A THEORETICAL STUDY AND COMPUTER SEARCH FOR BINARY SEQUENCES HAVING SPECIFIC AUTOCORRELATION FUNCTIONS

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THESIS

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SPECIFIC AUTOCORRELATION FUNCTIONS

by

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A Theoretical Study and Computer Search For Binary Sequences Having Specific Autocorrelation Functions

by

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ABSTRACT

Binary sequences find increasing use in electrical engineering applications of ranging, time measurement and communications. A property of interest in these applications is the autocorrelation function of the binary sequence or pair of sequences. Of the 2^n possible sequences of length n, only a few have usable autocorrelation functions. There is, to date, no procedure known which will provide the sequence having a specific autocorrelation function, except for very particular cases.

In this report, known properties of complementary sequences are reviewed. Almost complementary sequences are defined and the procedure to obtain them is outlined. A formula is derived for the number of different autocorrelation functions of the 2^n possible sequences of length n bits. A computer search is implemented with the objective of discovering sequences with desirable autocorrelation functions.

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LIST OF SYMBOLS

$g(t)$	Output of a matched filter.
$h(t)$	Impulse response of a matched filter.
n	Length of a binary sequence.
$v(t)$	Two level voltage.
A	Binary sequence A (capital letters are used to denote binary sequences).
K	Length of a pair of almost complementary sequences.
QPSK	Quadriphase phase-shift keying.
R	Binary sequence which remains the same when it is reversed and complemented.
$R_v(\tau)$	Autocorrelation function of the function $v(t)$.
$R_A(\tau)$	Autocorrelation function of the binary sequence A.
S	Binary sequence which remains the same when it is reversed.
$(T)_{even}$	Number of all different autocorrelation functions of all sequences of even length.
$(T)_{odd}$	Number of all different autocorrelation functions of all sequences of odd length.
V	Value of $v(t)$.
ε	Bit duration of a binary sequence.
τ	Time delay.
Σ	Summer.

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I. INTRODUCTION

This study is concerned with binary sequences and their autocorrelation functions. The objective is to obtain autocorrelation functions with sidelobe levels less than or equal to predetermined values. Sequences or groups of sequences which can provide this property are very attractive for use in systems whose performance depends on the autocorrelation function magnitude. Such systems are used in communications, ranging and time measurement. The problem is to find these sequences or "good" codes and the rules to construct them, if such rules exist.

A class of such codes are the complementary sequences. They are pairs of sequences with the characteristic that the sum of their autocorrelation functions is a waveform that has no sidelobes. These sequences were first considered by M. J. Golay [Ref. 1] and further investigated by S. Jauregui [Ref. 2]. They are used here to develop a technique for constructing another class of "good" codes, the almost complementary sequences.

A. PLAN OF THE RESEARCH

The efforts to solve the problem follow two directions:

(1) Experimentation with the binary sequences and their properties, to find the rules which give desirable autocorrelation functions.

(2) Computer search of binary sequences of several lengths to find the ones with small autocorrelation sidelobes.

As a result the following were achieved:

(1) Discovery of the almost complementary sequences. These are pairs of binary sequences which can be constructed using complementary sequences. Their autocorrelation functions when added have sidelobes of predetermined magnitude, polarity and position.

(2) Discovery of codes of lengths ≤ 20 which have autocorrelation functions with sidelobes less or equal to one. For example, for length $n = 20$, only three such codes were found and for $n = 15$, none. The computer programs used here can be used to select codes with any sidelobe levels.

Other results of interest are:

(1) Derivation of the formula for the number of the different autocorrelation functions of all sequences of length n.

(2) Construction of computer programs which can be used in other cases as well. For example, an algorithm for the automatic production of all possible binary numbers of length n was devised. This algorithm can be used to select codes having certain properties, such as codes with a fixed number of ones and zeros.

B. PLAN OF THE REPORT

Chapter II provides the necessary background by giving the definitions and basic properties of the binary sequence

and autocorrelation function. The formation of the autocorrelation function $R_v(\tau)$ of a two level function $v(t)$ is developed. Matched filters and their realizations are also discussed.

Complementary sequences and their basic properties are reviewed in Chapter III, to form the basis for the material on almost complementary sequences.

In Chapter IV, the almost complementary sequences are defined, and rules for their construction are given.

The formula for the number of different autocorrelation functions in all sequences of length n is developed in Chapter V.

Computer programs and their algorithms are discussed next in Chapter VI.

Chapter VII gives a few applications.

II. BACKGROUND

In this section some basic concepts are discussed and definitions given as background material.

A. BINARY SEQUENCE

A binary sequence is a list of elements each of which can have one of two distinct values. These values are usually represented either by +1 and -1 or by 1 and 0. Often when the +1 and -1 convention is used, the ones are omitted and only + and - are written.

For example, sequence A can be written:

$$A = ++-+ \text{ or } A = 1101 \text{ or } A = +1+1-1+1$$

The number of elements in a sequence is the length denoted here by n. For the above example, $n = 4$.

The voltage equivalent $v(t)$ of a binary sequence is a time waveform where 1 is represented by a voltage level +V and 0 by a voltage level -V.

For the sequence A = 1101, $v(t)$ is given in Fig. 1 where ε is the bit duration.

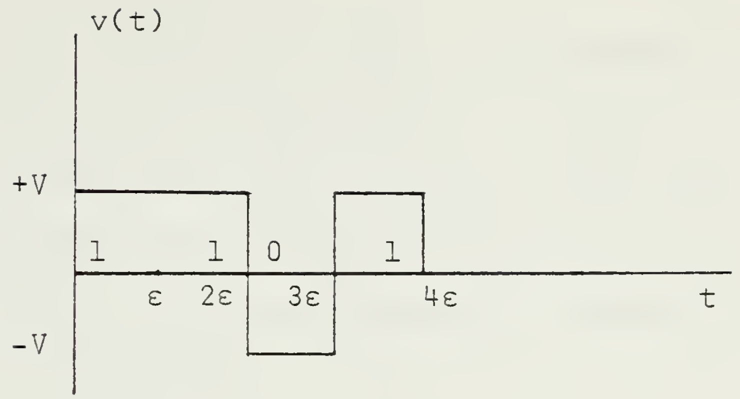


Fig. 1. Voltage equivalent of a binary sequence.

In this study the 1,0 notation is used. Also, when sequence is used, binary sequence is implied.

B. AUTOCORRELATION FUNCTION

The autocorrelation function of a two-level, time-limited voltage $v(\tau)$ is defined as the integral

$$R_v(\tau) = \int_{-\infty}^{+\infty} v(t)v(t-\tau)dt.$$

$R_v(\tau)$ is a measure of the similarity between a voltage or signal and its phase shifted version where all values of time delay τ are considered.

The way to find the autocorrelation function $R_v(\tau)$ of a digital sequence $v(t)$, is to "slide" the sequence past itself to the right and left and at each position form the product of the sequence and its shifted replica. Then the area of

the product waveform is taken and this gives the autocorrelation of the sequence at this position.

The procedure is illustrated in Fig. 2, by showing the complete steps for two "shift" positions $\tau = 0$ and $\tau = \varepsilon$.

It can be seen from the equation for $R_v(\tau)$ and from Fig. 2 that when $v(t)$ is a piece-wise constant function, $R_v(\tau)$ will be a piece-wise linear. The linear segments terminate at multiples of ε, a bit duration.

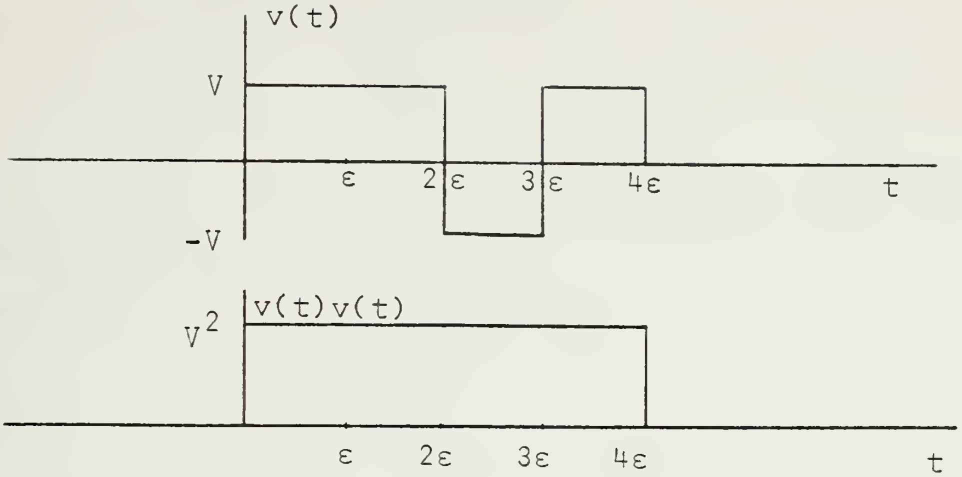
The autocorrelation is an even function which has its maximum value at $\tau = 0$. $R_v(0)$ gives the level of the main lobe. Secondary maxima are the sidelobe levels. For the example of Fig. 2, the mainlobe level is $R_v(0) = 4V^2\varepsilon$. Sidelobe levels are $R_v(\varepsilon) = -V^2\varepsilon$ and $R_v(3\varepsilon) = +V^2\varepsilon$.

Actually the shape of the autocorrelation function of a sequence $v(t)$ is obtained easier by letting $v = 1$ and $\varepsilon = 1$.

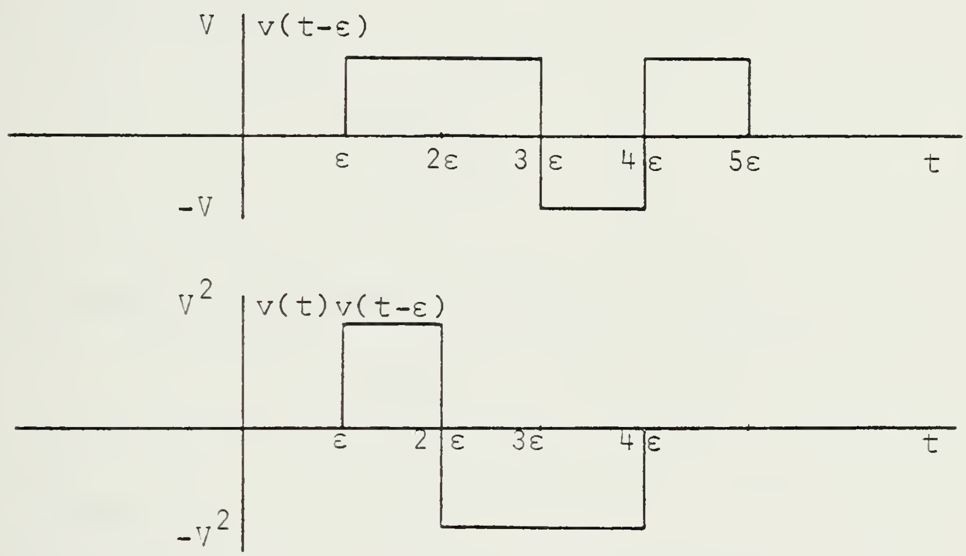
For example, to form the autocorrelation function of the sequence A = 1101, the sequence is written and its delayed version is placed beneath. For example, $\tau = 0$ gives

$$\begin{array}{c|} 1101 \\ 1101 \end{array}$$

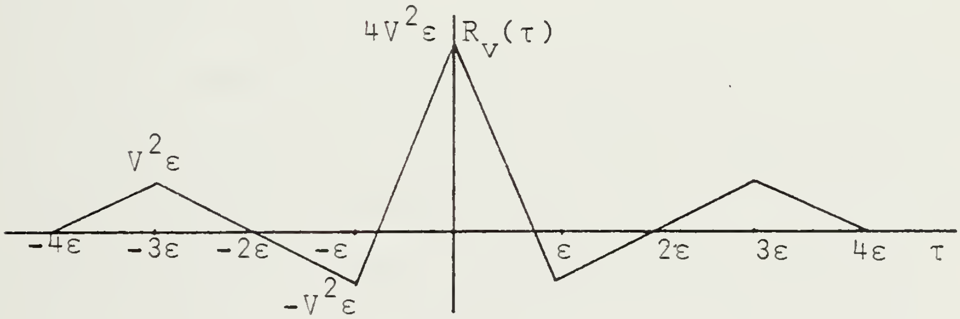
In each position the elements of these two similar sequences are compared. If they are the same (both zeros or ones) they form an agreement; if not (one zero, one one) they form a disagreement. The number of disagreements is subtracted from the number of agreements and the result is proportional to the autocorrelation function at this position. Here there are only four agreements and the result is 4.



(a) $\tau = 0,\ R_v(0) = 4V^2\varepsilon$



(b) $\tau = \varepsilon,\ R_v(\varepsilon) = -V^2\varepsilon$



(c) $R_v(\tau)$, for all τ

Fig. 2. Autocorrelation of a binary sequence.

Now a shift is made as follows

```
1101|
 110|1
```

and by the same method there are one agreement and two dis-agreements, so the result is -1.

Similarly, the next position is

```
1101|
  11|01
```

which gives 0.

The next position

```
1101|
   1|101
```

gives 1.

And finally, the last position

```
1101|
    |1101
```

always gives 0.

Since the autocorrelation is an even function, $R_v(\tau) = R_v(-\tau)$. If τ is considered to provide a shift to the future, $-\tau$ is a shift to the past.

It is not necessary for the values given by shifting to the left to be written, because they are the same with the ones resulting from shifting to the right. So, by convention the autocorrelation of the sequence A can be written

$$R_v(\tau) = 4,-1,0,1,0.$$

This convention will be followed in the rest of this report.

C. MATCHED FILTERS

Some interesting properties of matched filters will be listed here. These properties are derived in the literature [Ref. 3,4].

A matched filter is the best linear filter for detection of a pulse signal v(t) in noise. The impulse response h(t) of such a filter is a delayed, time inverted replica of the input. If v(t) is the input to the matched filter, its impulse response is

$$h(t) = Mv[-(t-to)], \quad t \geq 0$$
$$h(t) = 0 \qquad\qquad , \quad t < 0.$$

where M = an arbitrary constant

t_o = time delay inherent in the filter.

From linear system theory, the output g(t) of a matched filter is

$$g(t) = M\int_{-\infty}^{+\infty} v(t-\tau)v\,[-(\tau-t_o)]d\tau$$

Let $\lambda = t-\tau$ to obtain

$$g(t) = M\int_{-\infty}^{+\infty} v(\lambda)v(-t+\lambda+to)d\lambda$$

or $$g(t) = M\int_{-\infty}^{+\infty} v(\lambda)v(\lambda-\xi)d\lambda = M\ R_v(\xi)$$

where $\xi = t-to$.

The output is maximized when $\xi = 0$ or $t = t_o$.

It is concluded then, that the output g(t) of the matched

filter is the autocorrelation function of the input. Fig. 3 illustrates the concept.

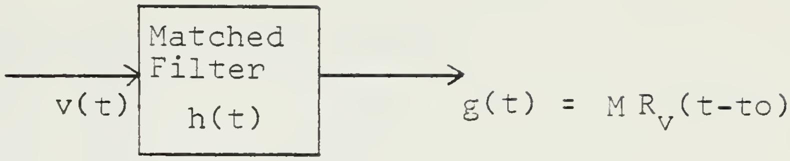
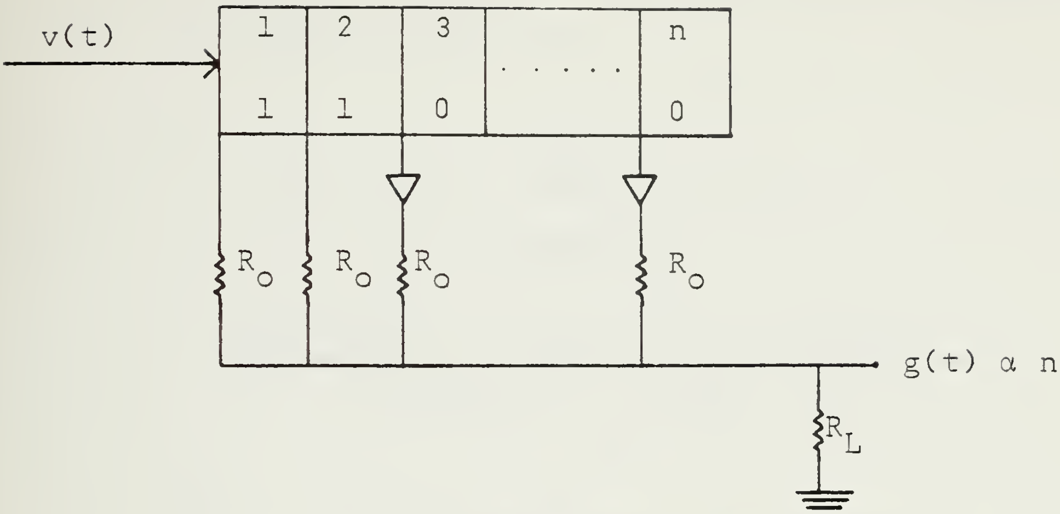


Fig. 3. Matched filter.

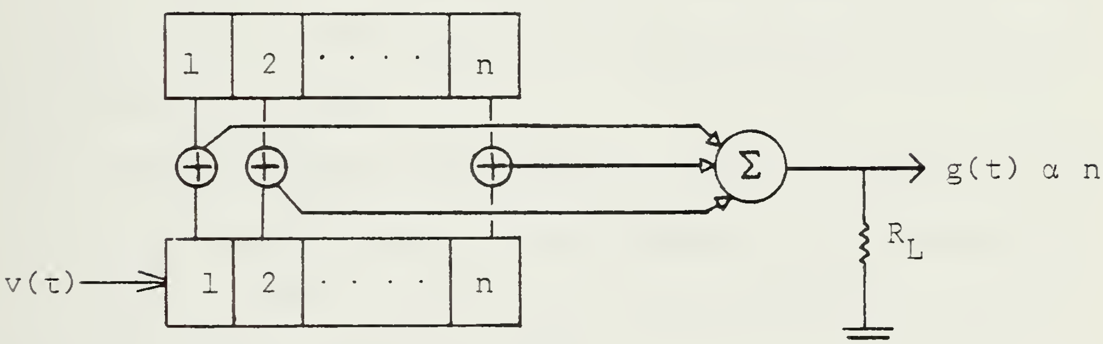
Matched filters for two-level voltages (binary sequences) can be realized by tapped delay lines or shift registers as shown in Fig. 4. The tapped delay line realization uses inverters at these positions where a zero element occurs in the sequence. The shift register realization uses a reference register where the original sequence is stored. This can be a read only memory (ROM) for example. Another register receives the input sequence v(t).

