AN INVESTIGATION OF THE POSSIBLE USES OF THE NORMAL MODE HELICAL ANTENNA IN THE TACTICAL ARMY.

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THESIS

AN INVESTIGATION OF THE POSSIBLE USES
OF THE NORMAL MODE HELICAL ANTENNA
IN THE TACTICAL ARMY

by

Michael Dean Vennum

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ABSTRACT

This paper briefly discusses the need for electrically short antennas in the combat maneuver units of the modern field army. A brief history of the work done in the field of helical antennas is presented. An approximate mathematical development is presented which describes the radiation fields of the thin normal mode helical antenna. From the radiation fields, the equations for the polarization ratio and the radiation resistance are developed. A possible design for a tactical VHF - FM antenna to cover the frequency range 30-76 MHZ is presented. The design technique is discussed in detail and a calculator program for computing the input impedance is given. The results of experimental testing are presented and recommendations for improvement are given.

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I. INTRODUCTION

A. BACKGROUND

In modern Army communications, the radio is used extensively. It is used in the millimeter wave and microwave region for satellite link communications. It is used at UHF for air to air voice, air to ground voice and multichannel telephone communications. At VHF and HF it is used primarily for tactical single channel voice or teletype communications. With modern camouflage techniques, very often the most difficult items of a command post to hide are the radio antennas so necessary to the command, control and fire direction of the unit. There are many reasons for this difficulty including the large number of these antennas and their large size at HF and low VHF. This paper concerns itself only with an attempt to reduce the size of these antennas.

It is possible with a highly mobile infantry battalion, to run the command post from the back of a jeep or smaller vehicle with only one or two radios. Even in a desert environment it is possible to hide this vehicle behind a small sand dune. It can be camouflaged against sight, sound and infrared from all directions including the air. The problem then becomes what to do with the radio antennas, the shortest of which rises 8-10 feet above the command post vehicle. Because of the limited transmitter power and limited amounts of coaxial cable available, it is not possible to move the antennas a safe distance away from the

radios. The next best solution is to make the antennas smaller and thus easier to hide.

B. HISTORY AND PURPOSE

The study of electrically small antennas has increased steadily since about 1947 when Wheeler [Ref. 26] published his paper on electrically small antennas. Work proceeded in many directions, however Kraus [Ref. 11] in 1949 investigated the helical antenna. This paper beiefly touched on the normal mode helix, but was mostly concerned with the axial or beam mode. In 1953, Kandoian and Sichak [Ref. 10] reported their findings on frequency tunable normal mode helical antennas and Chatterjee [Ref. 5] reported his findings on the conical helix. In 1958, Li [Refs. 14 and 15] authored two papers on the normal mode helical antenna. These papers suggested that a high efficiency antenna could be constructed and matched to a 50 ohm line for HF and low VHF frequencies. In 1964, Lain [Ref. 13] further investigated the normal mode helix and showed that several modes of radiation were possible from a single normal mode helix. About this time, the U.S. Army became interested in the use of small antennas. In 1965, Czerwinski [Ref. 6] reported on a method to shorten the standard mobile whip antenna by approximately 50%. Between 1965 and 1968, the ECOM labs [Ref. 22] worked on a design for a compact helical antenna for army helicopters. In 1976, ECOM held a conference [Ref. 7] on electrically small antennas. At this conference, the need for electrically small, efficient, rugged antennas for use by tactical units of the U.S. Army was again pointed out. The U.S. Navy had also been investigating the use of helical antennas as a means of improving shipboard communications and space utilization. In 1970, Smith [Ref. 20] reported on the

radiation efficiency of helical antennas and in 1971, [Ref. 21] on the proximity effect caused by parallel conductors. In 1974, Rockway [Refs. 17 and 18] reported on the Navy's efforts to develop a tunable normal mode helical antenna for shipboard use.

In all of these studies, the recuring problems are that the helix is a high Q circuit element and therefore narrow band which limits the usable frequency range and that the normal mode helix has a very low radiation resistance which leads to very low radiation efficiency and complicated matching networks. Li suggested that the second problem could be overcome by exciting the helix as a folded dipole or unipole. This would also increase the bandwidth somewhat. Kandoian and Sichak suggested overcoming the first problem by using a variable tap point on the helix. This approach was used by the Army in their helicopter antenna. It is felt that using both of these methods at the same time has not been adequately investigated and that they may prove to be a workable solution to the problem of an electrically small, efficient, tactical antenna for HF and low VHF.

It is the purpose of this paper to discuss the design and testing of a folded monopole helical normal mode antenna with a nominal radiation resistance and input resistance of 50 ohms. The criteria for the design are that the antenna cover the frequency range 30 - 76 MHZ, be no more than 24 inches in height and be self-resonant or nearly so. The rest of the paper is divided into four parts. The first part is the mathematical development which provides a basis for the development of the model. The second part is a description of the model including calculator program for the theoretical input impedance. The third is the experimental results and conclusions. The fourth is the recommendations.

II. MATHEMATICAL BASIS

In this section, the mathematical basis for the model is developed. The mathematics of a single electrical dipole antenna and a single magnetic dipole (current loop) antenna are developed. All the fundamental equations are developed from Maxwells Equations and the Biot-Savart Law. Wave propagation in free space is assumed. After developing the field equations for these antennas, the assumption is made that the thin helical antenna can be approximmated by a series of alternating electric and magnetic dipole antennas. Using this assumption, the fields for the thin helical antenna are found. From these field equations, the equations for radiation resistance and polarization ratio (axial ratio) are derived. A statement of the theoretical input impedance was derived by Li [Ref. 14] and is given without proof.

A. RADIATION FIELDS FROM AN ELECTRIC DIPOLE

Refer to figure 1.

From the Biot-Savart Law: $d\vec{H} = I d\vec{l} \times \hat{r}/4 r^2$ (E-1)

From Maxwells Equations: $\vec{B} = \mu_o \vec{H}$ therefore:

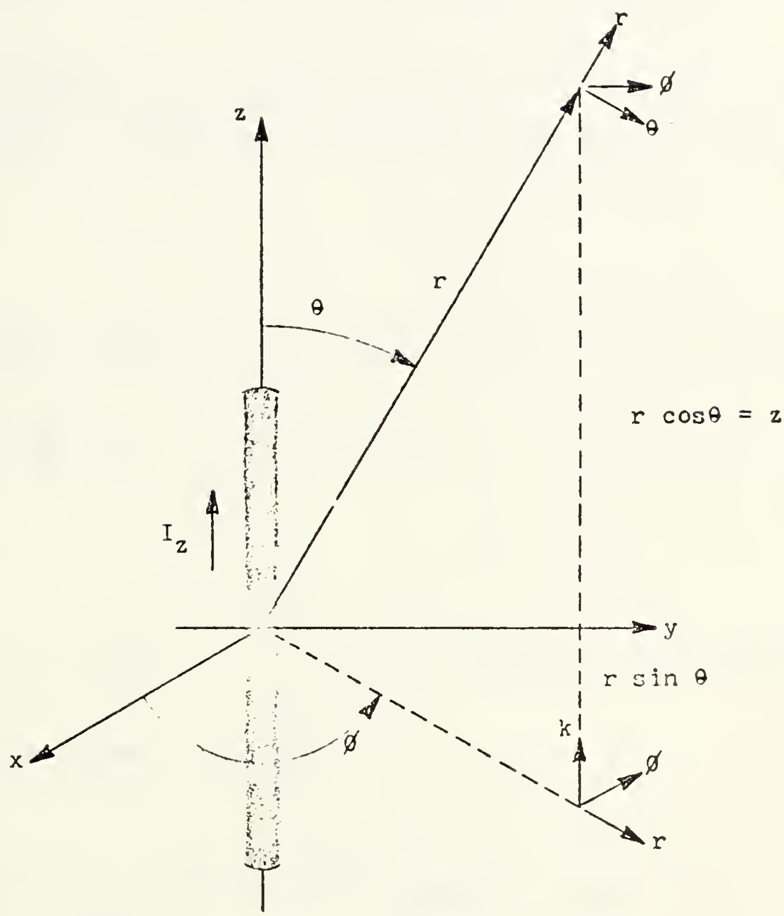

Figure 1 - ELECTRIC DIPOLE GEOMETRY

$d\vec{B} = \mu_o I d\vec{l} \times \hat{r}/4\ r^2$ or

$$\vec{B} = (\mu_o/4\pi)\int I dz\hat{k} \times \hat{r}/r^2 = \mu_o I l\hat{\phi}/4\pi r^2 = B_\phi\hat{\phi}$$

Since $\vec{\nabla} \times \vec{A} = \vec{B}$ then $B_\phi = (\partial A_r/\partial z) - (\partial A_z/\partial r)$

Since the current I is constrained to flow in the z direction, there is only a z component of A, ($A_r = A_\phi = 0$) [Ref. 4]

therefore $B_\phi = -A_z/r$ or $A_z = B_\phi dr$ thus

$$A_z = -(\mu_o Il/4\pi)\int dr/r^2 = \mu_o Il/4\pi r \text{ or}$$

$$\vec{A} = \mu_o Ilk/4\pi r \qquad \text{(E-2)}$$

Next set $I = I_o e^{jw(t-r/v)}$ and convert to spherical coordinates. $\hat{k} = \cos\theta\hat{r} - \sin\theta\hat{\theta}$

Therefore $\vec{A} = \mu_o I_o l e^{jw(t-r/v)}(\cos\theta\hat{r} - \sin\theta\hat{\theta})$ or

$$\vec{A} = A_z(\cos\theta\hat{r} - \sin\theta\hat{\theta}) \qquad \text{(E-3)}$$

but $A_z\cos\theta = A_r$ and $-A_z\sin\theta = A_\theta$, thus $\vec{A} = A_r\hat{r} + A_\theta\hat{\theta}$

From Maxwells equations: $\vec{H} = (\vec{\nabla} \times \vec{A})/\mu_o$ therefore

$$\vec{H} = 1/\mu_o \begin{vmatrix} \hat{r}/r^2\sin\theta & \hat{\theta}/r\sin\theta & \hat{\phi}/r \\ \partial/\partial r & \partial/\partial\theta & \partial/\partial\phi \\ A_r & rA_\theta & rA_\phi\sin\theta \end{vmatrix}$$

where $A_\phi = 0$

$$\vec{H} = (1/\mu_o)\{(\hat{r}/r^2\sin\theta)[-\partial(-rA_z\sin\theta)/\partial\phi\}$$
$$-(\hat{\theta}/r\sin\theta)[\partial(-A_z\cos\theta)/\partial\phi]$$
$$+(\hat{\phi}/r)[\{\partial(rA_z\sin\theta)/\partial r\} - \partial(A_z\cos\theta)/\partial\theta]\}$$

In this expression, the first two terms are derivatives of constants and therefore equal to zero which leaves only:

$\vec{H} = H_\phi\hat{\phi}$ and $H_\phi = (1/\mu_o r)[\{\partial(-rA_z\sin\theta)/\partial r\} - \partial(A_z\cos\theta)/\partial\theta]$

