DESIGN OF A MAGNETIC TAPE TRANSPORT SYSTEM EMPLOYING STEPPER MOTORS

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THESIS

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EMPLOYING STEPPER MOTORS

by

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December 1980

Thesis Advisor: G. J. Thaler

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) Design of a Magnetic Tape Transport System Employing Stepper Motors		5. TYPE OF REPORT & PERIOD COVERED Electrical Engineer December 1980
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Sotirios A. Tsavdaris		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Naval Postgraduate School Monterey, California 93940		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS Naval Postgraduate School Monterey, California 93940		12. REPORT DATE December 1980
		13. NUMBER OF PAGES 52
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office) Naval Postgraduate School Monterey, California 93940		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for Public Release; Distribution Unlimited

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Investigation and development of a magnetic tape transport system

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

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by

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Submitted in partial fulfillment of the requirements for the degree of

ELECTRICAL ENGINEER

from the

NAVAL POSTGRADUATE SCHOOL
December 1980

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A magnetic tape transport system employing stepper motors has been investigated and developed. Before the investigation of the system, a mathematical model for a four-phase permanent magnet stepper motor was developed and its behavior studied by computer simulation.

At the beginning, a system was proposed employing a single tension spring between the take-up motor and its reel to maintain a tape tension. From computer simulation it was found that such a scheme does not regulate tape tension within acceptable limits. A new scheme was proposed using two tension springs between the two motors and their reels. From the simulation it was shown that this scheme is acceptable since the tape tension is kept between the specified limits and the stepper motors execute the tape transfer commands without any failure.

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ACKNOWLEDGEMENT

The Author wishes to express his many thanks to Doctor G. J. Thaler for his encouragement, helpful suggestions and advice, communicated through many hours of deifying discussions.

I. INTRODUCTION

Variable reluctance and permanent magnet stepping motors are an important class of digital actuators which allow digital data to be converted directly into mechanical displacement. The motor and drive system converts each step command into a rotation or translation step of fixed size. At high stepping rates or underload or for some step sequences, the motor will fail to execute the step commands.

The limits on the motor response depend on: (1) load inertia, (2) load damping, (3) motor characteristics, (4) stator current, (5) load torque, (6) stepping sequence, and (7) driver characteristics.

Analytical models have been developed which describe the effects of stator winding self-inductance, mutual inductance and back EMF [1]-[2]. For a voltage source drive of the stator windings, the simplest model is fourth order. In order to get high stepping rates, drive circuit characteristics have approached those of an ideal current source. In these drives, the effect of stator inductances and BEMF voltages are minimized. Thus, these high-performance systems can be represented by a mathematical model which assumes the stator windings are driven by a current source. Such a model is a non-linear second order differential equation.

More information on the physical construction and operation of the stepping motors are included in References (3)-(6).

II. MATHEMATICAL MODEL OF STEPPING MOTOR

For a permanent magnet stepping motor with drive circuit characteristics approaching those of an ideal current source, the model is

$$J\ddot{\theta} + D\dot{\theta} + T(\theta) = T_L \tag{1}$$

where J = motor and load inertia, D = motor and load damping, θ = angle of the shaft, $T(\theta)$ = motor torque, and T_L = load torque. The torque of the motor depends on the angle of the shaft.

A torque angle $T(\theta)$ relation can be obtained by applying a steady torque to the motor and measuring the deflection of the rotor from its equilibrium with the windings energized. The applied torque necessary to deflect the motor to any given angle is a periodic function of the angle of rotation. Very often the torque-angle curve is very nearly sinusoidal. This is especially true for permanent magnet stepping motors whereas the variable reluctance motors tend to have a torque-angle characteristic which is much less sinusoidal but is, of course, periodic [7]. Thus

$$T(\theta) = TM \sin(A\theta) \tag{2}$$

The maximum applied torque, TM, is a linear function of the field current (I) and the number of windings excited.

$$TM = K \cdot (I) \tag{3}$$

This relation applies for most motor designs until stator saturation occurs. In this case, we assume no saturation, so equation (3) is valid. Thus equation (1) becomes:

$$J\ddot{\theta} + D\dot{\theta} + TM \cdot \sin(A\theta) = T_L \tag{4}$$

Normalizing (4) we get

$$\frac{J}{TM}\ddot{\theta} + \frac{D}{TM}\dot{\theta} + \sin(A\theta) = \bar{T}_L \tag{5}$$

where $\bar{T}_L$ is the ratio of load torque to the maximum available torque.

It is useful at this point to define a new angle variable which is independent of the motor design. Allow it to be called the electrical angle (X)

$$X = A \cdot \theta \tag{6}$$

By substituting this change in the variable into equation (5) we get

$$\frac{J}{ATM}\ddot{X} + \frac{D}{ATM}\dot{X} + \sin(X) = \bar{T}_L \tag{7}$$

It is clear that the equation (7) is a second order non-linear differential equation with a natural frequency and damping ratio which can be defined as

$$W_n = \left(\frac{ATM}{J}\right)^{\frac{1}{2}} \qquad \zeta = \frac{D}{2(AJTM)^{\frac{1}{2}}} \tag{8}$$

equation (7) can now be written:

$$\frac{\ddot{X}}{W_n^{\,2}} + \frac{2\zeta}{W_n}\dot{X} + \sin(X) = \bar{T}_L \tag{9}$$

It is useful, at this point, to change the time scale in equation (9) to make time dimensionless and so reduce the number of parameters associated with the motor. To do this, a new time, τ, is defined which is equal to real time multiplied by the system's natural frequency as shown in equation (10)

$$\tau = W_n\, t = \left(\frac{ATM}{J}\right)^{\frac{1}{2}} t \tag{10}$$

By making this substitution in the system differential equation, eq. (11)

results which is now dimensionless in both magnitude and time.

$$\frac{d^2X}{d\tau^2} + 2\zeta\frac{dX}{d\tau} + \sin(X) = \bar{T}_L \qquad (11)$$

From this equation it is seen that the form of the system response depends on only two parameters- the dimensionless load torque, $\bar{T}_L$, and the damping ratio, ζ.

Equation (11) describes the behavior of the motor for a fixed winding energization and load torque, for any given initial values of rotor angle and velocity.

A. THE PHASE PLANE

It is useful in non-linear equations of the form of eq. (11) to plot the normalized rotor velocity $(dX/d\tau)$ versus the normalized rotor position, X, for a fixed load.

Such plots can be found in reference (7). A solution to eq. (11) for a given set of initial conditions $(dX/d\tau)$ and X will be a curve or path in the $(dX/d\tau)$ versus X phase plane. When stepping, as the motor is subjected to a step command, which is a step change in the winding excitation; the rotor may be at any angular position and travel at any angular velocity. So, it is useful to look at the response of the motor when it has any arbitrary rotor initial position or velocity.

In the phase plane plots for $\bar{T}_L$ equal to zero [Ref. 7], it can be seen that the stable equilibrium points are located at $X = 0 \pm 2\eta\pi$ and the unstable equilibrium at $X = \pm\eta\pi$. Also, it can be seen that since there is a trajectory which goes through the point $(dX/D\tau) = 0$ and $X = -\pi$ and another which goes through $(dX/d\tau) = 0$, $X = +\pi$, these two trajectories are known as separatrices. They enter the two unstable equilibrium points. These separatrices are very critical to the behavior of the

stepping motor response. If the motor trajectory is forced (due to a step command) to cross a separatrix, then motor failure occurs. This is why it is necessary to be careful when stopping or starting the motor. If after the last step command the motor response ever crosses a separatrix, it will lose step.

It can be seen that changing the damping γ and the load torque the shape of the separatrix and the trajectories change. Also, it can be seen that the stable equilibrium points are shifted to the right by the load torque when it has positive value[7].

B. START-STOP PERFORMANCE

When we investigate the start-stop performance of a motor, we are really asking what happens when we subject the motor to a finite number of step commands applied at equal time intervals. We would like the motor to execute this burst of pulses without losing step. For this reason, we ask how short can the interval between the applications be, before the stepping motor will fail to execute these steps by losing or gaining a step. This question can be answered from the phase plane plots [Ref. 7] although in eq.(11) no stepping rate is implied.

When we give a step command to the motor, we want it to step through a rotor angle, θ_s, which is determined by the motor geometry and winding energization. To this angle θ_s corresponds an angle X_s equal to $A\theta_s$ due to equation (6).

For a four-phase motor the step angle, X, is one-quarter the period of the torque-angle characteristic curve; so that X_s is equal to $\pi/2$.

Consider X to be a position error and especially an error of the electrical angle. When a stepping command is applied due to the change

of the winding excitation, the equilibrium point jumps X_s radians to the right; so, the rotor will start rotating under the law of eq. (11) on a trajectory of the phase plane with initial conditions the values of $dX/d\tau$ and X immediately after the change. Thus, at any moment the value of X will indicate the electrical angle error. If after the application of a number of steps the value of X is zero, that means that the motor has executed the commands without failure.

Notice that in the case of a four-phase motor if the value of X is n times 2π that means that the motor has gained four times n steps of physical angle. This is true because in a four-phase motor for a given configuration of winding excitation, the equilibrium points are four physical angle-steps apart.

A graphical method is described in Reference 7 that illustrates the behavior of a four-phase stepping motor when six stepping commands are applied with a period of one time unit. The parameters of the motor are the damping factor γ equals 0.125 and the load torque $\overline{T}_L$ equals zero. It is shown that the motor is able to execute this burst of commands without any failure.

To verify the graphical method, a computer simulation was run. Equation (11) was taken as the model of the stepping motor under all the assumptions for which this equation is valid. It was assumed as before that γ equals 0.125, load torque, $\overline{T}_L$ equals zero and the period of the stepping commands equals one. The number of stepping commands was 24.

Figure 1 is the plot of the error of the electrical angle in radians and the speed in radians/unit time. It can be seen that eventually the average speed becomes 1.57 radians/unit time which corresponds to the required speed for one pulse occurring every unit time. It can be seen

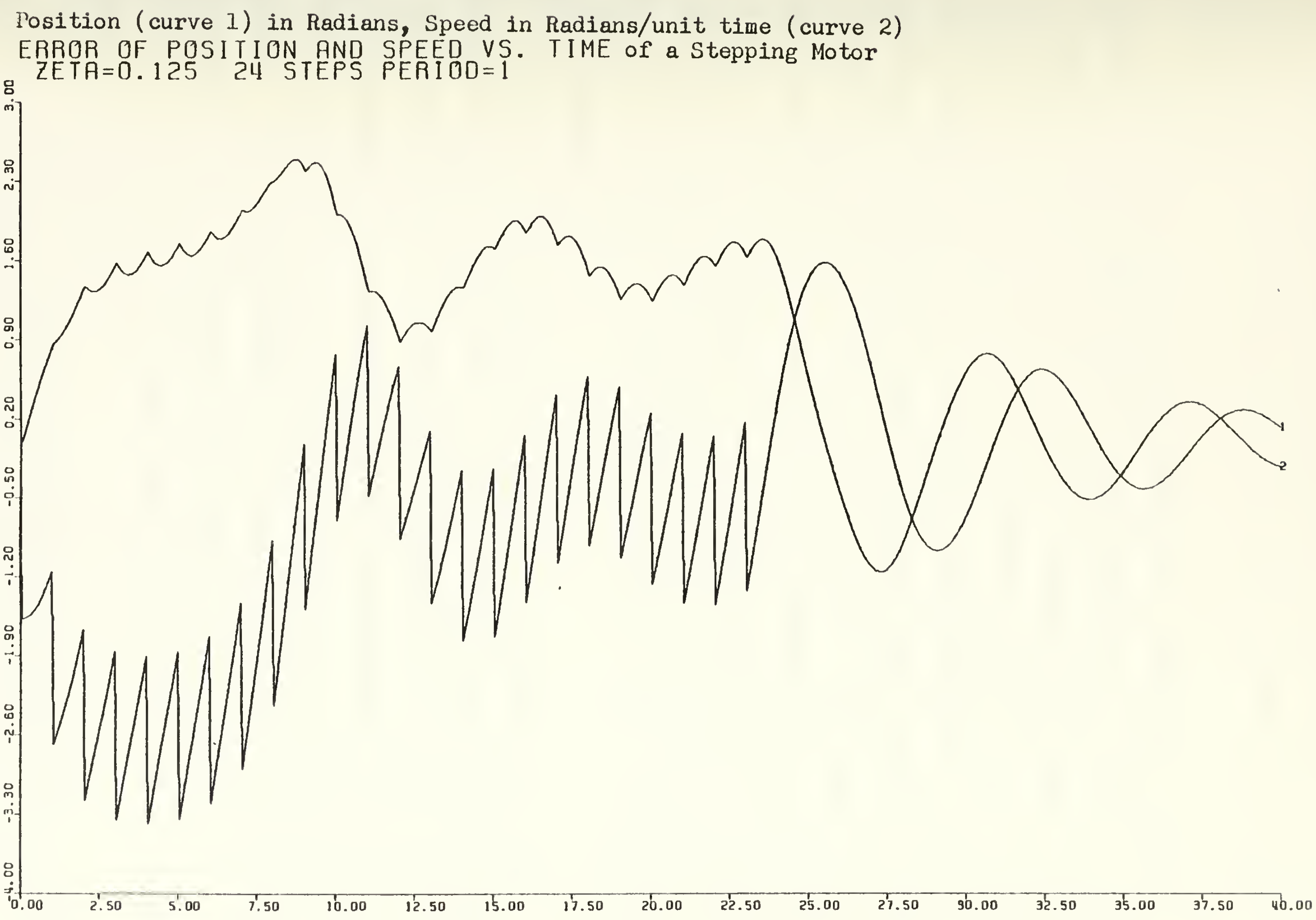
Position (curve 1) in Radians, Speed in Radians/unit time (curve 2)
ERROR OF POSITION AND SPEED VS. TIME of a Stepping Motor
ZETA=0.125 24 STEPS PERIOD=1
3.00
2.30
1.60
0.90
0.20
-0.50
-1.20
-1.90
-2.60
-3.30
-4.00
0.00
2.50
5.00
7.50
10.00
12.50
15.00
17.50
20.00
22.50
25.00
27.50
30.00
32.50
35.00
37.50
40.00
1
2