In both realizations, +1 units of current flow through the load resistor R_L for each element of the input sequence that agrees with the "stored" sequence. And -1 unit of current flows through R_L for each element of the input sequence that disagrees with the "stored" sequence. The net output current through R_L (and voltage across R_L) is proportional to the number of elements which agree less the number of elements which disagree. The output g(t) is, therefore, a measure of the autocorrelation function of the input sequence.

Since the systems of Fig. 4 perform a discrete comparison and summing instead of multiplication and integration, then g(t) is a discrete version of $R_V(\tau)$. For example, the sequence



(a) Tapped delay line



(b) Shift register

Fig. 4. Matched filter realizations.

1101 has the autocorrelation function shown in Fig. 2, whereas the output $g_d(t)$ of the discrete matched filter corresponding to 1101 is as shown in Fig. 5.

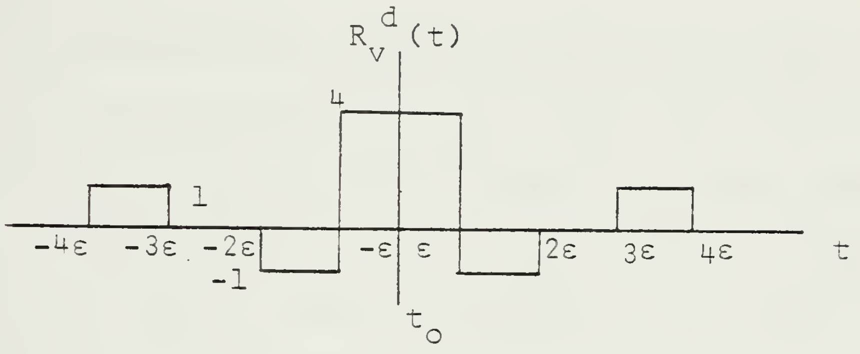


Fig. 5. Discrete matched filter output.

For a sequence of n elements, the peak output is n units of voltage. It is clear then, how signal detectability improves as n increases.

All binary sequences have autocorrelation functions with sidelobes of various values. The sequences of interest are the ones with either small or negative sidelobes. The problem is to find these sequences.

The next section addresses the issue of forming a pair of sequences or codes, which when processed with matched filters and the outputs added, yield a waveform with one mainlobe and no sidelobes. This scheme provides good detectability of binary sequences.

III. COMPLEMENTARY SEQUENCES

This section reviews complementary sequences and their basic properties.

A set of complementary series is defined as a pair of equally long, finite binary sequences which have the property that the number of pairs of like elements with any given separation in one series is equal to the number of pairs of unlike elements with the same separation in the other series.

For example the two series:

A = 00010010

B = 00011101

are complementary. In A there are three like elements (denoted by ℓ below) separated by one element.

$$0_{\ell}0_{\ell}0\ 1\ 0_{\ell}0\ 1\ 0$$

In B there are:

$$0\ 0\ 0_{u}1\ 1\ 1_{u}0_{u}1$$

three unlike elements (denoted by u below) separated by one element.

Similarly for all possible separations the number of like elements in A and unlike elements in B are as follows:

Separation	Number of Likes in A	Number of Unlikes in B
1	3	3
2	3	3
3	4	4
4	2	2
5	2	2
6	1	1
7	1	1

Series having the complementary property were conceived by Marcel J. E. Golay in connection with the optical problem of infrared multislit spectrometry [Ref. 1,5].

Complementary series have interesting autocorrelation functions. If the autocorrelation of each sequence is taken and these two autocorrelations summed, the result is zero for all τ except $\tau = 0$. At $\tau = 0$, the sum is twice that of either sequence. Therefore, the sum of the autocorrelation functions has one main lobe and no sidelobes.

For example, consider the sequences or codes

$$A = 00010010$$

$$B = 00011101$$

Sequence A has autocorrelation function:

$$R_A(\tau) = 8, -1, 0, 3, 0, 1, 0, 1, 0.$$

Sequence B has autocorrelation function:

$$R_B(\tau) = 8, 1, 0, -3, 0, -1, 0, -1, 0.$$

Their sum is:

$$\Sigma = R_A(\tau) + R_B(\tau) = 16, 0, 0, 0, 0, 0, 0, 0, 0.$$

This property can be treated in equation form as follows:

Let a_i and b_i ($i = 1, 2, ----, n$) be the elements of two sequences A and B each of length n. Assume a_i and b_i can be either +1 or -1. Then the respective values of the auto-correlation functions will be

$$c_j = \sum_{i=1}^{n-j} a_i \, a_{i+j} \text{ for } j \geq 0.$$

$$d_j = \sum_{i=1}^{n-j} b_i \, b_{i+j} \text{ for } j \geq 0.$$

Also $cj = c_{-j}$ for $j<0$.

$d_j = d_{-j}$ for $j<0$.

The necessary and sufficient condition for the pair of sequences to be complementary is:

$$c_j + d_j = 0 \text{ for } j \neq 0$$

and $c_j + d_j = 2n$ for $j = 0$.

where j ranges from $-n+1$ to $n-1$.

Or in expanded form,

$$\sum_{i=i}^{n-j} a_i \, a_{i+j} + \sum_{i-1}^{n-j} b_i \, b_{i+j} = 0 \text{ for } j \neq 0$$
$$= 2n \text{ for } j = 0.$$

When the elements of the sequence are 0 or 1, then the autocorrelation function is obtained by modulo-two addition. In this case the necessary and sufficient condition for the series to be complementary is that

$$\sum_{i=1}^{n-j} (a_i \oplus a_{i+j}) = \sum_{i=1}^{n-j} (b_i \oplus b_{i+j} \oplus 1) \text{ for all } j,\ 1 \leq j \leq n-1$$

This complementary property can be tested experimentally by using matched filters since the output of a filter matched to its input is the autocorrelation function of that input. This realization is shown in Figure 6. The output of the system of Figure 6 is shown in Figure 7 for the complementary sequences

A = 111-1

B = 11-11

A. GENERAL PROPERTIES

1. Number of Elements

The number of elements in two complementary series are equal. If they were different, the pair of extreme elements of the longest series would remain unmatched by an unlike pair of elements with the same spacing in the other series.

2. Symmetry

Two complementary series (A,B) are interchangeable (B,A); that is, one can take the place of the other. This results from the symmetry of the definition with respect to two complementary series.

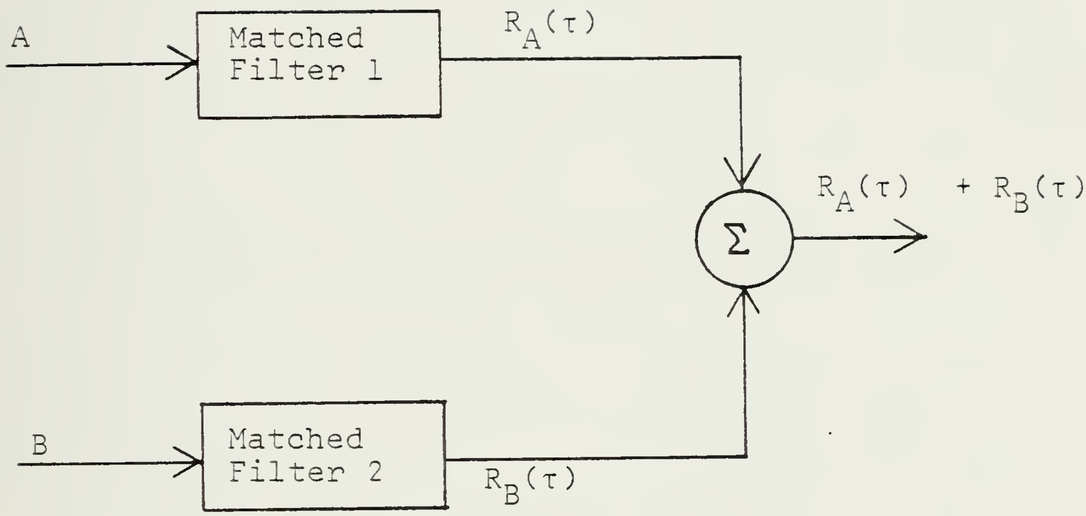


Fig. 6. Scheme for testing the complementary property.

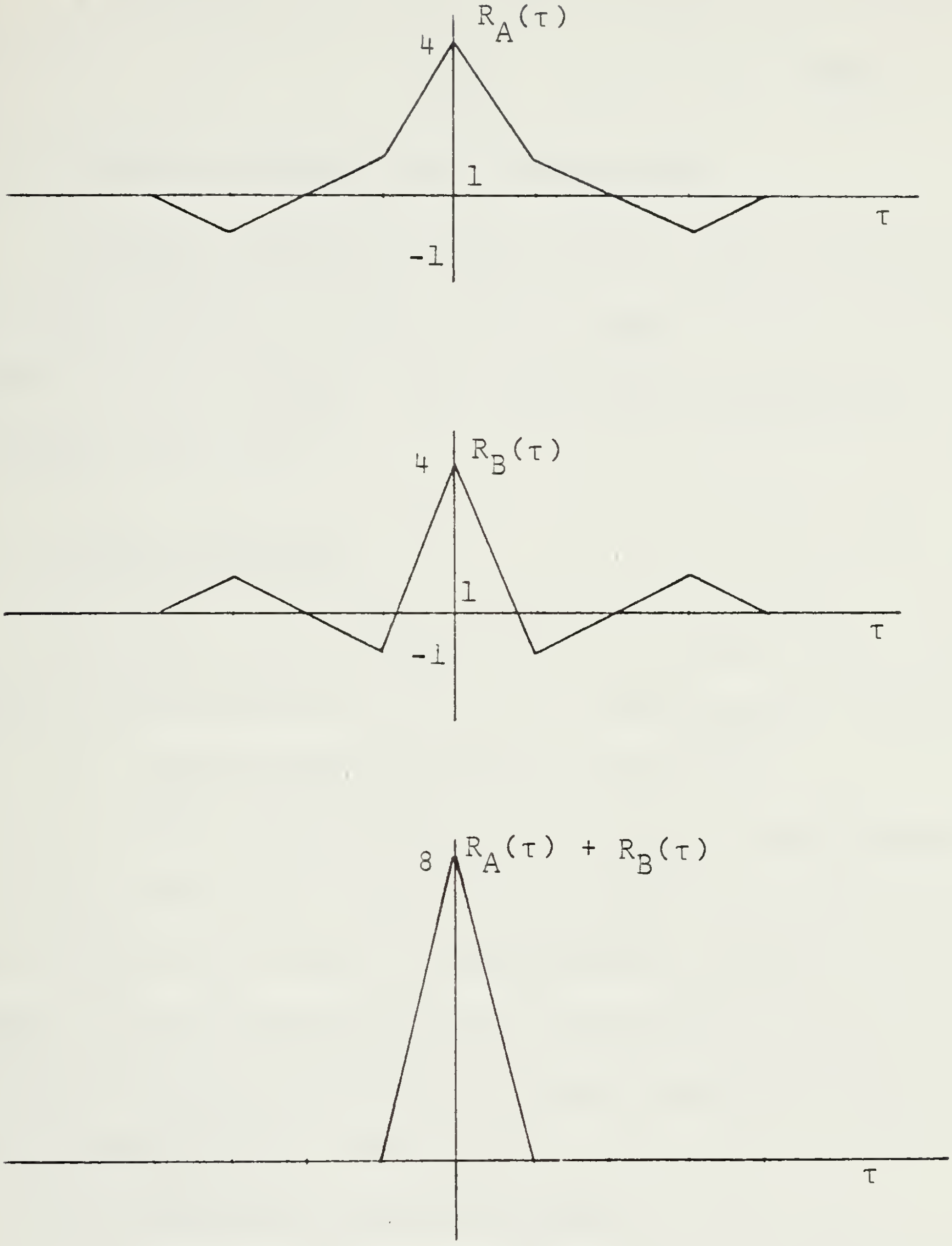


Fig. 7. Analog matched filter processing of two complementary sequences.

3. Sequence Length Even

A necessary condition for a sequence pair to be complementary is that their length n be an even number.

4. Sequence Length Sum of Two Squares

Another necessary condition for a pair of sequences to be complementary is that their length be the sum of the squares of two integers. The proof was developed by S. Jauregui [Ref. 4]:

5. Transformations

A single pair of complementary series can be the basis for the construction of 64 pairs of complementary series.

a. Order of Complementary Sequences

Denote the reverse of A by A_r. For example, if A = 1110, then A_r = 0111. The order of the elements of either or both of a pair of complementary series may be reversed. This follows from the fact that by reversing a sequence its autocorrelation function remains the same. The proof is developed in Section V.

b. Complementing the Sequence

Denote the complement of A by $\bar{A}$. For example, if A = 1110, then $\bar{A}$ = 0001. One or both of a pair of complementary sequences can be complemented—putting zeros in the place of ones and ones in the place of zeros, without affecting their complementary property. This follows from the fact that by

complementing a sequence, its autocorrelation function remains the same. Section V provides the proof.

c. Complementing Elements of Even Order

Denote the complement of the even order elements of A by $\bar{A}_e$. For example, if A = 1110, then $\bar{A}_e$ = 1011. Complementing the elements of even order in each sequence—putting zeros in the place of ones and ones in the place of zeros, does not affect their complementary property.

It is concluded from the above properties that a single pair (A,B) of complementary sequences can be the basis for the construction of $2^6 = 64$ pairs of complementary series (some of which might be identical) by either performing or not performing the following six operations:

a. Interchanging the sequences.

b. Reversing the first sequence.

c. Reversing the second sequence.

d. Complementing the first sequence.

e. Complementing the second sequence.

f. Complementing the elements of even order of each sequence.

As an example, consider the complementary pair A = 00010010 and B = 00011101:

Applying a gives B = 00011101 and A = 00010010.

Applying b gives A_r = 01001000 and B = 00011101.

Applying c gives A = 00010010 and B_r = 10111000.

Applying d gives $\bar{A}$ = 11101101 and B = 00011101.

Applying e gives $A = 00010010$ and $\bar{B} = 11100010$.

Applying f gives $\bar{A}_e = 01000111$ and $\bar{B}_e = 01001000$.

By applying the above properties properly, the original pair can be reproduced:

$B = 00011101$ and $A = 00010010$

$B_r = 10111000$ and $A = 00010010$

$B_r = 10111000$ and $A_r = 01001000$

$\bar{B}_r = 01000111$ and $A_r = 01001000$

$(\bar{\bar{B}}_r)_e = A = 00010010$ and $(\bar{A}_r)_e = B = 00011101$.

The last pair is the same as the original one.

6. Allowable Lengths

Since, as was mentioned before, the number of elements in complementary sequences must be even and equal to the sum of two squares, the allowable sequence lengths up to 50 are

2, 4, 8, 10, 16, 18, 20, 26, 32, 34, 36, 40, 50.

It has been verified by trial, though, that complementary sequences for length 18 do not exist.

7. Hamming Distance

The Hamming distance of two binary sequences A and B is defined as the number of positions in which these two binary sequences differ. This can be written in modulo two notation as follows:

$$D(A,B) = \sum_{i=1}^{i=n} a_i \oplus b_i$$

For example, the Hamming distance of the two sequences

$$A = 0100$$

$$B = 1111$$

is

$$D(A,B) = \sum_{i=1}^{i=4} a_i \oplus b_i = (1 + 0 + 1 + 1) = 3$$

Now for a complementary pair of sequences, it has been proven that their Hamming distance is always $= \frac{n}{2}$ [Ref. 2].

For example, the complementary pair of length $n = 10$

$$A = 1001010001$$

$$B = 1000000110$$

has Hamming distance

$$D(A,B) = \frac{n}{2} = 5.$$

8. Kernels

A Kernel is a basic length sequence which cannot be decomposed into shorter length sequences. The shortest possible complementary pair is 11 and 10. This pair or any of each transformation, which was mentioned before, is called a kernel of length two or a quad.

Some possible Kernel lengths are

2, 10, 18, 26, 34, 50.

It might be the case, though, that complementary pairs for some of them do not exist. For example, lengths n = 4 and n = 8 have complementary pairs, but are not Kernels because they can be constructed from n = 2 and n = 4 sequences respectively.

Among all the above mentioned Kernel lengths, it has been verified by M. J. Golay that n = 18 does not exist.

Also, it has been verified by S. Jauregui [Ref. 2] through exhaustive computer search that for n = 26 only the Kernel shown in Table I exists, not taking into account all allowable transformations.

For n = 34 a non-exhaustive computer search by S. Jauregui revealed no Kernel. An exhaustive computer search was not possible, due to the great computer time required.

It is possible, however, that a complete search for n = 34 could be achieved in the future, using new techniques. The following table shows the Kernels of n = 2, 10, 26, ignoring allowable transformations.

Table I

Kernels of Length 2, 10, 26

n	Number of Kernels	A Sequence	B Sequence
2	1	10	11
10	2	1001010001 0101000011	1000000110 0000100110
26	1	0100110111101011111 00111010	1011001000011111111 00111010

Note: (a) The possible Kernel of $n = 18$ does not exist.

(b) Partial computer search for $n = 34$ found no Kernel.

9. Number of Ones in Complementary Sequences

S. Jauregui showed [Ref. 2] that the equation

$$n = (n-p-q)^2 + (p-q)^2$$

holds for two complementary sequences A and B of length n where p is the number of ones in A and q the number of ones in B.

This leads to the conclusion that the number of ones in each of the sequences of a complementary pair cannot be arbitrary, but has to satisfy the above relation.

For example, for complementary sequences of length $n = 2$ the number of ones in A and B can be respectively

either (2, 1)

or (1, 0).

