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# Amplitude shading and phase weighting of a vertical linear array in the SOFAR channel by the linear minimum variance estimation technique.

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# NAVAL POSTGRADUATE SCHOOL
## Monterey, California

# THESIS

AMPLITUDE SHADING AND PHASE WEIGHTING OF A
VERTICAL LINEAR ARRAY IN THE SOFAR CHANNEL
BY THE LINEAR MINIMUM VARIANCE
ESTIMATION TECHNIQUE

by

Daniel Patrick McVicar

December 1983

Thesis Advisor: P. H. Moose

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# Amplitude Shading and Phase Weighting of a Vertical Linear Array in the SOFAR Channel by the Linear Minimum Variance Estimation Technique

by

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Submitted in partial fulfillment of the requirements for the degrees of

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## ABSTRACT

A single linear vertical passive array is used in the 'SOFAR' channel to determine the depth of a single underwater source at a constant range. The phase and amplitude weights applied to the array are determined by the linear minimum variance estimation technique. The resulting beam pattern is compared to the conventional time domain beamformer. It was found that the linear minimum variance estimation technique of amplitude shading and phase weighting was significantly superior to the conventional beamformer.

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# I. INTRODUCTION

There are many solutions to the problem of predicting the location of an underwater energy source. One common solution is the use of a passive hydrophone which detects the pressure waves radiating from the source. The hydrophone sensor is assumed to be omnidirectional and therefore incapable of estimating direction. To provide directionality a series of sensors are placed in a row to form a passive linear array.

A familiar method of determining directionality is time-domain beamforming. In this principle, it is assumed that the source is far enough away so that the pressure wave appears to be a plane wave when viewed at the site of the receiving array.

Thus a set of time delays are calculated for any direction of signal arrival, which, when applied to the receiver outputs causes them to be in phase and to reinforce when summed. The resultant angular response to signals arriving from other than the nominated direction is then a function of the array geometry, relative to the signal wavelength, and any weighting factors which have been applied to the receiver outputs. The effect is to generate a main receiving beam in the desired direction, with a series of undesired subsidiary sidelobes whose magnitude can be controlled to some extent by the choice of a suitable array geometry and the use of amplitude weightings on the receiver outputs.

In order to determine location, three or more such arrays separated by a known amount may be used.

This study is concerned with a single linear vertical passive array and the determination of the depth of a single

underwater source. The analysis is based on the following assumptions:

- The underwater source is emitting continuously and at a monochromatic frequency.
- Both the source and receiving hydrophones are stationary in space causing a constant range.
- The range is sufficiently long so that the channel is filled with R-R (refracted-refracted) rays.
- There is no distortion introduced in the propagating medium so that the signals received at each sensor are identical except for constant delays.
- The source signal and noise are independent and stationary gaussian random processes.
- The speed of sound profile is triangular and symmetric with the deep sound channel axis at 1000 meters. The velocity gradient is -0.017 meters/meter/sec above 1000 meters and +0.017 meters/meter/sec below 1000 meters. This profile gives a speed of sound at the surface of 1500 meters/sec.
- The speed of sound profile is constant in the horizontal plane.
- Only R-R rays are considered. All other rays have sufficient loss that their effect is negligible.

As opposed to conventional time-domain beamforming, this study makes no assumption of planar wave fronts at the receiver site. Therefore the time delays applied to each receiver will not, in general, be a linear function of depth.

Since it is desired to determine whether or not there is a source present at a specific depth the result will be a

binary decision. A "1" will indicate signal source present; a "0" will indicate signal source not present. For the constant range there will be "N" test depths investigated for the signal source. The number of hydrophones in the vertical array will be "L".

For a single source at a given depth, the travel time is calculated from the depth to each hydrophone. This travel time is converted into a phase delay for each hydrophone so that after summation from all hydrophones a maximum output is achieved. This output is then passed through a squaring device, an integrator, and a threshold and flip flop device to give a "1" binary output. If the signal source is at a different depth and the same previous phase delays are used for each hydrophone then the output will be somewhat less than the previous maximum. The difference in depth required to achieve a "0" binary output is the depth resolution of the system.

The travel times for each hydrophone are calculated for each of the "N" source depths to be considered. "N" will ordinarily be much greater than "L" so that the system will be overdetermined. An overdetermined system is one in which there are more equations than unknowns. The objective then is to calculate the phase angle and amplitude weight for each hydrophone so that a determination can be made indicating the presence or absence of a signal source at a given depth.

The method used to calculate the phase and amplitude weights is the linear minimum variance estimation technique. Linear minimum variance estimators are optimum when compared with all other estimators for gaussian problems. The method is directly applicable to overdetermined systems.

The output of the summer is calculated using the linear minimum variance amplitude and phase angles assuming a source at one of the "N" source depths and no source at the

others. The calculation is repeated for each of the depths. The result, when plotted against source depth, will be referred to as the "beam pattern" of the array in this report. (Although similar, it should not be interpreted as the angular response of an array as in the conventional definition of a beam pattern. The conventional definition loses much of its utility when the wavefronts are not planar.) Ideally the beam pattern will be maximum at the desired depth and very small at all other depths so that the binary "1" decision will be made for a source at the desired depth, and a "0" decision for sources at all others. This beam pattern is compared with the depth beam pattern of the conventional time-domain beamformer mentioned above. The purpose of this thesis is to determine, as an initial investigation, whether the linear minimum variance estimation technique, when applied to a linear vertical array, is useful in depth discrimination at long ranges in a 'SOFAR' type sound channel.

# II. GENERAL THEORY

## A. RAY ACOUSTICS

The propagation of sound in an elastic medium can be described mathematically by solutions of the wave equation using the appropriate boundary and medium conditions for a particular problem. The wave equation relating the accustic pressure 'p' to the coordinates 'x','y','z', and the time 't', may be written as

$$\frac{d^2p}{dt^2} = c^2 \quad \frac{d^2p}{dx^2} + \frac{d^2p}{dy^2} + \frac{d^2p}{dz^2} \tag{2.1}$$

where 'c' is a quantity that has the general significance of sound velocity and may vary with the coordinates.

One may approximate the solution of the wave equation using ray theory: its body of results and conclusions is called ray acoustics.

Officer [Ref. 1] describes the ray solution as a complete solution to any particular propagation problem within the validity of the approximation of the Eikonal equation to the wave equation. For these approximations to be valid neither the amplitude of the wave nor the speed of sound can change appreciably in distances comparable to a wavelength.

Thus the path of a ray through a medium in which the speed of sound varies with depth can be calculated by the application of Snell's law

$$\cos\theta/c = 1/c_0 = \text{a constant for any one ray} \tag{2.2}$$

where '$\theta$' is the angle of depression made with the horizontal at a depth where the speed of sound is '$c$', and '$c_0$' is the speed at a depth (real or extrapolated) where the ray would become horizontal.

In a medium in which the velocity of sound changes linearly with depth the sound rays can be shown to be arcs of circles, that is, to have a constant radius of curvature. Kinsler et al. [Ref. 2] give a simple and heuristic demonstation of the circularity of rays in a medium with a linear sound speed gradient '$g$'. The center of the circle which creates the arc lies at a depth where the sound speed extrapolates to zero. To understand this, consider a portion of a ray path with a local radius of curvature '$R$', as illustrated in Figure 2.1. Since the gradient '$g$' for this case is

$$g = \Delta c/\Delta z = (c_2 - c_1)/(d_2 - d_1) = (c_2 - c_1)/R(\cos\theta_1 - \cos\theta_2) \qquad (2.3)$$

where '$\Delta c$' is the change in sound speed and '$\Delta z$' is the change in depth. It can be seen that the radius of curvature is given by

$$R = -c_0/g = -c/(g \cos\theta) \qquad (2.4)$$

The ray path is therefore a circle when '$g$' is constant because '$R$' is then constant. The center of curvature of a circle lies at the depth where $\theta$ is 90 degrees, which corresponds to $c=0$. For the situation in Figure 2.1 the speed gradient is negative so that '$R$' is positive. If the speed gradient were positive '$R$' would be negative, and the path would curve upward.

Once the radius of curvature of each segment of a path is known the actual path can be traced graphically or

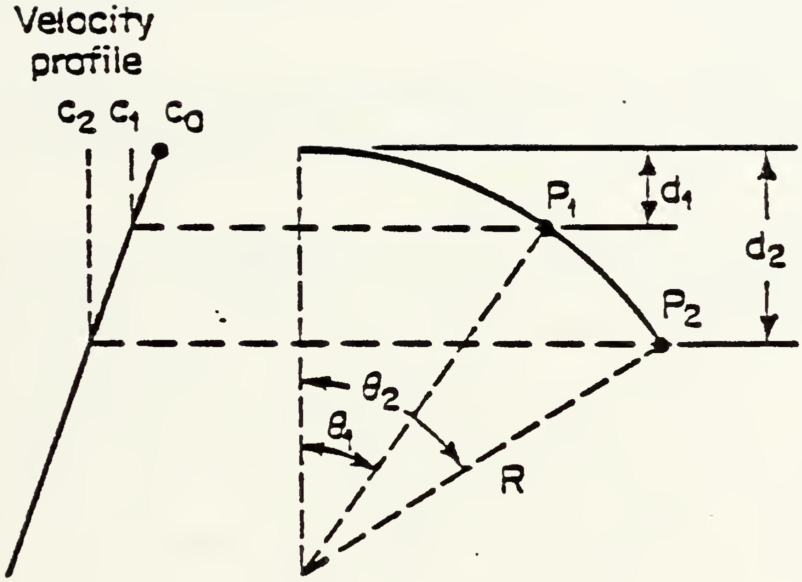


Figure 2.1 Circular Ray Path

computed. If the initial angle of depression of a ray is θ1, then by referring to the geometry of Figure 2.1 the changes in both range and depth are

$$\Delta r = c_1 (\sin(\theta 1) - \sin(\theta 2))/(g \cos(\theta 1)) \qquad (2.5)$$

$$\Delta z = c_1 (\cos(\theta 2) - \cos(\theta 1))/(g \cos(\theta 1)) \qquad (2.6)$$

The sign convention for these equations is: downward, to the right, and depression angles below the horizontal axis positive.

The symmetric triangular sound speed profile assumed in the introduction is similar to speed profiles encountered in the deep sound channel, sometimes called the SOFAR channel. The velocity minimum which occurs at the axis of the sound channel causes the sea to act like a kind of lens; above and below the minimum, the velocity gradient continually bends the sound rays toward the depth of minimum velocity. A portion of the power radiated by a source in the deep sound channel accordingly remains within the channel and encounters no acoustic losses by reflection from the surface and bottom. These rays are called R-R (refracted-refracted) and since they have very low transmission loss, very long ranges can be obtained from a source of moderate acoustic power output. Thus an energy source at a specific depth will propagate energy in all directions but only the direction which is toward the receiving array and for which the rays are R-R is of interest. To determine the range of depression angles which will yield R-R rays Snell's law is used to give

$$\theta max = arc\ cos(c1/(c1-(g\ d))) \qquad (2.7)$$

where '$\theta$max' is in radians and 'c1' is the speed of sound at the source depth 'd'.

An example is given in Figure 2.2 of a single ray trace propagating in the SOFAR channel to show how equations 2.5 and 2.6 are used in determining range and depth. The ray path is broken up into arcs of circles as shown in the figure, and then by paying close attention to the previously defined sign conventions, the change in range and depth is found. Being more specific, for arc 1, $\theta 1$ and $\theta 2$ are both positive; for arc 2, $\theta 1$ is positive and $\theta 2$ is zero; for arc 3, $\theta 1$ is zero and $\theta 2$ is negative; and for arc 4, $\theta 1$ is negative and $\theta 2$ is zero. By keeping a running total of all the depth and range changes it is possible to determine the total horizontal distance travelled and the depth at that distance.

For the speed of sound profile assumed in the introduction a computer generated ray plot is shown in Figure 2.3 for a source depth of 300 meters. As each ray propagates out from the source the triangular channel becomes filled with sound. If a receiving hydrophone is placed a great distance away, a number of refracted propagation paths will exist, each having a different travel time and crossing the channel axis at different intervals. The path with the greatest excursion from the axis will have the shortest travel time.

Officer [Ref. 1] shows that the travel time 't' of a ray, which is an arc of a circle, is given by

$$t = \frac{1}{g}\int_{\theta_1}^{\theta_2} \frac{d\theta}{\cos\theta} \qquad (2.8)$$

and the travel time for each arc is

$$t = -\frac{1}{g}\log_e\left[\frac{\tan((\pi/4)+(\theta_2/2))}{\tan((\pi/4)+(\theta_1/2))}\right] \tag{2.9}$$

Equation 2.9, applied using the same convention as equations 2.5 and 2.6, determines the total travel time of a ray in the deep sound channel.

## B. ARRAY MODEL

The receiving linear vertical array is presumed to consist of 'L' hydrophones as shown in Figure 2.4. It is assumed that the source is emitting energy at a constant frequency, 'f', and amplitude, 'A', regardless of the depth. The source signal at the source is $A\exp(j2\pi ft)$. The inherent received signal at the first hydrophone is

$$x1(t) = A'\exp(j2\pi f(t-t1)) \tag{2.10}$$

where 't1' is the travel time from the energy source to the first hydrophone and 'A'' is the amplitude of the signal at the range of the array. After passage through the amplitude weight 'a1' and a phase delay of '$\tau_1$', the signal on the first hydrophone at the input to the summer is

$$y1(t)=A'a1x1(t-\tau_1)=A'a1\exp(j2\pi f(t-t1-\tau_1)) \tag{2.11}$$

A time delay, for monochromatic signals corresponds to a phase shift

$$\theta = 2\pi ft \tag{2.12}$$

where '$\theta$' is the phase shift in radians and 't' is the time delay in seconds. Thus equation 2.11 can be written as

$$y1(t) = A'a1\exp(j(2\ ft-\phi 1-\theta 1)) \qquad (2.13)$$

where '$\phi 1=2\pi ft1$' is the phase delay due to the travel time from the source to the first hydrophone and '$\theta 1$' is the phase delay in the receiver on the first hydrophone.

Combining all the hydrophones in the array in a summer gives as an expression for the array output

$$Y(t) = \sum_{k=1}^{L} A'a_k \exp(j(2\pi ft-\phi_k-\theta_k)) \qquad (2.14)$$

where '$\phi_k$' represents the phase delay due to the travel time from the source to the "k th" hydrophone and '$\theta_k$' is the phase delay in the receiver on the "k th" hydrophone. If the amplitude of the energy source is normalized by setting $A'=1$, and equation 2.14 is written in terms of real and imaginary components, we have

$$Y(t)= \sum_{k=1}^{L} a_k \cos(-\phi_k-\theta_k)+j\sin(-\phi_k-\theta_k)\ \exp(j2\pi ft) \qquad (2.15)$$

When an energy source is at the "q th" depth, we wish to have each receiving hydrophone's phase delay cancel out the effect of the travel time from the source to it, such that '$-\phi_k-\theta_k$' is equal to a multiple of '$2\pi$'. This will put all signals into the summer in phase and thus maximize the signal gain for a source at the "q th" depth. From equation 2.15,

$$\sum_{k=1}^{L} (a_{kq} \cos(-\phi_{kq}-\theta_{kq})) = L \quad \text{(source present at q)} \qquad (2.16)$$

$$\sum_{k=1}^{L} (a_{kq} \sin(-\phi_{kq}-\theta_{kq})) = 0 \quad \text{(source present at q)} \qquad (2.17)$$

Note that the first subscript on the phase angle indicates the receiving hydrophone and the second subscript indicates the depth of the source. Thus '$\phi_{kq}$' would indicate the phase shift relating to the travel time from the "q th" test depth to the "k th" hydrophone in the receiving array.

It is desirable for 'Y(t)' to be a minimum value for sources at other than the depth being investigated. Thus for each of the other 'N-1' depths 'Y(t)' is set to zero. This gives 'N-1' equations for the real terms of 'Y(t)' set to zero

$$\sum_{k=1}^{L} a_{kq}\cos(-\phi_{km}-\theta_{kq})=0 \quad \text{(source absent; } m\neq q) \tag{2.18}$$

and 'N-1' equations for the imaginary terms of 'Y(t)' set to zero

$$\sum_{k=1}^{L} a_{kq}\sin(-\phi_{km}-\theta_{kq})=0 \quad \text{(source absent; } m\neq q) \tag{2.19}$$

By using elementary trigonometric identities, equation 2.16 (real terms with source present at "q th" depth) becomes

$$\sum_{k=1}^{L} a_{kq}\left[\cos(\phi_{kq})\cos(\theta_{kq})-\sin(\phi_{kq})\sin(\theta_{kq})\right]=L \tag{2.20}$$

Equation 2.18 (real terms with source absent for each of the other 'N-1' depths) becomes

$$\sum_{k=1}^{L} a_{kq}\left[\cos(\phi_{km})\cos(\theta_{kq})-\sin(\phi_{km})\sin(\theta_{kq})\right]=0 \quad m=1,2,\ldots N;\ m\neq q \tag{2.21}$$

Equation 2.17 (imaginary terms with source present at the "q th" depth) becomes

$$\sum_{k=1}^{L} -a_{kq}(\sin(\phi_{kq})\cos(\theta_{kq})+\cos(\phi_{kq})\sin(\theta_{kq}))=0 \tag{2.22}$$

and equation 2.19 (imaginary terms with source absent for each of the other 'N-1' depths) becomes

$$\sum_{k=1}^{L} -a_{kq}\left[\sin(\phi_{km})\cos(\theta_{kq})+\cos(\phi_{km})\sin(\theta_{kq})\right]=0 \quad m=1,2,\ldots N;\ m\neq q \tag{2.23}$$

Thus, there are a total of '2N' equations with '2L' unknowns.

In order to simplify, we put these '2N' equations into matrix form. Arbitrarily the real terms are made the first 'N' equations and the imaginary terms the second 'N' equations. The first real and first imaginary equation is at the lowest (shallowest) source depth and equations increase in order after that until the last real and last imaginary equation correspond to a source at the deepest depth. The resultant matrix equation becomes:

$$\left[\begin{array}{c} 0 \\ 0 \\ \vdots \\ 0 \\ L \\ 0 \\ \vdots \\ 0 \\ \hline 0 \\ 0 \\ \vdots \\ 0 \end{array}\right] = \left[\begin{array}{cccc|cccc} \cos\phi_{11} & \cos\phi_{21} & \cdots & \cos\phi_{L1} & -\sin\phi_{11} & -\sin\phi_{21} & \cdots & -\sin\phi_{L1} \\ \cos\phi_{12} & \cos\phi_{22} & \cdots & \cos\phi_{L2} & -\sin\phi_{12} & -\sin\phi_{22} & \cdots & -\sin\phi_{L2} \\ \vdots & & & & \vdots & & & \\ & & & & & & & \\ \cos\phi_{1q} & \cos\phi_{2q} & \cdots & \cos\phi_{Lq} & -\sin\phi_{1q} & -\sin\phi_{2q} & \cdots & -\sin\phi_{Lq} \\ & & & & & & & \\ \vdots & & & & \vdots & & & \\ \cos\phi_{1N} & \cos\phi_{2N} & \cdots & \cos\phi_{LN} & -\sin\phi_{1N} & -\sin\phi_{2N} & \cdots & -\sin\phi_{LN} \\ \hline -\sin\phi_{11} & -\sin\phi_{21} & \cdots & -\sin\phi_{L1} & -\cos\phi_{11} & -\cos\phi_{21} & \cdots & -\cos\phi_{L1} \\ -\sin\phi_{12} & -\sin\phi_{22} & \cdots & -\sin\phi_{L2} & -\cos\phi_{12} & -\cos\phi_{22} & \cdots & -\cos\phi_{L2} \\ \vdots & & & & \vdots & & & \\ -\sin\phi_{1N} & -\sin\phi_{2N} & \cdots & -\sin\phi_{LN} & -\cos\phi_{1N} & -\cos\phi_{2N} & \cdots & -\cos\phi_{LN} \end{array}\right] \left[\begin{array}{c} a_{1q}\cos\theta_{1q} \\ a_{2q}\cos\theta_{2q} \\ \vdots \\ \\ \\ \\ \\ a_{Lq}\cos\theta_{Lq} \\ \hline a_{1q}\sin\theta_{1q} \\ a_{2q}\sin\theta_{2q} \\ \vdots \\ a_{Lq}\sin\theta_{Lq} \end{array}\right] \tag{2.24}$$

Simplifying further, equation 2.24 becomes:

$$\begin{bmatrix} 0 \\ \cdot \\ \cdot \\ L \\ \cdot \\ \cdot \\ 0 \\ \hline \underline{0} \end{bmatrix} = \left[\begin{array}{c|c} \underline{\alpha} & -\underline{\beta} \\ \hline -\underline{\beta} & -\underline{\alpha} \end{array}\right] \left[\begin{array}{c} \underline{c} \\ \hline \underline{s} \end{array}\right] \tag{2.25}$$

where '$\underline{\alpha}$', '$\underline{\beta}$', '$\underline{c}$', and '$\underline{s}$' represent the appropriate submatrices.

Then, by noting that the multiplication of each element of a column by the same nonzero constant doesn't affect the solution, equation 2.25 becomes:

$$\begin{bmatrix} 0 \\ \cdot \\ \cdot \\ L \\ \cdot \\ \cdot \\ 0 \\ \hline \underline{0} \end{bmatrix} = \left[\begin{array}{c|c} \underline{\alpha} & \underline{\beta} \\ \hline -\underline{\beta} & \underline{\alpha} \end{array}\right] \left[\begin{array}{c} \underline{c} \\ \hline \underline{s} \end{array}\right] \tag{2.26}$$

Finally letting $\underline{Z}$, $\underline{A}$, and $\underline{\theta}$ represent the matrices in equation 2.26, we obtain

$$\underline{Z} = \underline{A}\underline{\theta} \tag{2.27}$$

where a matrix is denoted by a capitalized underlined letter.

In summary, $\underline{Z}$ is the desired response of the vertical array to the 'N' source test depths. $\underline{A}$ represents known travel times from each of the 'N' source depths to each of the 'L' receiving hydrophones. $\underline{\theta}$ is unknown. It is the phase and amplitude weighting which must be applied to the vertical array in order to realize $\underline{Z}$. $\underline{\theta}$ contains '2L' unknowns.

Equation 2.27 represents '2N' equations. Since this system of equations is overdetermined (N>L) an exact solution does not exist. In order to make the best estimate of $\underline{\theta}$ for the desired response, the linear minimum variance estimation technique is used.

