NAVAL POSTGRADUATE SCHOOL
Monterey, California

THESIS

THE VARIATION OF EARTH TIDES AS A
TRIGGERING MECHANISM IN EARTHQUAKES

by

Richard Arthur Corradini

Thesis Advisor: R. H. Shudde

September 1973

NAVAL POSTGRADUATE SCHOOL
Monterey, California

THESIS

THE VARIATION OF EARTH TIDES AS A
TRIGGERING MECHANISM IN EARTHQUAKES

by

Richard Arthur Corradini

Thesis Advisor: R. H. Shudde

September 1973

The Variation of Earth Tides as a Triggering Mechanism in Earthquakes

by

Richard Arthur Corradini
Captain, United States Army
B.S., University of Illinois, Urbana, 1965

Submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE IN OPERATIONS RESEARCH

from the

NAVAL POSTGRADUATE SCHOOL
September 1973

ABSTRACT

Earthquake data from Central California over the period January 1, 1969, to December 31, 1971, and from Western Nevada over the period August 31, 1954, to December 31, 1961, have been analyzed to investigate the hypothesis that the Earth tides may be the triggering mechanism for the occurrence of earthquakes. One method used compared the tidal components and the component rates-of-change at the time and location of actual earthquakes to those for random events uniformly generated over the same time period. A second method compared the differences in the Earth tides computed across the breadth of a fault zone at the time of actual earthquakes to the differences at the times of random events uniformly generated over the same time period. Statistical tests were performed to see if the distributions from the actual events were the same as the corresponding distributions from the randomly generated events.

TABLE OF CONTENTS

I. INTRODUCTION - - - - - - - - - - - - - - - - - - 5

II. OBJECTIVE - - - - - - - - - - - - - - - - - - - 7

III. ORGANIZATION - - - - - - - - - - - - - - - - - 8

IV. EARTH TIDE DISCUSSION - - - - - - - - - - - - - 14

V. ANALYSIS - 16

A. KOLMOGOROV-SMIRNOV TEST - - - - - - - - - 16

B. CHI-SQUARE TEST - - - - - - - - - - - - - 18

VI. RESULTS - 21

A. SAN ANDREAS - - - - - - - - - - - - - - - 21

B. DIXIE VALLEY - - - - - - - - - - - - - - - 22

VII. CONCLUSIONS - - - - - - - - - - - - - - - - - - 30

APPENDIX A. EARTH TIDE CALCULATIONS - - - - - - 32

BIBLIOGRAPHY - - - - - - - - - - - - - - - - - - - 36

INITIAL DISTRIBUTION LIST - - - - - - - - - - - - 38

FORM DD 1473 - - - - - - - - - - - - - - - - - - - 39

ACKNOWLEDGEMENTS

I wish to express my appreciation to Professors Rex Shudde and Donald Barr, Naval Postgraduate School, for their valuable guidance and criticisms which contributed greatly to the completion of this thesis. I would also like to extend my appreciation to the personnel of the National Center for Earthquake Research, Menlo Park, California, for providing the basic data used in this thesis, and to the staff of the W. R. Church Computer Center, Naval Postgraduate School, for their assistance.

[illegible]

[illegible]

I. INTRODUCTION

Every point on the surface of the Earth is subject to three types of forces: the force of gravity due to the attraction of the mass of the Earth, the centrifugal force due to the rotation of the Earth, and a force due to the attraction of extra-terrestrial bodies, mainly the Sun and the Moon. The resultant forces on the surface of the Earth are generally referred to as the Earth tide forces.

Because of the stresses that these Earth tides create in the crust of the Earth, it has been long felt that they may be a significant factor in the triggering of earthquakes. Several previous studies have been undertaken in which attempts were made to correlate the gravitational component of the Earth tides with the occurrence of earthquakes. In one study, Knopoff [Ref. 1] concluded that there was no correlation between this tidal component and the occurrence of the 3800 earthquakes in Southern California that he investigated. Shlien [Ref. 2] came to a similar conclusion in his study of several widely separated regions of the world.

A team of Russian researchers [Ref. 3] contend that these and other similar studies have suffered from two serious drawbacks. First, they considered only the gravitational component of the Earth tides. More important may be the tangential components which depend very strongly on the orientation of a fault in a region. In general, the extremes in the tangential components do not occur at the times of the extremes of the gravitational component. Second, they tried

[illegible]

to cover too large an area. Knopoff considered a region of 18 square degrees, and the regions in Shlien's study varied between 50 and 450 square degrees. Since tides are time dependent and periodic in nature, considerations of large regions have averaging effects on their variation, possibly masking dependencies present.

II. OBJECTIVE

The objective of this study was three-fold. Given a region of investigation and a specific time period: (1) compare the tidal forces from a series of actual earthquakes to these forces from a series of random events generated uniformly over the same time period, (2) compare the rates-of-change of the tidal forces from the actual events to those from the randomly generated events, and (3) compare the differences in each of the tidal forces over the breadth of the region from the actual events to those from the randomly generated events. These comparisons were used to test the hypothesis that the distribution of a force, rate, or difference from the actual events was the same as the corresponding distributions from the randomly generated events. Rejection of the hypothesis would indicate a difference in the distributions being compared. Thus, an evaluation can be made as to whether a significant dependency exists between a force, rate, or difference and the occurrence of earthquakes for the particular region being studied.