B. SYNTHESIS

If the sequences A,B are complementary, they can be used to generate other complementary pairs as follows:

(a) If $A = a_1\ a_2\ a_3 ----a_{n-1}\ a_n$

$B = b_1\ b_2\ b_3 ----b_{n-1}\ b_n$

are a complementary sequence pair, then the sequences

$$C = a_1\ a_2\ \text{----}a_n\ b_1\ b_2\ \text{----}b_n$$

$$D = a_1\ a_2\ \text{----}a_n\ \bar{b}_1\ \bar{b}_2\ \text{----}\bar{b}_n$$

are also complementary.

For example, consider the complementary pair:

A = 0001 with autocorrelation $R_A(\tau) = 4, 1, 0, -1, 0$

B = 0010 with autocorrelation $R_B(\tau) = 4, -1, 0, 1, 0.$

Then the sequences

C = 00010010

D = 00011101

are also complementary with autocorrelation functions

$R_C(\tau) = 8, -1, 0, 3, 0, 1, 0, 1, 0$

$R_D(\tau) = 8, 1, 0, -3, 0, -1, 0, -1, 0.$

(b) If $A = a_1\ a_2\ a_3\ \text{----}a_{n-1}\ a_n$

$B = b_1\ b_2\ b_3\ \text{----}b_{n-1}\ b_n$

are a complementary sequence pair, then the sequences

$$C = a_1\ b_1\ a_2\ b_2\ \text{----}a_n\ b_n$$

$$D = a_1\ \bar{b}_1\ a_2\ \bar{b}_2\ \text{----}a_n\ \bar{b}_n$$

are also complementary.

For example, consider the same sequences

A = 0001

B = 0010

The sequences

$$C = 00000110$$

$$D = 01010011$$

are also complementary with autocorrelation functions

$$R_C(\tau) = 8,\ 3,\ 0,\ 1,\ 0,\ -1,\ 0,\ 1,\ 0$$

$$R_D(\tau) = 8,\ -3,\ 0,\ -1,\ 0,\ 1,\ 0,\ -1,\ 0.$$

(c) If (A,B) (C,D) are two complementary sequences pairs, A of length n and C of length n, then the pair

$$V_1 = A^{c_1}\ A^{c_2}\ \text{----}A^{c_n}\ B^{d_1}\ B^{d_2}\ \text{----}B^{d_n}$$

$$V_2 = A^{d_n}\ \text{----}A^{d_1}\ B^{\bar{c}_n}\ \text{----}B^{\bar{c}_1}$$

is also complementary, where if an exponent is one the A or B sequence is left unchanged, whereas if the exponent is zero the A or B sequence is complemented.

For example, consider the complementary pairs

$$A = 11$$

$$B = 10$$

and

$$C = 00$$

$$D = 01.$$

Then the pair

$$v_1 = 00000110$$

$$v_2 = 11001010$$

is also complementary with autocorrelation functions

$$R_{v_1}(\tau) = 8,\ 3,\ 0,\ 1,\ 0,\ -1,\ 0,\ 1,\ 0.$$

$$R_{v_2}(\tau) = 8,\ -3,\ 0,\ -1,\ 0,\ 1,\ 0,\ -1,\ 0.$$

The above methods make possible the generation of complementary sequence pairs of greater lengths than the original ones. They can be applied in succession to generate very long sequences which are very useful in many applications.

For example such a complementary pair used in a communications system with matched filter processing like that of Fig. 6 can improve signal detectability in the presence of considerable noise since the summer output voltage will consist of a large main lobe and no sidelobes.

C. SUPPLEMENTARY AND CYCLIC COMPLEMENTARY SEQUENCES

Complementary sequences are subsets of two larger sets, namely supplementary and cyclic complementary sequences.

1. Supplementary Sequences

Consider two sequences

$$A = a_1\ a_2\ a_3\ \text{----}a_{n-1}\ a_n$$

$$B = b_1\ b_2\ b_3\ \text{----}b_{n-1}\ b_n$$

Now let

$$I = a_1\ a_3\ a_5 \text{----} a_{n-1}$$

$$II = a_n\ a_{n-2} \text{----} a_2$$

$$III = b_1\ b_3\ b_5 \text{----} b_{n-1}$$

$$IV = b_n\ b_{n-2} \text{----} b_2$$

The expression of the sequence pair A, B in the form (I, II, III, IV) is called sequence quadruple. Supplementary sequences are quadruples of sequences with the property that the total number of likes at each spacing equals the total number of unlikes at the same spacing.

In terms of their autocorrelation function, the sum of the four autocorrelation functions is zero any place but $\tau = 0$, where it is four times the length of the sequences. For example,

$$A = 1001010001$$

$$B = 1000000110$$

Writing in (I, II, III, IV) form gives

$$I = 10000$$

$$II = 10110$$

$$III = 10001$$

$$IV = 01000.$$

The new sequences (I, II, III, IV) have autocorrelation functions

$$R_I(\tau) = 5,\ 2,\ 1,\ 0,\ -1,\ 0$$

$$R_{II}(\tau) = 5,\ -2,\ -1,\ 2,\ -1,\ 0$$

$$R_{III}(\tau) = 5,\ 0,\ -1,\ -2,\ 1,\ 0$$

$$R_{IV}(\tau) = 5,\ 0,\ 1,\ 0,\ 1,\ 0.$$

The sum of the autocorrelation functions is

$$\Sigma = R_I(\tau) + R_{II}(\tau) + R_{III}(\tau) + R_{IV}(\tau)$$

$$= 20,\ 0,\ 0,\ 0,\ 0,\ 0.$$

In this example the sequences A, B are complementary. In general they do not have to be though, since the supplementary sequences are a larger set. This is illustrated in the following example.

Consider

$$I = 000100111$$

$$II = 000101001$$

$$III = 000101000$$

$$IV = 000110110$$

with autocorrelation functions

$$R_I(\tau) = 9,\ 2,\ -1,\ 0,\ 1,\ 0,\ -3,\ -2,\ -1,\ 0$$

$$R_{II}(\tau) = 9,\ -2,\ 1,\ 0,\ -1,\ 2,\ 1,\ 0,\ -1,\ 0$$

$$R_{III}(\tau) = 9,\ 0,\ 3,\ -2,\ 1,\ 0,\ 3,\ 2,\ 1,\ 0$$

$$R_{IV}(\tau) = 9,\ 0,\ -3,\ 2,\ -1,\ -2,\ -1,\ 0,\ 1,\ 0.$$

The sum is

$$\Sigma = R_I(\tau) + R_{II}(\tau) + R_{III}(\tau) + R_{IV}(\tau)$$

$$= 36,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0$$

which is zero except at the position $\tau = 0$, where it is 36.

From the sequences (I, II, III, IV) the sequences A, B can be constructed

A = 0100001100011010l0

B = 000101100111000000

In this example neither the A, B sequences nor the (I, II, III, IV) ones are complementary, which demonstrates the fact that supplementary sequences are a larger set.

Figure 8 gives another example of the supplementary property.

2. Cyclic Complementary Sequences

In general a cyclic sequence is a never ending periodic sequence of zeros and ones which has period of n elements. A cyclic complementary sequence pair is a pair of cyclic sequences, each of period n, where the number of likes in one sequence equals the number of unlikes in the other one, for all possible spacings. In equation form

$$C_j = \sum_{i=1}^{n} a_l \oplus a_{n-j+l} = \sum_{i+1}^{n} b_l \oplus b_{n-j+l} \oplus 1, \quad 1 \leq j \leq n-1.$$

In terms of autocorrelation functions the sum of the

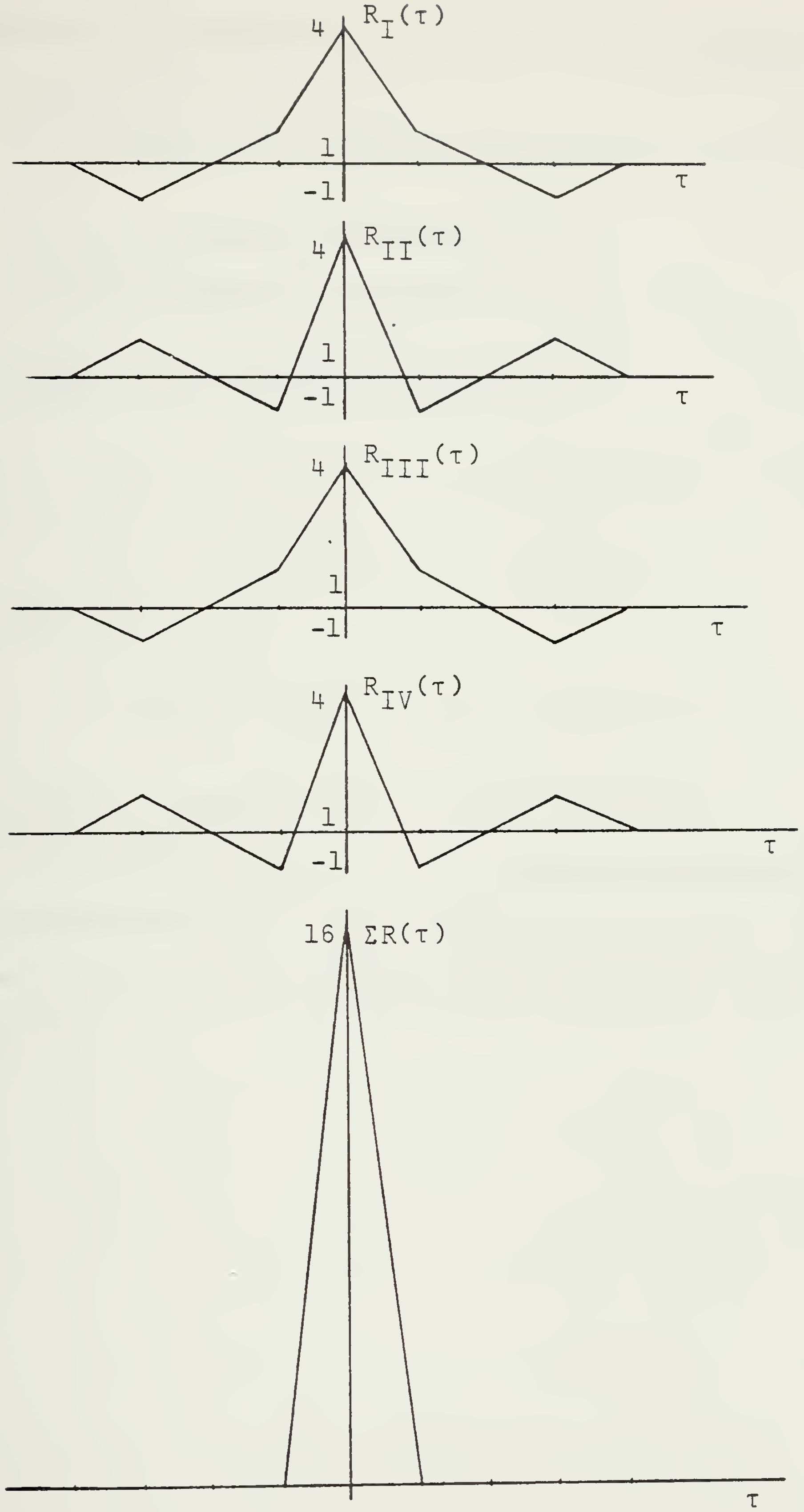


Fig. 8. The supplementary property.

two periodic autocorrelation functions is zero except at $\tau = kn$, where $k = 0, 1, 2, \ldots$.

For example, the cyclic complementary sequences

$$A = 01010011,\ 01010011,\ \ldots.$$

$$B = 00000110,\ 00000110,\ \ldots.$$

have periodic autocorrelation functions

$$R_A(\tau) = 8, -4, 0, 0, 0, 0, 0, -4, 8, \ldots.$$

$$R_B(\tau) = 8, 4, 0, 0, 0, 0, 0, 4, 8, \ldots.$$

The sum is

$$\Sigma = R_A(\tau) + R_B(\tau) = 16, 0, 0, 0, 0, 0, 0, 0, 16\ldots.$$

Figure 9 demonstrates the above example.

Complementary sequences are always supplementary and cyclic complementary, but the opposite is not always true. Supplementary and cyclic complementary sequences constitute a larger set.

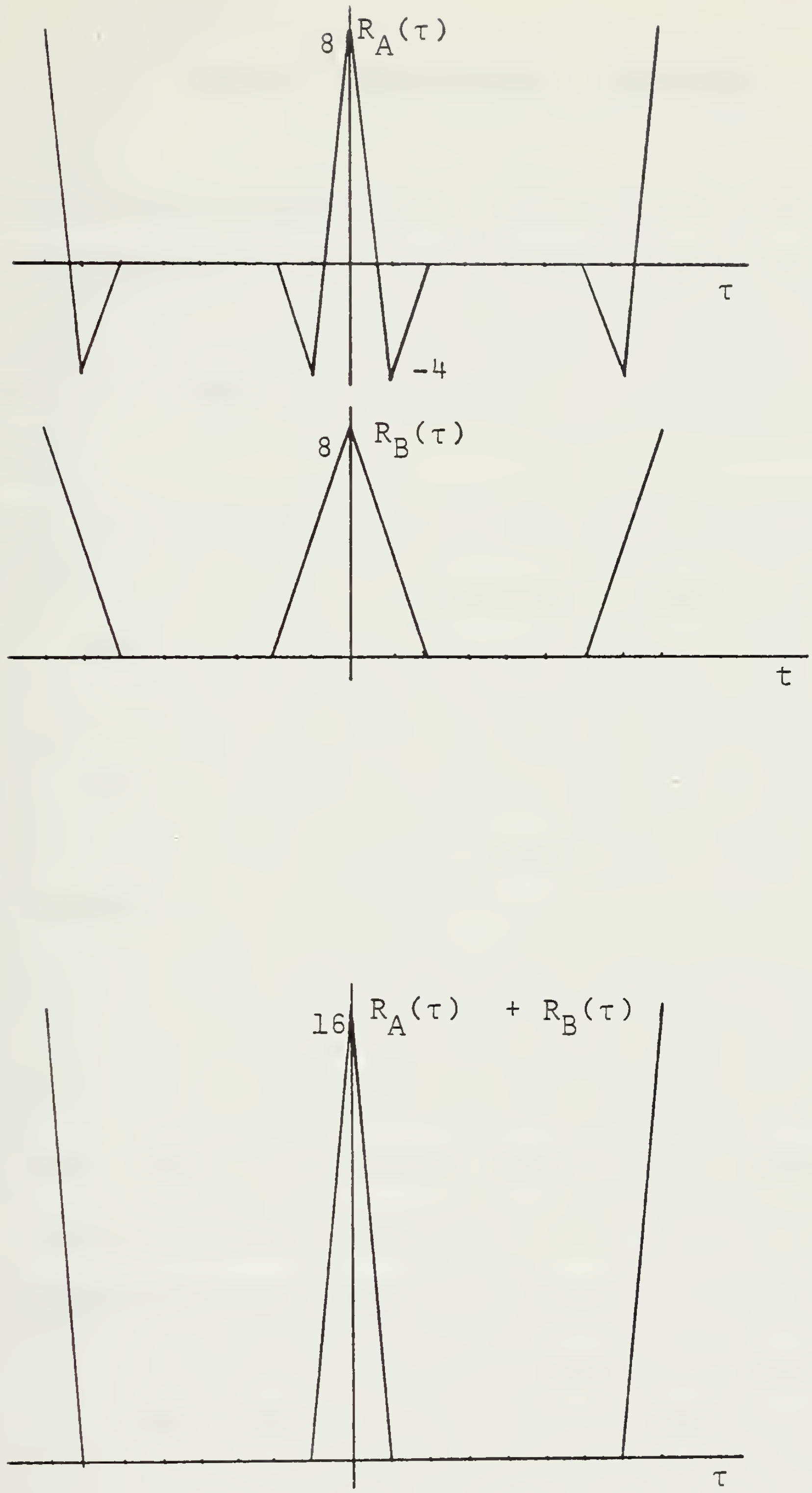


Fig. 9. The cyclic complementary property.

IV. ALMOST COMPLEMENTARY SEQUENCES

Complementary sequences are attractive for use in communications and ranging systems because the usable receiver output voltage has no sidelobes. Sequences providing "small" sidelobes compared to the main lobe may also be useful in some applications.

It may even be desirable to have a small sidelobe at a known position and level. For example, such a sidelobe can be used to measure doppler as shown in Section VII.

Two binary sequences whose summed autocorrelation functions exhibit two small sidelobes are called almost complementary in this report, since they exhibit properties similar to complementary sequences.

It should be noted that there are many sequences with small sidelobes, but the ones of interest here, are those for which certain rules hold. By applying these rules, the magnitude and position as well as polarity (positive or negative) of the sidelobe can be precisely predicted. By knowing these rules almost complementary sequences or codes can be constructed.

An extensive search for "good" almost complementary sequences was made. As a result, it is possible to list general rules for constructing almost complementary sequences with predictable sidelobe levels, polarity and position. There is a distinction between positive sidelobe and negative sidelobe sequences.

A. POSITIVE SIDELOBE SEQUENCES

Here two cases are developed.

1. Sidelobes $\pm \frac{1}{2}$ K Away

If A, B are complementary sequences of length n and the new sequences C and D of length K = 2n are formed as follows,

$$C = AA$$

$$D = BB$$

the sum of the autocorrelation functions of C and D gives only two positive sidelobes of magnitude half that of the main lobe at $\tau = \pm \frac{1}{2} K$.

For example, if n = 8, then K = 16 with the main lobe level of 16, and sidelobe levels of 8 at $\tau = \pm 4$.

For example, consider the two complementary sequences of length n = 4,

$$A = 0001$$

$$B = 1011,$$

and construct the almost complementary sequences

$$C = AA = 00010001$$

$$D = BB = 10111011$$

having autocorrelation functions

$$R_C(\tau) = 8, 1, 0, -1, 4, 1, 0, -1, 0$$

$$R_D(\tau) = 8, -1, 0, 1, 4, -1, 0, 1, 0.$$

Adding gives

$$\Sigma = R_C(\tau) + R_D(\tau) = 16, 0, 0, 0, 8, 0, 0, 0, 0.$$

So there are only two positive sidelobes with levels half the level of the main lobe and at a distance $\tau = \pm \frac{1}{2} K = \pm 4$. Figure 10 illustrates the above example.

