A CORRELATION STUDY OF SOME FACTORS EFFECTING SUBMARINE DETECTION BY DESTROYER MOUNTED SONARS

By

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THESIS

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ABSTRACT

A FORTRAN IV computer program was employed to conduct a statistical analysis of data collected during fleet anti-submarine warfare exercises. The object of the investigation was the identification of those variables which had greatest influence on a destroyer's ability to detect a submarine under certain conditions.

The variables were treated as a random vector arising from one of two multivariate normal populations with common covariance matrix. An artificial regression relation was formulated to facilitate development of a linear discriminant function in a subset of those variables found to be of dominant importance. This latter subset was identified by examination of multiple correlation coefficients.

The discriminant function was found to be seventy five per cent effective in classifying the experimental data correctly.

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I. INTRODUCTION

The sonar-equipped destroyer is the primary surface vessel employed by the U.S. Navy in antisubmarine warfare. Detection of the submarine necessarily precedes any offensive action the destroyer can take against it. It was the objective of the author to identify those variables which had the greatest influence on the effectiveness of a destroyer in detecting a submerged submarine during fleet exercises. Consideration was limited to detection capabilities while using sonar equipment in the active mode.

Reference 2 lists many of the large number of variables which influence the effectiveness of a destroyer in submarine detection. Certain of these variables can be regarded as control variables in the sense that the value of those variables can be chosen and fixed by the destroyer commander with a certain degree of freedom. For example, the destroyer's speed may range from zero to thirty knots. Furthermore, a particular destroyer can be equipped with one of several types of sonar system, and can be manned with differing numbers of sonar technicians of various skill levels. These latter variables serve as examples of control variables whose value can be adjusted by agencies exogenous to the ship. Similar comments can be made concerning the submarine involved in an encounter with a destroyer. No attempt was made by the author

to consider any variable as a control variable; all were regarded as completely random variables.

Based on personal experience in the Destroyer Force, the author chose a set of twenty-two variables whose examination was expected to be most fruitful for the purpose stated. Four groups of variables were considered: environmental factors, destroyer characteristics, submarine characteristics, and tactical factors. Table I lists the variables included in each group. Qualitative variables, such as destroyer hull type were assigned numerical codes. All values were recorded at the time detection was achieved, or - in those cases where detection did not occur - when the submarine had approached to a specified range. This range was specified by the U.S. Navy Antisubmarine Warfare Data Center (NADAC) as a standard for each sonar system. The standards are listed in Ref. 2.

Table I. Variables by Group

Environmental	Destroyer	Submarine	Tactical
Geographic location	Hull type	Hull type	Relative speed between opponents
Present weather	Hull number	Hull number	Distance between opponents
Sonic layer depth	Sonar type	Depth	Aspect angle of submarine
Wind speed	Speed	Speed	Relative bearing from destroyer to submarine
Sea state	Self noise		
Surface turbulence	Technician complement		
Bathythermograph profile type			
Water depth			

The data examined was supplied by NADAC on request, having been collected during sixteen fleet exercises conducted in the Atlantic and Pacific Oceans during the period 1962 to 1966, inclusive. The destroyer types considered included DLG, DL, DDG, DDR, DD, DEG and DE. Each destroyer was equipped with one of the following AN/SQS sonar systems: 23, 26, 29, or 30. Submarine types encountered included all types in the U.S. fleet during this period, ranging from the World War II vintage fleet submarine to the nuclear-powered ballistic missile submarine.

Each observation of the twenty-two variables was regarded as a random vector, $\underline{x}$, arising from one of two multivariate normal populations with common covariance matrix: $N(\underline{m}_1, V)$, or $N(\underline{m}_2, V)$. The former distribution representing an encounter which resulted in detection of the submarine by the destroyer, and the latter distribution applying otherwise.

Maximum likelihood estimators were used for the mean vectors, covariance matrix, and all other parameters employed in the analysis.

a FORTRAN IV computer program was prepared for analysis of the data and was utilized with the U.S. Naval Postgraduate School IBM-360 computer. A discussion of the program, a flow chart, and the program appear below.

In all, observations from ninety two encounters were included in the analysis. Twenty-seven encounters resulted in submarine detection by the destroyer, and sixty-five did not.

II. DISCUSSION

A. PRINCIPAL STEPS OF THE ANALYSIS

The following discussion is concerned with the principal concepts employed in the analysis, without regard to the computer programming techniques employed. A full discussion of the program appears later.

The linear discriminant function

$$\underline{x}'_j \; V^{-1} \; (\underline{m}_1 - \underline{m}_2) - \tfrac{1}{2}(\underline{m}_1 + \underline{m}_2)' \; V^{-1} \; (\underline{m}_1 - \underline{m}_2) = 0 \qquad (1)$$

is related to the artificial regression proposed by Fisher in Ref. 5

$$y_j = \underline{b}' \; (\underline{x}_j - \underline{c}). \qquad (2)$$

In the foregoing, $\underline{x}_j$ is the observation vector with j = 1, 2, ... , $n_1 + n_2$; there are n_1 observations in the first population with parameters ($\underline{m}_1$, V) and n_2 observations in the second population. The artificial dependent variable, y_j, takes the value $n_2/(n_1 + n_2)$ if $\underline{x}_j$ is from population one, and the value $-n_1/(n_1 + n_2)$ otherwise. The vector $\underline{c}$ is a constant vector with each element being the weighted grand mean of observations, i.e., $(n_1 \, \bar{\underline{x}}_1 + n_2 \, \bar{\underline{x}}_2)/(n_1 + n_2)$.

Equation (2) yields artificial regression coefficients, $\underline{b}$, which are proportional to the discriminant function coefficients, V^{-1} $(\underline{m}_1 - \underline{m}_2)$, of equation (1). (This may be verified by solution of the normal equations in the components of $\underline{b}$.)

Based on the above relationship between the discriminant and regression functions, equation (2) was utilized to determine a subset of variables which could be regarded as being of principal importance in the discriminant function. The technique employed is discussed in Ref. 7. A multiple correlation coefficient, R_{11}, was calculated from equation (1) using the full set of twenty-two independent variables. Then, each independent variable, x_i, was eliminated in iterative fashion and a corresponding multiple correlation coefficient, R_{2i}, was calculated based on the remaining twenty-one independent variables. The significance of each of the twenty-two variables in the regression was then tested according to the following scheme. If the following relationship was satisfied, then the deleted variable, x_i, was discarded as contributing little to the regression:

$$q(R_{11}^2 - R_{2i}^2)/(1 - R_{11}^2)(p_1 - p_2) \leq F \quad . \tag{2a}$$

In this relationship p_1 was the number of variables in the original regression (22), p_2 was the number in the reduced set (21), F was the $F_{.05}$ variate with $(p_1 - p_2)$ and q degrees of freedom, respectively; q had the value $(n_1 + n_2) - (p_1 + 1)$.

The above technique revealed that all but six of the original variables contributed little to the regression, and - therefore - to the discriminant function. The value of R_{11} was found to be 0.646 for the original variable set and had the value 0.528 for the six significant variables. The latter

value was calculated from a regression based on these six variables alone. That is, with a seventy-three per cent reduction in the number of independent variables, the correlation coefficient was reduced by only eighteen per cent. Table II lists the six significant variables in order of importance, the value of the correlation coefficient for the set of twenty-one variables remaining upon deletion of each of the significant variables, and the reduction in the multiple correlation coefficient which resulted upon deletion of the variable.

With the six significant variables thus identified, a linear discriminant function was constructed, and its properties tested. The specific form of equation (1) used at this point was

$$\underline{x}_j' \, \underline{d} - \frac{1}{2}k = 0, \qquad (3)$$

where $\underline{x}_j$ is the j^{th} 6 X 1 observation vector, $\underline{d}$ is the vector of discriminant coefficients and k is a constant. The relation between equations (1) and (3) is given by

$$\underline{d} = V^{-1} (\underline{m}_1 - \underline{m}_2)$$

$$k = (\underline{m}_1 + \underline{m}_2)' \, V^{-1} (\underline{m}_1 - \underline{m}_2) \quad .$$

With the value of all parameters calculated, equation (3) took the following form:

$$-0.165x_1 + 0.015x_2 + 0.411x_3 - 0.020x_4 + 0.506x_5 - 0.008x_6 \geq 8.28 \; . \qquad (4)$$

Table II. Significant Variables.

i	x_i	R_{2i}	Per cent Reduction in Correlation
5	Type of weather	0.565	12.5
2	Submarine hull number	0.604	6.5
6	Sonic layer depth	0.606	6.2
1	Geographic location	0.608	5.9
3	Submarine hull type	0.612	5.3
4	Submarine depth	0.617	4.5

An observation would be classified as arising from population one if the relation was satisfied, and as arising from population two otherwise. With the formulation of equation (4), the observations were then to be classified using the equation, and the results achieved to be considered a measure of the validity of the discriminant function.

Anderson discusses in Ref. 1 the probability of misclassification associated with a linear discriminant function. Under the assumption that the cost of an error of misclassification is equal for each of the two possible errors, the probability of misclassification, P(M), is given by

$$P(M) = P(Y \geq \sqrt{a/2}\ )$$

where Y is a standard normal variate and a takes the value of $(\underline{m}_1 - \underline{m}_2)\ V^{-1}\ (\underline{m}_1 - \underline{m}_2)$. From the above equation, it was found that the value of P(M) associated with equation (4) is 0.18.

Upon classifying the ninety-two observations of the six significant variables - using equation (4) - it was found that

eight (30%) of the twenty-seven cases where detection occurred were misclassified as cases of non-detection. Similarly, fourteen (22%) of the sixty-five cases of non-detection were misclassified. Thus, the overall error of misclassification was twenty-five per cent.

In view of the small sample sizes available, particularly in the case of population one (twenty-seven observations), the difference between the theoretical error of misclassification and the observed value was considered acceptable. The fact that the error of misclassification in each population decreased with sample size was viewed as an encouraging sign.

Consequently, it was concluded that the six variables cited above were, in fact, of dominant importance in destroyer - submarine encounters.

It was noted with interest that the character of these variables is such that the destroyer commander has virtually no means available to influence them to his advantage. There seems to be little direct action that could have been taken by the destroyer commander to increase significantly the likelihood of detecting his submerged opponent.

On the other hand, the submarine commander enjoys a distinct advantage. If his depth and location are chosen judiciously, and his submarine is of the preferred design, he may proceed with confidence of not being detected by his surface adversary.

B. THE COMPUTER PROGRAM

The computer program employed in the analysis contained four principal steps. First, the data were read into storage,

constants were defined, and the data were transformed into a form to facilitate subsequent calculations. Second, the multiple correlation coefficient for the twenty-two independent variables was calculated. Third, using the scheme described above, the significant variables were identified. Finally, the discriminant function was formulated and tested. The discussion below is concerned with the mechanics and logic of the entire program. A flow chart of the program is included as Appendix A, and is followed by a listing of the program itself.

The following subprograms which were prepared by the computer manufacturer formed a substantial part of the program: CORRE, ORDER, MINV, MULTR, DMATX. A full discussion of these subprograms may be found in Ref. 6. They are discussed below briefly where necessary to maintain continuity.

After storage requirements and constants are defined in the main program, control is passed to the subroutine DATA. Here, a DO loop is used to read the raw data into storage as a three-dimensional array X(I,J,K). (The indices of X are observation number, variable number, and population number respectively.) As each card of data is read, the values of four variables are calculated and the data are transformed to a one dimensional array XD. The latter step is required by the subsequent use of the manufacturer-supplied subprograms mention above.