## C. LINEAR MINIMUM VARIANCE METHOD

Equation 2.27 represents a noise free environment. If noise were present it would become

$$\underline{Z} = \underline{\underline{A}}\underline{\theta}+\underline{n} \tag{2.28}$$

with '$\underline{n}$' a "2N" element column vector. This represents the noise at each source depth. '$\underline{Z}$' is a linear function of '$\underline{\theta}$' and is called the oberservation. '$\underline{A}$' is a "2Nx2L" modulation or observation matrix which is known, and '$\underline{\theta}$' is the "2L" element random parameter vector which is to be estimated. Assume the first and second moments of '$\underline{\theta}$' and '$\underline{n}$' are given by

$$E(\underline{\theta}) = \underline{\mu}_\theta \qquad Var(\underline{\theta}) = \underline{V}_\theta \tag{2.29}$$

and

$$E(\underline{n}) = \underline{0} \qquad Var(\underline{n}) = \underline{V}_n \tag{2.30}$$

where 'E' represents expectation or first moment and 'Var' represents the covariance matrix. It is assumed that the parameter '$\underline{\theta}$' and the noise '$\underline{n}$' are uncorrelated.

A restriction imposed is that the estimate must be a weighted linear combination of the observations:

$$\hat{\underline{\theta}}_L = \underline{b}+\underline{B}\underline{Z} \tag{2.31}$$

where '^' indicates estimate. The objective is to select '$\underline{b}$' and '$\underline{B}$' in order to minimize the error variance. Such an estimator is the linear minimum variance estimator; it is the best, in the sense of minimum-error-variance linear estimators.

Another restriction is that the estimator be unbiased; in otherwords it is required that the expected value of the estimator '$\hat{\underline{\theta}}_L$' is equal to the expected value of the parameter '$\underline{\theta}$'. Thus,

$$E(\hat{\underline{\theta}}_L)=\underline{b}+\underline{B}E(\underline{z})=E(\underline{\theta})=\underline{\mu}_\theta \tag{2.32}$$

yielding

$$\underline{b}=\underline{\mu}_\theta-\underline{B}\underline{A}\underline{\mu}_\theta \tag{2.33}$$

Substituting this result in equation 2.31 gives for the unbiased linear estimator

$$\hat{\underline{\theta}}_L=\underline{\mu}_\theta+\underline{B}(\underline{Z}-\underline{A}\underline{\mu}_\theta) \tag{2.34}$$

Note that since the estimator is unbiased, the estimation error '$\underline{\theta}_E=\underline{\theta}-\hat{\underline{\theta}}_L$' is zero mean. The next step is to select '$\underline{B}$' in order to minimize the error variance. However, this optimization problem is ill-defined because the error variance is a matrix. Therefore in order to introduce a scalar goodness measure the sum of the variances of each component of '$\underline{\theta}$' is minimized. This is the sum of the main diagonal terms of the covariance matrix and is defined as the trace of the matrix.

$$\mathrm{tr}\,(\mathrm{Var}(\underline{\theta}_E)) = \sum_{n=1}^{2N} \mathrm{Var}((\underline{\theta}_E)_n) \tag{2.35}$$

where 'tr' indicates trace. '$\underline{B}$' is then selected to minimize the trace of the error variance, or

$$\min_{B} \mathrm{tr}\,(\mathrm{Var}(\underline{\theta}_E)) = \min_{B} \mathrm{tr}\,(E(\underline{\theta}_E\underline{\theta}_E^T)) \tag{2.36}$$

where '$^T$' indicates the transpose. The following problem is then obtained by substituting equation 2.34 into equation 2.36:

$$\min_{B} \mathrm{tr}\,(\mathrm{Var}(\underline{\theta}_E))=\min_{B} \mathrm{tr}(E((\underline{\theta}-\underline{u}_\theta-\underline{B}(\underline{Z}-\underline{Au}_\theta)(\underline{\theta}-\underline{u}_\theta\underline{B}(\underline{Z}-\underline{Au}_\theta)^T)) \tag{2.37}$$

It is well known [Ref. 3] that equation 2.37 is minimized when

$$\mathrm{Cov}\,(\underline{\theta},\underline{Z})-\underline{B}\mathrm{Var}\,(\underline{Z})=\underline{0} \tag{2.38}$$

where '$\mathrm{Cov}(\underline{\theta},\underline{Z})$' is the covariance matrix of the unknown parameters and the observations. Denoting the optimum filter by '$\underline{B}^*$', then if

$$\underline{B}^*=\mathrm{Cov}\,(\underline{\theta},\underline{Z})\,(\mathrm{Var}(\underline{Z}))^{-1} \tag{2.39}$$

a minimum is achieved for the sum of the squares of the errors.

Using equation 2.28 for '$\underline{Z}$', the covariance of '$\underline{\theta}$' and '$\underline{Z}$' becomes

$$\mathrm{Cov}\,(\underline{\theta},\underline{Z})=\mathrm{Cov}\,(\underline{\theta},\underline{A}\underline{\theta}+\underline{n})=\underline{V}_\theta\underline{A}^T \tag{2.40}$$

since '$\underline{\theta}$' and '$\underline{n}$' are uncorrelated. The variance of '$\underline{Z}$' is

$$\mathrm{Var}(\underline{Z}) = \mathrm{Var}(\underline{A}\underline{\theta} + \underline{n}) = \underline{A}\,\underline{V}_{\theta}\underline{A}^{T} + \underline{V}_{n} \tag{2.41}$$

Substituting these into equation 2.39 gives

$$\underline{B}^{*} = \underline{V}_{\theta}\underline{A}^{T}(\underline{A}\underline{V}_{\theta}\underline{A}^{T} + \underline{V}_{n})^{-1} \tag{2.42}$$

and the linear minimum variance estimator is

$$\hat{\underline{\theta}}_{LMV} = \underline{\mu}_{\theta} + \underline{V}_{\theta}\underline{A}^{T}(\underline{A}\underline{V}_{\theta}\underline{A}^{T} + \underline{V}_{n})^{-1}(\underline{Z} - \underline{A}\underline{\mu}_{\theta}) \tag{2.43}$$

By utilizing a matrix inversion lemma [Ref. 3] equation 2.43 becomes

$$\hat{\underline{\theta}}_{LMV} = (\underline{A}^{T}\underline{V}_{n}^{-1}\underline{A} + \underline{V}_{\theta}^{-1})^{-1}(\underline{A}^{T}\underline{V}_{n}^{-1}\underline{Z} + \underline{V}_{\theta}^{-1}\underline{\mu}_{\theta}) \tag{2.44}$$

The advantage of this equation over equation 2.43 is the size of the matrix to be inverted. In equation 2.43 the matrix has dimensionality '2N' while in equation 2.44 its dimenionality is only '2L'. Thus the advantages of the linear variance estimator are the ease with which they are derived, the mathematical tractability of the linear form, and the minimum amount of stochastic information required for development. An interesting characteristic is that the linear minimum variance estimate is the orthogonal projection of '$\underline{\theta}$' onto the space spanned by the observation '$\underline{Z}$'. Because of these factors this estimator is a popular form for estimating unknowns in overdetermined equations.

For this thesis it is assumed that the noise samples are uncorrelated and identically distributed so that:

$$\underline{V}_{n} = \sigma^{2}\underline{I} \tag{2.45}$$

No previous knowledge is assumed about '$\theta$'. This implies an infinite variance matrix which is represented as:

$$\underline{V}_{\theta}^{-1}=\underline{0} \qquad \text{and} \qquad \underline{\mu}_{\theta}=\underline{0} \tag{2.46}$$

The linear minimum variance estimate given by equation 2.44 is then

$$\hat{\underline{\theta}}_{LMV}=(\underline{A}^T\underline{A})^{-1}\underline{A}^T\underline{Z} \tag{2.47}$$

By determining '$\hat{\underline{\theta}}_{LMV}$' the phase and amplitude weights are found for a signal source on the 'q th' depth. Recall that

$$\hat{\underline{\theta}}_{LMV} = \begin{bmatrix} \hat{\theta}_1 \\ \hat{\theta}_2 \\ \cdot \\ \cdot \\ \hat{\theta}_L \\ \hline \hat{\theta}_{L+1} \\ \cdot \\ \cdot \\ \hat{\theta}_{2L} \end{bmatrix} = \begin{bmatrix} a_{1q}\cos\theta_{1q} \\ a_{2q}\cos\theta_{2q} \\ \cdot \\ \cdot \\ a_{Lq}\cos\theta_{Lq} \\ \hline a_{1q}\sin\theta_{1q} \\ \cdot \\ \cdot \\ a_{Lq}\sin\theta_{Lq} \end{bmatrix} \tag{2.48}$$

Upon solving, this equation gives for the phase delay

$$\hat{\theta}_{mq}=\arctan(\hat{\theta}_{L+m}/\hat{\theta}_m) \quad \text{where } m=1 \text{ to } L \tag{2.49}$$

and amplitude weight

$$\hat{a}_{mq}=\hat{\theta}_m/\cos(\hat{\theta}_{mq}) \quad \text{where } m=1 \text{ to } L \tag{2.50}$$

When these amplitude weights and phase delays are applied to the vertical linear array a resulting beam pattern is formed which in the absence of noise is:

$$\hat{\underline{Z}} = \underline{A}\hat{\underline{\theta}} \tag{2.51}$$

The resulting beam pattern '$\hat{\underline{Z}}$' can then be compared with the desired beam pattern '$\underline{Z}$', as well as with a conventional beam pattern '$\underline{Z}'$' obtained using linear phase shifts across the array aperture selected to "steer" the array to the dominant arrival angle for the selected source depth.

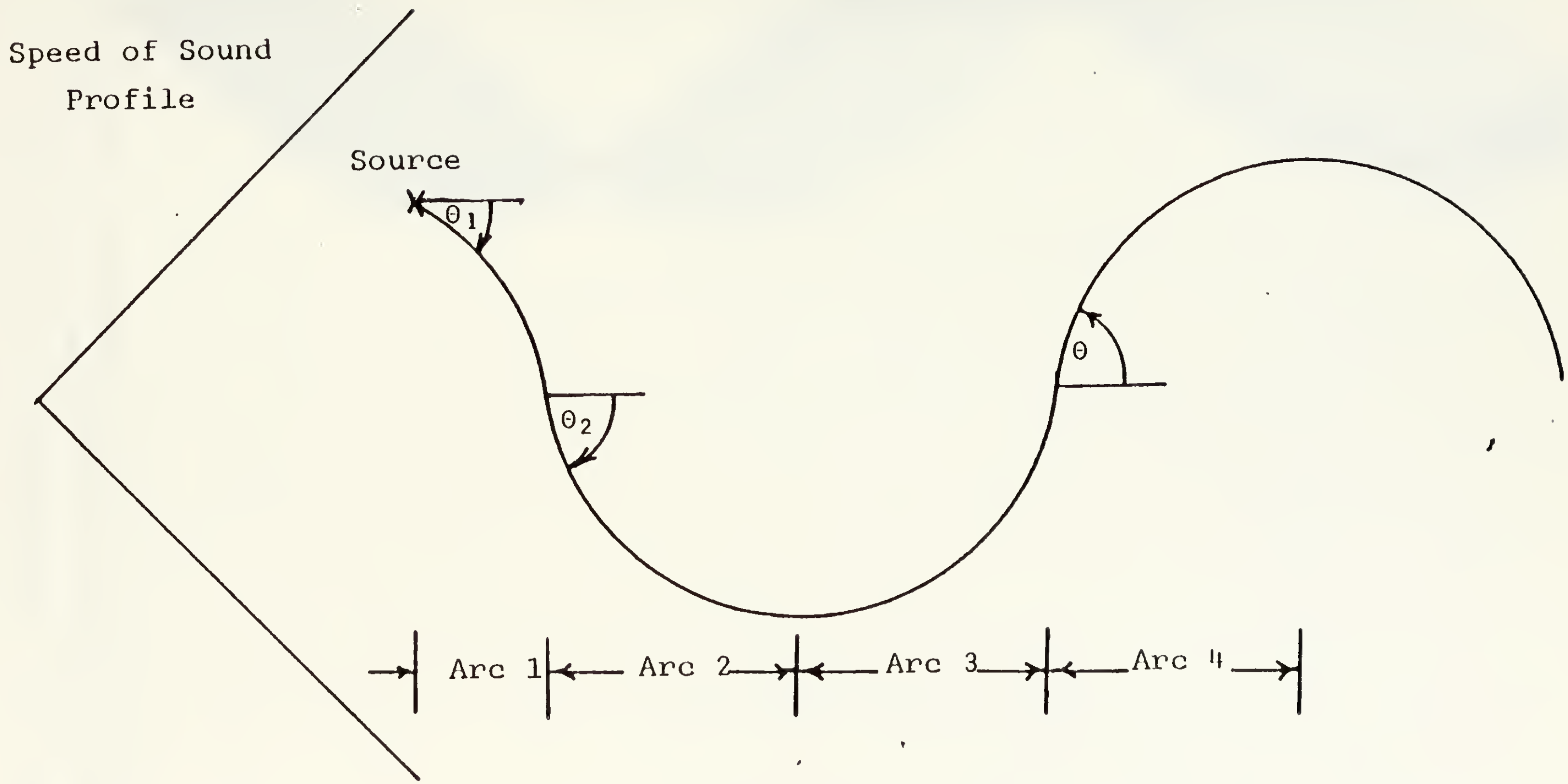


Figure 2.2 Single Ray Path Plot In Triangular SOFAR Channel

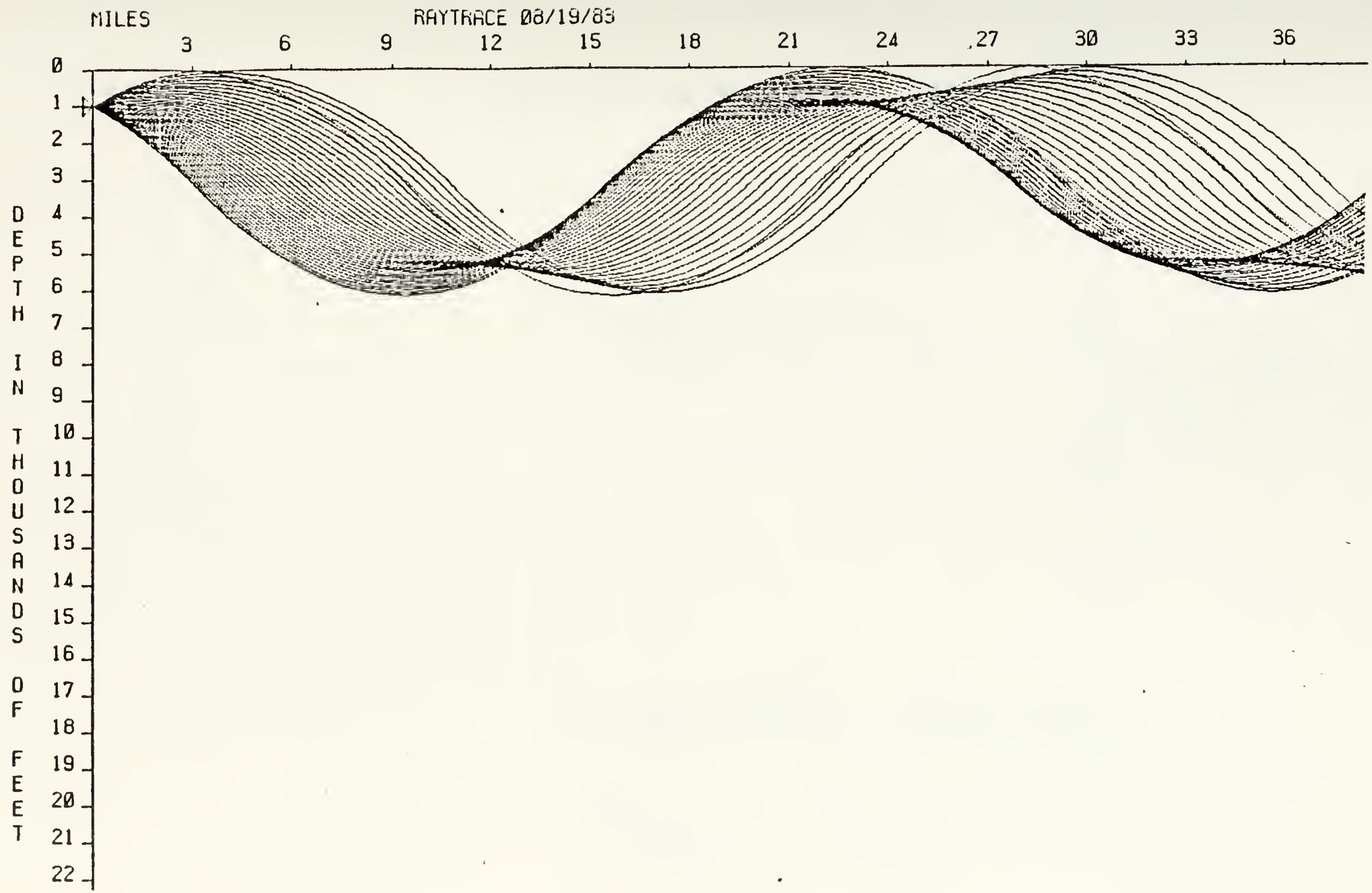


Figure 2.3 Ray Plot For Assumed Sound Channel

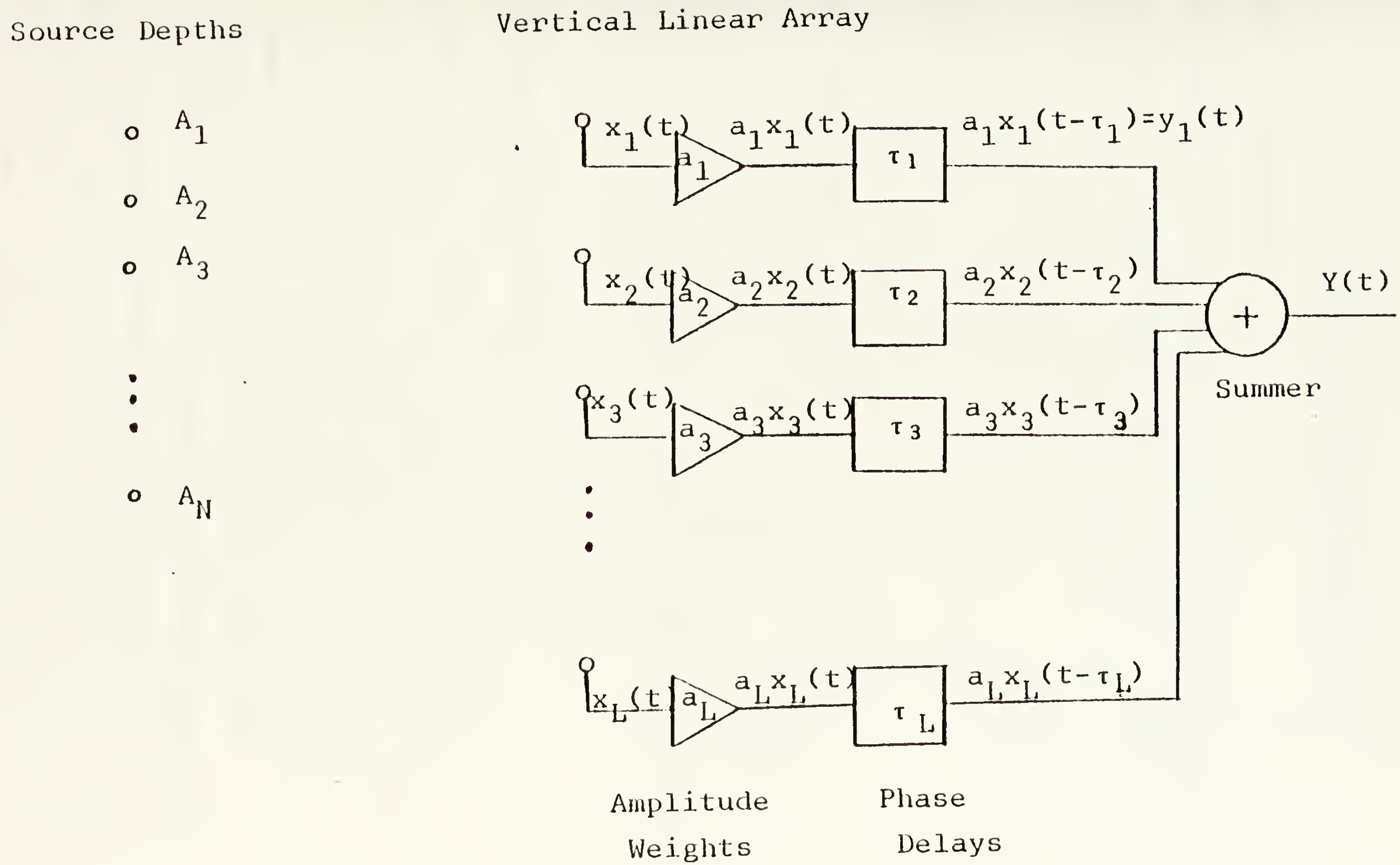


Figure 2.4 Array Model

# III. EXPERIMENTAL PROCEDURE

## A. BASIC ASSUMPTIONS

The speed of sound profile given in the Introduction was used to test the techinique, with 'L', the number of source depths chosen as 20. The source depths are at 220 meters and every 20 meters thereafter to 600 meters inclusive. 'N', the number of hydrophones used in the vertical array was chosen as 5. The hydrophones are placed at depths of 100, 200, 300, 400, and 500 meters. Thus, there are 40 equations, two for each source depth, and 10 unknowns, two for each hydrophone. A range of 200 kilometers (km) assures that the deep sound channel is filled with sound over the aperture of the vertical linear array. A frequency of 100 Hz provides good resolution in the beam pattern without introducing alias mainlobes at the selected range across the depths of investigation.

## B. '$\underline{A}$' MATRIX CALCULATION

Since the gradient of the sound velocity, 'g' is constant in the area of the source depths, and the speed of sound at the surface is known, equation 2.3 is used to solve for 'c2', the speed of sound at the source depth. Equation 2.7 is used for each source depth to calculate the maximum initial depression angle which yields R-R rays. In determining travel time, only depression angles from each source which are positive (downward) and yield R-R rays are used.

