OPTIMIZATION OF RING-STIFFENED CYLINDRICAL SHELLS FOR PRACTICAL HYDROSPACE APPLICATIONS

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By

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ABSTRACT

All present stiffened cylindrical shell design formulas for the case of external hydrostatic pressure were surveyed. This also included present design practices for allowances due to pressure hull imperfections, and actual test data when available. Formulations for all three basic hull failure modes were then selected, first for accuracy, and secondly for compatibility with both elastic-perfectly plastic and strain-hardening metals, whenever possible.

The formulas were then inserted in a logical flow pattern to design elastic-perfectly plastic scantlings for failure at the most efficient collapse mode. The process was programmed in FORTRAN IV and designed to iterate, varying several scantling parameters systematically. The "optimum" design, based on a simple hull weight/buoyancy

ratio was selected at the completion of the run.

Program inputs are: collapse depth, hull diameter, hull length, internal bulkhead spacing (specified or unspecified), framing (internal or external), and metal properties. Outputs are: shell thickness, typical frame spacing, typical frame size, heavy frame (bulkhead) spacing, and heavy frame size for each design, and an optimum design designation. Simple directions are given for conversion of the program to one compatible with strain-hardening metals.

Thesis Supervisor: J. Harvey Evans

Title; Professor of Naval Architecture

ACKNOWLEDGMENTS

I would like to gratefully acknowledge the help of two people, without whom this thesis could not have been successfully completed: my advisor, Professor J. Harvey Evans, and my wife, Connie.

To the former, my thanks are, first of all, for introducing me to such an interesting and applicable topic. Secondly, for his knowledgeable, patient manner, which made working for him a pleasure and an education.

To the latter, my thanks are not only for her enthusiastic typing job, for which she gave up many of her own activities. They are also for her solo performance as family manager during those particularly long and busy thesis work periods, when I was at my absent-minded worst.

I believe a project such as this can only be termed completely successful in one's own mind, when he has such people as these backing him up.

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LISTS OF SYMBOLS

USED COMMONLY IN THE TEXT

1. a: shorthand for expression $\frac{(1-\nu/2)\alpha}{\alpha+\beta+(1-\beta)F_1}$;used in ELNCK
2. D: displacement of pre-specified length of hull, tons
3. Dm: hull mean diameter (to shell mid-fiber), in.
4. E: Young's Modulus, psi
5. E_s: secant modulus, psi
6. E_t: tangent modulus, psi
7. L: unsupported length of plating between frames, in.
8. p: applied hydrostatic pressure, psi
9. R_m: hull mean radius, in.
10. t: shell thickness, in.
11. W: weight of pre-specified length of hull, tons
12. Υ: measure of beam-column effect (see chapter 2)
13. θ: shell fexibility parameter (see chapter 10)
14. λ: "thinness" factor, $\lambda = \sqrt[4]{\frac{(L/D_m)^2}{(t/D_m)^3}} \sqrt{\frac{\sigma_y}{E}}$
15. ν: Poisson's ratio
16. $\sigma_{b\phi}$: circumferential bending stress, midbay, psi
17. $\sigma_{m\,x}^{\;\phi}$: {circumferential / longitudianal} membrane stress, midbay, psi
18. σ_y: yield stress, psi

USED IN PROGRAM

19. A. same as (1) above.

20. AE: effective cross-sectional area of typical frame, sq. in.

21. AF: actual cross-sectional area of typical frame, sq. in.

22. AFH: actual cross-sectional area of heavy frame, sq. in.

23. ALFA: AE/(FS*T), used in definition of A.

24. B: typical frame web thickness, in.

25. BETA: B/FS, used in definition of A.

26. BH: heavy frame web thickness, in.

27. BHETA: Von Sanden-Gunther variable used in RNLDS (included so as not to be confused with BETA)

28. BS: Bulkhead spacing (i.e., heavy frame spacing), in.

29. CC: typical frame dimension parameter (see chapter 10 and Appendix A)

30. CCOP: typical frame moment of inertia parameter derived from CC.

31. DISP: same as (2) above

32. DM: same as (3) above

33. DN: distance from shell mid.-fiber to combined centroid of typical frame and effective length of shell plating, in.

34. DNH: same as (33), for heavy frames.

35. E, ESEC, ETAN: same as (4), (5) and (6) above.

36. EI: effective moment of inertia, of typical frame and effective length of shell plating, in.

37. EL: effective length of shell plating, in., for typical frames.

38. ELH: effective length of shell plating, in., for heavy frames.

39. F1, F2, F3, F4: Salerno-Pulos "F-functions" used in computing shell stresses.

40. FC: distance from shell mid-fiber to centroid of typical frame, in.

41. FCH: same as (40), for heavy frames.

42. FD: typical frame depth, in.

43. FI: typical frame moment of inertia, $in.^4$

44. FIH: typical heavy frame moment of inertia, in.

45. FS: typical frame spacing, in.

46. FW: typical flange width, in.

47. GAMA: same as (12) above.

48. GNU: Poisson's ratio

49. HULNTH: hull length, in.

50. NF: number of typical frames

51. PC: design collapse pressure, psi

52. PCG: general instability collapse pressure, psi, reduced for imperfections.

53. PCGE1: general instability elastic collapse pressure, psi

54. PCLE1: axisymmetric yield collapse pressure, psi

55. PCR: asymmetric buckling pressure, psi, reduced for imperfections.

56. PCRE or PCRE1: asymmetric elastic buckling pressure, psi

57. PEL: axisymmetric elastic buckling pressure, psi

58. PO: operating depth pressure, psi

59. PRE: P_e, chapter 3

60. PRI: P_i, chapter 3

61. RC: hull radius to centroid of heavy frame-effective length of shell plate combination, in.

62. RCG: same as (61), with typical frames

63. RF: hull radius to centroid of typical frame, in.

64. RHO: material density, lb/in.^{3}

65. RM: hull mean radius (to mid-fiber of shell), in.

66. RO: hull outer radius (to outer fiber of shell), in.

67. SIGY: yield stress, psi

68. SL: same as (7) above

69. T: same as (10) above

70. TF: typical frame flange thickness, in.

71. THETA: same as (13), above

72. VF: volume of typical ring frames; in^3

73. VFH: volume of heavy ring frames, in^3

74. WD: weight/displacement ratio

75. WDOPT: optimum WD

76. WT: same as (11) above

77. Z: input parameter specifying frame location
(i.e., 1.0 $\Rightarrow$ internal, anything else $\Rightarrow$ external)

INTRODUCTION

The amount of literature concerning the collapse of ring-stiffened cylindrical shells under hydrostatic pressure accumulated in the last fifty years is voluminous. This is understandable; the subject is very involved. To this day, an exact solution for all aspects of the problem does not exist. Good solutions for the different failure modes do exist, though, modified in varying amounts by empirical data. No attempt will be made here to list or summarize this knowledge. Many have already done this, and the finest review to date in this author's knowledge has been done by J. G. Pulos[16] for the Navy's former David Taylor Model Basin.

It appeared that one should be able to integrate this albeit incomplete, yet extensive knowledge with the use of present generation computer science. Hand calculations for only one geometrical combination of shell thickness, frame size, frame spacing, hull diameter, hull length, etc. are notoriously laborious, even for only one mode of failure. Submarine design processes using the hand technique would achieve adequate structures, but with little or no idea if anything better existed. Optimization, with the exception of a few combinations tried at a great cost in time, was out of the question: while similar submarines could be designed on past knowledge, different hull geometries or deeper operating depths meant a great deal of time and work. It was at the

suggestion of Professor Evans that the development of a computerized design optimization was undertaken. The basic design equations were there. The computing tools were readily available. All that needed to be done was an integration of the two into a practical, useable, and most of all, reliable (in terms of latest empirical data, if necessary) program.

Shortly after the start of the project, it was discovered that a smiliar program had just been completed[22] for the Naval Ships Research and Development Center (NSRDC). It is hoped that by using some different approaches and techniques, this program could be a valuable tool to use in conjunction with reference 22.

The description of the program development will be done by subprograms, each building on the other, and ending with the optimization scheme of the main program. While the program developed can be used only with elastic-perfectly plastic, isotropic materials (e.g., HY-80 steel), it can easily be modified so as to be applicable also to strain-hardening metals, such as HY-150 steel. Further discussion on this will follow later. Included also in the thesis will be various data obtained using the program or portions thereof in parametric-type studies.

It is recommended that while reading through the various chapters on subprograms, reference be made to figure 23 (chapter 10), which is a simplified main flow diagram for the entire optimization.

It is recommended that while reading through the various chapters on subsystems, reference is made to Figure 22 (chapter 10), which is a simplified model representing the entire organization.

CHAPTER ONE

BACKGROUND INFORMATION AND TERMINOLOGY

The entire hull design program is based on the three fundamental failure modes for ring-stiffened cylindrical shells. Therefore, in order that terminology remain consistently clear throughout the discussion, a brief description and catagorization of these modes follows. Refer to figure one for pictorial representations.

A. AXISYMMETRIC FAILURES BETWEEN STIFFENERS

All axisymmetric failures, whether elastic, plastic, or some mixture thereof, are characterized by one or more accordian-like pleats, or circumferential ripples between ring frames. For true axisymmetric failures, the stiffeners remain undeformed.

1. AXISYMMETRIC YIELD FAILURE. This type of failure occurs only with elastic-perfectly plastic (plateau-type stress-strain curve) materials. It is regarded, then, as almost totally a yield-type failure, although it is initiated partially by instability phenomena.
2. AXISYMMETRIC INELASTIC FAILURE. Inelastic failure is also a failure above the purely elastic range, but in strain-hardening materials. Thus, the failure is in the range where Young's modulus varies, and can intuitively be

considered a kind of combination elastic-plastic failure. Often this failure can be at a lower pressure than a pure buckling (elastic) failure, due to certain combinations of low modulus and hull geometry. The pure yield failure (A.1.) cannot occur in strain-hardening materials.

3. AXISYMMETRIC ELASTIC FAILURE. This is axisymmetric failure in the linearly elastic range. Theoretically, this could occur, given proper geometry, in either of the two above types of materials. Generally speaking, the required geometry is one of a thin shell relative to the hull diameter and depth. In reality, this failure is, at this writing, a mathematical phenomenon only. Other hull failure modes occur first, so this mode has never been achieved in actual testing. It is valuable, however, in determining effects of geometrical defects on collapse pressure.

B. ASYMMETRIC (LOBAR) BUCKLING FAILURES BETWEEN STIFFENERS:

All lobar buckling failures are characterized by lobes of buckling distributed partly or completely around the shell circumference. The frames remain intact.

1. ELASTIC OR INELASTIC MODES: This is primarily a buckling failure. The collapse pressure is

dictated by the modulus (elastic or inelastic) and hull geometry.

C. GENERAL INSTABILITY (SHELL AND FRAME) FAILURE.

General instability failures are characterized by failure of both frames and shell simultaneously, sometimes extending the entire length of the cylinder.

1. ELASTIC OR INELASTIC MODES. These are also buckling failures, except that lobes extend longitudinally as well as circumferentially. It is normally assumed that only a one-half wave extends between heavy frames (bulkheads), since these heavy frames are designed heavy enough not to deform at the general instability collapse pressure. This combination failure of frames and shell is not as well understood or formulated mathematically as the other two modes, especially in the inelastic region. Thus, larger safety factors are employed when checking hull designs in this mode.

It should be emphasized at the outset that the program to be described designs the stiffened cylindrical shells to fail in the axisymmetric yield failure mode. For the untested inelastic portion of the program, the design failure mode would be axisymmetric inelastic failure. This philosophy is used today in submarine design and is

advocated by most naval archietects. It has two basic reasons:

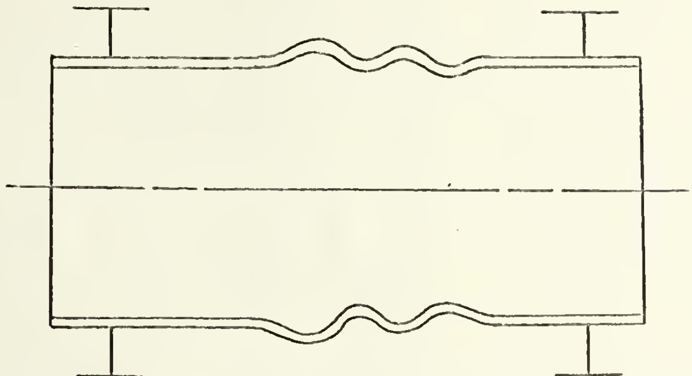
1) Failure by yielding of the shell utilizes full material (i.e., yield) strength. Stresses in buckling type failures are usually below yield stress, with failure depending mainly on the value of Young's modulus and geometry of the failing structure. More efficient structures should thus result from designing to a yielding failure.

2) Imperfections in construction (e.g., hull out-of-roundness) effect buckling failures far more seriously than yield failures.[2] Thus, hulls designed for yield failure would have less stringent requirements for building, and less chance of failure, given that imperfections might exist.

The optimization criterion used is a simple hull weight to hull displacement ratio, which appears to be the best general measure of design efficiency for this type of structure.

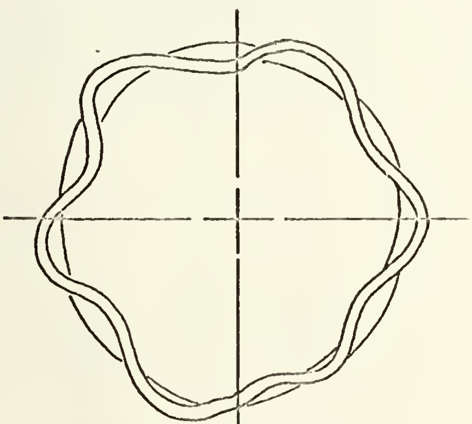
Figure 1.

FAILURE MODES: STIFFENED CYLINDRICAL SHELLS

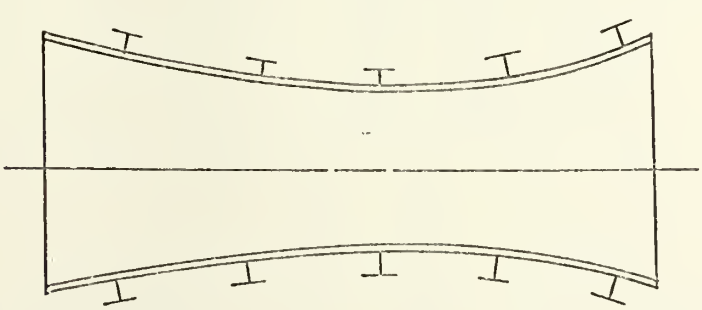


AXISYMMETRIC YIELDING (BETWEEN FRAMES)

COLLAPSE FORMULAS: LUNCHIK(DTMB# 1291,1393)



ASYMMETRIC(LOBAR)BUCKLING(BETWEEN FRAMES)
COLLAPSE FORMULA: REYNOLDS(DTMB#1392)



GENERAL INSTABILITY (CONCURRENT FAILURE OF SHELL AND FRAMES)

COLLAPSE FORMULA: KRENZKE-KIERNAN(DTMB# 1677)

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TABLE 1.
LIST OF PROGRAMS USED IN THE OPTIMIZATION

PROGRAM	TYPE	ARGUMENTS	COMMON BLOCKS	REMARKS
MAIN	MAIN	—	D,F	CONDUCTS OPTIMIZATION USING ALL SUBPROGRAMS LISTED BELOW
RNLDS	FUNCTION	T,SL	D,F	COMPUTES CRITICAL ASYMMETRIC (LOBAR) BUCKLING PRESSURE
THKNS	SUBROUTINE	TT,SL,FI,PC,*	D	CONVERGES ON THICKNESS TO SATISFY AXI SYMMETRIC FAILURE AT DESIGN DEPTH
STRTHK	SUBROUTINE	T,SL,FI,PC	D	ADJUSTS SHELL THICKNESS AS NECESSARY TO MEET 0.75 YIELD STRESS AT OPERATING DEPTH
GINST	SUBROUTINE	T,SL,FI,PC,PCG,N, BS,L,HULNTH	D,F,U	COMPUTES GENERAL INSTABILITY PRESSURE AND HEAVY FRAME SPACING
HVYFRM	SUBROUTINE	T,SL,FI,HULNTH,PC, PCG,BS,AFH,BH,AF, FCH	D,F,V	COMPUTES DIMENSIONS OF HEAVY FRAMES REQUIRED
WTDSP	FUNCTION	T,SL,FI,BS,AFH, FCH,AF	D,F	COMPUTES WEIGHT/DISPLACEMENT RATIO
ELNCK	SUBROUTINE	T,SL,FI,PC,P	D	COMPUTES CRITICAL AXISYMMETRIC YIELDING PRESSURE
FRAME	SUBROUTINE	T,SL,FI	D,F,U	COMPUTES FRAME/FRAME-SHELL CONSTNTS AND DIMENSIONS
PULOS	SUBROUTINE	T,SL,FI,P,F1,F2, F3,F4,A	D,U	COMPUTES SALERNO PULOS STRESS FUNCTIONS FOR GIVEN PRESSURE AND STIFFENED SHELL GEOMETRY(USED WITH ELNCK)

TABLE 1.(CONTINUED)

PROGRAM	TYPE	ARGUMENTS	COMMON BLOCKS	REMARKS
PULOS1	SUBROUTINE	T,SL,FI,P,F1,F2, F3,F4,A,*	D,U	SAME AS PULOS, EXCEPT USED IN GINST
PLNCK	FUNCTION	T,SL,FI,PC	D	COMPUTES CRITICAL AXISYMMETRIC BUCKLING PRESSURE
REDPR	FUNCTION	PRE,PRI	—	COMPUTES REDUCED PRESSURE DUE TO RESIDUAL STRESSES AND FABRICATION IMPERFECTIONS
KRNZK	SUBROUTINE	T,RM,FS,EI,RO, RCG,GAMMA,ESEC, ETAN,PCG,N	—	COMPUTES ELASTIC/INELASTIC GENERAL INSTABILITY COLLAPSE PRESSURE AND NUMBER OF CIRCUMFERENTIAL WAVES

CHAPTER TWO

HULL STRESSES

Numerous subprograms throughout the optimization require the values of various stresses in the hull. These stresses include circumferential and longitudinal stresses, at the frames and at midbay, at the inner and outer shell surfaces. Several solutions for stresses due to external hydrostatic pressure on ring-stiffened cylinders have appeared in the past. The most famous was due to the Germans, Von Sanden and Gunther, in 1920. Portions of their analysis are still in use today. In 1930, the Italian Viterbo modified their analysis to include the so-called stiffener-expansion effect (a result of axial stresses in the shell). Neither of these early analyses, however, included the "beam column" effect. This effect, introduced by Salerno and Pulos[18] in 1951, is caused by the interaction of longitudinal bending and longitudinal compression in the hull caused by the axial portion of the hydrostatic pressure acting on it. The Salerno-Pulos stresses are an exact solution, and the beam-column effect accounts for any non-linearities between pressure and strain in the cylinders. In all cases except one (see chapter four), the program uses the more accurate Salerno-Pulos (hereafter S-P) stresses.

The beam-column effect is represented in the S-P analysis by the parameter γ (hereafter: GAMA), where:

$$\gamma \equiv \frac{p}{p^*} = \frac{p}{2E}\sqrt{3(1-\nu^2)}\left(\frac{R_m}{t}\right)^2$$

p^* is defined[16,18] as the "critical load for axisymmetric elastic buckling of an unstiffened cylindrical shell under the action of uniform axial pressure." GAMA=0 corresponds to a zero beam-column effect (i.e., the stress solution of Von Sanden and Gunther, hereafter V-G). As GAMA grows larger, the beam-column effect, and thus the non-linearity between pressure and hull stresses, increases. When GAMA $\geq$ 1, then theoretically the between-frame failure mode shifts to axisymmetric elastic buckling (see chapter one for definition). As explained in Chapter 1, this appears to be a mathematical phenomenon only, for it has never been achieved in actual testing. Other modes of failure (e.g., lobar buckling) always appear to occur first, or else the axisymmetric failure is always accompained by some yielding.[13]

The S-P stresses are calculated by placing various combinations of S-P "F functions" (see reference 18 for expressions and curves) into the S-P stress equations. Formerly, this was a very laborious process, far more time-consuming than obtaining stresses through the simpler V-G equations. The curves developed by M.A. Krenzke and R. D. Short (included in reference 18) shortened the labor

considerably. The computerized solution makes it almost mandatory to use the superior S-P stress solution.

The "F functions" are obtained through the use of two basic subprograms (see figures 2, 3 and 4). Subroutine FRAME essentially takes shell thickness, unsupported shell length, and frame moment of inertia and manipulates them to obtain the S-P variables THETA, ALFA, and BETA (see Appendix A for method of obtaining frame dimensions). These variables are then transferred to the stress program PULOS via a COMMON statement. FRAME is separate from PULOS because in one program, (HVYFRM) PULOS is used in two different places with the same scantlings. All variables in FRAME are computed in accordance with the S-P stress analysis[18], with the exception of the expression for effective area, AE. Reference 18 lists this as (in program terms):

AE=AF*(RM/RF), or

AE=AF*(RM/RF)**2, depending on whether

the framing is internal or external, respectively. This is not strictly correct, and has since been refined by Short[24], who used the similar equation:

AE=AF*(RM/RF)**Q, where Q=1+ 2*GMU

This equation is good for either internally or externally framed cylinders with "reasonable" (i.e., suitable for this program's purposes) frame depth/shell radius ratios.

The subroutine PULOS is a straightforward adapation

of the S-P stress analysis. It produces the four "F functions" and the variable "a" (see reference 16), the various combinations of which are used in several parent programs to compute the hull stresses.

From the expression for GAMA, it can be seen that an input pressure is required for PULOS to compute its outputs. In other words, FRAME and PULOS can be used to calculate the hull stresses, given scantlings, material properties, and hydrostatic pressure. More often, however, the program is attempting to find a critical failure pressure. This results in two unknowns, the pressure and the stresses. GAMA thus becomes the result of a transcendental process, in which PULOS calculates stresses, and the pressure input is from a parent program dealing with a particular hull failure mode.

Great difficulties were experienced in this iterative process when GAMA≥1.0; this meant that ETA1 (see figures 3 or 4) became imaginary (i.e., the failure mode had shifted to axisymmetric elastic). To combat this, several methods were employed, each one being different for different parent programs. This is the reason for PULOS and PULOS 1. The methods of validly circumventing this pitfall will be explained separately under each individual failure mode subprogram.

CHART TITLE - SUBROUTINE FRAME(T,SL,FI)

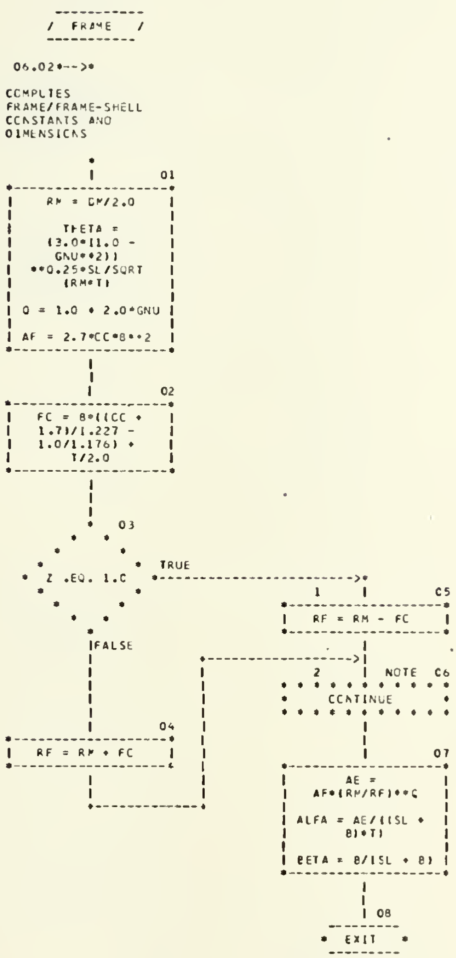


Figure 2

CHART TITLE - SUBRCUTINE PULOS(T,SL,F(,P,F1,F2,F3,F4,A)

```
 -----------
 /  PULOS  /
 -----------

11.04*-->*

COMPUTES
SALERNO-PULOS STRESS
FUNCTIONS FOR GIVEN
PRESSURE AND
STIFFENED CYLINDER
GEOMETRY

        |        01
*--------------------*
|   RM = DM/2.0      |
|                    |
|      GAMA =        |
|    F/(2.0*E)       |
| *SQRT(3.C*(1.0 -   |
|  GNU**2))*(RM/T)   |
|       **2          |
*--------------------*
        |
        *   02
      *   *
    *       *
(-) *          *
*-----*  GAMA - 1.0  *
|   *          *
|     *      *
|       *  *
|         *
|         |(0/+)
|         |
|   1     |          03
| *--------------------*
| |       T =          |
| |  SQRT(P/(1.9*E))   |
| |  *RM*(3.C*(1.0 -   |
| |  GNU**2))**0.25    |
| |                    |
| |    GAMA = 0.95     |
| *--------------------*
|         |
*---------->|
    2     | NOTE   04
* * * * * * * * * * * *
*       CONTINUE      *
* * * * * * * * * * * *
          |
          |            05
*--------------------*
| ETA1 = SQRT(1.0 -  |
|    GAMA)/2.0       |
|                    |
| ETA2 = SQRT(1.0 +  |
|    GAMA)/2.0       |
|                    |
|      E1TP =        |
| EXP(ETA1*THETA)    |
*--------------------*
          |
          |            06
*--------------------*
|  E1TN = 1.0/E1TP   |
|                    |
| SINH(X,Y) = (X -   |
|     Y)/2.0         |
|                    |
| COSH(X,Y) = (X +   |
|     Y)/2.0         |
*--------------------*
          |
          |            07
*--------------------*
|       C3 =         |
|  COSH(E1TP,E1TN)   |
| *SINH(E1TP,E1TN)   |
|      /ETA1         |
|                    |
|       C4 =         |
|  COS(ETA2*THETA)   |
| *SIN(ETA2*THETA)   |
|      /ETA2         |
|                    |
|  DELTA = C3 + C4   |
*--------------------*
          |
          *-----------> 08

          |            08
*--------------------*
|       F3 =         |
|  SQRT(3.0/0.91)    |
| *(-C3 + C4)/DELTA  |
|                    |
|       F1 =         |
| 4.0/THETA*((COSH   |
| (E1TP,E1TN))**2 -  |
| (COS(ETA2*THETA))  |
|   **2)/DELTA       |
*--------------------*
          |
          |            09
*--------------------*
|       C1 =         |
|  COSH(E1TP,E1TN)   |
| *SIN(ETA2*THETA)   |
|      /ETA2         |
|                    |
|       C2 =         |
|  SINH(E1TP,E1TN)   |
| *COS(ETA2*THETA)   |
|      /ETA1         |
|                    |
|   F2 = (C1 +       |
|    C2)/DELTA       |
*--------------------*
          |
          |            10
*--------------------*
|       F4 =         |
|  SQRT(3.0/0.91)    |
| *(C1 - C2)/DELTA   |
|                    |
|   A = (1.C -       |
|   GNU/2.0)         |
|  *ALFA/(ALFA +     |
|  BETA + (1.C -     |
|   BETA)*F1)        |
*--------------------*
          |
          | 11
      ---------
      * EXIT  *
      ---------
```

Figure 3

CHART TITLE - SUBROUTINE PULOS1(T,SL,F1,P,F1,F2,F3,F4,A,*)

```
 -----------
 / PULOS1 /
 -----------

13.09--->*

COMPUTES
SALERNO-PULOS STRESS
FUNCTIONS FOR GIVEN
PRESSURE AND
STIFFENED CYLINDER
GEOMETRY

C1
  RM = DM/2.0

  GAMA =
  P/(2.0*E)
  *SQRT(3.0*(1.0 -
  GNU**2))*(RM/T)
  **2

02
  GAMA .LT. 1.0   TRUE ---> 04 (10)
  FALSE
03
  * EXIT *
  RETURN 1

04 (10)
  ETA1 = SQRT(1.0 -
  GAMA)/2.0

  ETA2 = SQRT(1.0 +
  GAMA)/2.0

  E1TP =
  EXP(ETA1*THETA)

05
  E1TN = 1.0/E1TP

  SINH(X,Y) = (X -
  Y)/2.0

  COSH(X,Y) = (X +
  Y)/2.0

06
  C3 =
  COSH(E1TP,E1TN)
  *SINH(E1TP,E1TN)
  /ETA1

  C4 =
  COS(ETA2*THETA)
  *SIN(ETA2*THETA)
  /ETA2

  DELTA = C3 + C4

07
  F3 =
  SQRT(3.0/0.91)
  *(-C3 + C4)/DELTA

  F1 =
  4.0/THETA*((COSH
  (E1TP,E1TN))**2 -
  (COS(ETA2*THETA))
  **2)/DELTA

08
  C1 =
  COSH(E1TP,E1TN)
  *SIN(ETA2*THETA)
  /ETA2

  C2 =
  SINH(E1TP,E1TN)
  *COS(ETA2*THETA)
  /ETA1

  F2 = (C1 +
  C2)/DELTA

09
  F4 =
  SQRT(3.0/0.91)
  *(C1 - C2)/DELTA

  A = (1.0 -
  GNU/2.0)
  *ALFA/(ALFA +
  BETA + (1.0 -
  BETA)*F1)

10
  * EXIT *
```

Figure 4

CHAPTER THREE

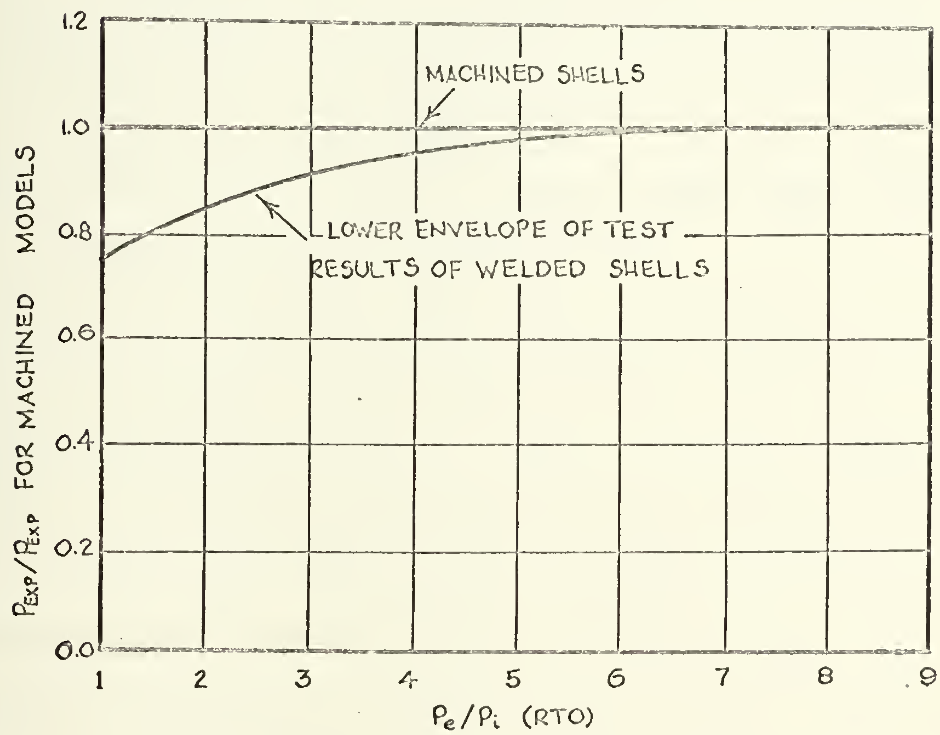
COLLAPSE PRESSURE REDUCTION

Realistically speaking, no cylinder that is manufactured today can be considered "perfect". There will always be some reduction in the collapse pressure due to manufacturing imperfections, such as shell or frame out-of-roundness, and to residual stresses from welding. If, however, a pressure hull is machined rather than rolled and welded, a structure that is "perfect" for all practical purposes may be attained. Of course, for large pressure hulls, the expense (or even impossibility) incurred due to size prohibits construction by machining only. Thus, some allowances must be made in scantling computation.

