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by

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ABSTRACT

Two-beam deckhouse theory including shear deflection is developed. Estimated value of deck shear-lag factor "r" is included in the theory. The longitudinal stress distribution is obtained using the developed theory and Bleich's method. The results are compared at midship and at the end of deckhouse. The agreement between the results at midship is reasonable. The difference between the results at the end of deckhouse is more distinguishable.

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NOMENCLATURE

A_1	cross sectional area of deckhouse
A_{1w}	section area of deckhouse (webs only)
A_2	cross sectional area of hull
A_{2w}	section area of hull (webs only)
a	Distance between centers of gravity of hull and deckhouse
I	total moment of inertia of structure cross section
I_A	factor for determining I
I_1	moment of inertia of deckhouse cross section
I_2	moment of inertia of hull cross section
K	deck stiffness
ℓ	length of deckhouse
L	length of hull
2b	beam of hull
M	constant moment
M_p	moment in the midship section due to the loads p_1 and p_2
(p_1+p_2)	equally distributed loads (load/unit length)
x_1	vertical distance from center of gravity of deckhouse cross-section
x_2	vertical distance from center of gravity of hull cross section
x	vertical distance from center of gravity of entire section
z	horizontal distance from amidship
y	transverse coordinate distance from centerline

α_1	ratio of the distance of center of gravity of deck-house from main deck
α_2	ratio of the distance of center of gravity of hull from main deck
E	Young's modulus
G	modulus of elasticity in shear ($= \frac{E}{2(1+\mu)}$)
μ	Poisson's ratio
σ_1	longitudinal stress in deckhouse
σ_2	longitudinal stress in hull
r	shear-lag factor

PARAMETERS USED FOR COMPUTATION

$$r = \frac{1}{2} \frac{1}{\cosh \frac{\pi b}{\ell}} \left(\frac{\pi y}{\ell} \sinh \frac{\pi y}{\ell} + 2 \cosh \frac{\pi y}{\ell} - \frac{\pi b}{\ell} \tanh \frac{\pi b}{\ell} \cosh \frac{\pi y}{\ell}\right)$$

$$I_A = a^2 \frac{A_1 A_2}{r A_1 + A_2}$$

$$I = I_1 + I_2 + I_A$$

$$E^* = \left(\frac{1}{2}\right)\left(\frac{E}{G}\right) \left[\frac{(I_1 + \alpha_1^2 I_A)^2}{A_1} + \frac{(\alpha_1 \alpha_2 I_A)^2}{A_2}\right]$$

$$F^* = \left(\frac{1}{2}\right)\left(\frac{E}{G}\right) \left[\frac{(r \alpha_1 \alpha_2 I_A)^2}{A_1} + \frac{(I_2 + r \alpha_2^2 I_A)^2}{A_2}\right]$$

$$\frac{G^*}{2} = \left(\frac{1}{2}\right)\left(\frac{E}{G}\right) (\alpha_1 \alpha_2 I_A) \left[\frac{r(I_1 + \alpha_1^2 I_A)}{A_1} + \frac{(I_2 + r \alpha_2^2 I_A}{A_2}\right.$$

Non-dimensional parameters

$$b = \frac{r \alpha_1 \alpha_2 I_A}{\ell^4}$$

$$a_{14} = \frac{I_1 + \alpha_1^2 I_A}{\ell^4}$$

$$a_{24} = \frac{I_2 + r \alpha_2^2 I_A}{\ell^4}$$

$$a = \left(\frac{1}{2}\right)\left(\frac{E}{G}\right)(b)\left(\frac{a_{14}}{A_1} + \frac{a_{24}}{r A_2}\right)$$

$$a_{16} = (\frac{1}{2})(\frac{E}{G})(\frac{a_{14}^2}{A_1} + \frac{b^2}{r^2 A_2})$$

$$a_{26} = (\frac{1}{2})(\frac{E}{G})(\frac{b^2}{A_1} + \frac{a_{24}^2}{A_2})$$

$$a^* = (a_{16}\, a_{26} - a^2)$$

$$a_3 = (-\, a_{16}\, a_{24} - a_{14}\, a_{26} + 2ab)/a^*$$

$$a_2 = (a_{14}\, a_{24} - b^2)/a^*$$

$$a_1 = \bar{K}(-\, a_{16} - a_{26} - 2a)/a^*$$

$$a_0 = \bar{K}(a_{14} + a_{24} + 2b)/a^*$$

$$f = \gamma_1^2 + \gamma_2^4 - 6\, \gamma_1^2\, \gamma_2^2$$

$$g = 4\gamma_1\, \gamma_2^3 - 4\, \gamma_1^3\, \gamma_2$$

$$h = \gamma_1^6 - \gamma_2^6 + 15\, \gamma_1^2\, \gamma_2^4 - 15\, \gamma_1^4\, \gamma_2^2$$

$$m = -6\, \gamma_1^5\, \gamma_2 - 6\, \gamma_1\, \gamma_2^5 + 20\, \gamma_1^3\, \gamma_2^3$$

$$n_1 = \gamma_1^2 - \gamma_2^2$$

$$n_2 = 2\, \gamma_1\, \gamma_2$$

$$r_1 = \gamma_1^3 - 3\, \gamma_1\, \gamma_2^2$$

$$r_2 = \gamma_2^3 - 3\, \gamma_1^2\, \gamma_2$$

$$s_1 = \gamma_1^5 + 5\, \gamma_1\, \gamma_2^4 - 10\, \gamma_1^3\, \gamma_2$$

$$s_2 = -5\, \gamma_1^4\, \gamma_2 - \gamma_2^5 + 10\, \gamma_1^2\, \gamma_2^3$$

$$t_1 = a_{16} r_1 + a r_1 \mu_1 - a r_2 \mu_2$$

$$t_2 = a_{16} r_2 + a r_1 \mu_2 + a r_2 \mu_1$$

$$t_3 = a r_1 + a_{26} r_1 \mu_1 - a_{26} r_2 \mu_2$$

$$t_4 = a r_2 + a_{26} r_1 \mu_2 + a_{26} r_2 \mu_1$$

$$t_5 = - a_{16} f - a f \mu_1 + a g \mu_2 + a_{14} n_1 + b n_1 \mu_1 + b n_2 \mu_2$$

$$t_6 = - a_{16} g - a f \mu_2 - a g \mu_1 - a_{14} n_2 + b n_1 \mu_2 - b n_2 \mu_1$$

$$t_7 = - af - a_{26} f \mu_1 + a_{26} g \mu_2 + b n_1 + a_{24} n_1 \mu_1 + a_{24} n_2 \mu_2$$

$$t_8 = - a g - a_{26} f \mu_2 - a_{26} g \mu_1 - b n_2 + a_{24} n_1 \mu_2 - a_{24} n_2 \mu_1$$

$$t_9 = a_{16} s_1 + a s_1 \mu_1 - a s_2 \mu_2 - a_{14} r_1 - b r_1 \mu_1 + b r_2 \mu_2$$

$$t_{10} = a_{16} s_2 + a s_1 \mu_2 + a s_2 \mu_1 - a_{14} r_2 - b r_1 \mu_2 - b r_2 \mu_1$$

$$v_1 = a_{16} - a \mu_3$$

$$v_2 = a_{16} - a \mu_4$$

$$v_3 = a - a_{26} \mu_3$$

$$v_4 = a - a_{26} \mu_4$$

$$v_5 = a_{14} - b \mu_3$$

$$v_6 = a_{14} - b \mu_4$$

$$v_7 = b - a_{24} \mu_3$$

$$v_8 = b - a_{24} \mu_4$$

$$v_9 = a_{14} + b$$

$$v_{10} = a_{24} + b$$

Second derivatives of the homogeneous part of the general solution,

$$\bar{y}_1'' = 2 c_2 + \sin \gamma_2 \bar{z} \sinh \gamma_1 \bar{z} (n_1 c_3 - n_2 c_4) + \cos \gamma_2 \bar{z} \cosh \gamma_1 \bar{z}(n_2 c_3 + n_1 c_4) + c_5 \gamma_3^2 \cosh \gamma_3 \bar{z} + c_6 \gamma_4^2 \cosh \gamma_4 \bar{z}$$

$$\bar{y}_2'' = 2 c_2 + \sin \gamma_2 \bar{z} \sinh \gamma_1 \bar{z}(n_1 c_3' - n_2 c_4') + \cos \gamma_2 \bar{z} \cosh \gamma_1 \bar{z}(n_2 c_3' + n_1 c_4') - \mu_3 c_5 \gamma_3^2 \cosh \gamma_3 \bar{z} - \mu_4 c_6 \gamma_4^2 \cosh \gamma_4 \bar{z}$$

Derivatives of the particular integral part of the general solutions:

$$\bar{y}_{1p}'' = \bar{y}_{2p}'' = \frac{\overline{p1} + \overline{p2}}{2(a_{14} + a_{24} + 2b)} \bar{z}^2$$

$$\bar{y}_{1p}''' = \bar{y}_{2p}''' = \frac{\overline{p1} + \overline{p2}}{(a_{14} + a_{24} + 2b)} \bar{z}$$

$$\bar{y}_{1p}^{IV} = \bar{y}_{2p}^{IV} = \frac{\overline{p1} + \overline{p2}}{(a_{14} + a_{24} + 2b)}$$

$$y_{1p}^{V} = y_{2p}^{V} = 0$$

$$y_{2p}^{VI} = y_{2p}^{VI} = 0$$

INTRODUCTION

In 1953, H. H. Bleich published a paper in the "Journal of Applied Mechanics" entitled "Non-Linear Distribution of Bending Stresses Due to Distortion of the Cross Section". In this paper he derived a viable analytical solution to the problem of hull-deckhouse interaction.

Basically, he considered the hull and the deckhouse as separate beams which are forced to act together by shearing forces and by vertical forces resisting relative displacements of the two beams. The case of constant cross section of the beam is treated, and it is assumed that Navier's hypothesis is valid for the deckhouse and hull separately. For two types of loading he considered (constant moment loading and equally distributed load), solutions were in a qualitative agreement with the test results at midship. As one moves away from amidships or the center of the deckhouse structure, solutions were departing from the reality.

It is proposed that including shear effects into Bleich's original two-beam deckhouse theory, it is possible to improve it, particularly at deckhouse ends. In order to confirm or disaprove this hypothesis, new theory which includes shear effects is developed, and the results are compared at midship and at the end of deckhouse.

The theorem of Minimum Potential Energy is used in the analysis. The variational procedure established two coupled six order differential equation systems and the natural boundary conditions. The boundary value problem is solved for the displacement and then the stress distributions.

FUNDAMENTAL ASSUMPTIONS

The two components of the combination of hull and deckhouse are treated as if each were a Navier beam which are forced to act together by shearing forces and by vertical forces.

It is assumed that the stiffness of bulkheads or deck beams resisting relative vertical displacement of the deckhouse is constant for the full length of the deckhouse, the magnitude of the stiffness being given by a spring constant K. In reality, the deck stiffness K vary along the deckhouse length due to the presence of structural bulkheads.

The possibility of having different materials for hull and deckhouse is not considered in this analysis.

Shear deformation is accounted for in the sides of both hull and deckhouse. For this purpose, the following equations are employed.

$$M_1 = - E\, I_1 (y_1'' + \frac{P_1}{A_{1w}\, G}) \qquad (1)$$

$$M_2 = -E\, I_2 (y_2'' + \frac{P_2}{A_{2w}\, G}) \qquad (2)$$

The longitudinal stress in the deck at the junction and the longitudinal stress in the deck-edge may differ. Bleich's theory neglects this. To include the shear-lag effect in the deck-edge and deckhouse side, the following assumption is used,

$$\sigma_1 = r\, \sigma_2 \qquad (3)$$

where r is the ratio of longitudinal deck stress at the junction to the stress at deck-edge.

Precise determination of "r" would require two-dimensional elastic analysis of the response of all plate elements of the section. For the present purpose, it is assumed that "r" has the same value it would have without the deckhouse, determined by a box-girder analysis of the hull alone, such as in Reference (2). According to this analysis, for a sinusoidal bending moment:

$$r = \frac{1}{2}\,\frac{1}{\cosh\frac{\pi b}{\ell}}\left(\frac{\pi y}{\ell}\sinh\frac{\pi y}{\ell} + 2\cosh\frac{\pi y}{\ell} - \frac{\pi b}{\ell}\tanh\frac{\pi b}{\ell}\cosh\frac{\pi y}{\ell}\right) \quad (4)$$

It is pointed out in Reference (4) by Shade, for a bending moment which is constant over the length of the deckhouse "r" should be taken as unity.

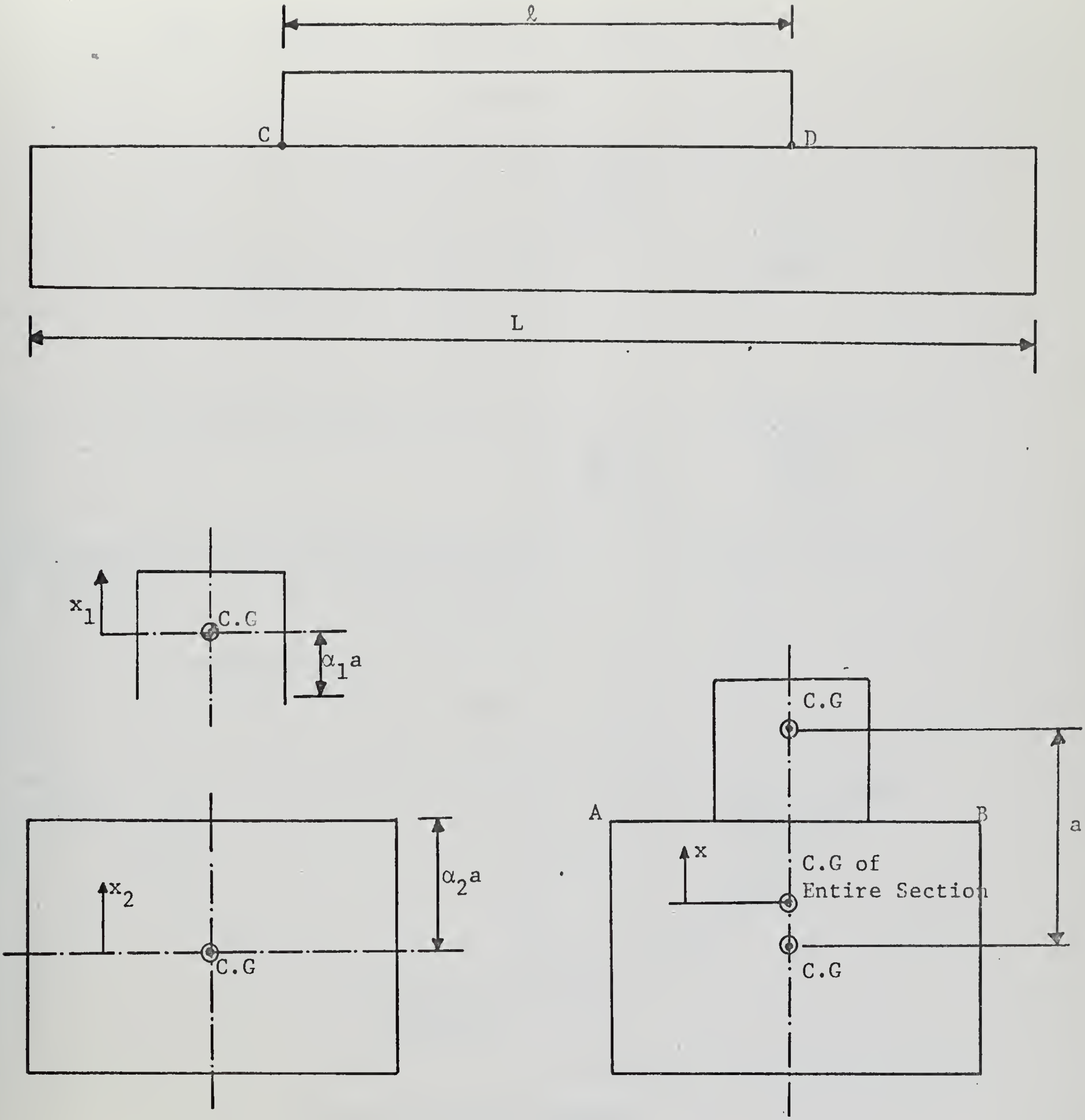
CHAPTER I - ANALYSIS OF A SIMPLIFIED TWO CELL STRUCTURE

Consider the problem of two separate beams forced to act together by horizontal shear forces and vertical forces acting at the junction of hull and deckhouse, (Figures 1 and 2). The vertical forces are due to elastic resistance of the deck framing or bulkheads against the motion of the super structure with respect to the hull. The system consists, therefore, essentially of a beam elastically supported by another beam, with a shear connection to enforce equal strains at deck level.

In this section the important simplifying assumption made is that the deck A-B, Figure 1, and its supports have no stiffness, and will not resist any relative vertical movements between hull and deckhouse. This simplification is not justified for any real ship system, but because of its relative simplicity it is easier to study the play of forces; the understanding gained is of value in treating the full problem in the following chapters.

