NAVAL POSTGRADUATE SCHOOL
Monterey, California

THESIS

AGGREGATION OF MEASURES OF EFFECTIVENESS WITH CONSTANT SUM SCALING METHOD AND MULTIPLE REGRESSION

by

Hyung Bae Kim

March 1979

Thesis Advisor: G. Lindsay

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with data from the U.S. Army Armor School. It was concluded that the aggregation method is feasible, and that for the tank system studied, the reciprocals of system characteristics give a good estimating equation for measuring tank system effectiveness.

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AGGREGATION OF MEASURES OF EFFECTIVENESS WITH CONSTANT SUM SCALING METHOD AND MULTIPLE REGRESSION

by

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ABSTRACT

This thesis explores a method of aggregating the measures of effectiveness of a weapon system from its characteristics. With this method, the constant sum method and multiple regression are used to develop a functional relationship between system effectiveness and system characteristics. As an example, a study of a tank weapon system was conducted with data from the U.S. Army Armor School. It was concluded that the aggregation method is feasible, and that for the tank system studied, the reciprocals of system characteristics give a good estimating equation for measuring tank system effectiveness.

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I. INTRODUCTION

We often find measurement problems in Operations Research that are difficult in that widely used concepts have not been made operational. How to measure the effectiveness of a weapon system is one of the most important tasks in military affairs. What is needed is some method to give answers to such questions as "Which tank is more effective?", or "How much better a M60A1 is than a T-62?"

We want to know how much better one weapon system is than another among similar systems. One way of doing this is through a functional relationship between system effectiveness and the characteristics of the system.

In this paper we will propose and demonstrate a way of structuring such a relationship using as values for the effectiveness of weapon systems values which originally came from military experts' judgements. Once we have found such a function, we would not necessarily require experts judgements again, since one can use the function to calculate the effectiveness of a proposed weapon system from its characteristics.

Chapter II gives the concept and general procedure of this approach to measuring effectiveness. For an illustrative problem we will discuss the selection of the systems and their major characteristics, the preparation of questionnaires and the selection of "expert" judges.

Using information from the judges, we will show in Chapter III how to compute the values of effectiveness of weapon systems using the constant sum method. In seeking a good functional relationship between systems effectiveness and system characteristics, we will use an APL computer program for multiple regression and explore various functional forms by transforming the data. This is the content of Chapter IV. In the final chapter, we will summarize major conclusions and observations during the course of this research.

It must be emphasized that although the illustration study presented here involves the effectiveness of a main battle tank, our purpose is to demonstrate a proposed approach to the measurement of systems effectiveness, and not to develop effectiveness relationships for main battle tanks. A study of the latter type would require resources in excess of those available for this work.

II. DESIGN AND DATA COLLECTION

This chapter describes the proposed method of finding a useful functional relationship between weapon system characteristics and the overall effectiveness of a system, and in particular describes and demonstrates the collection of data needed for this approach. First we will give the concept of the method and the general procedure we are going to follow. Then, using tanks as an example, we will discuss the selection of systems and their major characteristics, and comment about preparing questionnaires.

A. CONCEPT OF THE PROBLEM

The problem we are interested in is to calculate the overall effectiveness of a weapon system from its characteristics. The approach here will be to demonstrate how to estimate the effectiveness of a weapon system using a scaling method [1].

Every weapon system has its own characteristics and if we have a value for the overall effectiveness of that system, we should be able to obtain or fit a relationship expressing:

$$\text{Overall effectiveness of a weapon system} = f(X_1, X_2, \ldots, X_m) \qquad (1)$$

where

$X_1, X_2, \ldots, X_m$ are system characteristics.

The purpose of this paper is to show a procedure for obtaining values for overall effectiveness, and a way of determining the function f. Since we will be fitting functions to data, the more instances of the system we use, the better the functional relationship we can find.

Figure 1 shows the relationship between characteristics and system effectiveness.

SYSTEM INSTANCE	EFFECTIVENESS		CHARACTERISTICS $X_1, X_2, \ldots\ldots\ldots\ldots X_m$
1	Y_1		$x_{11}, x_{12} \cdots\cdots\cdots\cdots x_{1m}$
2	Y_2		$x_{21}, x_{22} \cdots\cdots\cdots\cdots x_{2m}$
3	Y_3	$\overset{f}{\longleftrightarrow}$	$x_{31}, x_{32} \cdots\cdots\cdots\cdots x_{3m}$
.	.		. .
.	.		. .
.	.		. .
.	.		. .
n	Y_n		$x_{n1}, x_{n2} \cdots\cdots\cdots\cdots x_{nm}$

FIG. 1 RELATIONSHIP BETWEEN EFFECTIVENESS AND CHARACTERISTICS

There are n instances of the system and thus n effectiveness values which have to be obtained. The system has m characteristics which we presume to relate to effectiveness, and assume $m < n$. If we express this figure in a mathematical equation we would write:

$$Y_i = f(X_{i1}, X_{i2}, \ldots, X_{im}) \qquad (2)$$

describing overall effectiveness of a system as a function of its characteristics, and we propose to show how to obtain values for the Y_i, and how to find a good fitting function f.

B. GENERAL STUDY PROCEDURE

The general study procedure is composed of 3 steps.

Step 1. Design of the study. In this step we have to select the instances of the system and the major characteristics of that system, and collect the data to provide characteristic values for each selected instance. A scaling method must then be chosen by which we can quantify information from judges about system effectiveness. After doing this, we prepare questionnaires and send them to selected judges to obtain information from them about the effectiveness of the system.

Step 2. Computation of the effectiveness of each instance of the system. The questionniares sent to the judges are to obtain expert judgement, which may be expressed as a ranking of systems or as a ratio scale value of effectiveness, according to the requirement of the scaling method which has been selected. Based on this information, the next thing to do is to use the scaling method to compute the effectiveness of each instance of the system.

Step 3. Determination of the functional relationship. Once we have the numberical values of effectiveness and the characteristics for each system, the next step is similar to

PROBLEM OBJECTIVE

STEP 1	STEP 2	STEP 3
DESIGN OF THE STUDY	COMPUTATION OF EFFECTIVENESS	DETERMINATION OF EQUATION
SELECTION OF SYSTEM	OBTAIN DATA FROM EXPERTS' JUDGEMENT	DETERMINATION OF EQUATION FROM MULTIPLE REGRESSION
SELECTION OF MAJOR CHARACTERISTICS		
COLLECTION OF DATA OF THE INSTANCE OF SYSTEM	COMPUTATION OF EFFECTIVENESS OF EACH SYSTEM	SELECTION OF THE BEST EQUATION
SELECTION OF SCALING METHOD		
PREPARATION OF QUESTIONNAIRE		
SELECTION OF JUDGES		

FIG. 2 PROCEDURE OF THE STUDY

finding a cost estimating equation [2]. Because of many characteristics and instances of the system, it seems reasonable to use the multiple regression technique to find a good estimating equation. Since we will not restrict ourselves to the linear equation case, there are many candidate functions available by transforming the data. Therefore we have to choose the best functional relationship by evaluating these candidate equations. This whole procedure is represented in Figure 2.

C. DETAILED PROCEDURE

We will discuss the elements of the design of the study in detail in this section. Steps 2 and 3 (computation of effectiveness and determination of a functional relationship) will be discussed in detail in Chapters III and IV.

1. Selection of the Competing Systems

Among various types of weapon systems we could use as the example in this paper, let's consider the tank weapon system. What kind of tank should we choose? It depends on what we are going to do with the tanks, and since we want to decide which tank system is good for battle, we shall choose Main Battle Tanks (MBTs) [3].

We shall define Main Battle Tanks (MBTs) as the greatest number of tanks a country has for battle. For this study six Main Battle Tanks were chosen:

M48A5	(Korea),
M60A1	(U.S.A.),
AMX-30	(France),
LEOPARD 1	(W. Germany),
T-62	(U.S.S.R.),

and

CENTURION (Mk.13)	(Great Britain).

The reason why these tanks are chosen is that (1) these are all Main Battle Tanks that are currently in service, and (2) they are very well-known tanks so that the experts can give knowledgeable information. If some of the tanks were older ones not in service or tanks under development, some judges might not be familiar with them.

As mentioned before, the more tanks used in the study, the better is the relationship that may be obtained. The reason for using only six MBTx here relates to the work-load that is placed on the judges. In the constant sum scaling method which we are going to use, each judge is asked to consider each possible pair of instances and split 100 points between the two instances in each pair. Thus for n instances to be scaled, n(n-1)/2 pairs must be considered by each judge. Therefore we have to consider the number of instances to be scaled because the greater the number of instances, the greater the workload asked of each judge and the less careful he might be in his rating and the smaller proportion of questionnaires we would expect to be returned.