The best overall method (i.e., including perfectly plastic and strain hardening materials) yet devised is presented as a graph in reference 10 (see figure 5). Here, the lower curve represents an envelope of numerous model test results conducted over the years at the Model Basin. It may be observed that "the factor which is all important in determining imperfection sensitivity is the margin of stability p_e/p_i, the abscissa on the plot. The lower the margin of stability, the greater the sensitivity to imperfections".[21] The reduction factor (REDFAC, or p_{EXP}/p_{EXP} for machined models) located on the ordinate is applied to

Figure 5.

A COMPARISON OF MODEL BASIN DATA ON MACHINED RING-STIFFENED CYLINDERS WITH WELDED CYLINDERS . . .



(Note: Above figure taken from reference 10)

PEXP: EXPERIMENTAL COLLAPSE PRESSURE

P_i : MINIMUM INELASTIC SHELL BUCKLE PRESSURE

P_e : ELASTIC SHELL BUCKLE PRESSURE ASSOCIATED WITH MINIMUM INELASTIC SHELL BUCKLE PRESSURE

collapse pressures of all modes in the optimization program (see figure 6).

The envelope curve is approximated by the equation:

REDFAC=0.64667+0.11367*RTO-0.00967*RTO**2, where

RTO=p_e/p_i

This equation and the general form of REDPR were adopted from a similar program in reference 22.

As mentioned above, this safety reduction factor is applicable not only to this optimization, but also to one for strain hardening metals as well. For this reason (i.e., the fact it is easily convertible to the strain hardening case) it was chosen above other existing out-of-roundness analyses for the strictly elastic buckling cases. No mathematical analysis presently exists for the inelastic case, due to its complexity.[21] Further discussion on this is contained in Chapter 13.

CHART TITLE - FUNCTION REDPR(PRE,PRI)

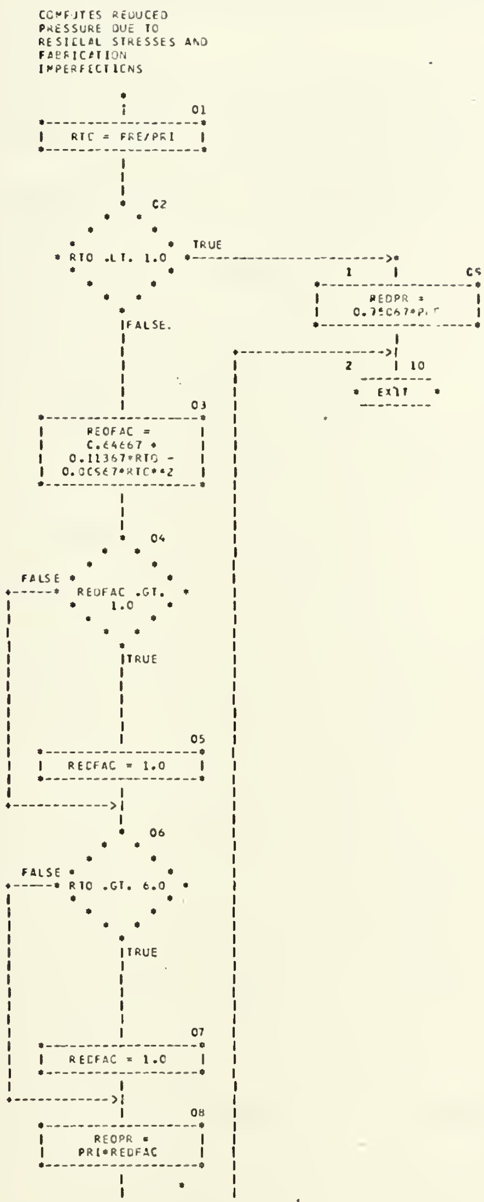


Figure 6

CHAPTER FOUR

ASYMMETRIC (LOBAR) BUCKLING

The most widely used formulation for this failure mode over the years has been the so-called "DTMB Instability Formula", developed by Windenburg and Trilling. The formula was good only for an elastically-perfectly plastic material. More recently, Reynolds[20] has developed a more generalized formulation which may be used for either elastic-perfectly plastic or strain-hardening materials. In reference 20, Reynolds recommended using the V-G stresses (longitudinal and circumferential at mid-bay). He stated that the accuracy of the analysis was not seriously impaired by not using the more cumbersome, yet more accurate S-P stresses. This is graphically borne out by the example given in figure 7. In any case, the theory correlates very closely with experiment, to within four percent[4].

The S-P stress programs (FRAME and PULOS) can easily be made common with any other subprogram. Originally it was decided that there would be redundancy involved if the less accurate V-G stresses were used for the Reynolds lobar buckling subprogram (RNLDS). Difficulties, however, were immediately encountered when the S-P stresses were utilized. RNLDS is used in the main program chiefly as a checking function. Since the hull scantlings, as mentioned in

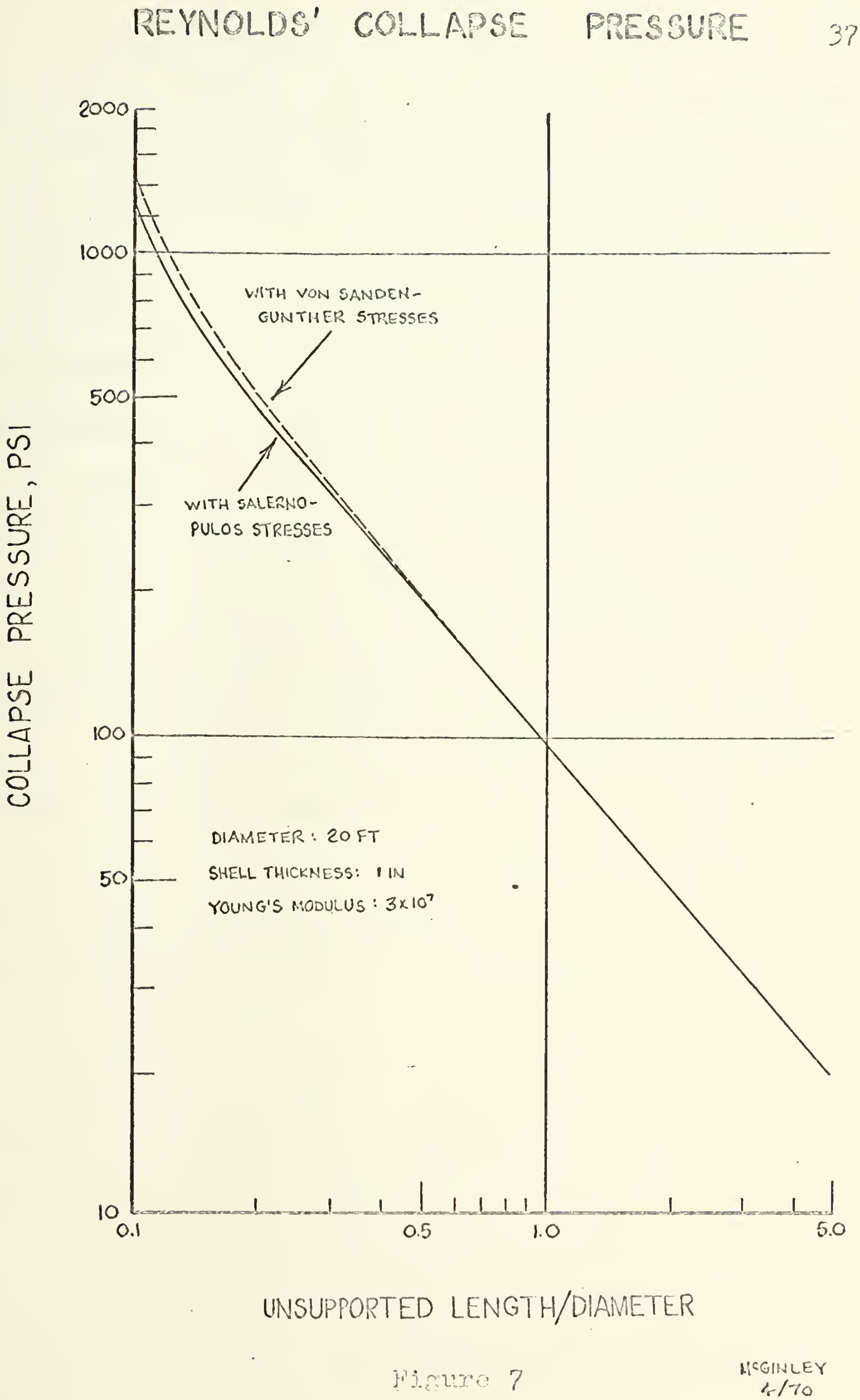

REYNOLDS' COLLAPSE PRESSURE
COLLAPSE PRESSURE, PSI
2000
1000
500
100
50
10
WITH VON SANDEN-GUNTHER STRESSES
WITH SALERNO-PULOS STRESSES
DIAMETER : 20 FT
SHELL THICKNESS: 1 IN
YOUNG'S MODULUS : 3x10^7
0.1
0.5
1.0
5.0
UNSUPPORTED LENGTH/DIAMETER
McGINLEY 4/70

Figure 7

chapter one, are designed for yield failure, RNLDS is used only to insure that the scantling set under scrutiny does not fail by lobar buckling. Thus, since scantling sets being examined will usually fail first by axisymmetric yielding, failure by lobar buckling sometimes occurs at much greater pressures. In many cases, failure by axisymmetric elastic buckling will occur at a pressure between failure by yield and failure by lobar buckling. As noted in chapter two, this phenomenon causes GAMA (beam-column effect) to increase in value over 1.0, resulting in imaginary values occuring within the PULOS subprograms. Efforts to "force" convergence of RNLDS by inserting, for instance, values of GAMA=0.95, or of taking only absolute values of the radical $\sqrt{1.0\text{-GAMA}}$, were only mildly successful. The value of final convergence in any case was not accurate, and certainly was not that of lobar collapse pressure.

For these reasons, then, V-G stresses were used in RNLDS (see figure 8), as originally recommended in reference 20. One distinct advantage of the V-G stresses is that they require no iteration for convergence. There is no separate stress program needed, and the V-G stresses are directly (and quickly) computed within RNLDS. Since (see chapter ten) RNLDS is used itself in an iterative process within the main program, this results in substantial savings

CHART TITLE - FUNCTICN RNLCS(T,SL)

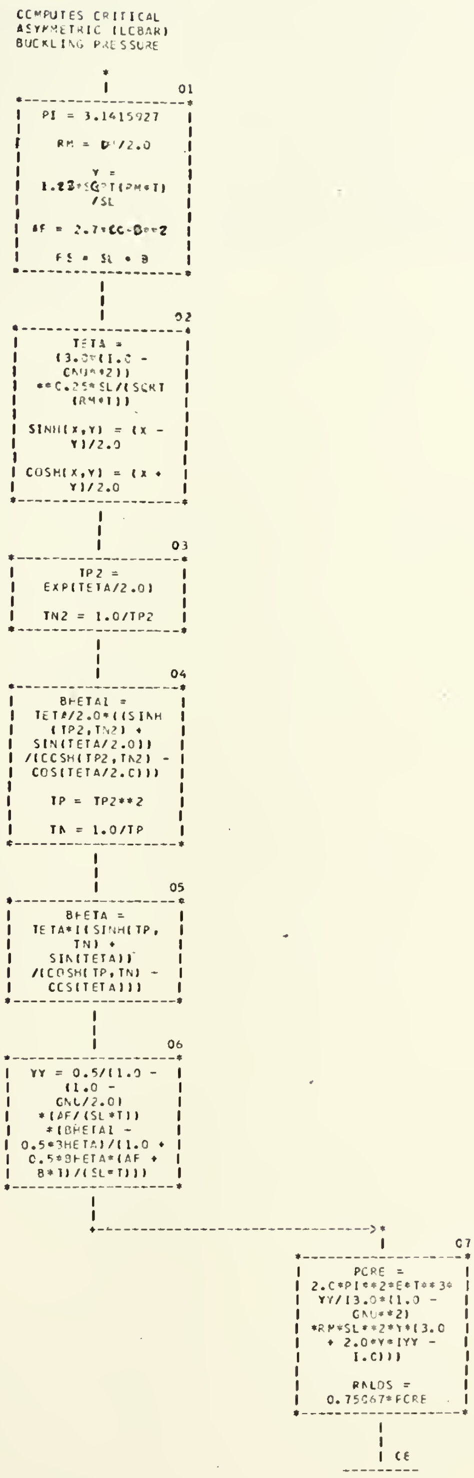


Figure 8

in computer time.

Function RMLDS, in this case for elastic-perfectly plastic material, predicts strictly an elastic type failure. Thus, the full pressure reduction from REDPR (see chapter three) is employed directly to compensate for imperfections and residual stresses.

CHAPTER FIVE

AXISYMMETRIC (YIELD) FAILURE

The real core of the optimization program is the axisymmetric failure mode, for the program designs its scantlings to fail by yield (see chapter one). Three subprograms in addition to FRAME, PULOS, and REDPR are included in this grouping: PLNCK, ELNCK and THKNS (see figures 9, 10, and 11).

The rather famous Von Sanden and Gunther formulas 92 and 92A (utilizing the maximum shell stress theory of Rankine at the frame and midbay, respectively) were used in design for many years. Recently, however, many more solutions have appeared. With the advent of S-P stresses[18], the stress analysis alone has improved in accuracy. The manner in which the stresses are used to predict collapse by axisymmetric yield varies greatly. Generally speaking, the maximum strain energy theory of Mises and Hencky provides the best manner of stress combination to predict failures within the various shell yield formulations. The point at which this is applied is also subject to discussion. Although it is generally agreed that the largest stresses actually occur at the frames, it is becoming evident that data indicate the best predictors use the mid-bay area as

the initiation point of yielding failures. The most frequently accurate analysis for axisymmetric yielding failure is that due to Lunchik[14]. Although his formulation has not been tested through complete ranges of geometries and depths, the tests that have been made indicate his solutions are at least as accurate as any others (to within 1% of actual failure pressure in many cases), and much better than 92 or 92A. In reference 14, Lunchik shows that very successful correlations were obtained with tests of ring-stiffened cylinders ranging from λ (thinness ratio)=0.41 to λ=0.70. He recommended his formulation, however, only for "cylinders where geometries are in the range of axisymmetric yielding", precisely the case in this program. Basically, Lunchik assumes a standard three-hinge failure mechanism, postulating that the frame plastic hinges fail first, and predicting the pressure at which the mid-bay hinge is complete. The basic difference between Lunchik's analysis and others is his computation of "plastic reserve strength". His structure does not fail when some outer hull fiber at mid-bay has reached yield stress. It fails only after this plasticity has progressed through the shell at that point far enough to produce a hinge and precipitate failure.

FUNCTION PINCA The use of Function REDPR (see chapter three) requires both an elastic and an inelastic collapse

CHART TITLE - FUNCTION PLNCK(T,SL,FT,PC)

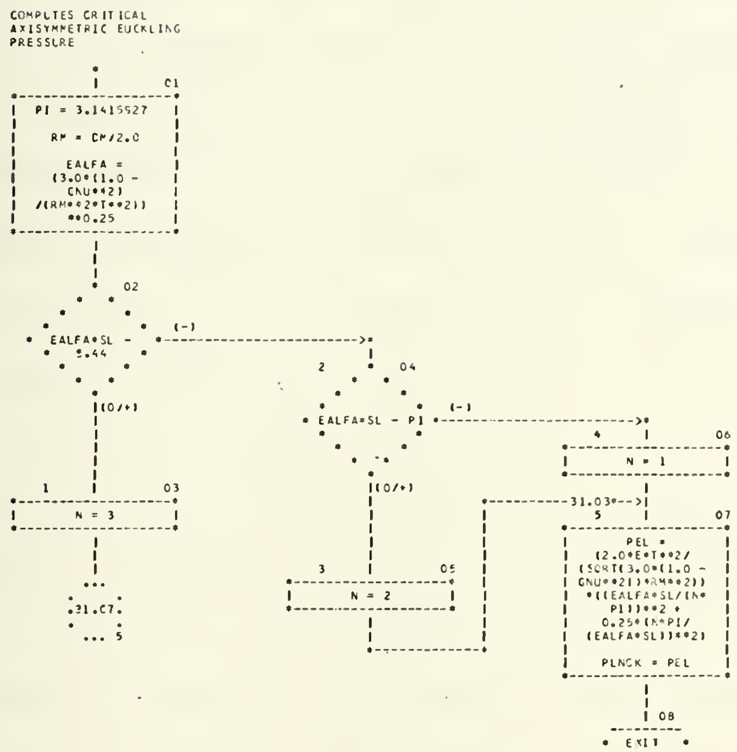


Figure 9

pressure for the particular failure mode under examination. For the elastic-perfectly plastic materials, there is no "inelastic failure pressure" as such, since no strain-hardening is involved. Thus, the yield failure pressure outlined above is substituted. For the solution of the elastic axisymmetric failure pressure, there remain two possibilities. One is the "exact" solution offered by the S-P stress analysis. When GAMA $\geq$ 1.0 the elastic axisymmetric mode occurs. However, the solution for this pressure is bound up in a new "F function" and requires a rather involved iteration. Because the REDPR method is approximate in any case, an exact elastic axisymmetric value is not required. Thus, the S-P solution was rejected in favor of Lunchik's inelastic axisymmetric analysis[13]. This analysis is very similar to Reynolds' analysis for asymmetric buckling (see chapter four), in that it can be applied in the strain hardening (inelastic case) using the secant and tangent moduli. By setting ETAN=0 and ESEC=E, the solution breaks down to one for elastic axisymmetric buckling. This is the pressure computed by Function PLNCK, and the process is taken directly from reference 13.

SUBROUTINE ELNCK The "inelastic" pressure used in REDPR (and also the pressure to be reduced itself) is computed in this subroutine. As explained above, Lunchik's

CHART TITLE - SUBROUTINE ELNCK(T,SL,F1,PC,P)

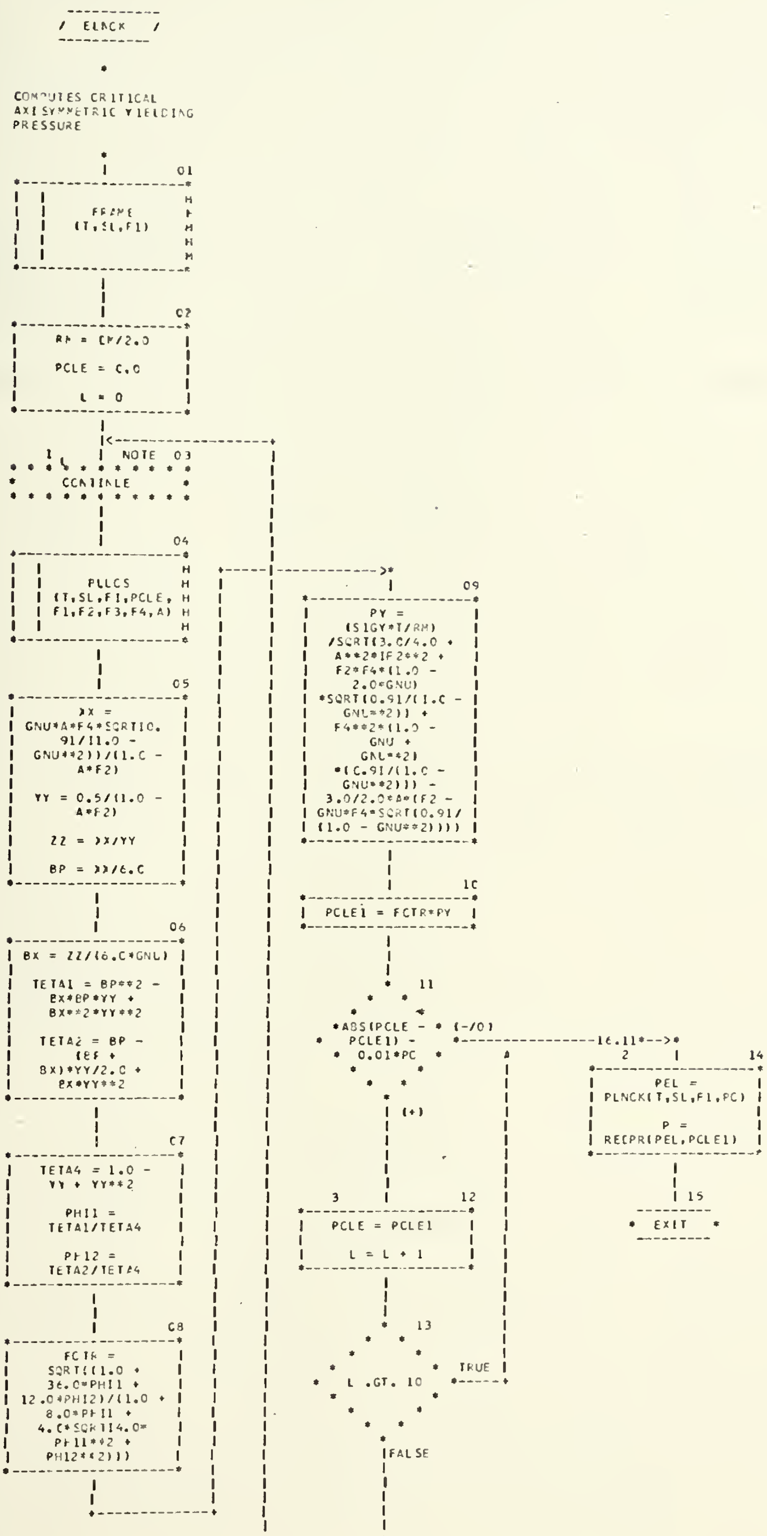


Figure 10

yield analysis for elastic-perfectly plastic materials[14] is used. S-P stresses are used in the formulation (see figure 10). This, of course, meant an iterative process, and occasionally convergence problems were encountered. If input geometries were satisfactory, convergence was accomplished in four or five cycles. Occasionally, the iterative program (THKNS) calling ELNCK jumped outside the range of convergent geometries in its search process. For such cases, an iteration limit of 10 was put into ELNCK. This meant that the pressure going back to THKNS was not entirely accurate, but good enough to continue with a search pattern to find a convergent geometry. An additional "safety valve" was built into Subroutine PULOS (see chapter two) to prevent GAMA $\geqslant$1.0 and thus producing imaginary values. If GAMA$\geqslant$1.0, GAMA was set equal to 0.95, and the shell thickness adjusted to achieve this. Thus, depending on entering geometries, occasionally shell thickness itself is adjusted within ELNCK. This is justifiable, in that the overall program is designing to a yield, not a buckling failure. Any buckling geometry, even if it could be converged upon, would not be desired. Lunchik's "yield pressure", PY (reference 16, equation 35) is the result of the S-P stresses obtained, and is the pressure at which yielding begins at the outer hull fiber, at mid-bay. His

plastic reserve strength ratio, PCLE1/PY (FCTR) is then computed and multiplied by PY to obtain Lunchik's final collapse pressure, PCLE1. The arrival at certain of Lunchik's equations, made somewhat confusing by a misprint in reference 14, is done in detail in Appendix B. Before sending the collapse pressure to THKNS, it is reduced for residual stresses and manufacturing defects by REDPR.

At this point (see figure 12), it is interesting to see, at least in one case, how the Lunchik and Reynolds analyses compare with the log-log plot of hoop stress vs. the Windenburg-Trilling formula presented in reference 8. For the particular hull diameter chosen, Reynolds' pressures follow Windenburg's almost exactly. Lunchik's pressures show hoop stress, at least in this case, to be rather conservative.

