FORECASTING THE SIZE DISTRIBUTION OF A POPULATION OF SAVINGS ACCOUNTS: A MARKOVIAN APPROACH

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THESIS

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ABSTRACT

The Markov chain model, with extensions to cover the phenomena of arrivals and departures, was applied to a population of savings accounts, in a savings institution, to forecast the size distribution, total number of accounts and total amount of savings of the population. The stochastic processes governing the behavior of the population were first assumed to be time stationary. This assumption was then relaxed and an econometric model was used to predict future values of the parameters of the non-stationary model. Both models were validated by comparing predicted size distributions, total number of accounts and total amount of savings against observed values. The chi square goodness of fit test was used in the comparison. The fundamental matrix of the stationary model was also used to predict the equilibrium distribution and related measures of the population.

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I. INTRODUCTION

A. PURPOSE

It is the purpose of this thesis to develop and evaluate two analytical models which can be used to forecast the structure of a population of savers and the level of savings of a savings institution. The population of savers is divided into a finite number of classes and the structure is the distribution of savers among the classes.

B. BACKGROUND

While it is difficult, if not impossible, to predict the future behavior of an individual it is believed that the aggregate behavior of a population is less erratic and, therefore, more amenable to analysis and prediction. Assuming that a large population has considerable inertia, current trends can be used to project into the future.

The rate of change of the structure and characteristics of a population can, at times, be considered to be dependent upon its size, external forces which affect the members of the population and the response to these forces.

In the case of the population of savers in savings institutions, it has been observed that members of this population are not very responsive to changes in economic conditions. Thus, during periods of constant rate of expansion or contraction in the business cycle, external forces

affecting this population may be considered to be constant and a time stationary Markov Chain model may be used to study the behavior of the population.

However, during turning points in the business cycle or periods of rapid economic changes, external forces may be sufficiently large to affect the savings pattern of the savers so that the stationarity assumption may no longer hold. Under these circumstances a more comprehensive model which takes into consideration the effects of external conditions on the behavior of the savers would be required. The major problem in constructing such a model would be in discovering the factors which affect the population, measuring the effect of these factors and the effects of interaction between various factors.

The effect of competition between various savings institutions for a larger share of the savers' market could not be modeled because of the lack of data. However, it is believed that, in the short run, the savers' market is in a state of equilibrium and the share of the market captured by a savings institution is relatively constant. Thus it can be assumed that competition does not affect the savers' behavior to such an extent that, not considering its effect, would render any model inadequate.

C. REVIEW OF MARKOV CHAIN MODELS

The basic model used in this study was introduced by A. A. Markov (1856-1922) around 1907. This model was first applied in economics to

the analysis of income and wage distributions by Solow [21] in 1951. The same model was used by Hart and Prais [12] in 1956 in a study of business concentration.

The model assumes that a population of entities can be classified into a finite number of classes. The population is observed at equidistant time points. The number of entities observed to move from one class to another is assumed to be generated by a stochastic process. The probability of transition is assumed to depend only on the class the entity is in, at the current time interval, and not on where it had previously been. This process of change can be completely described by a transition matrix, P, as shown below. The p_{ij} element is the probability that an entity currently in the ith class will be found in the jth class after one time period. If the stochastic process is time stationary then the matrix does not change with time.

P MATRIX

Ending in Class

		I	II	.	.	.	M
Beginning in Class	I	p_{11}	p_{12}	.	.	.	p_{1m}
	II	p_{21}	p_{22}	.	.	.	p_{2m}
	.	.					
	.	.					
	.	.					
	M	p_{m1}	p_{m2}	.	.	.	p_{mm}

In most of the research studies using this model the general procedure has been to observe some pattern of change over time and, assuming that the stochastic process is time stationary, estimate the transition probabilities and project the future change.

Projection of expected number of entities in each class can be computed as follows:

let the number of entities in each class at time t be $n_1^t, n_2^t, \dots n_m^t$. If the transition probabilities are known then the expected number of entities moving out of the ith class is $p_{i1}n_i^t, p_{i2}n_i^t, \dots p_{im}n_i^t$.

The expected number of entities in each class at time t+1 can be found by adding up all the entities that have moved into the class and those that did not move out. Thus

$$n_1^{t+1} = n_1^t p_{11} + n_2^t p_{21} + \dots n_m^t p_{m1}$$

$$n_2^{t+1} = n_2^t p_{21} + n^t p_{22} + \dots n_m^t p_{m2}$$

$$\vdots$$

$$n_m^{t+1} = n_m^t p_{m1} + n_m^t p_{m2} + \dots n_m^t p_{mm}$$

In matrix notation the above expressions can be compactly written as:

$$N^{t+1} = N^t \times P$$

where $N^t = (n_1^t \; n_2^t \quad \dots n_m^t)$, a 1 x m vector

$N^{t+1} = (n_1^{t+1} \; n_2^{t+1} \quad \dots n_m^{t+1})$, a 1 x m vector

P = matrix as defined earlier.

N^{t+2} can be computed by replacing N^t by N^{t+1} in the above expression. This is equivalent to multiplying N^t by P x P. The distribution after k periods can thus be obtained by multiplying N^t by P raised to the kth power.

This basic model has two major limitations. First, it assumes that the total number of entities in the system is fixed. This assumption has been violated frequently in practical applications of this model as changes due to entities entering the system, leaving it or losing identity by merging are the rule rather than exception. Second, the assumption that the stochastic process is time stationary is untenable over long periods. Changes in numerous exogenous variables such as wage rates, technology and legal requirements are likely to result in changes in the transition probabilities.

Adelman [1] in 1958, overcame the first limitation by introducing the concept of a reservoir of potential entrants, from which entrants may come and to which exants may go. There was an operational difficulty in estimating the size of the population of potential entrants. However, Adelman pointed out that the exact size of this population need not be known if one was dealing with the proportion of entities in each class rather than with the exact number of entities. She therefore assumed that the size of the reservoir to be 100,000. The reason given for this choice was that it must be large relative to the number of entities in the system.

Stanton and Kettunen [22] in 1967 confirmed Adelman's observation but went on to demonstrate that: "The number of potential entrants to an industry or to a population has a definite and measurable effect on subsequent projections made for that distribution when Markov processes are used." Thus, if the number of entities in each class is required, an arbitrary choice of the size of the population of entrants will not work.

Duncan and Lin [9] in 1972 proposed that arrivals could be treated as a separate stochastic process. The entry of an entity into the system is viewed as a two-stage process; first, arriving into the system, then entering into a particular class. One could then estimate the parameters of the entire process by observing the arrivals, the distribution of arrivals among the classes and the transitions between classes separately. He denoted the data by Z which was composed of the number of arrivals into each class (A) and the number of transitions between each class (X). The set of parameters of the process was denoted by $\theta = (P, p, \pi)$ where P was the transition matrix, p was the multinomial vector of probability of an arrival entering a particular class and π was the vector of parameters of the arrival distribution. The sampling distribution was then written as follows:

$$
\begin{aligned}
f_\theta(z) &= f_\theta(x,a) = f_\theta(x \mid A=a) f_\theta(a) \\
&= f_P(x \mid A=a) f_{(p,\pi)}(a)
\end{aligned}
$$

The likelihood function could then be written as

$$L_z(\theta) = L_{x \mid A=a}(P) \cdot L_a(p, \pi)$$

Three reasons were given for the importance of the factorization shown above:

"a. The first factor $L_{x|A=a}(P)$ depends on Z only through the transition counts;

b. The second factor $L_a(p, \pi\ )$ depends on Z only on the observed entries; and

c. Likelihood inference is reduced to two distinct and simpler problems."

Anderson and Goodman [2] in 1957 proposed a number of statistical tests for the following hypotheses

a. that the transition probabilities of a first order chain are constant;

b. that in case the transition probabilities are constant, they are specified numbers;

c. that the process is a uth Markov chain against the alternative it is rth but not uth order.

Because of the factorization of the likelihood function Duncan and Lin concluded that the methods of Anderson and Goodman are applicable to a system with changing number of entities.

Hallberg [11] in 1969 challenged one of the most demanding assumptions of the Markov chain model that the transition probabilities are constant regardless of time. He proposed to overcome this problem by relating transition probabilities to economic variables and to use these relations to predict future values of transition probabilities. For some

unknown reasons he regressed transition probabilities against the logarithms of exogenous variables. Some predicted transition probabilities did not fall within the range of zero to one range. He then suggested setting negative predictions to zero and to normalize each row of the transition matrix by dividing each element by the row sum.

D. REMARKS

Despite the limitations of the basic Markov chain model it has been successfully used in a variety of situations. The Duncan and Lin approach extends the basic model to include arrivals and departures. This can be done with little additional effort. To extend the model to cover the possibility of non-stationary transition probabilities is a considerably more difficult task. The first problem is acquiring a data base which is large enough to yield precise estimates of transition probabilities. The data must also span a long period so that the factors which affect the transition probabilities have an opportunity to vary. The second problem is to identify these factors and to obtain a functional relationship between transition probabilities. The third problem is to predict the future values of these factors. The prediction of the non-stationary Markov chain model is only as good as the prediction of these factors. The approach suggested by Hallberg can be improved by transforming the estimates of transition probabilities into logits (the logarithm of the estimates of odds of transition). This will ensure that the predicted transition probabilities are between zero and one.

The basic Markov chain model is used in this paper to model the behavior of a population of savers at a savings institution. The Duncan and Lin approach is used to treat the phenomena of entries and exits. A nonstationary Markov chain model (Model II of this paper) has also been developed. The parameters of the models were estimated with data from five quarters. The models were then validated with data from the following five quarters. The Chi-square Goodness of fit test was used to compare the predictive power of the two models.

II. MODEL OF A POPULATION OF SAVERS

A. GENERAL

The population of savers is first divided into m classes by the acount of savings each saver has in his savings account. Each saver is free to increase or decrease his savings and to leave the savings institution by closing his account. The population is observed periodically. A projection of the structure of the population and the amount of savings in each class, based on these observations, is desired. A Markov chain model can be used for this purpose provided the basic assumptions of the model are not violated.

B. ASSUMPTIONS

1. The probability that an account moves from class i to class j depends only on class i and does not depend on the past history of the account. This is obviously not true for an individual account but possibly holds for the population of a given class.

2. Each saver acts independently of other savers. If savers act in unison then a Markov model will fail as the assumption of independence is no longer valid. However, the assumption generally holds even if savers are affected by the same factors. The transition probabilities may shift because of these factors but the randomness in action of individual savers is still there.

3. The distribution of the size of accounts within a class is independent of the number of accounts that move in or out of that class. This assumption is not required for Markov model but is necessary if one has to determine the amount of savings from a knowledge of the number of accounts in each class. This assumption is generally true if the number of accounts in each class is large relative to the net change in each period. This assumption can be violated if the number of accounts in each class is small and if the class boundaries are wide.

4. The transition probabilities, arrival rate and the distribution of entrants among states are time stationary. This assumption may hold during periods of constant expansion or contraction of the business cycle. However, it is not expected to hold over long periods and during times when external forces change the saving pattern of savers. This assumption is relaxed in Model II where an attempt was made to discover their functional relationship with economic factors and other exogenous variables.

C. DESCRIPTION OF MODEL I

1. The Transition Matrix

Model I has only one stochastic process, the basic Markov chain model. The number of arrivals is considered to be constant and the proportion of arrivals entering each class is also constant.

Let m = total number of classes including one class of closed accounts

t = time, measured in periods, 0, 1, 2 . . . T

e_t = the accumulated number of accounts that have closed at time t

f'_t = $(f_2^t \; f_3^t \; . \; . \; . \; f_m^t)$

= number of accounts in each active class at t

c'_t = $(c_2^t \; c_3^t \; . \; . \; . \; c_m^t)$

= number of new accounts entering each active class at time t

$$P = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1m} \\ p_{21} & p_{22} & & p_{2m} \\ \vdots & & & \\ p_{m1} & p_{m2} & \cdots & p_{mm} \end{bmatrix}$$

Let class 1 be the class which contains all the closed accounts. It is assumed that an account in the inactive state will not re-enter the active states. Thus $p_{11} = 1.0$ and $p_{1j} = 0.0$, $j = 2 \; . \; . \; . \; m$

The expected number of accounts at time t can be computed from the following relationship 9 :

$$E(e_t \; f'_t) = (0 \; f'_0)P_t + (0 \; c') \sum_{j=0}^{t-1} P_j$$

where $t = 0, 1 \; . \; . \; . \; T$ and $P_0 = I$

The first term on the right of the equality sign is the expected number of accounts in each class at time t from the original population f'_0. The second term is the expected number of accounts in each class derived from those accounts which join the system at each period. Thus, the accounts that arrive by period 1 would have undergone t-1 periods of transition. Those that arrive by period 2 would have undergone t-2 periods of transition. Those arriving at time t would undergo no transition as $P_0 = I$.

As the stochastic process has been assumed to be time stationary the elements of the P matrix are constant and P_t is just the single period P matrix raised to the t^{th} power.

The expected total number of accounts in the system at time t is just the sum of the elements of f'_t.

If the size distribution of accounts within each class is constant over the period of prediction, then the amount of savings in each class can be estimated by multiplying the expected number of accounts by the average amount of savings in that class.

2. The Equilibrium Distribution

If prevailing conditions were to persist the structure of the population will reach an equilibrium in which the number of accounts leaving each class is balanced by an equal influx of accounts from the other classes. The limiting distribution is given by [9]:

$$\lim_{n \to \infty} E(e\, f') = (\infty \quad c'(I - Q)^{-1})$$

where Q is the sub-matrix of P obtained by removing the column of transition probabilities from the classes of active accounts (Class II to Class XI) into the class of closed accounts (Class I), and the row of transition probabilities of the class of closed accounts.

The matrix, $(I - Q)^{-1}$, is often called the fundamental matrix, denoted by M. The ij^{th} element of this matrix is the expected number of periods that a new account entering class i when it joins the system will spend in class j before closing.

The expected number of periods that a new account entering class i when it joins the system will remain in the system can be found by summing the i^{th} row of the fundamental matrix.

The above results and further treatment can be found in Chapter 3 of Ref. [13].

3. Prediction Interval for Single Step Transition

The predictions made with Model I are point estimates. They do not provide any information as to how close they could be to a future observation. A prediction interval which gives the range of values that a future observation would take say ninety percent of the time would be of greater value to a decision maker.

The number of accounts in each class is the sum of m binomial random variables. If the number of accounts in each class is large then the binomial random variables can be approximated by normal random variables. The sum of normal random variables is another normal random variable. Thus a prediction interval can be constructed using this

approximation. For one step transition the prediction interval can be easily constructed. However, for more than one step transitions the task of constructing a prediction interval becomes rather difficult. The problem is that after the first transition the number of accounts in each class becomes random and the expression for the unconditional variance of the number of accounts becomes quite unmanageable. The expressions for the variance of the number of accounts in each class, the total number of accounts, the amount of savings in each class and the total amount of savings for single step transition are listed below. The derivation can be found in Appendix A.

Let n_j^a be the number of accounts in class j at beginning of time period a

p_{ij} be the transition probability of an account from class i to j

N^a be the number of accounts in the system at beginning of time period a

Z_j^a be the amount of savings in class j at beginning of time period a

Z^a be the total amount of savings in the system at beginning of time period a

$$\mathrm{Var}(n_j^{a+1}) = \sum_{i=2}^{m} n_i^a p_{ij}(1 - p_{ij})$$

$$\mathrm{Var}(N^{a+1}) = \sum_{j=2}^{m} \mathrm{Var}(n_j^{a+1}) + 2 \sum_{j=2}^{m-1} \sum_{\substack{k=3 \\ j<k}}^{m} \mathrm{Cov}(n_j^{a+1}, n_k^{a+1})$$

$$\text{Cov}(n_j^{a+1}, n_k^{a+1}) = \sum_{\substack{i=2 \\ j \neq k}}^{m} - (n_i^a p_{ij} p_{ik})$$

Let z_{kj}^a be the amount of savings in an account which has moved into class j

$$\text{Var}(Z_j^{a+1}) = \sum_{i=2}^{m} n_i^a p_{ij} \text{Var}(z_{kj}) + E^2(z_{kj}) n_i^a p_{ij}(1 - p_{ij})$$

$$\text{Cov}(Z_j^{a+1}, Z_1^{a+1}) = \sum_{i=2}^{m} -(n_i^a p_{ij} p_{i1}) E(z_{kj}) E(z_{k1})$$

$$\text{Var}(Z^{a+1}) = \sum_{i=2}^{m} \text{Var}(Z_i^{a+1}) + 2 \sum_{j=2}^{m-1} \sum_{1=3}^{m} \text{Cov}(Z_j^{a+1}, Z_1^{a+1})$$

Using these expressions the prediction intervals for a single step transition are as follows:

90% Prediction Interval

of number of accounts in class i = $f_i \pm 1.645 \times (\text{Var}(n_i))^{1/2}$

of total number of accounts = $\sum_{j=2}^{m} f_j \pm 1.645 \times (\text{Var}(N))^{1/2}$

of amount of savings in class i = $s_i \pm 1.645 \times (\text{Var}(Z_i))^{1/2}$

of total amount of savings = $\sum_{j=2}^{m} s_j \pm 1.645 \times (\text{Var}(Z))^{1/2}$

where

f_i = expected number of accounts in class i = $E(n_i)$ after one period

s_i = expected amount of savings in class i = $E(Z_j)$ after one period

N = total number of accounts in the system after one period (random variable)

n_i = number of accounts in class i after one period (random variable)

Z_i = amount of savings in class i after one period (random variable)

Z = total amount of savings in the system after one period (random variable)

The model can be extended to cover the case of stochastic arrivals. Assuming the arrival process to be independent of the Markov chain process the expression for the number of accounts is the same as for the case of non-stochastic arrivals. The only difference is in replacing the vector of entrants (c') by the product of the expected number of arrivals and the multinomial vector of probability of entering each active class. Thus,

$$c' = E(R)\ (p_2\ p_3\ .\ .\ .\ p_m)$$

where R = random number of arrivals

p_i, i = 2, 3 . . . m = probability of an arrival entering class i

c' = vector of entrants into the active classes

The expressions for variance are changed to take into account the variability introduced by the additional stochastic processes.

Let e_j^{a+1} be the random number of entrants into class j at time period a+1

r^{a+1} be the number of arrivals at time period a+1

R be the random variable of arrivals

$$\mathrm{Var}(e_j^{a+1} \mid R = r^{a+1}) = r^{a+1} p_j (1 - p_j)$$

$$\mathrm{Var}(e_j^{a+1}) = p_j (1 - p_j) E(R) + p_j^2 \, \mathrm{Var}(R)$$

Since arrivals have been assumed to be independent of the accounts in the system

$$\mathrm{Var}(n_j^{a+1}) = \mathrm{Var}(e_j^{a+1}) + \sum_{i=2}^{m} n_i^a p_{ij} (1 - p_{ij})$$

$$\mathrm{Var}(N^{a+1}) = \sum_{j=2}^{m} \mathrm{Var}(n_j^{a+1}) + 2 \sum_{j=2}^{m-1} \sum_{k=3}^{m} \mathrm{Cov}(n_j^{a+1}, n_k^{a+1})$$

$$\mathrm{Cov}(n_j^{a+1}, n_k^{a+1}) = \sum_{\substack{i=2 \\ j \neq k}}^{m} - n_i^a p_{ij} p_{ik}$$

Let $E(Z_j)$ be the expected amount of savings in an account in class j

z_{kj} be the amount of savings in an account which has just entered class j

$$\mathrm{Var}(Z_j^{a+1}) = E(Z_j)^2 p_j (1 - p_j) + \sum_{i=2}^{m} n_i^a p_{ij} \mathrm{Var}(z_{kj}) + E(z_{kj})^2 n_i^a p_{ij} (1 - p_i)$$

$$\mathrm{Cov}(Z_j^{a+1}, Z_l^{a+1}) = \sum_{i=2}^{m} - (n^a p_{ij} p_{il}) E(z_{kj}) E(z_{kl})$$

$$\mathrm{Var}(Z^{a+1}) = \sum_{i=2}^{m} \mathrm{Var}(Z_i^{a+1}) + 2 \sum_{j=2}^{m-1} \sum_{l=3}^{m} \mathrm{Cov}(Z_j^{a+1}, Z_l^{a+1})$$

D. DESCRIPTION OF MODEL II

1. The Arrival Process

It was observed that the number of new accounts opened in each quarter was between seven hundred and one thousand. For such large arrival rates, an assumption that the arrival rate is normally distributed would be reasonable. However, it was felt that the arrival distribution could be affected by external factors like state of the national economy, seasonal effects and level of promotional or advertising activity of the savings institution. Thus the following linear econometric model was considered:

$$Ar = a_0 + a_1X_1 + a_2X_2 \; . \; . \; . \; a_{10}X_{10} + e$$

where

Ar	=	Number of new accounts opened in each quarter
X_1	=	Dummy variable for quarters of the year
X_2	=	California non-agricultural employment
X_3	=	Advertising and promotional expense of the savings institution
X_4	=	Prime commercial paper rate, 4 - 6 months
X_5	=	U. S. Government securities rate, 6 months
X_6	=	Corporation bonds rate
X_7	=	Wholesale price index
X_8	=	U. S. Government securities rate, 3 months
X_9	=	California personal income
X_{10}	=	U. S. total credit
e	=	Normally distributed random variable with zero mean and constant variance

The linear model was selected because of its simplicity and because of the lack of data required by more complex models.

2. The Size Distribution of New Accounts

The size distribution of new accounts may also change with time and external conditions. To model this change, the probabilities of new accounts entering each class were related to the same set of exogenous variables listed in sub-section 1. Direct application of least squares to the probabilities may yield predictions of future values that are outside the zero to one range. To overcome this potential area of difficulty the estimates of the probabilities were first transformed into logits.

3. Logit Analysis

Logit analysis is a special application of Econometrics to situations in which the dependent variable has a dichotomous character. The object is to estimate the probability of occurrence of a specified event given a set of prevailing conditions. For application in this study one looks for the probability that a new account enters a particular class and the probability that an account will move from one class to another, given a set of external conditions. Direct application of least squares may result in the prediction of probabilities outside the zero and one range. A monotonic transformation can overcome this difficulty. One simple transformation is to divide the relative frequency by one minus the relative frequency. This quantity is an estimate of the odds of the occurrence of the event. This transformation is still restrictive as the

new variable can take on only positive values. This problem can be overcome by taking the logarithm of this quantity. The logarithm of the estimated odds is termed the logit of an event. The model used to predict future values of the parameters of the entrants distribution and the transition probabilities of the transition matrix was as follows:

$$\text{Log}(p_i/(1 - p_i)) = a_0+a_1X_1+a_2X_2 + \ldots a_{10}X_{10} + e$$

$$\text{Log}(p_{ij}/(1 - p_{ij})) = b_0+b_1X_1b_2X_2 + \ldots b_{10}X_{10} + e$$

There is a further restriction that the sum of the probabilities of the entrants distribution must equal one and the row sum of the transition matrix should equal one too. The approach taken in this paper was to sum up these predicted probabilities and then divide each by the sum.

4. Transition Matrix of Model II

The transition matrix of Model II is allowed to change with external factors thus the t steps transition matrix is no longer the single step matrix raised to the t^{th} power but is the product of t matrices.

5. Predictions with Model II

To use Model II the first step would be to obtain predictions of future values of those factors that are in the regression equations. The parameters of the arrival process, entrance process and the transition probabilities are then predicted. The expected number of accounts in each class can then be computed by the following expression:

$$E(e_t\ f'_t) = (0\ f'_0) \prod_{j=0}^{t} P_j + \sum_{k=0}^{t-1} (0\ c'_j) \prod_{k=j}^{t-1} P_k$$

where

e_t = cumulative number of closed accounts

f'_t = $(f_2^t\ f_3^t\ \ldots\ f_m^t)$ = vector of number of accounts in each active class at time t, t=0, 1 ...T

P_j = Transition matrix at time j, j=0, 1, ...T

P_k = Transition matrix at time k, k=0, 1, ...T

c'_t = $E(Ar^t)(p_2^t\ p_3^t\ \ldots\ p_m^t)$

= Vector of expected number of entrants in each active class at time t.

$$E(N^t) = \sum_{j=2}^{m} f_j^t$$

where

$E(N^t)$ = Expected total number of active accounts in the system.

$E(Z_j^t)$ = $f_j^t \times E(z_j)$

where

$E(Z_j^t)$ = Expected total amount of savings in class j

$E(z_j)$ = Average amount of savings in each account in class j

$$E(Z^t) = \sum_{j=2}^{m} Z_j^t$$

where

$E(Z^t)$ = Expected total amount of savings in the system at time t.

III. THE DATA AND ESTIMATION OF PARAMETERS

A. DESCRIPTION OF DATA BASE

1. General

The data used in this study was obtained from the local branch of a savings institution. The population of passbook accounts was selected for study as it has greater mobility than other types of savings accounts.

The quarterly earnings ledgers for 1971, 1972 and the first two quarters of 1973 were made available for this study. The quarterly earnings ledgers contain the following information which have a bearing on this study:

1. Identification number of each active account.
2. Amount of savings as of the last day of each quarter.
3. Amount of earnings for the quarter.
4. Summary statistic of total number of active accounts.
5. Summary statistic of total amount of savings.
6. Summary statistic of total earnings withdrawn.
7. Summary statistic of total earnings accrued.

The basic Markov chain model requires the initial distribution of the subject population and the transition probability matrix for complete specificaion. A preliminary sample of two hundred accounts showed that seventy.-two percent of the population would have balances below two thousand dollars. A very large random sample would, therefore, be required to pick out the behavior of large accounts. It was decided to pick

a stratified sample instead. Thus, the sample of accounts examined consisted of three blocks of about two hundred each. The first consisted of accounts with balances exceeding ten thousand dollars on 31 March 1971. The second block consisted of accounts between two to ten thousand dollars and the third block consisted of accounts below two thousand dollars. The quarterly balance of each account was recorded. To determine the initial distribution of the population, the amount of savings of all the accounts with balances exceeding one thousand dollars on 31 March 1972 were recorded. The accounts were sorted by their order of magnitude and then divided into ten classes. The class intervals were selected to ensure that the amount of savings in each class was of the same order of magnitude. The first eight classes uniformly spanned the interval $1 - $15,999. The ninth class contained all accounts between $16,000 - $19,999 and the tenth class covered the range from $20,000 - $100,000. Accounts exceeding $100,000 were rare; there were six of them in the 31 March 1972 population. Including them in the largest class could result in an unstable mean of the amount of savings in that class; they were thus eliminated from the population. It is believed that these large accounts are important in the prediction of total acount of savings and should, therefore, be treated separately. For the purpose of this study the amount of savings for accounts exceeding $100,000 was considered to be unchanged over the period of observation.

2. Arrival Rate

The arrival rate was determined by taking the difference between the last identification numbers of consecutive quarters. This method failed to provide an accurate estimate of the arrival rate for Quarter IV-72. It was subsequently learned from the management that a block of about two hundred accounts were used to facilitate some financial transactions of newly arrived servicemen to Monterey. These accounts were subsequently closed. With this information the arrival rate for Quarter IV-72 was accordingly reduced.

3. The Size Distribution of New Accounts

The distribution of new accounts was estimated by taking a random sample of two hundred and fifty from the population of new accounts for each quarter.

4. The Validation Sample

To test if the models with parameters estimated from six hundred and twenty-two accounts could predict the behavior of the population, a sample comprising one-fourth of the accounts of Quarter I-73 was taken to be used as a base for comparison. A chi square test was performed to check if the predicted distribution fits the observation.

5. Summary Statistics

A second check on the predictive power of the model was made by comparing the total number of accounts and total amount of savings predicted for Quarters II-72 to II-73 against the summary statistics for these quantities.

6. Total Number of Accounts

It was found that the statistics for total number of active accounts included those that had been closed. It appeared that these accounts were purged from the records about once a year. As this information would be used as a check on the accuracy of prediction it had to be precise, thus, a page count of each quarters' ledger was conducted. The information on the total number of accounts and the arrival rate is shown in Table I.

TABLE I

TIME SERIES OF TOTAL NUMBER OF ACCOUNTS AND ARRIVAL RATE

QUARTER	# OF NEW ACCOUNTS	TOTAL # OF ACCOUNTS	MARGINAL CHANGE
I-71	UK	16895	UK
II-71	754	17059	+164
III-71	817	17181	+122
IV-71	599	17177	+ 96
I-72	778	17257	+ 80
II-72	860	17354	+ 97
III-72	791	17483	+129
IV-72	798	17752	+269
I-73	998	18013	+261
II-73	896	18087	+ 74

Nb: UK - Unknown

7. Total Amount of Savings

The trend in the total amount of savings was studied by fitting a least squares line through the observations. The data on total amount of savings are contained in Table II.

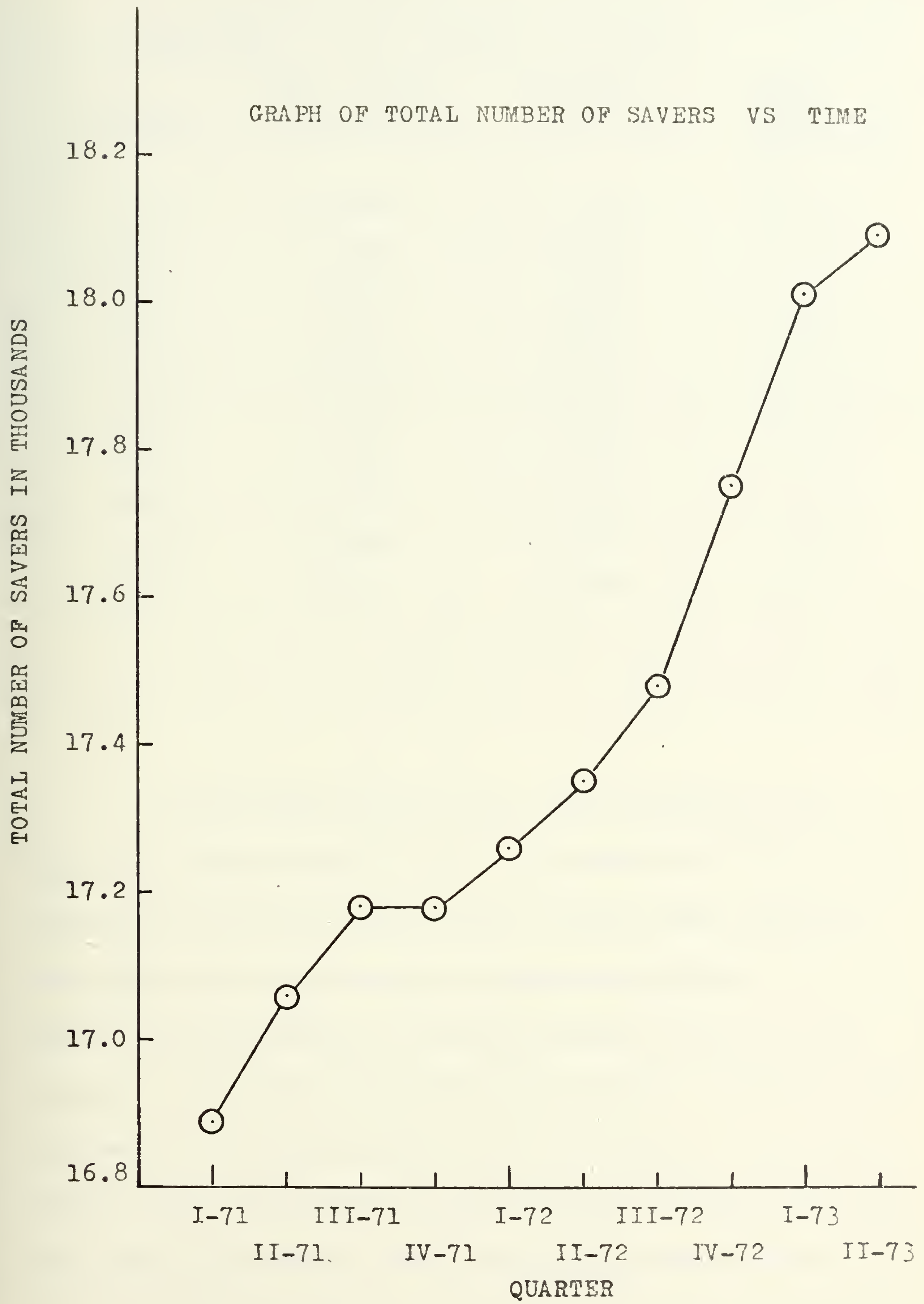
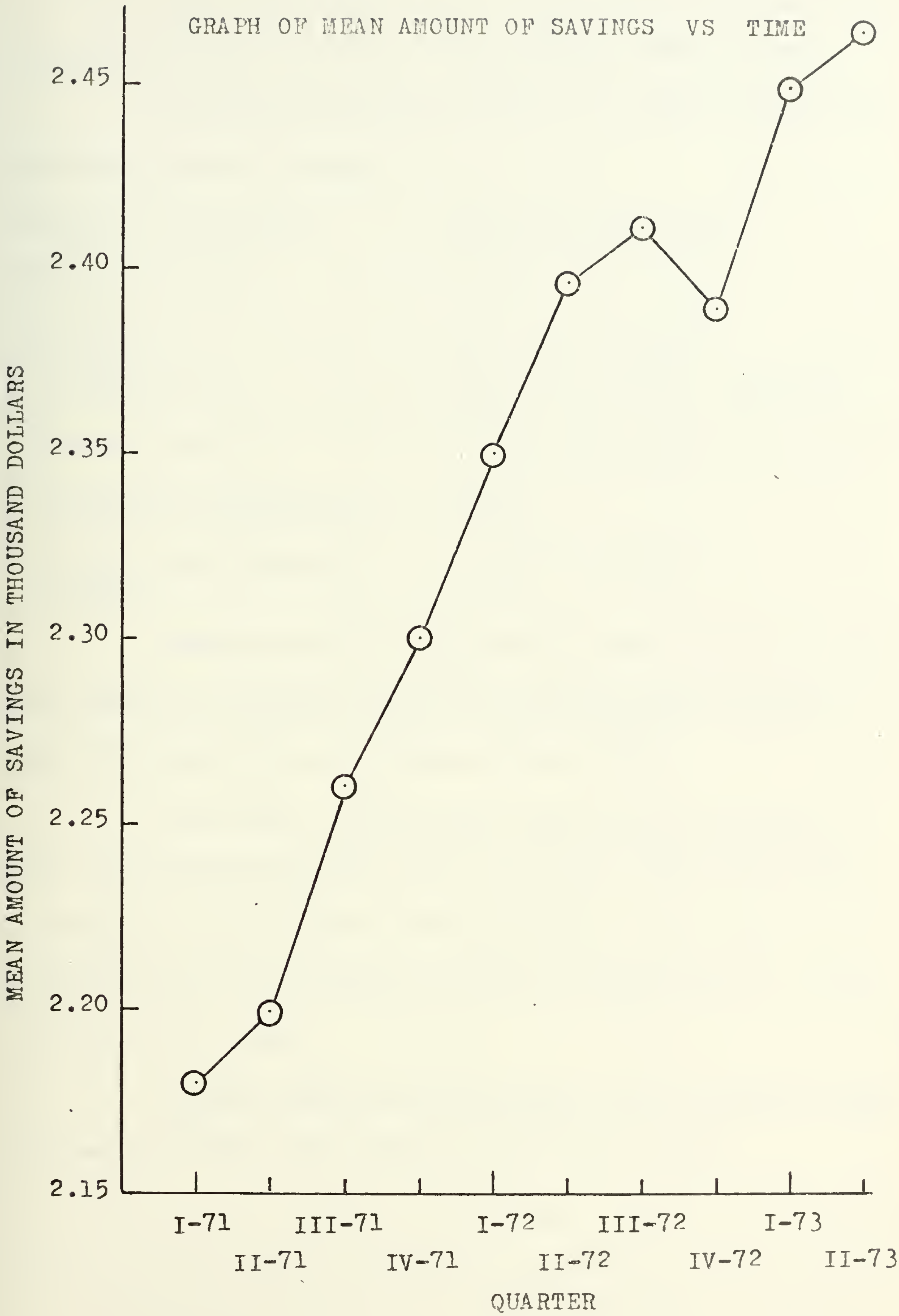

GRAPH OF TOTAL NUMBER OF SAVERS VS TIME
TOTAL NUMBER OF SAVERS IN THOUSANDS
18.2
18.0
17.8
17.6
17.4
17.2
17.0
16.8
I-71
II-71
III-71
IV-71
I-72
II-72
III-72
IV-72
I-73
II-73
QUARTER

TABLE II

TIME SERIES OF TOTAL AMOUNT OF SAVINGS

QUARTER	TOTAL AMOUNT OF SAVINGS ($M)	MARGINAL CHANGE ($M)	MEAN AMOUNT OF SAVINGS ($)
I-71	36.8345	UK	2180.20
II-71	37.5140	0.6795	2199.07
III-71	38.8286	1.3146	2259.97
IV-71	39.5192	0.6905	2300.70
I-72	40.5565	1.0374	2350.15
II-72	41.5743	1.0177	2395.66
III-72	42.1492	0.5749	2410.87
IV-72	42.4047	0.2555	2388.73
I-73	44.1283	1.7273	2449.80
II-73	44.5614	0.4431	2463.73

The standard deviation of the amount of savings in each account was estimated to be $5,314. The standard error of the mean was estimated to be $40.54. Using the t test, any two means differing by more than $66.86 are considered to be significantly different at the ten percent level of significance. Thus the hypothesis that the mean was constant over the period of observation was rejected. The average rate of increase in the mean was found to be 1.1158 percent. This increase could be partly accounted for by earnings accrued in the accounts. On the average, 95.01 percent of the quarterly earnings was retained in the


GRAPH OF MEAN AMOUNT OF SAVINGS VS TIME
MEAN AMOUNT OF SAVINGS IN THOUSAND DOLLARS
2.45
2.40
2.35
2.30
2.25
2.20
2.15
I-71
II-71
III-71
IV-71
I-72
II-72
III-72
IV-72
I-73
II-73
QUARTER

institution, thus a quarterly increase of 1.219 percent in the mean could be expected if there is no change in the structure of the population.

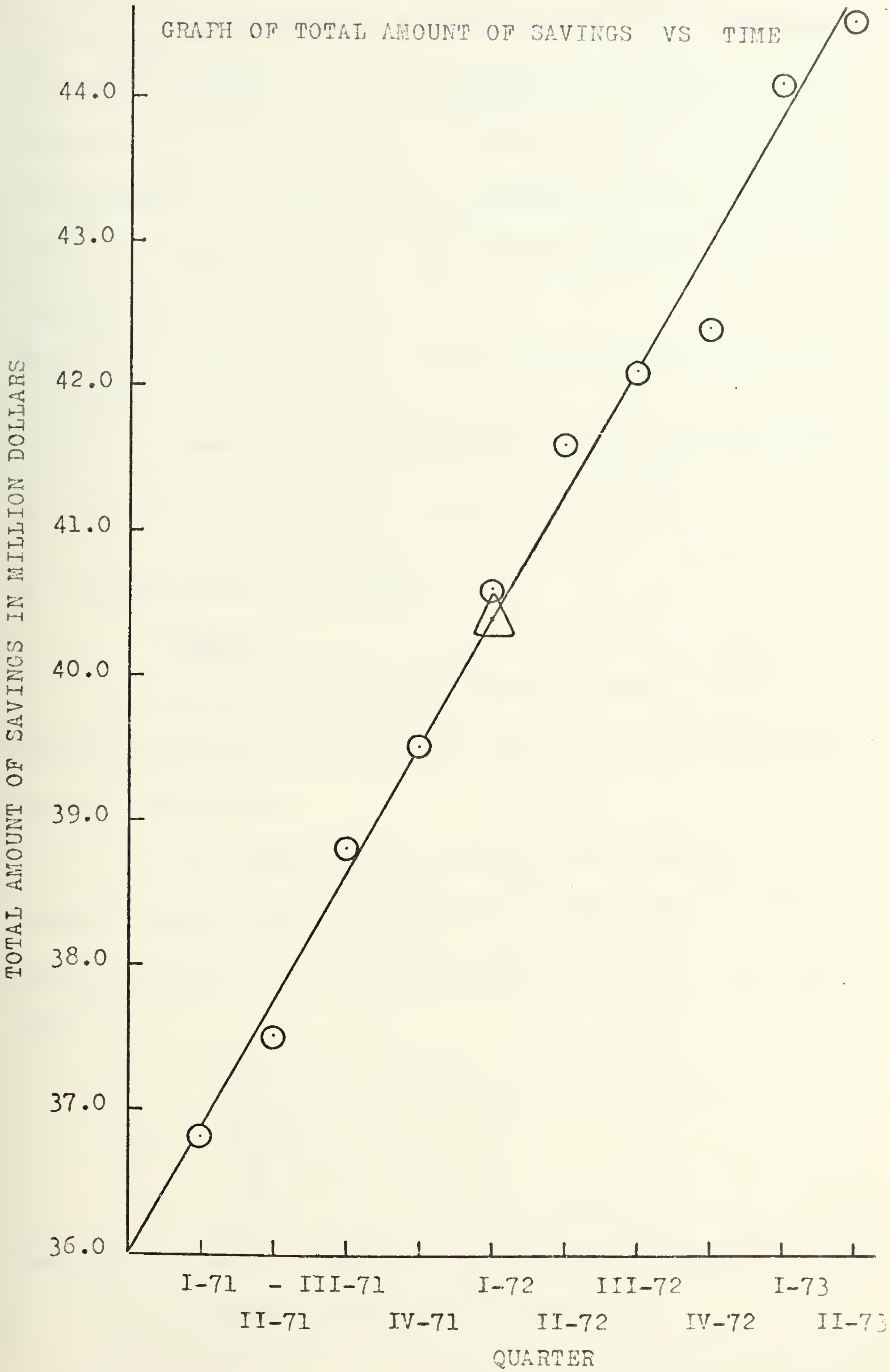
The following results were obtained by fitting the trend line to the total amount of savings:

(1) Mean of total savings	= 40.3899 million dollars
(2) Standard deviation	= 2.4153 million dollars
(3) Constant = a	= 36.011 million dollars
(4) Coefficient = b	= 0.876 million dollars per quarter
(5) Standard error of b	= 0.039 million dollars per quarter
(6) Coefficient of determination	= 0.986
(7) Standard error of dependent variable	= 0.286 million dollars

During the period of observation the total amount of savings was increasing at a constant rate of 0.876 million dollars per quarter. The annual growth rate based on this would be 8.675%.

It was found, on the average, that 95.01% of earnings was left in the accounts each quarter and so the annual growth rate caused by new accounts and increases in existing accounts less losses due to closing of accounts and reduction in levels of savings would be 8.675% -.9501 x 5.13% = 3.801%

A second regression was performed using the marginal change as dependent variable. The following results were obtained:


GRAPH OF TOTAL AMOUNT OF SAVINGS VS TIME
TOTAL AMOUNT OF SAVINGS IN MILLION DOLLARS
44.0
43.0
42.0
41.0
40.0
39.0
38.0
37.0
36.0
I-71
II-71
III-71
IV-71
I-72
II-72
III-72
IV-72
I-73
II-73
QUARTER

(1) Mean of first differences = 0.9117 million dollars per quarter

(2) Standard deviation = 0.4622 million dollars per quarter

(3) Constant = a = 0.823 million dollars per quarter

(4) Coefficient = b = 0.020 million dollars per quarter^2

(5) Standard error of b = 0.077 million dollars per quarter^2

(6) Coefficient of determination = 0.011

(7) Standard error of dependent variable = 0.460 million dollars per quarter

It was concluded that there was no trend in the net change of total savings in each quarter over the period of observation.

B. ESTIMATION OF PARAMETERS

1. Model I

The arrival rate can be estimated by adding up all the new accounts opened during the period of observation and dividing by the number of time periods.

The distribution of new accounts can be estimated by taking samples from each batch of new accounts, adding up the accounts entering each class and dividing by the total number of accounts in the sample. Thus:

$$\hat{p}_j = \frac{\sum_{t=1}^{T} e_j^t}{\sum_{t=1}^{T} r^t}$$

where $\hat{p}_j$ = maximum likelihood estimate of the probability of a new account entering the j^{th} class

e_j^t = number of new accounts entering the j^{th} class at time t

r^t = number of accounts in the sample of new accounts at time t

T = number of periods of observation

The average number of accounts entering each class can be found by:

$$c' = \hat{A}r(\hat{p}_2 \hat{p}_3 \ldots \hat{p}_m)$$

where

c' = average number of new accounts entering each class at each time period

$\hat{A}r$ = Maximum likelihood estimate of the arrival rate

The stationary transition probabilities can be estimated by the following 2 :

$$\hat{p}_{ij} = n_{ij}/n_{i} \cdot = \sum_{t=1}^{T} n_{ij}(t) \Big/ \sum_{k=1}^{m} \sum_{t=1}^{T} n_{ik}(t)$$

$$= \sum_{t=1}^{T} n_{ij}(t) \Big/ \sum_{t=1}^{T} n_{i}(t-1)$$

where

$\hat{p}_{ij}$ = Maximum likelihood estimate of the probability of transition from class i to class j in any one given period

n_{ij} = Total number of accounts that have moved from class i to class j over the period of observation (0 - T)

$n_{i.}$ = Total number of accounts that were in class i at the beginning of each period

$n_{ij}(t)$ = Number of accounts that moved from class i to class j during the period between t-1 and t

$n_{ik}(t)$ = Number of accounts that moved from class i to class k during the period between t-1 and t

$n_i(t-1)$ = Total number of accounts in class i at the time period (t-1)

Anderson and Goodman 2 showed that as n, the total number of entities in the system, tends to infinity the set $(n_{i.})^{1/2}(\hat{p}_{ij} - p_{ij})$ has a joint normal distribution with means 0, variances $p_{ij}(1 - p_{ij})$ and covariances $-\delta_{ig}\, p_{ij} p_{gh}$ where $\delta_{ig} = 0$ if $i \neq g$ and $\delta_{ii} = 1$.

This fact can be used to test if certain transition probabilities p_{ij} have specified values p^0_{ij} and if the transition probabilities are indeed stationary.

2. Statistical Tests

The chi square test of goodness of fit can be used to test hypotheses concerning transition probabilities. To test the hypothesis that $p_{ij} = p^0_{ij}$, $j = 1,2, \ldots m$, the quantity,

$$\sum_{j=1}^{m} n_{i.} \frac{(\hat{p}_{ij} - p^0_{ij})^2}{p^0_{ij}} ,$$

under the null hypothesis has an asymptotic chi square distribution with m-1 degrees of freedom. The null hypothesis is rejected if $\hat{p}_{ij}$ differs

from p^0_{ij} to such an extent that the above test statistic exceeds the $(1 - \alpha)$ percentile of the chi square distribution with m-1 degrees of freedom, where α is the level of significance.