Substituting for A_z from (E-3) and solving yields:

$$H_\phi = (I_o 1/4\pi r)e^{jw(t-r/v)}\sin\theta[(jw/rv) + 1/r^2] \qquad \text{(E-4)}$$

From Maxwells equations:

$$\vec{\nabla} \times \vec{H} = \partial\vec{D}/\partial t = jw\varepsilon_o\vec{E} \text{ or } \vec{E} = (1/jw\varepsilon_o)(\vec{\nabla} \times \vec{H})$$

$$\vec{E} = (1/jw\varepsilon_o) \begin{vmatrix} \hat{r}/r^2\sin\theta & \hat{\theta}/r\sin\theta & \hat{\phi}/r \\ \partial/\partial r & \partial/\partial\theta & \partial/\partial\phi \\ H_r & rH_\theta & rH_\phi\sin\theta \end{vmatrix}$$

where $H_r = H_\theta = 0$

$$\vec{E} = (1/jw\varepsilon_o)\{(\hat{r}/r^2\sin\theta)[\partial(rH_\phi\sin\theta)/\partial\theta]$$

$$-(\hat{\theta}/r\sin\theta)[\partial(rH_\phi\sin\theta)/\partial r]\}$$

substituting for H_ϕ from equation (E-4) and solving for E_r and E_θ separately yields:

$$E_r = (1/j\,w\varepsilon_o r^2\sin\theta)[(jw/rv)$$

$$+(1/r^2)](rI_o l/4)e^{jw(t - r/v)}[\partial(\sin^2\theta)/\partial\theta]$$

but $\sin^2\theta = 1/2 - (1/2)\cos 2\theta$ and $\partial(\cos 2\theta)/\partial\theta = -2\sin 2\theta$ and $\sin 2\theta = 2\sin\theta\cos\theta$

so $\partial(\sin^2\theta)/\partial\theta = -(1/2)(-2)2\sin\theta\cos\theta$ thus:

$$E_r = (I_o l/2\pi\varepsilon_o)e^{jw(t-r/v)}\cos\theta[(1/r^2 v) - (j/wr^3)] \qquad \text{(E-5)}$$

$$E_\theta = -I_o l\sin^2\theta e^{jwt}A/jw\varepsilon_o 4\pi r\sin\theta$$

where $\quad A = \partial\{re^{-jwr/v}[(jw/rv) + (1/r^2)]\}/\partial r$

$$\partial[j(w/v)e^{-jwr/v}]/\partial r = (w/v)^2 e^{-jwr/v}$$

$$\partial[(1/r)e^{-jwr/v}]/\partial r = -e^{-jwr/v}[(1/r^2) + (jw/rv)]$$

$$E_\theta = \{-I_o le^{jw(t-r/v)}\sin\theta/4\pi\varepsilon_o\}[-(jw/rv^2) + (j/wr^2) - (1/r^2 v)]$$

(E-6)

To summarize and put the equations in a more conventional form use $\beta = w/v$ and delete e^{jwt}. This yields:

$$E_r = [I_o le^{-j\beta r}\cos\theta/2\pi\varepsilon_o jwr^2][j\beta + (1/r)] \qquad \text{(E-7)}$$

$$E_\theta = I_o l e^{-j\beta r} \sin\theta/4\pi\epsilon_o jwr][-\beta^2 + (j\beta/r) + (1/r^2)]$$

(E-8)

$$H_\phi = I_o l e^{-j\beta r}[\sin\theta/4\pi r][j\beta + (1/r)] \qquad \text{(E-9)}$$

B. RADIATION FIELDS FROM A MAGNETIC DIPOLE

Refer to figure 2.

From equation (E-2) $d\vec{A} = (\mu_o/4\pi)(Id\vec{l}/R)$

$\vec{R} = \vec{r} - \vec{a}$ and $\vec{r} = r\sin\theta(\sin\phi\hat{j} + \cos\phi\hat{i}) + r\cos\theta\hat{k}$

and $\vec{a} = a\cos\phi'\hat{i} + a\sin\phi'\hat{j}$ therefore

$R = (r\sin\theta\cos\phi - a\cos\phi')i + (r\sin\theta\sin\phi - a\sin\phi')j + r\cos\theta k$

$$|\vec{R}| = (r^2\sin^2\theta\cos^2\phi + a^2\cos^2\phi' - 2ar\sin\theta\cos\phi\cos\phi' + r^2\sin^2\theta\sin^2\phi$$

$$+a^2\sin^2\phi - 2ar\sin\theta\sin\phi\sin\phi' + r^2\cos^2\theta)^{1/2}$$

which reduces to:

$$|\vec{R}| = [r^2 + a^2 - 2ar\sin\theta(\cos\phi\cos\phi' + \sin\phi\sin\phi')]^{1/2}$$

or

$$|\vec{R}| = [r^2 + a^2 - 2ar\sin\theta\cos(\phi-\phi')]^{1/2}$$

(M-1)

Let point P be in the xz - plane then $\phi = 0$ and $\cos(-\phi') = \cos\phi'$. Therefore equation (M-1) becomes

$$|\vec{R}| = (r^2 + a^2 - 2ar\sin\theta\cos\phi')^{1/2}$$

As before $I = I_o e^{jw(t-R/v)} = I_o e^{j(wt-\beta R)}$

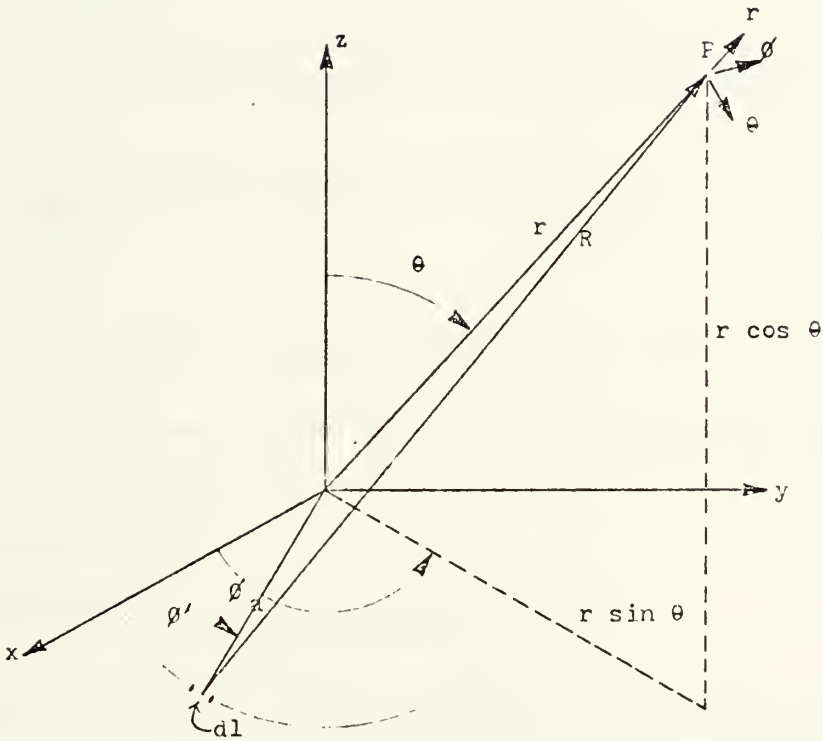

Figure 2 - MAGNETIC DIPOLE GEOMETRY

or in phasor form with e^{jwt} understood $I = I_o e^{-j\beta R}$

therefore $\quad d\vec{A} = (\mu I_o/4\pi)\int_0^{2\pi}(e^{-j\beta R}d\vec{l}/R)$

If r>>a then the a^2 term can be ignored, thus $|\vec{R}| = (r^2 - 2ar\sin\theta\cos\theta')^{1/2}$

or $\quad |\vec{R}| = r[1-2(a/r)\sin\theta\cos\phi']^{1/2}$

This can be approximated using the first two terms of the binomial series expansion of (1-x)

$(1-x)^n = 1-nx+[n(n-1)x^2/2]-[n(n-1)(n-2)x^3]/3]+\ldots$

$|\vec{R}| \approx r\{1-(1/2)[2(a/r)\sin\theta\cos\theta']\} = r[1-(a/r)\sin\theta\cos\theta']$

therefore

$$d\vec{A} = (\mu I_o/4\pi)\int_0^{2\pi} e^{-j\beta r[1-(a/r)\sin\theta\cos\phi']}\, d\vec{l}/[1-(a/r)\sin\theta\cos\phi'] \qquad \text{(M-2)}$$

$e^{j\beta a\sin\theta\cos\phi'}$ can be approximated by using the series expansion for e^x and again ignoring a^2 and higher powers.

$e^x = 1+x+(x^2/2)+(x^3/3)+\ldots$

where $x = a\sin\theta\cos\phi'$

thus $e^{j\beta a\sin\theta\cos\phi'} \approx 1+j\beta a\sin\theta\cos\phi'$.

Now $[1-(a/r)\sin\theta\cos\phi']^{-1}$ can be approximated using the first two terms of the binomial series expansion of $(1-x)^{-1}$

$(1-x)^{-1} \approx (1+x)^{+1}$

thus $[1-(a/r)\sin\theta\cos\phi']^{-1} \approx [1+(a/r)\sin\theta\cos\phi']$

Substituting all this back into Eqn (M-2) gives:

$$d\vec{A} = (\mu I_o/4\pi)\int_0^{2\pi}(1+j\beta a\sin\theta\cos\phi')[1+(a/r)\sin\theta\cos\phi']e^{-j\beta r}\, d\vec{l}/r$$

An expression for $d\vec{l}$ is now needed. Refer to figure 3 which is correct for very small dl.

