COMPUTER SOLUTION OF HALLEN'S INTEGRAL EQUATION ON MULTI-ELEMENT ARRAYS EMPLOYING THE TWO TERM APPROXIMATE CURRENT DISTRIBUTION

Charles William Schillinger

Library
Naval Postgraduate School
Monterey, California 93940

NAVAL POSTGRADUATE SCHOOL
Monterey, California

THESIS

Computer Solution of Hallen's Integral
Equation on Multi-Element Arrays Employing
the Two Term Approximate Current Distribution

by

Charles William Schillinger

Thesis Advisor: R. W. Adler

March 1973

T 154597

Approved for public release; distribution unlimited

Computer Solution of Hallen's Integral Equation on Multi-Element Arrays Employing the Two Term Approximate Current Distribution

by

Charles William Schillinger
Captain, United States Marine Corps
B.S.E.E., Pennsylvania State University, 1967

Submitted in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE IN ELECTRICAL ENGINEERING

from the

NAVAL POSTGRADUATE SCHOOL

Library
Naval Postgraduate School
Monterey, California 93940

ABSTRACT

The objective of this analytical study was to develop a rapid theoretical analysis on approximate half wavelength elements of an antenna configured in a co-planar, symmetrical array. A computer program was written employing the method of moments approach to the solution of Hallen's integral equation with an approximate two term entire domain expansion assumed for the current distributions. With the solution of the current distributions on each element, additional calculations were made for the input impedance and admittance values, field distributions, power gain, and a graphical output of the radiation pattern.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF DRAWINGS

ACKNOWLEDGMENTS

I wish to express my sincere appreciation to Professor R. W. Adler for suggesting this thesis topic and providing constant guidance and support throughout.

Mr. E. G. Neely and Mr. J. M. Callaghan provided considerable assistance in this project.

Finally, I wish to thank my wife, Barbara, for typing the preliminary drafts and providing suggestions, understanding and encouragement during this project.

I. INTRODUCTION

A. DEVELOPMENT OF THE STUDY

A goal in the theory of multi-element antenna design is to have a tool which easily analyzes the behavior of an array. The equations that describe the behavior of a one voltage-driven element with a simple sinusoidal current distribution are readily available in many texts [1]. However, upon the introduction of a second or third element with different physical dimensions--either voltage driven or parasitic--the behavior without the assumption of a simple sinusoidal current distribution becomes rather complex and lends itself to a computer oriented solution.

A large portion of the theory employed in the programming of this software package was presented and discussed in a seminar at the University of Mississippi, sponsored by the National Science Foundation in 1972 [2].

The computer program of this study was written in the XDS Fortran IV language. It was specifically to be used in the computer laboratory of the electircal engineering department at the Naval Postgraduate School, Monterey, California [3].

B. ANALYSIS OF THE PROBLEM

A brief synopsis of the theory of this study is as follows:

1. The Development of Hallen's Integral Equation

2. The Solving of Hallen's Equation for the Current Distribution by the Use of an Assumed Two Term Approximate Solution Employing the Method of Moments

3. <u>The Development of the Associated Equations for the Relative Vertical Plane Pattern, Relative Horizontal Plane Pattern, and the Gain of the Array</u>

4. <u>An Outlining of the Computer Programming Procedures</u>

C. OUTLINE OF THE RESULTS

An antenna array of from one to ten elements was modeled in free space. The simple physical parameters of the designed array are the input to the computer program. All calculations were made by the computer, producing an output of: (a) a listing of the current distribution on each element of the array; (b) the input impedance and admittance quantities for each of the voltage-driven elements; (c) a listing of the relative field distributions about the array; (d) a graphics output of the radiation patterns; and (e) the values of the gain in magnitude and in decibels.

D. SCOPE AND LIMITATIONS

The procedures outlined were completed for a symmetrical, co-planar array of from one to ten elements in free space with each element being approximately a half wavelength long. No boom effects were included. Each of the ten elements may be voltage-driven or an array of one voltage-driven element and nine parasitic (reflectors or directors) elements may be configured. The latter includes possible Yagi-Uda arrays. The elements may be spaced arbitrarily apart and the center-driven voltage may be any complex value. The radii of the elements are restricted to the thin wire approximation ($a < .01\ \lambda$). The graphical radiation pattern outputs were of the relative vertical and horizontal planes.

No optimality procedures were introduced. Further study in this area could be extended by introducing a finitely conducting ground plane to the model; include elements which are other than co-planar; or include elements which are not symmetrical about the axis, but are at angles to the axis.

II. FORMULATION OF HALLEN'S INTEGRAL EQUATION

A. THE DIFFERENTIAL EQUATION FOR THE VECTOR POTENTIAL FROM THE ELECTRIC FIELD INTENSITY

The basis of this study is the calculation of the approximated two-term current in the solution of Hallen's integral equation, as all other quantities are determined from this data. In this section, a brief discussion of the formulation of Hallen's integral equation is presented. The objective is to obtain an expression of the current distribution of a cylindrical center-fed voltage driven antenna in terms of its length and diameter.

Consider a voltage driven element as depicted in Figure 1. It is known that:

$$E'_Z = E_Z \tag{II-1}$$

$$E'_\rho = E_\rho \tag{II-2}$$

In this case, the electrical field intensity, $\overline{E}$, and the vector potential, $\overline{A}$, exist only in the Z-component. Therefore, the equation,

$$\overline{E} = \frac{-jc^2}{\omega} \nabla(\nabla\cdot\overline{A}) - j\omega\overline{A} \tag{II-3}$$

becomes,

$$\frac{d^2\overline{A}_Z}{dZ^2} + k^2\overline{A}_Z = \frac{jk^2\overline{E}_Z}{\omega} \tag{II-4}$$

Equation (II-4) is correct everywhere, where $\overline{A}_Z(x,y,z)$ is the vector potential and $\overline{E}_Z(x,y,z)$ is the electric field intensity. This equation is solved on each element of an array of co-planar elements in the YZ plane as seen in Figure 2. The radii of the elements in this figure are represented as 'a_n'.

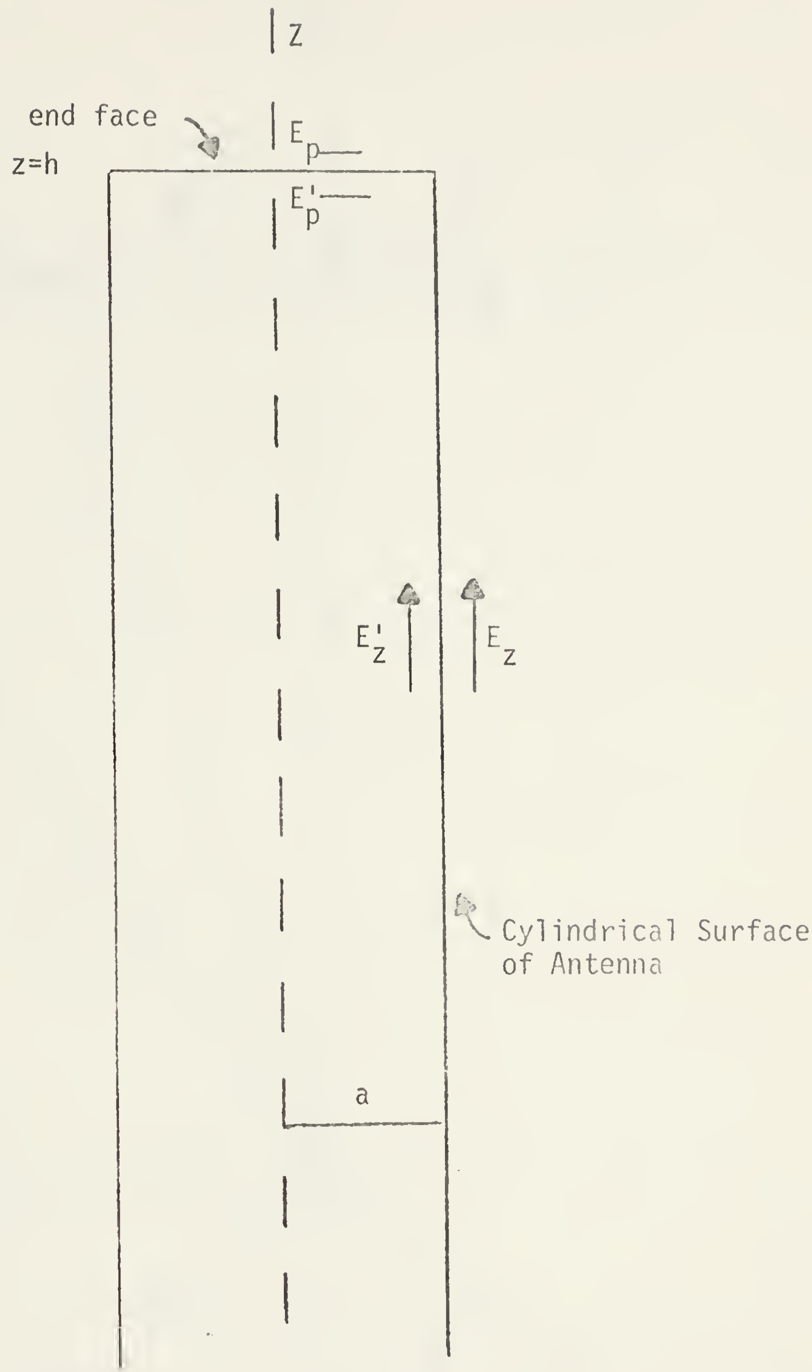

FIGURE 1. COMPONENTS OF THE ELECTRIC FIELD AT THE SURFACE OF THE ANTENNA

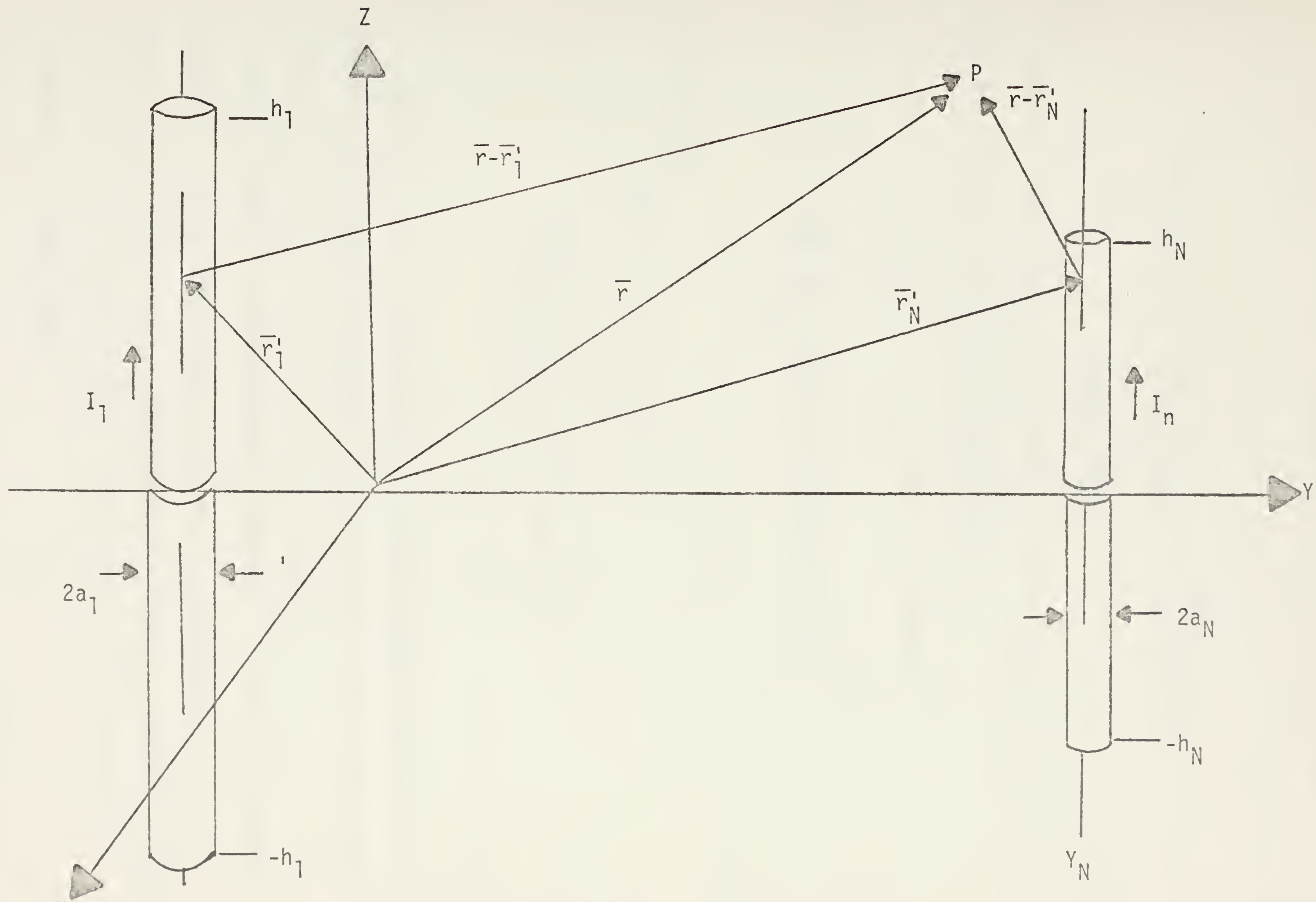

FIGURE 2. CONFIGURATION OF PARALLEL ELEMENTS IN THE Y Z PLANE

Evaluating (II-4) on the surface of each of the N elements produces:

$$\frac{d^2\overline{A}_Z(a_n,y_n,z)}{dZ^2} + k^2\overline{A}_Z(a_n,y_n,z) = \frac{jk^2\overline{E}_Z(a_n,y_n,z)}{\omega}$$

$$Z\varepsilon(-h_n, +h_n) \qquad \text{(II-5)}$$

Solving (II-5) with B_n and C_n being arbitrary constants and Z' being an arbitrary point on the 'nth' element gives:

$$\overline{A}_Z(a_n,y_n,z) = C_n\cos(kZ) + B_n\sin(kZ)$$

$$+\frac{jk}{\omega}\int_{S=Z'_n}^{Z} \overline{E}_Z(a_n,y_n,S)\sin(Z-S)k\,dS$$

$$n = 1, \ldots .N \qquad \text{(II-6)}$$

simplifying,

$$\overline{A}_Z(a_n,y_n,z) = C_n\cos(kZ)$$

$$+\frac{jk}{\omega}\int_{S=0}^{Z} \overline{E}_Z(a_n,y_n,S)\sin(Z-S)k\,dS$$

$$n = 1, \ldots .N \qquad \text{(II-7)}$$

Equation (II-7) is achieved with the requirements that the array of elements are perfectly symmetric in the YZ plane so that the scalar potential is zero at the center of each element.

Now an expression for the vector potential, $\overline{A}$, will be expressed in terms of the antenna current and the two expressions will be set equal to each other, for the formulation of Hallen's integral equation. The only unknowns of the equation will be the current distribution and a constant of integration.

B. THE DIFFERENTIAL EQUATION FOR THE VECTOR POTENTIAL FROM ELEMENT CURRENTS

At an arbitrary point, 'P', in Figure 2, the vector potential can be written:

$$\bar{A}(\bar{r}) = \sum_{i=1}^{N} \bar{A}_i(\bar{r}) \tag{II-8}$$

where $\bar{A}_i(\bar{r})$ is the vector potential due to the current on the 'ith' element and is expressed as:

$$\bar{A}_{i}(\bar{r}) = \frac{\mu}{4\pi\mu_Z} \int_{z_i'=-h_i}^{h_i} I_i(z_i') \frac{e^{-jk|\bar{r} - \bar{r}_i'|}}{|\bar{r} - \bar{r}_i'|} dz_i' \tag{II-9}$$

where,

$$|\bar{r} - \bar{r}_i'| = \left[x^2 + (y - y_i')^2 + (z - z_i')^2\right]^{\frac{1}{2}} \tag{II-10}$$

In the above equation, x, y, and z are the coordinates of the point 'P'. The y_i' and z_i' are the coordinates on the 'ith' element.

Substuting (II-9) and (II-10) into (II-8) produces (II-11).

$$\bar{A}_Z(\bar{r}) = \frac{\mu}{4\pi} \sum_{i=1}^{N} \int_{z_i'=-h_i}^{h_i} I_i(z_i') \frac{e^{-jk|\bar{r} - \bar{r}_i'|}}{|\bar{r} - \bar{r}_i'|} dz_i' \tag{II-11}$$

Equation (II-11) is the vector potential at an arbitrary point in space due to all N antenna currents. Evaluating (II-11) along the surface of the 'nth' element produces (II-12).

$$\bar{A}_Z(a_n, y_n, z) = \frac{\mu}{4\pi} \sum_{i=1}^{N} \int_{z_i'=-h_i}^{h_i} I_i(z_i') \frac{e^{-jkR'_{ni}}}{R'_{ni}} dz_i' \tag{II-12}$$

where:

$$R'_{ni} = \left[a_n^2 + (y_n - y_i')^2 + (z - z_i')^2\right]^{\frac{1}{2}} \tag{II-13}$$

C. HALLEN'S INTEGRAL EQUATION

Setting equation (II-7) and equation (II-12) equal to each other along each of the N elements results in Hallen's integral equation (II-14).

$$\frac{\mu}{4\pi} \sum_{i=1}^{N} \int_{Z'=-h_i}^{h_i} I_i(Z') \frac{e^{-jkR_{ni}(Z,Z')}}{R_{ni}(Z,Z')} dZ' =$$

$$C_n \cos(kZ) + \frac{jk}{\omega} \int_{S=0}^{Z} E_Z(a_n, y_n, S) \sin(Z-S)k \, dS$$

$$n = 1, \ldots N \qquad \text{(II-14)}$$

where:

$$R_{ni}(Z,Z') = \left[a_n^2 + (y_n - y_i)^2 + (Z - Z')^2 \right]^{\frac{1}{2}} \qquad \text{(II-15)}$$

Since the current in each element is an even function of Z, the limits of the integrals can be changed to zero and h_i. With this change the set of N equations in (II-14) becomes (II-16), where,

$$V_n(Z) = \frac{-jV_n}{60} \sin(kZ) \qquad \text{(II-17)}$$

$$t(z) = \frac{-4\pi}{\eta} \cos(kZ) = \frac{-1}{30} \cos(kZ) \qquad \text{(II-18)}$$

$$K_{ni}(Z,Z') = \frac{e^{-jkR_{ni}(Z,Z')}}{R_{ni}(Z,Z')} + \frac{e^{-jkR_{ni}(Z,Z')}}{R_{ni}(Z,-Z')} \qquad \text{(II-19)}$$

With this formulation of Hallen's integral equation, it was this study's objective to solve the N set of equations (II-16) for the unknowns of $I_1 \ldots I_N$ and the constants of integration $C_1 \ldots C_N$ as a function of the arbitrary distance along each element Z'.

$$\int_{Z'=0}^{h_1} I_1(Z')K_{11}(Z,Z')dZ' + \cdots + \int_{Z'=0}^{h_i} I_i(Z')K_{1i}(Z,Z')dZ' + \cdots + \int_{Z'=0}^{h_N} I_N(Z')K_{1N}(Z,Z')dZ' = -C_1 t(Z) + V_1(Z)$$

$$\vdots$$

$$\int_{Z'=0}^{h_1} I_1(Z')K_{n1}(Z,Z')dZ' + \cdots + \int_{Z'=0}^{h_i} I_i(Z')K_{ni}(Z,Z')dZ' + \cdots + \int_{Z'=0}^{h_N} I_N(Z')K_{nN}(Z,Z')dZ' = -C_n t(Z) + V_n(Z)$$

$$\vdots$$

$$\int_{Z'=0}^{h_1} I_1(Z')K_{N1}(Z,Z')dZ' + \cdots + \int_{Z'=0}^{h_i} I_i(Z')K_{Ni}(Z,Z')dZ' + \cdots + \int_{Z'=0}^{h_N} I_N(Z')K_{NN}(Z,Z')dZ' = -C_N t(Z) + V_N(Z)$$

(II-16)

III. THE THEORY OF THE TWO TERM APPROXIMATE SOLUTION TO HALLEN'S INTEGRAL EQUATION

A. TWO TERM APPROXIMATION

Even though the N set of equations (II-16) may look rather complicated, there exists only two sets of unknown quantities. Knowing the values of the parameters of an array of elements, the currents and constants of integration are easily determined.

The technique of the two-term solution assumes that the current on the element may be expressed as (III-1).

$$I_i(Z) = \sum_{p=1}^{P} b_p^i \sin\left[p\pi/2h_i \; (h_i - Z)\right] \quad P = 2 \qquad \text{(III-1)}$$

Expanding (III-1) produces (III-2).

$$I_i(Z) = b_1^1 \sin\left[\pi/2h_i \; (h_i - Z)\right] + b_2^1 \sin\left[2\pi/h_i \; (h_i - Z)\right] \qquad \text{(III-2)}$$

In equation (III-1), the 'i' represents the number of the element and the 'p' is the number of the coefficient in the sinusoidal expansion. This expansion sets the number of unknowns at two current coefficients and one constant of integration per element. Therefore, there are three unknowns per element in the set of equations (II-16). Applying (III-2) to (II-16) produces the set of N equations in (III-3). For an example, a three element array (N=3) would have nine unknowns in the set of three equations. They are, as seen in (III-3), b_1^1, b_2^1, b_1^2, b_2^2, b_1^3, b_2^3, C_1, C_2, and C_3.

B. METHOD OF MOMENTS

In order to determine these unkowns, the method of moments is employed [4]. The set of testing functions selected are a set of delta

$$
\begin{aligned}
& b_1^1\int_{Z'=0}^{h_1}\sin\frac{\pi}{2h_1}(h_1-Z')K_{11}(Z,Z')dZ'+b_2^1\int_{Z'=0}^{h_1}\sin\frac{\pi}{h_1}(h_1-Z')K_{11}(Z,Z')dZ'+\cdots+\\
& \quad b_1^i\int_{Z'=0}^{h_i}\sin\frac{\pi}{2h_i}(h_i-Z')K_{1i}(Z,Z')dZ'+b_2^i\int_{Z'=0}^{h_i}\sin\frac{\pi}{h_i}(h_i-Z')K_{1i}(Z,Z')dZ'+\cdots+\\
& \qquad b_1^N\int_{Z'=0}^{h_N}\sin\frac{\pi}{2h_N}(h_N-Z')K_{1N}(Z,Z')dZ'+b_2^N\int_{Z'=0}^{h_N}\sin\frac{\pi}{h_N}(h_N-Z')K_{1N}(Z,Z')dZ' = -C_1t(Z)+V_1(Z)\\
& \vdots\\
& b_1^1\int_{Z'=0}^{h_1}\sin\frac{\pi}{2h_1}(h_1-Z')K_{n1}(Z,Z')dZ'+b_2^1\int_{Z'=0}^{h_1}\sin\frac{\pi}{h_1}(h_1-Z')K_{n1}(Z,Z')dZ'+\cdots+\\
& \quad b_1^i\int_{Z'=0}^{h_i}\sin\frac{\pi}{2h_i}(h_i-Z')K_{ni}(Z,Z')dZ'+b_2^i\int_{Z'=0}^{h_i}\sin\frac{\pi}{h_i}(h_i-Z')K_{ni}(Z,Z')dZ'+\cdots+\\
& \qquad b_1^N\int_{Z'=0}^{h_N}\sin\frac{\pi}{2h_N}(h_N-Z')K_{1N}(Z,Z')dZ'+b_2^N\int_{Z'=0}^{h_N}\sin\frac{\pi}{h_N}(h_N-Z')K_{nN}(Z,Z')dZ' = -C_nt(Z)+V_n(Z)\\
& \vdots\\
& b_1^1\int_{Z'=0}^{h_1}\sin\frac{\pi}{2h_1}(h_1-Z')K_{N1}(Z,Z')dZ'+b_2^1\int_{Z'=0}^{h_1}\sin\frac{\pi}{h_1}(h_1-Z')K_{N1}(Z,Z')dZ'+\cdots+\\
& \quad b_1^i\int_{Z'=0}^{h_i}\sin\frac{\pi}{2h_i}(h_i-Z')K_{Ni}(Z,Z')dZ'+b_2^i\int_{Z'=0}^{h_i}\sin\frac{\pi}{h_i}(h_i-Z')K_{Ni}(Z,Z')dZ'+\cdots+\\
& \qquad b_1^N\int_{Z'=0}^{h_N}\sin\frac{\pi}{2h_N}(h_N-Z')K_{NN}(Z,Z')dZ'+b_2^N\int_{Z'=0}^{h_N}\sin\frac{\pi}{h_N}(h_N-Z')K_{NN}(Z,Z')dZ' = -C_Nt(Z)+V_N(Z)
\end{aligned}
$$

(III-3)

functions, which enables the usage of the point matching procedure. In this study, the selected number of matched points per element equal the number of unknowns per element, that is, three. The point matching process then produces three equations for every one of the N equations in (III-3). Each integral in (III-3) must be solved for Z equalling one of the designated matched points and Z' existing as the dummy variable of integration. The notation used in (III-3) is as follows:

$$S_{mp}^{ni} = \int_{Z'=0}^{h_i} \sin\left[p\pi/2h_i \ (h_i - Z')\right] K_{ni}(Z_m^n, Z') \ dZ' \qquad \text{(III-4)}$$

Here, the 'i' represents the number of the element, 'p' is the number of the coefficient in the sinusoidal expansion (in this study p=1 or p=2), and 'm' is the number of the matched point on a particular element, 'n'. Employing this notation and arranging all the unknowns to the left hand side in (III-3), produces (III-5), where there are three equations for every one in (III-3). In (III-5) as previously, the unknown quantities are the b_p^i coefficients and the C_n constants of integration. Consider again, the example of the three element array (N=3) with its nine unknowns. Equation (III-5) becomes nine simultaneous integral equations with nine unknowns. Every S_{mp}^{ni} becomes a solvable integral with the selection of the matched points (Z_m^n).