As the variables $n_{i.} (\hat{p}_{ij} - p^0_{ij})^2$ for different i are independent the summation over i is distributed as a chi square distribution with m(m - 1) degrees of freedom.

To test the hypothesis that the transition probabilities are stationary over the period of observation the following test statistic can be used 2 :

$$X^2 = \sum_{i=1}^{m} X_i^2 = \sum_{i=1}^{m} \sum_{j=1}^{m} \sum_{t=1}^{T} n_i(t-1) \left\{ p_{ij}(t) - p_{ij} \right\} / p_{ij}$$

where

$n_i(t-1)$ = total of entities in class i at time t-1

$\hat{p}_{ij}(t)$ = estimate of the transition probability at time t, obtained by counting the number of transitions from class i to class j and dividing by $n_i(t-1)$

$\hat{p}_{ij}$ = estimate of the transition probability from class i to class j

$$= \sum_{t-1}^{T} n_{ij}(t) \Big/ \sum_{t=0}^{T-1} n_i(t)$$

The asymptotic distribution of this test statistic is chi square with m(m-1) (T-1) degrees of freedom. The number of degrees of freedom is reduced from m(m-1)T by m(m-1), the number of parameters estimated.

The chi square test is based on a statistic which follows a chi square distribution when n, the total number of entities in the system, tends to infinity. Hence it has been customary of statistics text books to recommend that the smallest expected number of entities in each class should exceed five or ten. If this requirement is not met in the original classification then combination of neighboring classes, until the rule is satisfied, is recommended. Cochran 4 challenged this arbitrary rule claiming that the power of the test is reduced by pooling classes to conform to the rule. He found that for goodness of fit tests of bell shaped curves such as the normal distribution there is little disturbance to the five percent level when a single expectation is as low as 1/2. He continued stating that the result is also true for the one percent level if the number of degrees of freedom exceeds six and that two expectations may be as low as one may be allowed with negligible disturbance to the five percent level.

Using Cochran's results, classes with small expectations were pooled to ensure that the smallest expected number of entities in each class exceeded one and no more than two classes had expected numbers less than two. The number of degrees of freedom was reduced from m(m-1) (T-1) by the number of classes eliminated.

3. Model II

The predictor for arrival rate may be obtained by applying the method of least squares to the number of new accounts observed in each time period and the corresponding exogenous variables.

The distribution of new accounts is estimated in each period by dividing the number of new accounts entering each class by the total number of accounts in the sample.

The transition probabilities $p_{ij}(t)$ are estimated by dividing the number of accounts that moved from class i to class j at time t by the number of accounts in class i at time t-1.

These estimates are maximum likelihood estimates as in Model I. They can be transformed into logits and then regressed against the set of exogenous variables.

4. Estimation of Transition Probabilities

Each of the six hundred and twenty-two accounts was categorized in accordance with the classification given in Section A. 1. of this chapter. The number of accounts in each class for each quarter during the period of observation is presented in Table III. The relative fraction of accounts, obtained by dividing the number of accounts in each class by six hundred and twenty-two, is shown in Table IV.

It can be seen that twenty-seven percent of the accounts in the sample were closed after ten quarters. The proportion of active accounts in each class was found by dividing the number of accounts in each class by the total number of active accounts. The results are presented in Table V. The time series of amount of savings in each class is presented in Table VI.

A chi square test was performed to test if the distribution of active accounts had changed during the period of observation. The number

TABLE III

TIME SERIES OF DISTRIBUTION OF ACCOUNTS IN THE SAMPLE OF 622 ACCOUNTS

	I-71	II-71	III-71	IV-71	I-72	II-72	III-72	IV-72	I-73	II-73	SUM
I	0	17	38	58	84	102	119	132	145	170	865
II	166	182	175	174	168	169	160	153	148	141	1636
III	112	92	92	89	90	80	80	76	78	69	858
IV	52	60	65	62	55	51	50	57	53	45	550
V	45	50	35	36	37	37	38	30	30	31	369
VI	26	24	33	28	20	22	20	17	21	15	226
VII	88	76	65	61	60	57	50	49	44	46	596
VIII	42	38	34	29	30	21	27	23	23	25	292
IX	31	29	33	27	19	22	21	25	21	17	245
X	35	28	24	24	23	20	21	26	25	22	248
XI	25	26	28	34	36	41	36	34	34	41	335
SUM	622	622	622	622	622	622	622	622	622	622	6220

TABLE IV

TIME SERIES OF FRACTION OF ACCOUNTS IN EACH CLASS IN THE SAMPLE OF 622 ACCOUNTS

	I-71	II-71	III-71	IV-71	I-72	II-72	III-72	IV-72	I-73	II-73	CUM.
I	0.0	0.0273	0.0611	0.0932	0.1350	0.1640	0.1913	0.2122	0.2331	0.2733	0.1391
II	0.2669	0.2926	0.2814	0.2797	0.2701	0.2717	0.2572	0.2460	0.2379	0.2267	0.2630
III	0.1801	0.1479	0.1479	0.1431	0.1447	0.1286	0.1286	0.1222	0.1254	0.1109	0.1379
IV	0.0836	0.0965	0.1045	0.0997	0.0884	0.0820	0.0804	0.0916	0.0852	0.0723	0.0884
V	0.0723	0.0804	0.0563	0.0579	0.0595	0.0595	0.0611	0.0482	0.0482	0.0498	0.0593
VI	0.0418	0.0386	0.0531	0.0450	0.0322	0.0354	0.0322	0.0273	0.0338	0.0241	0.0363
VII	0.1415	0.1222	0.1045	0.0981	0.0965	0.0916	0.0804	0.0788	0.0707	0.0740	0.0958
VIII	0.0675	0.0611	0.0547	0.0466	0.0482	0.0338	0.0434	0.0370	0.0370	0.0402	0.0469
IX	0.0498	0.0466	0.0531	0.0434	0.0305	0.0354	0.0338	0.0402	0.0338	0.0273	0.0394
X	0.0563	0.0450	0.0386	0.0386	0.0370	0.0322	0.0338	0.0418	0.0402	0.0354	0.0399
XI	0.0402	0.0418	0.0450	0.0547	0.0579	0.0659	0.0579	0.0547	0.0547	0.0659	0.0539

TABLE V

TIME SERIES OF FRACTION OF ACTIVE ACCOUNTS IN EACH CLASS IN THE SAMPLE OF 622 ACCOUNTS

	I-71	II-71	III-71	IV-71	I-72	II-72	III-72	IV-72	I-73	II-73	CUM.
I	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
II	0.2669	0.3008	0.2997	0.3085	0.3123	0.3250	0.3181	0.3122	0.3103	0.3119	0.3055
III	0.1801	0.1521	0.1575	0.1578	0.1673	0.1538	0.1590	0.1551	0.1635	0.1527	0.1602
IV	0.0836	0.0992	0.1113	0.1099	0.1022	0.0981	0.0994	0.1163	0.1111	0.0996	0.1027
V	0.0723	0.0826	0.0599	0.0638	0.0688	0.0712	0.0755	0.0612	0.0629	0.0686	0.0689
VI	0.0418	0.0397	0.0565	0.0496	0.0372	0.0423	0.0398	0.0347	0.0440	0.0332	0.0422
VII	0.1415	0.1256	0.1113	0.1082	0.1115	0.1096	0.0994	0.1000	0.0922	0.1018	0.1113
VIII	0.0675	0.0628	0.0582	0.0514	0.0558	0.0404	0.0537	0.0469	0.0482	0.0553	0.0545
IX	0.0498	0.0479	0.0565	0.0479	0.0353	0.0423	0.0417	0.0510	0.0440	0.0376	0.0458
X	0.0563	0.0463	0.0411	0.0426	0.0428	0.0385	0.0417	0.0531	0.0524	0.0487	0.0463
XI	0.0402	0.0430	0.0479	0.0603	0.0669	0.0788	0.0716	0.0694	0.0713	0.0907	0.0626

TABLE VI

TIME SERIES OF AMOUNT OF SAVINGS IN EACH CLASS IN THE SAMPLE

	I-71	II-71	III-7	IV-71	I-72	II-72	III-72	IV-72	I-73	II-73
I	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
II	66082.	81908.	86410.	88719.	84655.	85201.	76517.	74296.	68827.	69300.
III	321198.	272850.	273748.	259730.	264756.	239441.	238997.	224544.	231257.	207472.
IV	260135.	298754.	320550.	308415.	278002.	251076.	241451.	274778.	259631.	218088.
V	305605.	342862.	241076.	251244.	256009.	256481.	262935.	215352.	206544.	215346.
VI	227472.	219393.	290953.	247839.	181335.	201731.	183567.	153325.	189416.	131926.
VII	949568.	816795.	699708.	660763.	649922.	610157.	536059.	520816.	467021.	492183.
VIII	546811.	494414.	445729.	374042.	388163.	276219.	354780.	298484.	299438.	325462.
IX	465098.	436453.	495717.	400968.	284605.	327455.	315938.	373113.	315862.	252917.
X	627300.	504324.	426658.	429558.	405656.	351882.	372054.	468395.	440700.	388406.
XI	705468.	779734.	752844.	944725.	979935.	1125332.	1054542.	1002154.	1031799.	1211984.
SUM	4474737.	4247487.	4033393.	3966003.	3773038.	3724975.	3636840.	3605257.	3510495.	3513084.

of degrees of freedom of the distribution of the chi square statistic is eighty-one and the ninetieth percentile of the distribution is 98.01. The chi square statistic was found to be 64.2. Thus, the null hypothesis that the distribution did not change with time could not be rejected. This result was rather surprising as it could imply that the probability of an account closing did not depend on the class it was in.

Each account was examined at each quarter to determine if it had made a transition to another class. The transitions were accumulated in a transition count matrix. The ij^{th} element of this matrix is the number of transitions from the i^{th} class to the j^{th} class in a given quarter. An example of a transition count matrix is shown in Table VII. The transition count matrices for other quarters are contained in Appendix B.

The estimate of each quarter's transition matrix was obtained by the method described earlier in this section. An example of the estimate of the transition matrix of Quarter II 71 is shown in Table VIII. The estimates for subsequent quarters are contained in Appendix C.

A cumulative transition count matrix was formed by adding successive transition count matrices. Thus the cumulative transition count matrix of Quarter I-72 is the sum of the transition count matrices of Quarters II-71, III-71, IV-71 and I-72. The cumulative transition count matrices are contained in Appendix D.

The time stationary estimate of the transition matrix was obtained by dividing each element of the cumulative transition count matrix by its row sum. For the sake of brevity the estimate of transition

TABLE VII

TRANSITION FREQUENCY MATRIX BETWEEN QUARTER 1 AND QUARTER 2

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	SUM
I	0	0	0	0	0	0	0	0	0	0	0	0
II	8	152	3	0	1	1	0	0	0	0	1	166
III	1	14	79	13	3	0	0	2	0	0	0	112
IV	1	4	3	37	5	0	0	1	0	0	1	52
V	1	0	2	4	33	2	3	0	0	0	0	45
VI	0	2	1	2	4	14	3	0	0	0	0	26
VII	2	5	3	1	1	5	64	7	0	0	0	88
VIII	2	2	1	0	1	0	3	22	8	3	0	42
IX	1	3	0	2	1	1	1	4	18	0	0	31
X	0	0	0	0	0	0	1	2	2	24	6	35
XI	1	0	0	1	1	1	1	0	1	1	18	25
SUM	17	182	92	60	50	24	76	38	29	28	26	622

TABLE VIII

ESTIMATE OF TRANSITION MATRIX BETWEEN QUARTER 1 AND QUARTER 2

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI
I	1.0000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
II	0.0482	0.9157	0.0181	0.0	0.0060	0.0060	0.0	0.0	0.0	0.0	0.0060
III	0.0089	0.1250	0.7054	0.1161	0.0268	0.0	0.0	0.0179	0.0	0.0	0.0
IV	0.0192	0.0769	0.0577	0.7115	0.0962	0.0	0.0	0.0192	0.0	0.0	0.0192
V	0.0222	0.0	0.0444	0.0889	0.7333	0.0444	0.0667	0.0	0.0	0.0	0.0
VI	0.0	0.0769	0.0385	0.0769	0.1538	0.5385	0.1154	0.0	0.0	0.0	0.0
VII	0.0227	0.0568	0.0341	0.0114	0.0114	0.0568	0.7273	0.0795	0.0	0.0	0.0
VIII	0.0476	0.0476	0.0238	0.0	0.0238	0.0	0.0714	0.5238	0.1905	0.0714	0.0
IX	0.0323	0.0968	0.0	0.0645	0.0323	0.0323	0.0323	0.1290	0.5806	0.0	0.0
X	0.0	0.0	0.0	0.0	0.0	0.0	0.0286	0.0571	0.0571	0.6857	0.1714
XI	0.0400	0.0	0.0	0.0400	0.0400	0.0400	0.0400	0.0	0.0400	0.0400	0.7200

matrices was termed CPM Z where Z was a Roman numeral indicating that the data used in the estimation came from the first Z quarter of the period of observation. Thus CPM V stands for the estimate of the stationary transition matrix using data from Quarter I-71 to Quarter I-72. CPM II through CPM X are contained in Appendix E.

5. <u>Test of Time Stationary Assumption</u>

It can be seen from the transition count matrices that there are a large number of elements with zero or one transition counts. The chi square test could not, therefore, be applied directly. The classes of each row were combined so that the smallest class had an expectation exceeding one count and no more than two classes had expectation of less than two counts. The following grouping was obtained:

Class	I	II	III	IV	V	VI	VII	VIII	IX	X	XI
II	.046	.883	.054	-	-	-	-	-	-	-	.017
III	.023	.110	.733	.104	.015	-	-	-	-	-	.015
IV	.040	.040	.075	.711	.089	-	-	-	-	-	.046
V	.050	-	.130	-	.672	.104	-	-	-	-	.046
VI	.081	-	.147	-	-	.536	-	-	-	-	.237
VII	.044	.087	-	-	-	-	.760	.084	-	-	.026
VIII	.064	-	-	.109	-	-	-	.611	.169	-	.049
IX	.075	-	-	-	.083	-	-	.083	.636	-	.123
X	.067	-	-	.102	-	-	-	-	-	.712	.120
XI	.042	-	-	-	.097	-	-	-	-	-	.861

The number of degrees of freedom for the above matrix is equal to the number of elements minus the number of linear constraints, (47-10). As the number of matrices is nine and the number of parameters

estimated in (47-10) the number of degrees of freedom for the distribution of the chi square statistic for the test of stationary transition probability matrix is (47-10) (9-1) = 296.

The rejection region for 10% level of significance is 328.6. The chi square statistic was found to be 288.7 thus the null hypothesis that the transition probabilities were stationary could not be rejected.

6. <u>The Initial Distribution of the Population</u>

The initial distribution of the population was determined by recording all accounts with balance exceeding one thousand dollars on 31 March 1972. The number of accounts below one thousand dollars was found by taking the difference between the total number of accounts and the number of accounts recorded. The mean and variance of the amount of savings in an account in each class were estimated from this sample. Table IX is a summary of the data obtained.

It can be seen that the estimate of the mean of each class, except for Classes II and XI is close to the midpoint of the respective class intervals. All the means are below the midpoints as there are more accounts at the lower end of each class. The estimates of variance of Classes II to IX are very close because the class intervals are the same and the distribution of accounts in each class has the same general shape. The estimates of variance for Classes X and XI show the importance of length of class interval on predictions of total amount of savings. The variance of the amount of savings of accounts in Classes X and XI

TABLE IX

SIZE DISTRIBUTION OF THE ENTIRE POPULATION OF ACCOUNTS AT QUARTER I-72

CLASS	INTERVAL ($)	NUMBER OF	MEAN ($)	VARIANCE
I	0	0	0	0
II	1 - 1999	12373	353	246544
III	2000 - 3999	1793	2837	310372
IV	4000 - 5999	1034	4916	317481
V	6000 - 7999	563	6855	328649
VI	8000 - 9999	366	8905	346948
VII	10000 - 11999	372	10757	362291
VIII	12000 - 13999	209	12920	329649
IX	14000 - 15999	153	14961	314260
X	16000 - 19999	183	17791	1355376
XI	20000 - 99999	205	27888	110502144
	100000	6	156558	2.983×10^9

can be reduced by the introduction of more classes to cover the same interval. However, this could lead to classes having smaller populations which may not possess the Markovian property.

This paper took the compromise in selecting class intervals such that each class had a minimum of one hundred and fifty accounts. The six accounts that exceeded $100,000 were considered to be unchanged during the period of observation. These accounts added up to $0.94 million. Thus the predicted amount of total savings could differ by one million dollars because of the action of a handful of savers.

7. <u>The Size Distribution of New Accounts</u>

Each new account of the samples of new accounts was classified according to the rule given in Section A. 1. of this chapter. The number of new accounts in each class for Quarter II-71 through Quarter II-73 is shown in Table XI.

The maximum likelihood estimate of the probability of a new account entering each class was obtained by dividing the number of new accounts in each class by the total number of new accounts. The quarterly estimates of the probability of a new account entering each class and the time stationary estimates are presented in Table XII.

A chi square test was performed to test the hypothesis that the probabilities were time stationary. The number of degrees of freedom of the distribution of the chi square statistic was seventy-two and the ninetieth percentile of the distribution is 87.84. The chi square statistic obtained was 68.8. Thus the null hypothesis could not be rejected at the ten percent level of significance.

As a further check a one way analysis of variance was performed. The results are as follows:

Total number of observations	=	2250
Average of all observations	=	2535.38
Standard error within groups	=	8732.41
Degrees of freedom	=	2241
Standard error between groups	=	11488.08
Degrees of freedom	=	8

F statistic = 1.73

Level of significance = 0.0865

Thus the null hypothesis that the mean amount of savings of new accounts is constant over the period of observation is rejected at the 10% level of significance.

The mean and standard deviation of the amount of savings of the samples of new accounts are as follows:

TABLE X

MEAN, STANDARD DEVIATION, MEDIAN, MAXIMUM VALUE AND MINIMUM VALUE OF SAMPLES OF NEW ACCOUNTS

Quarter	Mean ($)	Standard Deviation	Median	Maximum Value	Minimum Value
II-71	1671.34	3615.79	279.5	25000.	1.
III-71	1960.13	5038.32	301.5	52518.	1.
IV-71	2500.38	6561.85	300.0	50000.	1.
I-72	2169.10	5553.17	224.5	40000.	2.
II-72	3193.56	8641.02	340.50	103157.	1.
III-72	2812.04	8264.18	282.50	100032.	1.
IV-72	2271.53	7642.48	146.50	100000.	1.
I-73	4054.80	18161.52	238.5	200000.	1.
II-73	2185.53	6536.75	101.5	50000.	2.

Nb. sample size = 250

The Duncan's Multiple Range Test showed that the means of Quarters II-71, III-71, IV-71, I-72, IV-72 and II-73 are significantly different from that of Quarter I-73 at the ten percent level of significance. The means of Quarters II-71 and II-72 are also significantly different at the ten percent level of significance. The differences between the means of other quarters were not considered significant.

TABLE XI

TIME SERIES OF DISTRIBUTION OF SAMPLE OF NEW ACCOUNTS

	II-71	III-71	IV-71	I-72	II-72	III-72	IV-72	I-73	II-73	SUM
I	0	0	0	0	0	0	0	0	0	0
II	200	198	190	195	183	191	202	197	202	1758
III	16	19	21	17	17	20	15	13	15	153
IV	16	13	13	14	12	7	7	14	13	109
V	4	5	6	6	9	3	5	2	2	42
VI	4	2	4	3	2	4	3	4	2	28
VII	3	1	4	3	3	8	5	3	5	35
VIII	1	2	1	1	6	1	2	4	2	20
IX	0	2	3	4	5	5	3	3	0	25
X	3	5	0	1	1	3	1	2	1	17
XI	3	3	8	6	12	8	7	8	8	63
SUM	250	250	250	250	250	250	250	250	250	2250

TABLE XII

TIME SERIES OF ESTIMATE OF DISTRIBUTION CF NEW ACCOUNTS

	II-71	III-71	IV-71	I-72	II-72	III-72	IV-72	I-73	II-73	CUM.
I	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
II	0.8000	0.7920	0.7600	0.7800	0.7320	0.7640	0.8080	0.7880	0.8080	0.7813
III	0.0640	0.0760	0.0840	0.0680	0.0680	0.0800	0.0600	0.0520	0.0600	0.0680
IV	0.0640	0.0520	0.0520	0.0560	0.0480	0.0280	0.0280	0.0560	0.0520	0.0484
V	0.0160	0.0200	0.0240	0.0240	0.0360	0.0120	0.0200	0.0080	0.0080	0.0187
VI	0.0160	0.0080	0.0160	0.0120	0.0080	0.0160	0.0120	0.0160	0.0080	0.0124
VII	0.0120	0.0040	0.0160	0.0120	0.0120	0.0320	0.0200	0.0120	0.0200	0.0156
VIII	0.0040	0.0080	0.0040	0.0040	0.0240	0.0040	0.0080	0.0160	0.0080	0.0089
IX	0.0	0.0080	0.0120	0.0160	0.0200	0.0200	0.0120	0.0120	0.0	0.0111
X	0.0120	0.0200	0.0	0.0040	0.0040	0.0120	0.0040	0.0080	0.0040	0.0076
XI	0.0120	0.0120	0.0320	0.0240	0.0480	0.0320	0.0280	0.0320	0.0320	0.0280

TABLE XIII

AMOUNT OF SAVINGS IN EACH CLASS IN THE SAMPLE OF NEW ACCOUNTS

	II-71	III-71	IV-71	I-72	II-72	III-72	IV-72	I-73	II-73
I	0.	0.	0.	0.	0.	0.	0.	0.	0.
II	71127.	70044.	67374.	56644.	60215.	58335.	57673.	58228.	55155.
III	43235.	53277.	52603.	44149.	40571.	55191.	42711.	33332.	40603.
IV	77795.	58750.	63552.	67778.	55803.	32468.	33338.	65920.	59473.
V	26750.	34213.	41922.	40082.	58254.	20060.	33087.	13476.	14041.
VI	37166.	18870.	34774.	24800.	17480.	35225.	26041.	34742.	17553.
VII	30462.	10897.	41535.	31703.	30675.	83988.	50741.	31344.	52329.
VIII	12590.	25842.	13212.	12000.	77000.	12000.	26581.	53069.	25833.
IX	0.	29723.	44000.	58997.	73403.	71623.	43434.	43288.	0.
X	53711.	87865.	0.	19000.	16000.	53351.	17486.	35274.	16186.
XI	65000.	100552.	266122.	187122.	368989.	280768.	236791.	645026.	265209.
SUM	417836.	490033.	625094.	542275.	798390.	703009.	567883.	1013699.	546382.

The means are greatly influenced by the large accounts. The mean of Quarter I-73 would drop to $3267.87 if the $200000 account were deleted from the sample. This reduced mean will be significantly different from that of Quarter II-71 only.

Deleting accounts that were greater than $100000 from the samples reduced the means of Quarters II-72, III-72, IV-72 and I-73 to 2792.10, 2421.59, 1879.04 and 2292.24 respectively. The maximum difference between the means is 1120.76 which is considered insignificant at the ten percent level of significance.

8. Predictors of Transition Probabilities

The corresponding estimates of transition probabilities of each quarter were grouped together, transformed into logits and regressed against the following set of exogenous variables:

X_1 = Dummy variable for quarters of the year

X_2 = California non-agricultural employment

X_3 = Advertising and promotional expense of the savings institution

X_4 = Prime commercial paper rate, 4-6 months

X_5 = U. S. Government securities rate, 6 months

X_6 = Corporation bonds rate

X_7 = Wholesale price index lagged by one period

X_8 = U. S. Government securities rate, 3 months

X_9 = California personal income

X_{10} = U. S. total credit

The values of these variables are contained in Table XIV..

TABLE XIV

TIME SERIES OF EXOGENOUS VARIABLES USED IN THE REGRESSIONS

	II-71	III-71	IV-71	I-72	II-72	III-72	IV-72	I-73	II-73	
X_1 =	2	3	4	1	2	3	4	1	2	
X_2 =	6.87	6.93	7.00	7.14	7.20	7.24	7.34	7.42	7.50	$\times 10^6$
X_3 =	3.78	6.59	3.96	8.05	4.92	4.14	4.40	7.53	6.44	$\times \$10^4$
X_4 =	5.04	5.74	5.07	4.06	4.58	4.94	5.33	6.28	7.47	%
X_5 =	4.44	5.27	4.41	3.80	4.23	4.73	5.17	5.99	6.79	%
X_6 =	8.01	8.08	7.80	7.67	7.69	7.62	7.53	7.56	7.64	%
X_7 =	1.13	1.14	1.15	1.15	1.17	1.18	1.20	1.21	1.27	$\times 10^2$
X_8 =	4.24	5.00	4.23	3.44	3.77	4.22	4.86	5.70	*	%
X_9 =	0.94	0.95	0.97	1.00	1.02	1.02	1.05	1.06	*	$\times \$10^{11}$
X_{10} =	1.26	1.15	1.15	1.17	1.18	1.20	1.21	1.27	*	$\times \$10^{11}$

* These values were not available when the regressions were first performed. They were not recorded subsequently as the variables had been dropped from the regression.

Sources: (1) Federal Reserve Bulletin
(2) California Economic Indicators, June 1973

There was some difficulty in transforming the transition probabilities as a number of them was equal to zero and the logit of zero is minus infinity. The following rule was used to get around this problem:

1. If there are more than two estimates for $p_{ij}(t)$, t = II-71, III-71, . . . I-73, equal to zero, assume that $p_{ij}(t)$ is constant over the period of observation and use the time stationary estimate obtained for Model I. No regression will be performed for these elements.

2. If there are one or two zeros in the estimates, replace the zeros by the time stationary estimate and proceed with logit transformation and regression.

The number of transition probabilities removed by these rules was seventy-two. As there were one hundred and ten elements in the transition matrix that required estimation, application of these rules left a balance of thirty-eight elements for regression.

The transition matrix for Quarter II-73 was not included in the regression in order that it could be used to test the correctness of the predictors obtained with data from earlier periods. Thus, there were eight data points in the regressions instead of nine.

In the first regressions performed, it was found that X_8, U. S. Government securities rate, 3 months, X_9, California personal income and X_{10}, U. S. total credit were highly correlated with each other

and some of the other exogenous variables ($R \geqslant .98$). To reduce the problem of multicollinearity, these three variables were dropped from the regression equations.

The following criteria were used to determine if the variance of the logits of transition could be explained by the exogenous variables:

1. The F statistic obtained by the ratio of the estimate of the variance before and after the introduction of an independent variable must exceed 2.06, the eightieth percentile of the $F(7,6)$ distribution.
2. The coefficient of determination, R^2, must exceed 0.70.

Of the thirty-eight regressions only ten were found to be significant according to these criteria. As each row of the transition matrix would be divided by the sum of its elements these ten elements could cause significant changes to the transition matrix.

The predictors for the ten logits of transition, obtained by regression, are as follows:

$$L_{2\,1} = -1.919 + \underset{(0.049)}{0.085}X_1 - \underset{(0.085)}{0.300}X_5$$

$$L_{3\,2} = 9.386 - \underset{(0.150)}{0.350}X_5 - \underset{(3.375)}{8.488}X_7$$

$$L_{4\,4} = -0.374 + \underset{(0.045)}{0.086}X_1 + \underset{(0.078)}{0.217}X_4$$

$$L_{5\,5} = 2.627 - \underset{(0.107)}{0.402}X_5$$

$$L_{5\,6} = -11.091 + \underset{(0.066)}{0.223}X_3 + \underset{(3.653)}{6.620}X_7$$

$$L_{7\,2} = -3.001 - \underset{(0.171)}{0.498}X_1 - \underset{(0.119)}{0.257}X_3 + \underset{(0.220)}{0.361}X_4$$

$$L_{7\,7} = 9.163 + \underset{(0.020)}{0.136}X_1 - \underset{(0.233)}{0.734}X_6 - \underset{(1.535)}{2.264}X_7$$

$$L_{7\,9} = -5.784 + \underset{(0.073)}{0.143X_1} + \underset{(0.052)}{0.140X_3} + \underset{(0.094)}{0.130X_5}$$

$$L_{10\,11} = 7.557 - \underset{(0.080)}{0.382X_1} - \underset{(0.056)}{0.083X_3} - \underset{(2.275)}{7.066X_7}$$

$$L_{11\,10} = -6.469 + \underset{(0.087)}{0.758X_5}$$

These logits were then transformed back into probabilities by taking the anti-logarithms and dividing by one plus the anti-logarithms of the logits. Thus,

$$\hat{p}_{ij} = \exp(L_{ij})/(1 + \exp(L_{ij}))$$

The frequency of appearance of each exogenous variable is as follows:

VARIABLE	FREQUENCY
1	6
2	0
3	4
4	2
5	5
6	0
7	4

The estimates of transition probabilities that were found to vary significantly with the set of exogenous variables appeared to have a seasonal effect as the dummy variable appeared most frequently in the regressions.

An increase in X_5, U. S. Government securities rate, would result in an increase in the probability of an account to move from

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Nb. The number in brackets below each regression coefficient is the standard error of the coefficient.

Class XI to Class X. A possible explanation is that savers in Class XI will reduce their passbook account savings and invest in U. S. Government securities when the securities rate increases. However, a consistent set of explanations could not be given for the ten predictors so a non-casual approach had to be followed.

The transition probabilities without predictors were considered to be stationary during the period of observation. Thus the nonstationary matrix was formed by replacing ten elements of the estimate of the stationary matrix with predicted values. To ensure that each row add up to one, each element was divided by the rwo sum. Selected transition matrices used in Model II are contained in Appendix F.

A chi square test was performed to test if the predictors could predict the transition matrix for Quarter II-73. The predicted matrix was formed by replacing ten elements of the Quarter I-73 cumulative matrix with values obtained with the predictors and normalizing each row. The problem of small expected number of transitions in certain elements of the matrix was resolved by combininb classes of each row in the manner described in Section A. 5. The ninetieth percentile for the chi square distribution with 37 degrees of freedom is 48.84. The chi square statistic obtained in the test was 35.25, thus, the null hypothesis, that the predicted matrix and the observed matrix of Quarter II-73 were the same, could not be rejected.

9. <u>Predictors of Arrival Rate</u>

The number of new accounts opened in each quarter was regressed against the same set of exogenous variables listed in sub-section

8. The predictor of arrival rate, measured in thousands per quarter, was found to be as follows:

$$Ar = \underset{}{0.052} - \underset{(0.017)}{0.073X_1} + \underset{(0.029)}{0.094X_5}$$

The standard error of each coefficient is contained in the bracket below each coefficient. The square of the multiple correlation between the arrival rate and the exogenous variables, X_1 and X_5, was 0.846. The standard error of Ar before and after the regression was 0.7887 and 0.045.

According to this predictor, the number of new accounts opened per quarter decreases as the year progresses, as X_1, the dummy variable for quarters, takes on values 1, 2, 3 and 4 for the four quarters of the year. The number of new accounts opened would also increase as the U. S. Government securities rate increases. No apparent reasons could be found for this relationship. Predictions are compared with observations in the following table.

TABLE XV

PREDICTED ARRIVAL RATE AND ACTUAL RATE OBSERVED

QUARTER	PREDICTION	OBSERVATION
II-72	777	860
III-72	751	791
IV-72	719	798
I-73	1015	998
II-73	1017	896

10. Predictors of the Probabilities of a New Account Entering Each Class

The estimates of the probability of a new account entering each class obtained for Quarter II-71 through Quarter II-73 were collected together. They were transformed into logits and regressed against the set of exogenous variables listed in sub-section 8. Using the criteria given in sub-section 8 to determine if the exogenous variables in a regression could explain the variance of the logits, only four predictors were accepted. They are:

$$L_4 = \underset{}{2.217} - \underset{(0.029)}{0.082X_1} - \underset{(0.180)}{0.466X_2}$$

$$L_7 = 6.187 - \underset{(0.029)}{0.089X_3} - \underset{(0.246)}{0.979X_6}$$

$$L_9 = -4.482 - \underset{(0.049)}{0.184X_4} + \underset{(1.073)}{3.053X_7}$$

$$L_{10} = -10.725 + \underset{(0.094)}{0.184X_5} + \underset{(0.332)}{0.998X_6}$$

The standard error of each coefficient is contained in the bracket below each coefficient.

The logits are transformed back to estimates of probabilities by:

$$10^{(L_i)} / (1.0 + 10^{(L_i)})$$

Logarithms to the base of 10 were used in both the forward transformation and the inverse transformation. The base of the logarithm does not affect the results of the regressions.

Predictions of the number of new accounts in each class were checked by means of the chi square test. The number of degrees of freedom of the distribution was thirty and the ninetieth percentile of

the distribution is 40.26. The chi square statistic obtained was 36.87. Thus, the hypothesis that the predicted distributions matched the observations could not be rejected.

The predicted arrival distributions for Quarters II-72 to II-73 are contained in Appendix G.

IV. MODEL VALIDATION

A. VALIDATION OF MODEL I

1. Prediction of Sample Population Behavior

As there were no entries into the sample population changes to the structure were caused by accounts moving between classes and by accounts closing. Thus the basic Markov chain model could be used to model the behavior of this population.

It was decided to use the data from the five quarters, Quarter I-71 through Quarter I-72, to estimate the time stationary transition matrix and then use the matrix to predict the structure of the sample population for Quarter II-72 through Quarter II-73. Predictions could then be compared against observations and the chi-square test be used to determine the goodness of fit.

CPM V, the estimate of the time stationary transition matrix with the first five quarters' data, was used to predict the number of accounts in each class and the amount of savings in each class. The results of the predictions on the number of accounts is contained in Table XVI. The actual number observed and the chi-square statistic for each class are presented next to the predictions.

The predictions were expected to diverge more and more from observations as time progressed as errors would accumulate. The chi-square statistic for the first prediction was 3.49 and the value for the fifth prediction was 11.91. These correspond to the fourth percentile and

the seventieth percentile of the chi-square distribution with ten degrees of freedom. The predicted distribution after five quarters still provided a reasonably good fit to the observations.

The predicted amount of savings in each class and the actual amount observed are presented in Table XVII. The predictions did not match the observations as well as the predictions of number of accounts. The error in prediction of total amount of savings amounted to 10.6 percent after five quarters. The difference between predicted total amount of savings and the amount observed could be explained by the fact that the predicted number of accounts for the larger classes, class VII to class XI, were generally smaller than the number observed. The error in the number of accounts, though relatively insignificant in absolute magnitude, when multiplied by the average amount of savings would amount to a substantial sum. Thus the estimates of transition probabilities between classes with low average amount of savings per account and those with high average amount of savings per account would have to be precise to yield more accurate predictions of total amount of savings.

A relatively small number of large accounts can increase the variability of total amount of savings significantly. The error in prediction for Quarter II-73 amounted to about four hundred and fifty six thousand dollars. Of this amount four hundred and forty two thousand dollars were contributed by twenty two accounts in classes VIII, IX, X and XI. It would seem to appear that there is no easy way to reduce the variability in total amount of savings caused by this small group of savers.

TABLE XVI

TABLE OF PREDICTED NUMBER OF ACCOUNTS, ACTUAL NUMBER OBSERVED AND CHI-SQUARE STATISTICS
(MODEL I USING CPM V)

	QUARTER II-72			QUARTER III-72			QUARTER IV-72			QUARTER I-73			QUARTER II-73		
CLASS	PRED.	ACT.	CHI	PRED.	ACT.	CHI	PRED.	ACT.	CHI	PRED.	ACT.	CHI	PRED.	ACT.	CHI
0.	19.	18.	0.06	38.	35.	0.18	55.	48.	1.00	73.	61.	1.90	89.	86.	0.13
1999.	167.	169.	0.02	166.	160.	0.19	163.	153.	0.64	160.	148.	0.96	157.	141.	1.68
3999.	84.	80.	0.22	80.	80.	0.00	76.	76.	0.00	73.	78.	0.28	71.	69.	0.05
5999.	55.	51.	0.36	55.	50.	0.45	54.	57.	0.18	53.	53.	0.00	51.	45.	0.71
7999.	35.	37.	0.12	33.	38.	0.61	32.	30.	0.16	31.	30.	0.04	30.	31.	0.03
9999.	21.	22.	0.10	20.	20.	0.00	20.	17.	0.37	19.	21.	0.21	18.	15.	0.57
11999.	54.	57.	0.21	49.	50.	0.04	45.	49.	0.45	41.	44.	0.21	38.	46.	1.65
13999.	27.	21.	1.31	24.	27.	0.27	22.	23.	0.02	21.	23.	0.30	19.	25.	1.93
15999.	19.	22.	0.47	18.	21.	0.40	17.	25.	3.42	16.	21.	1.40	15.	17.	0.22
19999.	20.	20.	0.01	19.	21.	0.33	17.	26.	4.72	16.	25.	5.40	15.	22.	3.64
100000.	36.	41.	0.60	36.	36.	0.00	36.	34.	0.09	35.	34.	0.04	34.	41.	1.29
SUM	519.	520.	3.49	500.	503.	2.45	483.	490.	11.05	465.	477.	10.74	449.	452.	11.91

TABLE XVII

TABLE OF PREDICTED AMOUNT OF SAVINGS AND ACTUAL AMOUNT OBSERVED
(MODEL I USING CPM V)

	QUARTER II-72		QUARTER III-72		QUARTER IV-72		QUARTER I-73		QUARTER II-73	
CLASS	PRED.	ACT.	PRED.	ACT.	PRED.	ACT.	PRED.	ACT.	PRED.	ACT.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
1999.	80904.	85201.	80098.	76517.	78977.	74296.	77622.	68827.	76095.	69300.
3999.	242192.	239441.	229694.	238997.	219580.	224544.	210991.	231257.	203391.	207472.
5999.	275999.	251076.	273472.	241451.	268201.	274778.	261425.	259631.	253868.	218088.
7999.	242688.	256481.	232384.	262935.	223739.	215352.	215813.	206544.	208211.	215346.
9999.	185082.	201731.	182939.	183567.	177698.	153325.	171188.	189416.	164273.	131926.
11999.	584069.	610157.	529512.	536059.	484776.	520816.	447034.	467021.	414535.	492183.
13999.	352429.	276219.	319576.	354780.	291952.	298484.	268336.	299438.	247880.	325462.
15999.	287733.	327455.	277184.	315938.	262146.	373113.	245752.	315862.	229552.	252917.
19999.	368251.	351882.	333940.	372054.	306708.	468395.	284039.	440700.	264505.	388406.
100000.	1052621.	1027230.	1049706.	954255.	1037581.	906064.	1018738.	925321.	995050.	1116465.
SUM	3671967.	3626873.	3508503.	3534753.	3351354.	3509167.	3200935.	3404017.	3057358.	3417565.

If the time stationary assumption is not violated then it is legitimate to estimate the transition matrix with data from the entire period of observation. The increase in data should yield better estimates of transition probabilities. Thus CPM X, the transition matrix estimated with all ten quarters' data, was used in predicting the number of accounts and the amount of savings in each class. The results are presented in Appendix H.

To demonstrate the importance of data on predictions, CPM II, the transition matrix estimated with data from Quarter I-71 and Quarter II-71, was also used to predict the number of accounts and the amount of savings in each class. The results are also presented in Appendix H.

The chi-square statistics obtained using CPM V, CPM II and CPM X are compared in the following table:

TABLE XVIII

COMPARISON OF CHI SQUARE STATISTICS OBTAINED WITH CPM V, CPM II AND CPM X

			QUARTER		
MATRIX	II-72	III-72	IV-72	I-73	II-73
CPM V	3.49	2.45	11.05	10.74	11.91
CPM II	7.59	13.84	35.67	51.11	65.97
CPM X	3.26	1.12	5.93	5.03	3.57

The tenth percentile and the ninetieth percentile of the chi square distribution with ten degrees of freedom are as follows:

$$P_{10} = 4.87$$

$$P_{90} = 15.99$$

Using P_{90} as a criterion to determine if the fit is acceptable it could be seen that predictions with CPM V and CPM X passed the test for the entire period of prediction whereas predictions with CPM II were only acceptable for the first two periods.

The total amount of savings predicted using CPM V, CPM II and CPM X are compared in the following table:

TABLE XIX

COMPARISON OF TOTAL AMOUNT OF SAVINGS OBTAINED WITH CPM V, CPM II AND CPM X ($M)

QUARTER MATRIX	II-72	III-72	IV-72	I-73	II-73
CPM V	3.672	3.509	3.351	3.201	3.057
CPM II	3.553	3.293	3.060	2.851	2.664
CPM X	3.727	3.613	3.500	3.389	3.280
ACTUAL	3.627	3.535	3.509	3.404	3.418

The superiority of predictions with CPM X is apparent. The percentage error in predicting the total amount of savings of Quarter II-73 is 4.0 which is less than half of that obtained using CPM V. The importance of accurate estimates of transition probabilities is clearly demonstrated by the above comparisons.

2. Prediction of Behavior of Population

To predict the behavior of the entire population the model has to include the process of arrivals and entrants. As the sample size was small (about 3.5% of the population) it was decided to use the entire data base to estimate the transition matrix.

The average arrival rate (number of new accounts opened per quarter) was found to be 800.7 and the distribution of new accounts was estimated to be as follows:

CLASS	$\hat{p}_j$
II	0.7813
III	0.0680
IV	0.0484
V	0.0187
VI	0.0124
VII	0.0156
VIII	0.0089
IX	0.0111
X	0.0076
XI	0.0280

The estimates were obtained by adding up the number of new accounts in each class over the period of observation and dividing by the total number of new accounts sampled.

The number of accounts in each class was predicted by adding the expected number of accounts moving into or remaining in that class from the population of accounts already in the system and the number of new accounts entering that class. The expression used in the computation can be found in Section C of Chapter II.

The predicted total number of accounts and the total amount of savings are shown in the following table:

TABLE XX

PREDICTED TOTAL NUMBER OF ACCOUNTS AND TOTAL AMOUNT OF SAVINGS AND OBSERVED VALUES

	QUARTER	II-72	III-72	IV-72	I-73	II-73
TOTAL # OF ACCOUNTS	PRED.	17345	17447	17557	17664	17776
	ACT.	17354	17483	17485	17746	17820
TOTAL AMOUNT OF SAVINGS	PRED.	45.65	49.87	53.78	57.39	60.74
	ACT.	41.57	42.15	42.40	44.13	44.56

The maximum error in predicting the total number of accounts was 82 which was about half a percent of the total number of accounts. This indicated that the process of arrivals and the process of departures were probably as described by the model during the period of prediction.

The failure of the model to predict the total amount of savings could be due to the failure of the model to predict the structure of the population or a violation of the constant average amount of savings in each class assumption.

To test the hypothesis that the error in total amount of savings was caused by error in predicting the number of accounts in each class, a sample comprising one-fourth of the population at Quarter I-73 was taken and used to compare with the predicted structure of active accounts. The chi square test was used to determine the goodness of fit between the predicted distribution and the distribution of the sample.

The number of degrees of freedom of the distribution of the chi square statistic is eight and the ninetieth percentile of the distribution

is 13.36. The chi square statistic obtained was 111.0, thus, the null hypothesis that the predicted distribution and the distribution of the sample could be rejected.

In examining the chi square statistic of each class it was found that major sources of error came from Classes II and III, IV, V, VII and XI (Classes II and III had been combined to ease the burden of extracting data for the validation sample). It appeared that Classes IV, V, VII and IX became much larger at the expense of Classes II and III. This would account for the high predictions of total amount of savings.

Another check was made by taking the difference between the predicted number of accounts in the sample and the actual number of accounts in each class and multiplying by the respective average amount of savings of each class. The errors in the amount of savings in each class are shown in Table XXI.

If the validation sample could be taken as a good representation of the population then the error in prediction of the population could be estimated by multiplying the error in the amount of savings in the validation sample by four. Thus, the prediction of total amount of savings would be high by $11.2 million. The observed error of $13.3 million could therefore be considered to be mainly the result in errors in predicting the structure of the population.

Looking at the error in the prediction of amount of savings of each class, it can be seen that Class XI is a major contributor to the total error. It was suspected that the model failed because of sampling errors

TABLE XXI

ERRORS IN PREDICTING THE AMOUNT OF SAVINGS IN THE VALIDATION SAMPLE

CLASS	PREDICTED # OF A/C	ACTUAL # OF A/C	ERROR IN # OF A/C	ERROR IN AMOUNT OF SAVINGS
II & III	3435	3699	-264	-182759
IV	342	222	+120	+589920
V	180	144	+ 36	+246780
VI	95	103	- 8	- 71240
VII	138	93	+ 45	+484065
VIII	72	60	+ 12	+155040
IX	52	41	+ 11	+164571
X	48	50	- 2	- 35582
XI	122	70	+ 52	+1450175
TOTAL	4484*	4482	+ 2**	+2800971

* Should equal 4482. Discrepancy caused by rounding error

** Should equal 0. Discrepancy caused by rounding error

which resulted in estimating higher probabilities of transition between classes with low average amount of savings and those with large average amount of savings.

To check out this hypothesis the following changes were made to CPM X:

1. Accounts found to have made two or more transitions between Classes II, III, IV and V and Classes VIII, IX, X and XI were removed from the data base as these accounts would not be representative of the normal behavior of the population. Eight accounts were rejected

according to this rule and CPM X was recomputed with the remaining six hundred and fourteen accounts. This modified transition matrix was termed MOD I.

2. The 90% lower confidence limit was estimated for transition probabilities from Classes II, III, IV and V to higher classes. The Poisson distribution was used to approximate the binomial distribution in cases when the total number of transitions observed was below seven. The normal approximation was used when the number of transitions observed exceeded seven. This modification was applied to MOD I and termed MOD II.

3. Further adjustments were made to a few transition probabilities based on the results of the chi square fit using MOD I and MOD II. The rationale for the adjustments is as follows:

Since the data base of accounts is inadequate for estimation of population parameters, use the additional data available from the validation sample to correct the estimation of certain parameters. Hypothesize that the new matrix, termed MOD III, as the best estimate and proceed with the prediction of total number of accounts and total amount of savings in the institution. A good fit between predicted total amount of savings over the prediction interval would give support to the hypothesis.

MOD I, MOD II and MOD III are contained in Appendix E.

