A STUDY OF SUPERSONIC FLOW PAST VIBRATING PANELS AND SHELLS

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THESIS

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ABSTRACT

Supersonic flow past harmonically vibrating two-dimensional panels and cylindrical shells is analyzed using a linearized method-of-characteristics procedure. A detailed description of this method to solve the linearized unsteady potential equation is given and numerical results are presented to indicate the nature of the aerodynamic pressure distributions. Also, comparisons of the present results are made with earlier work by Nelson and Cunningham and by Anderson which is based upon quite different approaches.

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LIST OF SYMBOLS

A_1	See Eq. 5.2
a	Speed of sound
c	Free stream speed of sound
Cp	Coefficient of pressure
E	Energy
e	2.718282
h	Radial deflection
i	The square root of minus one
K	Reduced frequency
L	Cylinder length
l	Axial wave length
M	Free stream Mach number
m	Axial mode number of the cylinder
n	Circumferential mode number of the cylinder
P	Pressure
$\overline{p}$	Pressure perturbation ($P-P_\infty$)
q	Heat
R	Cylinder radius
r	Radial coordinate
S	Entropy
s	Increment of distance along a characteristic
T	Temperature
t	Time
U	Free stream velocity in x direction
$\overline{u}$	Velocity component in x direction
v	Velocity component in y direction

w	Velocity component in z direction
W	Total velocity
x,y,z	Rectangular coordinates
x,r,θ	Cylindrical coordinates
Z(x)	Radial deflection amplitude
α	Mach angle
ρ	Density
Φ	Non-dimensional velocity potential
φ	Perturbation velocity potential
ϕ	Velocity potential
ψ_1	Force phase angle

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I. INTRODUCTION

The determination of the pressure distribution over vibrating panels and shells is a prerequisite for the study of aeroelastic stability. This problem has been previously considered by a number of investigators, who applied either various asymptotic approximation techniques (e.g., piston theory) or operational methods to the solution of the linearized unsteady potential equation. For a recent comprehensive review of this subject refer to Reference 7. In the present study a quite different approach is developed; i.e., the method of characteristics is used to obtain a solution to the problem of supersonic flow past vibrating panels and shells.

The major assumptions in the mathematical model are that we are dealing with a perfect gas, that the flow is irrotational and supersonic, and that linearization is permissible. Reference 1 was used to help develop the linearized unsteady potential equation and the boundary conditions for panels and shells. These equations were then rewritten in cylindrical coordinates.

Steady supersonic flow past bodies of revolution was studied by several authors using linearized characteristics methods; i.e., Haack [Ref. 9], Sauer [Ref. 10], Oswatitsch and Erdmann [Ref. 11]. Teipel [Ref. 12] developed a characteristics procedure for two-dimensional supersonic flow past airfoils oscillating at arbitrary frequency using the

velocity perturbations and the velocity of sound perturbation as dependent variables. Platzer and Sherer [Ref. 13] gave an extension of the Oswatitsch-Erdmann method to slowly oscillating bodies of revolution.

In the present report a linearized characteristics method is presented to study supersonic flow past cylindrical shells vibrating at arbitrary frequency. The velocity potential function is introduced as the dependent variable and the linearized unsteady potential equation is solved subject to the proper boundary conditions. Although two-dimensional flow past vibrating panels is contained in this solution as a special case for infinite shell radius, two separate programs were written in Fortran IV code for the IBM 360 system, one for panels and one for shells. Thus, the panel program is complementary to Teipel's work in that it uses the velocity potential rather than the velocity perturbations as dependent variables. The method of singularities [Ref. 4] served as an independent method to verify the results. Anderson's work, on the other hand, was used to compare the method-of-characteristics predictions for cylindrical shells.

II. PROBLEM FORMULATION

In nearly all engineering studies some basic assumptions have to be made to formulate the problem. In this study this assumption is that the compressible medium with which we are dealing is a perfect gas. We therefore have the equation of state for a gas

$$P = \rho RT \tag{2.1}$$

Flow of a perfect gas is then completely described by expressing the ambient quantities P, ρ and T as well as the velocity components $\bar{u}$, v, w as a function of position x, y, z and time t. These six dependent variables are always related by the conditions of conservation of mass, momentum and energy as developed in Johns [Ref. 1].

Conservation of Mass

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} + \frac{\partial \rho}{\partial t} = 0 \tag{2.2}$$

Conservation of Momentum

$$\begin{aligned} -\frac{1}{\rho}\frac{\partial P}{\partial x} &= \frac{\partial \bar{u}}{\partial t} + \bar{u}\frac{\partial \bar{u}}{\partial x} + v\frac{\partial \bar{u}}{\partial y} + w\frac{\partial \bar{u}}{\partial z} \\ -\frac{1}{\rho}\frac{\partial P}{\partial y} &= \frac{\partial v}{\partial t} + \bar{u}\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} \\ -\frac{1}{\rho}\frac{\partial P}{\partial z} &= \frac{\partial w}{\partial t} + \bar{u}\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} \end{aligned} \tag{2.3}$$

Conservation of Energy

$$\frac{D}{Dt}\left[E + \frac{W^2}{2}\right] = -\frac{1}{\rho}\left[\frac{\partial}{\partial x}(\bar{u}P) + \frac{\partial}{\partial y}(vP) + \frac{\partial}{\partial z}(wP)\right] \qquad (2.4)$$

where

$$W^2 = \bar{u}^2 + v^2 + w^2. \qquad (2.5)$$

Equation 2.4 can also be stated as (See Ref. 1)

$$\frac{DS}{Dt} = 0 \qquad (2.6)$$

The condition of conservation of energy therefore reduces to the simple statement that the entropy of a fluid particle remains constant. For later convenience we now rewrite the continuity equation, Eq. 2.2

$$\frac{D\rho}{Dt} + \rho\left(\frac{\partial \bar{u}}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) = 0 \qquad (2.7)$$

Introducing the speed of sound

$$a^2 = \frac{\partial P}{\partial \rho} \qquad (2.8)$$

we have

$$\frac{D\rho}{Dt} = \frac{\partial \rho}{\partial P}\frac{DP}{Dt} = \frac{1}{a^2}\frac{DP}{Dt}$$

Therefore the continuity equation can also be written

$$\frac{DP}{Dt} = -\rho a^2\left(\frac{\partial \bar{u}}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right). \qquad (2.9)$$

The velocity potential is introduced to reduce the number of dependent variables. Its existence depends upon the condition of irrotationality of the flow. This condition

is expressed mathematically by the vanishing of the curl of the velocity vector and can be written in component form

$$\frac{\partial v}{\partial x} - \frac{\partial \bar{u}}{\partial y} = 0, \quad \frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} = 0, \quad \frac{\partial \bar{u}}{\partial z} - \frac{\partial w}{\partial x} = 0. \tag{2.10}$$

It can be shown by Kelvin's theorem [Ref. 2] that flows starting either from reservoirs or from parallel stream-lines and being irrotational initially will remain so. Only strong shock or intense heat will require a reformulation of this law. However, such effects are usually quite unimportant in aeroelastic considerations. Therefore, Eq. 2.10 is necessary and sufficient to derive the velocity components $\bar{u}$, v, w from the velocity potential ϕ such that

$$\bar{u} = \frac{\partial \phi}{\partial x}, \quad v = \frac{\partial \phi}{\partial y}, \quad w = \frac{\partial \phi}{\partial z}. \tag{2.11}$$

Therefore the continuity equation can be written

$$-\frac{1}{\rho}\frac{D\rho}{Dt} = \frac{\partial \bar{u}}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} \tag{2.12}$$

In vector form the Eulerian equations of motion can also be written as

$$\frac{\partial \vec{v}}{\partial t} + (\bar{v} \cdot \text{grad})\bar{v} = -\frac{1}{\rho}\,\text{grad}\, P. \tag{2.13}$$

This is equivalent in irrotational barotropic flow to

$$\frac{\partial}{\partial t}\,\text{grad}\,\phi + \text{grad}\left(\frac{W^2}{2}\right) = -\frac{1}{\rho}\,\text{grad}\, P \tag{2.14}$$

or

$$\text{grad}\left[\frac{\partial\phi}{\partial t} + \frac{W^2}{2} + \int\frac{dP}{\rho}\right] = 0$$

Integration therefore gives

$$\frac{\partial\phi}{\partial t} + \frac{W^2}{2} + \int\frac{dP}{\rho} = C(t) \tag{2.15}$$

Moreover, $C(t) = \frac{1}{2}U^2$ whenever the remote fluid motion consists of parallel streamlines with velocity U. The partial derivative of Eq. 2.15 with respect to time is

$$-\frac{\partial}{\partial t}\left[\frac{\partial\phi}{\partial t} + \frac{W^2}{2}\right] = \frac{\partial}{\partial t}\int\frac{dP}{\rho} = \frac{1}{\rho}\frac{\partial P}{\partial t} \tag{2.16}$$

In rewriting the continuity equation

$$\phi_{xx} + \phi_{yy} + \phi_{zz} + \frac{1}{a^2\rho}\left[\phi_x\frac{\partial P}{\partial x} + \phi_y\frac{\partial P}{\partial y} + \phi_z\frac{\partial P}{\partial z} + \frac{\partial P}{\partial t}\right] = 0 \tag{2.17}$$

one obtains therefore as a result of Eqs. 2.3 and 2.16

$$\left(1 - \frac{\phi_x^2}{a^2}\right)\phi_{xx} + \left(1 - \frac{\phi_y^2}{a^2}\right)\phi_{yy} + \left(1 - \frac{\phi_z^2}{a^2}\right)\phi_{zz} +$$

$$-\frac{2\phi_x\phi_y}{a^2}\phi_{xy} - \frac{2\phi_x\phi_y}{a^2}\phi_{xz} - \frac{2\phi_y\phi_z}{a}\phi_{yz} + \tag{2.18}$$

$$-\frac{2\phi_x}{a^2}\phi_{xt} - \frac{2\phi_y}{a^2}\phi_{yt} - \frac{2\phi_z}{a^2}\phi_{zt} - \frac{1}{a^2}\phi_{tt} = 0$$

Let a denote the speed of sound. For infinitely large speeds of sound (incompressible flow) the above equation reduces to Laplace's equation. Equation 2.18 is the

governing differential equation for general nonstationary, nonviscous, potential flow and is not limited to small disturbances. However, because of its nonlinear nature, solutions were found only in very few special cases. Therefore, it is necessary to introduce the small disturbance concept. Regarding all velocity disturbances small in comparison with U, a, and (U - a) and pressure differences and density changes small in comparison with main stream pressure and density, a perturbation potential φ is introduced so that

$$\phi = Ux + \varphi \qquad (2.19)$$

If a is expanded around its value for the undisturbed stream the procedure of retaining only first order terms leads to the following equation for the disturbance velocity potential

$$(1-M^2)\varphi_{xx} + \varphi_{yy} + \varphi_{zz} - \frac{2M}{C}\varphi_{xt} - \frac{1}{C^2}\varphi_{tt} = 0 \qquad (2.20)$$

where c is the constant value of the speed of sound in the uniform stream and M the constant free-stream Mach number. It is this linearized unsteady potential Eq. 2.20 which is being used in most aeroelastic analyses as the basic equation to predict the oscillatory pressures and generalized aerodynamic forces. However, it must be emphasized that its validity is restricted to purely subsonic or supersonic flows, as well as to sufficiently unsteady transonic flows.

Expressing Eq. 2.20 in cylindrical coordinates one has

$$\varphi_{xx} + \varphi_{rr} + \frac{1}{r}\varphi_r + \frac{1}{r^2}\varphi_{\theta\theta} - \frac{1}{C^2}\left[\varphi_{tt} + 2U\varphi_{xt} + U^2\varphi_{xx}\right] = 0 \qquad (2.21)$$

There are two main boundary conditions applicable to a vibrating cylinder in supersonic flow. The first is that the panel surface is impenetrable to the gas, i.e., the flow is tangential to the surface at each instant of time. The second condition is prescribed by the Sommerfeld radiation condition which requires waves to propagate away from sources of disturbance toward infinity. Furthermore, uniform supersonic flow must exist upstream of the panel leading edge, since no disturbances can propagate upstream.

If the equation of the surface of a body which is moving in a time-dependent manner is given by

$$F(x,y,z,t) = 0 \qquad (2.22)$$

then it can be shown that the flow tangency condition requires [Ref. 8]

$$\frac{DF}{Dt} = \frac{\partial F}{\partial t} + \bar{u}\frac{\partial F}{\partial x} + v\frac{\partial F}{\partial y} + w\frac{\partial F}{\partial z} = 0 \qquad (2.23)$$

This condition states that the rate of change of the numerical value of the function F is zero, when we follow the motion of a particular fluid element (substantial derivative), so that the element continually touches the surface F = 0. Consider a two-dimensional panel (Fig. 1a) whose chord is

aligned with the x-axis and whose upper surface is exposed to a uniform flow of velocity U in the positive x-direction. We then find that we can describe the body by

$$F = z - h(x,t) = 0 \tag{2.24}$$

Since $\frac{\partial F}{\partial z} = 1$ we have from the boundary condition

$$w = \frac{\partial h}{\partial t} + \bar{u}\,\frac{\partial h}{\partial x} \qquad \text{For } z = h(x,t) \tag{2.25}$$

Imposing the linear perturbation assumption Eq. 2.19 we have

$$\phi = Ux + \varphi \tag{2.19}$$

where the disturbance velocity components are obtained from

$$u = \frac{\partial \varphi}{\partial x}, \qquad w = \frac{\partial \varphi}{\partial z} \tag{2.26}$$

and satisfy the order-of-magnitude requirement

$$\bar{u},\ w \ll U \tag{2.27}$$

The terms $u\,\frac{dh}{\partial x}$ can then be neglected compared to the much larger term $U\,\frac{\partial h}{\partial x}$. This leads to

$$w = \frac{\partial h}{\partial t} + U\,\frac{\partial h}{\partial x} \tag{2.28}$$

It can be shown that it is possible to express the flow tangency condition in the plane z = 0. Taylor series expansion gives

$$w(x,z,t) = w(z,0^+,t) + h\,\frac{\partial w(x,0^+,t)}{\partial z} + \frac{h^2}{2!}\,\frac{\partial^2 w(x,0^+,t)}{\partial z^2} + - - - - + \tag{2.29}$$

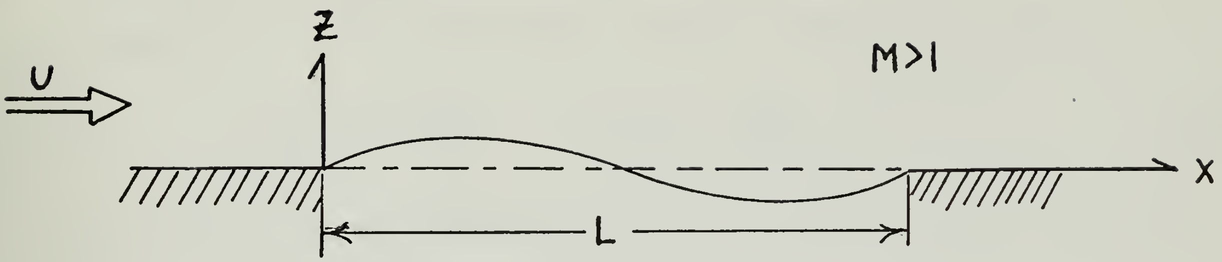

FIGURE 1a TWO-DIMENSIONAL PANEL

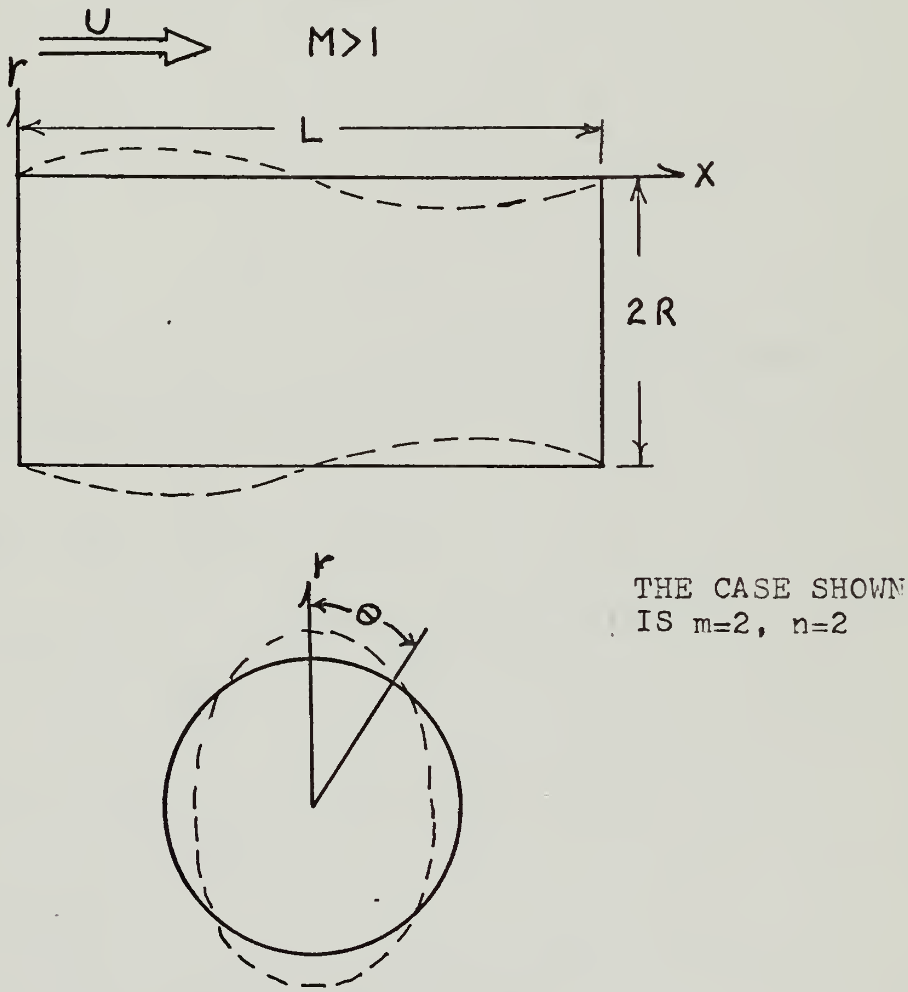

FIGURE 1b PROFILE AND CROSS SECTION
OF CYLINDRICAL SHELL

Using the same argument as above the product terms are neglected so that the flow tangency condition equals

$$w = \frac{\partial h}{\partial t} + U \frac{\partial h}{\partial x} \qquad \text{for } z = 0^{+} \tag{2.30}$$

A similar consideration for the cylindrical shell (Fig. 1b) whose vibration mode is given by

$$h(x,\theta,t) = Z(x) \cos n\theta \, e^{i\omega t} \tag{2.31}$$

results in the following boundary conditions

$$\left.\frac{\partial \varphi}{\partial r}\right|_{r=R} = U \left[\frac{\partial Z}{\partial x} + i\omega Z\right] \cos n\theta \, e^{i\omega t} \qquad \text{for } 0 < x < L$$

$$= 0 \qquad \text{for } x < 0 \tag{2.32}$$

The calculation of perturbation pressures and pressure coefficients begins with Eq. 2.15

$$\frac{\partial \phi}{\partial t} + \frac{W^2}{2} + \int \frac{dP}{\rho} = C\,(t)$$

For uniform parallel flow far upstream we have

$$C(t) = \frac{U^2}{2} .$$

Using again the perturbation assumption, Eq. 2.19

$$\phi = Ux + \varphi$$

and noting that

$$W^2 = \left(U + \frac{\partial \varphi}{\partial x}\right)^2 + \left(\frac{\partial \varphi}{\partial y}\right)^2 + \left(\frac{\partial \varphi}{\partial z}\right)^2 \tag{2.5}$$

we can approximate

$$\frac{\partial \phi}{\partial t} + \frac{W^2}{2} = \frac{\partial \varphi}{\partial t} + \frac{U^2}{2} + U \frac{\partial \varphi}{\partial x} \tag{2.33}$$

by dropping second order terms. Also the density-pressure relation is expanded as

$$\rho(p) = \rho_\infty + \left(\frac{d\rho}{dp}\right)_\infty \left(p-p_\infty\right) + \frac{1}{2!}\left(\frac{d^2\rho}{dp^2}\right)_\infty \left(p-p_\infty\right) + \ldots \qquad (2.34)$$

Therefore

$$\int_{p_\infty}^{p} \frac{dp}{\rho(p)} = \int_{p_\infty}^{p} \frac{dp}{\rho_\infty \left[1 + \left(\frac{d\rho}{dp}\right)_\infty \frac{p-p_\infty}{\rho_\infty}\right]} = \frac{1}{\rho_\infty}\int_{p_\infty}^{p} \left[1 - \frac{p-p_\infty}{p_\infty \gamma}\right] dp \doteq \frac{p-p_\infty}{\rho_\infty}$$

(2.35)

Hence, the perturbation pressure is related to the perturbation potential by

$$p-p_\infty = -\rho_\infty \cdot \left[\frac{\partial}{\partial t} + U\frac{\partial}{\partial x}\right] \qquad (2.36)$$

Writing this equation in pressure-coefficient form one has

$$Cp = \frac{p-p_\infty}{\frac{1}{2}\rho_\infty U^2} = -\frac{2}{U^2}\frac{\partial \varphi}{\partial t} - \frac{2}{U}\frac{\partial \varphi}{\partial x} \qquad (2.37)$$

Consistent with the assumed cylinder deflection, Eq. 2.31 a velocity potential is now assumed in the form

$$\varphi = \Phi(x,r) \text{ Cos } n\theta \, e^{i\omega t} \qquad (2.38)$$

Substituting this into the Eq. 2.21 results in

$$(1-M^2)\,\Phi_{xx} + \Phi_{rr} + \frac{1}{r}\Phi_r - \frac{n^2}{r^2}\Phi - \frac{2i\omega U}{a^2}\Phi_x + \frac{\omega^2}{a^2}\Phi = 0 \qquad (2.39)$$

This is the basic equation for $\Phi(x,r)$, which needs to be solved, subject to the boundary conditions

$$\left.\frac{\partial \Phi}{\partial r}\right|_{r=R} = U \frac{\partial Z}{\partial x} + i\omega Z \qquad \text{For } 0 < x < L$$

$$= 0 \qquad \text{For } x < 0 \qquad (2.40)$$

When non-dimensional coordinates $\bar{x} = \frac{x}{L}$, $\bar{r} = \frac{r}{L}$ and the reduced frequency $K = \frac{\omega L}{U}$ are introduced then it is found

$$(1-M^2)\Phi_{xx} + \Phi_{rr} + \frac{1}{r}\Phi_r - \frac{n^2}{r^2}\Phi - 2iKM^2\Phi_x + K^2M^2\Phi = 0 \qquad (2.41)$$

The bar has been omitted for simplicity. The boundary condition is

$$\left.\Phi_r\right|_{r=R} = \left[\frac{\partial Z}{\partial x} + iKZ\right] \qquad 0 \leq x \leq L$$

$$= 0 \qquad x < 0 \qquad (2.42)$$

Converting Cp to cylindrical coordinates and using the above non-dimensional coordinates one has

$$Cp = -2\left[iK\Phi - \Phi_x\right] \text{ Cos } n\theta e^{iKt} \qquad (2.43)$$

III. NUMERICAL TECHNIQUE

A. METHOD OF CHARACTERISTICS

Consider the following pair of simultaneous first order partial differential equations for the dependent variables u and v,

$$A_1 \frac{\partial u}{\partial x} + B_1 \frac{\partial u}{\partial y} + C_1 \frac{\partial v}{\partial x} + D_1 \frac{\partial v}{\partial y} = 0 \qquad (3.1)$$

$$A_2 \frac{\partial u}{\partial x} + B_2 \frac{\partial u}{\partial y} + C_2 \frac{\partial v}{\partial x} + D_2 \frac{\partial v}{\partial y} = 0$$

where the coefficients $A_1, \ldots, D_2$ are functions of u and v but not of x and y.