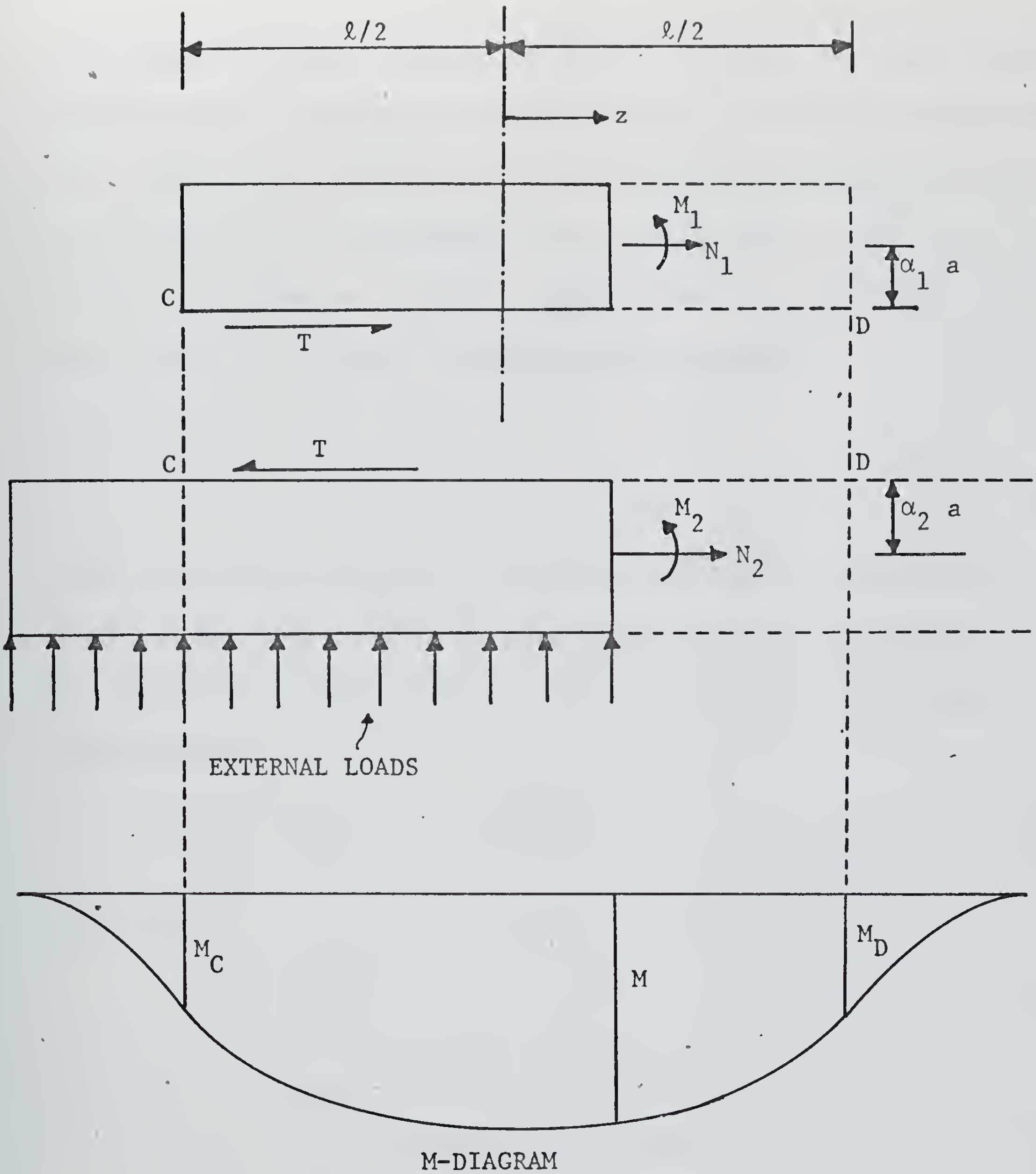
In the structure shown in Figure 1, the lower hollow box beam represents the hull and is of length L while the upper box, the deckhouse, is shorter and is of length ℓ. Both boxes are assumed to be of constant cross-section. The cross-sectional area and the moment of inertia of the deckhouse and hull are A_1, I_1, and A_2, I_2, respectively, and the distances of the respective centers of gravity from each other and from the deck are a, $\alpha_1 a$, and $\alpha_2 a$.

At a distance z in the free body diagram of Figure 2, the moment and direct forces in the deckhouse and hull are M_1, N_1, and M_2, N_2 respectively, with positive moments producing compression at the top of



TWO BEAM MODEL

FIGURE 1



TWO BEAM FREE BODY DIAGRAM

FIGURE 2

the respective units. Direct forces N_1 and N_2 are positive if they create tension. The external loads acting to the left of the section have a moment M. A shear force T of unknown magnitude will act on the underside of the deckhouse, and a similar force T will act in the opposite direction on the hull. Equilibrium of the portions of deckhouse and hull in Figure 2, requires the relations:

$$N_1 = -T, \quad M_1 = - a\,\alpha_1 T \tag{5a}$$

$$N_2 = T, \quad M_2 = M - a\,\alpha_2 T \tag{5b}$$

Owing to the assumption that Navier's hypothesis is valid for the deckhouse and hull separately, the stresses can be determined at points at a distance x_1 or x_2 from the respective center of gravities. In the deckhouse,

$$\sigma_1 = \frac{N_1}{A_1} - \frac{M_1}{I_1} x_1 \tag{6a}$$

in the hull,

$$\sigma_2 = \frac{N_2}{A_2} - \frac{M_2}{I_2} x_2 \tag{6b}$$

with tension stresses counted as positive.

At the junction of deckhouse and hull, the longitudinal stress in the deck at this junction and the longitudinal stress in the deck-edge may differ; simple beam theory implies that they are the same. Shear lag in the deck plating may modify this using the following relation,

$$\sigma_1 = r\,\sigma_2 \tag{3}$$

where r is the ratio of longitudinal deck stress at the junction to the stress at deck-edge.

Furnishing Equations (5) and (6) with $x_1 = -a\ \alpha_1$, $x_2 = a\ \alpha_2$ and using Equation (3), the value of T is found to be:

$$T = \frac{a\ \alpha_2\ I_1\ r}{\frac{I_1 I_2}{A_1} + \frac{I_1 I_2}{A_2}\ r + a^2(\alpha_2^2\ r\ I_1 + \alpha_1^2\ I_2)}\ M \qquad (7)$$

T was defined as the total horizontal shear force acting between the left end of the deckhouse and the section at z. According to Equation (7), T is proportional to the moment M, and the unit horizontal shear (dT/dz) which will be transferred by rivets or welds from the hull to the deckhouse, will be

$$\frac{dT}{dz} = \frac{a\ \alpha_2\ I_1\ r}{\frac{I_1 I_2}{A_1} + \frac{I_1 I_2}{A_2}\ r + a^2(\alpha_2^2\ r\ I_1 + \alpha_1^2\ I_2)}\ V \qquad (8)$$

where $V = \frac{dM}{dz}$, is the total shear force in the structure.

After finding the value of T using Equation (7), and introducing this into Equations (5) and (6), the longitudinal stresses σ_1 and σ_2 at any point can be calculated easily.

CHAPTER II - GENERAL ANALYSIS OF TWO-CELL STRUCTURE

Consider again the structure in Figure 1; differing from the treatment in the preceding section the assumption is made that any relative displacement of the deckhouse with respect to the hull will be resisted by the internal forces required to deflect bulkheads and transferse beams supporting the deckhouse. The deckhouse is considered as a beam elastically supported on the hull, and is further attached to the hull at deck level so as to enforce equal strains. External vertical loads and buoyancy will cause the structure to deflect, and this deflection can be described by the displacements y_1 and y_2 of the center lines of the deckhouse and hull respectively, (Figure 3). In order to exclude motions of the entire vessel as a rigid body, y_1 and y_2 are defined as the relative displacements measured from a straight line C-D rigidly connected to the hull. As a result of this definition the displacement y_2 of the centroid of the hull at points C and D must always be zero.

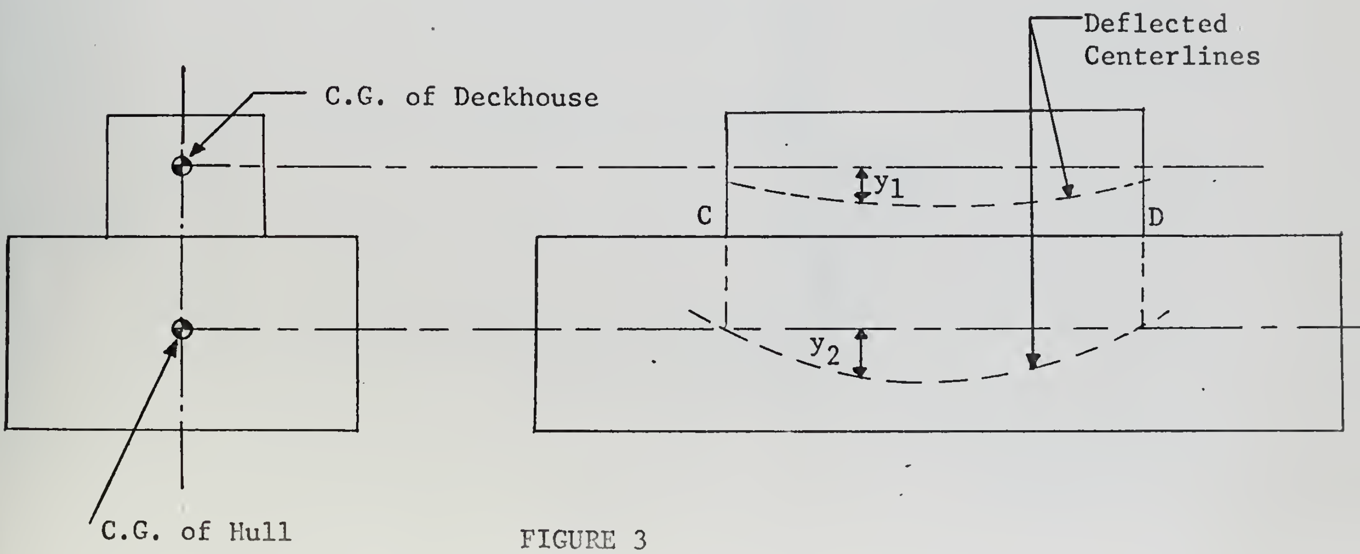


FIGURE 3

It is assumed that the stiffness of bulkheads or deck beams, resisting relative vertical displacements of the deckhouse, is constant for the full length of the deckhouse, the magnitude of the stiffness being given by a spring constant K. K is defined by Bleich as being the force per unit length of deckhouse required to produce a relative deflection equal to one unit of length. Therefore the vertical reaction between hull and deckhouse will be $K(y_1 - y_2)$ per unit of length.

The structure analysed here is shown in Figure 4. There are two beams having areas A_1 and A_2 and moments of inertia I_1 and I_2; and they are connected along C-D in such a way that both horizontal shear forces and vertical reactions can be transferred. It is assumed that, also, Navier's hypothesis to be valid for the hull and for the deckhouse separately.

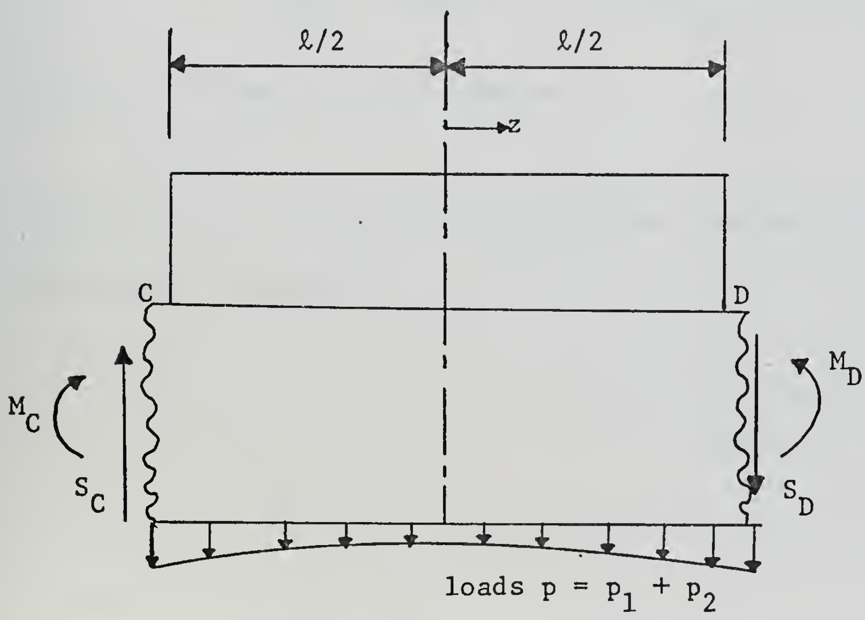


FIGURE 4

This structure will be under the action of vertical loads p_1 on the deckhouse, p_2 on the hull (which includes buoyancy), shear forces S_C, S_D and moments M_C and M_D.

Using the "theorem of stationary potential energy", the differential equations for the deflections y_1 and y_2 can be obtained. The total potential energy U consists of the internal strain energy V, and the potential energy U_w of the external forces. The total potential energy $U = V + U_w$ is[1],

$$U = \frac{1}{2}\int_{-\ell/2}^{\ell/2} [EI_1\, y_1''^2 + EI_2\, y_2''^2 + EI_A(\alpha_1 y_1'' + r\,\alpha_2 y_2''^{2})^2 + K(y_1 - y_2)^2$$

$$+ E\,E^*(y_1''')^2 + E\,F^*(y_2''')^2 + E\,G^*(y_1'''\, y_2''')$$

$$- 2p_1y_1 - 2p_2y_2]\, dz + [M\, y_2']_{-\ell/2}^{\ell/2} - [S\, y_2]_{-\ell/2}^{\ell/2} \qquad (9)$$

U will be minimum if the variation

$\delta U = 0$

Using the rules of calculus of variations, the set of two simultaneous equations are derived.[2]

---

[1] See Appendix I, for the derivation of V and U_w

[2] See Appendix II

$$- EE^* y_1^{V1} + E(I_1 + \alpha_1^2 I_A) y_1^{1V} + K y_1 - \frac{EG^*}{2} y_2^{V1} + E r\alpha_1 \alpha_2 I_A y_2^{1V}$$

$$- K y_2 = p_1 \qquad (10.a)$$

$$- \frac{EG^*}{2} y_1^{V1} + E r \alpha_1 \alpha_2 I_A y_1^{1V} - K y_1 - EF^* y_2^{V1} + E(I_2 + r^2 \alpha_2^2 I_A) y_2^{1V}$$

$$+ K y_2 = p_2 \qquad (10.b)$$

The calculus of variations method also furnish the boundary conditions required to determine the arbitrary constants which will appear in the general solutions of the differential equations. Because of symmetry, there are six boundary conditions instead of twelve. For $z = + \ell/2$ and $z = - \ell/2$

$$y_2 = 0 \qquad (11.a)$$

$$2EE^* y_1''' + EG^* y_2''' = 0 \qquad (11.b)$$

$$EG^* y_1''' + 2EF^* y_2''' = 0 \qquad (11.c)$$

$$-EE^* y_1^{1V} + E(I_1 + \alpha_1^2 I_A) y_1'' - \frac{EG^*}{2} y_2^{1V} + E r \alpha_1 \alpha_2 I_A y_2'' = 0 \qquad (11.d)$$

$$- \frac{EG^*}{2} y_1^{1V} + E r \alpha_1 \alpha_2 I_A y_1'' - EF^* y_2^{1V} + E(I_2 + r_2 \alpha_2^2 I_A) y_2'' = -M \qquad (11.e)$$

$$EE^* y_1^{V} - E(I_1 \alpha_1^2 I_A) y_1''' + \frac{EG^*}{2} y_2^{V} - E(r \alpha_1 \alpha_2 I_A) y_2''' = 0 \qquad (11.f)$$

II.A - NON-DIMENSIONALIZATION OF THE EQUATIONS

It is thought that employing non-dimensional equations and boundary conditions, the amount of algebra in the computation will be reduced and the results can be represented in a general form.

Non-dimensionalization is made in the manner that is given by the following equations.

$$y_i = \bar{y}_i\,\ell, \quad i = 1,2 \qquad (12.a)$$

$$z = \bar{z}\,\ell \qquad (12.b)$$

$$a = \bar{a}\,\ell \qquad (12.c)$$

$$A_i = \overline{A_i}\,\ell^2, \quad i = 1,2 \qquad (12.d)$$

$$I_i = \overline{I_i}\,\ell^4, \quad i = 1,2 \qquad (12.e)$$

$$p_i = p_i\,E\ell, \quad i = 1,2 \qquad (12.f)$$

$$M = \bar{M}\,E\,\ell^3 \qquad (12.g)$$

$$K = \bar{K}\,E \qquad (12.h)$$

The length of the deckhouse is selected as a characteristic length, because it is one of the most important parameters in the distribution of the longitudinal stresses.