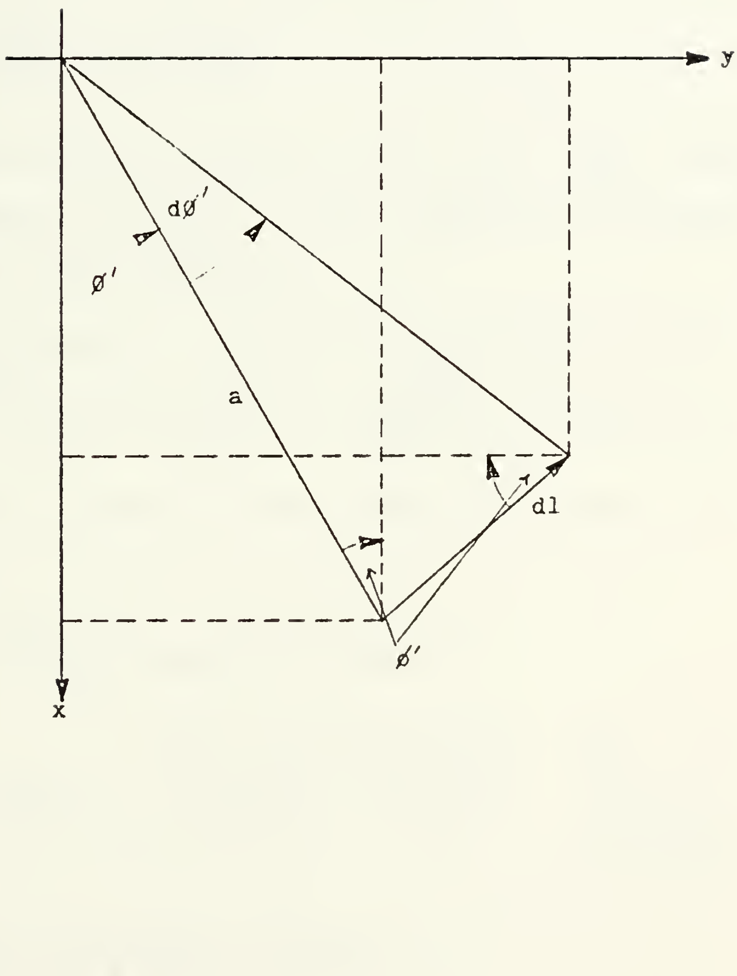

Figure 3 - APPROXIMATION FOR DL

$$d\vec{A} = (\mu I_o a e^{-j\beta r}/4\pi r)\int_0^{2\pi}[1+j\beta a\sin\theta\cos\phi' + (a/r)\sin\theta\cos\phi'$$

$$+j(\beta a^2/r)\sin^2\theta\cos^2\phi'](\cos\phi'\hat{j}-\sin\phi'\hat{i})\,d\phi'$$

Once again an approximation is made by ignoring the a^2 term.

$$d\vec{A} = (\mu I_o a e^{-j\beta r}/4\pi r)\int_0^{2\pi}[\cancelto{0}{\cos\phi'}\hat{j}-\cancelto{0}{\sin\phi'}\hat{i}+j\beta a\sin\theta\cos^2\phi'\hat{j}$$

$$+(a/r)\sin\theta\cos^2\phi'\hat{j}-j\beta a\sin\theta\cancelto{0}{\cos\phi\sin\phi'}\hat{i}$$

$$+(a/r)\sin\theta\cancelto{0}{\cos\phi'\sin\phi'}\hat{i}]\,d\phi'$$

The indicated terms integrate to zero when integrated over one complete period. The equation can be further reduced using the trig idenity:

$\cos^2 x = [(1/2)+\cos 2x/2]$ ($\cos 2\phi'$ integrates to zero)

$d\vec{A} = (\mu I_o a^2 e^{-j\beta r}/4\pi r)\sin\theta[j\beta+(1/r)]\int_0^{2\pi}\hat{j}\,d\phi'/2$ and since point P was assumed to be in the xz-plane, then $\hat{j} = \hat{\phi}$ and therefore

$$\vec{A} = [\mu I_o(\pi a^2)e^{-j\beta r}/4\pi r]\sin\theta[j\beta+(1/r)]\hat{\phi} \qquad \text{(M-3)}$$

$$\vec{H} = (\vec{\nabla}\times\vec{A})/\mu = (1/\mu)\begin{vmatrix} \hat{r}/r^2\sin\theta & \hat{\theta}/r\sin\theta & \hat{\phi}/r \\ \partial/\partial r & \partial/\partial\theta & \partial/\partial\phi \\ 0 & 0 & rA_\phi\sin\theta \end{vmatrix}$$

$H_r = (1/\mu r^2\sin\theta)\ \partial(rA_\phi\sin\theta)/\partial\theta$ and

$H_\theta = (-1/\mu r\sin\theta)\ \partial(rA_\phi\sin\theta)/\partial r$

$$H_r = [I_o(\pi a^2)e^{-j\beta r}\{j\beta+(1/r)\}/4\pi r^2\sin\theta]\ \partial(\sin^2\theta)/\partial\theta$$

using the trig identity $\sin^2 x = 1/2 - (1/2)\cos 2x$

$\partial(\sin^2\theta)/\partial\theta = (-1/2)(-\sin 2\theta)(2) = \sin 2\theta$

from another trig identity $\sin 2\theta = 2\sin\theta\cos\theta$

therefore $$H_r = I_o(\pi a^2)e^{-j\beta r}\cos\theta\{j\beta + (1/r)\}/2\pi r^2 \qquad \text{(M-4)}$$

$$H_\theta = (-1/\mu r\sin\theta)\,\partial\{r\sin^2\theta\mu I_o(\pi a^2)e^{-j\beta r}[j\beta + (1/r)]/4\pi r\}/\partial r$$

$$= [-I_o(\pi a^2)\sin\theta/4\pi r]\,\partial\{e^{-j\beta r}[j\beta + (1/r)]\}/\partial r$$

$$= [-I_o(\pi a^2)\sin\theta/4\pi r]\{j\beta(-j\beta)e^{-j\beta r} + [r(-j\beta)e^{-j\beta r} - e^{-j\beta r}]/r^2\}$$

$$= [-I_o(\pi a^2)\sin\theta/4\pi r]e^{-j\beta r}\{\beta^2 - (j\beta/r) - (1/r^2)\}$$

$$H_\theta = [I_o(\pi a^2)\sin\theta e^{-j\beta r}/4\pi r]\{(1/r^2) + (j\beta/r) - \beta^2\} \qquad \text{(M-5)}$$

$$\vec{\nabla}\times\vec{H} = \vec{J} + \partial\vec{D}/\partial t = \cancel{\vec{J}}^{\,0} + \varepsilon\partial\vec{E}/\partial t = jw\varepsilon\vec{E}$$

$$E = (1/jw\varepsilon)\begin{vmatrix} \hat{r}/r^2\sin\theta & \hat{\theta}/r\sin\theta & \hat{\phi}/r \\ \partial/\partial r & \partial/\partial\theta & \partial/\partial\phi \\ H_r & rH_\theta & 0 \end{vmatrix}$$

$$\vec{E} = (1/jw\varepsilon)\{-(\hat{r}/r^2\sin\theta)[\partial(rH_\theta)/\partial\phi]$$

$$+ (\hat{\theta}/r\sin\theta)[\partial(H_r)/\partial\phi]$$

$$+ (\hat{\phi}/r)[\partial(rH_\theta)/\partial r - \partial(H_r)/\partial\theta]\}$$

$\vec{E} = E_\phi\hat{\phi}$ and $E_\phi = (1/jw\varepsilon r)[\partial(rH_\theta)/\partial r - \partial(H_r)/\partial\theta]$

$$\partial(rH_\theta)/\partial r = [I_o(\pi a^2)\sin\theta/4\pi](\partial/\partial r)\{e^{-j\beta r}[(1/r^2) + (j\beta/r) - \beta^2]\}$$

$$= [I_o(\pi a^2)\sin\theta/4\pi]\{-j\beta e^{-j\beta r}[(1/r^2) + (j\beta/r) - \beta^2] + e^{-j\beta r}$$

$$[-(2/r^3) - (j\beta/r^2)]\}$$

$$[-(2/r^3)-(j/r^2)]\}$$

$$\partial(rH_\theta)/\partial r=[I_o(\pi a^2)\sin\theta/4\pi r]e^{-j\beta r}\{(-2j\beta/r^2)+(\beta^2/r)+j\beta^3$$

$$-(2/r^3)-(j\beta/r^2)\} \qquad \text{(M-6)}$$

$$\partial(H_r)/\partial\theta=I_o(\pi a^2)e^{-j\beta r}\{(j\beta/r^2)+1/r^3\}[\partial(\cos\theta)/\partial\theta]$$

$$\partial(H_r)/\partial\theta=[I_o(\pi a^2)e^{-j\beta r}/4\pi]\sin\theta\{(-2j\beta/r^2)-(2/r^3)\} \qquad \text{(M-7)}$$

therefore:

$$E_\phi=[I_o(\pi a^2)e^{-j\beta r}\sin\theta/4\pi jw\varepsilon r]\{(\beta^2/r)+j\beta^3\}$$

or by substitution of $\beta^2=(w/v)^2$ and $v^2=1/\mu\varepsilon$ the following is obtained

$$E_\phi=[-jw\mu I_o(\pi a^2)e^{-j\beta r}\sin\theta/4\pi r]\{j\beta+(1/r)\} \qquad \text{(M-8)}$$

The following is a summary of the field equations for the electric and magnetic diploes.

$$E_r=[I_o le^{-j\beta r}\cos\theta/2\pi jw\varepsilon r^2]\{j\beta+(1/r)\} \qquad \text{(E-7)}$$

$$E_\theta=[I_o le^{-j\beta r}\sin\theta/4\pi jw\varepsilon r]\{-\beta^2+(j\beta/r)+(1/r^2)\} \qquad \text{(E-8)}$$

$$H_\phi=[I_o le^{-j\beta r}\sin\theta/4\pi r]\{j\beta+(1/r)\} \qquad \text{(E-9)}$$

$$H_r=[I_o(\pi a^2)e^{-j\beta r}\cos\theta/2\pi r^2]\{j\beta+(1/r)\} \qquad \text{(M-4)}$$

$$H_\theta=[I_o(\pi a^2)e^{-j\beta r}\sin\theta/4\pi r]\{-\beta^2+(j\beta/r)+(1/r^2)\} \qquad \text{(M-5)}$$

$$E_\phi=[-jw\mu I_o(\pi a^2)e^{-j\beta r}\sin\theta/4\pi r]\{j\beta+(1/r)\} \qquad \text{(M-8)}$$

It should be noted how similar the fields are and by substituting the following relationships this similarity can be further demonstrated.

$$w = v\beta = \beta/\sqrt{\mu\varepsilon} \text{ and } \eta = \sqrt{\mu/\varepsilon}$$

$$E_r = [\eta I_o l e^{-j\beta r}\cos\theta/2\pi j\beta r^2]\{j\beta + (1/r)\} \quad \text{(E-7)}$$

$$E_\theta = [\eta I_o l e^{-j\beta r}\sin\theta/2\pi j\beta r]\{-\beta^2 + (j\beta/r) + (1/r^2)\} \quad \text{(E-8)}$$

$$H_\phi = [I_o l e^{-j\beta r}\sin\theta/4\pi r]\{j\beta + (1/r)\} \quad \text{(E-9)}$$

$$E_\phi = -j\beta\eta[I_o(\pi a^2)e^{-j\beta r}\sin\theta/4\pi r]\{j\beta + (1/r)\} \quad \text{(M-8)}$$

To determine the far field equations it must be realized that for r very large, the $1/r^2$ and $1/r^3$ terms become insignificant in relationship to the 1/r terms. This can also be strictly proven by use of Poyntings Theorem. With this in mind, equations E-7, E-8, E-9, M-4, M-5, and M-8 for the far fields reduce to:

$$E_r = 0 \quad \text{(E-7)}$$

$$E_\theta = [\eta I_o l e^{-j\beta r}\sin\theta/4\pi](j\beta/r) \quad \text{(E-8)}$$

$$H_\phi = [I_o l e^{-j\beta r}\sin\theta/4\pi](j\beta/r) \quad \text{(E-9)}$$

$$H_r = 0 \quad \text{(M-4)}$$

$$H_\theta = [I_o(\pi a^2)e^{-j\beta r}\sin\theta/4\pi](-\beta^2/r) \quad \text{(M-5)}$$

$$E_\phi = [\eta I_o(\pi a^2)e^{-j\beta r}\sin\theta/4\pi](\beta^2/r) \quad \text{(M-8)}$$

By Poyntings Theorem, $\vec{P}\text{ower} = \vec{P} = \mathrm{Re}\iint \vec{E}\times\vec{H}^*$. This will be in the radial direction for both electric and magnetic dipoles.