Since "the more judges the better" is particularly an axiom in scaling, tradeoffs may have to be made between the effort that will be required of a judge and the amount of confidence one wishes to have in the resulting scale. Our selection of six tanks (instances) requires $\frac{6(6-1)}{2} = 15$ pairs of tanks to be considered, and this number is thought reasonable for a judge to handle in a short period of time.

2. Selection of the Major Characteristics

There are of course many characteristics which affect the effectiveness of a system. For example, one source lists about 30 characteristics which are deemed relevant to the importance of a tank system [3].

Some of the characteristics have very similar values among various MBTs, and these characteristics should not make any substantial difference in the comparison of effectiveness among the competing systems. Thus we don't need to consider this kind of characteristic.

We have chosen 4 tank characteristics as follows:

1. speed
2. silhouette
3. Hp/ton
4. armor.

These characteristics are not necessarily the most important ones. For example, fire power is a very important consideration, but obtaining useful numerical data on fire power is very difficult due to a lack of a standard measurement criterion.

For fire power, we could consider the main gun, but all the selected Main Battle Tanks have a similar main gun with caliber 105 mm except T-62 which has 115 mm main gun. Therefore we have chosen the above characteristics as generally accepted important factors, which should serve well in our demonstration of a method for assessing system effectiveness.

3. Collection of Data

After selecting the system instances and the major characteristics for the study, we have to collect data for each characteristic. Table I shows the basic data [3] of the six Main Battle Tanks, which we will use. Clearly, one should use all the information sources available: manufacturer, military sources, or technical reports.

Table I. CHARACTERISTICS OF SIX MAIN BATTLE TANKS [3]

CHARACTERISTICS / SYSTEMS	ARMOR (mm on nose)	SILHOUETTE (height in m)	SPEED (km/hr)	H_p/ton
LEOPARD 1	70	2.64	65	20.7
M60A1	110	3.26	48.3	15.3
T-62	100	2.4	50	19.2
M48A5	110	3.09	48.2	15.9
CENTURION (Mk.13)	118	3.01	34.6	12.5
AMX-30	48	2.85	65	19.4

4. Selection of the Method

There are many scaling methods we could use for our study such as a numerical method, the constant sum method, a comparative method, or a categorical method. For our study we want to know how much better one system is than another. In other words, we want a ratio scale that can be used for directly comparing two systems.

Among those scaling methods the constant sum method will give a ratio scale which is easy to use. We can convert the information from judges about system effectiveness into a ratio scale. Therefore the constant sum method was chosen for the illustrative study of tank effectiveness.

5. Preparation of the Questionnaire

This is one of the most important parts of the analysis. The questionnaire should be prepared very carefully with a clear explanation of how to fill it out, together with information about the systems which the judges can use to assist them in their ratings, Since we are going to use the constant sum method to compute the effectiveness, the judges will be asked to make ratio scale judgements by splitting 100 points between the two instances represented by each possible pair of n tanks.[1] For example a judge

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[1]Alternately, of course, one could gather information from a judge by asking for a numerical effectiveness rating for each tank, so that a judge would assign a number to each tank reflecting how effective he thought it was. This process, however, is a very difficult rating task for the judge. Literature in psychometrics suggests that judges will reflect their feelings more accurately, and with more confidence, if they are given just two instances at a time to rate, as in the constant sum method [1].

might split the 100 points as

M60A1 80 T-62 20

if he thinks M60A1 is much better (4 times in this example) than the T-62 or he might split the 100 points as

M60A1 50 T-62 50

if he thinks they are equal in terms of combat effectiveness.

For our study we have six instances, and therefore there are 15 pairs to be presented to the judges (the questionnaire which was used is shown in Appendix A).

The questionnaire actually used consisted of two parts, seeking ratings on both overall tank effectiveness and on the contribution to tank effectiveness of various characteristics. Scaled values from the second, supplementary study are given in Appendix C and may be of interest to readers.

6. Selection of the Judges

There appears to be no rule or standard for designating individuals as "experts". It depends on common sense or military judgements.

We believe that the armor officers in the U.S. Army Armor school may be considered experts about tanks. Therefore we selected them as judges and, after obtaining the gracious cooperation of the Armor School, sent 50 questionnaires to them. The 50 questionnaires were completed by all ranks ranging from Lieutenant to flag officer, and all 50 questionnaires were returned, ready for analysis.

So far in this paper we have structured the approach, selected the systems we are interested in evaluating, collected data on those systems, selected an analytic method and prepared the questionnaires for the judges. In the next chapter we will describe how the overall effectiveness values were computed from the information provided by the judges.

III. COMPUTATION OF SYSTEM EFFECTIVENESS

In the previous chapter we showed how the model was organized, and how the questionnaires were prepared for the judges. In this chapter we will explain the constant sum scaling method of computing the overall effectiveness values for each system instance, using the information received from the judges.

A. CALCULATION OF THE EFFECTIVENESS OF EACH WEAPON SYSTEM USING THE CONSTANT SUM SCALING METHOD

The constant sum scaling method is designed to scale a property with a natural origin or an origin upon which the judges agree. Judges are asked to allocate 100 points between two instances, considering each possible pair of instances. Thus the resulting scale for instances of the property is taken as a ratio scale. In the following paragraphs we will explain how the scale values are obtained.

Let a_{ij} be the number of points out of 100 which a judge awards to instance j when it is compared to instance i. If we arrange a judge's responses in an array a_{ij}, there will be one array for each judge in which values on the diagonal would be set at 50 because comparing something with itself should be 50:50.

We could average these arrays over the N judges to produce an array $\overline{a_{ij}}$ where

$$\overline{a_{ij}} = \frac{\sum\limits_{\text{over all judges}} a_{ij}}{N} \qquad (3)$$

and the values on the diagonal remain set at 50. Table III shows the $\overline{a_{ij}}$ values for this study, as computed from the data collected from 50 judges (the 50 a_{ij} arrays of individual judges are omitted).

From Table III one may see, for example, that the average of the judges' 100-point split in overall tank effectiveness between the T-62 and the AMX-30 was 55.9 for the T-62 and 44.1 for the AMX-30.

Table III. $\overline{a_{ij}}$ Array Computed from Judges' Responses

	M48A5	M60A1	AMX-30	LEOPARD 1	T-62	CENTURION (Mk.13)
M48A5	50	56.94	45.1	64.22	54.7	45.4
M60A1	43.06	50	39.14	54.44	45.04	39.36
AMX-30	54.9	60.86	50	63.86	55.9	49.7
LEOPARD 1	35.78	45.56	36.14	50	41.46	37.54
T-62	45.3	54.96	44.1	58.54	50	44
CENTURION (Mk. 13)	54.6	60.64	50.3	62.46	56	50

The next step is to construct a new W_{ij} array where the entries are the ratios of the instance values across the diagonal, or

$$W_{ij} = \frac{\overline{a_{ij}}}{\overline{a_{ji}}} \qquad (4)$$

Of course the diagonal entries in the W_{ij} array should be 1.0. In this array, it is immediately apparent that the entry in the ith row and jth column is the reciprocal of that in the jth row, and ith column, i.e.,

$$W_{ij} = \frac{1}{W_{ji}} . \qquad (5)$$

Values of w_{ij} for the tank data were computed from the data in Table III, and are shown in Table IV.

Table IV. W_{ij} Array

	M48A5	M60A1	AMX-30	LEOPARD 1	T-62	CENTURION (Mk.13)
M48A5	1	1.322	.821	1.785	1.207	.831
M60A1	.756	1	.643	1.195	.819	.649
AMX-30	1.217	1.555	1	1.767	1.267	.988
LEOPARD 1	.557	.837	.566	1	.708	.601
T-62	.828	1.220	.789	1.412	1	.786
CENTURION (Mk. 13)	1.203	1.541	1.012	1.664	1.273	1

Since W_{ij} is the ratio of the average points awarded to j (when compared to i) to the average points awarded to i (when compared to j), then in general, if S_i and S_j are the

scale values we seek, W_{ij} is an estimate of the ratio S_j/S_i. Thus in terms of the Table IV data, for example, judges have indicated that they feel that the M60A1 tank is 1.322 times more effective than the M48A5 tank. The solution is overdetermined, however, since there are far more W_{ij} ratios (fifteen) than there are scale values to be estimated (six). For example, one could also compare the M60A1 and the M48A5 by comparing both against the Leopard 1, and in this case the M60A1 would be judged

$$\frac{0.837}{0.557} = 1.503$$

times better than the M48A5.