III. ORGANIZATION

For the purpose of this analysis, the tidal forces were resolved into five quantities. The orientation and construction of these quantities were as follows: The gravitational component was formed along the radius of the Earth at the earthquake epicenter, the normal component was formed perpendicular to both the gravitational component and the strike of a fault, the axial component was formed perpendicular to the gravitational component and parallel to the strike of a fault, the tangent magnitude was the resultant formed by the combination of the magnitudes of the normal and axial components, and the total magnitude was the resultant formed by the combination of the magnitudes of the gravitational, normal, and axial components. By observing how these five quantities changed over a small change in time, the rate-of-change of each quantity also was approximated. By calculating these tidal quantities on both sides of a fault zone, the differences in these quantities also were calculated.

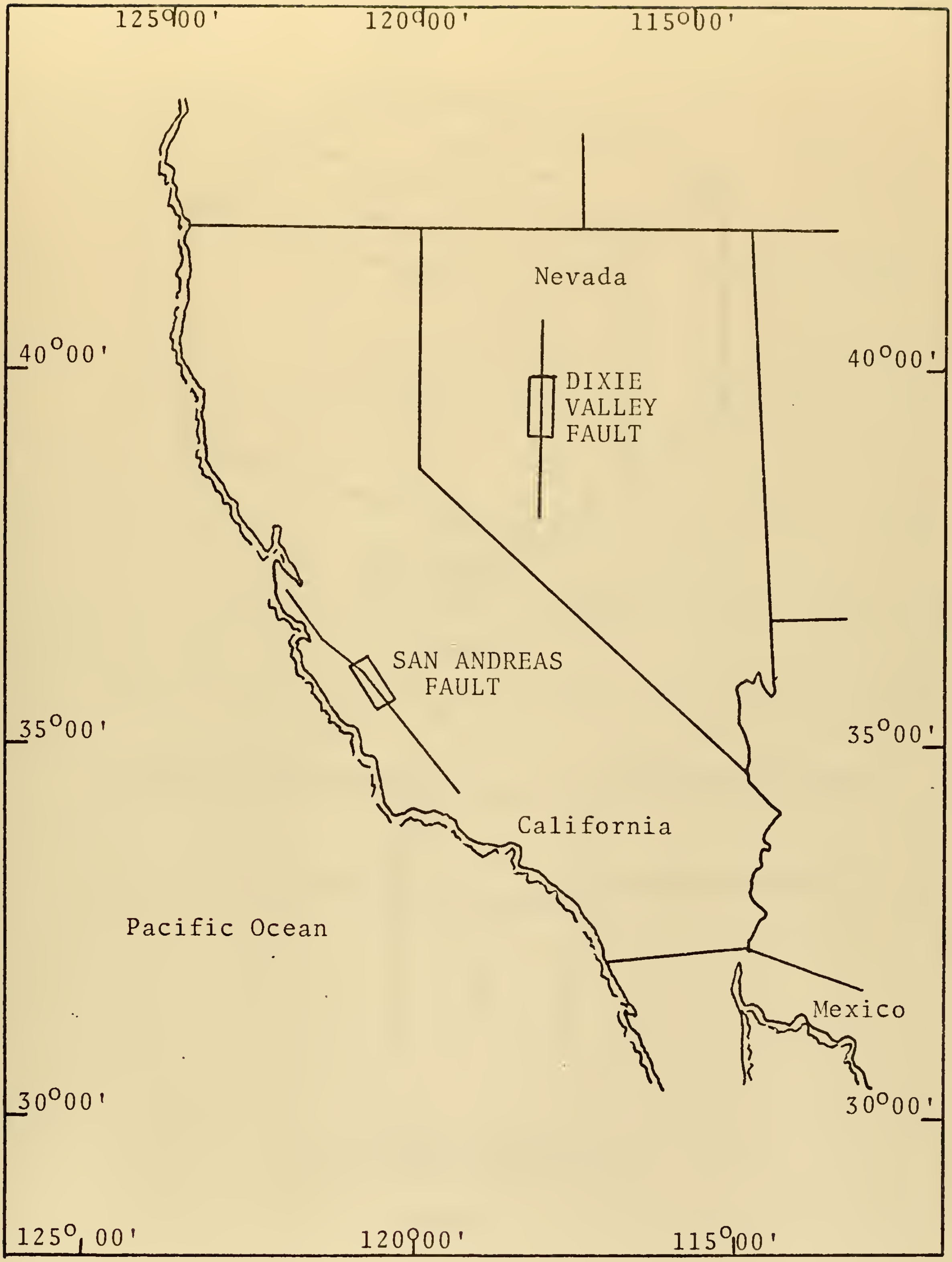
It should be noted here that the foregoing definitions of the normal and axial components of the tidal forces usually precludes their being exactly tangent to the surface of the Earth owing to the fact that the Earth in not a perfect sphere. Since this angular deviation from tangency was so small and was nearly constant over the small range in latitude of the regions investigated, no correction was made in calculating component values. This correction should cancel

out during the comparisons of component frequency distributions and, therefore, have little influence on the results of the comparisons.

In selecting regions for investigation, zones of high seismic activity were desired so that relatively small regions could be selected and still yield a sufficient number of earthquake occurrences to support a detailed analysis. The regions chosen were small sections of the San Andreas fault in Central California and the Dixie Valley fault in Western Nevada, both of which are located in the Western United States (Figure 1). These two regions were chosen because of their differing fault structure. The San Andreas fault is a strike-slip fault where the land mass to the southwest side of the fault moves northward in relation to the land mass on the northeast side of the fault (Figure 2). In the particular region of the San Andreas fault investigated, the strike of the fault is approximately 40 degrees west of north as measured from longitude 121 W. The Dixie Valley fault is a graben fault where the material in the center of the fault zone moves downward in relation to the sides of the fault (Figure 3). The strike of this fault is approximately parallel to longitude 118 W. Co-ordinates of these regions are given in Table 1.