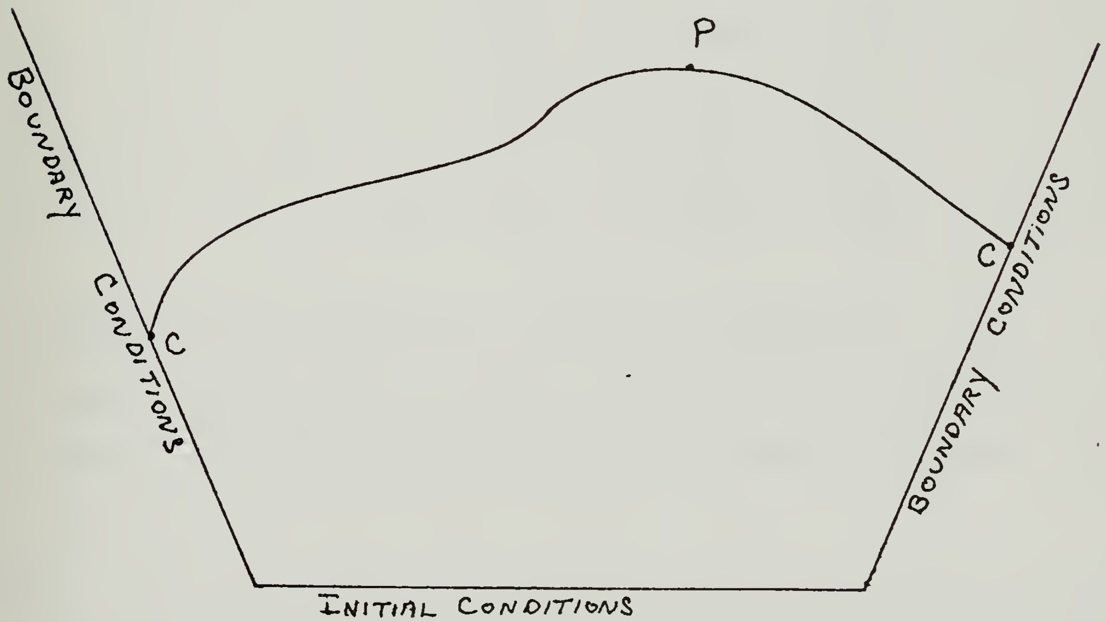

FIGURE 2

Assume that in Figure 1 the solution is known up to the curve CPC. At P one knows continuously differentiable values of u and v along the curve CPC as well as all their derivatives in the directions that lie below the curve. However, the question needs to be answered whether there is enough information to deduce the directional derivatives of u and v at P in directions that lie above the curve.

The directional derivative of u in the direction s is:

$$\frac{\partial u}{\partial s} = \frac{\partial u}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial u}{\partial y}\frac{\partial y}{\partial s} \qquad (3.2)$$

so that the directional derivative in any direction is known as soon as the derivatives $\partial u/\partial x$ and $\partial u/\partial y$ are known.

To find the values of $\partial u/\partial x$ and $\partial u/\partial y$, write out equations 3.1 at point P and the directional differentials[1] of u and v taken along CPC at P. In matrix form the four equations are:

$$\begin{bmatrix} A_1 & B_1 & C_1 & D_1 \\ A_2 & B_2 & C_2 & D_2 \\ dx & dy & 0 & 0 \\ 0 & 0 & dx & dy \end{bmatrix} \begin{bmatrix} \partial u/\partial x \\ \partial u/\partial y \\ \partial v/\partial x \\ \partial v/\partial y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ du \\ dv \end{bmatrix} \qquad (3.3)$$

With u and v known at P the coefficients $A_1, \ldots, D_2$ will also be known. When the directions of CPC are known then dx and dy can be found. If u and v are then known along

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[1]In Eq. 3.3 du is an abbreviation for $\partial u/\partial s$ ds, where $\partial u/\partial s$ is given by Figure 3.2 and s now measures distance along the curve CPC. The symbols dv, dx, and dy stand for similar abbreviations.

CPC, du and dv will be known. Equations 3.3 therefore constitute a pair of four simultaneous linear algebraic equations for the four first derivatives. If the determinant of this matrix is not zero, there is a unique solution. In this case satisfaction of these governing equations requires that the directional derivatives have the same value above and below the line CPC. If the matrix determinant had equaled zero, there would not be a unique solution and there could be discontinuities across the line CPC.

Expanding the determinant of Eqs. 3.3 and setting it equal to zero one has

$$(A_1C_2-A_2C_1)dy^2 - (A_1D_2-A_2D_1+B_1C_2-B_2C_1)dxdy + (B_1D_2-B_2D_1)dx^2 = 0 \qquad (3.4)$$

This is a quadratic equation for the slope dy/dx. If the direction of the curve CPC at P is such that it has a slope satisfying Eq. 3.4, then the derivatives of u and v are not uniquely determined by the values of u and v along the curve. Such a direction is called a characteristic direction. The quadratic Eq. 3.4 gives two real slopes, one real slope or a pair of complex slopes, depending upon whether the discriminant

$$(A_1D_2-A_2D_1+B_1C_2-B_2C_1)^2 - 4(A_1C_2-A_2C_1)(B_1D_2-B_2D_1) \qquad (3.5)$$

is positive, zero or negative. This is also the criterion for cataloging the system Eq. 3.1 as hyperbolic, parabolic, or elliptic. The system is hyperbolic at a point if there

are two real characteristic directions; it is parabolic if there is only a single characteristic direction; or it is elliptic if there are no real characteristic directions at the point.

The foregoing analysis can be repeated for the single-order quasi-linear equation,

$$a \frac{\partial^2\psi}{\partial x^2} + b\frac{\partial^2\psi}{\partial x\partial y} + c \frac{\partial^2\psi}{\partial y^2} = f \tag{3.6}$$

in which a, b, c, and f are functions of x, y, ψ, $\frac{\partial\psi}{\partial x}$, and $\partial\psi/\partial y$. The characteristic directions are determined from the quadratic

$$ady^2 - bdxdy + cdx^2 = 0 \tag{3.7}$$

and hence Eq. 3.7 is hyperbolic, parabolic or elliptic at a point according to whether the discriminant $b^2 - 4ac$ is positive, zero, or negative.

To determine the characteristic curves we return to the pair of first-order Eqs. 3.1 and suppose that the system is hyperbolic throughout the domain of interest. At every point there are two roots, $(dy/dx)\alpha$ and $(dy/dx)\beta$, to the quadratic Eqs. 3.4. A curve, which at each of its points has the slope, $(dy/dx)\alpha$, is said to be a α characteristic; A curve, whose slope is everywhere, $(dx/dy)\beta$, is said to be a β characteristic. Thus there are two families of characteristic curves filling the domain as shown in Figure 3.

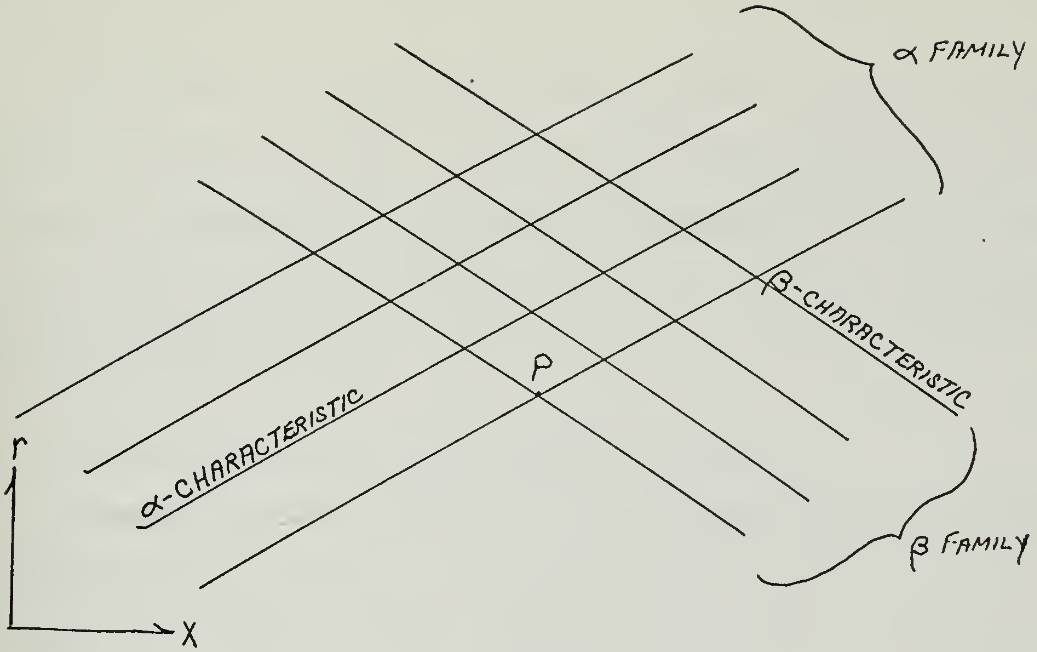


FIGURE 3

If we are considering a characteristic direction so that the determinant in Eq. 3.3 is zero, the right-hand column must be compatible with this if there are to be any solutions at all for the first derivatives; i.e., when the right-hand column is substituted for any of the columns on the left, the resulting determinant must also vanish. Replacing the fourth column on the left with the column on the right and setting the determinant equal to zero leads to

$$(A_1B_2-A_2B_1)du + \left[(A_1C_2-A_2C_1)\frac{dy}{dx} - (B_1C_2-B_2C_1)\right] dv = 0. \quad (3.8)$$

Insert $(dy/dx)\alpha$ into Eq. 3.8; it then becomes an ordinary differential equation of u and v along the characteristic. A similar equation can be obtained along a β characteristic.

A method of attack has been outlined for solving hyperbolic systems: first, locate the characteristic curves; second, integrate the ordinary differential Eq. 3.8 along the characteristic. This procedure is called the method of characteristics.

B. FORMULATION OF THE EQUATIONS PROGRAMMED

In Section II, D, Eq. 2.41 was developed as the partial differential equation for $\Phi(x,r)$ that needed to be solved subject to the boundary condition Eq. 2.42. If $M > 1$, the preceding equation is of the hyperbolic type and possesses real characteristics, which satisfy the ordinary differential equation

$$(M^2 - 1)dr^2 - dx^2 = 0 \tag{3.9}$$

Introduce along the characteristics the arc-length $ds^2 = M^2 dr^2$ then one has $X' = \frac{dx}{ds} = \frac{\sqrt{M^2-1}}{M}$, $r' = \frac{dr}{ds} = \frac{1}{M}$ (3.10)

Note: The angle between the characteristics and the x-axis is $\sin \alpha = \frac{1}{M}$ (3.11)

An arbitrary function $F(x,r)$ has the following derivative along a characteristic

$$\frac{dF}{ds} = F_x X' + F_r \cdot r' = \frac{\sqrt{M^2-1}}{M} F_x \pm \frac{1}{M} F_r \tag{3.12}$$

Denoting the derivative along the left-running characteristic with F_1, the derivative along the right-running characteristic with F_2 one has

$$F_1 = \frac{\sqrt{M^2 1}}{M} F_x + \frac{1}{M} F_r, \quad F_2 = \frac{\sqrt{M^2-1}}{M} F_x - \frac{1}{M} F_r \tag{3.13}$$

Solving for F_x, F_r results in

$$F_r = \frac{M}{2}(F_1 - F_2), \quad F_x = \frac{M}{2\sqrt{M^2-1}}(F_1 + F_2) \tag{3.14}$$

Call s_1 the arc-length of the left-running characteristic, then one has for the second derivatives

$$F_{12} = \frac{dF_1}{dS_2}, \quad F_{21} = \frac{dF_2}{dS_1} \tag{3.15}$$

Hence

$$F_{12} = (F_1)_x \cdot \frac{\sqrt{M^2-1}}{M} - (F_1)_r \cdot \frac{1}{M}$$

and

$$-F_{12} = \frac{1}{M^2}\left[(1-M^2)F_{xx} + F_{rr}\right] = -F_{21}$$

Therefore the basic equation for $\Phi(x,r)$ was written

$$-M^2\Phi_{12} + \frac{M}{2r}\left(\Phi_1 - \Phi_2\right) - \frac{ikM^3}{\sqrt{M^2-1}}\left(\Phi_1+\Phi_2\right) + \left(K^2M^2 - \frac{n^2}{r^2}\right)\Phi = 0 \tag{3.16}$$

or

$$\Phi_{12} = \frac{\Phi_1-\Phi_2}{2rM} - \frac{iKM}{\sqrt{M^2-1}}\left(\Phi_1 + \Phi_2\right) + \left(K^2 - \frac{n^2}{r^2M^2}\right)\Phi$$

This equation was then written in finite difference form.

Using the backward difference form of the finite difference method as developed in Wang Ref. 3 one obtained

$\left.\frac{\partial \phi}{\partial x}\right)_j = \frac{1}{\Delta S}\left(-\varphi_{j-1} + \varphi_j\right)$ Applying this technique to

Eq. 3.16 where the points A and B were on the right-running characteristic (See Fig. 3) then

$$\frac{\phi_1(B) - \phi_1(A)}{\Delta S} = \frac{\phi_1 - \phi_2}{2rM} - \frac{iKM}{\sqrt{M^2-1}}\left(\phi_1 + \phi_2\right) + \left(K^2 - \frac{n^2}{r^2M^2}\right)\Phi \qquad (3.17)$$

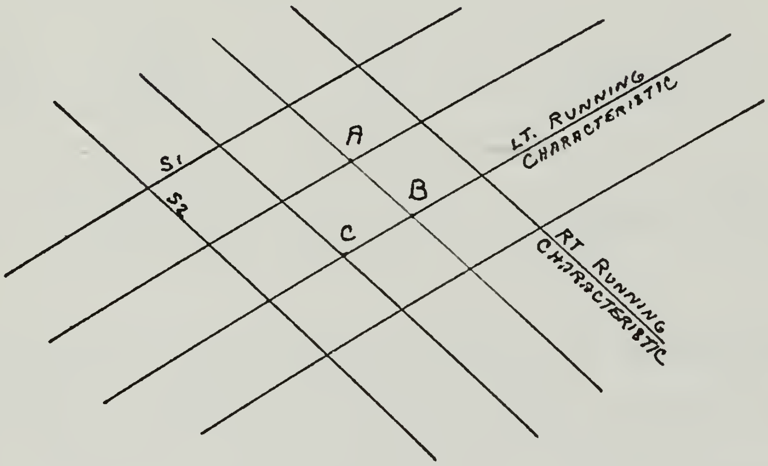


FIGURE 4

The values of Φ, Φ_1 and Φ_2 in Eq. 3.17 were not specified at a point. When this equation was first written these values were taken at A, the previous point. It was found in comparing the results in Chapter V that there was a considerable amount of error present. In investigating the source of these errors it was found that if the values for Φ, Φ_1 and Φ_2 were averaged between points A and B that these errors disappeared. The value for Φ at point B was not yet known but it was found by integrating along the characteristic. Using trapezoidal rule

$$\Phi(B) = \Phi(A) + \frac{1}{2}\left[\Phi_2(A) + \Phi_2(B)\right]\Delta S \qquad (3.18)$$

Substituting this into Eq. 3.17 where $\Phi(B)$ arises, and using the above averaging technique for Φ_1, Φ_2 and Φ results in

$$\Phi_1(B)\left[1-\frac{\Delta S}{4r(B)M}+\frac{iKM}{2\sqrt{M^2-1}}\Delta S\right]=\Phi_1(A)\left[1+\frac{\Delta S}{4r(A)M}-\frac{iKM}{2\sqrt{M^2-1}}\Delta S\right]-$$

$$\Phi_2(B)\left[\frac{\Delta S}{4r(B)M}+\frac{iKM}{2\sqrt{M^2-1}}\Delta S-\frac{\Delta S^2}{4}\left(K^2-\frac{n^2}{r^2(B)M^2}\right)\right]-$$

$$\Phi_2(A)\left[\frac{\Delta S}{4r(A)M}+\frac{iKM}{2\sqrt{M^2-1}}\Delta S-\frac{\Delta S^2}{4}\left(K^2-\frac{n^2}{r^2(A)M^2}\right)\right]+ \qquad (3.19)$$

$$\Phi(A)\left[K^2-\frac{n^2}{r^2(A)M^2}\right]\frac{\Delta S}{2}+\Phi(A)\left[K^2-\frac{n^2}{r^2(B)M^2}\right]\frac{\Delta S}{2}$$

Similarly to solve for Φ_2 at point B, the above procedure was used except now move along the left running characteristic and integrate Φ_1 along the left running characteristic to find $\Phi(B)$.

$$\Phi_2(B)\left[1-\frac{\Delta S}{4r(B)M}+\frac{iKM}{2\sqrt{M^2-1}}\Delta S\right]=\Phi_2(C)\left[1+\frac{\Delta S}{4r(C)M}-\frac{iKM}{2\sqrt{M^2-1}}\Delta S\right]-$$

$$\Phi_1(B)\left[\frac{\Delta S}{4r(B)M}+\frac{iKM}{2\sqrt{M^2-1}}\Delta S-\frac{\Delta S^2}{4}\left(K^2-\frac{n^2}{r^2(B)M^2}\right)\right]-$$

$$\Phi_1(C)\left[\frac{\Delta S}{4r(C)M}+\frac{iKM}{2\sqrt{M^2-1}}\Delta S-\frac{\Delta S^2}{4}\left(K^2-\frac{n^2}{r^2(C)M^2}\right)\right]+ \qquad (3.20)$$

$$\Phi(C)\left[K^2-\frac{n^2}{r^2(C)M^2}\right]\frac{\Delta S}{2}+\Phi(C)\left[K^2-\frac{n^2}{r^2(B)M^2}\right]\frac{\Delta S}{2}$$

Then there are two equations [3.19 and 3.20] in two unknowns, $\Phi_1(B)$ and $\Phi_2(B)$, which can be solved by using Cramer's rule.[2] The following abbreviations are defined as

---

[2] Crandall, S. H., <u>Engineering Analysis</u>, New York, 1956, p. 36.

$$A = 1 - \frac{\Delta S}{4r(B)M} + \frac{iKM}{2\sqrt{M^2-1}} \Delta S$$

$$A_1 = 1 + \frac{\Delta S}{4r(A)M} - \frac{iKM}{2\sqrt{M^2-1}} \Delta S$$

$$A_2 = 1 + \frac{\Delta S}{4r(C)M} - \frac{iKM}{2\sqrt{M^2-1}} \Delta S$$

$$B = \frac{\Delta S}{4r(B)M} + \frac{iKM}{2\sqrt{M^2-1}} \Delta S - \frac{\Delta S^2}{4} \quad K^2 - \frac{n^2}{r^2(B)M^2} \tag{3.21}$$

$$C = \Phi_1(A)A_1 + \Phi_2(A)\left[-\frac{\Delta S}{4r(A)M} - \frac{iKM}{2\sqrt{M^2-1}} + \frac{\Delta S^2}{4}\left(K^2 - \frac{n^2}{r^2(A)M^2}\right)\right]$$

$$+ \frac{1}{2}\ \Phi(A)\left[K^2 - \frac{n^2}{r^2(A)M^2}\right]\Delta S + \frac{1}{2}\ \Phi(A)\left[K^2 - \frac{n^2}{r^2(B)M^2}\right]\Delta S$$

$$D = \Phi_2(C)A_2 + \Phi_1(C)\left[-\frac{\Delta S}{4r(C)M} - \frac{iKM}{2\sqrt{M^2-1}} + \frac{\Delta S^2}{4}\left(K^2 - \frac{n^2}{r^2(C)M^2}\right)\right]$$

$$+ \frac{1}{2}\ \Phi(C)\left[K^2 - \frac{n^2}{r^2(C)M^2}\right]\Delta S + \frac{1}{2}\ \Phi(A)\left[K^2 - \frac{n^2}{r^2(B)M^2}\right]\Delta S$$

In terms of these abbreviations the two equations are

$$A\Phi_1(B) + B\,\Phi_2(B) = C$$

$$B\Phi_1(B) + A\,\Phi_2(B) = D \tag{3.22}$$

Then using Cramer's rule

$$\Phi_1(B) = CA - BD/A^2 - B^2$$

$$\Phi_2(B) = DA - CD/A^2 - B^2 \tag{3.23}$$

Φ_1 was integrated to find $\Phi(B)$ along the right running characteristic and as a check Φ_2 was also integrated along the left running characteristic using the trapezoidal rule

$$\Phi^1(B) = \Phi(A) + \frac{1}{2}\left[\Phi_2(A) + \Phi_2(B)\right] \Delta S$$

$$\Phi^2(B) = \Phi(C) + \frac{1}{2}\left[\Phi_1(C) + \Phi_1(B)\right] \Delta S \qquad (3.24)$$

The difference $\left|\Phi^1(B) - \Phi^2(B)\right|$ was a measure of the accuracy. If it becomes too large, the grid size must be made smaller. $\Phi^1(B)$ and $\Phi^2(B)$ were averaged together for the best results.

$$\Phi(B) = \frac{1}{2}\left[\Phi^1(B) + \Phi^2(B)\right] \qquad (3.25)$$

The above procedure will suffice for a general field point which is not subject to a boundary condition once the procedure has been initiated.