Beginning with the first source depth of 220 meters, an initial depression angle of 0.0 degrees is selected. The ray path is then calculated using equations 2.5 and 2.6 and broken into a series of arcs as in figure 2.2. Equation 2.9

is used to determine the travel time for each arc in the same manner. Each arc's horizontal range, travel time, and depth are summed. When the summed horizontal range reaches 200 km the summation process ends. The travel time and depth of the ray at this horizontal range is then known. The same procedure is repeated for an initial depression angle of 0.1 degrees and every increment of 0.1 degrees thereafter until the maximum depression from equation 2.7 is reached. The final ray path is at this maximum depression angle.

The same procedure is repeated for each of the other 19 source depths.

Thus, for each initial angle from each source depth there is a ray which has a travel time and a depth when it reaches the horizontal range of 200 km. Since it is the profile of the sound pressure wave which impinges on the vertical array which is of importance, a constant can be subtracted from these calculated travel times. This constant is selected to be the travel time for the source depth of 220 meters which has an initial depression angle of 0 degrees. It is subtracted from each of the travel times making the resultant travel times relative with respect to the ray which has a 0 degree depression angle from the 220 meter depth. The program and its listing which calculates the relative travel times and the depths of these rays at the horizontal range of 200 km is given in Appendix A.

A plot of the relative travel times versus depth for the 220 meter source depth is shown in Figure 3.1. Figure 3.2 displays the plot for the 380 meter source depth. Negative relative travel times in the plots indicate that the overall travel time is less than the reference. These rays arrive at the 200 km horizontal distance before the reference ray.

Since the receiving hydrophones are at set vertical positions (100, 200, 300, 400, and 500 meters), an

interpolation is done to determine relative travel times to them from each source depth. The interpolation program and its listing is in Appendix B. Sometimes more than one R-R ray travels from the source depth to a hydrophone. When this occurs, the ray which arrives first is used in the calculation of relative travel time to that hydrophone.

Equation 2.12 determines the phase shift relating to the relative travel times. The '$\underline{A}$' matrix is formed by taking the appropriate sine and cosine values as in equation 2.24. The '$\underline{A}$' matrix is '40 by 10'.

## C. '$\underline{Z}$' MATRIX

Referring to equation 2.26, the '$\underline{Z}$' matrix is a '40 by 1' column vector. It is the desired beam pattern. The bottom 20 rows give the imaginary terms and are set to zero. The top 20 rows represent the value of the real terms at each source depth. Therefore each of the top 20 rows is set to zero except for the row containing the source. It is set to 1. For example, if the source is at 220 meters then only the top row is set to 1. If the source is at 380 meters then only the ninth row is set to 1.

## D. RESULTING BEAM PATTERN

### 1. Using the Linear Minimum Variance Method

'$\hat{\underline{\theta}}_{LMV}$' is calculated using equation 2.47. The resulting beam pattern '$\hat{\underline{Z}}$' is calculated using equation 2.51. The program which calculates the '$\underline{A}$' matrix, uses it in determining '$\hat{\underline{\theta}}_{LMV}$', and then calculates '$\hat{\underline{Z}}$' is given in Appendix C. The program listing is also included.

## 2. Using Linear Phase Shifts

The conventional beam pattern is determined by using equation 2.51 where '$\hat{\underline{\theta}}$' is calculated by approximating the plot of relative travel time vs. depth by a straight line at the receiving hydrophone depths. For example Figure 3.3 represents this plot for the 380 meter source depth. The straight line is determined by a least squares linear regression which minimizes the sum of the squares of the deviations of the actual data points from the straight line of best fit. Note that only data points which are on the dominant curve are used in calculating the straight line. From the straight line, relative travel times to the receiving hydrophones are calculated. The relative travel times for the 380 meter source depth are given in Table I. They correspond to a plane wave arrival angle of 3.73 degrees. '$\hat{\underline{\theta}}$' is determined by converting these relative travel times to phase delays using equation 2.12 and then taking the appropriate sine and cosine values of these phase delays as in equation 2.24. The amplitude weights are initially assumed to be unity.

A second method for obtaining the conventional beam pattern is calculated by the same procedure except the amplitude weights which are determined by equation 2.50, the '$\hat{\underline{\theta}}_{LMV}$' amplitude amplitude weights, are applied to each hydrophone.

## TABLE I

Relative Travel Times For 380 Meter Source Depth

| Hydrophone Depth | Relative Travel Time |
|---|---|
| 100 meters | -0.07565509 sec. |
| 200 meters | -0.07131599 sec. |
| 300 meters | -0.06697690 sec. |
| 400 meters | -0.06263780 sec. |
| 500 meters | -0.05829871 sec. |

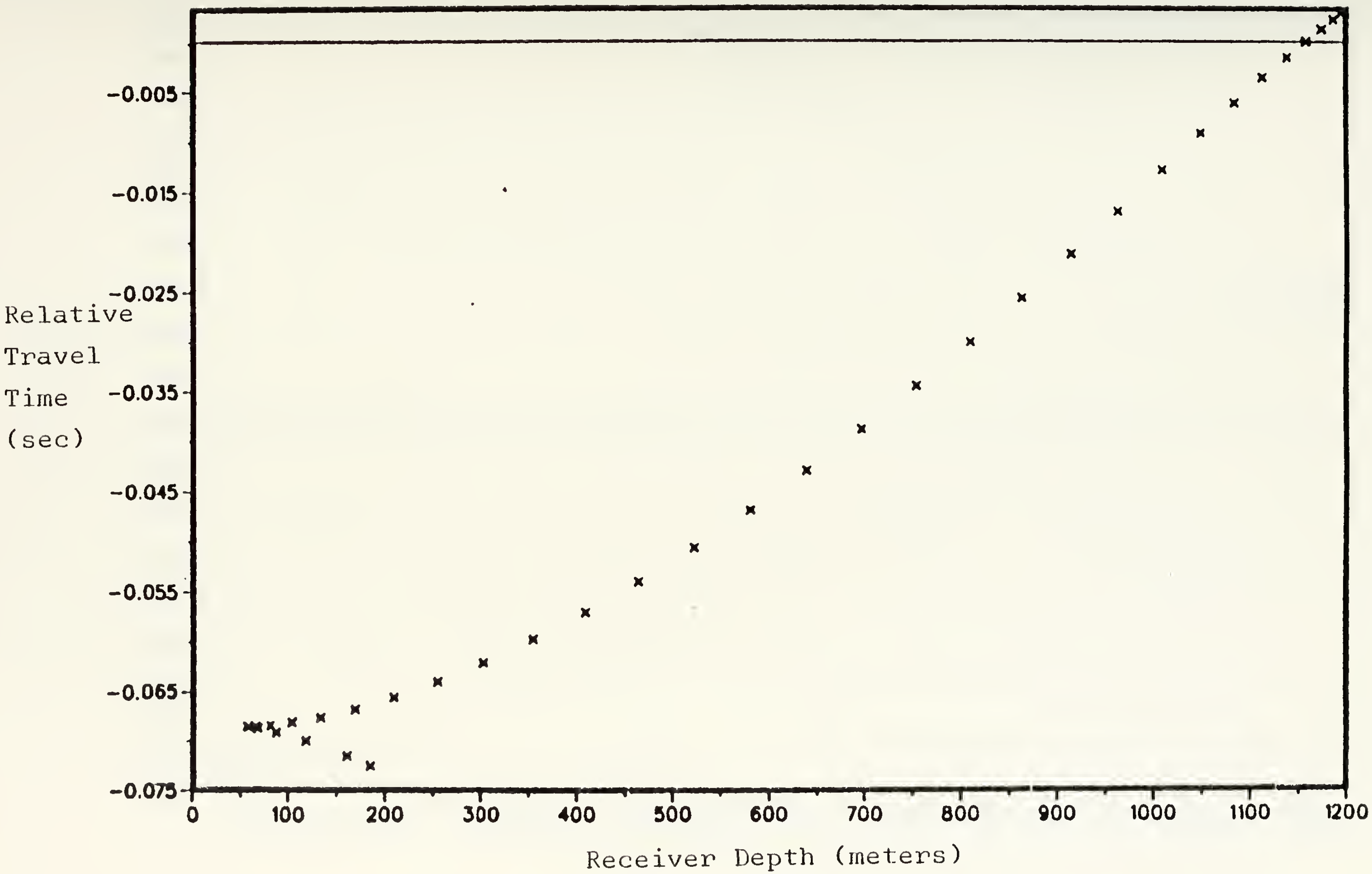


Figure 3.1 Relative Travel Time vs. Depth (220 meter source)

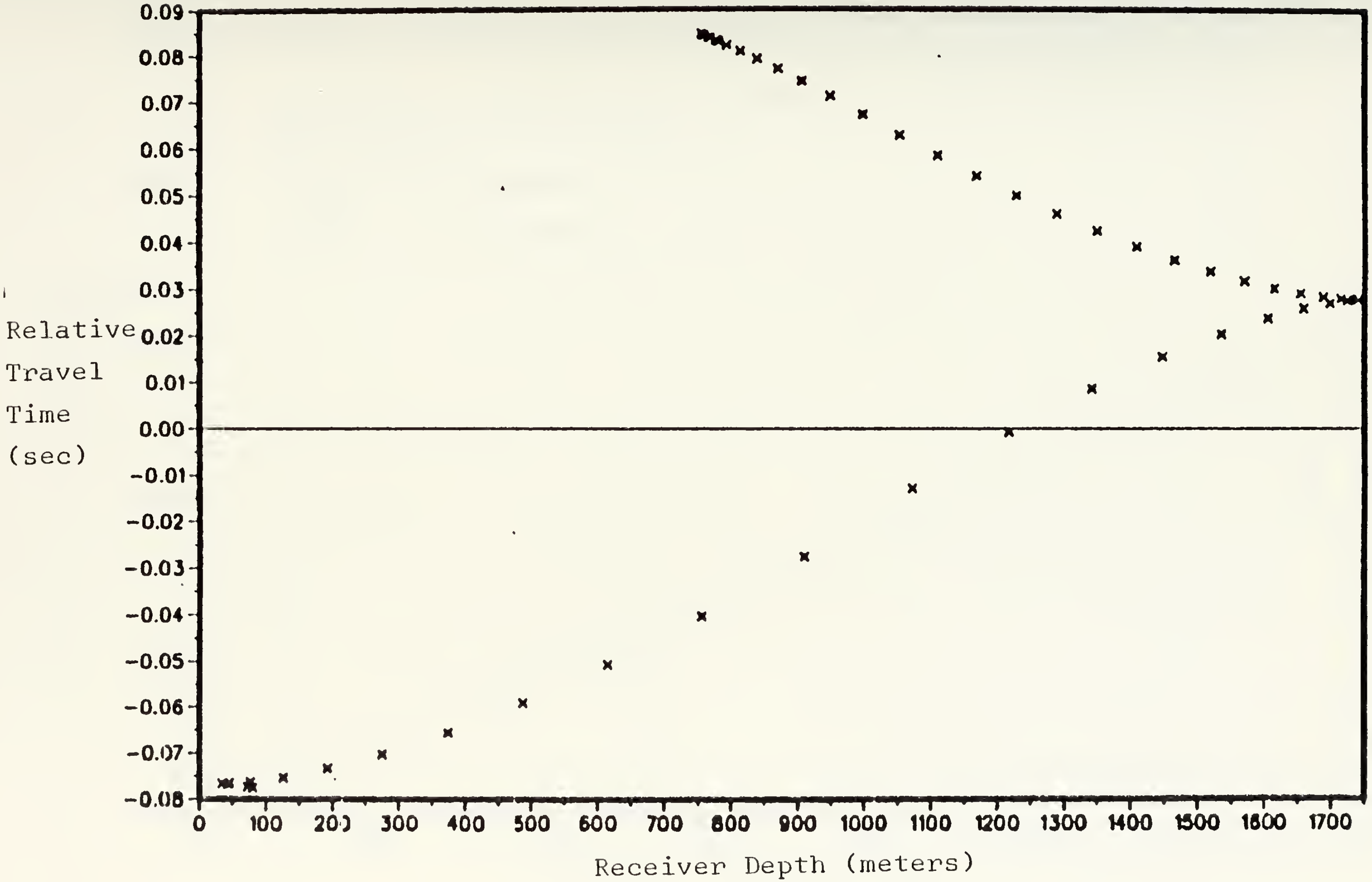


Figure 3.2 Relative Travel Time vs. Depth (380 meter source)

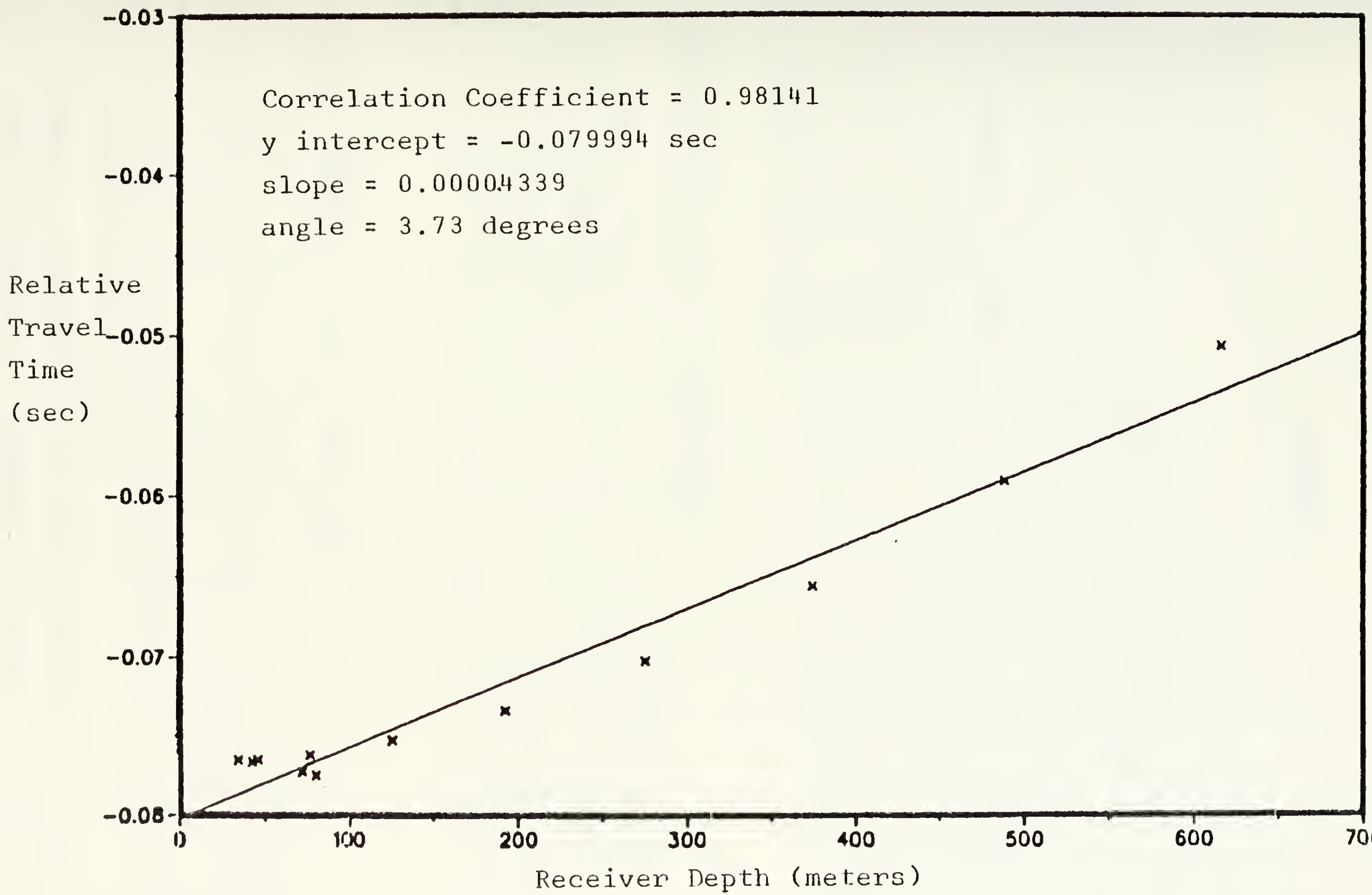


Figure 3.3 Straight Line Approx. of Rel. Trav. Time vs. Depth (source at 380 m)

# IV. RESULTS

## A. EXACT SOLUTION

An exact solution is derived for the beam pattern if the number of receiving hydrophones equal the number of source depths. For example, when the 5 receivers are used to discriminate between 5 source depths (220, 240, 260, 280, and 300 meters) there are 10 equations with 10 unknowns. Figure 4.1 is a plot of the resulting beam pattern with the source at the shallowest depth. Note that because of round-off errors in the 'IMSL' subroutines there is a small value for the resulting beam pattern at the non-energy source depths under investigation.

## B. FOUR DEPTHS WITH TWO RECEIVERS

For source depths at 220, 280, 300, and 320 meters with receiving hydrophones at 100 and 200 meters, there are 8 equations with 4 unknowns. Figure 4.2 is a plot of the resulting beam pattern with a source at the 220 meter depth. Figure 4.3 is the plot for the source at the 280 meter depth.

## C. TWENTY DEPTHS WITH TWO RECEIVERS

For all 20 source depths with receiving hydrophones at 100 and 200 meters, there are 40 equations with 4 unknowns. Figure 4.4 is a plot of the resulting beam pattern for a source at the 220 meter depth. Figure 4.5 is a plot for the source at the 380 meter depth.

In order to determine if the '$\hat{\underline{\theta}}$' calculated in this case is the best an alternate method is devised. Four source

depths (220, 240, 260 meters, and another source test depth) with the original source at the 220 meter depth and receivers at 100 and 200 meters are used. The beam pattern is calculated each time with a different source test depth substituted for the fourth source depth. The 5 best resulting beam patterns are selected along with the 8 source depths (220, 240, 260 meters, and the 5 test depths which created the 5 best beam patterns). Then, using these 8 source depths, '$\underline{\hat{\theta}}$' is determined for the two receivers. This '$\underline{\hat{\theta}}$' is applied to the two receivers and the beam pattern obtained for all 20 source depths.

The resulting beam pattern obtained by this alternate method isn't as good as the beam pattern obtained by using all 20 source depths in the determination of '$\underline{\hat{\theta}}$'.

## D. TWENTY DEPTHS WITH FIVE RECEIVERS

For all 20 source depths with all 5 receiving hydrophones there are 40 equations with 10 unknowns. Figure 4.6 is a plot of the resulting beam pattern with the source at the 220 meter depth. Figures 4.7, 4.8, and 4.9 are the plots for the source at the 360, 380, and 400 meter depths respectively.

## E. CONVENTIONAL BEAMFORMER

Figure 4.10 is a plot of the beam pattern for a conventional beamformer using linear phase shifts across the array with the source at the 380 meter depth and the amplitude weights set to unity. All 20 source depths and 5 receiving hydrophones are used. Note that in figure 4.10 that there is less than 1 db discrimation between each of the source depths. Figure 4.11 is the plot obtained for the amplitude weights set to values determined by equation 2.50.

## F. RANGE OF 250 KILOMETERS

The calculations were repeated for a range of 250 km using the same 20 source depths and 5 receiving hydrophones. Figures 4.12 and 4.13 represent the plots of relative travel times versus depth for the 220 and 380 meter source depths respectively. Figure 4.14 represents the straight line approximation of the relative travel times for the 380 meter depth. Note that in this figure the relative travel times are represented by two straight lines; the upper line represents linear phase shifts of the slower travel times for the conventional beamformer while the lower line represents linear phase shifts of the faster travel times. Tables II and III are the straight line interpolations of these slower and faster travel times which correspond to arrival angles of 4.04 and -3.16 degrees respectively. Figures 4.15 and 4.16 are the resulting beam patterns for the conventional beamformer for the slower travel times using unity amplitude weights and linear minimum variance amplitude weights respectively. Figures 4.17 and 4.18 are the beam patterns for the faster travel times.

Figures 4.19, 4.20, 4.21, and 4.22 represent plots of the beam pattern for the source at the 220, 340, 360, and 380 meter depths respectively.