We propose to handle this multiple estimate problem by a least squares approach over the estimates.

We could write

$$W_{ij} = \frac{S_j}{S_i} \tag{6}$$

by taking the log of both sides of (6), we have

$$\log S_j - \log S_i = \log W_{ij},$$

or

$$\log W_{ij} - (\log S_j - \log S_i) = 0 . \tag{7}$$

For the least-squares approach we wish to obtain values of S_j and S_i which will make

$$\log W_{ij} - (\log S_j - \log S_i)$$

close to zero over all instance pairs i,j. Thus we want to find values of $S_1, S_2, \ldots, S_n$ such that

$$Q = \sum_{i=1}^{n} \sum_{j=1}^{n} [\log W_{ij} - (\log S_j - \log S_i)]^2 \qquad (8)$$

in minimized.

Algebraically expanding (8), we have

$$Q = \sum_{i=1}^{n} \sum_{j=1}^{n} [\log^2 W_{ij} - 2 \log W_{ij} \log S_j + 2 \log W_{ij} \log S_i + \log^2 S_j - 2 \log S_j \log S_i + \log^2 S_i].$$

In order to solve for the values of S_j which will minimize Q, we take the n partial derivatives of Q with respect to S_j and set them equal to zero. Thus

$$\frac{\partial Q}{\partial S_j} = \sum_{i=1}^{n} \sum_{j=1}^{n} [\frac{-2 \log W_{ij}}{S_j} + \frac{2 \log S_j}{S_j} - \frac{2 \log S_i}{S_j}] = 0 ,$$

$$\sum_{i=1}^{n} \sum_{j=1}^{n} [-\log W_{ij} + \log S_j - \log S_i] = 0 ,$$

$$\sum_{i=1}^{n} \sum_{j=1}^{n} \log S_j = \sum_{i=1}^{n} \sum_{j=1}^{n} \log W_{ij} + \sum_{i=1}^{n} \sum_{j=1}^{n} \log S_i ,$$

and finally,

$$\log S_j = \frac{\sum_{i=1}^{n} \log W_{ij}}{n} + \frac{\sum_{i=1}^{n} \log S_i}{n}, \quad j = 1, 2, \ldots, n \tag{9}$$

Since the choice of a unit for the scale is arbitrary, we will choose one such that the average of the logs of the scale value is zero, or

$$\frac{\sum_{i=1}^{n} \log S_i}{n} = 0 .$$

Thus equation (9) becomes

$$\log S_j = \frac{\sum_{i=1}^{n} \log W_{ij}}{n}, \quad j = 1, 2, \ldots, n . \tag{10}$$

The estimated scale values, S_j, are given, of course, by the antilogarithms of

$$\frac{\sum_{i=1}^{n} \log W_{ij}}{n} .$$

If we take the antilogarithms of

$$\frac{\sum_{i=1}^{n} \log W_{ij}}{n},$$

equation (10) becomes

$$S_j = \prod_{i=1}^{n} (W_{ij})^{1/n}, \quad j = 1, 2, \ldots, n \qquad (11)$$

Therefore the scale value of instance j, S_j, derived from the least square approach can be interpreted simply as a Geometric mean of the jth column of the W_{ij} array. Using Table IV (W_{ij} array) and equation (11), we obtained S_j values as follows:

$$S_1 = 0.89$$
$$S_2 = 1.22$$
$$S_3 = 0.78$$
$$S_4 = 1.44$$
$$S_5 = 1.02$$
$$S_6 = 0.79$$

These ratio scale values for the overall effectiveness of six tank weapon systems will be discussed in the next section.

B. SYSTEM EFFECTIVENESS VALUES

At last we have ratio scale values for the relative effectiveness of each tank as shown in Table V.

Table V. EFFECTIVENESS OF SIX MAIN BATTLE TANKS

TANK	EFFECTIVENESS
M48A5	0.89
M60A1	1.22
AMX-30	0.78
LEOPARD 1	1.44
T-62	1.02
CENTURION (Mk. 13)	0.79

If we represent these effectiveness values graphically, they would appear as in Figure 3.

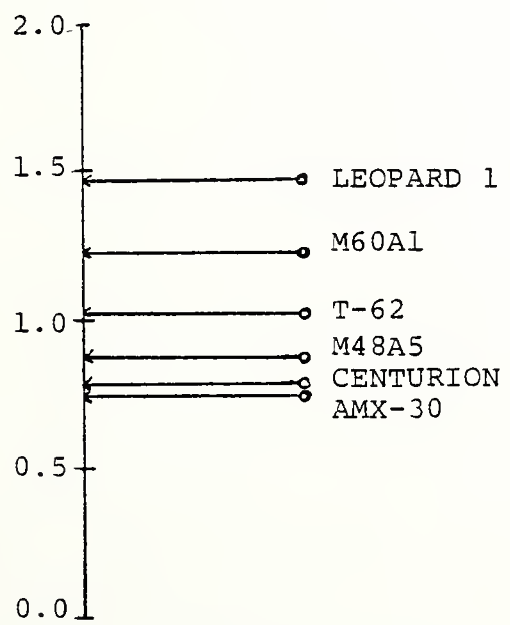

FIG. 3 EFFECTIVENESS OF SIX MAIN BATTLE TANKS

The effectiveness values calculated above and shown here are relative effectiveness values, and have no physical units.

An advantage of using a scaling method which requires that the judges provide ratio scale information is that the output is also a ratio scale. This is very advantageous in interpreting the results in that, for example, we can say the Leopard 1 is more effective (or better) than the AMX-30 by

$$\frac{1.44 - 0.78}{1.44} \times 100 = 46\% .$$

Such a statement would not be possible if the scale was interval or less. The effectiveness value by itself is meaningless, but because this is a ratio scale, we can compare the two systems directly by ratios, and can say how much better one is than the other.

In this chapter we have computed the effectiveness of six tank weapon systems using the constant sum scaling method. This scaling approach provides an effective way of computing the overall effectiveness of weapon systems. However, this use of a scaling method alone requires that we have to send questionnaires everytime we want to calculate the effectiveness of, say, a new or different Main Battle Tank. This is because the effectiveness we computed is based on the information given by the judges, but not directly on the system characteristics.

In the next chapter, we are going to find a functional relationship between the effectiveness computed from the information given by the judges and the characteristics of systems. Such a relationship could be used in subsequent analyses instead of sending questionnaires.

IV. DETERMINATION OF A FUNCTIONAL RELATIONSHIP FOR SYSTEM EFFECTIVENESS

In the previous chapter we computed the effectiveness of six Main Battle Tanks, and we have the characteristics of these systems from Chapter II. In this chapter, we are going to find a function which relates the characteristics to the effectiveness values. With the data shown in Table I (characteristics), tank characteristics can be thought of as explantory variables and the effectiveness can be thought of as a dependent variable for the multiple regression [2] analysis which will now be discussed.

A. SEARCHING FOR A FUNCTIONAL RELATIONSHIP BETWEEN EFFECTIVENESS AND SYSTEM CHARACTERISTICS USING MULTIPLE REGRESSION

Since there are six dependent variables (six Main Battle Tanks) and four explanatory variables (four characteristics), computational work [4] is simplified with a computer program for linear and non-linear multiple regression. There exists a very useful APL computer program for multiple regression named "REGRESS",[2] which will be used to find a good functional relationship between effectiveness and system characteristics.

The "REGRESS" program does a multiple regression analysis relating the dependent variables Y to a set of explanatory

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[2]See Appendix B.

variables X. Here Y is a vector of size n and the right hand argument X is an n x m matrix consisting of n observations on each of m variables, corresponding to the tank characteristic data in Table I.

Output consists of an ANOVA table, coefficient of determination R^2, Standard Error SE, regression coefficients b_j (the first coefficient is the constant term, a), and a vector of predicted Y values and residuals.