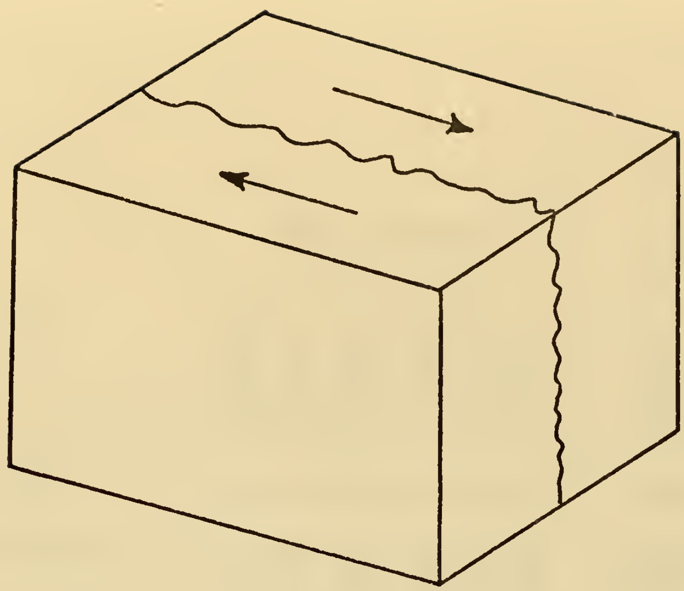
In choosing a time period over which earthquakes were to be analyzed, a large enough time period was needed so that bias due to the periodic fluctuation of the Earth tides (monthly, annually, etc.) would be damped or averaged out.

This was needed so that differences found in the comparison of these quantities would not be attributable to the cyclic nature of the tides. For the specified portion of the San Andreas fault, 1732 earthquakes of Richter magnitude 0.0 or greater occurred in the three year period from January 1, 1969, to December 31, 1971. For the portion of the Dixie Valley fault, the available data prior to 1962 was for shocks of magnitude 4.0 or greater, and from 1962 on, for shocks of magnitude 0.0 or greater. So as not to unduly bias the distribution of data with the increased frequency of recorded small shocks after 1962, only the time period through 1961 was considered. During the seven year period from August 31, 1954 (the start of detailed recording in this area), to December 31, 1961, 100 earthquakes of magnitude 4.0 or greater occurred.



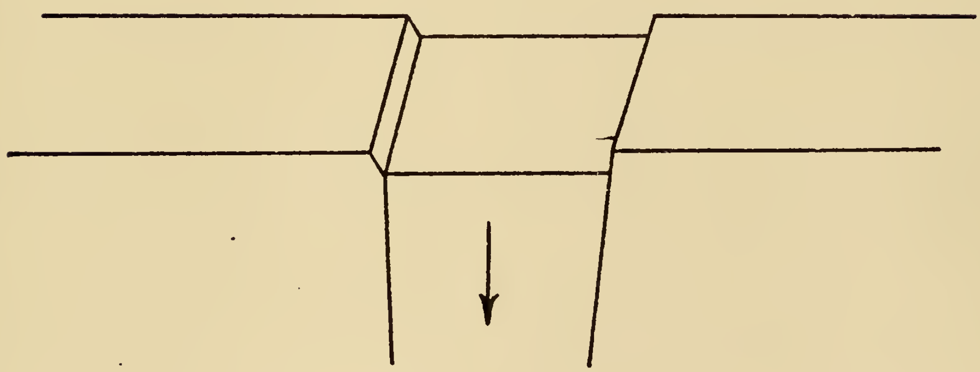
Fault Locations
Western United States

Figure 1



San Andreas
Strike-Slip Fault Structure

Figure 2



Dixie Valley
Graben Fault Structure

Figure 3

	San Andreas	Dixie Valley
Corners of Area	121 W 18, 36 N 54	117 W 48, 39 N 48
	121 W 30, 36 N 42	118 W 18, 39 N 48
	120 W 48, 36 N 00	118 W 18, 38 N 54
	120 W 36, 36 N 12	117 W 48, 38 N 54
Center of Area	121 W 03, 36 N 27	118 W 03, 39 N 21
Difference Points	121 W 36, 35 N 54	118 W 30, 39 N 21
	120 W 36, 36 N 54	117 W 30, 39 N 21
N-S Comparison	121 W 18, 36 N 54	118 W 03, 39 N 48
	120 W 48, 36 N 00	118 W 03, 38 N 54
W-E Comparison	121 W 30, 36 N 42	118 W 18, 39 N 21
	120 W 36, 36 N 12	117 W 48, 39 N 21

Area Co-ordinates

Table 1

IV. EARTH TIDE DISCUSSION

The actual tidal data were generated through the use of a computer routine (unpublished) by Professor R. H. Shudde, Naval Postgraduate School. Much work has been accomplished over the past few years on the subject of theoretical Earth tides. Melchior [Ref. 4] has one of the better discussions on the subject. He goes into great detail concerning the computation of the theoretical Earth tides using an equipotential surface argument. His procedure determines the locations of the Sun and the Moon in relation to the Earth at a particular time and establishes the potential surfaces for the tides. Then by specifying the location of a point on the Earth in relation to these surfaces, he determines the tides at that location. Shudde's program takes a different approach. It also determines the locations of the Sun and the Moon using the procedures in reference 5, but then at a particular point on the Earth, the tides are calculated directly using the standard gravitational force laws. For a more detailed description of the Earth tide calculations, see the discussion in Appendix A. It will suffice here to say that all Shudde's program requires for inputs are the time, date, and geographic co-ordinates of an earthquake occurrence. For this analysis, the times and locations for the actual occurrences were taken from references 6-8 and from data supplied by the National Center for Earthquake Research. The dates for the random occurrences were generated uniformly over the time periods

specified in Section III. The location for these random events was taken to be the center of the region being investigated.