**TABLE II**

**Slower Ray Travel Times For 250 km Range and 380 m Source**

| Hydrophone Depth | Relative Travel Time |
|---|---|
| 100 meters | -0.09949109 sec. |
| 200 meters | -0.09478615 sec. |
| 300 meters | -0.09008121 sec. |
| 400 meters | -0.08537627 sec. |
| 500 meters | -0.08067133 sec. |

**TABLE III**

Faster Travel Times For 250 km Range and 380 m Source

| Hydrophone Depth | Relative Travel Time |
|---|---|
| 100 meters | -0.10100745 sec. |
| 200 meters | -0.10468763 sec. |
| 300 meters | -0.10836781 sec. |
| 400 meters | -0.11204799 sec. |
| 500 meters | -0.11572817 sec. |

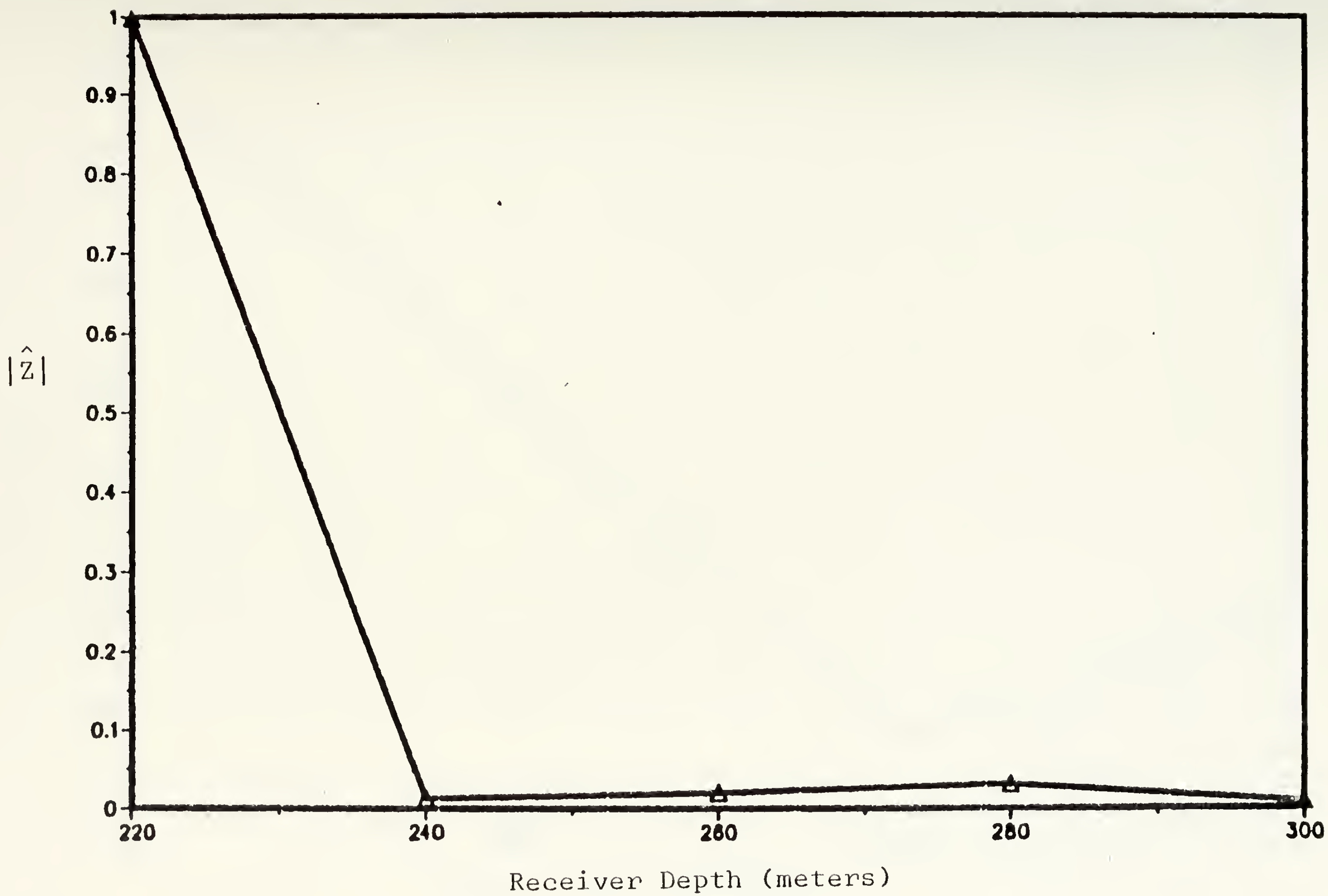


Figure 4.1 Beam Amplitude vs. Source Depth (5 depths 5 receivers)

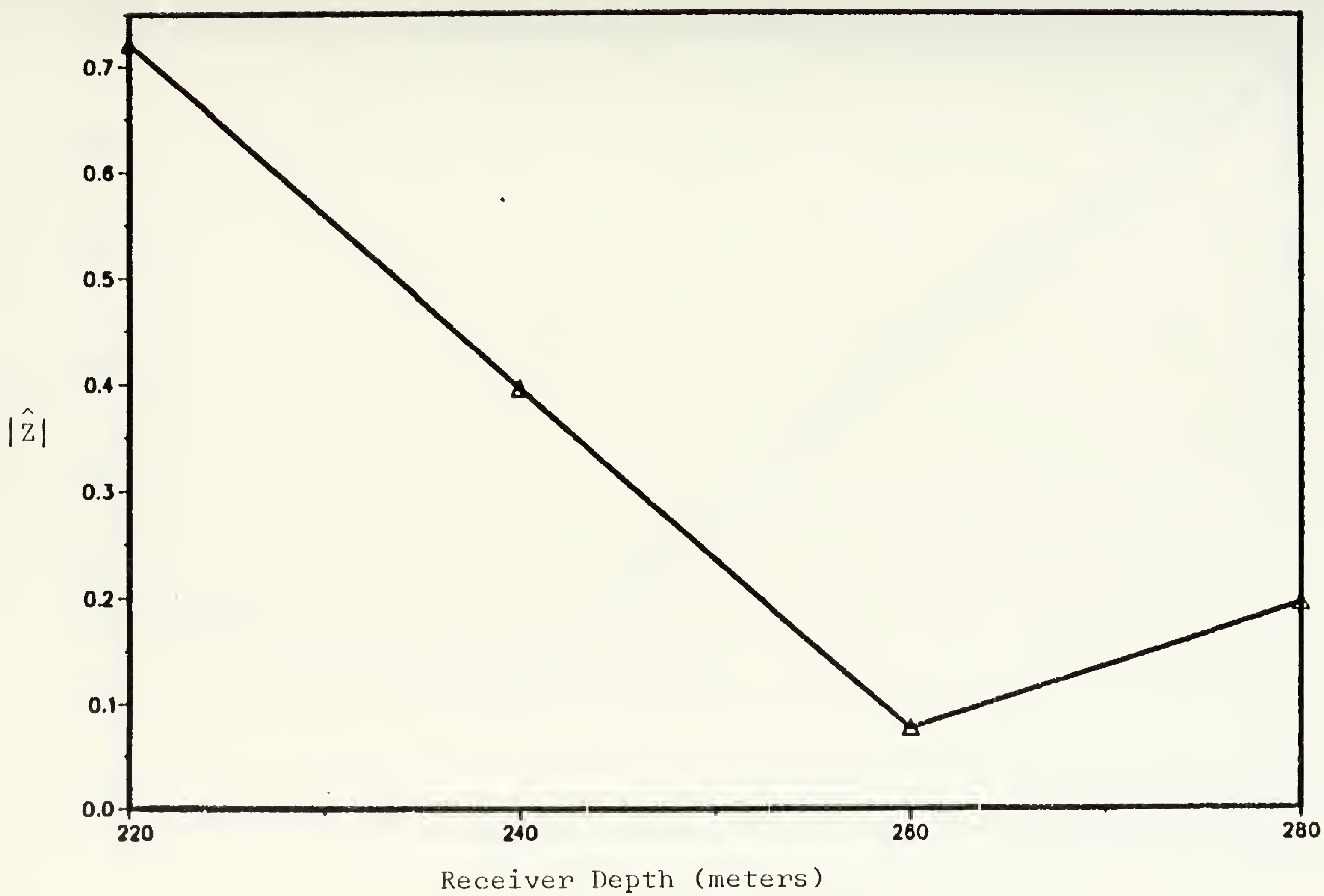


Figure 4.2 Beam Pattern (4 depths 2 receivers, source at 220 m)

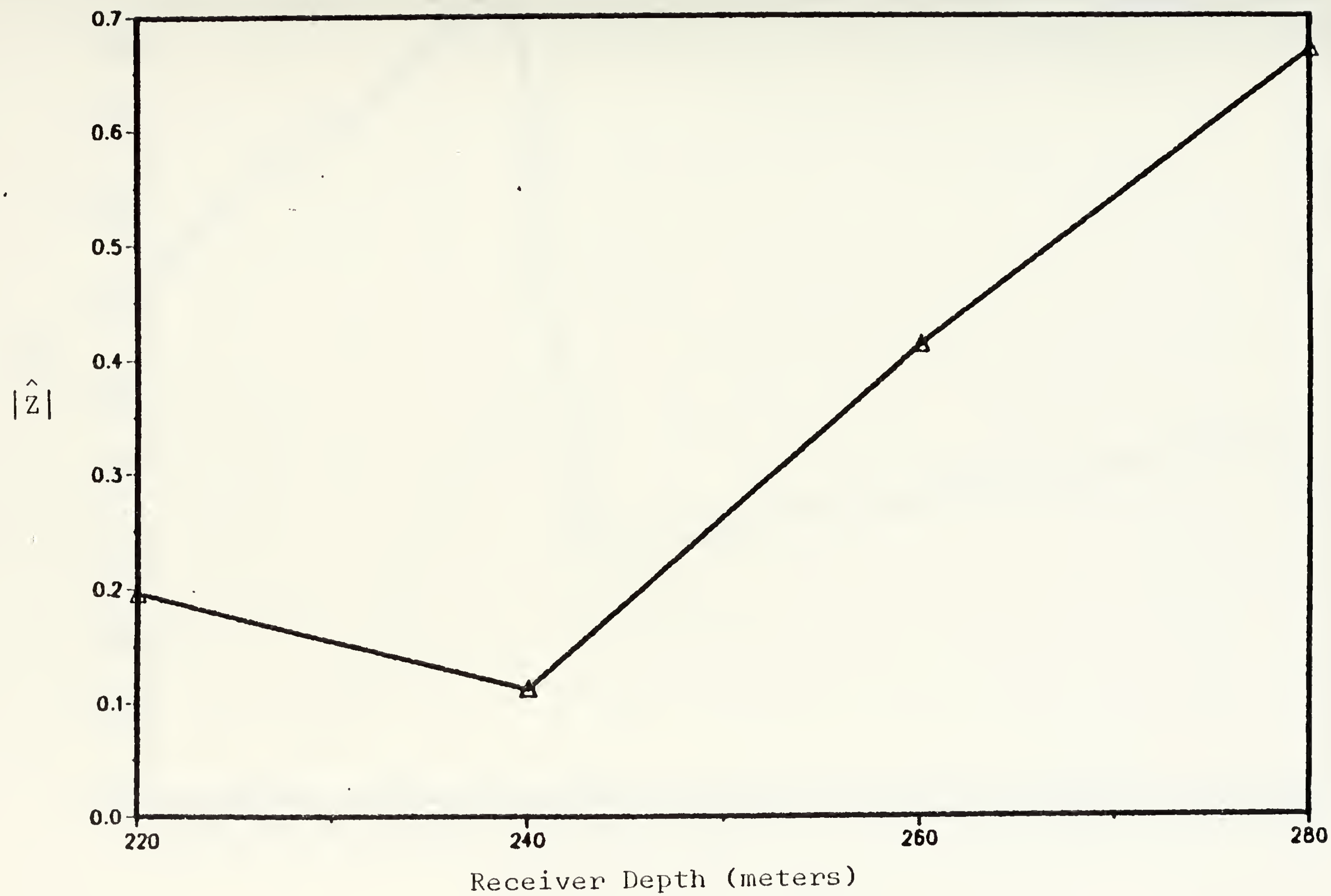


Figure 4.3 Beam Pattern (4 depths 2 receivers, source at 280 m)

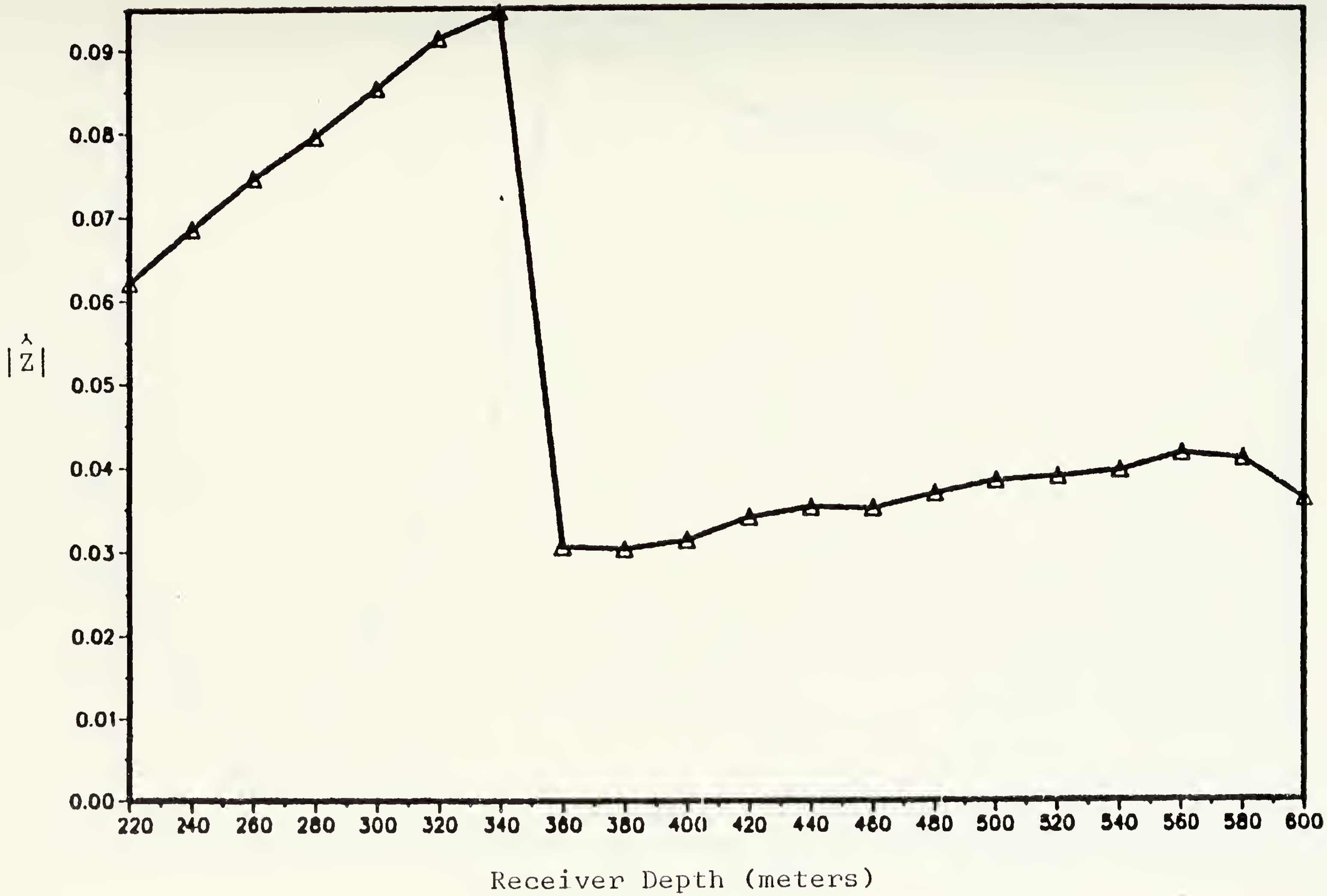


Figure 4.4 Beam Pattern (20 depths 2 receivers, source at 220 m)

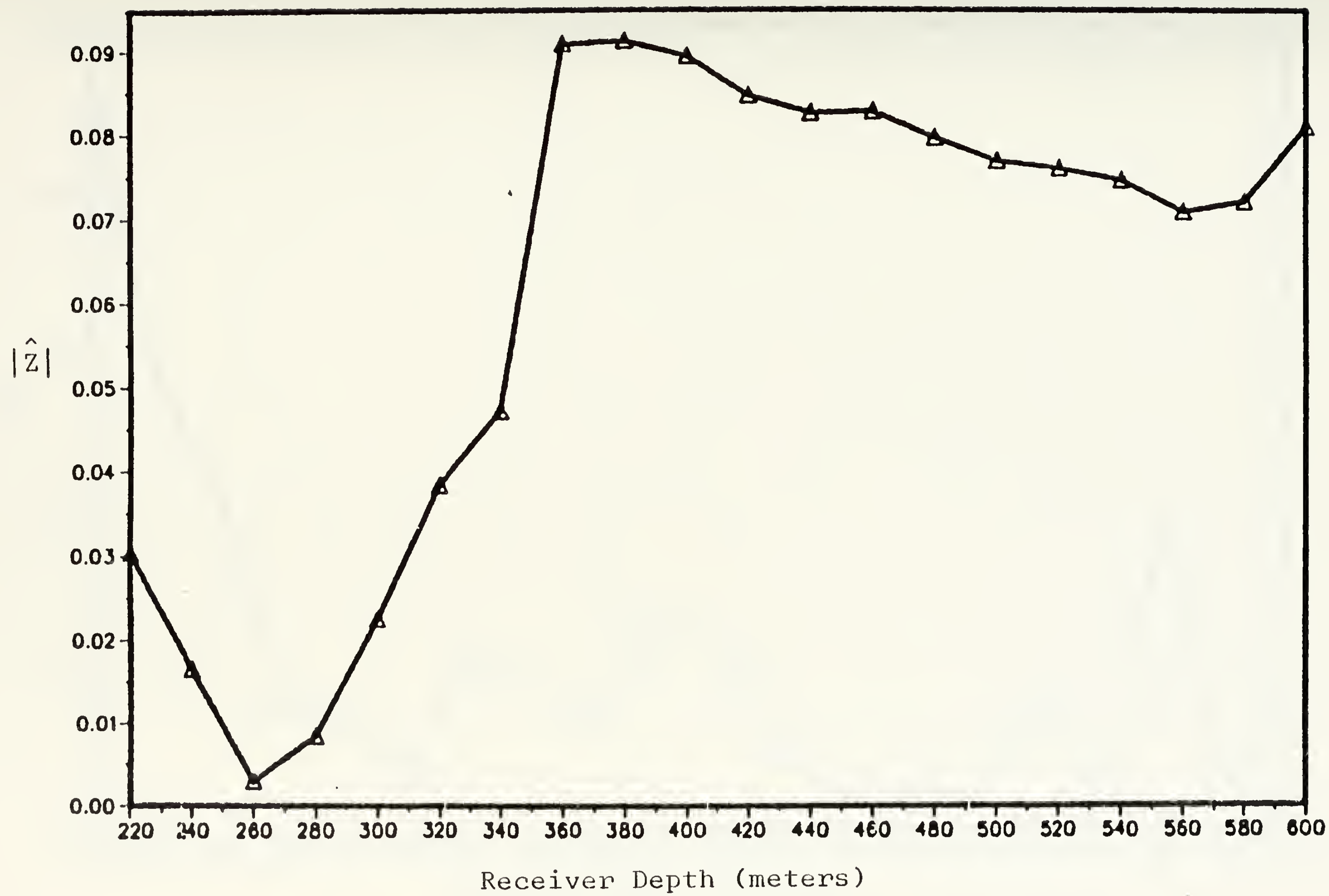


Figure 4.5 Beam Pattern (20 depths 2 receivers, source at 380 m)

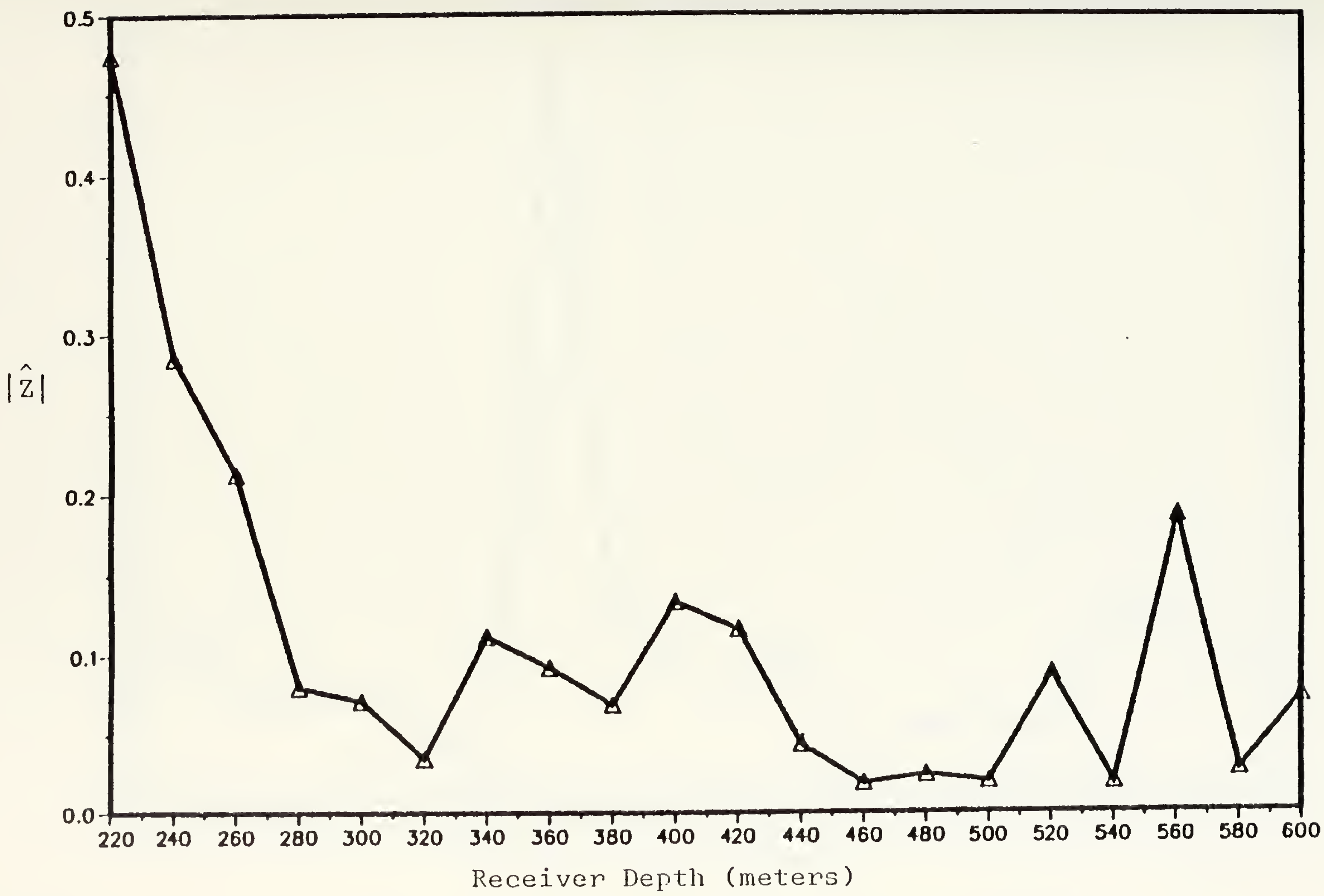


Figure 4.6 Beam Pattern (20 depths 5 receivers, source at 220 m)