On the initial Mach line it is known that $\Phi = 0$, $\Phi_1 = 0$, $\Delta\Phi_1 = 0$, therefore from Eqs. 3.20 and 3.21 (See Fig. 5).

$$A\Phi_2(E) = A_2\Phi_2(D)$$

$$\Phi_2(E) = \Phi_2(D)\,\frac{A_2}{A} \qquad (3.26)$$

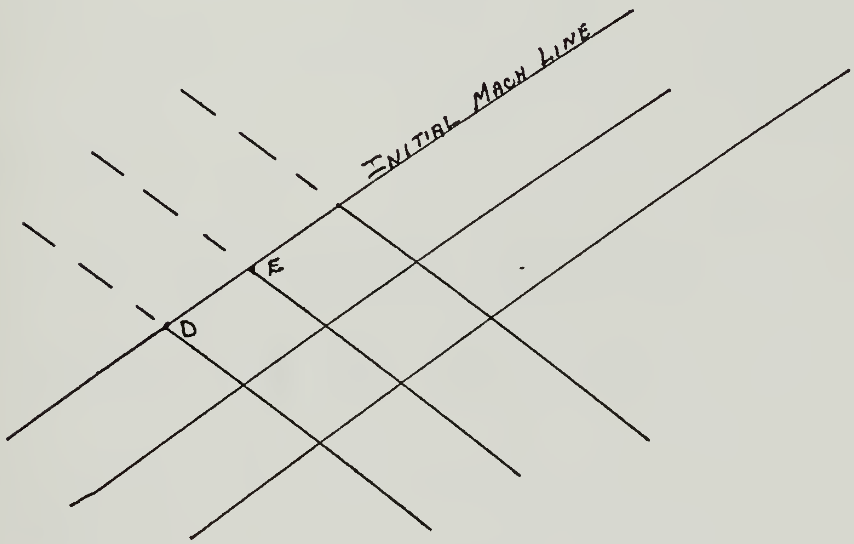


FIGURE 5

At the initial point on the initial Mach line the panel boundary condition also applied, i.e.,

$$\Phi_r \Big|_{r=R} = \left[\frac{\partial Z}{\partial x} + iKZ\right] ,$$

also from Eq. 3.14

$$\Phi_r = \frac{M}{2}\left(\Phi_1 - \Phi_2\right)$$

remembering that $\Phi_1 = 0$ and combining the two equations

$$\Phi_2 = -\frac{2}{M}\left[\frac{\partial Z}{\partial x} - iKZ\right] \tag{3.27}$$

On the panel from the above boundary condition (See Fig. 6)

$$\Phi_1(L) - \Phi_2(L) = -\frac{2}{M}\left[\frac{\partial Z}{\partial x} + iKZ\right] \tag{3.28}$$

and from Eqs. 3.19 and 3.21 $\Phi_1(L) + \Phi_2(L)$ was found on the right running characteristic and once again there exists a system of two equations in two unknowns.

$$\Phi_1(L) - \Phi_2(L) = -\frac{2}{M}\left[\frac{\partial Z}{\partial x} - iKZ\right] \tag{3.29}$$

$$A\Phi_1(L) + B\Phi_2(L) = C$$

Using Cramer's rule[3]

$$\Phi_1(L) = \left[C + B\,\frac{2}{M}\left(\frac{\partial Z}{\partial x} + iKZ\right)\right] \Big/ A+B$$

$$\Phi_2(L) = \left[-\,A\,\frac{2}{M}\left(\frac{\partial Z}{\partial x} + iKZ\right)\right] \Big/ A+B \tag{3.30}$$

---

[3] Ibid.

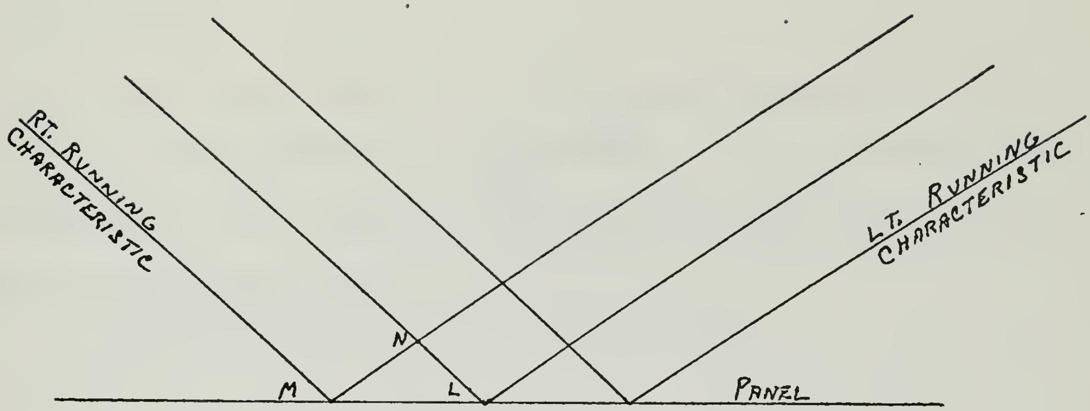


FIGURE 6

To write the program for a vibrating panel or the two-dimensional case, the radius of the cylinder r was set to infinity. Inspection of the basic equations programmed (Eq. 3.21) showed that r was always in the denominator, which causes these terms to equal zero. Therefore, terms containing r were neglected in the Fortran IV encoding of these equations. For the vibrating shell or three dimensional case the full equations as developed were Fortran IV encoded.

Also, it should be noted that the problem was formulated for an arbitrary wall deflection Z(x). However, in the numerical computations only the case of a sinsuoidal standing wave given by

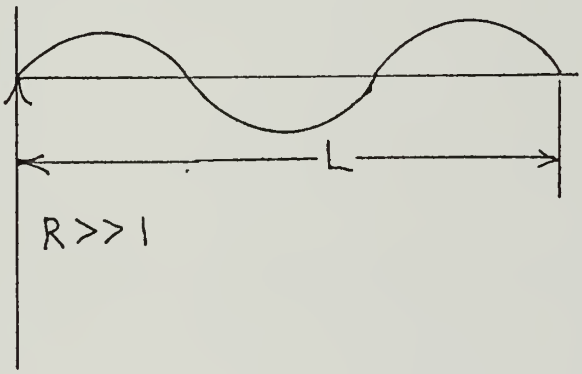
$$Z(x) = Z_o \text{Sin } (m\pi x) \tag{3.31}$$

was considered. The axial wave-length l is hence equal to L/2m where m is the number of axial half-waves in a given cylinder length L.

C. TEST CASES

The first test case considered a two-dimensional flat panel. If the cylinder radius is set at a very large value in relation to the length, then one has essentially a flat panel in supersonic flow.

When MFREQ is set at various values, NFREQ and K set at zero, one then has supersonic flow over a wavy wall as developed in Johns Ref. 1 .



Taking the wavy wall equations for Φ, Φ_r and Φ_x from Johns Ref. 1 and normalizing results in

$$\Phi = - \frac{1}{\sqrt{M^2-1}} \text{Sin } \left(x - \sqrt{M^2-1} \quad r\right)$$

$$\Phi_r = \pi \text{ Cos} \left[\pi \left(x - \sqrt{M^2-1} \quad r\right)\right] ,$$

$$\Phi_x = - \frac{\pi}{\sqrt{M^2-1}} \text{ Cos} \left[\pi \left(x - \sqrt{M^2-1} \quad r\right)\right] \qquad (3.31)$$

By using a large value of R in the three-dimensional case, it and the two-dimensional case results were then compared to the wavy wall results. These results were found to agree to the fifth significant figure. This case was also used to remove the majority of minor programming errors.

To remove any other hidden programming errors it was decided to set each variable equal to a finite value and to step through the first three characteristic lines by hand. The values were then compared to those computed by the computer. In this manner the rest of the programming bugs were found and removed.

IV. COMPUTER PROGRAM ORGANIZATION AND OPERATION

A. GENERAL ORGANIZATION

The numerical methods discussed in the foregoing sections have been coded as a FORTRAN IV computer program for the IBM 360 system.

The program consists of a main control section and four subsections or subroutines. An algorithm showing the program information flow and the interaction between the control section and the subroutines was listed in Figure 7. The control section reads in the input data, computes the characteristic grid, computes the initial condition, and computes several constants common to the subroutines. Then subroutine COMPRX is called to compute the coordinates of the next grid point to be calculated, see Figure 8 for a diagram of the characteristic grid used. Two "logical if" statements then determine if the point in question is located on either the initial Mach line, the panel, or at a general field point. Control is then passed to that subroutine to compute the velocity potential and its derivatives along the characteristics at that point. The coefficient of pressure is computed along the panel in subroutine PANPT and after the program has stepped down to the last point on the last characteristic, a listing of the coefficient of pressure at each panel point is printed out.

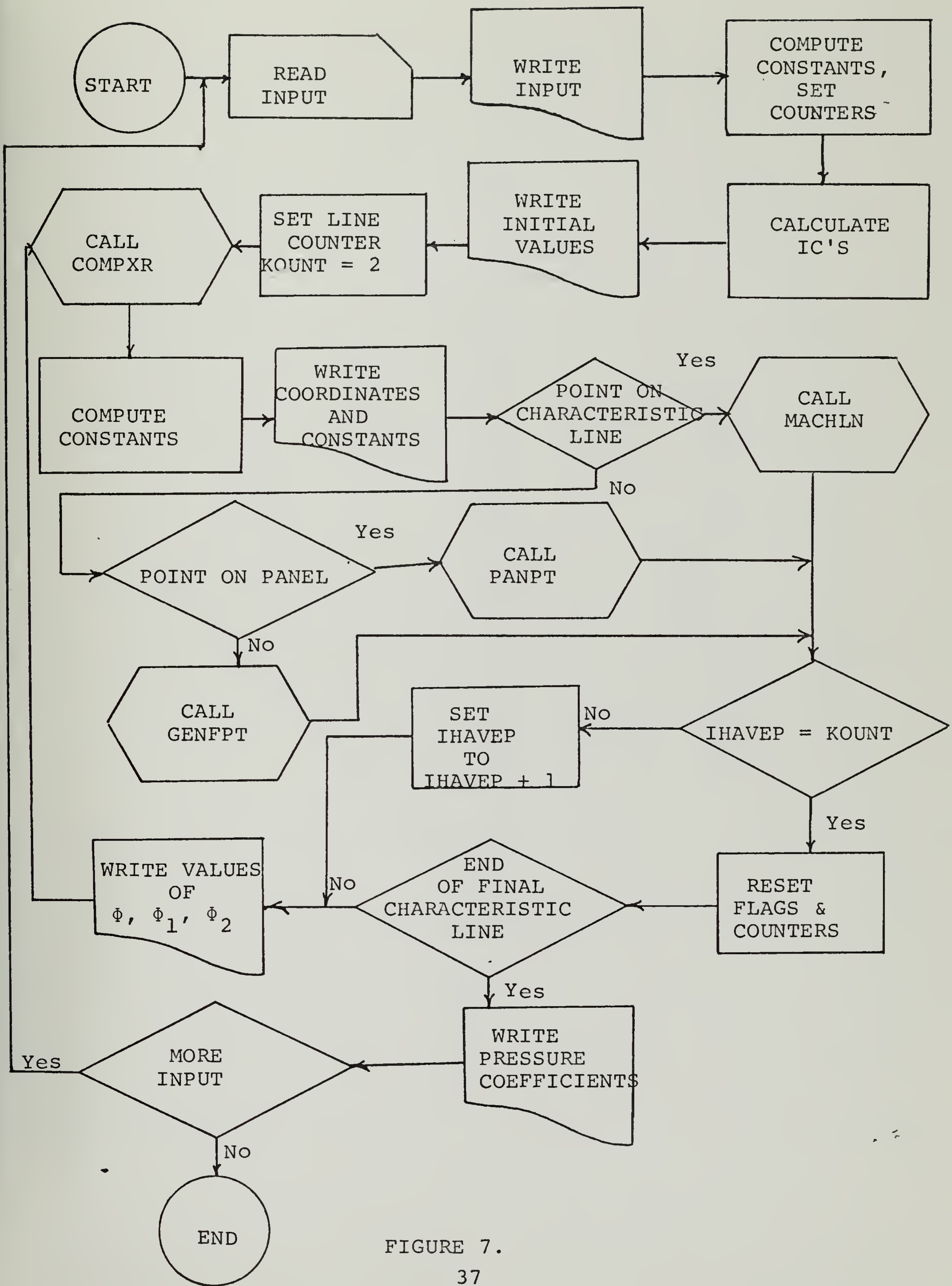

START
READ INPUT
WRITE INPUT
COMPUTE CONSTANTS, SET COUNTERS
CALCULATE IC'S
WRITE INITIAL VALUES
SET LINE COUNTER KOUNT = 2
CALL COMPXR
COMPUTE CONSTANTS
WRITE COORDINATES AND CONSTANTS
POINT ON CHARACTERISTIC LINE
Yes
CALL MACHLN
No
POINT ON PANEL
Yes
CALL PANPT
No
CALL GENFPT
SET IHAVEP TO IHAVEP + 1
No
IHAVEP = KOUNT
Yes
RESET FLAGS & COUNTERS
END OF FINAL CHARACTERISTIC LINE
No
WRITE VALUES OF Φ, Φ1, Φ2
Yes
WRITE PRESSURE COEFFICIENTS
MORE INPUT
Yes
No
END

FIGURE 7.

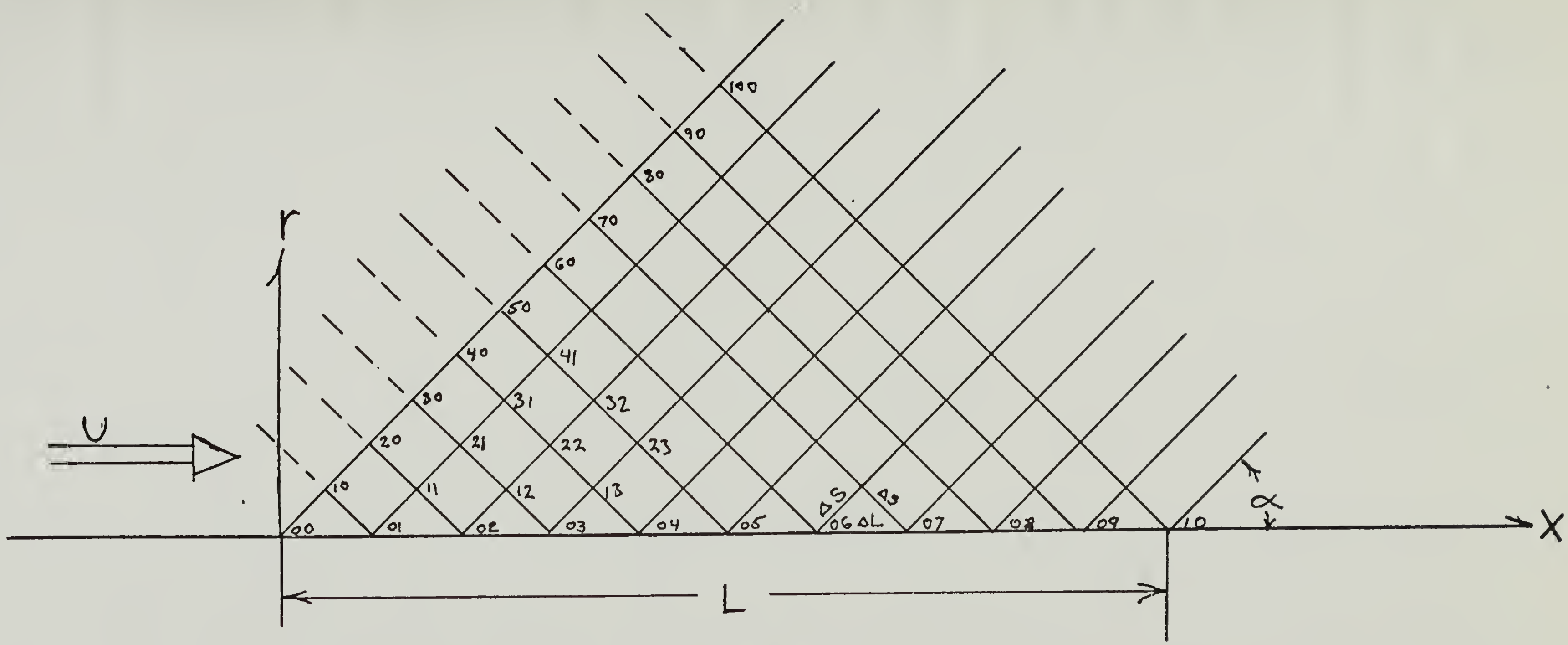


FIGURE 8

THE CHARACTERISTIC GRID SYSTEM

B. MAIN CONTROL PROGRAM OPERATION

The main program dimensions the storage requirements in common block form; then it reads three input data cards. The first card has the date of the run in 3A4 format. The second card reads in the fineness of the grid, number of this run, circumferential mode number, and the axial mode number in 4I5 format. The third card reads Mach number, the reduced frequency number, and the cylinder radius in 3F10.4 format. Input is then written to verify input data.

The main program then computes the Mach angle, step sizes, and zeros the counters. The values for the velocity potential and its derivatives and the coordinates of each point are stored as a n x 2 matrix. As the program steps from characteristic line to characteristic line, only those values for the line being computed and the line just computed are in storage. The initial values at the first point on the initial characteristic is computed using Eq. 3.27 and written out to aid in the program check-out. The line counter, "Kount", is then set at two and subroutine COMPXR is called to compute the values of x and r at the next point. The main program next computes various quantities dependent on x and r which are needed in the other subroutines. The values for x, r, and dependent quantities are printed out as a program debug aid. Two "logical if" statements next determine if the point under consideration is on the initial Mach line, at a general field point, or on the panel. The appropriate subroutine is then called to solve for the velocity potential and its derivatives along the

characteristics at that point. The next "logical if" statement determines if the program must switch lines or step along the present line. Two more "logical if" statements are used to determine if this solution is completed and if there is more data to be read and more solutions to be computed. If "numrun" is set at two, the program will look for more data; if "numrun" equals one, the program will terminate. Before it stops or reads more data the coefficient of pressure along the panel will be printed out for each panel point and an in-line graph, using the IBM scientific library program POLTP, will plot the real and imaginary parts of the coefficient of pressure.

C. SUBROUTINE COMPXR OPERATION

Information is passed between the calling program and the subroutines by means of the common storage. The first step in COMPXR is to determine if the point to be computed is on the initial Mach line or not. The coordinates of the last point computed are used and the step distance in the x and r directions either added to or subtracted from them to find the new x and r coordinates. The addition or subtraction depends upon whether we step up to a new characteristic line or down the present characteristic line.

D. SUBROUTINE MACHLN OPERATION

In subroutine MACHLN the boundary conditions of the equations developed in the foregoing section are enforced such that the velocity potential and its derivatives along the right running characteristic are set to zero. Equations 3.26 were coded in FORTRAN to solve for Φ_2 at the point in question. The value of Φ_2 and the variables used to solve for Φ_2 are printed out for the first 12 characteristic lines to aid in the program checkout and then control is returned to the main program.

E. OPERATION OF SUBROUTINE PANPT

In subroutine PANPT the boundary condition along the panel is enforced. The velocity potential and its derivatives along the characteristics are computed at the point in question, using Eq. 3.29. The real and imaginary parts of the coefficient of pressure are stored for future read out. Control is then returned to the main program.

F. OPERATION OF SUBROUTINE GENFPT

GENFPT computes the velocity potential and its first derivatives along the right and left running characteristics. It draws the necessary information from common storage and uses the Eqs. 3.21, 3.22, and 3.23, which were developed in the foregoing chapter. To aid in program check out the variables A, A_1, A_2, B, C, and D are printed out for the first 12 characteristic lines. Control is then returned to the main program.

V. RESULTS AND DISCUSSION

The case of a vibrating two-dimensional panel was chosen as the first test of the accuracy of the characteristics method. These results were compared to the results obtained by evaluating the method-of-singularities integrals developed in Ref. 4 for flutter of a two-dimensional panel with one surface exposed to supersonic flow. These integrals were evaluated by applying a 12-point Gaussian Quadrature. The results of these evaluations were available in unpublished notes by M. F. Platzer.

The comparisons for $M = \sqrt{2}$ at a reduced frequency of 0.5 and an axial mode number of one are shown in Figures 8-11. Figures 8 and 9 are plots of the real and imaginary parts of the coefficient of pressure obtained by the method of singularities. Figures 10 and 11 are plots of the real and imaginary parts of the Cp for the characteristics method. When these tests were first run it was found that the real parts of the Cp plots were identical, as shown in Figures 8 and 10. The imaginary parts of Cp, shown in Figures 9 and 11, did not agree. In rechecking the formulation of the equations it was found that the values of Φ, Φ_1 and Φ_2 had not been averaged as explained in Chapter II, Section B, where Eqs. 3.19 and 3.20 were formulated. When these average values were incorporated into the equations, the computer program rewritten and the program rerun, then the two plots were found to be identical for both the real and imaginary parts of the Cp.

Figures 12-14 are plots for a reduced frequency of two, an axial mode of four, and the same Mach number. These comparisons of the real and imaginary parts of the Cp were also found to be identical.

In attempting to verify the results of the vibrating shell program, Ref. 5 was used. Reference 5 includes a study of supersonic potential flow past an infinitely long vibrating cylinder. It was not known what effect the difference in an infinitely long cylinder vice a finite length cylinder would have on the outcome, but these were the only published results found that could be used for a comparison of the vibrating shell program.

Two parameters were used to compare the results of the two studies: the maximum amplitude, A_1, of the sinsuoidal Cp wave, and the phase angle, ψ_1, of this wave. Equation 14 in Ref. 5 is used to equate Cp to the value of A_1 listed in Figure 2a of Ref. 5.

$$\overline{p}(x,R,\theta) = \frac{Z_o}{R} \frac{\rho U^2}{M} \frac{m\pi R}{L} \text{Cos } n\theta \, A_1 \text{ Cos} \left(\frac{m\pi x}{L} + \psi_1\right)$$

$\overline{p}$ is defined as $\overline{p} = p - p_\infty$ where

$$C_p = \frac{p - p_\infty}{\frac{1}{2} \rho U^2} \qquad (5.1)$$

and the terms, Z_o, L, and Cos(x) were all taken as equal to one then

$$C_p = \frac{\bar{p}}{\frac{1}{2}\rho U^2} = \frac{2}{M} m\pi A_1$$

$$A_1 = C_p \frac{M}{2m\pi} \qquad (5.2)$$

Three different values of n were computed and the coefficients of pressure were listed in Figure 17 as a test case for this comparison. For the case n = 0, (See Figure 2a of Ref. 5) we find that for the ratio of axial wave length/cylinder radius $\ell/R = 3.33$, A_1 equals 1.05. Using Eq. 5.2 to determine A_1 for a maximum Cp of 5.63 in Figure 17, A_1 for this study would equal 1.044. Using this same procedure for n = 4 and 8

n = 4	A_1(ref 5) = 1.30	A_1 = 1.445
n = 8	A_1(ref 5) = 1.80	A_1 = 1.85.