Figure 1

also, that after the shut off of the motor, the speed and the error oscillate and eventually they become both zero, meaning that the motor did not fail.

If the rate of the input step sequences is increased so that the changes in step commands occur at time intervals of 0.8 time unit, then a different situation occurs. The same graphical method again is applied [Ref. 7] for this case and it can be seen that when the fourth step command is applied the error moves across a separatrix and now the motor moves along the trajectory which will actually cause a decrease in speed rather than an increase and so it fails to step. Notice that the motor would slow down and begin to move to a new equilibrium point 2π radians to the left. The application of another step would drive it further to the left, so that the motor would tend to stop. Thus, for a stepping period of 0.8 units, the motor cannot execute the burst of commands and will therefore fail.

Figure 2 is the computer simulation of the execution of 24 step commands 0.8 unit time apart. It can be seen that the speed during the first and second step increases, but when the third and later fourth step is applied, the speed decreases. After the fifth and until the last step, the speed oscillates (without stopping) around its equilibrium position. After the application of the last step the rotor oscillates and eventually stops. The value of the electrical angle error is -31.4 which is -5 times 2π. That means that the motor lost 5 times 4, 20 steps and it executed only 4.

The fact that the computer simulation gave the same results as the graphical investigation on the phase plane proved that eq. (11) is a

Position (curve 1) in Radians, Speed (curve 2) in Radians/unit time
ERROR OF POSITION AND SPEED VS. TIME of the Stepping Motor
ZETA=0.125 24 STEPS PERIOD=0.8

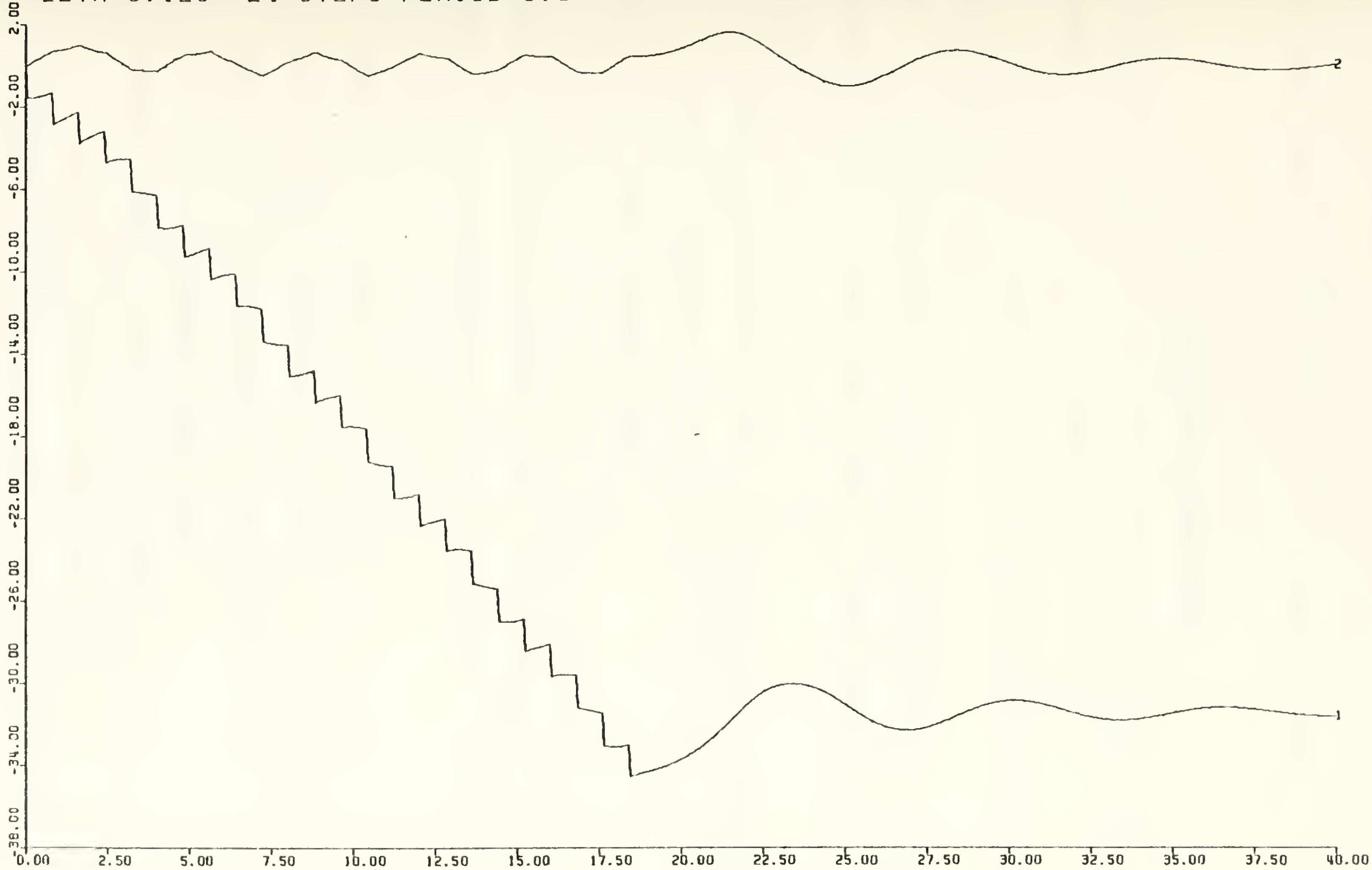

Figure 2

valid model of the stepping motor under the assumptions that the drive circuit approaches an ideal current source,which produces square wave current pulses and also that stator saturation does not occur.

III. STATEMENT OF THE PROBLEM

A transport system is to be developed for a magnetic tape memory. The concept is new and is as follows:

a) The tape will be stored in a cassette from which it will be withdrawn for recording.

b) In the memory system, the tape will be stationary and the heads will be moved for both writing and reading. A rough approximation is in Figure 3.

c) To advance the tape, each reel is to be driven by a stepper motor.

d) To maintain acceptable tape tension, the take-up reel will be connected to the motor through a spring.

e) Pulses applied to a motor can be counted, pulses due to reel motion can be counted by an optical sensor. The difference in count is a measure of spring deflection and thereby of tension, if the radius of the tape around the reel is known.

f) the desired tape tension limits are 0.2 to 0.6 pounds in order to maintain an air cushion between tape and heads. It is not acceptable that the tape tension ever be reduced to zero because the tape will be injured and possible the magnetic heads also.

An algorithm should be found for the control of the motion of the two reels, in order that the tape be transferred rapidly and smoothly. Also, step sequences should be developed so that the stepping motors can

Geometry of the System

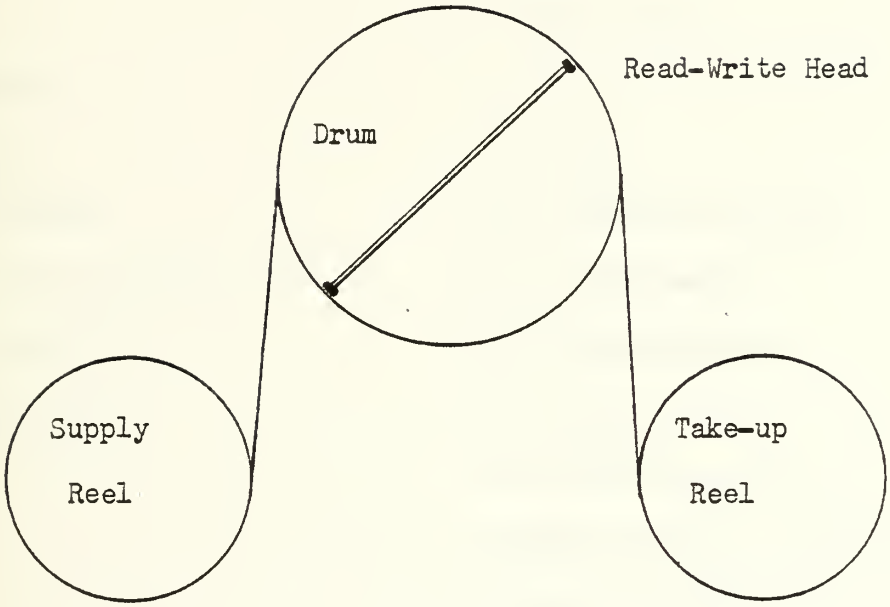

Figure 3

start and stop without any failure. This way, the request file, which will be located on a known portion of the tape, will be transferred quickly and placed around the drum.

Data and Specifications

Reel radius	:	0.461" (empty)
	:	1.1825" (full)
Drum diameter	:	4.3"
Drum speed	:	3100 RPM
Optical encoder	:	200 counts/revolution
Maximum reel start/stop rate	:	110 steps/second
Maximum slew rate	:	1000 steps/second
Stepping motor	:	four-phase
		200 steps/revolution
		Maximum torque = 1.055 in/lb
		Moment of inertia = 4.9 E-5 in/lb/sec^2
Moment of inertia of tension spring and take-up reel (empty)	:	1.04 E-4 in/lb/sec^2
Moment of inertia of reel (full)	:	1.34 E-4 in/lb/sec^2

IV. INVESTIGATION OF THE PROPOSED CONFIGURATION

The proposed scheme for the tape transport system is shown in Figure four. For simplicity and because it is not important in the following analysis, the drum and the optical encoder have not been included.

The main concept of the logic of the operation is that one motor will be the master and the other will be the slave. In the forward direction the master is the take-up motor. In rewind it is the supply motor. The following is a description of the operation of the system while in the forward direction.

When a command of a certain amount of steps has been applied, the gate A opens and clock pulses are applied to the drive circuit of the master (take-up). The gate A remains open until the number of applied pulses is the number of steps that have been requested. When the tension of the tape, which in a real system will be monitored through the optical encoder, exceeds a certain threshold, the gate B opens and asynchronous pulses will be applied to the drive circuit of the slave (supply motor) in order to relieve the tension. Gate B remains open during the time that the value of the tape tension is greater than the threshold. Since, in general, the radius of the tape on the supply and on the take-up reel are not the same, the two pulse trains cannot have the same frequency. Actually, the frequency of the pulse train applied on the supply reel should be R_1/R_2 times the frequency of the other pulse train; where R_1 and R_2 are the radii of the tape around the supply and take-up reels respectively.