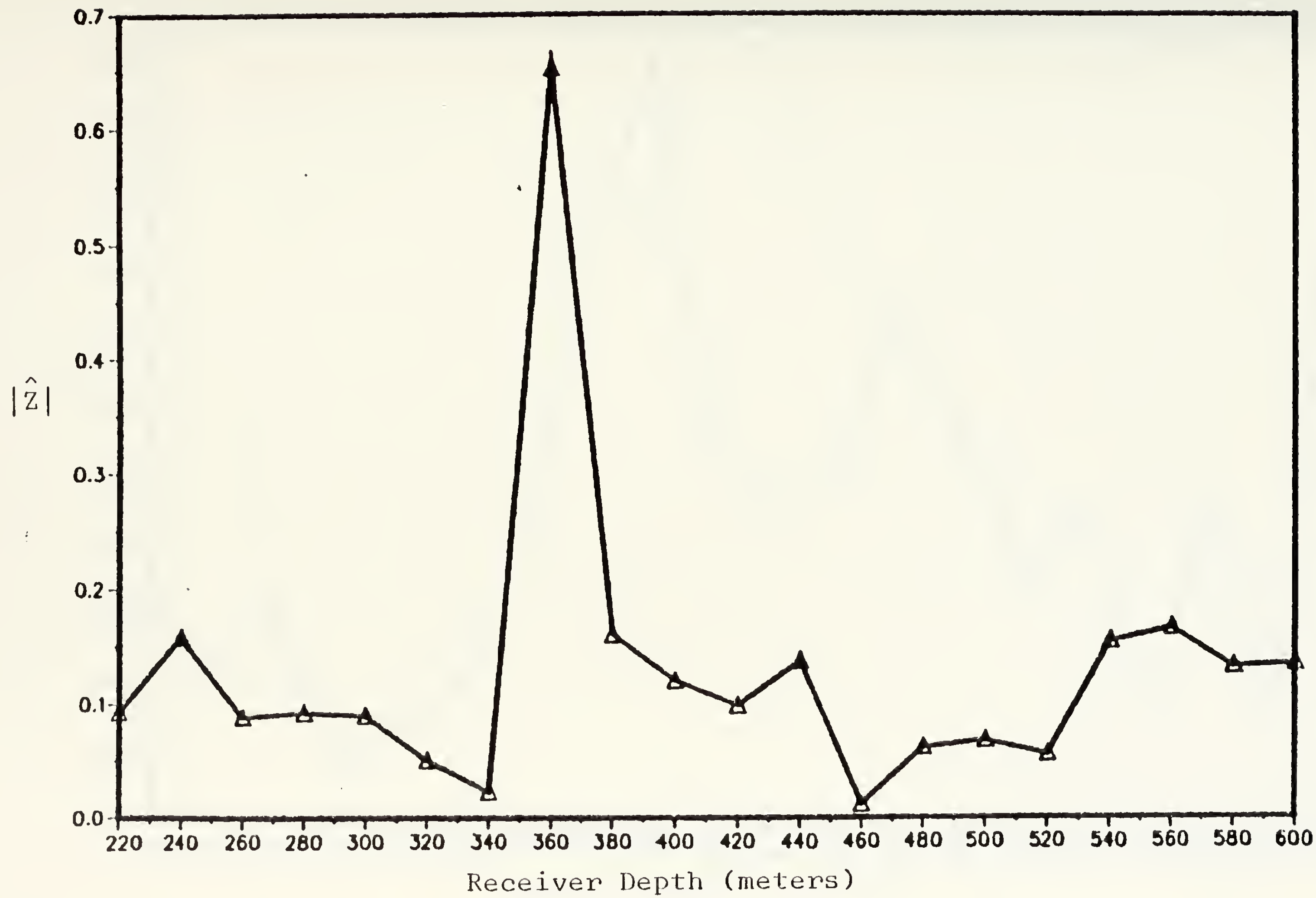


Figure 4.7 Beam Pattern (20 depths 5 receivers, source at 360 m)


Figure 4.8 Beam Pattern (20 depths 5 receivers, source at 380 m)

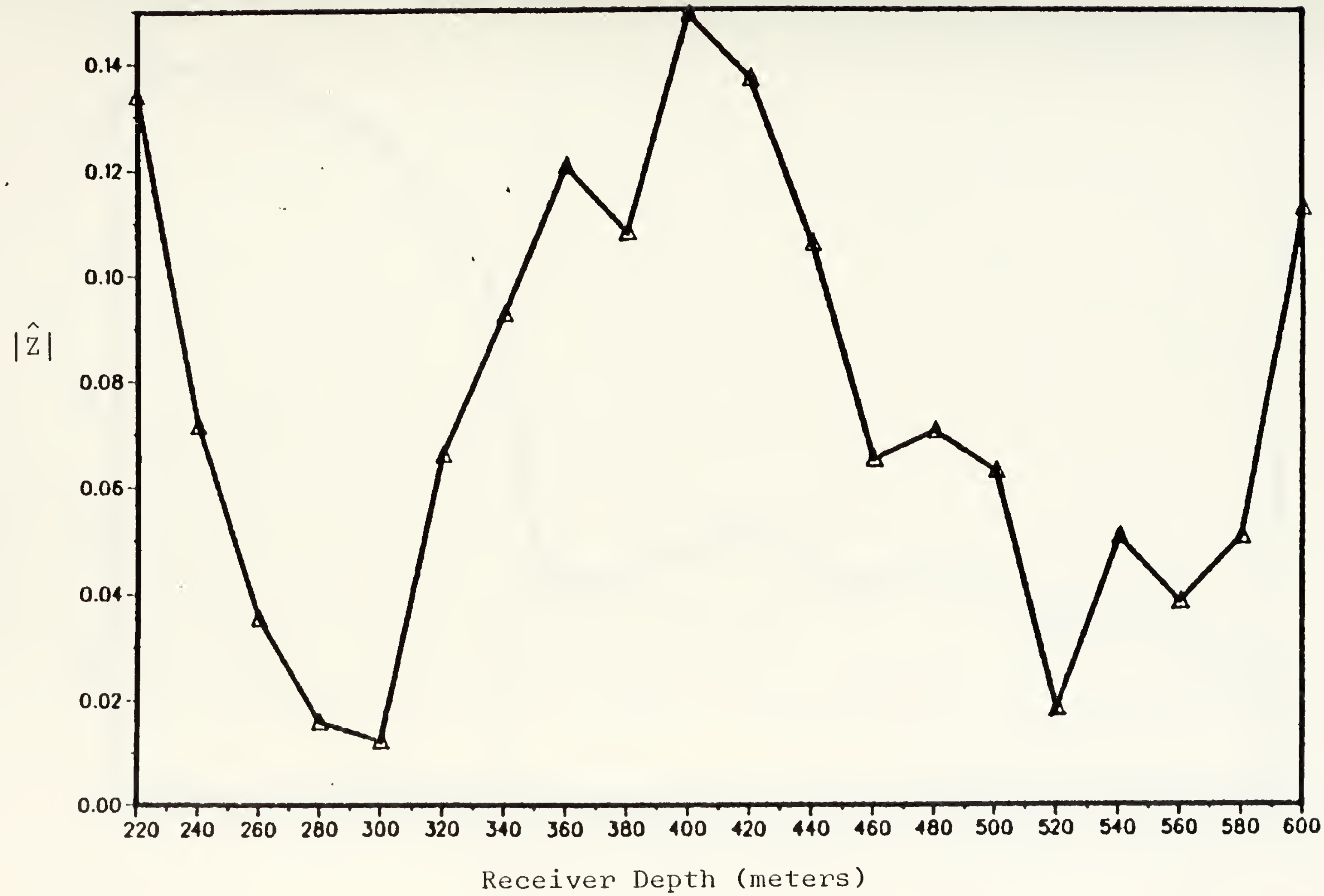


Figure 4.9 Beam Pattern (20 depths 5 receivers, source at 400 m)

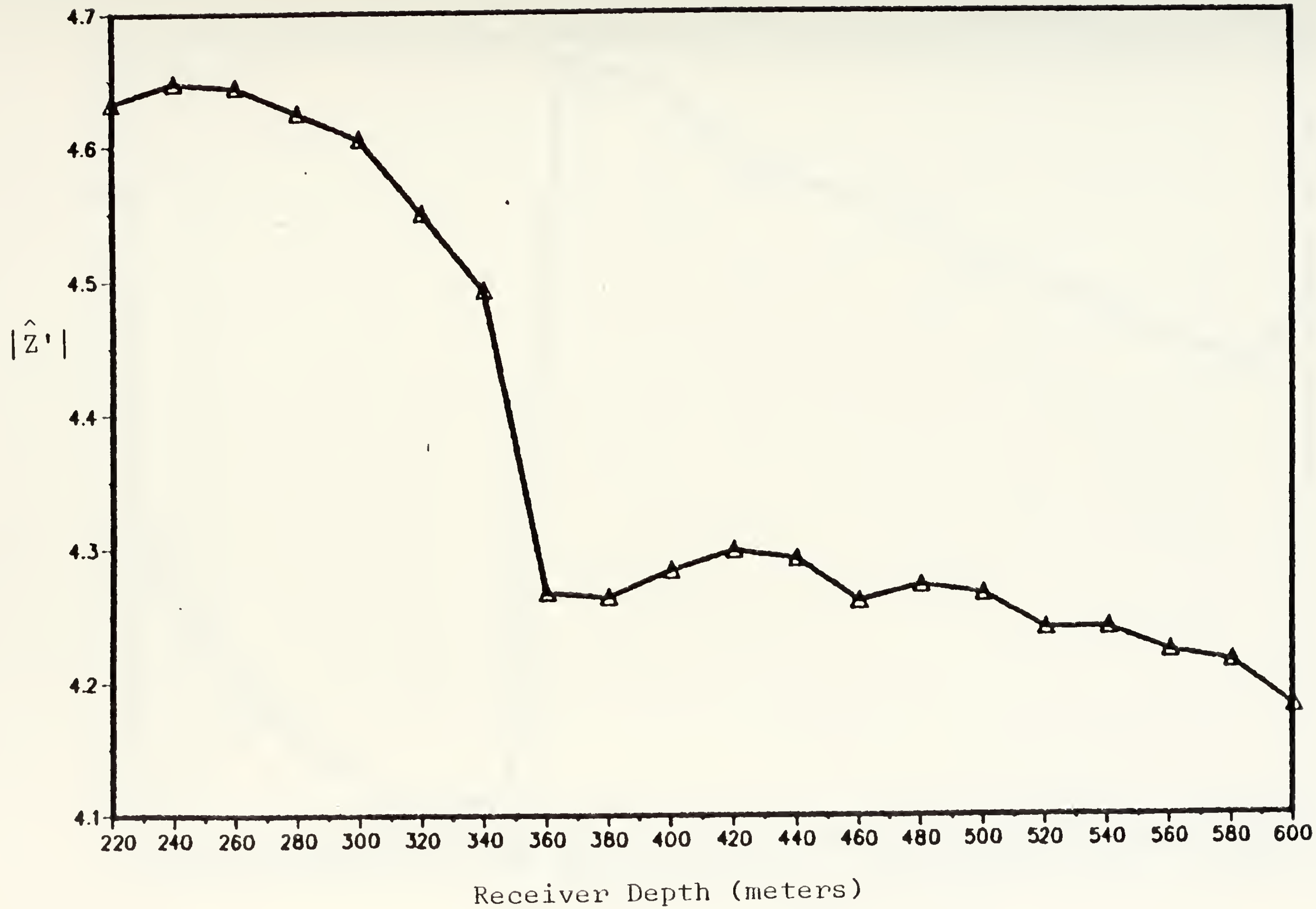


Figure 4.10 Conventional Beam Pattern (source at 380 meters)

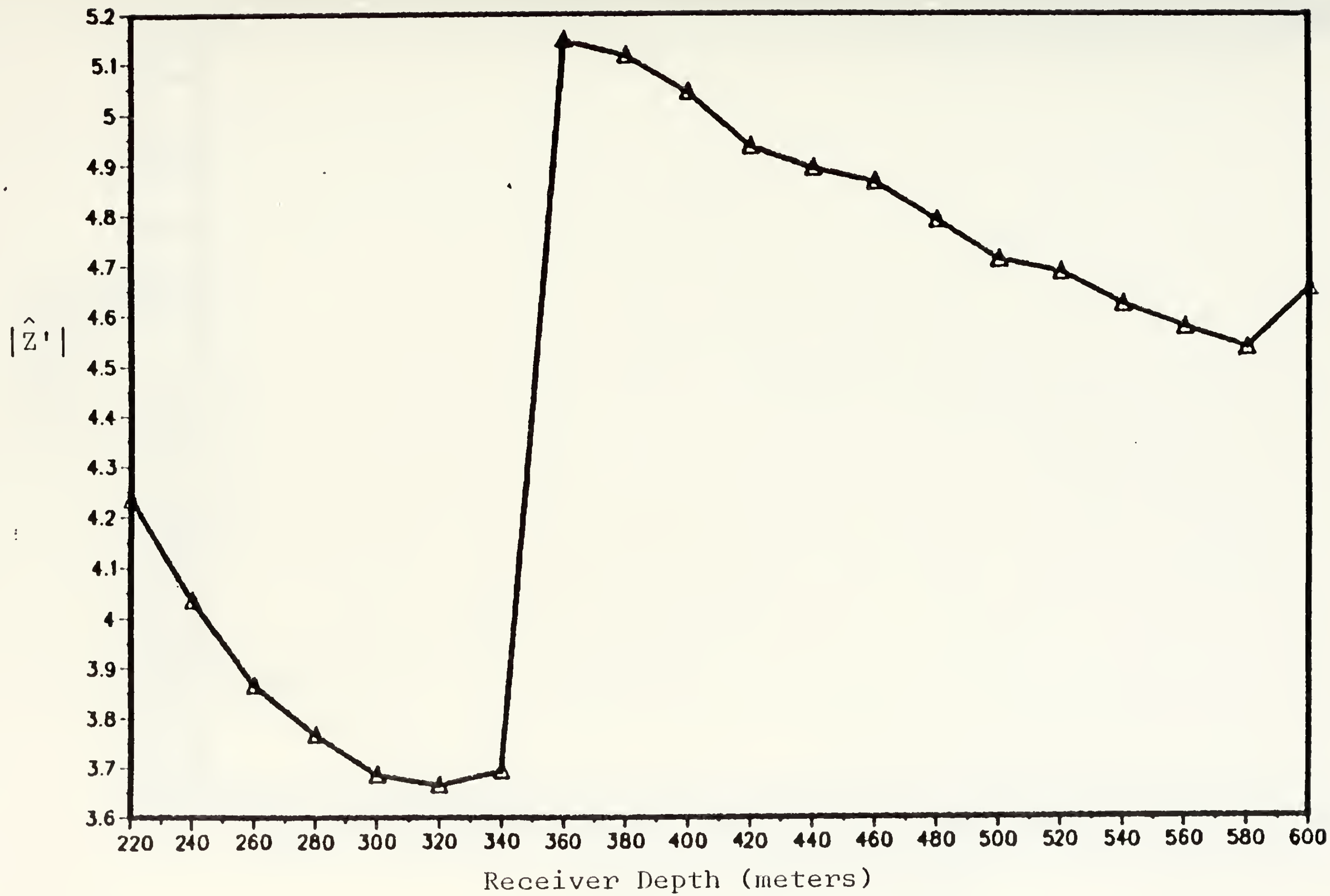


Figure 4.11 Conventional Beam Pattern (non-unity amp. wts. source at 380 m)

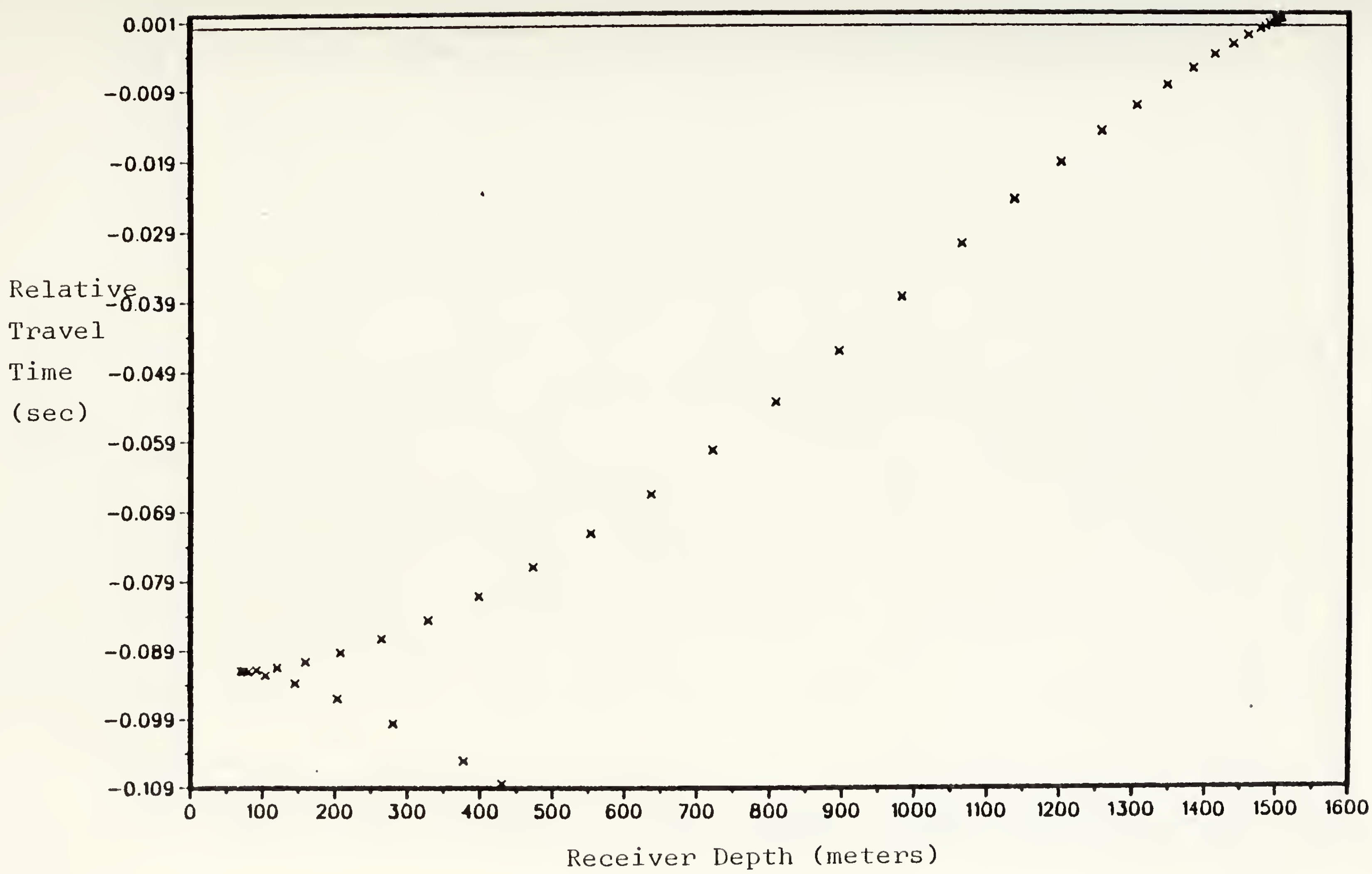


Figure 4.12 Relative Travel Time vs. Depth (range=250 km, source at 220 m)

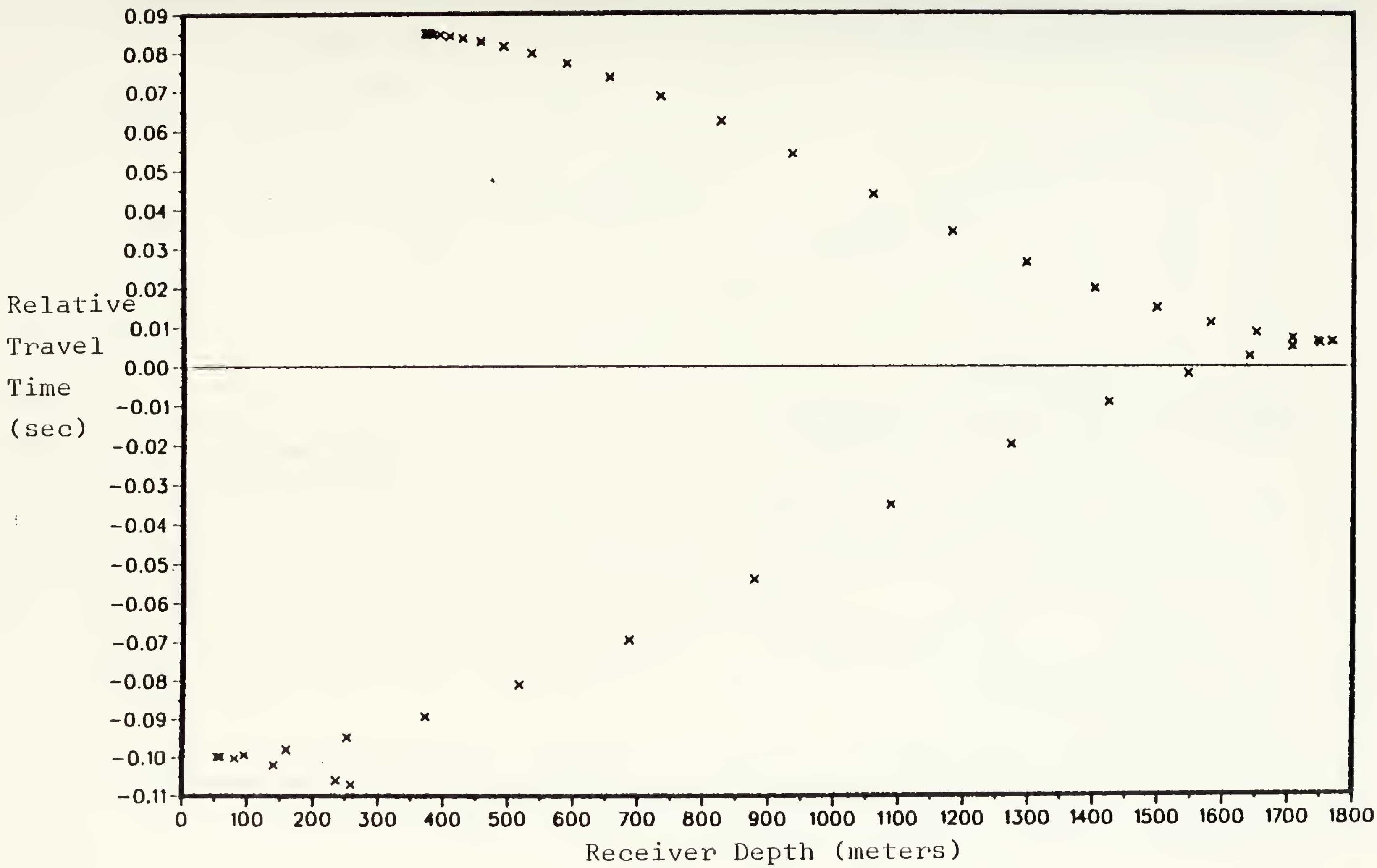


Figure 4.13 Relative Travel Time vs. Depth (range=250 km, source at 380 m)