The results obtained using the modified matrices are compared against predictions using CPM X in Tables XXII and XXIII.

TABLE XXII

COMPARISON OF PREDICTIONS OF SIZE DISTRIBUTION OF VALIDATION SAMPLE BY MODEL I, USING CPM X, MOD I, MOD II AND MOD III

		CPM X		MOD I		MOD II		MOD III	
CLASS	ACTUAL	PRED.	CHI	PRED.	CHI	PRED.	CHI	PRED.	CHI
II&III	3699	3435	20.3	3482	13.5	3614	2.0	3698	0.0
IV	222	342	41.8	337	39.2	302	21.3	216	0.2
V	144	180	7.0	178	6.6	156	1.0	148	0.1
VI	103	95	0.6	89	2.2	75	10.5	100	0.1
VII	93	138	14.7	133	12.0	117	5.1	99	0.4
VIII	60	72	1.8	64	0.3	50	2.0	55	0.5
IX	41	52	2.2	51	1.9	43	0.1	45	0.4
X	50	48	0.1	47	0.2	43	1.1	43	1.0
XI	70	122	22.4	101	9.6	82	1.8	78	0.9
TOTAL	4483		111.0		85.4		45.0		3.6

It can be seen from Table XXII that the structure of the predicted distribution changed substantially with each modification. The improvement in fit in the predicted distribution with each modification had a corresponding effect in the prediction of total amount of savings. However, the predicted total number of accounts were marginally degraded by each modification. The changes, however, were not considered to be significant as the percentage error was still of the order of less than one percent.

Though the modifications to the transition matrix improved the predictions they do not prove that the true transition matrix should be as specified by MOD III. However, with the amount of information available the best estimate of the transition matrix is MOD III. Although its ability to predict the structure of the population has not been put to a test, the accurate prediction of total amount of savings encourages one to believe that MOD III is close to the true matrix.

TABLE XXIII

MODEL I PREDICTIONS OF TOTAL NUMBER OF ACCOUNTS AND AMOUNT OF SAVINGS ($M) USING CPM X, MOD I, MOD II AND MOD III

	QUARTER	II-72	III-72	IV-72	I-73	II-73
TOTAL NUMBER OF ACCOUNTS	CPM X	17345	17447	17554	17664	17776
	MOD I	17336	17428	17525	17625	17726
	MOD II	17335	17424	17516	17609	17702
	MOD III	17329	17405	17408	17552	17622
	ACTUAL	17354	17483	17485	17746	17820
TOTAL AMOUNT OF SAVINGS	CPM X	45.65	49.87	53.78	57.39	60.74
	MOD I	44.64	47.97	51.06	53.97	56.67
	MOD II	43.00	44.80	46.43	47.96	49.38
	MOD III	41.94	42.74	43.48	44.16	44.79
	ACTUAL	41.57	42.15	42.40	44.13	44.56

3. Estimates of the Fundamental Matrix

The fundamental matrix $(I - Q)^{-1}$ was estimated by substituting Q from CPM X into the expression. It is displayed in Table XXIV.

The ij^{th} element of this matrix is the expected number of time periods that a new account beginning in Class i will spend in Class j before closing. Thus a new account joining, say, Class IV will on the average visit Class V for 2.4562 periods during its entire life in the system.

The expected total time a new account which joins Class i spends in the system is the sum of the ith row of the fundamental matrix, M.

The equilibrium distribution is obtained by multiplying the distribution of arrivals by M. The results obtained are presented in Table XXVI. Results obtained using MOD III are also presented.

The results are interesting in that they are predictions of the final state of the population if current conditions were to prevail. This state of equilibrium is reached when the number of new accounts opened per quarter balances the number of accounts closed, and the number of accounts moving out of each class is balanced by a corresponding number of accounts moving in from other classes. The Fundamental matrix obtained with CPM X predicts that the population will grow from 17251, at Quarter I-72, to a final value of 21734. The population of each class grows larger except for Class II. However, as noted earlier, CPM X did not predict the total amount of savings accurately; therefore, projection of the equilibrium distribution using it has little value except to constrast with the results obtained with MOD III.

TABLE XXIV

THE FUNDAMENTAL MATRIX OBTAINED WITH CPM X

CLASS	2	3	4	5	6	7	8	9	10	11
2	14.1651	4.0564	2.2670	1.2645	0.6762	1.1681	0.5977	0.4574	0.4965	1.2818
3	9.7766	7.5328	3.6642	1.8582	0.9443	1.5888	0.8242	0.6349	0.6667	1.6472
4	7.9666	4.3412	6.1576	2.4562	1.1924	1.9368	0.9650	0.7401	0.7914	1.9742
5	7.7207	4.1835	3.3245	4.8768	1.6663	2.6680	1.1286	0.9149	0.9289	2.0480
6	7.2862	3.5812	2.7516	2.0698	3.2617	3.7083	1.4298	1.1322	1.1057	2.4513
7	6.5715	3.1805	2.3220	1.6403	1.2173	6.4955	2.0078	1.4352	1.3419	2.4762
8	7.0330	3.4951	2.5243	1.7064	1.2096	3.3053	4.1641	2.4155	2.0418	3.2687
9	6.8526	3.3779	2.6515	1.8311	1.2361	2.8609	1.9047	4.1577	2.4828	3.5589
10	6.8307	3.3472	2.4810	1.6899	1.1062	2.4504	1.4142	1.4978	5.2143	5.5407
11	6.5118	3.0388	2.4376	1.7207	1.0875	2.5135	1.3747	1.2695	2.4559	10.3263

TABLE XXV

THE FUNDAMENTAL MATRIX OBTAINED WITH MOD III

CLASS	2	3	4	5	6	7	8	9	10	11
2	18.6320	2.1486	0.6975	0.5055	0.3487	0.2970	0.1789	0.1290	0.1120	0.1957
3	13.9483	7.6081	1.6522	1.0351	0.6766	0.5939	0.3616	0.2607	0.2275	0.4029
4	11.0942	4.0122	5.1017	2.0140	1.1712	0.9801	0.4717	0.3681	0.3376	0.6698
5	10.6964	3.8671	2.2258	4.5582	1.8111	1.6457	0.6521	0.5332	0.4679	0.8382
6	10.0411	3.2436	1.8418	1.8231	3.9264	2.2303	0.9076	0.7808	0.6739	1.2462
7	8.5780	2.7255	1.3918	1.3129	1.3184	5.9223	1.6855	1.2308	1.1034	1.7619
8	9.1408	3.0098	1.5279	1.3503	1.2855	2.6633	3.8411	2.2298	1.8308	2.5803
9	8.8689	2.9483	1.7134	1.5059	1.3269	2.1881	1.5975	4.0228	2.2698	2.8652
10	8.9842	2.8778	1.4858	1.3226	1.1002	1.7663	1.0778	1.3009	5.0798	4.9639
11	8.3009	2.5232	1.4782	1.3989	1.1390	1.8922	1.0193	1.0736	2.3608	10.1137

TABLE XXVI

ESTIMATES OF EQUILIBRIUM DISTRIBUTION AND AMOUNT OF SAVINGS IN EACH CLASS AND OBSERVED VALUES AT QUARTER I-72

CLASS	ARRIVAL	DIST. AT QTR I-72	EQUILIBRIUM DISTRIBUTION		AMOUNT OF SAVINGS AT QTR I-72 (MILLION DOLLARS)	PREDICTED TOTAL SAVINGS (MILLION DOLLARS)	
			CPM X	MOD III		CPM X	MOD III
II	626	12373	10271	13579	4.603	3.626	4.800
III	55	1793	3397	2160	5.087	9.638	6.127
IV	38	1034	2074	861	5.083	10.194	4.234
V	15	563	1179	616	3.859	8.080	4.221
VI	10	366	644	436	3.259	5.734	3.884
VII	12	372	1170	468	4.002	12.590	5.040
VIII	7	209	598	261	2.700	7.731	3.368
IX	9	153	484	224	2.289	7.241	3.350
X	6	183	543	240	3.256	9.654	4.271
XI	22	205	1374	518	5.717	38.323	14.447
TOTAL	800	17251	21734	19363	39.855	112.812	53.742

The Fundamental matrix obtained with MOD III produced rather believable kind of predictions. It predicted that the total number of accounts will grow to a maximum of 19363 and each class grows larger at the same time. The equilibrium amount of savings in the population will be $53.74 million. Thus, if current conditions will prevail the institution can expect a growth of another $10 million, from the current level of $44 million (as at 30 June 1973), in the passbook accounts.

The population under consideration, however, did not include accounts greater than $100,000. A separate study will therefore be required to predict the equilibrium number of accounts in this group of accounts which numbered six, at Quarter I-72.

The expected length of stay of accounts in the system are presented in the following table:

TABLE XXVII

EXPECTED LENGTH OF STAY IN THE SYSTEM COMPUTED WITH CPM X AND MOD III

CLASS	LENGTH OF STAY IN SYSTEM (QUARTERS)	
	CPM X	MOD III
II	26	23
III	29	27
IV	29	26
V	29	27
VI	29	27
VII	29	27
VIII	31	29
IX	31	29
X	32	30
XI	33	31

The expected length of stay in the system is almost constant for all the classes except for Classes II and XI. The conclusion that can be drawn from this observation is that the length of stay of a saver, in the system, is relatively indifferent to the amount of savings he started out with. The shorter life of accounts in Class II is a fact that has been noticed previously. The longer life of accounts in Class XI is contrary to expectation, as one would expect savers who do not have immediate need for such large sums, to transfer the passbook account into other types of savings account which yield higher earnings. The observation may be explained if these savers do not close their account when funds are transferred to other types of accounts. The length of stay would then reflect the length of time a saver wishes to remain a customer of the savings institution. The Fundamental matrix using CPM X predicts, on the average, lengths of stay of 29.8 periods whereas the Fundamental matrix using MOD III predicts 27.6 periods. The smaller total number of accounts predicted using MOD III can be explained by the fact that customers spend less time in the system.

Thus, the model shows that efforts to keep customers in the system are as important as attracting new customers into the system.

B. VALIDATION OF MODEL II

1. Prediction of Sample Population Behavior

The transition matrices used in predicting the behavior of the sample were estimated by the method described in Chapter II, Section B. 8. The elements of the transition matrices that did not have predictors were

taken from CPM V, the estimate of the time stationary transition matrix using data from the first five quarters. The predicted matrices are contained in Appendix F. The predicted number of accounts in each class was compared against the actual number observed. The chi square test was used to determine the goodness of fit between the predicted and observed distribution of accounts in the sample.

The results are presented in Appendix I. It was found that the predictions matched the observations very closely for the first four quarters. The chi square statistic of each of the first four quarters was less than 6.7. However, the predictions for the fifth quarter were extremely poor. The chi square statistic was 25.02. If the null hypothesis that the predicted and observed distributions are the same were true, then this chi square statistic would be obtained 0.5 percent of the time. The null hypothesis could thus be safely rejected at the 10% level of significance.

An investigation of the causes of the failure of the model to predict accurately for Quarter II-73 showed that the ten predictions of transition probabilities for Quarter II-73 had altered the transition matrix for Quarter II-73 substantially. Two exogenous variables X_4, prime commercial paper rate, 4-6 months and X_5, U. S. Government securites rate, 6 months, were considerably higher in Quarter II-73 than in the earlier quarters. Thus the predictors were used beyond the data base from which they were derived. This could lead to unexpected results.

To verify the hypothesis that Model II failed in Quarter II-73 because of the use of some predictors beyond the data base on which they

were derived, predictions were repeated using a matrix with predictors that had X_5 as an explanatory variable removed. The chi square statistic obtained with this modified matrix was 14.87, a substantial improvement from that obtained without the modification. The ninetieth percentile of the chi square distribution with ten degrees of freedom is 15.99. Thus the null hypothesis could not be rejected at the 10% level of significance. It was therefore concluded that hypothesis on the failure of the model is correct.

2. Prediction of Population Behavior

The complete Model II was used in the prediction of the behavior of the population. The predicted number of new accounts opened in each quarter was computed in Chapter III, Section B. 9. The predicted number of new accounts entering each class was presented in Chapter III, Section B. 10. The transition matrix used was the same as that used in the prediction of sample population behavior in sub-section 1.

With experience gained in earlier predictions with Model I, high predicted total amount of savings was expected. The modifications applied to the transition matrix of Model I were also applied to Model II. The predicted total number of accounts and total amount of savings are presented in Table XXVIII.

The total number of accounts predicted by Model II matched the observed values closely for Quarters II-72, III-72 and IV-72, but diverged quite widely by Quarter II-73. The predicted total amount of savings was high but the divergence increased substantially in Quarter II-73.

TABLE XXVIII

PREDICTED TOTAL NUMBER OF ACCOUNTS AND TOTAL AMOUNT OF SAVINGS ($M) BY MODEL II, USING CPM X, MOD I, MOD II AND MOD III

	QUARTER	II-72	III-72	IV-72	I-73	II-73
TOTAL NUMBER OF ACCOUNTS	CPM X	17307	17380	17448	17985	18547
	MOD I	17305	17374	17438	17973	18534
	MOD II	17304	17370	17430	17966	18531
	MOD III	17304	17364	17414	17953	18526
	ACTUAL	17354	17483	17485	17746	17820
TOTAL AMOUNT OF SAVINGS (MILLION DOLLARS)	CPM X	45.61	49.79	53.68	58.20	62.49
	MOD I	44.59	47.88	50.98	54.80	58.47
	MOD II	42.95	44.69	46.32	48.75	51.13
	MOD III	41.90	42.64	43.31	44.80	46.19
	ACTUAL	41.57	42.15	42.40	44.13	44.56

The hypothesis, that the model failed to yield accurate predictions because the predictors of transition probabilities were used beyond the range of data used to obtain the predictors, was put to another test by predicting with a transition matrix that had predictors with X_5 as explanatory variable removed. The predictions are presented in Table XXIX.

It can be seen that the predicted total number of accounts has improved considerably by this change to the transition matrices. The improvement to predictions of total amount of savings is not so pronounced.

The validation sample of 4483 accounts taken from the Quarter I-73 population was used to check if Model II predicted the population structure accurately. The predictions obtained with CPM X, MOD I, MOD II and MOD III are presented in Table XXX Predictions by Model II' are presented in Table XXXI.

TABLE XXIX
PREDICTED TOTAL NUMBER OF ACCOUNTS AND TOTAL AMOUNT OF SAVINGS BY MODEL II'

	QUARTER	II-72	III-72	IV-72	I-73	II-73
TOTAL NUMBER OF ACCOUNTS	CPM X	17320	17373	17404	17738	18068
	MOD I	17310	17354	17375	17697	18016
	MOD II	17310	17350	17365	17681	17992
	MOD III	17309	17343	17346	17645	17937
	ACTUAL	17354	17483	17485	17746	17820
TOTAL AMOUNT OF SAVINGS (MILLION DOLLARS)	CPM X	45.62	49.69	53.34	57.44	61.23
	MOD I	44.60	47.76	50.62	54.01	57.14
	MOD II	42.96	44.58	46.00	48.04	49.92
	MOD III	41.90	42.55	43.06	44.28	45.37
	ACTUAL	41.57	42.15	42.40	44.13	44.56

It can be seen that the predicted distribution improved with each modification. The error in predicting the total amount of savings can be attributed to the error in the prediction of number of accounts in each class. As an example, the error in predicting the number of accounts in Classes XI, VII and IV could account for $2.9 million in the prediction of total amount of savings for Quarter I-73 using MOD II.

Though the predicted distribution using MOD III fitted the observed distribution very closely, the error in predicting the number of accounts in Class XI could account for $0.67 million of the error in predicting the total amount of savings for the entire population. This again demonstrates the importance of accurate predictions of number of accounts in classes with large average amount of savings.

TABLE XXX

PREDICTED DISTRIBUTION OF ACCOUNTS IN THE VALIDATION SAMPLE, OBSERVED DISTRIBUTION AND CHI SQUARE STATISTICS BY MODEL II

		CPM X		MOD I		MOD II		MOD III	
CLASS	ACTUAL	PRED.	CHI	PRED.	CHI	PRED.	CHI	PRED.	CHI
II &III	3699	3438	19.8	3488	12.7	3622	1.6	3713	0.1
IV	222	348	45.7	342	42.2	304	22.3	213	0.4
V	144	159	1.4	158	1.2	137	0.4	128	2.0
VI	103	110	0.5	104	0.0	89	2.2	113	0.9
VII	93	134	12.6	128	9.5	113	3.5	96	0.1
VIII	60	71	1.7	64	0.2	50	2.2	54	0.6
IX	41	54	3.3	53	2.8	46	0.5	47	0.8
X	50	57	0.9	55	0.4	49	0.0	49	0.0
XI	70	111	15.0	92	5.1	74	0.1	70	0.0
	4483		100.8		74.0		33.0		4.9

TABLE XXXI

PREDICTED DISTRIBUTION OF ACCOUNTS IN THE VALIDATION SAMPLE, OBSERVED DISTRIBUTION AND CHI SQUARE STATISTICS BY MODEL II'

		CPM X		MOD I		MOD II		MOD III	
CLASS	ACTUAL	PRED.	CHI	PRED.	CHI	PRED.	CHI	PRED.	CHI
II&III	3699	3437	20.0	3485	13.1	3617	1.9	3702	0.0
IV	222	341	41.6	336	38.6	299	19.1	212	0.4
V	144	168	3.4	166	3.0	146	0.0	138	0.3
VI	103	107	0.2	102	0.0	86	3.2	111	0.5
VII	93	137	13.9	130	10.7	115	4.4	98	0.2
VIII	60	71	1.8	64	0.3	50	1.9	55	0.5
IX	41	53	2.7	52	2.3	45	0.3	47	0.7
X	50	50	0.0	48	0.1	44	0.7	45	0.7
XI	70	119	20.4	99	8.4	80	1.2	76	0.5
	4483		104.1		76.5		33.5		3.8

C. COMPARISON OF MODEL I AND MODEL II

1. Sample Population Behavior

The chi square statistics obtained in the test of goodness of fit between the predicted distributions and the observed distribution were used as a measure of the predictive power of the two models.

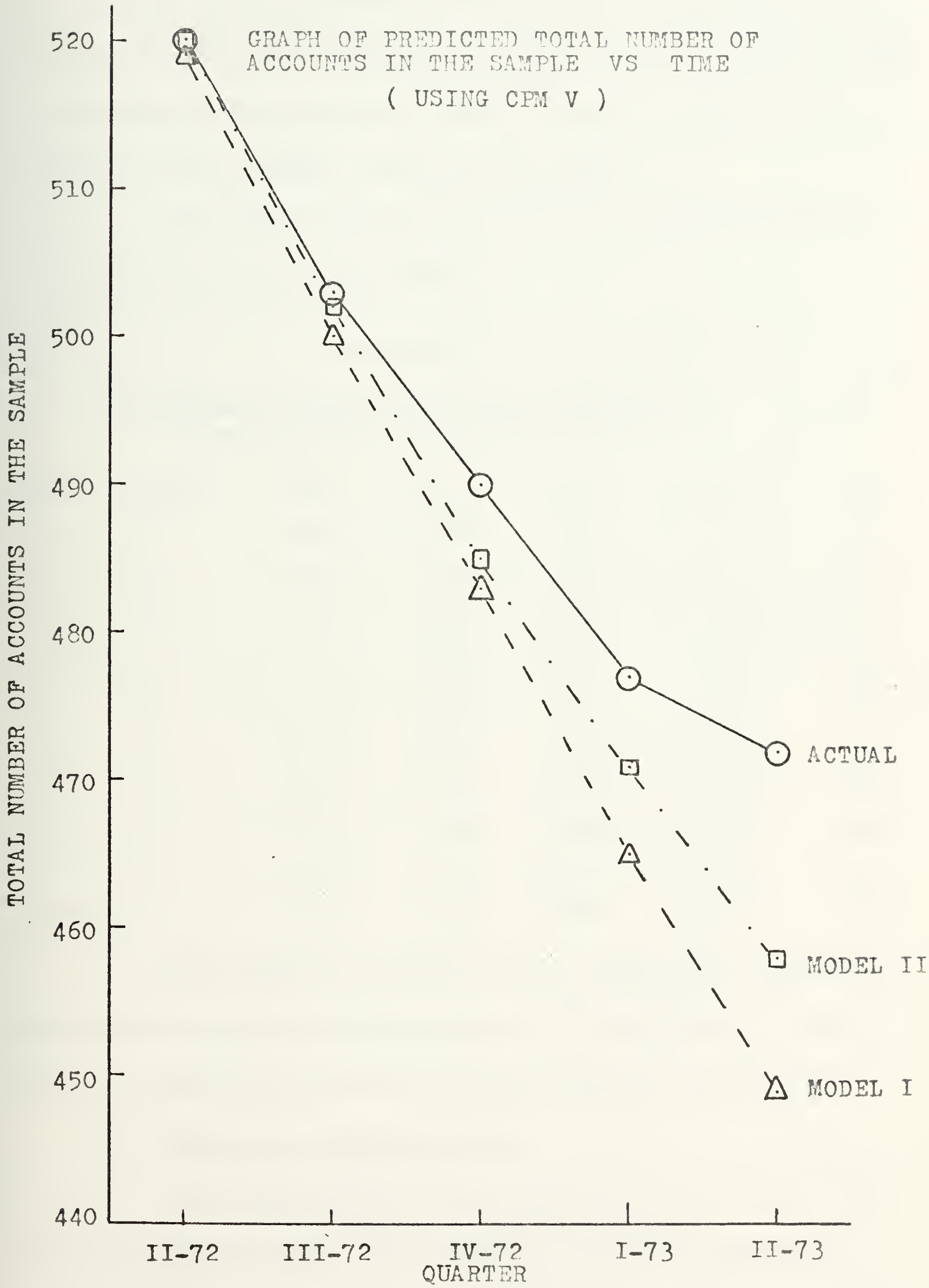
Model II' denotes Model II modified by the deletion of five predictors of transition probabilities which had X_5 as an explanatory variable. The chi square statistics obtained with Model I, Model II and Model II' are presented in Table XXXII.

TABLE XXXII

COMPARISON OF CHI SQUARE STATISTICS OBTAINED WITH MODELS I, II AND II'

		QUARTER				
CPM	MODEL	II-72	III-72	IV-72	I-73	II-73
V	I	3.49	2.45	11.05	10.74	11.91
V	II	3.60	1.98	6.70	4.35	25.02
V	II'	3.47	2.19	8.69	8.84	14.87
II	I	7.59	13.84	35.67	51.11	65.97
II	II	6.76	11.46	24.65	33.01	76.64
II	II'	6.99	12.83	30.73	43.05	68.39
X	I	3.26	1.12	5.93	5.03	3.57
X	II	2.97	0.94	3.82	1.26	15.92
X	II'	3.17	1.05	4.71	3.99	7.01

Except for Quarter II-73, Model II was generally superior to Model I. Model II' improved the predictions for Quarter II-73 but did not perform as well as Model II for the other quarters. The results were expected as Model II, having greater flexibility, should perform better



GRAPH OF PREDICTED TOTAL NUMBER OF ACCOUNTS IN THE SAMPLE VS TIME
(USING CPM V)
TOTAL NUMBER OF ACCOUNTS IN THE SAMPLE
520
510
500
490
480
470
460
450
440
II-72
III-72
IV-72
I-73
II-73
QUARTER
ACTUAL
MODEL II
MODEL I

under normal situations. Model II', with only five predicted elements in its transition matrix, would be expected to be less responsive to changes in external conditions, thus would not perform as well as Model II. Model I, being completely indifferent to external conditions, should be expected to be the poorest performer among the three models.

The predicted total amount of savings predicted by Models I, II and II' are presented in Table XXXIII.

TABLE XXXIII

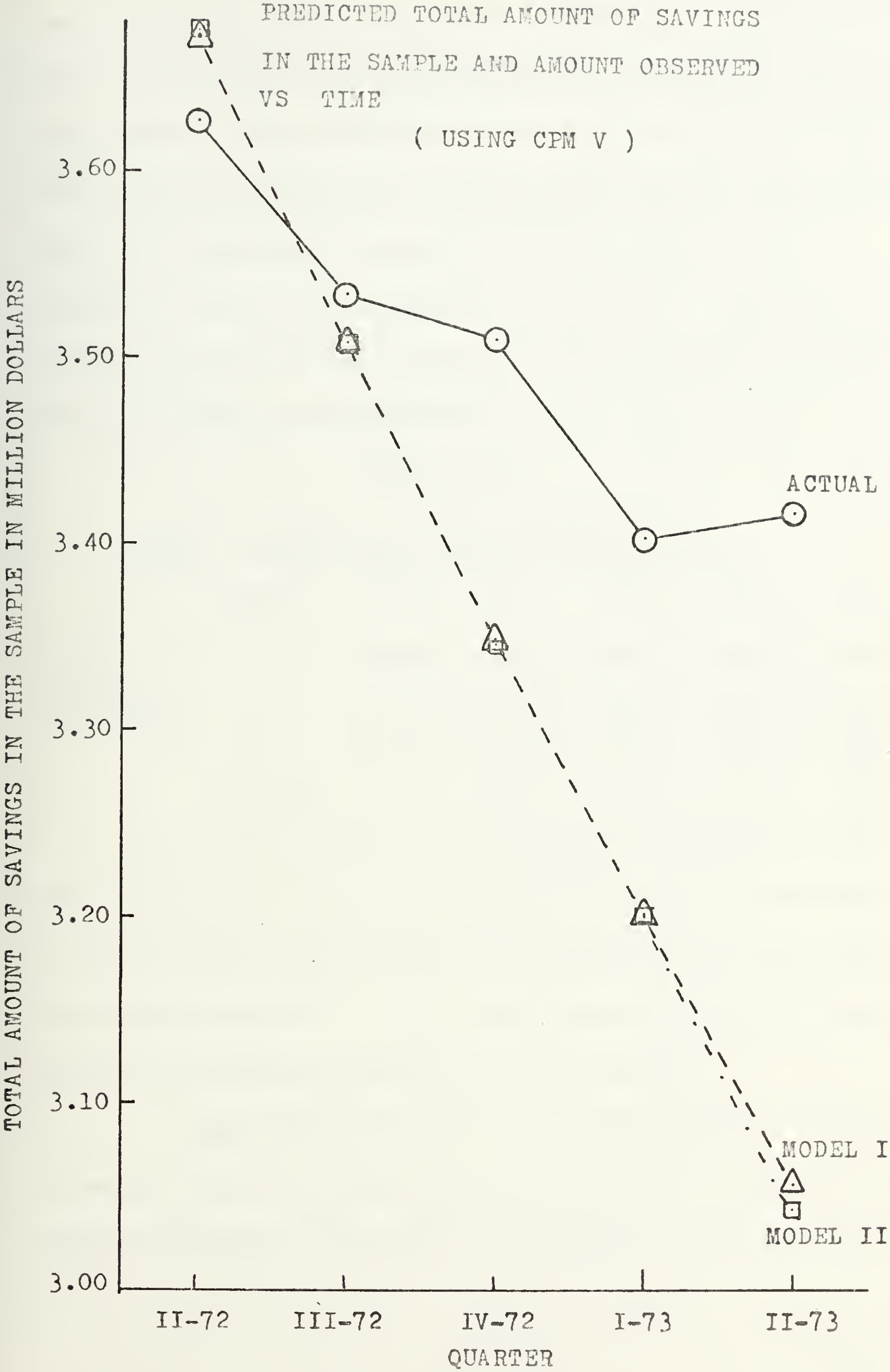
COMPARISON OF PREDICTED TOTAL AMOUNT OF SAVINGS ($M) BY MODELS I, II AND II'

CPM	MODEL	II-72	III-72	IV-72	I-73	II-73
V	I	3.672	3.509	3.351	3.201	3.057
V	II	3.674	3.507	3.346	3.201	3.043
V	II'	3.669	3.497	3.329	3.180	3.024
II	I	3.553	3.293	3.060	2.851	2.664
II	II	3.574	3.329	3.114	2.937	2.764
II	II'	3.567	3.315	3.092	2.902	2.713
X	I	3.727	3.613	3.500	3.389	3.280
X	II	3.729	3.609	3.488	3.373	3.234
X	II'	3.726	3.603	3.479	3.367	3.241
ACTUAL		3.627	3.535	3.509	3.404	3.418

The predictions between the three models were pretty close. In view of the variability of the predictions of total amount of savings it was not possible to state which of the three models performed better.

2. Behavior of Entire Population

Both models predicted total number of accounts very closely for the first three quarters. The performance of Model II deteriorated


PREDICTED TOTAL AMOUNT OF SAVINGS
IN THE SAMPLE AND AMOUNT OBSERVED
VS TIME
(USING CPM V)
TOTAL AMOUNT OF SAVINGS IN THE SAMPLE IN MILLION DOLLARS
3.60
3.50
3.40
3.30
3.20
3.10
3.00
ACTUAL
MODEL I
MODEL II
II-72
III-72
IV-72
I-73
II-73
QUARTER

badly in the fifth quarter, Quarter II-73. The failure of Model II in Quarter II-73 was attributed to the failure of the predictors of transition probabilities to predict beyond the data base from which they were derived. Predictions made with a matrix modified by the removal of predictors which had X_5 as an explanatory variable were closer to the actual value for Quarters I-73 and II-73 than predictions by Model II. Table XXXIV compares the total number of accounts predicted by Model I, Model II and Model II', Model II modified as described above.

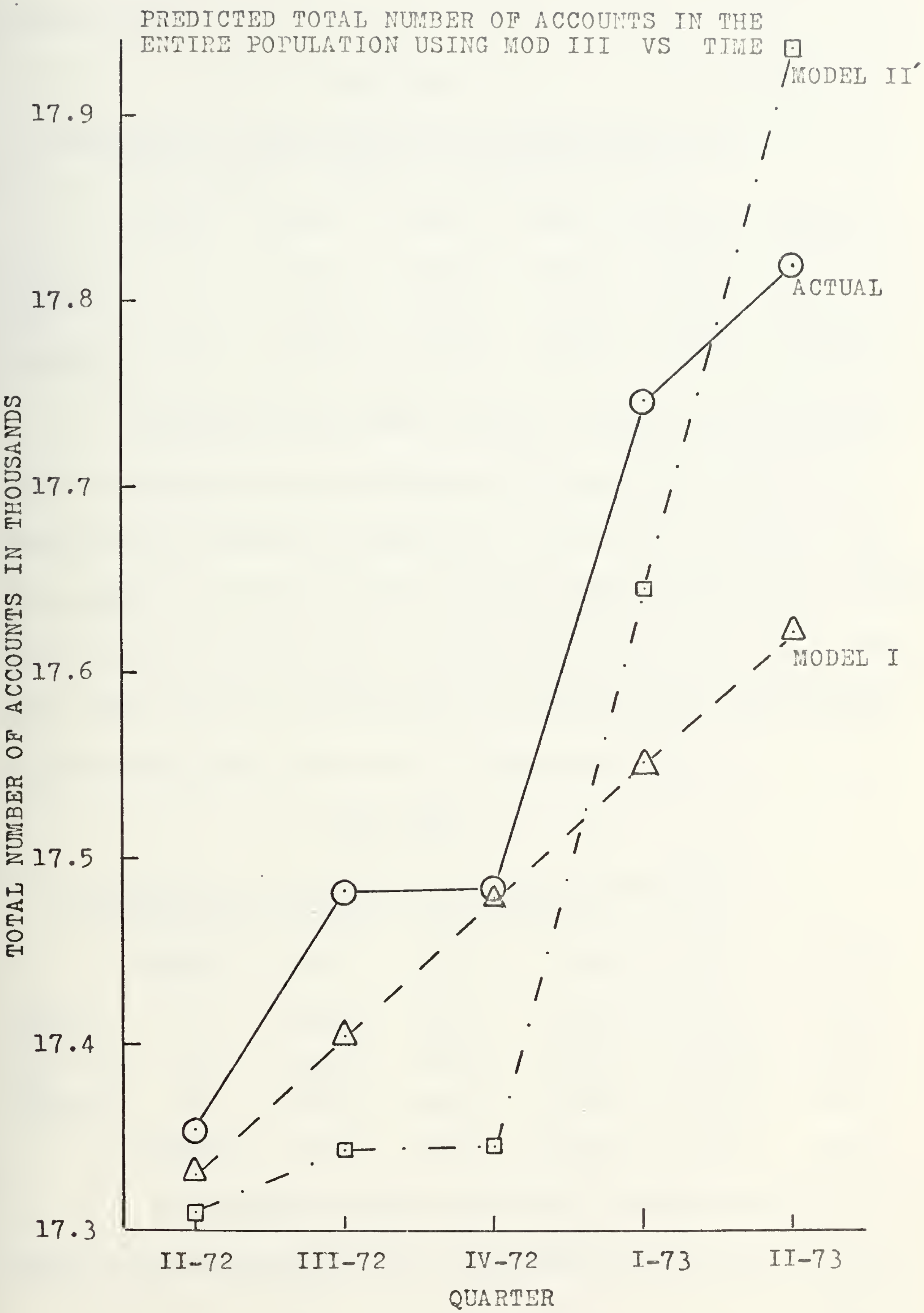
TABLE XXXIV

TOTAL NUMBER OF ACCOUNTS PREDICTED BY
MODEL I, MODEL II AND MODEL II' USING MOD III

	MODEL	II-72	II-72	IV-72	I-73	II-73
TOTAL NUMBER OF ACCOUNTS	I	17329	17405	17480	17552	17622
	II	17304	17364	17414	17953	18526
	II'	17309	17343	17346	17645	17937
	ACTUAL	17354	17483	17485	17746	17820

It can be seen that Model I predictions are closer to the observed values for the first three quarters. However, unlike Models II and II', Model I could not predict the sudden increase in the number of accounts in Quarter I-73. This, again, shows that Model I is applicable only when external conditions remain constant.

Both models were equally bad in predicting the total amount of savings. The cause for the failure was attributed to sampling errors. Similar modifications were made to the transition matrix of both models.


PREDICTED TOTAL NUMBER OF ACCOUNTS IN THE ENTIRE POPULATION USING MOD III VS TIME
TOTAL NUMBER OF ACCOUNTS IN THOUSANDS
17.9
17.8
17.7
17.6
17.5
17.4
17.3
MODEL II′
ACTUAL
MODEL I
II-72
III-72
IV-72
I-73
II-73
QUARTER

The improvement finally achieved was substantial as can be seen in the following table:

TABLE XXXV

COMPARISON OF TOTAL AMOUNT OF SAVINGS PREDICTED BY MODELS I, II AND II' FOR QUARTER II-73

	MODEL	CPM X	MOD I	MOD II	MOD III	ACTUAL
TOTAL	I	60.74	56.67	49.38	44.79	44.56
AMOUNT OF	II	62.49	58.47	51.13	46.19	44.56
SAVINGS	II'	61.23	57.14	49.92	45.37	44.56

Predictions using CPM X, MOD I and MOD II are so different from the observations that the difference between Model I and Model II' predictions are considered insignificant. In the case of predictions made using MOD III, the errors between prediction and observation are too small to discriminate between Model I and Model II' using just one point. Thus, Table XXXVI comparing the predictions of the three models using MOD III over the entire period of prediction, is presented below.

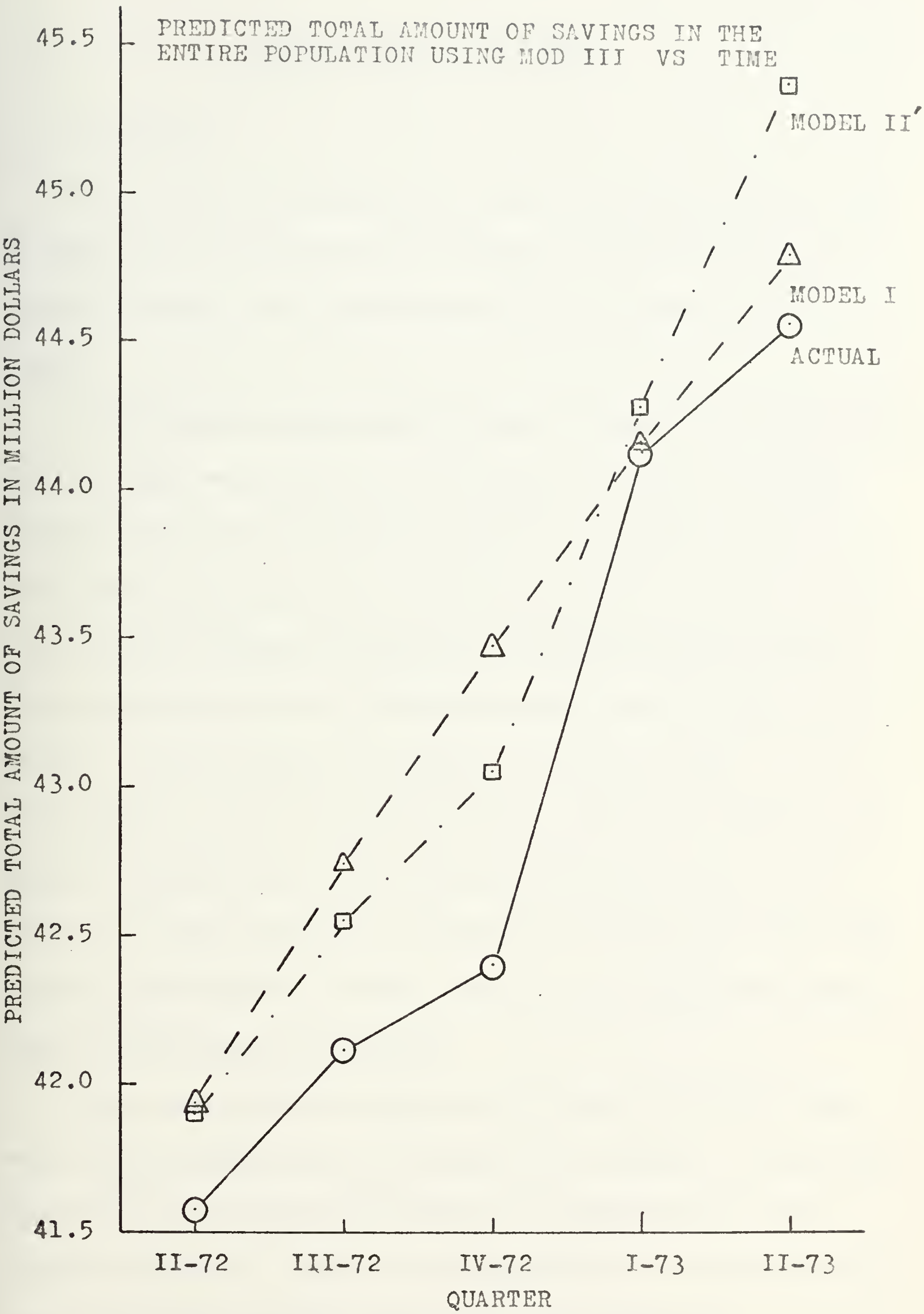
TABLE XXXVI

COMPARISON OF TOTAL AMOUNT OF SAVINGS PREDICTED BY MODELS I, II AND II' USING MOD III

	MODEL	II-72	III-72	IV-72	I-73	II-73
TOTAL	I	41.94	42.74	43.48	44.16	44.79
AMOUNT OF	II	41.90	42.64	43.31	44.80	46.19
SAVINGS	II'	41.90	42.55	43.06	44.28	45.37
	ACTUAL	41.57	42.15	42.40	44.13	44.56

The predictive power of each model in predicting the size distribution of the population could not be compared as the validation sample

was also used in estimating the parameters of MOD III. Thus, another sample would have to be taken to validate this capability of the two models. It is regrettable that this step could not be carried out at the time of the writing of this report because of lack of time. It is therefore proposed that the models be validated again at a later date.


PREDICTED TOTAL AMOUNT OF SAVINGS IN THE
ENTIRE POPULATION USING MOD III VS TIME
MODEL II'
MODEL I
ACTUAL
PREDICTED TOTAL AMOUNT OF SAVINGS IN MILLION DOLLARS
45.5
45.0
44.5
44.0
43.5
43.0
42.5
42.0
41.5
II-72
III-72
IV-72
I-73
II-73
QUARTER

V. SUMMARY AND CONCLUSIONS

A. SUMMARY

The purpose of this research has been to develop a model that can be used to study the structure of a population of savings accounts in a savings institution and to predict future levels of savings in the institution.

Two stochastic models were developed and evaluated in this study. The first model was based on the time stationary Markov chain model extended to cover the phenomena of opening and closing of accounts. The population was divided into ten classes and the continuous distribution of amount of savings of each account was idealized by a discrete distribution with ten classes. The classes were numbered from two to eleven. The class intervals of Classes II to IX were $2,000. Class X contained all accounts with balances between $16,000 and $19,999 and Class XI contained all accounts with balances between $20,000 and $100,000. Class I was used as a reservoir for all the accounts that had closed. The parameters of Model I were assumed to be constant over the period of observation and prediction.

The second model was based on the nonstationary Markov chain model. The parameters were not assumed to be constant. An econometric model was used to relate the estimates of the parameters to a set of exogenous variables. Predictors of the parameters, if found to be significant, were used to predict future values of the parameters.

By assuming that the mean of the amount of savings of accounts in each class remain constant with time the total amount of savings in each class could be computed by multiplying the number of accounts in each class by the mean.

The parameters of the two models were estimated with data obtained from the local branch of a savings institution. The level of savings of a stratified sample of 622 accounts were observed over a period of ten quarters, Quarter I-71 to Quarter II-73. Movements of accounts between classes were recorded as transitions between the respective classes. The transition probability matrix was estimated by dividing the number of transitions from each class by the total number of accounts in the class at the beginning of the quarter.

The total number of new accounts opened in each quarter of the period of observation was used to estimate the arrival rate or expected number of new accounts per quarter.

Two hundred and fifty new accounts were randomly selected each quarter. These were used to determine if the size distribution of new accounts had changed during the period of observation. These accounts were classified into the ten classes described earlier and the probability of a new account being in each class estimated. These estimates were transformed into logits and regressed against a set of exogenous variables. The regressions that were considered significant were used as predictors for future values of the probability of a new account entering a particular class.

The structure of the population of savings accounts for Quarter I-72 was determined and used as the initial distribution in predictions of the behavior of the population.

The chi square test was used to determine if the transition matrix had changed during the period of observation and if the predicted size distributions matched the observed distributions.

The parameters of Model I were estimated using data from the first five quarters. The model was then used to predict the size distribution of accounts of the sample and the amount of savings in the sample population.

The size distribution of the population of savings accounts was predicted using the distribution of the population at Quarter I-72 as the initial distribution. Total number of accounts and total amount of savings were also predicted.

Most of the parameters of Model II were estimated using data from the first five quarters. Of 110 transition probabilities 10 were found to vary significantly with the set of exogenous variables. Thus the transition matrix of Model II contained only ten predicted elements. The predictors were determined using data from the first eight quarters.

Model II was used to predict the size distribution of accounts in the sample and the amount of savings in the sample. It was then used to predict the behavior of the population.

A sample comprising one fourth of the population of Quarter I-73 was used to test if the size distribution predicted by both models were any good. Predicted total number of accounts and total amount of savings were also tested by comparison with actual values observed over the prediction horizon.

B. CONCLUSIONS

1. Model I

The hypothesis that the stochastic processes were stationary during the period of observation could not be rejected at the ten percent level of significance. Thus the assumption of stationarity could be considered to hold.

The predicted size distribution of the sample matched the observed distribution closely. The largest chi square statistic obtained was 11.91. This corresponded to the seventieth percentile of the chi square distribution with ten degrees of freedom. It was concluded that the sample of 622 accounts behaved as described by the Markov chain model.

The predicted total amount of savings differed from the actual amount by a maximum of ten percent. It was concluded that Model I could predict total amount of savings but the variability in the prediction could be rather large as a small number of savers with large accounts could cause large fluctuations in the total amount of savings.

Model I failed to predict the behavior of the population. The failure was attributed to errors in estimation of parameters of the transition

matrix. This observation was supported by the fact that predictions were substantially improved by changing the values of some transition probabilities. The additional data in the validation sample was used to adjust the estimates of a few transition probabilities. Predictions of total amount of savings made with this modified matrix were greatly improved. The maximum error was found to be half a percent. A good fit between predicted and total amount of savings by itself is not sufficient to indicate that the model has predicted the size distribution of the population correctly. However, as the predicted size distribution of the population of Quarter I-73 has been made to fit the observed distribution and if the structure of the population did not change drastically, over the period of observation, then it is plausible that the true transition matrix is not very different from the modified matrix. It is regrettable that time did not permit the drawing of further samples to validate the model so that a firmer conclusion could be reached.

The fundamental matrix, obtained from the 'best' estimate of the transition matrix, predicted that the maximum total number of accounts in the institution will be 19363, and the maximum total amount of savings contributed by accounts below $100,000 will be $53.74 million, if the conditions existing during the period of the data were to persist.

The average time an account remains opened was predicted to be 27.6 quarters, 6.9 years. The expected length of stay of an account, in the system, appeared to be independent of the amount of savings in the account when it first joined the system except if the amount was

less than $2,000 or more than $20,000. It was concluded that a saver's desire to remain a customer of the institution did not depend on his initial deposit.

A small increase in the expected length of stay of an account, in the system, could have a large effect on the total amount of savings. Thus efforts to keep customers contented and remain longer in the system are important.

2. Model II

The predicted size distributions of the sample were very close to the observed distribution for the first four periods. The maximum chi square statistic was 6.7 which is less than the thirtieth percentile of the chi square distribution with ten degrees of freedom. The chi square statistic for the fifth quarter, Quarter II-73 shot up to 25.02. An investigation showed that the model failed because five of the predictors of transition probability were used beyond the data base on which they were derived thus giving erroneous predictions for Quarter II-73. It was therefore concluded that Model II could predict accurately provided the predictors are not required to predict beyond the data base on which they were derived.

The maximum percentage of error in predicting the total amount of savings was about ten. The predictions were very close to the predictions made by Model I.

Model II fared no better than Model I in the prediction of population behavior and for the same reasons as stated earlier.

3. Discussion

Both models performed credibly in predicting the behavior of the sample of 622 accounts. This is encouraging as it leads one to conclude that a population of savers does possess the Markovian property.

Failure of the models to predict the behavior of the entire population correctly was attributed to errors in the estimation of parameters. This explanation is plausible, as modifications to the transition matrix, using additional data from the validation sample, yielded predictions of total amount of savings that were accurate to half a percent. As it is difficult to conceive, how a random sample could exhibit the Markovian behavior, with the population not possessing that characteristic, one is further led to believe in the above explanation.