Another example is

$$A = 0001001011100010$$

$$B = 0100011110110111$$

A, B are complementary of length n = 16. Constructing C = AA, D = BB gives

$$C = 00010010111000100001001011100010$$

$$B = 01000111101101110100011110110111$$

C, B are almost complementary of length K = 32 with auto-correlations

$$R_C(\tau) = 32, -1, 0, -1, 0, 3, 0, -5, 0, -1, 0, 9, 0, 1, 0, 1, 16, -1, 0, -1, 0, -1, 0, -1, 0, -3, 0, 5, 0, 1, 0, 1, 0.$$

$$R_D(\tau) = 32, 1, 0, 1, 0, -3, 0, 5, 0, 1, 0, -9, 0, -1, 0, -1, 16, +1, 0, +1, 0, +1, 0, +1, 0, 3, 0, -5, 0, -1, 0, -1, 0$$

and sum

$$\Sigma = R_C(\tau) + R_D(\tau) = 64, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 32, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0$$

with main lobe level 64 and positive sidelobe levels 32 at $\tau = \pm 16$ as predicted by the rule.

For this example, the IBM-360 computer was used.

2. <u>Sidelobes $\pm \frac{2}{3}$ K Away</u>

Let A, B be complementary sequences of length n. Let $\cancel{A}$ be a new sequence generated by taking only the $\frac{n}{2}$ first digits of A (truncating A after its $\frac{n}{2}$ first digits). Also let $\cancel{B}$ be a new sequence generated by taking only the $\frac{n}{2}$ first digits of B (truncating B after its $\frac{n}{2}$ first digits). Also let $\cancel{A}$, $\cancel{B}$ be complementary of length $\frac{n}{2}$. Then the new sequences of length $K = n + \frac{n}{2} = \frac{3n}{2}$ are

$$C = A\cancel{A}$$

$$D = B\cancel{B}$$

and C and D are almost complementary sequences. The sum of their autocorrelation functions has two positive sidelobes only of magnitude $\frac{1}{3}$ that of the main lobe and at $\tau = \pm \frac{2}{3}$ K. In this case there is an improvement relative to the previous case in that the sidelobes are smaller compared to the main lobe and farther removed from the main lobe.

The sequences A, B of length n can be constructed by using two complementary sequences of length $\frac{n}{2}$ according to the rule in Section III.B(a). Then the sequences $\not{A}$, $\not{B}$ are always complementary. For example, consider the two complementary sequences of length n = 8

$$A = 00011101$$

$$B = 01001000.$$

The new sequences of length 12 are

$$C = A\not{A} = 000111010001$$

$$D = B\not{B} = 010010000100$$

with autocorrelation functions

$$R_C(\tau) = 12,\ 1,\ 0,\ -5,\ 0,\ -5,\ 0,\ 1,\ 4,\ 1,\ 0,\ -1,\ 0.$$

$$R_D(\tau) = 12,\ -1,\ 0,\ 5,\ 0,\ 5,\ 0,\ -1,\ 4,\ -1,\ 0,\ 1,\ 0.$$

and sum

$$\Sigma = R_C(\tau) + R_D(\tau) = 24,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 8,\ 0,\ 0,\ 0,\ 0$$

with main lobe level 24 and sidelobe levels $\frac{1}{3}$ (24) = 8 at a distance $\tau = \pm 8$.

This example is illustrated in Fig. 11. Another example is

$$A = 01001000000111010100100011100010$$

$$B = 01001000000111011011011100011101$$

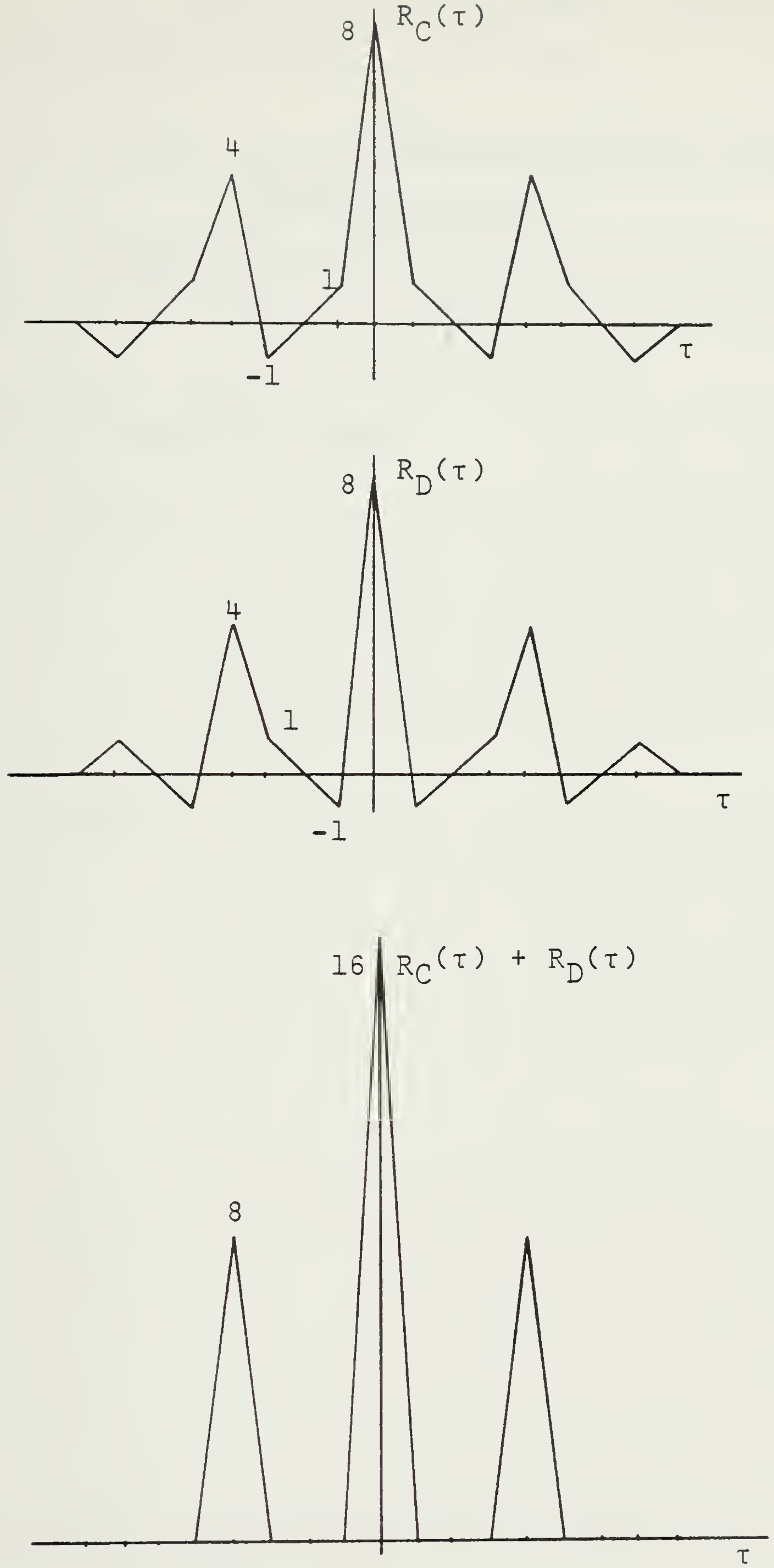


Fig. 10. Positive sidelobes at $\tau = \pm \frac{1}{2} K$.

A, B are complementary of length $n = 32$.

The new sequences of length $K = 48$ are

$$C = A\not{A} = 01001000000111010100100011100010$$
$$0100100000011101$$
$$D = B\not{B} = 10111000111011011011100000010010$$
$$1011100011101101$$

with autocorrelation functions

$$R_C(\tau) = 48,\ 1,\ 2,\ 3,\ 0,\ -1,\ -2,\ 1,\ 0,\ 3,\ -2,\ 5,\ 0,$$
$$+9,\ 2,\ -5,\ 0,\ -5,\ 2,\ 9,\ 0,\ 5,\ -2,\ 3,\ 0,\ 1,$$
$$-2,\ -1,\ 0,\ 3,\ 2,\ 1,\ 16,\ 1,\ 2,\ 3,\ 0,\ -1,\ -2,$$
$$1,\ 0,\ -1,\ -2,\ 1,\ 0,\ -3,\ 2,\ -1,\ 0.$$

$$R_D(\tau) = 48,\ -1,\ -2,\ -3,\ 0,\ 1,\ 2,\ -1,\ 0,\ -3,\ 2,\ -5,$$
$$0,\ -9,\ -2,\ 5,\ 0,\ 5,\ -2,\ -9,\ 0,\ -5,\ 2,\ -3,$$
$$0,\ -1,\ 2,\ 1,\ 0,\ -3,\ -2,\ -1,\ 16,\ -1,\ -2,\ -3,$$
$$0,\ 1,\ 2,\ -1,\ 0,\ 1,\ 2,\ -1,\ 0,\ 3,\ -2,\ 1,\ 0.$$

and sum

$$\Sigma = R_C(\tau) + R_D(\tau) = 96,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,$$
$$0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,$$
$$0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,$$
$$32,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,$$
$$0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0.$$

The main lobe has level 96 and the sidelobes have levels $\frac{1}{3}(96) = 32$, at a distance $\tau = \pm\frac{2}{3}48 = \pm 32$ as predicted by the rule.

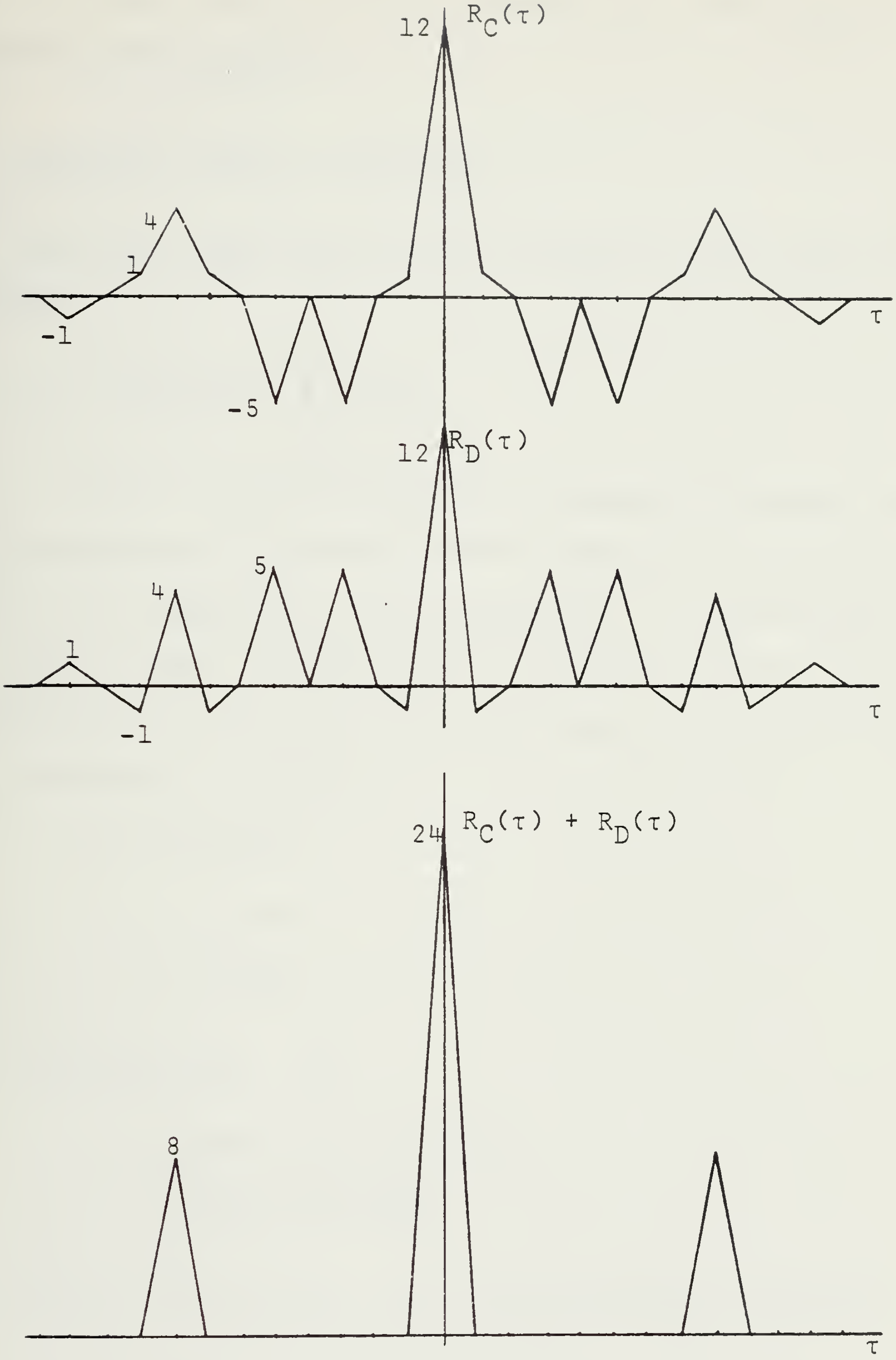


Fig. 11. Positive sidelobes at $\tau = \pm \frac{2}{3} K$.

Here also the IBM-360 computer was used because of the length of the sequence.

B. NEGATIVE SIDELOBE SEQUENCES

Here two cases are developed similar to the ones considered before.

1. Sidelobes $\pm \frac{1}{2}$ K Away

If A, B are complementary sequences of length n and the new sequences $C = A\bar{A}$ and $D = B\bar{B}$ of length $K = 2n$ are formed, where $\bar{A}$, $\bar{B}$ represent the complements of A, B respectively, then the sum of their autocorrelation functions gives two negative sidelobes only with magnitudes half that of the main lobe and at a distance $\tau = \pm \frac{1}{2} K$.

For example

$$A = 1011$$

$$B = 1110$$

are complementary. Then

$$C = A\bar{A} = 10110100 \text{ and}$$

$$D = B\bar{B} = 11100001$$

are almost complementary with autocorrelation functions

$$R_C(\tau) = 8,\ -3,\ 0,\ 3,\ -4,\ 1,\ 0,\ -1,\ 0$$

$$R_D(\tau) = 8,\ 3,\ 0,\ -3,\ -4,\ -1,\ 0,\ 1,\ 0$$

and sum

$$\Sigma = R_C(\tau) + R_D(\tau) = 16,\ 0,\ 0,\ 0,\ -8,\ 0,\ 0,\ 0,\ 0.$$

Here the sidelobes are negative with levels half that of the main lobe at $\tau = \pm \frac{1}{2} K = \pm \frac{1}{2} 9 = \pm 4$. This is illustrated in Fig. 12.

Another example, for which the IBM-360 computer was used is:

$$A = 0001001011100010$$

$$B = 0100011110110111$$

A, B are complementary of length n = 16. Constructing $C = A\bar{A}$, $D = B\bar{B}$ gives

$$C = 00010010111000101110110100011101$$

$$D = 01000111101101111011100001001000$$

Sequences C and D are almost complementary of length K = 32 with autocorrelation functions

$$\begin{aligned} R_C(\tau) = {} & 32, -3, 0, -3, 0, -7, 0, 1, 0, -5, 0, 11, 0, \\ & 3, 0, 3, -16, 1, 0, 1, 0, 1, 0, 1, 0, 3, 0, \\ & -5, 0, -1, 0, -1, 0 \\ R_D(\tau) = {} & 32, 3, 0, 3, 0, 7, 0, -1, 0, 5, 0, -11, \\ & 0, -3, 0, -3, -16, -1, 0, -1, 0, -1, 0, \\ & -1, 0, -3, 0, 5, 0, 1, 0, 1, 0 \end{aligned}$$

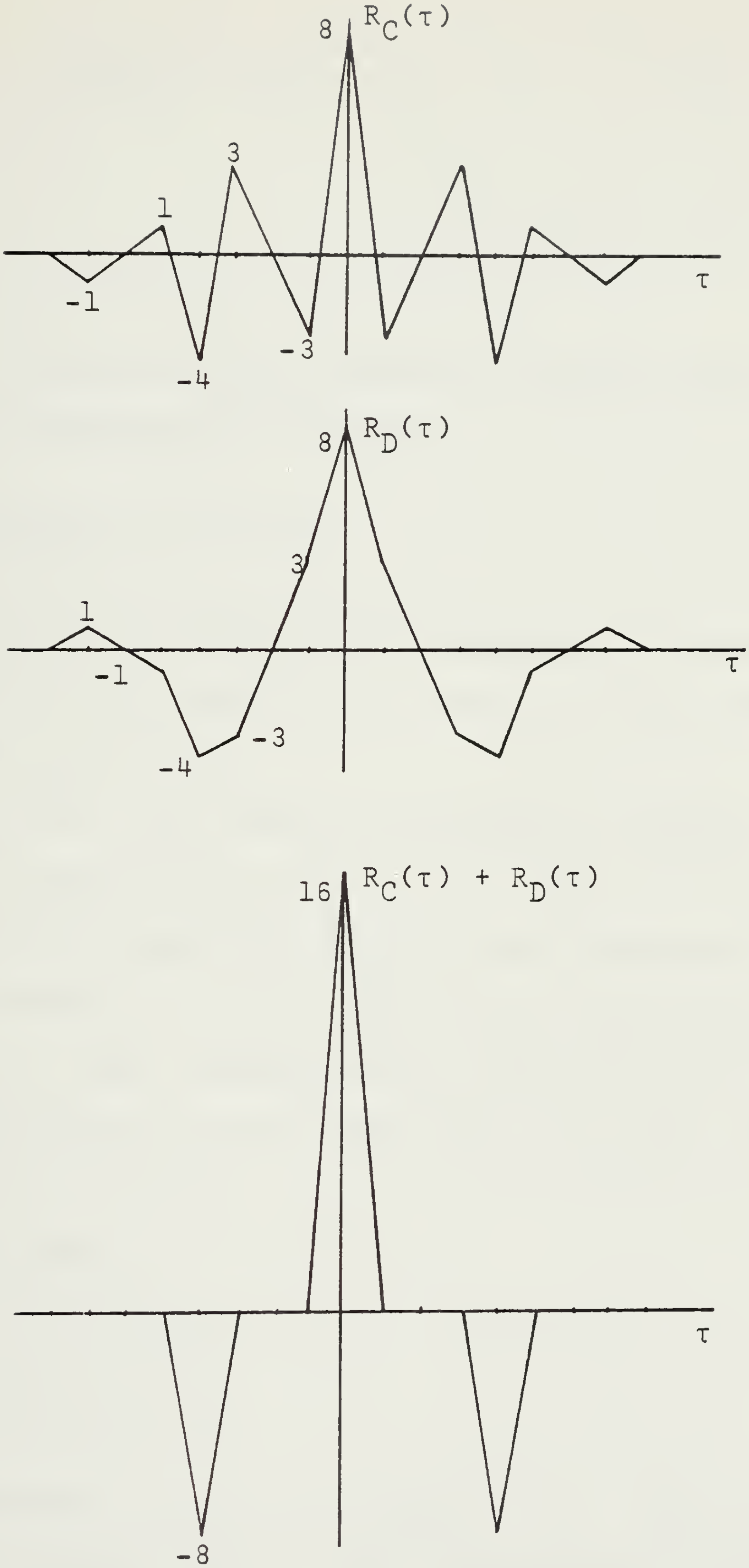


Fig. 12. Negative sidelobes at $\tau = \pm \frac{1}{2} K$.

and sum

$$\Sigma = R_C(\tau) + R_D(\tau) = 64, 0, 0, 0, 0, 0, 0, 0, 0, 0,$$
$$0, 0, 0, 0, 0, 0, -32, 0, 0, 0,$$
$$0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,$$
$$0, 0$$

with main lobe level 64 and negative sidelobe levels 32 at $\tau = \pm 16$ as predicted by the rule.