SUBROUTINE THKNS This subroutine uses ELNCK in an iterative process to converge on an exact shell thickness which will fail by axisymmetric yield at the desired collapse pressure. If THKNS cannot converge on a thickness, it is obvious that the input scantlings, despite changes in thickness, are such that failure by elastic axisymmetric buckling occurs before axisymmetric yield failure. Since the full strength of this metal is not then being used, this type of solution obviously is not desired. In such

CHART TITLE - SUBROUTINE THKNS(TT,SL,F1,PC,*)

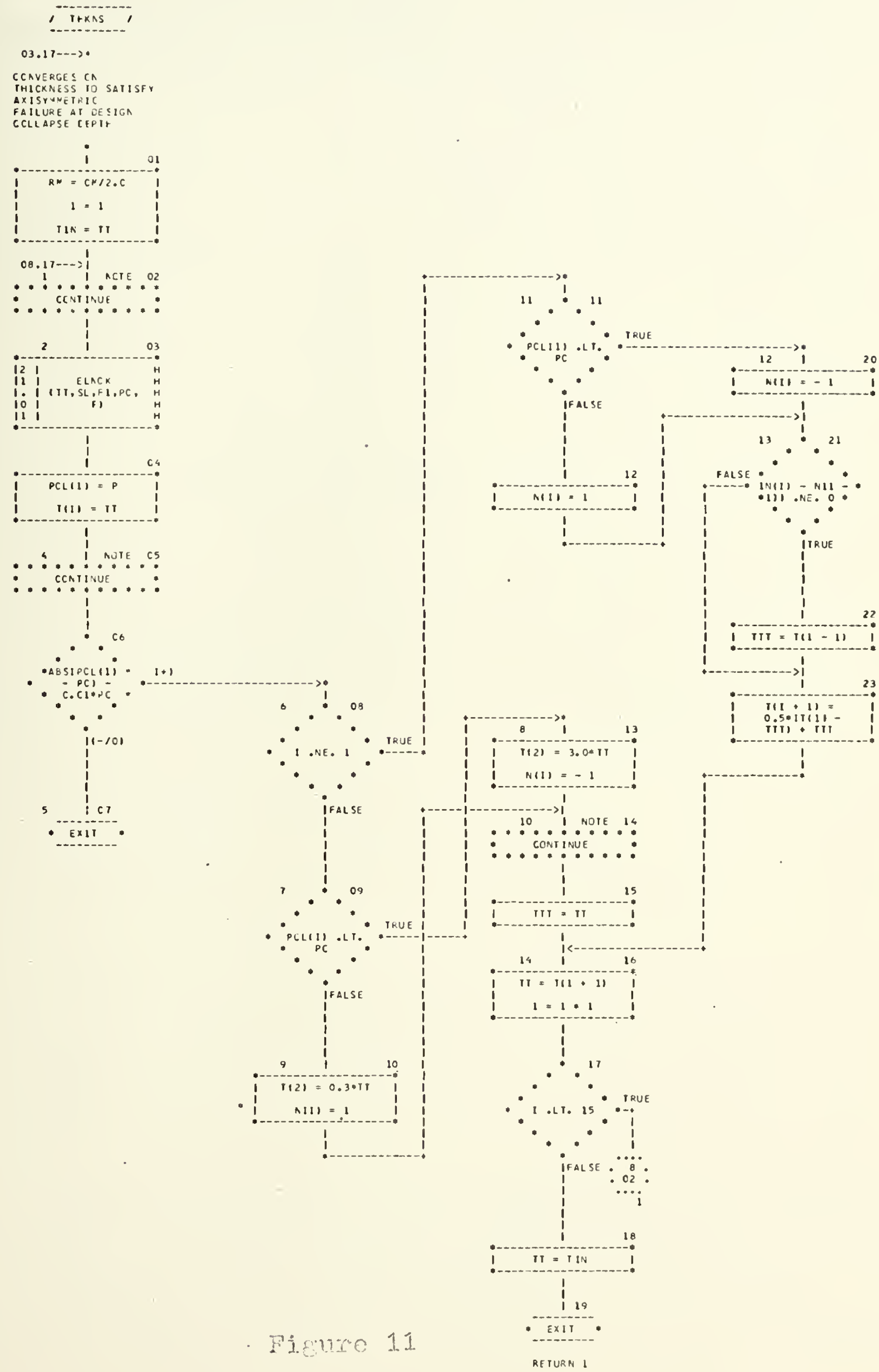


Figure 11

a case, THKNS transfers control to the next iteration loop of the main program (see chapter ten).

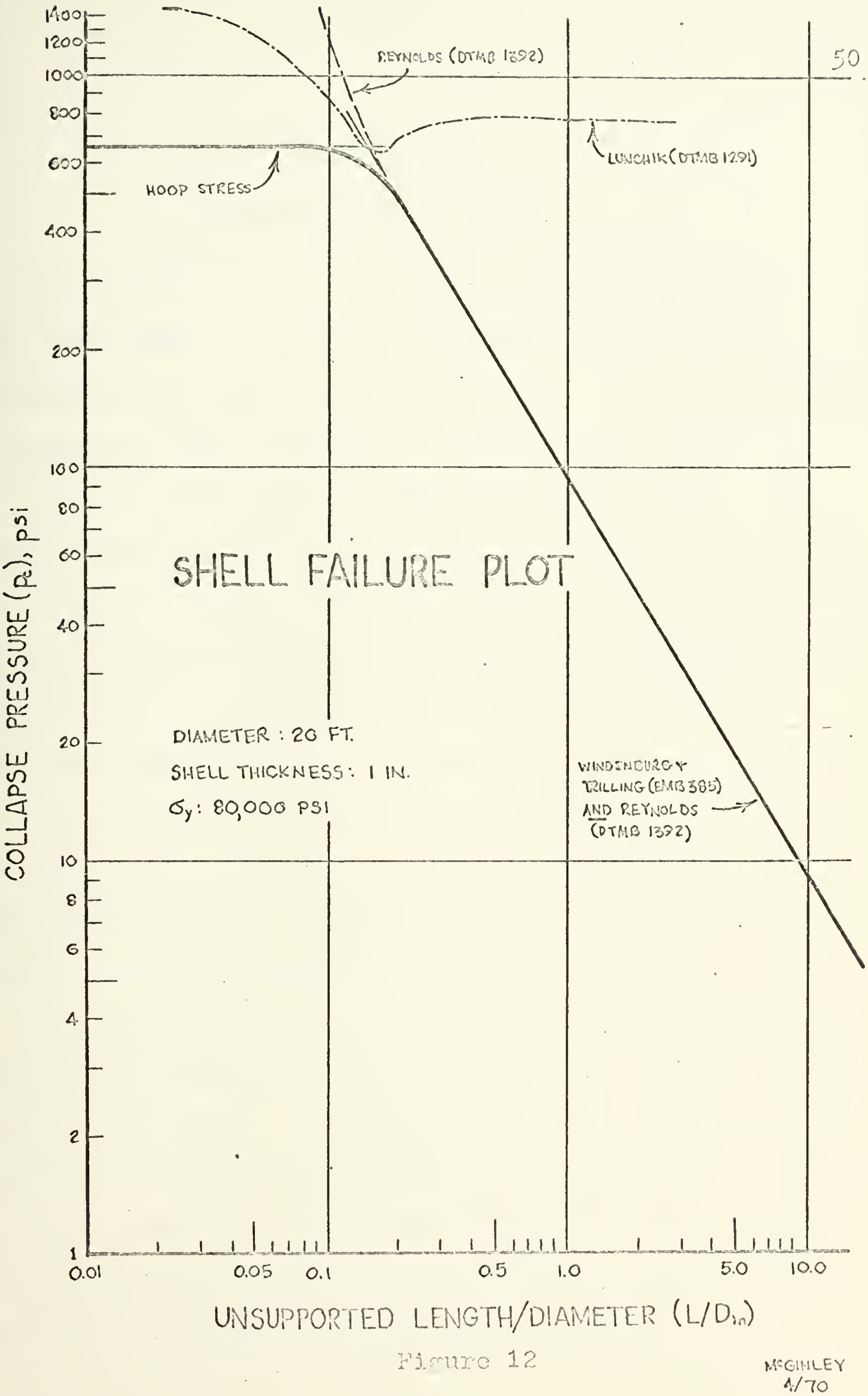

SHELL FAILURE PLOT
COLLAPSE PRESSURE (Pc), psi
UNSUPPORTED LENGTH/DIAMETER (L/Din)
REYNOLDS (DTMB 1392)
LUNCHIK (DTMB 1291)
HOOP STRESS
DIAMETER : 20 FT.
SHELL THICKNESS : 1 IN.
σy : 80,000 PSI
WINDENBURG & TRILLING (EMB 385) AND REYNOLDS (DTMB 1392)
1400
1200
1000
800
600
400
200
100
80
60
40
20
10
8
6
4
2
1
0.01
0.05
0.1
0.5
1.0
5.0
10.0
McGINLEY 4/70

Figure 12

CHAPTER SIX

HULL THICKNESS INCREASE

Generally speaking, the safety factor introduced by REDPR (see chapter three) to compensate for "imperfections and residual stresses" will be adequate. However, it should be remembered that the collapse pressures developed thus far all are "triggered" by stress values at midbay. The highest stresses actually encountered usually occur at the frame faying "flange". To account for this, stresses in that area are limited to 75% of yield stress at operating depth (assumed here to be 2/3 collapse depth) by subroutine STRTHK, as recommended in reference 10. This is done to account for such things as low-cycle fatigue, creep, stress corrosion, and to insure a reasonable stress level in the frame flange prior to collapse for those frames with an initial out-of-roundness[10].

Subroutine STRTHK (see figure 13) uses the S-P stress analysis[18] to compute all four stresses of interest in the shell at the frames: inner and outer plate surfaces, and longitudinal and circumferential stress directions. If the largest of these is greater than 75% yield stress at 2/3 collapse pressure, shell thickness is increased in increments of 5% until the criterion is met.

CHART TITLE - SUBROUTINE STRTHK(T,SL,FI,PC)

Figure 13

CHAPTER SEVEN

GENERAL INSTABILITY

The problem of finding accurate collapse pressures in the general instability mode has generally given analysts more trouble than the first two failure modes. The most accurate elastic general instability failure analysis was done by S. Kendrick in 1953 at Britain's Naval Construction Research Establishment, and is generally known as the "Kendrick Part III" solution[3]. One year later, A. R. Bryant, working in the same establishment, developed a far simpler approach[7] (Kendrick's was exceedingly complex). Bryant's solution was a single two-term equation which could be (and has been) used for design studies without extensive computerization, although its accuracy left something to be desired (it was non-conservative). Basically speaking, Bryant's equation incorporates the "split rigidities concept", where one term represents the contribution of the shell, and the other the contribution of the frames and a frame-length of shell plating. Although Kendrick's solution was put in a simple graphical form by Reynolds[19] (later extended by Ball[3]), it was rejected for use in this program for two reasons:

1) since the general instability pressure is used merely to check the solution designed for yield

failure, extreme accuracy was not required; and

2) Kendrick's solution is good only for elastic failure, and cannot be applied to strain-hardening materials. Since this program is designed to be easily converted to one which can also handle strain-hardening materials, Kendrick Part III was rejected.

Bryant's solution was also rejected for the second reason above. More recently, a very convenient and more accurate formula has been developed by Krenzke and Kiernan[11]. It is very similar in structure to Bryant's formula, but can be used for either ideally plastic or strain-hardening materials. At about the same time, a very similar solution was worked out by Lunchik at the Model Basin. Both the Krenzke-Kiernan and the Lunchik analyses give about the same results when compared with actual test data[6]. Krenzke-Kiernan was somewhat arbitrarily selected for this program, solely because it appeared to be referenced more often in the literature.

<u>SUBROUTINE KRNZK</u> This subroutine is called by the general instability subprogram, and essentially computes failure pressure using the Krenzke-Kiernan formula. It also returns the value of "N" (number of circumferential collapse lobes) which gives the lowest (most critical)

CHART TITLE - SUBROUTINE KRNZK(T,RM,FS,EI,RO,RCG,GAMMA,ESEC,ETAN,PCG,N)

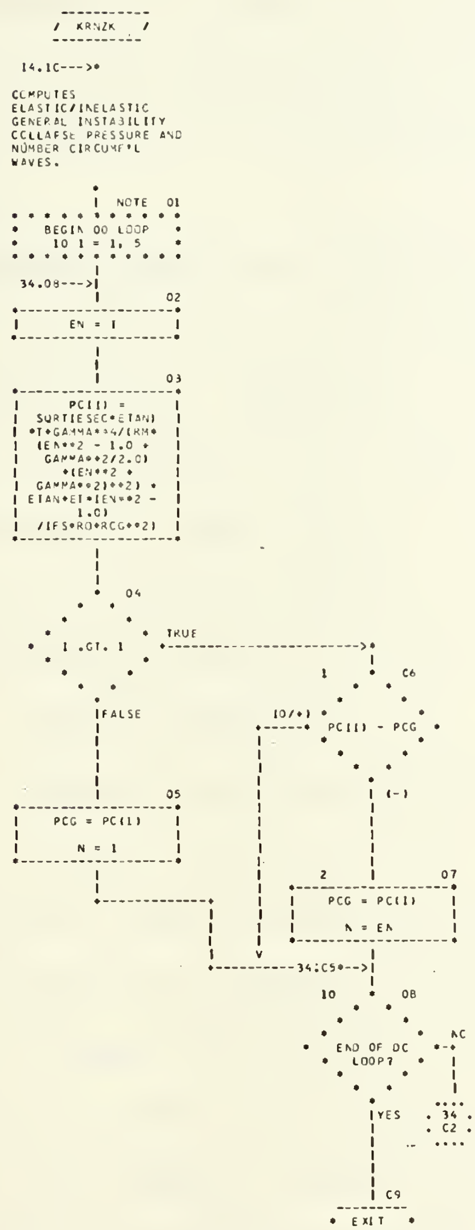


Figure 14

failure pressure. It might be noted that this program, like some others in the optimization, is designed for dual usage: by ideally plastic or by strain-hardening materials (see figure 14).

SUBROUTINE GINST (see figure 15) GINST tests the incoming scantling set from the main program for failure by general instability. Depending on whether the collapse pressure is too shallow (test failed) or too deep (design too conservative), heavy frame (i.e., bulkhead) spacing is adjusted so as to give a collapse pressure that is either:

1) greater than 1.05 times desired collapse depth (this requirement for a small safety margin is due to the uncertainties of general instability design, and comes from reference 10), or
2) less than 2 times desired collapse depth.

If, in this process, bulkhead spacing becomes longer than the entire pressure hull itself, BS is set equal to the hull length (HULNTH), and the requirement for heavy frames is dropped (see chapter eight on Subroutine HVYFRM). Incoming bulkhead spacing may either be declared or undeclared in the input data. If undeclared, starting bulkhead spacing is taken as approximately twice the hull diameter, but always a multiple of the small frame spacing.

The frame term in the Krenzke-Kiernan formula contains,

CHART TITLE - SUBROUTINE GINST(T,SL,FI,FC,PCG,N,BS,L,FULNTF)

```
                       -----------
                      /  GINST   /
                     -----------

                   04.04*-->*

                  COMPUTES GEN. INSTAB.
                  PRESSURE AND HEAVY
                  FRAME SPACING

                          *
                          I          01
                  *--------------------*
                  I  PI = 3.1415927    I
                  I                    I
                  I    RM = DM/2.0     I
                  *--------------------*
                          I
                          I
                          I          02
                  *--------------------*
                  I  I                 H
                  I  I    FRAME        H
                  I  I  (T,SL,FI)      H
                  I  I                 H
                  I  I                 H
                  *--------------------*
                          I
                          I
                          I          03
                  *--------------------*
                  I   FS = SL + B      I
                  *--------------------*
                          I
                          I
                          I
                          *    04
                        *   *
                      *       *
              TRUE  *           *
             *-----*  BS .NE. 0.0  *
             I      *           *
             I        *       *
             I          *   *
             I            *
             I            IFALSE
             I            I
             I            I
             I            I
             I            I
             I            I          05
             I    *--------------------*
             I    I       BS =         I
             I    I 2.0*(AINT(DM/FS)   I
             I    I    + 1.0)*FS       I
             I    I                    I
             I    I   PCGL = 0.0       I
             I    I                    I
             I    I     LL = 2         I
             I    *--------------------*
             I            I
             *----09.04*-->I
                    10    I   NOTE   06
                  * * * * * * * * * * *
                  *     CONTINUE      *
                  * * * * * * * * * * *
                          I
                          I
                          I          07
                  *--------------------*
                  I GAMMA = PI*RM/BS   I
                  I                    I
                  I   PCGE = 0.0       I
                  *--------------------*
                          I
                  10.12--->I
                      I   I   NOTE   08
                  * * * * * * * * * * *
                  *     CONTINUE      *
                  * * * * * * * * * * *
                          I
                          I
                          I          09
                  *--------------------*
                  I  I                 H
                  I  I    PULCS1       H
                  I  I (T,SL,FI,PCGE,  H
                  I  I FI,F2,F3,F4,A,  H
                  I  I     &99)        H
                  *--------------------*
                  I ALTERNATE RETURN   I
                  *--------------------*
                  I  99        10.01   I
                  *--------------------*
                          I
                          I
                          I          10
                  *--------------------*
                  I  EL = SL*FI + B    I
                  *--------------------*
                          I
                          I
                          I
                         ...
                        .   .
                        .10.02.
                        .   .
                         ... 98
```

Figure 15 a

CHART TITLE - SUBROUTINE GINST(T,SL,FI,PC,PCG,N,BS,L,HULNTH)

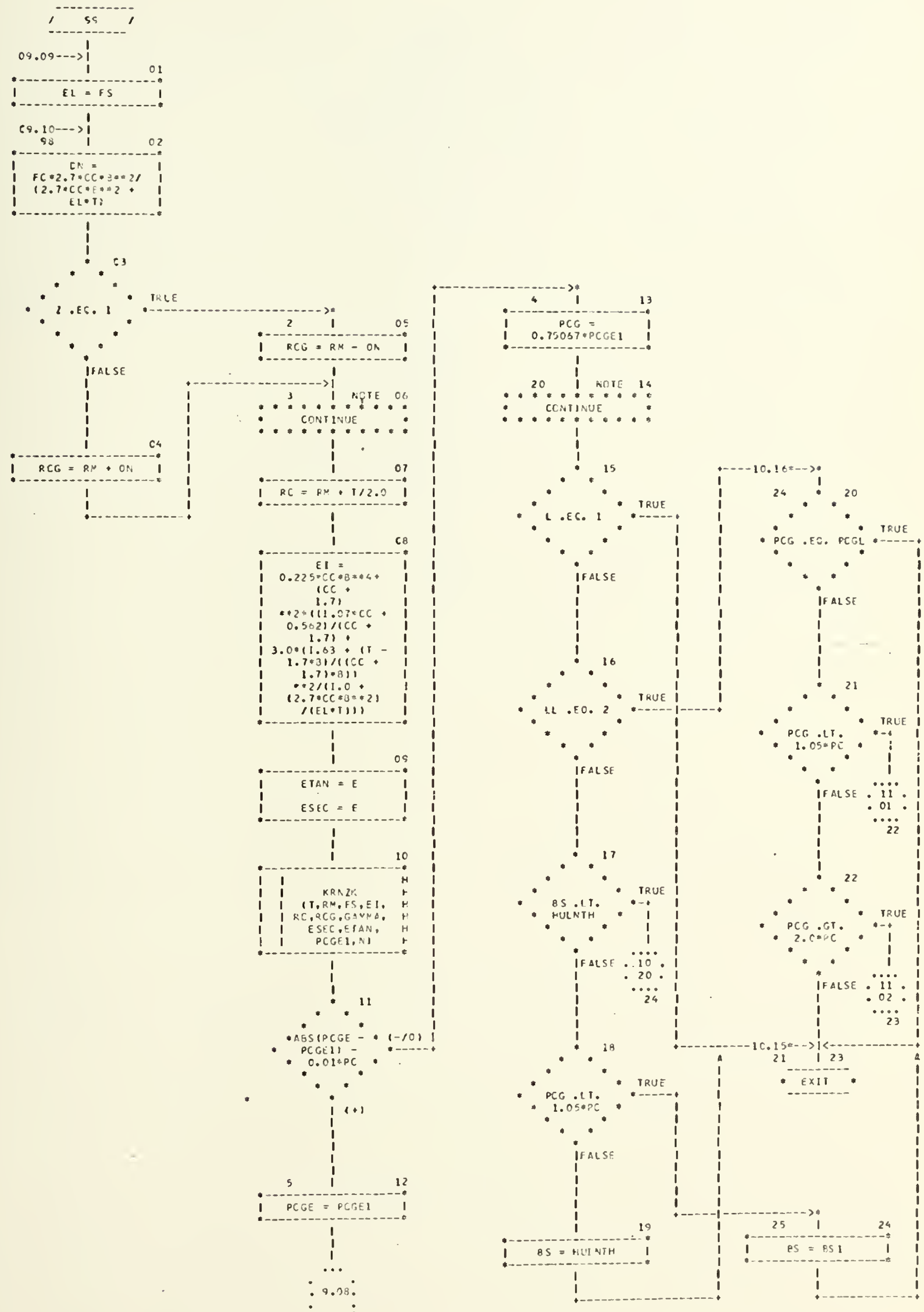


Figure 15 b

CHART TITLE - SUBROUTINE GINST(T,SL,FI,PC,PCG,N,BS,L,HULNTH)

```
           -----------
          /    22    /
          -----------
               |
   10.21--->|
               |              01
   *-------------------------*
   |     BS = BS - FS        |
   |                         |
   |     PCGL = PCG          |
   |                         |
   |      LL = 2             |
   *-------------------------*
               |
               |
               |
              ...
            .     .
            . 9.06.
            .     .
              ... 10


           -----------
          /    23    /
          -----------
               |
   10.22--->|
               |              02
   *-------------------------*
   |      BS1 = BS           |
   |                         |
   | BS = BS + 2.0*FS        |
   |                         |
   |      LL = 1             |
   |                         |
   |     PCGL = PCG          |
   *-------------------------*
               |
               |
               |
              ...
            .     .
            . 9.06.
            .     .
              ... 10
```

Figure 15 c

as one of its variables, the "effective moment of inertia", EI. This is defined as the combined moment of inertia of a small frame and its effective length, EL, of shell plating. This differs slightly from the Bryant formula, which merely uses a frame space (FS) of shell plating. The EL is determined via the S-P stress analysis[18], using the equation:

$$EL=SL*F1+B$$

The use of either EL or FS in the formula makes little difference in most cases; however, since all the tools were handy, the EL was used when possible. Problems of convergence, however, again developed in the PULOS subprogram. When general instability pressure became very large (and it does for many geometries), the value of GAMA (see chapter two) exceeded 1.0. Thus, a separate subprogram (PULOS1) was added, which set EL=FS when GAMA≥1.0. This is totally acceptable, since this case occurs only when the general instability collapse pressure is far too deep to worry about (its accuracy is only slightly degraded anyway). When successive iterations of GINST expand BS and bring this collapse pressure to shallower depths, EL can again be accurately computed by PULOS1.

CHAPTER EIGHT

HEAVY FRAMES

Subroutine HVYFRM (see figure 16) computes the minimum size of heavy frames needed to insure the general instability collapse pressure calculated by its preceeding subprogram GINST. Until recently, heavy frames were designed using the standard Lèvy formula. It was found after considerable testing, however, that this formula often gave unsafe estimates[17]. A new formula was derived at the Model Basin by Blumenberg in 1965, which agrees much more closely with tests. Results from testing indicated that the effectiveness of a particular size of heavy frames decreases as the cylinder is lengthened and also that the minimum size of heavy frames needed to localize failure between heavy frames is possibly not dependent upon their spacing[5]. Although adequate testing has not yet been published to positively confirm Blumenberg's formula, initial results show it is better than what was formerly used, and thus it was put into HVYFRM:

$$FIH= \frac{PCG*BS*RC**3}{((M**2-1.0)*ETAN)}, \text{ where:}$$

1) E may be substituted for ETAN with ideally plastic materials. ETAN actually is not in Blumenberg's formula, but was placed there by the author to make HVYFRM useful in an inelastic optimization.

CHART TITLE - SUBROUTINE HVYFRM(T,SL,FI,HULNTH,PC,PCG,BS,AFH,BH,AF,FCH)

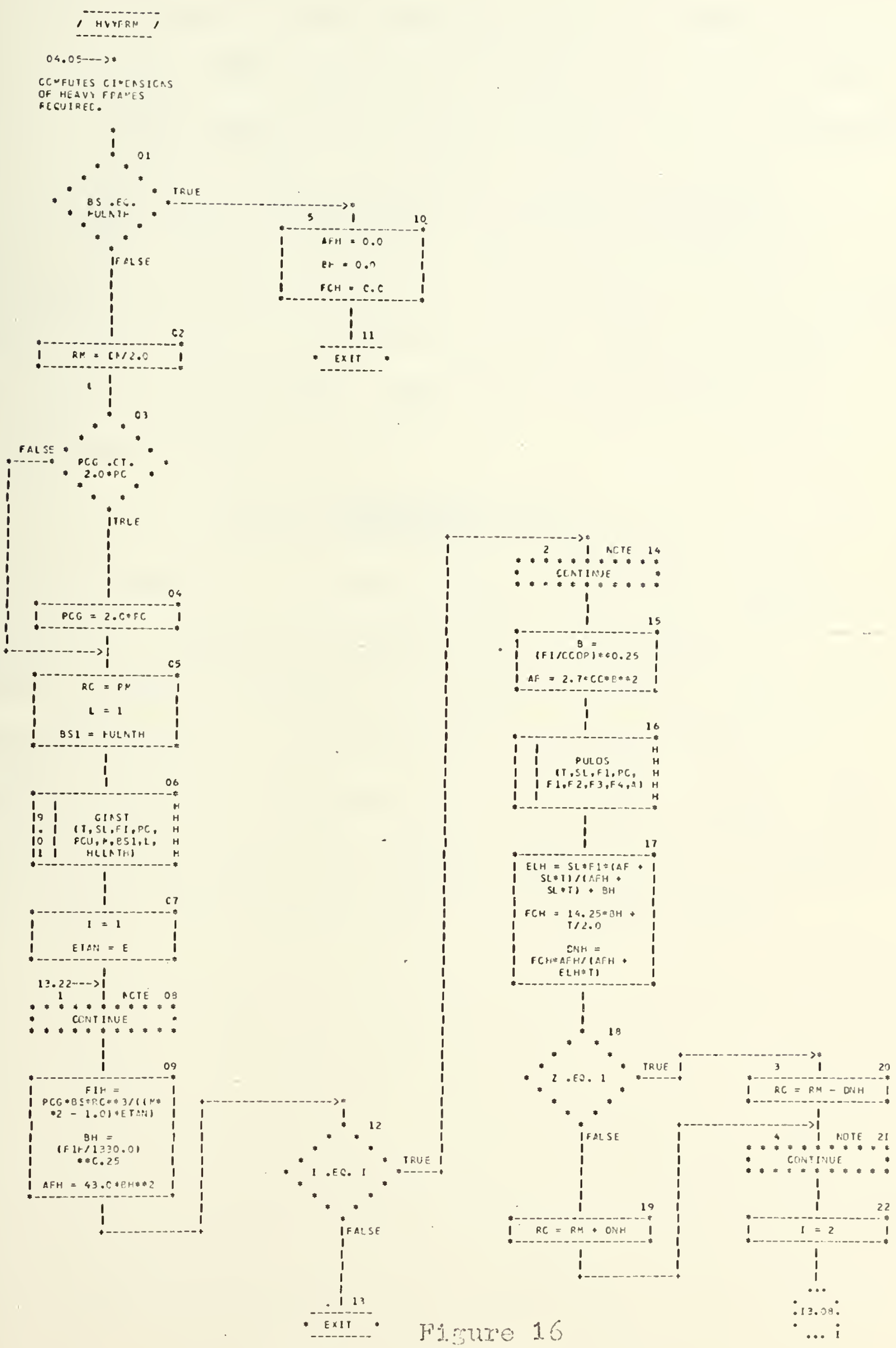


Figure 16

2) M is the critical buckling mode for elastic general instability failure of the cylinder with the heavy frames replaced by typical frames (i.e., a cylinder equal in length, but with no heavy frames).