V. ANALYSIS

The objective of this analysis was to test the hypothesis,

$$H_o: A = R$$

where A is the distribution of data from the actual events, and R is the distribution of the corresponding data from the randomly generated events. Rejection of this hypothesis for a particular force, rate, or difference would indicate a difference in the two distributions being compared and, hence, a dependency between that quantity and the occurrence of earthquakes.

The primary statistical test used to test this hypothesis was the Kolmogorov-Smirnov (K-S), Two-Sample test. This is a non-parametric test independent of distributional assumptions about the data other than the distribution be continuous. It is an exact test and operates by the comparison with the actual data rather than with grouped data. A second test used was the Chi-Square Goodness-of-Fit test which operates by the comparison of grouped data.

> The K-S statistic "possesses the advantage that it is an exact method, whereas the Chi-Square method requires a fairly large sample to justify the approximations that are needed to apply it" [Hoel, 1971].

The Chi-Square test was used because this is the common statistical test used in many studies. The K-S test was used because it is a more powerful test and was more appropriate to the data.

A. KOLMOGOROV-SMIRNOV, TWO-SAMPLE TEST

This test was implemented through the use of the IBM

computer subroutine, KOLM2 [Ref. 10], with a modified sort routine [Ref. 11]. Using these routines, for each of the five tidal, rate, and difference quantities, the distribution of data from the actual earthquakes in a region was compared to the distribution for that quantity calculated from the randomly generated events. The two sets of data for each distribution were each sorted into increasing order and formed into sample distribution functions. The maximum absolute ordinate difference between the two distributions was determined. A sample size adjustment factor was calculated by taking the square root of the ratio of the sample size product to the sample size sum. The K-S, two-sample statistic, Z, was then calculated by taking the product of the maximum difference and the adjustment factor. That is,

$$Z = \max \left| A(x) - S(x) \right| \times \sqrt{\frac{N_1 \times N_2}{N_1 + N_2}}$$

where A(x) is the distribution function from the actual events, S(x) is the distribution function from the random events, N_1 is the number of events in A(x), and N_2 is the number of events in S(x). The use of the adjustment factor is necessary in the K-S test. If not applied to the statistic as shown above, the inverse of the factor would need to be applied to the critical values listed in Table 2 to keep the test statistic consistent with the critical values. In this case, application of the adjustment factor to the statistic simplified the use of the computer routines and enabled the critical values to remain constant for a given level of significance.

B. CHI-SQUARE GOODNESS-OF-FIT TEST

For each tidal quantity, the maximum range covering both the tidal data from the actual earthquakes and the data from the randomly generated events were divided into 20 equal intervals and the data was grouped into these intervals. Adjacent intervals were collected together until the expected frequency in the collected intervals exceeded three [Ostle, 1963]. The Chi-Square statistic was then calculated as

$$\chi^2 = \sum_{i=1}^{m'} \frac{(O_i - E_i)^2}{E_i}$$

where O_i is the frequency in interval, i, from the actual events, E_i is the expected frequency in interval, i, and m' is the number of collected intervals. Selected critical values for the Chi-Square test are given in Table 3.

Level of Significance	Value of Z So Large as to Call for Rejection of the H_o
.10	1.22
.05	1.36
.025	1.48
.01	1.63
.005	1.73

Adapted from Beyer, p. 429.

Critical Values of the
K-S, Two-Sample, Statistic, Z

Table 2

Degrees of Freedom	.10	.05	.025	.01	.005
11	17.3	19.7	21.9	24.7	26.8
12	18.5	21.0	23.3	26.7	28.3
13	19.8	22.4	24.7	27.7	29.8
14	21.1	23.7	26.1	29.1	31.3
15	22.3	25.0	27.5	30.6	32.8
16	23.5	26.3	28.8	32.0	34.3
17	24.8	27.6	30.2	33.4	35.7
18	26.0	28.9	31.5	34.8	37.2
19	27.2	30.1	32.9	36.2	38.6
20	28.4	31.4	34.2	37.6	40.0

Levels of Significance

Adapted from Beyer, p. 294.

Critical Values of the
Chi-Square Statistic

Table 3

VI. RESULTS

In the analysis of the tidal data, some bias in the distributions of the data was expected due to the physical distance between the origin of the actual earthquakes and the geometric center of the region at which the random events were generated. Although this bias was felt to be minimal due to the small size of the regions and the large number of events, additional checks were performed to test the size of this bias. Random events were generated at the north, south, west, and east extremes of each region and the five tidal and five rate quantities were calculated at each location for each occurrence. The ten quantities at the north and south extremes were then compared using the K-S test, as were the ten quantities from the west and east extremes. The results are shown in Tables 4 and 5. The conclusion was that bias in the data due to the physical separation of the events was so slight as to have an insignificant influence on the final analysis. It must be stressed here that the significance of the following results pertains only to the particular region and the nature and orientation of the particular fault being analyzed.