C. MATRIX FORMATION

Rearranging (III-5) into matrix form produces (III-6), which is in the form AX=B. The "A" matrix must be filled in with the desired number of zeroes, depending on the number of elements, N. The 'A' matrix will always be square and be a 3N by 3N size matrix. The 'X' and 'B' matrices will always be a 3N by 1 size matrix. Clearly, the

$$
\begin{array}{l}
b_1^1 (S_{11}^{11}) + b_2^1 (S_{12}^{11}) + \cdots\cdots + b_1^i (S_{11}^{1i}) + b_2^i (S_{12}^{1i}) + \cdots\cdots + b_1^N (S_{11}^{1N}) + b_2^N (S_{12}^{1N}) + C_1 t(Z_1^1) = V_1 (Z_1^1) \\
b_1^1 (S_{21}^{11}) + b_2^1 (S_{22}^{11}) + \cdots\cdots + b_1^i (S_{21}^{1i}) + b_2^i (S_{22}^{1i}) + \cdots\cdots + b_1^N (S_{21}^{1N}) + b_2^N (S_{22}^{1N}) + C_1 t(Z_2^1) = V_1 (Z_2^1) \\
b_1^1 (S_{31}^{11}) + b_2^1 (S_{31}^{11}) + \cdots\cdots + b_1^i (S_{31}^{1i}) + b_2^i (S_{32}^{1i}) + \cdots\cdots + b_1^N (S_{31}^{1N}) + b_2^N (S_{32}^{1N}) + C_1 t(Z_3^1) = V_1 (Z_3^1) \\
\quad \vdots \\
b_1^1 (S_{11}^{n1}) + b_2^1 (S_{12}^{n1}) + \cdots\cdots + b_1^i (S_{11}^{ni}) + b_2^i (S_{12}^{ni}) + \cdots\cdots + b_1^N (S_{11}^{nN}) + b_2^N (S_{12}^{nN}) + C_n t(Z_1^n) = V_n (Z_1^n) \\
b_1^1 (S_{21}^{n1}) + b_2^1 (S_{22}^{n1}) + \cdots\cdots + b_1^i (S_{21}^{ni}) + b_2^i (S_{22}^{ni}) + \cdots\cdots + b_1^N (S_{21}^{nN}) + b_2^N (S_{22}^{nN}) + C_n t(Z_2^n) = V_n (Z_2^n) \\
b_1^1 (S_{31}^{n1}) + b_2^1 (S_{32}^{n1}) + \cdots\cdots + b_1^i (S_{31}^{ni}) + b_2^i (S_{32}^{ni}) + \cdots\cdots + b_1^N (S_{31}^{nN}) + b_2^N (S_{32}^{nN}) + C_n t(Z_3^n) = V_n (Z_3^n) \\
\quad \vdots \\
b_1^1 (S_{11}^{N1}) + b_2^1 (S_{12}^{N1}) + \cdots\cdots + b_1^i (S_{11}^{Ni}) + b_2^i (S_{12}^{Ni}) + \cdots\cdots + b_1^N (S_{11}^{NN}) + b_2^N (S_{12}^{NN}) + C_N t(Z_1^N) = V_N (Z_1^N) \\
b_1^1 (S_{21}^{N1}) + b_2^1 (S_{22}^{N1}) + \cdots\cdots + b_1^i (S_{21}^{Ni}) + b_2^i (S_{22}^{Ni}) + \cdots\cdots + b_1^N (S_{21}^{NN}) + b_2^N (S_{22}^{NN}) + C_N t(Z_2^N) = V_N (Z_2^N) \\
b_1^1 (S_{31}^{N1}) + b_2^1 (S_{32}^{N1}) + \cdots\cdots + b_1^i (S_{31}^{Ni}) + b_2^i (S_{32}^{Ni}) + \cdots\cdots + b_1^N (S_{31}^{NN}) + b_2^N (S_{32}^{NN}) + C_N t(Z_3^N) = V_N (Z_3^N)
\end{array}
\qquad \text{(III-5)}
$$

'A' and 'B' matrices contain all known quantities and the 'X' matrix contains all the unknown quantities.

$$
\begin{bmatrix}
S_{11}^{11} & S_{12}^{11} \cdots & S_{11}^{1i} & S_{12}^{1i} \cdots & S_{11}^{1N} & S_{12}^{1N} & t(Z_1^1) & 0 \cdots & \cdots & 0 \\
S_{21}^{11} & S_{22}^{11} \cdots & S_{21}^{1i} & S_{22}^{1i} \cdots & S_{21}^{1N} & S_{22}^{1N} & t(Z_2^1) & 0 \cdots & \cdots & 0 \\
S_{31}^{11} & S_{31}^{11} \cdots & S_{31}^{1i} & S_{32}^{1i} \cdots & S_{31}^{1N} & S_{32}^{1N} & t(Z_3^1) & 0 \cdots & \cdots & 0 \\
\vdots & & & & & & & & & \\
S_{11}^{n1} & S_{12}^{n1} \cdots & S_{11}^{ni} & S_{12}^{ni} \cdots & S_{11}^{nN} & S_{12}^{nN} & 0 \cdots 0 & t(Z_1^n) & 0 \cdots & 0 \\
S_{21}^{n1} & S_{22}^{n1} \cdots & S_{21}^{ni} & S_{22}^{ni} \cdots & S_{21}^{nN} & S_{22}^{nN} & 0 \cdots 0 & t(Z_2^n) & 0 \cdots & 0 \\
S_{31}^{n1} & S_{32}^{n1} \cdots & S_{31}^{ni} & S_{32}^{ni} \cdots & S_{31}^{nN} & S_{32}^{nN} & 0 \cdots 0 & t(Z_3^n) & 0 \cdots & 0 \\
\vdots & & & & & & & & & \\
S_{11}^{N1} & S_{12}^{N1} \cdots & S_{11}^{Ni} & S_{12}^{Ni} \cdots & S_{11}^{NN} & S_{12}^{NN} & 0 \cdots & \cdots & 0 & t(Z_1^N) \\
S_{21}^{N1} & S_{22}^{N1} \cdots & S_{21}^{Ni} & S_{22}^{Ni} \cdots & S_{21}^{NN} & S_{22}^{NN} & 0 \cdots & \cdots & 0 & t(Z_2^N) \\
S_{31}^{N1} & S_{32}^{N1} \cdots & S_{31}^{Ni} & S_{32}^{Ni} \cdots & S_{31}^{NN} & S_{32}^{NN} & 0 \cdots & \cdots & 0 & t(Z_3^N)
\end{bmatrix}
\times
\begin{bmatrix}
b_1^1 \\ b_2^1 \\ \vdots \\ b_1^i \\ b_2^i \\ \vdots \\ b_1^N \\ b_2^N \\ c_1 \\ \vdots \\ c_n \\ \vdots \\ c_N
\end{bmatrix}
=
\begin{bmatrix}
V_1(Z_1^1) \\ V_1(Z_2^1) \\ V_1(Z_3^1) \\ \vdots \\ V_n(Z_1^n) \\ V_n(Z_2^n) \\ V_n(Z_3^n) \\ \vdots \\ V_N(Z_1^N) \\ V_N(Z_2^N) \\ V_N(Z_3^N)
\end{bmatrix}
\qquad \text{(III-6)}
$$

IV. ASSOCIATED EQUATIONS OF THE STUDY

A. ADMITTANCE AND IMPEDANCE

The solving of the current distribution from the matrix equation (III-6) is the essence of this study. From the current distributions, the admittance and impedance quantities are found by using the terminal current (current at center of element, where the voltage is applied) and Ohm's Law.

B. RELATIVE VERTICAL AND HORIZONTAL PLANE PATTERNS

The calculations begin with the solving of the differential equation (IV-1) in the far field region [5].

$$dH_\phi = \frac{j(I) \sin \theta \, dZ}{2s\lambda} \tag{IV-1}$$

The representation of this equation is found in Figure (3). The relative vertical plane pattern E_θ in free space is found from (IV-1) by equation (IV-2).

$$E_\theta = ZH_\phi = 120\pi H_\phi \tag{IV-2}$$

The current distribution calculated previously is pictured in Figure (4). The approach used in solving (IV-1) was by superposition of the two currents. The current I_1^i may be substituted into (IV-1) as:

$$I_1^i \sin [k(L/2 \pm Z)] \, e^{j\omega(t - s/c)} \tag{IV-3}$$

and the current I_2^i as:

$$I_2^i \sin[2k(L/2 \pm Z)] \, e^{j\omega(t - s/c)} \tag{IV-4}$$

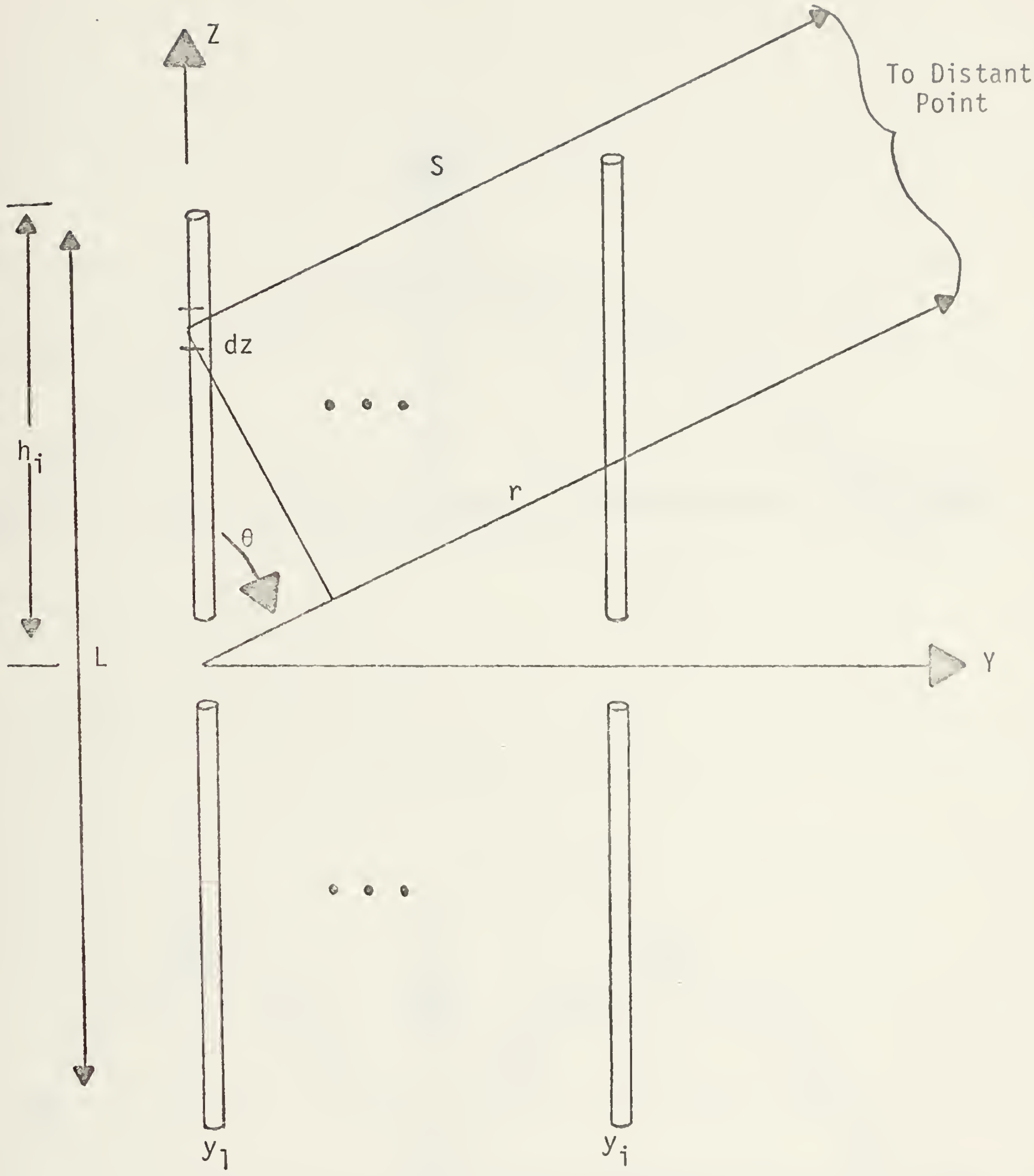

FIGURE 3. RELATIONS FOR SYMMETRICAL, THIN, LINEAR, CENTER-FED ANTENNA

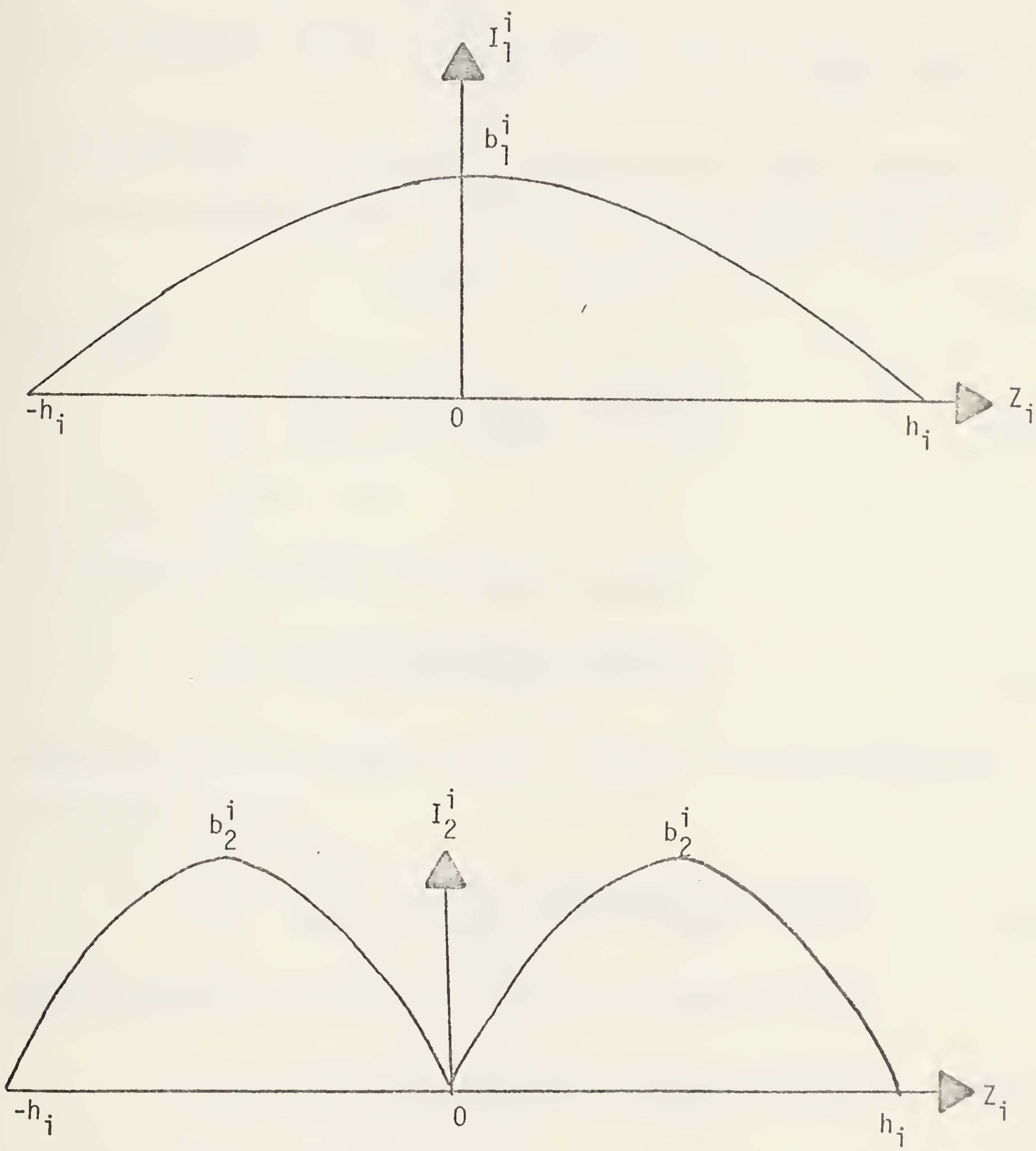

FIGURE 4. TWO TERM EXPANSION OF THE CURRENT

In Figure (3), the distance 's' to a distant point is found to be:

$$S = r - Z\cos\theta \qquad \text{(IV-5)}$$

Substituting (IV-3) and (IV-5) into (IV-1) produces an integral equation of the form in (IV-6).

$$\int e^{ax}\sin(c + bx)dx = \frac{e^{ax}}{a^2 + b^2}[a \sin(c + bx) - b \cos(c + bx)] \qquad \text{(IV-6)}$$

Carrying through the integration and making use of the trigonometric identity in (IV-7) and the relationship that h=L/2 from Figure (3),

$$\sin^2\theta = 1 - \cos^2\theta \qquad \text{(IV-7)}$$

the result is:

$$H_{\phi 1} = \frac{jI_1^{i'}}{2\pi r}\left[\frac{\cos(kh\cos\theta) - \cos(kh)}{\sin\theta}\right] \qquad \text{(IV-8)}$$

where $I_1^{i'} = I_1^{i}e^{j\omega(t - r/c)}$

Relating this to (IV-2) and setting r=1 gives:

$$E_{\theta 1} = j60I_1^{i'}\left[\frac{\cos(kh\cos\theta) - \cos(kh)}{\sin\theta}\right] \qquad \text{(IV-9)}$$

Employing (IV-4) and (IV-6) in (IV-1) produces the second components of the superposition.

$$H_{\phi 2} = \frac{jI_2^{i'}}{2\pi r}\left[\frac{2\sin\theta}{4 - \cos^2\theta}\right]\left[\frac{\cos(kh\cos\theta) - \cos(kh)}{\sin\theta}\right] \qquad \text{(IV-10)}$$

Relating (IV-10) to (IV-2) and setting r=1 produces (IV-11).

$$E_{\theta 2} = j60I_2^{i'}\left[\frac{2\sin\theta}{4 - \cos^2\theta}\right]\left[\frac{\cos(kh\cos\theta) - \cos(kh)}{\sin\theta}\right] \qquad \text{(IV-11)}$$

Now for multiple symmetrical elements, a phase term is introduced due to the separation of elements along the Y axis. In taking this into

account, the time term of $I_p^{i'}$ is dropped and with the substitution of $k=\omega/c$ and $r = y_i\sin\theta$, the form of $I_p^{i'}$ becomes that of (IV-12).

$$I_p^{i'} = I_p^i e^{jky_i\sin\theta} \qquad \text{(IV-12)}$$

Finally, substituting (IV-12) into (IV-11) and (IV-9), the far field pattern in the Y-Z axis is achieved, using the superposition equation (IV-13).

$$E_\theta = E_{\theta 1} + E_{\theta 2} \qquad \text{(IV-13)}$$

The relative horizontal plane pattern as depicted in Figure (5) is found with equation (IV-14).

$$E_\phi = j60(I_1^i + I_2^i)\, e^{jky_i\cos\phi} \qquad \text{(IV-14)}$$

C. POWER GAIN

The power gain was calculated for comparison with an isotropic source [6]. The gain is equal to the ratio of the power intensity to the power density. The gain is then expressed as in (IV-15).

$$G = \frac{4\pi W'}{W_{in}} \qquad \text{(IV-15)}$$

where from the Poynting vector,

$$W' = \frac{r^2}{120\pi} |E|^2 \qquad \text{(IV-16)}$$

With,

$$W_{in} = |I|^2 R_{in} \qquad \text{(IV-17)}$$

The gain with r=1 is expressed as in (IV-18).

$$G = \frac{|E|^2}{30|I|^2 R_{in}} \qquad \text{(IV-18)}$$

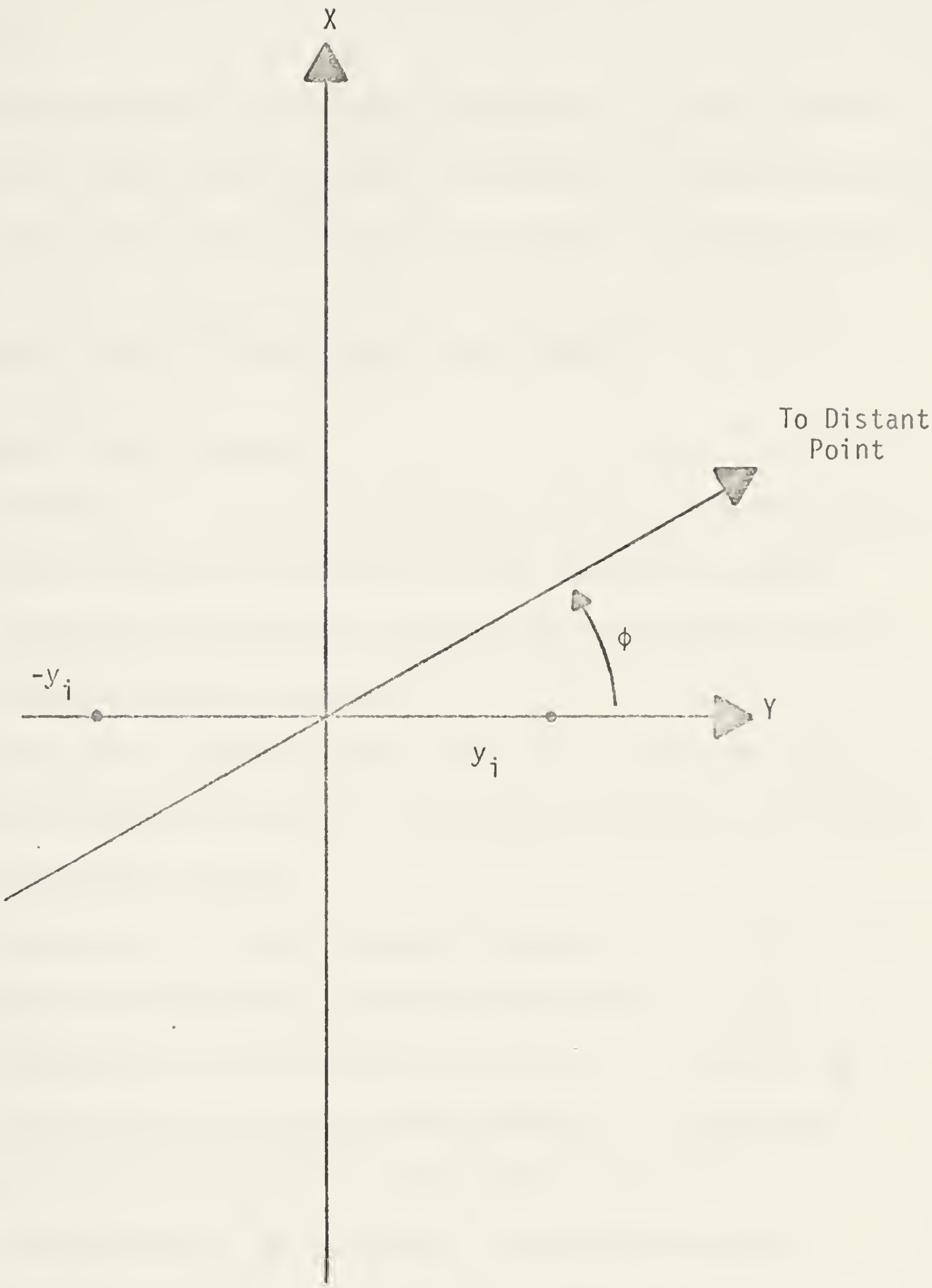

FIGURE 5. CONFIGURATION OF PARALLEL ELEMENTS IN THE XY PLANE

V. PROGRAMMING PROCEDURES

A. ANALYSIS

With the equations of the unknown available, the writing of the software which would produce computer solutions with graphical outputs began. The program consists of the main program, one function subprogram, and four subroutines. The following is a brief outline of the procedures used, in the writing of the program.