(see "List of Symbols" for other variables)

In computing RC, the radius to the combined centroid of the heavy frame and its effective length of shell plating, an ELH had to be determined. This was also provided in Blumenberg's report:

$$ELH = \frac{SL*F1*(AF+SL*T)}{AFH+SL*T} + BH$$

It can be seen to be a form of simple ratioing of the original EL formula developed in the S-P analysis[18]. Another feature of HVYFRM is to send a value of zero back to the main program if the heavy frame spacing computed by GINST is equal to (or greater than) the hull length. This, of course, would mean that heavy frames are not required, and that the hull end closures are used in their place.

CHAPTER NINE

WEIGHT-DISPLACEMENT RATIO

The simple weight/displacement ratio was selected for the optimization criterion. Not only was it the simplest to use, but it also seemed to be the most general, all-encompassing determinant of an optimum design. Due to the subprogram system utilized in the optimization, another form of criterion could easily be substituted if the need arose.

Essentially (see figure 17), WTDSP computes the weight of a hull section, the length of which is equal to one heavy frame spacing, splitting the heavy frame on each end in half. This weight is divided by the same hull's displacement in sea water, taking into account internal or external frames.

CHART TITLE - FUNCTION WTOSP(T,SL,FI,BS,AFH,FCH,AF)

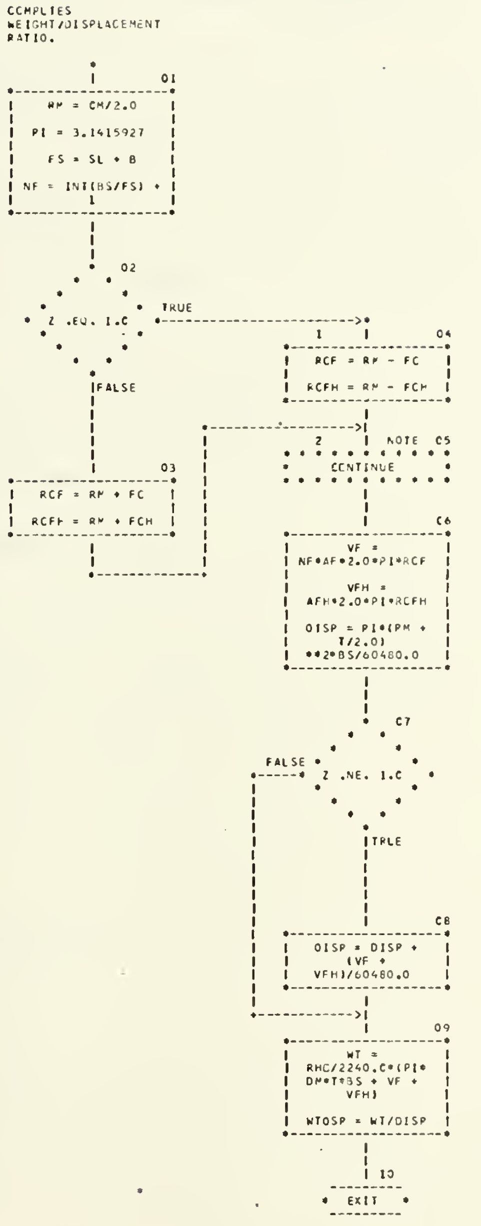


Figure 17

CHAPTER TEN

MAIN PROGRAM

Essentially, the main program performs very few calculations. Its primary task is manipulation of the various subprograms and iteration control of the entire process so as to arrive at an optimum design. See figure 23 for a basic, easier-to-follow main flow diagram. Figure 24 is the complete main program flow diagram. In addition, it computes "reasonable" entering scantlings for the iteration process, based on input data and proven design practice. The objective of this type of main program design was to obtain a system in which the various modes of failure and safety factors could be changed or interchanged easily as desired. As all failure modes are separated into subprograms, it is possible even to substitute an entirely different analysis for any individual failure mode. Safety factors, which so often are subject to modification due to new test data, are also easily replaced or changed.

The input data required (see Appendix D for format) are: overall hull length, bulkhead spacing (may or may not be given; if not, it computes an optimum BS), internal or external framing desired, Young's modulus, metal yield stress, metal density, and Poisson's ratio. In addition, as presently set up, the program may be iterated for various input values

of depth and/or hull diameter.

The depth input (in feet) is converted to a collapse pressure PC (in psi) requirement by using the equation of the mean line drawn in figure 18; this represents an average depth vs. psi curve for the various oceans included. The figure (minus the mean line) was taken from the Handbook of Ocean Engineering Tables, published by the U. S. Naval Oceanographic Office, and compiled by E. L. Bialek.

The frame constant CC is computed from the equation of the curve shown in figure 19. CC is used to determine frame proportions to be used for various depths. The curve is an average of proportions of many ring stiffeners used in present generation submersibles. The method of obtaining frame proportions is printed in the program output (see Appendix D). A more detailed explanation may be found in Appendix A.

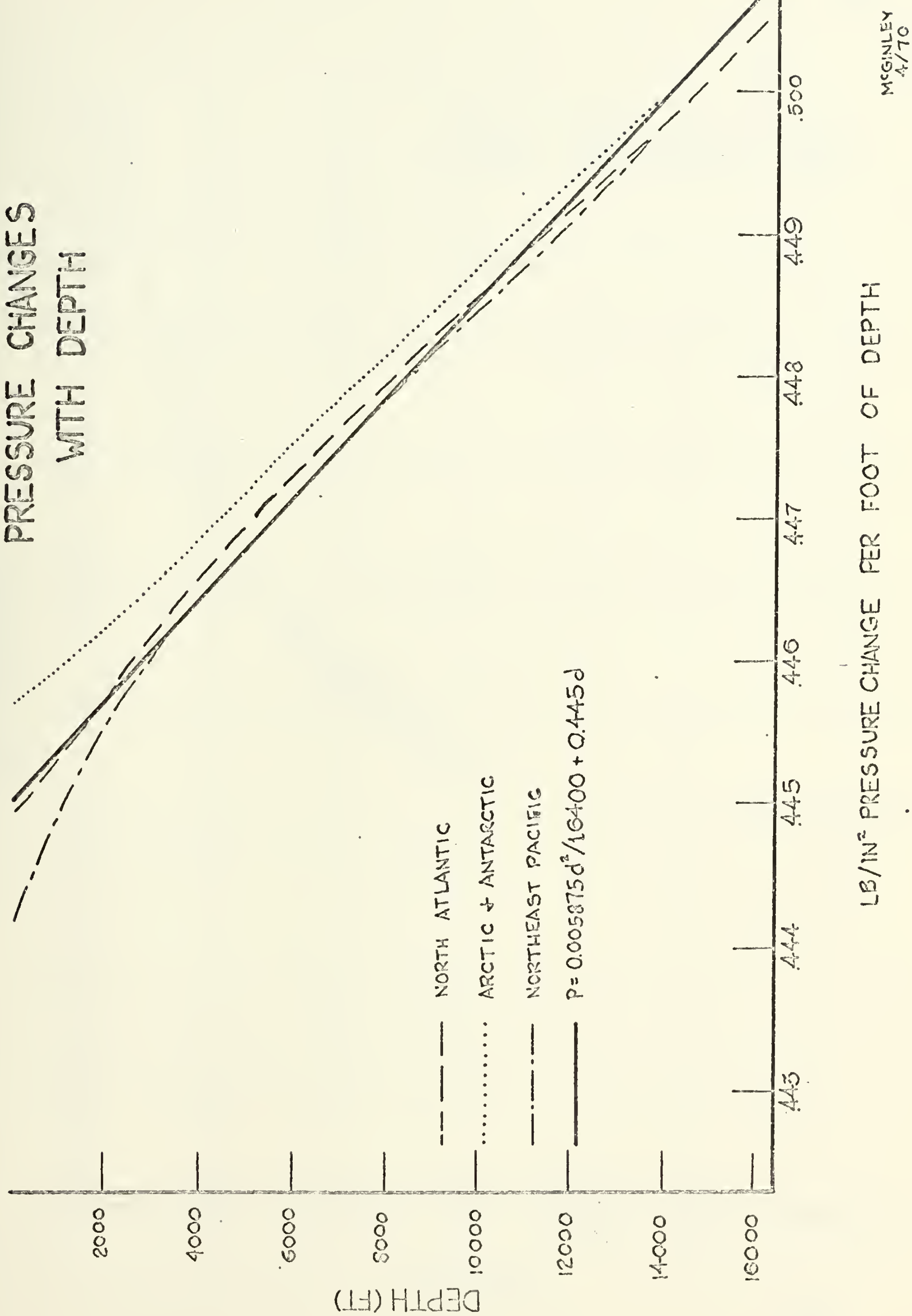
The program next computes its "datum point", or starting set of hull scantlings. Shell thickness, T, is computed from the simple hoop stress formula:

$$T = \frac{PC*DH}{2.0*SIGY}$$

Frame spacing, or, more accurately in this case, unsupported length of shell between frames (SL) is computed from the parameter θ, where:

$$\theta = \sqrt[4]{3(1-\nu^2)}\,\frac{L}{\sqrt{R_m t}}$$

Figure 18



PRESSURE CHANGES WITH DEPTH
NORTH ATLANTIC
ARCTIC + ANTARCTIC
NORTHEAST PACIFIC
P = 0.005875d²/16400 + 0.445d
DEPTH (FT)
2000
4000
6000
8000
10000
12000
14000
16000
.443
.444
.445
.446
.447
.448
.449
.500
LB/IN² PRESSURE CHANGE PER FOOT OF DEPTH
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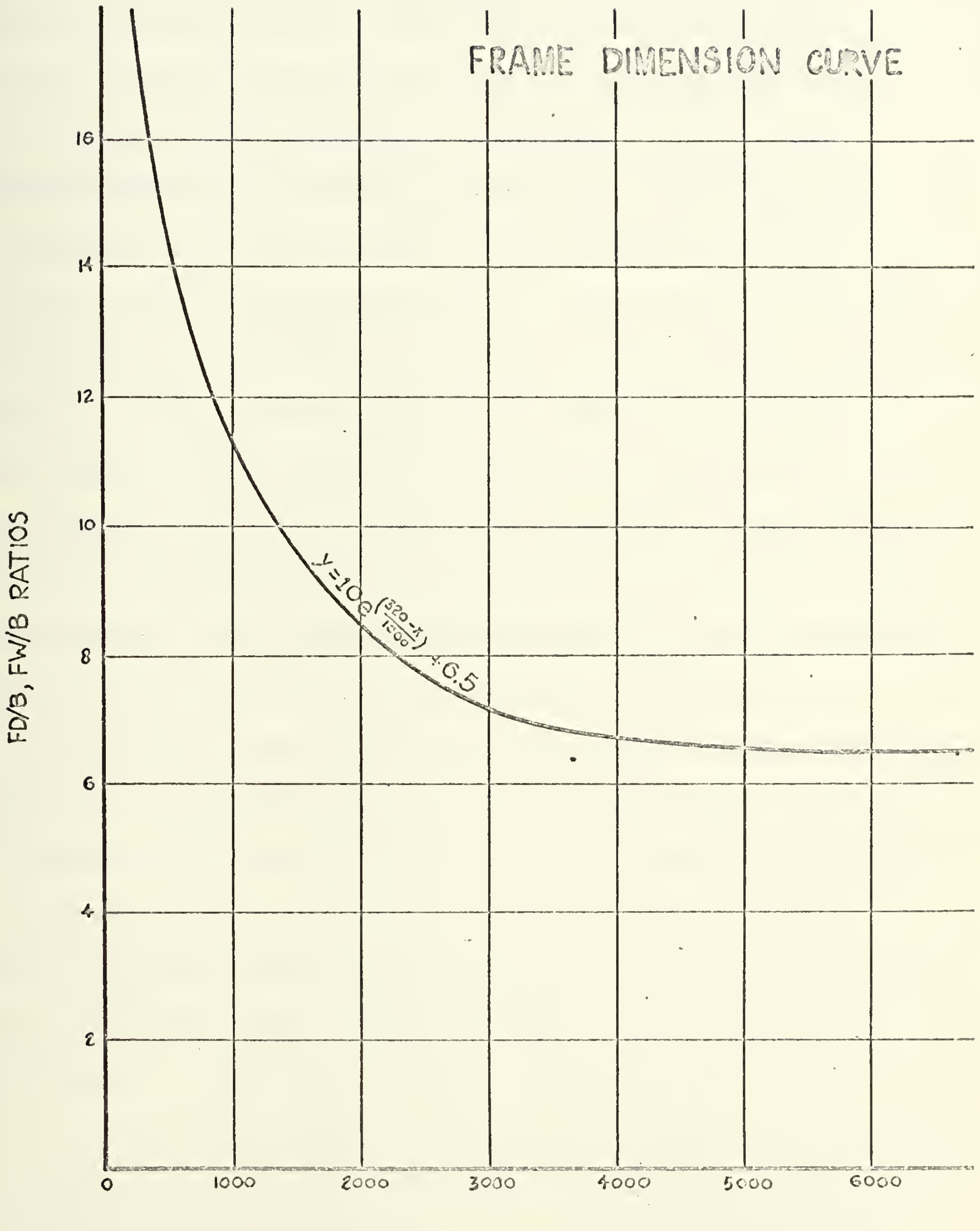

Figure 19

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Reference 6 lists the usual range of θ for present day submersibles as 1.0-2.5. The main program (see below) is set up so that it starts at a maxmium frame spacing and then decreases it in further iterations. Thus, a high starting value of θ would be desirable. After several calculations at various depths and geometries, a value of θ =5.0 was used for the datum point. This gives a wide range of SL values. The iterative process of the program produces values of θ that eventually go low enough to bracket the above range. Other methods of obtaining a starting SL were investigated, such as the combined solution of hoop stress and the Windenberg-Trilling equation, but all gave too wide a range (usually too large a frame spacing) as depth increased. Thus θ , which does not vary greatly, was selected.

A starting frame size is selected based on the ratio of frame cross-sectional area to the cross-sectional area of one unsupported length of shell plating. Reference 6 lists the normal range of this ratio to be 0.2-0.8. During rather extensive investigation, however, it was found that a ratio of greater than 0.5 gave frames that were grossly overdesigned. Thus, the starting ratio was set at 0.5:

AF=0.5*SL*T

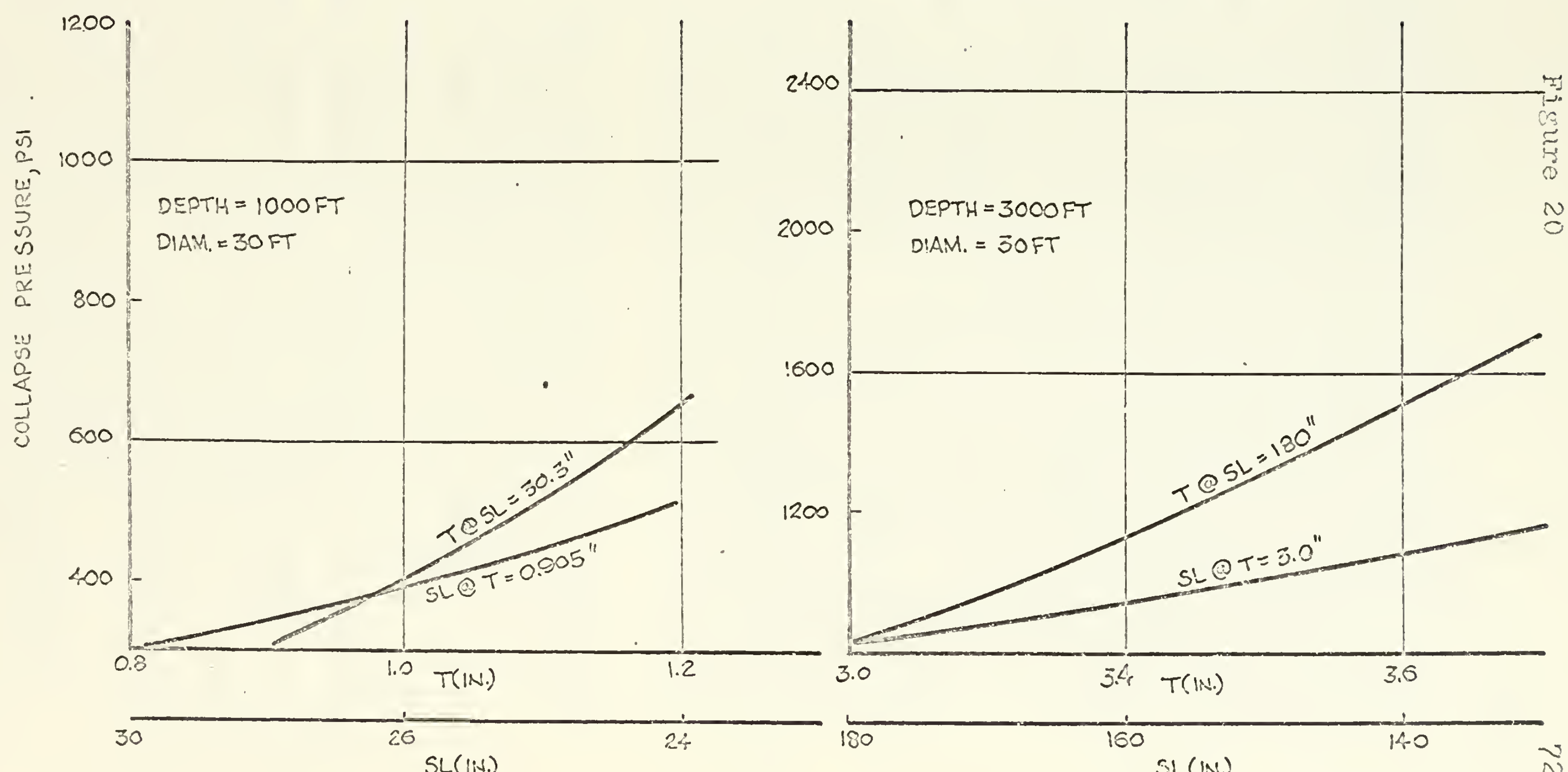
The frame moment of inertia is then computed in accordance with relations developed in Appendix A. Originally, it

was intended to select a starting frame size by a more accurate approach. However, the only formula which could be solved in anything approaching a closed form was far too conservative (i.e., Tokugawa's formula, reference 25). The advantage of a computerized approach, i.e., investigation of a wide range of variables at great speed and low cost, was the deciding factor in using the more random iterative method described above.

At this point the program goes into a double loop, iterating on SL (outer loop) and AF (inner loop). The scantlings are tested by RNLDS. If they do not fail in the lobar buckling mode at design collapse depth, the program goes on to THANS. If the scantlings fail RNLDS, SL is decreased by multiplying it by 0.9, and the RNLDS test is rerun. The decision to run RNLDS at this point was made in order to start as near the "shoulder" (or at least, not to the right of it) of figure 12 as possible, since this is generally acknowledged to be an area of optimum design. Also, the decision to vary SL, rather than T, to achieve a set of scantlings that would not fail the RNLDS test, was made for two reasons:

1) From figure 20, it can be seen that either T <u>or</u> SL could be successfully used to change RNLDS failure pressure, with T being slightly more effective.

RELATIVE EFFECTS OF T & SL ON REYNOLDS' LOBAR BUCKLING COLLAPSE PRESSURE AT TWO DEPTHS



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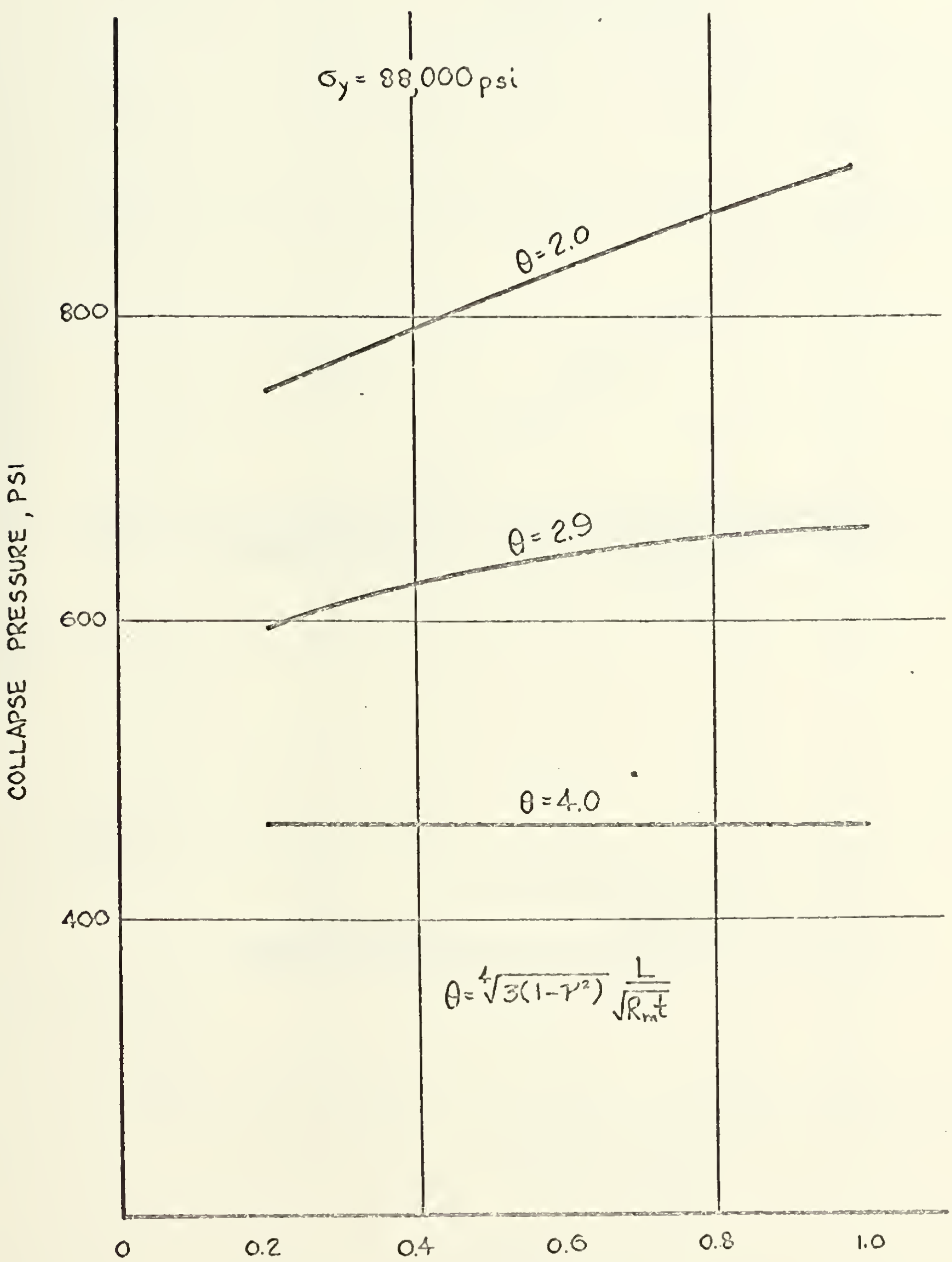
2) The next subprogram in line is THKNS. It is connected to RNLDS via another iteration loop. To have both programs converging on different values of T would cause endless loops in almost every case. Thus, RNLDS was iterated on SL.

The decision not to vary AF to change RNLDS failure pressure is justified by figure 21. The Nott buckling equation is almost identical to RNLDS, and thus can be considered the same for these qualitative investigations. It may be noted that for some geometries, AF has no effect at all on PCR; thus, AF was not used in the RNLDS loop. This same reasoning (see figure 22) was used in deciding against using an AF variation of any kind for convergence within the THKNS subprogram.

As stated above, once the scantling set (revised or unrevised) gets successfully past RNLDS, it goes on to THKNS. There, the hull thickness, T, is adjusted so as to have the shell fail by axisymmetric yield exactly (within 1%) at design collapse pressure. At that point, the scantlings with revised T are again looped back through RNLDS to insure against lobar buckling, and SL adjusted again as necessary. If SL has to be re-adjusted, the scantlings again go through THKNS. The process continues until either:

1) a set of the same scantlings pass without change through both RNLDS and THKNS, or

EFFECT OF FRAME SIZE ON ASYMMETRIC BUCKLING PRESSURE BY NOTT . . .



(NOTE: TAKEN FROM REFERENCE 15)

Figure 21

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EFFECT OF FRAME SIZE ON AXISYMMETRIC (YIELD) FAILURE PRESSURE BY LUNCHIK . . .

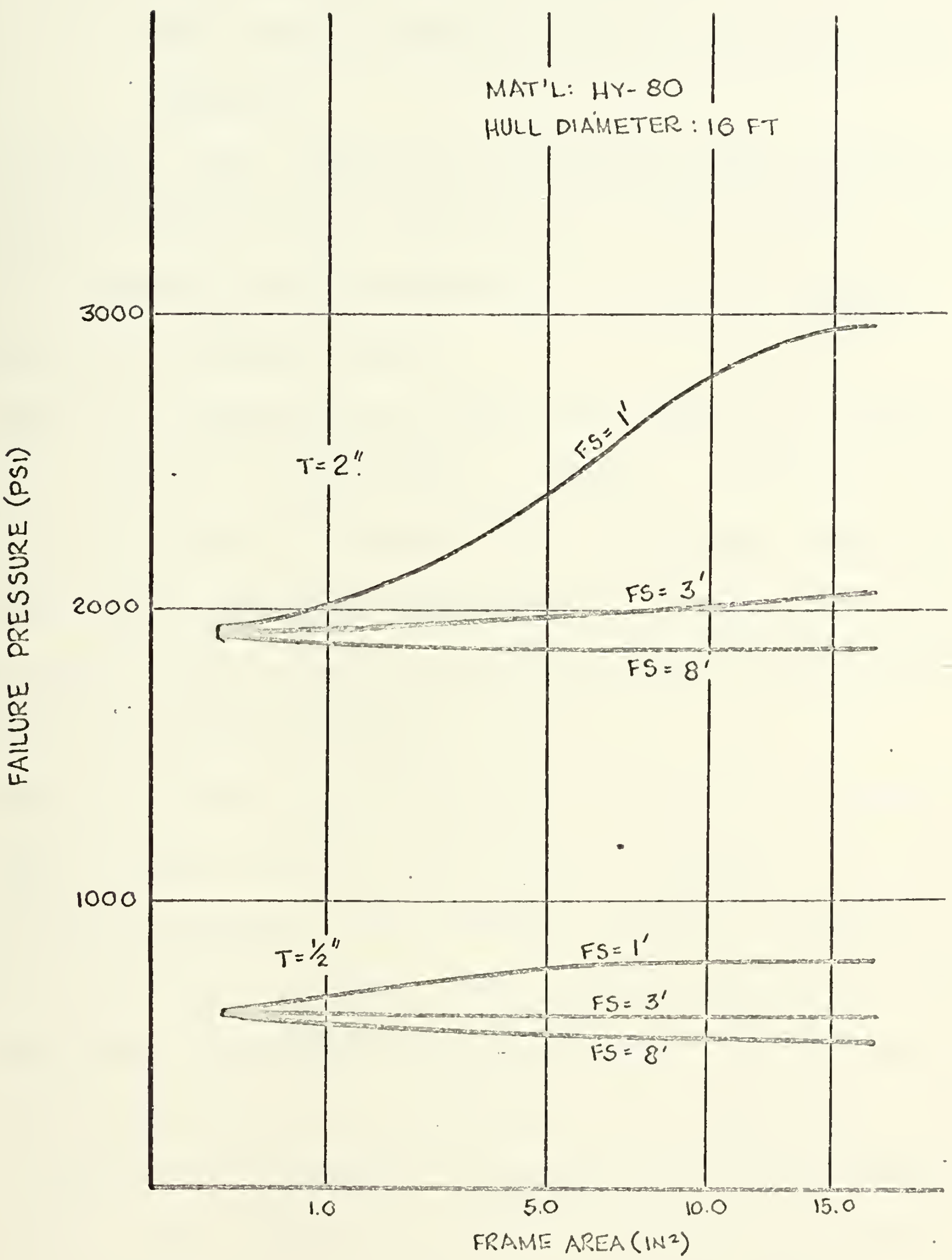

Figure 22

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2) SL becomes so small that there is less than four inches clearance between adjacent frame flanges.