C is the value of the spring constant for the tension spring. CT is

the value of the spring constant for the tape when we model it as a spring. The model of each stepping motor has been taken as that of eq. (11) under the same assumptions; therefore, the model of the system is:

$$(J_A + J_1)\,\ddot{\theta}_1 + D_1\dot{\theta}_1 + T_1 \sin\,(50\theta_1) - TF\,R_1 = 0 \qquad (12)$$

$$J_2\ddot{\theta}_2 + D_2\dot{\theta}_2 + C(\theta_2 - \theta_3) + TF\,R_2 = 0 \qquad (13)$$

$$J_A\ddot{\theta}_3 + D_3\dot{\theta}_3 + C(\theta_3 - \theta_2) + T_2 \sin(50\theta_3) = 0 \qquad (14)$$

$$TF = CT\,(R_2\theta_2 - R_1\theta_1) \qquad (15)$$

where J_A = moment of inertia of the motor, J_1 = moment of inertia of the supply reel, J_2 = moment of inertia of the take-up reel, D_1 = the damping of the supply motor and its reel, D_2 = the damping of the take-up reel, D_3 = the damping of the take-up motor, θ_1 = the physical angle of the rotor of the supply motor, θ_2 = the angle of the take-up reel, θ_3 = the physical angle of the rotor of the take-up motor, and TF = the tape tension.

If we call $X_1 = 50\ \theta_1$ and $X_3 = 50\ \theta_3$ for the same reason as was done in the modeling of the stepping motor, the model of the system becomes:

$$(J_A + J_1)\,\ddot{X}_1/50.0 + D_1 \cdot \dot{X}_1/50.0 + T_1 \sin(X_1) - TF \cdot R_1 = 0 \qquad (16)$$

$$J_2\ddot{\theta}_2 + D_2\dot{\theta}_2 + C(\theta_2 - \theta_3) + TF \cdot R_2 = 0 \qquad (17)$$

$$J_A\,\ddot{X}_3/50.0 + D_3\,\dot{X}_3/50.0 + C(\theta_3 - \theta_2) + T_2 \sin(X_3) = 0 \qquad (18)$$

$$TF = CT \cdot (R_2\theta_2 - R_1\theta_1) \qquad (19)$$

The above model of the system permits the tape tension (TF) to take positive (stretch) and negative (compression) values. Actually, the physical tape, of course, cannot take compression forces but since we will not permit the tape to become loose this model is acceptable and it is not wise to investigate a more realistic but more complicated model.

Single Tension Spring Tape Transport System

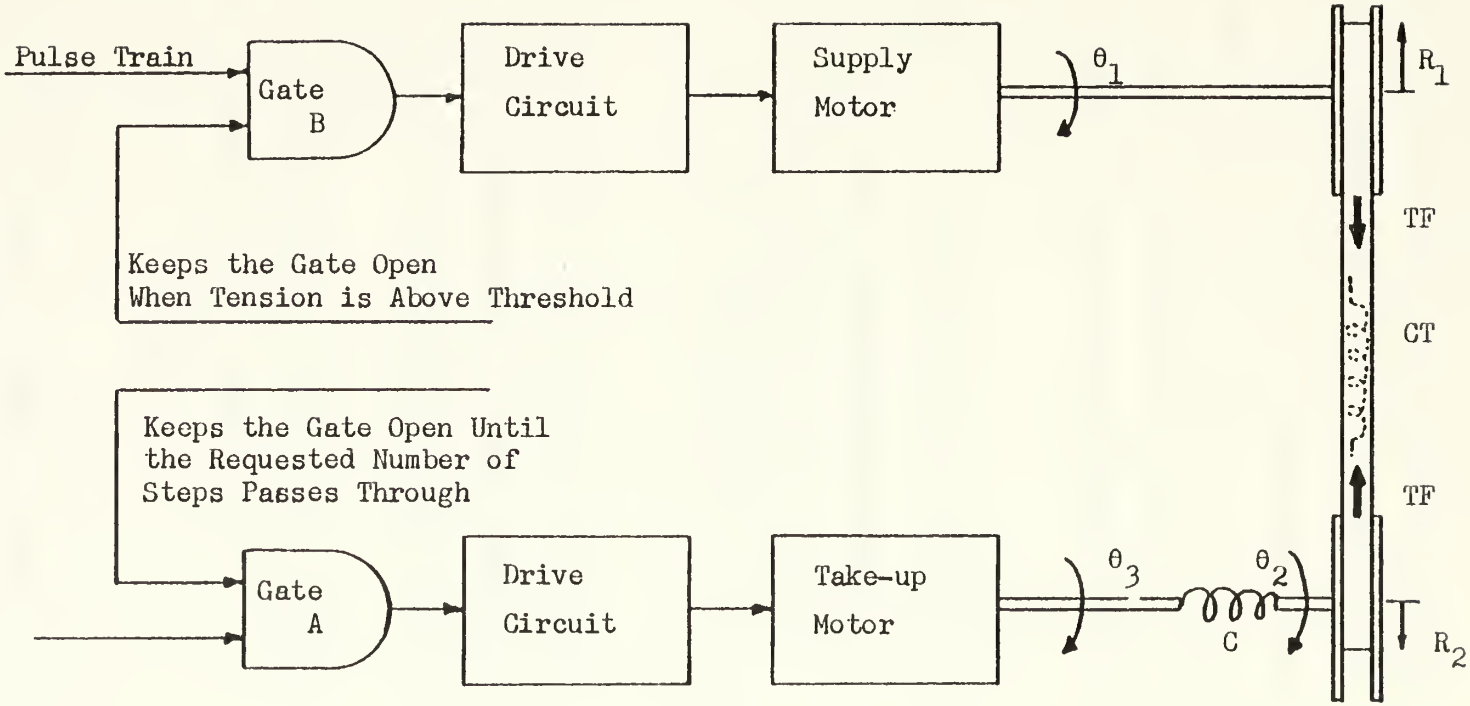

Figure 4

Also, it has been assumed that the values of J_1, J_2, R_1, R_2 do not change with time, since the intention is to transfer a small amount of tape from one reel to the other.

The simulation was run with all the tape on the supply reel. The rate of the stepping commands applied to the take-up reel is 500 steps per second. The threshold (TFL) for the tape-tension was set to the value of 0.4 pounds, which is in the middle of the tension limits. There was not any data for the values of D_1, D_2, D_3. It is known that a typical value of damping factors for the stepping motors is 0.125, so from eq. (8) it was found that:

$$D_3 = 2\gamma(AJ_AT_2)^{\frac{1}{2}} = (2)(0.125)(50\ 4.9\ E - 5\ 1.055)^{\frac{1}{2}} = 0.0127$$

For the value of D_2, two cases are simulated where $D_2 = 0.0$ and $D_2 = 0.005$. The value of D_1 was taken again from eq. (8) so that:

$$D_1 = 2\gamma(A(J_A + J_1)\ T_1)^{\frac{1}{2}} = (2)(0.125)(50(4.9\ E - 5 + 1.34\ E - 4)\ 1.055)^{\frac{1}{2}}$$

$$= 0.0245$$

At the beginning it was thought that the main problem would be to keep the take-up stepping motor from gaining or losing steps but simulation showed that the real problem was to keep the tape tension in the acceptable limits and non-negative.

The assumption was made that the tape can be considered a rigid element meaning that CT is very large. From eq. (19)

$$R_2\theta_2 = R_1\theta_1 \qquad \text{and} \qquad (20)$$

$$R_2\ddot{\theta}_2 = R_1\ddot{\theta}_1 \qquad (21)$$

which means that all of the tape that comes out from the supply reel goes to the take-up reel. But, due to eq. (17) and (21), at the moment of

stepping command to the supply motor the angle acceleration exhibits a discontinuity as does the tape-tension. Actually, when a stepping command was applied to motor number one (supply) the simulation showed that the tension was jumping from positive to negative values which was totally unacceptable.

A lot of attempts were made to solve this problem by increasing the threshold (TFL) of the tape tension from 0.4 to the upper bound 0.6 and using different values for the spring constant C. In spite of all these trials, the tension became negative when stepping commands were applied to the supply motor.

Then it was assumed that CT was not large. It was found that for a value of CT less than 10 pounds/inch the tape tension could be kept between the limits. In order to increase this limit of the 10 lb/in, the drive scheme of the supply motor was changed and immediately when the tension exceeded the threshold (TFL) a stepping command was applied to the supply motor. So, the value of TFL was set to the upper limit of 0.6 lb. This new scheme did not improve significantly the value of CT. Thus, the start-up problem appeared to be solved. (Note: independent calculations for the spring constant of 3/8" Mylar tape in tension indicates a spring constant of 9.7 lb/in, so the simulation value of 10 lb/in was considered close enough to demonstrate feasibility.)

However, the problem of stopping the system had not yet been considered. Simulation results showed that the tape tension dropped below zero. This was caused by the stored energy in the supply reel inertia. An attempt to solve the problem by having both motors shut off simultaneously was not a success. In figures 5 and 6 are seen plots of the

With Automatic Shut Off on Supply Motor, $D_2 = 0.005$
ERROR OF POSITION OF MOTORS #1 AND #2, Supply (curve 1), take-up (curve 2) in Radians
AND THE TAPE TENSION VS. TIME, Tension in lb/sec.

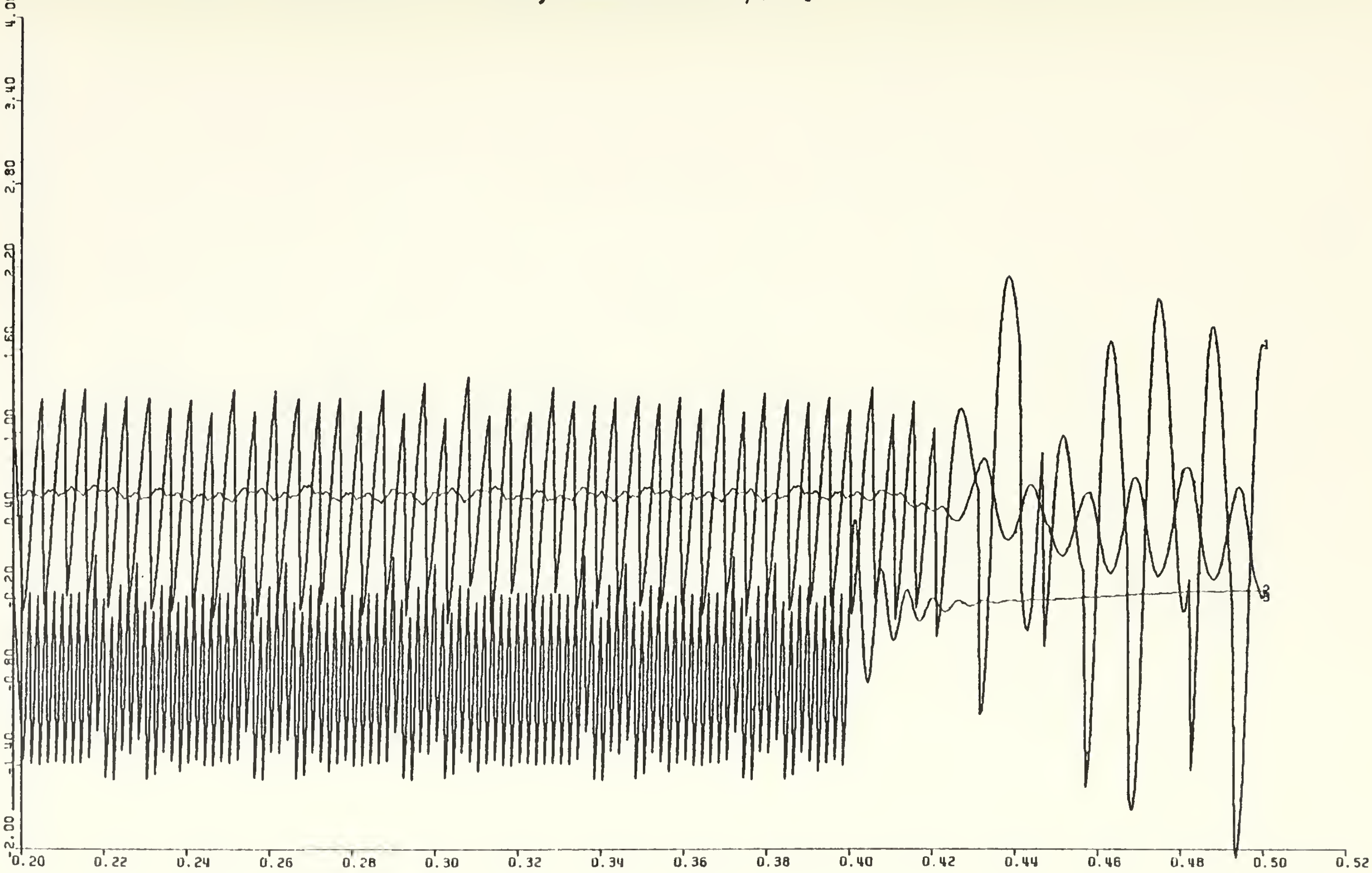

Figure 5

Automatic Shut off on Supply Motor, $D_2 = 0.0$
ERROR OF POSITION OF MOTORS #1 AND #2, Supply (curve 1), Take-up(curve 2) in Radians
AND THE TAPE TENSION VS. TIME, in seconds

Figure 6

Simultaneous Motor Shut off $D_2 = 0.005$

ERROR OF POSITION OF MOTORS #1 AND #2, Supply (curve 1), Take-up (curve 2) in Radians AND THE TAPE TENSION VS. TIME (curve 3) in lb/sec.