To compare the phase angle ψ_1, Figure 2b in Ref. 5 was entered for $\ell/R = 3.33$ and the three values of n taken previously. This shift, ψ_1, was calculated from results (Fig. 17) by determining the zero crossing point.

n = 0	59.8
n = 4	57.7
n = 8	48.9

Converting these zero crossing points to angular shifts, it was found that

$n = 0$	$\psi_1 = 4.8$	ψ_1 (ref 5) $= 5$
$n = 4$	$\psi_1 = 13.5$	ψ_1 (ref 5) $= 10$
$n = 8$	$\psi_1 = 55.4$	ψ_1 (ref 5) $= 60$

which compare favorably with Anderson's values listed in the third column.

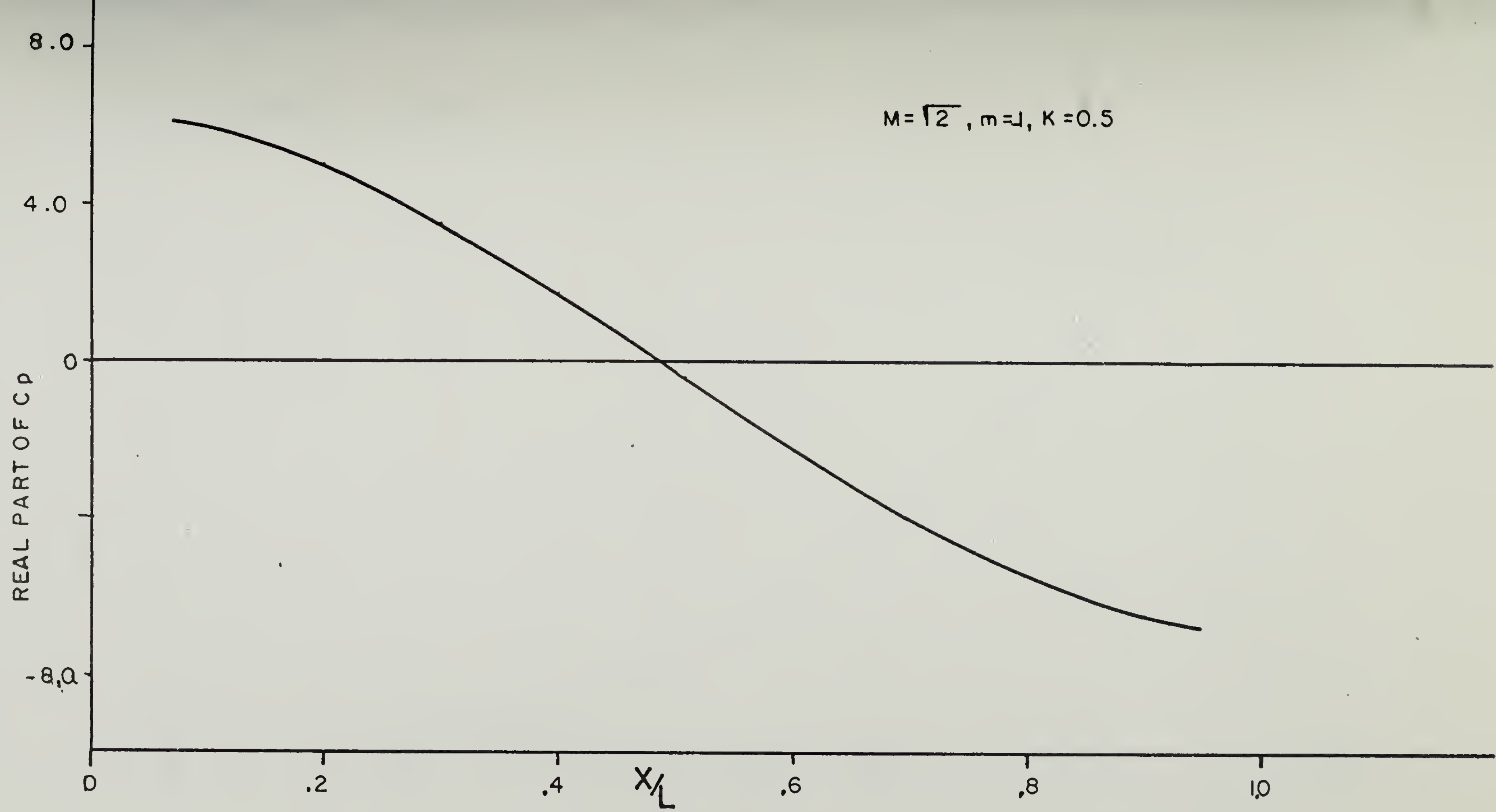


FIGURE 9

REAL PART OF C_p FOR A VIBRATING TWO-DIMENSIONAL FLAT PANEL

METHOD OF SINGULARITIES

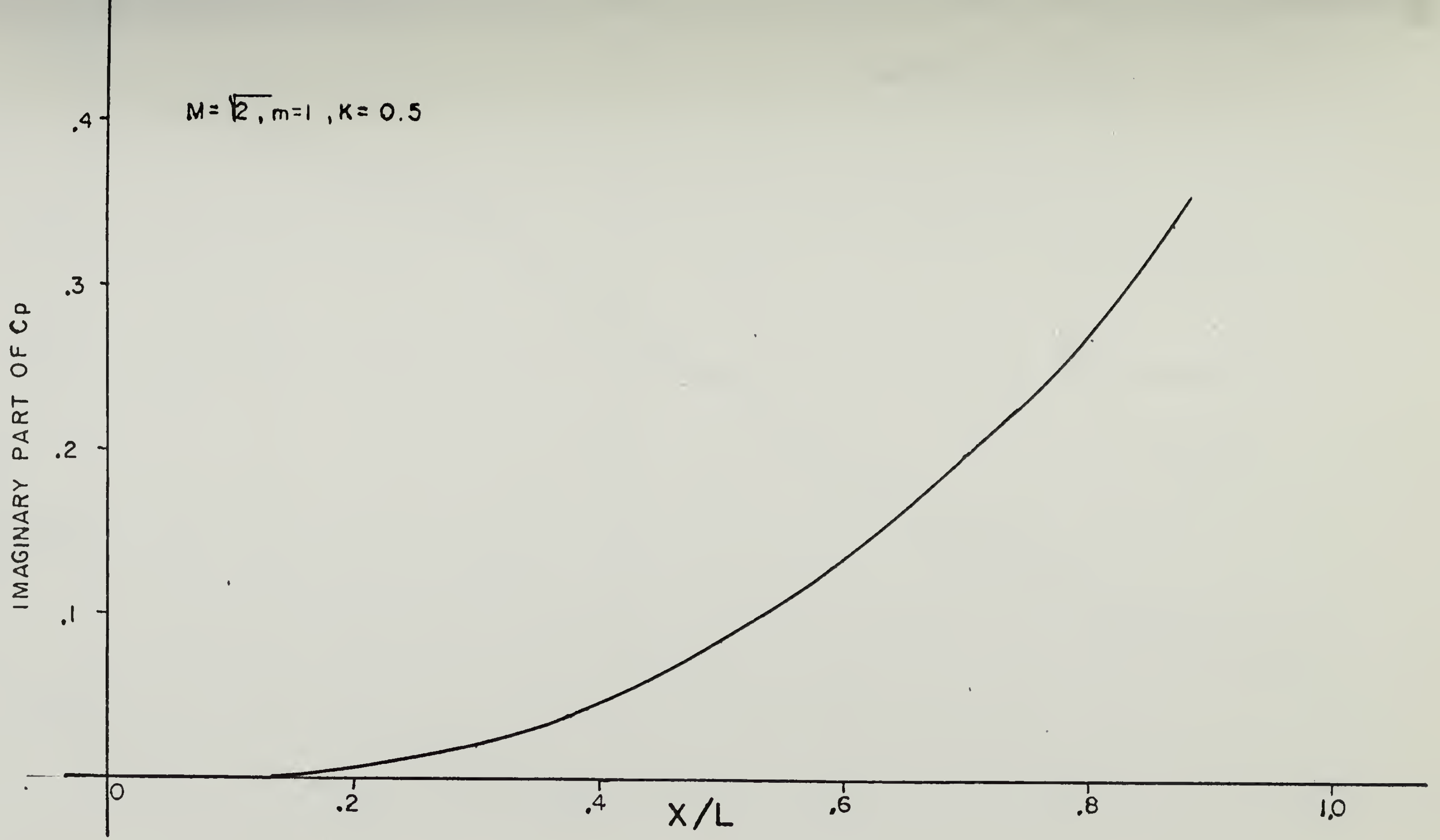


FIGURE 10

IMAGAINARY PART OF C_p FOR A VIBRATING TWO-DIMENSIONAL FLAT PANEL

METHOD OF SINGULARITIES

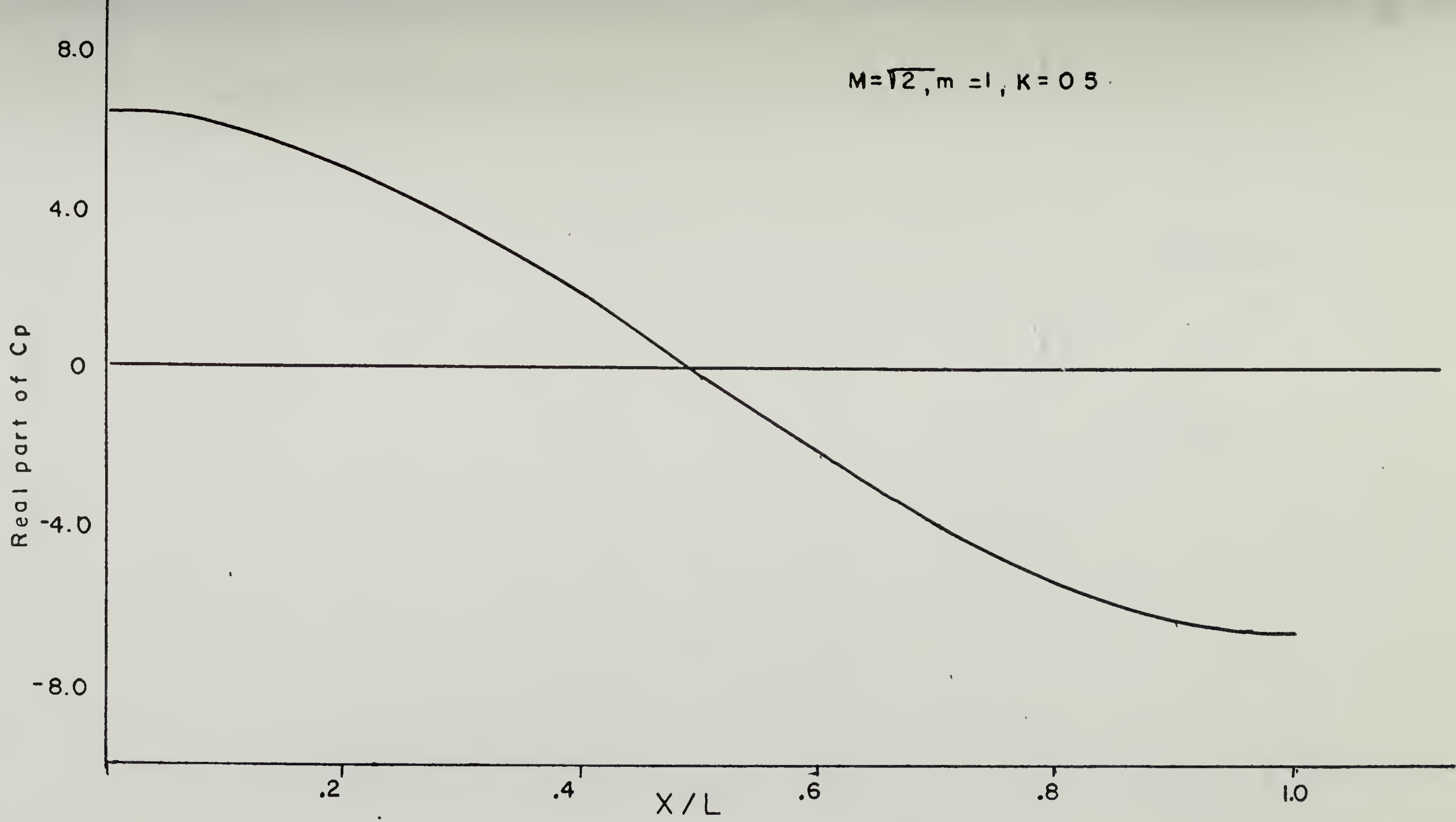


FIGURE 11

REAL PART OF C_p FOR A VIBRATING TWO-DIMENSIONAL FLAT PANEL

METHOD OF CHARACTERISTICS

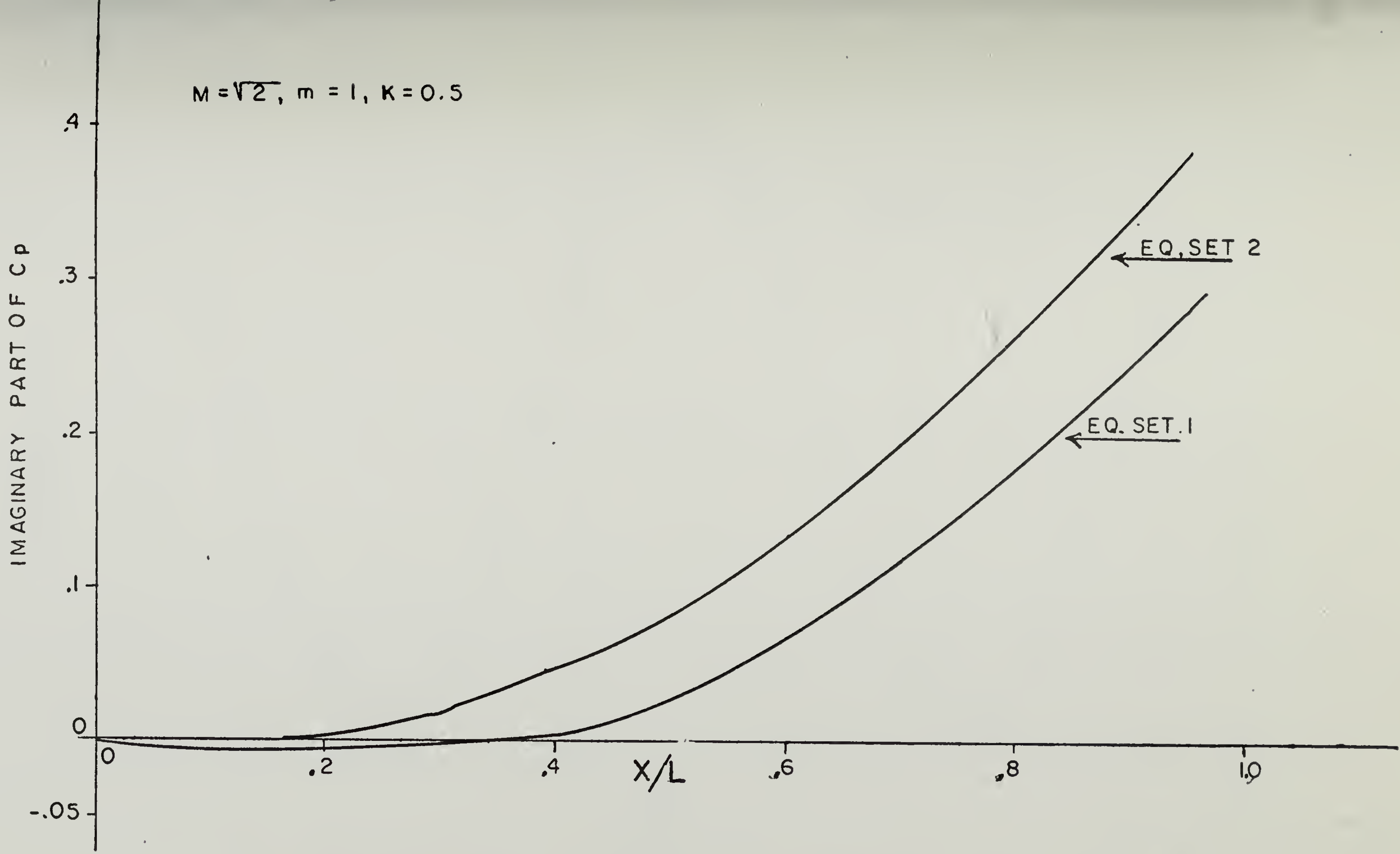


FIGURE 12

IMAGAINARY PART OF Cp FOR A VIBRATING TWO-DIMENSIONAL FLAT PANEL

METHOD OF CHARACTERISTICS

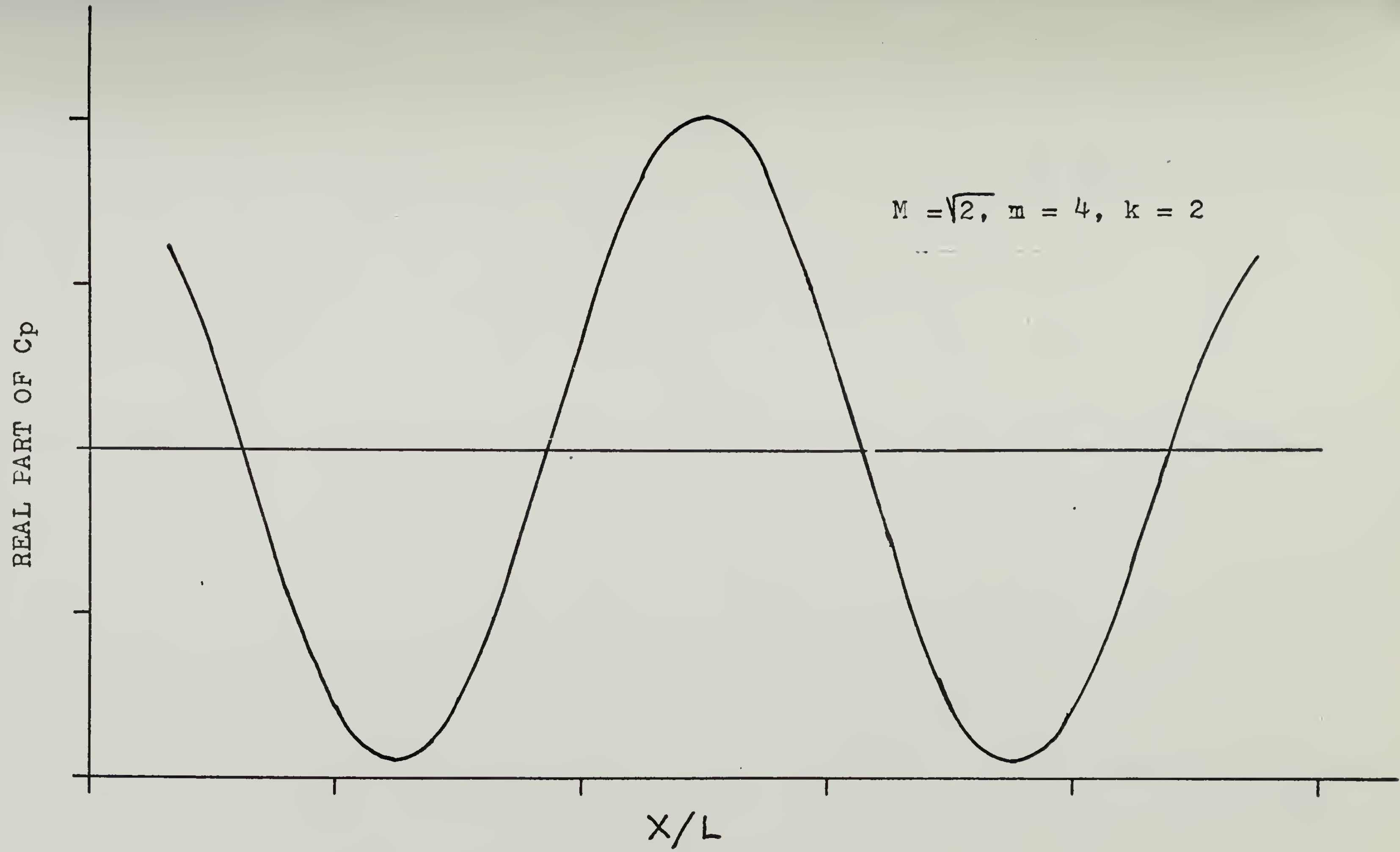


FIGURE 13

REAL PART OF Cp FOR A VIBRATING TWO-DIMENSIONAL FLAT PANEL

METHOD OF SINGULARITIES

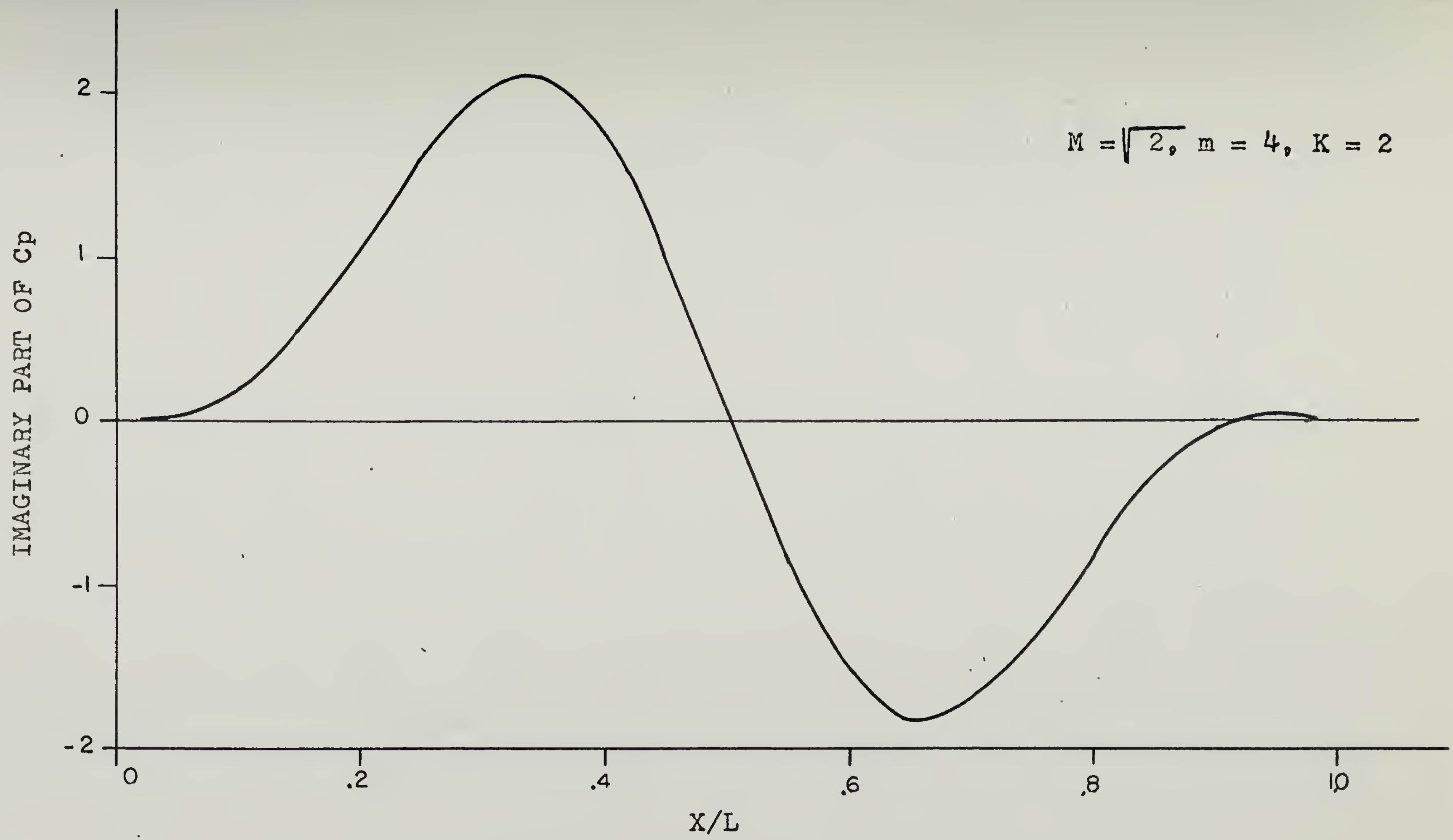


FIGURE 14

IMAGAINARY PART OF Cp FOR A VIBRATING TWO-DIMENSIONAL FLAT PANEL

METHOD OF SINGULARITIES

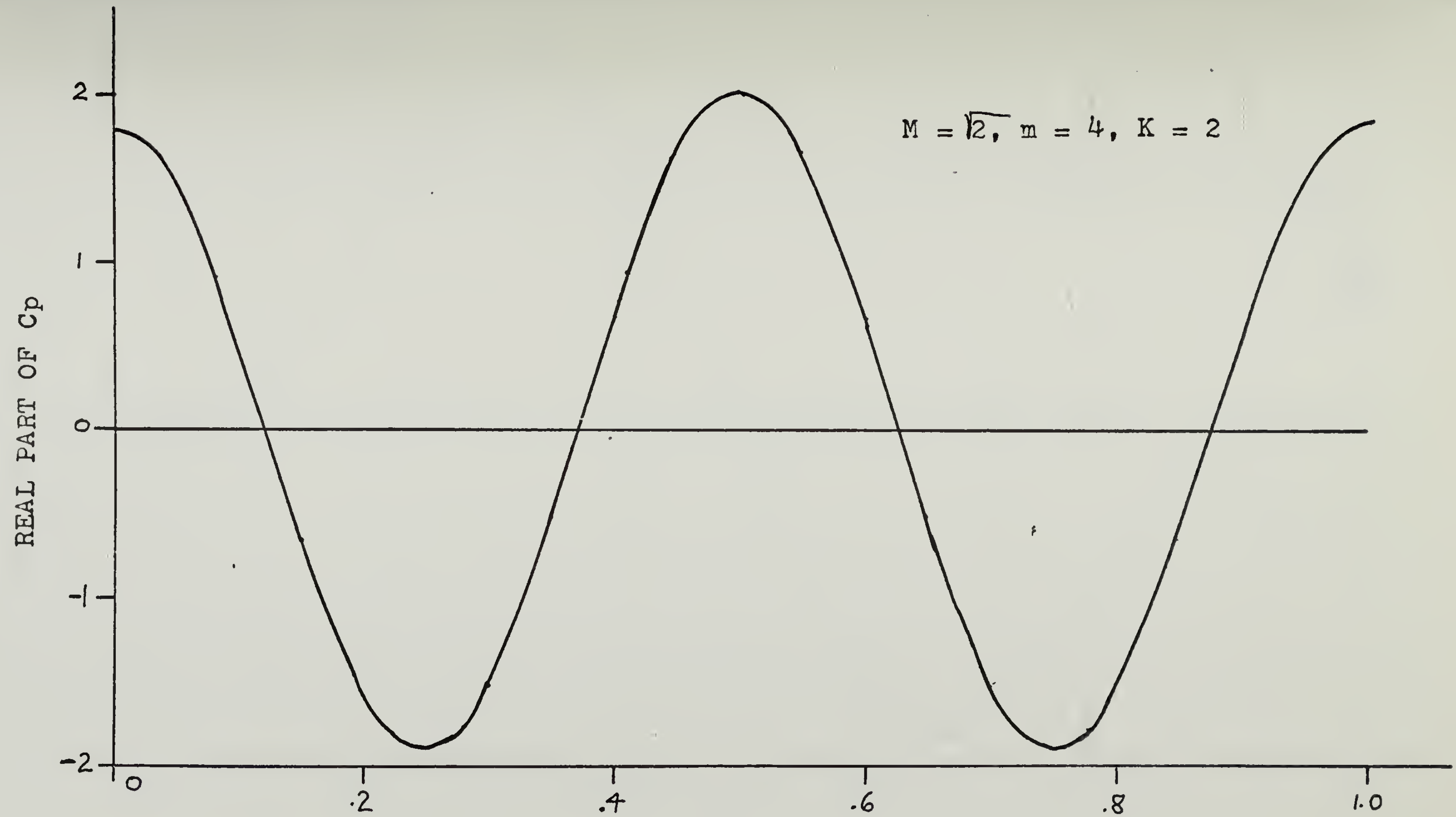


FIGURE 15

REAL PART OF C_p FOR A VIBRATING TWO-DIMENSIONAL FLAT PANEL

METHOD OF CHARACTERISTICS

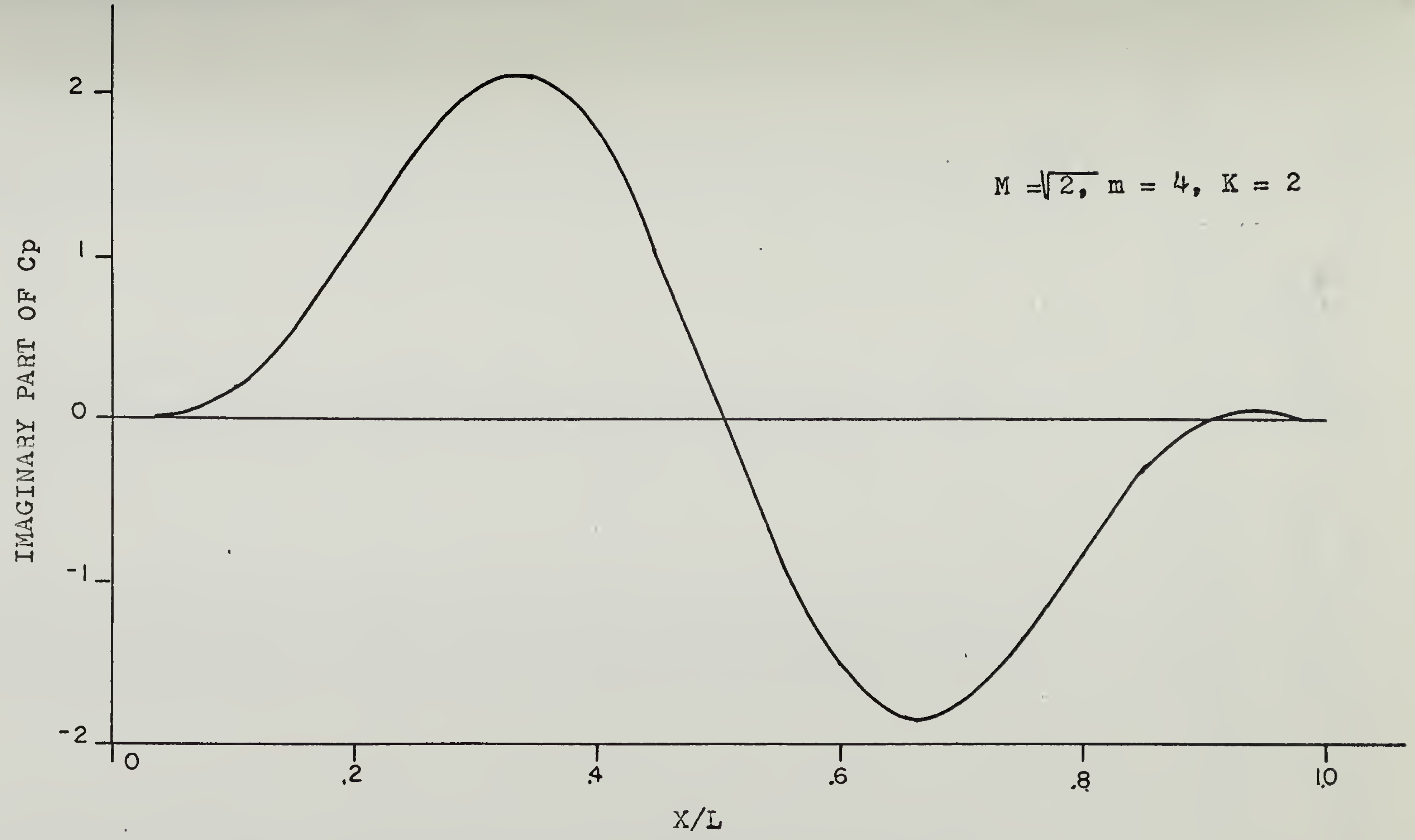


FIGURE 16

IMAGAINARY PART OF Cp FOR A VIBRATING TWO-DIMENSIONAL FLAT PANEL

METHOD OF CHARACTERISTICS

COEFFICIENT OF PRESSURE COMPUTER OUTPUT LISTING

FOR CASE OF VIBRATING SHELL

M = 3.5, m = 3, K = 0, r = 0.20 and n = 0, 1, 3, where L is divided into 120 grid points.

Grid Point	n = 0	n = 4	n = 8
1.	5.6198	5.6198	5.6198
2.	5.5677	5.5538	5.5642
3.	5.4813	5.4266	5.4676
4.	5.3613	5.2396	5.3307
5.	5.2083	4.9950	5.1546
6.	5.0233	4.6954	4.9404
7.	4.8075	4.3440	4.6897
8.	4.5621	3.9444	4.4041
9.	4.2887	3.5007	4.0857
10.	3.9890	3.0172	3.7366
11.	3.6647	2.4988	3.3591
12.	3.3180	1.9504	2.9559
13.	2.9509	1.3775	2.5295
14.	2.5657	0.7855	2.0828
15.	2.1648	0.1801	1.6188
16.	1.7507	-0.4329	1.1407
17.	1.3258	-1.0477	0.6515
18.	0.8929	-1.6586	0.1546
19.	0.4546	-2.2597	-0.3467
20.	0.0136	-2.8453	-0.8492
21.	-0.4273	-3.4100	-1.3495
22.	-0.8656	-3.9435	-1.8442
23.	-1.2985	-4.4557	-2.3301
24.	-1.7232	-4.9267	-2.8040
25.	-2.1373	-5.3572	-3.2626
26.	-2.5380	-5.7431	-3.7028
27.	-2.9230	-6.0806	-4.1218
28.	-3.2899	-6.3665	-4.5167
29.	-3.6365	-6.5980	-4.8848
30.	-3.9605	-6.7726	-5.2237
31.	-4.2600	-6.8886	-5.5309
32.	-4.5331	-6.9446	-5.8044
33.	-4.7782	-6.9398	-6.0423
34.	-4.9938	-6.8738	-6.2428
35.	-5.1785	-6.7468	-6.4046
36.	-5.3312	-6.5595	-6.5263
37.	-5.4509	-6.3132	-6.6071
38.	-5.5369	-6.0095	-6.6462
39.	-5.5887	-5.6507	-6.6432

Grid Point	n = 0	n = 4	n = 8
40.	-5.6059	-6.5979	-5.2394
41.	-5.5885	-6.5104	-4.7787
42.	-5.5366	-6.3810	-4.2721
43.	-5.4504	-6.2105	-3.7235