B. OUTLINE OF STEPS EMPLOYED

1. The number of arrays to be analyzed, 1 to 99, and the designated number of the graphical output device [7] to be used is read in.

2. The number of elements in the array to be analyzed, 1 to 10, and the wavelength in meters is read in.

3. The applied complex voltage, value of 'Y' along the Y-axis in meters, half length of element 'h' in meters, and radius 'a' in meters is read in for each element.

4. Constants are formed including the designation of the position of the matched points for the solution of the integral, S_{mp}^{ni}.

5. That portion of the 'A' matrix involving the integral S_{mp}^{ni} is solved. The integration routine employs Weddle's Rule [8] which approximates the function with a sixth degree polynomial.

6. The remainder of the 'A' matrix is then completed by appropriately filling in the zeroes and the function, $t(Z_m^n)$.

7. The 'B' matrix is completed by solving the function, $V_n(Z_m^n)$.

8. With the 'A' and 'B' matrices formed, the 'X' matrix is solved for by using a Gaussian Elimination routine [9] including the

decomposing of the 'A' matrix, the solving for the 'X' matrix, and then the attempt to improve the 'X' matrix for the most accurate solution possible.

9. With the solution of the 'X' matrix achieved, the values of the current are outputed starting at the center (Z=0) and for every tenth of the distance along each element until the end of the element ($Z=h_i$).

10. For every element which is voltage-driven, the input impedance and admittance values are outputed.

11. Starting at theta equal to zero degrees, and for every degree through 360 degrees, the values of $E_{\theta 1}$ and $E_{\theta 2}$ for each element are summed together.

12. These values are normalized for output and the appropriate XY values are calculated for the graphical computer.

13. Starting at phi equal to zero degrees and for every degree to 360 degrees, the values of E_{ϕ} for each element are summed together.

14. These values are normalized for output and the appropriate XY values are calculated for the graphical computer.

15. The XY coordinates of the relative patterns to be drawn are sent to the graphics computer for display.

16. The power gain in magnitude and decibels is computed and outputed.

VI. CONCLUSIONS

A. COMPARISON OF RESULTS

The parameters of two simple antenna configurations were used as input data. Results of this study, were then compared for degrees of accuracy.

B. EXAMPLE ONE. TWO SYMMETRICALLY VOLTAGE DRIVEN ELEMENTS

The first antenna consisted of two identical symmetrically voltage driven elements with a wavelength set equal to one meter (300MHz). Each element was driven with the complex voltage of 1+j0. The separation between the two elements was .01 meters (y_1=-0.005, y_2=0.005). Both elements were 0.50 meters in length (L=2h). The radius of each element was 0.000277 meters. Computer results produced by the program of this study for the current distribution are displayed in Figure 6. These results show good correlation with those of King's first-order theory [10] and a two term sinusoidal approximation as depicted by Butler [11].

With this study's calculated current distributions, the input admittances, input impedances, relative vertical pattern, relative horizontal pattern and power gain in magnitude and decibels were also calculated as shown in Table I.

C. EXAMPLE TWO. SIMPLE SYMMETRIC CENTER FED DIPOLE

The second example is a simple symmetric center-fed dipole, with a wavelength set equal to one meter (300MHz). The complex voltage applied was 1+j0. There is no separation involved in this example

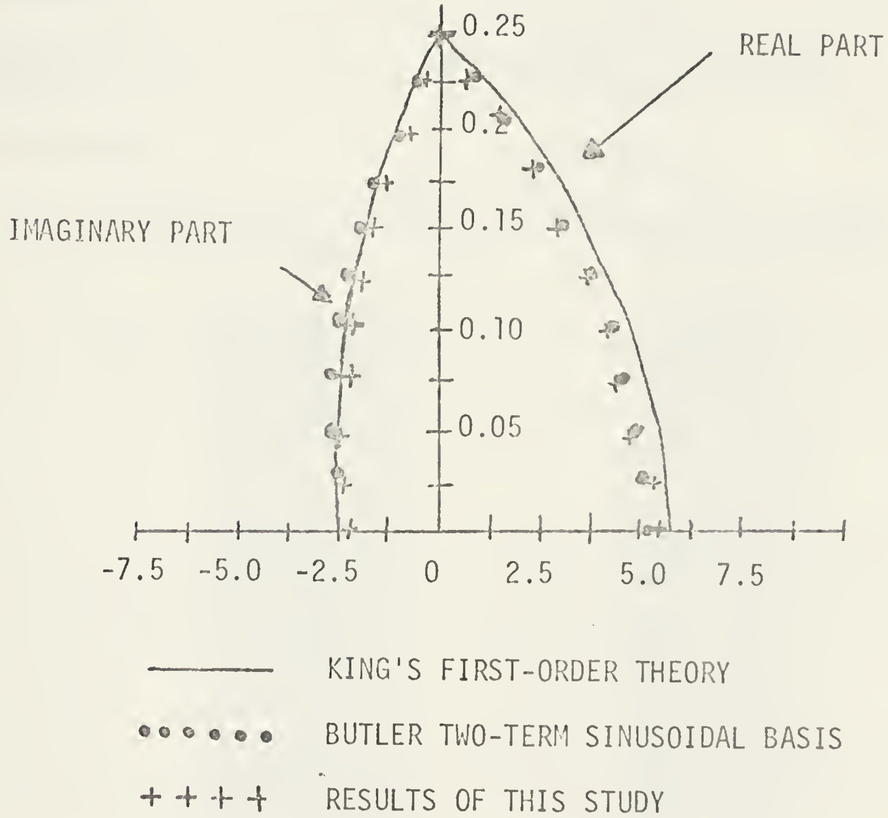

FIGURE 6. CURRENT DISTRIBUTION FOR TWO SYMMETRICALLY VOLTAGE DRIVEN ELEMENTS

REAL CURRENT (ma.)	IMAGINARY CURRENT (ma.)	DISTANCE ALONG ELEMENT FROM CENTER (meters)
5.28236974	-2.37995399	.000
5.16780772	-2.47847437	.025
4.92962568	-2.50660181	.050
4.57696180	-2.45519573	.075
4.12109759	-2.31881755	.100
3.57492582	-2.09652088	.125
2.95246973	-1.79220618	.150
2.26848157	-1.41511770	.175
1.53813553	-0.97857729	.200
0.77681740	-0.50012839	.225
0.00000000	-0.00000000	.250

INPUT ADMITTANCE REAL (milisiemens)	INPUT ADMITTANCE IMAGINARY (milisiemens)
5.28236974	-2.37995399

INPUT IMPEDANCE REAL (ohms)	INPUT IMPEDANCE IMAGINARY (ohms)
157.36507827509	70.90030883439

OMEGA (deg.)	RELATIVE VERT. PATTERN (absolute)	RELATIVE HORZ. PATTERN (absolute)
000	1.00000000000	.99950656036
010	.97786160215	.99952143819
020	.91417041942	.99956427764
030	.81632464159	.99962991265
040	.69429231999	.99971022788
050	.55865218871	.99979611278
060	.41741211316	.99987663247
070	.27633644475	.99994227433
080	.13729063227	.99998611978
090	.00000000000	1.00000000000
100	.13729063227	.99998511978
110	.27633644475	.99994227433
120	.41751211316	.99987663247
130	.55865218871	.99979611278
140	.69429231999	.99971022788
150	.81643564159	.99962991256
160	.91417041942	.99956427764
170	.97786160215	.99952143819
180	1.00000000000	.99950656036

The gain of the array is 3.09 = 4.90db.

TABLE I. COMPUTER OUTPUT FOR TWO SYMMETRICALLY VOLTAGE DRIVEN ELEMENTS

(y_1=0.0). Following the length and radii restrictions, the length of the element was 0.25 meters and the radius was 0.0069 meters.

Computer results produced by the program of this study, are displayed in Figure 7. These results are plotted along with a two term sinusoidal approximation, and a five term sinusoidal approximation as displayed by Butler [12] and measured data by Mack [13]. The additional data calculated as listed in Example one is shown in Table II.

D. RECOMMENDATIONS

More complicated arrays were also analyzed by the computer program of this study. These outputs yielded results which were closely related to those of other two term approximations for current distributions [14].

In Examples one and two, the matched points used were Z=0.0, Z=0.5h_i, and Z=h_i. It is recommended in a study by Darko Kajfez [15] that for the closest comparable results, using the two term approximation of this study, the matched points should be located at Z=0.0, Z=0.75h_i, and Z=h_i.

This study found that the number of increments that the integral function is divided into when being approximated by a sixth degree polynomial employing Weddle's Rule, will in some cases have an unwieldy effect in the solutions of the integrals. It is recommended that the integer, 75, be employed in the determination of the number of increments.

Recommendations for further study are to employ a different complex numerical integration routine with the theory of this study. Possibly additional programming could be written in order to allow interactive graphics for design and testing procedures.