If external conditions do not have much influence on the behavior of the population of savers then Model I, because of its simplicity, is the ideal model to use. Model I could still be used if the rate of change of the population behavior is slow. Transition probabilities could be estimated each quarter and exponential smoothing used to adjust the past estimates with this additional information. However, this model does not allow the use of additional information regarding the operating environment to improve the predictions.

Model II has not been given an opportunity to demonstrate its capability because of the limited data base. It has the advantage of improvement with additional knowledge of the operating environment. However, its main limitation is in the requirement of predictions of

values of exogenous variables to predict future values of the parameters of the model. Thus, predictions of Model II are only as good as predictions of exogenous variables. The success of the model, therefore, depends to a great extent on the judgement of the forecaster.

4. <u>Areas for Further Research</u>

The Markovian property of a population is an important population characteristic. The results observed in the application of the models to the sample should be verified using a larger number of accounts, preferably the entire population. A computerized bookkeeping system should be able to take on the additional task of counting the number of transitions between classes without much additional effort.

The variability of predictions in total amount of savings could be reduced if the movement of large accounts could be predicted. Accounts with a balance exceeding $100,000 could be the subject of another study.

The present study did not deal with the interaction between various types of accounts in a savings association. Movement of accounts between different types of accounts has an impact on the total amount of savings in the institution. This area merits further research especially if management desires to know the future level of savings of the whole institution.

The variance of the predictions for more than one period is difficult to derive as the elements of the transition matrix are sums of

products of normal random variables, when the sample size is large. An alternate approach would be to use the Monte Carlo method to obtain an estimate of the variance.

The specification of the econometric models used in predicting the transition probabilities, arrival rate and distribution of new accounts does not imply that the true relationships between parameters of the model and exogenous variables are as specified. This study has merely scratched the surface of the problem of identifying casual relationships between the parameters of the model and external factors. Further research in this area is necessary before reliable predictors can be developed for the parameters.

C. RECOMMENDATIONS

Model I can be turned into an operational tool with little effort. It is recommended that the parameters of the model be updated each quarter to reflect slight changes that may have taken place. If possible, the entire population be used to estimate the parameters.

Model II can be made operational only after further research has been conducted to determine the predictors of the parameters of the model.

APPENDIX A

DERIVATION OF THE VARIANCES OF NUMBER OF ACCOUNTS AND AMOUNT OF SAVINGS IN THE POPULATION FOR SINGLE STEP TRANSITION

(1) EXPECTATION, VARIANCE AND COVARIANCE OF RANDOM SUMS

Let N be an integer random variable

M be an integer random variable

X_i be i.i.d.

Y_j be i.i.d.

$$X = \sum_{i=1}^{N} X_i$$

$$Y = \sum_{j=1}^{M} Y_j$$

$$E(XY) = E(\sum_{i=1}^{N} X_i \sum_{j=1}^{M} Y_j)$$

$$= E(\sum_{i=1}^{N} \sum_{j=1}^{M} X_i, Y_j)$$

$$= E(MN)E(X_i Y_j)$$

$$Cov(X,Y) = E(XY) + E(X)E(Y)$$

$$= E(MN)E(X_i Y_j) + E(N)E(X_i)E(M)E(Y_j)$$

If X_i and Y_j are uncorrelated then

$$Cov(X,Y) = E(MN)E(X_i)E(Y_j) + E(N)E(M)E(X_i)E(Y_j)$$

$$= E(X_i)E(Y_j)(E(MN) + E(M)E(N))$$

$$= E(X_i)E(Y_j)Cov(M,N)$$

$$Var(X) = E^2(X_i)Var(N) + E(N)Var(X_i)$$

Note: $E(X) = E(N)E(X_i)$ can be derived as follows:

$$E(X) = \sum_{n=0}^{\infty} E(X|N=n)P(N=n)$$

$$= \sum_{n=0}^{\infty} nE(X)P(N=n)$$

$$= E(N)E(X)$$

(2) EXPECTATION AND VARIANCE OF NUMBER OF ACCOUNTS

Let n_i = number of accounts in the ith class at beginning of time period a.

p_{ij} = transition probability between classes i and j.

$i = 2, 3, \ldots m, \; j = 1, 2, \ldots m$

x_{ij} = number of transitions between classes i and j during period a.

n_j^{a+1} = number of accounts in the jth class at beginning of time period a+1.

N^{a+1} = total number of accounts in the system at beginning of time period a+1.

The assumption that accounts moving out of a class are distributed in accordance with a multinomial distribution with parameters (p_{i0} p_{i1} p_{i2} ... p_{im}) is implicit in the Markov chain model. If it can be further assumed that accounts moving out of different classes are independent then the following expressions could be obtained.

$$n_j^{a+1} = \sum_{i=2}^{m} x_{ij}$$

$$E(n_j^{a+1}) = \sum_{i=2}^{m} E(x_{ij})$$

$$= \sum_{i=2}^{m} n_i p_{ij}$$

$$Var(n_j^{a+1}) = \sum_{i=2}^{m} Var(x_{ij})$$

$\because$ $Cov(x_{ij}, x_{kj}) = 0$ by assumption of independence between accounts exiting from different classes

$$= \sum_{i=2}^{m} n_i p_{ij}(1 - p_{ij})$$

$$N^{a+1} = \sum_{j=2}^{m} n_j^{a+1}$$

$$E(N^{a+1}) = \sum_{j=2}^{m} E(n_j^{a+1})$$

$$= \sum_{j=2}^{m} \sum_{i=2}^{m} n_i p_{ij}$$

$$Var(N^{a+1}) = \sum_{j=2}^{m} Var(n_j^{a+1}) + 2 \sum_{\substack{j=2 \\ j \neq k}}^{m-1} \sum_{k=3}^{m} Cov(n_j^{a+1}, n_k^{a+1})$$

$$Cov(n_j^{a+1}, n_k^{a+1}) = Cov\left(\sum_{i=2}^{m} x_{ij}, \sum_{l=2}^{m} x_{lk} \right)$$

$$= E\left(\sum_{i=2}^{m} x_{ij} \sum_{l=2}^{m} x_{lk} \right) - E\left(\sum_{i=2}^{m} x_{ij} \right) E\left(\sum_{l=2}^{m} x_{lk} \right)$$

$$= \sum_{i=2}^{m} \sum_{l=2}^{m} E(x_{ij}x_{lk}) - \sum_{i=2}^{m} \sum_{l=2}^{m} E(x_{ij})E(x_{lk})$$

$$= \sum_{i=2}^{m} \sum_{l=2}^{m} E(x_{ij}x_{lk}) - E(x_{ik})E(x_{lk})$$

$$= \sum_{i=2}^{m} \sum_{l=2}^{m} Cov(x_{ij},x_{lk})$$

By assumption $Cov(x_{ij},x_{lk}) = 0$ if $i \neq 1$

$$Cov(n_j^{a+1}, n_j^{a+1}) = \sum_{i=2}^{m} Cov(x_{ij},x_{ik})$$

As x_{ij} and x_{ik} are multinomial random variables from the same distribution

$$Cov(x_{ij},x_{ik}) = - n_i p_{ij} p_{ik}$$

$$\therefore \quad Cov(n_j^{a+1}, n_k^{a+1}) = \sum_{\substack{i=2 \\ j \neq k}}^{m} - n_i p_{ij} p_{ik}$$

(3) <u>EXPECTATION AND VARIANCE OF AMOUNT OF SAVINGS</u>

Let z_{kj} = size of the kth account that has entered the jth class

Z_j^{a+1} = amount of savings in class j at the beginning of period a+1

Z^{a+1} = total amount of savings in the system at the beginning of period a+1

$$Z_j^{a+1} = \sum_{i=2}^{m} \sum_{k=1}^{x_{ij}} z_{kj}$$

$$E(Z_j^{a+1}) = \sum_{i=2}^{m} E\left(\sum_{k=1}^{x_{ij}} z_{kj} \right)$$

$$= \sum_{i=2}^{m} E(x_{ij})E(z_{kj}) \qquad \text{(using results from (1))}$$

$$Var(Z_j^{a+1}) = \sum_{i=2}^{m} Var\left(\sum_{k=1}^{x_{ij}} z_{kj}\right) + 2 \sum_{i=2}^{m-1} \sum_{l=3}^{m} Cov\left(\sum_{k=1}^{x_{ij}} z_{kj}, \sum_{k=1}^{x_{lj}} z_{kj}\right)$$

$$= \sum_{i=2}^{m} E(x_{ij})Var(z_{kj}) + E^2(z_{kj})Var(x_{ij})$$

$$+ 2 \sum_{i=2}^{m} \sum_{l=3}^{m} E^2(z_{kj})Cov(x_{ij}x_{lj})$$

$$= \sum_{i=2}^{m} n_i p_{ij} Var(z_{kj}) + E^2(z_{kj}) n_i p_{ij}(1 - p_{ij})$$

The covariance terms drop out as $Cov(x_{ij}, x_{lj}) = 0$ if $i \neq l$

$$Cov(Z_j^{a+1}, Z_l^{a+1}) = Cov\left(\sum_{i=2}^{m} \sum_{k=1}^{x_{ij}} z_{kj}, \sum_{n=2}^{m} \sum_{k=1}^{x_{nl}} z_{kl}\right)$$

$$= \sum_{i=2}^{m} \sum_{n=2}^{m} Cov\left(\sum_{k=1}^{x_{ij}} z_{kj}, \sum_{n=1}^{x_{nl}} z_{nl}\right)$$

$$= \sum_{i=2}^{m} E(z_{kj})E(z_{nl})Cov(x_{ij}, x_{il})$$

$$= \sum_{i=2}^{m} E(z_{kj})E(z_{nl})E(z_{nl})(-n_i p_{ij} p_{il})$$

$$Var(Z^{a+1}) = \sum_{j=2}^{m} Var(Z_j^{a+1}) + 2 \sum_{j=2}^{m-1} \sum_{l=3}^{m} Cov(Z_j^{a+1}, Z_l^{a+1})$$

APPENDIX B

TRANSITION FREQUENCY MATRIX BETWEEN QUARTER 2 AND QUARTER 3

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	SUM
I	0	0	0	0	0	0	0	0	0	0	0	0
II	7	157	14	3	1	0	0	0	0	0	0	182
III	4	11	64	9	0	2	0	0	1	0	1	92
IV	1	3	6	44	3	2	0	0	0	0	1	60
V	0	1	5	6	28	8	1	1	0	0	0	50
VI	0	0	1	0	0	16	6	0	0	0	1	24
VII	5	1	1	1	3	2	57	3	2	1	0	76
VIII	1	0	0	0	0	1	1	26	6	3	0	38
IX	0	0	0	2	0	0	0	4	21	2	0	29
X	2	1	1	0	0	2	0	0	3	16	3	28
XI	1	1	0	0	0	0	0	0	0	2	22	26
SUM	21	175	92	65	35	33	65	34	33	24	28	605

TRANSITION FREQUENCY MATRIX BETWEEN QUARTER 3 AND QUARTER 4

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	SUM
I	0	0	0	0	0	0	0	0	0	0	0	0
II	10	152	12	0	0	1	C	0	0	0	0	175
III	2	13	67	8	2	0	0	0	0	0	0	92
IV	2	3	3	49	5	2	0	0	0	0	1	65
V	2	2	3	1	25	2	0	0	0	0	0	35
VI	1	1	1	0	2	19	5	0	2	0	2	33
VII	1	1	1	1	0	2	52	6	1	0	0	65
VIII	0	0	2	0	0	0	2	22	5	1	2	34
IX	1	0	0	1	2	2	2	1	18	5	1	33
X	1	1	0	1	0	0	0	0	1	18	2	24
XI	0	1	0	1	0	0	0	0	0	0	26	28
SUM	20	174	89	62	36	28	61	29	27	24	34	584

TRANSITION FREQUENCY MATRIX BETWEEN QUARTER 4 AND QUARTER 5

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	SUM
I	0	0	0	0	0	0	0	0	0	0	0	C
II	9	147	15	1	0	0	C	1	0	0	1	174
III	2	14	64	9	0	0	C	0	0	0	0	89
IV	5	4	6	40	6	0	1	0	0	0	0	62
V	2	0	0	3	26	5	0	0	0	0	0	36
VI	2	2	3	0	3	13	5	0	0	0	0	28
VII	4	0	0	1	1	1	46	7	1	0	0	61
VIII	0	0	1	0	0	1	5	19	3	0	0	29
IX	1	0	1	0	0	0	3	2	15	4	1	27
X	1	0	0	0	0	0	0	1	0	18	4	24
XI	0	1	0	1	1	0	0	0	0	1	29	33
SUM	26	168	90	55	37	20	60	30	19	23	35	563

TRANSITION FREQUENCY MATRIX BETWEEN QUARTER 5 AND QUARTER 6

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	SUM
I	0	0	0	0	0	0	0	0	0	0	0	0
II	6	151	7	1	1	1	0	0	0	0	1	168
III	2	12	65	10	1	0	0	0	0	0	0	90
IV	3	2	3	36	4	4	0	1	0	0	2	55
V	1	0	1	2	28	4	1	0	0	0	0	37
VI	0	2	0	1	1	11	5	0	0	0	0	20
VII	6	1	1	0	0	1	45	3	1	1	1	60
VIII	0	1	1	0	0	0	5	17	5	1	0	30
IX	0	0	0	0	0	1	0	0	15	3	0	19
X	0	0	2	1	2	0	0	0	1	14	3	23
XI	0	0	0	0	0	0	1	0	0	1	33	35
SUM	18	169	80	51	37	22	57	21	22	20	40	537

TRANSITION FREQUENCY MATRIX BETWEEN QUARTER 6 AND QUARTER 7

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	SUM
I	0	0	0	0	0	0	C	0	0	0	0	0
II	8	147	11	1	1	1	C	0	0	0	0	169
III	1	8	62	7	2	0	C	0	0	0	0	80
IV	1	0	3	39	7	0	0	1	0	0	0	51
V	1	1	3	1	24	4	1	0	2	0	0	37
VI	1	1	0	0	3	13	4	0	0	0	0	22
VII	2	1	0	1	0	1	44	7	0	0	1	57
VIII	1	1	0	1	0	0	1	15	2	0	0	21
IX	0	0	1	0	1	0	C	1	16	3	0	22
X	1	1	0	0	0	1	C	1	0	16	0	20
XI	1	0	0	0	0	0	0	2	1	2	34	40
SUM	17	160	80	50	38	20	50	27	21	21	35	519

TRANSITION FREQUENCY MATRIX BETWEEN QUARTER 7 AND QUARTER 8

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	SUM
I	0	0	0	0	0	0	0	0	0	0	0	0
II	6	143	7	1	1	0	1	1	0	0	0	160
III	3	4	59	12	1	0	0	1	0	0	0	80
IV	1	2	5	37	3	1	0	1	0	0	0	50
V	1	2	2	5	23	4	0	0	0	1	0	38
VI	1	1	1	2	1	10	3	0	0	0	1	20
VII	0	0	0	0	0	1	41	5	1	0	2	50
VIII	0	1	2	0	0	0	3	12	8	1	0	27
IX	0	0	0	0	1	1	0	2	16	1	0	21
X	0	0	0	0	0	0	0	0	0	20	1	21
XI	1	0	0	0	0	0	1	1	0	3	29	35
SUM	13	153	76	57	30	17	49	23	25	26	33	502

TRANSITION FREQUENCY MATRIX BETWEEN QUARTER 8 AND QUARTER 9

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	SUM
I	0	0	0	0	0	0	C	0	0	0	0	0
II	4	141	7	0	1	0	0	0	0	0	0	153
III	1	4	59	8	2	1	0	0	0	0	1	76
IV	2	0	5	43	5	1	0	1	0	0	0	57
V	0	0	5	1	18	5	1	0	0	0	0	30
VI	0	0	0	1	1	9	5	1	0	0	0	17
VII	2	2	0	0	3	2	36	3	1	0	0	49
VIII	0	0	0	0	0	2	0	15	5	0	1	23
IX	2	0	0	0	0	1	1	3	15	3	0	25
X	1	1	2	0	0	0	1	0	0	18	3	26
XI	1	0	0	0	0	0	0	0	0	4	28	33
SUM	13	148	78	53	30	21	44	23	21	25	33	489

TRANSITION FREQUENCY MATRIX BETWEEN QUARTER 9 AND QUARTER 10

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	SUM
I	0	0	0	0	0	0	0	0	0	0	0	0
II	10	130	5	1	0	0	1	0	0	0	1	148
III	2	7	59	6	1	1	1	1	0	0	0	78
IV	4	2	4	34	7	1	0	0	0	0	1	53
V	1	2	0	0	22	1	3	0	0	0	1	30
VI	3	0	0	3	0	8	3	2	1	0	1	21
VII	2	0	1	0	0	2	33	5	0	1	0	44
VIII	1	0	0	0	0	1	2	15	3	0	1	23
IX	1	0	0	1	0	0	1	2	11	4	1	21
X	0	0	0	0	0	0	1	0	2	17	5	25
XI	1	0	0	0	1	1	1	0	0	0	29	33
SUM	25	141	69	45	31	15	46	25	17	22	40	476

APPENDIX C

ESTIMATE OF TRANSITION MATRIX BETWEEN QUARTER 2 AND QUARTER 3

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI
I	1.0000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
II	0.0385	0.8626	0.0769	0.0165	0.0055	0.0	0.0	0.0	0.0	0.0	0.0
III	0.0435	0.1196	0.6957	0.0978	0.0	0.0217	0.0	0.0	0.0109	0.0	0.0109
IV	0.0167	0.0500	0.1000	0.7333	0.0500	0.0333	0.0	0.0	0.0	0.0	0.0167
V	0.0	0.0200	0.1000	0.1200	0.5600	0.1600	0.0200	0.0200	0.0	0.0	0.0
VI	0.0	0.0	0.0417	0.0	0.0	0.6667	0.2500	0.0	0.0	0.0	0.0417
VII	0.0658	0.0132	0.0132	0.0132	0.0395	0.0263	0.7500	0.0395	0.0263	0.0132	0.0
VIII	0.0263	0.0	0.0	0.0	0.0	0.0263	0.0263	0.6842	0.1579	0.0789	0.0
IX	0.0	0.0	0.0	0.0690	0.0	0.0	0.0	0.1379	0.7241	0.0690	0.0
X	0.0714	0.0357	0.0357	0.0	0.0	0.0714	0.0	0.0	0.1071	0.5714	0.1071
XI	0.0385	0.0385	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0769	0.8462

ESTIMATE OF TRANSITION MATRIX BETWEEN QUARTER 3 AND QUARTER 4

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI
I	1.0000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
II	0.0571	0.8686	0.0686	0.0	0.0	0.0C57	0.0	0.0	0.0	0.0	0.0
III	0.C217	0.1413	0.7283	0.0870	0.0217	0.0	0.0	0.0	0.0	0.0	0.0
IV	0.0308	0.0462	0.0462	0.7538	0.0769	0.0308	0.0	0.0	0.0	0.0	0.0154
V	0.0571	0.0571	0.0857	0.0286	0.7143	0.0571	0.0	0.0	0.0	0.0	0.0
VI	0.0303	0.0303	0.0303	0.0	0.0606	0.5758	0.1515	0.0	0.0606	0.0	C.0606
VII	0.0154	0.0154	0.0154	0.0154	0.0	0.C308	0.8000	0.0923	0.0154	0.0	0.0
VIII	0.0	0.0	0.0588	0.0	0.0	0.0	0.0588	0.6471	C.1471	0.0294	0.0588
IX	0.0303	0.0	0.0	0.0303	0.0606	0.0606	0.0606	0.0303	0.5455	0.1515	0.0303
X	0.0417	0.0417	0.0	0.0417	0.0	0.0	0.0	0.0	0.0417	0.7500	0.0833
XI	0.0	0.0357	0.0	0.0357	0.0	0.0	0.0	0.0	0.0	0.0	0.9286

ESTIMATE OF TRANSITION MATRIX BETWEEN QUARTER 4 AND QUARTER 5

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI
I	1.0000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
II	0.0517	0.8448	0.0862	0.0057	0.0	0.0	0.0	0.0057	0.0	0.0	0.0057
III	0.0225	0.1573	0.7191	0.1011	0.0	0.C	0.0	0.0	0.0	0.0	0.0
IV	0.0806	0.0645	0.0968	0.6452	0.0968	0.0	0.0161	0.0	0.0	0.0	0.0
V	0.0556	0.0	0.0	0.0833	0.7222	0.1389	0.0	0.0	0.0	0.0	0.0
VI	0.C714	0.0714	0.1071	0.0	0.1071	0.4643	0.1786	0.0	0.0	0.0	0.0
VII	0.0656	0.0	0.0	0.0164	0.0164	0.0164	0.7541	0.1148	0.0164	0.0	0.0
VIII	0.C	0.0	0.0345	0.0	0.0	0.0345	0.1724	0.6552	0.1034	0.0	0.0
IX	0.0370	0.0	0.0370	0.0	0.0	0.0	0.1111	0.0741	0.5556	C.1481	0.0370
X	0.0417	0.0	0.0	0.0	0.0	0.0	0.0	0.0417	0.0	0.7500	0.1667
XI	0.0	0.0303	0.0	0.0303	0.0303	0.C	0.0	0.0	0.0	0.0303	0.8788

ESTIMATE OF TRANSITION MATRIX BETWEEN QUARTER 5 AND QUARTER 6

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI
I	1.0000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
II	0.0357	0.8988	0.0417	0.0060	0.0060	0.0060	0.0	0.0	0.0	0.0	0.0060
III	0.0222	0.1333	0.7222	0.1111	0.0111	0.0	0.0	0.0	0.0	0.0	0.0
IV	0.0545	0.0364	0.0545	0.6545	0.0727	0.0727	0.0	0.0182	0.0	0.0	0.0364
V	0.0270	0.0	0.0270	0.0541	0.7568	0.1081	0.0270	0.0	0.0	0.0	0.0
VI	0.0	0.1000	0.0	0.0500	0.0500	0.5500	0.2500	0.0	0.0	0.0	0.0
VII	0.1000	0.0167	0.0167	0.0	0.0	0.0167	0.7500	0.0500	0.0167	0.0167	0.0167
VIII	0.0	0.0333	0.0333	0.0	0.0	0.0	0.1667	0.5667	0.1667	0.0333	0.0
IX	0.0	0.0	0.0	0.0	0.0	0.0526	0.0	0.0	0.7895	0.1579	0.0
X	0.0	0.0	0.0870	0.0435	0.0870	0.0	0.0	0.0	0.0435	0.6087	0.1304
XI	0.0	0.0	0.0	0.0	0.0	0.0	0.0286	0.0	0.0	0.0286	0.9429

ESTIMATE OF TRANSITION MATRIX BETWEEN QUARTER 6 AND QUARTER 7

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI
I	1.0000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
II	0.0473	0.8698	0.0651	0.0059	0.0059	0.0059	0.0	0.0	0.0	0.0	0.0
III	0.0125	0.1000	0.7750	0.0875	0.0250	0.0	0.0	0.0	0.0	0.0	0.0
IV	0.0196	0.0	0.0588	0.7647	0.1373	0.0	0.0	0.0196	0.0	0.0	0.0
V	0.0270	0.0270	0.0811	0.0270	0.6486	0.1081	0.0270	0.0	0.0541	0.0	0.0
VI	0.0455	0.0455	0.0	0.0	0.1364	0.5909	0.1818	0.0	0.0	0.0	0.0
VII	0.0351	0.0175	0.0	0.0175	0.0	0.0175	0.7719	0.1228	0.0	0.0	0.0175
VIII	0.0476	0.0476	0.0	0.0476	0.0	0.0	0.0476	0.7143	0.0952	0.0	0.0
IX	0.0	0.0	0.0455	0.0	0.0455	0.0	0.0	0.0455	0.7273	0.1364	0.0
X	0.0500	0.0500	0.0	0.0	0.0	0.0500	0.0	0.0500	0.0	0.8000	0.0
XI	0.0250	0.0	0.0	0.0	0.0	0.0	0.0	0.0500	0.0250	0.0500	0.8500

ESTIMATE OF TRANSITION MATRIX BETWEEN QUARTER 7 AND QUARTER 8

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI
I	1.0000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
II	0.0375	0.8937	0.0437	0.0062	0.0062	0.0	0.0062	0.0062	0.0	0.0	0.0
III	0.0375	0.0500	0.7375	0.1500	0.0125	0.0	0.0	0.0125	0.0	0.0	0.0
IV	0.0200	0.0400	0.1000	0.7400	0.0600	0.0200	0.0	0.0200	0.0	0.0	0.0
V	0.0263	0.0526	0.0526	0.1316	0.6053	0.1053	0.0	0.0	0.0	0.0263	0.0
VI	0.0500	0.0500	0.0500	0.1000	0.0500	0.5000	0.1500	0.0	0.0	0.0	0.0500
VII	0.0	0.0	0.0	0.0	0.0	0.0200	0.8200	0.1000	0.0200	0.0	0.0400
VIII	0.0	0.0370	0.0741	0.0	0.0	0.0	0.1111	0.4444	0.2963	0.0370	0.0
IX	0.0	0.0	0.0	0.0	0.0476	0.0476	0.0	0.0952	0.7619	0.0476	0.0
X	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.9524	0.0476
XI	0.0286	0.0	0.0	0.0	0.0	0.0	0.0286	0.0286	0.0	0.0857	0.8286

ESTIMATE OF TRANSITION MATRIX BETWEEN QUARTER 8 AND QUARTER 9

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI
I	1.0000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
II	0.0261	0.9216	0.0458	0.0	0.0065	0.0	0.0	0.0	0.0	0.0	0.0
III	0.0132	0.0526	0.7763	0.1053	0.0263	0.0132	0.0	0.0	0.0	0.0	0.0132
IV	0.0351	0.0	0.0877	0.7544	0.0877	0.0175	0.0	0.0175	0.0	0.0	0.0
V	0.0	0.0	0.1667	0.0333	0.6000	0.1667	0.0333	0.0	0.0	0.0	0.0
VI	0.0	0.0	0.0	0.0588	0.0588	0.5294	0.2941	0.0588	0.0	0.0	0.0
VII	0.0408	0.0408	0.0	0.0	0.0612	0.0408	0.7347	0.0612	0.0204	0.0	0.0
VIII	0.0	0.0	0.0	0.0	0.0	0.0870	0.0	0.6522	0.2174	0.0	0.0435
IX	0.0800	0.0	0.0	0.0	0.0	0.0400	0.0400	0.1200	0.6000	0.1200	0.0
X	0.0385	0.0385	0.0769	0.0	0.0	0.0	0.0385	0.0	0.0	0.6923	0.1154
XI	0.0303	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.1212	0.8485

ESTIMATE OF TRANSITION MATRIX BETWEEN QUARTER 9 AND QUARTER 10

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI
I	1.0000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
II	0.0676	0.8784	0.0338	0.0068	0.0	0.0	0.0068	0.0	0.0	0.0	0.0068
III	0.0256	0.0897	0.7564	0.0769	0.0128	0.0128	0.0128	0.0128	0.0	0.0	0.0
IV	0.0755	0.0377	0.0755	0.6415	0.1321	0.0189	0.0	0.0	0.0	0.0	0.0189
V	0.0333	0.0667	0.0	0.0	0.7333	0.0333	0.1000	0.0	0.0	0.0	0.0333
VI	0.1429	0.0	0.0	0.1429	0.0	0.3810	0.1429	0.0952	0.0476	0.0	0.0476
VII	0.0455	0.0	0.0227	0.0	0.0	0.0455	0.7500	0.1136	0.0	0.0227	0.0
VIII	0.0435	0.0	0.0	0.0	0.0	0.0435	0.0870	0.6522	0.1304	0.0	0.0435
IX	0.0476	0.0	0.0	0.0476	0.0	0.0	0.0476	0.0952	0.5238	0.1905	0.0476
X	0.0	0.0	0.0	0.0	0.0	0.0	0.0400	0.0	0.0800	0.6800	0.2000
XI	0.0303	0.0	0.0	0.0	0.0303	0.0303	0.0303	0.0	0.0	0.0	0.8788

APPENDIX D

CUMULATIVE TRANSITION FREQUENCY MATRIX OF QUARTER 1 TO QUARTER 2

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	SUM
I	0	0	0	0	0	0	0	0	0	0	0	0
II	8	152	3	0	1	1	0	0	0	0	1	166
III	1	14	79	13	3	0	0	2	0	0	0	112
IV	1	4	3	37	5	0	0	1	0	0	1	52
V	1	0	2	4	33	2	3	0	0	0	0	45
VI	0	2	1	2	4	14	3	0	0	0	0	26
VII	2	5	3	1	1	5	64	7	0	0	0	88
VIII	2	2	1	0	1	0	3	22	8	3	0	42
IX	1	3	0	2	1	1	1	4	18	0	0	31
X	0	0	0	0	0	0	1	2	2	24	6	35
XI	1	0	0	1	1	1	1	0	1	1	18	25
SUM	17	182	92	60	50	24	76	38	29	28	26	622

CUMULATIVE TRANSITION FREQUENCY MATRIX OF QUARTER 1 TO QUARTER 3

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	SUM
I	0	0	0	0	0	0	0	0	0	0	0	0
II	15	309	17	3	2	1	0	0	0	0	1	348
III	5	25	143	22	3	2	0	2	1	0	1	204
IV	2	7	9	81	8	2	0	1	0	0	2	112
V	1	1	7	10	61	10	4	1	0	0	0	95
VI	0	2	2	2	4	30	9	0	0	0	1	50
VII	7	6	4	2	4	7	121	10	2	1	0	164
VIII	3	2	1	0	1	1	4	48	14	6	0	80
IX	1	3	0	4	1	1	1	8	39	2	0	60
X	2	1	1	0	0	2	1	2	5	40	9	63
XI	2	1	0	1	1	1	1	0	1	3	40	51
SUM	38	357	184	125	85	57	141	72	62	52	54	1227

CUMULATIVE TRANSITION FREQUENCY MATRIX OF QUARTER 1 TO QUARTER 4

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	SUM
I	0	0	0	0	0	0	0	0	0	0	0	0
II	25	461	29	3	2	2	0	0	0	0	1	523
III	7	38	210	30	5	2	0	2	1	0	1	296
IV	4	10	12	130	13	4	0	1	0	0	3	177
V	3	3	10	11	86	12	4	1	0	0	0	130
VI	1	3	3	2	6	49	14	0	2	0	3	83
VII	8	7	5	3	4	9	173	16	3	1	0	229
VIII	3	2	3	0	1	1	6	70	19	7	2	114
IX	2	3	0	5	3	3	3	9	57	7	1	93
X	3	2	1	1	0	2	1	2	6	58	11	87
XI	2	2	0	2	1	1	1	0	1	3	66	79
SUM	58	531	273	187	121	85	202	101	89	76	88	1811

CUMULATIVE TRANSITION FREQUENCY MATRIX OF QUARTER 1 TO QUARTER 5

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	SUM
I	0	0	0	0	0	0	0	0	0	0	0	0
II	34	608	44	4	2	2	0	1	0	0	2	697
III	9	52	274	39	5	2	0	2	1	0	1	385
IV	9	14	18	170	19	4	1	1	0	0	3	239
V	5	3	10	14	112	17	4	1	0	0	0	166
VI	3	5	6	2	9	62	19	0	2	0	3	111
VII	12	7	5	4	5	10	219	23	4	1	0	290
VIII	3	2	4	0	1	2	11	89	22	7	2	143
IX	3	3	1	5	3	3	6	11	72	11	2	120
X	4	2	1	1	0	2	1	3	6	76	15	111
XI	2	3	0	3	2	1	1	0	1	4	95	112
SUM	84	699	363	242	158	105	262	131	108	99	123	2374

CUMULATIVE TRANSITION FREQUENCY MATRIX OF QUARTER 1 TO QUARTER 6

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	SUM
I	0	0	0	0	0	0	0	0	0	0	0	0
II	40	759	51	5	3	3	0	1	0	0	3	865
III	11	64	339	49	6	2	0	2	1	0	1	475
IV	12	16	21	206	23	8	1	2	0	0	5	294
V	6	3	11	16	140	21	5	1	0	0	0	203
VI	3	7	6	3	10	73	24	0	2	0	3	131
VII	18	8	6	4	5	11	264	26	5	2	1	350
VIII	3	3	5	0	1	2	16	106	27	8	2	173
IX	3	3	1	5	3	4	6	11	87	14	2	139
X	4	2	3	2	2	2	1	3	7	90	18	134
XI	2	3	0	3	2	1	2	0	1	5	128	147
SUM	102	868	443	293	195	127	319	152	130	119	163	2911

CUMULATIVE TRANSITION FREQUENCY MATRIX OF QUARTER 1 TO QUARTER 7

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	SUM
I	0	0	0	0	0	0	0	0	0	0	0	C
II	48	906	62	6	4	4	C	1	0	0	3	1034
III	12	72	401	56	8	2	C	2	1	0	1	555
IV	13	16	24	245	30	8	1	3	0	0	5	345
V	7	4	14	17	164	25	6	1	2	0	0	240
VI	4	8	6	3	13	86	28	0	2	0	3	153
VII	20	9	6	5	5	12	308	33	5	2	2	407
VIII	4	4	5	1	1	2	17	121	29	8	2	194
IX	3	3	2	5	4	4	6	12	103	17	2	161
X	5	3	3	2	2	3	1	4	7	106	18	154
XI	3	3	0	3	2	1	2	2	2	7	162	187
SUM	119	1028	523	343	233	147	369	179	151	140	198	3430

CUMULATIVE TRANSITION FREQUENCY MATRIX CF QUARTER 1 TO QUARTER 8

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	SUM
I	0	0	0	0	0	0	0	0	0	0	0	0
II	54	1049	69	7	5	4	1	2	0	0	3	1194
III	15	76	460	68	9	2	0	3	1	0	1	635
IV	14	18	29	282	33	9	1	4	0	0	5	395
V	8	6	16	22	187	29	6	1	2	1	0	278
VI	5	9	7	5	14	96	31	0	2	0	4	173
VII	20	9	6	5	5	13	349	38	6	2	4	457
VIII	4	5	7	1	1	2	20	133	37	9	2	221
IX	3	3	2	5	5	5	6	14	119	18	2	182
X	5	3	3	2	2	3	1	4	7	126	19	175
XI	4	3	0	3	2	1	3	3	2	10	191	222
SUM	132	1181	599	400	263	164	418	202	176	166	231	3932

CUMULATIVE TRANSITION FREQUENCY MATRIX OF QUARTER 1 TC QUARTER 9

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	SUM
I	0	0	0	0	0	0	C	0	0	0	0	0
II	58	1190	76	7	6	4	1	2	0	0	3	1347
III	16	80	519	76	11	3	C	3	1	0	2	711
IV	16	18	34	325	38	10	1	5	0	0	5	452
V	8	6	21	23	205	34	7	1	2	1	0	308
VI	5	9	7	6	15	105	36	1	2	0	4	19C
VII	22	11	6	5	8	15	385	41	7	2	4	5C6
VIII	4	5	7	1	1	4	20	148	42	9	3	244
IX	5	3	2	5	5	6	7	17	134	21	2	207
X	6	4	5	2	2	3	2	4	7	144	22	201
XI	5	3	0	3	2	1	3	3	2	14	219	255
SUM	145	1329	677	453	293	185	462	225	197	191	264	4421

CUMULATIVE TRANSITION FREQUENCY MATRIX OF QUARTER 1 TO QUARTER 10

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	SUM
I	0	0	0	0	0	0	0	0	0	0	0	0
II	68	1320	81	8	6	4	2	2	0	0	4	1495
III	18	87	578	82	12	4	1	4	1	0	2	789
IV	20	20	38	359	45	11	1	5	0	0	6	505
V	9	8	21	23	227	35	10	1	2	1	1	338
VI	8	9	7	9	15	113	39	3	3	0	5	211
VII	24	11	7	5	8	17	418	46	7	3	4	550
VIII	5	5	7	1	1	5	22	163	45	9	4	267
IX	6	3	2	6	5	6	8	19	145	25	3	228
X	6	4	5	2	2	3	3	4	9	161	27	226
XI	6	3	0	3	3	2	4	3	2	14	248	288
SUM	170	1470	746	498	324	200	508	250	214	213	304	4897

APPENDIX E

CUMULATIVE TRANSITION MATRIX OF QUARTERS 1 TO QUARTER 2

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI
I	1.0C00	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
II	0.0482	0.9157	0.0181	0.0	0.0060	0.0060	0.0	0.0	0.0	0.0	0.0060
III	0.0089	0.1250	0.7054	0.1161	0.0268	0.0	0.0	0.0179	0.0	0.0	0.0
IV	0.0192	0.0769	0.0577	0.7115	0.0962	0.C	0.0	0.0192	0.0	0.0	0.0192
V	0.0222	0.0	0.0444	0.C889	0.7333	0.C444	0.0667	0.0	0.0	0.0	0.0
VI	0.0	0.0769	0.0385	0.C769	0.1538	0.5385	0.1154	0.0	0.0	0.0	0.0
VII	0.C227	0.0568	0.0341	0.0114	0.0114	0.0568	0.7273	0.0795	0.0	0.0	0.0
VIII	0.0476	0.0476	0.0238	0.0	0.0238	0.0	0.0714	0.5238	0.1905	0.C714	C.0
IX	0.0323	0.0968	0.0	0.0645	0.0323	0.0323	0.0323	0.1290	0.5806	0.0	0.0
X	0.0	0.0	0.0	0.0	0.0	0.0	0.0286	0.0571	0.0571	0.6857	0.1714
XI	0.0400	0.0	0.0	0.0400	0.0400	0.0400	0.04CC	0.0	0.0400	0.0400	0.7200

CUMULATIVE TRANSITION MATRIX OF QUARTERS 1 TO QUARTER 3

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI
I	1.0000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
II	0.0431	0.8879	0.0489	0.0086	0.0057	0.0029	0.0	0.0	0.0	0.0	0.0029
III	0.0245	0.1225	0.7010	0.1078	0.0147	0.0098	0.0	0.0098	0.0049	0.0	0.0049
IV	0.0179	0.0625	0.0804	0.7232	0.0714	0.0179	0.0	0.0089	0.0	0.0	0.0179
V	0.0105	0.0105	0.0737	0.1053	0.6421	0.1053	0.0421	0.0105	0.0	0.0	0.0
VI	0.0	0.0400	0.0400	0.0400	0.0800	0.6000	0.1800	0.0	0.0	0.0	0.0200
VII	0.0427	0.0366	0.0244	0.0122	0.0244	0.0427	0.7378	0.0610	0.0122	0.0061	0.0
VIII	0.0375	0.0250	0.0125	0.0	0.0125	0.0125	0.0500	0.6000	0.1750	0.0750	0.0
IX	0.0167	0.0500	0.0	0.0667	0.0167	0.0167	0.0167	0.1333	0.6500	0.0333	0.0
X	0.0317	0.0159	0.0159	0.0	0.0	0.0317	0.0159	0.0317	0.0794	0.6349	0.1429
XI	0.0392	0.0196	0.0	0.0196	0.0196	0.0196	0.0196	0.0	0.0196	0.0588	0.7843

CUMULATIVE TRANSITION MATRIX OF QUARTERS 1 TO QUARTER 4

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI
I	1.0000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
II	0.0478	0.8815	0.0554	0.0057	0.0038	0.0038	0.0	0.0	0.0	0.0	0.0019
III	0.0236	0.1284	0.7095	0.1014	0.0169	0.0068	0.0	0.0068	0.0034	0.0	0.0034
IV	0.0226	0.0565	0.0678	0.7345	0.0734	0.0226	0.0	0.0056	0.0	0.0	0.0169
V	0.0231	0.0231	0.0769	0.0846	0.6615	0.0923	0.0308	0.0077	0.0	0.0	0.0
VI	0.0120	0.0361	0.0361	0.0241	0.0723	0.5904	0.1687	0.0	0.0241	0.0	0.0361
VII	0.0349	0.0306	0.0218	0.0131	0.0175	0.0393	0.7555	0.0699	0.0131	0.0044	0.0
VIII	0.0263	0.0175	0.0263	0.0	0.0088	0.0088	0.0526	0.6140	0.1667	0.0614	0.0175
IX	0.0215	0.0323	0.0	0.0538	0.0323	0.0323	0.0323	0.0968	0.6129	0.0753	0.0108
X	0.0345	0.0230	0.0115	0.0115	0.0	0.0230	0.0115	0.0230	0.0690	0.6667	0.1264
XI	0.0253	0.0253	0.0	0.0253	0.0127	0.0127	0.0127	0.0	0.0127	0.0380	0.8354

CUMULATIVE TRANSITION MATRIX OF QUARTERS 1 TO QUARTER 5

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI
I	1.0000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
II	0.0488	0.8723	0.0631	0.0057	0.0029	0.0029	0.0	0.0014	0.0	0.0	0.0029
III	0.0234	0.1351	0.7117	0.1013	0.0130	0.0052	0.0	0.0052	0.0026	0.0	0.0026
IV	0.0377	0.0586	0.0753	0.7113	0.0795	0.0167	0.0042	0.0042	0.0	0.0	0.0126
V	0.0301	0.0181	0.0602	0.0843	0.6747	0.1024	0.0241	0.0060	0.0	0.0	0.0
VI	0.0270	0.0450	0.0541	0.0180	0.0811	0.5586	0.1712	0.0	0.0180	0.0	0.0270
VII	0.0414	0.0241	0.0172	0.0138	0.0172	0.0345	0.7552	0.0793	0.0138	0.0034	0.0
VIII	0.0210	0.0140	0.0280	0.0	0.0070	0.0140	0.0769	0.6224	0.1538	0.0490	0.0140
IX	0.0250	0.0250	0.0083	0.0417	0.0250	0.0250	0.0500	0.0917	0.6000	0.0517	0.0167
X	0.0360	0.0180	0.0090	0.0090	0.0	0.0180	0.0090	0.0270	0.0541	0.6847	0.1351
XI	0.0179	0.0268	0.0	0.0268	0.0179	0.0089	0.0089	0.0	0.0089	0.0357	0.8482

CUMULATIVE TRANSITION MATRIX OF QUARTERS 1 TC QUARTER 6

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI
I	1.0000	0.0	0.0	0.0	0.0	0.C	0.0	0.0	0.0	0.0	0.0
II	0.0462	0.8775	0.0590	0.0058	0.0035	0.CC35	0.0	0.0012	0.0	0.0	0.0035
III	0.0232	0.1347	0.7137	0.1032	0.0126	0.0042	0.0	0.0042	0.0021	0.0	0.0021
IV	0.0408	0.0544	0.0714	0.7007	0.0782	0.0272	0.0034	0.0068	0.0	0.0	0.0170
V	0.0296	0.0148	0.0542	0.0788	0.6897	0.1034	0.0246	0.0049	0.0	0.0	0.0
VI	0.0229	0.0534	0.0458	0.0229	0.0763	0.5573	0.1832	0.0	0.0153	0.0	0.0229
VII	0.0514	0.0229	0.0171	0.0114	0.0143	0.0314	0.7543	0.0743	0.0143	0.0C57	0.0029
VIII	0.0173	0.0173	0.0289	0.0	0.0058	0.0116	0.0925	0.6127	0.1561	0.0462	0.0116
IX	0.0216	0.0216	0.0072	0.0360	0.0216	0.0288	0.0432	0.0791	0.6259	0.1007	0.0144
X	0.0259	0.0149	0.0224	0.0149	0.0149	0.0149	0.0075	0.0224	0.0522	0.6716	0.1343
XI	0.0136	0.0204	0.0	0.0204	0.0136	0.0068	0.0136	0.0	0.0068	0.0340	0.8707

CUMULATIVE TRANSITION MATRIX OF QUARTERS 1 TO QUARTER 7

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI
I	1.0000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
II	0.0464	0.8762	0.0600	0.0058	0.0039	0.0039	0.0	0.0010	0.0	0.0	0.0029
III	0.0216	0.1297	0.7225	0.1009	0.0144	0.0036	0.0	0.0036	0.0018	0.0	0.0018
IV	0.0377	0.0464	0.0696	0.7101	0.0870	0.0232	0.0029	0.0087	0.0	0.0	0.0145
V	0.0292	0.0167	0.0583	0.0708	0.6833	0.1042	0.0250	0.0042	0.0083	0.0	0.0
VI	0.0261	0.0523	0.0392	0.0196	0.0850	0.5621	0.1830	0.0	0.0131	0.0	0.0196
VII	0.0491	0.0221	0.0147	0.0123	0.0123	0.0295	0.7568	0.0811	0.0123	0.0049	0.0049
VIII	0.0206	0.0206	0.0258	0.0052	0.0052	0.0103	0.0876	0.6237	0.1495	0.0412	0.0103
IX	0.0186	0.0186	0.0124	0.0311	0.0248	0.0248	0.0373	0.0745	0.6398	0.1056	0.0124
X	0.0325	0.0195	0.0195	0.0130	0.0130	0.0195	0.0065	0.0260	0.0455	0.6883	0.1169
XI	0.0160	0.0160	0.0	0.0160	0.0107	0.0053	0.0107	0.0107	0.0107	0.0374	0.8663

CUMULATIVE TRANSITION MATRIX OF QUARTERS 1 TO QUARTER 8

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI
I	1.0000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
II	0.0452	0.8786	0.0578	0.0059	0.0042	0.0034	0.0008	0.0017	0.0	0.0	0.0025
III	0.0236	0.1197	0.7244	0.1071	0.0142	0.0031	0.0	0.0047	0.0016	0.0	0.0016
IV	0.0354	0.0456	0.0734	0.7139	0.0835	0.0228	0.0025	0.0101	0.0	0.0	0.0127
V	0.0288	0.0216	0.0576	0.0791	0.6727	0.1043	0.0216	0.0036	0.0072	0.0036	0.0
VI	0.0289	0.0520	0.0405	0.0289	0.0809	0.5549	0.1792	0.0	0.0116	0.0	0.0231
VII	0.0438	0.0197	0.0131	0.0109	0.0109	0.0284	0.7637	0.0832	0.0131	0.0044	0.0088
VIII	0.0181	0.0226	0.0317	0.0045	0.0045	0.0090	0.0905	0.6018	0.1674	0.0407	0.0090
IX	0.0165	0.0165	0.0110	0.0275	0.0275	0.0275	0.0330	0.0769	0.6538	0.0989	0.0110
X	0.0286	0.0171	0.0171	0.0114	0.0114	0.0171	0.0057	0.0229	0.0400	0.7200	0.1086
XI	0.0180	0.0135	0.0	0.0135	0.0090	0.0045	0.0135	0.0135	0.0090	0.0450	0.8604