The first value calculated is that of the dependent variable y_j which is stored in X(I,23,K). Next the relative bearing from destroyer to submarine is converted to radian measure using the simple subprogram REL. With destroyer

course and submarine true bearing as input variables, REL calculates the relative bearing as the output variable RB. This value is then converted to radian measure and stored in X(I,12,K). Then, relative speed is computed from destroyer speed, submarine speed, and the angle between the two vessels' courses. This value is stored in X(I,5,K). The value of sonic layer depth is stored in X(I,22,K) based on the fleet exercise from which the observation was drawn. Finally, the destroyer's manning level in the sonar technician (ST) rating is used to calculate a parameter reflecting the rate distribution. This is accomplished by calculating a sum of weighted values and storing the result in X(I,13,K). For each destroyer, the number of chief petty officers is multiplied by six, first class petty officers by five, and so on, until a product is formed for all skill levels in the ST rating. The sum of these products is then taken as the measure of manning level for that destroyer.

With the data transformation completed, control returns to the main program and the multiple linear regression equations are formulated. The first subprogram called is CORRE. Given as input variables the number of observations NS, the number of variables M, and the one dimensional array of observations, XD - CORRE computes the vector of means CMEAN, the vector of standard deviations PS, the MxM matrix of sums of cross-products of deviations from means D, and the upper triangular portion of the MxM symmetric matrix of correlation coefficients R.

After the subscripts of the variables are stored in the array ISAVE in the order in which the variables are arranged in the array XD, control is passed to subroutine ORDER.

Given the number of variables, the matrix of correlation coefficients, the subscript number of the dependent variable, and the array ISAVE, subroutine ORDER computes the 22x22 matrix of correlation coefficients among independent variables R_{11}, and the vector of correlation coefficients between the dependent variable and the independent variables R_{12}.

Next, the subroutine MINV is called to calculate the inverse of R_{11}. This matrix plus the vector of means, the vector of standard deviations and the diagonal elements of the matrix D are then passed to the subroutine MULTR. This subprogram computes the vector of regression coefficients W and the multiple correlation coefficient ANS(2). (The vector W and ANS(2) correspond to the vector $\underline{b}$ of equation (2) and the variable R_{11} of the relation (2a) discussed earlier.)

A DO loop is then used to eliminate each independent variable from the regression, and twenty-two multiple corelation coefficients for each combination of twenty-one variables is computed. The test statistic of relation (2a) is computed for each variable x_j and stored in C(J). As each new multiple correlation coefficient is calculated, the value of the test statistic is stored temporarily in TEM3 for each test. This value is then compared to the value of the F(1,69) variate at the 5% significance level. If the variable with subscript J is found to be of little significance, then a zero

is stored in array MK, otherwise MK(J) will contain the value of the subscript.

On completion of the above, the values of the test statistic are arranged in order of magnitude in the array C, and deleted from the array ISAVE.

At this point the number of significant variables is checked. This step is necessitated by the fact that the array TST contains values of the $F(f_1,f_2)$ statistic with $f_1 = 1$ and $6 \leq f_2 \leq 14$. The statistic is used later in the program and the results of the program would be invalid if the number of significant variables were not within the range given for f_2. (The range chosen was based on preliminary trials with the program. It was expected that there would be six significant variables.)

The subprograms ORDER, MINV and MULTR are employed as before with the regression equations being based on the significant variables only. Deletion of the indices of the insignificant variables from the array ISAVE results in the appropriate adjustments in the subprograms. The test statistic TEM3 is compared to the appropriate F variate TST(IT), and the index is incremented. If the test is not satisfied, i.e., if the discarded variables have significance as a group (an unexpected result), a value of one is stored in ICK and this will result in a change of the flow of the program later.

A DO loop follows the above and rearranges the order of the variables in the array XD in preparation for calculation

of the discriminant function. Control then passes to the subroutine DMATX.

Given the number of (significant) variables L, the number of populations LX, the number of observations in each population N, and the array of data XD - DMATX calculates the vector of means XBAR and the common covariance matrix D. The inverse of D is then calculated by subroutine MINV and control passes to subroutine DISC. This latter subroutine is a simple FORTRAN IV equivalent of equation (1).

Next, each observation is classified using the discriminant function just formulated. The value of the function is tested against zero and the observation classified accordingly. For each population, the value of KOUNT is incremented each time an observation is improperly classified.

After all observations have been classified, the value of ICK is checked. As discussed above, if ICK has value one, the previously discarded variables have group significance. If this is the case, a warning to that effect is printed, the index of the most significant deleted variable is added to the array MK, control returns to statement number 45, the new group of significant variables are tested and a new discriminant function is found. If ICK has a value of zero, the discriminant function coefficients are printed and the program terminates.

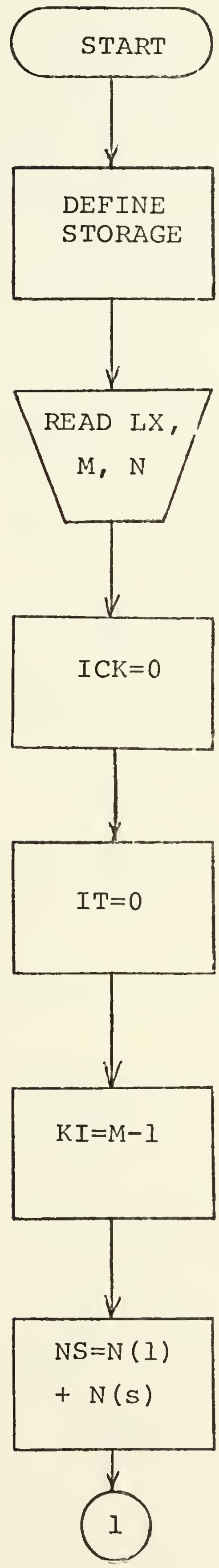
III. CONCLUSION

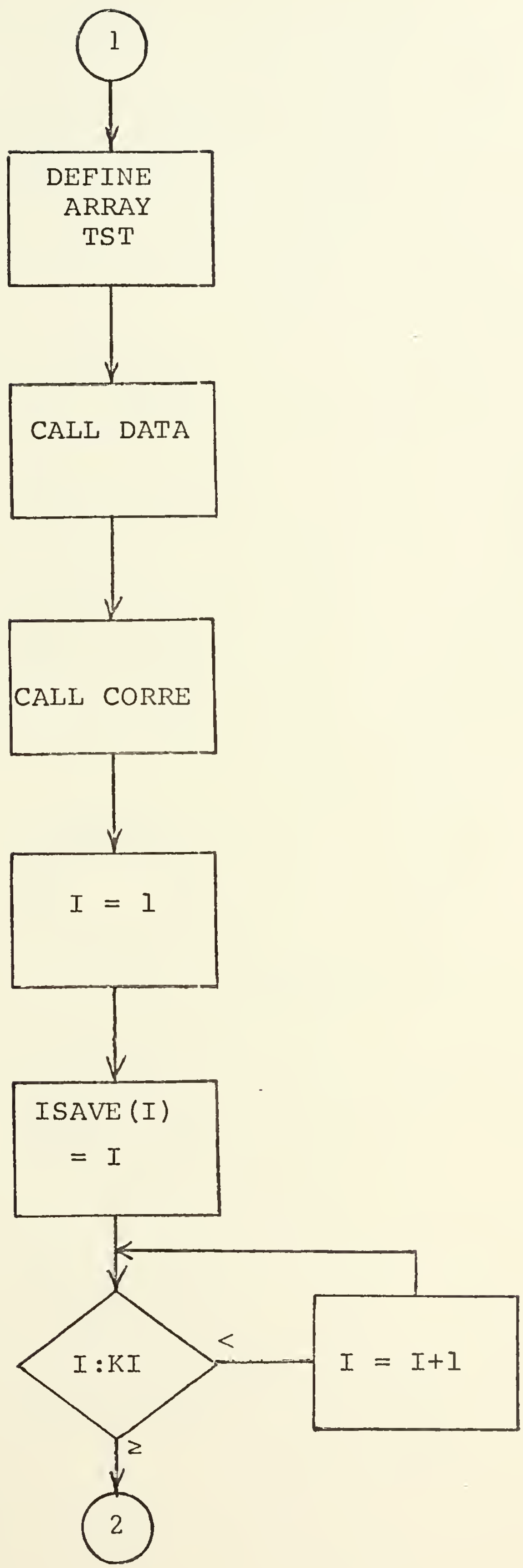
Analysis of data collected during fleet exercises conducted during the period 1962 to 1966 inclusive strongly suggests that the variables listed in Table II had a dominant influence on destroyer - submarine encounters. Specifically, the successful detection of a submarine by a destroyer utilizing a sonar of the type specified in the active mode was influenced more strongly by these six variables than by the remaining sixteen considered.

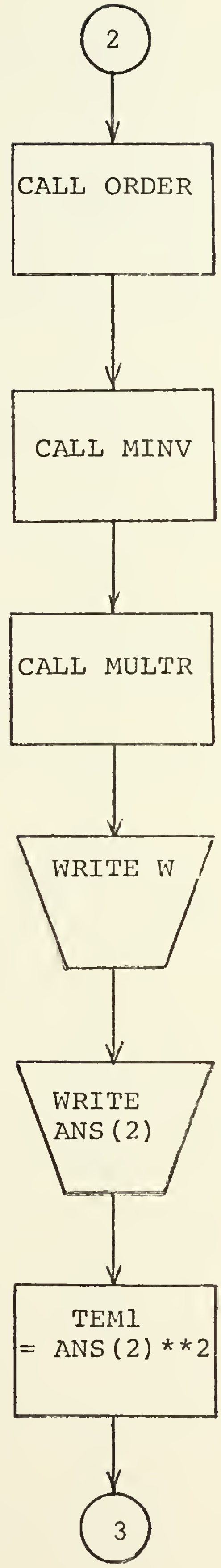
The discriminant function, equation (4), was 75% effective in classifying encounters and should be useful as a predictive tool in assessing the likelihood of detection under known environmental conditions and against known submarine types.

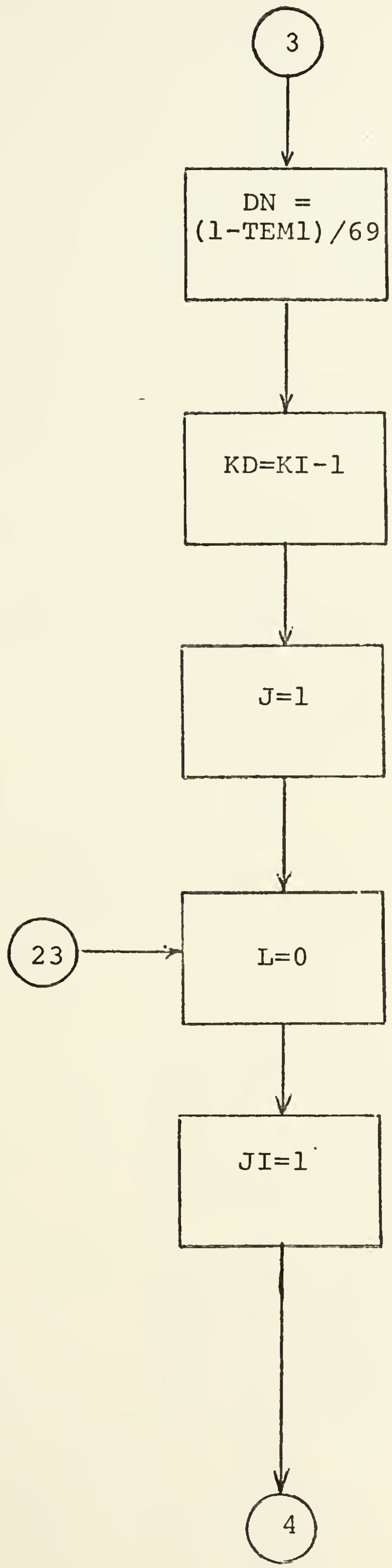
Consideration of the variables found to be significant indicates that the tactical advantage was with the submarine commander during encounters of the type considered.