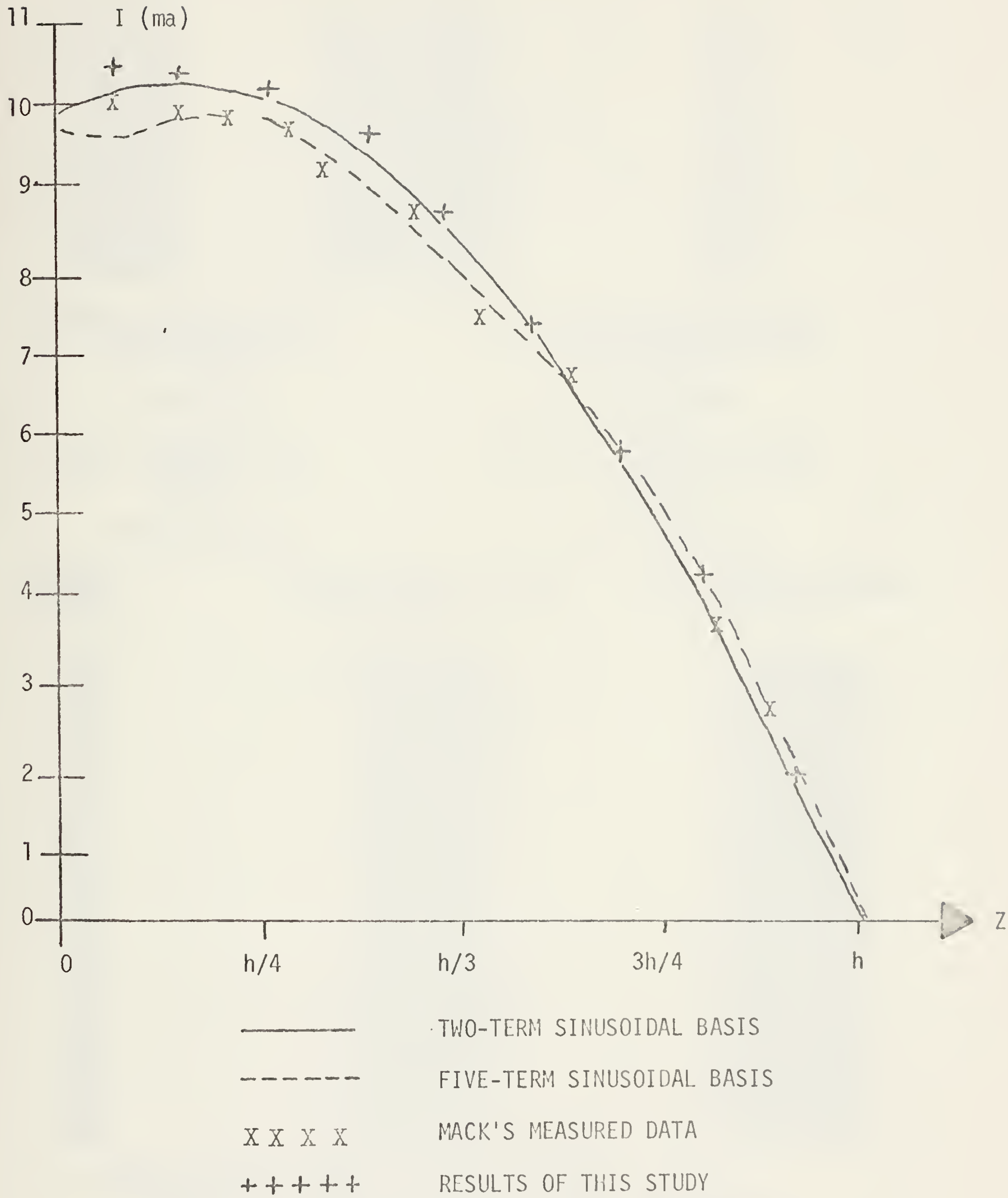

FIGURE 7. CURRENT DISTRIBUTION FOR SIMPLE SYMMETRIC CENTER-FED DIPOLE

MAGNITUDE (ma)	ARGUMENT (deg.)	DISTANCE ALONG ELEMENT FROM CENTER (meters)
10.06945176	-20.91494045546	.000
10.31137742	-23.88716574162	.025
10.29630974	-26.55377940962	.050
9.99144960	-28.92611960589	.075
9.37425727	-30.97158254089	.100
8.43725169	-32.68483783763	.125
7.19133077	-34.07240413432	.150
5.66716441	-35.13865883043	.175
3.91449351	-35.89288284490	.200
1.99942153	-36.34223309019	.225
0.00000000	-36.49146393058	.250

INPUT ADMITTANCE REAL (milisiemens)	IMPUT ADMITTANCE IMAGINARY (milisiemens)
9.40598988	-3.59460891

INPUT IMPEDANCE REAL (ohms)	INPUT IMPEDANCE IMAGINARY (ohms)
92.76685977913	35.45193909900

OMEGA (deg.)	RELATIVE VERT. PATTERN (absolute)	RELATIVE HORZ. PATTERN (absolute)
000	1.00000000000	1.00000000000
010	.97740371048	1.00000000000
020	.91249376631	1.00000000000
030	.81307695801	1.00000000000
040	.68974122014	1.00000000000
050	.55322994009	1.00000000000
060	.41223256076	1.00000000000
070	.27215639036	1.00000000000
080	.13498559880	1.00000000000
090	.00000000000	1.00000000000
100	.13498559880	1.00000000000
110	.27215639036	1.00000000000
120	.41223256076	1.00000000000
130	.55322994009	1.00000000000
140	.68974122014	1.00000000000
150	.81307695801	1.00000000000
160	.91249376631	1.00000000000
170	.97740371048	1.00000000000
180	1.00000000000	1.00000000000

The gain of the array is 1.64 = 2.15db.

TABLE II. COMPUTER OUTPUT FOR SIMPLE SYMMETRIC CENTER-FED DIPOLE

APPENDIX A

GUIDE TO OPERATION OF PROGRAM

Below is a step by step procedure to follow in order to operate the computer program, at the Naval Postgraduate School computer laboratory. The software requires the use of the digital computer (XDS9300) and the graphics computer (AGT).

A. FORMATION OF THE DECK

1. The first card is a 'BOOT' card which is found in the card stalls in the laboratory.

2. The second card is the 'JOB' card which is followed by the cards of the program through to the 'DATA' card as shown in the enclosed computer program printout.

3. Following the 'DATA' card is the initial data card which contains three digits in the first three columns of the card. The first two digits denote the number of arrays to be analyzed (01 to 99). The third digit denotes the assigned number of the graphics computer to be used (1 or 2).

4. After the initial data card, the input data of each of the arrays is formed. Each array will have one card plus as many cards as there are elements in the array. The first card of each array will have the number of elements in the array in the first two columns (01 to 10). In the next twenty columns, the value of the wavelength in meters is to be entered.

5. Following the 'wagelength' card, there is one card for each of the elements. The format for each of these cards is as follows.

Columns 01 to 15: Real value of complex voltage applied to element. Example 1.0

Columns 16 to 30: Imaginary value of complex voltage applied to element. Example -1.0

Columns 31 to 45: Value of the Y-axis, indicating the position of the element in meters. Example 0.0

Columns 46 to 60: Value of half length of element (h) in meters. Example 0.25

Columns 61 to 75: Value of radius of element in meters. Example 0.001

6. As an example, in order to run example one and two of section VI of this study, the cards after the 'DATA' card would appear as follows:

Card 1:	021				
or					
Card 1:	022				
Card 2:	02	1.0			
Card 3:	1.0	0.0	-.005	0.25	.000277
Card 4:	1.0	0.0	.005	0.25	.000277
Card 5:	01	1.0			
Card 6:	1.0	0.0	0.000	0.25	.0069

B. PREPARATION OF XDS9300 DIGITAL COMPUTER

1. Press IDLE switch at the XDS9300 control panel.

2. Hold the RESET switch depressed and press the POWER switch in order to energize the computer.

3. Turn switch at teletype to the 'on' position.

4. Press the READY button on the line printer.

C. PREPARATION OF THE AGT GRAPHICS COMPUTER

1. Press ON at the operator's control panel (OCP).

2. Wait 5 seconds, then press RESET on the OCP.

3. Press the ON/START switch at the disk drive of the AGT to be used.

4. Press HALT, RESET, and BTSP, in that order on the OCP.

5. Verify that the 'location counter' (behind the door of the left-most rack) indicates zero or all lights out. If not, press RESET and BTSP again.

6. Load a paper tape marked DISK BOOT LOADER (located behind the door of the left-most rack) into the tape reader of the AGT's teletype.

7. Verify that the knob on the front of the AGT's teletype is in the 'line' position.

8. Put the tape reader switch to 'start.'

9. Now verify that the 'location counter' indicates 110 (octal). If it is not, return to step 1 of this section and begin the procedure again.

10. When step 9 is satisfied, verify that the switches in the center of the OCP have the configuration 24000 77776 (octal).

11. Press HALT, RESET, RUN, and PULSE 1 on the OCP.

12. If the AGT's teletype does not respond now with a request for the operator to input MO/DA/YR, press HALT, RESET, RUN, BUTTON 29, and PULSE 1. If teletype does not respond now, return to step 1 of this section.

13. Type RESET("GATED",101)! on the AGT's teletype.

14. Type GATED! on the AGT's teletype.

15. Turn vertical and horizontal gain knobs fully clockwise.

D. READING IN PROGRAM DECK

1. Place cards in card hopper.
2. Push POWER ON button, at the card reader.
3. Push START button, at the card reader.
4. Push IDLE, RESET, RUN, and CARDS on the XDS9300 control panel.

```
THIS PROGRAM FOR ARRAY ANALYSIS WAS CONSTRUCTED BY CAPTAIN CHARLES SCHILLINGER
OF THE UNITED STATES MARINE CORPS IN PARTIAL FULFILLMENT OF HIS MASTER OF
SCIENCE DEGREE.
¬JOB
¬AGT
¬FORTRAN LS,GO
      DIMENSION REA(360), XX(720), YY(720), IGDIR(2),
     CFRAME(723), H(10), A(30,30), Y(10), ZZ(10,3), R(10),
     OLU(30,30), X(30), B(30), IPS(30), NMPPE(10), V(10),
     NC(10), REH(360)
      COMPLEX CF, CANS, CWEDF, CS, CI, A, X, LU, B, C, CSUM
      COMPLEX CCI, CECNE, CETWO, CEP, V
      INTEGER FRAME
      EQUIVALENCE (A(1,1), REA), (A(1,7), REH),
     E(A(1,13),XX), (LU(1,1),YY), (LU(1,13),FRAME)
      EXTERNAL CF, DECMP1, SOLVE1, IMPRV1
      COMMON I, LP, PI, H, Y, ZZ, CI, N, M, R, TOPI

THE VARIABLE, KILLE, IS THE NUMBER OF ARRAYS TO BE ANALYZED.
THE VARIABLE, IDEV, IS THE NUMBER OF THE AGT WHICH IS TO BE USED FOR THE
GRAPHICAL OUTPUT.

      READ(5,41) KILLE, IDEV
   41 FORMAT(I2,I1)
      DO 999, KILL=1,KILLE

THE VARIABLE, NN, IS THE NUMBER OF ELEMENTS IN THE ARRAY TO BE ANALYZED.
THE VARIABLE, AMBDA, IS THE VALUE OF THE WAVELENGTH IN METERS FOR THIS ARRAY.

      READ(5,40) NN, AMBDA
   40 FORMAT(I2, F20.13)

THE VARIABLES, V, Y, H, AND R ARE THE COMPLEX VOLTAGE APPLIED, SEPARATION
BETWEEN ELEMENTS, HALF LENGTH OF ELEMENT, AND RADIUS OF ELEMENT. ALL LENGTH
PARAMETERS MUST BE EXPRESSED IN METERS.

      READ (5,42) V(MA), Y(MA), H(MA), R(MA), MA=1,NN
   42 FORMAT(5F15.7)
      WRITE (6,22)
   22 FORMAT('1', 4X, 'VOLTAGE REAL', 6X,
     T'VOLTAGE IMAGINARY', 6X, 'SEPARATION', 9X,
     W'HALF-LENGTH', 12X, 'RADIUS')
      WRITE (6,23) V(MA), Y(MA), H(MA), R(MA), MA=1,NN
   23 FORMAT(5(F15.7, 5X))

CONSTANTS ARE FORMED.

      CCI=(0.0,1.0)
```

```
      TOPI=2.0*3.1415926535
      CNED=TOPI/360.0
      BETA=TOPI/AMBDA
      PI=3.1415926535
      LLP=2
       P=LLP
      CI=(0.0,-1.0)
      NS=(LLP+1)*NN
```

POSITION OF MATCHED POINTS ARE DETERMINED.

```
      F=3.0/4.0
      IF(NN.LT.3) F=0.5
      DO 202, KA=1,NN
      ZZ(KA,1)=0.0
      ZZ(KA,2)=H(KA)*F
      ZZ(KA,3)=H(KA)
  202 CONTINUE
```

THAT PORTION OF THE A MATRIX INVOLVING THE INTEGRAL FUNCTIONS ARE DERIVED.

```
      CANS=(0.0,0.0)
      IR=0
      IC=1
      DO 4, I=1, NN
      DO 3, LP=1, LLP
      DO 5, N=1,NN
      DO 2, M=1,3
      XU=H(I)
      NX=450
      IF(NN.EQ.10) NX=75
```

CWEDF IS THE SUBROUTINE PERFORMING THE NUMERICAL INTEGRATION.

```
      CALL CWEDF(CF, 0.0, XU, NX, CANS)
      IF (IR .EQ. NS) GO TO 20
      IR=IR+1
      GO TO 21
   20 IC=IC+1
      IR=1
   21 A(IR,IC)=CANS
    2 CONTINUE
    5 CONTINUE
    3 CONTINUE
    4 CONTINUE
```

THE REMAINDER OF THE A MATRIX IS COMPLETED.

```
      NTZER=0
      N=1
   14 DO 11, M=1,3
      IF(IR.EQ.NS) GO TO 12
      IR=IR+1
      GO TO 13
   12 IC=IC+1
      IR=1
   13 A(IR,IC)=-(COS(TOPI*ZZ(N,M)))/30.0
   11 CONTINUE
      NBZIC=NS-IR
      IF(NBZIC .EQ. 0) GO TO 16
      DO 10, KKK=1, NBZIC
      IR=IR+1
      A(IR,IC)=(0.0,0.0)
   10 CONTINUE
      IC=IC+1
      IR=0
      NTZER=3+NTZER
      DO 15, LLL=1, NTZER
      IR=IR+1
      A(IR,IC)=(0.0,0.0)
   15 CONTINUE
      N=N+1
      GO TO 14
```