CUMULATIVE TRANSITION MATRIX OF QUARTERS 1 TO QUARTER 9

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI
I	1.0000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
II	0.0431	0.8834	0.0564	0.0052	0.0045	0.0030	0.0007	0.0015	0.0	0.0	0.0022
III	0.0225	0.1125	0.7300	0.1069	0.0155	0.0042	0.0	0.0042	0.0014	0.0	0.0028
IV	0.0354	0.0398	0.0752	0.7190	0.0841	0.0221	0.0022	0.0111	0.0	0.0	0.0111
V	0.0260	0.0195	0.0682	0.0747	0.6656	0.1104	0.0227	0.0032	0.0065	0.0032	0.0
VI	0.0263	0.0474	0.0368	0.0316	0.0789	0.5526	0.1895	0.0053	0.0105	0.0	0.0211
VII	0.0435	0.0217	0.0119	0.0099	0.0158	0.0296	0.7609	0.0810	0.0138	0.0040	0.0079
VIII	0.0164	0.0205	0.0287	0.0041	0.0041	0.0164	0.0820	0.6066	0.1721	0.0369	0.0123
IX	0.0242	0.0145	0.0097	0.0242	0.0242	0.0290	0.0338	0.0821	0.6473	0.1014	0.0097
X	0.0299	0.0195	0.0249	0.0100	0.0100	0.0149	0.0100	0.0199	0.0348	0.7164	0.1095
XI	0.0196	0.0118	0.0	0.0118	0.0078	0.0039	0.0118	0.0118	0.0078	0.0549	0.8588

CUMULATIVE TRANSITION MATRIX OF QUARTERS 1 TO QUARTER 10

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI
I	1.0000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
II	0.0455	0.8829	0.0542	0.0054	0.0040	0.0027	0.0013	0.0013	0.0	0.0	0.0027
III	0.0228	0.1103	0.7326	0.1039	0.0152	0.0051	0.0013	0.0051	0.0013	0.0	0.0025
IV	0.0396	0.0396	0.0752	0.7109	0.0891	0.0218	0.0020	0.0099	0.0	0.0	0.0119
V	0.0266	0.0237	0.0621	0.0680	0.6716	0.1036	0.0296	0.0030	0.0059	0.0030	0.0030
VI	0.0379	0.0427	0.0332	0.0427	0.0711	0.5355	0.1848	0.0142	0.0142	0.0	0.0237
VII	0.0436	0.0200	0.0127	0.0091	0.0145	0.0309	0.7600	0.0836	0.0127	0.0055	0.0073
VIII	0.0187	0.0187	0.0262	0.0037	0.0037	0.0187	0.0824	0.6105	0.1685	0.0337	0.0150
IX	0.0263	0.0132	0.0088	0.0263	0.0219	0.0263	0.0351	0.0833	0.6360	0.1096	0.0132
X	0.0265	0.0177	0.0221	0.0088	0.0088	0.0133	0.0133	0.0177	0.0398	0.7124	0.1195
XI	0.0208	0.0104	0.0	0.0104	0.0104	0.0069	0.0139	0.0104	0.0069	0.0486	0.8611

MOD I

	I	II	III	IV	V	VI	V99	VIII	IX	X	XI
I	1.0000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
II	0.0461	0.8874	0.0536	0.0047	0.0040	0.0014	0.0007	0.0007	0.0	0.0	0.0014
III	0.0229	0.1094	0.7354	0.1031	0.0153	0.0051	0.0013	0.0051	0.0013	0.0	0.0013
IV	0.0403	0.0403	0.0766	0.7137	0.0887	0.0222	0.0020	0.0081	0.0	0.0	0.0081
V	0.0269	0.0240	0.0629	0.0689	0.6707	0.1018	0.0299	0.0030	0.0060	0.0030	0.0030
VI	0.0389	0.0389	0.0339	0.0437	0.0728	0.5485	0.1845	0.0146	0.0146	0.0	0.0097
VII	0.0440	0.0165	0.0128	0.0073	0.0147	0.0312	0.7670	0.0807	0.0128	0.0055	0.0073
VIII	0.0195	0.0156	0.0234	0.0039	0.0039	0.0195	0.0859	0.6094	0.1680	0.0352	0.0156
IX	0.0267	0.0044	0.0089	0.0267	0.0222	0.0267	0.0356	0.0844	0.6444	0.1067	0.0133
X	0.0269	0.0179	0.0224	0.0090	0.0090	0.0090	0.0135	0.0179	0.0404	0.7175	0.1166
XI	0.0214	0.0071	0.0	0.0071	0.0107	0.0071	0.0143	0.0071	0.0071	0.0500	0.8679

MOD II

	I	II	III	IV	V	VI	V99	VIII	IX	X	XI
I	1.0000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
II	0.0461	0.9018	0.0464	0.0026	0.0021	0.0004	0.0001	0.0001	0.0	0.0	0.0004
III	0.0229	0.1094	0.7630	0.0894	0.0106	0.0022	0.0001	0.0022	0.0001	0.0	0.0001
IV	0.0403	0.0403	0.0766	0.7249	0.0887	0.0222	0.00C2	0.0034	0.0	0.0	0.0034
V	0.0269	0.0240	0.0629	0.0689	0.6831	0.1018	0.0299	0.0003	0.0016	0.0C03	0.0003
VI	0.0389	0.0389	0.0339	0.0437	0.0728	0.5485	0.1845	0.C146	0.0146	0.0	0.0097
VII	0.0440	0.0165	0.0128	0.0073	0.0147	0.0312	0.767C	0.0807	0.0128	0.0055	0.0073
VIII	0.0195	0.0156	0.0234	0.0039	0.0039	0.C195	0.0859	0.6094	0.1680	0.0352	0.0156
IX	0.0267	0.0044	0.0089	0.0267	0.0222	0.0267	0.0356	0.0844	0.6444	0.1067	0.0133
X	0.C269	0.0179	0.0224	0.009C	0.0090	0.0090	0.0135	0.0179	0.0404	0.7175	0.1166
XI	0.0214	0.0071	0.0	0.0071	0.0107	0.0071	0.0143	0.0C71	0.0071	0.0500	0.8679

MOD III

	I	II	III	IV	V	VI	V99	VIII	IX	X	XI
I	1.0000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
II	0.0461	0.9302	0.0164	0.0026	0.0021	0.0014	0.0	0.0007	0.0	0.0	0.0
III	0.0229	0.1094	0.8072	0.0394	0.0106	0.0051	0.0001	0.0051	0.0001	0.0	0.0001
IV	0.0403	0.0403	0.0766	0.7249	0.0887	0.0222	0.0002	0.0034	0.0	0.0	0.0034
V	0.0269	0.0240	0.0629	0.0689	0.6831	0.1018	0.0255	0.0003	0.0016	0.0003	0.0003
VI	0.0389	0.0389	0.0339	0.0437	0.0728	0.6485	0.0845	0.0146	0.0146	0.0	0.0097
VII	0.0440	0.0165	0.0128	0.0073	0.0147	0.0312	0.7670	0.0807	0.0128	0.0055	0.0073
VIII	0.0195	0.0156	0.0234	0.0039	0.0039	0.0195	0.0859	0.6094	0.1680	0.0352	0.0156
IX	0.0267	0.0044	0.0089	0.0267	0.0222	0.0267	0.0356	0.0844	0.6444	0.1067	0.0133
X	0.0269	0.0179	0.0224	0.0090	0.0090	0.0090	0.0135	0.0179	0.0404	0.7175	0.1166
XI	0.0214	0.0071	0.0	0.0071	0.0107	0.0071	0.0143	0.0071	0.0071	0.0500	0.8679

APPENDIX F

PREDICTED TRANSITION MATRIX BETWEEN QUARTER 5 AND QUARTER 6

(MODEL II USING CPM V)

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI
I	1.0000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
II	0.0467	0.8742	0.0632	0.0057	0.0029	0.0029	0.0	0.0014	0.0	0.0	0.0029
III	0.0238	0.1187	0.7251	0.1032	0.0132	0.0053	0.0	0.0053	0.0026	0.0	0.0026
IV	0.0386	0.0600	0.0771	0.7044	0.0814	0.0171	0.0043	0.0043	0.0	0.0	0.0129
V	0.0291	0.0175	0.0582	0.0815	0.6924	0.0523	0.0233	0.0058	0.0	0.0	0.0
VI	0.0270	0.0450	0.0541	0.0180	0.0811	0.5586	0.1712	0.0	0.0180	0.0	0.0270
VII	0.0412	0.0263	0.0171	0.0137	0.0171	0.0343	0.7541	0.0789	0.0139	0.0034	0.0
VIII	0.0210	0.0140	0.0280	0.0	0.0070	0.0140	0.0769	0.6223	0.1538	0.0490	0.0140
IX	0.0250	0.0250	0.0083	0.0417	0.0250	0.0250	0.0500	0.0917	0.5999	0.0917	0.0167
X	0.0361	0.0181	0.0090	0.0090	0.0	0.0181	0.0090	0.0271	0.0543	0.6868	0.1325
XI	0.0179	0.0268	0.0	0.0268	0.0179	0.0089	0.0089	0.0	0.0089	0.0368	0.8472

PREDICTED TRANSITION MATRIX BETWEEN QUARTER 6 AND QUARTER 7

(MODEL II USING CPM V)

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI
I	1.0000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
II	0.0440	0.8767	0.0634	0.0057	0.0029	0.0029	0.0	0.0014	0.0	0.0	0.0029
III	0.0245	0.0951	0.7445	0.1060	0.0136	0.0054	0.0	0.0054	0.0027	0.0	0.0027
IV	0.0373	0.0580	0.0745	0.7144	0.0786	0.0165	0.0042	0.0042	0.0	0.0	0.0125
V	0.0306	0.0184	0.0612	0.0856	0.6846	0.0890	0.0245	0.0061	0.0	0.0	0.0
VI	0.0270	0.0450	0.0541	0.0180	0.0811	0.5586	0.1712	0.0	0.0180	0.0	0.0270
VII	0.0402	0.0217	0.0167	0.0134	0.0167	0.0335	0.7627	0.0769	0.0149	0.0033	0.0
VIII	0.0210	0.0140	0.0280	0.0	0.0070	0.0140	0.0769	0.6223	0.1538	0.0490	0.0140
IX	0.0250	0.0250	0.0083	0.0417	0.0250	0.0250	0.0500	0.0917	0.5999	0.0917	0.0167
X	0.0376	0.0188	0.0094	0.0094	0.0	0.0188	0.0094	0.0282	0.0565	0.7153	0.0965
XI	0.0176	0.0263	0.0	0.0263	0.0176	0.0087	0.0087	0.0	0.0087	0.0521	0.8338

PREDICTED TRANSITION MATRIX BETWEEN QUARTER 7 AND QUARTER 8

(MODEL II USING CPM V)

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI
I	1.0000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
II	0.0422	0.8784	0.0635	0.0057	0.0029	0.0029	0.0	0.0014	0.0	0.0	0.0029
III	0.0251	0.0740	0.7619	0.1084	0.0139	0.0056	0.0	0.0056	0.0028	0.0	0.0028
IV	0.0361	0.0561	0.0721	0.7234	0.0761	0.0160	0.0040	0.0040	0.0	0.0	0.0121
V	0.0314	0.0189	0.0628	0.0879	0.6610	0.1066	0.0251	0.0063	0.0	0.0	0.0
VI	0.0270	0.0450	0.0541	0.0180	0.0811	0.5586	0.1712	0.0	0.0180	0.0	0.0270
VII	0.0393	0.0140	0.0163	0.0131	0.0163	0.0328	0.7712	0.0753	0.0184	0.0032	0.0
VIII	0.0210	0.0140	0.0280	0.0	0.0070	0.0140	0.0769	0.6223	0.1538	0.0490	0.0140
IX	0.0250	0.0250	0.0083	0.0417	0.0250	0.0250	0.0500	0.0917	0.5999	0.0917	0.0167
X	0.0391	0.0195	0.0098	0.0098	0.0	0.0195	0.0098	0.0293	0.0587	0.7429	0.0617
XI	0.0173	0.0259	0.0	0.0259	0.0173	0.0086	0.0086	0.0	0.0086	0.0699	0.8182

PREDICTED TRANSITION MATRIX BETWEEN QUARTER 8 AND QUARTER 9

(MODEL II USING CPM V)

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI
I	1.0000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
II	0.0264	0.8928	0.0646	0.0058	0.0030	0.0030	0.0	0.0014	0.0	0.0	0.0030
III	0.0256	0.0521	0.7799	0.1110	0.0142	0.0057	0.0	0.0057	0.0028	0.0	0.0028
IV	0.0364	0.0567	0.0728	0.7208	0.0769	0.0161	0.0041	0.0041	0.0	0.0	0.0122
V	0.0308	0.0185	0.0616	0.0863	0.5676	0.2044	0.0247	0.0061	0.0	0.0	0.0
VI	0.0270	0.0450	0.0541	0.0180	0.0811	0.5586	0.1712	0.0	0.0180	0.0	0.0270
VII	0.0414	0.0404	0.0172	0.0138	0.0172	0.0345	0.7314	0.0792	0.0217	0.0034	0.0
VIII	0.0210	0.0140	0.0280	0.0	0.0070	0.0140	0.0769	0.6223	0.1538	0.0490	0.0140
IX	0.0250	0.0250	0.0083	0.0417	0.0250	0.0250	0.0500	0.0917	0.5999	0.0917	0.0167
X	0.0366	0.0183	0.0092	0.0092	0.0	0.0183	0.0092	0.0275	0.0551	0.6969	0.1198
XI	0.0164	0.0246	0.0	0.0246	0.0164	0.0082	0.0082	0.0	0.0082	0.1163	0.7773

PREDICTED TRANSITION MATRIX BETWEEN QUARTER 9 AND QUARTER 10

(MODEL II USING CPM V)

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI
I	1.0000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
II	0.0228	0.8962	0.0648	0.0059	0.0030	0.0030	0.0	0.0014	0.0	0.0	0.0030
III	0.0264	0.0254	0.8019	0.1141	0.0146	0.0059	0.0	0.0059	0.0029	0.0	0.0029
IV	0.0345	0.0536	0.0688	0.7360	0.0727	0.0153	0.0038	0.0038	0.0	0.0	0.0115
V	0.0327	0.0197	0.0654	0.0916	0.5154	0.2425	0.0262	0.0065	0.0	0.0	0.0
VI	0.0270	0.0450	0.0541	0.0180	0.0811	0.5586	0.1712	0.0	0.0180	0.0	0.0270
VII	0.0413	0.0494	0.0172	0.0138	0.0172	0.0344	0.7204	0.0792	0.0238	0.0034	0.0
VIII	0.0210	0.0140	0.0280	0.0	0.0070	0.0140	0.0769	0.6223	0.1538	0.0490	0.0140
IX	0.0250	0.0250	0.0083	0.0417	0.0250	0.0250	0.0500	0.0917	0.5999	0.0917	0.0167
X	0.0388	0.0194	0.0097	0.0097	0.0	0.0194	0.0097	0.0291	0.0584	0.7387	0.0670
XI	0.0152	0.0228	0.0	0.0228	0.0152	0.0076	0.0076	0.0	0.0076	0.1791	0.7220

PREDICTED TRANSITION MATRIX BETWEEN QUARTER 5 AND QUARTER 6

(MODEL II USING CPM X)

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI
I	1.0000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
II	0.0466	0.8819	0.0541	0.0054	0.0040	0.0027	0.0013	0.0013	0.0	0.0	0.0027
III	0.0227	0.1158	0.7280	0.1032	0.0151	0.0051	0.0013	0.0051	0.0013	0.0	0.0025
IV	0.0405	0.0405	0.0769	0.7042	0.0912	0.0223	0.0020	0.0101	0.0	0.0	0.0122
V	0.0257	0.0229	0.0599	0.0656	0.6910	0.0921	0.0286	0.0029	0.0057	0.0029	0.0029
VI	0.0379	0.0427	0.0332	0.0427	0.0711	0.5355	0.1848	0.0142	0.0142	0.0	0.0237
VII	0.0434	0.0262	0.0126	0.0091	0.0144	0.0307	0.7538	0.0831	0.0139	0.0055	0.0073
VIII	0.0187	0.0187	0.0262	0.0037	0.0037	0.0187	0.0824	0.6106	0.1685	0.0337	0.0150
IX	0.0263	0.0132	0.0088	0.0263	0.0219	0.0263	0.0351	0.0833	0.6360	0.1096	0.0132
X	0.0262	0.0175	0.0218	0.0087	0.0087	0.0131	0.0131	0.0175	0.0393	0.7036	0.1305
XI	0.0211	0.0105	0.0	0.0105	0.0105	0.0070	0.0141	0.0105	0.0070	0.0373	0.8715

PREDICTED TRANSITION MATRIX BETWEEN QUARTER 6 AND QUARTER 7

(MODEL II USING CPM X)

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI
I	1.0000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
II	0.0439	0.8844	0.0543	0.0054	0.0040	0.0027	0.0013	0.0013	0.0	0.0	0.0027
III	0.0232	0.0927	0.7470	0.1059	0.0155	0.0052	0.0013	0.0052	0.0013	0.0	0.0025
IV	0.0392	0.0392	0.0744	0.7142	0.0881	0.0216	0.0020	0.0098	0.0	0.0	0.0118
V	0.0270	0.0240	0.0630	0.0689	0.6831	0.0888	0.0300	0.0030	0.0060	0.0030	0.0030
VI	0.0379	0.0427	0.0332	0.0427	0.0711	0.5355	0.1848	0.0142	0.0142	0.0	0.0237
VII	0.0423	0.0217	0.0123	0.0088	0.0141	0.0300	0.7624	0.0811	0.0149	0.0053	0.0071
VIII	0.0187	0.0187	0.0262	0.0037	0.0037	0.0187	0.0824	0.6106	0.1685	0.0337	0.0150
IX	0.0263	0.0132	0.0088	0.0263	0.0219	0.0263	0.0351	0.0833	0.6360	0.1096	0.0132
X	0.0272	0.0182	0.0227	0.0090	0.0090	0.0137	0.0137	0.0182	0.0409	0.7323	0.0950
XI	0.0207	0.0104	0.0	0.0104	0.0104	0.0069	0.0138	0.0104	0.0069	0.0527	0.8575

PREDICTED TRANSITION MATRIX BETWEEN QUARTER 7 AND QUARTER 8

(MODEL II USING CPM X)

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI
I	1.0000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
II	0.0420	0.8861	0.0544	0.0054	0.0040	0.0027	0.0013	0.0013	0.0	0.0	0.0027
III	0.0238	0.0721	0.7640	0.1084	0.0159	0.0053	0.0014	0.0053	0.0014	0.0	0.0026
IV	0.0379	0.0379	0.0720	0.7232	0.0853	0.0209	0.0019	0.0095	0.0	0.0	0.0114
V	0.0277	0.0247	0.0646	0.0708	0.6596	0.1064	0.0308	0.0031	0.0061	0.0031	0.0031
VI	0.0379	0.0427	0.0332	0.0427	0.0711	0.5355	0.1848	0.0142	0.0142	0.0	0.0237
VII	0.0414	0.0140	0.0121	0.0086	0.0138	0.0293	0.7709	0.0793	0.0184	0.0052	0.0069
VIII	0.0187	0.0187	0.0262	0.0037	0.0037	0.0187	0.0824	0.6106	0.1685	0.0337	0.0150
IX	0.0263	0.0132	0.0088	0.0263	0.0219	0.0263	0.0351	0.0833	0.6360	0.1096	0.0132
X	0.0283	0.0189	0.0236	0.0094	0.0094	0.0142	0.0142	0.0189	0.0425	0.7601	0.0607
XI	0.0203	0.0102	0.0	0.0102	0.0102	0.0067	0.0136	0.0102	0.0067	0.0707	0.8412

PREDICTED TRANSITION MATRIX BETWEEN QUARTER 8 AND QUARTER 9

(MODEL II USING CPM X)

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI
I	1.0000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
II	0.0263	0.9006	0.0553	0.0055	0.0041	0.0028	0.0013	0.0013	0.0	0.0	0.0028
III	0.0243	0.0507	0.7816	0.1108	0.0162	0.0054	0.0014	0.0054	0.0014	0.0	0.0027
IV	0.0383	0.0383	0.0727	0.7206	0.0861	0.0211	0.0019	0.0096	0.0	0.0	0.0115
V	0.0272	0.0242	0.0634	0.0694	0.5663	0.2040	0.0302	0.0031	0.0060	0.0031	0.0031
VI	0.0379	0.0427	0.0332	0.0427	0.0711	0.5355	0.1848	0.0142	0.0142	0.0	0.0237
VII	0.0435	0.0403	0.0127	0.0091	0.0145	0.0309	0.7311	0.0835	0.0217	0.0055	0.0073
VIII	0.0187	0.0187	0.0262	0.0037	0.0037	0.0187	0.0824	0.6106	0.1685	0.0337	0.0150
IX	0.0263	0.0132	0.0088	0.0263	0.0219	0.0263	0.0351	0.0833	0.6360	0.1096	0.0132
X	0.0265	0.0177	0.0221	0.0088	0.0088	0.0133	0.0133	0.0177	0.0399	0.7137	0.1180
XI	0.0193	0.0096	0.0	0.0096	0.0096	0.0064	0.0129	0.0096	0.0064	0.1177	0.7987

PREDICTED TRANSITION MATRIX BETWEEN QUARTER 9 AND QUARTER 10

(MODEL II USING CPM X)

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI
I	1.0000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
II	0.0227	0.9040	0.0555	0.0055	0.0041	0.0028	0.0013	0.0013	0.0	0.0	0.0028
III	0.0250	0.0247	0.8030	0.1139	0.0167	0.0056	0.0014	0.0056	0.0014	0.0	0.0027
IV	0.0362	0.0362	0.0687	0.7358	0.0814	0.0199	0.0018	0.0090	0.0	0.0	0.0109
V	0.0288	0.0257	0.0673	0.0737	0.5142	0.2420	0.0321	0.0033	0.0064	0.0033	0.0033
VI	0.0379	0.0427	0.0332	0.0427	0.0711	0.5355	0.1848	0.0142	0.0142	0.0	0.0237
VII	0.0435	0.0494	0.0127	0.0091	0.0145	0.0308	0.7201	0.0834	0.0238	0.0055	0.0073
VIII	0.0187	0.0187	0.0262	0.0037	0.0037	0.0187	0.0824	0.6106	0.1685	0.0337	0.0150
IX	0.0263	0.0132	0.0088	0.0263	0.0219	0.0263	0.0351	0.0833	0.6360	0.1096	0.0132
X	0.0281	0.0188	0.0234	0.0093	0.0093	0.0141	0.0141	0.0188	0.0422	0.7559	0.0659
XI	0.0179	0.0090	0.0	0.0090	0.0090	0.0059	0.0120	0.0090	0.0059	0.1812	0.7413

PREDICTED TRANSITION MATRIX BETWEEN QUARTER 5 AND QUARTER 6

(MODEL II USING MOD III)

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI
I	1.0000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
II	0.0466	0.9302	0.0164	0.0026	0.0021	0.0014	0.0	0.0007	0.0	0.0	0.0
III	0.0227	0.1157	0.8015	0.0391	0.0105	0.0051	0.0001	0.0051	0.0001	0.0	0.0001
IV	0.0418	0.0418	0.0795	0.7144	0.0921	0.0230	0.0002	0.0035	0.0	0.0	0.0035
V	0.0262	0.0234	0.0612	0.0671	0.6976	0.0930	0.0291	0.0003	0.0016	0.0003	0.0003
VI	0.0389	0.0389	0.0339	0.0437	0.0728	0.6484	0.0845	0.0146	0.0146	0.0	0.0097
VII	0.0439	0.0263	0.0128	0.0073	0.0147	0.0311	0.7566	0.0806	0.0139	0.0055	0.0073
VIII	0.0195	0.0156	0.0234	0.0039	0.0039	0.0195	0.0859	0.6095	0.1680	0.0352	0.0156
IX	0.0267	0.0044	0.0089	0.0267	0.0222	0.0267	0.0356	0.0844	0.6444	0.1067	0.0133
X	0.0265	0.0176	0.0221	0.0089	0.0089	0.0089	0.0133	0.0176	0.0398	0.7065	0.1301
XI	0.0217	0.0072	0.0	0.0072	0.0108	0.0072	0.0145	0.0072	0.0072	0.0374	0.8796

PREDICTED TRANSITION MATRIX BETWEEN QUARTER 6 AND QUARTER 7

(MODEL II USING MOD III)

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI
I	1.0000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
II	0.0439	0.9328	0.0164	0.0026	0.0021	0.0014	0.0	0.0007	0.0	0.0	0.0
III	0.0233	0.0926	0.8224	0.0401	0.0108	0.0052	0.0001	0.0052	0.0001	0.0	0.0001
IV	0.0404	0.0404	0.0768	0.7242	0.0889	0.0223	0.0002	0.0034	0.0	0.0	0.0034
V	0.0275	0.0246	0.0644	0.0706	0.6900	0.0897	0.0306	0.0003	0.0016	0.0003	0.0003
VI	0.0389	0.0389	0.0339	0.0437	0.0728	0.6484	0.0845	0.0146	0.0146	0.0	0.0097
VII	0.0428	0.0218	0.0125	0.0071	0.0143	0.0304	0.7652	0.0785	0.0150	0.0054	0.0071
VIII	0.0195	0.0156	0.0234	0.0039	0.0039	0.0195	0.0859	0.6095	0.1680	0.0352	0.0156
IX	0.0267	0.0044	0.0089	0.0267	0.0222	0.0267	0.0356	0.0844	0.6444	0.1067	0.0133
X	0.0276	0.0183	0.0230	0.0092	0.0092	0.0092	0.0138	0.0183	0.0414	0.7352	0.0947
XI	0.0213	0.0071	0.0	0.0071	0.0107	0.0071	0.0143	0.0071	0.0071	0.0528	0.8655

PREDICTED TRANSITION MATRIX BETWEEN QUARTER 7 AND QUARTER 8

(MODEL II USING MOD III)

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI
I	1.0000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
II	0.0421	0.9346	0.0165	0.0026	0.0021	0.0014	0.0	0.0007	0.0	0.0	0.0
III	0.0239	0.0720	0.8411	0.0411	0.0110	0.0053	0.0001	0.0053	0.0001	0.0	0.0001
IV	0.0391	0.0391	0.0743	0.7330	0.0861	0.0215	0.0002	0.0033	0.0	0.0	0.0033
V	0.0283	0.0252	0.0661	0.0724	0.6664	0.1075	0.0314	0.0003	0.0017	0.0003	0.0003
VI	0.0389	0.0389	0.0339	0.0437	0.0728	0.6484	0.0845	0.0146	0.0146	0.0	0.0097
VII	0.0419	0.0141	0.0122	0.0070	0.0140	0.0297	0.7737	0.0769	0.0185	0.0052	0.0070
VIII	0.0195	0.0156	0.0234	0.0039	0.0039	0.0195	0.0859	0.6095	0.1680	0.0352	0.0156
IX	0.0267	0.0044	0.0089	0.0267	0.0222	0.0267	0.0356	0.0844	0.6444	0.1067	0.0133
X	0.0286	0.0190	0.0238	0.0096	0.0096	0.0096	0.0144	0.0190	0.0430	0.7630	0.0605
XI	0.0209	0.0069	0.0	0.0069	0.0105	0.0069	0.0140	0.0069	0.0069	0.0708	0.8490

PREDICTED TRANSITION MATRIX BETWEEN QUARTER 8 AND QUARTER 9

(MODEL II USING MOD III)

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI
I	1.0000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
II	0.0264	0.9500	0.0167	0.0027	0.0021	0.0014	0.0	0.0007	0.0	0.0	0.0
III	0.0244	0.0507	0.8604	0.0420	0.0113	0.0054	0.0001	0.0054	0.0001	0.0	0.0001
IV	0.0395	0.0395	0.0751	0.7305	0.0869	0.0218	0.0002	0.0033	0.0	0.0	0.0033
V	0.0278	0.0248	0.0649	0.0711	0.5721	0.2060	0.0308	0.0003	0.0017	0.0003	0.0003
VI	0.0389	0.0389	0.0339	0.0437	0.0728	0.6484	0.0845	0.0146	0.0146	0.0	0.0097
VII	0.0441	0.0405	0.0128	0.0073	0.0147	0.0313	0.7338	0.0809	0.0218	0.0055	0.0073
VIII	0.0195	0.0156	0.0234	0.0039	0.0039	0.0195	0.0859	0.6095	0.1680	0.0352	0.0156
IX	0.0267	0.0044	0.0089	0.0267	0.0222	0.0267	0.0356	0.0844	0.6444	0.1067	0.0133
X	0.0269	0.0179	0.0224	0.0090	0.0090	0.0090	0.0135	0.0179	0.0404	0.7166	0.1176
XI	0.0199	0.0066	0.0	0.0066	0.0099	0.0066	0.0133	0.0066	0.0066	0.1179	0.8061

PREDICTED TRANSITION MATRIX BETWEEN QUARTER 9 AND QUARTER 10

(MODEL II USING MOD III)

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI
I	1.0000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
II	0.0227	0.9535	0.0168	0.0027	0.0022	0.0014	0.0	0.0007	0.0	0.0	0.0
III	0.0251	0.0247	0.8840	0.0431	0.0116	0.0056	0.0001	0.0056	0.0001	0.0	0.0001
IV	0.0373	0.0373	0.0709	0.7453	0.0821	0.0206	0.0002	0.0031	0.0	0.0	0.0031
V	0.0295	0.0263	0.0689	0.0755	0.5197	0.2446	0.0328	0.0003	0.0018	0.0003	0.0003
VI	0.0389	0.0389	0.0339	0.0437	0.0728	0.6484	0.0845	0.0146	0.0146	0.0	0.0097
VII	0.0441	0.0495	0.0128	0.0073	0.0147	0.0313	0.7228	0.0808	0.0239	0.0055	0.0073
VIII	0.0195	0.0156	0.0234	0.0039	0.0039	0.0195	0.0859	0.6095	0.1680	0.0352	0.0156
IX	0.0267	0.0044	0.0089	0.0267	0.0222	0.0267	0.0356	0.0844	0.6444	0.1067	0.0133
X	0.0284	0.0189	0.0237	0.0095	0.0095	0.0095	0.0143	0.0189	0.0427	0.7588	0.0656
XI	0.0184	0.0061	0.0	0.0061	0.0092	0.0061	0.0123	0.0061	0.0061	0.1814	0.7480

PREDICTED TRANSITION MATRIX BETWEEN QUARTER 5 AND QUARTER 6

(MODEL II′ USING CPM V)

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI
I	1.0000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
II	0.0488	0.8723	0.0631	0.0057	0.0029	0.0029	0.0	0.0014	0.0	0.0	0.0029
III	0.0234	0.1351	0.7116	0.1013	0.0130	0.0052	0.0	0.0052	0.0026	0.0	0.0026
IV	0.0386	0.0600	0.0771	0.7044	0.0814	0.0171	0.0043	0.0043	0.0	0.0	0.0129
V	0.0303	0.0182	0.0606	0.0849	0.6795	0.0962	0.0243	0.0060	0.0	0.0	0.0
VI	0.0270	0.0450	0.0541	0.0180	0.0811	0.5586	0.1712	0.0	0.0180	0.0	0.0270
VII	0.0412	0.0263	0.0171	0.0137	0.0171	0.0343	0.7542	0.0789	0.0137	0.0034	0.0
VIII	0.0210	0.0140	0.0280	0.0	0.0070	0.0140	0.0769	0.6223	0.1538	0.0490	0.0140
IX	0.0250	0.0250	0.0083	0.0417	0.0250	0.0250	0.0500	0.0917	0.5999	0.0917	0.0167
X	0.0361	0.0181	0.0090	0.0090	0.0	0.0181	0.0090	0.0271	0.0543	0.6868	0.1325
XI	0.0179	0.0268	0.0	0.0268	0.0179	0.0089	0.0089	0.0	0.0089	0.0357	0.8482

PREDICTED TRANSITION MATRIX BETWEEN QUARTER 6 AND QUARTER 7

(MODEL II′ USING CPM V)

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI
I	1.0000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
II	0.0488	0.8723	0.0631	0.0057	0.0029	0.0029	0.0	0.0014	0.0	0.0	0.0029
III	0.0234	0.1351	0.7116	0.1013	0.0130	0.0052	0.0	0.0052	0.0026	0.0	0.0026
IV	0.0373	0.0580	0.0745	0.7144	0.0786	0.0165	0.0042	0.0042	0.0	0.0	0.0125
V	0.0306	0.0184	0.0611	0.0856	0.6849	0.0890	0.0245	0.0061	0.0	0.0	0.0
VI	0.0270	0.0450	0.0541	0.0180	0.0811	0.5586	0.1712	0.0	0.0180	0.0	0.0270
VII	0.0402	0.0218	0.0167	0.0134	0.0167	0.0335	0.7639	0.0771	0.0134	0.0033	0.0
VIII	0.0210	0.0140	0.0280	0.0	0.0070	0.0140	0.0769	0.6223	0.1538	0.0490	0.0140
IX	0.0250	0.0250	0.0083	0.0417	0.0250	0.0250	0.0500	0.0917	0.5999	0.0917	0.0167
X	0.0376	0.0188	0.0094	0.0094	0.0	0.0188	0.0094	0.0282	0.0565	0.7153	0.0965
XI	0.0179	0.0268	0.0	0.0268	0.0179	0.0089	0.0089	0.0	0.0089	0.0357	0.8482

PREDICTED TRANSITION MATRIX BETWEEN QUARTER 7 AND QUARTER 8

(MODEL II′ USING CPM V)

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI
I	1.0000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
II	0.0488	0.8723	0.0631	0.0057	0.0029	0.0029	0.0	0.0014	0.0	0.0	0.0029
III	0.0234	0.1351	0.7116	0.1013	0.0130	0.0052	0.0	0.0052	0.0026	0.0	0.0026
IV	0.0361	0.0561	0.0721	0.7234	0.0761	0.0160	0.0040	0.0040	0.0	0.0	0.0121
V	0.0301	0.0181	0.0602	0.0843	0.6749	0.1023	0.0241	0.0060	0.0	0.0	0.0
VI	0.0270	0.0450	0.0541	0.0180	0.0811	0.5586	0.1712	0.0	0.0180	0.0	0.0270
VII	0.0395	0.0141	0.0164	0.0132	0.0164	0.0329	0.7753	0.0757	0.0132	0.0032	0.0
VIII	0.0210	0.0140	0.0280	0.0	0.0070	0.0140	0.0769	0.6223	0.1538	0.0490	0.0140
IX	0.0250	0.0250	0.0083	0.0417	0.0250	0.0250	0.0500	0.0917	0.5999	0.0917	0.0167
X	0.0391	0.0195	0.0098	0.0098	0.0	0.0195	0.0098	0.0293	0.0587	0.7429	0.0617
XI	0.0179	0.0268	0.0	0.0268	0.0179	0.0089	0.0089	0.0	0.0089	0.0357	0.8482

PREDICTED TRANSITION MATRIX BETWEEN QUARTER 8 AND QUARTER 9

(MODEL II′ USING CPM V)

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI
I	1.0000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
II	0.0488	0.8723	0.0631	0.0057	0.0029	0.0029	0.0	0.0014	0.0	0.0	0.0029
III	0.0234	0.1351	0.7116	0.1013	0.0130	0.0052	0.0	0.0052	0.0026	0.0	0.0026
IV	0.0364	0.0567	0.0728	0.7208	0.0769	0.0161	0.0041	0.0041	0.0	0.0	0.0122
V	0.0274	0.0165	0.0549	0.0768	0.6149	0.1820	0.0220	0.0055	0.0	0.0	0.0
VI	0.0270	0.0450	0.0541	0.0180	0.0811	0.5586	0.1712	0.0	0.0180	0.0	0.0270
VII	0.0417	0.0407	0.0173	0.0139	0.0173	0.0347	0.7372	0.0798	0.0139	0.0034	0.0
VIII	0.0210	0.0140	0.0280	0.0	0.0070	0.0140	0.0769	0.6223	0.1538	0.0490	0.0140
IX	0.0250	0.0250	0.0083	0.0417	0.0250	0.0250	0.0500	0.0917	0.5999	0.0917	0.0167
X	0.0366	0.0183	0.0092	0.0092	0.0	0.0183	0.0092	0.0275	0.0551	0.6969	0.1198
XI	0.0179	0.0268	0.0	0.0268	0.0179	0.0089	0.0089	0.0	0.0089	0.0357	0.8482

PREDICTED TRANSITION MATRIX BETWEEN QUARTER 9 AND QUARTER 10

(MODEL II′ USING CPM V)

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI
I	1.0000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
II	0.0488	0.8723	0.0631	0.0057	0.0029	0.0029	0.0	0.0014	0.0	0.0	0.0029
III	0.0234	0.1351	0.7116	0.1013	0.0130	0.0052	0.0	0.0052	0.0026	0.0	0.0026
IV	0.0345	0.0536	0.0688	0.7360	0.0727	0.0153	0.0038	0.0038	0.0	0.0	0.0115
V	0.0269	0.0162	0.0537	0.0752	0.6020	0.1992	0.0215	0.0054	0.0	0.0	0.0
VI	0.0270	0.0450	0.0541	0.0180	0.0811	0.5586	0.1712	0.0	0.0180	0.0	0.0270
VII	0.0418	0.0499	0.0173	0.0139	0.0173	0.0348	0.7277	0.0800	0.0139	0.0034	0.0
VIII	0.0210	0.0140	0.0280	0.0	0.0070	0.0140	0.0769	0.6223	0.1538	0.0490	0.0140
IX	0.0250	0.0250	0.0083	0.0417	0.0250	0.0250	0.0500	0.0917	0.5999	0.0917	0.0167
X	0.0388	0.0194	0.0097	0.0097	0.0	0.0194	0.0097	0.0291	0.0584	0.7387	0.0670
XI	0.0179	0.0268	0.0	0.0268	0.0179	0.0089	0.0089	0.0	0.0089	0.0357	0.8482

PREDICTED TRANSITION MATRIX BETWEEN QUARTER 5 AND QUARTER 6

(MODEL II' USING CPM X)

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI
I	1.0000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
II	0.0455	0.8829	0.0542	0.0054	0.0040	0.0027	0.0013	0.0013	0.0	0.0	0.0027
III	0.0228	0.1103	0.7325	0.1039	0.0152	0.0051	0.0013	0.0051	0.0013	0.0	0.0025
IV	0.0405	0.0405	0.0769	0.7042	0.0912	0.0223	0.0020	0.0101	0.0	0.0	0.0122
V	0.0268	0.0239	0.0626	0.0685	0.6770	0.0963	0.0298	0.0030	0.0059	0.0030	0.0030
VI	0.0379	0.0427	0.0332	0.0427	0.0711	0.5355	0.1848	0.0142	0.0142	0.0	0.0237
VII	0.0434	0.0263	0.0126	0.0091	0.0144	0.0308	0.7548	0.0832	0.0126	0.0055	0.0073
VIII	0.0187	0.0187	0.0262	0.0037	0.0037	0.0187	0.0824	0.6106	0.1685	0.0337	0.0150
IX	0.0263	0.0132	0.0088	0.0263	0.0219	0.0263	0.0351	0.0833	0.6360	0.1096	0.0132
X	0.0262	0.0175	0.0218	0.0087	0.0087	0.0131	0.0131	0.0175	0.0393	0.7036	0.1305
XI	0.0208	0.0104	0.0	0.0104	0.0104	0.0069	0.0139	0.0104	0.0069	0.0486	0.8613

PREDICTED TRANSITION MATRIX BETWEEN QUARTER 6 AND QUARTER 7

(MODEL II' USING CPM X)

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI
I	1.0000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
II	0.0455	0.8829	0.0542	0.0054	0.0040	0.0027	0.0013	0.0013	0.0	0.0	0.0027
III	0.0228	0.1103	0.7325	0.1039	0.0152	0.0051	0.0013	0.0051	0.0013	0.0	0.0025
IV	0.0392	0.0392	0.0744	0.7142	0.0881	0.0216	0.0020	0.0098	0.0	0.0	0.0118
V	0.0270	0.0241	0.0631	0.0691	0.6824	0.0890	0.0301	0.0030	0.0060	0.0030	0.0030
VI	0.0379	0.0427	0.0332	0.0427	0.0711	0.5355	0.1848	0.0142	0.0142	0.0	0.0237
VII	0.0424	0.0218	0.0123	0.0088	0.0141	0.0300	0.7644	0.0813	0.0123	0.0053	0.0071
VIII	0.0187	0.0187	0.0262	0.0037	0.0037	0.0187	0.0824	0.6106	0.1685	0.0337	0.0150
IX	0.0263	0.0132	0.0088	0.0263	0.0219	0.0263	0.0351	0.0833	0.6360	0.1096	0.0132
X	0.0272	0.0182	0.0227	0.0090	0.0090	0.0137	0.0137	0.0182	0.0409	0.7323	0.0950
XI	0.0208	0.0104	0.0	0.0104	0.0104	0.0069	0.0139	0.0104	0.0069	0.0486	0.8613

PREDICTED TRANSITION MATRIX BETWEEN QUARTER 7 AND QUARTER 8

(MODEL II' USING CPM X)

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI
I	1.0000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
II	0.0455	0.8829	0.0542	0.0054	0.0040	0.0027	0.0013	0.0013	0.0	0.0	0.0027
III	0.0228	0.1103	0.7325	0.1039	0.0152	0.0051	0.0013	0.0051	0.0013	0.0	0.0025
IV	0.0379	0.0379	0.0720	0.7232	0.0853	0.0209	0.0019	0.0095	0.0	0.0	0.0114
V	0.0266	0.0237	0.0622	0.0681	0.6724	0.1024	0.0296	0.0030	0.0059	0.0030	0.0030
VI	0.0379	0.0427	0.0332	0.0427	0.0711	0.5355	0.1848	0.0142	0.0142	0.0	0.0237
VII	0.0416	0.0141	0.0121	0.0087	0.0138	0.0295	0.7759	0.0798	0.0121	0.0053	0.0070
VIII	0.0187	0.0187	0.0262	0.0037	0.0037	0.0187	0.0824	0.6106	0.1685	0.0337	0.0150
IX	0.0263	0.0132	0.0088	0.0263	0.0219	0.0263	0.0351	0.0833	0.6360	0.1096	0.0132
X	0.0283	0.0189	0.0236	0.0094	0.0094	0.0142	0.0142	0.0189	0.0425	0.7601	0.0607
XI	0.0208	0.0104	0.0	0.0104	0.0104	0.0069	0.0139	0.0104	0.0069	0.0486	0.8613

PREDICTED TRANSITION MATRIX BETWEEN QUARTER 8 AND QUARTER 9

(MODEL II′ USING CPM X)

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI
I	1.0000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
II	0.0455	0.8829	0.0542	0.0054	0.0040	0.0027	0.0013	0.0013	0.0	0.0	0.0027
III	0.0228	0.1103	0.7325	0.1039	0.0152	0.0051	0.0013	0.0051	0.0013	0.0	0.0025
IV	0.0383	0.0383	0.0727	0.7206	0.0861	0.0211	0.0019	0.0096	0.0	0.0	0.0115
V	0.0243	0.0216	0.0566	0.0620	0.6127	0.1822	0.0270	0.0027	0.0054	0.0027	0.0027
VI	0.0379	0.0427	0.0332	0.0427	0.0711	0.5355	0.1848	0.0142	0.0142	0.0	0.0237
VII	0.0439	0.0407	0.0128	0.0092	0.0146	0.0311	0.7377	0.0842	0.0128	0.0055	0.0074
VIII	0.0187	0.0187	0.0262	0.0037	0.0037	0.0187	0.0824	0.6106	0.1685	0.0337	0.0150
IX	0.0263	0.0132	0.0088	0.0263	0.0219	0.0263	0.0351	0.0833	0.6360	0.1096	0.0132
X	0.0265	0.0177	0.0221	0.0088	0.0088	0.0133	0.0133	0.0177	0.0399	0.7137	0.1180
XI	0.0208	0.0104	0.0	0.0104	0.0104	0.0069	0.0139	0.0104	0.0069	0.0486	0.8613

PREDICTED TRANSITION MATRIX BETWEEN QUARTER 9 AND QUARTER 10

(MODEL II' USING CPM X)

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI
I	1.0000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
II	0.0455	0.8829	0.0542	0.0054	0.0040	0.0027	0.0013	0.0013	0.0	0.0	0.0027
III	0.0228	0.1103	0.7325	0.1039	0.0152	0.0051	0.0013	0.0051	0.0013	0.0	0.0025
IV	0.0362	0.0362	0.0687	0.7358	0.0814	0.0199	0.0018	0.0090	0.0	0.0	0.0109
V	0.0238	0.0212	0.0555	0.0607	0.5998	0.1994	0.0264	0.0027	0.0053	0.0027	0.0027
VI	0.0379	0.0427	0.0332	0.0427	0.0711	0.5355	0.1848	0.0142	0.0142	0.0	0.0237
VII	0.0440	0.0499	0.0128	0.0092	0.0146	0.0312	0.7282	0.0844	0.0128	0.0056	0.0074
VIII	0.0187	0.0187	0.0262	0.0037	0.0037	0.0187	0.0824	0.6106	0.1685	0.0337	0.0150
IX	0.0263	0.0132	0.0088	0.0263	0.0219	0.0263	0.0351	0.0833	0.6360	0.1096	0.0132
X	0.0281	0.0188	0.0234	0.0093	0.0093	0.0141	0.0141	0.0188	0.0422	0.7559	0.0659
XI	0.0208	0.0104	0.0	0.0104	0.0104	0.0069	0.0139	0.0104	0.0069	0.0486	0.8613

PREDICTED TRANSITION MATRIX BETWEEN QUARTER 5 AND QUARTER 6

(MODEL II' USING MOD III)

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI
I	1.0000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
II	0.0461	0.9307	0.0164	0.0026	0.0021	0.0014	0.0	0.0007	0.0	0.0	0.0
III	0.0229	0.1094	0.8072	0.0394	0.0106	0.0051	0.0001	0.0051	0.0001	0.0	0.0001
IV	0.0418	0.0418	0.0795	0.7144	0.0921	0.0230	0.0002	0.0035	0.0	0.0	0.0035
V	0.0271	0.0242	0.0633	0.0693	0.6874	0.0961	0.0301	0.0003	0.0016	0.0003	0.0003
VI	0.0389	0.0389	0.0339	0.0437	0.0728	0.6484	0.0845	0.0146	0.0146	0.0	0.0097
VII	0.0440	0.0264	0.0128	0.0073	0.0147	0.0312	0.7575	0.0806	0.0128	0.0055	0.0073
VIII	0.0195	0.0156	0.0234	0.0039	0.0039	0.0195	0.0859	0.6095	0.1680	0.0352	0.0156
IX	0.0267	0.0044	0.0089	0.0267	0.0222	0.0267	0.0356	0.0844	0.6444	0.1067	0.0133
X	0.0265	0.0176	0.0221	0.0089	0.0089	0.0089	0.0133	0.0176	0.0398	0.7065	0.1301
XI	0.0214	0.0071	0.0	0.0071	0.0107	0.0071	0.0143	0.0071	0.0071	0.0500	0.8681