Electric dipole:

$$\vec{P} = \vec{E} \times \vec{H} = (E_\theta)(H_\emptyset)(\hat{\theta} \times \hat{\emptyset}) = H_\emptyset^2 \hat{r}$$

$$\vec{P} = (I_o l \sin\theta / 4\pi) e^{-j\beta r} (\beta/r)^2 \hat{r} \qquad \text{(P-1)}$$

Magnetic dipole:

$$\vec{P} = \vec{E} \times \vec{H} = (E_\emptyset)(H_\theta)(\hat{\emptyset} \times \hat{\theta}) = -\eta H_\theta^2 (-r) = \eta H_\theta^2 \hat{r}$$

$$\vec{P} = [I_o (\pi a^2) \sin\theta / 4\pi]^2 e^{-j\beta r} (\beta^2/r)^2 \hat{r} \qquad \text{(P-2)}$$

These represent the instaneous power density at point P. In order to find the total average radiated power these expressions must be integrated over the surface of a sphere.

$$S = \int_0^{\pi}\int_0^{2\pi} r\,d\theta\, r\sin\theta\, d\emptyset = 2\pi r^2 \int_0^{\pi} \sin\theta\, d\theta$$

Electric dipole:

$$P(\text{total}_e) = (1/2)[\eta I_o^2 l^2 e^{-2j\beta r}/16\pi^2](\beta^2/r^2)\, 2\pi r^2 \int_0^{\pi} \sin^3\theta\, d\theta$$

from a Trig identity: $\sin^3\theta = (1/4)(3\sin\theta - \sin 3\theta)$

$$P(\text{Total}_e) = \eta[I_o^2 l^2 e^{-2j\beta r}/16\pi]\beta^2 \{-(3/4)\cos\theta + (1/12)\cos 3\theta\}\Big|_0^{\pi}$$

$$P(\text{Total}_e) = \eta(I_o^2 l^2 e^{-2j\beta r}/12\pi)\beta^2$$

Using $\eta = 120\pi$ and $\beta = w/v = 2\pi f/v = 2\pi/\lambda$ the following is obtained

$$P(\text{Total}_e) = 40\pi^2 I_o^2 (l/\lambda)^2 e^{-2j\beta r} \qquad \text{(P-3)}$$

and since $P(\text{Total}_e) = (I/\sqrt{2})^2 R$ where $I = I_o e^{j(wt-\beta r)}$ we get

$$R(\text{rad}_e) = 80\pi^2 (l/\lambda)^2 \qquad \text{(P-4)}$$

Magnetic Dipole:

$$P(\text{Total}_m) = [(1/2)\eta I^2 (\pi a^2)^2 (\beta^2/r)^2/4\pi^2]\, 2\pi r^2 \int_0^{\pi} \sin^3\theta\, d\theta$$

which by the same process as above reduces to

$$P(Total_m)=160\pi^4 I^2 (\pi a^2/\lambda^2)^2 \quad \text{where } I=I_o e^{j(wt-\beta r)} \tag{P-5}$$

and again since $P(Total_m) = (I/\sqrt{2})^2 R$ we get

$$R(rad_m)=320\pi^4 (\pi a^2/\lambda^2)^2 = 80\pi^2 (4\pi^2) (A/\lambda^2)^2 \tag{P-6}$$

It must be remembered that all of the above development requires that
$r>>\lambda$, $r>>1$, $r>>a$, and $\lambda>>2\pi a$

C. RADIATION RESISTANCE OF ANTENNA

The following discussion is based on the work done by Li [Ref. 14]. From the previous work it is known that the radiation resistance of a single turn thin helical antenna can be approximated by $R(rad_e) + R(rad_m)$ which is

$$R_{rad}=80\pi^2[(1/\lambda)^2+4\pi^2(\pi a^2/\lambda)^2] \tag{H-1}$$

A more usual case is a helical antenna of many turns. A sample figure of a mathematical model based on individual electric and magnetic dipoles is shown in figure 4. The current distribution is the same as in the previous work although it is described in a different manner.

From the previous work it is known that the far electric field has only θ and $\emptyset$ components.

$$E_\theta^e = [I_o e^{-j\beta r} 1/4\pi]\sin\theta(j\beta/r) \tag{E-8}$$

$$E_\emptyset^m = [I_o A e^{-j\beta r}/4\pi]\sin\theta(\beta^2/r) \tag{M-8}$$

By substituting $\beta = 2\pi/\lambda$ and $l = P$ these become

$$E_\theta^e = \eta(I_o/2r)(jP/\lambda)e^{-j\beta r}\sin\theta \qquad \text{(H-2)}$$

$$E_\phi^m = \eta(I_o/2r)(2\pi A/\lambda^2)e^{-j\beta r}\sin\theta \qquad \text{(H-3)}$$

Now substituting $I_o = \hat{I}\cos(m\pi/N2)$ and $r = r - mL\cos\theta/N$ these equations become:

$$E_\theta^e = \eta(I/2r)(jP/\lambda)e^{-j\beta[r-(mL\cos\theta/N)]}\sin\theta\cos(m\pi/N2) \qquad \text{(H-4)}$$

$$E_\phi^m = \eta(I/2r)(2\pi A/\lambda^2)e^{-j\beta[r-mL\cos\theta/N)]}\sin\theta\cos(m\pi/N2) \qquad \text{(H-5)}$$

By using only the real part of Eulers relation the following is obtained:

$$E_\theta^e = (I/2r)(jP/\lambda)\cos(m\beta L\cos\theta/N)\cos(m\pi/N2)e^{-j\beta r}\sin\theta \qquad \text{(H-6)}$$

$$E_\phi^m = (I/2r)(2\pi A/\lambda^2)\cos(m\beta L\cos\theta/N)\cos(m\pi/N2)e^{-j\beta r}\sin\theta \qquad \text{(H-7)}$$

The electric field for the entire antenna is thus:

$$2\sum_{m=1}^{N} E_\theta^e + E_\phi^m$$

The sum of the cosine terms can be evaluated using the following formulas:

a. $\cos A\cos B = (1/2)\cos(A-B) + (1/2)\cos(A+B)$

b. $\sum_{m=1}^{N}\cos(ma) = \{[\sin a(N+1/2)]/[2\sin(a/2)]\} - 1/2$

this gives $2\sum_{m=1}^{N}\cos(m\pi/N2)\cos(m\beta L\cos\theta/N) =$

$\{\sin[(1+(1/2N))T]/2\sin(T/2N)\}$ +

$\{\sin[(1+(1/2N))T']/2\sin(T'/2N)\} - 1$

where $T = (\pi/2)-\beta L\cos\theta$ and $T' = (\pi/2)+ L\cos\theta$.

If N>>1 then the 1/2N term in the numerator can be ignored and the sine function in the denominator can be replaced by its argument. Using these approximatons and (sin a)(sin b) = (1/2) sin(a+b) + (1/2)sin(a-b) we get

$$2\sum_{m=1}^{N}\cos(m\pi/N2)\cos(m\beta L\cos 0/N) =$$

$$\{N\cos(\beta L\cos\theta)/[(\pi/2)^2-(\beta L\cos\theta)^2]\}-1$$

therefore:

$$\vec{E}_T=\eta(\hat{I}/2r)[\{N\pi\cos(\beta L\cos\theta)/[(\pi/2)^2-(\beta L\cos\theta)^2]\}-1]\sin\theta\, e^{-j\beta r}$$

$$[j(P/\lambda)\hat{\theta} + 2\pi(A/\lambda^2)\hat{\phi}] \qquad \text{(H-8)}$$

From this equation we can find the axial ratio which describes the polarization ellipse for the antenna. The radiation resistance can also be found.

1. Polarization

$$\text{Axial ratio}=AR=[(E_\theta^e)/(E_\phi^m) = (P/\lambda)/2\pi(A/\lambda^2) = 2P\lambda/(\pi D)^2 \qquad \text{(H-9)}$$

AR>>1: Vertical Polarization

AR<<1: Horizontal Polarization

AR = 1: Circular Polarization

2. Radiation Resistance

By Poyntings Theorem

$$\vec{P} = (1/2)\iint \vec{E} \times \vec{H}^{*} = (1/2)\iint (\vec{E} \times \vec{E}^{*})/\eta$$

Therefore $\vec{P} = 2\pi\int_0^{\pi} r^2 \, (\vec{E}x\vec{E}/2)\sin\theta d\theta$

$$R_H = (2P/\hat{I}^2 = 2\pi/\hat{I}^2)\int_0^{\pi} (r^2 \eta I^2/4r^2\eta)$$

$$[\{N\pi\cos(\beta L\cos\theta)/[(\pi/2)^2 - (\beta L\cos\theta)^2]\} - 1]^2 \sin^3\theta$$

$$[(P/\lambda)^2 + (2\pi)^2 (A/\lambda^2)^2]d\theta$$

by letting $\cos\theta = u$ then $du = -\sin\theta$ the following results:

$$R_H = (\pi/2)\,[(P/\lambda)^2 + 4\pi^2 (A/\lambda^2)^2]\int_0^{\pi} [\{N\pi\cos\beta Lu/[(\pi/2)^2$$

$$-(\beta Lu)^2]\} - 1]^2 (1-u^2)du$$

Let $f(u) = N\cos\beta Lu/[(\pi/2^2 - (\beta Lu)^2]$

$$= (4N/\pi)\cos\beta Lu[1-(2\beta Lu/\pi)^2]^{-1}$$

and expand using $\cos y = 1-(y^2/2!)+(y^4/4!)-(y^6/6!)+ \ldots$

with $y = \beta Lu$ and $(1-x)^{-1} = 1+x+x^2+x^3+x^4+ \ldots$

with $x = (2\beta Lu/\pi)^2$. Doing this the following is obtained

for $f(u)$: $f(u) = (4N/\pi)[1-.0947(\beta Lu)^2 + .0033(\beta Lu)^4 - \ldots]$ or

$f(u) \approx 4N/\pi$ for $\beta Lu < 1$

therefore:

$$R_H = (\pi/2)\eta[(P/\lambda)^2 + 4\pi^2 (A/\lambda^2)^2][(4N/\pi) - 1]^2 (4/3)$$

or with some slight simplification:

$$R_{H(dipole)} = 80\pi^2[(4N/\pi) - 1]^2[(P/\lambda)^2 + 4\pi^2 (A/\lambda^2)^2] \qquad \text{(H-10)}$$

It should be noted that the only difference between equation H-10 and the sum of equations P-4 and P-6 is the

term $([4N/\pi\gamma]-1)^2$. This should not be unexpected since this term only involves N, the number of turns of wire in the antenna. This equation can be further simplified if the helix is very thin (basic assumption) and N>>1. In that case $R_H \approx 1280(L/\lambda)^2$. (H-11)

It can be shown that for a monopole antenna above a perfect ground, the power radiated is approximately 1/2 the power radiated by a dipole in free space. Therefore it would be expected that R_H for a monopole would be $(1/2)R_H$ for a dipole. Therefore

$$R_{H(mono)} = 40\pi^2[(P/\lambda)^2 + 4\pi^2(A/\lambda^2)^2][(4N/\pi\gamma)-1]^2 \qquad \text{(H-12)}$$

Knowing the radiation resistance, the desired frequency, the axial ratio desired and the loss resistance of the type of wire to be used it is possible to design a simple effective helical antenna. A method for doing this will be described later. A more accurate and more complicated design method involves considering the actual theoretical input impedance of the antenna.