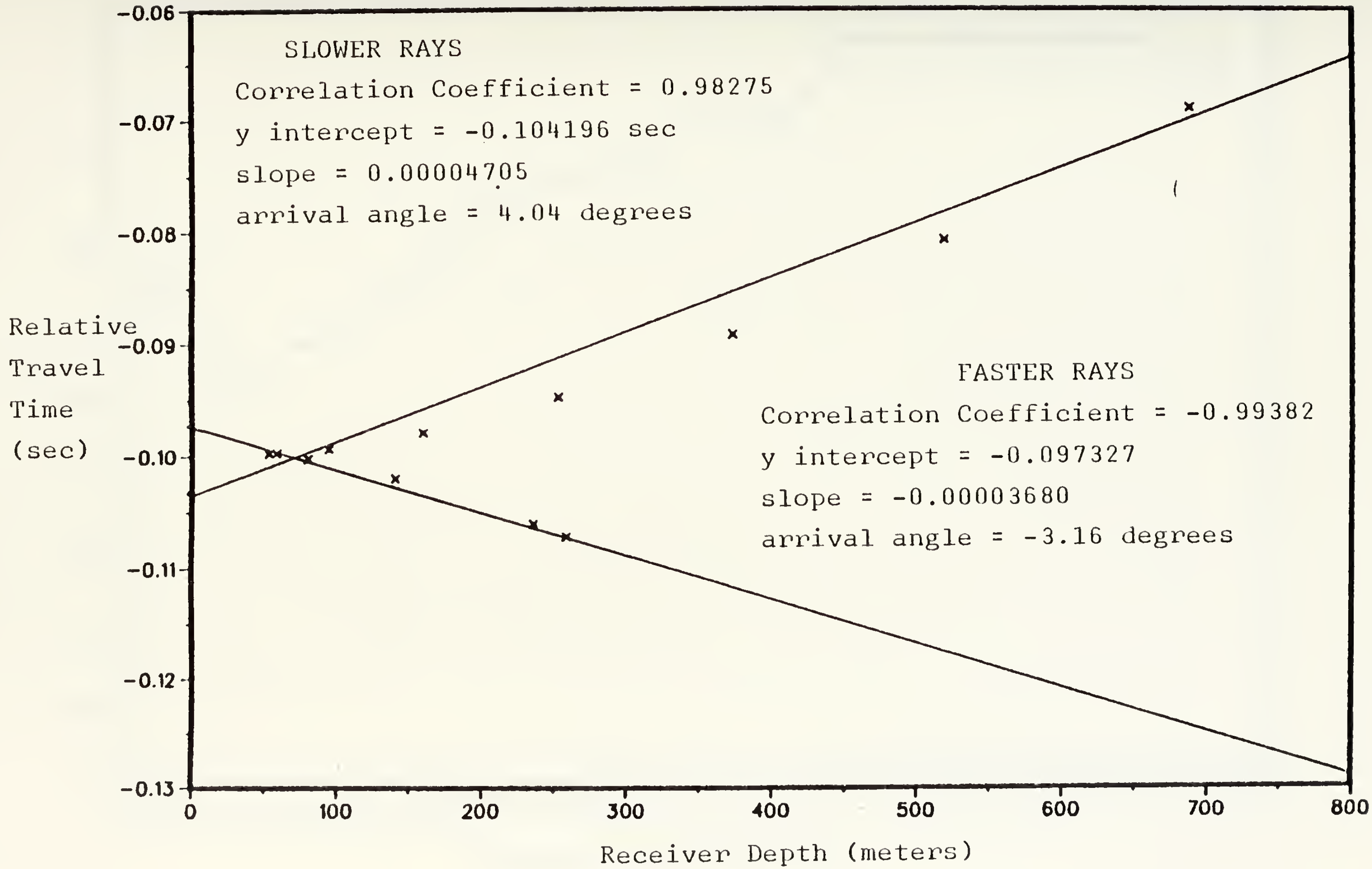


Figure 4.14 St. Line Approx. of Rel. Trav. Time vs. Depth (R=250 km, d=380 m)

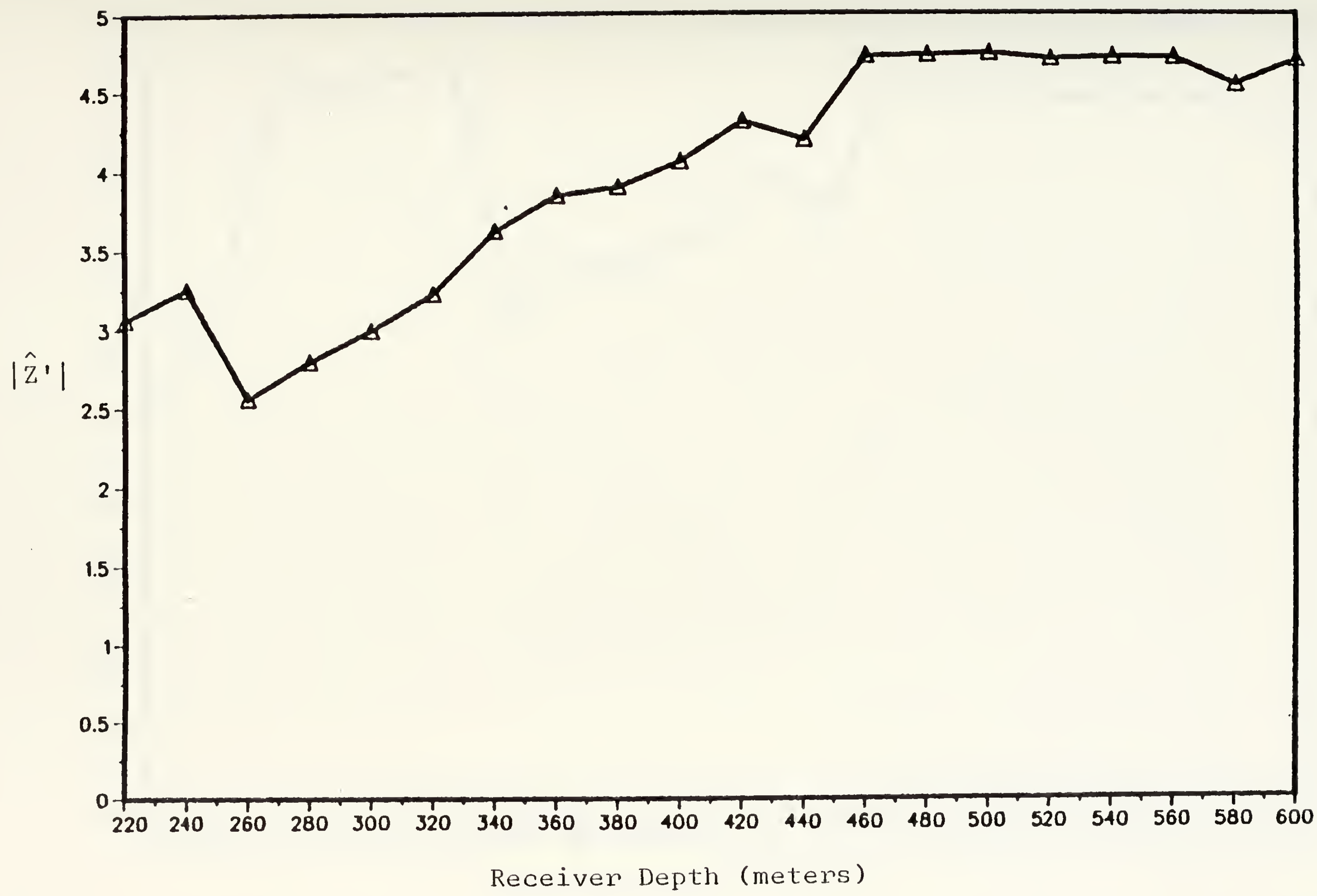


Figure 4.15 Conventional Beam Pattern (R=250 km, d=380 m, slower times, a=1)


Figure 4.16 Conventional Beam Pattern (R=250 km, d=380 m, slower times, LMV a)

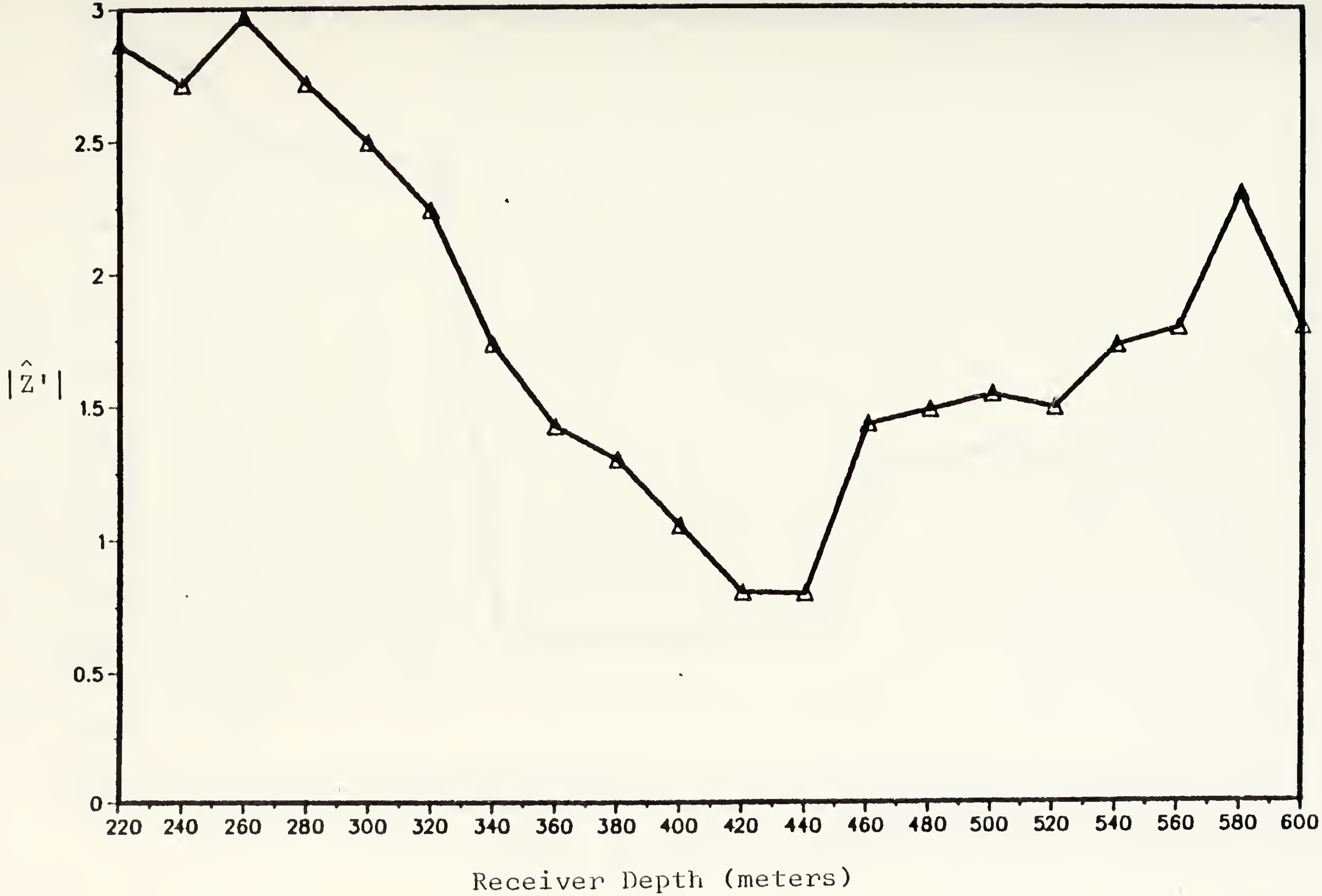


Figure 4.17 Conventional Beam Pattern (R=250 km, d=380 m, faster times, a=1)

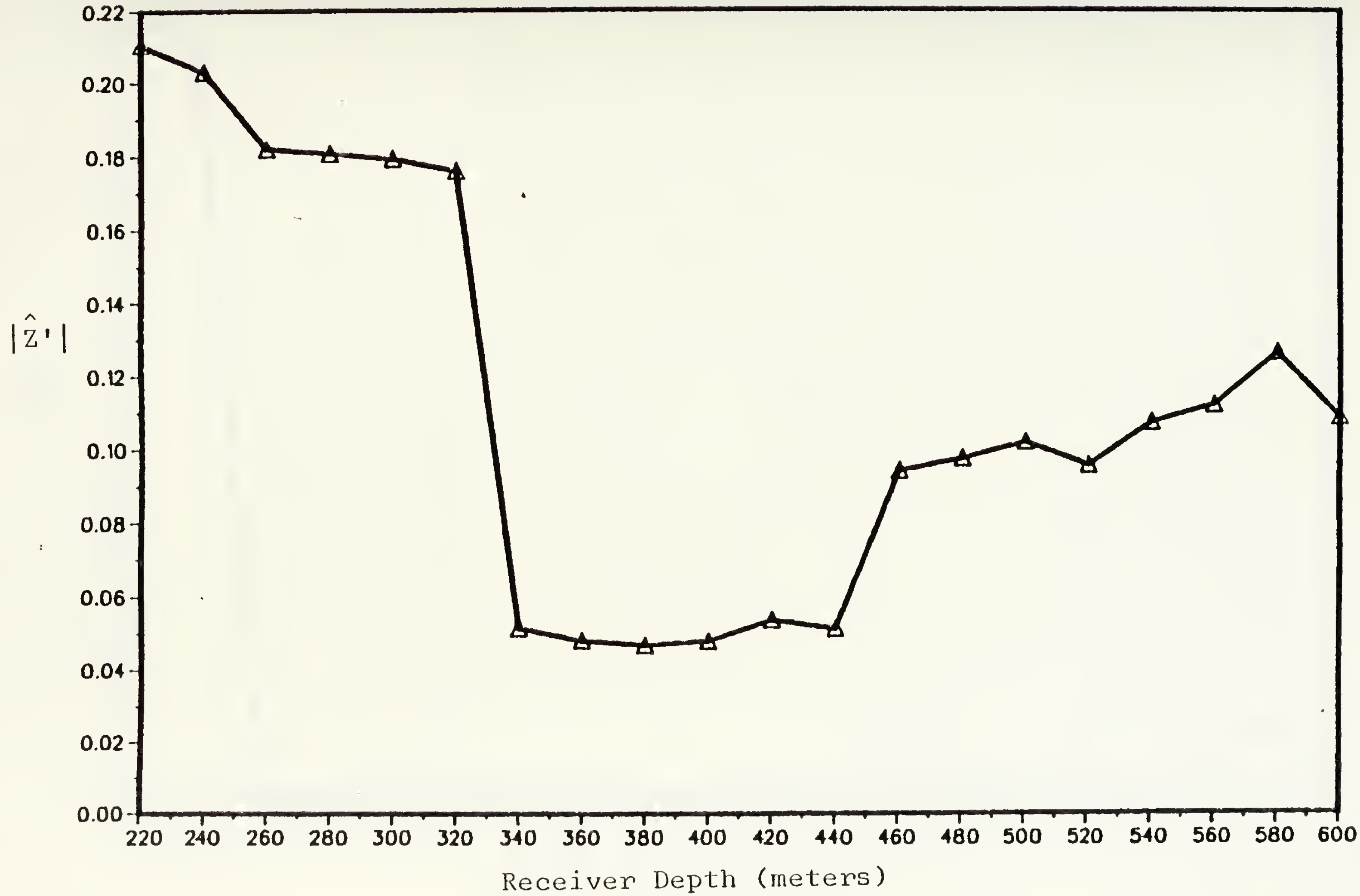


Figure 4.18 Conventional Beam Pattern (R=250 km, d=380 m, faster times, LMV a)

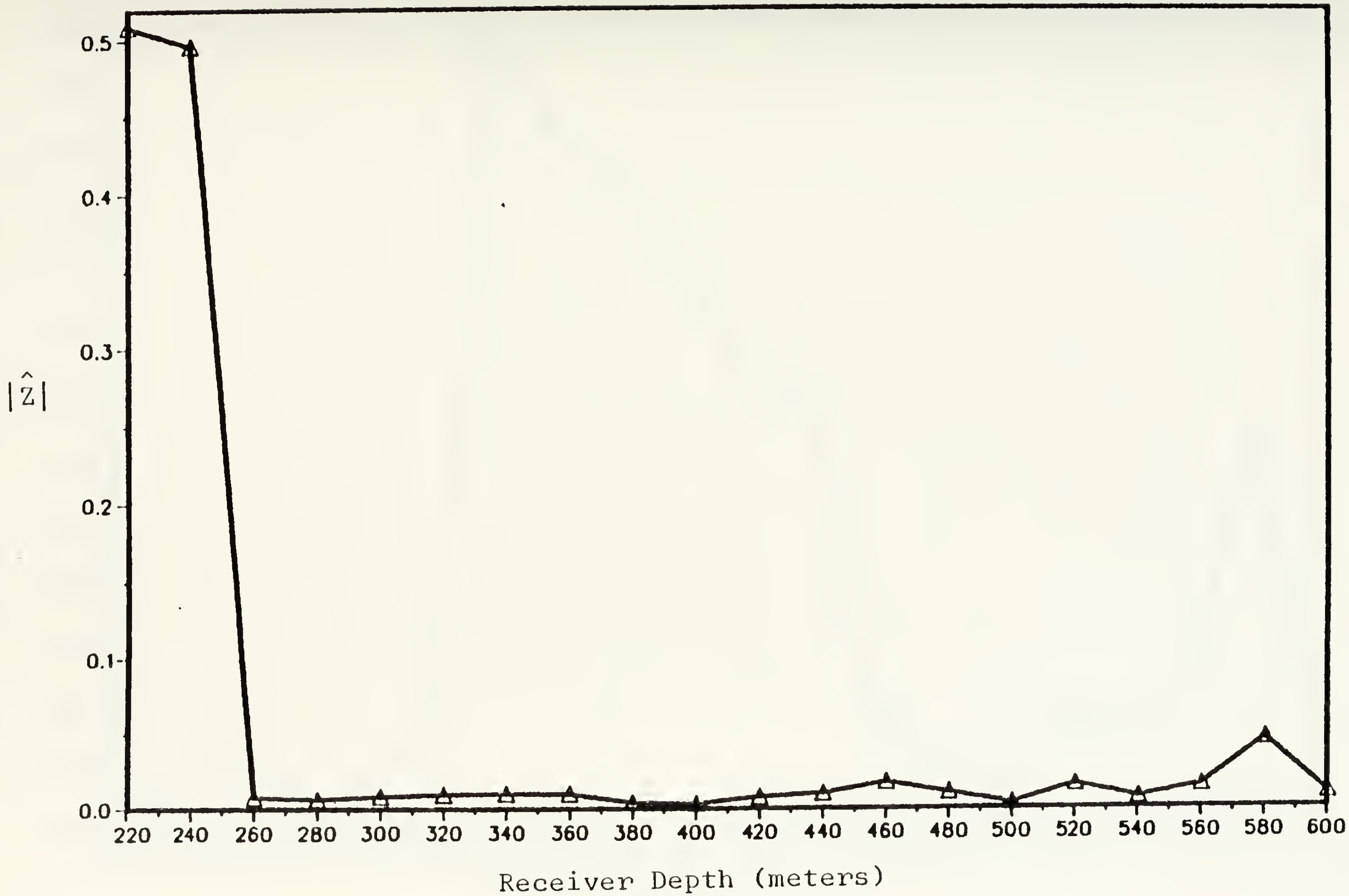


Figure 4.19 Beam Pattern (R=250 km, 20 depths, 5 receivers, source at 220 m)


Figure 4.20 Beam Pattern (R=250 km, 20 depths, 5 receivers, source at 340 m)


Figure 4.21 Beam Pattern (R=250 km, 20 depths, 5 receivers, source at 360 m)

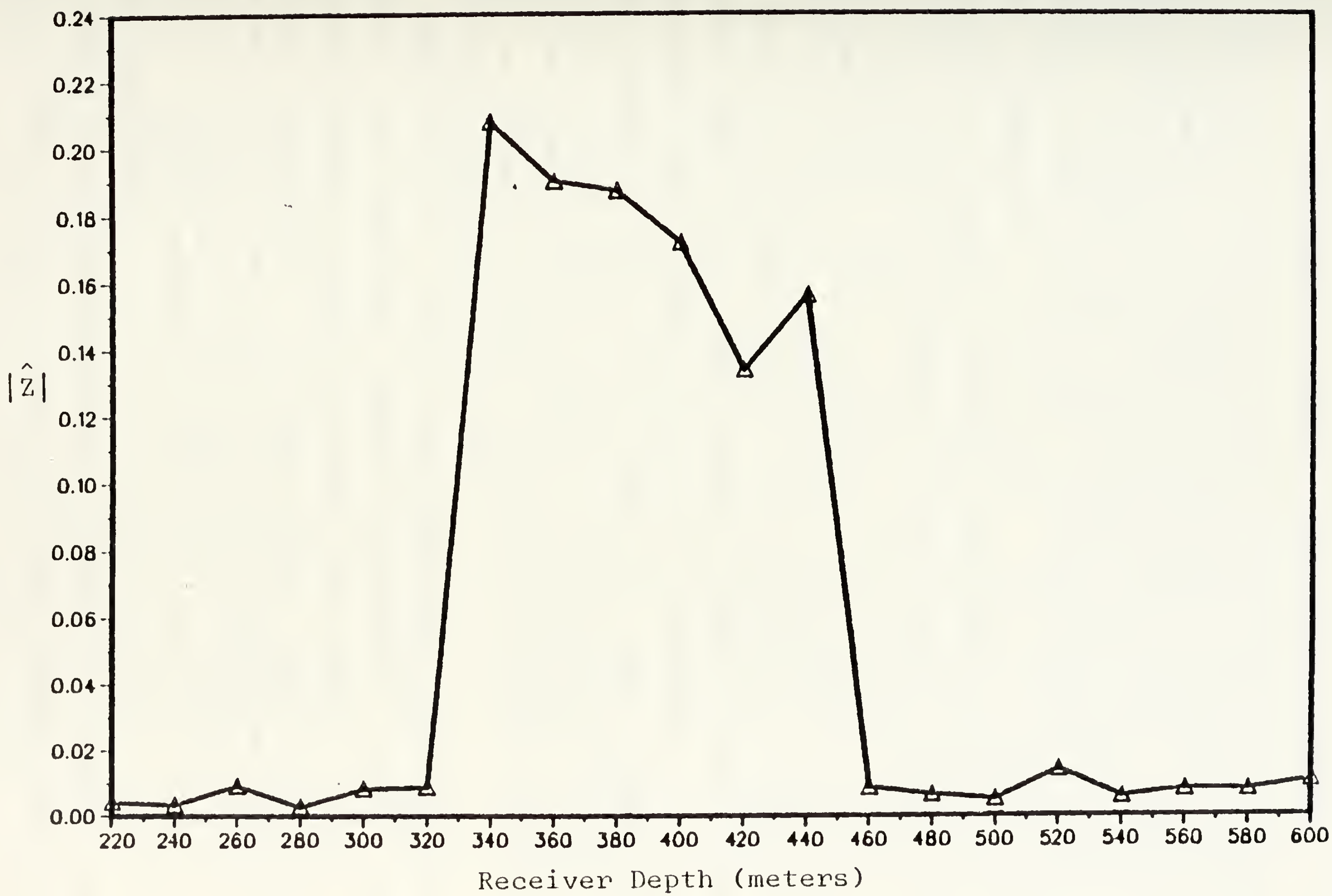


Figure 4.22 Beam Pattern (R=250 km, 20 depths, 5 receivers, source at 380 m)

## V. DISCUSSION OF RESULTS AND RECOMMENDATIONS

For many of the cases examined, the linear minimum variance estimation technique gives high resolution in the "depth beam pattern" for sources at long range. Figures 4.2 and 4.3 represent beam patterns for four depths with two receivers where the main lobe is at the desired source depth with a beamwidth of 30 meters. For 20 depths with 2 receivers the system is highly overdetermined and the beam pattern portrayed in figure 4.4 has its main lobe 80 meters from the source depth. At the source depth the strength of the beam pattern is 3.6 db down. For figure 4.5 the main lobe is on the source depth with a beamwidth of 250 meters.

