1954

Theory and design of counters and counting circuits.

Hinkle, W. L.

Monterey, California: U.S. Naval Postgraduate School

THEORY AND DESIGN OF COUNTERS AND COUNTING CIRCUITS

W. L. HINKLE

Artisan Gold Lettering & Smith Bindery

593 - 15th Street   Oakland, Calif.   GLencourt 1-9827

DIRECTIONS FOR BINDING

BIND IN
(CIRCLE ONE)

BUCKRAM
COLOR NO. 8854

FABRIKOID
COLOR ____

LEATHER
COLOR ____

LETTERING ~~ON~~ shelf BACK TO BE EXACTLY AS PRINTED HERE.

HINKLE

1954

Thesis
H57

OTHER INSTRUCTIONS

Letter in gold. Letter on the front cover:

THEORY AND DESIGN OF COUNTERS

AND COUNTING CIRCUITS

W. L. HINKLE

This work is accepted as fulfilling

the thesis requirements for the degree of

MASTER OF SCIENCE

IN

ENGINEERING ELECTRONICS

from the

United States Naval Postgraduate School

PREFACE

The original idea for this paper is the result of research and experiment performed at Sanders Associates Incorporated, Nashua, New Hampshire, during a ten week industrial experience tour from the United States Naval Postgraduate School in Monterey, California, in January, February, and March 1954. One of the projects to which the author was assigned during this period was to attempt to develop a high speed counter suitable for use in a "tinkertoy" type construction to increase the frequency range of the existing laboratory instrument. While searching the literature to learn as much as possible about the subject, it was noticed that no survey of the field had apparently ever been made; and from this observation this paper had its beginnings.

The author wishes to acknowledge the assistance and advice of Mr. Morton E. Goulder, Mr. Peter Clark, Mr. Russel Hawes, and many others of the engineering staff of Sanders Associates without whose help this paper would not have been possible.

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

1. Summary

The major purpose of this paper is to bring together and to supplement the more important elements of counter circuit theory and design. While it is no doubt true that a search of the literature will reveal literally thousands of bi-stable multivibrator circuits, there is a decided lack of information on the design of such circuits and of their use in counter circuitry. While this paper consists mainly of conclusions drawn from the work of others, it is the hope of the author that anyone interested in counter circuitry might, from this paper, gain enough information to understand thoroughly the basic principals involved and to be able to design such circuits with the required speed of response, stability, output voltage, etc.

While there are numerous types of basic counters such as mechanical, energy storage devices, and magnetic circuits, this paper will be restricted to the electronic bi-stable multivibrator type counter element. In Chapter I the action of the basic circuit is described, and it is shown how to connect the basic elements as either binary or ring counter chains. Next it is shown how to use feedback in order to change the natural counting base of the binary chains to any desired value, a method for indicating the residual count is discussed, and the chapter is concluded with the development of a design chart for triode bi-stable multivibrator circuits. Chapter II discusses the general problem of high speed circuits and includes information on the triggering of bi-stable circuits. A more

quantitative analysis of the change of state process in a multivibrator circuit is given in order to better understand the factors which limit the speed of response. It is shown next how the introduction of clamping diodes may increase the operating speed, and the design of circuits employing these diode clamping circuits is discussed.

2. Some general background on electronic counters.

Although the time since automatic counting devices made their appearance on the laboratory and industrial scene can be measured in decades, the high speed electronic counter, and most particularly the decade counter, is a comparatively recent innovation. The electronic bi-stable circuit is no newcomer to the electronic field, going back to the original paper of Eccles and Jordan in 1919, but its use as a practical counter element was rather slow in developing. This circuit was used in many pieces of apparatus in which it lost its identity, but its use as a basis for binary-decade counters was probably first made practical by Potter in 1944 when he introduced a practical four tube decade counter using the Eccles-Jordan "flip-flop" as a binary element, and forcing these binary elements to count to the base ten by means of feedback loops. In some circles the high speed counter circuit, regardless of manufacturer, is still termed the "Potter Counter".

The major factor which seems to have held up the development of these high speed counting circuits is the fact that the bi-stable circuit is a binary counter element, and count indication in the binary system had to be converted into the normal decade system

before the information was useful. This difficulty was first combated by means of the so-called "ring" circuits, but this system was wasteful of tubes, space, and power. It required a dual tube for each digit in the counting base; ten dual triodes, for example, being required to count to the base ten.

With the introduction of a means to force the much more desireable binary element to count to the desired base of our numeral system, the counter began to play an ever increasing role in solving heretofore difficult problems. While the original circuit as introduced by Potter would function at an input rate of approximately 100kc, the newer techniques of today make possible instruments which will count directly to as high as a 10mc input rate and indicate the total count every one-thousandth of a second. The uses to which these instruments can be applied are practically limitless, running from such simple tasks as measuring the quantities of screws and nuts being packaged, to direct measurement of oscillator frequency and stability to within .00001%, to the measurement of the number of atomic disintegrations per second in nuclear experiments.