Substituting Equation (12) into Equations (10) and (11), the following non-dimensional equations and boundary conditions are derived.

$$-a_{16}\,\bar{y}_1^{VI} + a_{14}\,\bar{y}_1^{IV} + \bar{K}\,\overline{y_1} - a\,\bar{y}_2^{VI} + b\,\bar{y}_2^{IV} - \bar{K}\,\overline{y_2} = \overline{p_1} \qquad (13.a)$$

$$-a\,\bar{y}_1^{VI} + b\,\bar{y}_1^{IV} - \bar{K}\,\overline{y_1} - a_{26}\,\bar{y}_2^{VI} + a_{24}\,\bar{y}_2^{IV} + \bar{K}\,\overline{y_2} = \overline{p_2} \qquad (13.b)$$

Boundary conditions for $\bar{z} = + \ell/2$ and $\bar{z} = - \ell/2$,

$$\bar{y}_2 = 0 \tag{14.a}$$

$$a_{16}\, \bar{y}_1''' + a\, \bar{y}_2''' = 0 \tag{14.b}$$

$$a\, \bar{y}_1''' + a_{26}\, \bar{y}_2''' = 0 \tag{14.c}$$

$$-a_{16}\, \bar{y}_1^{IV} + a_{14}\, \bar{y}_1'' - a\, \bar{y}_2^{IV} + b\, \bar{y}_2'' = 0 \tag{14.d}$$

$$-a\, \bar{y}_1^{IV} + b\, \bar{y}_1'' = a_{26}\, \bar{y}_2^{IV} + a_{24}\, \bar{y}_2'' = -\bar{M} \tag{14.e}$$

$$a_{16}\, \bar{y}_1^{V} - a_{14}\, \bar{y}_1''' + a\, \bar{y}_2^{V} - b\, \bar{y}_2''' = 0 \tag{14.f}$$

Expressions for non-dimensional coefficients for the equations and boundary conditions are given in the section entitled "Parameters Used for Computation".

CHAPTER III - SOLUTION OF DIFFERENTIAL EQUATION FOR CONSTANT MOMENT M

Considering the simple case that the loads p_1, p_2 and the shears S_c and S_D are zero, the only loads being $M_c = M_D = M$, Equations (13) are then homogeneous.

Setting the determinant of the coefficients of the differential equations equal to zero, the charactericits equation will be derived, and it will be in the following form.

$$r^4 \left(r^8 + a_3 r^6 + a_2 r^4 + a_1 r^2 + a_0\right) = 0 \tag{15}$$

where, a_3, a_2, a_1, and a_0 are known constants. The roots of the characteristic equation is found for the models with different dimensions, and it is seen that the roots were always in the following manner.

$$r_1 = r_2 = r_3 = r_4 = 0 \tag{16.a}$$

$$r_5 = - \gamma_3 \tag{16.b}$$

$$r_6 = \gamma_3 \tag{16.c}$$

$$r_7 = - \gamma_4 \tag{16.d}$$

$$r_8 = \gamma_4 \tag{16.e}$$

$$r_9 = - \gamma_1 - i \gamma_2 \tag{16.f}$$

$$r_{10} = - \gamma_1 + i \gamma_2 \tag{16.g}$$

$$r_{11} = \gamma_1 - i \gamma_2 \tag{16.h}$$

$$r_{12} = \gamma_1 + i \gamma_2 \tag{16.i}$$

Then, keeping in mind that the problem considered is symmetrical with respect to the origin of the co-ordinate z, and using only symmetrical functions, the general symmetrical solution will contain only six arbitrary constants. This general solution is,

$$\bar{y}_1 = c_1 + c_2\,\bar{z}^2 + c_3 \sin\gamma_2\,\bar{z}\,\sinh\gamma_1\,\bar{z} + c_4 \cos\gamma_2\,\bar{z}\,\cosh\gamma_1\,\bar{z}$$
$$+ c_5 \cosh\gamma_3\,\bar{z} + c_6 \cosh\gamma_4\,\bar{z} \qquad (17.a)$$

$$\bar{y}_2 = c_1 + c_2\,\bar{z}^2 + c_3' \sin\gamma_2\,\bar{z}\,\sinh\gamma_1 + c_4' \cos\gamma_2\,\bar{z}\,\cosh\gamma_1\,\bar{z}$$
$$- \mu_3\,c_5 \cosh\gamma_3\,\bar{z} - \mu_4\,c_6 \cosh\gamma_4\,\bar{z} \qquad (17.b)$$

where,

$$c_3' = \mu_1\,c_3 + \mu_2\,c_4 \qquad (18.a)$$

$$c_4' = -\,\mu_2\,c_3 + \mu_1\,c_4 \qquad (18.b)$$

and, μ_1, μ_2, μ_3, μ_4 are given in the following expressions,

$$\mu_1 = \frac{\begin{vmatrix} (a_{16}h - a_{14}f - \bar{K}) & -(am - bg) \\ -(a_{16}m - a_{14}g) & (-ah + bf - \bar{K}) \end{vmatrix}}{\begin{vmatrix} (-ah + bf - \bar{K}) & -(am - bg) \\ (am - bg) & (-ah + bf - \bar{K}) \end{vmatrix}} \qquad (18.c)$$

or,

$$= \frac{\begin{vmatrix} (ah - bf + \bar{K}) & -(a_{26}m - a_{24}g) \\ -(am - bg) & (-a_{26}h + a_{24}f + \bar{K}) \end{vmatrix}}{\begin{vmatrix} (-a_{26}h + a_{24}f + \bar{K}) & -(a_{26}m - a_{24}g) \\ (a_{26}m - a_{24}g) & (-a_{26}h + a_{24}f + \bar{K}) \end{vmatrix}} \qquad (18.d)$$

$$\mu_2 = \frac{\begin{vmatrix} (a_{16}m - a_{14}g) & -(am - bg) \\ (a_{16}h - a_{14}f - \bar{K}) & (-ah + bf - \bar{K}) \end{vmatrix}}{\begin{vmatrix} (-ah + bf - \bar{K}) & -(am - bg) \\ (am - bg) & (-ah + bf - \bar{K}) \end{vmatrix}} \qquad (18.e)$$

or,

$$= \frac{\begin{vmatrix} (am - bg) & -(a_{26}m - a_{24}g) \\ (ah - bf + \bar{K}) & (-a_{26}h + a_{24}f + \bar{K}) \end{vmatrix}}{\begin{vmatrix} (-a_{26}h + a_{24}f + \bar{K}) & -(a_{26}m - a_{24}g) \\ (a_{26}m - a_{24}g) & (-a_{26}h + a_{24}f + \bar{K}) \end{vmatrix}} \qquad (18.f)$$

$$\mu_3 = \frac{(-a_{16}\,\gamma_3^6 + a_{14}\,\gamma_3^4 + \bar{K})}{(-a\,\gamma_3^6 + b\,\gamma_3^4 - \bar{K})} \quad \text{or} \quad = \frac{(-a\,\gamma_3^6 + b\,\gamma_3^4 - \bar{K})}{(-a_{26}\,\gamma_3^6 + a_{24}\,\gamma_3^4 + \bar{K})} \qquad (18.g)$$

$$\mu_4 = \frac{(- a_{16}\,\gamma_4^6 + a_{14}\,\gamma_4^4 + \bar{K})}{(- a\,\gamma_4^6 + b\,\gamma_4^4 - \bar{K})} \quad \text{or} \quad = \frac{(-a\,\gamma_4^6 + b\,\gamma_4^4 - \bar{K})}{(- a_{26}\,\gamma_4^6 + a_{24}\,\gamma_4^4 + \bar{K})} \tag{18.h}$$

Introduction of Equations (17) into the boundary conditions, leads to the equations to find the arbitrary constants given in TABLE (1).

TABLE 1

$$c_1 + c_2\bar{z}^2 + \sin\gamma_2 z \sinh\gamma_1 z(\mu_1 c_3 + \mu_2 c_4) + \cos\gamma_2\bar{z}\cosh\gamma_1\bar{z}(-\mu_2 c_3 + \mu_1 c_4) - \mu_3 c_5 \cosh\gamma_3\bar{z}$$
$$-\mu_4 c_6 \cosh\gamma_4\bar{z} = 0$$

$$\sin\gamma_2\bar{z}\cosh\gamma_1\bar{z}(t_1 c_3 + t_2 c_4) + \cos\gamma_2\bar{z}\sinh\gamma_1\bar{z}(-t_2 c_3 + t_1 c_4) + \sinh\gamma_3\bar{z}\cdot v_1\gamma_1^3 c_5 + \sinh\gamma_4\bar{z}\cdot v_2\gamma_4^3\cdot c_6 = 0$$

$$\sin\gamma_2\bar{z}\cosh\gamma_1\bar{z}(t_3 c_3 + t_4 c_4) + \cos\gamma_2\bar{z}\sinh\gamma_1\bar{z}(-t_4 c_3 + t_3 c_4) + \sinh\gamma_3\bar{z}\cdot v_3\gamma_3^3\cdot c_5 + \sinh\gamma_4\bar{z}\cdot v_4\gamma_4^3\cdot c_6 = 0$$

$$\sin\gamma_2\bar{z}\sinh\gamma_1\bar{z}(t_5 c_3 + t_6 c_4) + \cos\gamma_2\bar{z}\cosh\gamma_1\bar{z}(-t_1 c_6 + t_5 c_4) + \cosh\gamma_3\bar{z}c_5(-v_1\gamma_3^4 + v_5\gamma_3^2)$$
$$+ \cosh\gamma_4\bar{z}\cdot c_6(-v_2\gamma_4^4 + v_6\gamma_4^2) + 2\,v_9\,c_2 = 0$$

$$\sin\gamma_2\bar{z}\sinh\gamma_1\bar{z}(t_7 c_3 + t_8 c_4) + \cos\gamma_2\bar{z}\cosh\gamma_1\bar{z}(-t_8 c_3 + t_7 c_4) + \cosh\gamma_3\bar{z}c_5(-v_3\gamma_3^4 + v_7\gamma_3^2)$$
$$+ \cosh\gamma_4\bar{z}\cdot c_6(-v_4\gamma_4^4 + v_8\,v_4^2) + 2v_{10}\,c_2 = -\bar{M}$$

$$\sin\gamma_2\bar{z}\cosh\gamma_1\bar{z}(t_9 c_3 + t_{10} c_4) + \cos\gamma_2\bar{z}\sinh\gamma_1\bar{z}(-t_{10} c_3 + t_9 c_4) + \sinh\gamma_3\bar{z}\cdot c_5(+v_1\gamma_3^5 - v_5\gamma_3^3)$$
$$+ \sinh\gamma_4\bar{z}\cdot c_6(v_2\gamma_4^5 - v_6\gamma_4^3) = 0$$

where $\bar{z} = 0.5$

After finding the general coefficients, the stresses at any point along the deckhouse can be computed from the expressions for the moments M_1, M_2, and direct forces N_1, N_2,

$$M_1 = - E I_1 y_1'' \qquad (19.a)$$

$$M_2 = - E I_2 y_2'' \qquad (19.b)$$

Because, in this case $p_1 = p_2 = 0$.

$$N_1 = - N_2 = \frac{E I_A}{a} (\alpha_1 y_1'' + r \alpha_2 y_2'') \qquad (20)$$

Here, it is important to notice that the solutions for $\overline{y_1}$ and $\overline{y_2}$ will be non-dimensional.

CHAPTER IV - SOLUTION OF DIFFERENTIAL EQUATION FOR FOR EQUALLY DISTRIBUTED LOADS

In this section the case is considered of equally distributed loads p_1 and p_2, acting on deckhouse and hull, respectively, while the moments at the end of the deckhouse are $M_C = M_D = 0$. Equilibrium requires external shear forces

$$S_C = - S_D = \frac{\ell}{2} (p_1 + p_2) \tag{21}$$

at the ends C and D. The moment in the midship section due to the loads p1 and p2 are

$$M_p = \frac{p_1 + p_2}{8} \ell^2 \tag{22}$$

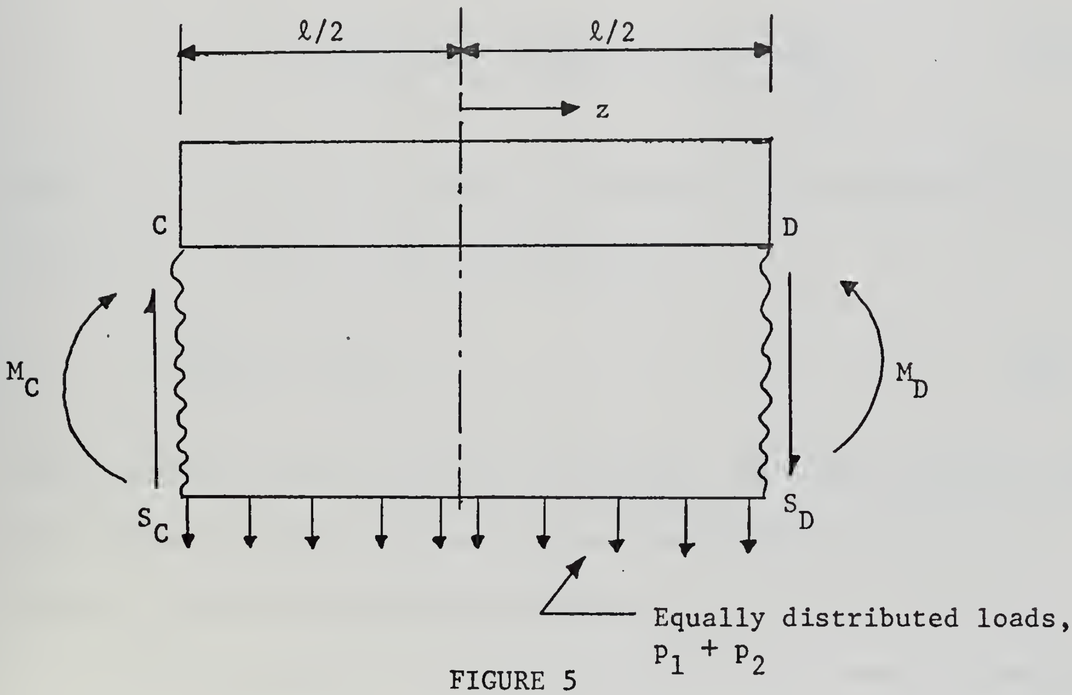


FIGURE 5

The loading being symmetrical, the general symmetrical solutions of Equation (13) are,

$$\bar{y}_1 = c_1 + c_2 \bar{z}^2 + c_3 \sin \gamma_2 \bar{z} \sinh \gamma_1 \bar{z} + c_4 \cos \gamma_2 \bar{z} \cosh \gamma_1 \bar{z}$$

$$+ c_5 \cosh \gamma_3 \bar{z} + c_6 \cosh \gamma_4 \bar{z}$$

$$+ \frac{(\overline{P_1} + \overline{P_2})}{24(a_{14} + a_{24} + 2b)} \bar{z}^4 + \frac{\overline{P_1}}{(1 + \mu_5)\bar{K}} \qquad (23.a)$$

$$\bar{y}_2 = c_1 + c_2 \bar{z}^2 + c_3' \sin \gamma_2 \bar{z} \sinh \gamma_1 \bar{z} + c_4' \cos \gamma_2 \bar{z} \cosh \gamma_1 \bar{z}$$

$$- \mu_3 c_5 \cosh \gamma_3 \bar{z} - \mu_4 c_6 \cosh \gamma_4 \bar{z}$$

$$+ \frac{(p_1 + p_2)}{24(a_{14} + a_{24} + 2b)} \bar{z}^4 + \frac{5\, p_2}{(1 + \mu_5)\bar{K}} \qquad (23.b)$$

where c_3', c_4', μ_1, μ_2, μ_3, and μ_4 as given in the Equations (18) for the case of constant moment loading, and

$$\mu_5 = \frac{a_{14} + b}{a_{24} + b} = \frac{v_9}{v_{10}} \qquad (24)$$

But it should be mentioned that the value of "r" shear-lag factor will be different in this case, so all the coefficients will not, in general, have the same numerical value.

Introducing general solutions Equations (23) into the boundary conditions Equations (14) will lead us to the same kind of equations given in Table 1. But, there will be some more terms, because of the

particular integral parts of the general solutions. These additional terms can be determined easily, using the particular integral parts of the general solutions.

After finding the general coefficients, the further computation to find the stresses at any point along the deckhouse follows the pattern for the preceding section. The only difference being that the additional terms appear in the equations for M_1 and M_2 due to distributed loads $\overline{p_1}$ and $\overline{p_2}$ on the deckhouse and hull, respectively.

$$M_1 = - E I_1 (y_1'' + \frac{p_1}{A_{1w} G}) \tag{25}$$

$$M_2 = - E I_2 (y_2'' + \frac{p_2}{A_{2w} G}) \tag{26}$$

$$N_1 = - N_2 = \frac{EI_A}{a} (\alpha_1 y_1'' + r \alpha_2 y_2'') \tag{27}$$

CHAPTER V - TOTAL SOLUTION OF DIFFERENTIAL EQUATION

Considering the total loading for the system shown in Figure (5), being equally distributed loads p_1 and p_2, acting on deckhouse and hull respectively, while the moments at the end of deckhouse are $M_C = M_D = M$, it is possible to find the total solution

The differential equations for this system are Equations (13) and the boundary conditions are given by Equations (14) for $(+\frac{\ell}{2})$ and $(-\frac{\ell}{2})$. The problem considered is symmetrical with respect to the origin of the co-ordinate z. The general symmetrical solutions of Equations (13) can be given by Equations (23.a) and (23.b) for $\overline{y_1}$ and $\overline{y_2}$, respectively. The coefficients in the solutions $(c_3', c_4', \mu_1, \mu_2, \mu_3, \mu_4, \mu_5)$ are given by Equations (18) and (24).