The mathematics for deriving the input impedance is much more complicated and has been worked out by Li and therefore his results will be accepted and used here.

$Z_{in} = (30/\sin^2\beta L)(\beta/\beta_o)(R + jX)$ where R and X are as follows:

$$R = 2\mathrm{cin}(\beta+\beta_o)L - 2\mathrm{cin}(\beta-\beta_o)L + \cos 2\beta L[2\mathrm{cin}(\beta+\beta_o)L - 2\mathrm{cin}(\beta-\beta_o)L$$
$$-\mathrm{cin}(\beta+\beta_o)2L + \mathrm{cin}(\beta-\beta_o)2L] + \sin 2\beta L[2\mathrm{Si}(\beta-\beta_o) - 2\mathrm{Si}(\beta+\beta_o)L$$

$$+Si(\beta+\beta_o)2L-Si(\beta+\beta_o)2L]-(\beta_o/\beta)[(4/3)(\beta/k)^2(\cos\beta_o L$$

$$-\cos\beta L)^2+(8/3)(k/\beta)^2(1-\cos\beta L)^2]$$

$$X = 2Si(\beta+\beta_o)L+2Si(\beta-\beta_o)L+\cos 2\beta L[2Si(\beta-\beta_o)L+2Si(\beta+\beta_o)L$$

$$-Si(\beta+\beta_o)2L-Si(\beta-\beta_o)2L]+\sin 2\beta L[2cin(\beta+\beta_o)+2cin(\beta-\beta_o)L$$

$$-cin(\beta+\beta_o)2L-cin(\beta-\beta_o)2L-2Log(L/a)]$$

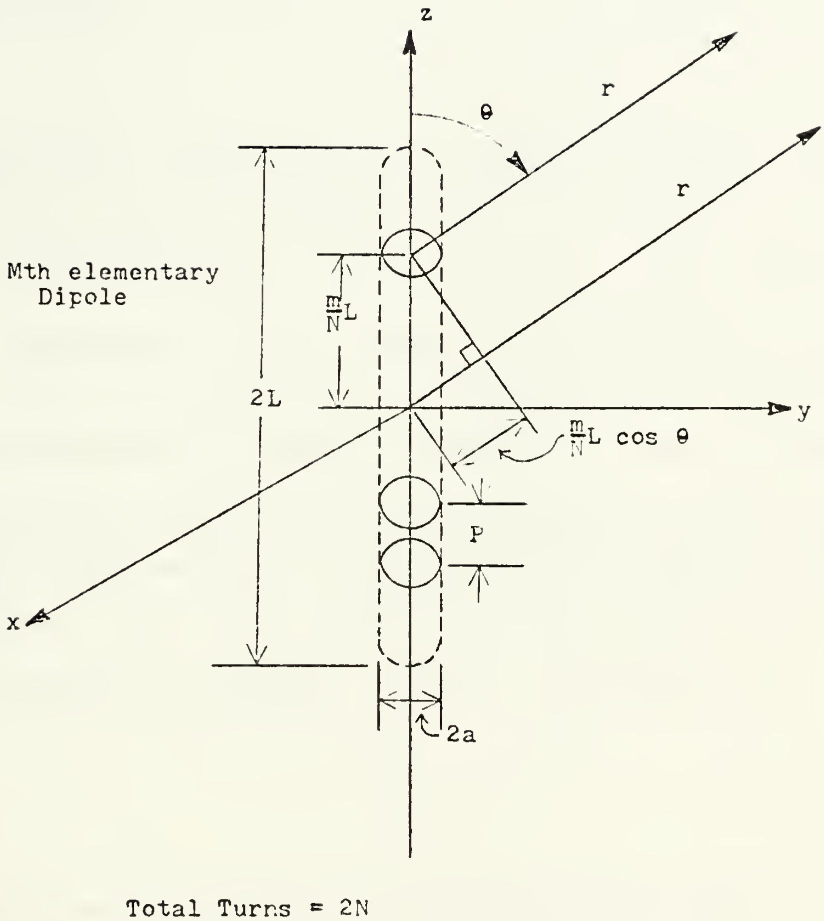

Figure 4 - APPROXIMATE ANTENNA GEOMETRY

III. DESIGN METHOD FOR CONSTRUCTION

A. DESIGN METHOD

The radiation resistance for a folded dipole increases approximately by the square of the number of folded elements. Therefore for 5 elements $R_{rad} = 25\ R_{rad}$ (dipole) and from equation (H-11) $R \simeq 1280(L/\lambda)^2$ for a dipole. A dipole may be modeled using a unipole over a perfectly conducting ground plane and this technique is used here. It can be shown that because a unipole only radiates in half space, it has 1/2 the radiation resistance of a dipole. Thus from this and equation (H-11) $R_H \simeq 640(L/\lambda)^2$ for a monopole.

For the sake of simplicity of design and ease of testing with currently available tactical radios, it was decided to design an antenna with an input resistance of 50 ohms. As a first approximation, the input resistance is assumed to be equal to the radiation resistance plus the loss resistance of the wire used to construct the antenna. If large wire is used then the loss resistance can be kept small and $R_i \simeq R_{rad} = R_H = 640(L/\lambda)^2$. The helical antenna normally

has a low radiation resistance, thus to satisfy the condition above it was decided to design the antenna as a 5 element folded monopole.

Thus $R_i \approx 25(640(L/\lambda)^2 = 16000(L/\lambda)^2$

The range of frequencies for this antenna was to be 30.0 - 76.0 MHZ, thus λ = 10m - 3.95m substituting into equation (D-1) and solving for L gives $\lambda(50/16000)^{1/2} = L = 22.01$ inches for 30 MHZ and 8.69 inches for 76 MHZ.

Now that the basic length has been determined, it is now necessary to determine the diameter and pitch of the helix. Consider a single section of the folded helical monopole as shown in Figure 5. All 5 wire elements are shown and "P" is called the pitch of the helix.

It has been shown experimentally [Ref. 21] that the optimum spacing between elements is 1.5 times the element diameter. At this point any increase in element diameter to reduce skin resistance is more than offset by an increase in resistance due to proximity effects. Knowing this it is found that $P = 12.5D_{wire}$. This means to determine the pitch, the optimum wire size must first be determined.

Equation (H-11) assumed N >> 1 and NP = L. Using this and Smiths relationship leads to $D_{wire} = L/(12.5N)$ The next logical question is for what range of values of N is the approximation leading to equation (H-11), valid? Another method of stating this is at what minimum value of N does $[(4N/\pi)-1]^2 \approx (4N/\pi)^2$. The answer to that depends on

the amount of error that can be tolerated in the rough design. It can be shown that for

error < 1%	N > 159
error < 5%	N > 33
error < 10%	N > 17

Because this is an initial design reasonable error can be tolerated in the interest of construction ease. Therefore, N is chosen to be 22 which gives a pitch of 1 inch and a wire diameter of .08 inch. This means AWG 12 wire can be used, but for safety sake in keeping at least a 1.5 ratio between diameter and spacing AWG 13 wire was used.

To determine the helix diameter use equation (H-9). For proper system compatibility with existing tactical VHF-FM antennas, vertical polarization is chosen.

$$1 \ll AR = 2P\lambda/(\pi D)^2 \text{ or } D = (1/\pi)(2P\lambda/AR)^{1/2}$$

As a practical matter, how great should AR be required to be? From a practical point of view on the average, most tactical VHF-FM antennas are at least 5^o from true vertical when they are in use. For this reason it is not necessary to design for an AR which would yield vertical polarization closer than $\pm 5^o$. From simple trigonometry

$$\theta = \tan^{-1}\left|E^m_\phi / E^e_\theta\right| = \tan^{-1}(\pi D)^2/2P\lambda$$

Which when solved for D gives $D = (1/\pi)(2P\lambda \tan\theta)^{1/2}$.

Thus for 76 MHZ and $\theta = 5^o$; D = 1.66 inches. For ease of manufacture the diameter of 1.5 inches was chosen. This gives $\theta = 1.62^o$ from vertical at 30 MHZ and 4.1^o from

vertical at 76 MHZ.

The specifications for the model to be tested are as follows:

Length	22 inches
Number of turns	22
Helix diameter	1.5 inches
Pitch	1 inch
Wire size	AWG 13

The length of the actual model was 24 inches. This was done to allow for making a capacitive top hat and to allow for experimental changes that might become necessary.

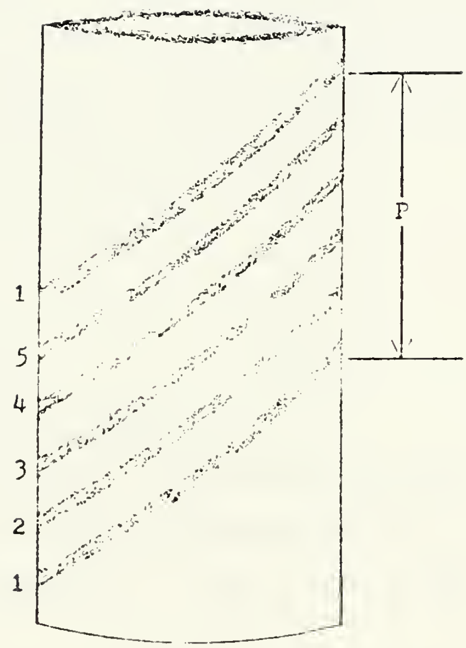

Figure 5 - SECTION OF ANTENNA

B. HP-67 CALCULATOR PROGRAM FOR INPUT IMPEDANCE

In order to properly use this program, the following data must be supplied and stored as directed: frequency in Hz in register 1, length of the monopole in register 2, radius of the form in register 3, the pitch of the helix in register 4, 30.00 in register 5, 4/3 in register 6, 6.283185308 in register 7, 5.319774201EEX-10 in register D, 12.5 in register E,and 11811 in register I. The program and data require two complete cards. The following are the instructions for running the program:

1. Load side 2 of card 2 (data)
2. Load sides 1 and 2 of card 1.
3. Input test values
   a. frequency in Hz
   b. length in inches
4. Begin the program by pressing A.
5. Insert side 1 of card 2 into the reader. (The program will pause and read this card automatically.)
6. Press R/S for output in the following order (stack review) 0,R,X,C.
7. Start over with step 2 for new test problem.