In the latter case, the loop is skipped without print-out, and the next iteration is started, much the same as what occurs if THKNS cannot internally converge on a shell thickness (see chapter six).

Once the scantling set gets past THKNS, T is again adjusted upward, if necessary, by STRTHK. A loop to re-test back through RNLDS and THKNS is not used here, and would serve no useful purpose, since T's from THKNS and STRTHK could rarely ever be made to converge (i.e., the largest T from both programs is used).

Prior to entering GINST, the integer L is set equal to 0. This, used as one of the entering arguments for GINST, is a control variable. L=0 insures normal (iterative BS design) operation of GINST. L=1 is usely solely to obtain a single-pass value of M for HVYFRM (see chapter eight).

The scantling set then enters and is tested by GINST. BS is adjusted as necessary to insure PCG falls between 1.05 PC and 2PC, the only exception being if BS becomes greater than total hull length (see chapter seven). The reason that BS, rather than T, SL, or AF, is used here to converge on a satisfactory PCG, is because in virtually any

case, regardless of depth or geometrical proportions, BS has a much smaller effect on hull weight for the same PCG reduction or increase. Since hull weight already had been optimized for shell yield failure (the desired situation), any further T, SL, or AF changes would most probably take the design seriously off the optimum.

Once past GINST, the scantlings pass through HVYFRM to obtain a suitable heavy frame design and WTDSP to calculate that particular scantling set's relative efficiency ratio.

At this point, values of the scantlings are printed out. Generally speaking, the print-out of each line of scantlings can be said to be optimized on thickness of the shell, although certainly other factors (SL, BS, etc.) change as necessary to keep the design on a yield failure basis.

Once out of the first loop cycle, AF is decreased by multiplying by 0.8. This is done for ten cycles, so that the original AF is reduced to 0.108*AF in the last cycle. SL is reduced in the same manner in the outer loop, and as described above, also may be reduced within each iteration as necessary to pass RNLDS. At the end of both loops (100 major iterations) the program terminates for that set of input parameters, and prints out the optimum (i.e., smallest) weight/displacement ratio.

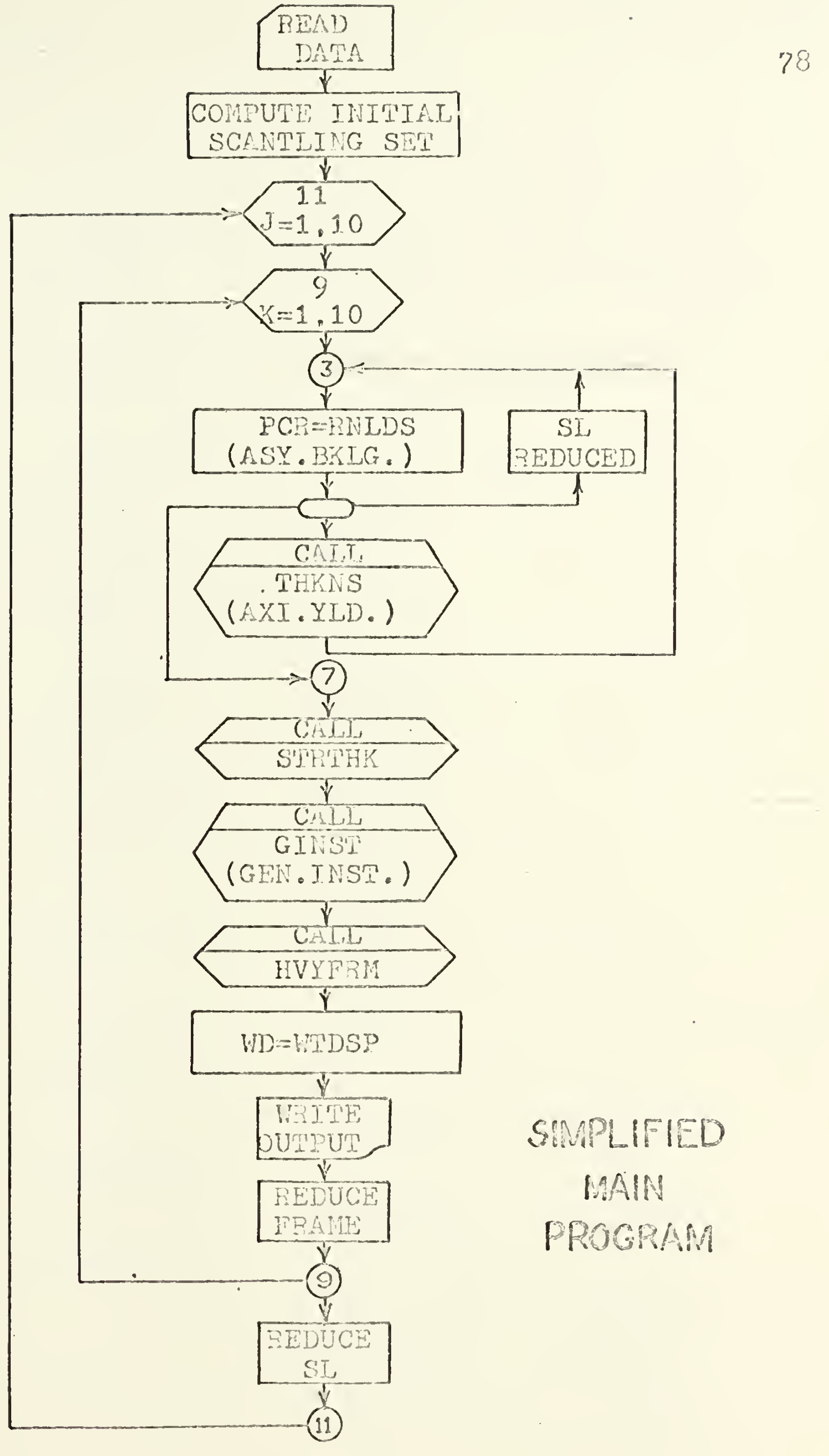
READ DATA
COMPUTE INITIAL SCANTLING SET
11 J=1,10
9 K=1,10
3
PCR=RNLDS (ASY.BKLG.)
SL REDUCED
CALL THKNS (AXI.YLD.)
7
CALL STRTHK
CALL GINST (GEN.INST.)
CALL HVYFRM
WD=WTDSP
WRITE OUTPUT
REDUCE FRAME
9
REDUCE SL
11
SIMPLIFIED MAIN PROGRAM
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Figure 23

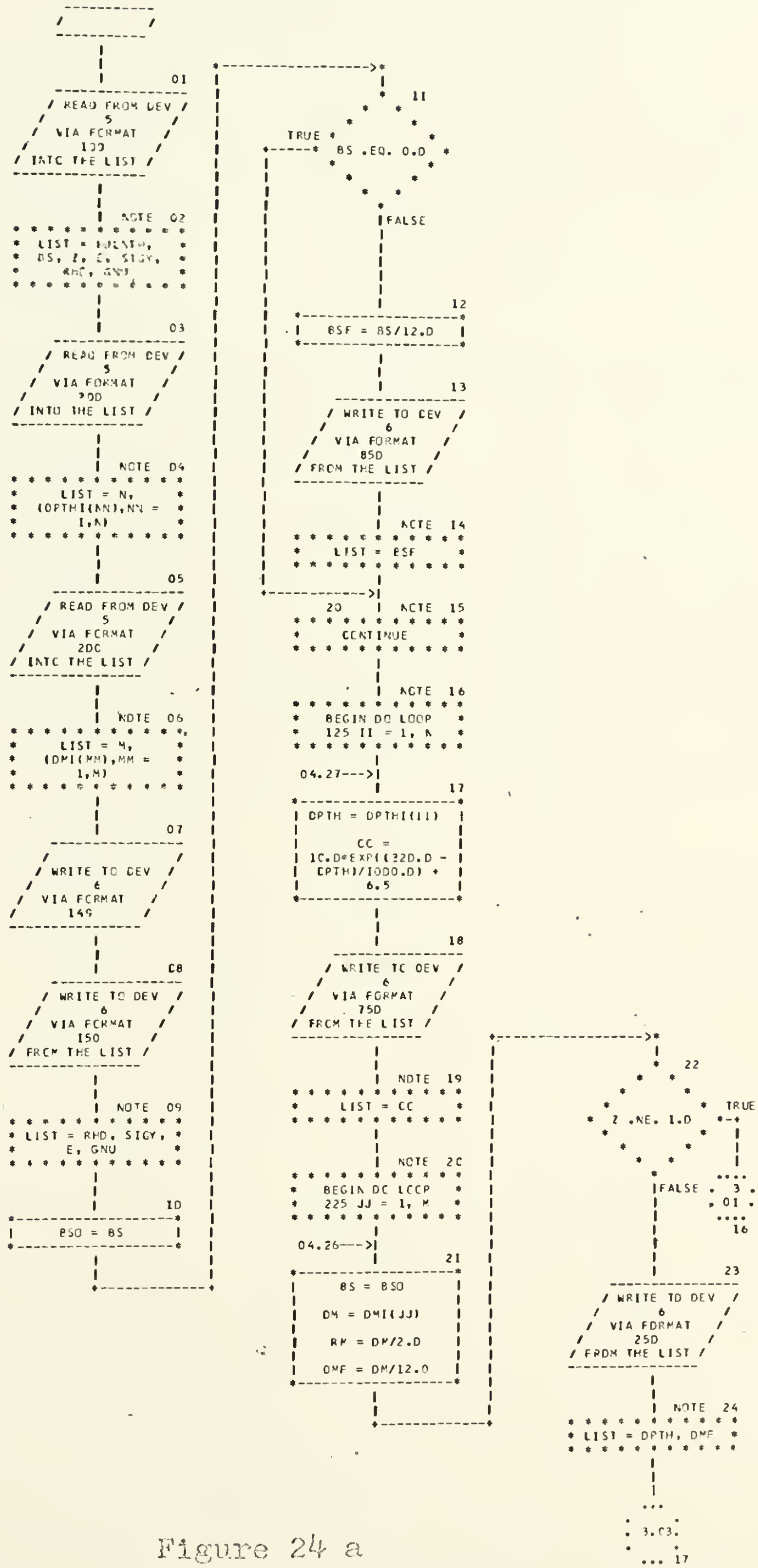
01
READ FROM DEV 5 VIA FORMAT 100 INTO THE LIST
NOTE 02
03
READ FROM DEV 5 VIA FORMAT 200 INTO THE LIST
NOTE 04
LIST = N, (DPTHI(NN),NN = 1,N)
05
READ FROM DEV 5 VIA FORMAT 200 INTO THE LIST
NOTE 06
LIST = M, (DMI(MM),MM = 1,M)
07
WRITE TO DEV 6 VIA FORMAT 149
08
WRITE TO DEV 6 VIA FORMAT 150 FROM THE LIST
NOTE 09
LIST = RHO, SIGY, E, GNU
10
BSO = BS
11
BS .EQ. 0.D
TRUE
FALSE
12
BSF = BS/12.D
13
WRITE TO DEV 6 VIA FORMAT 85D FROM THE LIST
NOTE 14
LIST = BSF
20
NOTE 15
CONTINUE
NOTE 16
BEGIN DO LOOP 125 II = 1, N
04.27
17
DPTH = DPTHI(II)
CC = 10.D*EXP((32D.D - DPTH)/1000.D) + 6.5
18
WRITE TO DEV 6 VIA FORMAT 75D FROM THE LIST
NOTE 19
LIST = CC
NOTE 20
BEGIN DO LOOP 225 JJ = 1, M
04.26
21
BS = BSO
DM = DMI(JJ)
RM = DM/2.D
DMF = DM/12.0
22
Z .NE. 1.D
TRUE
FALSE
3.01 16
23
WRITE TO DEV 6 VIA FORMAT 25D FROM THE LIST
NOTE 24
LIST = DPTH, DMF
3.03 17

Figure 24 a

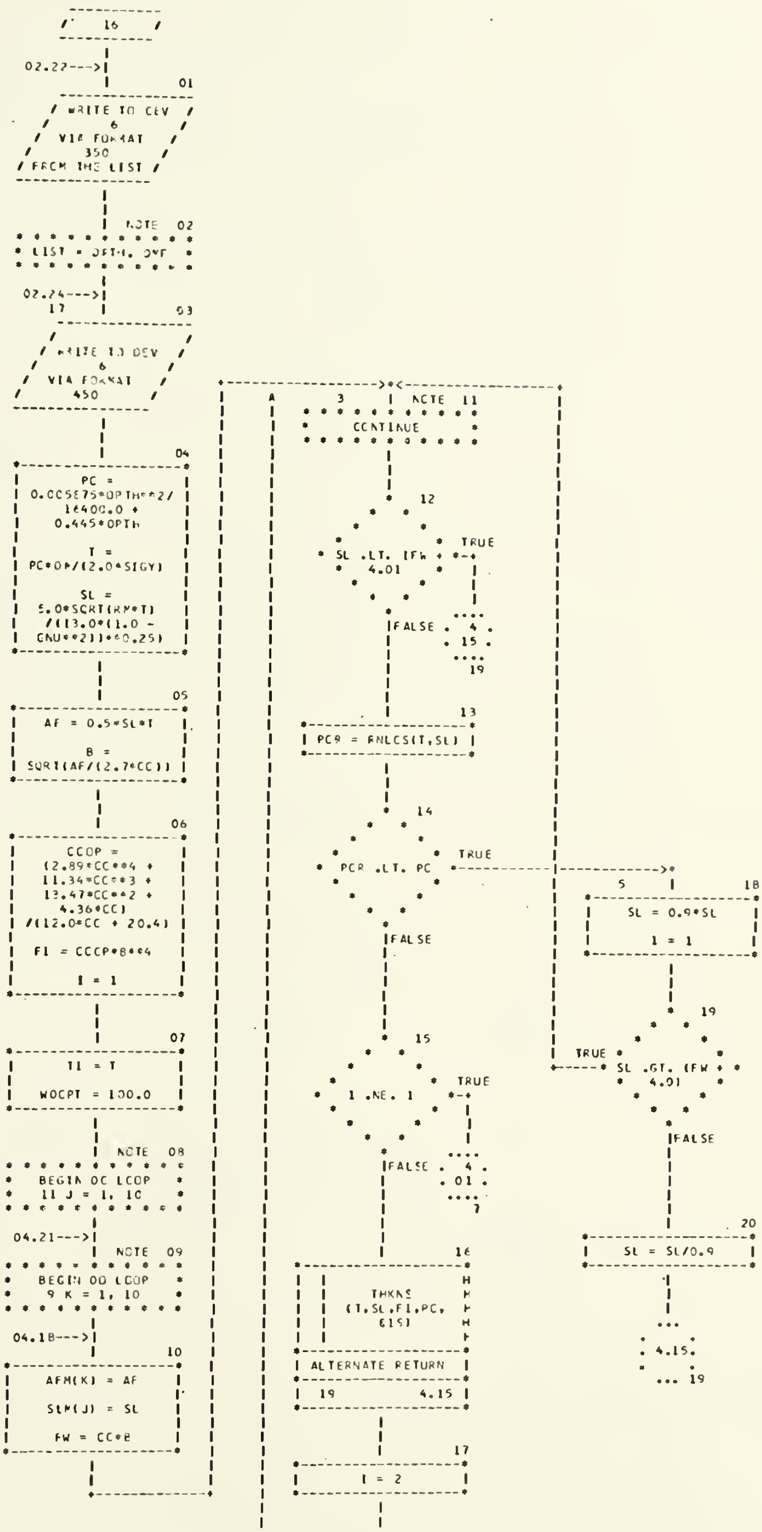
16
02.22--->
01
WRITE TO DEV 6 VIA FORMAT 350 FROM THE LIST
NOTE 02
LIST = OPTH, OVF
02.24--->
17
03
WRITE TO DEV 6 VIA FORMAT 450
04
PC = 0.005875*OPTH**2/ 16400.0 + 0.445*OPTH
T = PC*OP/(2.0*SIGY)
SL = 5.0*SQRT(RM*T) /((3.0*(1.0 - CNU**2))**0.25)
05
AF = 0.5*SL*T
B = SQRT(AF/(2.7*CC))
06
CCOP = (2.89*CC**4 + 11.34*CC**3 + 13.47*CC**2 + 4.36*CC) /(12.0*CC + 20.4)
FI = CCOP*B**4
I = 1
07
TI = T
WOCPT = 100.0
NOTE 08
BEGIN DO LOOP 11 J = 1, 10
04.21--->
NOTE 09
BEGIN DO LOOP 9 K = 1, 10
04.18--->
10
AFM(K) = AF
SLM(J) = SL
FW = CC*B
A
3
NOTE 11
CONTINUE
12
SL .LT. (FW + 4.0)
TRUE
FALSE
4. 15. 19
13
PCR = FNLCS(T,SL)
14
PCR .LT. PC
TRUE
FALSE
15
I .NE. 1
TRUE
FALSE
4. 01. 7
16
THKNS (T,SL,FI,PC, &15)
ALTERNATE RETURN
19
4.15
17
I = 2
5
18
SL = 0.9*SL
I = 1
19
TRUE
SL .GT. (FW + 4.0)
FALSE
20
SL = SL/0.9
4.15. 19

Figure 24b

CHART TITLE - PROCEDURES

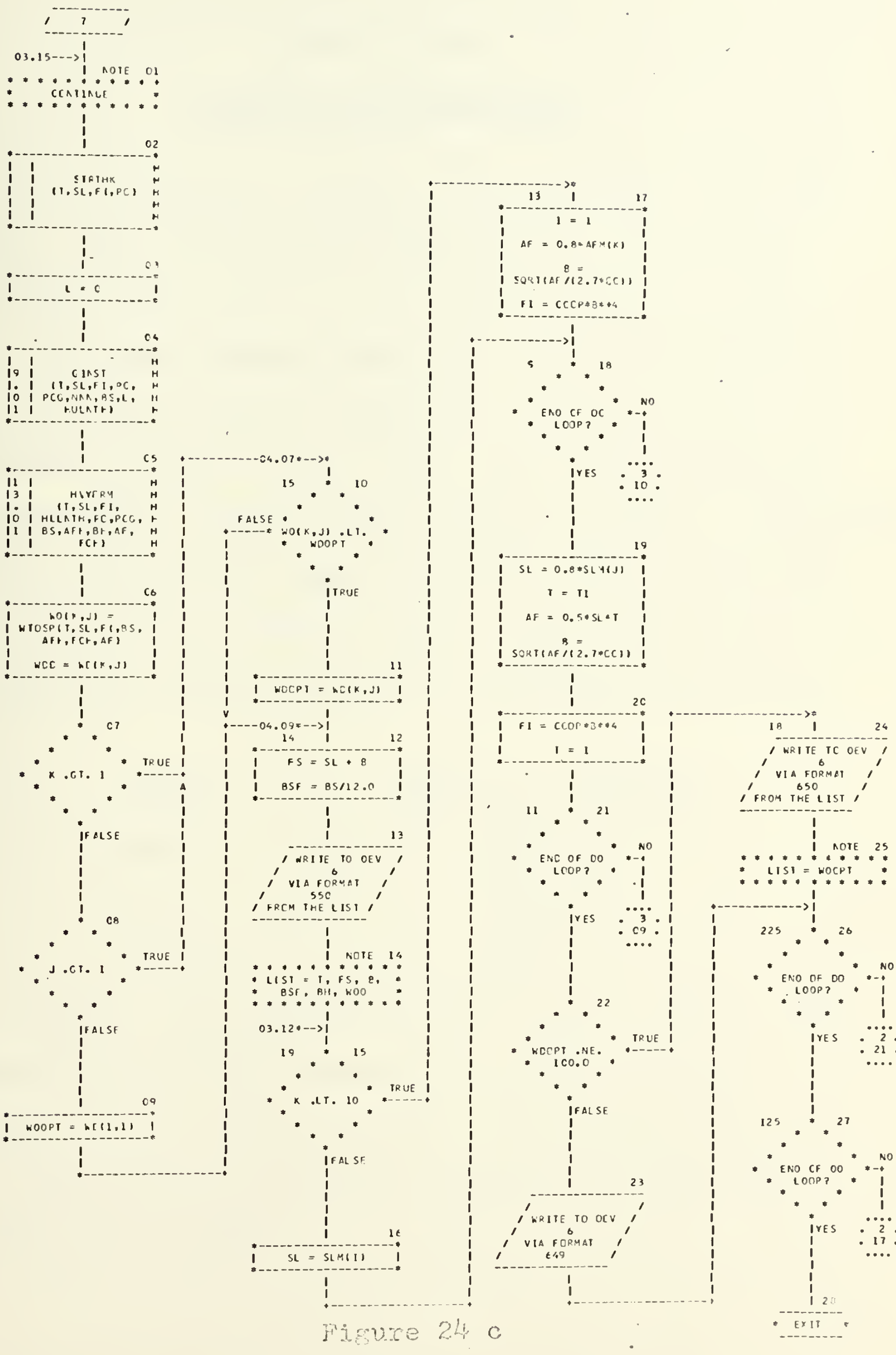

Figure 24 c

CHAPTER ELEVEN

RESULTS AND CONCLUSIONS

Various parametric studies were performed with the program, once it was checked against some contemporary submersible hulls to see if it indeed was "in the ball park". The most obvious study was to see how the W/D ratio varied with depth (figure 25). This was done for three common steels in use today, all with approximately "ideally plastic" stress-strain characteristics. The points obtained plotted into smooth curves on the semi-log plot used. It is fairly obvious that if a W/D greater than 0.5 is considered unsatisfactory, the following limits would apply:

STEEL	LIMIT FOR W/D ≤5.0
HTS	3500 FT.
HY-80	6200 FT.
HY-100	7400 FT.

A second study was conducted to observe how the W/D ratio varied with hull diameter, all other factors (including depth) being constant. It was found that it varied very little if at all, as can be seen from the following set of data:

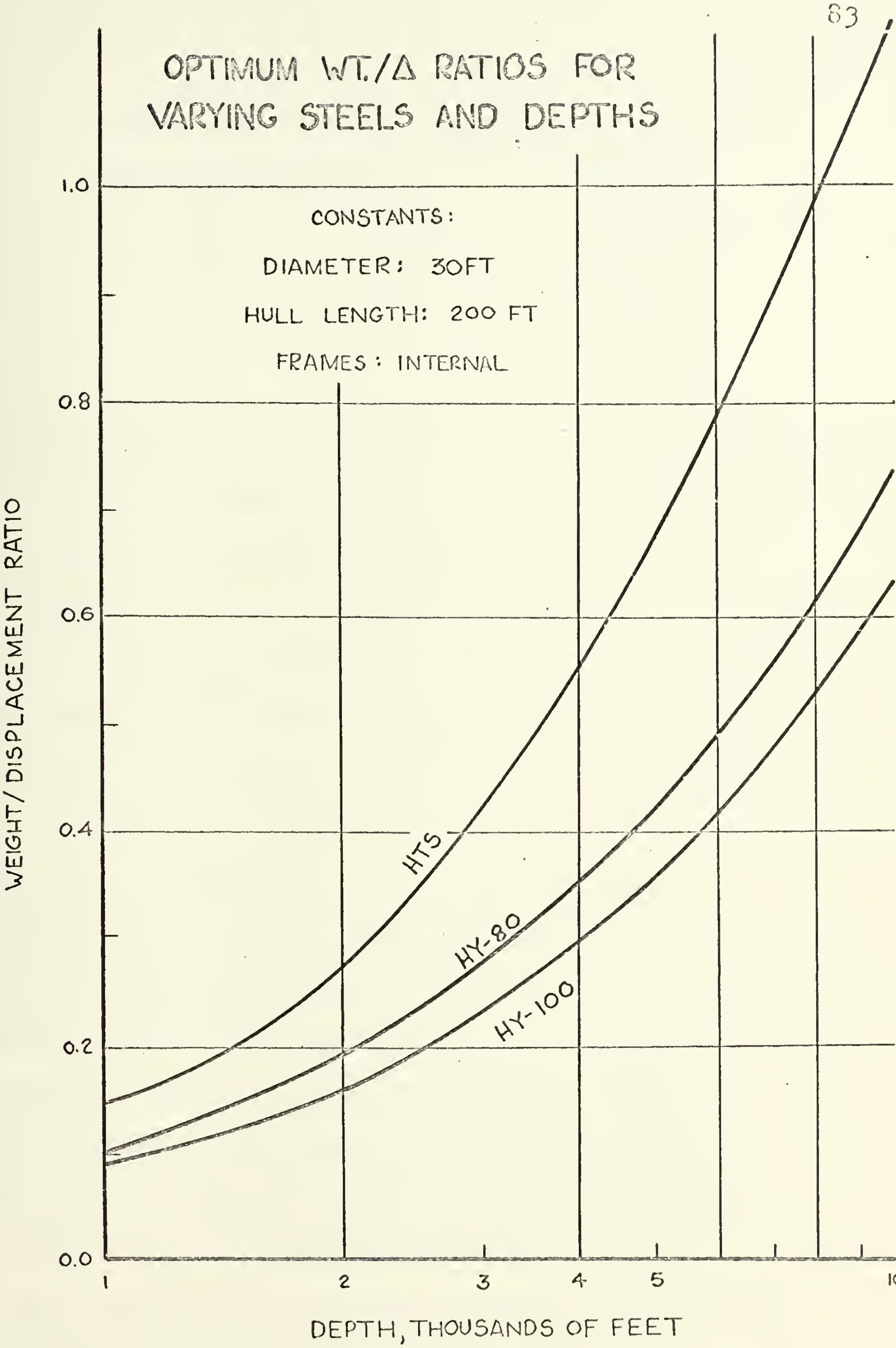

Figure 25

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Given: HY 80 steel, collapse depth=5000 ft.:

HULL DIAMETER (FT.)	W/D RATIO
20	0.417
30	0.418
50	0.417
80	0.409

It could tentatively be said then, that for a given depth, hull efficiency (W/D) will be approximately the same regardless of how large the hull diameter may be.

Figure 26 shows the result of plotting the variance of W/D with metal yield stress. This also plotted into a smooth curve in this case. It is probable, however, that for higher yield strength (and therefore, strain-hardening) metals requiring a slightly different analysis, the curve would have a sharp break.

The third plot attempted (figure 27) at first appeared to be a hopeless scattering of data, but after some analysis revealed rather interesting results. The points plotted were taken from a single optimization (one diameter, one depth) such as in the example print-out in Appendix D. A card computing thinness factor was inserted in the program and printed out with the regular data.