But, it must be remembered that the values of the general coefficients $(c_1, c_2, c_3, c_4, c_5, c_6)$ will be different numerically than the values found for the two cases considered before.

In the above approach to get the total solution, an additional assumption is made: Deck shear-lag factor "r" is considered to be constant over the length of the deckhouse and is given by Equation (4), even though there is an applied constant bending moment at the end of deckhouse.

Another approach to get the total solution is to make superposition to the solutions found for constant moment and equally distributed loading cases.

Total solution of differential equation is found by using both approachs explained above, and the results are compared in the

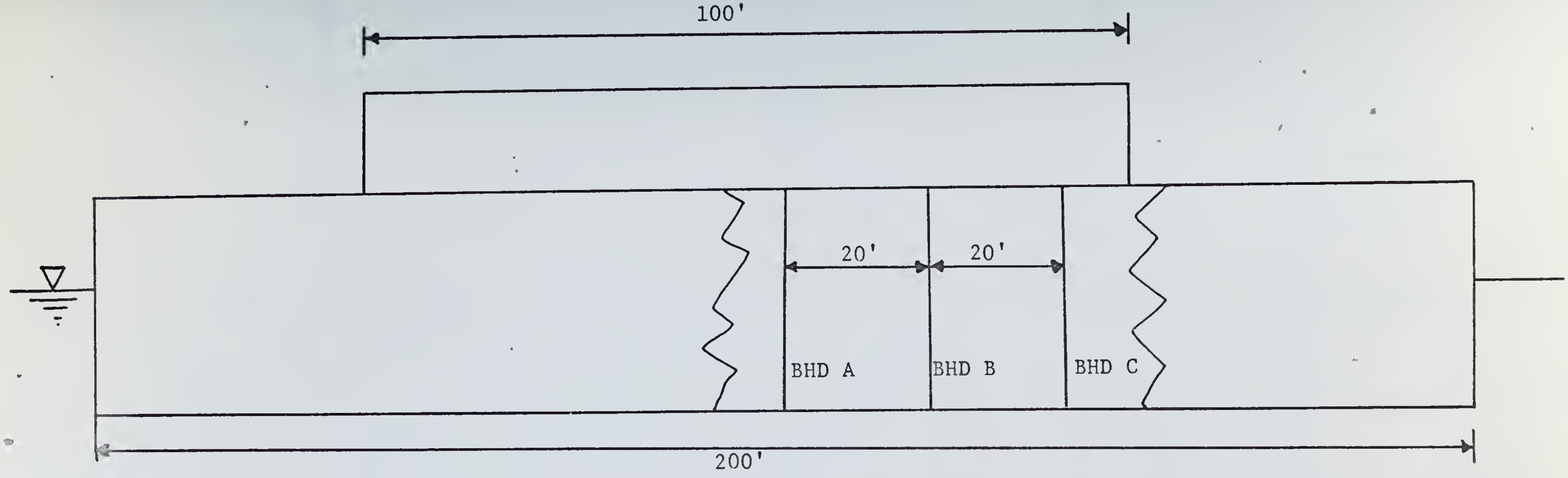
following chapter.

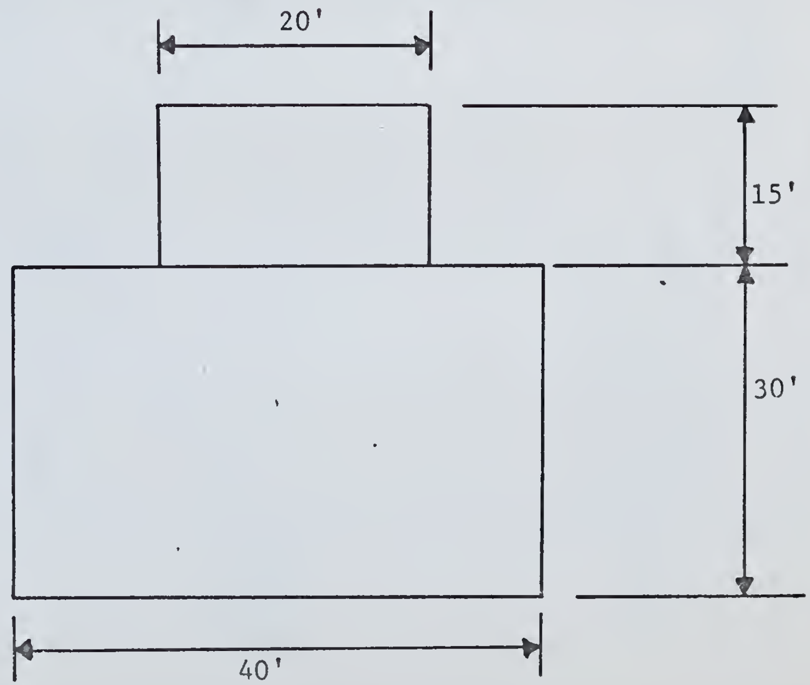
CHAPTER VI - MODEL USED FOR COMPUTATION

For this analysis a model was selected with the dimensions as shown in Figure (6). This model can be assumed as short deckhouse, so it will be possible to see more pronounced shear effects. Bulkheads are placed at equally spaced distances of 20 feet in the hull section. The thickness of the hull box girder plating is 0.5 inches and the thickness of the deckhouse plating and bulkheads is 0.25 inches. The material constants include a Young's modulus of 30×10^6, modulus of elasticity in shear of 11.5×10^6 and a Poisson's ratio of 0.3.

The model is assumed to have a 15 foot draft with a corresponding hydrostatic upward distributed force of 17.143 tons/ft. The internal loading is arranged so as to provide equilibrium and a resultant symmetrical loading. Shear and moment diagrams for the total model are also provided in Figure (7).

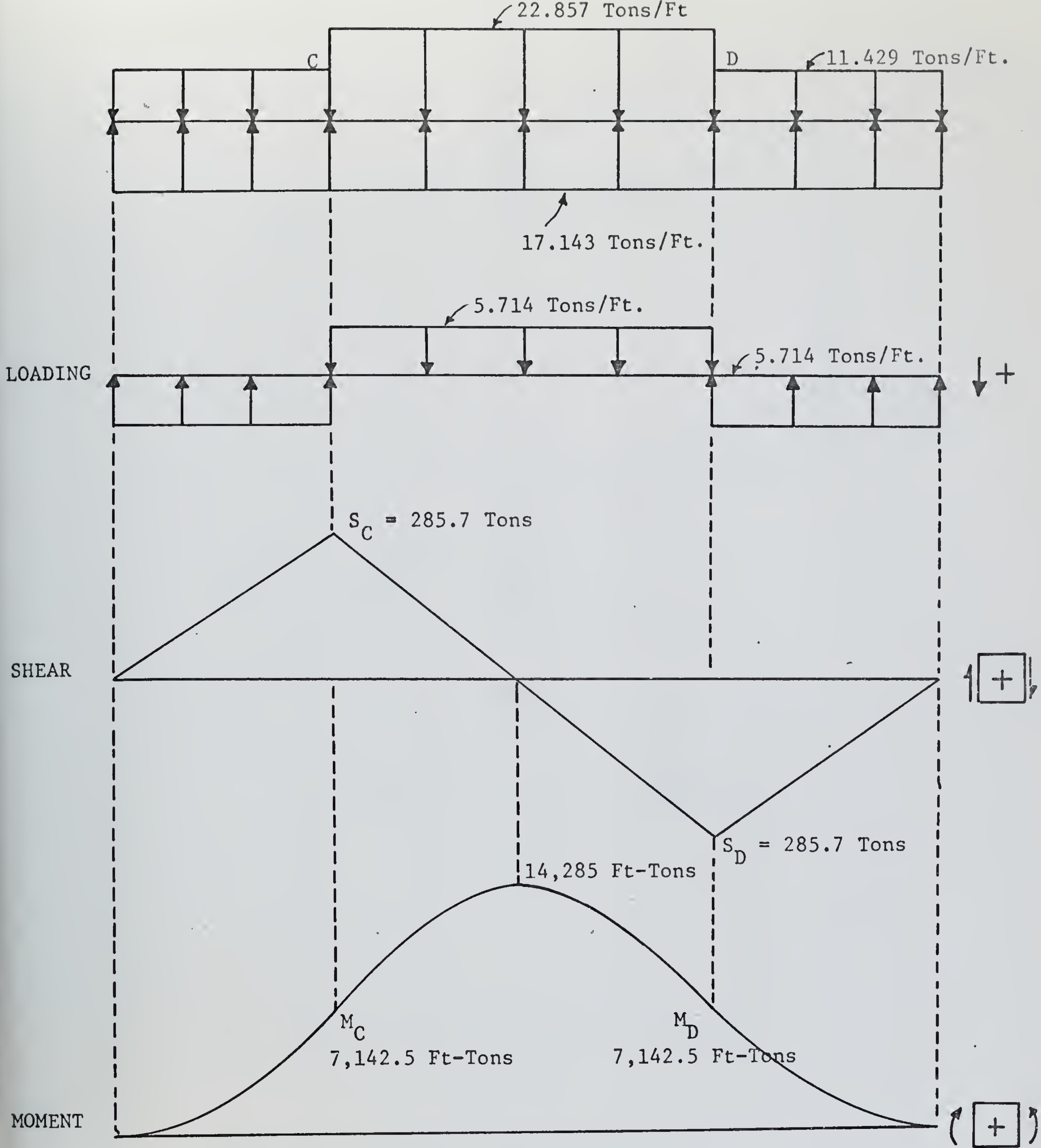
In a model with bulkheads, the main difficulty is the determination of the deck stiffness or spring constant (K). Since K was defined as the force per unit length of deckhouse required to produce a relative deflection equal to one unit of length, it is apparent that the value of K will, in reality, vary along the deckhouse length due to the emplacement of structural bulkheads. In order to simplify the use of the method, however, an average value of K must be determined. To achieve this end, the same approach that was used in Reference (3) is followed. A symmetrical portion of the hull structure is modelled to include a bulkhead and attached deck and bottom plating, Figure (8).





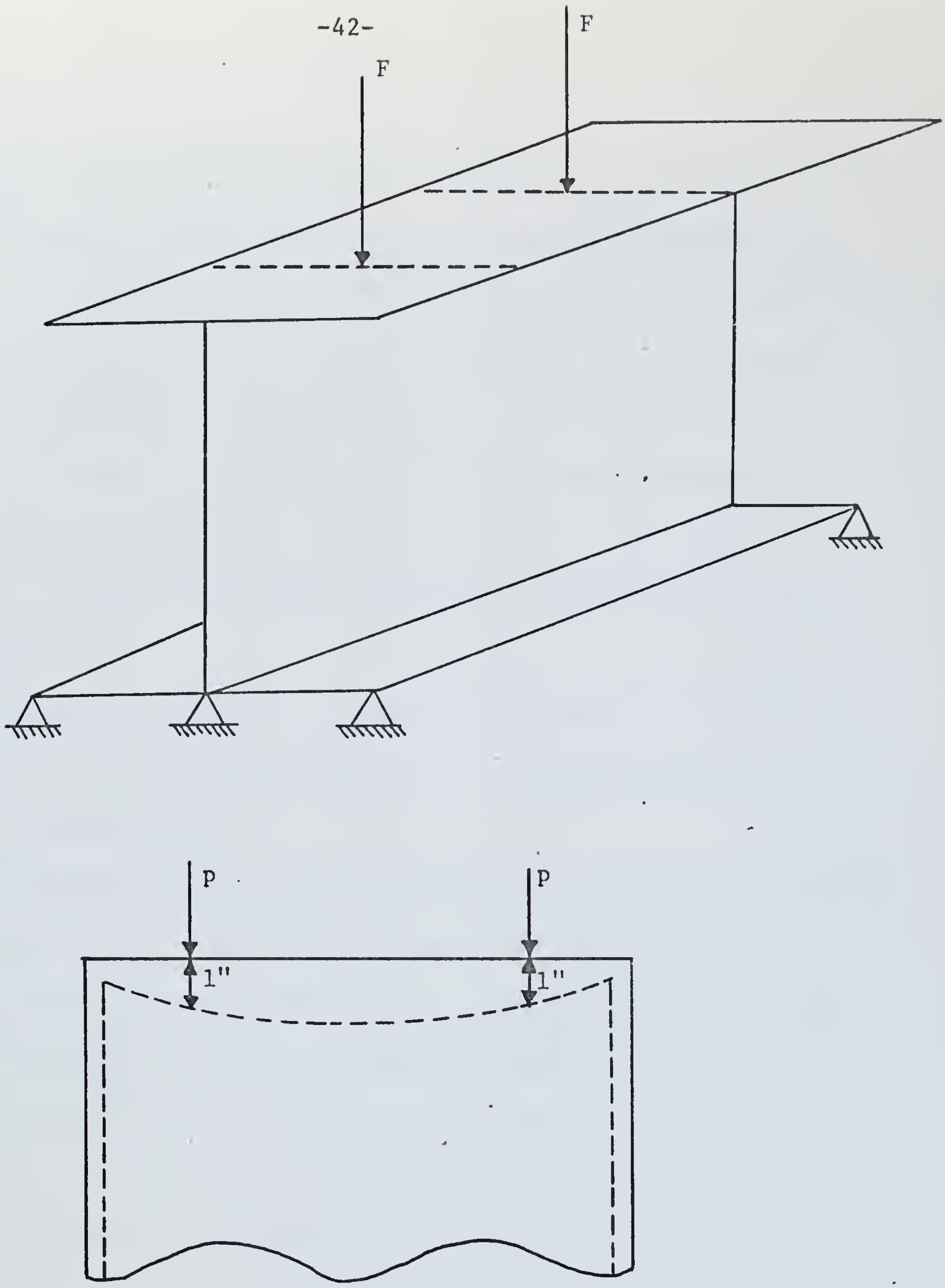
DIMENSIONS OF MODEL
WITH BULKHEADS

FIGURE 6



LOADING, SHEAR AND MOMENT DIAGRAM FOR
MODEL WITH BULKHEADS

FIGURE 7



SYMMETRICAL STRUCTURE FOR EVALUATION OF K

FIGURE 8

This I-beam type structure is simply supported on its end and allowed to deflect under vertical forces applied to the hull-deckhouse connections. In the analytical procedure used for determining the forces needed to deflect the hull-deckhouse connections 1 inch, it must be kept in mind that although the deflection due to shear forces is negligible in most cases, in short deep metal beams the deflection caused by shear may become a significant portion of the total deflection. In this case shear contributes a major portion to the total deflection.

This analytic approach to K yielded a value of 1.58×10^4 psi for the present model under consideration. Sample calculations are provided in Appendix III.

In Reference (3), as a further check on K, a STRUDL program using 'PSR' elements on the same model presented in Figure (8) was run. Arbitrary forces (F) were applied at the locations indicated, and the resulting deflection at the hull-deckhouse connection noted. The force was then scaled for a deflection of 1 inch. P was obtained by dividing F by the width of the flange (240 inches). Using Equation $K = 2P$, K was found to have value of 1.53×10^4 psi. As it is said in Reference (3), the disparity between the STRUDL K and the analytical K can be attributed to the fact that in the analytical approach the deflection calculations apply to the neutral axis of the beam only.

The value of K that is used in the computations was the value found by employing STRUDL program.

VI.A - PRESENTATION OF THE RESULTS

Presentation of the stress distributions for Bleich's method and for the method developed are shown in Figures (9), (10), (11), (12), (13), (14), and (15) on the following pages. Figures (9) and (10) show the comparison of longitudinal stress distribution for constant moment loading at midship and at the end of deckhouse, respectively. The comparison of the results for equally distributed loading are shown in Figures (11) and (12) at midship and at the end of the deckhouse, respectively. Figures (13), (14), and (15) show the comparison of stress distributions for the total solutions.

For the computation of the results, the model shown in Figure (6) is used for both methods.

"ACCESS II" Primer Operations in Linear Algebra for the Interdata Computer in Joint Computer Facility is used to find the roots of the characteristic equations and to solve the general coefficient matrices.

The explanation about Bleich's method can be found in References (1) and (3).