44.	-5.3305	-5.9996	-3.1371
45.	-5.1777	-5.7495	-2.5175
46.	-4.9929	-5.4616	-1.8695
47.	-4.7773	-5.1375	-1.1981
48.	-4.5320	-4.7791	-0.5086
49.	-4.2588	-4.3884	-0.1934
50.	-3.9592	-3.9678	-0.9027
51.	-3.6351	-3.5196	-1.6137
52.	-3.2886	-3.0466	-2.3205
53.	-2.9216	-2.5516	-3.0178
54.	-2.5366	-2.0375	-3.7000
55.	-2.1359	-1.5074	-4.3616
56.	-1.7219	-0.9644	-4.9975
57.	-1.2972	-0.4119	-5.6025
58.	-0.8645	0.1467	-6.1719
59.	-0.4263	0.7082	-6.7010
60.	0.0145	1.2692	-7.1856
61.	0.4554	1.8261	7.6218
62.	0.8935	2.3757	8.0060
63.	1.3262	2.9145	8.3350
64.	1.7508	3.4393	8.6062
65.	2.1647	3.9467	8.8171
66.	2.5653	4.4338	8.9659
67.	2.9502	4.8975	9.0512
68.	3.3170	5.3350	9.0720
69.	3.6633	5.7435	9.0280
70.	3.9872	6.1205	8.9191
71.	4.2866	6.4636	8.7458
72.	4.5596	6.7708	8.5092
73.	4.8046	7.0400	8.2107
74.	5.0200	7.2697	7.8522
75.	5.2046	7.4582	7.4361
76.	5.3571	7.6045	6.9652
77.	5.4767	7.7075	6.4427
78.	5.5626	7.7666	5.8721
79.	5.6142	7.7813	5.2573
80.	5.6313	7.7514	4.6027
81.	5.6138	7.6771	3.9126
82.	5.5617	7.5588	3.1920
83.	5.4755	7.3971	2.4459
84.	5.3555	7.1929	1.6793
85.	5.2025	6.9474	0.8978

Grid Point	n = 0	n = 4	n = 8
86.	5.0176	6.6619	0.1068
87.	4.8018	6.3383	-0.6882
88.	4.5565	5.9783	-1.4817
89.	4.2831	5.5841	-2.2681
90.	3.9834	5.1580	-3.0419
91.	3.6592	4.7026	-3.7977
92.	3.3125	4.2205	-4.5302
93.	2.9455	3.7146	-5.2342
94.	2.5604	3.1879	-5.9048
95.	2.1595	2.6436	-6.5374
96.	1.7454	2.0848	-7.1275
97.	1.3206	1.5150	-7.6710
98.	0.8878	0.9376	-8.1640
99.	0.4495	0.3559	-8.6031
100.	0.0085	-0.2264	-8.9853
101.	-0.4324	-0.8061	-9.3078
102.	-0.8706	-1.3795	-9.5684
103.	-1.3034	-1.9434	-9.7652
104.	-1.7282	-2.4942	-9.8968
105.	-2.1421	-3.0288	-9.9623
106.	-2.5429	-3.5439	-9.9611
107.	-2.9278	-4.0364	-9.8932
108.	-3.2947	-4.5035	-9.7591
109.	-3.6412	-4.9423	-9.5595
110.	-3.9652	-5.3503	-9.2957
111.	-4.2646	-5.7250	-8.9696
112.	-4.5378	-6.0642	-8.5833
113.	-4.7828	-6.3660	-8.1393
114.	-4.9984	-6.6285	-7.6406
115.	-5.1830	-6.8502	-7.0906
116.	-5.3356	-7.0299	-6.4928
117.	-5.4553	-7.1665	-5.8513
118.	-5.5413	-7.2593	-5.1703
119.	-5.5931	-7.3078	-4.4545
120.	-5.6103	-7.3117	-3.7084

VI. SUMMARY AND CONCLUSIONS

A linearized method of characteristics procedure was developed to compute supersonic flow past vibrating panels and shells. Good agreement was obtained when comparing with previous results by Nelson and Cunningham [Ref. 4] and Anderson [Ref. 5], who used quite different approaches to this problem.

However, more work needs to be done to further verify the cylindrical shell computations. In particular, the computations of the generalized aerodynamic forces presented by Dowell and Widnall [Ref. 6] should be verified. For this purpose the limiting case of vanishing shell radius (slender body theory) must be studied. This requires a reformulation of the flow boundary condition.

Also, it should be noted that the characteristics method allows the incorporation of quite arbitrary vibration modes (not restricted to sinsuoidal modes) and hence should prove to be a quite versatile tool for aeroelastic analyses in the supersonic flow regime.