PREDICTED TRANSITION MATRIX BETWEEN QUARTER 6 AND QUARTER 7

(MODEL II′ USING MOD III)

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI
I	1.0000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
II	0.0461	0.9307	0.0164	0.0026	0.0021	0.0014	0.0	0.0007	0.0	0.0	0.0
III	0.0229	0.1094	0.8072	0.0394	0.0106	0.0051	0.0001	0.0051	0.0001	0.0	0.0001
IV	0.0404	0.0404	0.0768	0.7242	0.0889	0.0223	0.0002	0.0034	0.0	0.0	0.0034
V	0.0273	0.0243	0.0638	0.0699	0.6929	0.0889	0.0303	0.0003	0.0016	0.0003	0.0003
VI	0.0389	0.0389	0.0339	0.0437	0.0728	0.6484	0.0845	0.0146	0.0146	0.0	0.0097
VII	0.0429	0.0219	0.0125	0.0071	0.0143	0.0304	0.7671	0.0787	0.0125	0.0054	0.0071
VIII	0.0195	0.0156	0.0234	0.0039	0.0039	0.0195	0.0859	0.6095	0.1680	0.0352	0.0156
IX	0.0267	0.0044	0.0089	0.0267	0.0222	0.0267	0.0356	0.0844	0.6444	0.1067	0.0133
X	0.0276	0.0183	0.0230	0.0092	0.0092	0.0092	0.0138	0.0183	0.0414	0.7352	0.0947
XI	0.0214	0.0071	0.0	0.0071	0.0107	0.0071	0.0143	0.0071	0.0071	0.0500	0.8681

PREDICTED TRANSITION MATRIX BETWEEN QUARTER 7 AND QUARTER 8

(MODEL II′ USING MOD III)

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI
I	1.0000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
II	0.0461	0.9307	0.0164	0.0026	0.0021	0.0014	0.0	0.0007	0.0	0.0	0.0
III	0.0229	0.1094	0.8072	0.0394	0.0106	0.0051	0.0001	0.0051	0.0001	0.0	0.0001
IV	0.0391	0.0391	0.0743	0.7330	0.0861	0.0215	0.0002	0.0033	0.0	0.0	0.0033
V	0.0269	0.0240	0.0629	0.0689	0.6828	0.1022	0.0299	0.0003	0.0016	0.0003	0.0003
VI	0.0389	0.0389	0.0339	0.0437	0.0728	0.6484	0.0845	0.0146	0.0146	0.0	0.0097
VII	0.0422	0.0142	0.0123	0.0070	0.0141	0.0299	0.7785	0.0773	0.0123	0.0053	0.0070
VIII	0.0195	0.0156	0.0234	0.0039	0.0039	0.0195	0.0859	0.6095	0.1680	0.0352	0.0156
IX	0.0267	0.0044	0.0089	0.0267	0.0222	0.0267	0.0356	0.0844	0.6444	0.1067	0.0133
X	0.0286	0.0190	0.0238	0.0096	0.0096	0.0096	0.0144	0.0190	0.0430	0.7630	0.0605
XI	0.0214	0.0071	0.0	0.0071	0.0107	0.0071	0.0143	0.0071	0.0071	0.0500	0.8681

PREDICTED TRANSITION MATRIX BETWEEN QUARTER 8 AND QUARTER 9

(MODEL II' USING MOD III)

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI
I	1.0000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
II	0.0461	0.9307	0.0164	0.0026	0.0021	0.0014	0.0	0.0007	0.0	0.0	0.0
III	0.0229	0.1094	0.8072	0.0394	0.0106	0.0051	0.0001	0.0051	0.0001	0.0	0.0001
IV	0.0395	0.0395	0.0751	0.7305	0.0869	0.0218	0.0002	0.0033	0.0	0.0	0.0033
V	0.0245	0.0219	0.0573	0.0628	0.6222	0.1819	0.0272	0.0003	0.0015	0.0003	0.0003
VI	0.0389	0.0389	0.0339	0.0437	0.0728	0.6484	0.0845	0.0146	0.0146	0.0	0.0097
VII	0.0445	0.0409	0.0129	0.0074	0.0149	0.0315	0.7404	0.0816	0.0129	0.0056	0.0074
VIII	0.0195	0.0156	0.0234	0.0039	0.0039	0.0195	0.0859	0.6095	0.1680	0.0352	0.0156
IX	0.0267	0.0044	0.0089	0.0267	0.0222	0.0267	0.0356	0.0844	0.6444	0.1067	0.0133
X	0.0269	0.0179	0.0224	0.0090	0.0090	0.0090	0.0135	0.0179	0.0404	0.7166	0.1176
XI	0.0214	0.0071	0.0	0.0071	0.0107	0.0071	0.0143	0.0071	0.0071	0.0500	0.8681

PREDICTED TRANSITION MATRIX BETWEEN QUARTER 9 AND QUARTER 10

(MODEL II' USING MOD III)

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI
I	1.0000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
II	0.0461	0.9307	0.0164	0.0026	0.0021	0.0014	0.0	0.0007	0.0	0.0	0.0
III	0.0229	0.1094	0.8072	0.0394	0.0106	0.0051	0.0001	0.0051	0.0001	0.0	0.0001
IV	0.0373	0.0373	0.0709	0.7453	0.0821	0.0206	0.0002	0.0031	0.0	0.0	0.0031
V	0.0240	0.0214	0.0561	0.0614	0.6091	0.1991	0.0267	0.0003	0.0014	0.0003	0.0003
VI	0.0389	0.0389	0.0339	0.0437	0.0728	0.6484	0.0845	0.0146	0.0146	0.0	0.0097
VII	0.0446	0.0501	0.0130	0.0074	0.0149	0.0316	0.7308	0.0817	0.0130	0.0056	0.0074
VIII	0.0195	0.0156	0.0234	0.0039	0.0039	0.0195	0.0859	0.6095	0.1680	0.0352	0.0156
IX	0.0267	0.0044	0.0089	0.0267	0.0222	0.0267	0.0356	0.0844	0.6444	0.1067	0.0133
X	0.0284	0.0189	0.0237	0.0095	0.0095	0.0095	0.0143	0.0189	0.0427	0.7588	0.0656
XI	0.0214	0.0071	0.0	0.0071	0.0107	0.0071	0.0143	0.0071	0.0071	0.0500	0.8681

PREDICTED NUMBER OF NEW ACCOUNTS IN THE SAMPLE, NUMBER OBSERVED AND CHI SQUARE STATISTICS
(MODEL II)

	QUARTER II-72			QUARTER III-72			QUARTER IV-72			QUARTER I-73			QUARTER II-73		
CLASS	PRED.	ACT.	CHI	PRED.	ACT.	CHI	PRED.	ACT.	CHI	PRED.	ACT.	CHI	PRED.	ACT.	CHI
1999.	195.	184.	0.57	195.	191.	0.10	197.	201.	0.09	196.	197.	0.00	197.	202.	0.11
3999.	17.	17.	0.00	17.	20.	0.53	17.	15.	0.26	17.	14.	0.56	17.	15.	0.27
5999.	12.	11.	0.06	9.	7.	0.65	7.	7.	0.01	11.	14.	0.58	9.	13.	1.94
7999.	5.	9.	4.05	5.	3.	0.60	5.	5.	0.02	5.	2.	1.55	5.	2.	1.57
9999.	3.	2.	0.38	3.	4.	0.26	3.	3.	0.00	3.	4.	0.25	3.	2.	0.41
11999.	4.	3.	0.28	6.	8.	1.05	7.	5.	0.35	3.	3.	0.02	3.	5.	0.76
13999.	2.	6.	6.46	2.	1.	0.68	2.	2.	0.03	2.	4.	1.39	2.	2.	0.03
15999.	4.	5.	0.11	4.	5.	0.22	4.	3.	0.21	3.	3.	0.01	3.	0.	2.62
19999.	1.	1.	0.08	1.	3.	1.83	1.	1.	0.10	2.	2.	0.00	4.	1.	1.79
100000.	7.	12.	3.63	7.	8.	0.14	7.	8.	0.13	7.	7.	0.00	7.	8.	0.12
SUM	250.	250.	15.62	250.	250.	6.05	250.	250.	1.20	250.	250.	4.36	250.	250.	9.64

CHI SQUARE STATISTICS = 36.871

APPENDIX H

TABLE OF PREDICTED NUMBER OF ACCOUNTS, ACTUAL NUMBER OBSERVED AND CHI-SQUARE STATISTICS

(MODEL I USING CPM II)

	QUARTER II-72			QUARTER III-72			QUARTER IV-72			QUARTER I-73			QUARTER II-73		
CLASS	PRED.	ACT.	CHI	PRED.	ACT.	CHI	PRED.	ACT.	CHI	PRED.	ACT.	CHI	PRED.	ACT.	CHI
0.	16.	18.	0.36	31.	35.	0.44	47.	48.	0.03	62.	61.	0.03	77.	86.	0.96
1999.	178.	169.	0.41	184.	160.	3.13	188.	153.	6.56	190.	148.	9.45	191.	141.	13.21
3999.	75.	80.	0.35	64.	80.	3.76	57.	76.	6.27	52.	78.	13.27	48.	69.	9.35
5999.	58.	51.	0.79	58.	50.	1.14	57.	57.	0.00	56.	53.	0.12	54.	45.	1.37
7999.	42.	37.	0.68	46.	38.	1.32	48.	30.	6.53	48.	30.	6.98	48.	31.	6.21
9999.	19.	22.	0.51	18.	20.	0.21	17.	17.	0.01	17.	21.	1.12	16.	15.	0.07
11999.	53.	57.	0.26	48.	50.	0.07	44.	49.	0.54	41.	44.	0.24	38.	46.	1.63
13999.	27.	21.	1.30	24.	27.	0.28	22.	23.	0.03	20.	23.	0.34	19.	25.	2.11
15999.	19.	22.	0.32	19.	21.	0.25	18.	25.	3.05	16.	21.	1.34	15.	17.	0.27
19999.	19.	20.	0.02	16.	21.	1.25	14.	26.	9.87	12.	25.	13.02	11.	22.	11.52
100000.	32.	41.	2.58	28.	36.	1.99	26.	34.	2.80	23.	34.	5.20	21.	41.	19.28
SUM	522.	520.	7.59	507.	503.	13.84	491.	490.	35.67	476.	477.	51.11	461.	452.	65.97

TABLE OF PREDICTED NUMBER OF ACCOUNTS, ACTUAL NUMBER CBSERVED AND CHI-SQUARE STATISTICS

(MODEL I USING CPM X)

	QUARTER II-72			QUARTER III-72			QUARTER IV-72			QUARTER I-73			QUARTER II-73		
CLASS	PRED.	ACT.	CHI	PRED.	ACT.	CHI	PRED.	ACT.	CHI	PRED.	ACT.	CHI	PRED.	ACT.	CHI
0.	19.	18.	0.02	37.	35.	0.08	54.	48.	0.69	71.	61.	1.40	87.	86.	0.02
1999.	165.	169.	0.10	161.	160.	0.01	158.	153.	0.13	154.	148.	0.20	149.	141.	0.46
3999.	84.	80.	0.23	80.	80.	0.00	76.	76.	0.00	72.	78.	0.42	69.	69.	0.00
5999.	54.	51.	0.22	53.	50.	0.21	52.	57.	0.51	50.	53.	0.16	48.	45.	0.25
7999.	35.	37.	0.09	34.	38.	0.53	33.	30.	0.19	31.	30.	0.06	30.	31.	0.02
9999.	20.	22.	0.17	20.	20.	0.00	19.	17.	0.24	18.	21.	0.34	18.	15.	0.44
11999.	55.	57.	0.09	51.	50.	0.01	47.	49.	0.08	44.	44.	0.00	41.	46.	0.50
13999.	27.	21.	1.46	25.	27.	0.12	24.	23.	0.01	22.	23.	0.04	21.	25.	0.83
15999.	20.	22.	0.27	20.	21.	0.11	19.	25.	1.91	18.	21.	0.41	17.	17.	0.01
19999.	22.	20.	0.13	21.	21.	0.00	20.	26.	1.85	19.	25.	1.72	19.	22.	0.62
100000.	37.	41.	0.48	37.	36.	0.04	37.	34.	0.31	37.	34.	0.29	37.	41.	0.42
SUM	519.	520.	3.26	501.	503.	1.12	484.	490.	5.93	467.	477.	5.03	451.	452.	3.57

TABLE OF PREDICTED AMOUNT OF SAVINGS AND ACTUAL AMOUNT OBSERVED

(MODEL I USING CPM II)

	QUARTER II-72		QUARTER III-72		QUARTER IV-72		QUARTER I-73		QUARTER II-73	
CLASS	PRED.	ACT.	PRED.	ACT.	PRED.	ACT.	PRED.	ACT.	PRED.	ACT.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
1999.	85893.	85201.	89029.	76517.	91022.	74296.	92126.	68827.	92535.	69300.
3999.	215003.	239441.	185005.	238997.	163911.	224544.	148708.	231257.	137404.	207472.
5999.	287399.	251076.	289341.	241451.	284674.	274778.	276479.	259631.	266557.	218088.
7999.	293990.	256481.	317489.	262935.	330488.	215352.	335690.	206544.	335250.	215346.
9999.	170197.	201731.	162668.	183567.	156250.	153325.	150307.	189416.	144570.	131926.
11999.	579978.	610157.	524027.	536059.	480156.	520816.	444637.	467021.	414925.	492183.
13999.	351982.	276219.	319124.	354780.	290941.	298484.	266348.	299438.	244788.	325462.
15999.	295289.	327455.	285163.	315938.	267453.	373113.	247194.	315862.	226900.	252917.
19999.	348506.	351882.	296574.	372054.	255249.	468395.	222032.	440700.	195038.	388406.
100000.	924925.	1027230.	825032.	954255.	740132.	906064.	667803.	925321.	606047.	1116465.
SUM	3553161.	3626873.	3293448.	3534753.	3060272.	3509167.	2851319.	3404017.	2664014.	3417565.

TABLE OF PREDICTED AMOUNT OF SAVINGS AND ACTUAL AMOUNT OBSERVED

(MODEL I USING CPM X)

	QUARTER II-72		QUARTER III-72		QUARTER IV-72		QUARTER I-73		QUARTER II-73	
CLASS	PRED.	ACT.	PRED.	ACT.	PRED.	ACT.	PRED.	ACT.	PRED.	ACT.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
1999.	79810.	85201.	78102.	76517.	76236.	74296.	74268.	68827.	72239.	69300.
3999.	242247.	239441.	229022.	238997.	217872.	224544.	208185.	231257.	199550.	207472.
5999.	270993.	251076.	265289.	241451.	257927.	274778.	249745.	259631.	241247.	218088.
7999.	244136.	256481.	234216.	262935.	225549.	215352.	217489.	206544.	209762.	215346.
9999.	181385.	201731.	177996.	183567.	172676.	153325.	166620.	189416.	160379.	131926.
11999.	596467.	610157.	550595.	536059.	512382.	520816.	479787.	467021.	451466.	492183.
13999.	357171.	276219.	330007.	354780.	307852.	298484.	289054.	299438.	272660.	325462.
15999.	298082.	327455.	295843.	315938.	287478.	373113.	276392.	315862.	264362.	252917.
19999.	390213.	351882.	372950.	372054.	358983.	468395.	346629.	440700.	335081.	388406.
100000.	1066267.	1027230.	1078826.	954255.	1083084.	906064.	1080770.	925321.	1073173.	1116465.
SUM	3726769.	3626873.	3612845.	3534753.	3500038.	3509167.	3388938.	3404017.	3279915.	3417565.

APPENDIX I

TABLE OF PREDICTED NUMBER OF ACCOUNTS, ACTUAL NUMBER OBSERVED AND CHI-SQUARE STATISTICS
(MODEL II USING CPM II)

	QUARTER II-72			QUARTER III-72			QUARTER IV-72			QUARTER I-73			QUARTER II-73		
CLASS	PRED.	ACT.	CHI	PRED.	ACT.	CHI	PRED.	ACT.	CHI	PRED.	ACT.	CHI	PRED.	ACT.	CHI
0.	15.	18.	0.45	30.	35.	0.78	44.	48.	0.31	55.	61.	0.59	65.	86.	6.60
1999.	175.	169.	0.23	179.	160.	1.97	179.	153.	3.87	182.	148.	6.37	184.	141.	9.91
3999.	75.	80.	0.28	67.	80.	2.70	61.	76.	3.55	58.	78.	6.60	57.	69.	2.45
5999.	57.	51.	0.70	58.	50.	1.23	59.	57.	0.07	58.	53.	0.49	59.	45.	3.13
7999.	41.	37.	0.39	43.	38.	0.58	43.	30.	3.82	38.	30.	1.86	34.	31.	0.26
9999.	21.	22.	0.09	21.	20.	0.02	21.	17.	0.83	25.	21.	0.76	28.	15.	5.92
11999.	54.	57.	0.12	51.	50.	0.01	48.	49.	0.01	44.	44.	0.00	40.	46.	0.78
13999.	27.	21.	1.31	25.	27.	0.23	23.	23.	0.00	22.	23.	0.09	20.	25.	1.00
15999.	20.	22.	0.13	20.	21.	0.03	20.	25.	1.51	19.	21.	0.30	18.	17.	0.03
19999.	20.	20.	0.00	18.	21.	0.38	18.	26.	3.74	17.	25.	3.66	18.	22.	0.85
100000.	31.	41.	3.04	26.	36.	3.53	22.	34.	6.95	19.	34.	12.29	15.	41.	45.70
SUM	523.	520.	6.76	508.	503.	11.46	494.	490.	24.65	483.	477.	33.01	473.	452.	76.64

TABLE OF PREDICTED NUMBER OF ACCOUNTS, ACTUAL NUMBER CBSERVED AND CHI-SQUARE STATISTICS
(MODEL II USING CPM V)

	QUARTER II-72			QUARTER III-72			QUARTER IV-72			QUARTER I-73			QUARTER II-73		
CLASS	PRED.	ACT.	CHI	PRED.	ACT.	CHI	PRED.	ACT.	CHI	PREC.	ACT.	CHI	PRED.	ACT.	CHI
0.	19.	18.	0.03	36.	35.	0.06	53.	48.	0.53	67.	61.	0.57	80.	86.	C.44
1999.	166.	169.	0.05	162.	160.	0.02	156.	153.	0.05	152.	148.	0.10	147.	141.	C.22
3999.	86.	80.	0.36	84.	80.	0.16	83.	76.	C.60	84.	78.	0.39	85.	69.	3.17
5999.	55.	51.	0.31	56.	50.	C.55	56.	57.	0.02	56.	53.	0.16	57.	45.	2.43
7999.	36.	37.	0.04	34.	38.	C.40	32.	30.	0.14	28.	30.	0.19	24.	31.	2.35
9999.	20.	22.	0.16	20.	20.	C.01	20.	17.	0.33	22.	21.	0.08	24.	15.	3.20
11999.	54.	57.	0.22	49.	50.	0.02	45.	49.	0.27	41.	44.	0.27	37.	46.	2.22
13999.	27.	21.	1.31	24.	27.	0.29	22.	23.	0.03	21.	23.	0.24	19.	25.	1.62
15999.	19.	22.	0.47	18.	21.	0.36	18.	25.	2.97	17.	21.	0.93	16.	17.	0.02
19999.	21.	20.	0.01	20.	21.	0.07	20.	26.	1.67	21.	25.	0.89	23.	22.	C.08
100000.	36.	41.	0.62	35.	36.	C.04	32.	34.	0.09	30.	34.	0.52	26.	41.	9.27
SUM	519.	520.	3.60	502.	503.	1.98	485.	490.	6.70	471.	477.	4.35	458.	452.	25.02

TABLE OF PREDICTED NUMBER OF ACCOUNTS, ACTUAL NUMBER OBSERVED AND CHI-SQUARE STATISTICS

(MODEL II USING CPM X)

	QUARTER II-72			QUARTER III-72			QUARTER IV-72			QUARTER I-73			QUARTER II-73		
CLASS	PRED.	ACT.	CHI	PRED.	ACT.	CHI	PRED.	ACT.	CHI	PRED.	ACT.	CHI	PRED.	ACT.	CHI
0.	19.	18.	0.04	37.	35.	0.07	53.	48.	0.54	67.	61.	0.58	80.	86.	0.43
1999.	166.	169.	0.07	161.	160.	0.00	154.	153.	0.01	150.	148.	0.02	144.	141.	0.08
3999.	84.	80.	0.18	81.	80.	0.01	79.	76.	0.11	79.	78.	0.01	80.	69.	1.39
5999.	54.	51.	0.16	53.	50.	0.21	53.	57.	0.30	52.	53.	0.01	53.	45.	1.16
7999.	36.	37.	0.03	35.	38.	0.34	32.	30.	0.18	28.	30.	0.16	24.	31.	2.24
9999.	20.	22.	0.27	19.	20.	0.05	19.	17.	0.18	22.	21.	0.02	23.	15.	2.77
11999.	54.	57.	0.13	50.	50.	0.00	47.	49.	0.06	43.	44.	0.03	39.	46.	1.12
13999.	27.	21.	1.45	25.	27.	0.15	23.	23.	0.00	22.	23.	0.05	21.	25.	0.84
15999.	20.	22.	0.26	20.	21.	0.09	19.	25.	1.62	19.	21.	0.22	19.	17.	0.13
19999.	21.	20.	0.05	21.	21.	0.00	22.	26.	0.78	23.	25.	0.17	27.	22.	0.80
100000.	37.	41.	0.34	37.	36.	0.03	35.	34.	0.04	33.	34.	0.01	29.	41.	4.96
SUM	519.	520.	2.97	501.	503.	0.94	485.	490.	3.82	471.	477.	1.26	458.	452.	15.92

TABLE OF PREDICTED NUMBER OF ACCOUNTS, ACTUAL NUMBER OBSERVED AND CHI-SQUARE STATISTICS

(MODEL II USING MOD III)

	QUARTER II-72			QUARTER III-72			QUARTER IV-72		
CLASS	PRED.	ACT.	CHI	PRED.	ACT.	CHI	PRED.	ACT.	CHI
0.	19.	18.	0.06	37.	35.	0.11	54.	48.	0.69
1999.	173.	169.	0.11	176.	160.	1.37	175.	153.	2.78
3999.	84.	80.	0.22	81.	80.	0.01	79.	76.	0.11
5999.	48.	51.	0.17	44.	50.	0.94	41.	57.	6.70
7999.	36.	37.	0.05	34.	38.	0.57	31.	30.	0.02
9999.	22.	22.	0.00	22.	20.	0.21	22.	17.	1.30
11999.	52.	57.	0.43	47.	50.	0.23	43.	49.	0.96
13999.	26.	21.	1.12	24.	27.	0.50	21.	23.	0.14
15999.	20.	22.	0.28	19.	21.	0.13	19.	25.	2.01
19999.	21.	20.	0.05	21.	21.	0.00	21.	26.	0.99
100000.	36.	41.	0.63	35.	36.	0.04	32.	34.	0.11
SUM	519.	520.	3.13	501.	503.	4.12	484.	490.	15.81

	QUARTER I-73			QUARTER II-73		
CLASS	PRED.	ACT.	CHI	PRED.	ACT.	CHI
0.	68.	61.	0.72	81.	86.	0.33
1999.	176.	148.	4.53	176.	141.	6.92
3999.	78.	78.	0.00	79.	69.	1.30
5999.	38.	53.	6.07	36.	45.	2.08
7999.	26.	30.	0.73	21.	31.	4.68
9999.	25.	21.	0.66	26.	15.	4.94
11999.	37.	44.	1.19	33.	46.	5.15
13999.	20.	23.	0.58	18.	25.	2.58
15999.	18.	21.	0.46	17.	17.	0.01
19999.	22.	25.	0.37	25.	22.	0.36
100000.	30.	34.	0.63	25.	41.	10.51
SUM	470.	477.	15.96	457.	452.	38.86

TABLE OF PREDICTED AMOUNT OF SAVINGS AND ACTUAL AMOUNT OBSERVED

(MODEL II USING CPM II)

	QUARTER II-72		QUARTER III-72		QUARTER IV-72		QUARTER I-73		QUARTER II-73	
CLASS	PRED.	ACT.	PRED.	ACT.	PRED.	ACT.	PRED.	ACT.	PRED.	ACT.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
1999.	84842.	85201.	86487.	76517.	86768.	74296.	88085.	68827.	88863.	69300.
3999.	216531.	239441.	191231.	238997.	175870.	224544.	167610.	231257.	164144.	207472.
5999.	285410.	251076.	291048.	241451.	293411.	274778.	290373.	259631.	291209.	218088.
7999.	284583.	256481.	298392.	262935.	296907.	215352.	266754.	206544.	235818.	215346.
9999.	185809.	201731.	186369.	183567.	190997.	153325.	228991.	189416.	250974.	131926.
11999.	592547.	610157.	552454.	536059.	524837.	520816.	478057.	467021.	439897.	492183.
13999.	352338.	276219.	322017.	354780.	298553.	298484.	282949.	299438.	267752.	325462.
15999.	308650.	327455.	306457.	315938.	296374.	373113.	282012.	315862.	268132.	252917.
19999.	358209.	351882.	330795.	372054.	321149.	468395.	307744.	440700.	325649.	388406.
100000.	905477.	1027230.	763605.	954255.	629165.	906064.	544672.	925321.	431729.	1116465.
SUM	3574393.	3626873.	3328854.	3534753.	3114029.	3509167.	2937245.	3404017.	2764163.	3417565.

TABLE OF PREDICTED AMOUNT OF SAVINGS AND ACTUAL AMOUNT OBSERVED
(MODEL II USING CPM V)

	QUARTER II-72		QUARTER III-72		QUARTER IV-72		QUARTER I-73		QUARTER II-73	
CLASS	PRED.	ACT.	PRED.	ACT.	PRED.	ACT.	PRED.	ACT.	PRED.	ACT.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
1999.	80435.	85201.	78383.	76517.	75435.	74296.	73509.	68827.	70948.	69300.
3999.	245766.	239441.	240197.	238997.	238548.	224544.	240393.	231257.	245440.	207472.
5999.	274433.	251076.	276186.	241451.	278772.	274778.	278573.	259631.	282356.	218088.
7999.	248058.	256481.	237937.	262935.	223129.	215352.	192201.	206544.	163466.	215346.
9999.	181905.	201731.	177255.	183567.	176296.	153325.	200961.	189416.	213775.	131926.
11999.	583105.	610157.	532844.	536059.	495201.	520816.	442919.	467021.	402165.	492183.
13999.	352207.	276219.	318440.	354780.	290839.	298484.	271318.	299438.	253707.	325462.
15999.	287887.	327455.	278929.	315938.	268718.	373113.	257691.	315862.	249099.	252917.
19999.	369844.	351882.	356886.	372054.	363718.	468395.	372814.	440700.	421223.	388406.
100000.	1050483.	1027230.	1009933.	954255.	935743.	906064.	870411.	925321.	741603.	1116465.
SUM	3674122.	3626873.	3506988.	3534753.	3346397.	3509167.	3200788.	3404017.	3043777.	3417565.

TABLE OF PREDICTED AMOUNT OF SAVINGS AND ACTUAL AMOUNT OBSERVED
(MODEL II USING CPM X)

	QUARTER II-72		QUARTER III-72		QUARTER IV-72		QUARTER I-73		QUARTER II-73	
CLASS	PRED.	ACT.	PRED.	ACT.	PRED.	ACT.	PRED.	ACT.	PRED.	ACT.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
1999.	80160.	85201.	77824.	76517.	74623.	74296.	72508.	68827.	69895.	69300.
3999.	241034.	239441.	231636.	238997.	226890.	224544.	225950.	231257.	228368.	207472.
5999.	268410.	251076.	265286.	241451.	263942.	274778.	260965.	259631.	262942.	218088.
7999.	249815.	256481.	240007.	262935.	225004.	215352.	193677.	206544.	164551.	215346.
9999.	177672.	201731.	171309.	183567.	169862.	153325.	194707.	189416.	207096.	131926.
11999.	592063.	610157.	548364.	536059.	515901.	520816.	466678.	467021.	428573.	492183.
13999.	356926.	276219.	327711.	354780.	303791.	298484.	287601.	299438.	272358.	325462.
15999.	298896.	327455.	298146.	315938.	293767.	373113.	286963.	315862.	280837.	252917.
19999.	379160.	351882.	375943.	372054.	393671.	468395.	414490.	440700.	479054.	388406.
100000.	1084614.	1027230.	1072865.	954255.	1020259.	906064.	969024.	925321.	840305.	1116465.
SUM	3728749.	3626873.	3609083.	3534753.	3487709.	3509167.	3372561.	3404017.	3233976.	3417565.

TABLE OF PREDICTED AMOUNT OF SAVINGS AND ACTUAL AMOUNT OBSERVED

(MODEL II USING MOD III)

CLASS	QUARTER II-72 PRED.	QUARTER II-72 ACT.	QUARTER III-72 PRED.	QUARTER III-72 ACT.	QUARTER IV-72 PRED.	QUARTER IV-72 ACT.	QUARTER I-73 PRED.	QUARTER I-73 ACT.	QUARTER II-73 PRED.	QUARTER II-73 ACT.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
1999.	83908.	85201.	84928.	76517.	84689.	74296.	85272.	68827.	85093.	69300.
3999.	242216.	239441.	232249.	238997.	226629.	224544.	224921.	231257.	227222.	207472.
5999.	239499.	251076.	217004.	241451.	201635.	274778.	188320.	259631.	180704.	218088.
7999.	247314.	256481.	233166.	262935.	213447.	215352.	178030.	206544.	146193.	215346.
9999.	196414.	201731.	199736.	183567.	201937.	153325.	226060.	189416.	238238.	131926.
11999.	569071.	610157.	508717.	536059.	463757.	520816.	406412.	467021.	358918.	492183.
13999.	345967.	276219.	308398.	354780.	278124.	298484.	256540.	299438.	237478.	325462.
15999.	297584.	327455.	293799.	315938.	285383.	373113.	274180.	315862.	263498.	252917.
19999.	378490.	351882.	372492.	372054.	385407.	468395.	398438.	440700.	450314.	388406.
100000.	1049426.	1027230.	1007669.	954255.	930285.	906064.	859408.	925321.	719673.	1116465.
SUM	3649886.	3626873.	3458155.	3534753.	3271291.	3509167.	3097580.	3404017.	2907329.	3417565.

TABLE OF PREDICTED NUMBER OF ACCOUNTS, ACTUAL NUMBER OBSERVED AND CHI-SQUARE STATISTICS

(MODEL II′ USING CPM II)

	QUARTER II-72			QUARTER III-72			QUARTER IV-72			QUARTER I-73			QUARTER II-73		
CLASS	PRED.	ACT.	CHI	PRED.	ACT.	CHI	PRED.	ACT.	CHI	PRED.	ACT.	CHI	PRED.	ACT.	CHI
0.	16.	18.	0.37	31.	35.	0.49	46.	48.	0.06	61.	61.	0.00	76.	86.	1.37
1999.	176.	169.	0.26	181.	160.	2.36	183.	153.	4.93	185.	148.	7.55	187.	141.	11.25
3999.	75.	80.	0.35	64.	80.	3.85	57.	76.	6.57	51.	78.	13.95	47.	69.	10.26
5999.	57.	51.	0.68	58.	50.	1.05	57.	57.	0.00	56.	53.	0.12	54.	45.	1.61
7999.	41.	37.	0.42	44.	38.	0.76	44.	30.	4.70	42.	30.	3.41	40.	31.	1.87
9999.	21.	22.	0.10	21.	20.	0.01	21.	17.	0.71	24.	21.	0.46	26.	15.	4.73
11999.	55.	57.	0.07	52.	50.	0.05	49.	49.	0.00	45.	44.	0.03	42.	46.	0.48
13999.	27.	21.	1.33	25.	27.	0.23	23.	23.	0.01	21.	23.	0.15	20.	25.	1.40
15999.	20.	22.	0.31	19.	21.	0.21	18.	25.	2.78	17.	21.	1.14	15.	17.	0.15
19999.	20.	20.	0.00	18.	21.	0.48	17.	26.	5.02	15.	25.	7.22	14.	22.	5.11
100000.	31.	41.	3.10	27.	36.	3.33	22.	34.	5.94	20.	34.	9.03	18.	41.	30.16
SUM	522.	520.	6.99	507.	503.	12.83	492.	490.	30.73	477.	477.	43.05	462.	452.	68.39

TABLE OF PREDICTED NUMBER OF ACCOUNTS, ACTUAL NUMBER OBSERVED AND CHI-SQUARE STATISTICS
(MODEL II′ USING CPM V)

	QUARTER II-72			QUARTER III-72			QUARTER IV-72			QUARTER I-73			QUARTER II-73		
CLASS	PRED.	ACT.	CHI	PRED.	ACT.	CHI	PRED.	ACT.	CHI	PRED.	ACT.	CHI	PRED.	ACT.	CHI
0.	19.	18.	0.07	38.	35.	0.18	55.	48.	0.98	73.	61.	1.84	89.	86.	0.11
1999.	167.	169.	0.01	166.	160.	0.19	163.	153.	0.57	161.	148.	0.98	158.	141.	1.86
3999.	84.	80.	0.23	80.	80.	0.00	76.	76.	0.00	73.	78.	0.33	70.	69.	0.02
5999.	55.	51.	0.31	55.	50.	0.44	55.	57.	0.11	53.	53.	0.00	52.	45.	1.04
7999.	35.	37.	0.09	34.	38.	0.49	32.	30.	0.16	29.	30.	0.03	26.	31.	0.82
9999.	20.	22.	0.14	20.	20.	0.01	19.	17.	0.28	21.	21.	0.01	22.	15.	2.30
11999.	54.	57.	0.22	49.	50.	0.02	46.	49.	0.23	41.	44.	0.22	37.	46.	2.09
13999.	27.	21.	1.31	24.	27.	0.29	22.	23.	0.03	21.	23.	0.29	19.	25.	1.87
15999.	19.	22.	0.47	18.	21.	0.39	17.	25.	3.34	16.	21.	1.33	15.	17.	0.17
19999.	20.	20.	0.01	19.	21.	0.17	19.	26.	2.99	17.	25.	3.80	16.	22.	1.93
10000C.	36.	41.	0.61	35.	36.	0.01	34.	34.	0.00	33.	34.	0.01	32.	41.	2.67
SUM	519.	520.	3.47	500.	503.	2.19	483.	490.	8.69	465.	477.	8.84	449.	452.	14.87

TABLE OF PREDICTED NUMBER OF ACCOUNTS, ACTUAL NUMBER OBSERVED AND CHI-SQUARE STATISTICS

(MODEL II′ USING CPM X)

	QUARTER II-72			QUARTER III-72			QUARTER IV-72			QUARTER I-73			QUARTER II-73		
CLASS	PRED.	ACT.	CHI	PRED.	ACT.	CHI	PRED.	ACT.	CHI	PRED.	ACT.	CHI	PRED.	ACT.	CHI
0.	19.	18.	0.03	37.	35.	0.08	54.	48.	0.66	71.	61.	1.33	87.	86.	0.01
1999.	165.	169.	0.08	162.	160.	0.02	158.	153.	0.13	154.	148.	0.26	151.	141.	0.69
3999.	84.	80.	0.24	80.	80.	0.00	76.	76.	0.00	72.	78.	0.47	69.	69.	0.00
5999.	54.	51.	0.18	53.	50.	0.20	52.	57.	0.39	51.	53.	0.08	50.	45.	0.50
7999.	35.	37.	0.06	34.	38.	0.42	33.	30.	0.20	29.	30.	0.01	27.	31.	0.72
9999.	20.	22.	0.23	19.	20.	0.05	19.	17.	0.15	21.	21.	0.00	22.	15.	2.00
11999.	54.	57.	0.12	51.	50.	0.01	48.	49.	0.03	43.	44.	0.01	40.	46.	0.97
13999.	27.	21.	1.46	25.	27.	0.15	23.	23.	0.00	22.	23.	0.05	21.	25.	0.87
15999.	20.	22.	0.28	20.	21.	0.11	19.	25.	1.91	18.	21.	0.42	17.	17.	0.01
19999.	21.	20.	0.10	21.	21.	0.00	21.	26.	1.13	20.	25.	1.24	20.	22.	0.21
100000.	37.	41.	0.41	37.	36.	0.02	36.	34.	0.09	36.	34.	0.12	35.	41.	1.03
SUM	519.	520.	3.17	501.	503.	1.05	484.	490.	4.71	467.	477.	3.99	451.	452.	7.01

TABLE OF PREDICTED NUMBER OF ACCOUNTS, ACTUAL NUMBER OBSERVED AND CHI-SQUARE STATISTICS
(MODEL II′ USING MOD III)

	QUARTER II-72			QUARTER III-72			QUARTER IV-72			QUARTER I-73			QUARTER II-73		
CLASS	PRED.	ACT.	CHI	PRED.	ACT.	CHI	PRED.	ACT.	CHI	PRED.	ACT.	CHI	PRED.	ACT.	CHI
0.	19.	18.	0.05	37.	35.	0.15	55.	48.	0.90	72.	61.	1.76	89.	86.	0.10
1999.	173.	169.	0.09	176.	160.	1.49	178.	153.	3.48	180.	148.	5.59	181.	141.	8.76
3999.	85.	80.	0.29	80.	80.	0.00	75.	76.	0.00	71.	78.	0.69	67.	69.	0.07
5999.	48.	51.	0.16	44.	50.	0.94	40.	57.	7.00	37.	53.	6.86	35.	45.	2.95
7999.	35.	37.	0.08	33.	38.	0.62	31.	30.	0.04	27.	30.	0.26	24.	31.	1.94
9999.	22.	22.	0.00	22.	20.	0.21	22.	17.	1.22	24.	21.	0.44	25.	15.	4.01
11999.	52.	57.	0.41	47.	50.	0.21	43.	49.	0.87	38.	44.	1.04	33.	46.	4.74
13999.	26.	21.	1.13	24.	27.	0.49	21.	23.	0.14	20.	23.	0.59	18.	25.	2.65
15999.	20.	22.	0.30	19.	21.	0.16	18.	25.	2.31	17.	21.	0.71	17.	17.	0.01
19999.	21.	20.	0.10	21.	21.	0.00	21.	26.	1.29	20.	25.	1.54	19.	22.	0.42
100000.	36.	41.	0.75	35.	36.	0.06	33.	34.	0.06	32.	34.	0.12	30.	41.	3.83
SUM	519.	520.	3.37	501.	503.	4.33	483.	490.	17.32	466.	477.	19.60	449.	452.	29.47

TABLE OF PREDICTED AMOUNT OF SAVINGS AND ACTUAL AMOUNT OBSERVED

(MODEL II' USING CPM II)

	QUARTER II-72		QUARTER III-72		QUARTER IV-72		QUARTER I-73		QUARTER II-73	
CLASS	PRED.	ACT.	PRED.	ACT.	PRED.	ACT.	PRED.	ACT.	PRED.	ACT.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
1999.	85059.	85201.	87408.	76517.	88560.	74296.	89703.	68827.	90401.	69300.
3999.	214973.	239441.	184558.	238997.	162815.	224544.	147201.	231257.	135050.	207472.
5999.	284737.	251076.	287604.	241451.	285602.	274778.	276750.	259631.	270438.	218088.
7999.	285733.	256481.	303616.	262935.	308452.	215352.	291218.	206544.	274856.	215346.
9999.	185680.	201731.	185096.	183567.	188036.	153325.	219316.	189416.	235402.	131926.
11999.	598881.	610157.	562326.	536059.	537544.	520816.	491558.	467021.	452220.	492183.
13999.	353015.	276219.	321685.	354780.	296198.	298484.	277817.	299438.	258258.	325462.
15999.	296097.	327455.	287439.	315938.	271591.	373113.	251981.	315862.	234219.	252917.
19999.	360160.	351882.	325338.	372054.	302752.	468395.	264693.	440700.	245832.	388406.
100000.	903077.	1027230.	770406.	954255.	650525.	906064.	591675.	925321.	516199.	1116465.
SUM	3567409.	3626873.	3315473.	3534753.	3092072.	3509167.	2901908.	3404017.	2712873.	3417565.

TABLE OF PREDICTED AMOUNT OF SAVINGS AND ACTUAL AMOUNT OBSERVED

(MODEL II′ USING CPM V)

	QUARTER II-72		QUARTER III-72		QUARTER IV-72		QUARTER I-73		QUARTER II-73	
CLASS	PRED.	ACT.	PRED.	ACT.	PRED.	ACT.	PRED.	ACT.	PRED.	ACT.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
1999.	81006.	85201.	80115.	76517.	78693.	74296.	77665.	68827.	76512.	69300.
3999.	242485.	239441.	229767.	238997.	219053.	224544.	209803.	231257.	201271.	207472.
5999.	274201.	251076.	273337.	241451.	271411.	274778.	264863.	259631.	260594.	218088.
7999.	244589.	256481.	235320.	262935.	223797.	215352.	201517.	206544.	182918.	215346.
9999.	183113.	201731.	177346.	183567.	174304.	153325.	192850.	189416.	195589.	131926.
11999.	583569.	610157.	534031.	536059.	497857.	520816.	446343.	467021.	404829.	492183.
13999.	352211.	276219.	318152.	354780.	289896.	298484.	269037.	299438.	248980.	325462.
15999.	287705.	327455.	277506.	315938.	263273.	373113.	247486.	315862.	233223.	252917.
19999.	369111.	351882.	345681.	372054.	334145.	468395.	305628.	440700.	294847.	388406.
100000.	1051362.	1027230.	1025416.	954255.	977343.	906064.	964583.	925321.	920818.	1116465.
SUM	3669350.	3626873.	3496669.	3534753.	3329768.	3509167.	3179770.	3404017.	3023579.	3417565.