For all 20 source depths with 5 hydrophone receivers the results range from a beam pattern having its main lobe on the source depth (figure 4.7) with a beamwidth of 20 meters and a secondary lobe 12.1 db down to a beam pattern having its main lobe 40 meters from the source depth (figure 4.22). For the beam pattern in figure 4.22 the width of the main lobe is 80 meters and the strength of the beam pattern at the source depth is 0.8 db down.

When using the conventional beamformer, the beam pattern results with amplitude weights determined by the linear minimum variance estimation method were superior to the beam pattern determined by using unity amplitude weights. However in all cases the conventional beamformer was significantly inferior to the beam pattern determined by the linear minimum variance estimation technique in terms of depth resolution.

The linear minimum variance estimation technique could not be used for all ranges with the chosen sound speed profile. For ranges of 187 km and 235 km the relative

travel times produce an '$\underline{A}$' matrix which is so ill-conditioned that the 'IMSL' subroutine 'LINV2F' cannot determine an accurate inverse of '$\underline{A}^T\underline{A}$'.

There is sufficient evidence from this initial investigation that the linear minimum variance estimation technique applied to a long linear vertical array can yield high resolution depth information about passive sources at very long ranges. However, further investigation is needed before any field tests are in order. Recommendations for further research are:

1. The use of a more realistic speed of sound profile, preferably one actually characteristic of the 'SOFAR' channel.
2. The use of more hydrophones in the vertical array. As more receivers are used the beam pattern should approach the desired beam pattern more closely (a less overdetermined system should yield smaller total minimum mean square error).
3. Investigation into causes for the peak response falling at other than the desired depth and techniques for correcting this problem when using the linear minimum variance estimation techinique.
4. Alternate assumption for choosing the ray paths to include in the '$\underline{A}$' matrix. For example one might choose the rays with the greatest intensity instead of the shortest travel time.
5. The possibility of estimating range as well as the depth of a passive source with linear minimum variance estimation techniques.

In conclusion, the linear minimum variance estimation technique of beamforming was significantly superior to the conventional beamformer. High resolution depth information about passive sources at long ranges is provided.

## APPENDIX A

## RELATIVE TRAVEL TIME CALCULATION

This program calculates the relative travel times from each source depth to the horizontal range and the depth of the ray at this range. To save paper, only the two shallowest source depths are listed.

```
C     PROGRAM WHICH CALCULATES RELATIVE TRAVEL TIMES AND DEPTHS OF
C     THESE RAYS AT THE SELECTED RANGE
C
C     DESCRIPTION OF IMPORTANT PARAMETERS
C
C*     F3 - MAXIMUM UPWARD ANGLE WHICH YIELDS R-R RAY
C*     CDEP - CHANGE IN VERTICAL DEPTH
C*     RANGE - HORIZONTAL RANGE FROM SOURCE TO RECEIVER
C*     RAN - VALUE OF SUMMED HORIZONTAL RANGE OF PRESENT RAY
C*     DEPTH - VALUE OF SUMMED DEPTH OF PRESENT RAY
C*     C1 - SPEED OF SOUND AT PRESENT DEPTH
C*     TIME - VALUE OF SUMMED TIME OF PRESENT RAY
C*     X1 - COSINE OF INITIAL DEPRESSION ANGLE OF ARC OF PRESENT RAY
C*     Y1 - SINE OF INITIAL DEPRESSION ANGLE OF ARC OF PRESENT RAY
C*     X2 - COSINE OF FINAL DEPRESSION ANGLE OF ARC OF PRESENT RAY
C*     Y2 - SINE OF FINAL DEPRESSION ANGLE OF ARC OF PRESENT RAY
C*     REFTI - TOTAL TRAVEL TIME OF THE REFERENCE RAY
C*     G - VELOCITY GRADIENT
C
C     THE REFERENCE RAY IS AT THE SHALLOWEST SOURCE DEPTH AND HAS AN
C     INITIAL DEPRESSION ANGLE OF 0 DEGREES
C
C     DEPRESSION ANGLES WHICH ARE DOWNWARD WITH RESPECT TO THE
C     HORIZONTAL AXIS ARE POSITIVE
C
       DIMENSION X(600),Y(600)
       DOUBLE PRECISION A1,X2,F2,C1,D,C,TIME,T,PI,Y1,X1,T1,G,Y2,F1,R6,P,
     *R11,CDEP,RAN,RANGE,ANGLE,DEPTH,D1,DRAN,R2,R3,T3,ERAN,T4,FRAN,R4,
     *R5,T5,T6,GRAN,HRAN,D2,D3,D4,D5,D6,R1A,DEGRE,X3,F3,D7,D8,D9,D10,
     *T7,T8,T9,T10,RAN1,RAN2,RAN3,RAN4,TI,X,Y,RANIN,REFTI
C
C      SET INITIAL CONDITIONS
C
       RANIN=0.0D0
       K1=1
3      DEPTH=200.0D0
5      DEPTH=DEPTH+20.0D0
       RANGE=20000.0D0
       G=-0.017D0
       I=0
       N=0
       K=0
       RAN=0.0D0
       R1A=0.0D0
       ANGLE=0.0D0
       C1=1500.0D0+G*DEPTH
       A1=ANGLE
       R=RANGE
```

```
      D=DEPTH
      C=C1
      TIME=0.0D0
      T=TIME
C
C   CALCULATE INITIAL MAXIMUM UPWARD DEPRESSION ANGLE
C
      X3=C1/((G*(-D))+C1)
66    F3=-DARCOS(X3)
      WRITE(6,600)
      WRITE(8,600)
600   FORMAT(3X,'DEPTH',3X,'RELATIVE TRAVEL TIME',2X,'INITIAL ANGLE')
      PI=3.14159265358980
      P=PI/4.0D0
C
C    CALCULATE HORIZONTAL RANGE OF ARC 1
C
      Y1=DSIN(A1)
      X1=DCOS(A1)
88    G=-0.017D0
      X2=(((1000.0D0-DEPTH)*G*X1)/C1)+X1
      F2=DARCOS(X2)
      Y2=DSIN(F2)
      RAN=(C1*(Y1-Y2))/(G*X1)+RAN
      R1=RAN
      IF(RANGE-RAN)11,11,13
C
C  IF HORIZONTAL RANGE OF ARC 1 IS LESS THAN TOTAL HORIZONTAL RANGE
C  THEN CONTINUE PROGRAM, OTHERWISE EXIT TO 11
C
C    CALCULATE TRAVEL TIME OF ARC 1
C
13    IF(TIME)83,83,84
83    TIME=(-1.0D0/G)*DLOG((DTAN(P+(F2/2.0D0)))/DTAN(P+(A1/2.0D0)))
      GO TO 79
84    TIME=(-1.0D0/G)*DLOG((DTAN(P+(F2/2.0D0)))/DTAN(P))+TIME
C
C     CALCULATE AND ADD ARC 2 TO SUM OF PRESENT HORIZONTAL RANGE
C
79    T1=TIME
      Y1=Y2
      X1=X2
      CDEP=1000.0D0-DEPTH
      C1=C1+G*CDEP
      DEPTH=DEPTH+CDEP
      D1=DEPTH
      G=0.017D0
      CDEP=C1*(1.0D0-X1)/(G*X1)
```

```
      CRAN=(C1*Y1)/(G*X1)
      RAN=RAN+CRAN
      R11=RAN
      IF(RANGE-RAN)12,12,14
C
C     IF HORIZONTAL RANGE OF ARC 1 AND 2 IS LESS THAN THE TOTAL
C     RANGE THEN CONTINUE PROGRAM, OTHERWISE EXIT TO 12
C
C     CALCULATE SUMMED TRAVEL TIME
C
14    C1=C1+G*CDEF
      DEPTH=DEPTH+CDEP
      D2=DEPTH
      TIME=TIME+(-1.0D0/G)*DLOG((DTAN(P))/(DTAN(P+(F2/2.0D0))))
C
C     ADD ARC 3
C
      T2=TIME
      CDEP=-CDEP
      X2=((CDEP*G)/C1)+1.0D0
      F2=-DARCOS(X2)
      Y2=DSIN(F2)
      DRAN=(C1*(-Y2))/G
      RAN=RAN+DRAN
      R2=RAN
      IF(RANGE-RAN)21,21,22
C
C     ADD ARC 4
C
 22   C1=C1+G*CDEF
      DEPTH=DEPTH+CDEP
      D3=DEPTH
      TIME=TIME+(-1.0D0/G)*DLOG((DTAN(P+(F2/2.0D0)))/(DTAN(P)))
      T3=TIME
      G=-0.017D0
      X1=X2
      Y1=Y2
      CDEP=(C1*(1.0D0-X1))/(G*X1)
      ERAN=C1*Y1/(G*X1)
      F1=DARSIN(Y1)
      RAN=RAN+ERAN
      R3=RAN
      IF(RANGE-RAN)41,41,42
C
C     ADD ARC 5
C
42    C1=C1+G*CDEF
      DEPTH=DEPTH+CDEP
```

```
      D4=DEPTH
      TIME=TIME+(-1.0D0/G)*DLOG((DTAN(P))/(DTAN(P+(F1/2.0D0))))
      T4=TIME
      CDEP=-CDEP
      X2=((CDEP*G)/C1)+1.0D0
      F2=DARCOS(X2)
      Y2=DSIN(F2)
      FRAN=(C1*(-Y2))/G
      RAN=RAN+FRAN
      R4=RAN
      IF(RANGE-RAN)51,51,52
C
C     ADD ARC 6
C
52    C1=C1+G*CDEF
      DEPTH=DEPTH+CDEP
      D5=DEPTH
      TIME=TIME+(-1.0D0/G)*DLOG((DTAN(P+F2/2.0D0))/(DTAN(P)))
      T5=TIME
      X1=X2
      Y1=Y2
      G=0.017D0
      CDEP=(C1*(1.0D0-X1))/(G*X1)
      GRAN=(C1*Y1)/(G*X1)
      RAN=GRAN+RAN
      R5=RAN
      IF(RANGE-RAN)61,61,62
C
C     ADD ARC 7
C
62    C1=C1+G*CDEF
      DEPTH=DEPTH+CDEP
      D6=DEPTH
      TIME=TIME+(-1.0D0/G)*DLOG((DTAN(P))/(DTAN(P+(F2/2.0D0))))
      T6=TIME
      CDEP=-CDEP
      X2=(CDEP*G)/C1+1.0D0
      F2=-DARCOS(X2)
      Y2=DSIN(F2)
      HRAN=(C1*(-Y2))/G
      RAN=RAN+HRAN
      R6=RAN
      IF(RANGE-RAN)71,71,92
C
C     ADD ARC 8
C
92    C1=C1+G*CDEF
      DEPTH=DEPTH+CDEP
```

```
      D7=DEPTH                                                          THE01930
      TIME=TIME+(-1.0D0/G)*DLOG((DTAN(P+(F2/2.0D0)))/(DTAN(P)))         THE01940
      T7=TIME                                                           THE01950
      G=-0.017D0                                                        THE01960
      X1=X2                                                             THE01970
      Y1=Y2                                                             THE01980
      CDEP=(C1*(1.0D0-X1))/(G*X1)                                       THE01990
      RAN1=C1*Y1/(G*X1)                                                 THE02000
      F1=DARSIN(Y1)                                                     THE02010
      RAN=RAN+RAN1                                                      THE02020
      R7=RAN                                                            THE02030
      IF(RANGE-RAN)101,101,102                                          THE02040
C                                                                       THE02050
C     ADD ARC 9                                                         THE02060
C                                                                       THE02070
102   C1=C1+G*CDEP                                                      THE02080
      DEPTH=DEPTH+CDEP                                                  THE02090
      D8=DEPTH                                                          THE02100
      TIME=TIME+(-1.0D0/G)*DLOG((DTAN(P))/(DTAN(P+(F1/2.0D0))))         THE02110
      T8=TIME                                                           THE02120
      CDEP=-CDEP                                                        THE02130
      X2=((CDEP*G)/C1)+1.0D0                                            THE02140
      F2=DARCOS(X2)                                                     THE02150
      Y2=DSIN(F2)                                                       THE02160
      RAN2=(C1*(-Y2))/G                                                 THE02170
      RAN=RAN+RAN2                                                      THE02180
      R8=RAN                                                            THE02190
      IF(RANGE-RAN)111,111,112                                          THE02200
C                                                                       THE02210
C     ADD ARC 10                                                        THE02220
C                                                                       THE02230
112   C1=C1+G*CDEP                                                      THE02240
      DEPTH=DEPTH+CDEP                                                  THE02250
      D9=DEPTH                                                          THE02260
      TIME=TIME+(-1.0D0/G)*DLOG((DTAN(P+F2/2.0D0))/(DTAN(P)))           THE02270
      T9=TIME                                                           THE02280
      X1=X2                                                             THE02290
      Y1=Y2                                                             THE02300
      G=0.017D0                                                         THE02310
      CDEP=(C1*(1.0D0-X1))/(G*X1)                                       THE02320
      RAN3=(C1*Y1)/(G*X1)                                               THE02330
      RAN=RAN+RAN3                                                      THE02340
      R9=RAN                                                            THE02350
      IF(RANGE-RAN)121,121,122                                          THE02360
C                                                                       THE02370
C     ADD ARC 11                                                        THE02380
C                                                                       THE02390
122   C1=C1+G*CDEP                                                      THE02400
```

```
      DEPTH=DEPTH+CDEP
      D10=DEPTH
      TIME=TIME+(-1.0D0/G)*DLOG((DTAN(P))/(DTAN(P+(F2/2.0D0))))
      T10=TIME
      CDEP=-CDEP
      X2=(CDEP*G)/C1+1.0D0
      F2=-DARCOS(X2)
      Y2=DSIN(F2)
      RAN4=(C1*(-Y2))/G
      RAN=RAN+RAN4
      R6=RAN
      IF(RANGE-RAN)131,131,92
C
C     CONTINUE ADDING ARCS IF SUMMED HORIZONTAL RANGE IS LESS THAN
C     TOTAL RANGE, OTHERWISE EXIT TO 131
C
C
C     CALCULATES TRAVEL TIME OF REFERENCE RAY
C
11    Y2=Y1-((RANGE-R1A)*G*X1)/C1
      F2=DARSIN(Y2)
      X2=DCOS(F2)
      CDEP=C1*(X2-X1)/(G*X1)
      DEPTH=DEPTH+CDEP
      IF(TIME)7,7,8
8     TIME=(-1.0D0/G)*DLOG((DTAN(P+(F2/2.0D0)))/DTAN(P))+TIME
      GO TO 32
7     TIME=(-1.0D0/G)*DLOG((DTAN(P+(F2/2.0D0)))/(DTAN(P+(A1/2.0D0))))
32    DEGRE=A1*180.0D0/PI
      N=N+1
      IF(A1)72,29,72
29    K1=K1-1
      IF(K1)72,811,72
811   REFTI=TIME
      GO TO 72
C
C     SUBROUTINE TO CALCULATE DEPTH AND TIME IF HORIZONTAL RANGE OF
C     ARC 1 IS GREATER THAN SOURCE-RECEIVER RANGE
C
12    F1=F2
      Y2=Y1-((RANGE-R1)*G*X1)/C1
      F2=DARSIN(Y2)
      X2=DCOS(F2)
      CDEP=C1*(X2-X1)/(G*X1)
      TIME=T1+(-1.0D0/G)*DLOG((DTAN(P+(F2/2.0D0)))/DTAN(P+(F1/2.0D0)))
      DEPTH=D1+CDEP
      GO TO 32
C
```

```
C       SUBROUTINE TO CALCULATE DEPTH AND TIME IF ADDITION OF RANGE OF
C       ARC 2 CAUSES SUMMED RANGE TO BE GREATER THAN SOURCE-RECEIVER
C       RANGE
C
21      Y2=-(RANGE-R11)*G/C1
        F2=DARSIN(Y2)
        X2=DCOS(F2)
        CDEP=C1*(X2-1.0D0)/G
        TIME=T2+(-1.0D0/G)*DLOG((DTAN(P+(F2/2.0D0)))/DTAN(P))
        DEPTH=D2+CDEP
        GO TO 32
C
C       USE IF ARC 3 CAUSES OVERFLOW
C
41      Y2=Y1-((RANGE-R2)*G*X1)/C1
        F2=DARSIN(Y2)
        X2=DCOS(F2)
        CDEP=C1*(X2-X1)/(G*X1)
        TIME=T3+(-1.0D0/G)*DLOG((DTAN(P+(F2/2.0D0)))/DTAN(P+(F1/2.0D0)))
        DEPTH=D3+CDEP
        GO TO 32
C
C       USE IF ARC 4 CAUSES OVERFLOW
C
51      Y2=-(RANGE-R3)*G/C1
        F2=DARSIN(Y2)
        X2=DCOS(F2)
        CDEP=C1*(X2-1.0D0)/G
        TIME=T4+(-1.0D0/G)*DLOG((DTAN(P+(F2/2.0D0)))/DTAN(P))
        DEPTH=D4+CDEP
        GO TO 32
C
C       USE IF ARC 5 CAUSES OVERFLOW
C
61      F1=F2
        Y2=Y1-((RANGE-R4)*G*X1)/C1
        F2=DARSIN(Y2)
        X2=DCOS(F2)
        CDEP=C1*(X2-X1)/(G*X1)
        TIME=T5+(-1.0D0/G)*DLOG((DTAN(P+(F2/2.0D0)))/DTAN(P+(F1/2.0D0)))
        DEPTH=D5+CDEP
        GO TO 32
C
C       USE IF ARC 6 CAUSES OVERFLOW
C
71      Y2=-(RANGE-R5)*G/C1
        F2=DARSIN(Y2)
        X2=DCOS(F2)
```

```
      CDEP=C1*(X2-1.0D0)/G
      TIME=T6+(-1.0D0/G)*DLOG((DTAN(P+(F2/2.0D0)))/DTAN(P))
      DEPTH=D6+CDEP
      GO TO 32
C
C     USE IF ARC 7 CAUSES OVERFLOW
C
101   Y2=Y1-((RANGE-R6)*G*X1)/C1
      F2=DARSIN(Y2)
      X2=DCOS(F2)
      CDEP=C1*(X2-X1)/(G*X1)
      TIME=T7+(-1.0D0/G)*DLOG((DTAN(P+(F2/2.0D0)))/DTAN(P+(F1/2.0D0)))
      DEPTH=D7+CDEP
      GO TO 32
C
C     USE IF ARC 8 CAUSES OVERFLOW
C
111   Y2=-(RANGE-R7)*G/C1
      F2=DARSIN(Y2)
      X2=DCOS(F2)
      CDEP=C1*(X2-1.0D0)/G
      TIME=T8+(-1.0D0/G)*DLOG((DTAN(P+(F2/2.0D0)))/DTAN(P))
      DEPTH=D8+CDEP
      GO TO 32
C
C     USE IF ARC 9 CAUSES OVERFLOW
C
121   F1=F2
      Y2=Y1-((RANGE-R8)*G*X1)/C1
      F2=DARSIN(Y2)
      X2=DCOS(F2)
      CDEP=C1*(X2-X1)/(G*X1)
      TIME=T9+(-1.0D0/G)*DLOG((DTAN(P+(F2/2.0D0)))/DTAN(P+(F1/2.0D0)))
      DEPTH=D9+CDEP
      GO TO 32
C
C     USE IF ARC 10 CAUSES OVERFLOW
C
131   Y2=-(RANGE-R9)*G/C1
      F2=DARSIN(Y2)
      X2=DCOS(F2)
      CDEP=C1*(X2-1.0D0)/G
      TIME=T10+(-1.0D0/G)*DLOG((DTAN(P+(F2/2.0D0)))/DTAN(P))
      DEPTH=D10+CDEP
      GO TO 32
C
C     DEPTH, RELATIVE TIME, AND INITIAL DEPRESSION ANGLE OF RAY ARE
C     PUT IN A LISTING
```

```
C
72    K=K+1
      Y(K)=TIME-REFTI
      X(K)=DEPTH
      WRITE(6,901)X(K),Y(K),DEGRE
      WRITE(8,901)X(K),Y(K),DEGRE
901   FORMAT(F10.4,F14.7,9X,F12.8)
C
C     INCREMENT THE INITIAL DEPRESSION ANGLE BY 0.1 DEGREES AND RESET
C     INITIAL CONDITIONS.  REPEAT THE OPERATION UNTIL MAXIMUM R-R RAY
C     INITIAL DEPRESSION ANGLE IS REACHED. LAST RAY FROM SOURCE DEPTH
C     IS AT THIS ANGLE.
C
      IF(A1)98,73,73
73    A1=A1+(PI*0.1D0)/180.0D0
      RANGE=R
      DEPTH=D
      C1=C
      RAN=0.0D0
      TIME=0.0D0
      Y1=DSIN(A1)
      X1=DCOS(A1)
      IF(A1+F3)88,88,97
97    I=I+1
      A1=-F3
      Y1=DSIN(A1)
      X1=DCOS(A1)
      IF(I-2)88,99,99
C
C
C     THE FOLLOWING SUBROUTINE IS FOR NEGATIVE (UPWARD) INITIAL
C     DEPRESSION ANGLES
C
96    A1=ANGLE
98    RANGE=R
      DEPTH=D
      C1=C
      A1=A1-(PI*0.1D0)/180.0D0
      Y1=DSIN(A1)
      X1=DCOS(A1)
      IF(A1+(-F3))93,53,53
 93   I=I+1
      A1=F3
      Y1=DSIN(A1)
      X1=DCOS(A1)
      IF(I-4)53,99,99
53    G=-0.017D0
      R1A=C1*Y1/(G*X1)
```

```
      IF(RANGE-R1A)81,81,82
82    CDEP=C1*(1.0D0-X1)/(G*X1)
      RAN=R1A
      TIME=(-1.0D0/G)*DLOG((DTAN(P))/DTAN(P+(A1/2.0D0)))
      X1=1.0D0
      Y1=0.0D0
      DEPTH=CDEP+DEPTH
      C1=C1+G*CDEP
      GO TO 88
81    Y2=Y1-(RANGE*G*X1)/C1
      F2=DARSIN(Y2)
      X2=DCOS(F2)
      TIME=(-1.0D0/G)*DLOG((DTAN(P+(F2/2.0D0)))/DTAN(P+(A1/2.0D0)))
      CDEP=C1*(X2-X1)/(G*X1)
      DEPTH=DEPTH+CDEP
      GO TO 32
C
C     K IS THE NUMBER OF RAYS CALCULATED FOR EACH SOURCE DEPTH
C
99    CONTINUE
      WRITE(6,991)DEPTH,K
      WRITE(8,991)DEPTH,K
991   FORMAT(1X,'NUMBER OF RAYS FOR THE ',F7.2,' METER DEPTH IS',1X,I3)
      WRITE(6,178)
      WRITE(8,178)
178   FORMAT(1X,'                ',1X)
531   CONTINUE
C
C     INCREMENT THE SOURCE DEPTH 20 METERS UNTIL 600 METERS
C     IS EXCEEDED
C
      IF(DEPTH-600.0D0)5,9,9
9     CONTINUE
      STOP
      END

  DEPTH     RELATIVE TRAVEL TIME   INITIAL ANGLE
 1157.4079       0.0                 0.0
 1173.6307       0.0012725           0.10000000
 1185.6802       0.0022367           0.20000000
 1193.6744       0.0028523           0.30000000
 1197.7002       0.0031453           0.40000000
 1197.8063       0.0031655           0.50000000
 1193.9812       0.0028606           0.60000000
 1186.1832       0.0022450           0.70000000
 1174.3260       0.0013153           0.80000000
 1158.2790       0.0000505           0.90000000
 1137.8606      -0.0016306           1.00000000
```

```
1112.8688    -0.0037000          1.10000000                          THE04810
1083.0441    -0.0062330          1.20000000                          THE04820
1048.1012    -0.0092386          1.30000000                          THE04830
1007.6893    -0.0128722          1.40000000                          THE04840
 962.3588    -0.0169717          1.50000000                          THE04850
 913.8646    -0.0212090          1.60000000                          THE04860
 862.6216    -0.0255669          1.70000000                          THE04870
 809.0476    -0.0299738          1.80000000                          THE04880
 753.5937    -0.0343231          1.90000000                          THE04890
 696.7140    -0.0386434          2.00000000                          THE04900
 638.9217    -0.0427604          2.10000000                          THE04910
 580.7177    -0.0467210          2.20000000                          THE04920
 522.6453    -0.0504554          2.30000000                          THE04930
 465.2722    -0.0538628          2.40000000                          THE04940
 409.1732    -0.0569386          2.50000000                          THE04950
 354.9462    -0.0596485          2.60000000                          THE04960
 303.2019    -0.0620132          2.70000000                          THE04970
 254.5751    -0.0639836          2.80000000                          THE04980
 209.7138    -0.0655253          2.90000000                          THE04990
 169.2698    -0.0667447          3.00000000                          THE05000
 133.9208    -0.0675970          3.10000000                          THE05010
 104.3509    -0.0680958          3.20000000                          THE05020
  81.2518    -0.0683973          3.30000000                          THE05030
  65.3323    -0.0684673          3.40000000                          THE05040
  57.3071    -0.0684959          3.50000000                          THE05050
  57.9020    -0.0685065          3.60000000                          THE05060
  67.8515    -0.0686560          3.70000000                          THE05070
  87.8990    -0.0690906          3.80000000                          THE05080
 118.7971    -0.0699827          3.90000000                          THE05090
 161.3067    -0.0715197          4.00000000                          THE05100
 185.4389    -0.0725201          4.04685864                          THE05110
NUMBER OF RAYS FOR THE  220.00 METER DEPTH IS  42                    THE05120
                                                                     THE05130
  DEPTH     RELATIVE TRAVEL TIME  INITIAL ANGLE                      THE05140
1420.4553     0.0184080          0.0                                 THE05150
1432.3934     0.0190909          0.10000000                          THE05160
1441.2554     0.0195765          0.20000000                          THE05170
1447.1769     0.0199448          0.30000000                          THE05180
1450.2256     0.0200907          0.40000000                          THE05190
1450.4492     0.0201025          0.50000000                          THE05200
1447.8389     0.0199679          0.60000000                          THE05210
1442.3401     0.0196584          0.70000000                          THE05220
1433.8582     0.0191709          0.80000000                          THE05230
1422.2537     0.0184861          0.90000000                          THE05240
1407.3428     0.0175682          1.00000000                          THE05250
1388.9098     0.0164489          1.10000000                          THE05260
1366.6786     0.0150164          1.20000000                          THE05270
1340.3486     0.0132680          1.30000000                          THE05280
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| | | | |
|---|---|---|---|
| 1309.5720 | 0.0111410 | 1.40000000 | THE05290 |
| 1273.9739 | 0.0086388 | 1.50000000 | THE05300 |
| 1233.1130 | 0.0055768 | 1.60000000 | THE05310 |
| 1186.5432 | 0.0019620 | 1.70000000 | THE05320 |
| 1133.7675 | -0.0023034 | 1.80000000 | THE05330 |
| 1074.2521 | -0.0073385 | 1.90000000 | THE05340 |
| 1007.4265 | -0.0132848 | 2.00000000 | THE05350 |
| 935.3854 | -0.0197747 | 2.10000000 | THE05360 |
| 862.1978 | -0.0260831 | 2.20000000 | THE05370 |
| 788.4706 | -0.0320888 | 2.30000000 | THE05380 |
| 714.7754 | -0.0378035 | 2.40000000 | THE05390 |
| 641.7250 | -0.0431235 | 2.50000000 | THE05400 |
| 569.9417 | -0.0479999 | 2.60000000 | THE05410 |
| 500.0637 | -0.0523972 | 2.70000000 | THE05420 |
| 432.7447 | -0.0562938 | 2.80000000 | THE05430 |
| 368.6572 | -0.0596407 | 2.90000000 | THE05440 |
| 308.4811 | -0.0624466 | 3.00000000 | THE05450 |
| 252.9088 | -0.0647361 | 3.10000000 | THE05460 |
| 202.6509 | -0.0664672 | 3.20000000 | THE05470 |
| 158.4175 | -0.0677407 | 3.30000000 | THE05480 |
| 120.9356 | -0.0685919 | 3.40000000 | THE05490 |
| 90.9398 | -0.0690749 | 3.50000000 | THE05500 |
| 69.1725 | -0.0692636 | 3.60000000 | THE05510 |
| 56.3833 | -0.0693339 | 3.70000000 | THE05520 |
| 53.3308 | -0.0693250 | 3.80000000 | THE05530 |
| 60.7798 | -0.0694228 | 3.90000000 | THE05540 |
| 79.5029 | -0.0697976 | 4.00000000 | THE05550 |
| 110.2802 | -0.0706456 | 4.10000000 | THE05560 |
| 153.8989 | -0.0721825 | 4.20000000 | THE05570 |
| 167.9190 | -0.0727444 | 4.22688592 | THE05580 |

NUMBER OF RAYS FOR THE 240.00 METER DEPTH IS 44 THE05590

THE05600

# APPENDIX B

## INTERPOLATION OF RELATIVE TRAVEL TIME CALCULATIONS

This program does an interpolation of the output data generated in Appendix A. The relative travel times are interpolated for the receiving hydrophone depths.