2. Sidelobes $\pm \frac{2}{3}$ K Away

Let A, B be complementary sequences of length n. Let $\bar{A}$ be a new sequence generated by taking only the complement of the $\frac{n}{2}$ first digits of A.

Let $\bar{B}$ be a new sequence generated by taking only the complement of the $\frac{n}{2}$ first digits of B. Also, let $\bar{A}$, $\bar{B}$ be complementary of length $\frac{n}{2}$. Then the new sequences $C = A\bar{A}$ and $D = B\bar{B}$ of length $K = n + \frac{n}{2} = \frac{3n}{2}$ will be almost complementary and the sum of their autocorrelation functions will have two negative sidelobes only with magnitudes one-third that of the main lobe at $\tau = \pm \frac{2}{3}$ K.

For example,

A = 11101101

B = 10111000

are complementary sequences of length n = 8. The new sequences $C = A\bar{A}$ = 111011010001 and $D = B\bar{B}$ = 101110000100 of length K = 12 are almost complementary with autocorrelations

$$R_C(\tau) = 12,\ -1,\ 0,\ 1,\ 0,\ -1,\ 0,\ 1,\ -4,\ -1,\ 0,\ 1,\ 0$$

$$R_D(\tau) = 12,\ 1,\ 0,\ -1,\ 0,\ 1,\ 0,\ -1,\ -4,\ 1,\ 0,\ -1,\ 0$$

and sum

$$\Sigma = R_C(\tau) + R_D(\tau) = 24,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ -8,\ 0,\ 0,\ 0,\ 0$$

The only sidelobes are negative with level $\frac{1}{3}(24) = 8$ at $\tau = \pm \frac{2}{3} K = \pm \frac{2}{3}(12) = \pm 8$. This is illustrated in Fig. 13.

Another example obtained with the use of the IBM-360 computer is

A = 01001000000111010100100011100010

B = 01001000000111011011011100011101

A, B are complementary of length n = 32. The new sequences of length K = 48 are

A = A$\bar{A}$ = 0100100000011101010010001110001010110
11111100010

B = B$\bar{B}$ = 1011100011101101101110000001001001000
11100010010.

The sequences C and D are almost complementary with autocorrelations

$$R_C(\tau) = 48,\ -1,\ 6,\ -3,\ 0,\ 1,\ -6,\ -1,\ 0,\ 1,\ -6,\ -1,\ 0,\ 3,\ 6,\ 1,\ 0,\ -1,\ -6,\ -3,\ 0,\ 1,\ 6,\ -1,\ 0,\ 1,\ 6,\ -1,\ 0,\ 3,\ -6,\ 1,\ -16,\ -1,\ -2,\ -3,\ 0,\ 1,\ 2,\ -1,\ 0,\ 1,\ 2,\ -1,\ 0,\ 3,\ -2,\ 1,\ 0$$

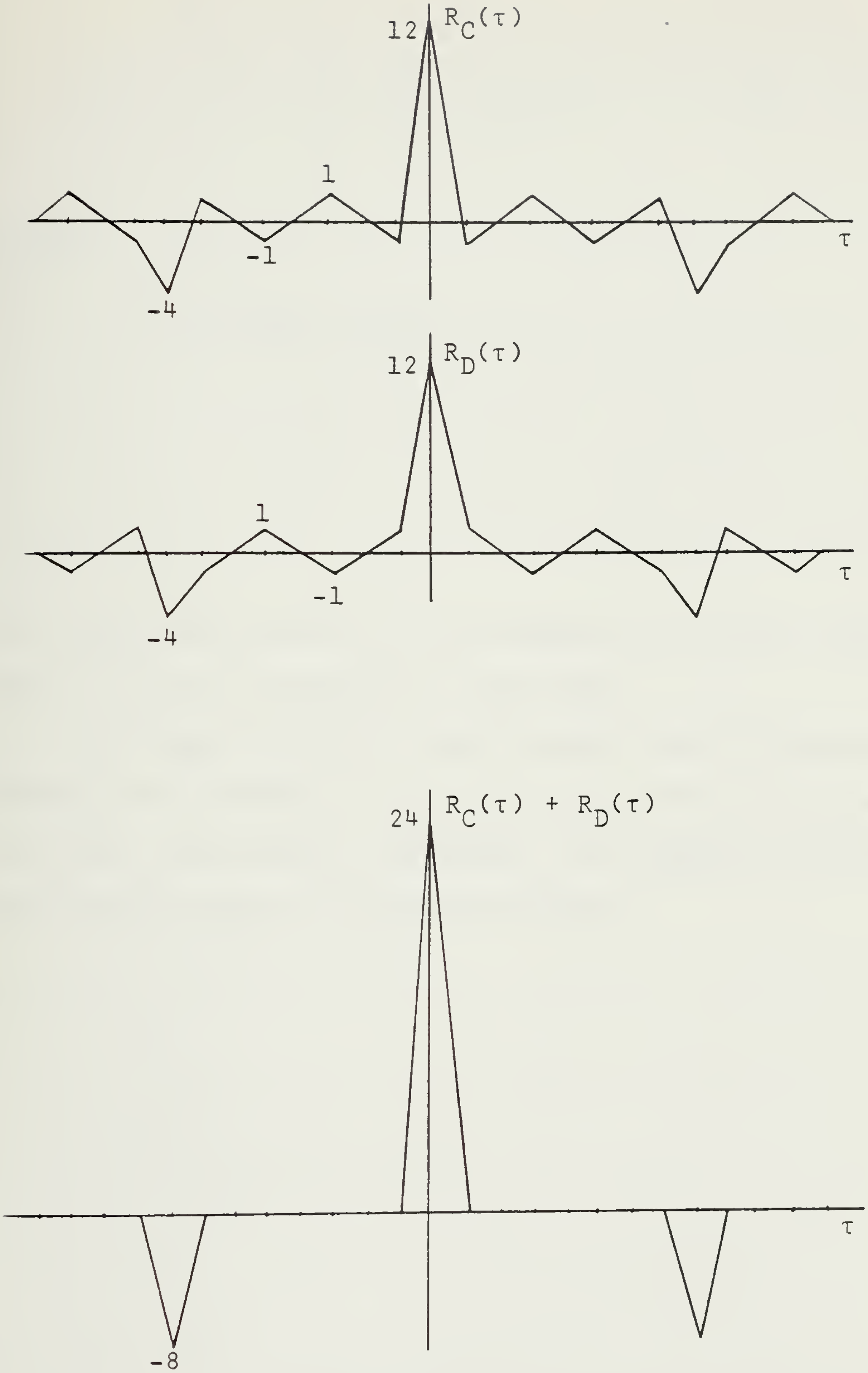


Fig. 13. Negative sidelobes at $\tau = \pm \frac{2}{3} K$.

$$R_D(\tau) = 48, 1, -6, 3, 0, -1, 6, 1, 0, -1, 6, 1, 0,$$
$$-3, -6, -1, 0, 1, 6, 3, 0, -1, -6, 1, 0,$$
$$-1, -6, 1, 0, -3, 6, -1, -16, 1, 2, 3, 0,$$
$$-1, -2, 1, 0, -1, -2, 1, 0, -3, 2, -1, 0$$

and sum

$$\Sigma = R_C(\tau) + R_D(\tau) = 96, 0, 0, 0, 0, 0, 0, 0, 0, 0,$$
$$0, 0, 0, 0, 0, 0, 0, 0, 0, 0,$$
$$0, 0, 0, 0, 0, 0, 0, 0, 0, 0,$$
$$0, 0, -32, 0, 0, 0, 0, 0, 0,$$
$$0, 0, 0, 0, 0, 0, 0, 0, 0, 0$$

with a main lobe level of 96 and negative sidelobe levels of 32 at $\tau = \pm 32$ as predicted by the rule.

In signal detection applications almost complementary sequences with negative sidelobes offer better noise immunity than those with positive sidelobes. The negative sidelobes can be removed with an envelope detector.

V. NUMBER OF DIFFERENT AUTOCORRELATION FUNCTIONS IN ALL BINARY SEQUENCES OF FIXED LENGTH

Binary sequences useful for communications or ranging purposes have autocorrelation functions with small sidelobes. Small can be defined in terms of a predetermined level.

How can these sequences be found? At present, the only way to find these sequences is to form the autocorrelation functions of all possible sequences of a given length and then select the desirable ones. This is a tedious task specially for long sequences, since there are 2^n possible binary sequences of length n. However, as shown in this section, many of these 2^n sequences have the same autocorrelation function.

In general there are four sequences of given length which have the same autocorrelation function:

1. The sequence itself.
2. The sequence obtained by reversing the original sequence.
3. The sequence obtained by complementing the original sequence.
4. The sequence obtained by complementing and reversing the original sequence.

For example, consider the sequence of length n = 4:

$$A = 1101$$

Reversing A gives

$$A_r = 1011 = B.$$

If A is complemented, results in

$$\bar{A} = 0010 = C.$$

If A is reversed and complemented, gives

$$\bar{A}_r = 0100 = D.$$

All these sequences A, B, C, and D have the same autocorrelation function

$$R_A(\tau) = R_B(\tau) = R_C(\tau) = R_D(\tau) = 4, -1, 0, 1, 0.$$

These results are easy to prove. Consider each case separately.

1. When a sequence is reversed and its autocorrelation function taken, this is exactly the same as if the autocorrelation function of the original sequence was taken, since the formation of the autocorrelation function can be considered as being accomplished by "sliding" the sequence past itself either to the right or to the left. So, "sliding" to the right for A is equivalent to "sliding" to the left for A_r.

2. The autocorrelation function of a sequence is generated by forming products and adding them. The general form of one of these products is

$a_i \; a_j$

a_i and a_j can have the values

$$a_i = 1 \text{ or } -1$$

$$a_j = 1 \text{ or } -1.$$

Here the values 1 and -1 are used since multiplication is considered in the formation of the autocorrelation function (if modulo 2 addition were considered, the values 1 and 0 would be used).

Now the possible values of the product $a_i \; a_j$ are

$$a_i \; a_j = (1)(1) = 1$$
$$\text{or } a_i \; a_j = (1)(-1) = -1$$
$$\text{or } a_i \; a_j = (-1)(1) = -1$$
$$\text{or } a_i \; a_j = (-1)(-1) = 1.$$

If the sequence is complemented, 1 is replaced by -1, and -1 by 1. So the possible values of the product $a_i \; a_j$ are respectively

$$a_i \; a_j = (-1)(-1) = 1$$
$$\text{or } a_i \; a_j = (1)(-1) = -1$$
$$\text{or } a_i \; a_j = (1)(-1) = -1$$
$$\text{or } a_i \; a_j = (1)(1) = 1.$$

The values of the product $a_i \; a_j$ are the same as before for all i and j and so, the autocorrelation function remains the same.

3. Since the autocorrelation function is the same if the sequence is reversed or complemented, it follows that it will be also the same if the sequence is reversed and complemented.

Some sequences are their own reverse. These sequences are called here symmetric and will be denoted by the letter S.

For example the sequence A = 1001, when reversed gives A_r = 1001 = A. In this case only the sequence itself and its complement need to be considered. The sequences A = 1001 and $\bar{A}$ = 0110 have the same autocorrelation function.

For some sequences, complementing and reversing gives the same sequence. These sequences are called here R sequences.

For example, the sequence A = 000111 when reversed and complemented gives $\bar{A}_r$ = 000111 = A. In this case only the sequence itself and its reverse need to be considered. The sequences 000111 and 111000 have the same autocorrelation function.

It is assumed that the number of sequences having the same autocorrelation function is either two or four. It has not been shown though that there does not exist any other number of sequences such as 3 or 5 or 6, etc., that have the same autocorrelation function. In this work, it has been verified by exhaustive computer search for sequences of various lengths, that the number of sequences having the same autocorrelation function is either two or four.

When an R sequence is reversed and complemented, the ones of the R sequence become the zeros of the reverse complement.

So, an R sequence has always the same number of ones and zeros. Therefore, an R sequence is always of even length.

For example, for length n = 3, there will not be any R sequence, since n is odd.

So far it has been established that:

(1) There are only four sequences with the same autocorrelation function, provided these sequences are not S or R.

(2) There are only two sequences with the same autocorrelation function, if these sequences are R or S.

(3) S sequences can be of any length.

(4) R sequences can be only of even length.

For example, all the sequences of length n = 2 and their autocorrelation functions are:

Sequence	Autocorrelations
00	2, 1, 0
01	2, -1, 0
10	2, -1, 0
11	2, 1, 0

Here there are two S sequences, 00 and 11, with the same autocorrelation function = 2, 1, 0. Also, there are two R sequences, 01 and 10, with the same autocorrelation function = 2, -1, 0.

Another example of length n = 3 is:

Sequence	Autocorrelation
000	3, 2, 1, 0
001	3, 0, -1, 0
010	3, -2, 1, 0
011	3, 0, -1, 0
100	3, 0, -1, 0
101	3, -2, 1, 0
110	3, 0, -1, 0
111	3, 2, 1, 0

Here there are four sequences which are neither R nor S, 001, 011, 100. 110 with the same autocorrelation function 3, 0, -1, 0. Also there are four S sequences: 000, 111, 010, 101. The sequences 000 and 111 have the same autocorrelation function 3, 2, 1, 0 and the sequences 010 and 101 have the same autocorrelation function 3, -2, 1, 0. In this example there is no R sequence since n = 3 is odd.

Next the exact number of R and S sequences will be established among all the possible sequences of length n. The maximum number of binary sequences of length n is given by 2^n. The sequences of n = 1 are 0 and 1. The number of S sequences is two. So S = 2 here;(0, 1). For n = 2 all sequences are 00, 01, 10, 11. Here S = 2:(00, 11), R = 2: (01, 10), and S + R = 4. For n = 3, the possible sequences are 000, 001, 010, 011, 100, 101, 110, 111. Here S = 4:(000, 010, 101, 111), R = 0 and S + R = 4. Going to n = 4, it can be seen that S = 4, R = 4 and S + R = 8.

The relation between n and S and R can be derived by considering the mechanism of moving from a sequence of length n to the next one of length n + 1.

For example, the sequences of n = 2 are formed by taking the sequences of length n = 1 and adding in front of each of them a zero and a one, one at a time, so the number of sequences for n = 1 is doubled and all the possible sequences of length n = 2 are formed. By doing so, the following can be noted:

(a) When moving from n even to n+1 which is odd, the S + R sequences in n is equal to the S sequences in n+1.

(b) When moving from n odd to n+1 which is even, the S in n is half the S + R in n+1.

For example, for n = 4 there are S + R = 8 sequences and for n = 5 there are S = 8, but moving to n = 6 gives S + R = 16.

So, moving from an even length to the next keeps the number S + R, but moving from an odd length to the next doubles the S + R number.

Using formulas it can be written:

(a) $n = \text{even}, \ (S + R)_n = (S)_{n+1}$

(b) $n = \text{odd}, \ (S)_n = \frac{1}{2}(S + R)_{n+1}$

Now, considering the following short table giving the S + R terms of all possible sequences up to length n = 10, the relation between n and S + R can be obtained.

n	$S + R$
1	2
2	4
3	4
4	8
5	8
6	16
7	16
8	32
9	32
10	64

For n even, $S + R = 2^{\frac{n+2}{2}}$ and for

n odd, $S + R = 2^{\frac{n+1}{2}}$.

The number of different autocorrelation functions contained in all possible sequences of length n is derived as follows.

If this number is denoted by T, two cases are considered.

(a) n is even. Here all the possible sequences number 2^n. Also the number of S and R sequences is $S + R = 2^{\frac{n+2}{2}}$. By deducting $S + R$ from 2^n, a number of sequences equal to $2^n - 2^{\frac{n+2}{2}}$ is obtained. It was established before that since these remaining $2^n - 2^{\frac{n+2}{2}}$ sequences contain no R or S sequences, they have $\frac{2^n - 2^{\frac{n+2}{2}}}{4}$ different autocorrelation functions. The $2^{\frac{n+2}{2}}$ S and R sequences have $\frac{2^{\frac{n+2}{2}}}{2}$ different autocorrelation

functions. So finally, all the 2^n sequences have

$$\frac{2^n - 2^{\frac{n+2}{2}}}{4} + \frac{2^{\frac{n+2}{2}}}{2}$$ different correlation functions. By re-arranging this result, the final formula for n even is derived

$$(T)_{even} = 2^{n-2} + \frac{2^{n/2}}{2} .$$

(b) n is odd. Similarly all possible sequences here number 2^n. Also, $S + R = 2^{\frac{n+1}{2}}$. Deducting the $2^{\frac{n+1}{2}}$ S + R sequences from 2^n gives $2^n - 2^{\frac{n+1}{2}}$ sequences with $\frac{2^n - 2^{\frac{n+1}{2}}}{4}$ different autocorrelation functions. So, all the 2^n sequences have $\frac{2^n - 2^{\frac{n+1}{2}}}{4} + \frac{2^{\frac{n+1}{2}}}{2}$ different autocorrelation functions

or in the final form

$$(T)_{odd} = 2^{n-2} + \frac{2^{\frac{n-1}{2}}}{2} .$$

Two examples are taken

(a) n = 7

$$(T)_{odd} = 2^5 + \frac{2^3}{2} = 36.$$

So the $2^7 = 128$ possible sequences of length 7 give only 36 different autocorrelation functions.