APPENDIX A: VIBRATING PANEL COMPUTER PROGRAM

```
C
C          'LINEARIZED METHOD OF CHARACTERISTICS'
C       FOR SUPERSONIC FLOW PAST A VIBRATING PANEL
C
C
C
C     DEFINITION OF TERMS USED
C
C
C     MACH IS FREE STREAM MACH NUMBER
C     K IS REDUCED DIMENSIONLESS FREQUENCY
C     MFREQ IS THE AXIAL MODE OF THE CYLINDER
C     MANGL IS THE MACH ANGLE
C     FINGRD  IS THE FINENESS OF THE GRID
C     DELTAS IS THE STEP SIZE ALONG THE MACH LINES
C     ISWTCH IS THE SWITCH VARIABLE FOR CHOOSING THE LINE ON WHICH
C     TO WORK
C     JSWTCH IS OPPOSITE OF ISWTCH.IF ISWTCH=1,JSWTCH=2 OR VICE-VERSA.
C     XLENGT  IS THE LENGHT OF INITIAL MACH LINE
C     MLINE IS THE MAX NUMBER OF PTS. ON THE INTIAL MACH LINE.
C     QPHI IS THE VELOCITY POTENIAL
C     PHK IS THE DERIVATIVE OF THE VEL. POT. ALONG THE LEFT RUNNNING
C     CHARACTERISTIC
C     PHI IS THE DERIVATIVE OF THE VEL. POT. ALONG THE RIGHT RUNNING
C     CHARACTERISTIC
C     PX IS THE DERIVATIVE  OF THE VEL. POT . IN THE X DIRECTION
C     PR IS THE DERIVATIVE OF THE VEL. POT. IN THE R DIRECTION
C     CP IS THE COEFFICENT OF PRESSURE
C     REALCP IS THE REAL PART OF CP
C     AMAGCP IS THE IMAGINARY PART OF CP
C
C     RUN IN DOUBLE PRECISION
C
      IMPLICIT REAL*8 (A-H,O-Z,$)
      DIMENSION DATE(3),YYY(90)
      COMMON DPHII,DPHIK,PHI(400,2),PHK(400,2),QPHI(400,2),IPHI1,IPHI2,
     4IKM,IKW,PARW,W,II,REALCP(400),CP(400),AMAGCP(400),
     1    MACH,K,MANGL,RADIUS,DELTAS,XLENGT,BETA,DSTSTR,HDSTRL,TRNGLH,
     2  X(400,2),R(400,2),NFREQ,FINGRD,ISWTCH,JSWTCH,ILINE,MLINE,JLINE,
     3  MFREQ,IHAVEP,KOUNT
C
      INTEGER FINGRD
      REAL*8 MACH,MANGL,K
      COMPLEX*16
     1        DPHII,DPHIK,PHI,IKM,CP,PX ,IPHI1,IPHI2,IKW,PHK,II,QPHI
     1 ,PR
C
```

```
C      PRINT NAME OF PROGRAM AND DATE OF RUN
C
       READ (5,10)DATE
 10    FORMAT (3A4)
 11    FORMAT (1H1///,40X,'LINEARIZED METHOD OF CHARACTERISTICS'//40X,
      1'FOR SUPERSONIC FLOW PAST A VIBRATING CYLINDRICAL SHELL'//10X
      2'DATE OF RUN-',3A4///)
       WRITE (6,11) DATE
C
C      READ INPUT
C
 200   READ (5,1) FINGRD ,NUMRUN,MFREQ
       READ (5,2) MACH,K
 1     FORMAT (3I5)
 2     FORMAT (2F10.4)
C
C      COMPUTE CONSTANTS
C
       II=(0.0,1.0)
       BETA=DSQRT(MACH**2-1.0)
       MANGL=DARSIN(1.0/MACH)
       XLENGT=1./(2.*DCOS(MANGL))
       HEIGHT=DSIN(MANGL)*XLENGT
       DELTAS=XLENGT /FINGRD
       DSTSTR=DELTAS*DCOS(MANGL)*2.0
       HDSTRL=.5*DSTSTR
       MLINE=FINGRD
       TRNGLH=HDSTRL*DTAN(MANGL)
C
       DO 31 KK=1,400
        CP(KK)=(0.,0.)
 31    CONTINUE
C
C
C      SET COUNTERS
C
 20    ISWTCH=2
       JSWTCH=1
       IHAVEP=1
C
C      COMPUTE INTIAL VALUES
C
C
       I=IHAVEP
       QPHI(I,JSWTCH)=(0.,0.)
       PHK(I,JSWTCH)=(0.,0.)
       X(IHAVEP,JSWTCH)=0.0
       R(IHAVEP,JSWTCH)=0
```

```
      PI=3.141592654
      PARW=MFREQ*PI*DCOS(MFREQ*PI*X(IHAVEP,JSWTCH)  )
      QR=PARW
      PHI(I,JSWTCH)=-2*QR/MACH
C     WRITE OUT INPUT
C
C
      WRITE (6,12) MACH,K,FINGRD,MFREQ
 12   FORMAT (1H0,20X,'FREESTREAM MACH NUMBER =',F9.6//20X,'REDUCED FREQ
     1=',F9.5//20X,'GRID FINENESS IS = ',
     2 I2,//' ',20X,'AXAIAL MODE NO. M = ',I2)
C
C     WRITE INTIAL VALUES
C
      WRITE (6,57) QPHI(1,1),PHI(1,1),PHK(1,1) ,X(1,1),R(1,1),PARW
 57    FORMAT (//'0',20X,'INTIAL VALUE OF QPHI =',2E10.5,5X,'PHI =',
     1 2E10.4,3X,'PHK =',2E10.4,/' ',20X,'X =',E10.3,2X,'R =',E8.3
     2,'PARW =',E10.3,///)
C
C     SET COUNTER UP FOR EACH LINE
C
      KOUNT=2
 100  WRITE (6,59) KOUNT
 59   FORMAT (' ','KOUNT =',I5)
C
C     COMPXR COMUTES THE VALUE FOR X AND R GIVEN IHAVEP,ISWTCH
C
 104   CALL COMPXR
      PI=3.141592654
      PARW=MFREQ*PI*DCOS(MFREQ*PI*X(IHAVEP,ISWTCH)          )
      W=DSIN(MFREQ*PI*X(IHAVEP,ISWTCH))
      XKM=(K*MACH)/BETA
      IKM=II*XKM
      XKW=K*W
      IKW=II*XKW
      IF (KOUNT.GT.12) GO TO 1001
      WRITE (6,58) PARW,W,IKM,IKW
 58   FORMAT (' ',20X,'CONSTANTS AT NEW POINT',/' ',20X,'PARW =',
     1 E16.5,3X,' W =',E10.5,/' ',20X,'IKM & IKW =',2(2E10.5,5X))
C
C     THE PROGRM NOW GOES TO 30 IF POINT IS ON INITIAL MACH LINE.
C
 1001  IF (IHAVEP.EQ.1) GO TO 30
C
C     TEST IF WE ARE AT A PANEL PT.
C
      IF  (IHAVEP.EQ.KOUNT) GO TO 40
      GO TO 60
```

```
C
 30     CALL MACHLN
       GO TO 50
C
 40    CALL PANPNT
       GO TO 50
C
 60    CALL GENFPT
C
C       WRITE OUT PUT
C
       IF (KOUNT.GT.12) GO TO 1000
 50    A=IHAVEP
       B=ISWTCH
       PX=(MACH/(2.0*BETA))*(PHK(A,B)+ PHI(A,B))
       PR=(MACH/2.0)*(PHK(A,B)-PHI(A,B))
 22    WRITE (6,61) IHAVEP,KOUNT,PX,PR
 61    FORMAT ('0' ,'IHAVEP FOR RUN    =',I3,3X,'KOUNT =',I3,
      12X,'PX =',2F15.8,5X,'PR =',2F15.8)
       WRITE (6,52) X(A,B),R(A,B),PHI(A,B),PHK(A,B),QPHI(A,B)
 52    FORMAT (' ',1X,2F14.5,3X,2F15.8,1X,2F15.8,1X,2F15.8//)
C
C
C
 1000    IF (IHAVEP.EQ.KOUNT) GO TO 25
       GO TO 35
C
C      INCREMENT FOR NEXT LINES
C
 25    IF (ISWTCH.EQ.1.) GO TO 103
       ISWTCH=1
       JSWTCH=2
       IHAVEP=1
       GO TO 108
C
C      SWITCH LINES HERE SO FIRST OLD (ILINE) BECOMES LAST NEW ONE
C      ZERO LINE COUNTER
C
 103   ISWTCH=2
       JSWTCH=1
       IHAVEP=1
       GO TO 108
C
C
C      AT 35 INCREMENT ALONG PRESENT LINE
C
 35    IHAVEP=IHAVEP+1
       GO TO 104
```

```
108    KOUNT=KOUNT+1
      IF ((KOUNT-1).GT.MLINE) GO TO  101
      GO TO 100
101    WRITE (6,39)
39    FORMAT (////,' ',10X,'CP')
      DO 37 KK=1,MLINE
      YYY(KK)=KK
      WRITE (6,36) CP(KK)
36    FORMAT (' ',10X,2F18.8)
37    CONTINUE
      NNN=MLINE
      MMM=0
      IF (NUMRUN.EQ.2) GO TO 200
      STOP
      END
C
C
```

```
      SUBROUTINE COMPXR
       IMPLICIT REAL*8 (A-H,O-Z,$)
      REAL*8 MACH,MANGL,K,KW
C
C     COMPUTES THE X AND R CORRDINATES
C
      COMMON DPHII,DPHIK,PHI(400,2),PHK(400,2),QPHI(400,2),IPHI1,IPHI2,
     4IKM,IKW,PARW,W,II,REALCP(400),CP(400),AMAGCP(400),
     1    MACH,K,MANGL,RADIUS,DELTAS,XLENGT,BETA,DSTSTR,HDSTRL,TRNGLH,
     2  X(400,2),R(400,2),NFREQ,FINGRD,ISWTCH,JSWTCH,ILINE,MLINE,JLINE,
     3  MFREQ,IHAVEP
      COMPLEX*16
     1        DPHII,DPHIK,PHI,IKM,CP,PX ,IPHI1,IPHI2,IKW,PHK,II,QPHI
      INTEGER FINGRD
C
C     TEST FOR POINT ON MACH LINE
C
      IF (IHAVEP.EQ.1) GO TO 10
      I=IHAVEP
      J=IHAVEP-1
      X(I,ISWTCH)=X(J      ,ISWTCH)+HDSTRL
      R(I,ISWTCH)=R(J      ,ISWTCH)        -TRNGLH
      GO TO 6
   10 I=IHAVEP
      X(I,ISWTCH)=X(I      ,JSWTCH)+HDSTRL
      R(I,ISWTCH)=R(I      ,JSWTCH)+TRNGLH
    6 RETURN
      END
C
C
```

```
      SUBROUTINE MACHLN
       IMPLICIT REAL*8 (A-H,O-Z,$)
      COMMON DPHII,DPHIK,PHI(400,2),PHK(400,2),QPHI(400,2),IPHI1,IPHI2,
     4IKM,IKW,PARW,W,II,REALCP(400),CP(400),AMAGCP(400),
     1     MACH,K,MANGL,RADIUS,DELTAS,XLENGT,BETA,DSTSTR,HDSTRL,TRNGLH,
     2  X(400,2),R(400,2),NFREQ,FINGRD,ISWTCH,JSWTCH,ILINE,MLINE,JLINE,
     3  MFREQ,IHAVEP,KOUNT
      REAL*8 MACH,MANGL,K,KW
      COMPLEX*16
     1A,B,C,D,DPHII,DPHIK,PHI,IKM,CP,PX ,IPHI1,IPHI2,IKW,PHK,II,QPHI
      INTEGER FINGRD
      I=IHAVEP
      PHK(I,ISWTCH)=(0.,0.)
      A=1+((II*K*MACH)/(2.*BETA))*DELTAS
      PHI(I,ISWTCH)=PHI(I,JSWTCH)*((1-((II*K*MACH)/(2.*BETA))*DELTAS)/A)
      QPHI(I,ISWTCH)=(0.,0.)
      WRITE (6,1) QPHI(I,ISWTCH),PHI(I,ISWTCH),PHK(I,ISWTCH),A
    1 FORMAT ('0','LEAVING MACHLN     QPHI =',2E15.5,5X,'PHI =',2E15.5,
     1  /5X,'PHK =',2E15.5,5X,'A =',2E16.8)
      RETURN
      END
C
```

```
      SUBROUTINE GENFPT
       IMPLICIT REAL*8 (A-H,O-Z,$)
C
C     GENFPT COMPUTES PHI'S AT A GENERAL FIELD POINT
C
      COMMON DPHII,DPHIK,PHI(400,2),PHK(400,2),QPHI(400,2),IPHI1,IPHI2,
     4IKM,IKW,PARW,W,II,REALCP(400),CP(400),AMAGCP(400),
     1    MACH,K,MANGL,RADIUS,DELTAS,XLENGT,BETA,DSTSTR,HDSTRL,TRNGLH,
     2 X(400,2),R(400,2),NFREQ,FINGRD,ISWTCH,JSWTCH,ILINE,MLINE,JLINE,
     3 MFREQ,IHAVEP,KOUNT
      REAL*8 MACH,MANGL,K,KW
      COMPLEX*16
     1A,B,C,D,DPHII,DPHIK,PHI,IKM,CP,PX ,IPHI1,IPHI2,IKW,PHK,II,QPHI
      INTEGER FINGRD
C
      I=IHAVEP
      J=IHAVEP -1
C
      A=1+((II*K*MACH)/(2.*BETA))*DELTAS
      B=-(K**2/4.0)*DELTAS**2+((II*K*MACH)/(2.*BETA))*DELTAS
      C=PHK(J,ISWTCH)* (1-((II*K*MACH)/(2.*BETA))*DELTAS)+PHI(J,ISWTCH)*
     1 ((K**2/4.0)*DELTAS**2-((II*K*MACH)/(2.*BETA))*DELTAS)+K**2*
     2 DELTAS*QPHI(J,ISWTCH)
      D=PHI(I,JSWTCH)* (1-((II*K*MACH)/(2.*BETA))*DELTAS)+PHK(I,JSWTCH)*
     1 ((K**2/4.0)*DELTAS**2-((II*K*MACH)/(2.*BETA))*DELTAS)+K**2*
     2 DELTAS*QPHI(I,JSWTCH)
      PHK(I,ISWTCH)=(C*A-B*D)/(A**2-B**2)
      PHI(I,ISWTCH)=(D*A-C*B)/(A**2-B**2)
      IPHI1=QPHI(I,JSWTCH)+0.5*(PHK(I,JSWTCH)+PHK(I,ISWTCH))*DELTAS
      IPHI2=QPHI(J,ISWTCH)+0.5*(PHI(J,ISWTCH)+PHI(I,ISWTCH))*DELTAS
C
C     AVERAGE THE TWO
C
      QPHI(I,ISWTCH)=0.5*(IPHI1+IPHI2)
      IF (KOUNT.GT.12) GO TO 15
      WRITE (6,10) A,B,C,D
   10 FORMAT (' ',3X,'A =',2F16.8,5X,'B =',2F16.8,/
     1 ' ',5X,'C =',2F16.8,5X,'D =',2F16.8)
   15 RETURN
      END
C
```

```
      SUBROUTINE PANPNT
       IMPLICIT REAL*8 (A-H,O-Z,$)
C
C     PANPNT COMPUTES THE VALUES OF PHI'S AND QHPI AT A PANEL POINT
C
      COMMON DPHII,DPHIK,PHI(400,2),PHK(400,2),QPHI(400,2),IPHI1,IPHI2,
     4IKM,IKW,PARW,W,II,REALCP(400),CP(400),AMAGCP(400),
     1    MACH,K,MANGL,RADIUS,DELTAS,XLENGT,BETA,DSTSTR,HDSTRL,TRNGLH,
     2  X(400,2),R(400,2),NFREQ,FINGRD,ISWTCH,JSWTCH,ILINE,MLINE,JLINE,
     3  MFREQ,IHAVEP,KOUNT
      REAL*8 MACH,MANGL,K,KW
      COMPLEX*16
     1A,B,C,D,DPHII,DPHIK,PHI,IKM,CP,PX  ,IPHI1,IPHI2,IKW,PHK,II,QPHI
      INTEGER FINGRD
C
      I=IHAVEP
      J=IHAVEP-1
      A=1+((II*K*MACH)/(2.*BETA))*DELTAS
      B=-(K**2/4.0)*DELTAS**2+((II*K*MACH)/(2.*BETA))*DELTAS
      C=PHK(J,ISWTCH)* (1-((II*K*MACH)/(2.*BETA))*DELTAS)+PHI(J,ISWTCH)*
     1  ((K**2/4.0)*DELTAS**2-((II*K*MACH)/(2.*BETA))*DELTAS)+K**2*
     2  DELTAS*QPHI(J,ISWTCH)
      PHK(I,ISWTCH)=      (C+B*(2.0/MACH)*(PARW+IKW))/(A+B)
      PHI(I,ISWTCH)=      (-A*(2.0/MACH)*(PARW+IKW)+C)/(A+B)
C
      QPHI(I    ,ISWTCH)=QPHI(J,ISWTCH)+0.5*(PHI(J,ISWTCH)+PHI(I     ,
     1  ISWTCH))*DELTAS
      PX=(MACH/(2.0*BETA))*(PHK(I,ISWTCH)+PHI(I,ISWTCH))
      CP(KOUNT)=-2.0*(II*K*QPHI(I,ISWTCH)+PX)
      AMAGCP(KOUNT)=AIMAG(CP(KOUNT))
      REALCP(KOUNT)=REAL(CP(KOUNT))
      WRITE (6,1) QPHI(I,ISWTCH),PHI(I,ISWTCH),PHK(I,ISWTCH),C
 1    FORMAT ('0','LEAVING PANPNT    QPHI =',2E15.5,5X,'PHI =',2E15.5,
     U /5X,'PHK =',2E15.5,5X,'C =',2D16.8)
      RETURN
      END
```