"CONSTANT MOMENT LOADING"

COMPARISON OF LONGITUDINAL STRESS DISTRIBUTION AT MIDSHIP

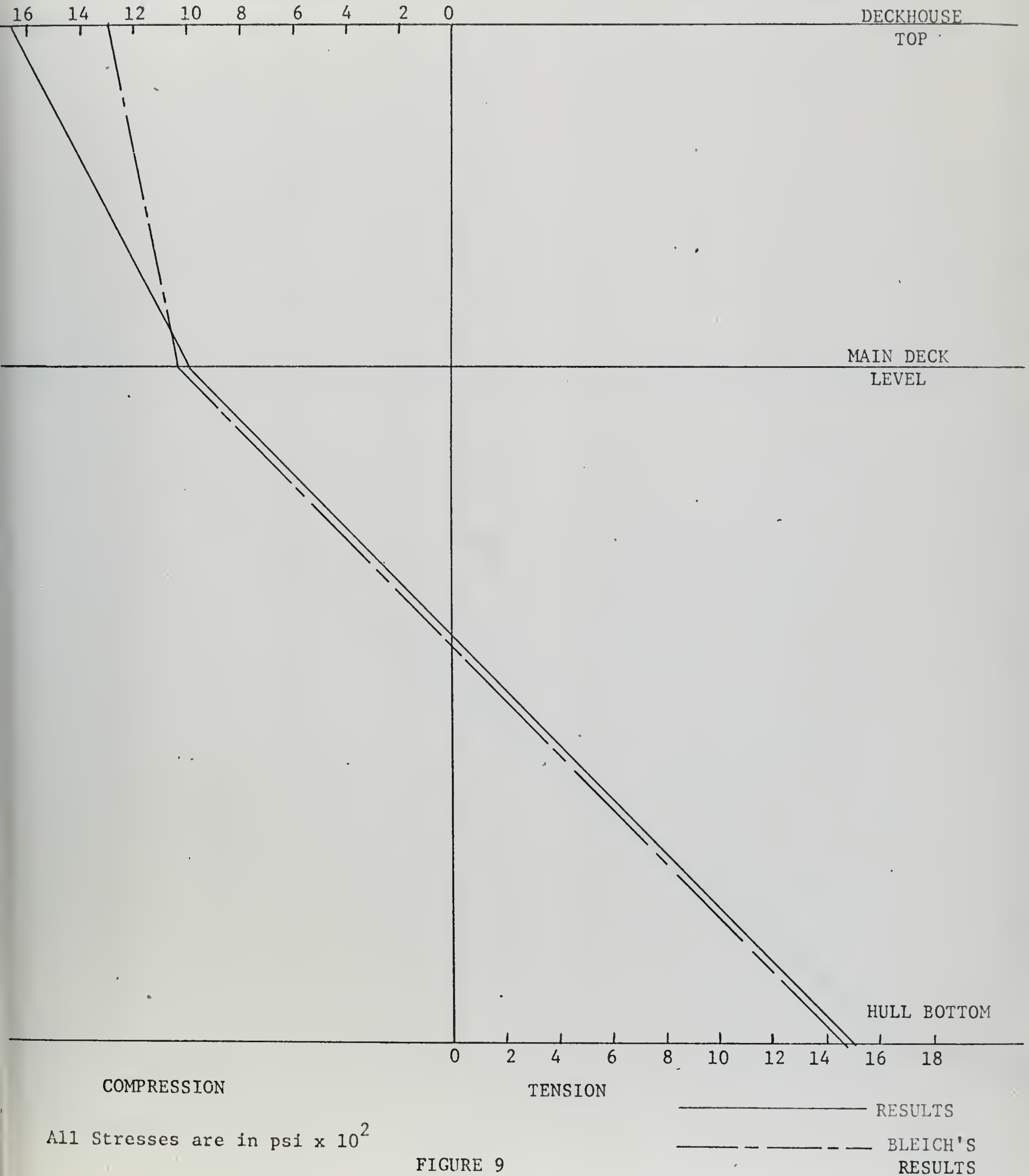
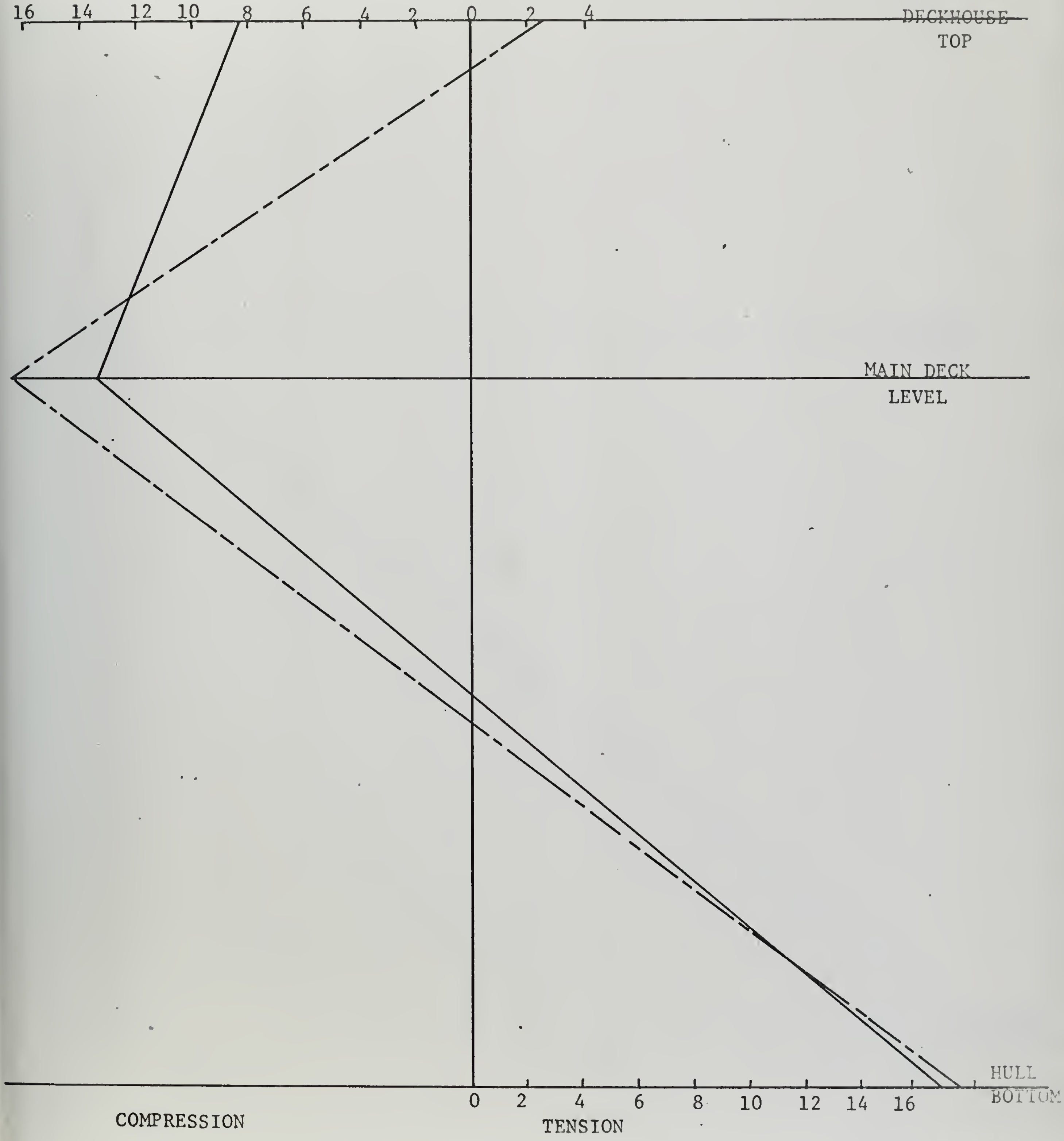


FIGURE 9

"CONSTANT MOMENT LOADING"

COMPARISON OF LONGITUDINAL STRESS DISTRIBUTION AT THE END OF DECKHOUSE



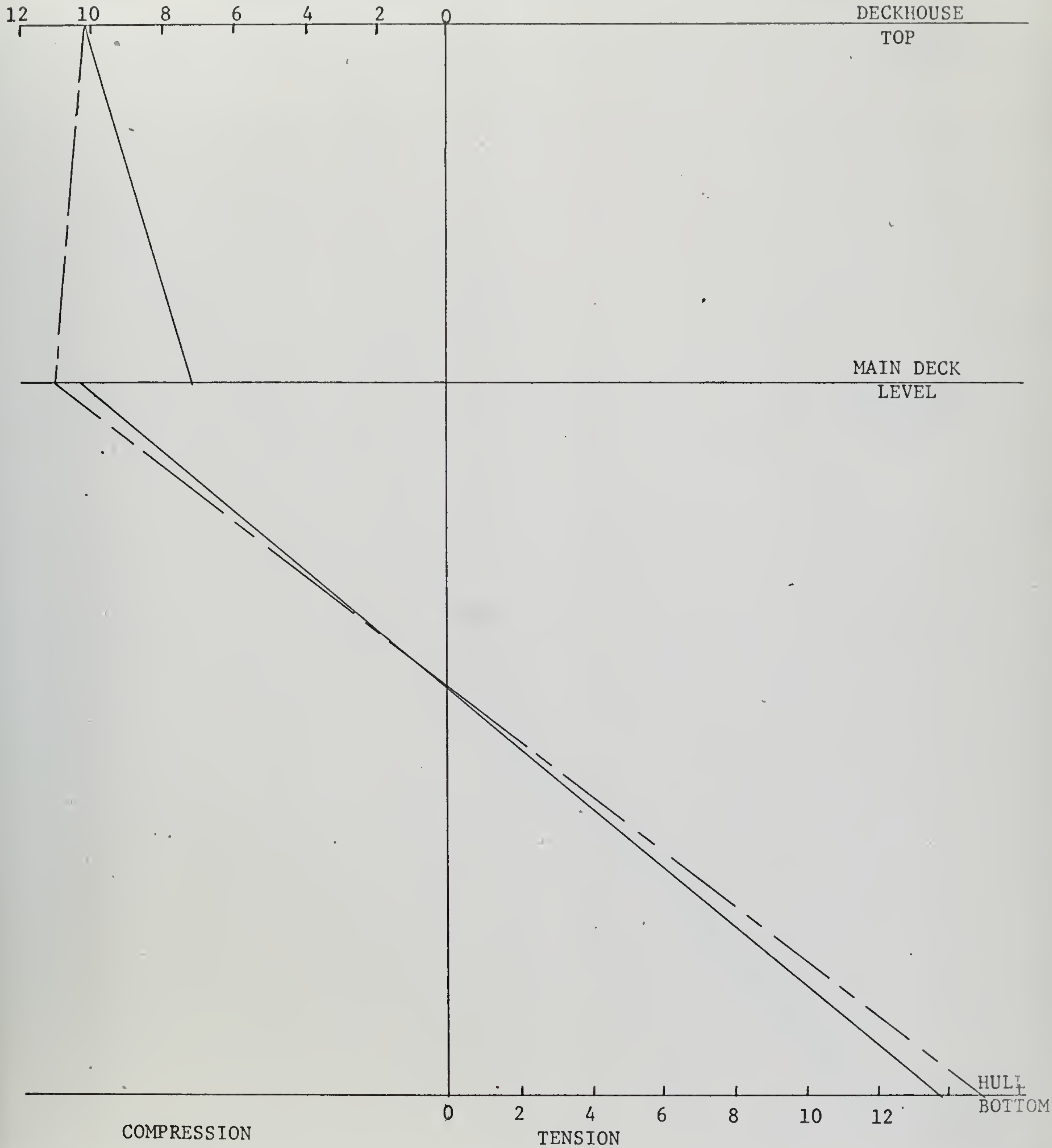
All stresses are in psi x 10^2

FIGURE 10

BLEICH'S RESULTS

"EQUALLY DISTRIBUTED LOADING"

COMPARISON OF LONGITUDINAL STRESS DISTRIBUTION AT MIDSHIP

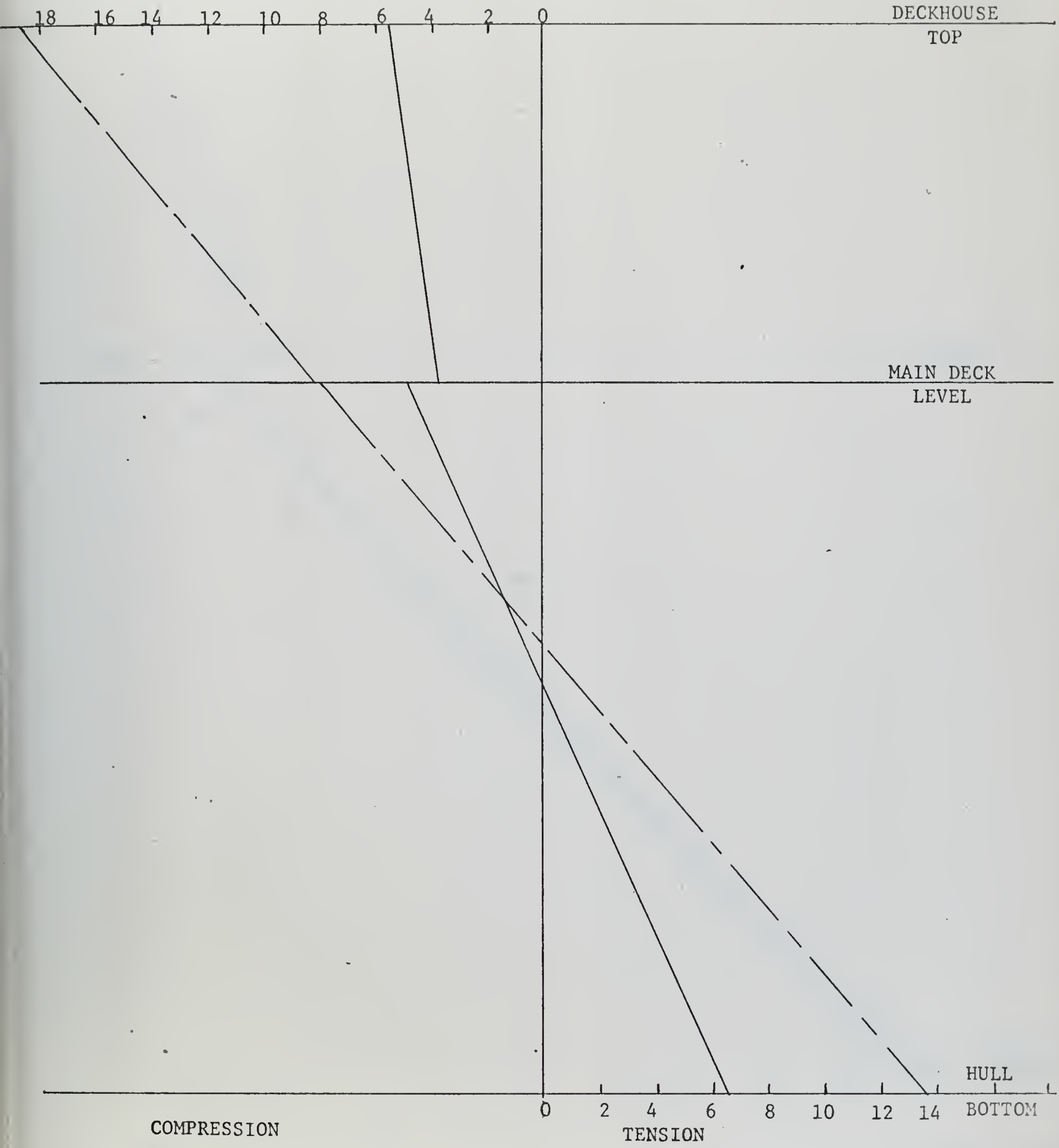


All stresses are in psi x 10^2

FIGURE 11

"EQUALLY DISTRIBUTED LOADING"