To solve for the radiation resistance, only card 2 is needed. Read both sides of card 2.

1. Store monopole length.
2. Store the frequency (MHZ) in register B.
3. Press B.

STEP	KEY ENTRY	KEY CODE
001	f(LBL) C	31 25 13
	RCL 8	34 08
	RCL 9	34 09
	+	61
	RCL 2	34 02
	x	71
	h(RTN)	35 22
	f(LBL) D	31 25 14
	RCL 8	34 08
010	RCL 9	34 09
	-	51
	RCL 2	34 02
	x	71
	h(RTN)	35 22
	g(LBLf) a	32 25 11
	0	00
	STO 2	33 02
	RCL A	34 11
	STO 0	33 00
020	h(f?) 1	35 71 01
	STO 2	33 02
	h(f?) 1	35 71 01
	1	01
	h(f?) 0	35 71 00
	0	00
	STO 1	33 01
	g(LBLf) b	32 25 12
	RCL 1	34 01
	2	02
030	x	71

STEP	KEY ENTRY	KEY CODE
031	h(f?) 1	35 71 01
	1	01
	h(f?) 0	35 71 00
	2	02
	+	61
	RCL 0	34 00
	h(X⇄Y)	35 52
	h(y^x)	35 63
	h(last x)	35 82
040	÷	81
	h(lastx)	35 82
	h(n!)	35 81
	÷	81
	1	01
	CHS	42
	RCL 1	34 01
	h(y^x)	35 63
	x	71
	STO+2	33 61 02
050	9	09
	RCL 1	34 01
	g(x=y)	32 51
	GTO 1	22 01
	1	01
	STO+1	33 61 01
	GTO fb	22 31 12
	f(LBL) 1	31 25 01
	RCL 2	34 02
	h(RTN)	35 22
060	f(LBL) A	31 25 11

STEP	KEY ENTRY	KEY CODE
061	RCL 1	34 01
	RCL D	34 14
	x	71
	STO 8	33 08
	h (↑r)	35 73
	2	02
	x	71
	RCL 2	34 02
	4	04
070	x	71
	÷	81
	STO 8	33 08
	h (pause)	35 72
	RCL 8	34 08
	g (x^2)	32 54
	RCL 9	34 09
	g (x^2)	32 54
	-	51
	STO 0	33 00
080	h (SF) 0	35 51 00
	f (GSB) C	31 22 13
	STO A	33 11
	f (P⇄S)	31 42
	g (GSBf) a	32 22 11
	2	02
	x	71
	STO 3	33 03
	h (CF) 0	35 61 00
	h (SF) 1	35 51 01
090	g (GSBf) a	32 22 11

STEP	KEY ENTRY	KEY CODE
091	2	02
	x	71
	STO 8	33 08
	f (P⇄S)	31 42
	f (GSBf) D	31 22 14
	STO A	33 11
	f (P⇄S)	31 42
	g (GSBf) a	32 22 11
	2	02
100	x	71
	STO 7	33 07
	h (CF) 1	35 61 01
	h (SF) 0	35 51 00
	g (GSBf) a	32 22 11
	2	02
	x	71
	STO 4	33 04
	f (P⇄S)	31 42
	f (GSB) C	31 22 13
110	2	02
	x	71
	STO A	33 11
	f (P⇄S)	31 42
	g (GSBf) a	32 22 11
	STO 5	33 05
	h (CF) 0	35 61 00
	h (SF) 1	35 51 01
	g (GSBf) a	32 22 11
	STO 9	33 09
120	f (P⇄S)	31 42

STEP	KEY ENTRY	KEY CODE
121	f(GSBf) D	31 22 14
	2	02
	x	71
	STO A	33 11
	f(P⇄S)	31 42
	g(GSBf) a	32 22 11
	STO B	33 12
	h(CF) 1	35 61 01
	h(SF) 0	35 51 00
130	g(GSBf) a	32 22 11
	STO 6	33 06
	CHS	42
	RCL 5	34 05
	-	51
	RCL 4	34 04
	+	61
	RCL 3	34 03
	+	61
	f(P S)	31 42
140	RCL 2	34 02
	RCL 3	34 03
	÷	81
	f(log)	31 53
	2	02
	x	71
	-	51
	RCL 8	34 08
	RCL 2	34 02
	2	02
150	x	71
151	x	71
	STO A	33 11
	f(P⇄S)	31 42
	h(rad)	35 42
	f(sin)	31 62
	x	71
	RCL 7	34 07
	RCL 8	34 08
	+	61
160	RCL 9	34 09
	-	51
	RCL B	34 12
	-	51
	RCL A	34 11
	f(cos)	31 63
	x	71
	+	61
	RCL 7	34 07
	RCL 8	34 08
170	+	61
	+	61
	STO C	33 13
	RCL 3	34 03
	RCL 4	34 04
	-	51
	RCL 5	34 05
	-	51
	RCL 6	34 06
	+	61
180	RCL A	34 11

STEP	KEY ENTRY	KEY CODE
181	f(cos)	31 63
	x	71
	RCL 7	34 07
	RCL 8	34 08
	-	51
	RCL 9	34 09
	+	61
	RCL B	34 12
	-	51
190	RCL A	34 11
	f(sin)	31 62
	x	71
	+	61
	RCL 3	34 03
	RCL 4	34 04
	-	51
	+	61
	f(P⇄S)	31 42
	RCL 9	34 09
200	RCL 2	34 02
	x	71
	f(cos)	31 63
	RCL 8	34 08
	RCL 2	34 02
	x	71
	f(cos)	31 63
	-	51
	g(x^2)	32 54
	RCL 6	34 06
210	x	71

STEP	KEY ENTRY	KEY CODE
211	RCL 8	34 08
	g(x^2)	32 54
	RCL 0	34 00
		81
	x	71
	RCL 8	34 08
	RCL 2	34 02
	x	71
	f(cos)	31 63
220	CHS	42
	1	01
	+	61
	g(x^2)	32 54
	h(pause)	35 72
	RCL 6	34 06
	x	71
	2	02
	x	71
	RCL 0	34 00
230	x	71
	RCL 8	34 08
	g(x^2)	32 54
	÷	81
	+	61
	RCL 9	34 09
	RCL 8	34 08
	÷	81
	x	71
	-	51
240	STO A	33 11

STEP	KEY ENTRY	KEY CODE
241	RCL C	34 13
	h(x⇄y)	35 52
	h(rad)	35 42
	g(→P)	32 72
	RCL 5	34 05
	RCL 8	34 08
	x	71
	x	71
	RCL 9	34 09
250	÷	81
	RCL 8	34 08
	RCL 2	34 02
	x	71
	f(sin)	31 62
	g(x^2)	32 54
	÷	81
	f(→R)	31 72
	h(eng)	35 23
	DSP 2	23 02
260	RCL E	34 15
	x	71
	h(x⇄y)	35 52
	RCL E	34 15
	x	71
	↑	41
	↑	41
	RCL 1	34 01
	x	71
	RCL 7	34 07
270	x	71

STEP	KEY ENTRY	KEY CODE
271	h(1/x)	35 62
	0	00
	R ↓	35 53
	R/ S	84
	g(stk)	32 84
	R/ S	84
	f(LBL)B	31 25 12
	h(RCI)	35 34
	÷	81
280	g(x^2)	32 54
	STO A	33 11
	RCL 3	34 03
	g(x^2)	32 54
	h(π)	35 73
	x	71
	x	71
	g(x^2)	32 54
	h(π)	35 73
	2	02
290	x	71
	g(x^2)	34 54
	x	71
	RCL A	34 11
	RCL 4	34 04
	x	71
	+	61
	RCL 2	34 02
	RCL 4	34 04
	÷	81
300	h(π)	35 73

STEP	KEY ENTRY	KEY CODE
301	÷	81
	4	04
	x	71
	1	01
	-	51
	g(x^2)	32 54
	x	71
	h(π)	35 73
	g(x^2)	32 54
310	EEX	43
	3	03
	x	71
	x	71
	h(RTN)	35 22

CALCULATIONS OF THEORETICAL Z_i AND CAPACITANCE

NEEDED FOR RESONANCE

30 MHZ

Length	24	23	22.25	22	21	20	19	18	17	16
Resist.	59.1	54.3	50.8	49.7	45.3	41.1	37.1	33.3	29.7	26.4
React.	5.41k	5.67k	5.88k	5.95k	6.26k	6.60k	6.97k	7.38k	7.84k	8.35k
Capac.	.980uf	.935uf	.902uf	.891uf	.847uf	.804uf	.761uf	.719uf	.677uf	.635uf

35 MHZ

Length	22	21	20	19	18	17	16	15	14	13
Resist.	67.5	61.5	55.9	50.4	45.3	40.4	35.8	31.5	27.5	28.7
React.	5.03k	5.29k	5.58k	5.91k	6.26k	6.66k	7.10k	7.60k	8.18k	8.83k
Capac.	.905uf	.859uf	.814uf	.770uf	.726uf	.683uf	.640uf	.598uf	.556uf	.515uf

40 MHZ

Length	19	18	17	16.5	16	15	13	12	11
Resist.	65.8	59.1	52.7	49.7	46.7	41.1	30.9	26.4	22.2
React.	5.01k	5.41k	5.76k	5.95k	6.16k	6.60k	7.68k	8.35k	9.15k
Capac.	.780uf	.735uf	.690uf	.668uf	.646uf	.603uf	.518uf	.476uf	.435uf

45 MHZ

Length	16	15	14.75	14	12	11	10	9	8
Resist.	59.1	52.0	50.3	45.3	33.3	28.0	23.2	18.8	14.8
React.	5.41k	5.81k	5.92k	6.26k	7.38k	8.09k	8.94k	9.97k	11.2k
Capac.	.653uf	.609uf	.598uf	.565uf	.479uf	.437uf	.396uf	.355uf	.314uf

Figure 6 - THEORETICAL Z_I 30-45 MHZ

50 MHZ

Length	15	14	13.25	13	12	11	10	9	8
Resist.	64.1	55.9	50.1	48.2	41.1	34.6	28.6	23.2	18.3
React.	5.18k	5.58k	5.93k	6.05k	6.60k	7.24k	8.00k	8.94k	10.1k
Capac.	.615uf	.570uf	.537uf	.526uf	.482uf	.440uf	.398uf	.356uf	.315uf

55 MHZ

Length	14	12.75	12	11	10	9	8	7
Resist.	67.5	56.1	49.7	41.8	34.6	28.0	22.2	17.0
React.	5.03k	5.58k	5.95k	6.54k	7.24k	8.09k	9.15k	10.5k
Capac.	.576uf	.519uf	.486uf	.442uf	.400uf	.358uf	.316uf	.276uf

60 MHZ

Length	13	12	11	10	9	8	7
Resist.	69.2	59.1	49.7	41.1	33.3	26.3	20.2
React.	4.95k	5.41k	5.95k	6.60k	7.38k	8.35k	9.60k
Capac.	.535uf	.490uf	.445uf	.402uf	.359uf	.318uf	.276uf