For the analysis, we used the computer program "REGRESS" on APL by taking

(1) a linear combination of the characteristics, or

$$Y_i = a + \sum_{j=1}^{m} b_j X_{ij} ,$$

(2) a linear combination of logs of the characteristics, or

$$Y_i = a + \sum_{j=1}^{m} b_j \log X_{ij} ,$$

(3) a linear combination of logs of both the characteristics and effectiveness, or

$$Y_i = \exp[a + \sum_{j=1}^{m} b_j \log X_{ij}] ,$$

(4) a linear combination of reciprocal of the characteristics, or

$$Y_i = a + \sum_{j=1}^{m} b_j \left(\frac{1}{X_{ij}}\right) ,$$

(5) a linear combination of reciprocal of both the characteristics and the effectiveness, or

$$Y_i = \left[a + \sum_{j=1}^{m} b_j \left(\frac{1}{X_{ij}}\right)\right]^{-1} ,$$

(6) a linear combination of square root of the characteristics, or

$$Y_i = a + \sum_{j=1}^{m} b_j \sqrt{X_{ij}} ,$$

(7) a linear combination of reciprocal of square root of the characteristics, or

$$Y_i = a + \sum_{j=1}^{m} b_j \left(\frac{1}{\sqrt{X_{ij}}}\right) ,$$

and finally

(8) a linear combination of square root of both the characteristics and effectiveness, or

$$Y_i = [a + \sum_{j=1}^{m} b_j \sqrt{X_{ij}}]^2 .$$

From the computer output (see Appendix B), the results were obtained as shown in Table VI. Here R^2, the coefficient of determination, shows the proportion of total variance accounted for by the estimating equation as a measure of dispersion, and thus a bigger R^2 is better. The third column shows the standard error which is defined as the square root of the unexplained variance of the dependent variables Y. Therefore the smaller the standard error, the better the estimating equation. The F-ratio is defined as

$$F = \frac{\text{Regression mean square}}{\text{Residual mean square}} . \qquad (12)$$

This F-statistic is used to test whether the incremental improvement associated with the addition of a variable is significant. Thus the larger the F-value the better. The last column shows the coefficient of variation which relates the standard error (SE), to the mean of the dependent variables Y's, or

$$CV = \frac{SE}{\bar{Y}} . \qquad (13)$$

Table VI. SUMMARY OF RESULTS

REGRESSION	R^2	SE	F	CV
$Y_i = a + \sum_{j=1}^{m} b_j X_{ij}$	0.783	0.267	0.955	0.261
$Y_i = a + \sum_{j=1}^{m} b_j \log X_{ij}$	0.888	0.196	1.978	0.192
$Y_i = \exp[a + \sum_{j=1}^{m} b_j \log X_{ij}]$	0.877	0.194	0.1775	0.189
$Y_i = a + \sum_{j=1}^{m} b_j (\frac{1}{X_{ij}})$	0.980	0.083	12.24	0.081
$Y_i = [a + \sum_{j=1}^{m} b_j (\frac{1}{X_{ij}})]^{-1}$	0.958	0.110	5.704	0.107
$Y_i = a + \sum_{j=1}^{m} b_j \sqrt{X_{ij}}$	0.838	0.236	1.29	0.231
$Y_i = a + \sum_{j=1}^{m} b_j \frac{1}{\sqrt{X_{ij}}}$	0.937	0.147	3.947	0.143
$Y_i = [a + \sum_{j=1}^{m} b_j \sqrt{X_{ij}}]^2$	0.833	0.116	1.249	0.113

Y_i : Effectiveness
a : Constant term
b_j : Coefficients
X_{ij} : Characteristic Data (Table I)

This value is used in comparing one standard error with another; a lower CV value is better.

B. SELECTION OF THE BEST EQUATION

Looking at Table VI, the largest measure of dispersion (R^2) is 0.98. The smallest standard error is 0.083, the highest F-ratio is 12.24 and the smallest Coefficient of Variation (CV) is 0.081. Fortunately, all of these best values for measures of fit occur when we linearly combine the reciprocals of the data. Here the Coefficient of Variation (CV) is 8.1%, which tells us the estimating equation is fitted very well in this case.

The coefficients[3] that provided the good fitting function were

-11.62
-74.15
27.93
-622.54
271.94

Therefore the best estimating relationship developed among those forms investigated is

$$\text{EFFECTIVENESS} = 11.62 - \frac{74.15}{\text{ARMOR}} + \frac{27.93}{\text{SILHOUETTE}} - \frac{622.54}{\text{SPEED}} + \frac{271.94}{\text{Hp/hr}} . \qquad (14)$$

---

[3] See Appendix B.

The closeness of the fit of this function to the total effectiveness values furnished by the judges is shown, for individual points, in Table VII.

Table VII. JUDGE EFFECTIVENESS VS FUNCTION EFFECTIVENESS

TANK	JUDGE EFFECTIVENESS	FUNCTION EFFECTIVNESS	DEVIATION	PERCENT DEVIATION
LEOPARD 1	1.44	1.46665249	-0.266524897	1.85%
M60A1	1.22	1.165318586	0.0546814318	4.48%
T-62	1.02	0.9953316577	0.0246683423	2.42%
M48A5	0.89	0.9391371068	-0.0491371068	5.52%
CENTURION (Mk. 13)	0.78	0.8004486225	-0.0104486225	1.32%
AMX-30	0.78	0.773111555	0.0068884449	0.88%

We can see from Table VII that the percent deviations of effectiveness for the six weapon systems are all less than 5.52%. This suggests that the estimating function of reciprocals of the data fits quite well the data upon which it was developed.

Let's look at the results from another point of view. A common practice in attempting to evaluate the effectiveness of a weapon system is to use a simple linear combination of the characteristic values, with coefficients determined by any of several rather arbitrary ways. One approach which is rarely undertaken is to do as was done in this paper, using a least square fit with effectiveness values obtained from experts' judgement.

Figure 4 shows the scatter plot of judge effectiveness vs function effectiveness estimated from a simple linear function of the characteristics, and Figure 5 shows the scatter plot of judge effectiveness vs function effectiveness estimated by taking the reciprocal of the characteristics.

It can be seen at a glance that the function effectiveness estimated by taking reciprocal of the characteristics is much closer (better) to the judge effectiveness than function effectiveness estimated by just taking a linear combination of the characteristics.

One possible reason why an estimating function using reciprocals of the characteristics is better fitted than the common procedure of evaluating the effectiveness of a weapon system using a simple linear combination of the characteristics, is that a property like system effectiveness may possess diminishing marginal returns with respect to increasing characteristic values. In the best fitting equation (reciprocals) the partial derivative of effectiveness with respect to a characteristic value (with a negative coefficient) was then the reciprocal of the square of the characteristic value, hence diminishing marginal returns.

We have now developed a functional relationship between the characteristics and the effectiveness of tank systems as intended at the beginning of the study, and found that the best estimating equation among those considered occurred when a linear combination of a reciprocal of the characteristics was used.

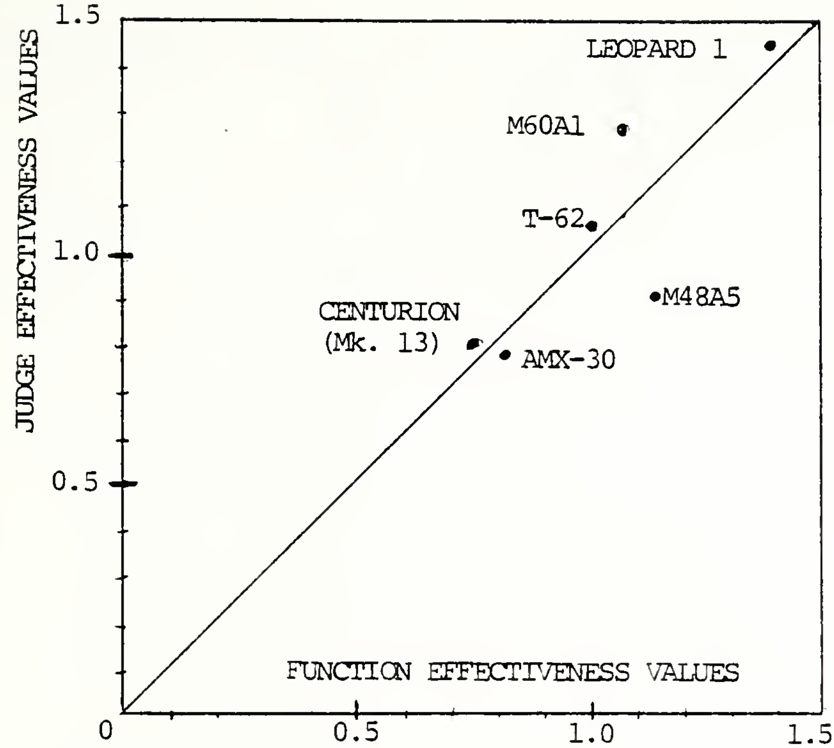


FIG. 4. JUDGE EFFECTIVENESS VALUES VS FUNCTION EFFECTIVENESS VALUES DETERMINED BY A LINEAR COMBINATION OF CHARACTERISTIC VALUES