COMPARISON OF LONGITUDINAL STRESS DISTRIBUTION AT THE END OF DECKHOUSE



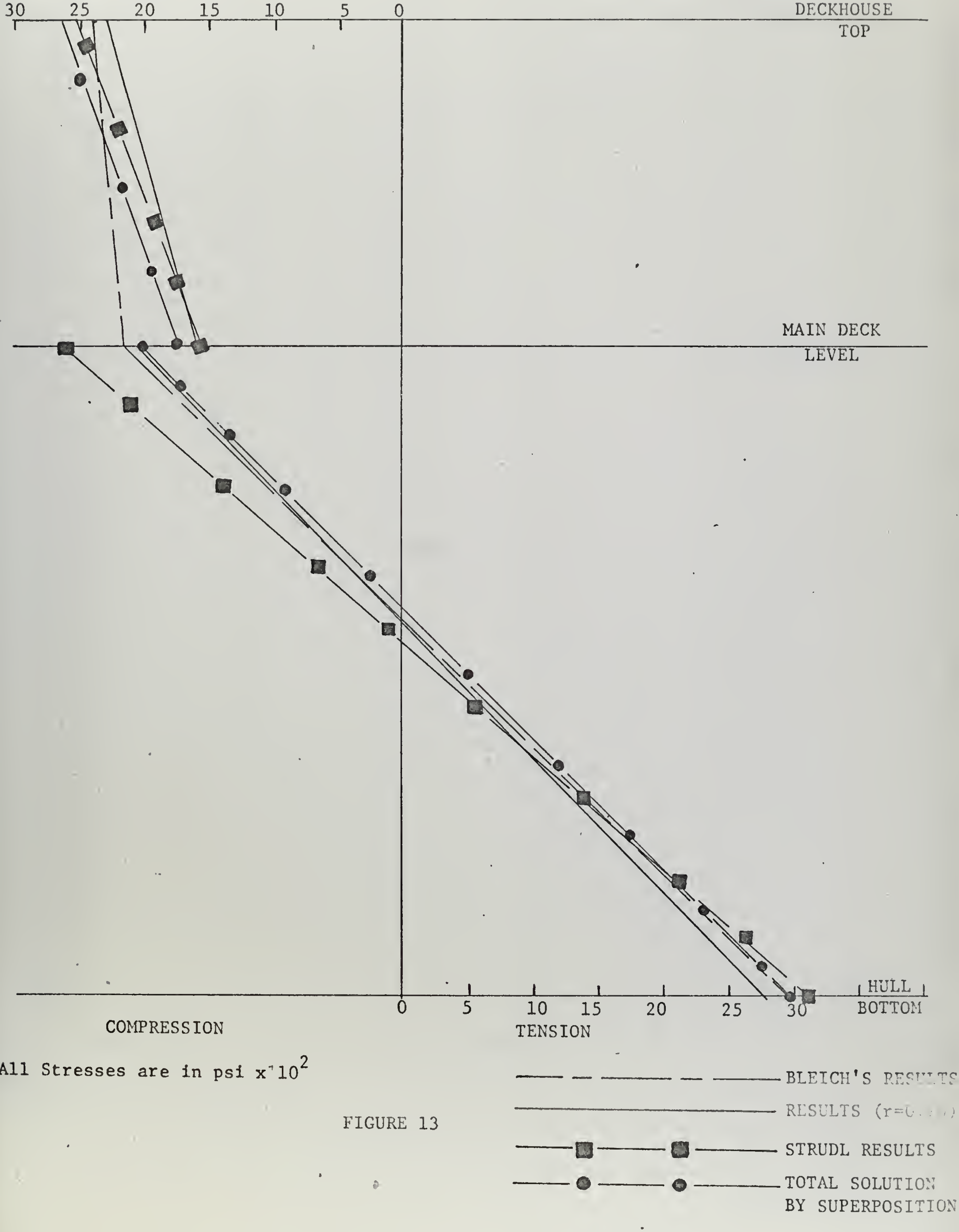
All Stresses are in psi x 10^2

FIGURE 12

BLEICH'S RESULTS

"TOTAL SOLUTION"

COMPARISON OF LONGITUDINAL STRESS DISTRIBUTION AT MIDSHIP

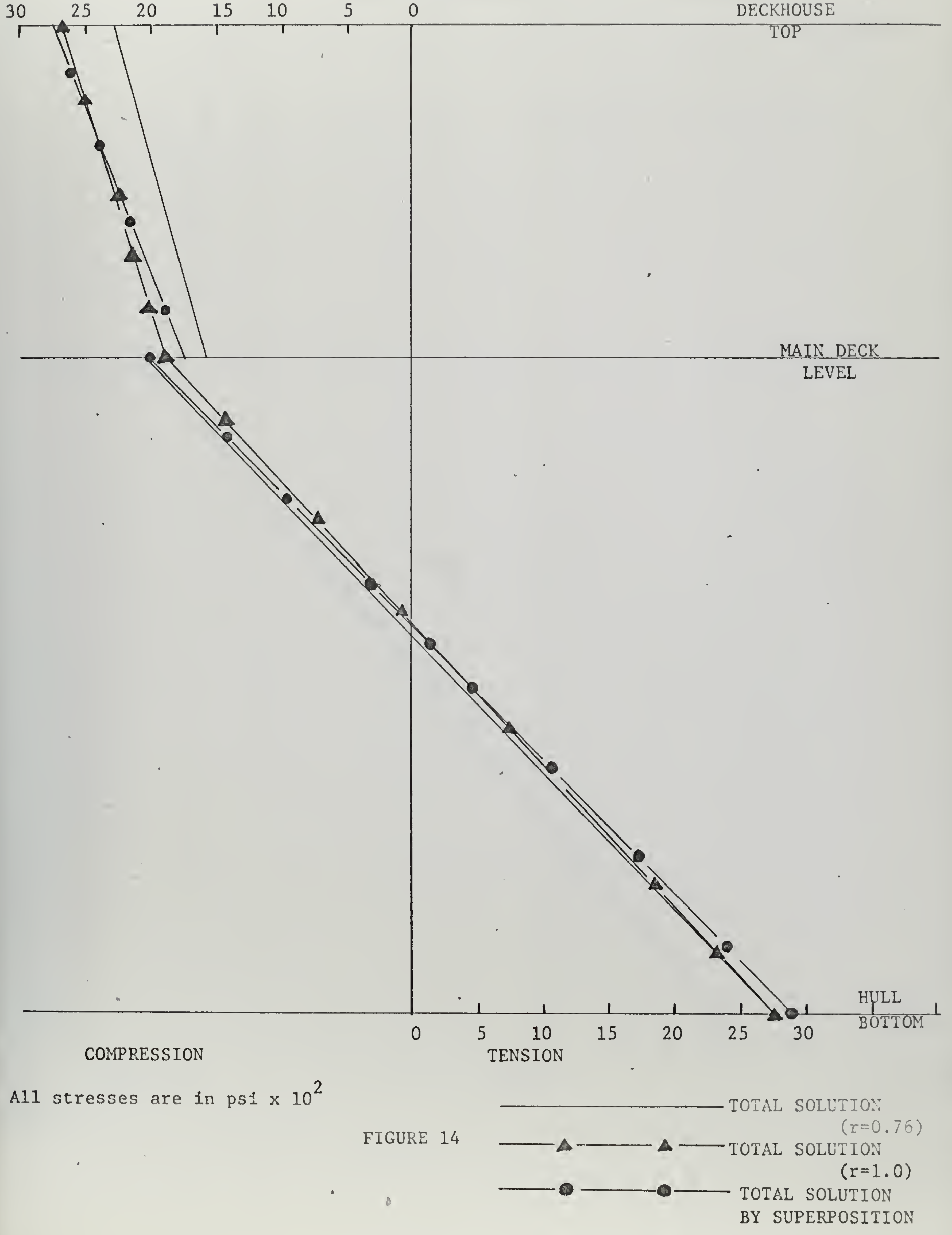


All Stresses are in psi x 10^2

FIGURE 13

"TOTAL SOLUTION"

COMPARISON OF THE RESULTS AT MIDSHIP

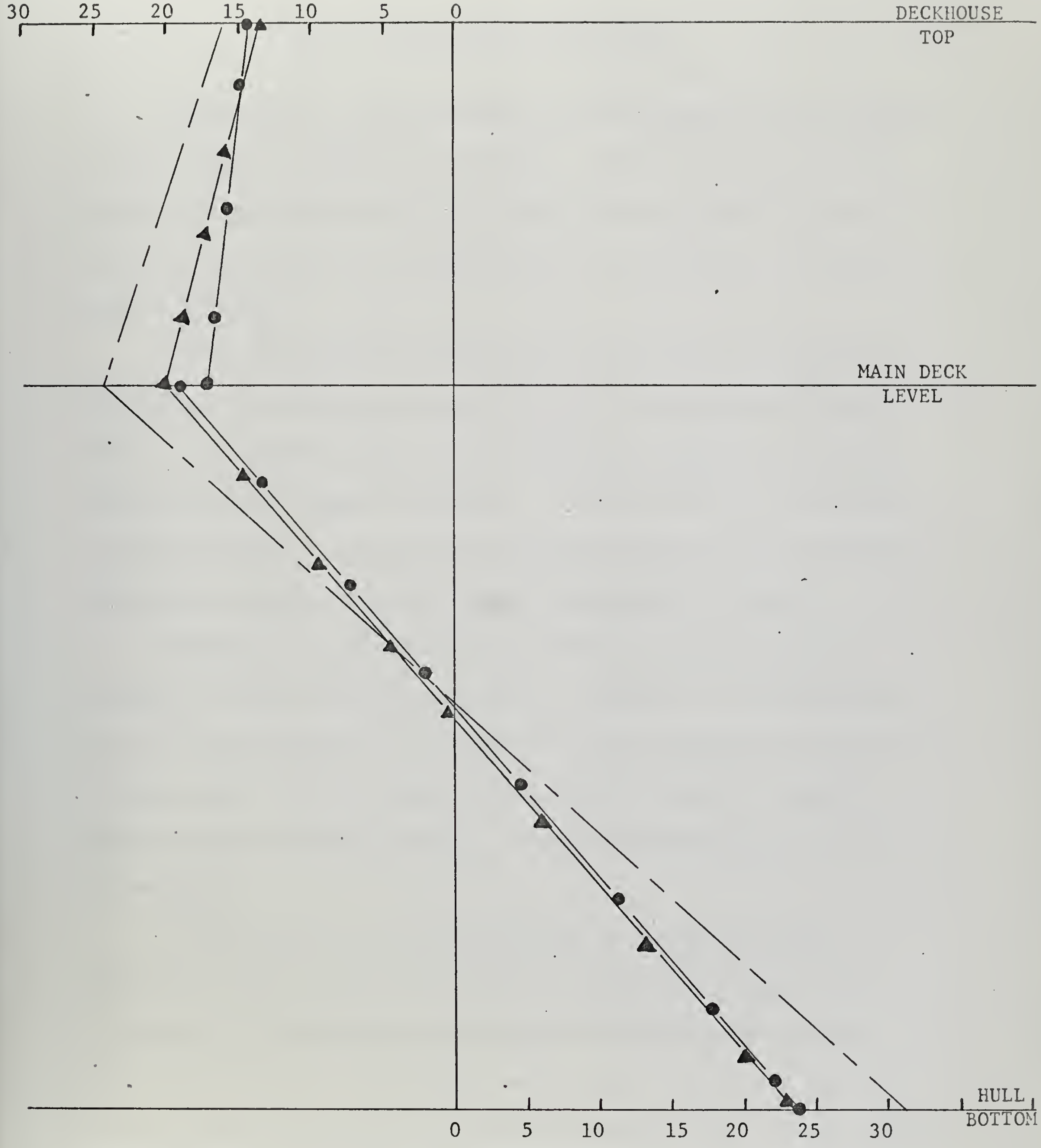


All stresses are in psi x 10^2

FIGURE 14

"TOTAL SOLUTION"

COMPARISON OF LONGITUDINAL STRESS DISTRIBUTION AT THE END OF DECKHOUSE



COMPRESSION

TENSION

All stresses are in psi x 10^2

FIGURE 15

BLEICH'S RE

RESULTS BY SUPERPOSITION

TOTAL SOLUTION (r=1.0)

VI.B - COMPARISON OF THE METHODS

In Figure (9), the longitudinal stress distributions at midship found by using two methods were plotted. In this figure, it can be easily seen that both results are almost the same except 400 psi difference in stresses at deckhouse top. The difference is 50 psi at main deck level.

Figure (10) shows the comparison of two methods at the end of deckhouse. In this case, the difference in stresses at deck level is 320 psi. At deckhouse top, there is a big gap between the values of stresses found by using two methods. At hull bottom, the difference is about 30 psi. But, the value of stress found by using the theory developed is smaller than the value found by Bleich's method.

Figures (11) and (12) show the comparison of longitudinal stresses for equally distributed load. Looking at the results shown by solid line in Figure (11), shear-lag effect at main deck level can be seen easily. All the values for stresses are smaller than the results found by Bleich's method. But, the differences are not so big at mid ship.

In Figure (12), it is not difficult to recognize the big differences between the results found by using both methods. The differences are approximately 1290 psi, at deckhouse top, and 690 psi at hull bottom. At main deck level, there is about 310 psi and 400 psi difference between the results at hullside and at the junction of main deck and deckhouse, respectively.

Figure (13) shows the comparison of the total solution at midship. The strudl results were taken from Reference (3). The agreement among the results is reasonable. The stress distributions were transformed into moments and checked against equilibrium condition; that is, the values of the moments obtained corresponded to the value on the bending moment diagram, except for strudl results and for the total solution using r = 0.76, strudl results give 9% bigger than the moment at midship, and the total solution with r = 0.76 gives 6% smaller bending moment. This was expected, because shear-lag factor r = 0.76 is not true for the whole system with applied constant bending moment at the end of deckhouse.

In Figure (14), the total solutions found by using r = 0.76, r = 1.0 and the total solution by superposition were compared. In this figure, it is seen that the total solution found by using r = 1.0 takes the average of the stresses at deck level.

Comparison of the total solutions at the end of deckhouse were shown in Figure (15). The values of the stresses found by using the theory developed were always smaller than Bleich's results.

As a conclusion of the comparison of the methods, inclusion of the shear effects into two beam theory did not change the values of the stresses at midship. But, at the end of deckhouse, more distinguishable differences were found.

CONCLUSIONS AND RECOMMENDATIONS

Even though the method which includes shear effects requires more elaborate work, it is seen that it is possible to use it for computing the longitudinal stresses at any point along the deckhouse. There seems to be no reason to ignore these effects in design procedures for short and moderate deckhouses where the shear effects could be pronounced.

After examining the comparison of the results found by using both methods at midship, it is possible to conclude that there is some indication to use Bleich's method for design purposes in the computation of longitudinal stresses at midship. The simplicity in his computation is a prime factor. But, for the stress solution at the end of the deckhouse where more distinguishable differences were found by employing both methods, the inclusion of shear effects into two-beam theory being used is necessary. The results may become more realistic.

In the theory developed, the shear-lag effects in the deck represented by "r" is important only in the case of equally distributed load. Application of Equation (4) is a very simplified means of estimating "r", because it was determined by a box-girder analysis of the hull alone in Reference (2). Even though it may require a much more elaborate analysis, it is recommended to improve on the estimation of the shear-lag parameter.

In this analysis, the stiffness of bulkheads or deck beams resisting relative vertical displacements of the deckhouse was assumed

constant for the full-length of the deckhouse. It is known that the deck stiffness K vary along the deckhouse length due to the presence of structural transverse bulkheads. The inclusion of this variation in K in the two-beam deckhouse theory needs further investigation.

Finally, it is recommended to built a physical model similar to the one considered in this theoretical analysis. Then the strain and deflection measurement under similar loading conditions could be employed for verification of the results.

APPENDIX - I

I.A - DERIVATION OF STRAIN ENERGY OF STRUCTURE

Denoting by ε_1 and ε_2 the average longitudinal strain in the deckhouse and hull, respectively, the strain energy of the longitudinal stresses will be as follows:

Deckhouse:

$$\frac{E}{2}\int_{-\ell/2}^{\ell/2} (A_1\,\varepsilon_1^2 + I_1\,y_1''^2)\,dz$$

Hull:

$$\frac{E}{2}\int_{-\ell/2}^{\ell/2} (A_2\,\varepsilon_2^2 + I_2\,y_2''^2)\,dz$$

The strains are counted positive if they represent elongation.

In addition to the strain energy of the longitudinal stresses, there will be energy stored in the bulkheads or deck beams which resist the relative vertical displacements of deckhouse and hull; this part of the strain energy can be expressed by the spring constant K in the form

$$\frac{1}{2}\int_{-\ell/2}^{\ell/2} K(y_1 - y_2)^2\,dz$$

The strain energy due to the shear deflection caused by vertical shear forces will be as follows:

Deckhouse:

$$\frac{1}{2}\int_{-\ell/2}^{\ell/2} \frac{k_1^*\,V_1^2}{A_1 G}\,dz$$

Hull:

$$\frac{1}{2}\int_{-\ell/2}^{\ell/2} \frac{k_2^* v_2^2}{A_2 G} dz$$

where, k_1^* and k_2^* are shear deflection constants and for wide flange box girders, can be assumed that they are equal to 1.0.