```
C     PROGRAM WHICH INTERPOLATES RELATIVE TRAVEL TIMES TO
C     RECEIVER DEPTHS
C
C      DESCRIPTION OF IMPORTANT PARAMETERS
C
C*      K - NUMBER OF RAYS FOR EACH DEPTH
C*      X - DEPTH WHERE RAY REACHES HORIZONTAL RANGE
C*      Y - RELATIVE TRAVEL TIME OF EACH RAY
C*      U - RECEIVING HYDROPHONE DEPTH
C*      S - RELATIVE TRAVEL TIME TO EACH HYDROPHONE
C
C
$JOB             1493P,TIME=56
      DIMENSION X(68),Y(68),IR(68),WK(68),C(67,3),IC(67),U(5),S(5),K(20)
      IJ=0
2     IJ=IJ+1
C
C      R,X,AND Y ARE THE OUTPUTS FROM THE PROGRAM IN APPENDIX A.
C     THESE ARE THE INPUTS TO THIS PROGRAM.  NOTE THAT WHEN MORE THAN
C     ONE RAY TERMINATES AT THE SAME DEPTH FOR A GIVEN SOURCE DEPTH
C     THEN THE RAY WHICH ARRIVES FIRST IS USED IN THE INTERPOLATION
C     PROGRAM.  TO INSURE THAT THE OTHER NONAPPLICABLE RAYS DO NOT
C     INTERFERE IN THE INTERPOLATION PROCESS THEIR TERMINATING DEPTHS
C     ARE MADE ARBITRARILY LARGE
C
      READ(5,88)K(IJ)
88    FORMAT(I3)
      NX=K(IJ)
      DO 100 I=1,NX
      READ(5,10)X(I),Y(I)
10    FORMAT(F10.4,F14.7)
100   CONTINUE
      K1=K(IJ)-1
      I1=0
6     I1=I1+1
      IR(I1)=I1
      IF(I1-NX)6,9,9
9     CONTINUE
C
C      INPUT VECTORS TO BE INTERPOLATED MUST BE IN ASCENDING ORDER.
C      IMSL ROUTINES 'VSRTR' AND 'VSRTU' DO THIS.  IMSL ROUTINE
C      'ICSCCU' INTERPOLATES THE SLICE COEFFICIENT
C
      CALL VSRTR(X,NX,IR)
      CALL VSRTU(Y,1,1,NX,0,IR,WK)
      CALL ICSCCU(X,Y,NX,C,K1,IER)
C
C      RECEIVING HYDROPHONE DEPTHS ARE SELECTED.  IMSL ROUTINE 'ICSEVU'
```

```
C     EVALUATES THE SLICE AT THESE POINTS
C
      U(1)=100.0
      U(2)=200.0
      U(3)=300.0
      U(4)=400.0
      U(5)=500.0
      CALL ICSEVU(X,Y,NX,C,K1,U,S,5,IER)
C
C     OUTPUT ARE TRAVEL TIMES PRINTED BY ROW.  THE FIRST ROW IS FOR
C     THE SHALLOWEST SOURCE DEPTH AND LAST ROW IS FOR THE DEEPEST
C     SOURCE DEPTH.  THE FIRST COLUMN IS FOR THE SHALLOWEST RECEIVING
C     HYDROPHONE AND THE FINAL COLUMN IS FOR THE DEEPEST HYDROPHONE.
C
      WRITE(6,101)(S(I),I=1,5)
      WRITE(7,101)(S(I),I=1,5)
101   FORMAT(5F14.7)
C
C     PROGRAM TERMINATES WHEN NUMBER OF RAYS FOR A SOURCE DEPTH EQUALS
C     THE NUMBER OF RAYS FOR THE DEEPEST SOURCE DEPTH
C
      IF(K(IJ)-68)2,21,21
21    CONTINUE
      STOP
      END
$ENTRY
   -0.0694169    -0.0658337    -0.0621516    -0.0574149    -0.0518334
   -0.0703276    -0.0665503    -0.0628172    -0.0580503    -0.0524011
   -0.0713123    -0.0673456    -0.0635689    -0.0587241    -0.0530590
   -0.0723230    -0.0681941    -0.0643641    -0.0595105    -0.0537945
   -0.0733907    -0.0690646    -0.0652127    -0.0603237    -0.0546137
   -0.0745428    -0.0699940    -0.0661212    -0.0612039    -0.0554598
   -0.0756658    -0.0709912    -0.0670775    -0.0621397    -0.0563615
   -0.0747207    -0.0720147    -0.0680565    -0.0631501    -0.0572667
   -0.0758055    -0.0731056    -0.0691335    -0.0641502    -0.0583075
   -0.0769579    -0.0742301    -0.0701951    -0.0652131    -0.0593780
   -0.0781758    -0.0753760    -0.0713190    -0.0663189    -0.0604718
   -0.0793883    -0.0765576    -0.0724953    -0.0674813    -0.0616073
   -0.0806178    -0.0777900    -0.0737091    -0.0686366    -0.0627376
   -0.0819192    -0.0790446    -0.0749412    -0.0698603    -0.0639641
   -0.0832829    -0.0803677    -0.0762265    -0.0711268    -0.0652111
   -0.0846354    -0.0817073    -0.0775561    -0.0724360    -0.0664785
   -0.0860246    -0.0830759    -0.0788832    -0.0737404    -0.0677993
   -0.0874630    -0.0844590    -0.0802819    -0.0751390    -0.0691366
   -0.0888928    -0.0859060    -0.0816601    -0.0764943    -0.0705184
   -0.0902550    -0.0873991    -0.0830757    -0.0779512    -0.0719424
```

# APPENDIX C

## RESULTING BEAM PATTERN FOR CALCULATED WEIGHTS

This program uses the output of Appendix B to calculate the '<u>A</u>' matrix. From this, the amplitude and phase weights are determined by the linear minumum variance estimation technique. These weights are applied to the array and the beam pattern is obtained.

```
C      PROGRAM WHICH DOES THE FOLLOWING TASKS:
C      * CALCULATES THE 'A' MATRIX FROM RELATIVE TRAVEL TIMES
C      * CALCULATES ANS WHICH REPRESENTS THE UNKNOWN - THE AMPLITUDE
C        AND PHASE WEIGHTS TO BE APPLIED TO THE ARRAY
C      * USES 'ANS' AND 'A' TO CALCULATE THE RESULTING BEAM PATTERN
C
C
C      DESCRIPTION OF IMPORTANT PARAMETERS
C
C      FREQ - FREQUENCY IN HZ.
C      Z - DESIRED BEAM PATTERN (REAL AND IMAGINARY PARTS SEPARATE)
C      ANS - THE UNKNOWN, PHASE AND AMPLITUDE WEIGHTS DETERMINED BY
C            LINEAR MINIMUM VARIANCE METHOD
C      Y - RESULTING BEAM PATTERN (REAL AND IMAGINARY PARTS SEPARATE)
C      XX - RESULTING BEAM PATTERN (REAL AND IMAGINARY PARTS COMBINED)
C      F - RELATIVE TRAVEL TIMES CALCULATED IN APPENDIX B
C      W - WEIGHTING MATRIX
C      X1 - SOURCE DEPTH
C
C
$JOB            1493P,TIME=56
       DIMENSION A(40,10),F(20,5),ANG(20,5),W(40,40),Z(40,1),C(10,40),
      *D(10,10),E(10,1),DINV(10,10),WKAREA(150),ANS(10,1),
      *B(20,20),Y(40,1),XX(20,1),X1(20,1)
C
C      SET INITIAL CONDITIONS
C
       J1=0
       I1=0
       I20=0
       I19=0
       I6=0
       I5=0
       J5=0
       PI=3.14159265358980
       FREQ=100.00
       P=2.0*PI*FREQ
C
C      READ IN THE RELATIVE TRAVEL TIMES DERIVED IN APPENDIX B
C
       READ(5,10)((F(I,J),J=1,5),I=1,20)
10     FORMAT(5F14.7)
C
C      CALCULATE THE 'A' MATRIX.
C      IF THERE IS NOT A SOUND SIGNAL AT A RECEIVER DEPTH THEN SET THE
C      SINE AND COSINE TO ZERO.
C
8      J1=J1+1
```

```
7      I1=I1+1                                                         MA900490
       IF(F(I1,J1).EQ.0)GO TO 6                                        MA900500
       ANG(I1,J1)=F*F(I1,J1)                                           MA900510
       A(I1,J1)=COS(ANG(I1,J1))                                        MA900520
       A(I1,J1+5)=SIN(ANG(I1,J1))                                      MA900530
       A(I1+20,J1)=-A(I1,J1+5)                                         MA900540
       A(I1+20,J1+5)=A(I1,J1)                                          MA900550
       IF(I1-20)7,2,2                                                  MA900560
2      I1=0                                                            MA900570
       IF(J1-5)8,72,72                                                 MA900580
6      A(I1,J1)=0                                                      MA900590
       A(I1,J1+5)=0                                                    MA900600
       A(I1+20,J1)=0                                                   MA900610
       A(I1+20,J1+5)=0                                                 MA900620
       IF(I1-20)7,2,2                                                  MA900630
C                                                                      MA900640
C      SET WEIGHTING MATRIX TO IDENTITY MATRIX.                        MA900650
C      SET DESIRED BEAM PATTERN 'Z'.                                   MA900660
C                                                                      MA900670
72     I5=I5+1                                                         MA900680
52     J5=J5+1                                                         MA900690
       W(I5,J5)=0.0                                                    MA900700
       IF(J5-40)52,53,53                                               MA900710
53     J5=0                                                            MA900720
       IF(I5-40)72,75,75                                               MA900730
75     I6=I6+1                                                         MA900740
       Z(I6,1)=0.0                                                     MA900750
       W(I6,I6)=1.00                                                   MA900760
       IF(I6-40)75,76,76                                               MA900770
76     W(1,1)=1.00                                                     MA900780
       Z(1,1)=1.00                                                     MA900790
C                                                                      MA900800
C      THE ENERGY SOURCE IS SELECTED TO BE ON THE SHALLOWEST DEPTH     MA900810
C                                                                      MA900820
C      THE IMSL ROUTINE SUMMARY                                        MA900830
C      * VMULFM - TRANSPOSE OF FIRST MATRIX TIMES SECOND MATRIX        MA900840
C      * VMULFF - FIRST MATRIX TIMES SECOND MATRIX                     MA900850
C      * LINV2F - INVERSE OF A MATRIX                                  MA900860
C                                                                      MA900870
 17    CONTINUE                                                        MA900880
       CALL VMULFM(A,W,40,10,40,40,40,C,10,IER1)                       MA900890
       CALL VMULFF(C,A,10,40,10,10,40,D,10,IER2)                       MA900900
       CALL VMULFF(C,Z,10,40,1,10,40,E,10,IER3)                        MA900910
       CALL LINV2F(D,10,10,DINV,3,WKAREA,IER4)                         MA900920
       CALL VMULFF(DINV,E,10,10,1,10,10,ANS,10,IER5)                   MA900930
C                                                                      MA900940
C      PRINT CALCULATED PHASE AND AMPLITUDE WEIGHTS                    MA900950
C                                                                      MA900960
```

```
      WRITE(6,999)
999   FORMAT(1X,'ANS MATRIX (PHASE AND AMPLITUDE WEIGHTS) FOLLOWS')
      WRITE(6,158)(ANS(I8,1),I8=1,10)
158   FORMAT(F14.7)
C
C     APPLYING THESE PHASE AND AMPLITUDE WEIGHTS, THE RESULTING BEAM
C     PATTERN IS CALCULATED (REAL AND IMAGINARY PARTS SEPARATE)
C
760   CALL VMULFF(A,ANS, 40,10,1,40,10,Y,40,IER2)
C
C     THE REAL AND IMAGINARY PARTS OF THE RESULTING BEAM PATTERN
C     ARE COMBINED TO GIVE ABSOLUTE VALUE.
C
987   I20=I20+1
      XX(I20,1)=SQRT(Y(I20,1)*Y(I20,1)+Y(I20+20,1)*Y(I20+20,1))
      IF(I20-20)987,654,654
654   I19=I19+1
      X1(I19,1)=20.0+I19*20.0
      IF(I19-20)654,655,655
C
C     PRINT SOURCE DEPTH AND ABSOLUTE VALUE OF BEAM PATTERN AT
C     THAT DEPTH
C
655   WRITE(6,988)(X1(I,1),XX(I,1),I=1,20)
      WRITE(9,988)(X1(I,1),XX(I,1),I=1,20)
988   FORMAT(F6.2,' METER DEPTH HAS STRENGTH OF ',F14.7)
172   CONTINUE
      STOP
      END
$ENTRY

 ANS MATRIX (PHASE AND AMPLITUDE WEIGHTS) FOLLOWS
     -0.0050524
      2.2758930
      0.5812412
     -3.6065380
     -4.3302970
      0.2429787
     -1.3739130
     -3.0745440
      2.1446630
      0.7122661
```

```
220.00 METER DEPTH HAS STRENGTH OF      0.4743061               MA901450
240.00 METER DEPTH HAS STRENGTH OF      0.2850570               MA901460
260.00 METER DEPTH HAS STRENGTH OF      0.2128582               MA901470
280.00 METER DEPTH HAS STRENGTH OF      0.0804181               MA901480
300.00 METER DEPTH HAS STRENGTH OF      0.0716692               MA901490
320.00 METER DEPTH HAS STRENGTH OF      0.0349326               MA901500
340.00 METER DEPTH HAS STRENGTH OF      0.1124682               MA901510
360.00 METER DEPTH HAS STRENGTH OF      0.0930375               MA901520
380.00 METER DEPTH HAS STRENGTH OF      0.0693972               MA901530
400.00 METER DEPTH HAS STRENGTH OF      0.1343069               MA901540
420.00 METER DEPTH HAS STRENGTH OF      0.1172233               MA901550
440.00 METER DEPTH HAS STRENGTH OF      0.0437957               MA901560
460.00 METER DEPTH HAS STRENGTH OF      0.0185341               MA901570
480.00 METER DEPTH HAS STRENGTH OF      0.0239681               MA901580
500.00 METER DEPTH HAS STRENGTH OF      0.0197991               MA901590
520.00 METER DEPTH HAS STRENGTH OF      0.0891870               MA901600
540.00 METER DEPTH HAS STRENGTH OF      0.0190372               MA901610
560.00 METER DEPTH HAS STRENGTH OF      0.1875219               MA901620
580.00 METER DEPTH HAS STRENGTH OF      0.0271981               MA901630
600.00 METER DEPTH HAS STRENGTH OF      0.0740827               MA901640
                                                                MA901650
```

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Thesis
M272 McVicar
c.1 Amplitude shading and phase weighting of a vertical linear array in the SOFAR channel by the linear minimum variance estimation technique.