65 MHZ

Length	12	11	10.25	9	8	7
Resist.	29.2	58.3	50.6	39.1	30.9	23.7
React.	4.95k	5.46k	5.89k	6.78k	7.68k	8.83k
Capac.	.494uf	.449uf	.415uf	.361uf	.319uf	.277uf

70 MHZ

Length	11	10	9.5	9	8	7
Resist.	67.5	55.9	50.4	45.3	35.8	24.5
React.	5.03k	5.58k	5.91k	6.26k	7.10k	8.18k
Capac.	.452uf	.407uf	.385uf	.363uf	.320uf	.278uf

Figure 7 - THEORETICAL Z_I 50-70 MHZ

75 MHZ

Length	10.5	10	9	8.75	8	7
Resist.	70.6	64.1	52.0	49.1	41.1	31.5
React.	4.90k	5.18k	5.81k	5.99k	6.60k	7.60k
Capac.	.433uf	.410uf	.365uf	.354uf	.322uf	.280uf

50 OHM TUNING

Freq.	30	35	40	45	50	55	60	65	70	75
Length	22	19	16.5	14.75	13.25	12	11	10.25	9.5	8.75
Resist.	49.7	50.4	49.7	50.3	50.1	49.7	49.7	50.6	50.4	49.1
React.	5.95k	5.91k	5.95k	5.92k	5.93k	5.95k	5.95k	5.89k	5.91k	5.99k
Capac.	.891uf	.770uf	.668uf	.598uf	.537uf	.486uf	.445uf	.415uf	.385uf	.354uf

Figure 8 - THEORETICAL Z_I 75 MHZ AND 50 OHM TUNING

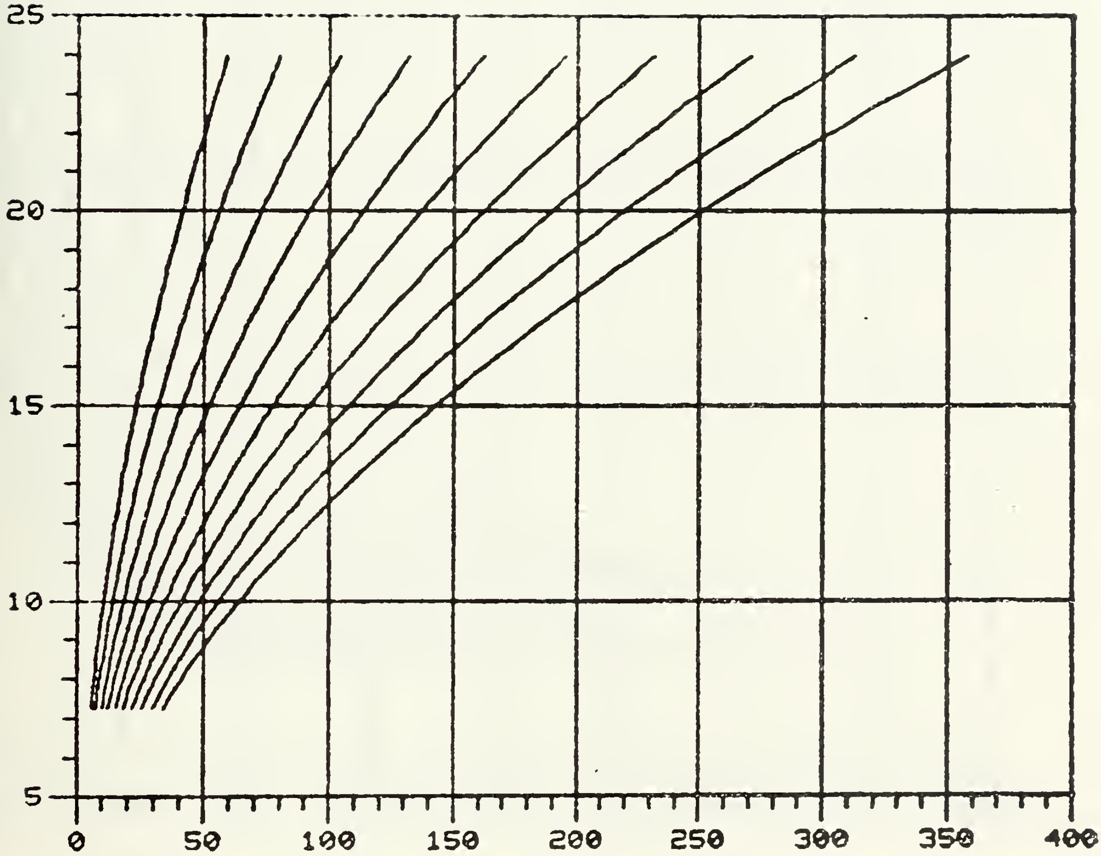

Figure 9 - GRAPH OF REAL Z_I

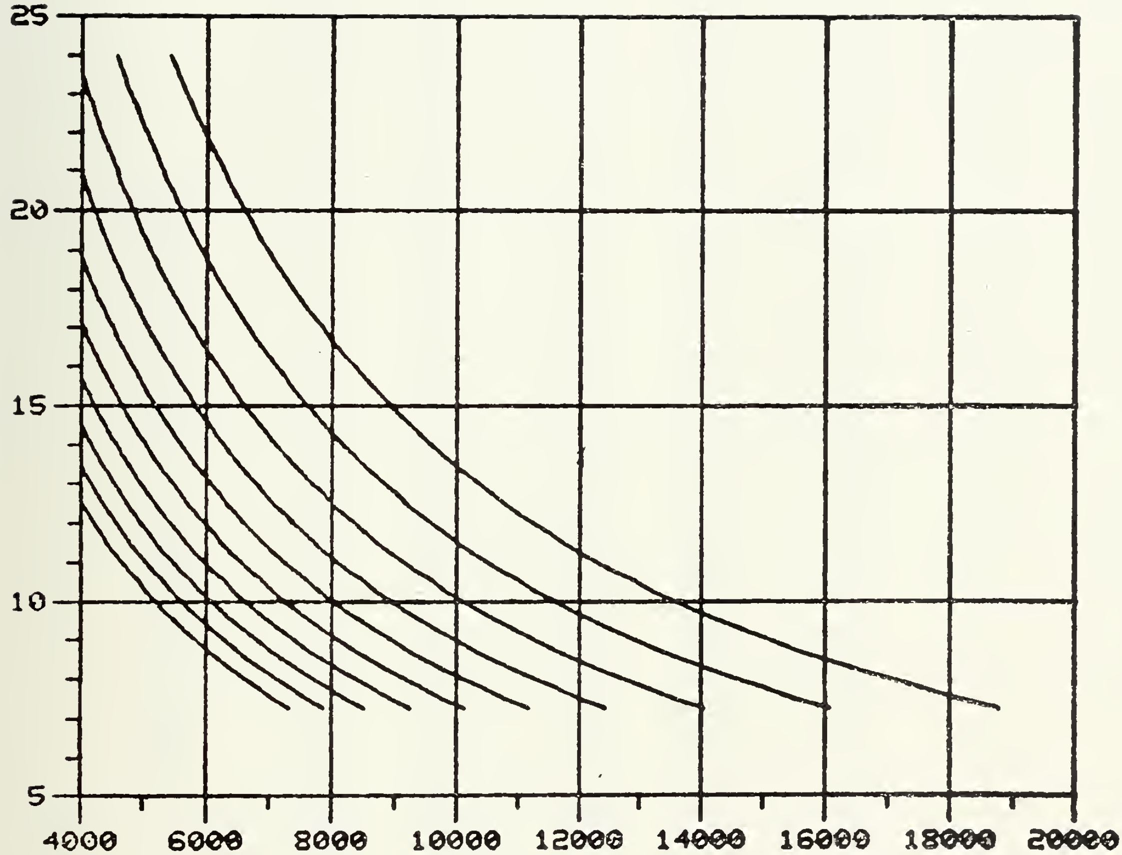

Figure 10 - GRAPH OF IMAGINARY Z_I

TABLE OF THEORETICAL RADIATION RESISTANCE

30 MHZ											
Length	24	23	22	21.7	21	20	19	18	17	16	15
R_H	55.67	50.98	46.49	45.19	42.21	38.14	34.27	30.61	27.16	23.91	20.87
35 MHZ											
Length	22	21	20	19	18.55	18	17	16	15	14	13
R_H	63.3	57.48	51.93	46.67	44.39	41.68	36.98	32.56	28.42	24.56	20.99
40 MHZ											
Length	19	18	17	16.25	16	15	14	13	12	11	10
R_H	60.97	54.46	48.32	43.95	42.54	37.13	32.09	27.42	23.11	19.17	15.6
45 MHZ											
Length	16	15	14.41	14	13	12	11	10	9	8	7
R_H	53.86	47.01	43.19	40.63	34.71	29.26	24.28	19.76	15.7	12.11	8.99
50 MHZ											
Length	14.5	14	13	12	11	10	9	8	7	6	
R_H	54.05	50.18	42.87	36.14	29.98	24.4	19.39	14.96	11.1	7.81	
55 MHZ											
Length	13.5	13	12	11.8	11	10	9	8	7	6	5
R_H	56.24	51.9	43.75	42.21	36.3	29.54	23.47	18.11	13.44	9.46	6.18

Figure 11 - THEORETICAL RADIATION RESISTANCE 30-55 MHZ

60 MHZ

Length	12	11	10.83	10	9	8	7
R_H	52.1	43.22	41.79	35.17	27.95	21.56	16.0

65 MHZ

Length	11	10	9	8	7
R_H	50.75	41.3	32.82	25.32	18.79

70 MHZ

Length	10.5	10	9.3	9	8	7
R_H	53.27	47.93	40.92	38.09	29.38	21.8

75 MHZ

Length	10	9	8.66	8	7
R_H	55.05	43.75	40.2	33.75	25.04

THEORETICAL EFFICIENCY AT R_E Z_i = 50 = R_H/R_e Z_i

Freq	30	35	40	45	50	55	60	65	70	75
	.90	.89	.88	.87	.85	.85	.83	.82	.81	.80

Figure 12 - THEORETICAL RADIATION RESISTANCE 60-75 MHZ

IV. EXPERIMENTAL RESULTS

A. MODEL CONSTRUCTION

The model was constructed based on the design previously given. A cylinder of 1.5 inch plumbing PVC tubing 24 inches long was grooved five times for a helix of 1 inch pitch. The starting points were placed every 72^{o} around the cylinder. .25 inch from both the top and bottom of the cylinder, .125 inch holes were drilled in each of the grooves to allow the wire to enter and exit the form. AWG 13 bare copper wire was used for the antenna. All five wires were shorted at the top using a solid piece of printed circuit board. Later this top short was changed to a loop of AWG 14 tinned copper wire. This change caused no change in the results, but did facilitate making experimental changes.

The antenna was mounted on a 6x5 foot 16 Ounce solid copper ground plane. The ground plane was constructed by placing 2 copper sheets 3x5 feet on a plywood platform 5 x 6 feet. The copper sheets were nailed to the plywood and copper tape was used to make a good electrical connection between the two sheets. The antenna was mounted in the center of this ground plane and four of the antenna conductors were soldered to the ground plane. The fifth conductor was passed through an insulator to a coaxial connector where it was soldered to the center connector.