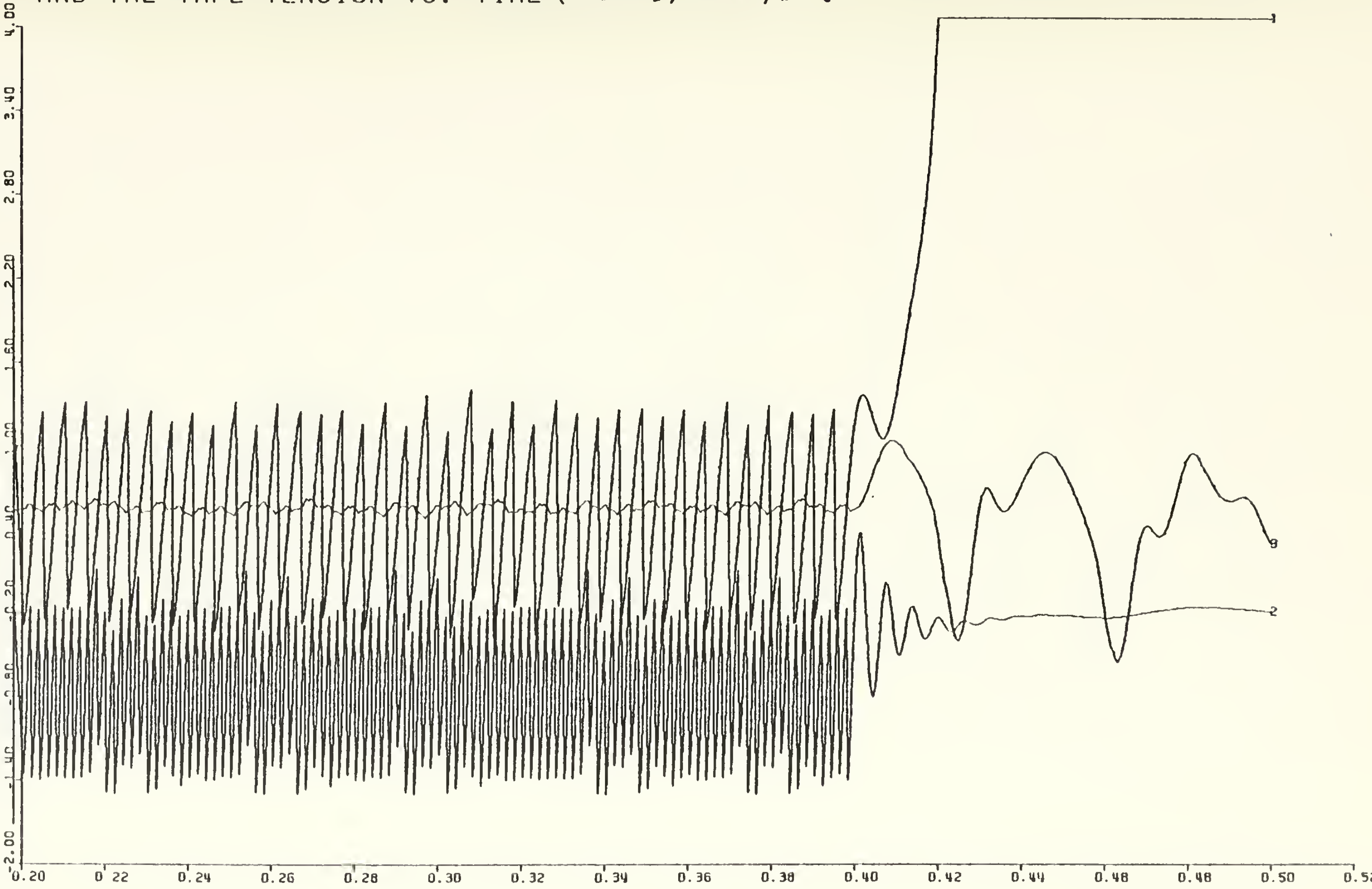

Figure 7

Simultaneous Motor Shut off $D_2 = 0.0$

ERROR OF POSITION OF MOTORS #1 AND #2, Supply(curve 1), Take-up(curve 2) in Radians AND THE TAPE TENSION VS. TIME(curve 3) in lb.

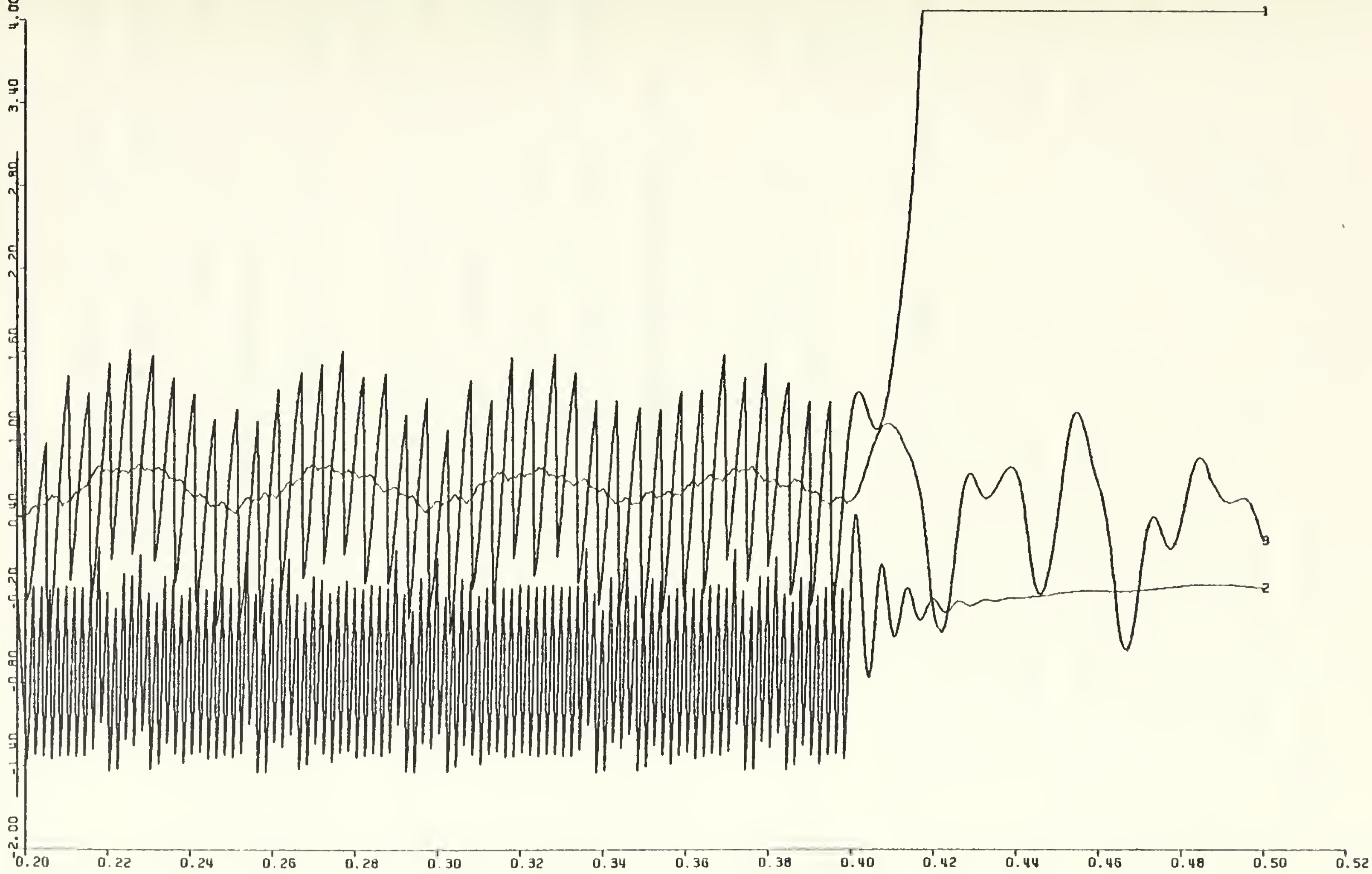

Figure 8

error of the electrical angle of both motors and the tape tension, when we shut off only the take-up motor and the supply motor is left to be shut off by itself when the tension stops exceeding the threshold.

In Figures 6 and 8 there is damping on the take-up reel. It can be seen that the tension fluctuates more in that case. Notice, that when both motors are shut off and since all the tape is on the supply reel the tape tension produces a moment high enough to drive the supply motor and force it to gain steps. Since the main interest on these plots is the fluctuations of the tape tension, it was preferred that a scale show these fluctuations clearly but clips the error of electrical angle of the supply motor.

In order to keep the tape tension in the limits during the shut off period also, a spring was added between the supply motor and its reel.

V. INVESTIGATION OF THE SYSTEM WITH SPRINGS ON BOTH MOTORS

In the investigation of the previous system it was shown that when the stepping commands had been completed and the take-up motor was shut off, the tape tension started to oscillate and to take negative values. In a real system that means that the tape is getting loose and damage would result.

In order to solve this problem, a spring with the same spring constant was added between the supply motor and its reel. The system configuration is shown in Figure 9. Notice that the only difference from the previous scheme is the second tension spring.

The model, then, of this system is:

$$J_A\ddot{\theta}_1 + D_1\dot{\theta}_1 + C(\theta_1 - \theta_2) + T_1\sin(50\,\theta_1) = 0 \qquad (22)$$