A. SAN ANDREAS

The K-S, two-sample statistics for each of the five tidal, five rate, and five difference quantities acting in the San Andreas region are shown in Table 6 and the Chi-Square statistics for these quantities are shown in Table 7.

Referring to Tables 2 and 3 for a 0.05 level of significance, the normal tidal and gravitational rate quantities were statistically significant under both tests. None of the tidal differences was statistically significant at this level in either test.

B. DIXIE VALLEY

The K-S, two-sample statistics for each of the five tidal, five rate, and five difference quantities acting in the Dixie Valley region are shown in Table 8 and the Chi-Square statistics for those quantities are shown in Table 9. Referring to Tables 2 and 3 for a 0.05 level of significance, the normal rate quantity was statistically significant under both tests. None of the tidal difference quantities was significant at this level in either test.

After much of this analysis had been completed, a similarity between the methods of calculating the rate and difference quantities was observed. The rate quantities were calculated by computing the tidal quantities at a location, allowing a small unit of time to elapse, and then computing the tidal quantities again. The change in the tidal quantity divided by the change in time was used as its rate of change. Because of the rotation of the Earth, changes in time correspond to changes in location at a fixed latitude. A lapse of four minutes is equivalent to a distance change of one degree in longitude. For small time lapses, the distance considered moved is negligible. For larger time lapses, the calculations can be considered as being at two separated locations. The

difference quantities were calculated by computing simultaneously the tidal quantities at two locations separated by one degree in longitude. Since both procedures involve the calculation of components at separated locations, there should have been a correlation between the test results on the rate quantities and the test tesults on the difference quantities. Comparison of the San Andreas results in Table 6 showed some similarity but the interpretation was clouded in that the tangential components in this region were rotated through an angle of 40 degrees in order to align them with the strike of the fault. For the region in which the tangential components had not been rotated, comparison of the Dixie Valley results in Table 8 showed a marked agreement in the order of the significance of the results. Therefore, these interpretations of the results could have been made from a single analysis of the rate quantities without doing the difference calculations.

Component	K-S Statistic N-S	W-E
Gravitational Tide	.77	.44
Normal Tide	.33	.25
Axial Tide	.39	.28
Tangent Magnitude	.32	.23
Total Magnitude	.39	.25
Gravitational Tide Rate	.42	.36
Normal Tide Rate	.21	.20
Axial Tide Rate	.25	.23
Tangent Magnitude Rate	.34	.29
Total Magnitude Rate	.53	.42

Results of Bias Check in Tidal Comparisons
N-S and W-E Extremes of the San Andreas Area
Using the K-S Statistic

Table 4

Component	K-S Statistic N-S	W-E
Gravitational Tide	.91	.17
Normal Tide	.28	.16
Axial Tide	.48	.20
Tangent Magnitude	.32	.16
Total Magnitude	.36	.15
Gravitational Tide Rate	.46	.17
Normal Tide Rate	.29	.17
Axial Tide Rate	.26	.15
Tangent Magnitude Rate	.30	.20
Total Magnitude Rate	.61	.16

Results of Bias Check in Tidal Comparisons
N-S and W-E Extremes of the Dixie Valley Area
Using the K-S Statistic

Table 5

Component	Statistic
Gravitational Tide	1.1411
Normal Tide	1.5038
Axial Tide	1.1776
Tangent Magnitude	1.3344
Total Magnitude	0.8658
Gravitational Tide Rate	1.6515
Normal Tide Rate	0.6126
Axial Tide Rate	0.8469
Tangent Magnitude Rate	0.9541
Total Magnitude Rate	1.2273
Gravitational Tide Difference	1.1724
Normal Tide Difference	1.2752
Axial Tide Difference	0.9117
Tangent Magnitude Difference	1.2259
Total Magnitude Difference	1.1985

Results of Actual Tidal Comparisons in the San Andreas Area Using the K-S Statistic

Table 6

Component	Statistic	Degrees of Freedom
Gravitational Tide	40.22	19
Normal Tide	96.84	18
Axial Tide	36.99	17
Tangent Magnitude	23.37	18
Total Magnitude	34.33	18
Gravitational Tide Rate	114.21	19
Normal Tide Rate	38.38	18
Axial Tide Rate	23.48	17
Tangent Magnitude Rate	38.21	18
Total Magnitude Rate	81.64	17

Results of Actual Tidal Comparisons in the San Andreas Area Using the Chi-Square Statistic

Table 7

Component	Statistic
Gravitational Tide	1.2218
Normal Tide	0.6278
Axial Tide	0.8792
Tangent Magnitude	0.7129
Total Magnitude	0.6139
Gravitational Tide Rate	0.6535
Normal Tide Rate	1.4119
Axial Tide Rate	0.5901
Tangent Magnitude Rate	0.4673
Total Magnitude Rate	0.9228
Gravitational Tide Difference	0.5743
Normal Tide Difference	1.2654
Axial Tide Difference	0.6040
Tangent Magnitude Difference	0.5822
Total Magnitude Difference	1.1605

Results of Actual Tidal Comparisons in the Dixie Valley Area Using the K-S Statistic

Table 8

Component	Statistic	Degrees of Freedom
Gravitational Tide	11.22	13
Normal Tide	6.90	14
Axial Tide	26.30	14
Tangent Magnitude	15.04	15
Total Magnitude	10.47	11
Gravitational Tide Rate	7.34	14
Normal Tide Rate	25.47	14
Axial Tide Rate	5.99	15
Tangent Magnitude Rate	3.91	12
Total Magnitude Rate	13.72	13

Results of Actual Tidal Comparisons in the Dixie Valley Area Using the Chi-Square Statistic

Table 9

VII. CONCLUSIONS

Results of the analysis indicate that, even at high levels of significance, there exist components of the Earth tides and their rates-of-change that are significantly related to the occurrence of earthquakes and that these components vary with the nature and orientation of the fault being investigated.