The total strain energy V is

$$V = \frac{E}{2}\int_{-\ell/2}^{\ell/2} [A_1 \varepsilon_1^2 + I_1 y_1''^2 + A_2 \varepsilon_2^2 + I_2 y_2''^2 + \frac{K}{E}(y_1 - y_2)^2 + \frac{1}{2EG A_1} v_2^2 + \frac{1}{2EG A_2} v_2^2] dz \qquad (a)$$

The stresses in the deckhouse and hull can be expressed by the average strains ε_1 and ε_2, and by the second derivatives of the deflections y_1'' and y_2''

Deckhouse: $\sigma_1 = E\varepsilon_1 + E\, y_1''\, x_1$

Hull: $\sigma_2 = E\varepsilon_2 + E\, y_2''\, x_2$

Using the relationship given in Equation (3) in Chapter I for the longitudinal stress in the deck at the junction, and the longitudinal stress in the deck-edge,

$$\varepsilon_1 - r\,\varepsilon_2 = a\alpha_1\, y_1'' + r\, a\, \alpha_2\, y_2'' \qquad (b)$$

Further, the longitudinal resultant of all stresses in the deckhouse N_1, must be equal to $EA_1\, \varepsilon_1$, and, similarly, $N_2 = EA_2\, \varepsilon_2$. The

resultant of all longitudinal forces, $N_1 + N_2$, must vanish.

$$N_1 + N_2 = E(A_1 \varepsilon_1 + A_2 \varepsilon_2) = 0 \qquad \text{(c)}$$

By means of Equations (a) and (b), ε_1 and ε_2 can be expressed as follows:

$$\varepsilon_1 = \frac{a A_2}{rA_1 + A_2} (\alpha_1 y_1'' + r \alpha_2 y_2'') \qquad \text{(d)}$$

$$\varepsilon_2 = - \frac{a A_1}{rA_1 + A_2} (\alpha_1 y_1'' + r \alpha_2 y_2'') \qquad \text{(e)}$$

Substituting expressions for ε_1 and ε_2, and v_1 and v_2 (Equations l, and m, in I-C) into the Equation (a) for total strain energy V,

$$V = \frac{E}{2} \int_{-\ell/2}^{\ell/2} [I_1 y_1''^2 + I_2 y_2''^2 + I_A(\alpha_1 y_1'' + r \alpha_2 y_2'')^2 + \frac{K}{E}(y_1 - y_2)^2$$

$$+ E^* y_1'''^2 + F^* y_2'''^2 + G^* y_1''' y_2'''] dz \qquad \text{(f)}$$

I.B - POTENTIAL ENERGY U_w OF EXTERNAL FORCES

Counting p_1 and p_2 positive if acting downward, their potential energy is,

$$- \int_{-\ell/2}^{\ell/2} (p_1 \, y_1 + p_2 \, y_2) dz$$

The shear forces S_c and S_D and the moments M_c and M_D act immediately outside points C and D, and their potential energy will depend on the vertical displacements y_{2c} and y_{2D}, and on the rotations of the end surfaces of the hull, y'_{2c}, and y'_{2D}.

Taking into account the direction of the shears and moments shown in Figure 6 in Chapter II, the potential energy will be;

$$- S_D \, y_{2D} + S_c \, y_{2c} + M_D \, y'_{2D} - M_c \, y'_{2c} = - [Sy_2]_{-\ell/2}^{\ell/2} + [My'_2]_{-\ell/2}^{\ell/2}$$

and the potential energy U_w of the external load is;

$$U_w = [My'_2]_{-\ell/2}^{\ell/2} - [Sy_2]_{-\ell/2}^{\ell/2} - \int_{-\ell/2}^{\ell/2} (p1 \, y_1 + p2 \, y_2) \, dz \qquad (g)$$

I.C - EXPRESSION FOR LONGITUDINAL FORCES AND HORIZONTAL AND VERTICAL SHEARS

The resultants N_1 and N_2 of the longitudinal stresses in the deckhouse and hull are $N_1 = EA_1 \varepsilon_1$, and $N_2 = EA_2 \varepsilon_2$, and using Equations (d) and (e)

$$N_1 = \frac{EI_A}{a} (\alpha_1 y_1'' + r \alpha_2 y_2'') \qquad \text{(h)}$$

$$N_2 = - \frac{EI_A}{a} (\alpha_1 y_1'' + r \alpha_2 y_2'') \qquad \text{(i)}$$

where, $I_A = a^2 \dfrac{A_1 A_2}{(r A_1 + A_2)}$

T being the total shear force from the left end of the deckhouse to a point having the co-ordinate z, equilibrium requires,

$$T = - N_1 = - \frac{EI_A}{a} (\alpha_1 y_1'' + r \alpha_2 y_2'') \qquad \text{(j)}$$

and the unit horizontal shear will be,

$$\frac{dT}{dz} = - \frac{EI_A}{a} (\alpha_1 y_1''' + r \alpha_2 y_2''') \qquad \text{(k)}$$

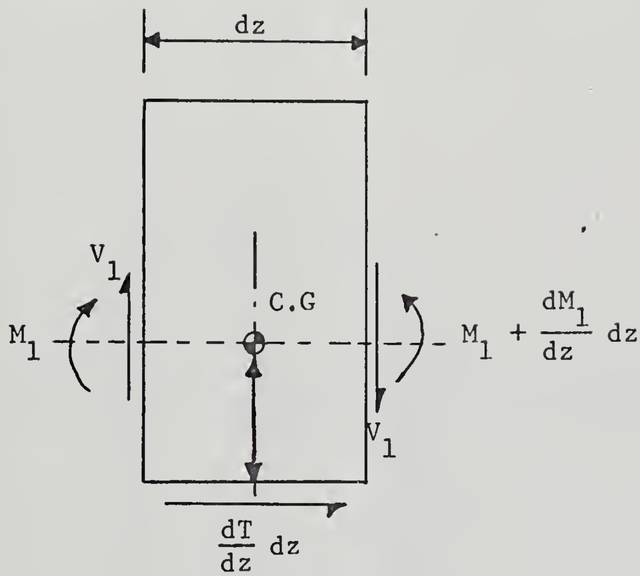


FIGURE A

To obtain the expression for the vertical shear V_1, in the deckhouse, considering an element of the deckhouse of length dz, as in above Figure A.

Equilibrium of moments with respect to the centroid requires,

$$V_1 - a\,\alpha_1 \frac{dT}{dz} = \frac{dM_1}{dz}$$

Substituting $\frac{dT}{dz}$ from the above Equation (k) and using

$$M_1 = -\,EI_1 \left(y_1'' + \frac{p_1}{A_{1w}G}\right)$$

$$V_1 = -\,E(I_1 + \alpha_1^2 I_A)y_1''' - \alpha_1\,\alpha_2\, r\, EI_A\, y_2''' \qquad (1)$$

The vertical shear force in the hull, considering Figure B below, similarly:

$$V_2 = - \alpha_1 \alpha_2 EI_A y_1''' - E(I_2 + r \alpha_2^2 I_A) y_2''' \qquad (m)$$

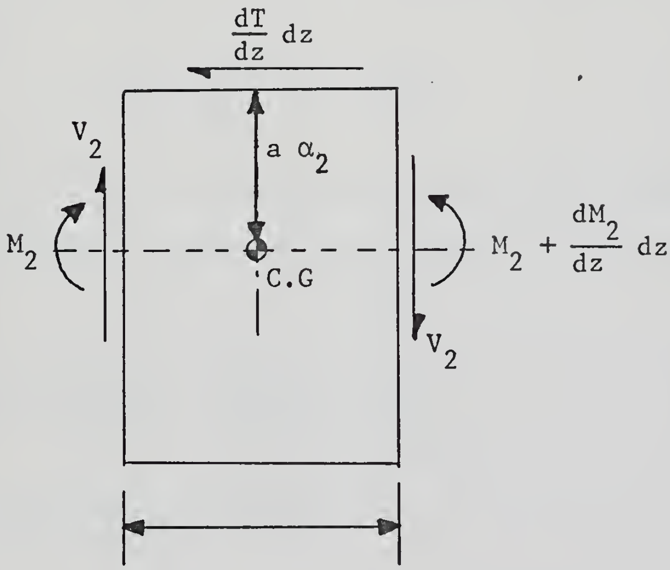


FIGURE B

In the derivation of the Equations (l) and (m), attention must be paid to the following point; when the derivative of $\frac{dM_i}{dz}$ (for i = 1,2) is taken, the contribution from the second term in the moment equation will be zero in both cases. Because, in the following chapters, the cases considered for solving the equations are constant moment loading ($p_1 = p_2 = 0$), and equally distributed loading ($\frac{dp_1}{dz} = \frac{dp_2}{dz} = 0$).

APPENDIX II - APPLICATION OF CALCULUS OF VARIATIONS

The problem considered was in the following manner,

$$U = \delta \int_{-\ell/2}^{\ell/2} F(z, y_1, y_1', y_1'', y_1''', y_2, y_2', y_2'', y_2''')\, dz$$

$$+ [M\, y_2']_{-\ell/2}^{\ell/2} - [S\, y_2]_{-\ell/2}^{\ell/2}$$

To get the set of two simultaneous differential equations, Euler-lagrange equation is employed for y_1 and y_2, in the following manner,

$$Fy - \frac{d}{dx} Fy' + \frac{d^2}{dx^2} Fy'' - \frac{d^3}{dx^3} Fy''' = 0$$

where,

$$Fy = \frac{\partial F}{\partial y}, \quad Fy' = \frac{\partial F}{\partial y'} \ldots\text{.etc.}$$

To get the natural boundary condition equations, the following equations are used for y_1 and y_2

$$\eta'' \left[\frac{\partial F}{\partial y'''}\right] = 0$$

$$\eta' \left[\frac{\partial F}{\partial y''} - \frac{d}{dx}\left(\frac{\partial F}{\partial y'''}\right)\right. = 0$$

$$\eta\left[\frac{\partial F}{\partial y'} - \frac{d}{dx}\left(\frac{\partial F}{\partial y''}\right) + \frac{d^2}{dx^2}\left(\frac{\partial F}{\partial y'''}\right)\right] = 0$$

APPENDIX III - CALCULATIONS FOR DETERMINING K

The following is an analytical appraoch to the determination of the spring constant (K) for a model with bulkheads spaced 20 feet apart.

Basic Nomenclature

A	Area of beam cross section
K'	Factor depending on shape of beam cross section
p	Distributed load at hull-deckhouse connection
V	Vertical shear due to actual forces
v	Vertical shear due to load of one pound acting at the section where the deflection is to be determined
Y_M	Deflection due to internal moments
Y_τ	Deflection due to shear
Y_T	Total deflection
x	Distance along length of beam
< >	Indicates singularity functions

The equation for the deflection due to the internal moments (Y_M) is calculated through the use of singularity functions. The expression for the deflection due to shear (Y_τ) is obtained through the use of the method of unit loads as described in Reference (11). The total deflection (Y_T) is then expressed as the sum of Y_M and Y_τ and set equal to 1 inch. The equation is solved for p (the distributed load acting on the hull-deckhouse connections). The expression for K is twice the value of p.

Deflection Due to Moment Only

$$EI \frac{d^2Y_M}{dx^2} = 240 \, [p_x - p <x - 120>^1 - p <x - 360>^1]$$

$$EIY_M = 240 \, [\frac{x^3}{6} - \frac{<x - 120>^3}{6} - \frac{<x - 360>^3}{6} + c_1 x + c_2]$$

B.C $\quad Y_M = 0 \quad$ when $\quad x = 0, \quad 480''$

$E = 30 \times 10^6$ psi

$I = 8{,}748{,}005 \text{ in}^4$

$Y_M = - (2.105 \times 10^{-5}) p \quad$ at $\quad x = 120''$

Deflection Due to Shear

$$Y_\tau = - \frac{1}{K'} \int \frac{V_v}{AG} \, dx \qquad \text{(method of unit loads)}$$

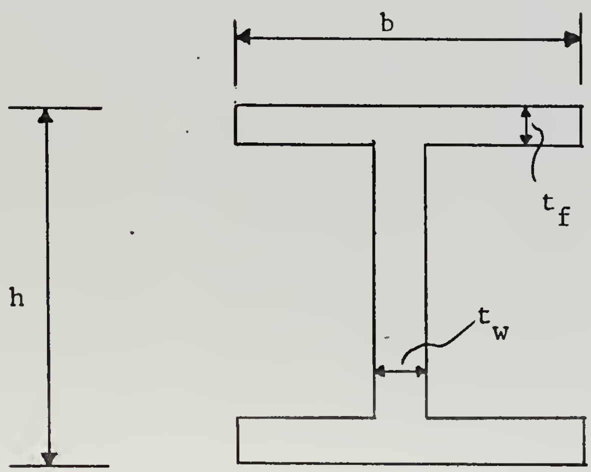
$$Y_\tau = - \frac{1}{K'} \, \frac{p(240)x}{AG}$$

$$K' = \frac{10(1 + \gamma)(1 + 3m)^2}{6(2 + 12m + 25m^2 + 15m^3) + (11 + 66m + 135m^2 + 90m^3) + 30mn^2(1 + m) + 5 \gamma mn^2(8 + 9m)}$$

where

$$m = \frac{2b \, t_f}{h \, t_w}, \qquad h = \frac{b}{n}$$

I-BEAM CROSS SECTION



$t_f = 0.5"$; $t_w = 0.25"$; $h = 360"$; $b = 240"$

$m = 2.66$; $n = 0.666$

$K' = 0.262$

It is customary to assume that only webs of structural shapes, such as channels and I beams resist shearing stresses, because shear stresses are small in flanges.

$$Y_\tau = -(10.55 \times 10^{-5})p$$

$$Y_T = Y_M + Y_\tau = 1"$$

$$1" = -(12.655 \times 10^{-5})p$$

$$p = 0.7902 \times 10^4$$

$$K = 2p$$

$$K = (1.58 \times 10^4)\text{psi}$$

APPENDIX IV - SAMPLE CALCULATIONS

IV.A - FOR CONSTANT MOMENT LOADING

$a = 306''$, $A_1 = 150 \text{ in}^2$, $A_2 = 840 \text{ in}^2$

$I_1 = 534{,}600 \text{ in}^4$, $I_2 = 19{,}440{,}000 \text{ in}^4$, $I_A = 11{,}917{,}309 \text{ in}^4$

$I = 31{,}891{,}909 \text{ in}^4$

$M = M_p = 19.2 \times 10^7$ in-lbs

$K = 1.53 \times 10^4$

$\alpha_1 = 0.4118$, $\alpha_2 = 0.5882$, $r = 1.0$ (shear-lag factor)

$b = (1.392) \times 10^{-6}$, $a_{14} = (1.232) \times 10^{-6}$, $a_{24} = (1.136) \times 10^{-5}$

$a = (5.685) \times 10^{-8}$, $a_{16} = (2.335) \times 10^{-8}$, $a_{26} = (3.129) \times 10^{-7}$

$$r^4(r^8 + a_3 r^6 + a_2 r^4 + a_1 r^2 + a_0) = 0$$

$a_3 = -(1.208) \times 10^2$, $a_2 = (2.959) \times 10^3$, $a_1 = -(5.630) \times 10^4$

$a_0 = (1.924) \times 10^6$

Roots of the characteristic equation are as follows:

$r_1 = r_2 = r_3 = r_4 = 0$

$r_5 = -r_6 = -\gamma_3 = -5.843$

$r_7 = -r_8 = -\gamma_4 = -9.656$

$r_9 = -\gamma_1 - i\,\gamma_2 = -3.266 - i\,3.731$

$r_{10} = -\gamma_1 + i\,\gamma_2 = -3.266 + i\,3.731$

$r_{11} = \gamma_1 - i\,\gamma_2 = 3.266 - i\,3.731$

$r_{12} = \gamma_1 + i\,\gamma_2 = 3.266 + i\,3.731$

$f = -(583.35)$, $g = (158.58)$, $h = 5762.89$, $m = 13{,}700.88$

$n_1 = (-)\ 3.253$, $n_2 = 24.370$, $r_1 = (-)\ 101.55$, $r_2 = (-)\ 67.45$

$S_1 = 2236.18$, $S_2 = 2694.43$

$\mu_1 = (-)\ 0.20974$, $\mu_2 = (-)\ 0.00449$, $\mu_3 = (-)\ 0.88467$

$\mu_4 = 0.22336$

$t_1 = (-)\ 1.166 \times 10^{-6}$, $t_2 = (-)\ 7.449 \times 10^{-7}$, $t_3 = 7.982 \times 10^{-7}$

$t_4 = 7.359 \times 10^{-7}$, $t_5 = 3.413 \times 10^{-6}$, $t_6 = (-)\ 39.09 \times 10^{-6}$

$t_7 = (-)\ 3.371 \times 10^{-6}$, $t_8 = 24.899 \times 10^{-6}$, $t_9 = 12.216 \times 10^{-5}$

$t_{10} = 9.302 \times 10^{-5}$

$v_1 = 7.364 \times 10^{-8}$, $v_2 = 1.065 \times 10^{-8}$, $v_3 = 33.375 \times 10^{-8}$

$v_4 = (-)(1.305) \times 10^{-8}$, $v_5 = 2.463 \times 10^{-6}$, $v_6 = 0.921 \times 10^{-6}$

$v_7 = 11.444 \times 10^{-6}$, $v_8 = (-)\ 1.146 \times 10^{-6}$, $v_9 = 2.624 \times 10^{-6}$

$v_{10} = 12.755 \times 10^{-6}$

For the B.C's equations; $(\bar{z} = 0.5)$

$\sin \gamma_2\bar{z} \sinh \gamma_1\bar{z} = 2.34891$

$\cos \gamma_2\bar{z} \cosh \gamma_1\bar{z} = (-)\ 0.76987$

$\sin \gamma_2\bar{z} \cosh \gamma_1\bar{z} = 2.53634$

$\cos \gamma_2\bar{z} \sinh \gamma_1\bar{z} = (-)\ 0.71298$

$\sinh \gamma_3\bar{z} = 9.257$

$\cosh \gamma_3\bar{z} = 9.404$

$\sinh \gamma_4\bar{z} = 62.540$

$\cosh \gamma_4\bar{z} = 62.548$

Substituting everything into the B C's equations shown in Table 1, the following matrix is derived,

$$\begin{bmatrix} 1.0 & 0.250 & -0.49611 & 0.1509 & 8.3199 & -13.970 \\ 0.0 & 0.0 & -3.5181 & -1.049 & 136.010 & 599.812 \\ 0.0 & 0.0 & 2.549 & 1.297 & 616.40 & -735.20 \\ 0.0 & 5.249 & -22.08 & -94.45 & -16.22 & -418.6 \\ 0.0 & 25.51 & 11.25 & 56.73 & 16.12 & 416.3 \\ 0.0 & 0.0 & 37.62 & 14.88 & 9.321 & 769.2 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \end{bmatrix} = \begin{bmatrix} 0.0 \\ 0.0 \\ 0.0 \\ 0.0 \\ -3.704 \times 10^{-3} \\ 0.0 \end{bmatrix}$$

Results from the solution of above matrix,

$c_1 = 3.494 \times 10^{-5}$

$c_2 = -1.290 \times 10^{-4}$

$c_3 = 3.070 \times 10^{-6}$

$c_4 = -7.899 \times 10^{-6}$

$c_5 = 7.021 \times 10^{-9}$

$c_6 = 2.549 \times 10^{-9}$

The value of c_1 is not used in the calculations, as it was pointed out by Bleich in Reference (1); it describes only a rigid-body motion of the structure not required for the purpose of this analysis.