(b) n = 10

$$(T)_{even} = 2^8 + \frac{2^5}{2} = 272.$$

So, by using sequences 10 bits long, at most 272 different autocorrelation functions can be obtained from the $2^{10} = 1024$ possible sequences.

VI. COMPUTER SEARCH FOR "GOOD CODES

In this section the computer programs for obtaining sequences with "small" autocorrelation sidelobes are discussed. These sequences or "good" codes are found by applying mainly the results of the previous section. Because of the large number of different sequences of length n for even modest values of n, it is necessary to use a digital computer to search for "good" codes. Practically, an exhaustive search is limited to $n \approx 20$ with present digital computers.

For small lengths (up to $n = 10$) it is possible that a programmable calculator can be used to find the autocorrelation function of one sequence at a time. This was done with a TI-59 programmable hand calculator by storing each element of the sequence in a memory location. Then the autocorrelation function was formed by multiplication of the proper elements in each position. In this case, +1 and -1 is used for the elements of the sequence.

The algorithm for computing the autocorrelation function of a sequence on a large computer (IBM-360) is constructed. This is given in Program 1 on page 111. In this program the autocorrelation function of only one sequence can be computed. The sequence has to be punched on a computer card. A small modification gives Program 2 on page 112 which gives the option to find the autocorrelation functions of any number of sequences. Each of these sequences has to be punched on a separate card.

In these computer programs and also all the next ones, the sequences are represented with zeros and ones. The algorithm for finding the autocorrelation function has been made by comparing the number of like and unlike elements in every position.

To avoid punching the sequences on the cards, a program is created to generate automatically all the possible 2^n sequences at length n. This is accomplished by counting in binary from 0 to 2^n and thus generating all the binary numbers of length n. This program is combined with the program for the computation of the autocorrelation function. So, every time a sequence is generated, its autocorrelation function is formed. This is Program 3 on page 113.

Since only the different autocorrelation functions are of interest here, a program is written to select only those codes having different autocorrelation functions. To understand the operation of this program, consider the following example.

Take all the sequences of length n = 4 and the corresponding autocorrelation functions.

Sequence	Autocorrelation
0000	4, 3, 2, 1, 0
0001	4, 1, 0, -1, 0
0010	4, -1, 0, 1, 0
0011	4, 1, -2, -1, 0
0100	4, -1, 0, 1, 0
0101	4, -3, 2, -1, 0
0110	4, -1, -2, 1, 0
0111	4, 1, 0, -1, 0
1000	4, 1, 0, -1, 0
1001	4, -1, -2, 1, 0
1010	4, -3, 2, -1, 0
1011	4, -1, 0, 1, 0
1100	4, 1, -2, -1, 0
1101	4, -1, 0, 1, 0
1110	4, 1, 0, -1, 0
1111	4, 3, 2, 1, 0

There are a total of $2^4 = 16$ sequences. After the eighth sequence 0111, the other sequences are complements of the first eight ones. So, they give no new autocorrelation function, and therefore they do not need to be taken under consideration. In the first eight sequences there are two pairs with the same autocorrelation function 0001 and 0111 and also 0010 and 0100. So, only six different autocorrelation functions remain. The general way to proceed is to take each sequence in the first $\frac{2^n}{2}$, reverse it, complement it and reverse complement it and then keep only the original sequence

and reject the others, since they have the same autocorrelation function. Here two cases are considered.

(a) The sequence ends in 0.

In that case its complement and reverse complement will start with one, so they belong to the sequences after the first $\frac{2^n}{2}$ ones and need not to be considered. In that case only the reverse of the sequence is taken.

(b) The sequence ends in 1.

In that case its reverse and complement will start with one, so they belong to the sequences after the first $\frac{2^n}{2}$ ones and need not to be considered. In that case only the reverse complement is taken.

Now, the program takes each sequence as it is generated and reverses it if it ends in 0, or reverse complements if it ends in 1. If the resulting sequence represents a smaller binary number than the original one, this means that the resulting sequence was generated before and its autocorrelation function already taken, so there is no need to be taken again, and the program goes to the next sequence. The procedure is repeated until all the first $\frac{2^n}{2}$ sequences are finished. This way only the codes having different autocorrelation functions are listed in the printed output.

For example, in the case of n = 4 the result is as follows:

Sequence	Autocorrelation
0000	4, 3, 2, 1, 0
0001	4, 1, 0, -1, 0
0010	4, -1, 0, 1, 0
0011	4, 1, -2, -1, 0
0101	4, -3, 2, -1, 0
0110	4, -1, -2, 1, 0

And the number of different autocorrelation functions is 6, which is in agreement with the formula

$$(A)_{even} = 2^{n-2} + \frac{2^{n/2}}{2} = 2^2 + \frac{2^2}{2} = 6$$

The program which generates automatically the sequences of length n and computes only the different autocorrelation functions is Program 4 on page 114. An example for n = 10 is given on page 84.

All the different autocorrelation functions are not needed. Only those with small sidelobes. So a filtering procedure has to be introduced in the program to keep only those autocorrelations which have sidelobes equal or smaller than a predetermined level. Program 5 on page 115 generates automatically the sequences of length n and prints only those different autocorrelation functions with sidelobe levels equal or less than 1. For this case the lengths n = 10, 11, 12, 13, 15, 17 and 20 were examined, and the number of different autocorrelation functions found with sidelobe levels equal or less than 1 were, respectively, 11, 1, 16, 31, 0, 40, 3.

It is interesting to note that for n=11 there is only one such autocorrelation which is a Barker code (a sequence with sidelobes between +1 and -1). For n=15 no such codes exist, and for n=20 only three were found.

For the case n=20 it was not possible to make an exhaustive search because of the computer time required. Using 30 minutes of computer time, only three such autocorrelation functions were found. It is estimated that about 2 hours of computer time is required to make an exhaustive search.

It is evident that for lengths greater than n=20 even with a large computer an exhaustive search is impractical.

There is though a way to search regions of big sequences. Computer Program 6 on page 116 was used to search for a region of the n=20 case with no new results.

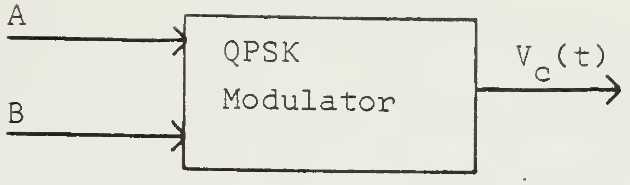
The results for the cases n=10, 11, 12, 13, 17, 20 are listed on pages 96, 98, 99, 101, 105, 110, respectively.

VII. APPLICATIONS

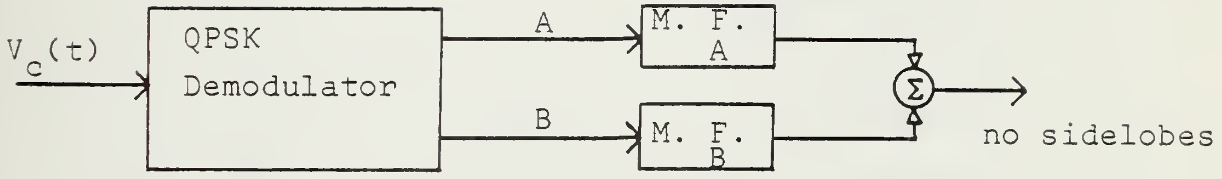
Complementary sequences, almost complementary sequences and codes with small sidelobes can be used in communications, ranging and spread spectrum systems. This section lists some possible applications.

Two complementary sequences can be transmitted simultaneously using a quadriphase phase-shift keying (QPSK), for example. Two complementary sequences A and B are applied to a QPSK modulator. The output $V_c(t)$ is a sine wave with a phase which can have one of four values. In the receiving system, $V_c(t)$ is demodulated. The outputs of the demodulator are the sequences A and B. Each sequence is processed by a matched filter and the outputs of the two matched filters added. Then the output of the summer will have one main lobe and no sidelobes. Fig. 14 illustrates this system.

Almost complementary sequences with positive sidelobes can be used to measure doppler. In Fig. 15 such a scheme is shown. The almost complementary sequences A and B of length K give after summing positive sidelobes at $\tau = \pm \frac{2}{3} K$ from the main lobe. Each of them is processed by a matched filter and the two outputs of the matched filters added. The output of the summer is connected to a counter in such a way that the first pulse (main lobe) enables the counter and the second pulse (sidelobe) inhibits the counter. When doppler is

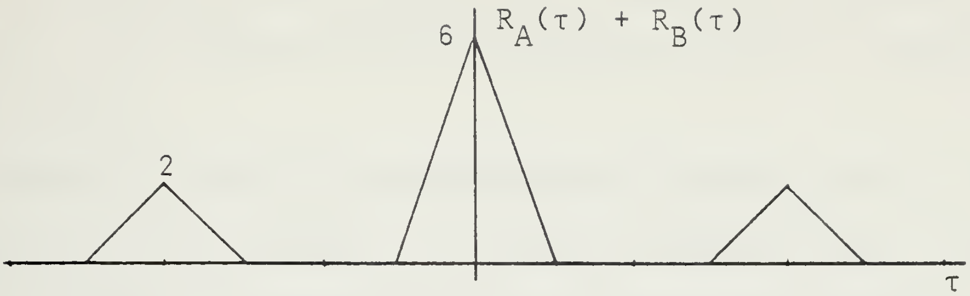


(a) Transmitter

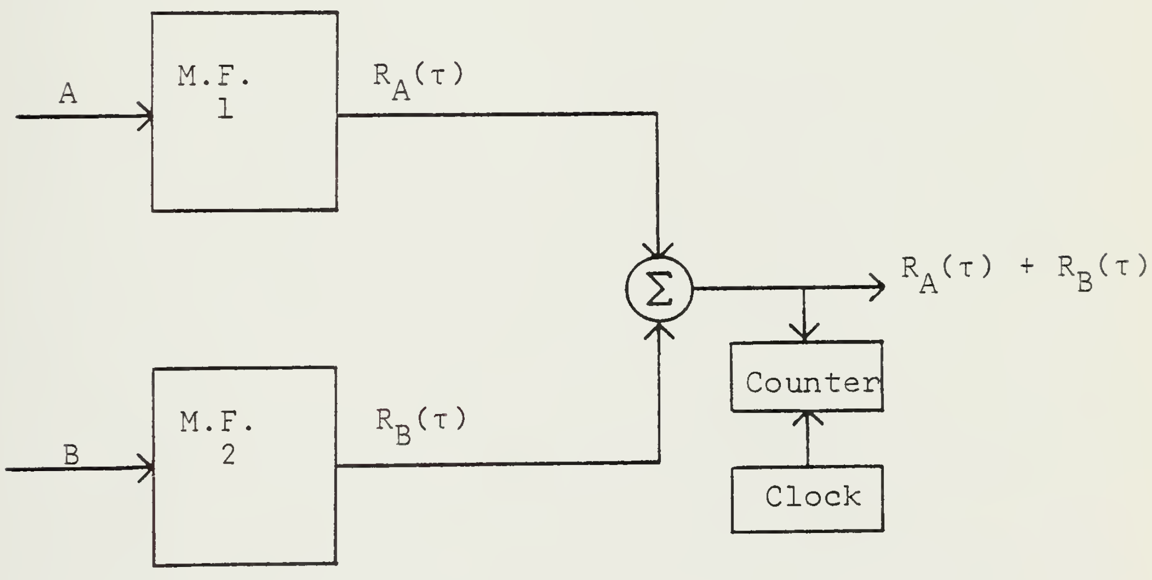


(b) Receiver

Fig. 14. QPSK System.



(a) Sum of the autocorrelations.



(b) Receiving system.

Fig. 15. System to measure doppler.

introduced to the system, the distance between the main lobe and the sidelobe will change by an amount proportional to the doppler. The contents of the counter is, then, a measure of the doppler.

Codes with small autocorrelation sidelobes can be used in spread spectrum systems. They can be used as "chip" sequences and as means for synchronizing the remote oscillators in such systems. They can also be used in surveillance and ranging systems because they provide accurate and unambiguous time measurements.

VIII. RESULTS AND CONCLUSION

The study presented in this thesis had as a main objective the discovery of means of generating "good" codes. Good codes imply binary sequences having autocorrelation functions with small sidelobes or no sidelobes at all.

The results of the search for good codes in this study are the following:

(1) Almost complementary sequences were discovered. These are constructed by using complementary sequences. The sum of the autocorrelation functions has two sidelobes only of predictable level, polarity and position.

(2) Computer programs for obtaining sequences with autocorrelation functions having sidelobe levels less than predetermined desirable levels were prepared and used. These computer programs reveal that for sequences of length $n = 15$, there are no codes with sidelobes less than or equal to 1. There is only one such code for $n = 11$ (which is a Barker code) and for $n = 20$ three such codes were found without exhausting all possibilities. For lengths greater than $n \approx 20$, an exhaustive computer search is impractical. A partial search can be made though by searching regions of these sequences with a special computer program.

The search for good codes resulted also in the following:

(1) A formula which gives the number of the different autocorrelation functions for all possible sequences of length n.

(2) A computer program which can generate automatically all sequences of length n and give their different autocorrelation functions.

Some application of these codes were also considered.

There are some suggestions for further research.

(1) Supplementary and cyclic complementary sequences and their properties could be further investigated and applications developed.

(2) Applications of the sequences presented here could be implemented with hardware.

APPENDIX A

COMPUTER OUTPUTS AND PROGRAMS

Six computer programs are included in this report and seven computer outputs which are the results of the search for the "good" codes. In the programs, the sequence length is denoted by L.

Program 1 computes the autocorrelation function of one sequence, by comparison of the elements of the original sequence and its shifted replica. The length L of the sequence and the sequence (CODE (I)) have to be punched on separate cards.

Program 2 computes the autocorrelation function of any number of sequences of the same length. The number of the sequences has to be specified in the loop DO 150. The length of the sequences and each of the different sequences have to be punched on separate cards.

Program 3 generates all the possible sequences of length L and computes their autocorrelation function. Only the length L has to be punched on a card. Everything else is done automatically. Thus, punching of the sequences on cards is avoided.

Program 4 computes only the different autocorrelation functions of all sequences of length L. This is done by the algorithm explained in Section VI. The program produces

everything automatically and only the length L has to be specified and punched.

Program 5 computes the different autocorrelation functions of all sequences of length L which have sidelobes less or equal to one. The length L has to be specified.

Program 6 searches only a region of all sequences of length L and computes the different autocorrelation functions with sidelobes less or equal to one which exist in this region. Here the length L and the starting sequence (SCODE (I)) have to be specified and punched on different cards each. All the sequences before the starting sequence are ignored. This was done because for long sequences, a complete computer search is impractical.

There is a computer output which lists the different autocorrelation functions for all sequences of length 10. The rest of the outputs list the "good" codes and their autocorrelation functions for lengths 10, 11, 12, 13, 17 and 20. The codes listed in all computer outputs are printed with the most significant bit on the right.

PRINCIPAL VARIABLES USED

CODE	binary sequence
COR	autocorrelation function of a binary sequence
L	length of a binary sequence
ICODE	variable used to complement a binary sequence
JCODE	variable used to reverse a binary sequence
SCODE	starting sequence when a region of the binary sequence of length L is searched

LIST OF ALL DIFFERENT AUTOCORRELATION FUNCTIONS
FOR ALL SEQUENCES OF LENGTH 10

SEQUENCE										AUTOCORRELATION										
0	0	0	0	0	0	0	0	0	0	10	9	8	7	6	5	4	3	2	1	0
1	0	0	0	0	0	0	0	0	0	10	7	6	5	4	3	2	1	0	-1	0
0	1	0	0	0	0	0	0	0	0	10	5	6	5	4	3	2	1	0	1	0
1	1	0	0	0	0	0	0	0	0	10	7	4	3	2	1	0	-1	-2	-1	0
0	0	1	0	0	0	0	0	0	0	10	5	4	5	4	3	2	1	2	1	0
1	0	1	0	0	0	0	0	0	0	10	3	6	3	2	1	0	-1	0	-1	0
0	1	1	0	0	0	0	0	0	0	10	5	2	3	2	1	0	-1	0	1	0
1	1	1	0	0	0	0	0	0	0	10	7	4	1	0	-1	-2	-3	-2	-1	0
0	0	0	1	0	0	0	0	0	0	10	5	4	3	4	3	2	3	2	1	0
1	0	0	1	0	0	0	0	0	0	10	3	2	5	2	1	0	1	0	-1	0
0	1	0	1	0	0	0	0	0	0	10	1	6	1	2	1	0	1	0	1	0
1	1	0	1	0	0	0	0	0	0	10	3	4	3	0	-1	-2	-1	-2	-1	0
0	0	1	1	0	0	0	0	0	0	10	5	0	1	2	1	0	1	2	1	0
1	0	1	1	0	0	0	0	0	0	10	3	2	3	0	-1	-2	-1	0	-1	0
0	1	1	1	0	0	0	0	0	0	10	5	2	-1	0	-1	-2	-1	0	1	0
1	1	1	1	0	0	0	0	0	0	10	7	4	1	-2	-3	-4	-3	-2	-1	0
0	0	0	0	1	0	0	0	0	0	10	5	4	3	2	3	4	3	2	1	0
1	0	0	0	1	0	0	0	0	0	10	3	2	1	4	1	2	1	0	-1	0
0	1	0	0	1	0	0	0	0	0	10	1	2	5	0	1	2	1	0	1	0
1	1	0	0	1	0	0	0	0	0	10	3	0	3	2	-1	0	-1	-2	-1	0