The significance of the normal tidal component on the strike-slip San Andreas fault gives credence to the theory that the occurrence of earthquakes in this fault zone may result from the lessening of the frictional force acting between the two vertical faces of the fault. Since the frictional force acting between two surfaces is a function of the normal force acting on those surfaces, a lessening of the normal force could decrease the frictional force enough to allow the accumulated stress in the fault to be released.

The relationship between the rate-of-change of a tidal force and the difference in a force over the breadth of a fault has been discussed previously. Although no difference quantities were considered significant at a 0.05 level of significance, reference to Tables 2, 6 and 8 reveal that these quantities did differ considerably and could have been considered significant at lower levels of significance. The significance of the gravitational rate in the region of the San Andreas fault indicates a differential in the vertical forces acting on opposite sides of the fault. This significance is difficult to interpret in view of the strike-slip nature of

the fault. Because of the proximity of this fault to the Pacific Ocean, this significance may indicate the presence of ocean tidal loading effects on the occurrence of earthquakes in this region. The significance of the normal rate in the region of the Dixie Valley fault indicates a differential in the east-west force acting on opposite sides of this fault. This interpretation is consistent with the nature of the Dixie Valley fault in that the downward slippage of the center of the fault zone occurs with the separation of the sides of the fault.

APPENDIX A

A. OBSERVATIONS ON ASSOCIATED LEGENDRE POLYNOMIALS.

The generating function for the associated Legendre polynomial of the m^{th} order is

$$\frac{(2m)!\,(1-x^2)^{\frac{m}{2}}\,y^m}{2^m m!\,(1-2xy+y^2)^{m+\frac{1}{2}}} = \sum_{\ell=m}^{\infty} y^{\ell}\cdot P_\ell^m(x) \qquad (1)$$

Other useful relationships are

$$P_\ell^m(x) = (1-x)^{\frac{m}{2}} \frac{d^m}{dx^m} P_\ell(x) \qquad (2)$$

$$P_{\ell+1}(x) = \frac{1}{\ell+1}\left[(2\ell+1)(x)P_\ell(x) - \ell\cdot P_{\ell-1}(x)\right] \qquad (3)$$

Letting m=1 in (1) and rearranging terms,

$$(1-2xy+y^2)^{-\frac{3}{2}} = (1-x^2)^{-\frac{1}{2}} \sum_{\ell=1}^{\infty} y^{\ell-1}\cdot P_\ell'(x) \qquad (4)$$

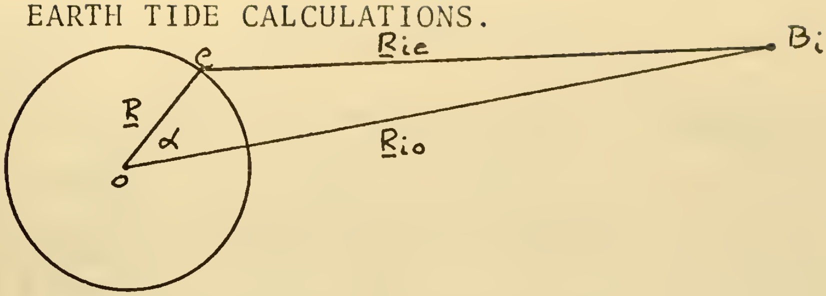
From (2),

$$\begin{aligned}(1-x^2)^{-1/2} P_1'(x) &= 1 \\ (1-x^2)^{-\frac{1}{2}} P_2'(x) &= 3x \\ (1-x^2)^{-\frac{1}{2}} P_3'(x) &= \tfrac{3}{2}(5x^2-1) \\ (1-x^2)^{-1/2} P_4'(x) &= \tfrac{5}{2}(7x^3-3x) \\ &\vdots\end{aligned} \qquad (5)$$

From (3),

$$\begin{aligned}P_0(x) &= 1 \\ P_1(x) &= x \\ P_2(x) &= \tfrac{1}{2}(3x^2-1) \\ P_3(x) &= \tfrac{1}{2}(5x^3-3x) \\ P_4(x) &= \tfrac{1}{8}(35x^4-30x^2+3) \\ &\vdots\end{aligned} \qquad (6)$$

B. EARTH TIDE CALCULATIONS.



$\underline{R}$ = Earth radius vector to epicenter

a = Scalar magnitude of $\underline{R}$

$\underline{R}_{ie}$ = Distance vector from epicenter to B_i

r_{ie} = Scalar magnitude of $\underline{R}_{ie}$

$\underline{R}_{io}$ = Distance vector from origin to B_i

r_{io} = Scalar magnitude of $\underline{R}_{io}$

B_i = Extra-terrestrial body, i=1 (moon), 2 (Sun)

$\underline{F}_{ie}$ = Force vector between the origin and B_i

$\underline{F}_{io}$ = Force vector between the origin and B_i

α_i = Angle between $\underline{R}$ and $\underline{R}_{io}$

m_i = mass of B_i

From Newton's laws of gravitation,

$$\underline{F}_{ie} = \frac{k \cdot m_i \cdot \underline{R}_{ie}}{r_{ie}^3} \qquad \underline{F}_{io} = \frac{k \cdot m_i \cdot \underline{R}_{io}}{r_{io}^3}$$