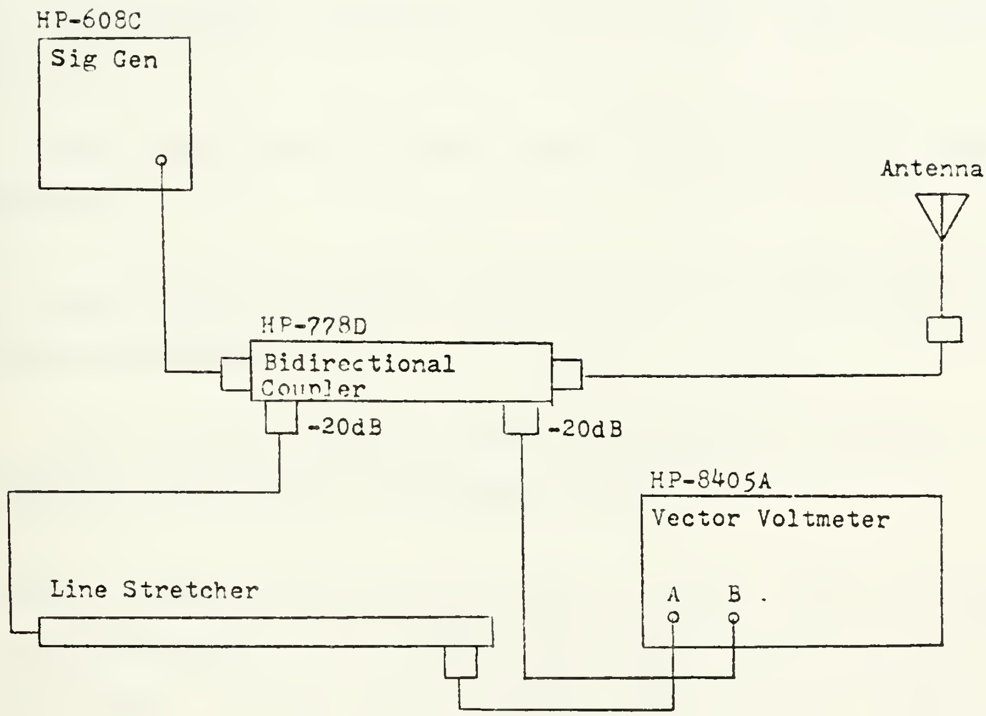

Figure 13 - EXPERIMENTAL SET UP

B. RESULTS AND CONCLUSIONS

The measurement of input impedance, resonant frequencies and VSWR was accomplished using a Hewlett Packard 8405A Vector Voltmeter. The experimental test set up is shown in figure 10. As a result of the testing, several interesting results were discovered which were not predicted from the mathematical development.

a. The antenna exhibited very small real radiation resistance.

b. The inductive reactance was not nearly as high as predicted.

c. The lowest resonant frequency was 10 MHZ higher than for which designed.

d. The antenna could be made to resonate properly only by shorting the turns by physical touch.

It was found that the real part of the input impedance was approximately 1.75 ohms using only the driven element. This was close to the 2 ohm design. When the antenna was excited using the driven element and the number three element as a folded element the real part of the input impedance increased to 6 ohms. This was just as the theory predicted. As more folded elements were excited, the real part of the radiation resistance began to decrease instead of increasing as was predicted. This unexpected result cannot be attributed to any drastic increase in ohmic losses, but must be attributed to the proximity effect discussed by Smith [Ref. 22]. In his work Smith did not

discuss the subject of input impedance directly, but was mostly interested in finding an optimum value of d/D where d = wire diameter and D = wire spacing. It was at this value where the decrease in skin resistance loss due to using larger wires would be just counteracted by the increase in proximity losses because of the closer spacing.

It is well known that for a folded dipole to work, the spacing between elements must be close. Array theory also states that as the spacing between elements gets closer, the radiation resistance goes down. For many specific antennas which have been thoroughly studied, guide values for element spacing have been developed. For this antenna, no such guide values exist. Based on the experimental results thus far attained, it would appear that $.0005\lambda$ is too close and $.001\lambda$ may be satisfactory.

The antenna, as designed was to exhibit a very large inductive reactance on the order of kiloohms. When tested, it was found that the antenna exhibited a reactance which was sometimes 0.1 times the predicted value. It is felt that this effect was the cause of the antenna resonating off frequency. Once again it is felt that the proximity effect which alters the current distribution in systems of parallel conductors was the cause. All the theory for this antenna was based on all elements having equal uniform current distributions. It is felt that the proximity effect prevented this condition from existing by causing the current distribution to be different from wire to wire.

It is felt that the reason the antenna could be properly loaded when physically shorted by hand was that the human body became part of the antenna, thus lengthening it. The internal capacitance of the body also acted as a distributed capacitive load to tune out the inductive reactance of the antenna.

V. RECOMMENDATIONS

The initial design of this antenna did not approach the characteristics predicted by the mathematics. This should not, however, discourage further testing of similar designs. Li [Ref. 15] did build and test a single frequency normal mode 4 wire helical antenna. He used 300 ohm T.V. lead-in wire and designed for 120 MHZ. Using T.V. lead-in, his antenna was not bothered by proximity effects which caused the failure of the design discussed here.

There are several directions future study could go. Bailey [Ref. 3] showed how the input impedance of folded dipoles using parallel elements and carrying opposite currents varied greatly with element spacing. He also showed that for the same element size, if all the elements carry current in the same direction, then the input impedance is nearly independent of element spacing. This should be investigated by driving all 5 elements directly.

Figure 11 was extrapolated from data given in the ARRL Antenna Book [Ref. 2]. The degree of accuracy for very small spacings is unknown. It is known that standard T.V. lead-in uses AWG 12 or AWG 14 wire separated by about 0.5 inch to produce 300 ohm lead-in wire. It can be seen that AWG 13 wire at 0.5 inch spacing gives approximately 315 ohms input impedance. It is therefore assumed that the extrapolation is good at least to this point. The lack of the proximity effect has been suggested as one reason for

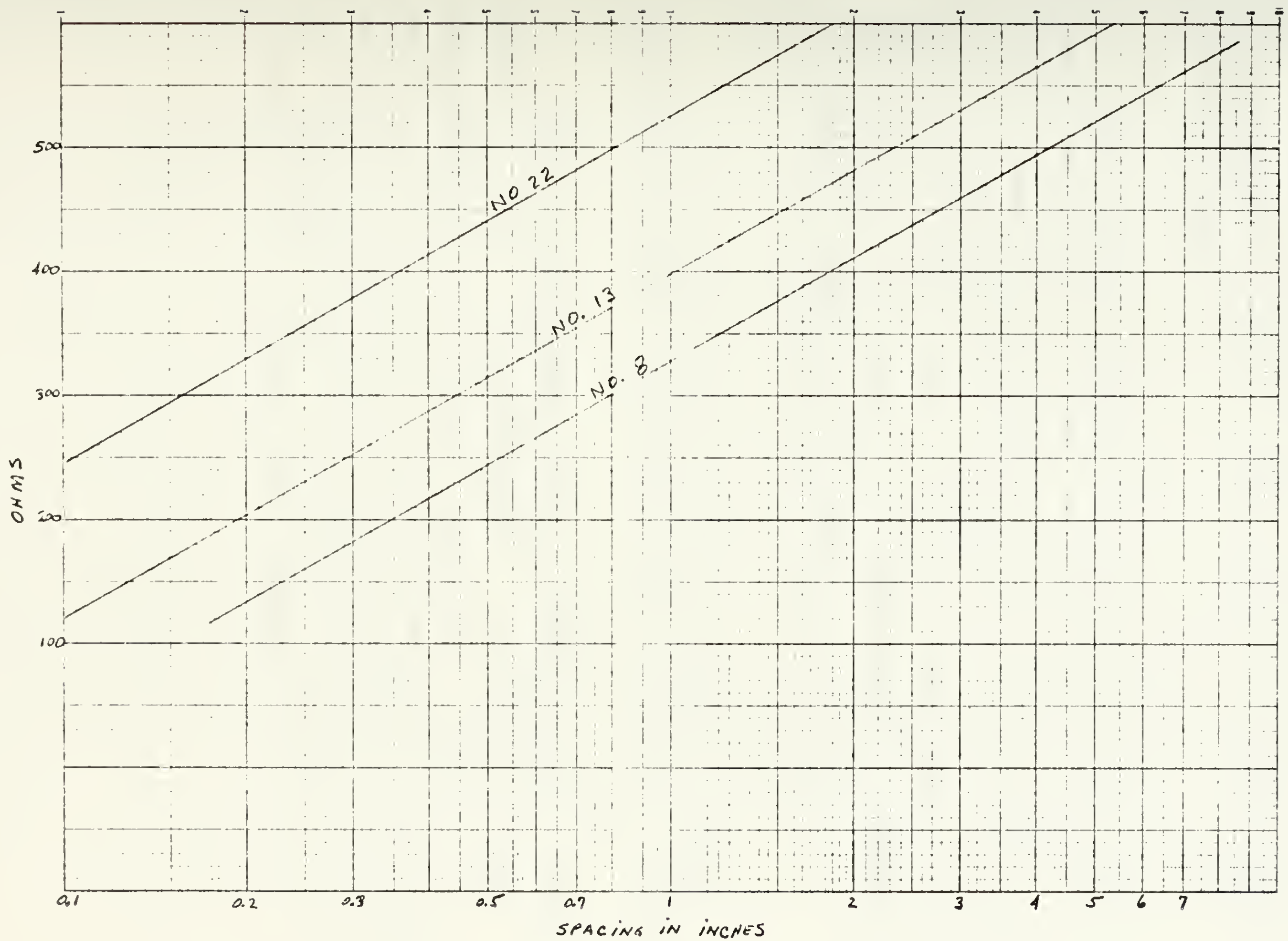

Figure 14 - INPUT IMPEDANCE VS. SPACING

Li's success. Another may be that the antenna was made with wire which had moderately high characteristic impedance. Another direction for future study should be the use of smaller wire such as AWG 22 spaced at 0.5 inch. This will give high characteristic impedance as well as eliminating the proximity effect. This would of course double the size of the antenna.

Still another method of improving the imput impedance characteristics of the antenna is to use a thin driven element and large folded elements. This could be done by using a combination of thin wire and copper tubing. If this approach is taken then it will be necessary to increase the diameter of the antenna to a minimum of 3 inches in order to make smooth bends in the tubing. This will not change the normal mode characteristics of the antenna.

A final possibility to be studied is the effect of combining any or all of the above mentioned techniques.

Work on this style thin normal mode helical antenna should not be abandoned. A similar model has been demonstrated at a higher frequency. There is a pressing need in the Army and Marine Corps for an electrically short efficient antenna for the tactical VHF-FM radio system. If the problems can be overcome, the system has tremendous potential for improving the shipboard H-F antenna system.

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