0	0	1	0	1	0	0	0	0	0	10	1	4	1	0	1	2	1	2	1	0
1	0	1	0	1	0	0	0	0	0	10	-1	6	-1	2	-1	0	-1	0	-1	0
0	1	1	0	1	0	0	0	0	0	10	1	2	3	-2	-1	0	-1	0	1	0
1	1	1	0	1	0	0	0	0	0	10	3	4	1	0	-3	-2	-3	-2	-1	0
0	0	0	1	1	0	0	0	0	0	10	5	0	-1	0	1	2	3	2	1	0
1	0	0	1	1	0	0	0	0	0	10	3	-2	1	2	-1	0	1	0	-1	0
0	1	0	1	1	0	0	0	0	0	10	1	2	1	-2	-1	0	1	0	1	0
1	1	0	1	1	0	0	0	0	0	10	3	0	3	0	-3	-2	-1	-2	-1	0
0	0	1	1	1	0	0	0	0	0	10	5	0	-3	-2	-1	0	1	2	1	0
1	0	1	1	1	0	0	0	0	0	10	3	2	-1	0	-3	-2	-1	0	-1	0
0	1	1	1	1	0	0	0	0	0	10	5	2	-1	-4	-3	-2	-1	0	1	0
1	1	1	1	1	0	0	0	0	0	10	7	4	1	-2	-5	-4	-3	-2	-1	0
1	0	0	0	0	1	0	0	0	0	10	3	2	1	0	5	2	1	0	-1	0
0	1	0	0	0	1	0	0	0	0	10	1	2	1	4	1	2	1	0	1	0
1	1	0	0	0	1	0	0	0	0	10	3	0	-1	2	3	0	-1	-2	-1	0
0	0	1	0	0	1	0	0	0	0	10	1	0	5	0	1	2	1	2	1	0
1	0	1	0	0	1	0	0	0	0	10	-1	2	3	-2	3	0	-1	0	-1	0
0	1	1	0	0	1	0	0	0	0	10	1	-2	3	2	-1	0	-1	0	1	0
1	1	1	0	0	1	0	0	0	0	10	3	0	1	0	1	-2	-3	-2	-1	0
0	0	0	1	0	1	0	0	0	0	10	1	4	-1	0	1	2	3	2	1	0
1	0	0	1	0	1	0	0	0	0	10	-1	2	1	-2	3	0	1	0	-1	0
0	1	0	1	0	1	0	0	0	0	10	-3	6	-3	2	-1	0	1	0	1	0
1	1	0	1	0	1	0	0	0	0	10	-1	4	-1	0	1	-2	-1	-2	-1	0
0	0	1	1	0	1	0	0	0	0	10	1	0	1	-2	-1	0	1	2	1	0

1	0	1	1	0	1	0	0	0	0	10	-1	2	3	-4	1	-2	-1	0	-1	0
0	1	1	1	0	1	0	0	0	0	10	1	2	-1	0	-3	-2	-1	0	1	0
1	1	1	1	0	1	0	0	0	0	10	3	4	1	-2	-1	-4	-3	-2	-1	0
0	0	0	0	1	1	0	0	0	0	10	5	0	-1	-2	1	4	3	2	1	0
1	0	0	0	1	1	0	0	0	0	10	3	-2	-3	0	3	2	1	0	-1	0
0	1	0	0	1	1	0	0	0	0	10	1	-2	1	0	-1	2	1	0	1	0
1	1	0	0	1	1	0	0	0	0	10	3	-4	-1	2	1	0	-1	-2	-1	0
0	0	1	0	1	1	0	0	0	0	10	1	0	1	-4	-1	2	1	2	1	0
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0	1	1	1	0	1	0	0	1	0	10	-3	0	1	-2	-1	0	1	-2	1	0
1	0	0	0	1	1	0	0	1	0	10	-1	-4	-1	2	1	0	-1	2	-1	0
0	1	0	0	1	1	0	0	1	0	10	-3	-4	3	2	-3	0	3	-2	1	0
1	0	1	0	1	1	0	0	1	0	10	-5	0	1	0	-1	2	-3	2	-1	0
0	1	1	0	1	1	0	0	1	0	10	-3	-4	5	0	-5	2	1	-2	1	0
1	0	0	1	1	1	0	0	1	0	10	-1	-4	-1	0	3	-2	-1	2	-1	0
0	1	0	1	1	1	0	0	1	0	10	-3	0	-1	0	-1	-2	3	-2	1	0
0	1	1	1	1	1	0	0	1	0	10	1	0	1	-2	-3	0	1	-2	1	0
1	0	0	0	0	0	1	0	1	0	10	-1	4	-1	0	-1	2	-1	2	-1	0
1	0	1	0	0	0	1	0	1	0	10	-5	4	-3	2	-3	4	-3	2	-1	0
0	1	1	0	0	0	1	0	1	0	10	-3	0	-3	2	1	0	1	-2	1	0
1	0	0	1	0	0	1	0	1	0	10	-5	0	3	-2	1	0	-1	2	-1	0
0	1	0	1	0	0	1	0	1	0	10	-7	4	-1	-2	5	-4	3	-2	1	0
0	1	1	1	0	0	1	0	1	0	10	-3	0	-3	0	3	-2	1	-2	1	0

1	0	0	0	1	0	1	0	1	0	10	-5	4	-5	4	-3	2	-1	2	-1	0
1	0	1	0	1	0	1	0	1	0	10	-9	8	-7	6	-5	4	-3	2	-1	0
0	1	1	0	1	0	1	0	1	0	10	-7	4	-3	2	-1	0	1	-2	1	0
1	0	0	1	1	0	1	0	1	0	10	-5	0	-1	2	-1	0	-1	2	-1	0
0	1	0	1	1	0	1	0	1	0	10	-7	4	-1	-2	3	-4	3	-2	1	0
0	1	1	1	1	0	1	0	1	0	10	-3	4	-3	0	1	-2	1	-2	1	0
1	0	0	0	0	1	1	0	1	0	10	-1	0	-1	-4	1	2	-1	2	-1	0
1	0	1	0	0	1	1	0	1	0	10	-5	0	1	-2	-1	4	-3	2	-1	0
0	1	1	0	0	1	1	0	1	0	10	-3	-4	1	2	-1	0	1	-2	1	0
1	0	0	1	0	1	1	0	1	0	10	-5	0	3	-6	3	0	-1	2	-1	0
0	1	1	1	0	1	1	0	1	0	10	-3	0	1	0	1	-2	1	-2	1	0
1	0	0	0	1	1	1	0	1	0	10	-1	0	-5	0	-1	2	-1	2	-1	0
0	1	1	0	1	1	1	0	1	0	10	-3	0	1	2	-3	0	1	-2	1	0
1	0	0	1	1	1	1	0	1	0	10	-1	0	-1	-2	1	0	-1	2	-1	0
0	1	0	1	1	1	1	0	1	0	10	-3	4	-1	-2	1	-4	3	-2	1	0
0	1	1	1	1	1	1	0	1	0	10	1	4	1	0	-1	-2	1	-2	1	0
1	0	0	0	0	0	0	1	1	0	10	3	0	1	0	-1	-2	1	2	-1	0
0	1	1	0	0	0	0	1	1	0	10	1	-4	-1	-2	1	4	-1	-2	1	0
1	0	0	1	0	0	0	1	1	0	10	-1	-4	1	2	1	-4	1	2	-1	0
0	1	1	1	0	0	0	1	1	0	10	1	-4	-5	0	3	2	-1	-2	1	0
1	0	0	0	1	0	0	1	1	0	10	-1	-4	1	4	-3	-2	1	2	-1	0
0	1	1	0	1	0	0	1	1	0	10	-3	-4	3	-2	-1	4	-1	-2	1	0
1	0	0	1	1	0	0	1	1	0	10	-1	-8	1	6	-1	-4	1	2	-1	0
0	1	1	1	1	0	0	1	1	0	10	1	-4	-1	0	1	2	-1	-2	1	0

1	0	0	0	0	1	0	1	1	0	10	-1	0	1	-4	1	-2	1	2	-1	0
1	0	0	1	0	1	0	1	1	0	10	-5	0	1	-2	3	-4	1	2	-1	0
0	1	1	1	0	1	0	1	1	0	10	-3	0	-1	0	1	2	-1	-2	1	0
1	0	0	0	1	1	0	1	1	0	10	-1	-4	1	0	-1	-2	1	2	-1	0
0	1	1	0	1	1	0	1	1	0	10	-3	-4	7	-2	-3	4	-1	-2	1	0
0	1	1	1	1	1	0	1	1	0	10	1	0	3	0	-1	2	-1	-2	1	0
1	0	0	0	0	0	1	1	1	0	10	3	0	-3	-2	-3	0	1	2	-1	0
0	1	1	1	0	0	1	1	1	0	10	1	-4	-5	2	5	0	-1	-2	1	0
1	0	0	0	1	0	1	1	1	0	10	-1	0	-3	2	-5	0	1	2	-1	0
0	1	1	1	1	0	1	1	1	0	10	1	0	-1	2	3	0	-1	-2	1	0
1	0	0	0	0	1	1	1	1	0	10	3	0	-3	-6	-1	0	1	2	-1	0
0	1	1	1	1	1	1	1	1	0	10	5	4	3	2	1	0	-1	-2	1	0

NOTE: THERE ARE 272 DIFFERENT AUTOCORRELATION FUNCTIONS

```
LIST OF THE DIFFERENT AUTOCORRELATION FUNCTIONS
OF ALL SEQUENCES OF LENGTH 10
WITH SIDELOBES LESS OR EQUAL TO 1

THE FIRST LINE IS THE SEQUENCE
THE SECOND IS THE AUTOCORRELATION

 0  1  0  0  1  1  1  0  0  0
10  1 -2 -3 -2  1  0  1  0  1  0

 0  1  1  0  1  1  1  0  0  0
10  1 -2 -1  0 -1 -2 -1  0  1  0

 0  1  1  1  1  0  0  1  0  0
10  1 -2  1 -2 -1  0 -3  0  1  0

 0  1  1  1  1  0  1  1  0  0
10  1 -2  1  0  1 -2 -3  0  1  0

 1  0  1  1  0  1  1  1  0  0
10 -1 -2  1  0  1 -2  1  0 -1  0

 0  1  1  0  1  1  1  1  0  0
10  1 -2  1 -2  1  0 -3  0  1  0

 0  1  1  1  0  0  0  0  1  0
10  1  0 -3 -2  1  0  1 -2  1  0

 0  1  1  1  1  0  0  0  1  0
```

10	1	0	-3	-2	-1	0	1	-2	1	0
0	1	1	1	0	1	0	0	1	0	
10	-3	0	1	-2	-1	0	1	-2	1	0
0	1	1	1	1	1	0	0	1	0	
10	1	0	1	-2	-3	0	1	-2	1	0
0	1	1	1	0	1	1	0	1	0	
10	-3	0	1	0	1	-2	1	-2	1	0

LIST OF ALL DIFFERENT AUTOCORRELATION FUNCTIONS
OF ALL SEQUENCES OF LENGTH 11
WITH SIDELOBES LESS OR EQUAL TO 1

THE FIRST LINE IS THE SEQUENCE
THE SECOND THE AUTOCORRELATION

```
 1  0  1  1  0  1  1  1  0  0  0
11  0 -1  0 -1  0 -1  0 -1  0 -1  0
```

NOTE: ONLY ONE SEQUENCE WAS FOUND

LIST OF THE DIFFERENT AUTOCORRELATION FUNCTIONS
OF ALL SEQUENCES OF LENGTH 12
WITH SIDELOBES LESS OR EQUAL TO 1

THE FIRST LINE IS THE SEQUENCE
THE SECOND IS THE AUTOCORRELATION

1	1	0	0	1	1	0	1	0	0	0	0	
12	1	-2	1	0	-1	0	1	0	-1	-2	-1	0
1	0	0	1	1	1	0	1	0	0	0	0	
12	1	0	-1	0	-1	-4	1	0	1	0	-1	0
1	0	1	1	0	1	1	1	0	0	0	0	
12	1	0	1	-2	-1	-2	1	-2	-1	0	-1	0
0	1	1	1	0	1	0	0	1	0	0	0	
12	-1	0	1	0	-1	0	1	-4	-1	0	1	0
1	0	1	1	1	1	0	0	1	0	0	0	
12	1	0	1	0	1	-2	-5	0	-1	0	-1	0
0	1	1	1	0	1	1	0	1	0	0	0	
12	-1	0	1	0	-1	-2	1	-4	-1	0	1	0
0	1	0	0	1	0	1	1	1	0	0	0	
12	-1	0	-1	-4	-1	0	1	0	1	0	1	0
0	1	0	1	1	0	1	1	1	0	0	0	

12	-1	0	-1	-2	1	-2	-1	-2	1	0	1	0
1	1	0	1	1	0	1	1	1	0	0	0	
12	1	-2	1	0	-1	0	1	0	-1	-2	-1	0
1	0	1	1	0	1	1	1	1	0	0	0	
12	1	0	1	-4	1	0	-1	0	-1	0	-1	0
1	0	1	1	1	0	0	0	0	1	0	0	
12	1	0	-1	0	1	0	-1	-4	1	0	-1	0
1	0	1	1	1	1	0	0	0	1	0	0	
12	1	0	-1	0	1	-2	-1	-4	1	0	-1	0
0	1	0	1	0	1	1	0	1	1	0	0	
12	-5	0	1	0	-1	-2	1	0	-1	0	1	0
0	1	0	1	1	1	1	0	1	1	0	0	
12	-1	0	1	0	1	-4	-1	0	-1	0	1	0
0	1	1	1	0	0	0	1	0	0	1	0	
12	-1	-2	-1	0	1	0	-1	0	1	-2	1	0
0	1	1	0	0	1	1	1	1	0	1	0	
12	-1	-2	-1	0	1	0	-1	0	1	-2	1	0

LIST OF THE DIFFERENT AUTOCORRELATION FUNCTIONS
OF ALL SEQUENCES OF LENGTH 13
WITH SIDELOBES LESS OR EQUAL TO 1

THE FIRST LINE IS THE SEQUENCE
THE SECOND IS THE AUTOCORRELATION

0	1	0	1	0	0	1	1	0	0	0	0	0	
13	0	1	0	1	0	1	0	1	0	1	0	1	0
0	1	0	1	1	1	0	0	1	0	0	0	0	
13	0	1	0	1	0	-3	0	-3	0	1	0	1	0
1	0	0	1	1	0	1	0	1	0	0	0	0	
13	-2	1	0	1	0	-1	-2	1	0	1	0	-1	0
1	0	1	0	1	0	0	1	1	0	0	0	0	
13	-2	1	-2	1	0	1	0	1	0	-1	0	-1	0
0	1	0	1	1	0	0	1	1	0	0	0	0	
13	0	-3	0	1	0	1	0	-3	0	1	0	1	0
0	1	0	1	1	1	0	1	1	0	0	0	0	
13	0	1	0	1	-4	-1	-2	-3	0	1	0	1	0
0	1	1	0	1	0	1	1	1	0	0	0	0	
13	0	1	-2	-3	0	1	0	-3	0	-1	0	1	0
0	1	0	1	1	0	1	1	1	0	0	0	0	

13	0	1	0	-3	0	-3	0	-3	0	1	0	1	0
0	1	0	1	1	1	1	0	0	1	0	0	0	
13	0	1	0	-1	0	-3	-4	1	-2	1	0	1	0
0	1	1	1	0	0	1	1	0	1	0	0	0	
13	0	-3	-2	1	0	1	0	1	-4	-1	0	1	0
0	1	0	1	0	1	1	0	1	1	0	0	0	
13	-4	1	0	-1	0	-3	0	1	-2	1	0	1	0
0	1	0	0	1	1	1	0	1	1	0	0	0	
13	0	-3	0	1	0	-7	0	1	0	1	0	1	0
0	1	0	0	1	0	0	1	1	1	0	0	0	
13	0	-3	0	-3	0	1	0	1	0	1	0	1	0
0	1	0	0	1	1	0	1	1	1	0	0	0	
13	0	-3	0	1	-4	-1	-2	1	0	1	0	1	0
0	1	0	1	0	0	1	1	1	1	0	0	0	
13	0	1	-4	-5	0	1	0	1	-2	1	0	1	0
0	1	0	0	1	0	1	1	1	1	0	0	0	
13	0	1	0	-7	0	-3	0	1	0	1	0	1	0
0	1	1	1	0	1	0	0	0	0	1	0	0	
13	0	1	0	1	0	-3	0	1	0	-3	0	1	0

0	1	1	1	1	0	1	0	0	0	1	0	0	
13	0	1	0	1	-2	-3	0	-1	0	-3	0	1	0
0	1	1	1	0	0	0	1	0	0	1	0	0	
13	0	-3	0	1	0	1	0	1	0	-3	0	1	0
0	1	1	1	0	1	0	1	0	0	1	0	0	
13	-4	1	0	1	0	-1	-2	1	0	-3	0	1	0
1	0	1	0	1	1	1	1	0	0	1	0	0	
13	-2	1	0	-1	0	-1	-2	1	-2	1	0	-1	0
0	1	1	1	1	1	0	0	1	0	1	0	0	
13	0	1	0	-3	0	1	0	-3	0	-3	0	1	0
1	0	1	1	1	0	0	1	1	0	1	0	0	
13	-2	-3	0	1	0	1	0	1	-4	1	0	-1	0
0	1	1	1	1	0	1	1	1	0	1	0	0	
13	0	1	0	1	0	1	0	-3	0	-3	0	1	0
0	1	1	0	1	1	0	0	1	1	1	0	0	
13	0	-7	0	1	0	1	0	1	0	-3	0	1	0
1	0	1	0	1	0	0	1	1	1	1	0	0	
13	-2	1	-4	-3	0	1	0	1	0	1	0	-1	0
0	1	1	0	1	0	1	1	1	1	1	0	0	

13	0	1	0	-3	0	1	0	1	0	-3	0	1	0
0	1	1	1	0	1	1	0	0	0	0	1	0	
13	0	-1	0	1	0	-3	-4	1	0	1	-2	1	0
0	1	1	1	1	0	0	1	0	0	0	1	0	
13	0	-1	0	1	-2	-1	-2	-1	0	1	-2	1	0
0	1	1	1	1	0	0	1	1	0	0	1	0	
13	0	-5	0	1	0	1	0	-3	0	1	-2	1	0
0	1	1	0	0	1	1	1	1	1	0	1	0	
13	0	-1	0	1	-2	1	0	-1	0	1	-2	1	0