Two Tension Spring Tape Transport System

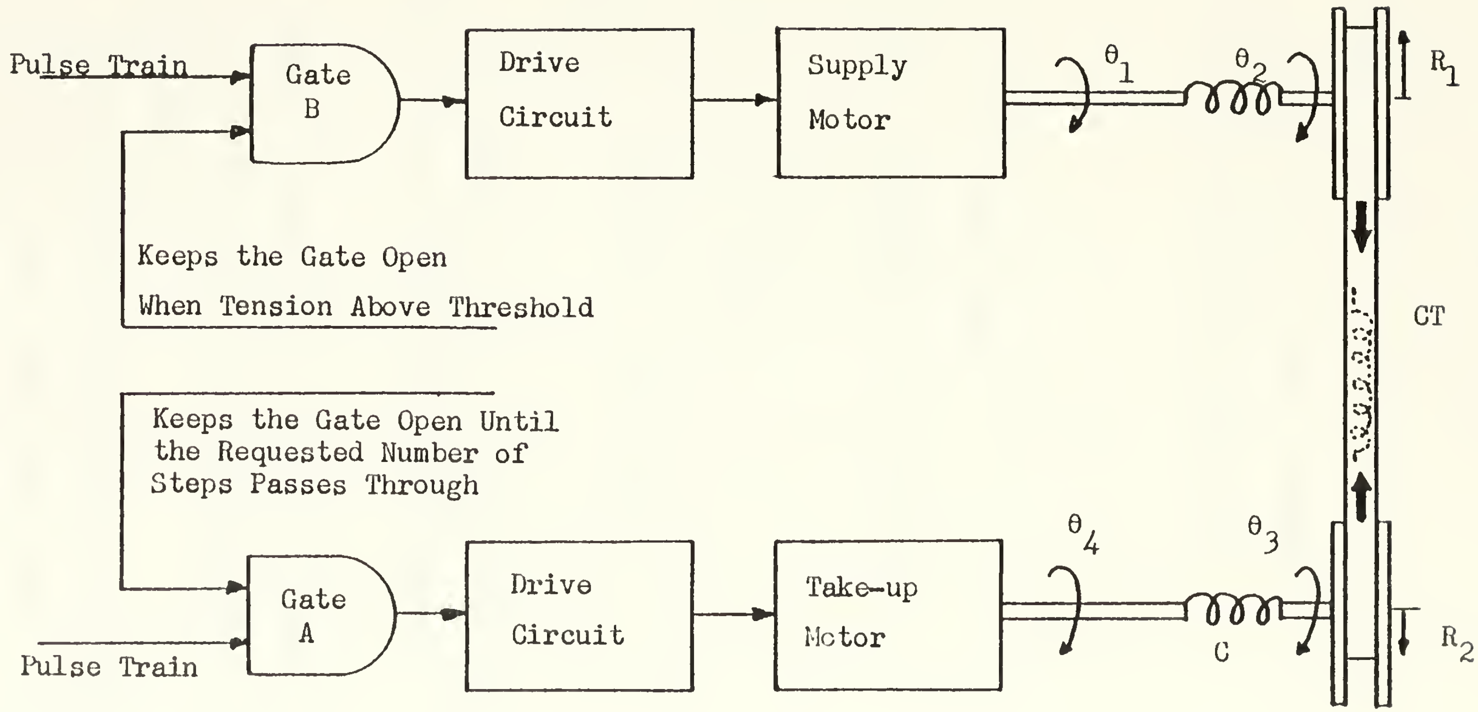

Figure 9

$$J_1\ddot{\theta}_2 + D_2\dot{\theta}_2 - TF\cdot R_1 + C(\theta_2 - \theta_1) = 0 \tag{23}$$

$$J_2\ddot{\theta}_3 + D_3\dot{\theta}_3 + TF\cdot R_2 + C(\theta_3 - \theta_4) = 0 \tag{24}$$

$$J_A\ddot{\theta}_4 + D_4\dot{\theta}_4 + C(\theta_4 - \theta_3) + T_2\sin(50\ \theta_4) = 0 \tag{25}$$

$$TF = CT\ (R_2\theta_3 - R_1\theta_2) \tag{26}$$

where the J_A, J_1, J_2, R_1, R_2 are the same as before.

If the following new variables are added, then,

$$X_1 = 50\ \theta_1 \tag{28}$$

$$X_3 = 50\ \theta_4 \tag{29}$$

Then the equations of the model become:

$$J_A\ \ddot{X}_1/50.0 + D_1\dot{X}_1/50.0 + T_1\sin(X_1) + C(\theta_2 - \theta_1) = 0 \tag{30}$$

$$J_1\ddot{\theta}_2 + D_2\dot{\theta}_2 - TF\cdot R_1 + C(\theta_2 - \theta_1) = 0 \tag{31}$$

$$J_2\ddot{\theta}_3 + D_3\dot{\theta}_3 + TF\cdot R_2 + C(\theta_3 - \theta_4) = 0 \tag{32}$$

$$J_A\ddot{X}_3/50.0 + D_4\dot{X}_3/50.0 + T_2\sin(X_3) + C(\theta_4 - \theta_3) = 0 \tag{33}$$

$$TF = CT\cdot(R_2\theta_3 - R_1\theta_2) \tag{34}$$

It was assumed, at the beginning, that all the tape is on the supply reel. This was simulated and a few hundred steps were applied to the take-up motor.

Because the amount of tape that is transfered is relatively small, it was assumed again for simplicity that the inertias and radii of the reels do not change with time.

It was found that, when there was no tension on the tape, it was possible to apply a stepping command burst with a rate of 1000 steps per second right from the beginning and the motor was able to execute them

without any failure. Then another test was executed.

One hundred steps were applied on the take-up reel in order to establish a pre-tension of a value of about 0.8 lb. After pausing a certain time in order for the system to settle down a few hundred steps were applied. In this case, if they are applied at a rate of 1000 per second, it was found that the take-up motor fails and cannot execute them.

Instead, the first six steps were applied at a rate of 500 steps per second and the following at a rate of 1000 per second. In this way, the take-up motor executed them without any failure. Also, the effect of slowing down the take-up motor for 30 steps before its shut off was studied. It was found that this procedure provides a smaller amplitude of oscillation of the tension after the shut off. This is shown in Figures 12 and 13. The last 30 steps were applied at a rate of 500 per second.

From Figures 10, 11, 12, and 13 it can be seen that the tape tension is kept very well within the limits without any significant oscillations. Except in the case where it was assumed that the damping D_2 and D_3 of the supply and take-up reels were equal to zero. This is shown in Figure 14.

Then it was assumed that almost all the tape is on the take-up reel and we want to transfer a few more. In this case, the supply motor must have a speed of 2.565 times more than the take-up motor. It was found that if we establish a tension on the tape and then apply stepping commands at a rate of 500 per second then the supply motor fails to start and that as a result, the take-up motor fails also.

Then the commands were applied at a rate of 357 per second and the

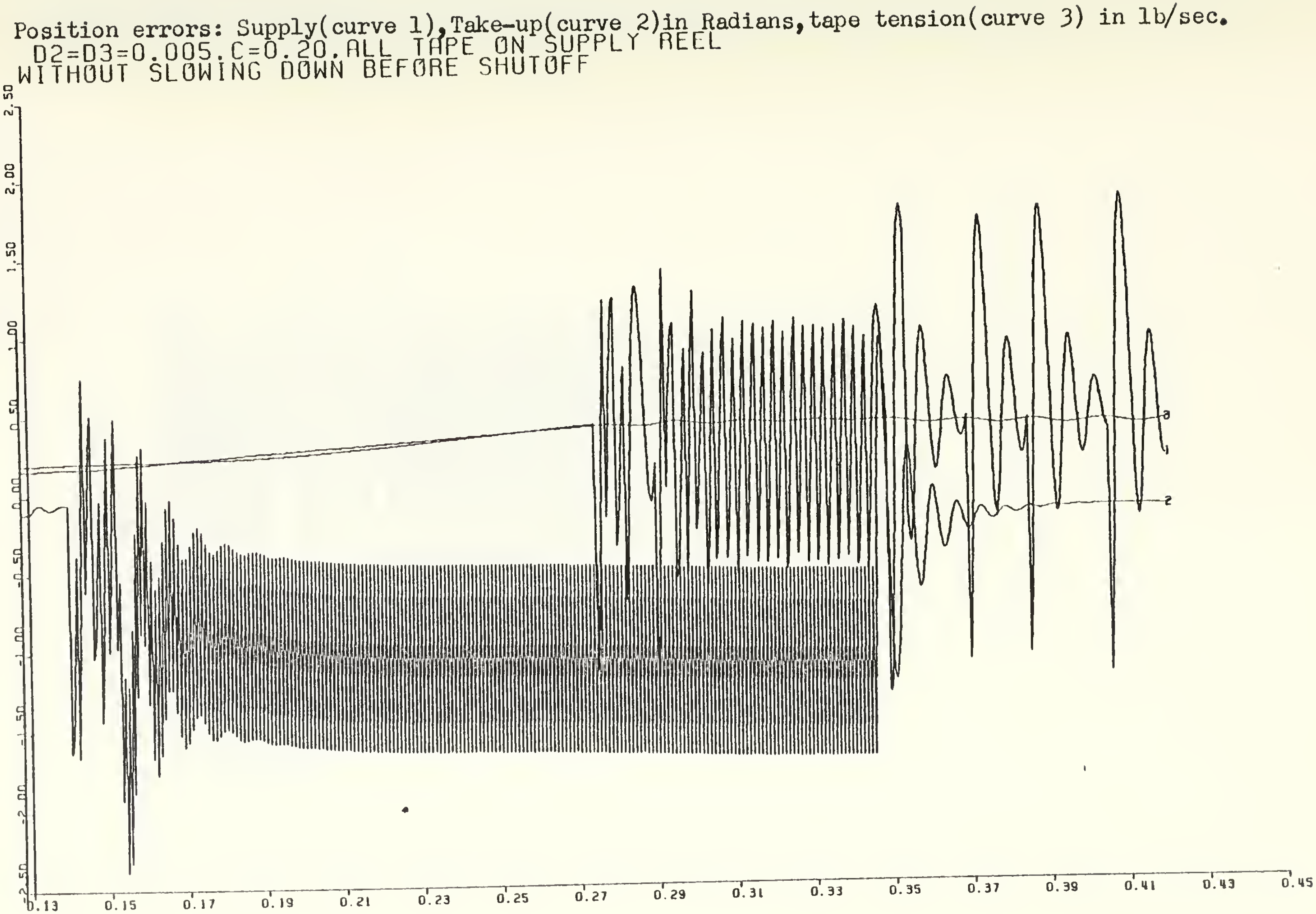
Position errors: Supply(curve 1),Take-up(curve 2)in Radians,tape tension(curve 3) in lb/sec.
D2=D3=0.005.C=0.20.ALL TAPE ON SUPPLY REEL
WITHOUT SLOWING DOWN BEFORE SHUTOFF
2.50
2.00
1.50
1.00
0.50
0.00
-0.50
-1.00
-1.50
-2.00
-2.50
0.13
0.15
0.17
0.19
0.21
0.23
0.25
0.27
0.29
0.31
0.33
0.35
0.37
0.39
0.41
0.43
0.45
3
1
2

Figure 10

Position errors: Supply(curve 1), Take-up(curve 2) in Radians, tape tension(curve 3) in lb/sec.
D2=D3=0.005. C=0.40. ALL TAPE ON SUPPLY REEL
WITHOUT SLOWING DOWN BEFORE SHUTOFF

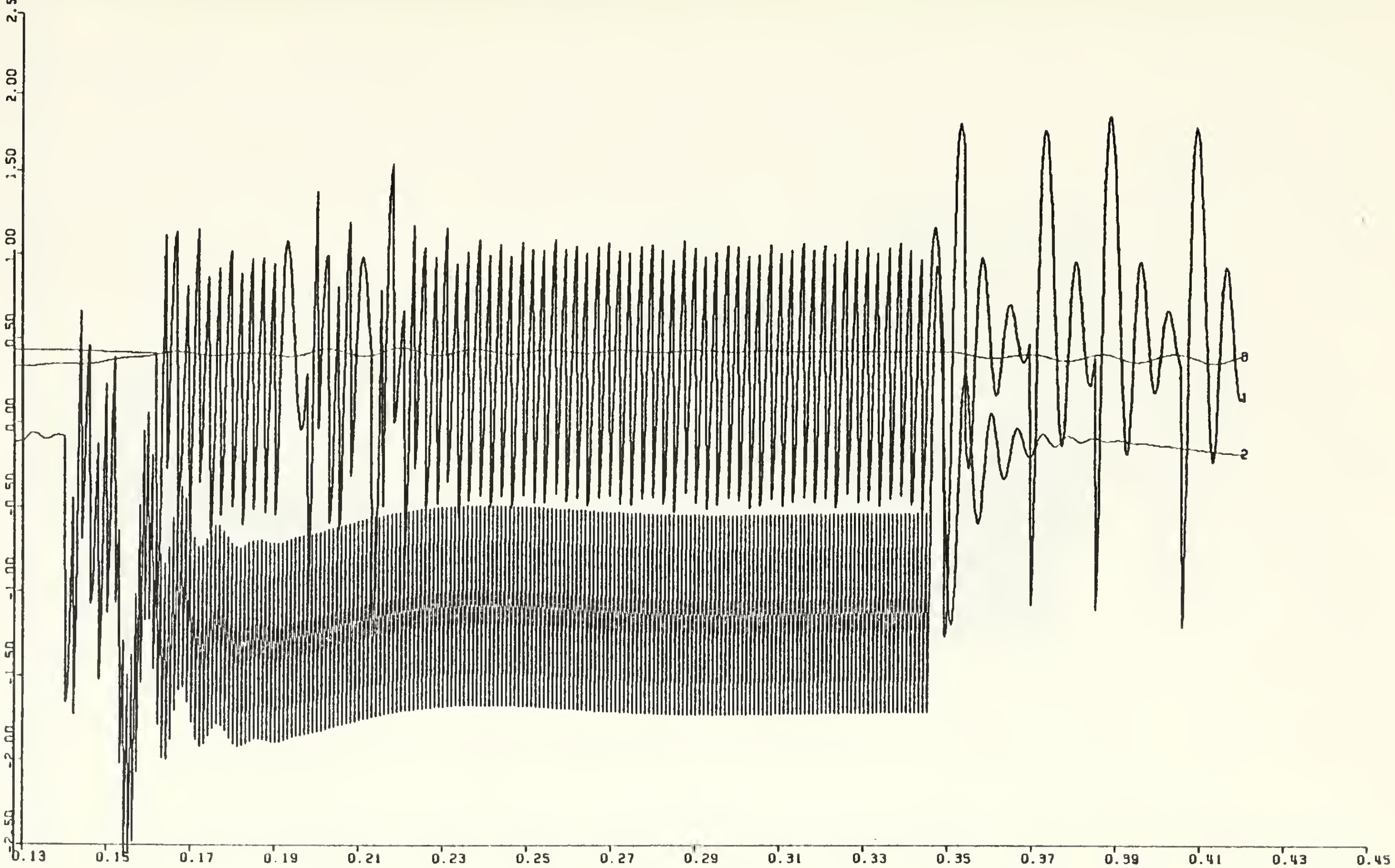

Figure 11

Position errors: Supply(curve 1), take-up(curve 2) in Radians, tape tension(curve 3) in lb/sec.