THE FORMATION OF THE B MATRIX BEGINS HERE.

```
   16 II=0
      DO 6, N=1, NN
      DO 7, M=1,3
      II=II+1
      B(II)=CI*V(N)*(SIN(TOPI*ZZ(N,M)))/60.0
    7 CONTINUE
    6 CONTINUE
```

THE SOLUTION OF THE X MATRIX BEGINS WITH THE SUBROUTINE DECMP1.
DECMP1 DECOMPOSES THE A MATRIX FOR THE GAUASSIAN ELIMINATION PROCESS.

```
      CALL DECMP1 (NS, A, 30, LU, IPS, 100S, 100S)
```

SOLVE1 SOLVES FOR THE X MATRIX WITH THE DECOMPOSED A MATRIX AND THE B MATRIX.

```
      CALL SOLVE1 (NS, LU, 30, B, X, IPS)
```

IMPRV1 COMPLETES THE SOLUTION OF THE X MATRIX BY ATTEMPTING TO IMPROVE THE
VALUES OF THE X MATRIX TO THE PRECISION OF THE MACHINE.

```
      CALL IMPRV1 (NS, A, 30, B, X, LU, IPS)
C PREPARATION FOR THE OUTPUTTING OF THE IMPEDANCE AND ADMITTANCE VALUES.
  100 N=0
      KBAR=NS-NN-LLP+1
      LAS=1
      LAST=0
      DO 1, NUM=1,NN
      V(NUM)=CABS(V(NUM))
      IF(V(NUM).EQ.0.0) GO TO 1
      LAST=LAST+1
      NMPPE(LAST)=NUM
    1 CONTINUE
C THE SOLVING OF THE CURRENT DISTRIBUTION AND ITS OUTPUT BEGINS HERE.
      DO 30, I=1,KBAR,LLP
      N=N+1
      Z=0.0
      KOUNT=0
   31 C(N)=(X(I)*SIN((PI*(H(N)-Z))/(2.0*H(N))))+
     B(X(I+1)*SIN((TOPI*(H(N)-Z))/(2.0*H(N))))
      ARG=(ATAN2(AIMAG(C(N)), REAL(C(N))))*57.2957795
      RE=CABS(C(N))
      IF(N.EQ.NMPPE(LAS).AND.KOUNT.EQ.0) GO TO 8
      GO TO 9
    8 B(1)=C(N)/V(N)
      B(2)=1/B(1)
      RESIS=ABS(REAL(B(2)))
      GAY=RE*RE*RESIS
    9 KOUNT=KOUNT+1
      IF(KOUNT.EQ.1) GO TO 299
  297 WRITE (6,32) N, C(N), RE, ARG, Z
   32 FORMAT (9X, I2, 12X, F16.11, 2X, F16.11, 5X, F16.11,
     T4X, F16.11, 10X, F16.11)
      GO TO 298
  299 WRITE(6,300)
  300 FORMAT('0', 1X, 'NUMBER OF ELEMENT', 5X,
     C'REAL CURRENT', 5X, 'IMAGINARY CURRENT', 5X,
     O'MAGNITUDE', 8X, 'ARGUMENT', 9X,
     C 'DISTANCE FROM CENTER OF ELEMENT')
      GO TO 297
  298 IF(KOUNT.EQ.11.AND.N.EQ.NMPPE(LAS)) GO TO 17
      IF(KOUNT.EQ.11) GO TO 30
      Z=Z+(.1*H(N))
      GO TO 31
   17 WRITE(6,18)
```

```
   18  FORMAT('0', 1X, 'NUMBER OF ELEMENT', 3X,
      F'INPUT ADMITTANCE REAL', 3X,
      I'INPUT ADMITTANCE IMAGINARY', 3X,
      G'INPUT IMPEDANCE REAL', 3X,
      H'INPUT IMPEDANCE IMAGINARY')
       WRITE (6,19) N, B(1), B(2)
   19  FORMAT(9X, I2, 14X, F16.11, 11X, F16.11, 10X, F16.11,
      C11X, F16.11)
       LAS=LAS+1
   30  CONTINUE
```

THE SOLVING FOR THE RELATIVE VERTICAL PATTERN BEGINS HERE.

```
       VORM=0.0
       DO 101, MC=1,360
       THETA=(MC-1)*ONED
       IF (MC .EQ. 1) GO TO 107
       IF (MC .EQ. 181) GO TO 107
       CSUM =(0.0,0.0)
       MU=1
       DO 43 LI=1,KBAR,2
       AA=(BETA*H(MU))
       BB=AA*COS(THETA)
       CSUM=(X(LI)*(COS(BETA*Y(MU)*SIN(THETA))+CCI*SIN(BETA
      L*Y(MU)*SIN(THETA))))*(COS(BB)-COS(AA))+CSUM
       MU=MU+1
   43  CONTINUE
       CEONE=CCI*60.0*CSUM/SIN(THETA)
       GO TO 108
  107  CEONE=(0.0, 0.0)
  108  CSUM=(0.0,0.0)
       MU=1
       DO 44 LI=1,KBAR,2
       CSUM=(X(LI+1)*(COS(BETA*Y(MU)*SIN(THETA))+CCI*SIN(BETA
      A*Y(MU)*SIN(THETA))))*(COS(BB)-COS(2.0*AA))+CSUM
       MU=MU+1
   44  CONTINUE
       CETWO=CCI*60.0*CSUM*2.0*SIN(THETA)/(4.0-(COS(THETA)*
      JCOS(THETA)))
       CEP=CEONE+CETWO
       REA(MC)=CABS(CEP)
       THETA=(PI/2.0)-((MC-1)*ONED)
```

THE VALUES OF X AND Y FOR THE GRAPHICS COMPUTER ARE FORMED HERE.

```
       XX(MC)=REA(MC)*COS(THETA)
       YY(MC)=REA(MC)*SIN(THETA)
       VORM=AMAX(ABS(REA(MC)), VORM)
```

```
  101 CONTINUE
THE SOLVING FOR THE RELATIVE HORIZONTAL PATTERN BEGINS HERE.
      HORM=0.0
      INE=361
      DO 400, INA=1,360
      REA(INA)=REA(INA)/VCRM
      PHI=(INA-1)*ONED
      CEONE=(0.0,0.0)
      IND=1
      DO 401, INB=1,NN
      CEONE=CEONE+(X(IND)+X(IND+1))*CCI*120.0*
     H(COS(BETA*Y(INB)*COS(PHI))+CCI*SIN(BETA*Y(INB)*COS
     O(PHI)))
      IND=IND+2
  401 CONTINUE
      REH(INA)=CABS(CEONE)
THE VALUES OF X AND Y FOR THE GRAPHICS COMPUTER ARE FORMED HERE.
      XX(INE)=REH(INA)*COS(PHI)
      YY(INE)=REH(INA)*SIN(PHI)
      HORM=AMAX(ABS(REH(INA)), HORM)
      INE=INE+1
  400 CONTINUE
      DO 402, INF=1,360
      REH(INF)=REH(INF)/HORM
  402 CONTINUE
THE OUTPUT OF THE RELATIVE PATTERN VALUES BEGINS HERE.
      WRITE(6,404)
  404 FORMAT('1', 1X, 'OMEGA', 30X,
     F'RELATIVE VERTICAL PATTERN', 30X,
     O'RELATIVE HORIZONTAL PATTERN')
      INY=91
      DO 405, INZ=1,360,10
      OMEGA=(INZ-1)
      WRITE(6,406) OMEGA, REA(INY), REH(INZ)
  406 FORMAT(2X, I3, 36X, F16.11, 41X, F16.11)
      INY=INY-10
      IF(INY.LT.1) INY=351
  405 CONTINUE
      XL=0.0
      YL=0.0
      DO 106, IV=1,360,2
      YL=AMAX(ABS(YY(IV)), ABS(YY(IV+1)), YL)
```

```
      XL=AMAX(ABS(XX(IV)), ABS(XX(IV+1)), XL)
  106 CONTINUE
      HUGE=AMAX(XL,YL)
      XL=0.0
      YL=0.0
      DO 407, IV=361,720,2
      YL=AMAX(ABS(YY(IV)), ABS(YY(IV+1)), YL)
      XL=AMAX(ABS(XX(IV)), ABS(XX(IV+1)), XL)
  407 CONTINUE
      BUGE=AMAX(XL,YL)
```

DGINIT IS THE SUBROUTINE WHICH WILL CONNECT THE 9300 TO THE GRAPHICS COMPUTER.

```
      CALL DGINIT(IDEV,IGDIR,2,IER)
      IF(IER .NE. 0) OUTPUT(101) IER, 'DGINIT1'
      FRAME(1)=IHEAD(0,10)
      FRAME(2)=IPACK(XX(1)*.5/HUGE, (YY(1)*.5/HUGE)+.51, 0)
      DO 104, IK=3,361
      FRAME(IK)=IPACK(XX(IK-1)*.5/HUGE, (YY(IK-1)*.5/HUGE)+
     Z.51, 1)
  104 CONTINUE
      FRAME(362)=IPACK(XX(1)*.5/HUGE, (YY(1)*.5/HUGE)+.51,1)
      FRAME(363)=IPACK(XX(361)*.5/BUGE, (YY(361)*.5/BUGE)-
     Y.51, 0)
      DO 403, ING=364,722
      FRAME(ING)=IPACK(XX(ING-2)*.5/BUGE, (YY(ING-2)*.5/
     XBUGE)-.51, 1)
  403 CONTINUE
      FRAME(723)=IPACK(XX(361)*.5/BUGE, (YY(361)*.5/BUGE)-
     W.51, 1)
      CALL GRAPHO (IDEV, FRAME, 723, 1, IER)
      IF(IER .NE. 0) OUTPUT(101) IER, 'FRAME'
```

THE VALUE OF THE POWER GAIN IS CALCULATED AND OUTPUTED HERE.

```
      HORM=0.5*HORM
      HALL=AMAX(HORM,VORM)
      HAIN=HALL*HALL/(30.0*GAY)
      GAIN=10.0*ALOG10(HAIN)
      WRITE (6,997) HAIN,GAIN
  997 FORMAT('0', 1X, 'THE GAIN OF THE ARRAY IS', F16.11,
     61X, 'WHICH IN DB IS', F16.11)
  999 CONTINUE
      STOP
```