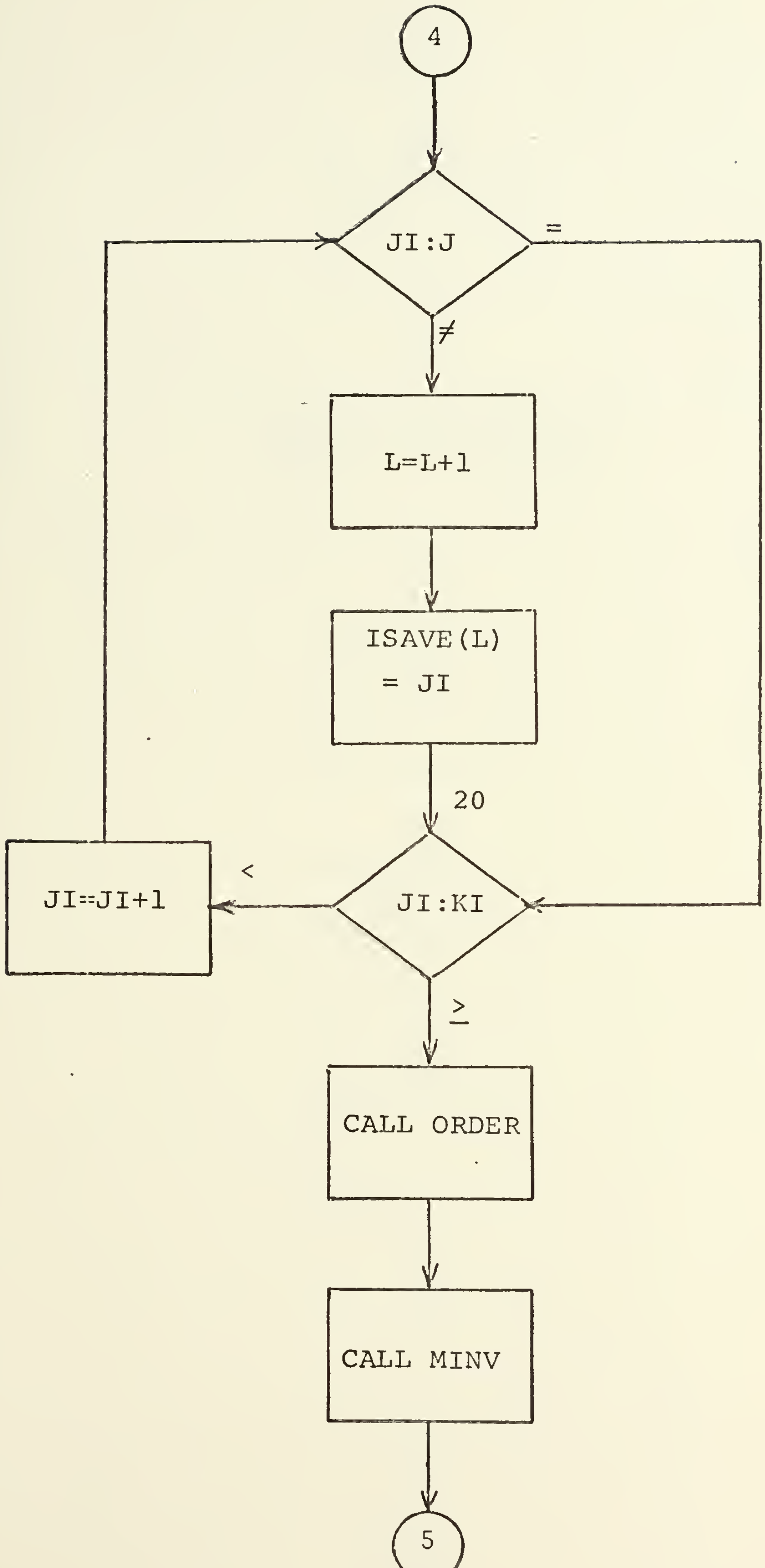
FLOW CHART OF COMPUTER PROGRAM

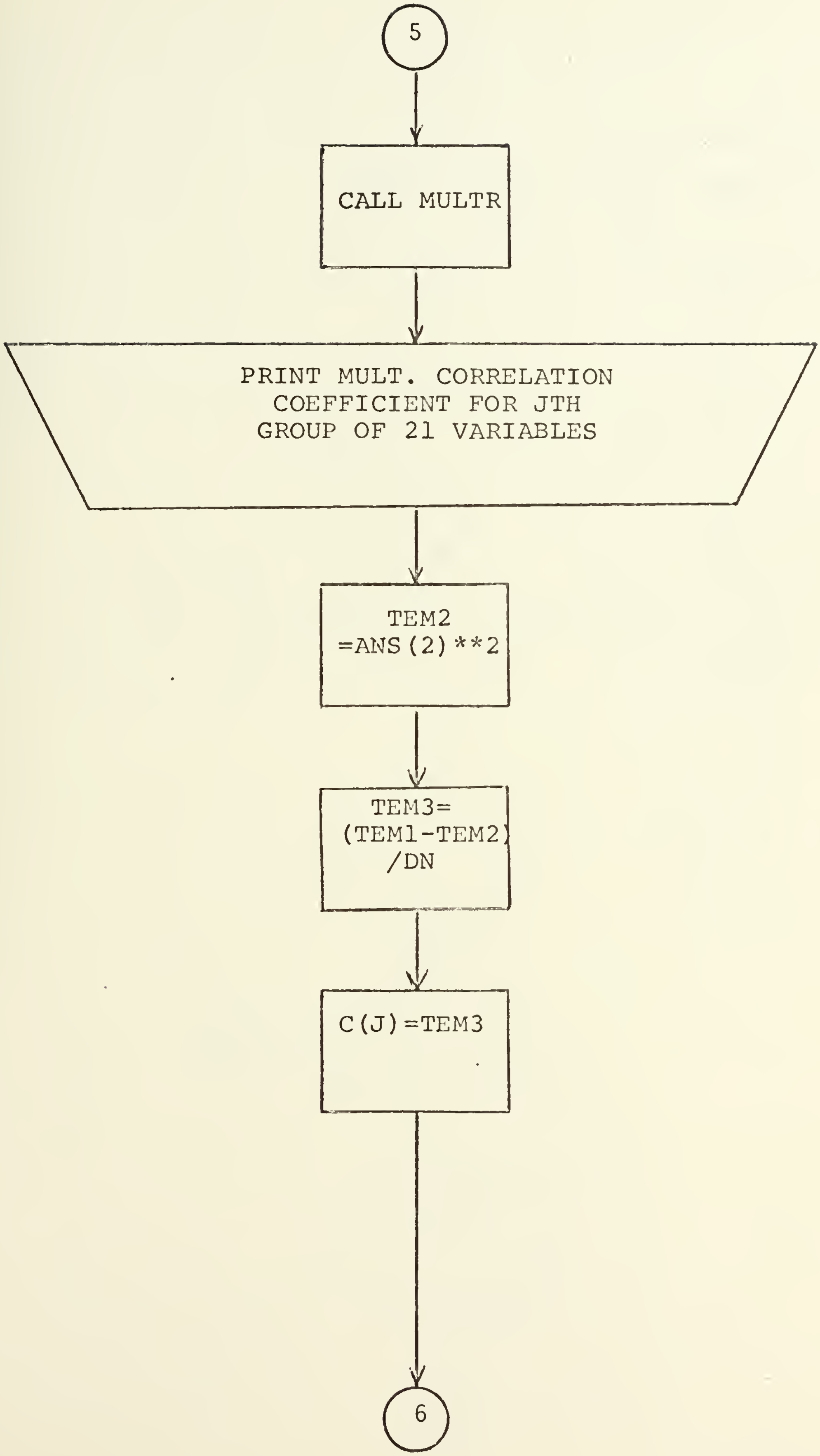


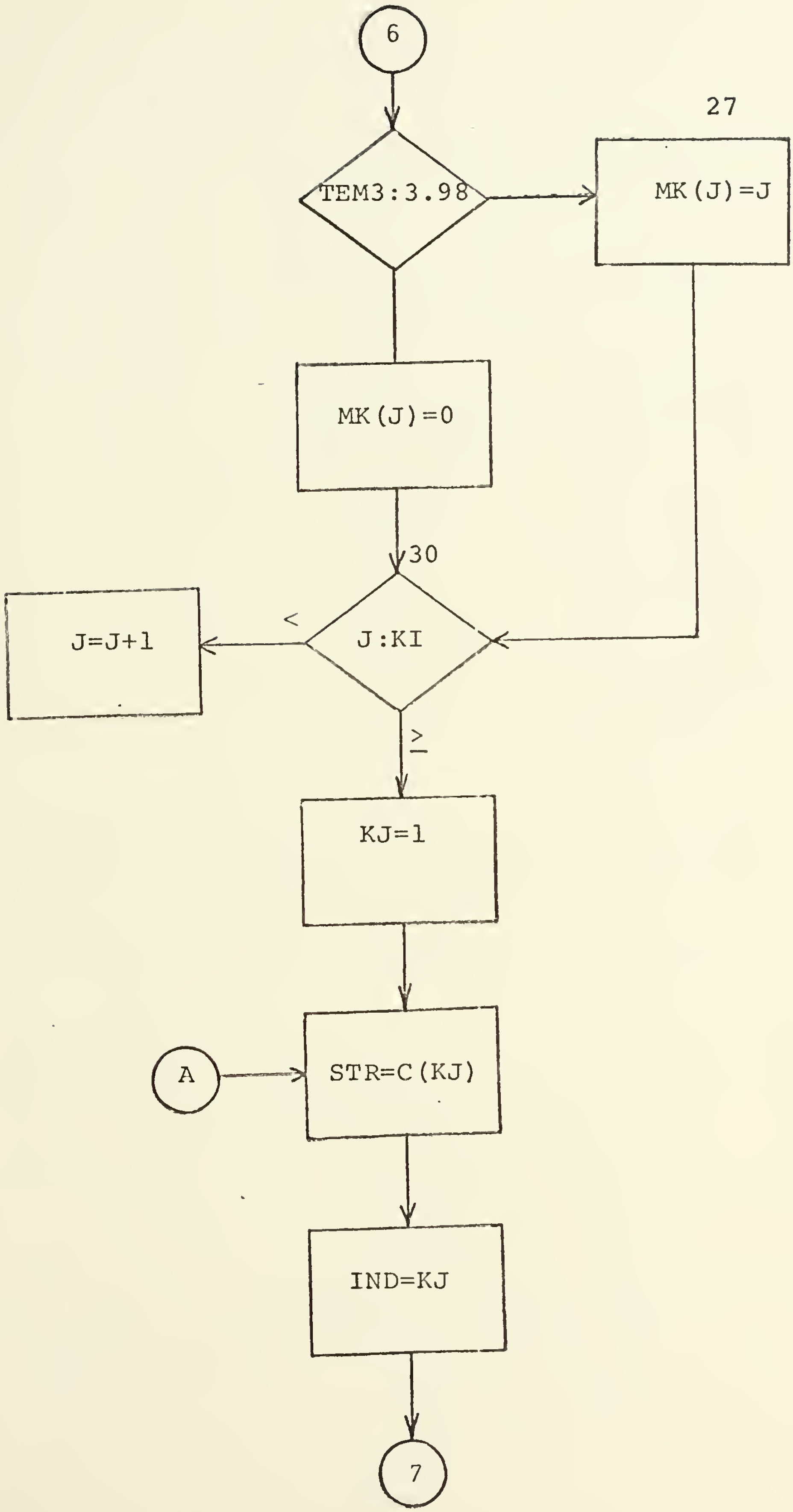

1
DEFINE ARRAY TST
CALL DATA
CALL CORRE
I = 1
ISAVE(I) = I
I:KI
<
I = I+1
≥
2

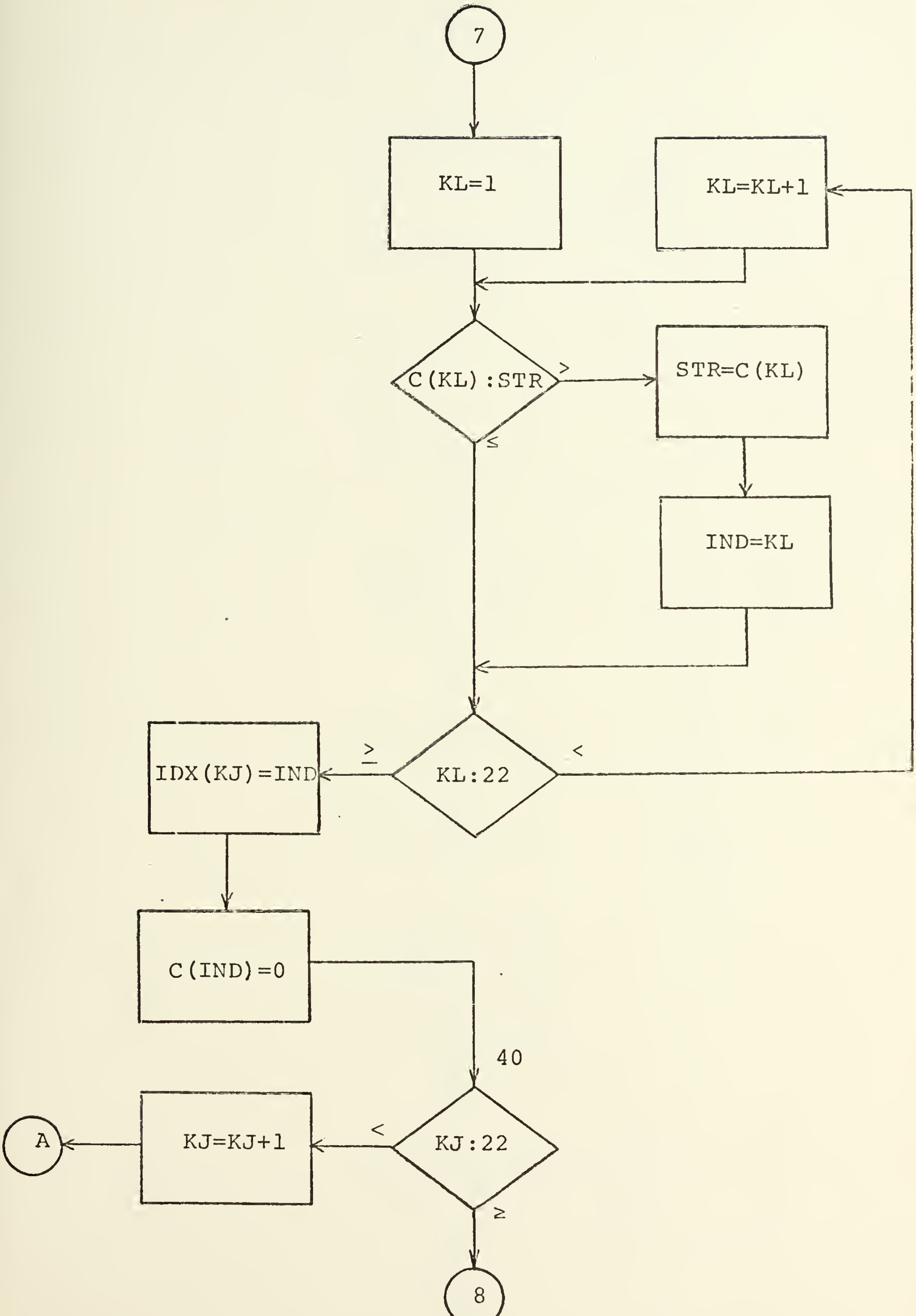

2
CALL ORDER
CALL MINV
CALL MULTR
WRITE W
WRITE
ANS(2)
TEM1
= ANS(2)**2
3