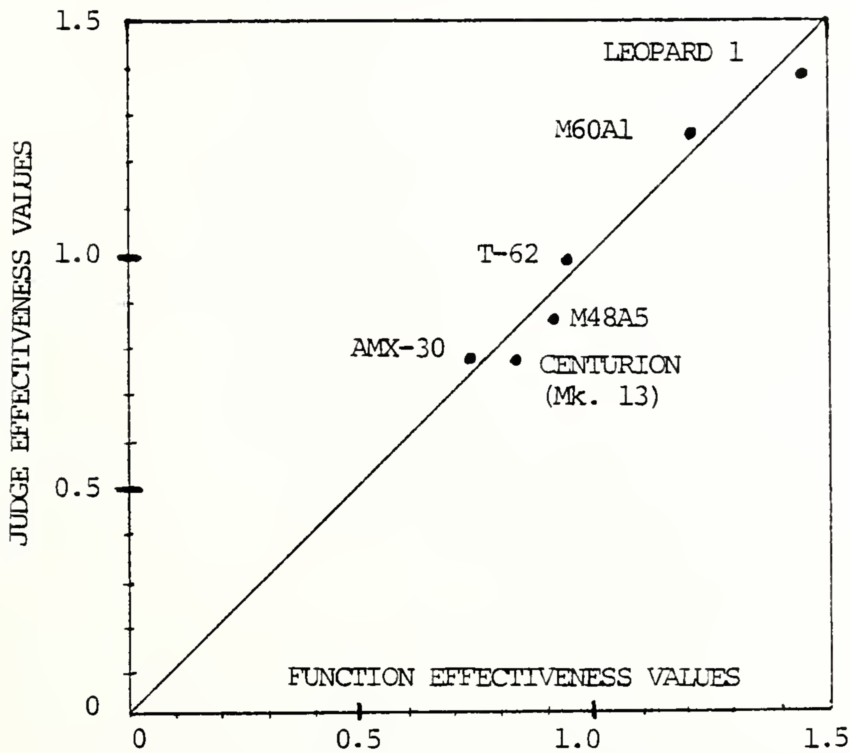


FIG. 5. JUDGE EFFECTIVENESS VALUS VS FUNCTION EFFECTIVENESS VALUES DETERMINED BY A LINEAR COMBINATION OF RECIPROCAL OF CHARACTERISTIC VALUES

The functional relationship developed in this manner could at least assist military planners in two ways: (1) in assessing the impact on effectiveness of a modification of characteristics of an existing tank, and (2) in evaluating the effectiveness of a new (and perhaps unbuilt) tank.

We will summarize the results of our study and propose some recommendations for further study in the next chapter.

V. CONCLUSIONS AND RECOMMENDATIONS

In this chapter we will give some conclusions from the study, and some suggestions for further study.

A. CONCLUSIONS

The principal purpose of this study was to determine whether we could compute the overall effectiveness of a weapon system from its characteristics, to establish the existence of functions which could be used to relate system characteristics and effectiveness, and to identify the best estimating relationship. To do this we proposed a procedure, sent appropriate questionnaires, and computed the overall effectiveness values for tank weapon systems by using the constant sum scaling method. Then, using multiple regression, we found functional relationships as in Table VI and evaluating these results we finally found that the best estimating equation occurred when we took the reciprocal, i.e.,

$$Y_i \text{ (Effectiveness)} = a + \sum_{j=1}^{m} b_j \left(\frac{1}{X_{ij}}\right)$$

where

a = -11.6119 is an intercept,

b_j = (-74.149,29.926,-622.536,291.944), and

X_{ij} are characteristics (Table I).

This is quite an interesting result because most people use a linear function in general.

This approach is felt to have merit as a way of finding an overall MOE because it is based on the opinion of many experts. A conspicuous limitation in the tank example used here is that this study did not include tank characteristics relating to fire power, primarily because of the difficulty of data collection.

The scaling for valuing system effectiveness is, of course, independent of the number of characteristics or presence of data on those characteristics. This provides, however, effectiveness values only for the instances listed in the questionnaire. Development of the functional relationships between characteristics and effectiveness, unfortunately, requires more data points (instances on the questionnaire) than characteristics if the function-finding approach used here is employed.

B. RECOMMENDATIONS FOR FURTHER STUDY

We have shown how to develop a model by which one can compute the system effectiveness directly from its characteristics. However we feel that we don't know how good our model is. To test our model, we suggest the following method which could be part of a further study.

(1) Select instances A.B.C.D.E (or any number of instances)

(2) Send questionnaires on A.B.C.D. (excluding instance E.).

(3) Find the best function

$$Y_i = f(X_1, X_2, \ldots, X_m)$$

(4) Predict the Y_E value using the above estimating equation.

(5) Send questionnaires on A.B.C.D.E (including the excluded instance).

(6) Compute the Y_E value using the constant sum scaling method and expert opinion.

(7) Comparing those two effectiveness values computed in step (4) and (6), provides a test of our model.

It is hoped that the work presented here will be useful to both those interested in tank effectiveness measurement and those concerned with multiple MOE research problems.

APPENDIX A

QUESTIONNAIRE

PROBLEM DESCRIPTION

A fundamental problem in Operations Research is that of satisfactorily combining various MOEs for a system into a single measure of overall effectiveness. A variety of methods have been used out of necessity, with the weighted sum of MOEs being perhaps the most frequent expedient. However, both the values of the weights and the notion of adding the MOEs are usually difficult to justify, and this approach often yields results which differ sharply from expert judgment.

The premise of our study at NPS is that individuals with expertise regarding the phenomenon (in our case, tanks) possess judgmental ways of combining MOEs which are far superior to the analyst's simple weighted average. The difficulty is that the expert usually cannot tell us how he knows, say, that on the basis of given data Tank A is superior to Tank B.

Our investigation seeks a procedure to approximate the way experts put MOEs together to form a single measure of effectiveness. This approach crosses disciplinary lines in that we use scaling techniques from the behavioral sciences together with nonlinear regression methods from statistics.

The heart of the approach, of course, is data from knowledgeable people, obtained through a specialized questionnaire. The research is still in its infancy, and the current effort seeks a small amount of data about a reduced set of MOEs, for use in development and assessment of this approach.

Degree Candidate: H.B. Kim, Lt, Col., Korean Army
Adviser: G.F. Lindsay, Assoc. Prof. of Operations Research, Naval Postgraduate School.
24 October, 1978

EFFECTIVENESS OF A TANK WEAPONS SYSTEM

A study is being made of various measures of effectiveness applicable to a tank, and how they relate to overall effectiveness. Judgments reflecting your experience and expertise are solicited.

Both parts of this questionnaire have purposefully been kept short, and we ask only two or three minutes of your time.

A. Importance of Tank Effectiveness Factors.

Please split 100 points within each pair listed below, assigning a higher number to a factor you think is more important. For example, if you think that fire power is much more important than armor, you might split the 100 points as follows:

Fire Power __80__, Armor __20__

Or, if you thought them to be equal in importance, you would write:

Fire Power __50__, Armor __50__ .

Omit pairs you feel unable to rate.

1. Range, Road ______, Fire Power ______
2. Range, Road ______, HP/ton ______
3. Range, Road ______, Armor ______
4. Range, Road ______, Speed ______
5. Range, Road ______, Silhouette ______
6. Speed ______, Fire Power ______
7. Speed ______, HP/ton ______
8. Speed ______, Armor ______
9. Speed ______, Silhouette ______
10. Fire Power ______, HP/ton ______
11. Fire Power ______, Armor ______
12. Fire Power ______, Silhouette ______
13. HP/ton ______, Armor ______
14. HP/ton ______, Silhouette ______
15. Armor ______, Silhouette ______

B. Overall Combat Effectiveness of Existing Tank Weapons Systems

FACTORS	M48A1	M60A1	AMX-30	LEOPARD 1	T-62	CENTURION (Mk.13)
RANGE, ROAD (km)	482	500	650	600	500	190
FIRE POWER (main gun)	105	105	105	105	115	105
SPEED (km/hr)	48.2	48.3	65	65	50	34.6
HP/ton	15.9	15.3	19.4	20.7	19.2	12.5
ARMOR (mm on nose)	110	110	40	70	100	118
SILHOUETTE (height in m)	3.09	3.26	2.85	2.64	2.4	3.01

Some characteristics of six current tanks are shown above. Please split 100 points within each pair listed below, in terms of the overall effectiveness of the tank weapons system.