The "SL loop" is actually a series of W/D points computed for scantling sets with the <u>same frame spacing</u>,

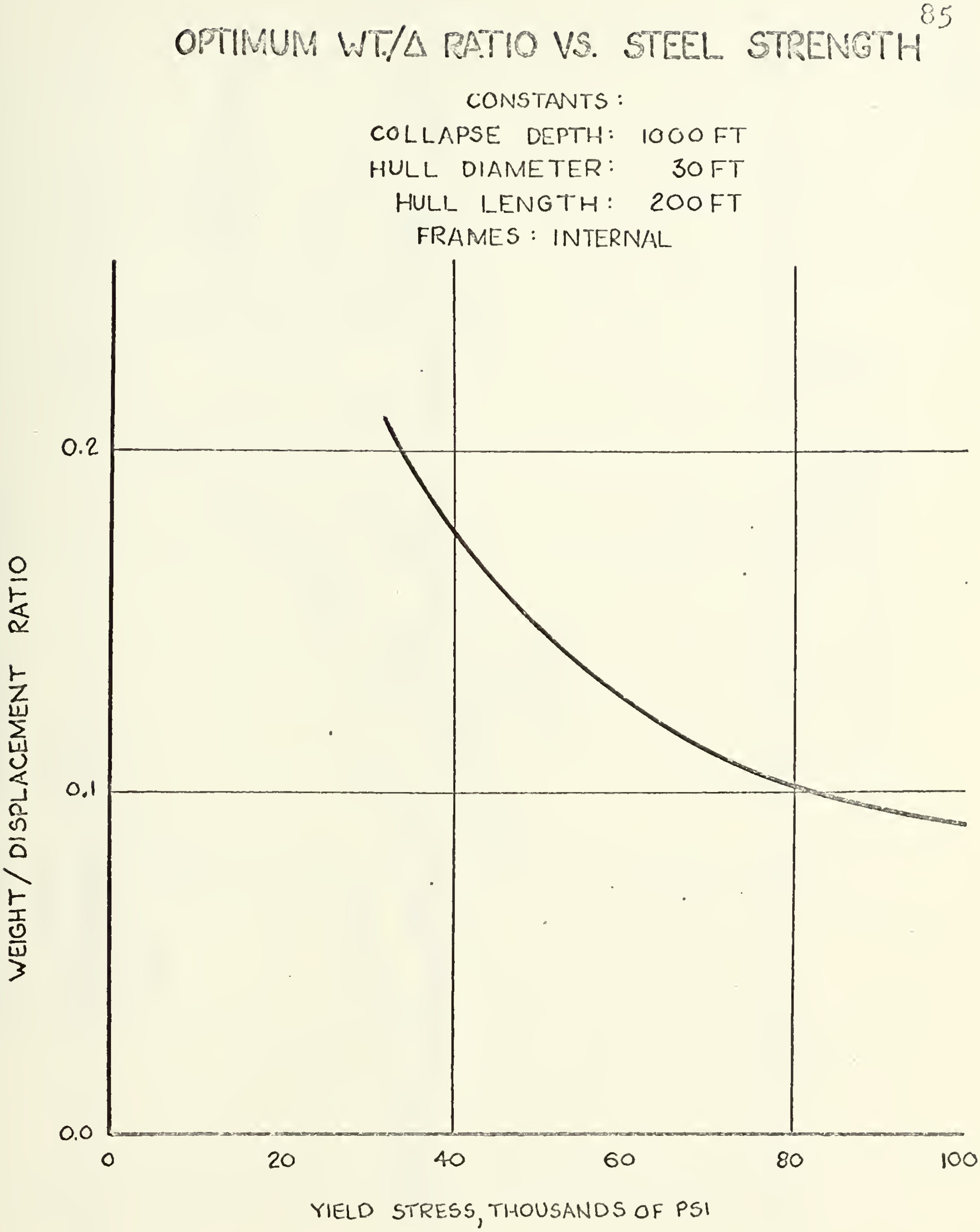

Figure 26

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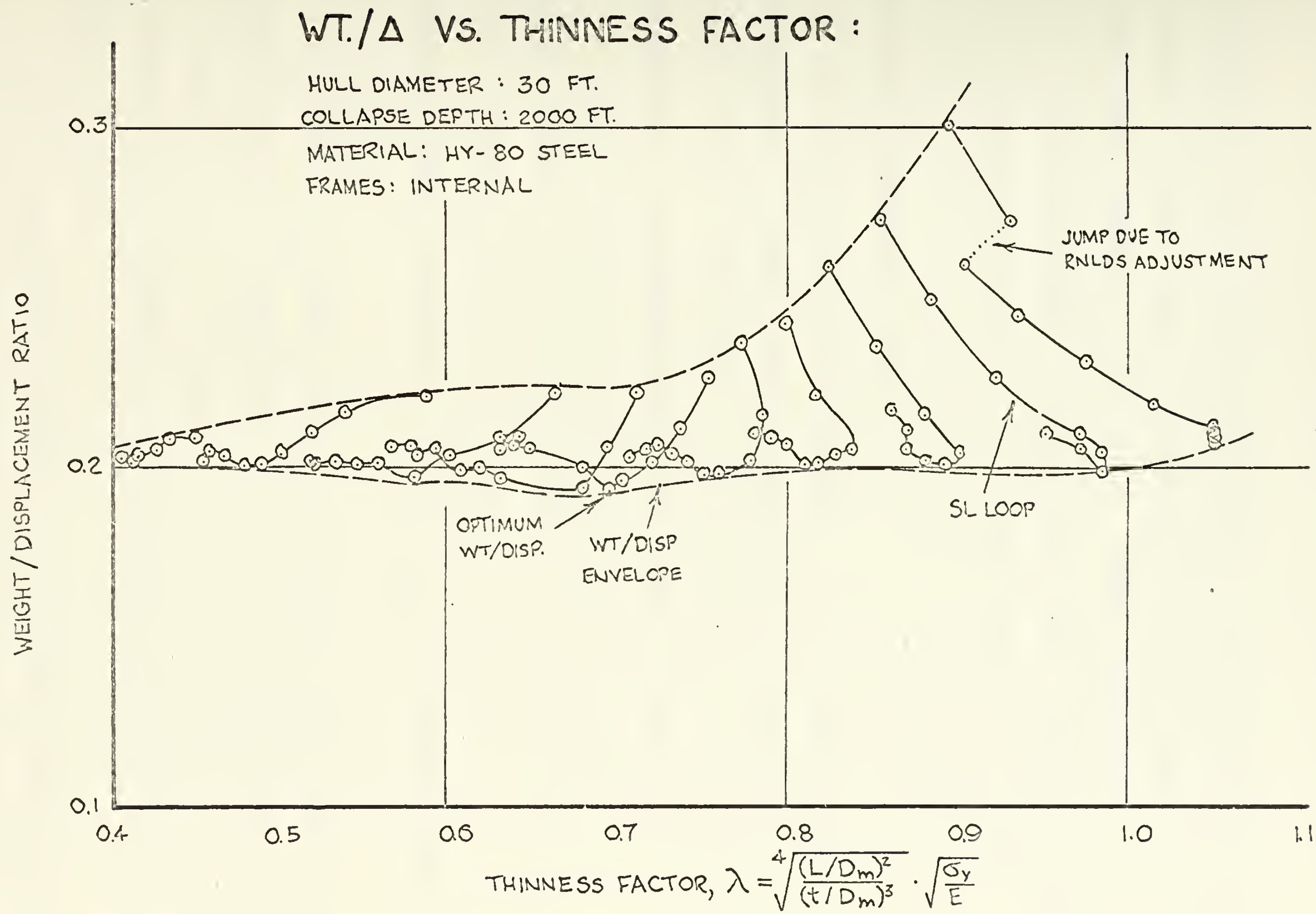
Figure 27



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buy varying frame size as an input. In each case, it is strongly evident that a saddle point, or optimum W/D for that frame spacing, was reached. This means that for at least this particular set of inputs, the program's frame size (AF) iteration range was large enough to bracket optimum values. One of these frame spacing minimums, then, was the optimum W/D ratio.

The "W/D envelope" encloses all W/D values computed in the program. There seems to be good indication that the "optimum W/D" indicated is a true optimum, since the lower portion of the envelope rises on either side of it.

Another indication given by the plot is that the program gives a much wider variation in W/D ratios with larger frame spacings, for varying frame sizes. As frame spacing becomes smaller, the W/D ratios produced become more "convergent".

Perhaps the most obvious conclusion from figure 26 is that λ alone is certainly not an accurate indicator of optimum W/D ratio, although it could be utilized (after extensive data gathering) to indicate the general area in which to design.

It is interesting to note that average computation time for each optimum computed was less than one-half minute. Also, it was interesting to note that the optimum solutions contained as scantlings, generally

speaking, smaller frames, larger heavy frame spacing, and slightly thicker shells than submersibles of present design. Evidently, this results in better W/D ratios.

One of the major advantages of this program and its general method of computation and print-out (see Appendix C) is that it gives a great variety of alternatives from which to choose. For instance, perhaps the optimum W/D ratio for a certain hull configuration and depth contained scantlings which gave a very small frame spacing. Analytically, this may give a superior W/D ratio. But practically speaking, the cost of fabricating and installing a great number of frames may be out of the question. Thus (particularly if the submersible is not critically weight-limited), the table of printouts may be "browsed" for more attractive scantling sets: ones with acceptible W/D ratios, but with inherently lower construction costs. It should be repeated here that each of these printed lines are not mere random choices. Each line, prior to print-out, has already been through one of the main hurdles of the optimization program. The design of shell thickness and frame spacing combination to give failure by axisymmetric yield, and thus most efficient use of the material's inherent strength, is completed prior to printing each line of the answer table. Thus, the program may be used not only

as a structural optimization, but also is an indispensible reference for any economic analysis of a particular design.

CHAPTER TWELVE

RECOMMENDATIONS

This Program.

1. Ranges of Variation. One of the most common questions after a particular run completion was "I wonder if this is really the optimum, or didn't I go far enough in frame (or frame spacing) variation?" It is believed that plots such as figure 27 for each run could probably give a definitive answer in most cases. Not only can one tell if each set of frames ran through an optimum, but it should also usually be possible to tell if the "W/D envelope" passed through its optimum. This may be a rather painful way to assure one's self that his run covered all the territory that was necessary, but at this writing, it appears to be the surest way. As an alternative, some rather extensive studies could be conducted using various depth and other inputs to test range validities. Once determined, the program's range of AF and SL variation, governed by the indices K and J, respectively, could be controlled appropriately. From a rather cursory inspection by the author, it appeared that in runs conducted

in data-gathering for this paper, the ranges of J=K=10 were adequate in all but a few questionable cases.

2. Effects of Fabrication Procedures. It is rather obvious that without any doubt, the greatest shortcoming in the optimization is in its allowances for such relative "unknowns" as residual welding stresses, low-cycle fatigue, shell or frame out-of-roundness, etc. The gross approximations made by REDPR and STRTHK certainly fall far short of the accuracy of the various failure mode analyses. There is no doubt that more basic research must be accomplished in this region before a completely dependable optimization program could be achieved. As it stands now, the safety factors built into the program could fall into two categories:

   a) Completely safe design, in which all scantlings are overdesigned to the extent that design "optimization" is almost useless.

   b) more realistic (i.e., lower) safety factors based on scanty experimental data, which is not universally applicable, and thus might be considered unsafe.

It is hoped that the safety devices employed in this program adhere to a "middle of the road" policy. REDPR, it is believed, results in somewhat of an overdesign for the two between-frame failure modes, but in underdesign for the general instability mode. This underdesign is hopefully picked up in STRTHK, which, as mentioned in Chapter 6, insures "a reasonable stress level in the frame flange prior to collapse for those frames with an initial out-of-roundness"[10].

The only way, it appears, to resolve these questions, is in extensive testing of models with deliberate, measurable defects of all types. The most needed data of this type is in the failure mode most effected by defects: general instability. It is believed by some authors[2] that the out of roundness analyses developed thus far are overly pessimisitic when applied to full sized submersibles.

Future Programs.

The most obvious extension of this program is into strain-hardening materials. Because of the program's general characteristics (i.e., a main flow control program manipulating subprograms which actually do pressure calculations), it can

be easily transformed with very few changes other than subprogram additions.

The need for such a transformation is obvious when one observes hull steels projected for future (and in many cases, present) usage. Any steel with a yield strength greater than 100,000 psi can be considered to be a strain-hardening material. Details of the method of transforming the present program into one suitable for use with high-strength steels may be found in Appendix E.

APPENDIX A

FRAME DIMENSIONS

APPENDIX A

FRAME DIMENSIONS

TYPICAL FRAMES:

In order to arrive at reasonable frame cross-sections, averages of a great variety of frame scantlings used in a number of submersibles at various depths were obtained (see figure 19, chapter 10). The common denominator for all of these frames is the web thickness, B. This method was taken from Adamchak in reference 1, who did the same thing for surface ship frames.

The ratios used in figure 19 were taken from the following, derived from the averages computed (see figure 28):

Flange width=FW=CC*B, where CC=y of figure 19.

Flange depth=FD=FW=CC*B

Flange thickness=1.7*B

In addition to the above, the remaining terms in figure 28 are defined as follows:

FC: Distance of frame centroid from plate neutral axis

DN: Distance of combined centroid from plate neutral axis

The various formulas for frame and combined frame-plate moments of inertia and for center of gyration were taken

T-SECTION
DIMENSION IDENTIFICATION

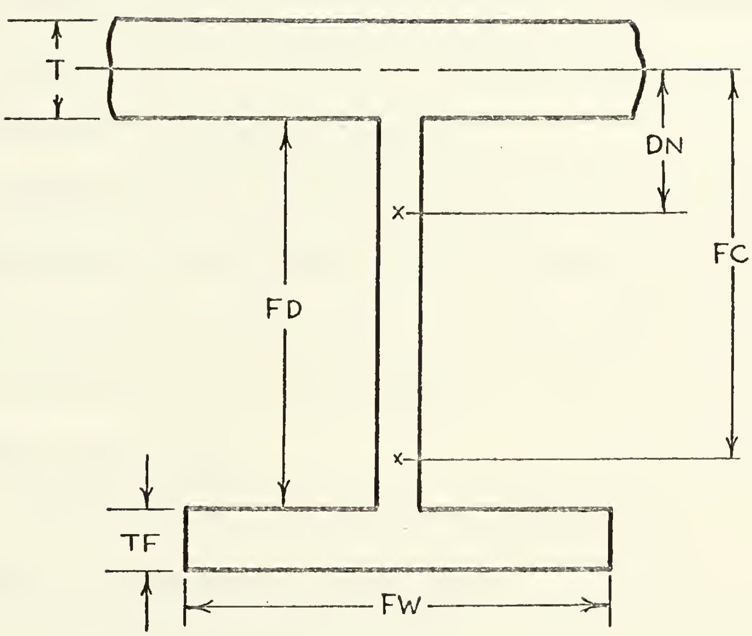

Figure 28

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from a general development of these in reference 19. They are listed in easily programmed form, and according to reference 19, differences from values of actual standard production frames due to fillets, etc., are always less than two per cent.

The combination of these two methods enables the computer to easily arrive at any variety of T-stiffener characteristics, each of which is proportioned according to present submersible design practice.

After substituting the expressions for FD, FW, TF and B into the equations for T-frames in reference 19, the following expressions result for typical (i.e., not heavy) frames, and are used in the program:

Frame area, AF:

```
AF=FW*TF+FD*B = 2.7*CC*B**2
```

Frame moment of inertia, FI:

```
FI = CCOP*B**4, where:
CCOP= (2.89*CC**4+11.34*CC**3+13.47*CC**2
        +4.36*CC)/(12.0*CC+20.4)
```

Effective frame-plate moment of inertia (using effective length of shell plating, EL, developed in reference 18), EI:

```
EI= 0.225*CC*B**4*(CC+1.7)**2*((1.07*CC+0.562)/
    (CC+1.7)+3.0*(1.63+(T-1.7*B)/((CC+1.7)*B))**2/
    (1.0+(2.7*CC*B**2)/(EL*T)))
```

FC:

FC=B*((CC+1.7)/1.227-1.0/1.176)+T/2.0

DN:

DN=FC*2.7*CC*B**2/(2.7*CC*B**2+EL*T)

HEAVY FRAMES:

Heavy frames were averaged in the same manner as above, although less data exists. The relationships obtained did not depend upon depth, and were determined to be as follows:

FDH=17.0*BH

FWH=13.0*BH

TFH=2.0*BH

CCOP=1330.0

Heavy frame area, AFH:

AFH=43.0*BH**2

Heavy frame moment of inertia, FIH:

FIH=1330.0*B**4

FCH:

FCH=14.25*BH+T/2.0

DNH:

DNH=FCH*AFH/(AFH+ELH*T), where

ELH=effective length of shell plate next to the heavy frame (see chapter 8)

EIH was not needed in the program.

APPENDIX B

LUNCHIK'S PLASTIC HINGE ANALYSIS

APPENDIX B

LUNCHIK'S PLASTIC HINGE ANALYSIS

(Refer to reference 14 for this discussion)

Lunchik's final Pc/Py equation is developed through the use of his parameters B and K. There are not solvable except by assumptions which Lunchik makes. Two of his assumptions (i.e., $k_\phi = \mu k_x$ and $\sigma_{mx}/\sigma_{m\phi} = K_x/K_\phi$) are easily worked out from substitutions in identities from his paper. However, one is less clear (i.e., $\sigma_{b\phi}/\sigma_{m\phi} = 6B_\phi$), particularly because B_ϕ is misprinted as β_ϕ in reference 14.

From Lunchik's definitions[14] in his analysis of a one unit square element:

(1) $M_\phi = k_\phi p \equiv$ circumferential edge moment

(2) $N_\phi = K_\phi ph \equiv$ circumferential compressive membrane force

(3) $B_\phi = \dfrac{k_\phi}{h^2 K_\phi}$

(4) $\sigma_{m\phi} = K_\phi p \equiv$ circumferential membrane stress

Assuming (i.e., approximating) the circumferential bending stress to be elastic:

$$\sigma_{b\phi} = \frac{M_\phi}{I} \cdot y = \frac{12}{bh^3} M_\phi \cdot \frac{h}{2} = \frac{6M_\phi}{bh^2} = \frac{6M_\phi}{(1)\,h^2} \qquad (5)$$

From (2) and (4):

$$N_\phi = \sigma_{m\phi} h \Rightarrow \sigma_{m\phi} = \frac{N_\phi}{h} \qquad (6)$$

Using (5) and (6):

$$\frac{\sigma_{b\phi}}{\sigma_{m\phi}} = \frac{6M_\phi}{h^2} \cdot \frac{h}{N_\phi} = \frac{6k_\phi p}{h \cdot K_\phi ph} = \frac{6k_\phi}{h^2 K_\phi} = 6B_\phi \qquad \text{Q.E.D.}$$

To further re-arrange these assumptions to obtain expressions for B_x and B_ϕ to use in program ELNCK:

(7) $B_\phi = \frac{1}{6}\sigma_{b\phi}/\sigma_{m\phi}$; (8) $K_x/K_\phi = \sigma_{mx}/\sigma_{m\phi}$

$$k_\phi = h^2 K_\phi B_\phi = h^2 K_\phi \left(\frac{1}{6}\sigma_{b\phi}/\sigma_{m\phi}\right)$$

$$B_x = \frac{k_x}{h^2 K_x} = \frac{k_\phi/\mu}{h^2 K_x} = \frac{h^2 K_\phi\left(\frac{1}{6}\sigma_{b\phi}/\sigma_{m\phi}\right)}{\mu h^2 K_x} = \frac{1}{6\mu}\left(\frac{\sigma_{b\phi}}{\sigma_{mx}}\right) \qquad (9)$$

(7), (8), and (9) are then put into Lunchik's equations (12), (13), and (15) for θ_1, θ_2 and θ_4 , and these in turn are substituted in his (25) and (26), which are used to solve for the plastic reserve stength ratio Pc/Py (FCTR in program ELNCK).