D2=D3=0.005.C=0.20.ALL TAPE ON SUPPLY REEL
SLOW DOWN BEFORE SHUTOFF

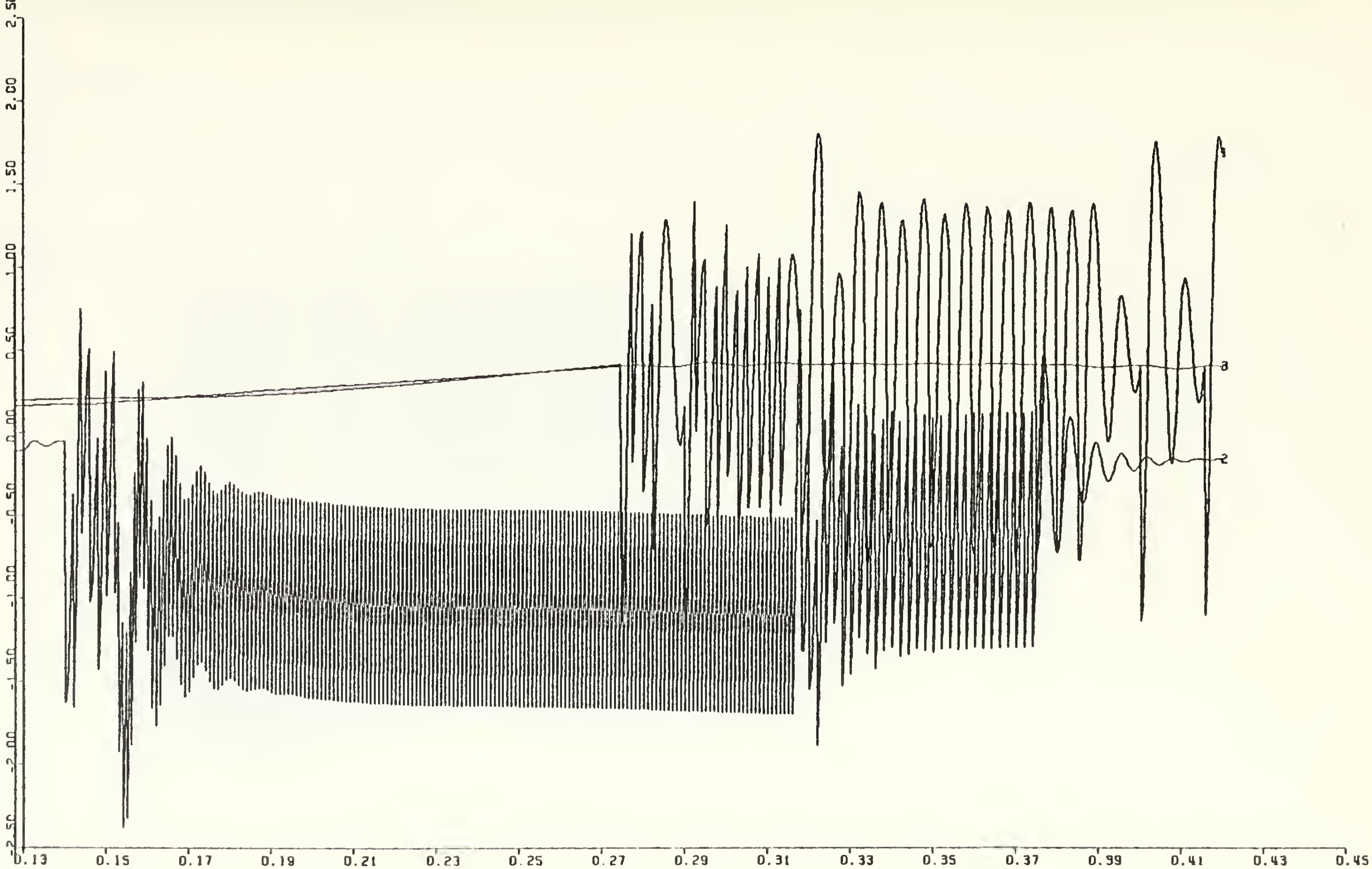

Figure 12

Position errors: Supply(curve 1), take-up(curve 2) in Radians, tape tension(curve 3) in lb/sec.
D2=D3=0.005.C=0.40.ALL TAPE ON SUPPLY REEL
SLOW DOWN BEFORE SHUTOFF

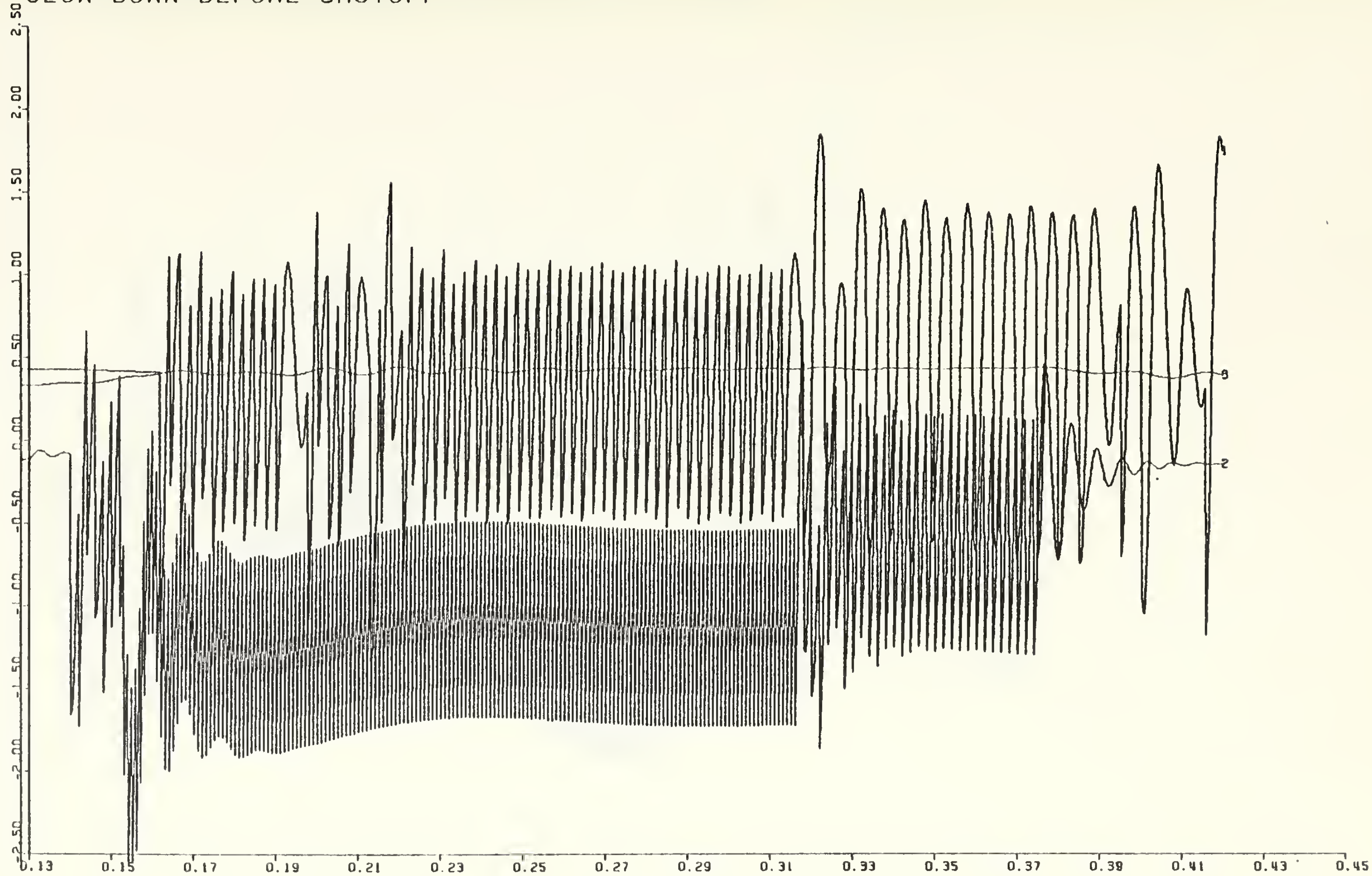

Figure 13

Position errors: Supply(curve 1),Take-up(curve 2) in Radians,tape tension(curve 3) in lb/sec.
D2=D3=0.000.C=0.40.ALL TAPE ON SUPPLY REEL
SLOW DOWN BEFORE SHUTOFF

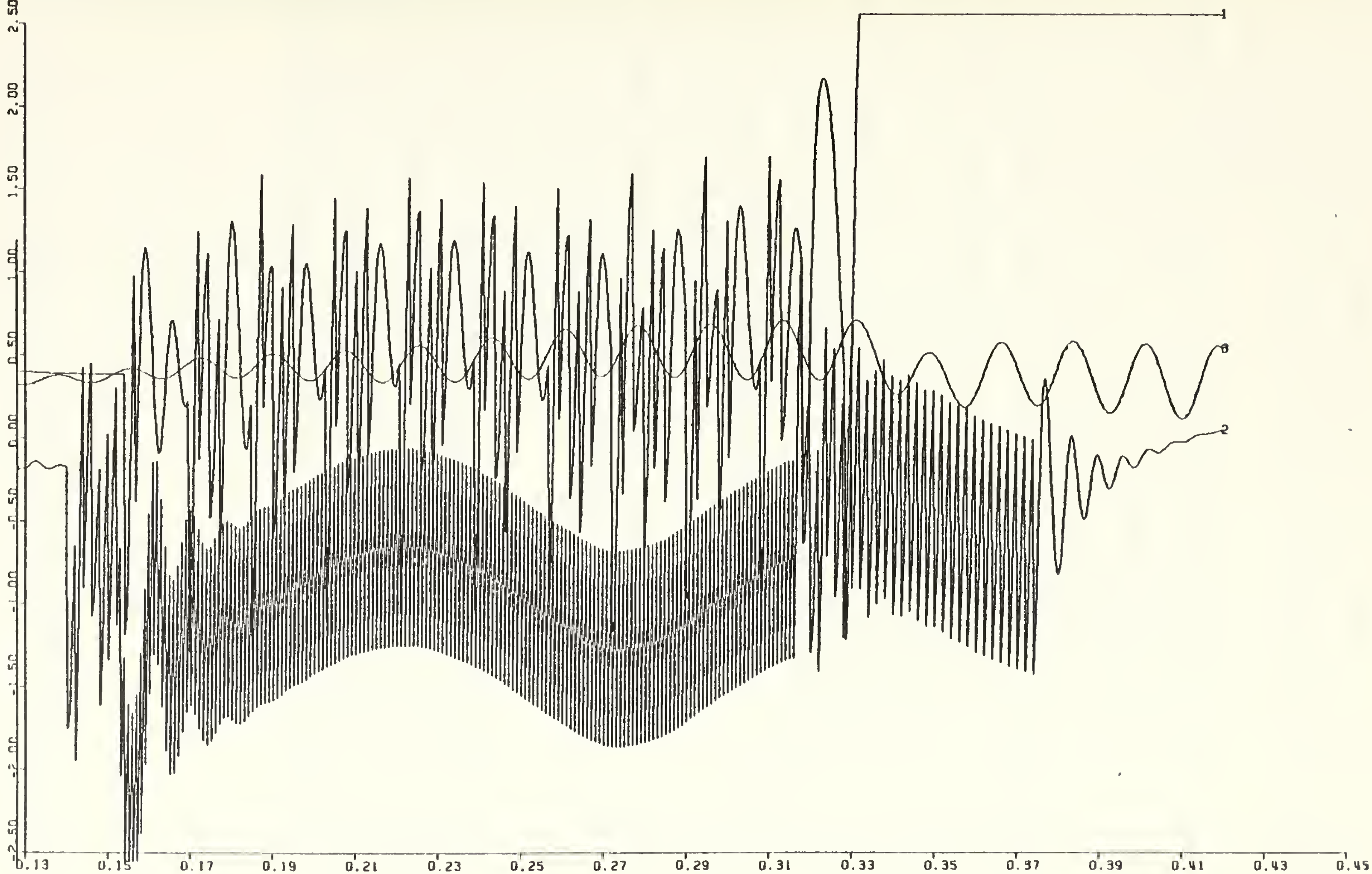

Figure 14

supply motor was able to start. During the transient time, as is shown in Figures 15 and 16, the supply motor loses 4 steps but this is not so important since the tape tension is kept in the limits and the take-up does not fail at all.