LIST OF ALL DIFFERENT AUTOCORRELATION
FUNCTIONS OF THE SEQUENCES OF LENGTH 17
WITH SIDELOBES LESS OR EQUAL TO 1

THE FIRST LINE IS THE SEQUENCE
THE SECOND LINE IS THE AUTOCORRELATION

```
 0  1  0  1  1  1  0  0  1  1  0  1  0  0  0  0  0
17  0  1  0  1 -2  1  0  1 -4  1 -4 -1  0  1  0  1  0

 0  1  0  1  0  0  1  1  1  0  1  1  0  0  0  0  0
17  0  1  0  1  0 -7  0 -3  0  1  0  1  0  1  0  1  0

 1  0  1  0  1  1  1  0  0  1  1  0  1  0  0  0  0
17 -2  1 -2  1  0  1  0  1 -4  1 -4  1  0 -1  0 -1  0

 0  1  0  1  1  1  1  0  0  1  1  0  1  0  0  0  0
17  0  1  0 -3  0  1  0  1  0 -7  0 -3  0  1  0  1  0

 0  1  0  1  0  1  0  0  1  1  0  1  1  0  0  0  0
17 -4  1  0  1 -6  1  0  1  0  1  0 -1  0  1  0  1  0

 1  0  1  0  1  0  1  1  0  0  1  1  1  0  0  0  0
17 -2  1 -6  1  0  1  0  1 -4  1  0  1  0 -1  0 -1  0

 0  1  0  1  1  0  1  1  0  0  1  1  1  0  0  0  0
17  0 -3  0 -3  0  1  0  1  0 -3  0 -3  0  1  0  1  0

 0  1  1  0  1  0  1  1  1  0  1  1  1  0  0  0  0
```

17 0 1 -2 1 0 1 -4 -3 0 1 0 -3 0 -1 0 1 0

0 1 0 1 1 0 1 1 1 0 1 1 1 0 0 0 0
17 0 1 0 1 0 -3 0 -3 0 -3 0 -3 0 1 0 1 0

0 1 0 1 0 1 1 0 0 1 1 1 1 0 0 0 0
17 0 1 -4 -3 -2 1 0 1 0 -3 0 -1 0 1 0 1 0

0 1 1 1 0 1 0 1 1 1 1 0 0 1 0 0 0
17 0 1 -2 1 0 1 -2 1 -4 -1 0 1 -4 -1 0 1 0

0 1 1 1 1 1 0 0 1 1 0 1 0 1 0 0 0
17 0 1 -2 1 -2 1 0 1 0 1 -4 -1 -4 -1 0 1 0

0 1 0 1 1 1 0 0 1 1 1 1 0 1 0 0 0
17 0 1 -4 -1 0 -1 0 1 0 1 -6 1 -2 1 0 1 0

0 1 0 0 1 1 0 1 1 0 0 0 1 1 0 0 0
17 0 -7 0 1 0 1 0 1 0 -3 0 1 0 1 0 1 0

0 1 0 0 1 1 1 1 1 0 1 0 1 1 0 0 0
17 0 1 0 -3 0 1 0 -3 0 -7 0 1 0 1 0 1 0

0 1 0 0 1 0 0 1 1 0 0 1 1 1 0 0 0
17 0 -7 0 1 0 1 0 -3 0 1 0 1 0 1 0 1 0

0 1 0 0 1 0 0 0 1 1 0 1 1 1 0 0 0
17 0 -3 0 1 0 -7 0 1 0 1 0 1 0 1 0 1 0

0	1	0	1	0	1	1	0	1	1	0	1	1	1	0	0	0	
17	-4	1	0	-1	0	-1	0	1	-4	1	-2	1	-2	1	0	1	0
0	1	0	0	1	0	0	1	1	1	0	1	1	1	0	0	0	
17	0	-3	0	1	0	-3	-4	-1	-2	1	0	1	0	1	0	1	0
0	1	0	1	0	0	1	1	0	0	1	1	1	1	0	0	0	
17	0	-3	-4	-1	0	1	0	-3	0	1	0	1	-2	1	0	1	0
0	1	0	0	1	0	1	1	1	0	1	1	1	1	0	0	0	
17	0	1	0	-3	0	1	0	-7	0	-3	0	1	0	1	0	1	0
0	1	0	1	0	0	1	0	0	1	1	1	1	1	0	0	0	
17	0	1	0	-5	-4	-3	0	1	0	1	0	1	-2	1	0	1	0
0	1	0	0	1	0	1	0	0	1	1	1	1	1	0	0	0	
17	0	1	0	-7	0	-3	0	1	0	-3	0	1	0	1	0	1	0
0	1	0	1	0	0	1	1	0	1	1	1	1	1	0	0	0	
17	0	1	0	-1	-4	1	-4	-1	-2	1	0	1	-2	1	0	1	0
0	1	1	1	1	1	0	0	0	1	0	1	0	0	1	0	0	
17	0	1	0	-3	-2	1	0	1	0	1	-4	-1	0	-3	0	1	0
1	0	1	1	1	0	1	1	0	0	0	0	1	0	1	0	0	
17	-2	1	0	1	0	-1	0	1	-4	1	-2	1	-4	1	0	-1	0
0	1	1	1	1	1	0	1	1	0	0	0	1	0	1	0	0	

17	0	1	0	1	0	1	0	-7	0	1	0	-3	0	-3	0	1	0
0	1	1	1	1	1	0	0	0	1	0	0	1	0	1	0	0	
17	0	1	0	-3	0	-3	0	1	0	1	0	-3	0	-3	0	1	0
0	1	1	1	1	1	0	0	1	1	0	0	1	0	1	0	0	
17	0	-3	0	1	0	1	0	-3	0	1	0	-3	0	-3	0	1	0
1	0	1	1	1	1	1	0	0	1	1	0	1	0	1	0	0	
17	-2	1	0	1	0	1	0	1	0	1	-4	1	-4	1	0	-1	0
0	1	0	1	0	1	1	1	1	1	0	0	0	1	1	0	0	
17	0	1	-6	1	0	1	-2	1	-4	-1	0	1	0	-1	0	1	0
0	1	0	1	0	1	0	0	1	1	1	1	0	1	1	0	0	
17	-4	1	-2	1	0	-1	-4	1	0	1	-2	1	0	-1	0	1	0
0	1	1	0	1	1	0	1	0	0	0	0	1	1	1	0	0	
17	0	-3	0	-7	0	1	0	1	0	1	0	1	0	-3	0	1	0
1	0	1	0	1	0	1	0	0	1	0	0	1	1	1	0	0	
17	-6	1	-4	1	0	-1	0	1	0	1	-2	1	0	1	0	-1	0
1	0	1	0	1	0	0	1	1	0	0	1	1	1	1	0	0	
17	-2	-3	-4	1	0	1	0	-3	0	1	0	1	0	1	0	-1	0
0	1	0	1	0	1	0	0	1	1	0	1	1	1	1	0	0	
17	-4	1	-2	1	-2	1	-4	1	0	1	0	-1	0	-1	0	1	0

0	1	1	0	1	0	1	1	0	0	1	1	1	1	1	0	0	
17	0	-3	0	-3	0	1	0	1	0	1	0	1	0	-3	0	1	0
0	1	1	1	0	0	1	1	1	1	0	1	0	0	0	1	0	
17	0	-1	-4	1	0	1	-2	1	0	-1	-4	1	0	1	-2	1	0
0	1	1	1	0	1	0	1	1	0	1	1	0	0	0	1	0	
17	-4	-1	0	1	0	-1	0	1	0	1	-6	1	0	1	-2	1	0
0	1	1	0	0	1	0	0	1	1	1	1	0	1	0	1	0	
17	-4	-1	0	1	-4	-1	0	1	0	1	-2	1	0	1	-2	1	0

PARTIAL LIST OF THE DIFFERENT AUTOCORRELATION FUNCTIONS
OF THE SEQUENCES OF LENGTH 20
WITH SIDELOBES LESS OR EQUAL TO 1

THE FIRST LINE IS THE SEQUENCE
THE SECOND THE AUTOCORRELATION

```
 1  0  0  1  1  1  0  0  1  0  1  1  1  0  1  0  0  0  0  0
20  1  0 -1  0 -3  0 -1  0 -1  0  1  0 -3  0 -1  0  1  0 -1  0

 0  1  1  0  1  1  0  0  1  1  1  1  0  1  0  1  0  0  0  0
20 -1  0  1  0 -1  0  1 -4 -1 -2  1  0 -1  0 -3  0 -1  0  1  0

 1  0  1  1  0  1  0  0  1  1  1  1  1  0  0  0  1  0  0  0
20  1  0  1 -4 -1  0 -1  0 -1 -2  1  0  1  0 -3  0 -1  0 -1  0
```

NOTE: THE SEARCH DID NOT EXHAUST ALL THE SEQUENCES,
DUE TO THE GREAT TIME REQUIRED.
ONLY THREE SEQUENCES WERE FOUND

PROGRAM #1

COMPUTATION OF THE AUTOCORRELATION FUNCTION
OF ONE BINARY SEQUENCE

```
      INTEGER CODE(50),COR(50)
      READ(5,10)L
   10 FORMAT(I2)
      READ(5,1000) (CODE(I),I=1,48)
 1000 FORMAT(50I1)
      DO 40 I=1,L
      N=0
      DO 30 J=I,L
      IF(CODE(J).EQ.CODE(J-(I-1))) GO TO 20
      N=N-1
      GO TO 30
   20 N=N+1
   30 CONTINUE
   40 COR(I)=N
      COR(L+1)=0
      K=L+1
      WRITE(6,100) (CODE(I),I=1,L)
      WRITE(6,100) (COR(I),I=1,K)
  100 FORMAT(//,' ',30(I3,1X))
      END
```

PROGRAM #2

COMPUTATION OF THE AUTOCORRELATION FUNCTION
OF M BINARY SEQUENCES

```
      INTEGER CODE(50),COR(50)
      READ(5,10)L
   10 FORMAT(I2)
      DO 150 M=1,4
      READ(5,1000) (CODE(I),I=1,48)
 1000 FORMAT(50I1)
      DO 40 I=1,L
      N=0
      DO 30 J=I,L
      IF(CODE(J).EQ.CODE(J-(I-1))) GO TO 20
      N=N-1
      GO TO 30
   20 N=N+1
   30 CONTINUE
   40 COR(I)=N
      COR(L+1)=0
      K=L+1
      WRITE(6,100) (CODE(I),I=1,L)
      WRITE(6,100) (COR(I),I=1,K)
  100 FORMAT(//,' ',30(I3,1X))
  150 CONTINUE
      END
```

```
      PROGRAM #3

      AUTOMATIC PRODUCTION OF ALL SEQUENCES
      OF LENGTH L AND COMPUTATION OF
      THEIR AUTOCORRELATION FUNCTIONS

      INTEGER CODE(20),COR(20)
      READ(5,10)L
   10 FORMAT(I2)
      NN=2**L

      GENERATE SEQUENCES

      DO 100 II=1,NN
      DO 20 I=1,20
   20 CODE(I)=0
      NI=II-1
      DO 30 M=1,L
      JJ=L-M
      IF(NI.LT.2**JJ) GO TO 30
      CODE(JJ+1)=1
      NI=NI-2**JJ
   30 CONTINUE

      COMPUTE AUTOCORRELATIONS

      DO 60 I=1,L
      N=0
      DO 50 J=I,L
      IF(CODE(J).EQ.CODE(J-(I-1))) GO TO 40
      N=N-1
      GO TO 50
   40 N=N+1
   50 CONTINUE
   60 COR(I)=N
      COR(L+1)=0
      K=L+1
      WRITE(6,200) (CODE(I),I=1,L)
      WRITE(6,200) (COR(I),I=1,K)
  100 CONTINUE
  200 FORMAT(//,' ',20(I2,2X))
      END
```

```
      PROGRAM #4

      AUTOMATIC CALCULATION OF THE DIFFERENT
      AUTOCORRELATION FUNCTIONS IN ALL POSSIBLE
      SEQUENCES OF LENGTH L

      INTEGER CODE(20),COR(21),ICODE(20),JCODE(20)
      READ(5,10)L
   10 FORMAT(I2)
      NN=2**(L-1)

      GENERATE SEQUENCES

      DO 100 II=1,NN
      DO 20 I=1,20
   20 CODE(I)=0
      NI=II-1
      DO 30 M=1,L
      JJ=L-M
      IF(NI.LT.2**JJ) GO TO 30
      CODE(JJ+1)=1
      NI=NI-2**JJ
   30 CONTINUE

      TEST IF SEQUENCE ENDS IN 0 OR 1

      DO 31 M=1,L
   31 ICODE(M)=CODE(M)
      IF(CODE(1).EQ.0) GO TO 33

      COMPLEMENT SEQUENCE

      DO 32 M=1,L
      ICODE(M)=ICODE(M)+1
      IF(ICODE(M).GT.1) ICODE(M)=0
   32 CONTINUE

      REVERSE SEQUENCE

   33 DO 34 M=1,L
   34 JCODE(L-M+1)=ICODE(M)
      DO 35 M=1,L
      IF(JCODE(L-M+1)-CODE(L-M+1))100,35,36
   35 CONTINUE
   36 CONTINUE

      COMPUTE AUTOCORRELATION

      DO 60 I=1,L
      N=0
      DO 50 J=I,L
      IF(CODE(J).EQ.CODE(J-(I-1))) GO TO 40
      N=N-1
      GO TO 50
   40 N=N+1
   50 CONTINUE
   60 COR(I)=N
      COR(L+1)=0
      K=L+1
   70 WRITE(6,200) (CODE(I),I=1,L),(COR(I),I=1,K)
  100 CONTINUE
  200 FORMAT(/,' ',41(I2,1X))
      END
```

```
      PROGRAM #5

      COMPUTATION OF THE DIFFERENT
      AUTOCORRELATION FUNCTIONS
      OF ALL SEQUENCES OF LENGTH L
      WITH SIDELOBES LESS OR EQUAL TO 1

      INTEGER CODE(20),COR(21),ICODE(20),JCODE(20)
      READ(5,10)L
   10 FORMAT(I2)
      NN=2**(L-1)

      GENERATE SEQUENCES

      DO 100 II=1,NN
      DO 20 I=1,20
   20 CODE(I)=0
      NI=II-1
      DO 30 M=1,L
      JJ=L-M
      IF(NI.LT.2**JJ) GO TO 30
      CODE(JJ+1)=1
      NI=NI-2**JJ
   30 CONTINUE

      TEST IF SEQUENCE ENDS IN 0 OR 1

      DO 31 M=1,L
   31 ICODE(M)=CODE(M)
      IF(CODE(1).EQ.0) GO TO 33

      COMPLEMENT SEQUENCE

      DO 32 M=1,L
      ICODE(M)=ICODE(M)+1
      IF(ICODE(M).GT.1) ICODE(M)=0
   32 CONTINUE

      REVERSE SEQUENCE

   33 DO 34 M=1,L
   34 JCODE(L-M+1)=ICODE(M)
      DO 35 M=1,L
      IF(JCODE(L-M+1)-CODE(L-M+1))100,35,36
   35 CONTINUE
   36 CONTINUE

      COMPUTE AUTOCORRELATION

      DO 60 I=1,L
      N=0
      DO 50 J=I,L
      IF(CODE(J).EQ.CODE(J-(I-1))) GO TO 40
      N=N-1
      GO TO 50
   40 N=N+1
   50 CONTINUE

      KEEP SEQUENCES WITH
      SIDELOBES LESS OR EQUAL TO 1

      IF(N.EQ.L) GO TO 55
      IF(N.GT.1) GO TO 100
   55 COR(I)=N
   60 COR(I)=N
      COR(L+1)=0
      K=L+1
   70 WRITE(6,200) (CODE(I),I=1,L),(COR(I),I=1,K)
  100 CONTINUE
  200 FORMAT(/,' ',41(I2,1X))
      END
```

```
      PROGRAM #6

      COMPUTATION OF DIFFERENT
      AUTOCORRELATION FUNCTIONS FOR A
      REGION OF THE SEQUENCES OF LENGTH L
      WITH SIDELOBES LESS OR EQUAL TO 1

      THE SEARCH STARTS AT THE
      SEQUENCE SCODE(I),I=1,L

      INTEGER CODE(20),COR(21),ICODE(20),JCODE(20),SCODE(20)
      READ(5,10)L
   10 FORMAT(I2)

      READ STARTING SEQUENCE

      READ(5,11) (SCODE(I),I=1,L)
   11 FORMAT(20I1)
      ISTART=0
      DO 12 I=1,L
   12 ISTART=ISTART+SCODE(I)*2**(I-1)
      NN=2**(L-1)-1

      GENERATE SEQUENCES

      DO 100 II=ISTART,NN
      DO 20 I=1,20
   20 CODE(I)=0
      NI=II
      DO 30 M=1,L
      JJ=L-M
      IF(NI.LT.2**JJ) GO TO 30
      CODE(JJ+1)=1
      NI=NI-2**JJ
   30 CONTINUE

      TEST IF SEQUENCE ENDS IN 0 OR 1

      DO 31 M=1,L
   31 ICODE(M)=CODE(M)
      IF(CODE(1).EQ.0) GO TO 33

      COMPLEMENT SEQUENCE

      DO 32 M=1,L
      ICODE(M)=ICODE(M)+1
      IF(ICODE(M).GT.1) ICODE(M)=0
   32 CONTINUE

      REVERSE SEQUENCE

   33 DO 34 M=1,L
   34 JCODE(L-M+1)=ICODE(M)
      DO 35 M=1,L
      IF(JCODE(L-M+1)-CODE(L-M+1))100,35,36
   35 CONTINUE
   36 CONTINUE

      COMPUTE AUTOCORRELATION

      DO 60 I=1,L
      N=0
      DO 50 J=I,L
      IF(CODE(J).EQ.CODE(J-(I-1))) GO TO 40
      N=N-1
      GO TO 50
   40 N=N+1
   50 CONTINUE

      KEEP AUTOCORRELATIONS
      WITH SIDELOBES LESS OR EQUAL TO 1
```

```
      IF(N.EQ.L) GO TO 55
      IF(N.GT.1) GO TO 100
 55   COR(I)=N
 60   COR(I)=N
      COR(L+1)=0
      K=L+1
 70   WRITE(6,200) (CODE(I),I=1,L),(COR(I),I=1,K)
100   CONTINUE
200   FORMAT(/,' ',41(I2,1X))
      END
```

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