The total tidal force vector, $\underline{T}$, is the sum over all bodies

$$\underline{T} = \sum_{i=1}^{2} \underline{T}_i$$

where $\underline{T}_i = \underline{F}_{io} - \underline{F}_{ie} = k \cdot m_i \left[\frac{\underline{R}_{io}}{r_{io}^3} - \frac{\underline{R}_{ie}}{r_{ie}^3} \right]$

Since $\underline{R}_{ie} = \underline{R}_{io} - \underline{R}$, $\underline{T}_i = \frac{k \cdot m_i}{r_{io}^3} \left[\underline{R}_{io} - (\underline{R}_{io} - \underline{R}) \left(\frac{r_{io}}{r_{ie}} \right)^3 \right]$ (7)

With $r_{ie}^2 = r_{io}^2 + a^2 - 2a \cdot r_{io} \cos(\alpha_i)$, the expression $\left(\frac{r_{io}}{r_{ie}} \right)^3$ can be written as

$$\left(\frac{r_{io}}{r_{ie}}\right)^3 = \left[1 - 2\left(\frac{a}{r_{io}}\right)\cos(\alpha_i) + \left(\frac{a}{r_{io}}\right)^2\right]^{-\frac{3}{2}} \tag{8}$$

Letting $x_i = \cos(\alpha_i)$, and $y_i = \frac{a}{r_{io}}$, a comparison of (4) with (8) yields,

$$\left(\frac{r_{io}}{r_{ie}}\right)^3 = (1-x^2)^{-\frac{1}{2}} \sum_{\ell=1}^{\infty} y^{\ell-1} \cdot P_\ell'(x)$$

$$= \sum_{\ell=1}^{\infty} \left(\frac{a}{r_{io}}\right)^{\ell-1} \frac{P_\ell'(\cos(\alpha_i))}{\sin(\alpha_i)}$$

$$= \sum_{\ell=1}^{\infty} \left(\frac{a}{r_{io}}\right)^{\ell-1} \frac{d}{d\cos(\alpha_i)} P_\ell(\cos(\alpha_i)) \tag{9}$$

Substituting (9) into (7) yields,

$$\underline{T}_i = \frac{k \cdot m_i}{r_{io}^3}\left[\underline{R}_{io} - (\underline{R}_{io} - \underline{R}) \sum_{\ell=1}^{\infty} \left(\frac{a}{r_{io}}\right)^{\ell-1} \frac{d}{d\cos(\alpha_i)} P_\ell(\cos(\alpha_i))\right]$$

Letting t_n be the component of $\underline{T}_i$ in the direction of $\underline{N}$, a unit vector in the plane normal to $\underline{R}$, and letting β_i be the angle between $\underline{N}$ and $\underline{T}_i$, results in the general expression,

$t_n = \underline{N} \cdot \underline{T}_i$, and after rearranging terms,

$$= -\frac{k \cdot m_i}{r_{io}^2} \cos(\beta_i) \sum_{\ell=2}^{\infty} \left(\frac{a}{r_{io}}\right)^{\ell-1} \frac{d}{d\cos(\alpha_i)} P_\ell(\cos(\alpha_i)) \tag{10}$$

Expanding terms in (10) and substituting values from (5),

$$t_n = -\frac{k \cdot m_i}{r_{io}^2} \cos(\beta_i) \left[\left(\frac{a}{r_{io}}\right) \frac{d}{d\cos(\alpha_i)} P_2(\cos(\alpha_i)) + \left(\frac{a}{r_{io}}\right)^2 \frac{d}{d\cos(\alpha_i)} P_3(\cos(\alpha_i)) + \cdots\right]$$

$$= -\frac{k \cdot m_i \cdot a}{r_{io}^3} \cos(\beta_i)\left[3\cos(\alpha_i) + \frac{3}{2}\left(\frac{a}{r_{io}}\right)\left(5\cos^2(\alpha_i) - 1\right) + \cdots\right] \tag{11}$$

Noting that $\cos(\alpha_i) = \frac{\underline{R} \cdot \underline{R}_{io}}{a \cdot r_{io}} = \frac{x_i}{r_{io}}$,

if $\underline{N}$ is taken to be the north(z) direction, (11) gives t_z

with $\cos(\beta_i) = \frac{z_i}{r_{io}}$, and if $\underline{N}$ is taken to be the east(y) direction, (11) gives t_y with $\cos(\beta_i) = \frac{y_i}{r_{io}}$. If the co-ordinate system is oriented so that $\underline{R}$ is in the x-direction, then $t_x = t_a = \frac{\underline{R} \cdot \underline{T}_i}{a}$

$$= \frac{K \cdot m_i}{a \cdot r_{io}^3} \left\{ a \cdot r_{io} \cdot \cos(\alpha_i) - (a \cdot r_{io} \cdot \cos(\alpha_i) - a^2) \left[1 + \sum_{\ell=2}^{\infty} \left(\frac{a}{r_{io}}\right)^{\ell-1} \cdot \frac{d}{d\cos(\alpha_i)} P_\ell(\cos(\alpha_i)) \right] \right\}$$

$$= \frac{k \cdot m_i \cdot a}{r_{io}^3} \left\{ 1 + \left[\left(\frac{a}{r_{io}}\right) - \cos(\alpha_i) \right] \sum_{\ell=2}^{\infty} \left(\frac{a}{r_{io}}\right)^{\ell-2} \frac{d}{d\cos(\alpha_i)} P_\ell(\cos(\alpha_i)) \right\} \quad (12)$$

Expanding a few terms of (12), collecting coefficients of $\left(\frac{a}{r_{io}}\right)^j$, where j=0,1, and substituting values from (6),

$$t_x = -\frac{K \cdot m_i a}{r_{io}^3} \left[3\cos^2(\alpha_i) - 1 + \frac{3}{2}\left(\frac{a}{r_{io}}\right)\left(5\cos^3(\alpha_i) - 3\cos(\alpha_i)\right) + \cdots \right]$$

with $\cos(\alpha_i) = \frac{x_i}{r_{io}}$. For tangential components in other than the y and z directions, $\underline{N}$ can be any unit vector in the plane normal to the vector $\underline{R}$.