TABLE OF PREDICTED AMOUNT OF SAVINGS AND ACTUAL AMOUNT OBSERVED

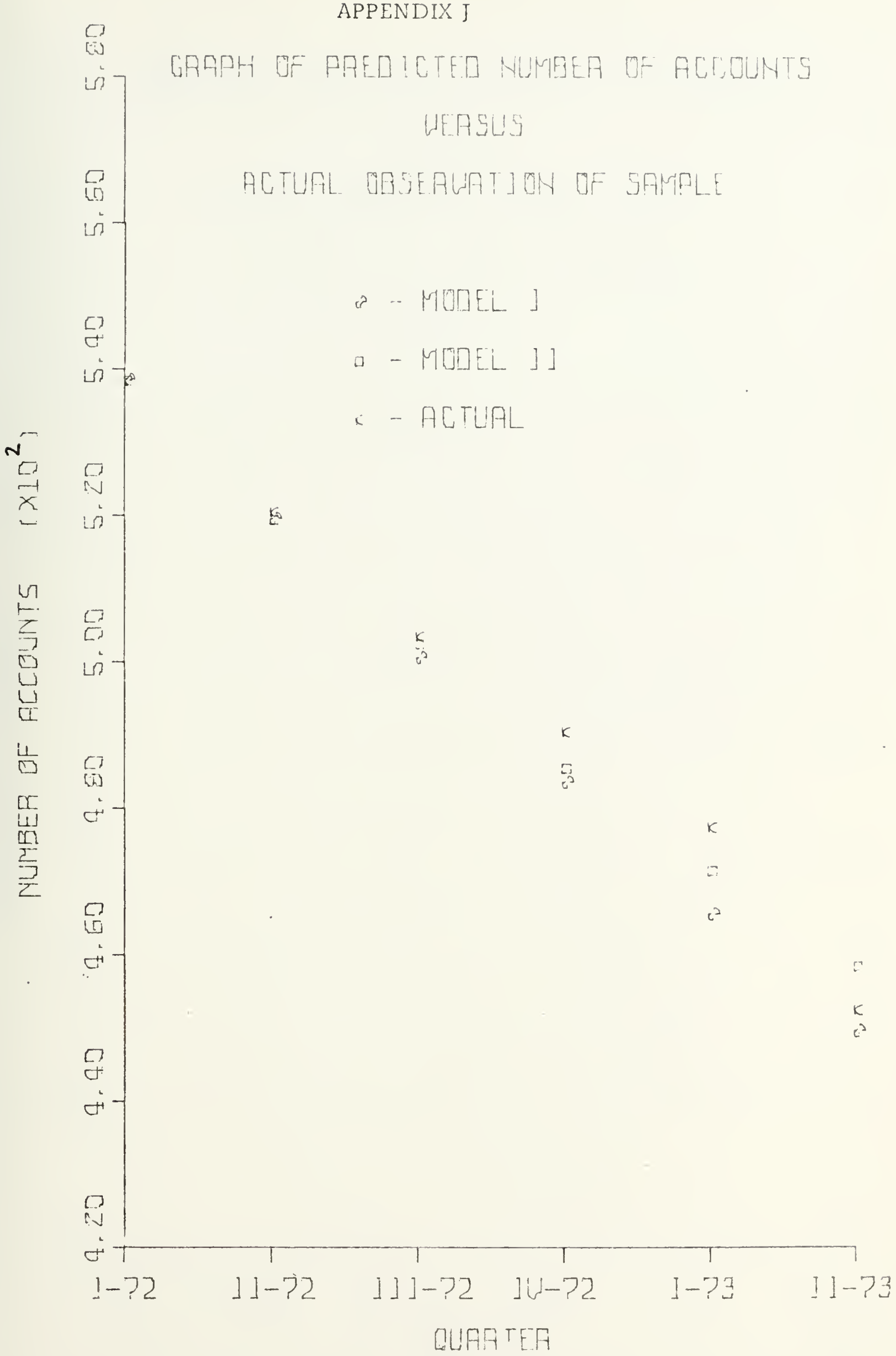
(MODEL II′ USING CPM X)

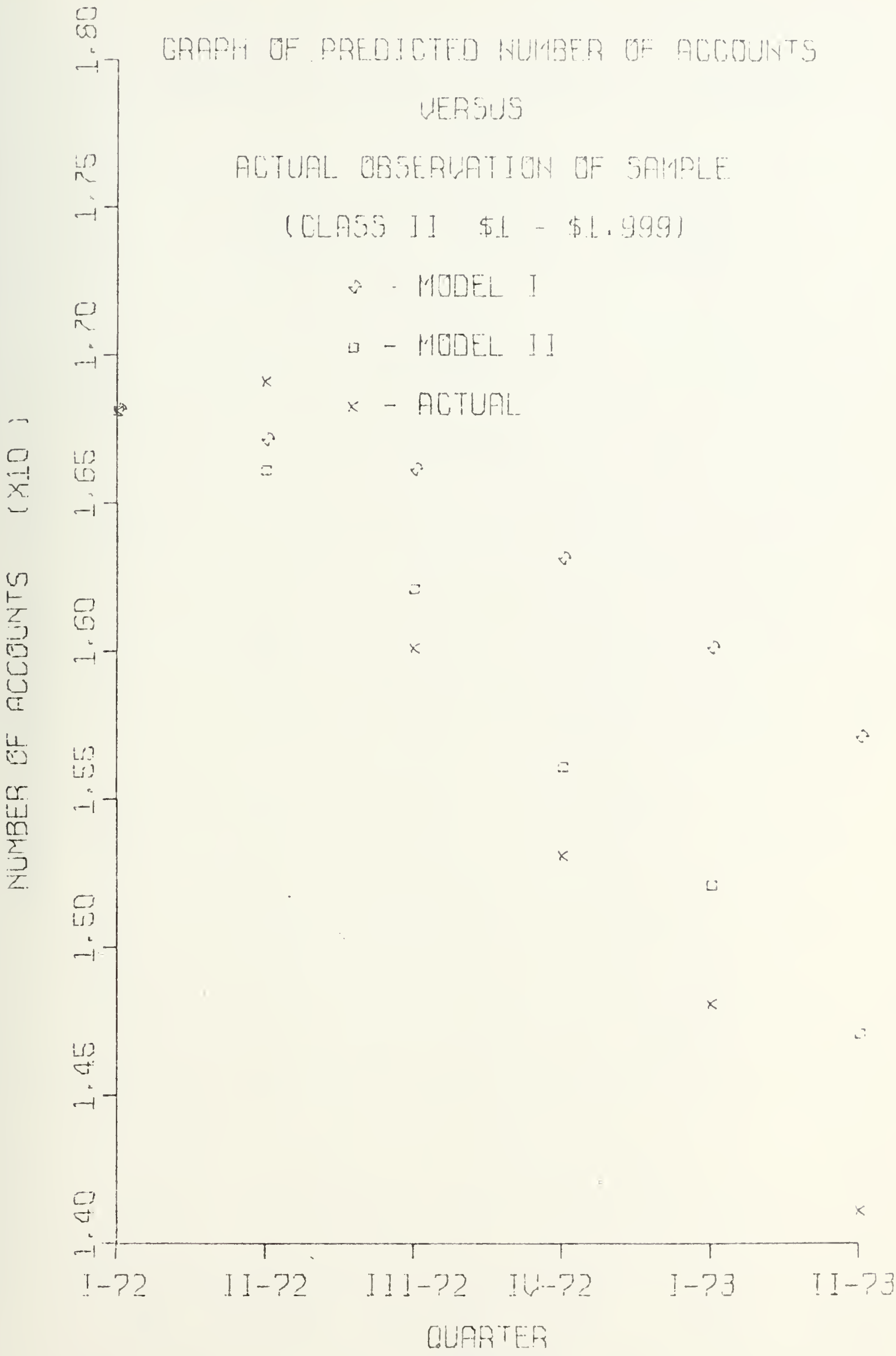
	QUARTER II-72		QUARTER III-72		QUARTER IV-72		QUARTER I-73		QUARTER II-73	
CLASS	PRED.	ACT.	PRED.	ACT.	PRED.	ACT.	PRED.	ACT.	PRED.	ACT.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
1999.	80017.	85201.	78325.	76517.	76251.	74296.	74697.	68827.	73149.	69300.
3999.	242530.	239441.	229162.	238997.	217574.	224544.	207267.	231257.	197626.	207472.
5999.	269231.	251076.	265165.	241451.	261102.	274778.	253443.	259631.	248804.	218088.
7999.	246274.	256481.	237479.	262935.	226048.	215352.	203629.	206544.	184708.	215346.
9999.	179083.	201731.	171774.	183567.	168492.	153325.	187594.	189416.	194412.	131926.
11999.	593133.	610157.	550518.	536059.	519849.	520816.	471720.	467021.	433276.	492183.
13999.	357052.	276219.	327989.	354780.	304110.	298484.	287669.	299438.	271500.	325462.
15999.	297904.	327455.	295667.	315938.	287431.	373113.	276065.	315862.	264940.	252917.
19999.	386573.	351882.	378143.	372054.	380112.	468395.	360620.	440700.	359234.	388406.
100000.	1074166.	1027230.	1069087.	954255.	1038370.	906064.	1044706.	925321.	1013678.	1116465.
SUM	3725961.	3626873.	3603307.	3534753.	3479335.	3509167.	3367407.	3404017.	3241323.	3417565.

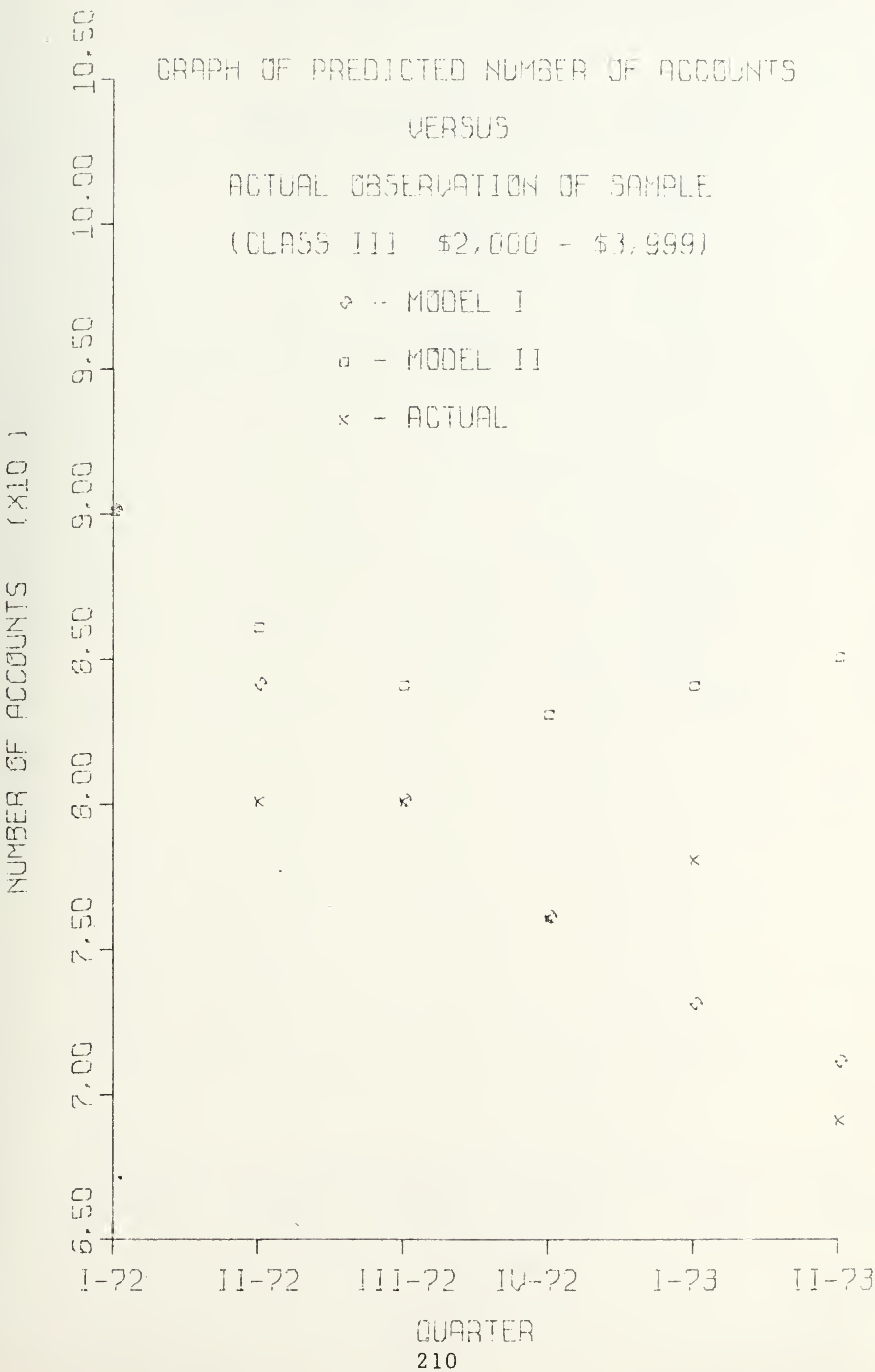
TABLE OF PREDICTED AMOUNT OF SAVINGS AND ACTUAL AMOUNT OBSERVED
(MODEL II' USING MOD III)

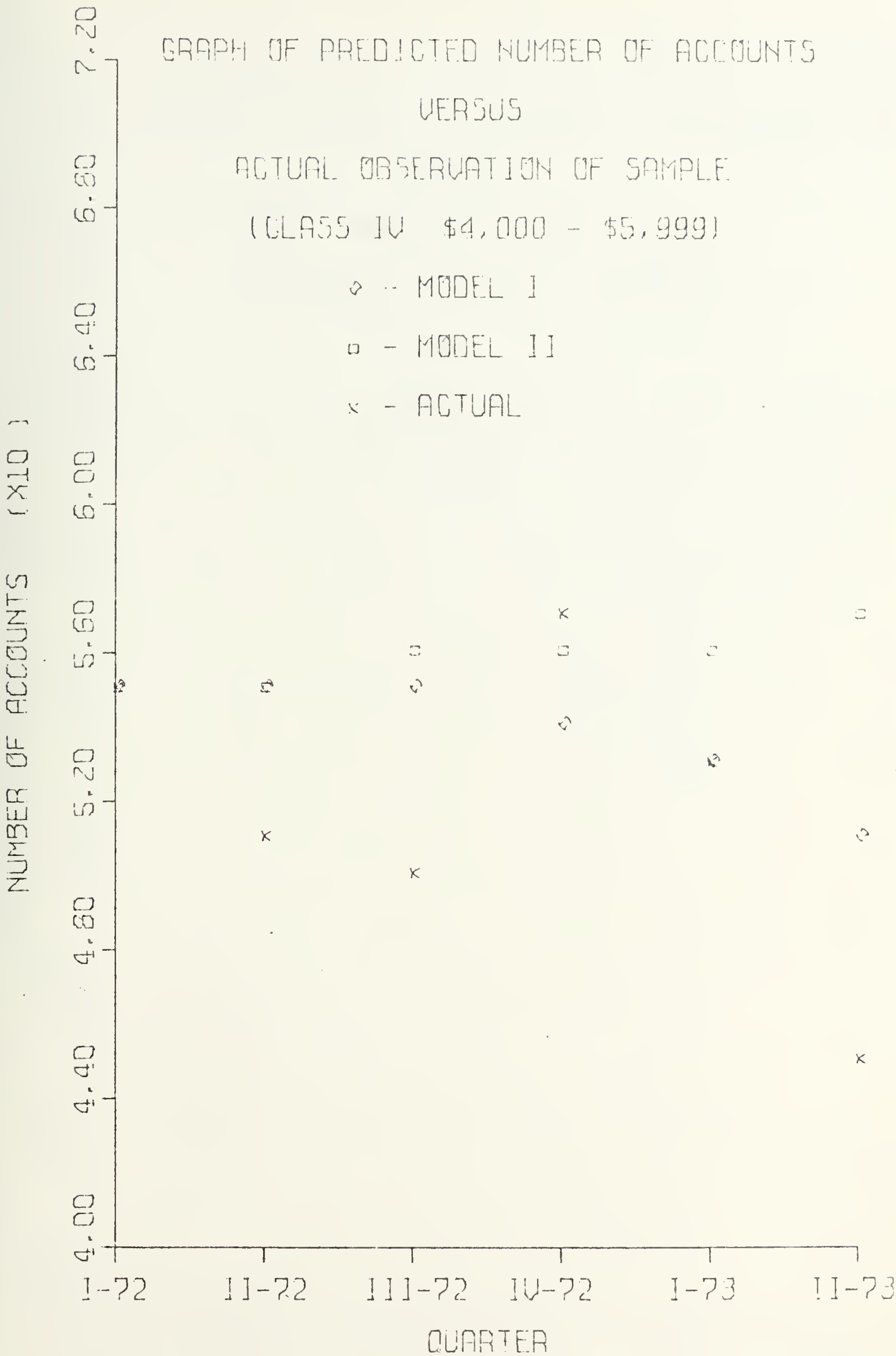
	QUARTER II-72		QUARTER III-72		QUARTER IV-72		QUARTER I-73		QUARTER II-73	
CLASS	PRED.	ACT.	PRED.	ACT.	PRED.	ACT.	PRED.	ACT.	PRED.	ACT.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
1999.	83685.	85201.	85258.	76517.	86066.	74296.	86945.	68827.	87473.	69300.
3999.	243922.	239441.	229848.	238997.	216502.	224544.	203936.	231257.	191893.	207472.
5999.	240024.	251076.	216966.	241451.	200158.	274778.	184381.	259631.	173461.	218088.
7999.	244733.	256481.	232154.	262935.	216354.	215352.	189788.	206544.	167602.	215346.
9999.	197471.	201731.	199877.	183567.	200231.	153325.	218717.	189416.	225422.	131926.
11999.	569950.	610157.	510351.	536059.	466980.	520816.	410812.	467021.	363781.	492183.
13999.	346043.	276219.	308616.	354780.	278350.	298484.	256454.	299438.	236365.	325462.
15999.	296546.	327455.	291472.	315938.	279717.	373113.	264746.	315862.	250033.	252917.
19999.	386696.	351882.	376206.	372054.	374740.	468395.	351332.	440700.	345352.	388406.
100000.	1037409.	1027230.	1001237.	954255.	944258.	906064.	927075.	925321.	876164.	1116465.
SUM	3646475.	3626873.	3451983.	3534753.	3263351.	3509167.	3094184.	3404017.	2917544.	3417565.

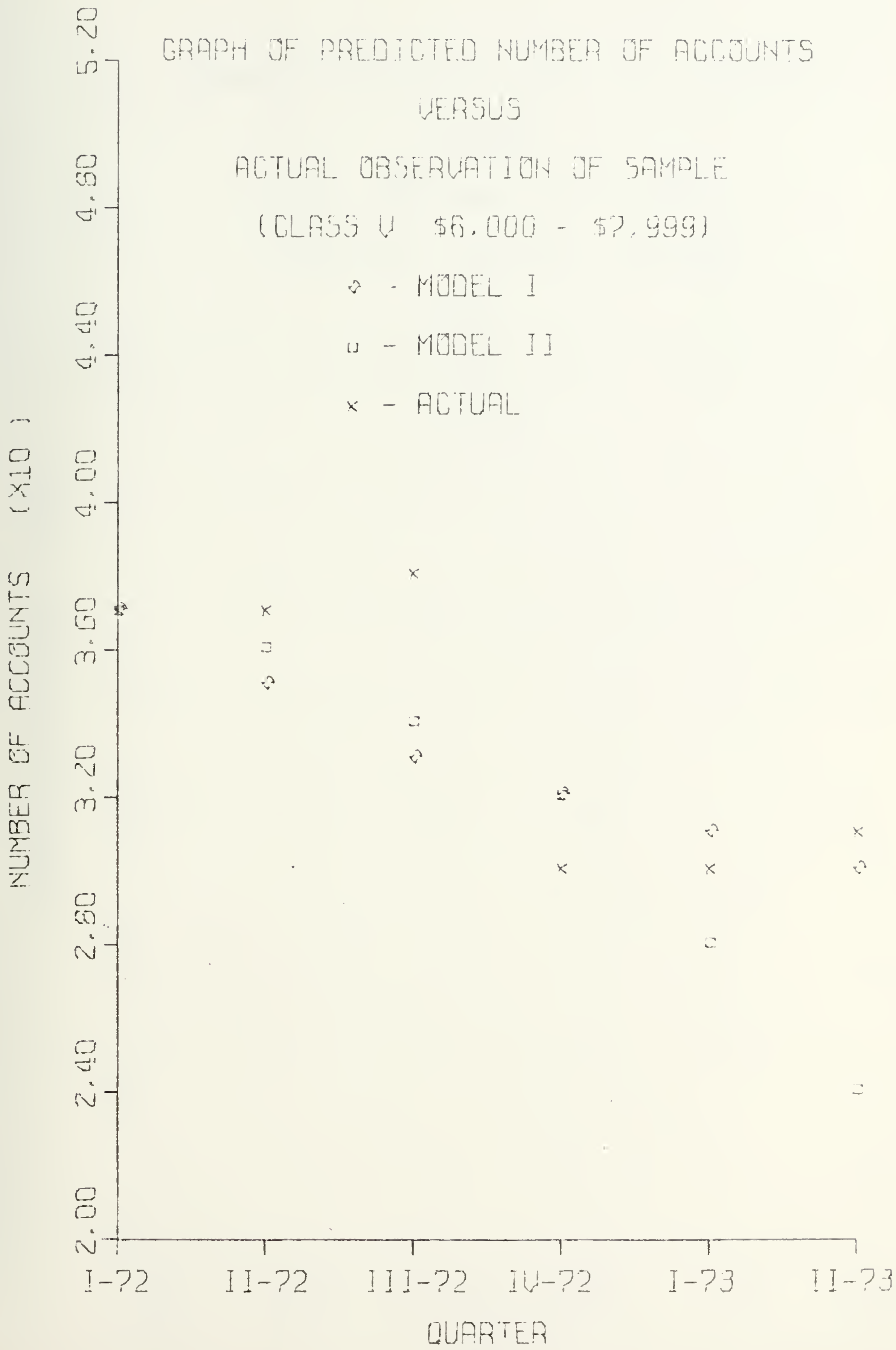
APPENDIX J

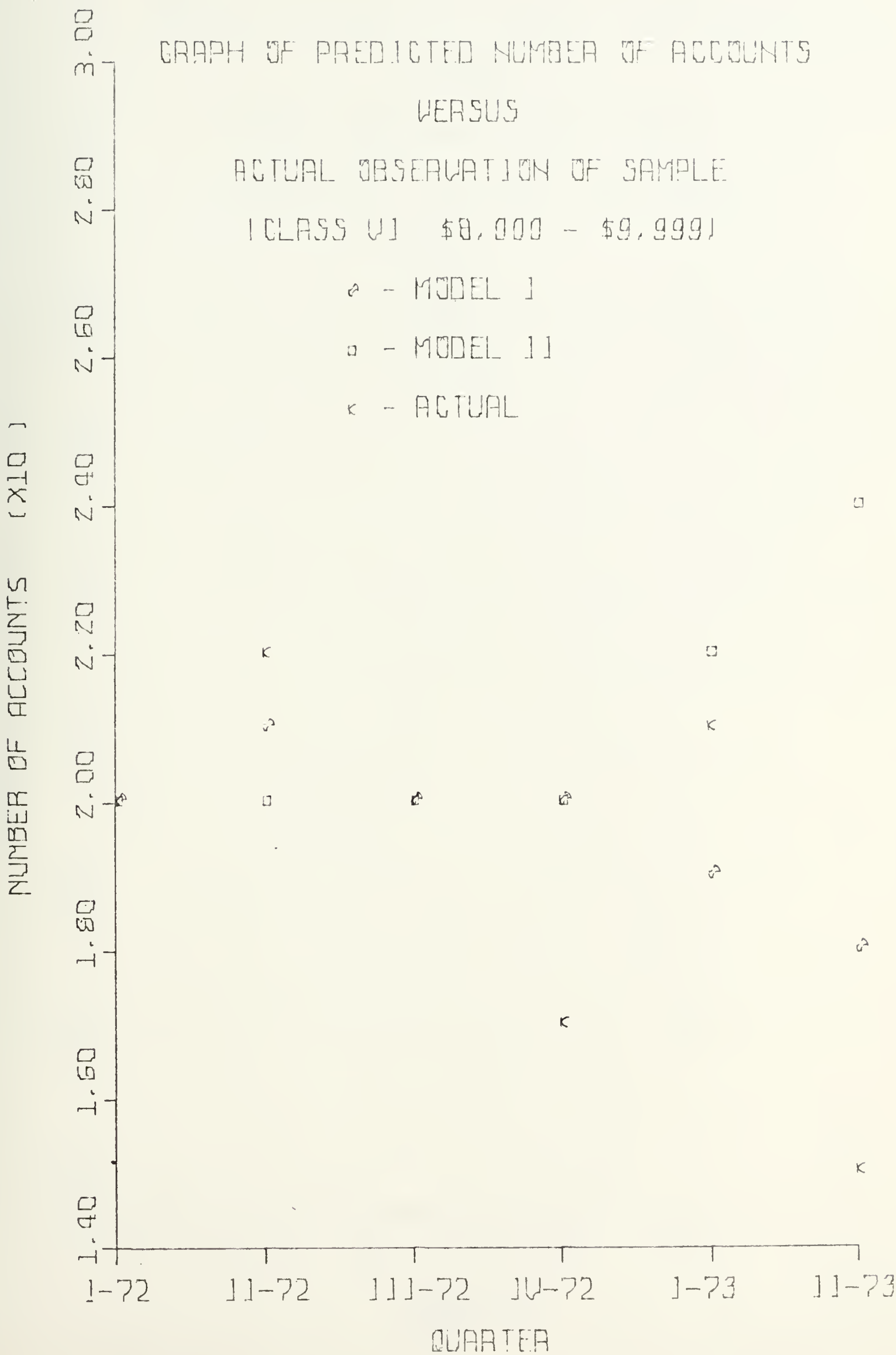

GRAPH OF PREDICTED NUMBER OF ACCOUNTS
VERSUS
ACTUAL OBSERVATION OF SAMPLE
- MODEL I
- MODEL II
- ACTUAL
NUMBER OF ACCOUNTS (X10^2)
5.80
5.60
5.40
5.20
5.00
4.80
4.60
4.40
4.20
I-72
II-72
III-72
IV-72
I-73
II-73
QUARTER


GRAPH OF PREDICTED NUMBER OF ACCOUNTS
VERSUS
ACTUAL OBSERVATION OF SAMPLE
(CLASS II $1 - $1,999)
- MODEL I
- MODEL II
- ACTUAL
NUMBER OF ACCOUNTS (X10)
1.40
1.45
1.50
1.55
1.60
1.65
1.70
1.75
1.80
I-72
II-72
III-72
IV-72
I-73
II-73
QUARTER

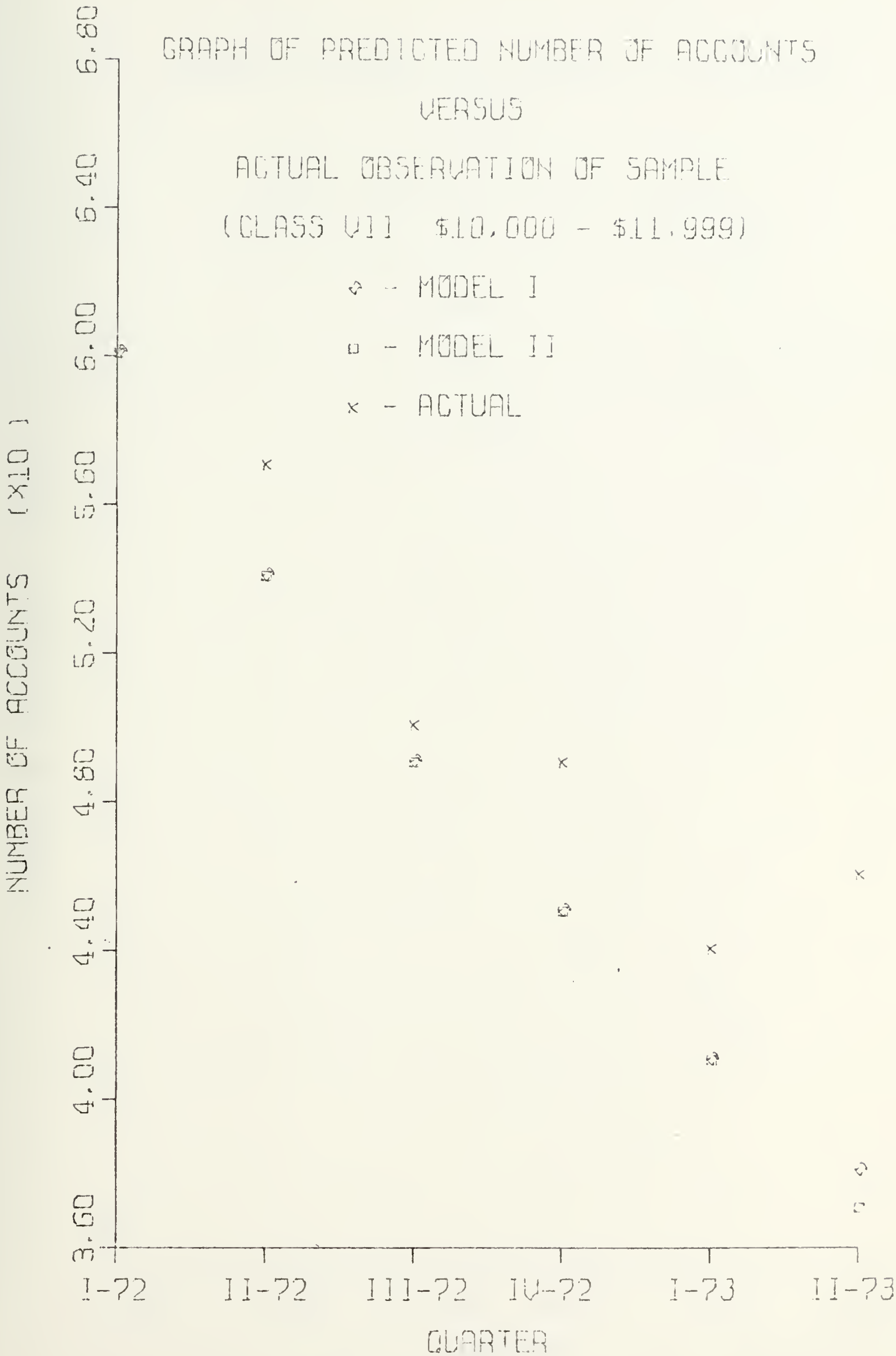

GRAPH OF PREDICTED NUMBER OF ACCOUNTS
VERSUS
ACTUAL OBSERVATION OF SAMPLE
(CLASS III $2,000 - $3,999)
◇ - MODEL I
□ - MODEL II
× - ACTUAL
NUMBER OF ACCOUNTS (X10)
10.50
10.00
9.50
9.00
8.50
8.00
7.50
7.00
6.50
I-72
II-72
III-72
IV-72
I-73
II-73
QUARTER

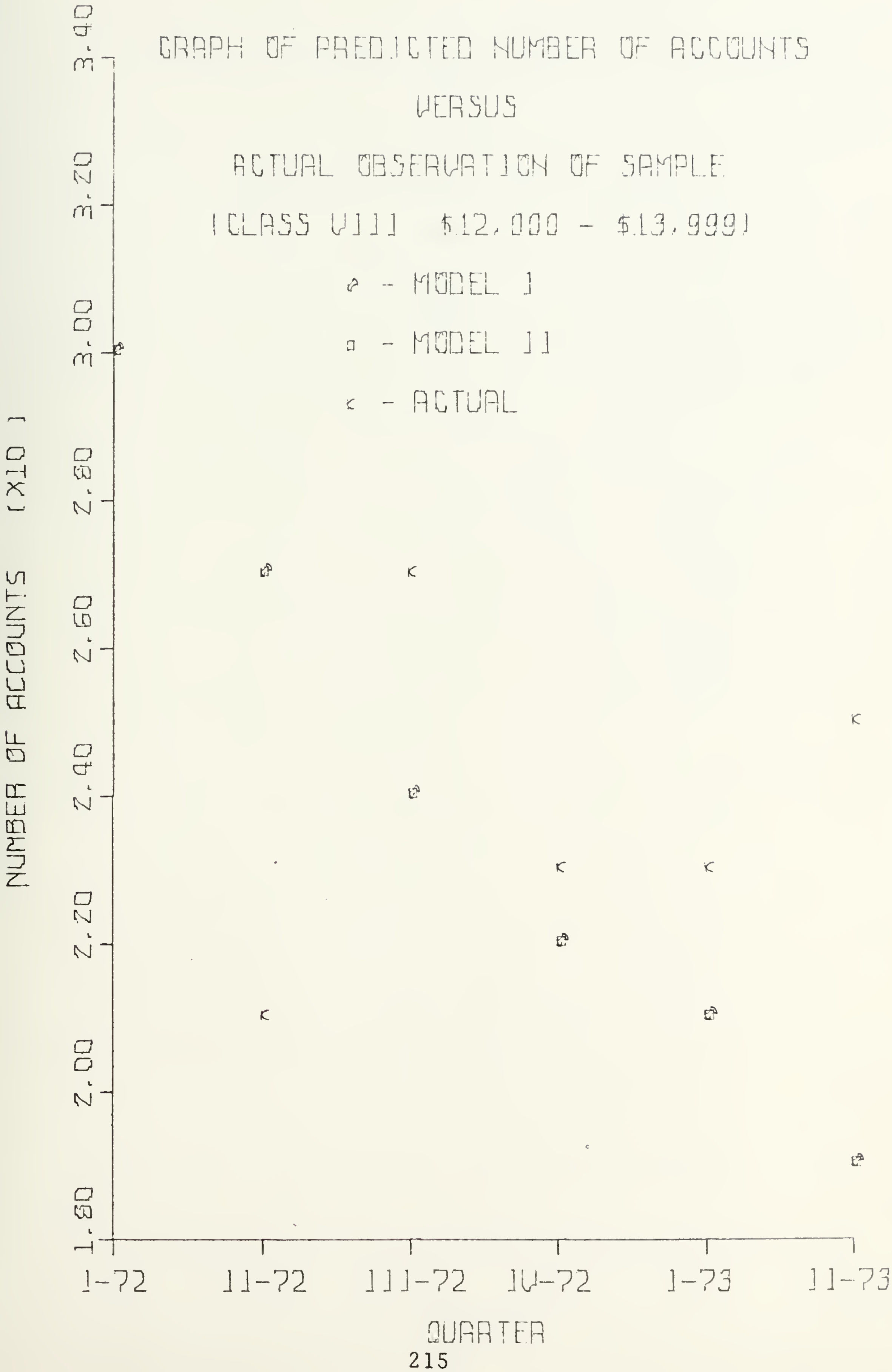

GRAPH OF PREDICTED NUMBER OF ACCOUNTS
VERSUS
ACTUAL OBSERVATION OF SAMPLE
(CLASS IV $4,000 - $5,999)
◇ - MODEL I
□ - MODEL II
x - ACTUAL
NUMBER OF ACCOUNTS (X10)
7.20
6.80
6.40
6.00
5.60
5.20
4.80
4.40
4.00
I-72
II-72
III-72
IV-72
I-73
II-73
QUARTER

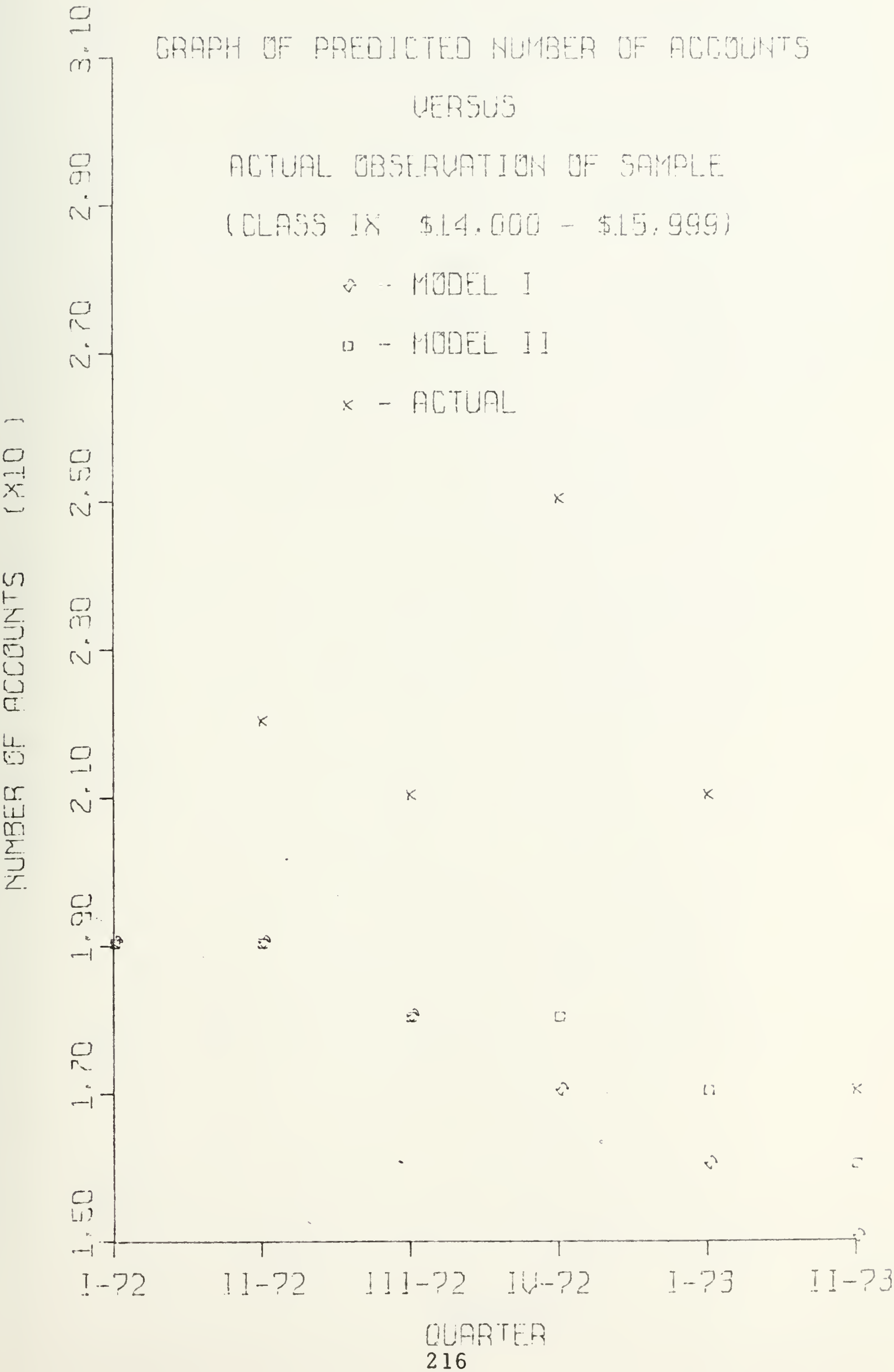

GRAPH OF PREDICTED NUMBER OF ACCOUNTS
VERSUS
ACTUAL OBSERVATION OF SAMPLE
(CLASS V $6,000 - $7,999)
◇ - MODEL I
□ - MODEL II
× - ACTUAL
NUMBER OF ACCOUNTS (X10)
5.20
4.80
4.40
4.00
3.60
3.20
2.80
2.40
2.00
I-72
II-72
III-72
IV-72
I-73
II-73
QUARTER

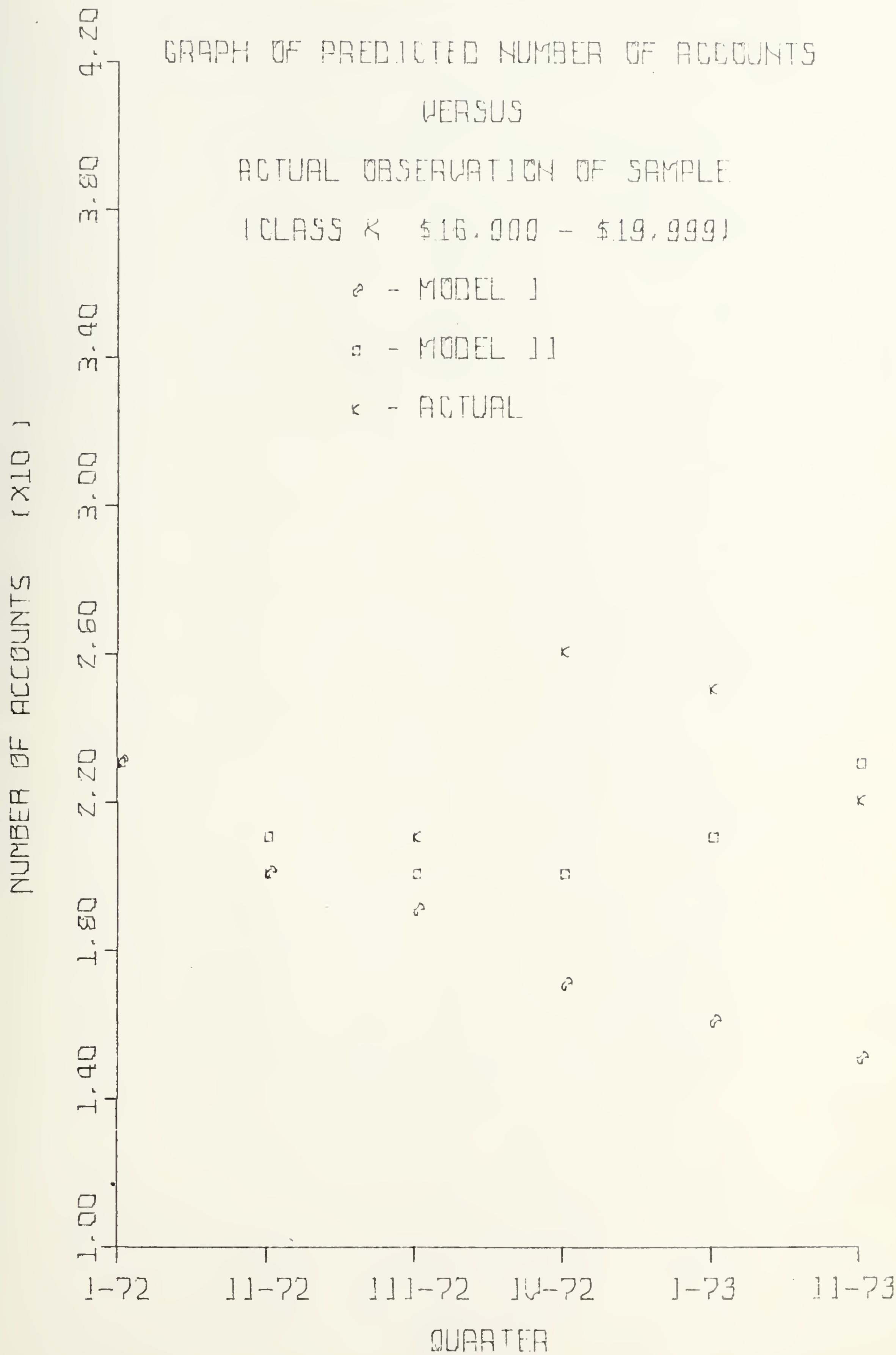


GRAPH OF PREDICTED NUMBER OF ACCOUNTS
VERSUS
ACTUAL OBSERVATION OF SAMPLE
(CLASS V) $8,000 - $9,999)
- MODEL I
- MODEL II
- ACTUAL
NUMBER OF ACCOUNTS (X10)
3.00
2.80
2.60
2.40
2.20
2.00
1.80
1.60
1.40
I-72
II-72
III-72
IV-72
I-73
II-73
QUARTER

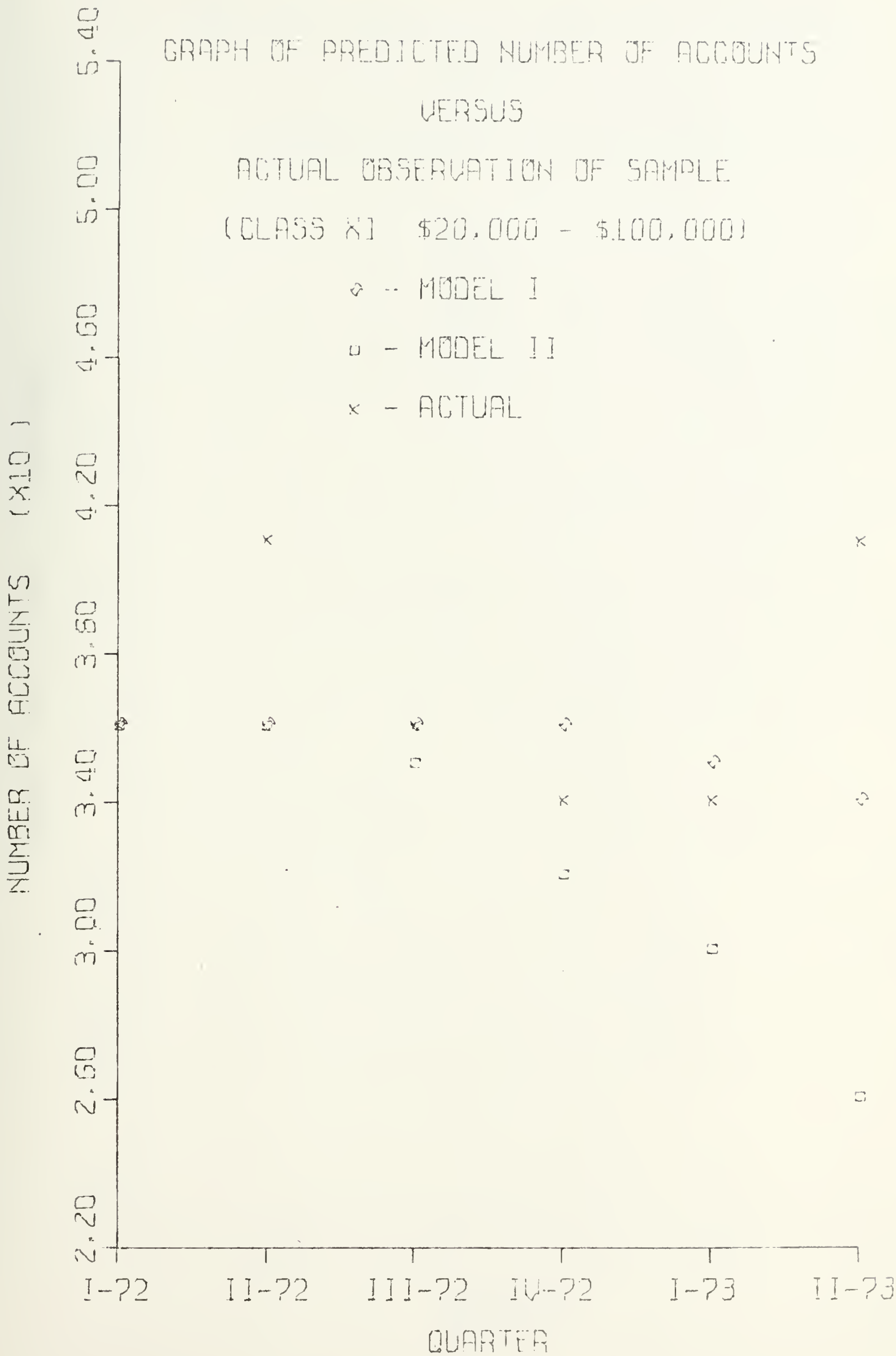

GRAPH OF PREDICTED NUMBER OF ACCOUNTS
VERSUS
ACTUAL OBSERVATION OF SAMPLE
(CLASS VII) $10,000 - $11,999)
- MODEL I
- MODEL II
x - ACTUAL
NUMBER OF ACCOUNTS (X10)
6.80
6.40
6.00
5.60
5.20
4.80
4.40
4.00
3.60
I-72
II-72
III-72
IV-72
I-73
II-73
QUARTER


GRAPH OF PREDICTED NUMBER OF ACCOUNTS
VERSUS
ACTUAL OBSERVATION OF SAMPLE
(CLASS VIII $12,000 - $13,999)
- MODEL I
- MODEL II
- ACTUAL
NUMBER OF ACCOUNTS (X10)
3.40
3.20
3.00
2.80
2.60
2.40
2.20
2.00
1.80
I-72
II-72
III-72
IV-72
I-73
II-73
QUARTER


GRAPH OF PREDICTED NUMBER OF ACCOUNTS
VERSUS
ACTUAL OBSERVATION OF SAMPLE
(CLASS IX $14,000 - $15,999)
◇ - MODEL I
□ - MODEL II
× - ACTUAL
NUMBER OF ACCOUNTS (X10)
3.10
2.90
2.70
2.50
2.30
2.10
1.90
1.70
1.50
I-72
II-72
III-72
IV-72
I-73
II-73
QUARTER