1. M48A5 ______, M60A1 ______
2. M48A5 ______, AMX-30 ______
3. M48A5 ______, LEOPARD 1 ______
4. M48A5 ______, T-62 ______
5. M48A5 ______, CENTURION ______
6. T-62 ______, M60A1 ______
7. T-62 ______, AMX-30 ______
8. T-62 ______, LEOPARD 1 ______
9. T-62 ______, CENTURION ______
10. AMX-30 ______, M60A1 ______
11. AMX-30 ______, LEOPARD 1 ______
12. AMX-30 ______, CENTURION ______
13. CENTURION ______, M60A1 ______
14. CENTURION ______, LEOPARD 1 ______
15. LEOPARD 1 ______, M60A1 ______

Thank you very much for your time and cooperation.

APPENDIX B

MULTIPLE REGRESSION OUTPUT ON APL

```
      ∇REGRESS[⎕]∇
    ∇ Z←Y REGRESS X;N;K;C;XPXINV;XPY;BETA;RSS;TSS;S2;ESS;WID;DEP
[1]    X←(2↑(⍴X),1)⍴X
[2]    X←(0,1-△INTERCEPT)↓1,X
[3]    XPXINV←⌹(⍉X)+.×X
[4]    BETA←XPXINV+.×XPY←(⍉X)+.×Y
[5]    RSS←((⍉BETA)+.×XPY)-C←((+/Y)*2)÷N←⍴,Y
[6]    ESS←(TSS←((⍉Y)+.×Y)-C)-RSS
[7]    S2←,ESS÷(N-1)-K←(⍴,BETA)-△INTERCEPT
[8]    CR
[9]    '                             ANOVA'
[10]   C̲W̲←'SOURCE,DF,SUM SQUARES,MEAN SQUARE,F-RATIO'
[11]   ' '
[12]   '⍞REGRESSION⍞,I4,BE16.4' FMT(K),(,RSS),(,RSS÷K),(,RSS÷K)÷S2
[13]   C̲W̲←''
[14]   '⍞  RESIDUAL⍞,I4,BE16.4' FMT((N-1)-K),(,ESS),S2,0
[15]   '⍞TOTAL      ⍞,I4,BE16.4' FMT(N-1),(,TSS),0,0
[16]   ' '
[17]   'R SQUARE:  ';,RSS÷TSS
[18]   'STD ERROR: ';,S2*0.5
[19]   C̲W̲←'COEFFICIENTS,T STATISTICS'
[20]   'F15.4' FMT⍉(2,⍴,BETA)⍴(,BETA),(,BETA)÷(1 1 ⍉V←S2×XPXINV)*0.
5
[21]   'DO YOU WANT A PRINTOUT OF THE VARIANCE-COVARIANCE MATRIX?'
[22]   →A1×⍳'Y'≠1↑⍞
[23]   'VARIANCE-COVARIANCE MATRIX: ',C̲W̲←''
[24]   'E12.4' FMT V
[25]  A1:'DURBIN-WATSON: ';(+/((1↓,C)-(¯1↓,C))*2)÷+/(,C←Y-X+.×BETA)*2
[26]   Z←⍉(2,N)⍴(,X+.×BETA),,C
[27]  B1:'DO YOU WANT TO FORECAST A VALUE FOR Y?'
[28]   →C1×⍳'Y'≠1↑⍞
[29]   'ENTER X VECTOR (';K;' VALUES)'
[30]   'FORECAST OF Y VALUE: ';(C←(1-△INTERCEPT)↓1,⎕)+.×BETA
[31]   'VARIANCE OF FORECAST ERROR: ';S2×1+C+.×XPXINV+.×⍉C
[32]   →B1
[33]  C1:'DO YOU WANT TO SCAT RESIDUALS VS. PREDICTED Y?'
[34]   →0×⍳'N'=1↑⍞
[35]   DEP←0.5×WID←⌊/70,(⌈/((0.75×N),30))
[36]   SCAT Z
    ∇
```

X

70	2.64	65	20.7
110	3.26	48.3	15.3
100	2.4	50	19.2
110	3.09	48.2	15.9
118	3.01	34.6	12.5
40	2.85	65	19.4

Y

1.44	1.22	1.02	0.89	0.79	0.78

Y REGRESS X

1

ANOVA

SOURCE	DF	SUM SQUARES	MEAN SQUARE	F-RATIO
REGRESSION	4	2.7245E¯1	6.8112E¯2	9.5547E¯1
RESIDUAL	1	7.1286E¯2	7.1286E¯2	
TOTAL	5	3.4373E¯1		

R SQUARE: 0.7926120129

STD ERROR: 0.2669946893

COEFFICIENTS	T STATISTICS
2.3863	0.3699
0.0251	1.4535
¯1.6073	¯0.7715
0.1405	1.0579
¯0.3682	¯0.764

DO YOU WANT A PRINTOUT OF THE VARIANCE-COVARIANCE MATRIX?

Y

VARIANCE-COVARIANCE MATRIX:

4.1616E+1	7.3596E¯2	¯1.2816E+1	7.3682E¯1	¯2.8942E00
7.3596E¯2	2.9907E¯4	¯3.0556E¯2	2.1268E¯3	¯7.1847E¯3
¯1.2816E+1	¯3.0556E¯2	4.3399E00	¯2.6553E¯1	9.8432E¯1
7.3682E¯1	2.1268E¯3	¯2.6553E¯1	1.7633E¯2	¯6.3026E¯2
¯2.8942E00	¯7.1847E¯3	9.8432E¯1	¯6.3026E¯2	2.3233E¯1

DURBIN-WATSON: 2.006601651

DO YOU WANT TO FORECAST A VALUE FOR Y?

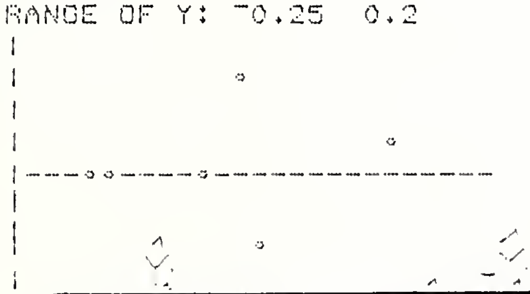
N

DO YOU WANT TO SCAT RESIDUALS VS. PREDICTED Y?