For $\bar{z} = 0$ (Midship)

$$\frac{N_1}{A_1} = -\ 1481.439 \text{ psi}$$

$$\frac{M_1}{I_1} x_1 = (-)\ 494.586 \text{ psi}; \quad x_1 = -\ 126.01'' \text{ (main-deck level)}$$

$$\frac{M_1}{I_1} x_1 = +\ 211.948 \text{ psi}; \quad x_1 = 54.0'' \text{ (deckhouse top).}$$

so; using $\sigma_1 = \frac{N_1}{A_1} - \frac{M_1}{I_1} x_1$

$$\sigma_1 = -\ 1481.439 + 494.586 = -\ 986.85 \text{ psi (main-deck level)}$$

$$\sigma_1 = -\ 1481.439 - 211.948 = -\ 1693.38 \text{ psi (deckhouse top)}$$

$$\frac{N_2}{A_2} = 264.542 \text{ psi}$$

$$\frac{M_2}{I_2} x_2 = 1251.396 \text{ psi}; \quad x_2 = 179.989'' \text{ (main-deck level)}$$

$$\frac{M_2}{I_2} x_2 = -\ 1251.396 \text{ psi}; \quad x_2 = -\ 179.989'' \text{ (hull-bottom)}$$

Using, $\sigma_2 = \frac{N_2}{A_2} - \frac{M_2}{I_2} x_2$

$$\sigma_2 = 264.542 - 1251.396 = -\ 986.54 \text{ psi (main-deck level)}$$

$$\sigma_2 = 264.542 + 1251.396 = 1515.938 \text{ psi (hull-bottom)}$$

<u>For $\bar{z} = 0.5$</u> (At the end of deckhouse)

$$\frac{N_1}{A_1} = -\ 982.177 \text{ psi}$$

$$\frac{M_1}{I_1} x_1 = 348.770 \text{ psi}; \quad x_1 = -\ 126.01'' \quad \text{(main-deck level)}$$

$$\frac{M_1}{I_1} x_1 = -\ 149.461 \text{ psi}; \quad x_1 = 54'' \quad \text{(deckhouse top)}$$

$$\sigma_1 = -\ 982.177 - 348.770 = -\ 1330.94 \text{ psi} \quad \text{(main deck level)}$$

$$\sigma_1 = -\ 982.177 + 149.461 = -\ 832.71 \text{ psi} \quad \text{(deckhouse top)}$$

$$\frac{N_2}{A_2} = 175.388 \text{ psi}$$

$$\frac{M_2}{I_2} x_2 = 1506.336 \text{ psi}; \quad x_2 = 179.989'' \quad \text{(main-deck level)}$$

$$\frac{M_2}{I_2} x_2 = -\ 1506.336 \text{ psi}; \quad x_2 = -\ 179.989'' \quad \text{(hull-bottom)}$$

$$\sigma_2 = 175.388 - 1506.336 = -\ 1330.948 \text{ psi} \quad \text{(main deck level)}$$

$$\sigma_2 - 175.388 + 1506.336 = 1681.724 \text{ psi} \quad \text{(hull-bottom)}$$

IV.B - FOR EQUALLY DISTRIBUTED LOADING

All the values of parameter for mathematical model will be the same except,

$r = 0.76$ (shear-lag factor)

$$I_A = a^2 \frac{A_1 A_2}{rA_1 + A_2} = 1.23670 \times 10^7 \text{ in}^4$$

$$I = I_1 + I_2 + I_A = 32{,}341{,}600 \text{ in}^4$$

$b = 1.09790 \times 10^{-6}$, $a_{14} - 1.26918 \times 10^{-6}$, $a_{24} = 1.09432 \times 10^{-5}$

$a = 5.28018 \times 10^{-8}$, $a_{16} = 2.48364 \times 10^{-8}$, $a_{26} = 2.82865 \times 10^{-7}$

Coefficients in the characteristic equation,

$a_3 = - 1.21261 \times 10^2$, $a_2 = 2.98730 \times 10^3$

$a_1 = - 4.96456 \times 10^4$, $a_0 = 1.730691 \times 10^6$

Roots of the characteristic equation are as follows.

$r_1 = r_2 = r_3 = r_4 = 0$

$r_5 = - r_6 = - \gamma_3 = - 5.882$

$r_7 = - r_8 = - \gamma_4 = - 9.624$

$r_9 = - \gamma_1 - i\, \gamma_2 = - 3.183 - i\, 3.621$

$r_{10} = - \gamma_1 + i\, \gamma_2 = - 3.183 + i\, 3.621$

$r_{11} = \gamma_1 - i\, \gamma_2 = 3.183 - i\, 3.621$

$r_{12} = \gamma_1 + i\, \gamma_2 = 3.183 + i\, 3.621$

$f = -522.480, \quad g = 137.393, \quad h = 4724.149$

$m = 11{,}634.397$

$n_1 = -2.980, \quad n_2 = 23.051$

$r_1 = -92.955, \quad r_2 = -62.581$

$S_1 = 1895.044, \quad S_2 = 2329.223$

$\mu_1 = -0.20766, \quad \mu_2 = -0.01375, \quad \mu_3 = -0.72377$

$\mu_4 = 0.25226$

$t_1 = -1.289 \times 10^{-6}, \quad t_2 = -8.006 \times 10^{-7}, \quad t_3 = 3.085 \times 10^{-7}$

$t_4 = 7.331 \times 10^{-7}, \quad t_5 = 3.697 \times 10^{-6}, \quad t_6 = -26.240 \times 10^{-7}$

$t_7 = -3.617 \times 10^{-6}, \quad t_8 = 26.307 \times 10^{-6}, \quad t_9 = 12.570 \times 10^{-5}$

$t_{10} = 10.272 \times 10^{-5}$

$v_1 = 6.305 \times 10^{-8}, \quad v_2 = 1.151 \times 10^{-8}, \quad v_3 = 25.753 \times 10^{-8}$

$v_4 = -1.855 \times 10^{-8}, \quad v_5 = 2.063 \times 10^{-6}, \quad v_6 = 0.991 \times 10^{-6}$

$v_7 = 9.018 \times 10^{-6}, \quad v_8 = -1.667 \times 10^{-6}, \quad v_9 = 2.367 \times 10^{-6}$

$v_{10} = 12.041 \times 10^{-6}$

For the B.C's equations; $(\bar{z} = 0.5)$

$\sin \gamma_2 \bar{z} \sinh \gamma_1 \bar{z} = 2.2854$

$\cos \gamma_2 \bar{z} \cosh \gamma_1 \bar{z} = -0.6060$

$\sin \gamma_2 \bar{z} \cosh \gamma_1 \bar{z} = 2.4832$

$\cosh \gamma_2 \bar{z} \sinh \gamma_1 \bar{z} = -0.5578$

$\sinh \gamma_3 \bar{z} = 9.43150$

$\cosh \gamma_3 \bar{z} = 9.48439$

$\sinh \gamma_4 \bar{z} = 61.517$

$\cosh \gamma_4 \bar{z} = 61.526$

Substituting everything into the B C's equations shown in Table 1, the following matrix is derived.

$$\begin{bmatrix} 1.0 & 0.250 & -0.4829 & 0.0944 & 6.864 & -15.55 \\ 0.0 & 0.0 & -3.648 & -1.268 & 121.0 & 631.2 \\ 0.0 & 0.0 & 11.75 & 16.48 & 4943.0 & -10170. \\ 0.0 & 4.734 & -7.452 & -62.61 & -38.86 & -427.8 \\ 0.0 & 24.08 & 7.676 & 62.31 & 35.38 & 291.5 \\ 0.0 & 0.0 & 36.94 & 18.50 & 22.73 & 411.7 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \end{bmatrix} = \begin{bmatrix} 2.322 \times 10^{-5} \\ -8.066 \times 10^{-5} \\ -3.487 \times 10^{-3} \\ -4.539 \times 10^{-4} \\ -2.430 \times 10^{-3} \\ 2.459 \times 10^{-4} \end{bmatrix}$$

Results from the solution of above matrix,

$c_1 = 5.678 \times 10^{-5}$

$c_2 = -1.000 \times 10^{-4}$

$c_3 \doteq 7.168 \times 10^{-6}$

$c_4 = -1.015 \times 10^{-6}$

$c_5 = -6.462 \times 10^{-7}$

$c_6 = 3.548 \times 10^{-8}$

The value of c_1 is not included in the computations. As explained before, it describes only a rigid-body motion of the structure, not required for the purposes of this analysis.

<u>For $\bar{z} = 0$</u> (Midship)

$$\frac{N_1}{A_1} = -\ 957.677 \text{ psi}$$

$$\frac{M_1}{I_1} x_1 = -\ 200.573 \text{ psi}; \quad x_1 = -\ 126.01'' \quad \text{(main deck level)}$$

$$\frac{M_1}{I_1} x_1 = 85.592 \text{ psi}; \quad x_1 = 54'' \quad \text{(deckhouse top)}$$

using, $\sigma_1 = \frac{N_1}{A_1} - \frac{M_1}{I_1} x_1$

$$\sigma_1 = -\ 957.677 + 200.573 = -\ 759.103 \text{ psi} \quad \text{(main deck level)}$$

$$\sigma_1 = -\ 957.677 - 85.592 = -\ 1045.62 \text{ psi} \quad \text{(deckhouse top)}$$

$$\frac{N_2}{A_2} = +\ 171.371 \text{ psi}$$

$$\frac{M_2}{I_2} x_2 = 1224.315 \text{ psi}; \quad x_2 = 179.989'' \quad \text{(main deck level)}$$

$$\frac{M_2}{I_2} x_2 = -\ 1224.315 \text{ psi}; \quad x_2 = -\ 179.989'' \quad \text{(hull-bottom)}$$

$$\sigma_2 = \frac{N_2}{A_2} - \frac{M_2}{I_2} x_2$$

$$\sigma_2 = 171.371 - 1224.315 = -\ 1052.94 \text{ psi} \quad \text{(main deck level)}$$

$$\sigma_2 = 171.371 + 1224.315 = 1395.686 \text{ psi} \quad \text{(hull-bottom)}$$

For $\bar{z} = 0.5$ (At the end of deckhouse)

$$\frac{N_1}{A_1} = -\ 520.778 \text{ psi}$$

$$\frac{M_1}{I_1} x_1 = -\ 148.120 \text{ psi}; \quad x_1 = -\ 126.01'' \quad \text{(main deck level)}$$

$$\frac{M_1}{I_1} x_1 = 63.474 \text{ psi}; \quad x_1 = 54'' \quad \text{(deckhouse top)}$$

$$\sigma_1 = -520.778 + 148.120 = -\ 372.658 \text{ psi} \quad \text{(main deck level)}$$

$$\sigma_1 = -\ 520.778 - 63.474 = -\ 584.252 \text{ psi} \quad \text{(deckhouse top)}$$

$$\frac{N_2}{A_2} = 92.996 \text{ psi}$$

$$\frac{M_2}{I_2} x_2 = 583.474 \text{ psi}; \quad x_2 = 179.989'' \quad \text{(main deck level)}$$

$$\frac{M_2}{I_2} x_2 = -\ 583.474 \text{ psi}; \quad x_2 = -\ 179.989'' \quad \text{(hull-bottom)}$$

$$\sigma_2 = \frac{N_2}{A_2} - \frac{M_2}{I_2} x_2$$

$$\sigma_2 = 92.996 - 583.474 = -\ 490.478 \text{ psi} \quad \text{(main-deck level)}$$

$$\sigma_2 = 92.996 + 583.474 = 676.470 \text{ psi} \quad \text{(hull-bottom)}$$

IV.C - FOR TOTAL LOADING

All the values of parameter for mathematical model will be the same except,

$r = 0.76$ (shear-lag factor)

$I_A = 1.23670 \times 10^7 \text{ in}^4$

$I = I_1 + I_2 + I_A = 32{,}341{,}600 \text{ in}^4$

The other coefficients and parameters will be the same as for equally distributed loading case.

Substituting everything into the B.C's equations, the following matrix will be derived

$$\begin{bmatrix} 1.0 & 0.250 & -0.4829 & 0.0944 & 6.864 & -15.15 \\ 0.0 & 0.0 & -3.648 & -1.268 & 121.0 & 631.2 \\ 0.0 & 0.0 & 11.75 & 16.48 & 4943.0 & -10170.0 \\ 0.0 & 4.734 & -7.452 & -62.21 & -38.86 & -427.8 \\ 0.0 & 24.08 & 7.676 & 62.31 & 35.38 & 291.5 \\ 0.0 & 0.0 & 36.94 & 18.50 & 22.73 & 411.7 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \end{bmatrix} = \begin{bmatrix} 2.322 \times 10^{-5} \\ -8.066 \times 10^{-5} \\ -3.487 \times 10^{-3} \\ -4.539 \times 10^{-4} \\ -6.134 \times 10^{-3} \\ 2.459 \times 10^{-4} \end{bmatrix}$$

Results from the solution of above matrix,

$c_1 = 9.244 \times 10^{-5}$

$c_2 = -\ 2.286 \times 10^{-4}$

$c_3 = 1.234 \times 10^{-5}$

$c_4 = -\ 1.145 \times 10^{-5}$

$c_5 = -\ 6.169 \times 10^{-7}$

$c_6 = 3.878 \times 10^{-8}$

For $\bar{z} = 0$ (Midship)

$$\frac{N_1}{A_1} = -\ 2049.499 \text{ psi}$$

$$\frac{M_1}{I_1} x_1 = -\ 492.638 \text{ psi}; \quad x_1 = -\ 126.01'' \quad \text{(main deck level)}$$

$$\frac{M_1}{I_1} x_1 = +\ 211.113 \text{ psi}; \quad x_1 = 54'' \quad \text{(deckhouse top)}$$

$$\sigma_1 = -\ 2049.499 + 492.638 = -\ 1556.861 \text{ psi} \quad \text{(main deck level)}$$

$$\sigma_1 = -\ 2049.499 - 211.113 = -\ 2260.61 \text{ psi} \quad \text{(deckhouse top)}$$

$$\frac{N_2}{A_2} = 365.982 \text{ psi}$$

$$\frac{M_2}{I_2} x_2 = 2414.481 \text{ psi}; \quad x_2 = +\ 179.989'' \quad \text{(main deck level)}$$

$$\frac{M_2}{I_2} x_2 = -\ 3414.481 \text{ psi}; \quad x_2 = -\ 179.989 \quad \text{(hull-bottom)}$$

$$\sigma_2 = 365.982 - 2414.481 = -\ 2048.49 \text{ psi} \quad \text{(main deck level)}$$

$$\sigma_2 = 365.982 + 2414.481 = 2780.46 \text{ psi} \quad \text{(hull-bottom)}$$

For $\bar{z} = 0.5$ (At the end of deckhouse)

$$\frac{N_1}{A_1} = -\ 1035.029 \text{ psi}$$

$$\frac{M_1}{I_1} x_1 = 463.73 \text{ psi}; \quad x_1 = 126.01'' \quad \text{(main deck level)}$$

$$\frac{M_1}{I_1} x_1 = -\ 198.725 \text{ psi}; \quad x_1 = 54'' \quad \text{(deckhouse top)}$$

$\sigma_1 = -\ 1035.029 - 463.730 = -\ 1498.759$ psi (main deck level)

$\sigma_1 = -\ 1035.029 + 198.725 = -\ 836.30$ psi (deckhouse top)

$$\frac{N_2}{A_2} = 184.826 \text{ psi}$$

$\frac{M_2}{I_2} x_2 = 2156.883$ psi; $x_2 = 179.989''$ (main deck level)

$\frac{M_2}{I_2} x_2 = -\ 2156.883$ psi; $\bar{x}_2 = 179.989''$ (hull-bottom)

$\sigma_2 = 184.826 - 2156.883 = -\ 1972.05$ psi (main deck level)

$\sigma_2 = 184.826 + 2156.883 = 2341.709$ psi (hull-bottom)

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