APPENDIX B: VIBRATING SHELL COMPUTER PROGRAM

```
C
C           'LINEARIZED METHOD OF CHARACTERISTICS'
C     FOR   SUPERSONIC FLOW PAST A VIBRATING CYLINDRICAL SHELL
C
C
C
C     DEFINITION OF TERMS USED
C
C
C     MACH IS FREE STREAM MACH NUMBER
C     K IS REDUCED DIMENSIONLESS FREQUENCY
C     NFREQ IS THE RADIAL MODE OF THE CYLINDER
C     MFREQ IS THE AXIAL MODE OF THE CYLINDER
C     MANGL IS THE MACH ANGLE
C     FINGRD  IS THE FINENESS OF THE GRID
C     RADIUS IS THE RADIUS OF THE CYLINDER
C     DELTAS IS THE STEP SIZE ALONG THE MACH LINES
C     ISWTCH IS THE SWITCH VARIABLE FOR CHOOSING THE LINE ON WHICH
C     TO WORK.
C     JSWTCH IS OPPOSITE OF ISWTCH.IF ISWTCH=1,JSWTCH=2 OR VICE-VERSA.
C     XLENGT  IS THE LENGHT OF INITIAL MACH LINE
C     MLINE IS THE MAX NUMBER OF PTS. ON THE INTIAL MACH LINE.
C     QPHI IS THE VELOCITY POTENTIAL
C     PHK IS THE DERIVATIVE OF THE VEL. POT. ALONG THE LEFT RUNNNING
C     CHARACTERISTIC
C     PHI IS THE DERIVATIVE OF THE VEL. POT. ALONG THE RIGHT RUNNING
C     CHARACTERISTIC
C     PX IS THE DERIVATIVE OF THE VEL. POT. IN THE X DIRECTION
C     PR IS THE DERIVATIVE OF THE VEL. POT. IN THE R DIRECTION
C     CP IS THE COEFFICENT OF PRESSURE
C     REALCP IS THE REAL PART OF CP
C     AMAGCP IS THE IMAGINARY PART OF CP
C
C     RUN IN DOUBLE PRECISION
C
       IMPLICIT REAL*8 (A-H,O-Z,$)
      DIMENSION DATE(3),YYY(90)
      COMMON DPHII,DPHIK,PHI(400,2),PHK(400,2),QPHI(400,2),IPHI1,IPHI2,
     4IKM,IKW,PARW,W,II,REALCP(400),CP(400),AMAGCP(400),
     1    MACH,K,MANGL,RADIUS,DELTAS,XLENGT,BETA,DSTSTR,HDSTRL,TRNGLH,
     2  X(400,2),R(400,2),NFREQ,FINGRD,ISWTCH,JSWTCH,ILINE,MLINE,JLINE,
     3  MFREQ,IHAVEP,KOUNT
C
      INTEGER FINGRD
      REAL*8 MACH,MANGL,K
      COMPLEX*16
     1        DPHII,DPHIK,PHI,IKM,CP,PX ,IPHI1,IPHI2,IKW,PHK,II,QPHI
     1 ,PR
```

```
C
C      PRINT NAME OF PROGRAM AND DATE OF RUN
C
       READ (5,10)DATE
 10    FORMAT (3A4)
  11   FORMAT (1H1///,40X,'LINEARIZED METHOD OF CHARACTERISTICS'//40X,
      1'FOR SUPERSONIC FLOW PAST A VIBRATING CYLINDRICAL SHELL'//10X
      2'DATE OF RUN-',3A4///)
       WRITE (6,11) DATE
C
C      READ INPUT
C
 200   READ (5,1) FINGRD ,NUMRUN,NFREQ,MFREQ,GRAPH
       READ (5,2) MACH,K,RADIUS
 1     FORMAT (5I5)
 2     FORMAT (3F10.4)
C
C      COMPUTE CONSTANTS
C
       II=(0.0,1.0)
       BETA=DSQRT(MACH**2-1.0)
       MANGL=DARSIN(1.0/MACH)
       XLENGT=DCOS(MANGL)
       HEIGHT=DSIN(MANGL)*XLENGT
       DELTAS=XLENGT /FINGRD
       DSTSTR=DELTAS*DCOS(MANGL)*2.0
       HDSTRL=.5*DSTSTR
       MLINE=FINGRD
       TRNGLH=HDSTRL*DTAN(MANGL)
C
C
C      WRITE OUT INPUT
C
       WRITE (6,12) MACH,K,NFREQ,FINGRD ,RADIUS ,MFREQ
 12    FORMAT (1H0,10X,'FREESTREAM MACH NUMBER =',F9.6//10X,'REDUCED FREQ
      1=',F9.5//10X,'RAD MODE NO. N   =',I2//10X,'GRID FINENESS IS = ',
      2 I2  //10X,'CYLINDER RADIUS IS =',F9.2//10X,'AXIAL MODE NO. M   ='
      3,I2)
       DO 31 KK=1,400
        CP(KK)=(0.,0.)
       REALCP(KK)=(0.,0.)
       AMAGCP(KK)=(0.,0.)
 31    CONTINUE
C
C
C      SET COUNTERS
C
 20    ISWTCH=2
```

```
      JSWTCH=1
      IHAVEP=1
C
C     COMPUTE INTIAL VALUES
C
C
      I=IHAVEP
      QPHI(I,JSWTCH)=(0.,0.)
      PHK(I,JSWTCH)=(0.,0.)
      X(IHAVEP,JSWTCH)=0.0
      R(IHAVEP,JSWTCH)=RADIUS
      PI=3.141592654
      PARW=MFREQ*PI*DCOS(MFREQ*PI*X(IHAVEP,JSWTCH)  )
      PHI(I,JSWTCH)=-2*PARW/MACH
C
C     WRITE INTIAL VALUES
C
      WRITE (6,57) QPHI(1,1),PHI(1,1),PHK(1,1) ,X(1,1),R(1,1),PARW,QR
 57    FORMAT (//'0','INTIAL VALUE OF QPHI =',2E10.5,5X,'PHI =',2E15.5,
     15X,'PHK =',2E10.5/5X,'X =',E10.4,5X,'R =',E10.4,10X,'PARW =',
     1 E10.4,5X,'QR =',2E15.4///)
C
C     SET COUNTER UP FOR EACH LINE
C
      KOUNT=2
 100  WRITE (6,59) KOUNT
 59   FORMAT (' ','KOUNT =',I5)
C
C     COMPXR COMUTES THE VALUE FOR X AND R GIVEN IHAVEP,ISWTCH
C
 104   CALL COMPXR
      PI=3.141592654
      PARW=MFREQ*PI*DCOS(MFREQ*PI*X(IHAVEP,ISWTCH)          )
      W=DSIN(MFREQ*PI*X(IHAVEP,ISWTCH))
      XKM=(K*MACH)/BETA
      IKM=II*XKM
      XKW=K*W
      IKW=II*XKW
      IF (KOUNT.GT.12) GO TO 1001
      WRITE(6,58)     PARW,W,XKM,IKM,XKW,IKW
 58   FORMAT (' ',5X,'PARW =',E18.8,5X,'W =',E18.8,5X,'XKM =',E16.8/
     1  ' ',4X,'IKM =',2E10.5,2X,'KW =',E18.8,2X,'IKW =',2E10.5)
C
C     THE PROGRM NOW GOES TO 30 IF POINT IS ON INITIAL MACH LINE.
C
 1001  IF (IHAVEP.EQ.1) GO TO 30
C
C     TEST IF WE ARE AT A PANEL PT.
```

```
C
      IF  (IHAVEP.EQ.KOUNT) GO TO 40
      GO TO 60
C
 30    CALL MACHLN
      GO TO 50
C
 40   CALL PANPNT
      GO TO 50
C
 60   CALL GENFPT
C
C      WRITE OUT PUT
C
      IF (KOUNT.GT.12) GO TO 1000
 50   A=IHAVEP
      B=ISWTCH
      PX=(MACH/(2.0*BETA))*(PHK(A,B)+ PHI(A,B))
      PR=(MACH/2.0)*(PHK(A,B)-PHI(A,B))
 22   WRITE (6,61) IHAVEP,KOUNT,PX,PR
 61   FORMAT ('0' ,'IHAVEP FOR RUN    =',I3,3X,'KOUNT =',I3,
     12X,'PX =',2F15.8,5X,'PR =',2F15.8)
      WRITE (6,52) X(A,B),R(A,B),PHI(A,B),PHK(A,B),QPHI(A,B)
 52   FORMAT (' ',1X,2F14.5,3X,2F15.8,1X,2F15.8,1X,2F15.8//)
C
C
C
 1000   IF (IHAVEP.EQ.KOUNT) GO TO 25
      GO TO 35
C
C     INCREMENT FOR NEXT LINES
C
 25   IF (ISWTCH.EQ.1.) GO TO 103
      ISWTCH=1
      JSWTCH=2
      IHAVEP=1
      GO TO 108
C
C     SWITCH LINES HERE SO FIRST OLD (ILINE) BECOMES LAST NEW ONE
C     ZERO LINE COUNTER
C
 103  ISWTCH=2
      JSWTCH=1
      IHAVEP=1
      GO TO 108
C
C
C     AT 35 INCREMENT ALONG PRESENT LINE
```

```
C
 35    IHAVEP=IHAVEP+1
       GO TO 104
 108    KOUNT=KOUNT+1
       IF ((KOUNT-1).GT.MLINE) GO TO  101
       GO TO 100
 101    WRITE (6,39)
 39    FORMAT (////,' ',10X,'CP')
       DO 37 KK=1,MLINE
       YYY(KK)=KK
       WRITE (6,36) CP(KK)
 36    FORMAT (' ',10X,2F18.8)
 37    CONTINUE
       IF (GRAPH.EQ.0.0) GO TO 1006
       NNN=MLINE
       MMM=0
       CALL  PLOTP(YYY,REALCP,NNN,MMM)
       MMM=0
       CALL  PLOTP(YYY,AMAGCP,NNN,MMM)
 1006  IF (NUMRUN.EQ.2) GO TO 200
       STOP
       END
```

```
      SUBROUTINE COMPXR
       IMPLICIT REAL*8 (A-H,O-Z,$)
      REAL*8 MACH,MANGL,K,KW
C
C     COMPUTES THE X AND R CORRDINATES
C
      COMMON DPHII,DPHIK,PHI(400,2),PHK(400,2),QPHI(400,2),IPHI1,IPHI2,
     4IKM,IKW,PARW,W,II,REALCP(400),CP(400),AMAGCP(400),
     1    MACH,K,MANGL,RADIUS,DELTAS,XLENGT,BETA,DSTSTR,HDSTRL,TRNGLH,
     2  X(400,2),R(400,2),NFREQ,FINGRD,ISWTCH,JSWTCH,ILINE,MLINE,JLINE,
     3  MFREQ,IHAVEP
      COMPLEX*16
     1        DPHII,DPHIK,PHI,IKM,CP,PX ,IPHI1,IPHI2,IKW,PHK,II,QPHI
       INTEGER FINGRD
C
C     TEST FOR POINT ON MACH LINE
C
      IF (IHAVEP.EQ.1) GO TO 10
      I=IHAVEP
      J=IHAVEP-1
      X(I,ISWTCH)=X(J     ,ISWTCH)+HDSTRL
      R(I,ISWTCH)=R(J     ,ISWTCH)        -TRNGLH
      GO TO 6
   10 I=IHAVEP
      X(I,ISWTCH)=X(I     ,JSWTCH)+HDSTRL
      R(I,ISWTCH)=R(I     ,JSWTCH)+TRNGLH
    6 RETURN
      END'
C
C
```

```
      SUBROUTINE MACHLN
       IMPLICIT REAL*8 (A-H,O-Z,$)
C
C
C     SUBROUTINE MACHLN COMPUTES THE VALUE OF PHI AND ITS DERIVITIVES
C     ON THE INITIAL MACH LINE
C
C
      CCMMON DPHII,DPHIK,PHI(400,2),PHK(400,2),QPHI(400,2),IPHI1,IPHI2,
     4IKM,IKW,PARW,W,II,REALCP(400),CP(400),AMAGCP(400),
     1    MACH,K,MANGL,RADIUS,DELTAS,XLENGT,BETA,DSTSTR,HDSTRL,TRNGLH,
     2  X(400,2),R(400,2),NFREQ,FINGRD,ISWTCH,JSWTCH,ILINE,MLINE,JLINE,
     3  MFREQ,IHAVEP,KOUNT
      REAL*8 MACH,MANGL,K,KW
      COMPLEX*16                  AA2,
     1A,B,C,D,DPHII,DPHIK,PHI,IKM,CP,PX ,IPHI1,IPHI2,IKW,PHK,II,QPHI
      INTEGER FINGRD
      I=IHAVEP
      N=NFREQ
C
C     INFORCE THE BOUNDARY CONDITIONS
C
      PHK(I,ISWTCH)=(0.,0.)
      QPHI(I,ISWTCH)=(0.,0.)
C
      A=1-DELTAS/(4.*R(I,ISWTCH)*MACH)+0.5*IKM*DELTAS
      AA2=1.+DELTAS/ (4.*R(I,JSWTCH)*MACH)-0.5*IKM*DELTAS
      PHI(I,ISWTCH)=PHI(I,JSWTCH)*(A/AA2)
C
C     OUTPUT
C
      WRITE (6,1) QPHI(I,ISWTCH),PHI(I,ISWTCH),PHK(I,ISWTCH),A
    1 FORMAT ('0','LEAVING MACHLN    QPHI =',2E15.5,5X,'PHI =',2E15.5,
     1  /5X,'PHK =',2E15.5,5X,'A =',2E16.8)
      RETURN
      END
C
```

```
      SUBROUTINE GENFPT
       IMPLICIT REAL*8 (A-H,O-Z,$)
C
C
C     GENFPT COMPUTES PHI AND ITS DERIVITIVES AT A GENERAL FIELD PT.
C
C
      COMMON DPHII,DPHIK,PHI(400,2),PHK(400,2),QPHI(400,2),IPHI1,IPHI2,
     4IKM,IKW,PARW,W,II,REALCP(400),CP(400),AMAGCP(400),
     1    MACH,K,MANGL,RADIUS,DELTAS,XLENGT,BETA,DSTSTR,HDSTRL,TRNGLH,
     2  X(400,2),R(400,2),NFREQ,FINGRD,ISWTCH,JSWTCH,ILINE,MLINE,JLINE,
     3  MFREQ,IHAVEP,KOUNT
      REAL*8 MACH,MANGL,K,KW
      COMPLEX*16                 AA1,AA2,
     1A,B,C,D,DPHII,DPHIK,PHI,IKM,CP,PX ,IPHI1,IPHI2,IKW,PHK,II,QPHI
      INTEGER FINGRD
C
      I=IHAVEP
      J=IHAVEP -1
      N=NFREQ
C
      A=1-DELTAS/(4.*R(I,ISWTCH)*MACH)+0.5*IKM*DELTAS
C
      AA1=1+DELTAS/(4.*R(J,ISWTCH)*MACH)-0.5*IKM*DELTAS
C
      AA2=1.+DELTAS/ (4.*R(I,JSWTCH)*MACH)-0.5*IKM*DELTAS
C
      B=DELTAS/(4.*R(I,ISWTCH)*MACH)+0.5*IKM*DELTAS-0.25*DELTAS**2*
     1  (K**2-(N**2/R(I,ISWTCH)**2))
C
C
      C=PHK(J,ISWTCH)*AA1+PHI(J,ISWTCH)*(-DELTAS/(4.*R(J,ISWTCH)*MACH)
     1 -0.5*IKM*DELTAS+0.25*DELTAS**2*(K**2-(N**2/(R(I,ISWTCH)**2*MACH**
     22)))+  QPHI(J,ISWTCH)*DELTAS*0.5*(K**2- N**2/(R(J,ISWTCH)**2*MACH
     3**2))+   QPHI(J,ISWTCH)*DELTAS*0.5*(K**2- N**2/(R(I,ISWTCH)**2*
     3**2))+   QPHI(J,ISWTCH)*DELTAS*0.5*(K**2- N**2/(R(I,ISWTCH)**2*
     4 MACH**2))
C
      D=PHI(I,JSWTCH)*AA2+PHK(I,JSWTCH)*(-DELTAS/(4.*R(I,JSWTCH)*MACH)
     1 -0.5*IKM*DELTAS+0.25*DELTAS**2*(K**2-(N**2/R(I,ISWTCH)**2)))+
     22)))+  QPHI(I,JSWTCH)*DELTAS*0.5*(K**2- N**2/(R(I,JSWTCH)**2*MACH
     3**2))+   QPHI(I,JSWTCH)*DELTAS*0.5*(K**2- N**2/(R(I,ISWTCH)**2*
     4 MACH**2))
C
```

```
C     CRAMERS RULE
C
      PHK(I,ISWTCH)=(C*A-B*D)/(A**2-B**2)
      PHI(I,ISWTCH)=(D*A-C*B)/(A**2-B**2)
C
C     INTERGATE
C
      IPHI1=QPHI(I,JSWTCH)+0.5*(PHK(I,JSWTCH)+PHK(I,ISWTCH))*DELTAS
      IPHI2=QPHI(J,ISWTCH)+0.5*(PHI(J,ISWTCH)+PHI(I,ISWTCH))*DELTAS
C
C     AVERAGE THE TWO
C
      QPHI(I,ISWTCH)=0.5*(IPHI1+IPHI2)
      IF (KOUNT.GT.12) GO TO 15
C
C     OUTPUT
C
      WRITE (6,10) A,B,C,D ,AA1,AA2
   10 FORMAT (' ',3X,'A =',2F16.8,5X,'B =',2F16.8,/
     1 ' ',5X,'C =',2F16.8,5X,'D =',2F16.8,/' ',5X,'AA1 =',2E18.8,5X,
     2  'AA2 =',2E18.8)
   15 RETURN
      END
C

      SUBROUTINE PANPNT
       IMPLICIT REAL*8 (A-H,O-Z,$)
C
C
C     PANPT COMPUTES THE VALUES OF PHI AND ITS DERIVITIVES AT A PANEL
C     POINT
C
C
      COMMON DPHII,DPHIK,PHI(400,2),PHK(400,2),QPHI(400,2),IPHI1,IPHI2,
     4IKM,IKW,PARW,W,II,REALCP(400),CP(400),AMAGCP(400),
     1    MACH,K,MANGL,RADIUS,DELTAS,XLENGT,BETA,DSTSTR,HDSTRL,TRNGLH,
     2  X(400,2),R(400,2),NFREQ,FINGRD,ISWTCH,JSWTCH,ILINE,MLINE,JLINE,
     3  MFREQ,IHAVEP,KOUNT
      REAL*8 MACH,MANGL,K,KW
      COMPLEX*16                         AA1,AA2
     1A,B,C,D,DPHII,DPHIK,PHI,IKM,CP,PX ,IPHI1,IPHI2,IKW,PHK,II,QPHI
      INTEGER FINGRD
C
      N=NFREQ
      I=IHAVEP
      J=IHAVEP-1
      A=1-DELTAS/(4.*R(I,ISWTCH)*MACH)+0.5*IKM*DELTAS
```

```
      AA1=1+DELTAS/(4.*R(J,ISWTCH)*MACH)-0.5*IKM*DELTAS
      B=DELTAS/(4.*R(I,ISWTCH)*MACH)+0.5*IKM*DELTAS-0.25*DELTAS**2*
     1  (K**2-(N**2/R(I,ISWTCH)**2))
C
      C=PHK(J,ISWTCH)*AA1+PHI(J,ISWTCH)*(-DELTAS/(4.*R(J,ISWTCH)*MACH)
     1 -0.5*IKM*DELTAS+0.25*DELTAS**2*(K**2-(N**2/(R(I,ISWTCH)**2*MACH**
     22)))+  QPHI(J,ISWTCH)*DELTAS*0.5*(K**2-  N**2/(R(J,ISWTCH)**2*MACH
     4 MACH**2))
C
C     CRAMERS RULE
C
      PHK(I,ISWTCH)=      (C+B*(2.0/MACH)*(PARW+IKW))/(A+B)
      PHI(I,ISWTCH)=      (-A*(2.0/MACH)*(PARW+IKW)+C)/(A+B)
C
      QPHI(I     ,ISWTCH)=QPHI(J,ISWTCH)+0.5*(PHI(J,ISWTCH)+PHI(I     ,
     1  ISWTCH))*DELTAS
C
C
C     COMPUTES THE CP ON THE PANEL
C
C
      PX=(MACH/(2.0*BETA))*(PHK(I,ISWTCH)+PHI(I,ISWTCH))
      CP(KOUNT)=-2.0*(II*K*QPHI(I,ISWTCH)+PX)
      AMAGCP(KOUNT)=AIMAG(CP(KOUNT))
      REALCP(KOUNT)=REAL(CP(KOUNT))
C
C     OUTPUT
C
      WRITE (6,1) QPHI(I,ISWTCH),PHI(I,ISWTCH),PHK(I,ISWTCH),C
 1    FORMAT ('0','LEAVING PANPNT     QPHI =',2E15.5,5X,'PHI =',2E15.5,
     U /5X,'PHK =',2E15.5,5X,'C =',2D16.8)
      RETURN
      END
```

APPENDIX C: VIBRATING PANEL SAMPLE COMPUTER OUTPUT

```
                         LINEARIZED METHOD OF CHARACTERISTICS
                         FOR SUPERSONIC FLOW PAST A VIBRATING PANEL
DATE OF RUN-22 FEB 1972


 FREESTREAM MACH NUMBER = 1.414213
REDUCED FREQ=  2.00000
GRID FINENESS IS = 60
 AXAIAL MODE NO. M =  4


 INTIAL VALUE OF CPHI =0.0     0.0      PHI =-.1777D 020.0             PHK =0.0      0.0
 X = 0.0          R =.100D 01PARW = 0.126D 02


 KOUNT =      2
 CONSTANTS AT NEW POINT
 PARW =      0.12498D 02     W =.10453D 00
 IKM & IKW =.0          .28284D 01       .0          .20906D 00
 MACHLN A =  0.10000000D 01  0.16666680D-01

 IHAVEP FOR RUN   =  1   KOUNT =  2
 PX =    -12.55940122     0.41876336      PR =     12.55939123    -0.41876303
          0.00833        1.00833      -17.7616685      0.5922206
          0.0             0.0             0.0            0.0

 CONSTANTS AT NEW POINT
 PARW =      0.12292D 02     W =.20791D 00
 IKM & IKW =.0          .28284D 01       .0          .41582D 00
 PANPT  C =  0.74034507D-02  0.29611030D 00
```

```
IHAVEP FOR RUN    =  2    KOUNT =  2
PX =    -12.27098406      0.41197224       PR =     12.29176527      0.41582338
         0.01667         1.00000       -17.3684866    -0.0027234
          0.01470          0.58534         -0.20701        0.00347

KOUNT =     3
CONSTANTS AT NEW POINT
PARW =      0.12292D 02     W =.20791D 00
IKM & IKW =.0          .28284D 01       .0          .41582D 00
MACHLN A =  0.10000000D 01  0.16666680D-01

IHAVEP FOR RUN    =  1    KOUNT =  3
PX =    -12.53847078      0.83706156       PR =     12.53846081     -0.83706090
         0.01667         1.01667       -17.7320684      1.1837833
          0.0              0.0             0.0             0.0

CONSTANTS AT NEW POINT
PARW =      0.11951D 02     W =.30902D 00
IKM & IKW =.0          .28284D 01       .0          .61803D 00
GENFPT  A =        1.00000000        0.01666668B =     -0.00013889      0.01666668
C =        0.01726695        0.29569912  D =     -17.36853268       0.28675163

IHAVEP FOR RUN    =  2    KOUNT =  3
PX =    -12.24356674      0.82008816       PR =     12.29190471      0.00632595
         0.02500         1.00833       -17.3491983      0.5754165
          0.03419          0.58436         -0.20672        0.01037

CONSTANTS AT NEW POINT
PARW =      0.11480D 02     W =.40674D 00
IKM & IKW =.0          .28284D 01       .0          .81347D 00
PANPT  C =  0.41362261D-01  0.87351513D 00

IHAVEP FOR RUN    =  3    KOUNT =  3
PX =    -11.39791587      0.80179271       PR =     11.47995080      0.81347329
         0.03333         1.00000       -16.1770947    -0.0082599
          0.05801          1.14217         -0.40427        0.01371

KOUNT =     4
CONSTANTS AT NEW POINT
PARW =      0.11951D 02     W =.30902D 00
IKM & IKW =.0          .28284D 01       .0          .61803D 00
MACHLN A =  0.10000000D 01  0.16666680D-01
```

```
IHAVEP FOR RUN    =  1    KOUNT =  4
PX =    -12.50361256        1.25442995      PR =      12.50360262    -1.25442895
          0.02500         1.02500       -17.6827714      1.7740311
          0.0              0.0              0.0              0.0

CONSTANTS  AT  NEW  POINT
PARW =       0.11480D 02     W =.40674D 00
IKM & IKW =.0            .28284D 01      .0           .81347D 00
GENFPT   A =        1.00000000       0.01666668B =     -0.00013889      0.01666668
C =        0.02711127        0.29495949  D =     -17.33960864       0.86457012

IHAVEP FOR RUN    =  2    KOUNT =  4
PX =    -12.20258943        1.22683527      PR =      12.27841520    -0.40271945
          0.03333         1.01667       -17.3106844      1.1522690
          0.05362          0.58274         -0.20620         0.01724

CONSTANTS  AT  NEW  POINT
PARW =       0.10883D 02     W =.50000D 00
IKM & IKW =.0            .28284D 01      .0           .10000D 01
GENFPT   A =        1.00000000       0.01666668B =     -0.00013889      0.01666668
C =        0.07041592        0.87132823  D =     -16.17724585       0.26119654

IHAVEP FOR RUN    =  3    KOUNT =  4
PX =    -11.35151762        1.17936428      PR =      11.48723180     0.43136818
          0.04167         1.00833       -16.1494346      0.5289127
          0.09597          1.13896         -0.40337         0.02715

CONSTANTS  AT  NEW  POINT
PARW =       0.10166D 02     W =.58779D 00
IKM & IKW =.0            .28284D 01      .0           .11756D 01
PANPT   C =   0.10251085D 00   0.14078709D 01

IHAVEP FOR RUN    =  4    KOUNT =  4
PX =     -9.98596471        1.14832144      PR =      10.16640739     1.17557050
          0.05000         1.00000       -14.2498790     -0.0192687
          0.12760          1.64324         -0.58250         0.03015

KOUNT =      5
CONSTANTS  AT  NEW  POINT
PARW =       0.11480D 02     W =.40674D 00
IKM & IKW =.0            .28284D 01      .0           .81347D 00
MACHLN A =   0.10000000D 01   0.16666680D-01
```

```
IHAVEP FOR RUN    =  1    KOUNT =  5
PX =    -12.45486528      1.67040491      PR =     12.45485537     -1.67040358
         0.03333        1.03333       -17.6138324       2.3623083
          0.0             0.0             0.0             0.0

CONSTANTS AT NEW POINT
PARW =      0.10883D 02     W =.50000D 00
IKM & IKW =.0           .28284D 01      .0          .10000D 01
GENFPT  A =       1.00000000       0.01666668B =     -0.00013889      0.01666668
C =       0.03692547       0.29389221  D =     -17.29148057       1.44078067

IHAVEP FOR RUN    =  2    KOUNT =  5
PX =    -12.14811285      1.63176349      PR =     12.25132697     -0.81085967
         0.04167        1.02500       -17.2530094       1.7271952
          0.07299         0.58047        -0.20545         0.02410

CONSTANTS AT NEW POINT
PARW =      0.10166D 02     W =.58779D 00
IKM & IKW =.0           .28284D 01      .0          .11756D 01
GENFPT  A =       1.00000000       0.01666668B =     -0.00013889      0.01666668
C =       0.09936982       0.86817572  D =     -16.14063830       0.79790866

IHAVEP FOR RUN    =  3    KOUNT =  5
PX =    -11.29263803      1.55473753      PR =     11.48182061      0.04967939
         0.05000        1.01667       -16.1039742       1.0642364
          0.13378         1.13449        -0.40201         0.04055

CONSTANTS AT NEW POINT
PARW =      0.93386D 01     W =.66913D 00
IKM & IKW =.0           .28284D 01      .0          .13383D 01
GENFPT  A =       1.00000000       0.01666668B =     -0.00013889      0.01666668
C =       0.14923647       1.40272298  D =     -14.25025424       0.21775254

IHAVEP FOR RUN    =  4    KOUNT =  5
PX =     -9.92307994      1.47682439      PR =     10.18055951      0.83778395
         0.05833        1.00833       -14.2154199       0.4518692
          0.18207         1.63668        -0.58067         0.04948

CONSTANTS AT NEW POINT
PARW =      0.84085D 01     W =.74314D 00
IKM & IKW =.0           .28284D 01      .0          .14863D 01
PANPT   C =  0.18753280D 00  0.18729597D 01
```

```
IHAVEP FOR RUN    =  5    KOUNT =  5
PX =     -8.09788429       1.43239740       PR =       8.40854319       1.48628965
        0.06667        1.00000        -11.6718069     -0.0381084
         0.21967          2.06382         -0.73321          0.05192
```

COEFFICENT OF PRESSURE

1.0	0.0	0.0
2.0	24.55586270	0.00408154
3.0	22.85066452	0.01351091
4.0	20.09254127	0.03334298
5.0	16.40344123	0.06806011
6.0	11.94654108	0.12131901
7.0	6.91909287	0.19573567
8.0	1.54377602	0.29271826
9.0	-3.94106736	0.41235615
10.0	-9.29188676	0.55337020
11.0	-14.27060977	0.71312712
12.0	-18.65509318	0.88771781
13.0	-22.24888160	1.07209644
14.0	-24.88984639	1.26027485
15.0	-26.45732668	1.44556368
16.0	-26.87745992	1.62085036
17.0	-26.12646822	1.77890147
18.0	-24.23175624	1.91267680
19.0	-21.27077195	2.01564102
20.0	-17.36767944	2.08205951
21.0	-12.68798847	2.10726521
22.0	-7.43137495	2.08788457
23.0	-1.82300542	2.02201259
24.0	3.89625584	1.90932880
25.0	9.48032952	1.75114890
26.0	14.68863637	1.55040919
27.0	19.29657652	1.31158411
28.0	23.10531564	1.04053992
29.0	25.95045047	0.74433023
30.0	27.70917520	0.43094193
31.0	28.30563578	0.10900179
32.0	27.71423823	-0.21254394
33.0	25.96076634	-0.52476371
34.0	23.12126002	-0.81907450
35.0	19.31870316	-1.08756155
36.0	14.71766553	-1.32327050
37.0	9.51713284	-1.52045979
38.0	3.94183787	-1.67480302
39.0	-1.76752874	-1.78353270