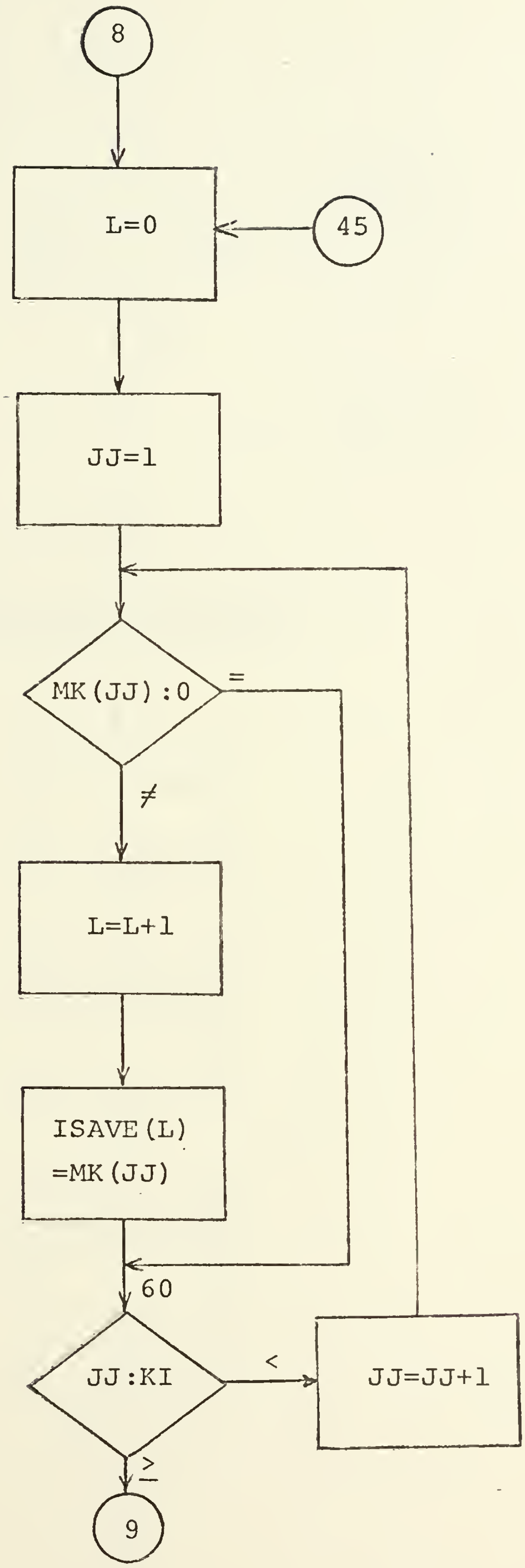

3
DN =
(1-TEM1)/69
KD=KI-1
J=1
23
L=0
JI=1
4

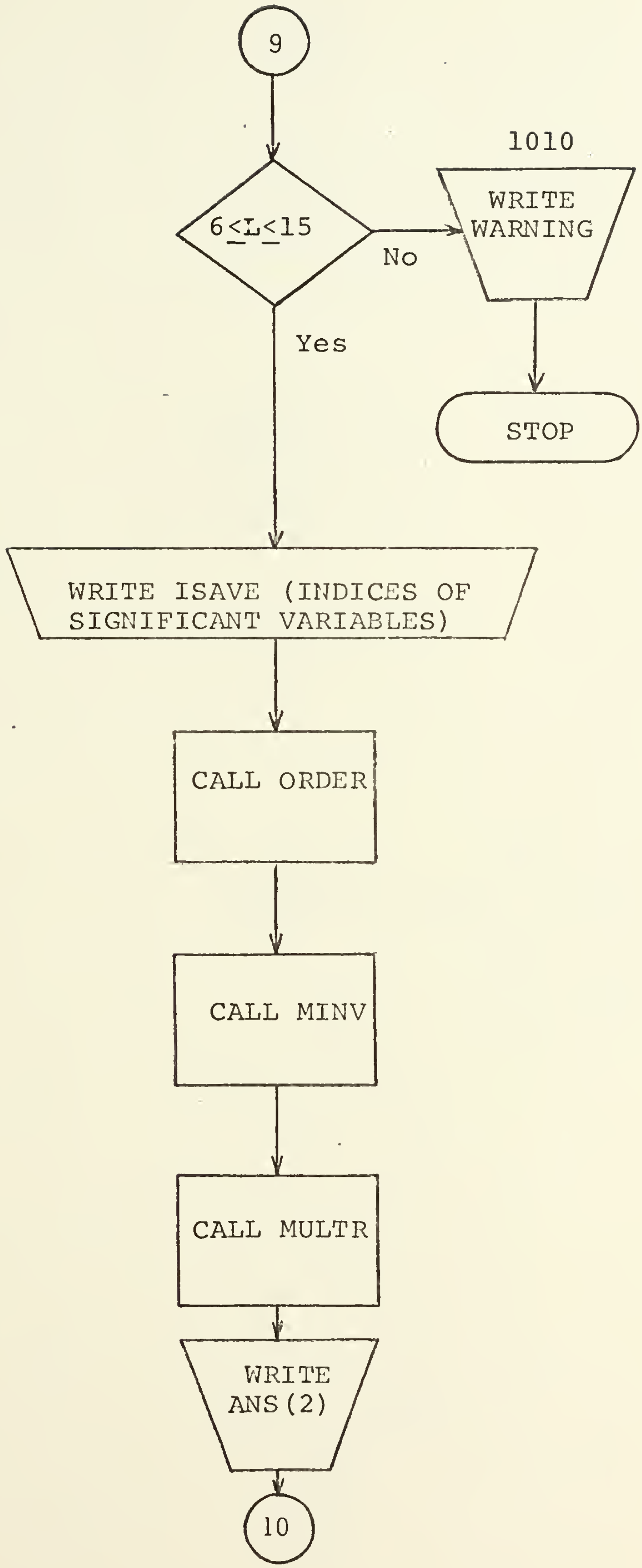

4
JI:J
=
≠
L=L+1
ISAVE(L)
= JI
20
JI=JI+1
<
JI:KI
≥
CALL ORDER
CALL MINV
5

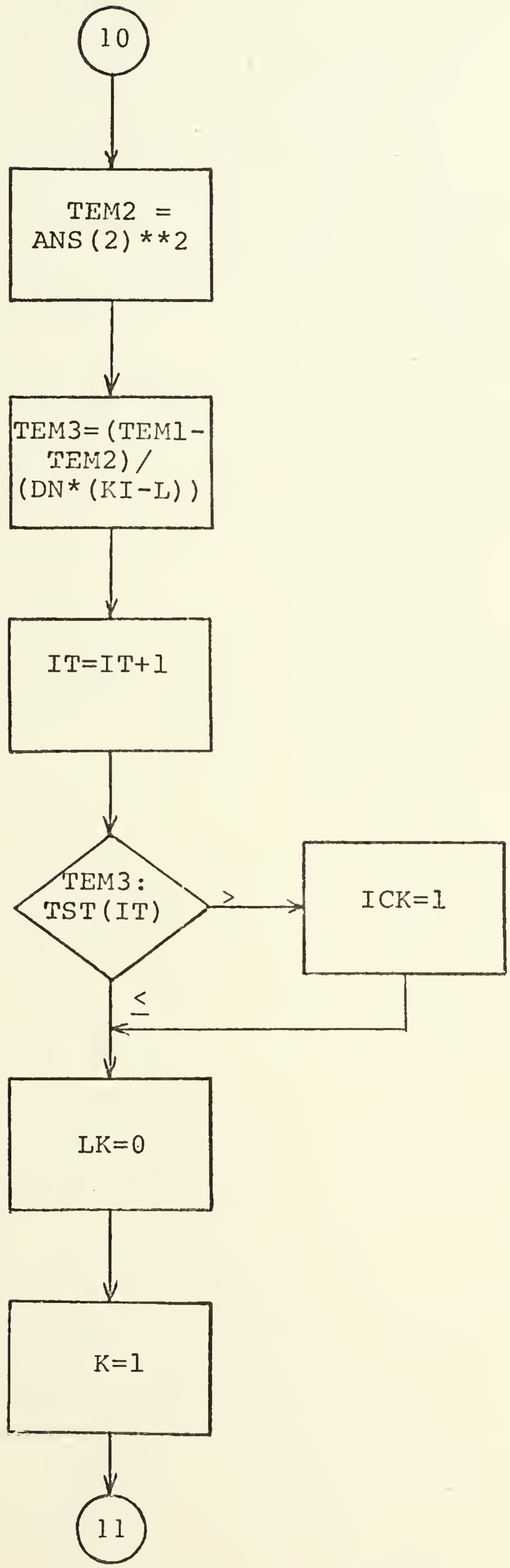

5
CALL MULTR
PRINT MULT. CORRELATION COEFFICIENT FOR JTH GROUP OF 21 VARIABLES
TEM2 =ANS(2)**2
TEM3= (TEM1-TEM2) /DN
C(J)=TEM3
6