1. Principles and the basic binary circuit.

That resistance-coupled multivibrators were suitable for counter use was pointed out by Grosdoff in 1946 when he listed the following properties of such a circuit which would make them useful:

1. The resistance-coupled or untuned multivibrator has two stable positions which can be reversed by the application of a driving pulse.

2. There is an upper limit in the frequency response but no lower limit for a multivibrator having certain parameters.

3. The multivibrator is equally responsive to either a periodic or a random rate of count.

4. The multivibrator can store and indicate but one count, and only one meaning can be attached to that count.

In order to demonstrate the use of such multivibrators as counters, and to lay the foundation for understanding the various types of circuits to follow, consider the following four basic triode multivibrators shown as fig. 1:

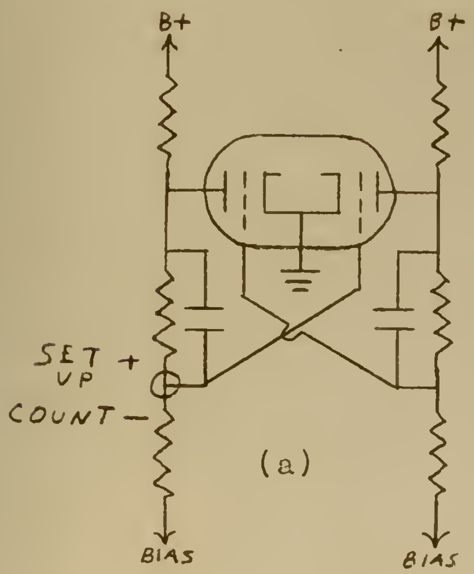
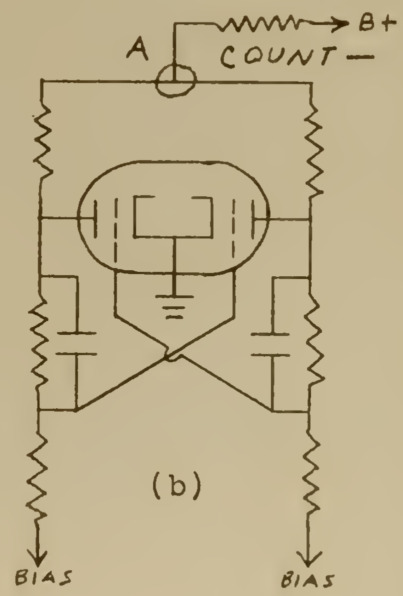


Figure 1



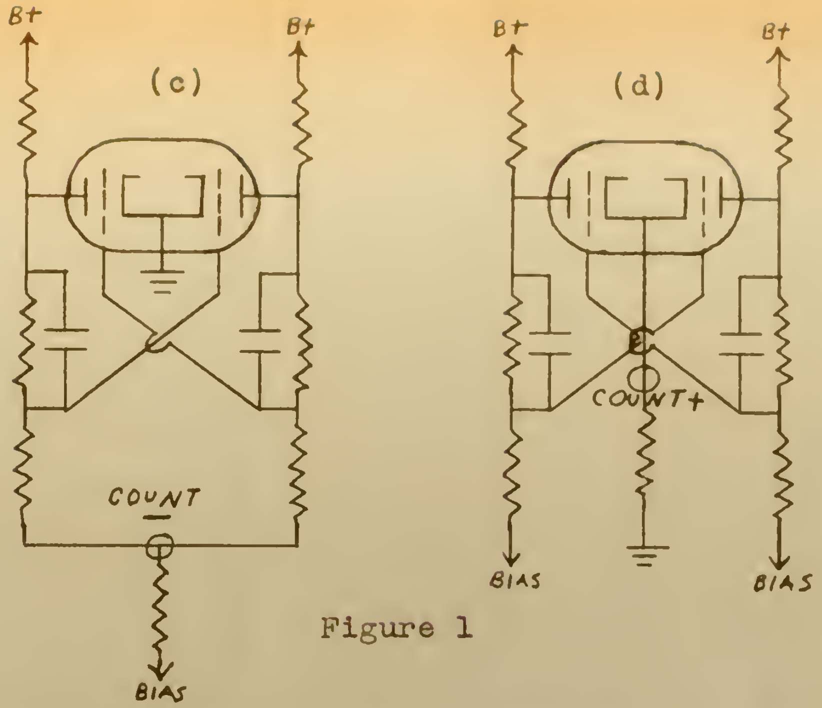


Figure 1

Starting with these four basic types of multivibrators, we may now proceed to a discussion of the circuits and theory involved in using these multivibrators as counters.