40.0	-7.36480060	-1.84551983
41.0	-12.60905291	-1.86128623
42.0	-17.27508720	-1.83294939
43.0	-21.16322567	-1.76410272
44.0	-24.10798772	-1.65963661
45.0	-25.98527008	-1.52550839
46.0	-26.71771716	-1.36847141
47.0	-26.27804790	-1.19577514
48.0	-24.69019390	-1.01484935
49.0	-22.02820014	-0.83298627
50.0	-18.41293660	-0.65703434
51.0	-14.00676536	-0.49311690
52.0	-9.00639664	-0.34638770
53.0	-3.63424668	-0.22083373
54.0	1.87132446	-0.11913365
55.0	7.26646797	-0.04257745
56.0	12.31253691	0.00894953
57.0	16.78654483	0.03691793
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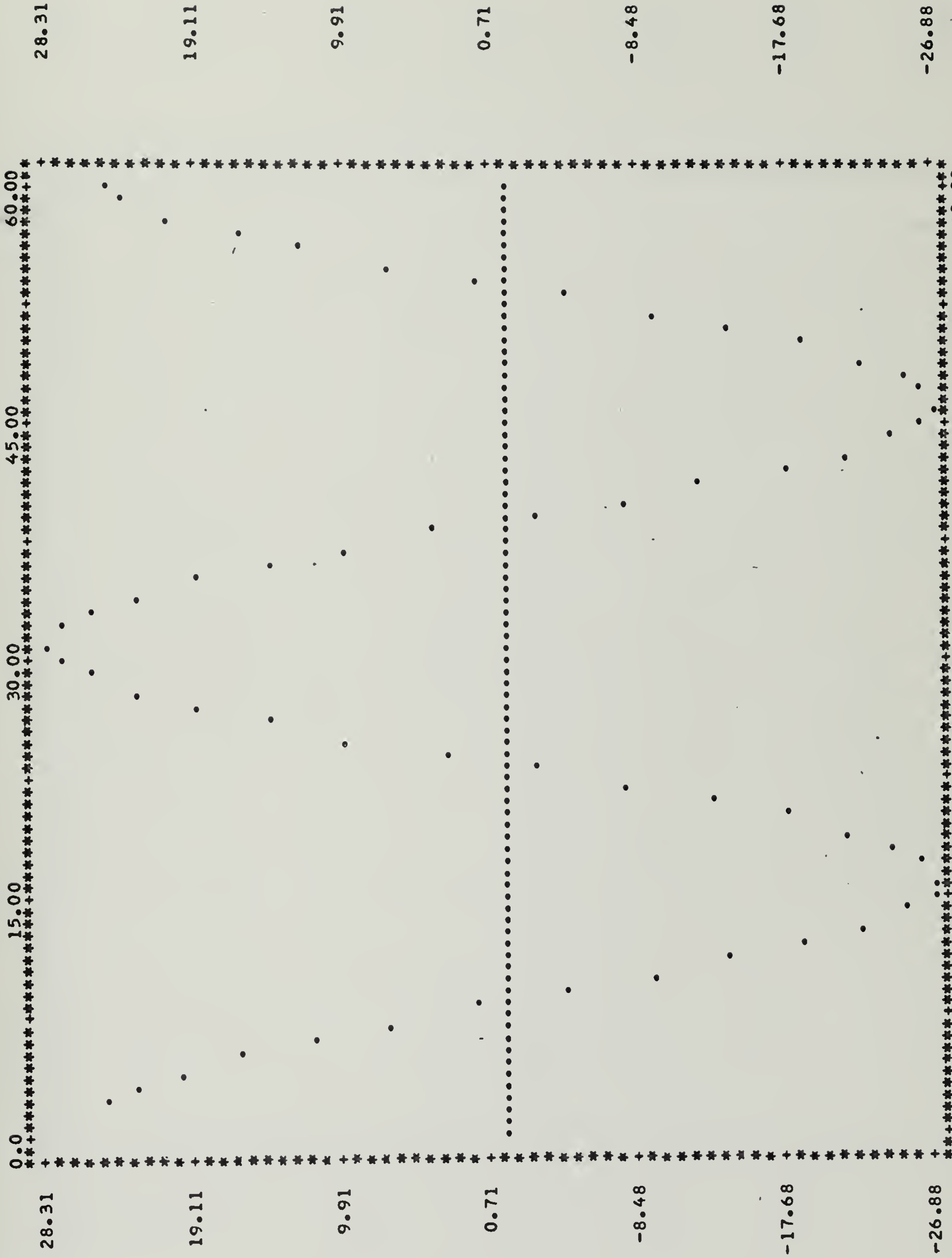
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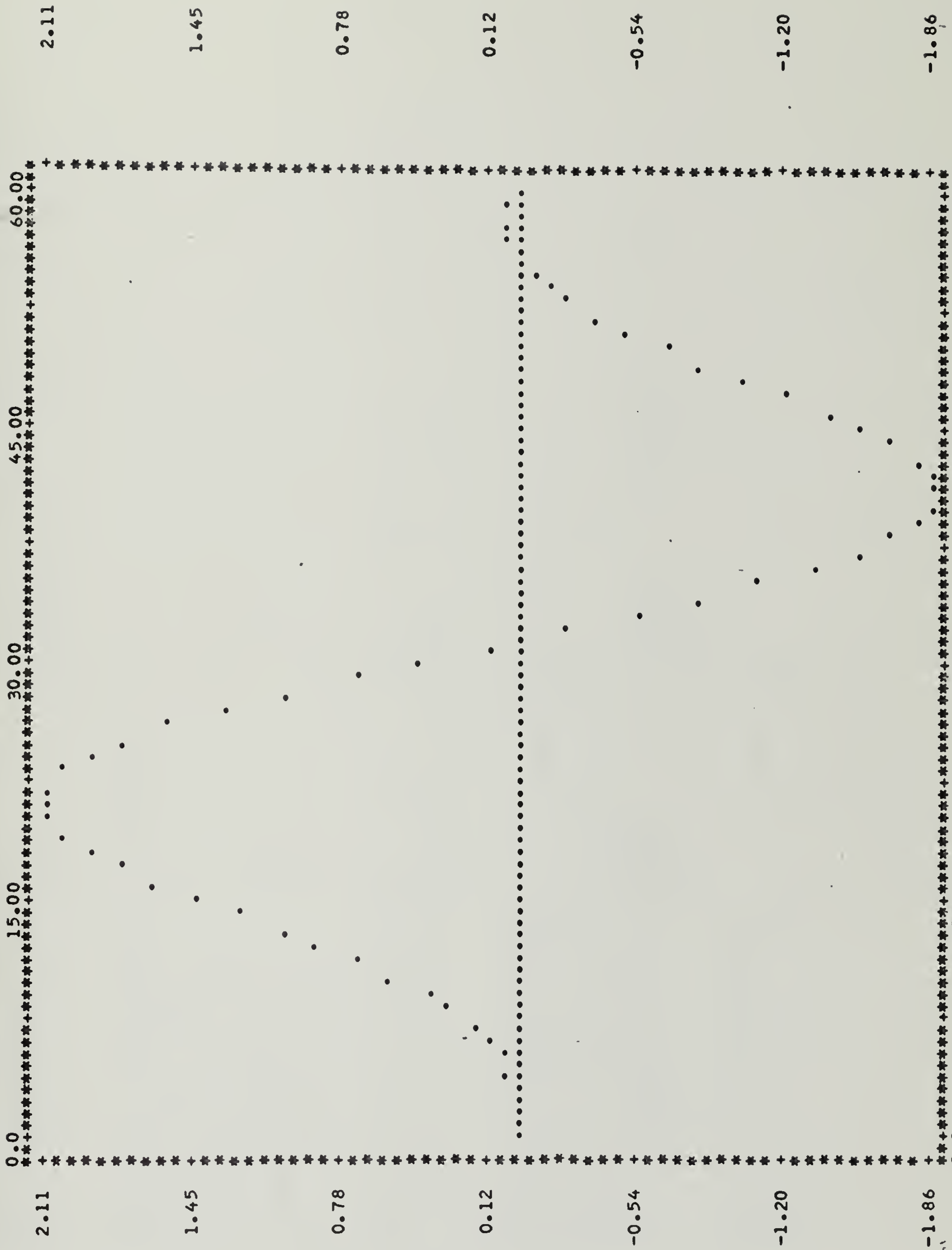
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30.00
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60.00

APPENDIX D: VIBRATING SHELL SAMPLE COMPUTER OUTPUT

```
                    LINEARIZED METHOD OF CHARACTERISTICS
                    FOR SUPERSONIC FLOW PAST A VIBRATING CYLINDRICAL SHELL
DATE OF RUN-26 FEB 1972


 FREESTREAM MACH NUMBER = 3.500000
REDUCED FREQ=  0.0
RAD MODE NO. N    = 0
GRID FINENESS IS = **
CYLINDER RADIUS IS =     0.20
AXIAL MODE NO. M    = 3

 INTIAL VALUE OF QPHI =0.0    0.0     PHI =-.5386D 010.0          PHK =0.0     0.0
 X = 0.0          R =.2000 00PARW = 0.942D 01

 KCUNT =     2
 CONSTANTS AT NEW POINT
 PARW =      0.94175D 01    W =.39260D-01
 IKM & IKW =.0          .0             .C          .0
 MACHLN A =  0.99845676D 00  0.0

 IHAVEP FOR RUN   =  1   KCUNT =  2
 PX =    -2.81863905      0.0            PR =      9.45400278     0.0
       0.00417         0.20124       -5.4022873      0.0
        0.0             0.0            0.0            0.0

 CONSTANTS AT NEW POINT
 PARW =      0.93957D 01    W =.78459D-01
 IKM & IKW =.0          .0             .0          .0
 PANPNT C =  0.83370229D-02  0.0
```

```
IHAVEP FOR RUN    =  2    KOUNT =  2
PX =     -2.78386441      0.0                 PR =       9.39572452      0.0
          0.00833       0.20000          -5.3523113       0.0
          0.01667         0.0              -0.02338         0.0


KOUNT =      3
CONSTANTS  AT  NEW  POINT
PARW =        0.93957D 01      W =.78459D-01
IKM & IKW =.0            .0                 .0           .0
MACHLN  A =  0.99846623D 00  0.0

IHAVEP FOR RUN    =  1    KOUNT =  3
PX =     -2.82732536      0.0                 PR =       9.48313754      0.0
          0.00833       0.20248          -5.4189357       0.0
          0.0             0.0                0.0            0.0


CONSTANTS AT NEW POINT
PARW =        0.93594D 01      W =.11754D 00
IKM & IKW =.0            .0                 .0           .0
GENEPT   A =         0.99845676        0.0          B =      0.00154324       0.0
C =         0.00831141        0.0           D =      -5.36064842       0.0

IHAVEP FOR RUN    =  2    KOUNT =  3
PX =     -2.79257752      0.0                 PR =       9.42476904      0.0
          0.01250       0.20124          -5.3689597       0.0
          0.01662         0.0              -0.02338         0.0


CONSTANTS AT NEW POINT
PARW =        0.93087D 01      W =.15643D 00
IKM & IKW =.0            .0                 .0           .0
PANPNT C =  0.24933896D-01  0.0

IHAVEP FOR RUN    =  3    KOUNT =  3
PX =     -2.74069336      0.0                 PR =       9.30874331      0.0
          0.01667       0.20000          -5.2860881       0.0
          0.03319         0.0              -0.04654         0.0


KOUNT =      4
CONSTANTS  AT  NEW  POINT
PARW =        0.93594D 01      W =.11754D 00
IKM & IKW =.0            .0                 .0           .0
MACHLN  A =  0.99847558D 00  0.0

IHAVEP FOR RUN    =  1    KOUNT =  4
PX =     -2.83598506      0.0                 PR =       9.51218306      0.0
```

```
        0.01250        0.20373        -5.4355332      0.0
         0.0            0.0             0.0             0.0

CONSTANTS AT NEW POINT
PARW =        0.93087D 01    W =.15643D 00
IKM & IKW =.0           .0              .0          .0
GENFPT  A =        0.99846623       0.0         B =     0.00153377     0.0
C =         0.00828603        0.0          D =      -5.37727090     0.0

IHAVEP FOR RUN    =  2   KOUNT =   4
PX =     -2.80126353      0.0            PR =      9.45372434      0.0
        0.01667         0.20248        -5.3855565     0.0
         0.01657         0.0             -0.02342       0.0

CONSTANTS AT NEW POINT
PARW =        0.92437D 01    W =.19509D 00
IKM & IKW =.0           .0              .0          .0
GENFPT  A =        0.99845676       0.0         B =     0.00154324     0.0
C =         0.02485729        0.0          D =      -5.29434799     0.0

IHAVEP FOR RUN    =  3   KOUNT =   4
PX =     -2.74935253      0.0            PR =      9.33742903      0.0
        0.02083         0.20124        -5.3025822     0.0
         0.03309         0.0             -0.04653       0.0

CONSTANTS AT NEW POINT
PARW =        0.91644D 01    W =.23345D 00
IKM & IKW =.0           .0              .0          .0
PANPNT C =   0.41325736D-01   0.0

IHAVEP FOR RUN    =  4   KOUNT =   4
PX =     -2.68067855      0.0            PR =      9.16437060      0.0
        0.02500         0.20000        -5.1873257     0.0
         0.04946         0.0             -0.06933       0.0

KOUNT =       5
CONSTANTS AT NEW POINT
PARW =        0.93087D 01    W =.15643D 00
IKM & IKW =.0           .0              .0          .0
MACHLN A =   0.99848482D 00   0.0

IHAVEP FOR RUN    =  1   KOUNT =   5
PX =     -2.84461840      0.0            PR =      9.54114016      0.0
```

```
        0.01667         0.20497          -5.4520801      0.0
         0.0             0.0              0.0             0.0

CONSTANTS  AT  NEW  POINT
PARW =       0.92437D 01     W =.19509D 00
IKM & IKW =.0          .0                .0          .0
GENFPT   A =       0.99847558         0.0         B =      0.00152442      0.0
C =        0.00826088        0.0             D =       -5.39384216      0.0

IHAVEP FOR RUN   =  2   KOUNT =  5
PX =     -2.80992270      0.0                PR =      9.48259122      0.0
        0.02083         0.20373          -5.4021024      0.0
         0.01652         0.0               -0.02347       0.0

CONSTANTS  AT  NEW  POINT
PARW =       0.91644D 01     W =.23345D 00
IKM & IKW =.0          .0                .0          .0
GENFPT   A =       0.99846623         0.0         B =      0.00153377      0.0
C =        0.02478139        0.0             D =       -5.31081644      0.0

IHAVEP FOR RUN   =  3   KOUNT =  5
PX =     -2.75798453      0.0                PR =      9.36602689      0.0
        0.02500         0.20248          -5.3190252      0.0
         0.03299         0.0               -0.04658       0.0

CONSTANTS  AT  NEW  POINT
PARW =       0.90709D 01     W =.27144D 00
IKM & IKW =.0          .0                .0          .0
GENFPT   A =       0.99845676         0.0         B =      0.00154324      0.0
C =        0.04119893        0.0             D =       -5.19545746      0.0

IHAVEP FOR RUN   =  4   KOUNT =  5
PX =     -2.68923024      0.0                PR =      9.19252120      0.0
        0.02917         0.20124          -5.2035639      0.0
         0.04931         0.0               -0.06929       0.0

CONSTANTS  AT  NEW  POINT
PARW =       0.89635D 01     W =.30902D 00
IKM & IKW =.0          .0                .0          .0
PANPNT C =  0.57411798D-01  0.0
```

```
IHAVEP FOR RUN   =  5   KOUNT =  5
PX =     -2.60418968     0.0              PR =      8.96349650      0.0
        0.03333        0.20000        -5.0566326     0.0
         0.06537         0.0            -0.09159        0.0
```

1.0	5.61985179	0.0
2.0	5.56772883	0.0
3.0	5.48138671	0.0
4.0	5.36135710	0.0
5.0	5.20837936	0.0
6.0	5.02339600	0.0
7.0	4.80754686	0.0
8.0	4.56216209	0.0
9.0	4.28875394	0.0
10.0	3.98900744	0.0
11.0	3.66477001	0.0
12.0	3.31804007	0.0
13.0	2.95095473	0.0
14.0	2.56577658	0.0
15.0	2.16487980	0.0
16.0	1.75073545	0.0
17.0	1.32589630	0.0
18.0	0.89298105	0.0
19.0	0.45465819	0.0
20.0	0.01362957	0.0
21.0	-0.42738628	0.0
22.0	-0.86567090	0.0
23.0	-1.29852267	0.0
24.0	-1.72327345	0.0
25.0	-2.13730505	0.0
26.0	-2.53806536	0.0
27.0	-2.92308408	0.0
28.0	-3.28998796	0.0
29.0	-3.63651544	0.0
30.0	-3.96053057	0.0
31.0	-4.26003619	0.0
32.0	-4.53318625	0.0
33.0	-4.77829719	0.0
34.0	-4.99385832	0.0
35.0	-5.17854110	0.0
36.0	-5.33120741	0.0
37.0	-5.45091646	0.0
38.0	-5.53693070	0.0
39.0	-5.58872029	0.0

40.0	-5.60596640	0.0
41.0	-5.58856315	0.0
42.0	-5.53661830	0.0
43.0	-5.45045257	0.0
44.0	-5.33059765	0.0
45.0	-5.17779292	0.0
46.0	-4.99298092	0.0
47.0	-4.77730152	0.0
48.0	-4.53208488	0.0
49.0	-4.25884328	0.0
50.0	-3.95926178	0.0
51.0	-3.63518781	0.0
52.0	-3.28861982	0.0
53.0	-2.92169493	0.0
54.0	-2.53667577	0.0
55.0	-2.13593651	0.0
56.0	-1.72194827	0.0
57.0	-1.29726381	0.0
58.0	-0.86450186	0.0
59.0	-0.42633093	0.0
60.0	0.01454712	0.0
61.0	0.45541373	0.0
62.0	0.89355043	0.0
63.0	1.32625557	0.0
64.0	1.75086101	0.0
65.0	2.16474854	0.0
66.0	2.56536602	0.0
67.0	2.95024315	0.0
68.0	3.31700666	0.0
69.0	3.66339496	0.0
70.0	3.98727210	0.0
71.0	4.28664090	0.0
72.0	4.55965530	0.0
73.0	4.80463172	0.0
74.0	5.02005944	0.0
75.0	5.20460994	0.0
76.0	5.35714505	0.0
77.0	5.47672401	0.0
78.0	5.56260922	0.0
79.0	5.61427083	0.0
80.0	5.63139000	0.0
81.0	5.61386085	0.0
82.0	5.56179113	0.0
83.0	5.47550153	0.0
84.0	5.35552372	0.0
85.0	5.20259710	0.0

86.0	5.01766417	0.0
87.0	4.80186480	0.0
88.0	4.55652915	0.0
89.0	4.28316947	0.0
90.0	3.98347081	0.0
91.0	3.65928060	0.0
92.0	3.31259727	0.0
93.0	2.94555794	0.0
94.0	2.56042522	0.0
95.0	2.15957328	0.0
96.0	1.74547322	0.0
97.0	1.32067779	0.0
98.0	0.88780571	0.0
99.0	0.44952549	0.0
100.0	0.00853898	0.0
101.0	-0.43243528	0.0
102.0	-0.87067882	0.0
103.0	-1.30349001	0.0
104.0	-1.72820071	0.0
105.0	-2.14219271	0.0
106.0	-2.54291390	0.0
107.0	-2.92789396	0.0
108.0	-3.29475966	0.0
109.0	-3.64124940	0.0
110.0	-3.96522724	0.0
111.0	-4.26469601	0.0
112.0	-4.53780966	0.0
113.0	-4.78288461	0.0
114.0	-4.99841016	0.0
115.0	-5.18305778	0.0
116.0	-5.33568933	0.0
117.0	-5.45536403	0.0
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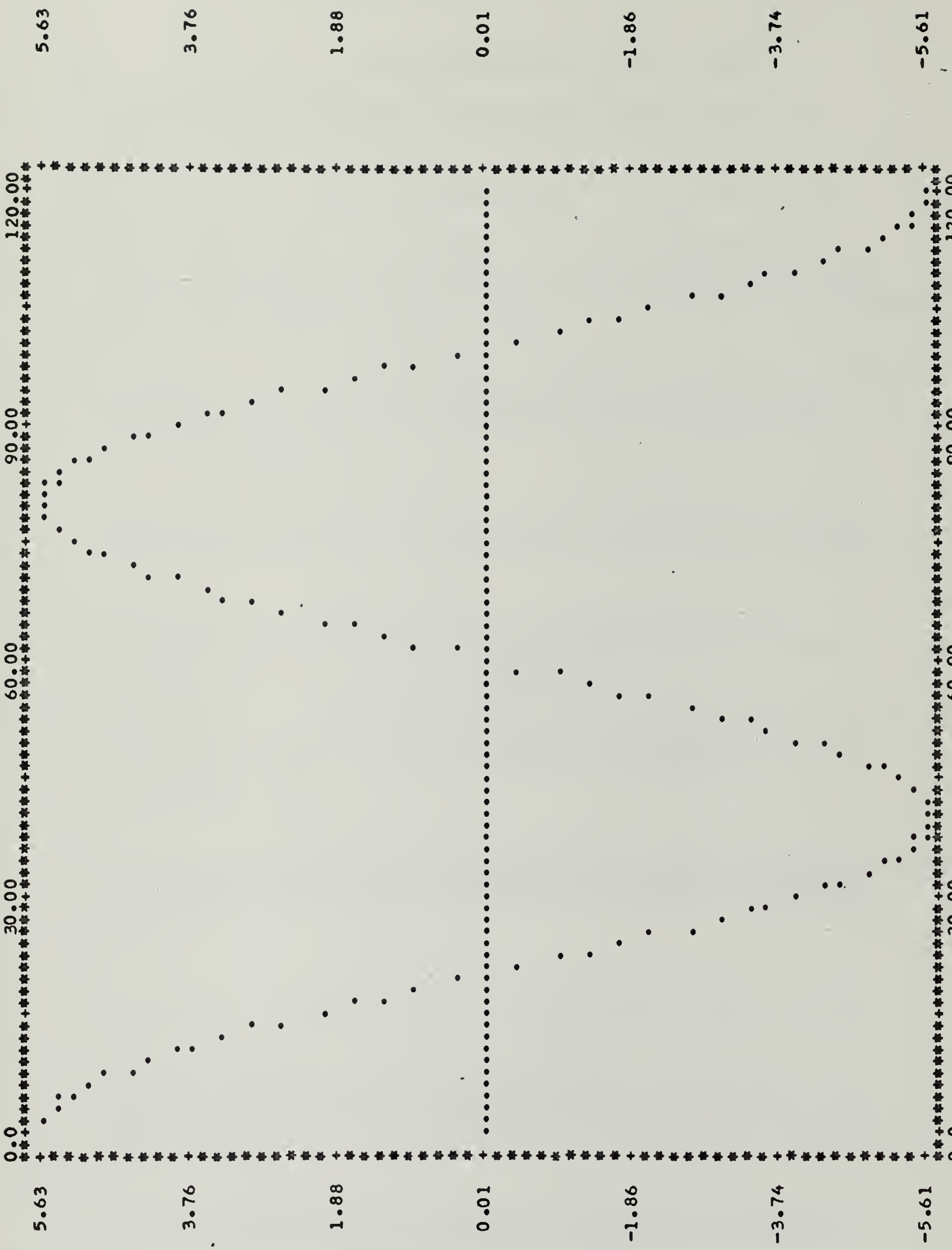
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Supersonic flow past harmonically vibrating two-dimensional panels and cylindrical shells is analyzed using a linearized method-of-characteristics procedure. A detailed description of this method to solve the linearized unsteady potential equation is given and numerical results are presented to indicate the nature of the aerodynamic pressure distributions. Also, comparisons of the present results are made with earlier work by Nelson and Cunningham and by Anderson which is based upon quite different approaches.

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