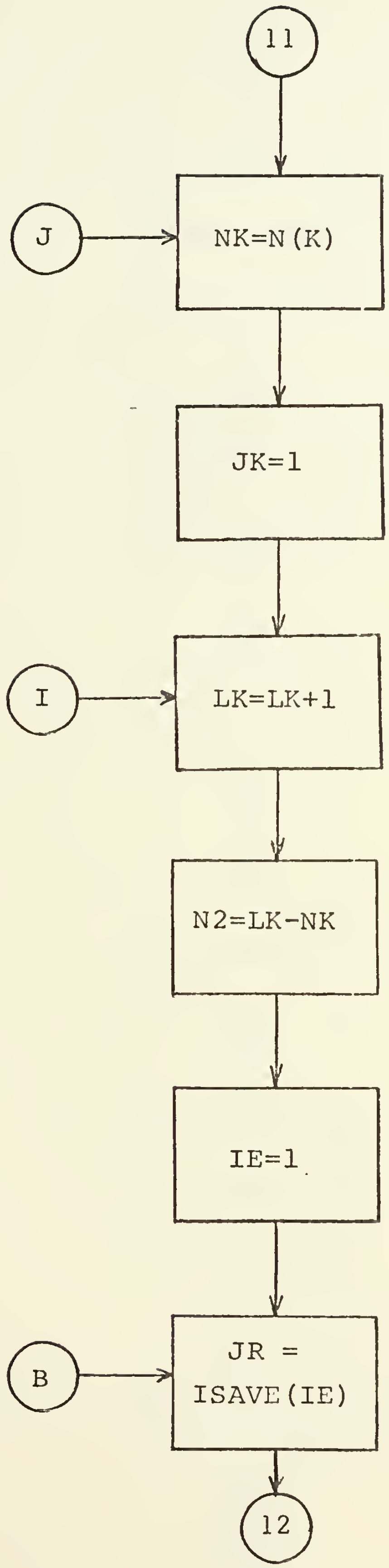

6
27
TEM3:3.98
MK(J)=J
MK(J)=0
30
<
J=J+1
J:KI
≥
KJ=1
A
STR=C(KJ)
IND=KJ
7

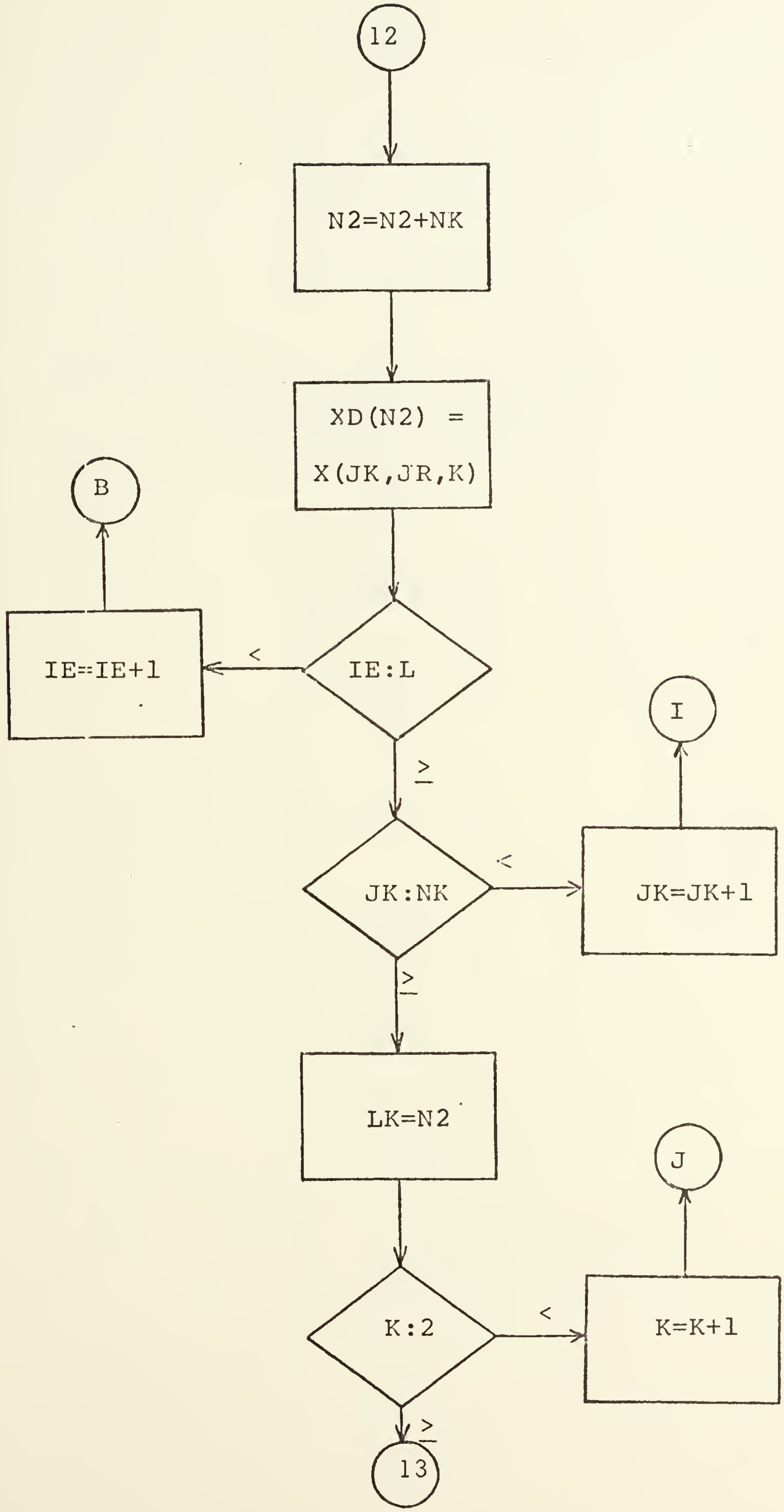

7
KL=1
KL=KL+1
C(KL):STR
>
STR=C(KL)
≤
IND=KL
≥
IDX(KJ)=IND
KL:22
<
C(IND)=0
40
A
KJ=KJ+1
<
KJ:22
≥
8

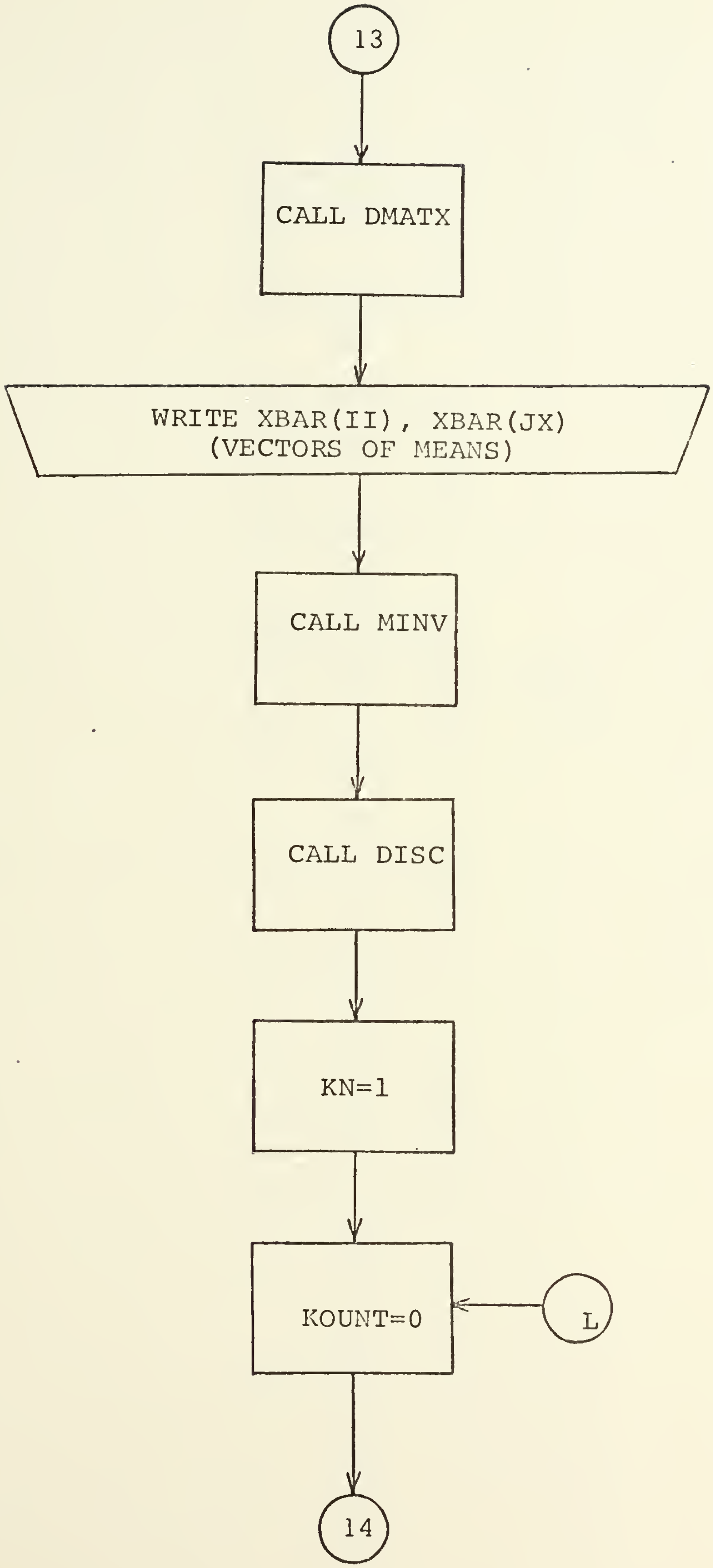

8
L=0
45
JJ=1
MK(JJ):0
=
≠
L=L+1
ISAVE(L)
=MK(JJ)
60
JJ:KI
<
JJ=JJ+1
≥
9

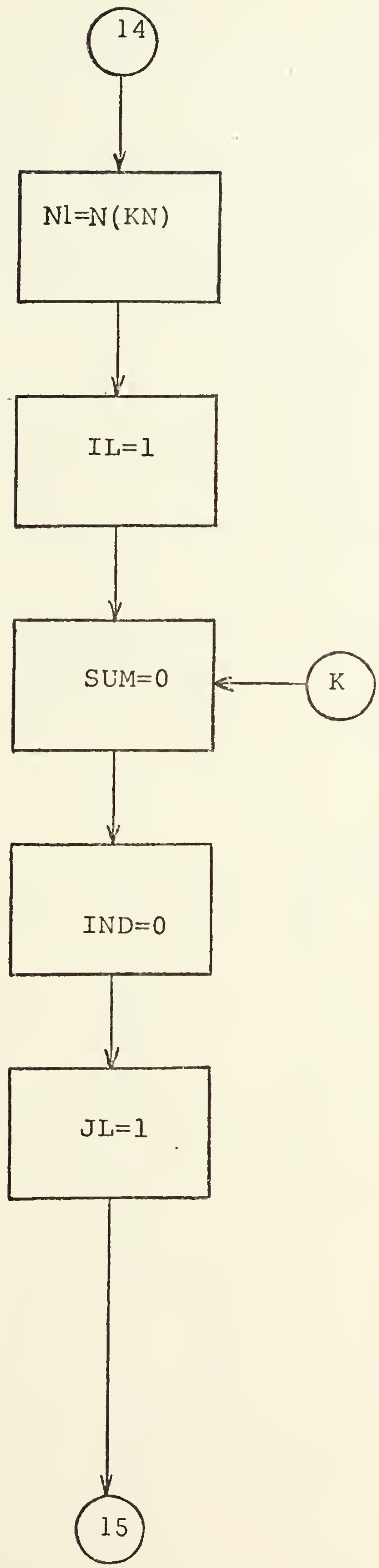

9
6≤L≤15
No
1010
WRITE WARNING
STOP
Yes
WRITE ISAVE (INDICES OF SIGNIFICANT VARIABLES)
CALL ORDER
CALL MINV
CALL MULTR
WRITE ANS(2)
10