THIS IS THE END OF THE MAIN PROGRAM.

```
      END
```

```
      FUNCTION CF(Z)
C THIS IS THE FUNCTION TO BE INTEGRATED IN THE FORMING OF THE A MATRIX.
      COMPLEX CI, CF, CEXP
      DIMENSION H(10), Y(10), ZZ(10,3), R(10)
      COMMON I, LP, PI, H, Y, ZZ, CI, N, M, R, TOPI
      P=LP
      AA=(R(N)**2)+((Y(N)-Y(I))**2)
      CF=(SIN(((P*PI)/(2.0*H(I)))*(H(I)-Z)))*(((CEXP((CI*
     MTOPI)*(SQRT
     B(AA+(ZZ(N,M)-Z)**2))))/(SQRT(AA+(ZZ(N,M)-Z)**2)))+
     A((CEXP((CI*TOPI)*(SQRT(AA+(ZZ(N,M)+Z)**2))))/
     R(SQRT(AA+(ZZ(N,M)+Z)**2))))
      RETURN
C THIS IS THE END OF THE FUNCTION SUBROUTINE.
      END
```

```
      SUBROUTINE CWEDF (CF,XL,XU,NX,CANS)
C
CWEDF IS A SUBROUTINE WHICH WILL NUMERICALLY INTEGRATE A USER SUPPLIED FUNCTION
BETWEEN SPECIFIED LIMITS.  PRPARED BY MICHAEL G. HARRISON
CF IS THE NAME OF THE FUNCTION TO BE INTEGRATED.
XL IS THE LOWER LIMIT OF INTEGRATION.
XU IS THE UPPER LIMIT OF INTEGRATION.
NX IS THE APPROXIMATE NUMBER OF NODES AT WHICH THE FUNCTION IS TO BE EVALUATED.
CANS IS THE RESULT OF THE INTEGRATION.

      DIMENSION CW(6)
      COMPLEX CANS,CF
      DATA CW/82.,216.,27.,272.,27.,216./
      N=((NX+4)/6)*6+1
      DX=(XU-XL)/FLOAT(N-1)
      DXDX=DBLE(DX)
      NWIX=N/6
      X=XL
      CANS=-CF(X)*41.0
      DO 800 MX=1,NWIX
      DO 700 KX=1,6
      CANS=CANS+CW(KX)*CF(X)
      XX=DBLE(X)
      X=SNGL(XX+DXDX)
  700 CONTINUE
  800 CONTINUE
      CANS=(CANS+41.0*CF(X))*DX/140.0
      RETURN

THIS IS THE END OF THE INTEGRATION SUBROUTINE.

      END
```

```
      SUBROUTINE DECMP1(N, A, IDIM, LU, IPS, M, L)

C     DECMP1 IS A SUBRCUTINE WHICH DECUMPOSES THE A MATRIX INTO A TRIANGULAR L & U, SO
C     THAT L*U = A.  PREPARED BY JOHN H. WELSCH
C     IPS IS THE ROW PIVOT VECTOR.

      DIMENSION A(IDIM,N), IPS(N), LU(IDIM,N), SCALES(100)
      COMPLEX A, LU, EM, PIVOT
      DO 5 I = 1,N
         IPS(I) = I
         ROWNRM = 0.0
         DO 2 J = 1,N
            LU(I,J) = A(I,J)
      ROWNRM=AMAX1(ROWNRM,CABS(LU(I,J)))
   2  CONTINUE

C     TEST FOR MATRIX WITH ZERC ROW.

      IF(ROWNRM.EQ.0) RETURN M
         SCALES(I) = 1.0/ROWNRM
    5 CONTINUE

C     GAUSSIAN ELIMINATION WITH PARTIAL PIVOTING.

      NM1 = N-1
      DO 17 K = 1,NM1
         BIG = 0.0
         DO 11 I = K,N
            IP = IPS(I)
      SIZE=CABS(LU(IP,K))*SCALES(IP)
            IF(SIZE.LE.BIG) GO TO 11
  10           BIG = SIZE
               IDXPIV = I
  11     CONTINUE
      IF(BIG.EQ.0) RETURN L
         IF(IDXPIV.EQ.K) GO TO 15
            J = IPS(K)
            IPS(K) = IPS(IDXPIV)
            IPS(IDXPIV) = J
  15     KP = IPS(K)
         PIVOT = LU(KP,K)
         KP1 = K+1
         DO 16 I = KP1,N
            IP = IPS(I)
            EM = LU(IP,K)/PIVOT
            LU(IP,K) = EM
```

```
          DO 16 J = KP1,N
             LU(IP,J) = LU(IP,J) - EM*LU(KP,J)
   16     CONTINUE
   17 CONTINUE
      IF(CABS(LU(IPS(N),N)).EQ.0) RETURN L
      RETURN
C THIS IS THE END OF THE DECOMPOSE SUBROUTINE.
      END
```

```
      SUBROUTINE SOLVE1(N, LU, IDIM, B, X, IPS)
C
C  SOLVE1 IS A SUBROUTINE WHICH SOLVES FOR THE X MATRIX USING THE LU MATRIX FROM
C  THE SUBROUTINE DECMP1.  PREPARED BY JOHN H. WELSCH
C  IPS IS THE ROW INTERCHANGE VECTOR FROM THE SUBROUTINE DECMP1.
C
      COMPLEX LU, B, X, SUM
      DIMENSION IPS(N), LU(IDIM,N), B(N), X(N)
      NP1 = N+1
      X(1) = B(IPS(1))
      DO 2 I = 2,N
         IP = IPS(I)
         IM1 = I-1
      SUM=(0.0,0.0)
         DO 1 J = 1,IM1
    1       SUM = SUM + LU(IP,J)*X(J)
    2 X(I) = B(IP) - SUM
      X(N) = X(N)/LU(IPS(N),N)
      DO 4 IBACK = 2,N
      I = NP1-IBACK
         IP = IPS(I)
         IP1 = I+1
      SUM=(0.0,0.0)
         DO 3 J = IP1,N
    3       SUM = SUM + LU(IP,J)*X(J)
    4 X(I) = (X(I)-SUM)/LU(IP,I)
      RETURN
C
C  THIS IS THE END OF THE SOLVE SUBROUTINE.
C
      END
```

```
      SUBROUTINE IMPRV1 (N, A, IDIM, B, X, LU, IPS)
C IMPRV1 IS A SUBROUTINE WHICH IMPROVES X TO MACHINE ACCURACY.
C PREPARED BY CHARLES W. SCHILLINGER
C A IS THE ORIGINAL A MATRIX FROM THE MAIN PROGRAM.
C B IS THE ORIGINAL B MATRIX FROM THE MAIN PROGRAM.
C X IS THE SOLUTION FROM THE SUBROUTINE SOLVE1.
      COMPLEX A, LU, B, X, R, DX, T, SUM, DBL, OX
      DOUBLE PRECISION DB, D, SCH, SCHI, WIZ, WIZA, SUMR,
     DSUMJ
      DIMENSION A(IDIM,N), IPS(N), LU(IDIM,N), B(N), X(N),
     CR(30), DX(30), OX(30), WORST(13)
      EXTERNAL SOLVE1
      XNORM = 0.0
      DO 1 I = 1,N
      OX(I)=X(I)
    1     XNORM = AMAX1(XNORM,CABS(X(I)))
      IF(XNORM.EQ.0.0) RETURN
    3 DO 2, ILP=1,13
          DO 5 I = 1,N
      SUMJ=0.0
      SUMR=0.0
             DO 4 J = 1,N
      DB=REAL(X(J))
      D=AIMAG(X(J))
      SCH=REAL(A(I,J))
      SCHI=AIMAG(A(I,J))
      SUMR=SUMR+SCH*DB
    4 SUMJ=SUMJ+SCHI*D
      WIZ=REAL(B(I))
      WIZA=AIMAG(B(I))
      SUMR=WIZ-SUMR
      SUMJ=WIZA-SUMJ
      SUMRS=SUMR
      SUMJS=SUMJ
      WORST(ILP)=AMAX(ABS(SUMRS), ABS(SUMJS))
      IF(WORST(ILP).EQ.0.0) RETURN
    5 R(I)=CMPLX(SUMRS,SUMJS)
      IF(ILP.EQ.1) GO TO 7
      IF(WORST(ILP).GT.WORST(ILP-1)) GO TO 11
      DO 12, NIL=1,N
      OX(NIL)=X(NIL)
   12 CONTINUE
    7     CALL SOLVE1(N, LU, IDIM, R, DX, IPS)
          DO 6 I = 1,N
```

```
            X(I) = X(I) + DX(I)
  6       CONTINUE
 2    CCNTINUE
 11   DO 10, IL=1,N
      X(IL)=DX(IL)
 10   CCNTINUE
 9    RETURN

THIS IS THE END OF THE IMPROVE SUBROUTINE.

      END
¬LOAD XR, MAP
¬DATA
```

BIBLIOGRAPHY

1. Kraus, John D., Antennas, p. 127-132, 1st edition, McGraw Hill, 1950.

2. Butler, C.M., Study of Current Distribution in Arrays of Parallel, Co-Planar Wire Elements, presented to the seminar at the University of Mississippi, Oxford, Mississippi, summer 1972.

3. DeLaura, R.D., Electrical Engineering Computer Laboratory, compiled for the electrical engineering computer laboratory at the Naval Postgraduate School, Monterey, California, 10 February 1972.

4. Butler, C.M. and Pearson, L.W., Moment Solution for the Currents on an Array of Parallel Symmetrically Driven Dipoles, presented to the seminar at the University of Mississippi, Oxford, Mississippi, summer 1972.

5. Kraus, John D., Antennas, p. 140-142, 1st edition, McGraw Hill, 1950.

6. Kennedy, Edward E., A Computer Model for Rapid Solutions and Visual CRT Display of Radiation Patterns for Arbitrarily Orientable Yagi-UDA Arrays Operating Over Lossy Ground or in Ship Ocean Environments, p. 31, Master of Science, Naval Postgraduate School, Monterey, California, 1972.

7. DeLaura, R.D., Electrical Engineering Computer Laboratory, compiled for the electrical engineering computer laboratory at the Naval Postgraduate School, Monterey, California, p. III-24.

8. Scheid, Francis, Theory and Problems of Numerical Analysis, p. 123, McGraw Hill, 1968.

9. Dorn, W.S. and McCracken, D.D., Numerical Methods and Fortran Programming, p. 231-238, 3rd edition, John Wiley and Sons, Inc.,1965.

10. King, R.W., Theory of Linear Antennas, p. 276, Harvard University Press, 1956.

11. Butler, C.M., Study of Current Distribution in Arrays of Parallel, Co-Planar Wire Elements, presented to the seminar at the University of Mississippi, Oxford, Mississippi, summer 1972, p. M-24.

12. Butler, C.M., Integral Equation Solution Methods, presented to the seminar at the University of Mississippi, Oxford Mississippi, summer 1972.

13. Craft Laboratory Report Numbers 381-386, A Study of Circular Arrays, Harvard University, 1963.

14. King, R.W., Mack, R.B. and Sandler, S.S., Arrays of Cylindrical Dipoles, p. 226-230, Cambridge University Press, 1968.

15. Kajfez, Darko, Analysis of Yagi Antenna by Moment Method, presented to the seminar at the University of Mississippi, Oxford, Mississippi, summer 1972.

INITIAL DISTRIBUTION LIST

		No. Copies
1.	Defense Documentation Center Cameron Station Alexandria, Virginia 22314	2
2.	Library, Code 0212 Naval Postgraduate School Monterey, California 93940	2
3.	Asst. Professor R. W. Adler Code 52Ab Naval Postgraduate School Monterey, California 93940	1
4.	Capt. Charles W. Schillinger 7338 N. Oconto Avenue Chicago, Illinois 60648	1

Security Classification

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

ORIGINATING ACTIVITY *(Corporate author)* Naval Postgraduate School Monterey, California 93940	2a. REPORT SECURITY CLASSIFICATION Unclassified 2b. GROUP

REPORT TITLE
Computer Solution of Hallen's Integral Equation on Multi-Element Arrays Employing the Two Term Approximate Current Distribution

DESCRIPTIVE NOTES *(Type of report and, inclusive dates)*
Master's Thesis: March 1973

AUTHOR(S) *(First name, middle initial, last name)*
Charles William Schillinger

REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
March 1973	60	15

. CONTRACT OR GRANT NO.	9a. ORIGINATOR'S REPORT NUMBER(S)
. PROJECT NO.	
c.	9b. OTHER REPORT NO(S) *(Any other numbers that may be assigned this report)*
d.	

0. DISTRIBUTION STATEMENT

Approved for public release; distribution unlimited.

1. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY Naval Postgraduate School Monterey, California 93940

3. ABSTRACT

The objective of this analytical study was to develop a rapid theoretical analysis on approximate half wavelength elements of an antenna configured in a co-planar, symmetrical array. A computer program was written employing the method of moments approach to the solution of Hallen's integral equation with an approximate two term entire domain expansion assumed for the current distributions. With the solution of the current distributions on each element, additional calculations were made for the input impedance and admittance values, field distributions, power gain, and a graphical output of the radiation pattern.

Security Classification

KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Two Term Current Distributions of Multi-Element Arrays						
Yagi-Uda Arrays						
Radiation Patterns						
Input Impedances and Admittances for Arrays						
Gain of Arrays						

DD FORM 1 NOV 65 1473 (BACK)

S/N 0101-807-6821

Security Classification

A-31409

17 NOV 82 29006

144020
Thesis
S33673 Schillinger
c.1 Computer solution of
Hallen's integral equation on multi-element
arrays employing the two
term approximate current
distribution.

17 NOV 82 29006

Thesis 144020
S33673 Schillinger
c.1 Computer solution of
Hallen's integral equation on multi-element
arrays employing the two
term approximate current
distribution.

thesS33673
Computer solution of Hallen's integral e
3 2768 001 00427 8
DUDLEY KNOX LIBRARY