Figures 15 and 16 show two simulations with spring constant equal to 0.2 and 0.4 respectively. Since the take-up motor turns at a low speed, slow down was not applied before its shutting off. The two springs were chosen to have to same spring constant in order that the system would be symmetrical with respect to the direction of the tape transfer. In this way, the same strategy of controlling the motors can be used no matter if the tape is moved forward or in reverse. For this reason, transport of the tape was investigated for one direction only.

In the system that had only one spring, it is easier to drive the take-up motor than the supply motor since the load at the first is decoupled through the spring. So, two different strategies of sequence of rates of command application should be developed, according to the direction of the tape transport.

It was found that as far as the tape tension was concerned, the value of the spring constant was not very critical. The tape tension remained within the limits and with very small fluctuations, for values of spring constants equal to 0.2 and 0.4 inches per pound per radian.

Position errors: Supply(curve 1), take-up(curve 2) in Radians, tape tension(curve 3) in lb/sec.
D2=D3=0.005.C=0.20.ALL TAPE ON TAKE UP REEL
WITHOUT SLOWING DOWN BEFORE SHUTOFF

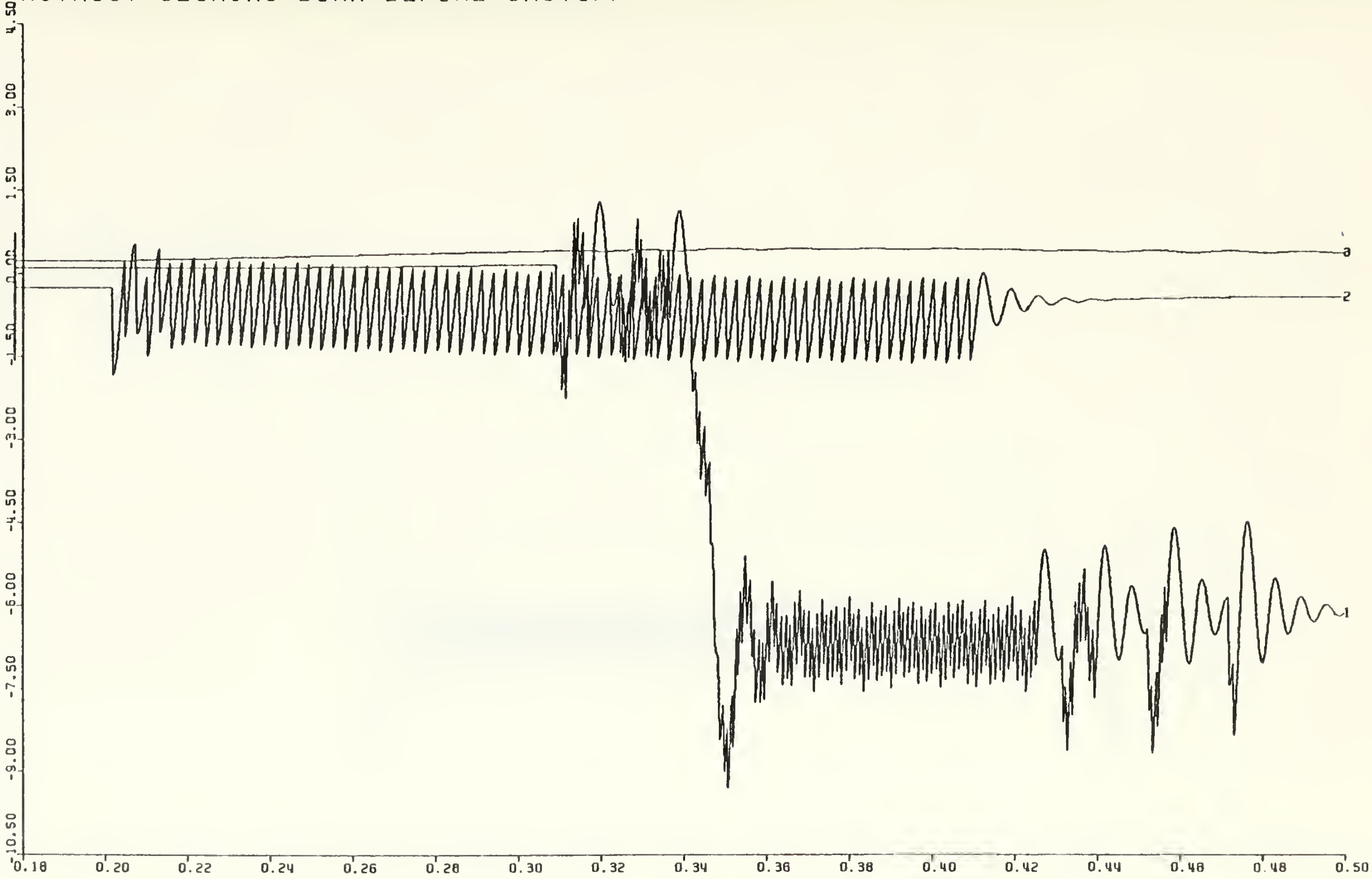

Figure 15

Position errors: Supply(curve 1), Take-up(curve 2) in Radians, tape tension(curve 3) in lb/sec.
D2=D3=0.005.C=0.40.ALL TAPE ON TAKE UP REEL
WITHOUT SLOWING DOWN BEFORE SHUTOFF

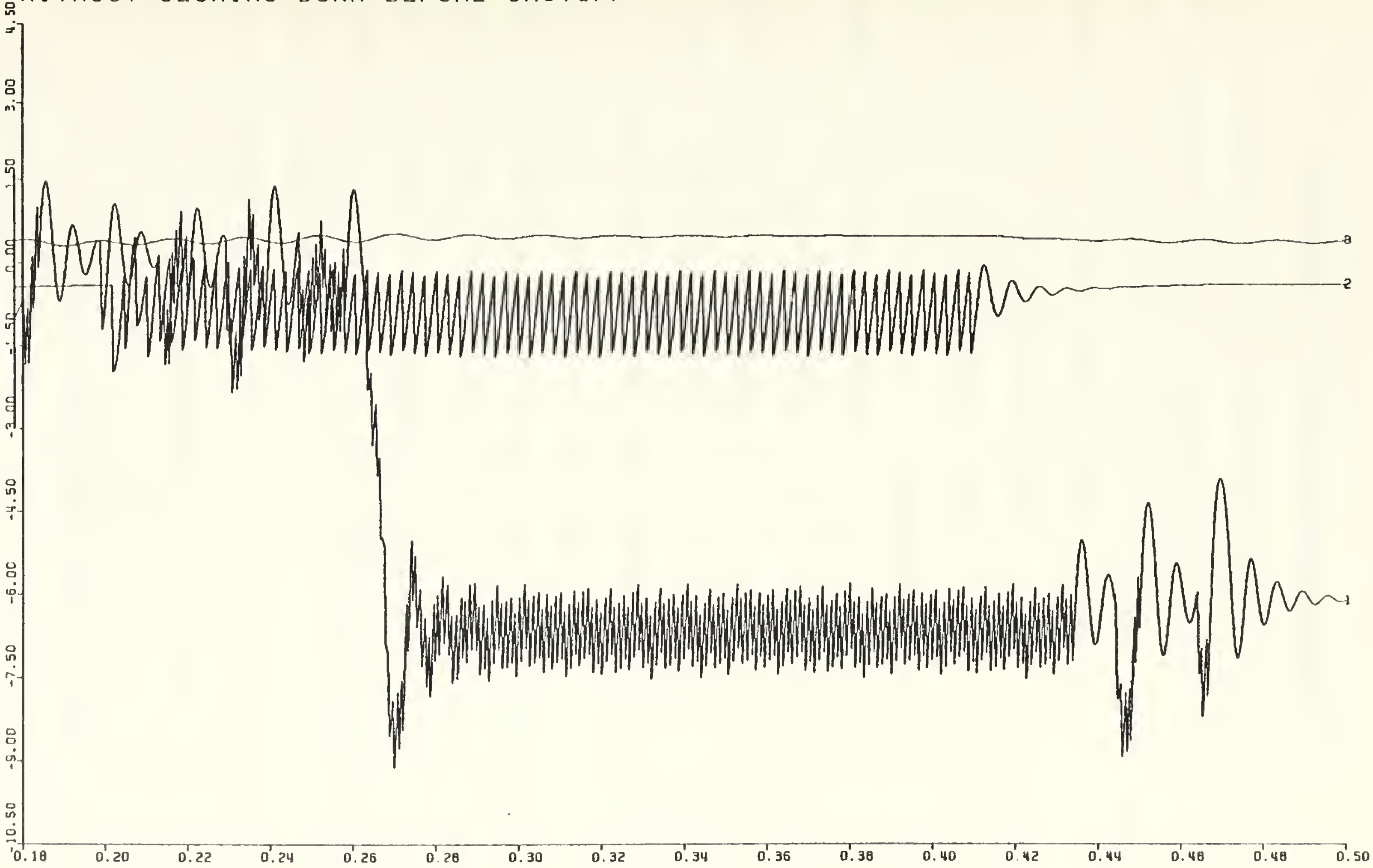

Figure 16

VI. CONCLUSION AND FUTURE STUDIES RECOMMENDATIONS

The investigation and the computer simulation of the magnetic tape transport system employing two tension springs proved that such a scheme fulfills the specifications that were set for a quick tape transfer and the maintenance of the tape tension between desired limits.

The knowledge of the value of the tape tension at any instant of time is essential for the algorithm of the operation and control of the system. In a real system, an optical encoder is proposed to be used on the take-up reel in order to measure the tape tension. Pulses applied to the motor can be counted; pulses due to reel motion can be counted by the optical sensor, so their difference is a measure of spring deflection and thereby of tension if the radius of the tape is known. Thus, the take-up reel should not vibrate too much because it will cause an incorrect estimation of the value of the tension.

Fortunately, the vibration in front of the optical sensor will give an estimation of less tension than its real value and the supply motor will not turn so the tape is not going to get loose. However, accumulative errors can cause the tape tension to maintain higher and higher values until the tape is deformed.

The investigation of this problem is very critical, since the control of the system is based on the assumption that the correct value of the tension is known. Any accumulation of error more than 0.2 lb, if the threshold is set to 0.4 lb, will cause the tension to exceed the upper limit. This problem was not investigated due to lack of time.

When all the tape was on the supply reel, the take-up motor could be

driven at a speed of 1000 steps per second without any failure. When all the tape was on the take-up reel, its maximum speed was found to be 357 steps per second. The optimum speed of the take-up motor between the above two extreme cases should be investigated in order to have the fastest tape transport.

It is recommended that the speed of the take-up reel should be 1000 steps per second until half of the amount of tape has been transferred from the supply to the take-up reel. Then the speed of the take-up motor should be reduced piece-wise from 1000 to 357 steps per second until all the tape has been transferred to the take-up reel.