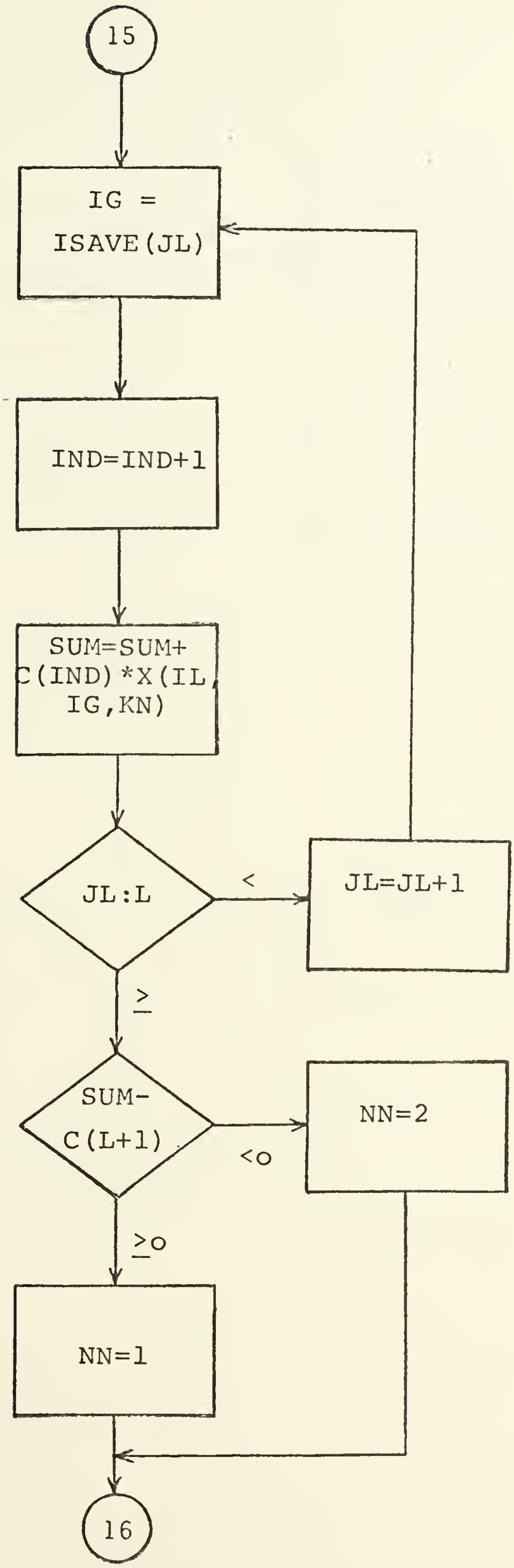

10
TEM2 =
ANS(2)**2
TEM3=(TEM1-
TEM2)/
(DN*(KI-L))
IT=IT+1
TEM3:
TST(IT)
>
ICK=1
≤
LK=0
K=1
11

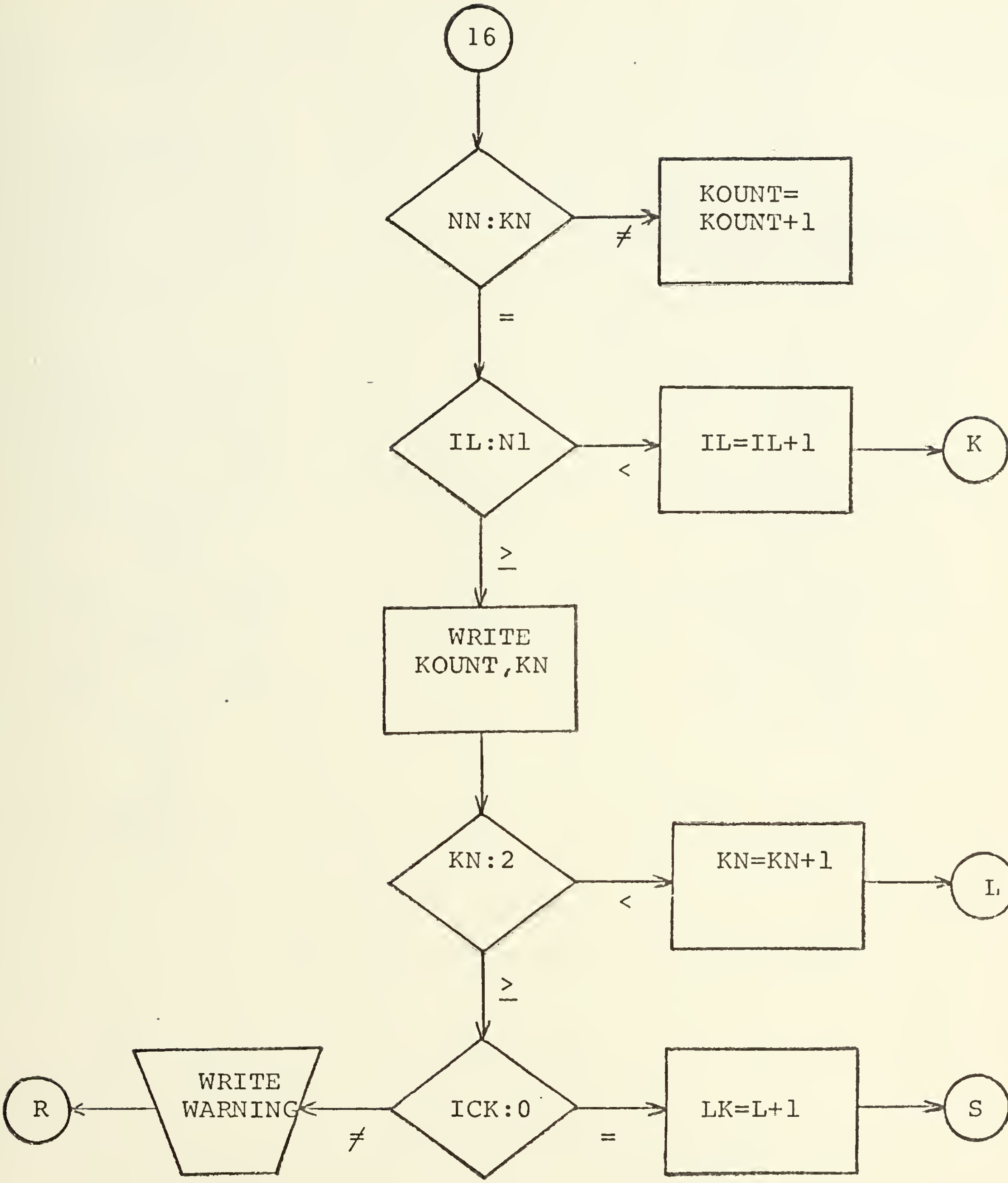

11
J
NK=N(K)
JK=1
I
LK=LK+1
N2=LK-NK
IE=1
B
JR =
ISAVE(IE)
12

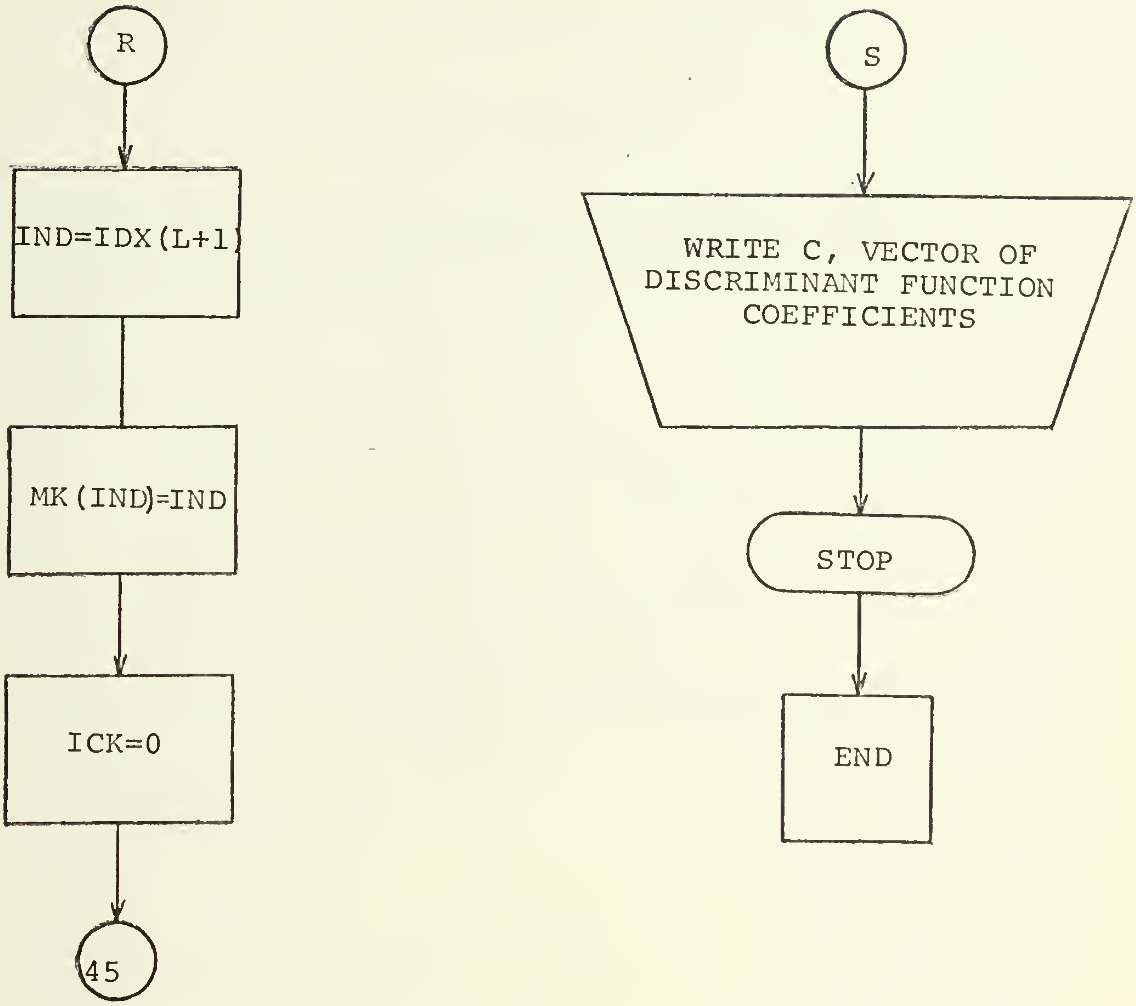

12
N2=N2+NK
XD(N2) =
X(JK,JR,K)
B
IE=IE+1
<
IE:L
≥
I
JK:NK
<
JK=JK+1
≥
LK=N2
J
K:2
<
K=K+1
≥
13


13
CALL DMATX
WRITE XBAR(II), XBAR(JX)
(VECTORS OF MEANS)
CALL MINV
CALL DISC
KN=1
KOUNT=0
L
14


14
N1=N(KN)
IL=1
SUM=0
K
IND=0
JL=1
15


15
IG =
ISAVE(JL)
IND=IND+1
SUM=SUM+
C(IND)*X(IL,
IG,KN)
JL:L
<
JL=JL+1
≥
SUM-
C(L+1)
<o
NN=2
≥o
NN=1
16


16
NN:KN
≠
KOUNT=
KOUNT+1
=
IL:N1
<
IL=IL+1
K
≥
WRITE
KOUNT,KN
KN:2
<
KN=KN+1
L
≥
ICK:0
=
LK=L+1
S
≠
WRITE
WARNING
R


R
IND=IDX(L+1)
MK(IND)=IND
ICK=0
45
S
WRITE C, VECTOR OF DISCRIMINANT FUNCTION COEFFICIENTS
STOP
END

COMPUTER PROGRAM

```
      DIMENSION X( 65,23,2),XBAR(44), D(529), CMEAN(23),
     /XD(2116),PS(23),N(2),R(529),B(23),W(23),T(23),
     /ISAVE(23),R11(484),R12(22),SW(23),ANS(10),MK(22),
     /IDX(22),TST(10),C(23)
      READ(5,1) LX,M,N
      ICK=0
      IT=0
      KI=M-1
      NS=N(1)+N(2)
      DATA TST/2.36,2.39,2.42,2.45,2.49,2.55,2.60,2.68,
     /2.80,2.95/
C
C    TRANSFORM RAW DATA FOR MULTIPLE REGRESSION ANALYSIS
C
      CALL DATA(M,N,NS,X,XD)
      CALL CORRE(NS,M,1,XD,CMEAN,PS,D,R,B,W,T)
      DO 10 I=1,KI
   10 ISAVE(I)=I
      CALL ORDER(M,R,M,KI,ISAVE,R11,R12)
      CALL MINV(R11,KI,DT,W,T)
      CALL MULTR(NS,KI,CMEAN,PS,B,R11,R12,ISAVE,W,SW,T,ANS)
      WRITE(6,800)
      WRITE(6,501) (W(IA),IA=1,KI)
      WRITE(6,810)
      WRITE(6,501) ANS(2)
      TEM1=(ANS(2))**2.
      DN=(1.-TEM1)/69.
      KD=KI-1
      DO 30 J=1,KI
      L=0
      DO 20 JI=1,KI
      IF(JI.EQ.J) GO TO 20
      L=L+1
      ISAVE(L)=JI
   20 CONTINUE
      CALL ORDER(M,R,M,KD,ISAVE,R11,R12)
      CALL MINV(R11,KD,DT,W,T)
      CALL MULTR(NS,KD,CMEAN,PS,B,R11,R12,ISAVE,W,SW,T,ANS)
      WRITE(6,820) J
      WRITE(6,501) ANS(2)
      TEM2=(ANS(2))**2.
      TEM3=(TEM1-TEM2)/DN
      C(J)=TEM3
      IF(TEM3.GT.3.98) GO TO 27
   25 MK(J)=0
C
C     MK(J)=0 IMPLIES THAT X(J) DOES NOT CONTRIBUTE
C    SIGNIFICANTLY TO THE CORRELATION COEFFICIENT.
C
      GO TO 30
   27 MK(J)=J
   30 CONTINUE
      DO 40 KJ=1,22
      STR=C(KJ)
      IND=KJ
      DO 35 KL=1,22
      IF(C(KL).LE.STR) GO TO 35
      STR=C(KL)
      IND=KL
   35 CONTINUE
      IDX(KJ)=IND
   40 C(IND)=0.
C
C     IDX CONTAINS THE INDICES OF THE ''INDEPENDENT''
```

```
C   VARIABLES ORDERED WITH RESPECT TO SIGNIFICANCE OF
C   THE VARIABLE'S CONTRIBUTION TO THE CORRELATION
C   COEFFICIENT
C
   45 L=0
      DO 60 JJ=1,KI
      IF(MK(JJ).EQ.0) GO TO 60
      L=L+1
      ISAVE(L)=MK(JJ)
   60 CONTINUE
      IF( (L.LT.6).OR.(L.GT.15) ) GO TO 1010
      WRITE(6,830)
      WRITE(6,2) (ISAVE(IV),IV=1,L)
      CALL ORDER(M,R,M,L,ISAVE,R11,R12)
      CALL MINV(R11,L,DT,W,T)
      CALL MULTR(NS,L,CMEAN,PS,B,R11,R12,ISAVE,W,SW,T,ANS)
      WRITE(6,840)
      WRITE(6,501) ANS(2)
      TEM2=(ANS(2))**2.
      TEM3=(TEM1-TEM2)/(DN*(KI-L))
      IT=IT+1
      IF(TEM3.GT.TST(IT) ) ICK=1
C
C    DISCRIMINANT FUNCTION IS FORMED BASED ON THE
C   SIGNIFICANT ''INDEPENDENT'' VARIABLES.
C
      LK=0
      DO 80 K=1,2
      NK=N(K)
      DO 70 JK=1,NK
      LK=LK+1
      N2=LK-NK
      DO 70 IE=1,L
      JR=ISAVE(IE)
      N2=N2+NK
   70 XD(N2)=X(JK,JR,K)
   80 LK=N2
      CALL DMATX(LX,L,N,XD,XBAR,D,T)
      WRITE(6,995)
      DO 77 II=1,L
      JX=II+L
   77 WRITE(6,999) XBAR(II),XBAR(JX)
      CALL MINV(D,L,DT,T,C)
      CALL DISC(XBAR,L,D,T,C)
C
C     INPUT DATA IS CLASSIFIED ACCORDING TO THE CURRENT
C   DISCRIMINANT FUNCTION
C
      DO 81 KN=1,2
      KOUNT=0
      N1=N(KN)
      DO 85 IL=1,N1
      SUM=0.
      IND=0
      DO 79 JL=1,L
      IG=ISAVE(JL)
      IND=IND+1
   79 SUM=SUM+C(IND)*X(IL,IG,KN)
      IF(SUM-C(L+1)) 4,5,5
    4 NN=2
      GO TO 88
    5 NN=1
   88 IF(NN.NE.KN) KOUNT=KOUNT+1
   85 CONTINUE
   81 WRITE(6,601) KOUNT,KN
      IF(ICK.EQ.0) GO TO 1000
   90 WRITE(6,91)
      IND=IDX(L+1)
      MK(IND)=IND
      ICK=0
      GO TO 45
 1000 LK=L+1
```

```
      WRITE(6,500)
      WRITE(6,501) (C(JS),JS=1,LK)
      STOP
 1010 WRITE(6,1015) L
 1015 FORMAT('WARNING','L=',I10)
      STOP
    1 FORMAT(4I4)
    2 FORMAT(10X,20I4)
   91 FORMAT(10X,'DISCARDED  VARIABLES HAVE GROUP ',
     /'SIGNIFICANCE')
  500 FORMAT(25X,'DISCRIMINANT FUNCTION COEFFICIENTS' )
  501 FORMAT(8E15.3)
  601 FORMAT(1HC,I35, 4X,'MISCLASSIFICATIONS IN GROUP',I6)
  800 FORMAT(1HC,10X,'STANDARD REGRESSION COEFFICIENTS')