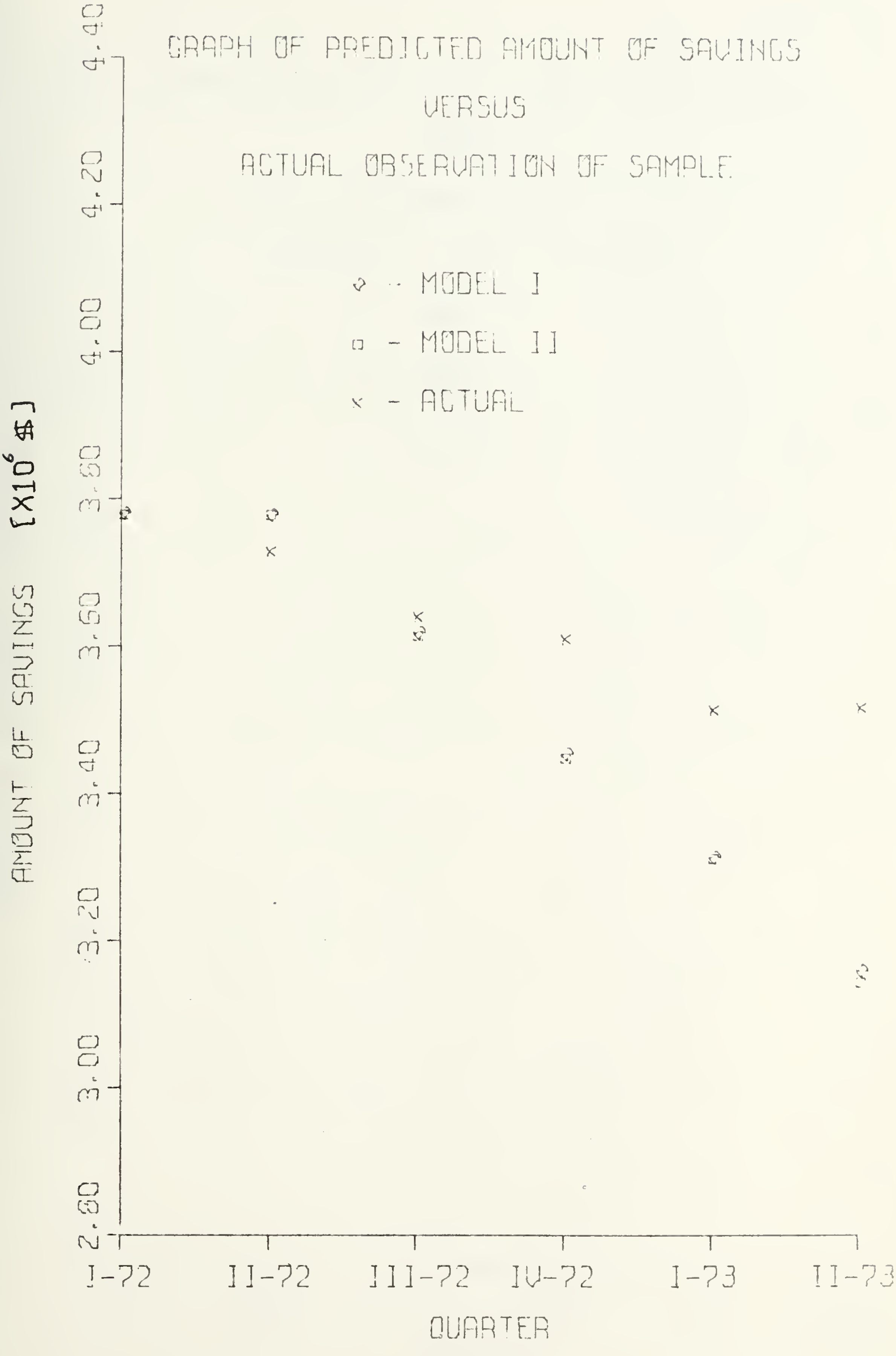


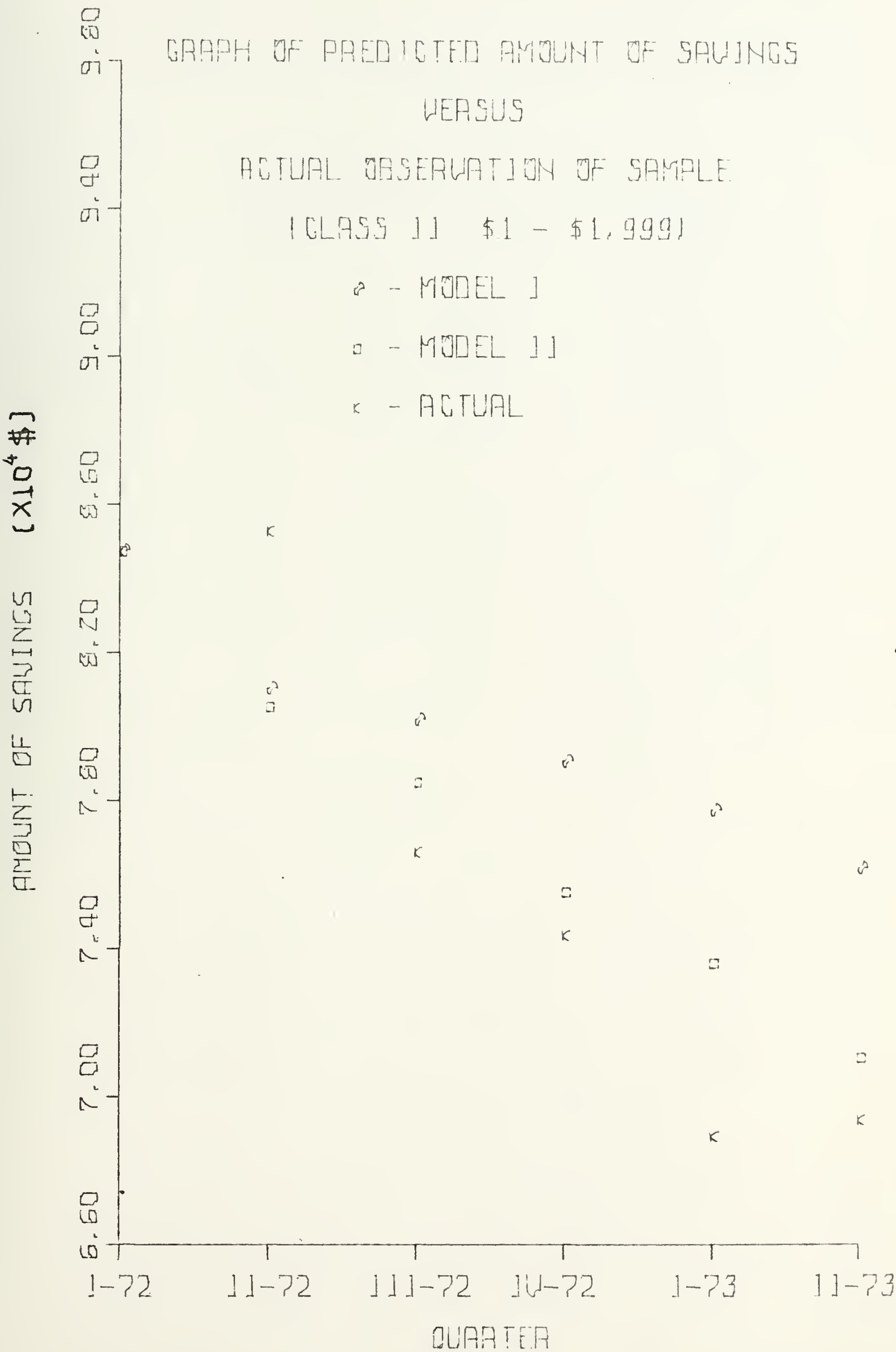
GRAPH OF PREDICTED NUMBER OF ACCOUNTS
VERSUS
ACTUAL OBSERVATION OF SAMPLE
(CLASS K $16,000 - $19,999)
- MODEL I
- MODEL II
- ACTUAL
NUMBER OF ACCOUNTS (X10)
1.00
1.40
1.80
2.20
2.60
3.00
3.40
3.80
4.20
I-72
II-72
III-72
IV-72
I-73
II-73
QUARTER

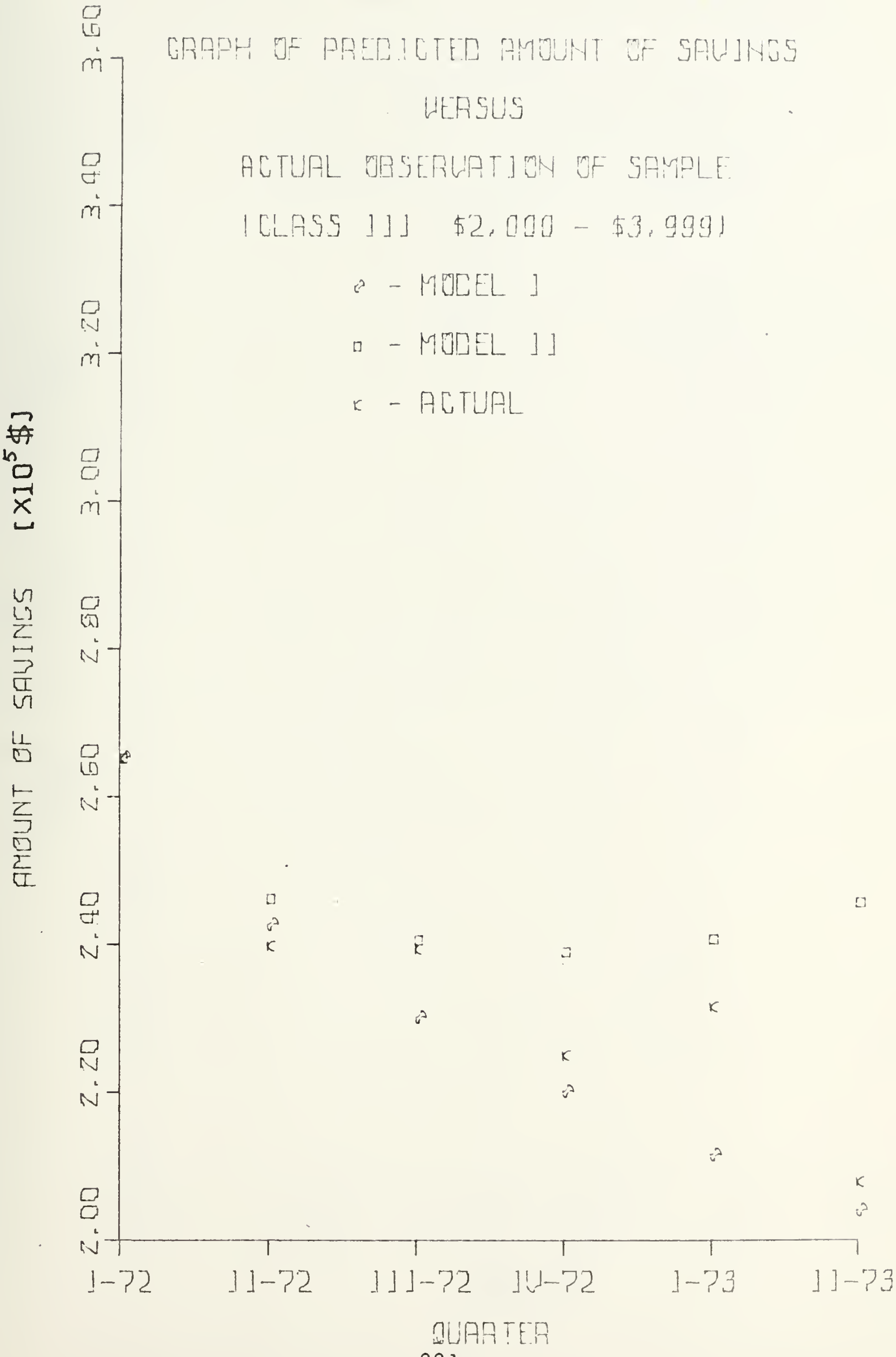


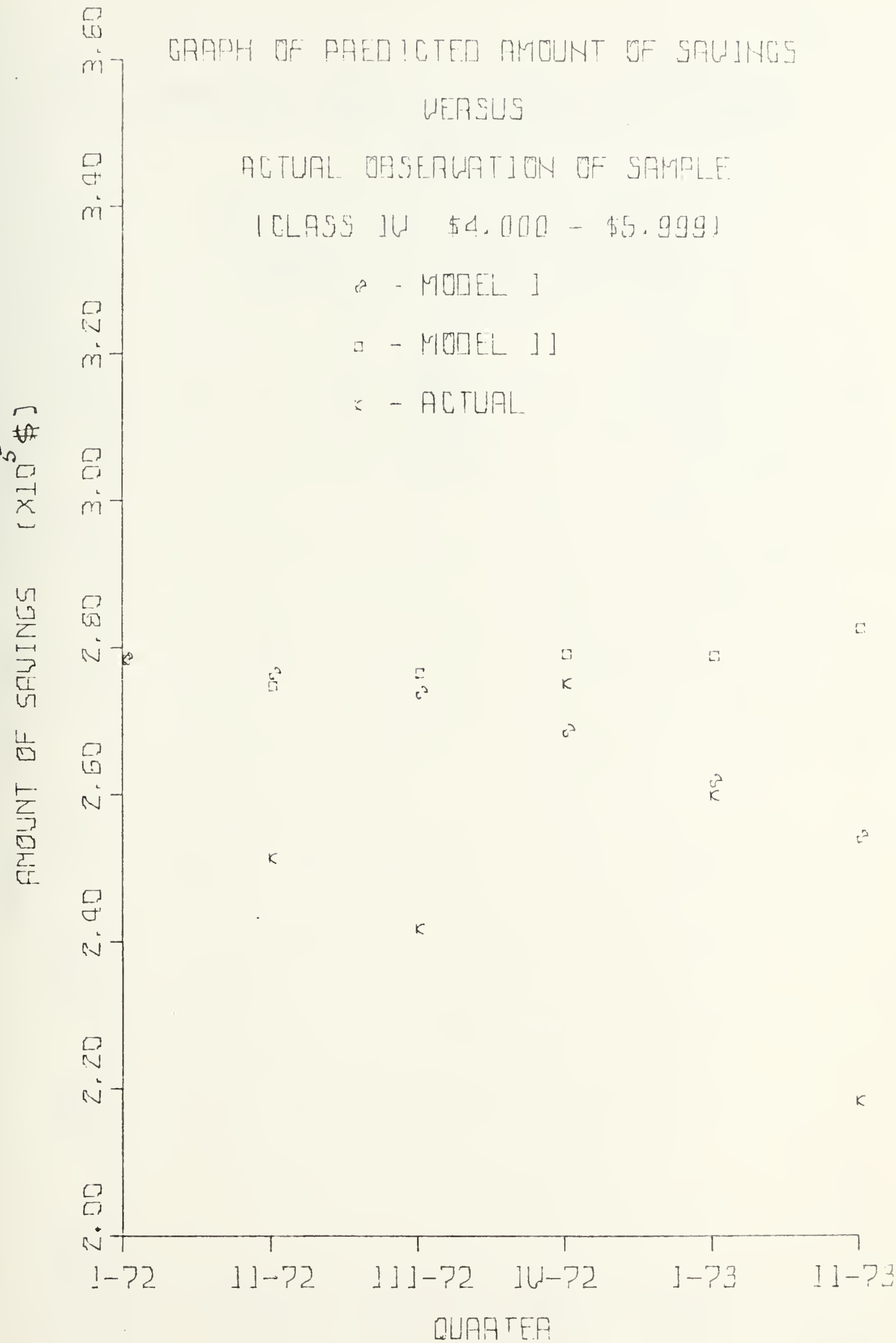
GRAPH OF PREDICTED NUMBER OF ACCOUNTS
VERSUS
ACTUAL OBSERVATION OF SAMPLE
(CLASS XI $20,000 - $100,000)
◇ - MODEL I
□ - MODEL II
× - ACTUAL
NUMBER OF ACCOUNTS (X10)
5.40
5.00
4.60
4.20
3.80
3.40
3.00
2.60
2.20
I-72
II-72
III-72
IV-72
I-73
II-73
QUARTER

APPENDIX K

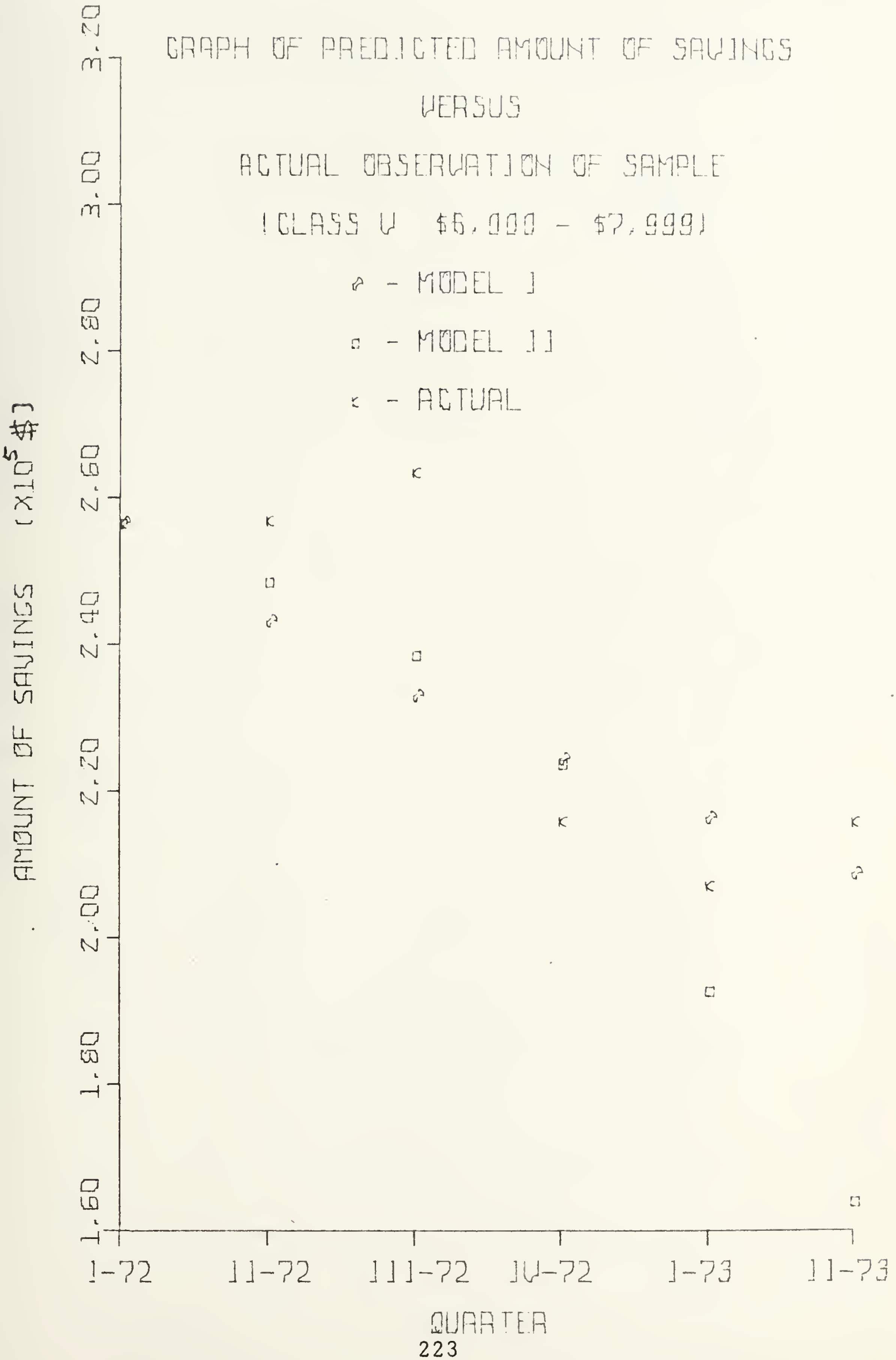

GRAPH OF PREDICTED AMOUNT OF SAVINGS
VERSUS
ACTUAL OBSERVATION OF SAMPLE
◇ - MODEL I
□ - MODEL II
× - ACTUAL
AMOUNT OF SAVINGS [X10^6 $]
4.40
4.20
4.00
3.80
3.60
3.40
3.20
3.00
2.80
I-72
II-72
III-72
IV-72
I-73
II-73
QUARTER

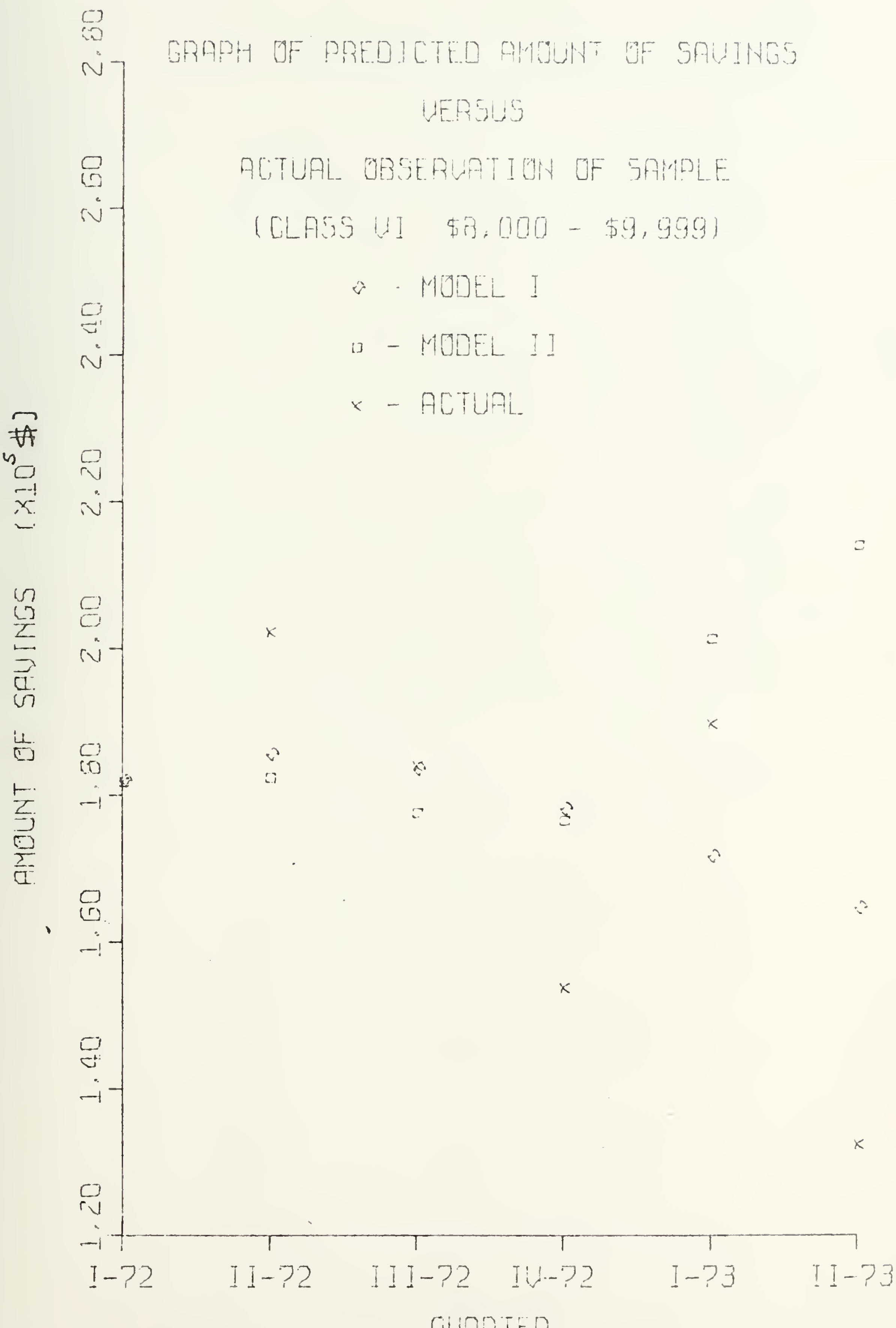

GRAPH OF PREDICTED AMOUNT OF SAVINGS
VERSUS
ACTUAL OBSERVATION OF SAMPLE
(CLASS II $1 - $1,999)
- MODEL I
- MODEL II
- ACTUAL
AMOUNT OF SAVINGS (X10⁴ $)
9.80
9.40
9.00
8.60
8.20
7.80
7.40
7.00
6.60
I-72
II-72
III-72
IV-72
I-73
II-73
QUARTER

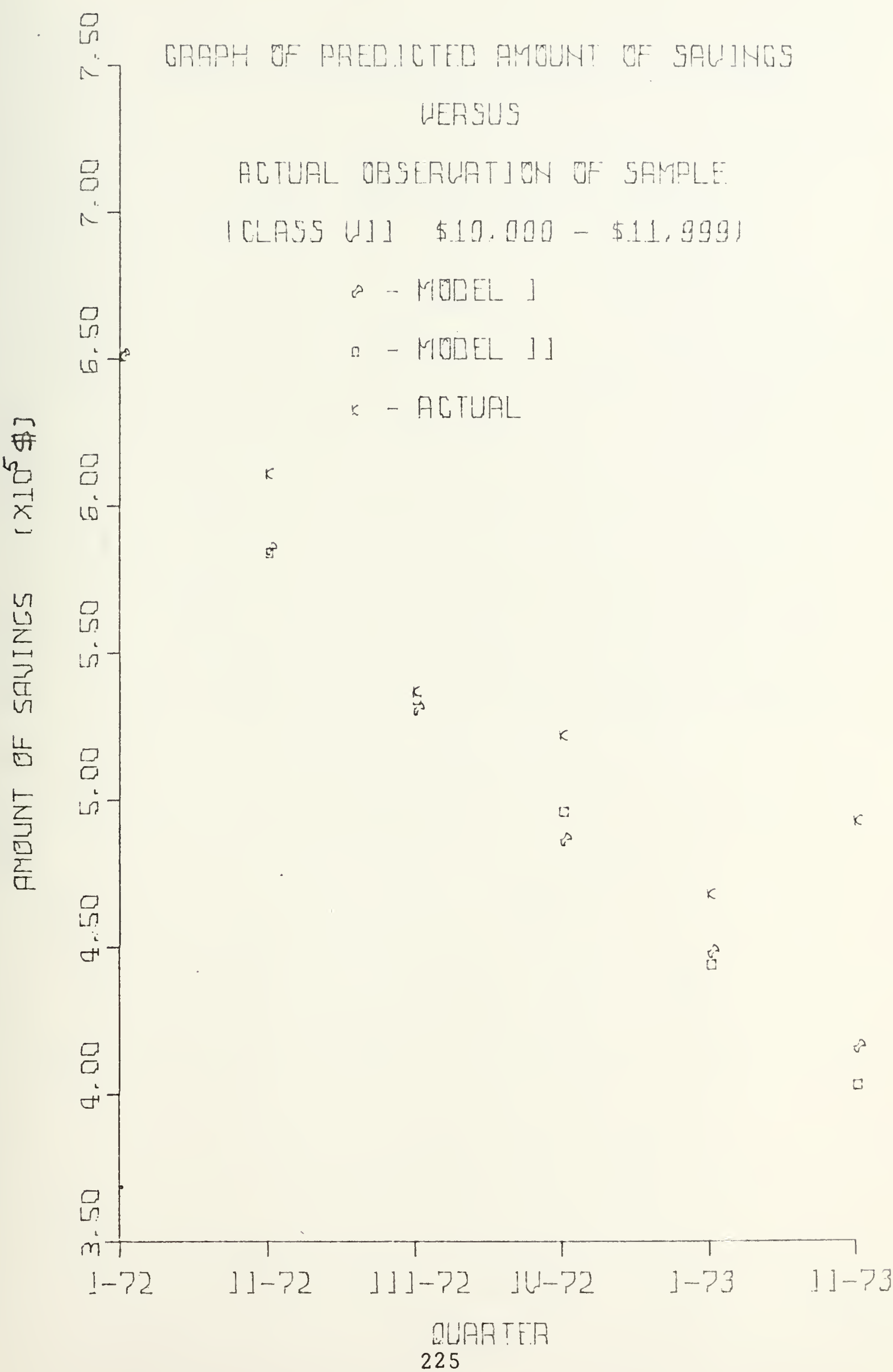

GRAPH OF PREDICTED AMOUNT OF SAVINGS
VERSUS
ACTUAL OBSERVATION OF SAMPLE
(CLASS III $2,000 - $3,999)
- MODEL I
- MODEL II
- ACTUAL
AMOUNT OF SAVINGS [X10^5 $]
3.60
3.40
3.20
3.00
2.80
2.60
2.40
2.20
2.00
I-72
II-72
III-72
IV-72
I-73
II-73
QUARTER



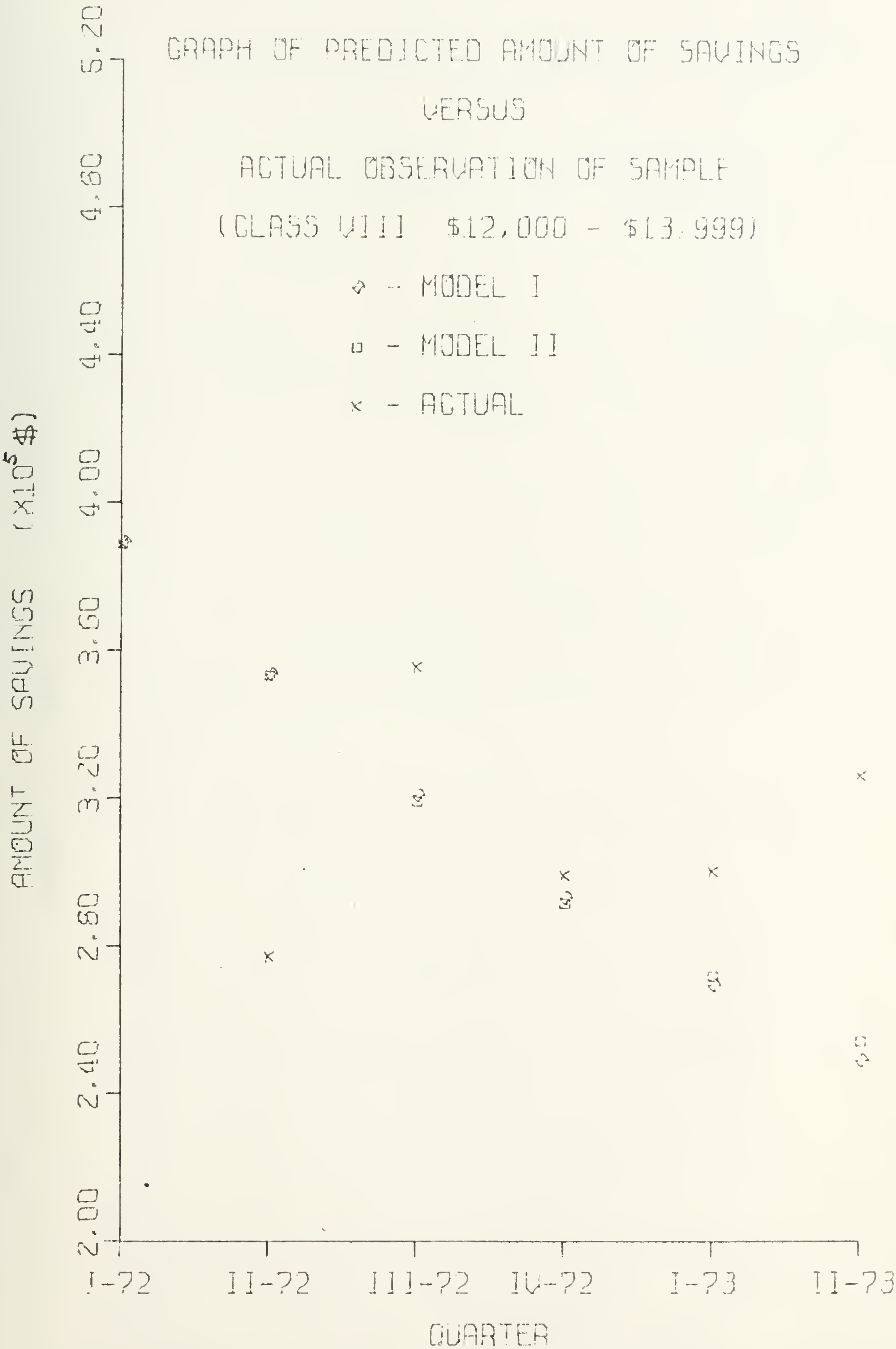
GRAPH OF PREDICTED AMOUNT OF SAVINGS
VERSUS
ACTUAL OBSERVATION OF SAMPLE
(CLASS IV $4,000 - $5,999)
- MODEL I
- MODEL II
- ACTUAL
AMOUNT OF SAVINGS (X10^5 $)
2.00
2.20
2.40
2.60
2.80
3.00
3.20
3.40
3.60
I-72
II-72
III-72
IV-72
I-73
II-73
QUARTER

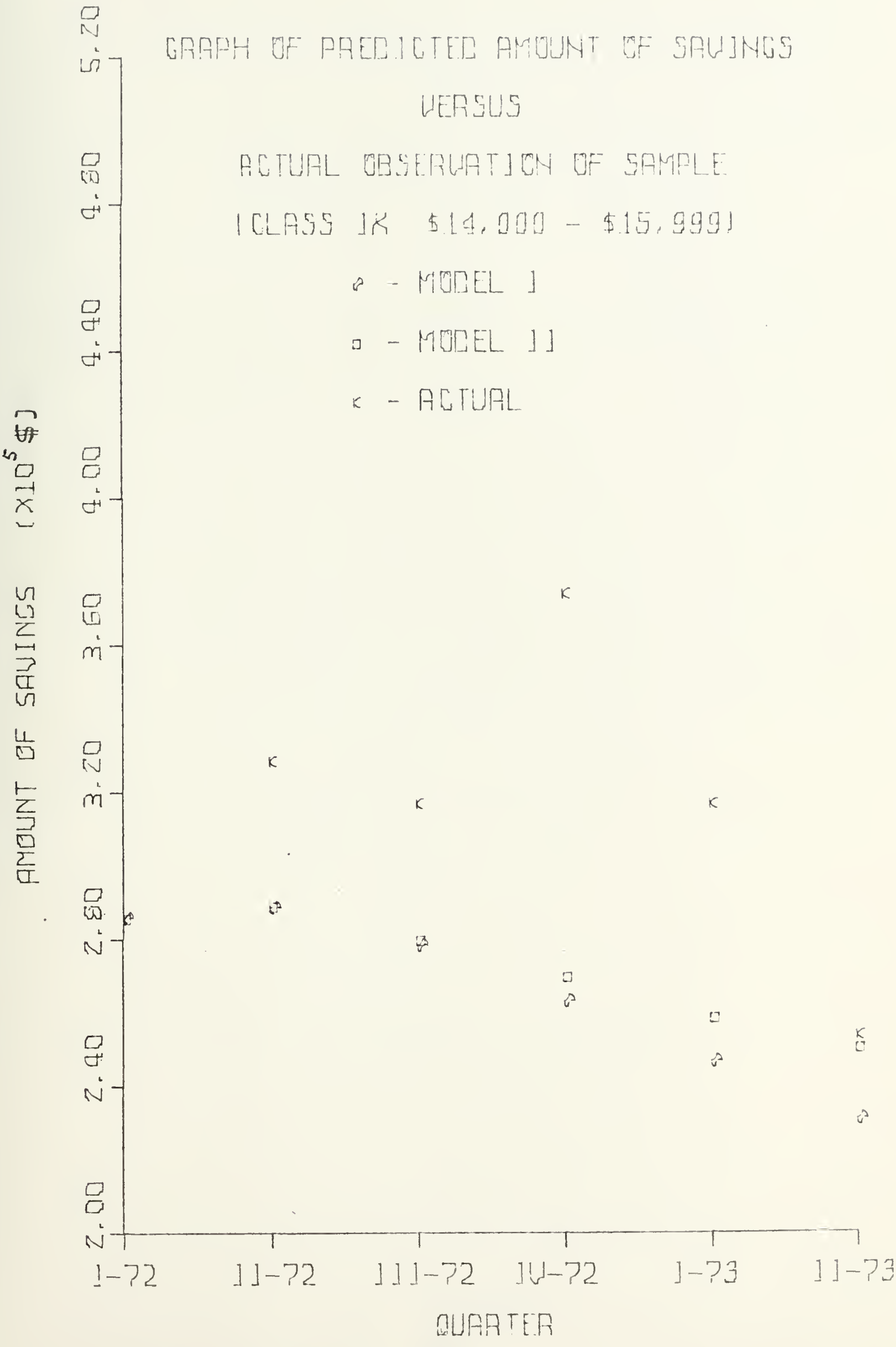

GRAPH OF PREDICTED AMOUNT OF SAVINGS
VERSUS
ACTUAL OBSERVATION OF SAMPLE
(CLASS V $6,000 - $7,999)
- MODEL I
- MODEL II
- ACTUAL
AMOUNT OF SAVINGS (X10^5 $)
3.20
3.00
2.80
2.60
2.40
2.20
2.00
1.80
1.60
I-72
II-72
III-72
IV-72
I-73
II-73
QUARTER


GRAPH OF PREDICTED AMOUNT OF SAVINGS
VERSUS
ACTUAL OBSERVATION OF SAMPLE
(CLASS VI $8,000 - $9,999)
◇ - MODEL I
□ - MODEL II
× - ACTUAL
AMOUNT OF SAVINGS (X10^5 $)
2.80
2.60
2.40
2.20
2.00
1.80
1.60
1.40
1.20
I-72
II-72
III-72
IV-72
I-73
II-73
QUARTER

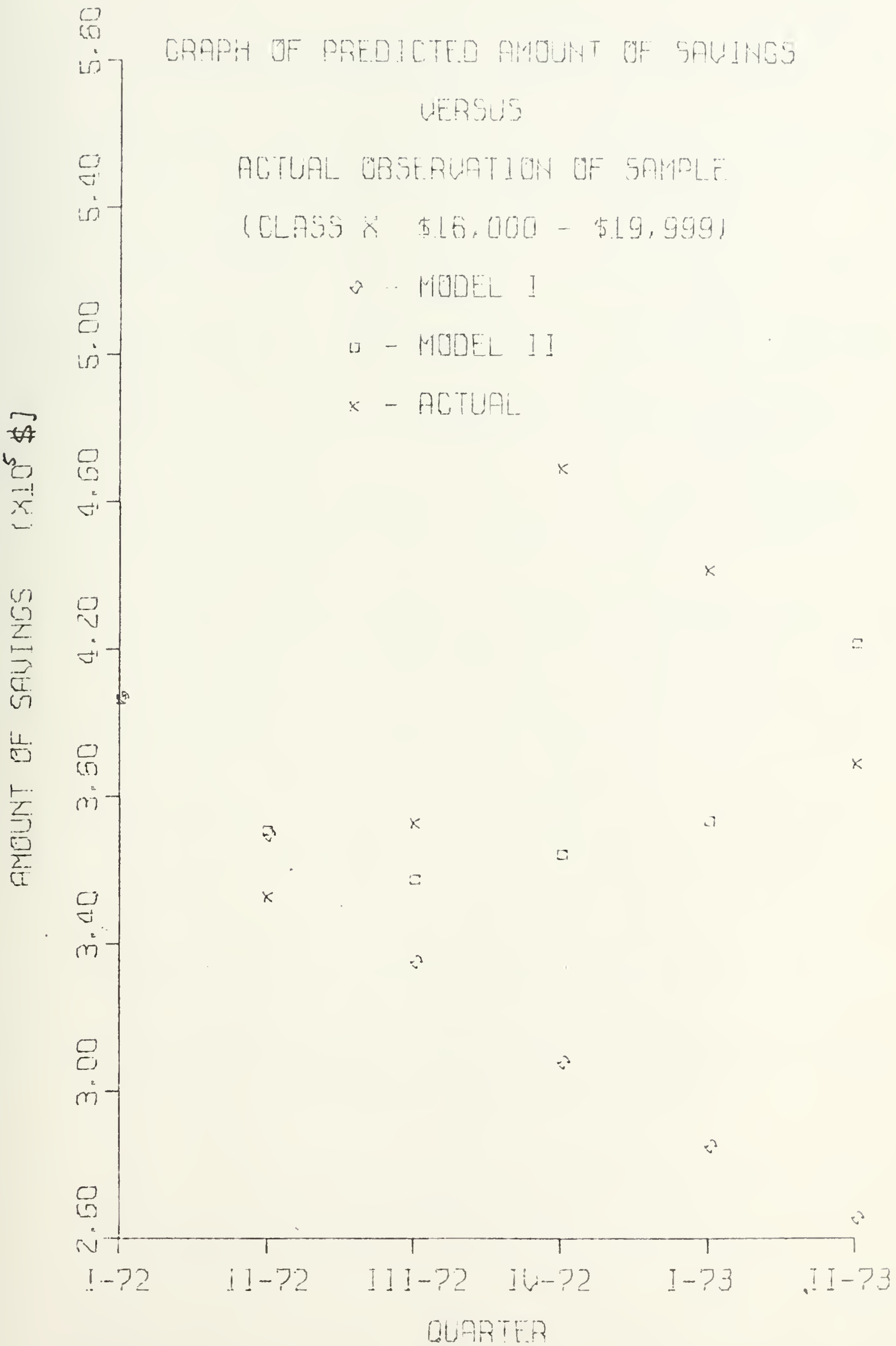


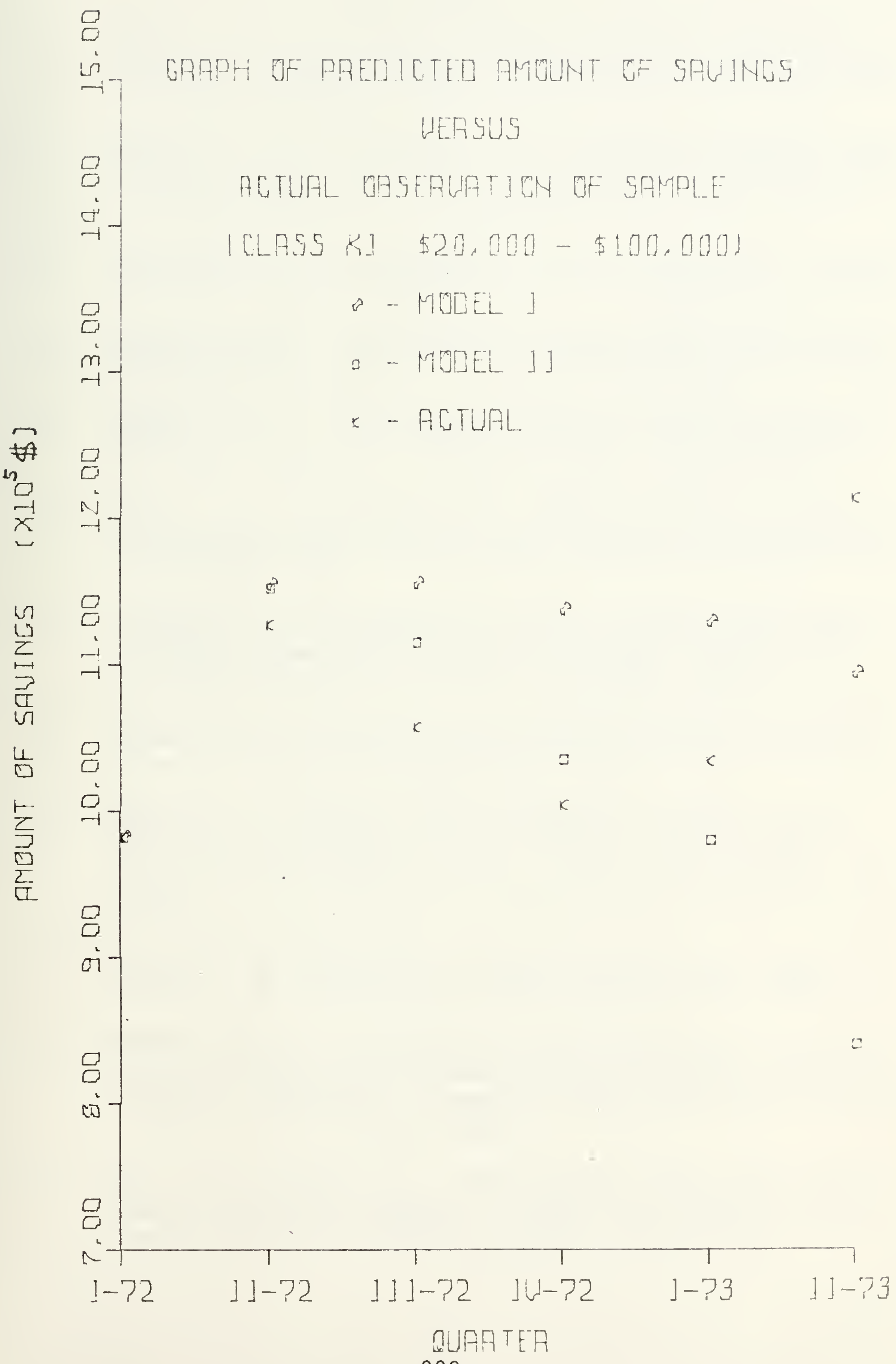
GRAPH OF PREDICTED AMOUNT OF SAVINGS
VERSUS
ACTUAL OBSERVATION OF SAMPLE
(CLASS VII $10,000 - $11,999)
- MODEL I
- MODEL II
- ACTUAL
AMOUNT OF SAVINGS (X10^5 $)
7.50
7.00
6.50
6.00
5.50
5.00
4.50
4.00
3.50
I-72
II-72
III-72
IV-72
I-73
II-73
QUARTER


GRAPH OF PREDICTED AMOUNT OF SAVINGS
VERSUS
ACTUAL OBSERVATION OF SAMPLE
(CLASS VIII $12,000 - $13,999)
◇ - MODEL I
□ - MODEL II
x - ACTUAL
AMOUNT OF SAVINGS (X10⁵ $)
5.20
4.80
4.40
4.00
3.60
3.20
2.80
2.40
2.00
I-72
II-72
III-72
IV-72
I-73
II-73
QUARTER



GRAPH OF PREDICTED AMOUNT OF SAVINGS
VERSUS
ACTUAL OBSERVATION OF SAMPLE
(CLASS IK $14,000 - $15,999)
- MODEL I
- MODEL II
- ACTUAL
AMOUNT OF SAVINGS (X10^5 $)
5.20
4.80
4.40
4.00
3.60
3.20
2.80
2.40
2.00
I-72
II-72
III-72
IV-72
I-73
II-73
QUARTER


GRAPH OF PREDICTED AMOUNT OF SAVINGS
VERSUS
ACTUAL OBSERVATION OF SAMPLE
(CLASS X $16,000 - $19,999)
◇ - MODEL I
□ - MODEL II
x - ACTUAL
AMOUNT OF SAVINGS (X10⁵ $)
5.80
5.40
5.00
4.60
4.20
3.80
3.40
3.00
2.60
I-72
II-72
III-72
IV-72
I-73
II-73
QUARTER


GRAPH OF PREDICTED AMOUNT OF SAVINGS
VERSUS
ACTUAL OBSERVATION OF SAMPLE
(CLASS K) $20,000 - $100,000)
- MODEL I
- MODEL II
- ACTUAL
AMOUNT OF SAVINGS (X10^5 $)
7.00
8.00
9.00
10.00
11.00
12.00
13.00
14.00
15.00
I-72
II-72
III-72
IV-72
I-73
II-73
QUARTER

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The Markov chain model, with extensions to cover the phenomena of arrivals and departures, was applied to a population of savings accounts, in a savings institution, to forecast the size distribution, total number of accounts and total amount of savings of the population. The stochastic processes governing the behavior of the population were first assumed to be time stationary. This assumption was then relaxed and an econometric model was used to predict

20. (Cont'd)

future values of the parameters of the nonstationary model. Both models were validated by comparing predicted size distributions, total number of accounts and total amount of savings against observed values. The chi square goodness of fit test was used in the comparison. The fundamental matrix of the stationary model was also used to predict the equilibrium distribution and related measures of the population.

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