APPENDIX C

PROGRAM LISTING

```
C     E.S.MCGINLEY,II  MAIN PROGRAM: STIFFENED CYLINDRICAL
C     SHELL OPTIMIZATION.
      COMMON /D/E,GNU,DM,Z,CC,SIGY,CCOP,RHO/F/B,FC
      DIMENSION AFM(10),SLM(10),WD(10,10)
      DIMENSION DMI(100), DPTHI(100)
      READ (5,100) HULNTH,BS,Z,E,SIGY,RHO,GNU
      READ (5,200) N, (DPTHI(NN), NN=1,N)
      READ (5,200) M,(DMI(MM), MM=1,M)
  100 FORMAT (7F10.0)
  200 FORMAT (I2,7F10.0/(7F10.0))
      WRITE (6,149)
  149 FORMAT (//////////,32X,' E.S.MCGINLEY,II'////)
      WRITE(6,150) RHO,SIGY,E,GNU
  150 FORMAT(16X,' RING-STIFFENED CYLINDRICAL SHELL',
     1' OPTIMIZATION'///,28X,' ...INPUT PARAMETERS...'//,10X
     2.' MATERIAL DENSITY='F5.3' LBS/CU.IN.',1X,
     3' YIELD STRESS='F7.0' PSI'//,13X,' YOUNGS MODULUS='E8.
     41' PSI',3X,' POISSONS RATIO='F4.2)
      BSO=BS
      IF(BS.EQ.0.0) GO TO 20
      BSF=BS/12.0
      WRITE(6,850) BSF
  850 FORMAT('0',28X,' BULKHEAD SPACING='F5.1' FT.')
   20 CONTINUE
      DO 125 II=1,N
      DPTH=DPTHI(II)
      CC=10.0*EXP((320.0-DPTH)/1000.0)+6.5
      WRITE(6,750) CC
  750 FORMAT('1',12X,' *FRAME DIMENSIONS MAY BE OBTAINED BY',
     1' USING THE FRAME'/,20X,' CONSTANT B(WEB THICKNESS)',
     2' AS FOLLOWS:'///,26X,' ...LIGHT(TYPICAL) FRAMES...'//
     3,23X,' FRAME DEPTH=FLANGE WIDTH= 'F4.1' X B'//,26X,
     4' FLANGE THICKNESS= 1.7 X B'///,29X,' ...HEAVY',
     5' FRAMES...'//,28X,' FRAME DEPTH= 17 X B'//,27X,
     6' FLANGE WIDTH= 13 X B'//,26X,' FLANGE THICKNESS=',
     7' 2 X B')
```

```
      DO 225 JJ=1,M
      BS=BSO
      DM=DMI(JJ)
      RM=DM/2.0
      DMF=DM/12.0
      IF(Z.NE.1.0) GO TO 16
      WRITE(6,250) DPTH,DMF
  250 FORMAT(///,12X,  ' DEPTH='F7.1' FT.',1X,' DIAMETER='F5
     1.1' FT.',2X,' FRAMES:INTERNAL'///)
      GO TO 17
   16 WRITE(6,350) DPTH,DMF
  350 FORMAT('1             DEPTH='F7.1' FT.',1X,' DIAMETER='F4
     1.1' FT.',2X,' FRAMES:EXTERNAL'///)
   17 WRITE(6,450)
  450 FORMAT(29X,' ...DESIGN OPTIONS...'//,21X,' TYPICAL',
     1'   TYPICAL    HEAVY     HEAVY    WEIGHT/'/,12X,
     2' SHELL     FRAME     FRAME     FRAME     FRAME  ',
     3' DISPLACEMENT'/,10X,' THICKNESS  SPACING  CONSTANT*',
     4'  SPACING  CONSTANT*  RATIO'/,12X,' (IN.)      (IN.)',
     5'     (IN.)      (FT.)      (IN.)'//)
      PC=0.005875*DPTH**2/16400.0+0.445*DPTH
      T=PC*DM/(2.0*SIGY)
      SL=5.0*SQRT(RM*T)/((3.0*(1.0-GNU**2))**0.25)
      AF=0.5*SL*T
      B=SQRT(AF/(2.7*CC))
      CCOP=(2.89*CC**4+11.34*CC**3+13.47*CC**2+4.36*CC)/
     1(12.0*CC+20.4)
      FI=CCOP*B**4
      I=1
      T1=T
      WDOPT=100.0
      DO 11 J=1,10
      DO 9 K=1,10
      AFM(K)=AF
      SLM(J)=SL
      FW=CC*B
```

```
  3 CONTINUE
    IF(SL.LT.(FW+4.0)) GO TO 19
    PCR=RNLDS(T,SL)
    IF(PCR.LT.PC) GO TO 5
    IF(I.NE.1) GO TO 7
    CALL THKNS (T,SL,FI,PC,&19)
    I=2
    GO TO 3
  5 SL=0.9*SL
    I=1
    IF(SL.GT.(FW+4.0))GO TO 3
    SL=SL/0.9
    GO TO 19
  7 CONTINUE
    CALL STRTHK(T,SL,FI,PC)
    L=0
    CALL GINST(T,SL,FI,PC,PCG,NNN,BS,L,HULNTH)
    CALL HVYFRM(T,SL,FI,HULNTH,PC,PCG,BS,AFH,BH,AF,FCH)
    WD(K,J)=WTDSP(T,SL,FI,BS,AFH,FCH,AF)
    WDD=WD(K,J)
    IF(K.GT.1) GO TO 15
    IF(J.GT.1) GO TO 15
    WDOPT=WD(1,1)
    GO TO 14
 15 IF(WD(K,J).LT.WDOPT) WDOPT=WD(K,J)
 14 FS=SL+B
    BSF=BS/12.0
    WRITE (6,550) T,FS,B,BSF,BH,WDD
550 FORMAT(11X,F7.4,4X,F6.2,4X,F6.3,4X,F7.2,4X,F6.3,5X,F5.
   13)
 19 IF(K.LT.10) GO TO 13
    SL=SLM(1)
    GO TO 9
 13 I=1
    AF=0.8*AFM(K)
    B=SQRT(AF/(2.7*CC))
```

```
      FI=CCCP*B**4
    9 CCNTINUE
      SL=0.8*SLM(J)
      T=T1
      AF=0.5*SL*T
      B=SQRT(AF/(2.7*CC))
      FI=CCCP*B**4
      I=1
   11 CONTINUE
      IF(WDOPT.NE.100.0) GO TO 18
      WRITE(6,649)
  649 FORMAT(///17X,' RECOMMEND LOWER YIELD STRENGTH MATERIAL')
      GO TO 225
   18 WRITE(6,650) WDOPT
  650 FORMAT(///17X,' OPTIMUM WEIGHT/DISPLACEMENT RATIO:'F5.
     13)
  225 CONTINUE
  125 CONTINUE
      CALL EXIT
      END
```

```
      FUNCTICN RNLDS(T,SL)
C     COMPUTES CRITICAL ASYMMETRIC (LOBAR) BUCKLING PRESSURE
      CCMMCN/D/E,GNU,DM,Z,CC,SIGY,CCOP,RHO
      COMMON/F/B,FC
      PI=3.1415927
      RM=DM/2.0
      Y=1.23*SQRT(RM*T)/SL
      AF=2.7*CC*B**2
      FS=SL+B
      TETA=(3.0*(1.0-GNU**2))**0.25*SL/(SQRT(RM*T))
      SINH(X,Y)=(X-Y)/2.0
      COSH(X,Y)=(X+Y)/2.0
      TP2=EXP(TETA/2.0)
      TN2=1.0/TP2
      BHETA1=TETA/2.0*((SINH(TP2,TN2)+SIN(TETA/2.0))/(COSH(T
     1P2,TN2)-COS(TETA/2.0)))
      TP=TP2**2
      TN=1.0/TP
      BHETA=TETA*((SINH(TP,TN)+SIN(TETA))/(COSH(TP,TN)-COS(T
     1ETA)))
      YY=0.5/(1.0-(1.0-GNU/2.0)*(AF/(SL*T))*(BHETA1-0.5*BHET
     1A)/(1.0+0.5*BHETA*(AF+B*T)/(SL*T)))
      PCRE =2.0*PI**2*E*T**3*YY/(3.0*(1.0-GNU**2)*RM*SL**2*Y
     1*(3.0+2.0*Y*(YY-1.0)))
      RNLDS=0.75067*PCRE
      RETURN
      END
```

```
      SUBROUTINE THKNS (TT,SL,FI,PC,*)
C     CCNVERCES CN THICKNESS TO SATISFY AXISYMMETRIC
C     FAILURE AT DESIGN CCLLAPSE DEPTH
      DIMENSICN T(100),PCL(100),N(100)
      CCMMCN/D/E,GNU,DM,Z,CC,SIGY,CCCP,RHO
      RM=DM/2.C
      I=1
      TIN=TT
    1 CCNTINUE
    2 CALL ELNCK(TT,SL,FI,PC,P)
      PCL(I)=P
      T(I)=TT
    4 CCNTINUE
      IF(ABS(PCL(I)-PC)-C.01*PC) 5,5,6
    5 RETURN
    6 IF(I.NE.1) GO TO 11
    7 IF(PCL(I).LT.PC) GO TC 8
    9 T(2)=C.3*TT
      N(I)=1
      GO TC 10
    8 T(2)=3.C*TT
      N(I)=-1
   10 CCNTINUE
      TTT=TT
   14 TT=T(I+1)
      I=I+1
      IF(I.LT.15) GC TO 1
      TT=TIN
      RETURN 1
   11 IF(PCL(I).LT.PC) GO TO 12
      N(I)=1
      GC TC 13
   12 N(I)=-1
   13 IF((N(I)-N(I-1)).NE.0) TTT=T(I-1)
      T(I+1)=C.5*(T(I)-TTT)+TTT
      GC TC 14
      END
```

```
      SUBROLTINE STRTHK(T,SL,FI,PC)
C     ADJUSTS SHELL THICKNESS AS NECESSARY TO MEET C.75
C     YIELD STRESS CRITERION AT CPERATING DEPTH.
      CCMMCN/D/E,GNU,DM,Z,CC,SIGY,CCCP,RHC
      SIGAL=C.75*SIGY
      PC=2.0/3.0*PC
      RM=DM/2.0
    1 CONTINUE
      CALL FRAME (T,SL,FI)
      CALL PULOS (T,SL,FI,PC,F1,F2,F3,F4,A)
      SIGC=A*F3*SQRT(C.91/(1.0-GNU**2))
      SIGU=-PO*RM/T
      S1=ABS(SIGU*(0.5+SIGC))
      S2=ABS(SIGU*(0.5-SIGC))
      S3=ABS(SIGU*(1.0-A+GNU*SIGC))
      S4=ABS(SIGU*(1.0-A-GNU*SIGC))
      SIGMX=AMAX1(S1,S2,S3,S4)
      IF((SIGMX-SIGAL).LT.1.C) GC TC 2
      T=T+0.05*T
      GO TO 1
    2 RETURN
      END
```

```
      SUBROUTINE GINST (T,SL,FI,PC,PCG,N,BS,L,HULNTH)
C     COMPUTES GEN. INSTAB. PRESSURE AND HEAVY FRAME SPACING
      COMMON/D/E,GNU,DM,Z,CC,SIGY,CCOP,RHO/F/B,FC
      COMMON/L/THETA,ALFA,BETA
      DIMENSION BHS(10)
      PI=3.1415927
      RM=DM/2.0
      CALL FRAME(T,SL,FI)
      FS=SL+B
      IF(BS.NE.0.0) GO TO 10
      BS=2.0*(AINT(DM/FS)+1.0)*FS
      PCGL=0.0
      LL=2
   10 CONTINUE
      GAMMA=PI*RM/BS
      PCGE=0.0
    1 CONTINUE
      CALL PULOS1 (T,SL,FI,PCGE,F1,F2,F3,F4,A,&99)
      EL=SL*F1+B
      GO TO 98
   99 EL=FS
   98 DN=FC*2.7*CC*B**2/(2.7*CC*B**2+EL*T)
      IF(Z.EQ.1) GO TO 2
      RCG=RM+DN
      GO TO 3
    2 RCG=RM-DN
    3 CONTINUE
      RO=RM+T/2.0
      EI=0.225*CC*B**4*(CC+1.7)**2*((1.07*CC+0.562)/(CC+1.7)
     1+3.0*(1.63+(T-1.7*B)/((CC+1.7)*B))**2/(1.0+(2.7*CC*B**
     22)/(EL*T)))
      ETAN=E
      ESEC=E
      CALL KRNZK(T,RM,FS,EI,RO,RCG,GAMMA,ESEC,ETAN,PCGE1,N)
      IF(ABS(PCGE-PCGE1)-0.01*PC) 4,4,5
    5 PCGE=PCGE1
```

```
      GO TO 1
    4 PCG=0.75067*PCGE1
   20 CONTINUE
      IF(L.EQ.1) GO TO 21
      IF(LL.EQ.2) GO TO 24
      IF(BS.LT.HULNTH) GO TO 24
      IF(PCG.LT.1.05*PC) GO TO 25
      BS=HULNTH
      GO TO 21
   25 BS=BS1
      GO TO 21
   24 IF(PCG.EQ.PCGL) GO TO 21
      IF(PCG.LT.1.05*PC) GO TO 22
      IF(PCG.GT.2.0*PC) GO TO 23
   21 RETURN
   22 BS=BS-FS
      PCGL=PCG
      LL=2
      GO TO 10
   23 BS1=BS
      BS=BS+2.0*FS
      LL=1
      PCGL=PCG
      GO TO 10
      END
```

```
      SUBROUTINE HVYFRM(T,SL,FI,HULNTH,PC,PCG,BS,AFH,BH,
     1AF,FCH)
C     COMPUTES DIMENSIONS OF HEAVY FRAMES REQUIRED.
      COMMON/D/E,GNU,DM,Z,CC,SIGY,CCOP,RHO/F/B,FC
      COMMON/V/ETAN
      IF(BS.EQ.HULNTH) GO TO 5
      RM=DM/2.0
      IF(PCG.GT.2.0*PC) PCG=2.0*PC
      RC=RM
      L=1
      BS1=HULNTH
      CALL GINST(T,SL,FI,PC,PCU,M,BS1,L,HULNTH)
      I=1
      ETAN=E
    1 CONTINUE
      FIH=PCG*BS*RC**3/((M**2-1.0)*ETAN)
      BH=(FIH/1330.0)**0.25
      AFH=43.0*BH**2
      IF(I.EQ.1) GO TO 2
      RETURN
    2 CONTINUE
      B=(FI/CCOP)**0.25
      AF=2.7*CC*B**2
      CALL PULOS(T,SL,FI,PC,F1,F2,F3,F4,A)
      ELH=SL*F1*(AF+SL*T)/(AFH+SL*T)+BH
      FCH=14.25*BH+T/2.0
      DNH=FCH*AFH/(AFH+ELH*T)
      IF(Z.EQ.1) GO TO 3
      RC=RM+DNH
      GO TO 4
    3 RC=RM-DNH
    4 CONTINUE
      I=2
      GO TO 1
    5 AFH=0.0
      BH=0.0
```

```
FCH=C.0
RETURN
END
```

```
      FUNCTION WTDSP (T,SL,FI,BS,AFH,FCH,AF)
C     COMPUTES WEIGHT/DISPLACEMENT RATIO.
      COMMON/C/E,GNU,DM,Z,CC,SIGY,CCOP,RHO/F/B,FC
      RM=DM/2.0
      PI=3.1415927
      FS=SL+B
      NF=INT(BS/FS)+1
      IF(Z.EQ.1.0) GO TO 1
      RCF=RM+FC
      RCFH=RM+FCH
      GO TO 2
    1 RCF=RM-FC
      RCFH=RM-FCH
    2 CONTINUE
      VF=NF*AF*2.0*PI*RCF
      VFH=AFH*2.0*PI*RCFH
      DISP=PI*(RM+T/2.0)**2*BS/60480.0
      IF(Z.NE.1.0) DISP=DISP+(VF+VFH)/60480.0
      WT=RHO/2240.0*(PI*DM*T*BS+VF+VFH)
      WTDSP=WT/DISP
      RETURN
      END
```

```
      SUBROUTINE ELNCK(T,SL,FI,PC,P)
C     COMPUTES CRITICAL AXISYMMETRIC YIELDING PRESSURE
      COMMON/D/E,GNU,DM,Z,CC,SIGY,CCOP,RHO
      CALL FRAME(T, SL, FI)
      RM=DM/2.0
      PCLE=0.0
      L=0
    1 CONTINUE
      CALL PULOS(T,SL,FI,PCLE,F1,F2,F3,F4,A)
      XX=GNU*A*F4*SQRT(0.91/(1.0-GNU**2))/(1.0-A*F2)
      YY=0.5/(1.0-A*F2)
      ZZ=XX/YY
      BP=XX/6.0
      BX=ZZ/(6.0*GNU)
      TETA1=BP**2-BX*BP*YY+BX**2*YY**2
      TETA2=BP-(BP+BX)*YY/2.0+BX*YY**2
      TETA4=1.0-YY+YY**2
      PHI1=TETA1/TETA4
      PHI2=TETA2/TETA4
      FCTR=SQRT((1.0+36.0*PHI1+12.0*PHI2)/(1.0+8.0*PHI1+
     14.0*SQRT(4.0*PHI1**2+PHI2**2)))
      PY=(SIGY*T    /RM)/SQRT(3.0/4.0+A**2*(F2**2+F2*F4*(1.0-
     12.0*GNU)*SQRT(0.91/(1.0-GNU**2))+F4**2*(1.0-GNU+GNU**2
     2)*(0.91/(1.0-GNU**2)))-3.0/2.0*A*(F2-GNU*F4*SQRT(0.91/
     3(1.0-GNU**2))))
      PCLE1=FCTR*PY
      IF(ABS(PCLE-PCLE1)-0.01*PC) 2,2,3
    3 PCLE=PCLE1
      L=L+1
      IF (L.GT.10) GO TO 2
      GO TO 1
    2 PEL=PLNCK(T,SL,FI,PC)
      P=REDPR(PEL,PCLE1)
      RETURN
      END
```

```
      SUBROUTINE FRAME (T,SL,FI)
C     COMPUTES FRAME/FRAME-SHELL CONSTANTS AND DIMENSIONS
      COMMON/U/THETA,ALFA,BETA/D/E,GNU,DM,Z,CC,SIGY,CCCP,RHO
      COMMON/F/B,FC
      RM=DM/2.0
      THETA=(3.0*(1.0-GNU**2))**0.25*SL/SQRT(RM*T)
      Q=1.0+2.0*GNU
      AF=2.7*CC*B**2
      FC=B*((CC+1.7)/1.227-1.0/1.176)+T/2.0
      IF(Z.EQ.1.0) GO TO 1
      RF=RM+FC
      GO TO 2
    1 RF=RM-FC
    2 CONTINUE
      AE=AF*(RM/RF)**Q
      ALFA=AE/((SL+B)*T)
      BETA=B/(SL+B)
      RETURN
      END
```

```
      SUBROUTINE PULOS(T,SL,FI,P,F1,F2,F3,F4,A)
C     COMPUTES SALERNO-PULOS STRESS FUNCTIONS FOR GIVEN
C     PRESSURE AND STIFFENED CYLINDER GEOMETRY
      COMMON/U/THETA,ALFA,BETA/C/E,GNU,DM,Z,CC,SIGY,CCCP,RHO
      RM=DM/2.0
      GAMA=P/(2.0*E)*SQRT(3.0*(1.0-GNU**2))*(RM/T)**2
      IF(GAMA-1.0) 2,1,1
    1 T=SQRT(P/(1.9*E))*RM*(3.0*(1.0-GNU**2))**0.25
      GAMA=0.95
    2 CONTINUE
      ETA1=SQRT(1.0-GAMA)/2.0
      ETA2=SQRT(1.0+GAMA)/2.0
      E1TP=EXP(ETA1*THETA)
      E1TN=1.0/E1TP
      SINH(X,Y)=(X-Y)/2.0
      COSH(X,Y)=(X+Y)/2.0
      C3=COSH(E1TP,E1TN)*SINH(E1TP,E1TN)/ETA1
      C4=COS(ETA2*THETA)*SIN(ETA2*THETA)/ETA2
      DELTA=C3+C4
      F3=SQRT(3.0/0.91)*(-C3+C4)/DELTA
      F1=4.0/THETA*((COSH(E1TP,E1TN))**2-(COS(ETA2*THETA))**
     12)/DELTA
      C1=COSH(E1TP,E1TN)*SIN(ETA2*THETA)/ETA2
      C2=SINH(E1TP,E1TN)*COS(ETA2*THETA)/ETA1
      F2=(C1+C2)/DELTA
      F4=SQRT(3.0/0.91)*(C1-C2)/DELTA
      A=(1.0-GNU/2.0)*ALFA/(ALFA+BETA+(1.0-BETA)*F1)
      RETURN
      END
```

```
      SUBROUTINE PULOS1 (T,SL,FI,P,F1,F2,F3,F4,A,*)
C     COMPUTES SALERNO-PULOS STRESS FUNCTIONS FOR GIVEN
C     PRESSURE AND STIFFENED CYLINDER GEOMETRY
      COMMON/U/THETA,ALFA,BETA/C/E,GNU,DM,Z,CC,SIGY,CCCP,RHO
      RM=DM/2.0
      GAMA=P/(2.0*E)*SQRT(3.0*(1.0-GNU**2))*(RM/T)**2
      IF(GAMA.LT.1.0) GO TO 10
      RETURN 1
   10 ETA1=SQRT(1.0-GAMA)/2.0
      ETA2=SQRT(1.0+GAMA)/2.0
      E1TP=EXP(ETA1*THETA)
      E1TN=1.0/E1TP
      SINH(X,Y)=(X-Y)/2.0
      COSH(X,Y)=(X+Y)/2.0
      C3=COSH(E1TP,E1TN)*SINH(E1TP,E1TN)/ETA1
      C4=COS(ETA2*THETA)*SIN(ETA2*THETA)/ETA2
      DELTA=C3+C4
      F3=SQRT(3.0/0.91)*(-C3+C4)/DELTA
      F1=4.0/THETA*((COSH(E1TP,E1TN))**2-(COS(ETA2*THETA))**
     12)/DELTA
      C1=COSH(E1TP,E1TN)*SIN(ETA2*THETA)/ETA2
      C2=SINH(E1TP,E1TN)*COS(ETA2*THETA)/ETA1
      F2=(C1+C2)/DELTA
      F4=SQRT(3.0/0.91)*(C1-C2)/DELTA
      A=(1.0-GNU/2.0)*ALFA/(ALFA+BETA+(1.0-BETA)*F1)
      RETURN
      END
```

```
      FUNCTION PLNCK(T,SL,FI,PC)
C     COMPUTES CRITICAL AXISYMMETRIC BUCKLING PRESSURE
      COMMON/D/E,GNU,DM,Z,CC,SIGY,CCCP,RHO
      PI=3.1415927
      RM=DM/2.0
      EALFA=(3.0*(1.0-GNU**2)/(RM**2*T**2))**0.25
      IF(EALFA*SL-5.44) 2,1,1
    1 N=3
      GO TO 5
    2 IF(EALFA*SL-PI) 4,3,3
    3 N=2
      GO TO 5
    4 N=1
    5 PEL=(2.0*E*T**2/(SQRT(3.0*(1.0-GNU**2))*RM**2))*
     1((EALFA*SL/(N*PI))**2+0.25*(N*PI/(EALFA*SL))**2)
      PLNCK=PEL
      RETURN
      END
```

```
      FUNCTICN REDPR(PRE,PRI)
C     CCMPUTES REDUCED PRESSURE CUE TO RESICUAL STRESSES AND
C     FABRICATION IMPERFECTIONS
      RTC=PRE/PRI
      IF(RTO.LT.1.0) GC TC 1
      REDFAC=0.64667+0.11367*RTO-C.00967*RTO**2
      IF(REDFAC.GT.1.0)REDFAC=1.0
      IF(RTC.GT.6.0) REDFAC=1.0
      REDPR=PRI*REDFAC
      CC TO 2
    1 REDPR=0.75067*PRE
    2 RETURN
      END
```

```
      SUBROUTINE KRNZK(T,RM,FS,EI,RC,RCG,GAMMA,ESEC,ETAN,
     1PCG,N)
C     COMPUTES ELASTIC/INELASTIC GENERAL INSTABILITY
C     COLLAPSE PRESSURE AND NUMBER CIRCUMF'L WAVES.
      DIMENSION PC(5)
      DO 10 I=1,5
      EN=I
      PC(I)=SQRT(ESEC*ETAN)*T*GAMMA**4/(RM*(EN**2-1.0+GAMMA
     1**2/2.0)*(EN**2+GAMMA**2)**2)+ETAN*EI*(EN**2-1.0)/
     2(FS*RC*RCG**2)
      IF(I.GT.1) GO TO 1
      PCG=PC(1)
      N=1
      GO TO 10
    1 IF(PC(I)-PCG) 2,10,10
    2 PCG=PC(I)
      N=EN
   10 CONTINUE
      RETURN
      END
```

APPENDIX D

SAMPLE INPUT CARDS

SAMPLE OUTPUT

FIRST INPUT DATA CARD

```
    READ (5,100) HULNTH, BS, Z, E, SIGY, RHO, GNU
100 FORMAT (7F10.0)
```

EXAMPLE

HULNTH: 240 FT

BS: UNSPECIFIED (0.0)

Z: INTERNAL FRAMES (1.0)

E: $30X10^6$

SIGY: 80000 PSI

RHO: 0.285 LB/IN^3

GNU: 0.3

2880.0 0.0 1.0 30.0E680000.0 0.285 0.3

Figure 29

SECOND INPUT DATA CARD

```
    READ (5,200) N, (DPTHI(NN), NN=1,N)
200 FORMAT (I2, 7F10.0/(7F10.0))
```

EXAMPLE

N=3 (DEPTHS) :   2000 FT, 5000 FT, 10000 FT

32000.0   5000.0   10000.0

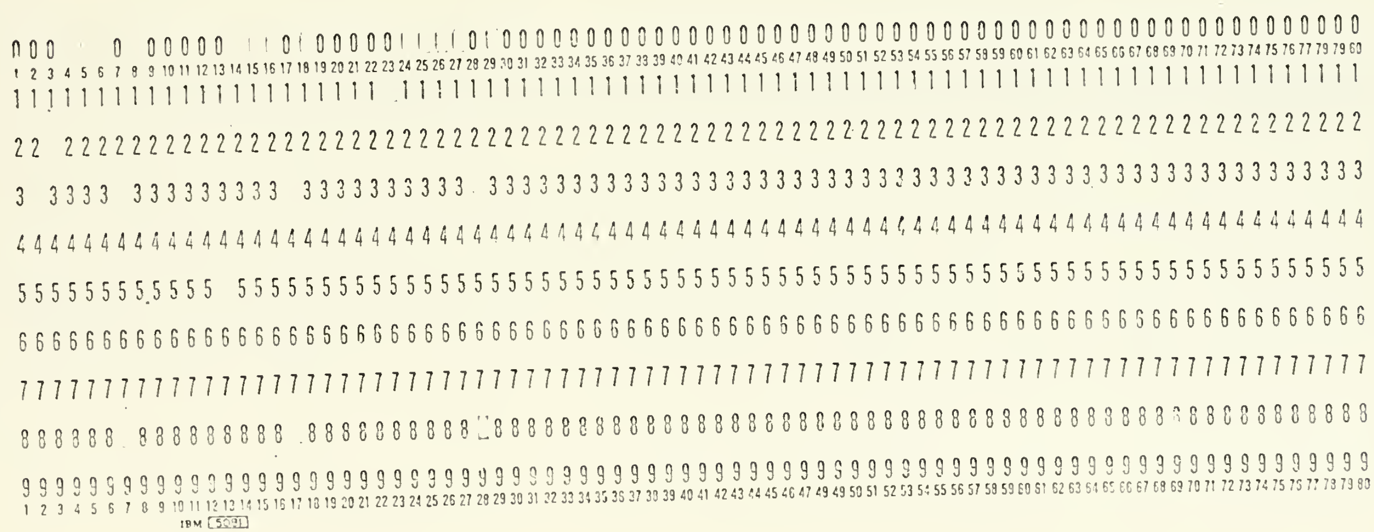


Figure 30

THIRD INPUT DATA CARD

```
    READ (5,200) M, (DMI(MM), MM=1,M)
200 FORMAT (I2, 7F10.0/(7F10.0))
```

EXAMPLE

M=4 (DIAMETERS) : 20 FT, 30 FT, 50 FT, 80 FT

4240.0 360.0 600.0 960.0

Figure 31

E.S.MCGINLEY,II

RING-STIFFENED CYLINDRICAL SHELL OPTIMIZATION

...INPUT PARAMETERS...

MATERIAL DENSITY=0.285 LBS/CU.IN. YIELD STRESS=100000. PSI

YOUNGS MODULUS= 0.3E 08 PSI POISSONS RATIO=0.30

*FRAME DIMENSIONS MAY BE OBTAINED BY USING THE FRAME CONSTANT B(WEB THICKNESS) AS FOLLOWS:

...LIGHT(TYPICAL) FRAMES...

FRAME DEPTH=FLANGE WIDTH= 6.6 X B

FLANGE THICKNESS= 1.7 X B

...HEAVY FRAMES...

FRAME DEPTH= 17 X B

FLANGE WIDTH= 13 X B

FLANGE THICKNESS= 2 X B

DEPTH= 5000.0 FT. DIAMETER= 30.0 FT. FRAMES:INTERNAL

...DESIGN OPTIONS...

SHELL THICKNESS (IN.)	TYPICAL FRAME SPACING (IN.)	TYPICAL FRAME CONSTANT* (IN.)	HEAVY FRAME SPACING (FT.)	HEAVY FRAME CONSTANT* (IN.)	WEIGHT/ DISPLACEMENT RATIO
5.0350	108.09	3.438	200.00	0.000	0.565
4.5805	107.73	3.075	200.00	0.000	0.501
4.5324	107.40	2.750	200.00	0.000	0.474
4.2713	107.11	2.460	200.00	0.000	0.435
4.0395	106.85	2.200	198.16	2.981	0.411
3.8703	106.62	1.968	73.77	2.664	0.399
3.6163	106.41	1.760	47.17	2.463	0.378
3.6163	106.22	1.574	38.32	2.354	0.374
3.6163	106.06	1.408	20.64	2.215	0.393
3.5768	105.91	1.260	20.64	2.084	0.378
4.6269	86.80	3.075	200.00	0.000	0.531
4.3938	86.47	2.750	200.00	0.000	0.484

4.1962	86.18	2.460	200.00	0.000	0.446
4.1138	85.92	2.200	194.23	3.082	0.433
3.7526	85.69	1.968	79.98	2.677	0.401
3.7247	85.48	1.760	65.73	2.651	0.390
3.5618	85.29	1.574	44.41	2.399	0.374
3.5618	85.13	1.408	30.22	2.285	0.378
3.5228	84.98	1.260	23.14	2.154	0.376
3.5228	84.85	1.127	16.07	2.014	0.385
4.2901	69.73	2.750	200.00	0.000	0.505
4.0739	69.44	2.460	200.00	0.000	0.460
3.8908	69.18	2.200	200.00	0.000	0.422
3.7266	68.94	1.968	190.39	2.956	0.401
3.5705	68.74	1.760	70.10	2.652	0.388
3.4143	68.55	1.574	58.67	2.514	0.365
3.4143	68.38	1.408	35.88	2.354	0.368
3.4143	68.24	1.260	24.51	2.198	0.372
3.4143	68.10	1.127	18.83	2.074	0.374
3.5850	67.98	1.008	13.17	1.998	0.402
4.0094	56.04	2.460	200.00	0.000	0.482
3.8607	55.78	2.200	200.00	0.000	0.444
3.6871	55.55	1.968	199.97	3.068	0.416
3.5315	55.34	1.760	80.06	2.660	0.396
3.2611	55.16	1.574	66.27	2.596	0.362
3.3121	54.99	1.408	43.36	2.385	0.362
3.4397	54.84	1.260	34.22	2.320	0.369
3.4536	54.71	1.127	25.10	2.153	0.370
3.4677	54.59	1.008	16.01	2.020	0.383
3.5216	54.48	0.901	11.47	1.971	0.404
3.6047	45.06	2.200	94.08	3.014	0.474
3.4426	44.83	1.968	94.08	2.845	0.430
3.3388	44.62	1.760	86.65	2.684	0.398
3.0832	44.44	1.574	68.13	2.590	0.362
3.2879	44.27	1.408	64.44	2.537	0.364
3.3768	44.12	1.260	38.70	2.335	0.369
3.3905	43.99	1.127	31.37	2.222	0.365
3.4822	43.87	1.008	20.40	2.097	0.380
3.4963	43.77	0.901	16.75	1.971	0.379
3.5105	43.67	0.806	13.12	1.921	0.388
3.2294	36.26	1.968	67.50	2.849	0.449
2.9829	36.05	1.760	67.50	2.708	0.398
2.9176	35.87	1.574	67.50	2.603	0.367
3.1114	35.70	1.408	67.50	2.549	0.363
3.1955	35.55	1.260	43.80	2.359	0.362
3.2819	35.42	1.127	34.95	2.286	0.362
3.3706	35.30	1.008	26.12	2.130	0.366
3.3843	35.19	0.901	17.33	2.002	0.374
3.4757	35.10	0.806	14.40	1.928	0.382
3.4899	35.01	0.721	11.48	1.902	0.392
2.8816	29.19	1.760	60.14	2.755	0.420
2.8272	29.01	1.574	60.14	2.661	0.385

2.9036	28.84	1.408	60.14	2.600	0.370
3.0154	28.69	1.260	55.36	2.441	0.360
3.0969	28.56	1.127	38.70	2.282	0.358
3.1806	28.44	1.008	29.22	2.171	0.360
3.2666	28.33	0.901	22.13	2.071	0.365
3.4439	28.24	0.806	17.43	1.949	0.378
3.3788	28.15	0.721	12.73	1.890	0.382
3.4313	28.08	0.645	12.73	1.848	0.378
2.6059	23.52	1.574	55.86	2.683	0.399
2.6770	23.35	1.408	55.86	2.626	0.375
2.8109	23.21	1.260	55.86	2.488	0.360
2.8869	23.07	1.127	40.47	2.301	0.356
3.0312	22.95	1.008	34.74	2.238	0.355
3.1131	22.85	0.901	25.22	2.086	0.357
3.1973	22.75	0.806	17.63	1.969	0.367
3.2837	22.67	0.721	13.85	1.894	0.376
3.3725	22.59	0.645	11.97	1.865	0.384
3.4249	22.52	0.577	11.97	1.832	0.381
2.3498	18.97	1.408	37.26	2.531	0.389
2.5830	18.82	1.260	37.26	2.429	0.376
2.7121	18.68	1.127	37.26	2.327	0.360
2.8477	18.56	1.008	35.71	2.246	0.355
2.9901	18.46	0.901	28.02	2.130	0.356
3.1396	18.36	0.806	21.90	2.044	0.363
3.2245	18.28	0.721	15.81	1.914	0.371
3.3117	18.20	0.645	14.29	1.855	0.373
3.4012	18.13	0.577	12.78	1.830	0.379
3.4150	18.07	0.516	11.27	1.821	0.383
2.3059	15.31	1.260	31.68	2.425	0.387
2.5725	15.17	1.127	31.68	2.357	0.382
2.7011	15.05	1.008	31.68	2.283	0.366
2.8362	14.95	0.901	31.68	2.195	0.355
2.9780	14.85	0.806	24.25	2.059	0.357
3.1269	14.77	0.721	18.10	1.954	0.366
3.2114	14.69	0.645	14.43	1.874	0.372
3.2982	14.62	0.577	13.21	1.833	0.374
3.3873	14.56	0.516	12.00	1.817	0.380
3.4011	14.51	0.461	12.00	1.789	0.374

OPTIMUM WEIGHT/DISPLACEMENT RATIO:0.355

APPENDIX E

METHOD OF PROGRAM CONVERSION TO ACCOMMODATE STRAIN-HARDENING MATERIALS

APPENDIX E

OUTLINE OF METHOD FOR CONVERTING TO AN OPTIMIZATION PROGRAM FOR STRAIN-HARDENING MATERIALS

Strain-hardening metals differ from elastic perfectly plastic metals, in that above the yield point, the stress is not a constant value as strain increases, but increases (usually at progressively slower rates) as strain increases. This means that true plastic flow is never achieved; the metal continues to retain a modulus of some value, albeit smaller than the original constant value. This results in a type of combination elastic-plastic buckling failure in stiffened cylindrical shells, termed inelastic failure. It is particularly important to analyze subersibles constructed of strain-hardening materials with strain-hardening analyses[11,13,20]. Normally, a submarine hull is designed to have some plastic yielding (i.e., initially beginning in the hull adjacent to the frame flanges) somewhere between operating and collapse depth; indeed, there must be yielding prior to collapse depth in order for the hull to crush there. With ideally plastic materials, once the yielding point is reached, generally speaking, buckling failure is ruled out (or it would have occurred earlier, due to hull geometry). With strain-hardening materials, however, once the hull begins to plastically deform,

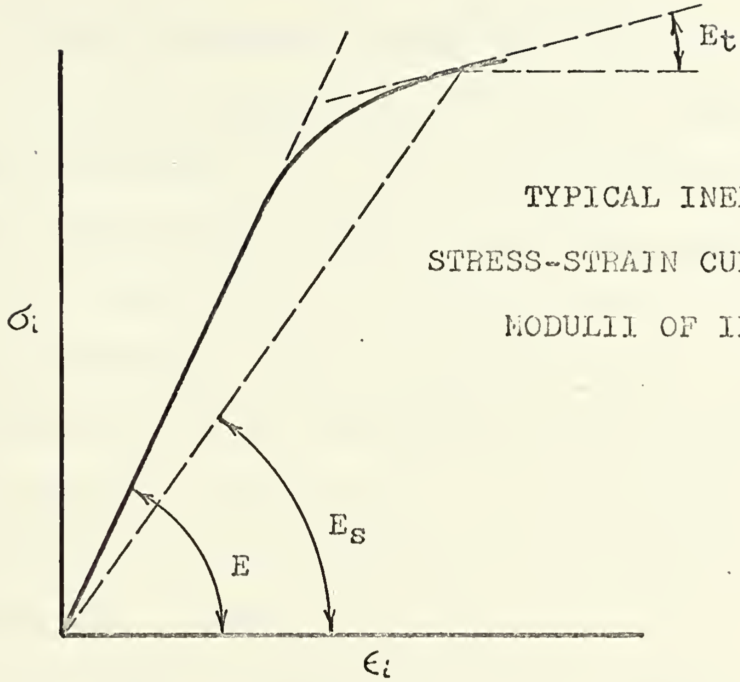
buckling failure is not ruled out. In fact, there is a strong possibility that the buckling failure (which will occur) will happen at a lower pressure than the yield failure calculated by an ideally plastic analysis, due to the reduced metal modulus. Thus, to use an ideally plastic collapse pressure analysis on a strain hardening material might give dangerously overoptimistic failure predictions, particularly if the metal has a very high yield point.