  810 FORMAT(1HC,'MULTIPLE CORRELATION COEFFICIENT')
  820 FORMAT(1HC,'MULTIPLE CORRELATION COEFFICIENT FOR ',
     /'GROUP',I4)
  830 FORMAT(1H0,'INDICES OF VARIABLES MAKING SIGNIFICANT ',
     /'CONTRIBUTION TO THE CORRELATION COEFFICIENT')
  840 FORMAT(1HC,'CORRELATION COEFFICIENT FOR SIGNIFICANT ',
     /'VARIABLES')
  995 FORMAT(28X,'MEAN VECTOR ONE',25X,'MEAN VECTOR TWO')
  999 FORMAT(2E40.3)
              END


      SUBROUTINE REL(PHI1,PHI2,PHI)
      PHI=PHI1-PHI2
      IF(PHI.LT.0.0) PHI=PHI+360.
      RETURN
          END


      SUBROUTINE DISC (XBAR,M,D,PS,C)
      DIMENSION XBAR(1),PS(1),C(1),D(1)
      SUM=0.
      DO 76 J=1,M
      MC=J+M
   76 PS(J) =XBAR(J)-XBAR(MC)
      IJ=C
      DO 77 I=1,M
      C(I)=0.
      DO 77 L=1,M
      IJ=IJ+1
   77 C(I)=C(I)+PS(L)*D(IJ)
      DO 78 K=1,M
      MC=K+M
   78 SUM=SUM+(XBAR(K)+XBAR(MC))*C(K)
      C(M+1)=  (SUM/2.)
      RETURN
      END


      SUBROUTINE MINV(A,N,D,L,M)
      DIMENSION A(1),L(1),M(1)
      D=1.0
      NK=-N
      DO 80 K=1,N
      NK=NK+N
      L(K)=K
      M(K)=K
      KK=NK+K
      BIGA=A(KK)
      DO 20 J=K,N
      IZ=N*(J-1)
      DO 20 I=K,N
      IJ=IZ+I
   10 IF( ABS(BIGA)- ABS(A(IJ))) 15,20,20
   15 BIGA=A(IJ)
      L(K)=I
      M(K)=J
   20 CONTINUE
```

```
      J=L(K)
      IF(J-K) 35,35,25
   25 KI=K-N
      DO 30 I=1,N
      KI=KI+N
      HOLD=-A(KI)
      JI=KI-K+J
      A(KI)=A(JI)
   30 A(JI) =HOLD
   35 I=M(K)
      IF(I-K) 45,45,38
   38 JP=N*(I-1)
      DO 40 J=1,N
      JK=NK+J
      JI=JP+J
      HOLD=-A(JK)
      A(JK)=A(JI)
   40 A(JI) =HOLD
   45 IF(BIGA) 48,46,48
   46 D=0.0
      WRITE(6,400)
  400 FORMAT(40X,'SINGULAR INPUT TO MINV')
      RETURN
   48 DO 55 I=1,N
      IF(I-K) 50,55,50
   50 IK=NK+I
      A(IK)=A(IK)/(-BIGA)
   55 CONTINUE
      DO 65 I=1,N
      IK=NK+I
      HOLD=A(IK)
      IJ=I-N
      DO 65 J=1,N
      IJ=IJ+N
      IF(I-K) 60,65,60
   60 IF(J-K) 62,65,62
   62 KJ=IJ-I+K
      A(IJ)=HOLD*A(KJ)+A(IJ)
   65 CONTINUE
      KJ=K-N
      DO 75 J=1,N
      KJ=KJ+N
      IF(J-K) 70,75,70
   70 A(KJ)=A(KJ)/BIGA
   75 CONTINUE
      D=D*BIGA
      A(KK)=1.0/BIGA
   80 CONTINUE
      K=N
  100 K=(K-1)
      IF(K) 150,150,105
  105 I=L(K)
      IF(I-K) 120,120,108
  108 JQ=N*(K-1)
      JR=N*(I-1)
      DO 110 J=1,N
      JK=JQ+J
      HOLD=A(JK)
      JI=JR+J
      A(JK)=-A(JI)
  110 A(JI) =HOLD
  120 J=M(K)
      IF(J-K) 100,100,125
  125 KI=K-N
      DO 130 I=1,N
      KI=KI+N
      HOLD=A(KI)
      JI=KI-K+J
      A(KI)=-A(JI)
  130 A(JI) =HOLD
      GO TO 100
  150 RETURN
```

```
      END


      SUBROUTINE DMATX (K,M,N,X,XBAR,D,CMEAN)
      DIMENSION N(1),X(1),XBAR(1),D(1),CMEAN(1)
      MM=M*M
      DO 100 I=1,MM
  100 D(I)=0.0
      N4=0
      L=0
      LM=0
      DO 160 NG=1,K
      N1=N(NG)
      FN=N1
      DO 130 J=1,M
      LM=LM+1
      XBAR(LM)=0.0
      DO 120 I=1,N1
      L=L+1
  120 XBAR(LM)=XBAR(LM)+X(L)
  130 XBAR(LM)=XBAR(LM)/FN
      LMEAN=LM-M
      DO 150 I=1,N1
      LL=N4+I-N1
      DO 140 J=1,M
      LL=LL+N1
      N2=LMEAN+J
  140 CMEAN(J)=X(LL)-XBAR(N2)
      LL=0
      DO 150 J=1,M
      DO 150 JJ=1,M
      LL=LL+1
  150 D(LL)=D(LL)+CMEAN(J)*CMEAN(JJ)
  160 N4=N4+N1*M
      LL=-K
      DO 170 I=1,K
  170 LL=LL+N(I)
      FN=LL
      DO 180 I=1,MM
  180 D(I)=D(I)/FN
      RETURN
      END


      SUBROUTINE DATA(M,N,NS,X,XD)
      DIMENSION N(2),X(65,23,2),SOD(8,66),SLD(16),XD(2116),
     /C(2)
      READ (5,200) SOD
C     SOD IS AN INPUT MATRIX CONTAINING DATA RELATING
C   TO MANNING STRENGTH IN THE ST RATING FOR EACH
C   DESTROYER PARTICIPATING IN THE EXERCISES .
C
C
C     THE INPUT MATRIX SLD CONTAINS SONIC LAYER DEPTH
C   DATA FOR EACH EXERCISE.
      READ (5,100) SLD
C
      C(1)=FLOAT(N(2))/FLOAT(NS)
      C(2)=-FLOAT(N(1))/FLOAT(NS)
C
C
C     CONVERT 180 DEGREES TO RADIAN MEASURE
C
      DEN= 180./3.141593
      L=0
      DO 68 K=1,2
      NK=N(K)
      DO 61 I=1,NK
      L=L+1
      N2=L-NS
      READ(5,102) (X(I,J,K),J=1, M)
      X(I,23,K)=C(K)
```

```
C
C     CONVERT GEOGRAPHIC DATA TO RELATIVE DATA.
C
      CALL REL( X(I,12,K),X(I,5,K),RB )
      X(I,12,K)=RB/DEN
      CALL REL(X(I,13,K),X(I,5,K),OUT)
      OUT=OUT/DEN
      X( I,5,K )= ( X(I,6,K)**2.+X(I,14,K)**2.-2.*X(I,6,K)*
     /X(I,14,K)*COS(OUT) )**0.5
      DO 63 KK=1,16
      Y=FLOAT(KK)
      IF(X(I,1,K).EQ.Y) GO TO 62
      GO TO 63
   62 X(I,22,K)=SLD(KK)
   63 CONTINUE
      DO 64 KD=1,150
      IF((X(I, 1,K).EQ.SOD(1,KD)).AND.(X(I,3,K).EQ.SOD(2,KD)
     /)) GO TO 7
   64 CONTINUE
    7 P=6.
      X(I,13,K)=0.
      DO 74 KX=3,8
      X(I,13,K)=X(I,13,K)+P*SOD(KX,KD)
   74 P=P-1.