Y

RANGE OF X: 0.6 1.6

RANGE OF Y: ¯0.25 0.2



1.410903046	0.02909695409
1.062459362	0.1575406384
0.9959965683	0.02400343169
1.100700476	¯0.2107004762
0.7719477676	0.01805223238
0.7979927803	¯0.0179927803

```
      Y REGRESS *X
1
                                   ANOVA

     SOURCE   DF       SUM SQUARES       MEAN SQUARE           F-RATIO
REGRESSION     4        3.0517E¯1         7.6292E¯2         1.9782E00
  RESIDUAL     1        3.8565E¯2         3.8565E¯2
TOTAL          5        3.4373E¯1

R SQUARE:   0.8878042559
STD ERROR:  0.1963807962
   COEFFICIENTS    T STATISTICS
        ¯4.6385        ¯1.0011
         1.5055         2.3534
        ¯7.2268        ¯1.4786
         9.6399         1.7563
       ¯11.0647        ¯1.5005
DO YOU WANT A PRINTOUT OF THE VARIANCE-COVARIANCE MATRIX?
Y
VARIANCE-COVARIANCE MATRIX:
   2.1470E+1   ¯9.4373E¯2   ¯1.1615E+1    9.0073E00   ¯1.5628E+1
  ¯9.4373E¯2    4.0924E¯1   ¯2.5707E00    3.1039E00   ¯3.9644E00
  ¯1.1615E+1   ¯2.5707E00    2.3888E+1   ¯2.6094E+1    3.5500E+1
   9.0073E00    3.1039E00   ¯2.6094E+1    3.0126E+1   ¯4.0196E+1
  ¯1.5628E+1   ¯3.9644E00    3.5500E+1   ¯4.0196E+1    5.4374E+1
DURBIN-WATSON: 2.080517414
DO YOU WANT TO FORECAST A VALUE FOR Y?
N
DO YOU WANT TO SCAT RESIDUALS VS. PREDICTED Y?
Y
RANGE OF X: 0.6   1.6
RANGE OF Y: ¯0.15  0.15
|
|          o
|
|
|
|        o
|---oo--------------------
|                   o
|
|
|
|
|         o
  1.455072367       ¯0.01507236729
  1.093099258        0.1269007416
  0.9840070802       0.03599291975
  1.034541009       ¯0.1445410085
  0.7961936605      ¯0.006193660469
  0.777086625        0.002913375008
```

```
      (⍟Y) REGRESS (⍟X)
 1
                                    ANOVA

      SOURCE    DF       SUM SQUARES        MEAN SQUARE           F-RATIO
 REGRESSION      4         2.6622E¯1         6.6555E¯2         1.7746E00
   RESIDUAL      1         3.7504E¯2         3.7504E¯2
 TOTAL           5         3.0373E¯1

 R SQUARE:  0.8765198175
 STD ERROR: 0.1936598687
    COEFFICIENTS    T STATISTICS
         ¯6.0277         ¯1.3192
          1.3669          2.1668
         ¯5.9679         ¯1.2382
          8.1447          1.5047
         ¯9.1104         ¯1.2529
 DO YOU WANT A PRINTOUT OF THE VARIANCE-COVARIANCE MATRIX?
 Y
 VARIANCE-COVARIANCE MATRIX:
    2.0879E+1   ¯9.1776E¯2   ¯1.1295E+1    8.7594E00   ¯1.5198E+1
   ¯9.1776E¯2    3.9798E¯1   ¯2.5000E00    3.0185E00   ¯3.8553E00
   ¯1.1295E+1   ¯2.5000E00    2.3231E+1   ¯2.5376E+1    3.4523E+1
    8.7594E00    3.0185E00   ¯2.5376E+1    2.9297E+1   ¯3.9090E+1
   ¯1.5198E+1   ¯3.8553E00    3.4523E+1   ¯3.9090E+1    5.2878E+1
 DURBIN-WATSON: 2.080517414
 DO YOU WANT TO FORECAST A VALUE FOR Y?
 N
 DO YOU WANT TO SCAT RESIDUALS VS. PREDICTED Y?
 Y
 RANGE OF X: ¯0.4  0.4
 RANGE OF Y: ¯0.15  0.15
                     |
                     |   o
                     |
                     |
                     |
                   o |
 -----oo-------------|--------------
                     |          o
                     |
                     |
                     |
                     |
                     | o

  0.3795066478        ¯0.01486353417
  0.07370837312        0.1251424856
 ¯0.01569159746        0.03549422475
  0.02600452402       ¯0.1425383403
 ¯0.2296144885        ¯0.006107845058
 ¯0.2513343684         0.002873009127
```

```
     TX←1÷X

          Y REGRESS TX
1
                              ANOVA

    SOURCE   DF       SUM SQUARES      MEAN SQUARE          F-RATIO
REGRESSION    4         3.3685E¯1        8.4213E¯2        1.2240E+1
  RESIDUAL    1         6.8800E¯3        6.8800E¯3
TOTAL         5         3.4373E¯1

R SQUARE:   0.9799844232
STD ERROR: 0.08294589169
   COEFFICIENTS    T STATISTICS
       ¯11.6119         ¯3.4369
       ¯74.1488         ¯6.2379
        27.9259          4.2011
      ¯622.5395         ¯4.6658
       271.9441          4.2671
DO YOU WANT A PRINTOUT OF THE VARIANCE-COVARIANCE MATRIX?
Y
VARIANCE-COVARIANCE MATRIX:
    1.1415E1     2.7450E1    ¯2.2374E1     4.3981E2    ¯2.1281E2
    2.7450E1     1.4130E2    ¯5.7579E1     1.2390E3    ¯5.6655E2
   ¯2.2374E1    ¯5.7579E1     4.4187E1    ¯8.7077E2     4.1895E2
    4.3981E2     1.2390E3    ¯8.7077E2     1.7803E4    ¯8.4757E3
   ¯2.1281E2    ¯5.6655E2     4.1895E2    ¯8.4757E3     4.0615E3
DURBIN-WATSON: 2.145430874
DO YOU WANT TO FORECAST A VALUE FOR Y?
N
DO YOU WANT TO SCAT RESIDUALS VS. PREDICTED Y?
Y
RANGE OF X: 0.6  1.6
RANGE OF Y: ¯0.06  0.06
|
|             o
|
|
|         o
|   o
|-------------------------
|    o
|
|                     o
|
|       o
|
  1.46665249        ¯0.02665248971
  1.165318568        0.05468143179
  0.9953316577       0.02466834227
  0.9391371068      ¯0.04913710681
  0.8004486225      ¯0.01044862252
  0.773111555        0.006888444973
```

(1÷Y) REGRESS (1÷X)

1

ANOVA

SOURCE	DF	SUM SQUARES	MEAN SQUARE	F-RATIO
REGRESSION	4	2.7511E¯1	6.8777E¯2	5.7039E00
RESIDUAL	1	1.2058E¯2	1.2058E¯2	
TOTAL	5	2.8717E¯1		

R SQUARE: 0.9580105484

STD ERROR: 0.1098086393

COEFFICIENTS	T STATISTICS
9.9859	2.2326
65.7943	4.181
¯20.6599	¯2.3477
471.4091	2.6688
¯200.6539	¯2.3783

DO YOU WANT A PRINTOUT OF THE VARIANCE-COVARIANCE MATRIX?

Y

VARIANCE-COVARIANCE MATRIX:

2.0005E1	4.8109E1	¯3.9213E1	7.7082E2	¯3.7296E2
4.8109E1	2.4764E2	¯1.0091E2	2.1714E3	¯9.9294E2
¯3.9213E1	¯1.0091E2	7.7442E1	¯1.5261E3	7.3424E2
7.7082E2	2.1714E3	¯1.5261E3	3.1201E4	¯1.4854E4
¯3.7296E2	¯9.9294E2	7.3424E2	¯1.4854E4	7.1182E3

DURBIN-WATSON: 2.145430874

DO YOU WANT TO FORECAST A VALUE FOR Y?

N

DO YOU WANT TO SCAT RESIDUALS VS. PREDICTED Y?

Y

RANGE OF X: 0.6 1.4

RANGE OF Y: ¯0.08 0.08

```
|
|                 o
| o
|                        o
|-------------------------o----
|
|               o
|
|          o
```

0.6591603144	0.03528413003
0.8920626197	¯0.07239048852
1.013049556	¯0.03265739921
1.05854492	0.06505058572
1.251990285	0.01383249969
1.29117061	¯0.00911932772

Y REGRESS X*.5

1

ANOVA

SOURCE	DF	SUM SQUARES	MEAN SQUARE	F-RATIO
REGRESSION	4	2.8793E¯1	7.1984E¯2	1.2900E00
RESIDUAL	1	5.5799E¯2	5.5799E¯2	
TOTAL	5	3.4373E¯1		

R SQUARE: 0.8376671578
STD ERROR: 0.2362185618

COEFFICIENTS	T STATISTICS
9.6981	0.7228
0.3986	1.8048
¯7.0421	¯1.0655
2.3585	1.3374
¯4.2137	¯1.0744

DO YOU WANT A PRINTOUT OF THE VARIANCE-COVARIANCE MATRIX?
Y
VARIANCE-COVARIANCE MATRIX:

1.8003E+2	2.1507E00	¯8.6857E+1	2.1746E+1	¯5.0610E+1
2.1507E00	4.8783E¯2	¯1.2281E00	3.5430E¯1	¯7.4316E¯1
¯8.6857E+1	¯1.2281E00	4.3683E+1	¯1.1273E+1	2.5479E+1
2.1746E+1	3.5430E¯1	¯1.1273E+1	3.1101E00	¯6.8470E00
¯5.0610E+1	¯7.4316E¯1	2.5479E+1	¯6.8470E00	1.5381E+1

DURBIN-WATSON: 2.038119584
DO YOU WANT TO FORECAST A VALUE FOR Y?
N
DO YOU WANT TO SCAT RESIDUALS VS. PREDICTED Y?
Y
RANGE OF X: 0.6 1.6
RANGE OF Y: ¯0.2 0.15

```
|         o
|
|
|
|
|       o
|----2------------o-----
|
|
|
|
|
|
|         o
|
```