BIBLIOGRAPHY

1. Knopoff, L., "Earth Tides as a Triggering Mechanism for Earthquakes," Bulletin of the Seismological Society of America, v. 54(6), p. 1865-1870, December, 1964.

2. Shlien, S., "Earthquake-Tide Correlation," Geophysical Journal of the Royal Astronomical Society, v. 28, p. 27-34, 1972.

3. Pariyskiy, N. N., Artamasova, G. N., and Kramer, M. V., "On the Problem of the Role of Tidal Stresses as the Trigger Mechanism in Earthquakes," Akademiia Nauk SSSR, Inst. Fiziki Zemli (Izdatel'stvo "Nauka"), p. 62-63, 1970.

4. Melchior, P. J., The Earth Tides, p. 458, Pergamon Press, 1966.

5. Air Force Cambridge Research Laboratories Report AFRLL-71-0049, Prediction and Analysis of Solar Eclipse Circumstances, by W. Williams, Jr., 25 March 1971.

6. Lee, W. H. K., and others, Catalog of Earthquakes along the San Andreas Fault System in Central California for the Year 1969, National Center for Earthquake research, 1972.

7. Lee, W. H. K., and others, Catalog of Earthquakes along the San Andreas Fault System in Central California for the Year 1970, National Center for Earthquake Research, 1972.

8. Lee, W. H. K., and others, Catalog of Earthquakes along the San Andreas Fault System in Central California for the Year 1971, National Center for Earthquake Research, 1972.

9. Hoel, P. G., Port, S. C., and Stone, C. J., Introduction to Statistical Theory, p. 168-171, Houghton-Mifflin Co., 1971.

10. IBM Application Program, GH 20-0205-4, System/360 Scientific Subroutine Package, Version III, Programmer's Manual, 360A-CM-03X, 5th ed., p. 65-67, August, 1970.

11. Boothrood, J., "Shellsort," Collected Algorithms from the Communications of the Association for Computing Machinery, 1959.

12. Ostle, B., Statistics in Research, 2d ed., p. 126-127, The Iowa State College Press, 1963.

13. Beyer, W. H., Handbook of Tables for Probability and Statistics, 2d ed., p. 427-429, The Chemical Rubber Co., 1968.

14. Goguel, J., Tectonics, p. 124-128, W. H. Freeman Co., 1962.

INITIAL DISTRIBUTION LIST

		No. of Copies
1.	Defense Documentation Center Cameron Station Alexandria, Virginia, 22314	2
2.	Library, Code 0212 Naval Postgraduate School Monterey, California, 93940	2
3.	Assoc. Professor R. H. Shudde, Code 55Su Department of Operations Research and Administrative Sciences Naval Postgraduate School Monterey, California, 93940	1
4.	Assoc. Professor D. R. Barr, Code 55Bn Department of Operations Research and Administrative Sciences Naval Postgraduate School Monterey, California, 93940	1
5.	Chief of Naval Personnel Pers 11b Department of the Navy Washington, D. C., 20370	1
6.	Department of Operations Research and Administrative Sciences Naval Postgraduate School Monterey, California, 93940	1
7.	Dr. M. D. Wood, Ph.D. National Center for Earthquake Research 345 Middlefield Road Menlo Park, California, 94025	1
8.	CPT Richard A. Corradini, USA Engineer Strategic Studies Group 6500 Brooks Lane Washington, D. C., 20315	2

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) The Variation of Earth Tides as a Triggering Mechanism in Earthquakes		5. TYPE OF REPORT & PERIOD COVERED Master's Thesis; September 1973
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Richard Arthur Corradini		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Naval Postgraduate School Monterey, California, 93940		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS Naval Postgraduate School Monterey, California, 93940		12. REPORT DATE September 1973
		13. NUMBER OF PAGES 40
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office) Naval Postgraduate School Monterey, California, 93940		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Earthquakes
Earth Tides
Tidal Variation
Triggering Mechanism

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Earthquake data from Central California over the period January 1, 1969, to December 31, 1971, and from Western Nevada over the period August 31, 1954, to December 31, 1961, have been analyzed to investigate the hypothesis that the Earth tides may be the triggering mechanism for the occurrence of earthquakes. One method used compared the tidal components and the component rates-of-change at the time and location of actual earthquakes

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE
(Page 1) S/N 0102-014-6601

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

Block 20 continued

to those for random events uniformly generated over the same time period. A second method compared the differences in the Earth tides computed across the breadth of a fault zone at the time of actual earthquakes to the differences at the times of random events uniformly generated over the same time period. Statistical tests were performed to see if the distributions from the actual events were the same as the corresponding distributions from the randomly generated events.

Thesis 146539
C7554 Corradini
c.2 The variation of earth
tides as a triggering
mechanism in earthquakes.