITLE FREQUENCY RESPONSE
TITLE BODE
D COMPLEX S,N,D,G,D1,D2,D3
D COMMON/CAREA/S,N,C,G,D1,D2,D3
INTGER NPLCT
CONST NPLCT=1
CONTRL FINTIM=1.0, DELT=1.0E-2, DELS=1.0E-2
PRINT 4.0E-3, W, PHI, RE, IM, MDB, MAG
DYNAMIC
  RW=RAMP(C.0)
  LOGW=3.0+RW
  W=10.**LOGW
  S=CMPLX(C.,W)
  D1=S*(C.2*S+1.0)*(0.003333*S+1.0)
  D2=(S*(C.4E-7*S+0.6324E-8)+1.0)*(S*(0.25E-6*S+
  C.5E-5)+1.0)
*-----*
* COMPLEX POLES AT FREQUENCY 2800 HZ WITH D.RATIO 0.1 *
*-----*
  D3=(S*(127.551E-9*S+71.428E-6)+1.0)
  D=D1*D2*D3
*-----*
* IMAGINARY ZERCS AT FREQUENCY 1900 HZ *
*-----*
  N=2300.0*(0.1*S+1.0)*(S*(277.008E-9*S+0.0)+1.0)
  G=N/D
  RE=REAL(G)
  IM=AIMAG(G)
  PHI=57.3*ATAN2(IM,RE)
  IF(PHI.GT.C.) PHI=PHI-360
  MAG=CABS(G)
  MDB=20.*ALCG10(MAG)
  GREAL=LIMIT(-2., 2., RE)
  GIMAG=LIMIT(-2., 2., IM)
  PHI180=PHI+180
SAMPLE
  CALL DRWG(1,1,LOGW,MDB)
  CALL DRWG(2,1,LOGW,PHI)
  CALL DRWG(1,2,LOGW,0.0)
  CALL DRWG(2,2,LOGW,-180.)
TERMINAL
  CALL ENCRW(NPLCT)
END
STOP

```



```

*TIME RESPONSE FOR A SYSTEM COMPENSATED USING
*IMAGINARY ZERCS @ COMPLEX POLES
ERRCR=IN-CUT
IN=STEP(C.C)
X8=ERRCR*2300.C
X7=REALPL(C.0,C.3333E-2,X8)
X6=LEDLAG(0.1,C.2,X7)
X5=CMPLXPL(C.0,0.C,5.0E-3,2000.0,X6)
X4=4.0E+6*X5
X3=CMPLXPL(0.0,0.0,15.81E-6,5000.C,X4)
X2=25.CE+6*X3
STORAGE DEN(3),NUM(3)
TABLE DEN(1-3)=83.333E-6,173.611E-9,1.0,...
NUM(1-3)=0.0,277.008E-9,1.0
X1=TRANSF(2,DEN,2,NUM,X2)
CUT=INTGPL(C.C,X1)
TIMER FINTIM=C.5,CUTDEL=C.005
PRTPLT CUT
LABEL COMPENSATION WITH I. ZERCS C. POLES
LABEL POLES AT 2400 ZERCS AT 1900 D.R.=0.1
OUTPUT TIME,CUT
LABEL S. CONSTANT CULAKIS
LABEL POLES AT 2400 ZERCS AT 1900 D.R.=0.1
PAGE XYPLCT
END
STCP

```





```

TITLE FREQUENCY RESPONSE
TITLE COMPENSATION WITH COMPLEX POLES
D COMPLEX S,N,D,G,D1,D2,D3
D COMMON/CAREA/S,N,C,G,D1,D2,D3
INTEGER NPLOT
CONST NPLOT=1
CONTRL FINTIM=1.2,DELT=0.00061,DELS=0.00061
PRINT 4.0E-3,W,PHI,RE,IM,MOB,MAG
DYNAMIC
RW=RAMP(C.0)
LCGW=2.3+RW
W=10.**LCGW
S=CMPLX(C.,W)
D1=S*(0.2*S+1.0)*(0.003333*S+1.0)
D2=(S*(C.4E-7*S+0.6324E-8)+1.0)*(S*(0.25E-6*S+,...
C.5E-5)+1.0
*-----*
* COMPLEX POLES AT FREQUENCY 1900 HZ WITH D.RATIO 0.1 *
*-----*
D3=(S*(277.008E-9*S+10.526E-5)+1.0)
D=D1*D2*D3
N=2300.0*(0.1*S+1.0)
G=N/D
RE=REAL(G)
IM=AIMAG(G)
PHI=57.3*ATAN2(IM,RE)
IF(PHI.GT.C.) PHI=PHI-36C
MAG=CABS(G)
MDB=2C.*ALCG1C(MAG)
* GREAL=LIMIT(-2., 2., RE)
* GIMAG=LIMIT(-2., 2., IM)
PHI18C=PHI+18C
SAMPLE
CALL DRWG(1,1,LOGW,MDB)
CALL DRWG(2,1,LOGW,PHI)
CALL DRWG(1,2,LOGW,0.0)
CALL DRWG(2,2,LOGW,-180.)
CALL DRWG(3,1,RE,IM)
CALL DRWG(3,2,RE,C.0)
TERMINAL
CALL ENDRW(NPLOT)
END
STOP

```