1.434688413	0.005311587208
1.073082645	0.1469173555
0.988212396	0.03178760399
1.071996205	¯0.181996205
0.7859517492	0.00404825083
0.7860685924	¯0.006068592406

```
      Y REGRESS (1÷X*.5)
1
                              ANOVA

    SOURCE    DF      SUM SQUARES      MEAN SQUARE          F-RATIO
REGRESSION     4        3.2224E¯1        8.0559E¯2        3.7474E00
  RESIDUAL     1        2.1497E¯2        2.1497E¯2
TOTAL          5        3.4373E¯1

R SQUARE:  0.9374595357
STD ERROR: 0.1466193789
   COEFFICIENTS     T STATISTICS
       ¯18.7584          ¯1.7598
       ¯21.7109          ¯3.367
        28.7162           2.2021
      ¯155.6577          ¯2.5165
       111.2614           2.2382
DO YOU WANT A PRINTOUT OF THE VARIANCE-COVARIANCE MATRIX?
Y
VARIANCE-COVARIANCE MATRIX:
    1.1362E2     5.0221E1    ¯1.3817E2     6.3622E2    ¯5.2118E2
    5.0221E1     4.1580E1    ¯6.6284E1     3.3699E2    ¯2.5875E2
   ¯1.3817E2    ¯6.6284E1     1.7005E2    ¯7.8844E2     6.3979E2
    6.3622E2     3.3699E2    ¯7.8844E2     3.8260E3    ¯3.0602E3
   ¯5.2118E2    ¯2.5875E2     6.3979E2    ¯3.0602E3     2.4711E3
DURBIN-WATSON: 2.122115702
DO YOU WANT TO FORECAST A VALUE FOR Y?
N
DO YOU WANT TO SCAT RESIDUALS VS. PREDICTED Y?
Y
RANGE OF X: 0.6  1.6
RANGE OF Y: ¯0.1  0.1
|            o
|
|
|
|         o
|
|---o---------------------
|    o
|                    o
|
|
|
|         o
  1.467833212     ¯0.02783321172
  1.12315161       0.09684839049
  0.9852532576     0.03474674236
  0.9897234947    ¯0.09972349474
  0.8015763548    ¯0.01157635483
  0.7724620716     0.007537928422
```

```
      (Y*.5) REGRESS (X*.5)
1
                                    ANOVA

     SOURCE     DF        SUM SQUARES        MEAN SQUARE           F-RATIO
REGRESSION       4         6.6737E¯2          1.6684E¯2          1.2494E00
  RESIDUAL       1         1.3354E¯2          1.3354E¯2
TOTAL            5         8.0091E¯2

R SQUARE:   0.8332679272
STD ERROR: 0.1155581755
   COEFFICIENTS    T STATISTICS
         4.6632          0.7104
         0.1884          1.7438
        ¯3.1607         ¯0.9776
         1.076           1.2472
        ¯1.8862         ¯0.9831
DO YOU WANT A PRINTOUT OF THE VARIANCE-COVARIANCE MATRIX?
Y
VARIANCE-COVARIANCE MATRIX:
   4.3085E+1    5.1469E¯1   ¯2.0786E+1    5.2041E00   ¯1.2112E+1
   5.1469E¯1    1.1675E¯2   ¯2.9389E¯1    8.4790E¯2   ¯1.7785E¯1
  ¯2.0786E+1   ¯2.9389E¯1    1.0454E+1   ¯2.6978E00    6.0975E00
   5.2041E00    8.4790E¯2   ¯2.6978E00    7.4430E¯1   ¯1.6386E00
  ¯1.2112E+1   ¯1.7785E¯1    6.0975E00   ¯1.6386E00    3.6810E00
DURBIN-WATSON: 2.038119584
DO YOU WANT TO FORECAST A VALUE FOR Y?
N
DO YOU WANT TO SCAT RESIDUALS VS. PREDICTED Y?
Y
RANGE OF X: 0.85   1.2
RANGE OF Y: ¯0.1   0.1
|
|
|              o
|
|
|           o
|--2------------------------o
|
|
|
|
|              o
|
 1.19740157          0.002598429701
 1.032664097         0.07187200443
 0.9943999906        0.01555050326
 1.032430699        ¯0.08903258577
 0.8868390365        0.001980405251
 0.8861448434       ¯0.002968756811
```

APPENDIX C

COMPUTATION OF A RATIO SCALE OF THE IMPORTANCE OF SIX MAJOR FACTORS OF A TANK WEAPON SYSTEM, USING THE CONSTANT SUM SCALING METHOD

As a supplementary study, we computed a ratio scale of the importance of six major factors of a tank weapon system. The factors were:

Road Range,

Fire Power,

Speed,

Hp/ton,

Armor, and

Silhouette.

As we did in Chapters II and III for system effectiveness, we followed the same procedure, i.e., we selected the major factors, collected data, selected the constant sum scaling method, sent questionnaires together with the questionnaires for tank weapon system (Appendix A), and computed the ratio scale values about the importance of each of the six major factors.

The $\overline{a_{ij}}$ array was computed from the 50 judges responses and we computed the W_{ij} array from this $\overline{a_{ij}}$ array by taking the ratio of the average points awarded to instance j (when compared to instance i) to the average points awarded to instance i (when compared to instance j). After constructing

the W_{ij} array we used equation (11), i.e.,

$$S_j = \sum_{i=1}^{n} (W_{ij})^{1/n} , \quad j = 1,2, \ldots, n$$

to get the ratio scale values of factors. The ratio scale values computed are shown in Table C, and represented graphically in Figure a.

Table A. $\overline{a_{ij}}$ ARRAY COMPUTED FROM JUDGES RESPONSES

	RANGE, ROAD	FIRE POWER	SPEED	HP/TON	ARMOR	SILHOUETTE
RANGE, ROAD	50	75	58.7	59.8	66	63.3
FIRE POWER	25	50	32.5	30.7	38.3	36.6
SPEED	41.3	69.5	50	43.1	58.7	55.6
HP/TON	42.2	63.3	50.9	50	59.3	54.7
ARMOR	34	61.9	41.3	42.7	50	55.5
SILHOUETTE	36.4	63.4	44.4	45.3	54.5	50

Table B W_{ij} ARRAY

	RANGE, ROAD	FIRE POWER	SPEED	HP/TON	ARMOR	SILHOUETTE
RANGE, ROAD	1	3	1.421	1.369	1.941	1.949
FIRE POWER	.333	1	.481	.443	.621	.599
SPEED	.703	2.099	1	.965	1.421	1.252
HP/TON	.730	2.259	1.037	1	1.342	1.209
ARMOR	.515	1.611	.703	.745	1	.835
SILHOUETTE	.592	1.932	.798	.828	1.198	1
S_j =	.61	1.84	.86	.84	1.18	1.04

Table C RATIO SCALE OF SIX MAJOR FACTORS OF TANK WPN. SYS.

FACTORS	RATIO SCALE
FIRE POWER	1.84
ARMOR	1.18
SILHOUETTE	1.04
SPEED	0.86
HP/TON	0.84
RANGE, ROAD	0.61

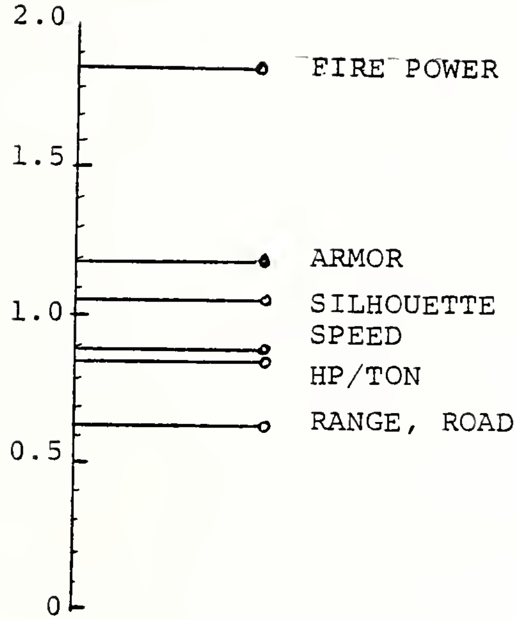


FIG. a RATIO SCALE OF SIX MAJOR FACTORS OF TANK WEAPON SYSTEM

LIST OF REFERENCES

1. Torgerson, Warren, S., Theory and Methods of Scaling, Wiley, 1976.

2. Batchelder, C.A., An Introduction to Equipment Cost Estimating, The RAND Corporation.

3. Foss, Christopher F., Jane's World Armoured Fighting Vehicles, St. Martin's Press, 1977.

4. McNeil, Donald R., Interactive Data Analysis, Wiley, 1976.

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