Other algorithms can be investigated if the motors are to be driven at higher speeds and without restricting the non-negative tape tension limits, as is the case for slewing.

An algorithm that can be investigated is that after pre-tension and with and appropriate starting stepping sequence, both motors start simultaneously.

When the tape tension exceeds an upper threshold, the supply motor should be accelerated to relieve the tension. When to tension exceeds a lower threshold, the supply motor should be decelerated to build up the tension again.

```
//SOTIRIS       EXEC  DSL
//DSL.INPUT     DD   *
*
TITLE ** STEPPING MOTOR SIMULATION **
*
PARAM ZETA=.125,PI=3.1415926,K=0.0,A=1.0
INTGER NPLOT
      NPLOT=1
      THETA=PI/2.0
*
      NSTEP=24.0
*   NSTEP IS THE NUMBER OF  STEPS WE WANT THE MOTOR
*   TO EXECUTE
*
*    EQUATIONS OF THE MODEL
*
      X1DOT=X2
      X1=INTGRL(0.0,X1DOT)
      X2DOT=-2.0*ZETA*X2-SIN(X1)
      X2=INTGRL(0.0,X2DOT)
*
DYNAMIC
*
*    GENERATION OF INPUT PULSES
      P1=0.05
      P2=0.8
      P=IMPULS(P1,P2)
      IF(P.GE.1.0)X1=X1-THETA*A
*
      K=K+P
      IF(K.GE.NSTEP)A=0.0
*    THE K COUNTS THE NUMBER OF STEPS THAT HAVE BEEN
*   APPLIED ON THE MOTOR
*    THE APPICATION OF STEPS STOPS WHEN K BECOMES EQUAL
*   TO NSTEP
*
CONTRL FINTIM=40.0, DELT=0.05,DELS=0.05
SAMPLE
      CALL DRWG(1,1,TIME,X1)
      CALL DRWG(1,2,TIME,X2)
TERMINAL
      CALL ENDRW(NPLOT)
END
STOP
//C.SYSPRINT DD DSN=,SYSOUT=A
//PLOT.STEPLIB DD DSN=C0044.Q,UNIT=3330,VOL=SER=DISK02,DISP=SHR
//PLOT.PLOTPARM DD *
 &PLOT SCALE=0.5,LYNES=1000 &END
```

```
//PLOT.SYSIN DD *
ERROR OF POSITION AND SPEED VS. TIME
 ZETA=0.125  24 STEPS PERIOD=0.8                                                    5
0.0        2.5        -30.0      4.0        16.0       10.0
```

```
//SOTIRIS       EXEC  DSL
//DSL.INPUT     DD  *
TITLE ** ONE SPRING TAPE TRANSPORT SYSTEM **
*
PARAM JA=0.000049,...
J1=0.000134,...
J2=0.000104,...
R1=1.1825,R2=0.461,...
C=.20,CT=10.,D1=.0245,D2=.000,D3=.0127,TFL=.4,T1=1.055,T2=1.055,...
B=0.0,W=0.0,Q=0.0,...
A=1.0,PI=3.1415927,...
NSTEP=200.0
*   NSTEP IS THE NUMBER OF STEPS WE WANT THE MOTOR
*  TO EXECUTE
INTGER NPLOT
      NPLOT=1
      THETA=PI/2.0
*   EQUATIONS OF THE MODEL
INTEG     RKSFX
      X1DOT=X2
      X1=INTGRL(0.0,X1DOT)
      X2DOT=(50./(JA+J1))*(-D1*X2/50.0-T1*SIN(X1)+CT*(R2*TH2-R1*TH1)...
*R1)
      X2=INTGRL(0.0,X2DOT)
      X3DOT=X4
      X3=INTGRL(0.0,X3DOT)
      X4DOT=(-(D3/50.0)*X4-C*(TH3-TH2)-T2*SIN(X3))*(50.0/JA)
      X4=INTGRL(0.0,X4DOT)
      TH2AX=(-CT*(R2*TH2-R1*TH1)*R2-C*(TH2-TH3)-D2*TH2DOT)/J2
      TH2DOT=INTGRL(0.0,TH2AX)
      TH2=INTGRL(0.0,TH2DOT)
      TF=CT*(R2*TH2-R1*TH1)
*
DYNAMIC
*   GENERATION OF THE INPUT PULSES TO MOTOR NUMBER 2
      P1=0.00025
      P2=0.002
      P=IMPULS(P1,P2)
*   GENERATION OF THE INPUT PULSES TO MOTOR NUMBER 1
      L1=0.00025
      L2=(R1/R2)*P2
      L=IMPULS(L1,L2)
*   THE ANGLE OF THE SHAFT #(TH#) IN RADIANS AS A FUNCTION
*  OF THE ELECTRICAL ANGLE X#
      TH1=(W*THETA+X1)/50.
      TH3=(Q*THETA+X3)/50.
*   A IS A GATE THAT PASSES ONLY NSTEP( THE NUMBER OF
*  REQUESTED STEPS ) STEPS
```

```
      Z=P*A
*    WHEN THE NEXT PHASE OF THE MOTOR  # 2 IS ENERGIZED (DUE TO A STEP COMMAND)
*   THE ELECTRICAL ANGLE IS REDUCED BY THETA
      IF(Z.GE.1.0) X3=X3-THETA
*    THE Q COUNTS THE NUMBER OF STEPS THAT HAVE BEEN APPLIED
*   TO THE MOTOR # 2
      IF(Z.GE.1.0) Q=Q+1.0
*
*    B IS A GATE OPEN ONLY IF THE TENSION OF THE TAPE (TF)
*   IS GREATER THAN A LIMIT TFL
*
      IF(TF.LT.TFL) B=0.0
      IF(TF.GE.TFL) B=1.0
      M=B*L
*    IF BOTH MOTORS STOP THE SAME TIME THIS CARD MUST BE REPLACED BY
*     M=B*A*L
*
*    THE SAME AS BEFOR BUT FOR THE MOTOR NUMBER 1
      IF(M.GE.1.0) X1=X1-THETA
      IF(M.GE.1.0) W=W+1.0
      K=K+P
*    THE K COUNTS THE NUMBER OF STEPS THAT HAVE BEEN APPLIED
*   TO THE MOTOR # 2
      IF(K.GE.NSTEP) A=0.0
CONTRL FINTIM=0.5,DELT=0.0001
PRINT 0.00025,X1,X3,TF,X4DOT,P1,P,K
END
STOP
```

```
//SOTIRIS       EXEC  DSL
//DSL.INPUT     DD   *
TITLE ** TWO SPRING TAPE TRANSPORT SYSTEM **
*
PARAM JA=.000049,...
J1=0.000134,...
J2=0.000104,...
D1=0.0127,D2=0.005,D3=0.005,D4=0.0127,TFL=0.4,...
R1=1.1825,R2=0.461,...
C=0.40,CT=10.0,...
T1=1.055,T2=1.055,P1=0.00010,...
A=1.0,K=0.0,PI=3.1415927,W=0.0,Q=0.0,...
NSTEP=100.0
*    NSTEP IS THE NUMBER OF STEPS WE WANT THE MOTOR
*   TO EXECUTE
*
*    THE 100 STEP COMMAND IS GIVEN TO ESTABLISH TENSION
*   ON THE TAPE
*
INTGER NPLOT
      NPLOT=1
      THETA=PI/2.0
*    EQUATIONS OF THE MODEL
INTEG      RKSFX
      X1DOT=X2
      X1=INTGRL(0.0,X1DOT)
      X2DOT=-(D1*X2/50.0+T1*SIN(X1)+C*(TH1-TH2))*( 50.0/JA)
      X2=INTGRL(0.0,X2DOT)
      TH2=INTGRL(0.0,TH2DOT)
      TH2DOT=INTGRL(0.0,TH2AX)
      TH2AX=-(D2*TH2DOT-CT*(R2*TH3-R1*TH2)*R1+C*(TH2-TH1))/J1
      X3DOT=X4
      X3=INTGRL(0.0,X3DOT)
      X4DOT=-(D4*X4/50.0+T2*SIN(X3)+C*(TH4-TH3))*(50.0/JA)
      X4=INTGRL(0.0,X4DOT)
      TH3=INTGRL(0.0,TH3DOT)
      TH3DOT=INTGRL(0.0,TH3AX)
      TH3AX=-(D3*TH3DOT+CT*(R2*TH3-R1*TH2)*R2+C*(TH3-TH4))/J2
      TF=CT*(R2*TH3-R1*TH2)
*
DYNAMIC
      R2STR=NSTEP-30.
*
*    30 STEPS BEFORE SHUT OFF THE MOTOR IS SLOWED DOWN
*   IF WE DO NOT WANT IT TO SLOW DOWN THEN THIS CARD MUST
*   BE REPLACED BY
*     R2STR=NSTEP
*
```

```
*    GENERATION OF THE INPUT PULSES TO MOTOR NUMBER 2
       IF(K.GT.106..OR.K.LT.R2STR)P2=0.001
       IF(K.LE.106..OR.K.GE.R2STR)P2=0.002
       IF(K.LE.100.) P2=0.001
       P=IMPULS(P1,P2)
*    GENERATION OF THE INPUT PULSES TO MOTOR NUMBER 1
       L1=0.00025
       L2=(R1/R2)*P2
       L=IMPULS(L1,L2)
*
*    THE ANGLE OF THE SHAFT #(TH#) IN RADIANS AS A FUNCTION
*   OF THE ELECTRICAL ANGLE X#
       TH1=(W*THETA+X1)/50.0
       TH4=(Q*THETA+X3)/50.0
*    A IS A GATE THAT PASSES ONLY NSTEP( THE NUMBER OF
*   REQUESTED STEPS ) STEPS
       Z=P*A
*    WHEN THE NEXT PHASE OF THE MOTOR  # 2 IS ENERGIZED (DUE TO A STEP COMMAND)
*   THE ELECTRICAL ANGLE IS REDUCED BY THETA
       IF(Z.GE.1.0) X3=X3-(THETA)
       IF(Z.GE.1.0) Q=Q+1.0
*    THE Q COUNTS THE NUMBER OF STEPS THAT HAVE BEEN APPLIED
*   TO THE MOTOR # 2
*
*    B IS A GATE OPEN ONLY IF THE TENSION OF THE TAPE (TF)
*   IS GREATER THAN A LIMIT TFL
       IF(TF.LT.TFL) B=0.0
       IF(TF.GE.TFL) B=1.0
*
       M=B*L
*    THE SAME AS BEFOR BUT FOR THE MOTOR NUMBER 1
       IF(M.GE.1.0) X1=X1-THETA
       IF(M.GE.1.0) W=W+1.0
*
       K=K+P*A
*    THE K COUNTS THE NUMBER OF STEPS THAT HAVE BEEN APPLIED
*   TO THE MOTOR # 2
       IF(K.GE.NSTEP) A=0.0
       IF(K.LT.NSTEP) A=1.0
       IF(TIME.GT.0.1400) NSTEP=300.
*
*    SINCE TAPE TENSION HAS BEEN ESTABLISHED A COMMAND
*   FOR 200 MORE STEPS IS GIVEN
CONTRL FINTIM=.50,DELT=0.000050,DELS=0.00030
SAMPLE
       CALL DRWG(1,1,TIME,X1)
       CALL DRWG(1,2,TIME,X3)
       CALL DRWG(1,3,TIME,TF)
```

```
TERMINAL
      CALL ENDRW(NPLOT)
END
STOP
//C.SYSPRINT DD DSN=,SYSOUT=A
//PLOT.STEPLIB DD DSN=C0044.Q,UNIT=3330,VOL=SER=DISK02,DISP=SHR
//PLOT.PLOTPARM DD *
 &PLOT SCALE=0.5,LYNES=1000 &END
*   WIHTOUT THE 'LYNES=1000'ON THIS CARD THE BUFFER OF
*  THE VERSATEC PLOTER IS OVERFLOWED
//PLOT.SYSIN DD *
 D2=D3=0.005,C=0.40,ALL TAPE ON SUPPLY REEL
SLOW DOWN BEFORE SHUTOFF                                                5
0.180     0.02      -10.5     1.5       16.0      10.0
```

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Thesis
T8228 Tsavdaris
c.1 Design of a magnetic
tape transport system
employing stepper
motors.