      DO 61 JJ=1,M
      N2=N2+NS
   61 XD(N2)=X(I,JJ,K)
   68 L=NK
  100 FORMAT(7F10.0)
  102 FORMAT(3F10.0,10X,3F10.0,/,7F10.0,/,7F10.0,/,F10.0,
     /10X,2F10.0)
  200 FORMAT(F4.0,F6.0,6F10.0)
      RETURN
      END


      SUBROUTINE CORRE (N,M,IO,X,XBAR,STD,RX,R,B,D,T)
      DIMENSION X(1),XBAR(1),STD(1),RX(1),R(1),B(1),D(1),
     /T(1)
      DO 100 J=1,M
      B(J)=0.0
  100 T(J)=0.0
      K=(M*M+M)/2
      DO 102 I=1,K
  102 R(I)=0.0
      FN=N
      L=0
      IF(IO) 105, 127, 105
  105 DO 108 J=1,M
      DO 107 I=1,N
      L=L+1
  107 T(J)=T(J)+X(L)
      XBAR(J)=T(J)
  108 T(J)=T(J)/FN
      DO 115 I=1,N
      JK=0
      L=I-N
      DO 110 J=1,M
      L=L+N
      D(J)=X(L)-T(J)
  110 B(J)=B(J)+D(J)
      DO 115 J=1,M
      DO 115 K=1,J
      JK=JK+1
  115 R(JK)=R(JK)+D(J)*D(K)
      GO TO 205
  127 IF(N-M) 130, 130, 135
  130 KK=N
      GO TO 137
  135 KK=M
  137 DO 140 I=1,KK
      CALL DATA (M,D)
```

```
      DO 140  J=1,M
      T(J)=T(J)+D(J)
      L=L+1
140   RX(L)=D(J)
      FKK=KK
      DO 150  J=1,M
      XBAR(J)=T(J)
150   T(J)=T(J)/FKK
      L=0
      DO 180  I=1,KK
      JK=0
      DO 170  J=1,M
      L=L+1
170   D(J)=RX(L)-T(J)
      DO 180  J=1,M
      B(J)=B(J)+D(J)
      DO 180  K=1,J
      JK=JK+1
180   R(JK)=R(JK)+D(J)*D(K)
      IF(N-KK) 205, 205, 185
185   KK=N-KK
      DO 200  I=1,KK
      JK=0
      CALL DATA (M,D)
      DO 190  J=1,M
      XBAR(J)=XBAR(J)+D(J)
      D(J)=D(J)-T(J)
190   B(J)=B(J)+D(J)
      DO 200  J=1,M
      DO 200  K=1,J
      JK=JK+1
200   R(JK)=R(JK)+D(J)*D(K)
205   JK=0
      DO 210  J=1,M
      XBAR(J)=XBAR(J)/FN
      DO 210  K=1,J
      JK=JK+1
210   R(JK)=R(JK)-B(J)*B(K)/FN
      JK=0
      DO 220  J=1,M
      JK=JK+J
220   STD(J)= SQRT( ABS(R(JK)))
      DO 230  J=1,M
      DO 230  K=J,M
      JK=J+(K*K-K)/2
      L=M*(J-1)+K
      RX(L)=R(JK)
      L=M*(K-1)+J
      RX(L)=R(JK)
      IF(STD(J)*STD(K)) 225, 222, 225
222   R(JK)=0.0
      GO TO 230
225   R(JK)=R(JK)/(STD(J)*STD(K))
230   CONTINUE
      FN=SQRT(FN-1.0)
      DO 240  J=1,M
240   STD(J)=STD(J)/FN
      L=-M
      DO 250  I=1,M
      L=L+M+1
250   B(I)=RX(L)
      RETURN
      END


      SUBROUTINE ORDER (M,R,NDEP,K,ISAVE,RX,RY)
      DIMENSION R(1),ISAVE(1),RX(1),RY(1)
      MM=0
      DO 130  J=1,K
      L2=ISAVE(J)
      IF(NDEP-L2) 122, 123, 123
122   L=NDEP+(L2*L2-L2)/2
```

```
      GO TO 125
  123 L=L2+(NDEP*NDEP-NDEP)/2
  125 RY(J)=R(L)
      DO 130 I=1,K
      L1=ISAVE(I)
      IF(L1-L2) 127, 128, 128
  127 L=L1+(L2*L2-L2)/2
      GO TO 129
  128 L=L2+(L1*L1-L1)/2
  129 MM=MM+1
  130 RX(MM)=R(L)
      ISAVE(K+1)=NDEP
      RETURN
      END


      SUBROUTINE MULTR (N,K,XBAR,STD,D,RX,RY,ISAVE,B,SB,T,
     /ANS)
      DIMENSION XBAR(1),STD(1),D(1),RX(1),RY(1),ISAVE(1),
     /T(1),ANS(1),B(1),SB(1)
      MM=K+1
      DO 100 J=1,K
  100 B(J)=0.0
      DO 110 J=1,K
      L1=K*(J-1)
      DO 110 I=1,K
      L=L1+I
  110 B(J)=B(J)+RY(I)*RX(L)
      RM=0.0
      BO=0.0
      L1=ISAVE(MM)
      DO 120 I=1,K
      RM=RM+B(I)*RY(I)
      L=ISAVE(I)
      B(I)=B(I)*(STD(L1)/STD(L))
  120 BO=BO+B(I)*XBAR(L)
      BO=XBAR(L1)-BO
      SSAR=RM*D(L1)
  122 RM= SQRT( ABS(RM))
      SSDR=D(L1)-SSAR
      FN=N-K-1
      SY=SSDR/FN
      DO 130 J=1,K
      L1=K*(J-1)+J
      L=ISAVE(J)
  125 SB(J)= SQRT( ABS((RX(L1)/D(L))*SY))
  130 T(J)=B(J)/SB(J)
  135 SY= SQRT( ABS(SY))
      FK=K
      SSARM=SSAR/FK
      SSDRM=SSDR/FN
      F=SSARM/SSDRM
      ANS(1)=BO
      ANS(2)=RM
      ANS(3)=SY
      ANS(4)=SSAR
      ANS(5)=FK
      ANS(6)=SSARM
      ANS(7)=SSDR
      ANS(8)=FN
      ANS(9)=SSDRM
      ANS(10)=F
      RETURN
      END
```

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13. ABSTRACT

A FORTRAN IV computer program was employed to conduct a statistical analysis of data collected during fleet antisubmarine warfare exercises. The object of the investigation was the identification of those variables which had greatest influence on a destroyer's ability to detect a submarine under certain conditions.

The variables were treated as a random vector arising from one of two multivariate normal populations with common covariance matrix. An artificial regression relation was formulated to facilitate development of a linear discriminant function in a subset of those variables found to be of dominant importance. This latter subset was identified by examination of multiple correlation coefficients.

The discriminant function was found to be seventy five per cent effective in classifying the experimental data correctly.

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