For convenience in analyses, strain hardening materials' stress-strain curves are characterized not only by E, but also by E_t (ETAN, tangent modulus) and E_s (ESEC, secant modulus). See figure 32.

Because with this type of stress-strain curve, the moduli are always dependant upon the stress state when above yield stress, it is necessary for all strain-hardening collapse pressures to be computed using an iterative process. It is generally the approach to the solution of this process, and an example pressure analysis by Reynolds[20] that will comprise the rest of Appendix E.

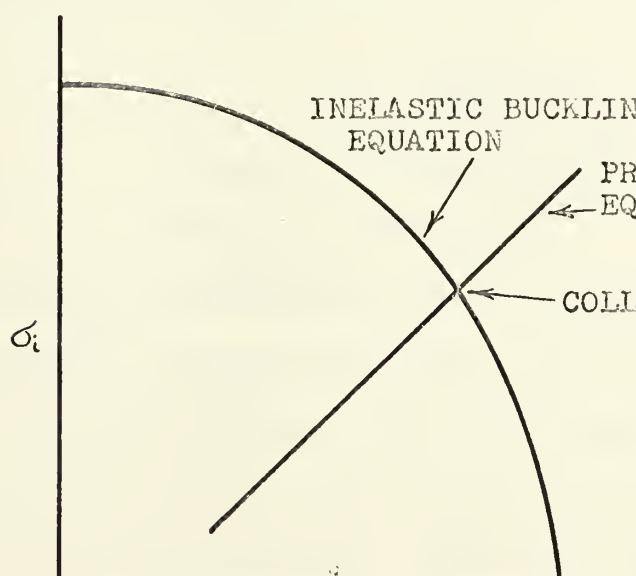
Essentially, the critical collapse pressure is obtained when the buckling equation (depending upon buckling mode, references 11, 13, or 20) is solved simultaneously with the pre-buckling equation (stress intensity as a function of pressure), where:

Figure 32



TYPICAL INELASTIC STRESS-STRAIN CURVE SHOWING MODULII OF INTEREST

Figure 33



GRAPHICAL DETERMINATION OF INELASTIC BUCKLING PRESSURE

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Stress intensity= $\sigma_i = \sqrt{\sigma_x^2 + \sigma_s^2 - \sigma_x \sigma_s}$ (see figure 33)
Either V-G or S-P stress theories could be used to calculate the stress intensity, S. V-G is less accurate, but S-P might give convergence problems, depending upon hull geometry and depth at which stress is calculated.

One difficulty, that of finding a way of describing a strain-hardening stress-strain curve with a minimum of input data, is solved in reference 23. In this method, the entire curve may be approximated with extreme accuracy by using only four inputs (see figure 34): E, σ_y, σ_a and σ_b . By manipulating some of the Romberg-Osgood equations, it is possible to obtain E_t and E_s, given any value of stress, via the following expressions:

$$n = 1 + \frac{0.3853}{\log_{10}(\sigma_a/\sigma_b)}$$

$$E/E_t = 1 + 0.42857n(\sigma/\sigma_a)^{n-1}$$

$$E/E_s = 1 + 0.42857(\sigma/\sigma_a)^{n-1}$$

This will be assumed to be the content of a subprogram called Subroutine ROMOS (see figure 35).

Both the pre-buckling equilibrium equation and the buckling equation will be approximated in the region of interest with straight lines. This, and some of the following methods of determining the intersection of the two equations, were patterned generally after similar methods used in reference 22.

METHOD OF OBTAINING ROMBERG-OSGOOD

INPUT PARAMETERS FOR COMPUTING E_s AND E_t

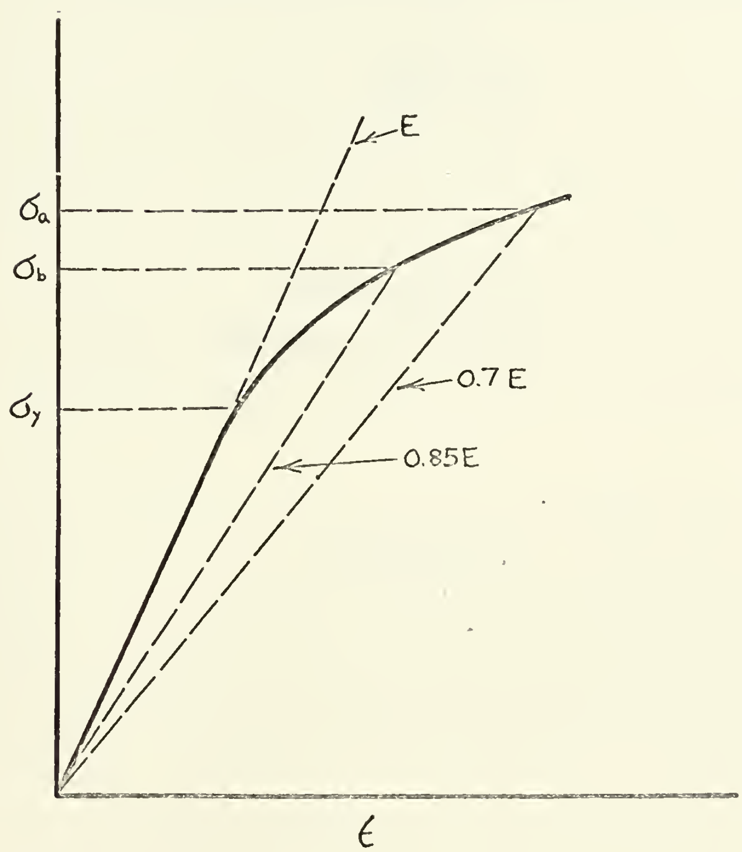

Figure 34

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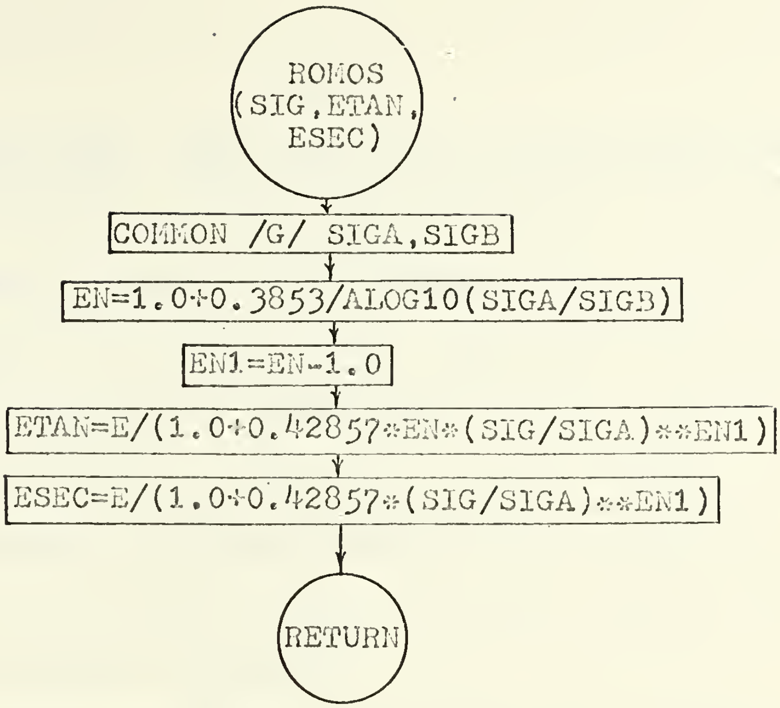


FLOW DIAGRAM:

SUBROUTINE ROMOS

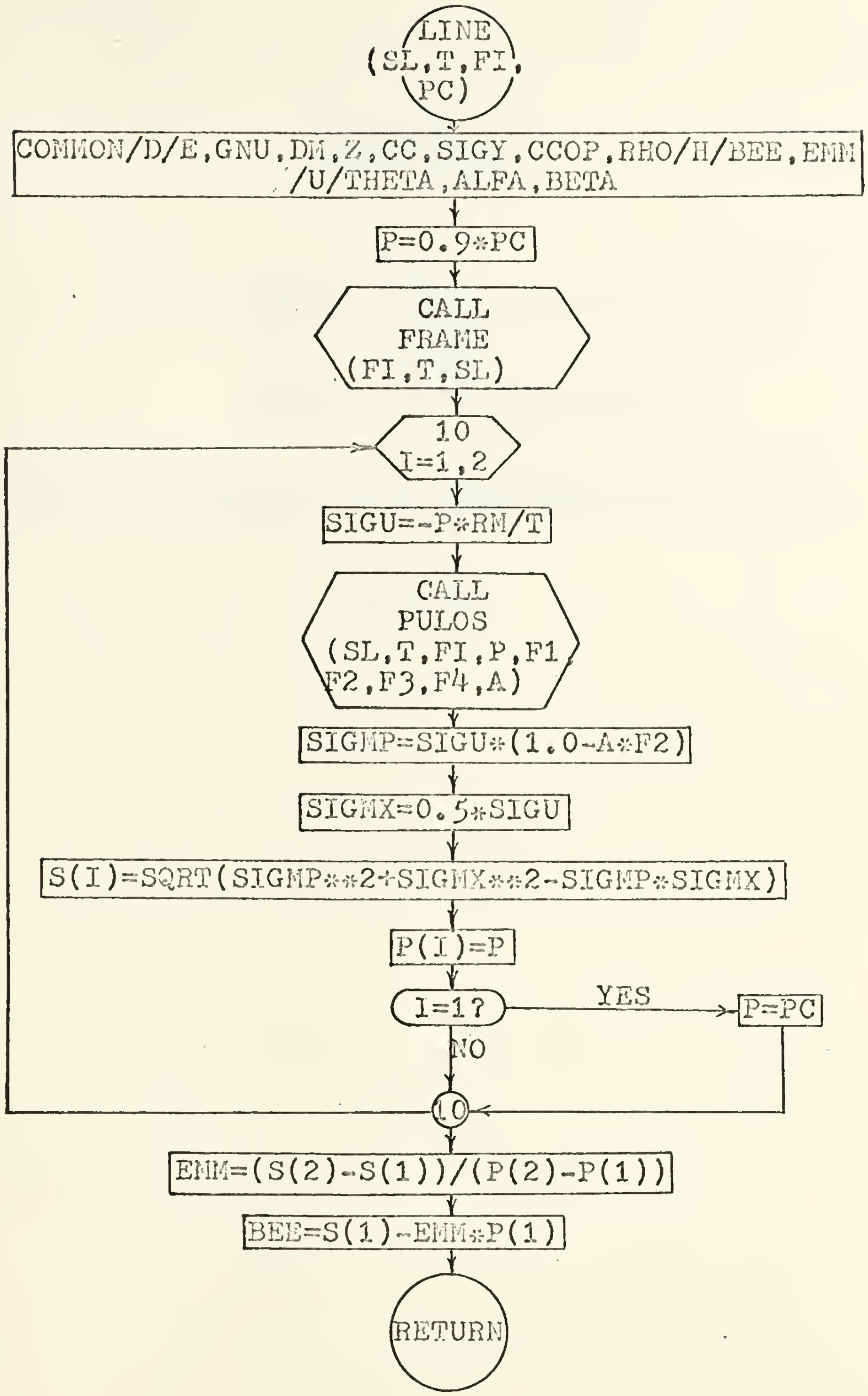
Figure 35

Subroutine LINE. This subroutine computes EMM, the line slope, and BEE, the line stress intercept of the pre-buckling equation, using the S-P analysis (see figure 36). The values given by LINE will change only with scantling changes. This subprogram would constitute the only deviation of the main flow program from the perfectly plastic case. It would be placed in the main program directly after point 3 (i.e., prior to the RMLDS call). It would also be used within THRNS whenever T changes.

The following discussion will involve the solution of the asymmetric inelastic buckling mode as developed by Reynolds in reference 20. The same general iterative process would be utilized in the solution of the other two modes of hull failure.

Function PCRP. See figure 37. This small subprogram merely computes asymmetric inelastic buckling pressure, using as inputs ETAN and ESEC (computed by ROMOS), T, SL, and PCRE1 (computed as PCRE by the elastic portion of PCRP's calling program, RMLDS). The pressure is computed using Reynolds' equations outlined in reference 20 (for a simplified presentation, however, see reference 12, which gives the axisymmetric mode also).

Subroutine RKPT. See figure 38. This subprogram obtains the intersection of the pre-buckling equilibrium equation with the line determined by the two input pressures (PMS1



FLOW DIAGRAM: SUBROUTINE LINE

Figure 36

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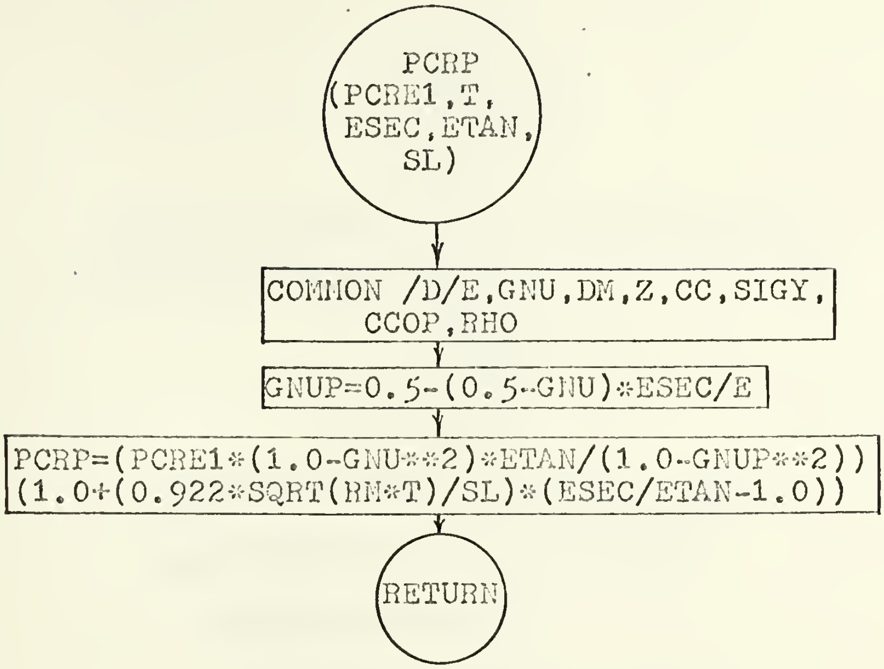
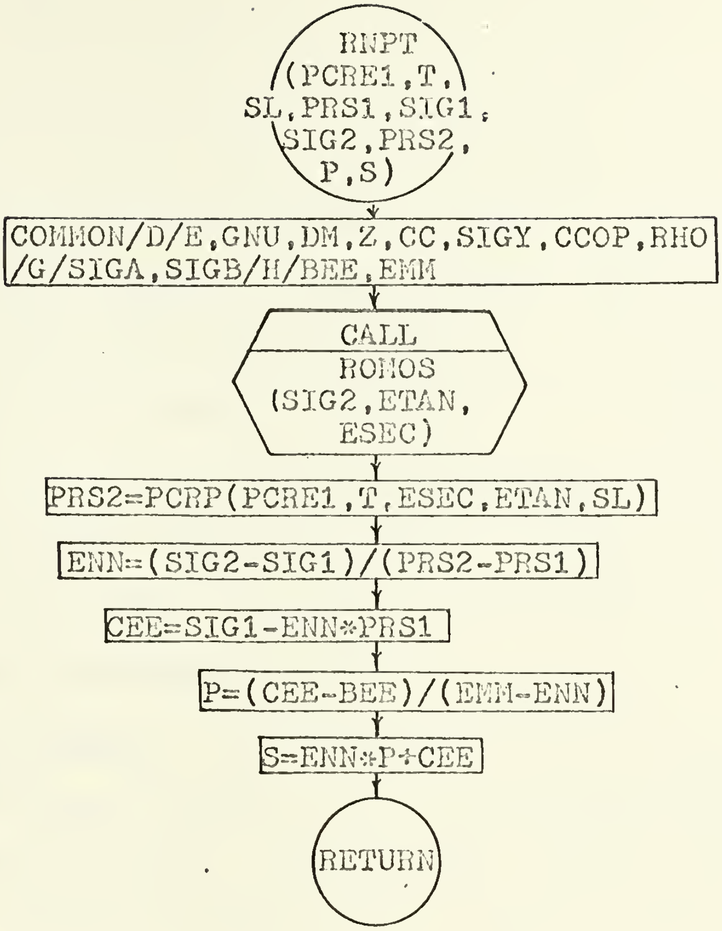


FLOW DIAGRAM:

FUNCTION PCRP

Figure 37



FLOW DIAGRAM:

SUBROUTINE RNPT

Figure 38

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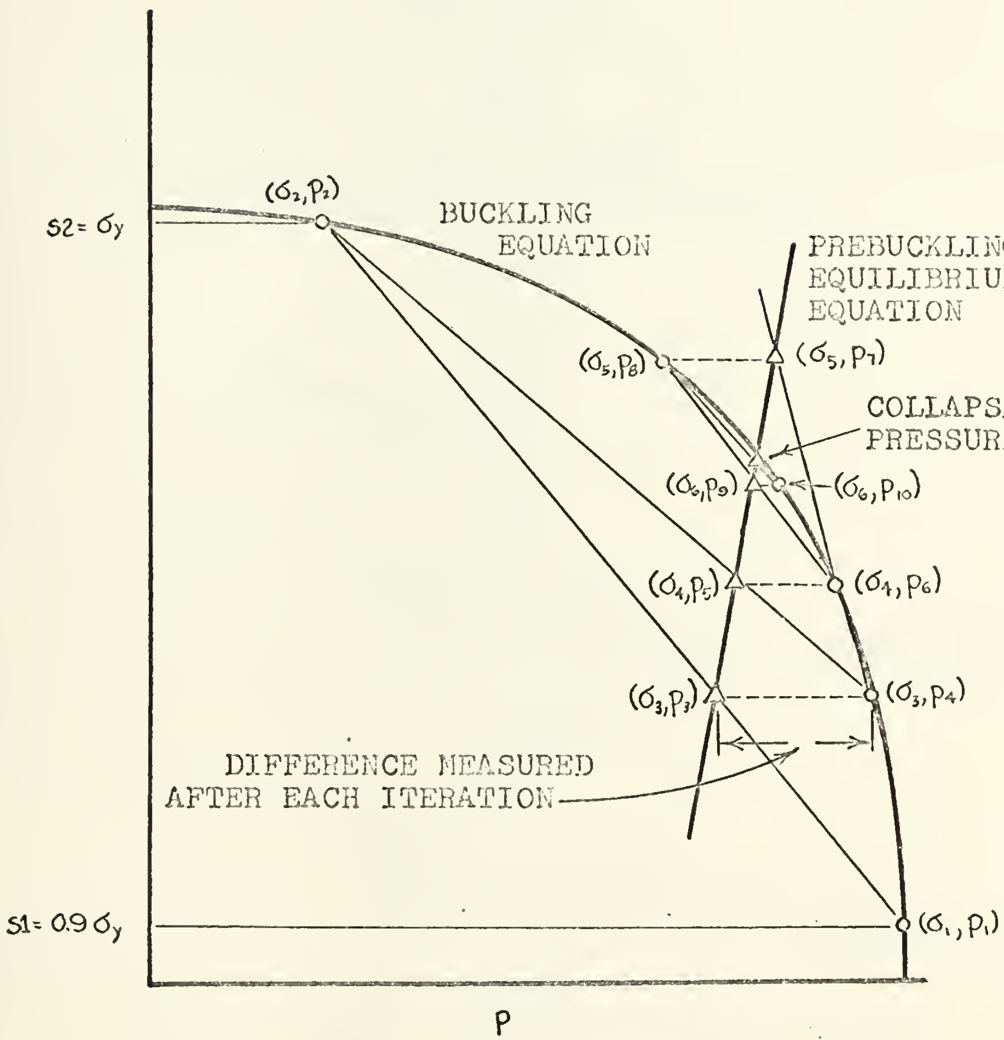
and PRS2) and stress intensities (SIG1 and SIG2) in terms of the intersection coordinates, S and P. The values EMM and BEE, which determine the pre-buckling equation, are piped into RNPT via a COMMON statement, along with other normal program data inputs, including SIGA and SIGB. Input arguments include SIG1, SIG2 and PRS1, which along with the inelastic buckling pressure at point 2, PRS2 (computed by RNPT from input PCRE1), describe end points of the buckling equation line approximation.

Function RNLDS. This subprogram is the same as RNLDS used in the ideally plastic case (i.e., computation of elastic lobar buckling pressure, PCRE1), with the addition of a programmed iteration (figure 39, as used in reference 22) for the inelastic portion. The first two stresses used in the iteration are SIGY and 0.9*SIGY. When the difference of computed inelastic failure pressure and assumed stress at which failure will occur in the next iteration becomes less than one half of one per cent of the present predicted failure pressure, convergence is assumed. The iteration sequence may be followed by using the diagram of the inelastic portion of RNLDS (figure 40) with the plot (figure 39).

As noted before, identical procedures would be followed in the axisymmetric and general instability cases. The main program would be unaltered with the exception, again as noted

ITERATIVE METHOD USED TO CONVERGE
ON INELASTIC COLLAPSE PRESSURE . . .



COMPUTED INELASTIC FAILURE PRESSURES

ASSUMED STRESS AT WHICH FAILURE WILL
OCCUR IN THE NEXT ITERATION

NOTE: METHOD TAKEN FROM REFERENCE 22

Figure 39

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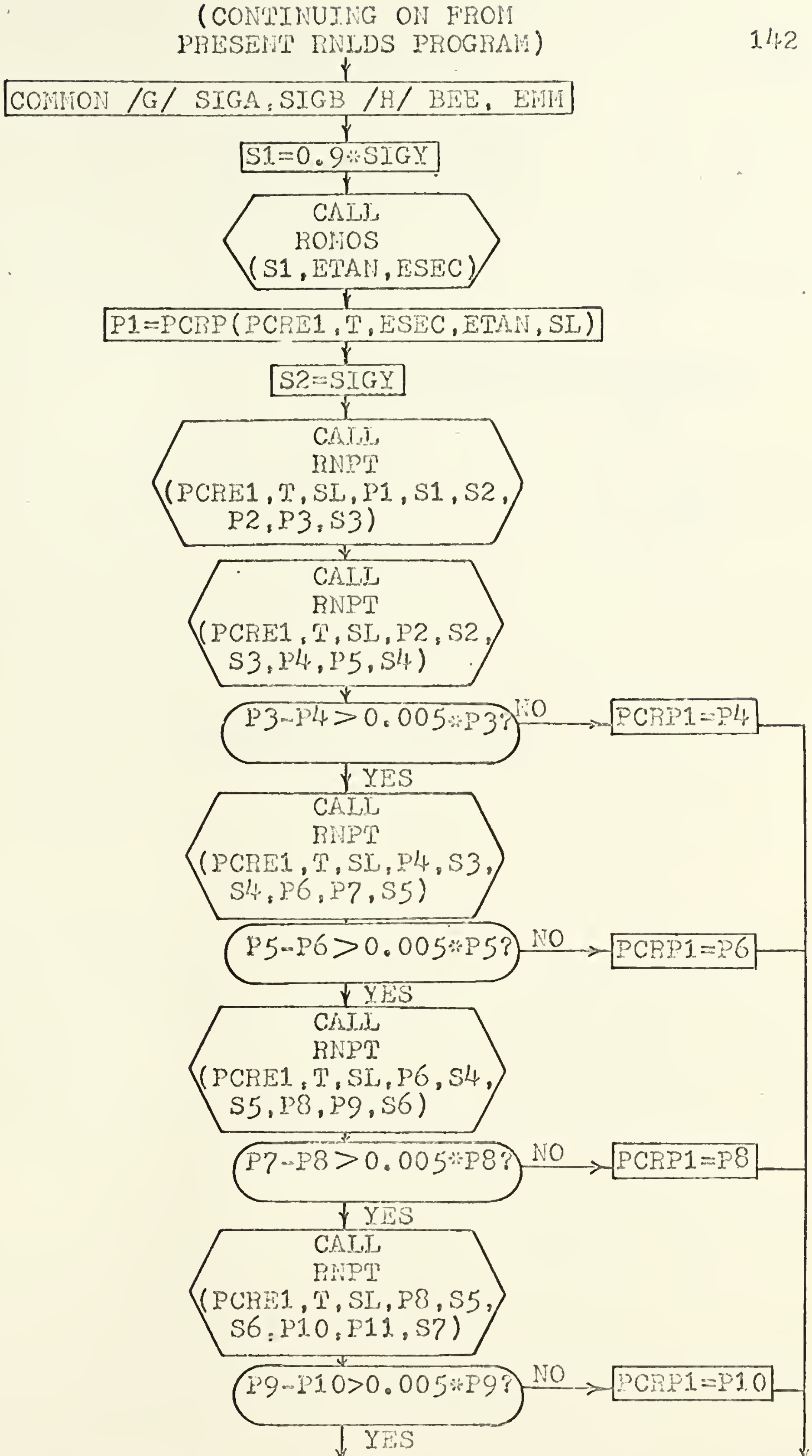

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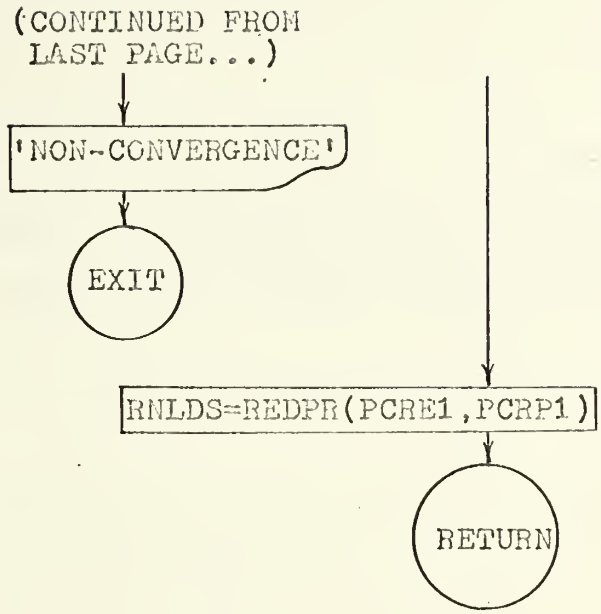
Figure 40



(Figure 40 Continued)

before, of insertion of subprogram SIZE. It is obvious that an optimization using the inelastic analysis would take much longer (perhaps by five or six times) than the ideally plastic one.

before, of insertion of subprogram LINE. It is obvious that an optimization using the inelastic analysis would take much longer (perhaps by five or six times) than the ideally plastic case.

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