```

TITLE STEP RESPONSE OF THE COMPENSATED SYSTEM WITH C. POLES
INTEGER NPLCT
CONST NPLCT=1
CCNST K1=25.0E+6,P1=31.6E-6,P2=5000.0
CONST K2=4.0E+6,P3=5.0E-3,P4=2000.0
CONST K3=4.2025E+6,P5=0.3333E-2,P6=0.1,P7=0.2,P8=0.05
CONST P9=2300.0,K4=2300.0
INTEG RKSF X
DERIVATIVE
E=R-Y
R=STEP(C.0)
Q=CMPLXPL(C.0,0.0,P1,P2,K1*E)
P=CMPLXPL(0.0,C.0,P3,P4,K2*Q)
M=REALFL(C.0,P5,P)
L=L*CLAG(C.0,P6,P7,M)
YDCT=CMPLXPL(C.0,0.0,P8,P9,L*K3)
Y=INTGRL(C.0,YDCT*K4)
CONTRL FINTIM=0.7,DELTA=2.0E-4,DELS=4.0E-4
PRINT C.01,E,C,YDCT,L,Y
SAMPLE
TERMINAL CALL DRWG(1,1,TIME,Y)
CALL ENCRW(NPLCT)
END
STOP

```



```

TITLE= FREQUENCY RESPONSE
TITLE= BCODE
D      COMPLEX S,N,C,G,C1,D2,D3
D      COMMON/CAREA/S,N,C,G,D1,D2,D3
INTEGER CUR,NPLOT
CONST CUR=1,NPLOT=1
PARAM K1=206.611E-9,K2=90.905E-6
CONTROL FINTIM=1.0,DELS=6.25E-4
PRINT 4.0E-3,W,PHI,RE,IM,MDB,MAG
DYNAMIC
      RW=RAMP(C.0)
      LOGW=3.C+RW
      W=10.**LOGW
      S=CMPLX(C.,W)
      D1=S*(C.2*S+1.C)*(0.003333*S+1.C)
      D2=(S*(C.4E-7*S+0.6324E-8)+1.0)*(S*(C.25E-6*S+0.5E-5)+
*-----
*      COMPLEX PCLES AT FREQUENCY 2200,2300,2500 HZ WITH D.RATIO
*-----
      D3=(S*(K1*S+K2)+1.C)
      C=C1*D2*D3
      N=2300.C*(C.1+S+1.C)
      G=N/D
      RE=REAL(G)
      IM=AIMAG(G)
      PHI=57.3*ATAN2(IM,RE)
      IF(PHI.GT.C.) PHI=PHI-36C
      MAG=CABS(G)
      MDB=2C.*ALOG10(MAG)
      GREAL=LIMIT(-2., 2., RE)
      GIMAG=LIMIT(-2., 2., IM)
      PHI180=PHI+18C
SAMPLE
      CALL DRWG(1,CUR,LOGW,MDB)
      CALL DRWG(2,CUR,LOGW,PHI)
      CALL DRWG(1,4,LOGW,0.0)
      CALL DRWG(2,4,LOGW,-180.)
TERMINAL
      IF(CUR.EQ.3) CALL ENDRW(NPLOT)
      CUR=CUR+1
END
PARAM K1=189.035E-9,K2=86.956E-6
END
PARAM K1=160.000E-9,K2=80.000E-6
END
STOP

```



## LIST OF REFERENCES

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