Perhaps the simplest type of circuit for counting use is the binary system in which any of the multivibrators (b), (c), or (d) might be used. Let us first consider in detail the operation of the multivibrator shown at (b). An inspection of this circuit shows that it is a modified Eccles-Jordan "flip-flop", commonly called a "scale-of-two", in which there exists two independent steady state conditions. If we consider that the left half of the triode is conducting, then the low voltage at the left plate acting through the voltage divider composed of R_1 and R_2 will keep the grid of the right half of the triode at a low enough potential so that the right half is cut-off; provided only that the circuit constants are correctly proportioned according to the values of voltages and tubes used.

If now a negative trigger is introduced at point "A", both plates and both grids will follow the trigger down, the exact amount of movement of these elements determined by the voltage divider effect of the resistances of the circuit. Since the grid of the right half of the triode was assumed already below cut-off, the trigger will have no effect upon this half of the triode. The grid of the left half, however, is at approximately zero bias, and as the trigger takes this grid below the cathode, plate current decreases raising the voltage at the plate. This rise in plate voltage is coupled to the right hand grid by means of the "speed-up" condenser, and if this plate voltage rise is great enough to bring the right hand grid above cut-off regeneration occurs as follows: With the right hand grid rising above cut-off the plate voltage of the right hand section begins to fall; this falling voltage is coupled to the left hand grid, driving it in the same direction as did the negative trigger. This results in an even further increase in right hand plate voltage, and the tube will end up in its second stable condition; i.e. the right hand section conducting and the left hand section cut off. The basic action of the other three multivibrators is exactly the same except for the point of application and polarity of the required trigger. The significance of these differences will be discussed later.

Two important factors in the design of such bi-stable multivibrators can be gathered from the above explanation. First, there is a minimum trigger voltage below which no regeneration can occur. This is evident from the fact that the negative trigger must by itself cause a rise in plate voltage of the conducting tube of sufficient amplitude to bring

the grid of the off tube above cut-off. The minimum trigger necessary will depend upon the voltages used, the value of the plate load resistors, and upon the tube characteristics. Second, depending upon the voltages used for plate supply and bias, the resistors must be proportioned in such a manner that when one half of the tube is off the voltage divider action will keep the opposite grid at or above zero bias, and when one plate is low with its grid at zero bias the voltage divider action will keep the opposite grid below cut-off. This last criterion is very easy to meet since the plate voltage variation between zero and cut-off bias is usually of sufficient magnitude so that the voltage division ratio is not particularly critical. Using only these two design factors, a circuit can be built which will operate quite satisfactorily; but such problems as stability, speed of response, trigger shape, and sensitivity to changes of tubes are factors which will have to be investigated later to see their modifying effect upon the design. Of primary importance at this stage of the explanation is basic understanding of how such multivibrators may be used as counters, and the fine points of design shall be left until after such a basic understanding has been acquired.

Let us now consider the waveshapes present at various points of the multivibrator diagrammed at (b) as shown below for four successive negative input triggers.

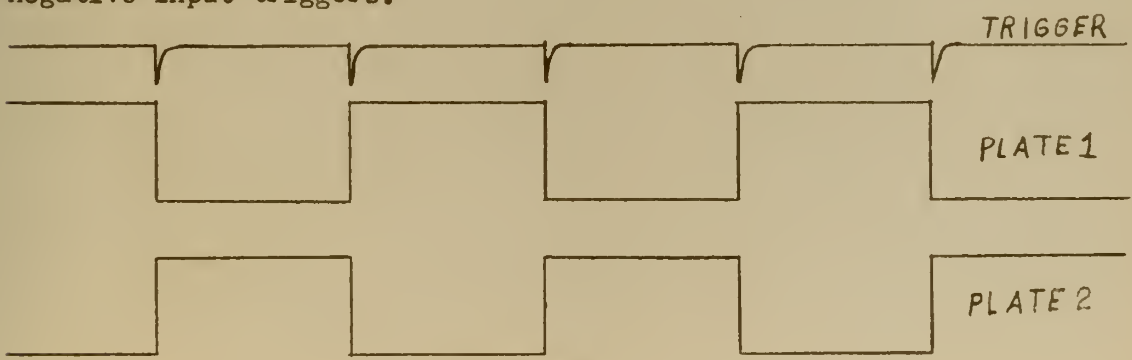


Figure 2

It is evident from an inspection of these waveforms that each plate makes one negative and one positive excursion for each two applied trigger pulses. This immediately suggests that the negative excursion could be coupled to another multivibrator stage as a trigger, and the number of output negative pulses would be exactly half the number of applied input pulses. Any number of such multivibrators could be connected in series using the plate wave of one stage to trigger the next, and each stage would show an output exactly half its input. In this manner the number of output pulses would decrease by factors of 2, 4, 8, ----- 2^n, where n is the number of stages so connected. This is the so-called series or binary connection and forms the basis of most of the counting circuits used in modern computing systems.

2. Ring counters and their disadvantages.

The multivibrator shown at (a) in fig. 1 is the basic unit used in the unitary or parallel counting chains, and while the operation of the basic multivibrator is the same as for that described, its use with other circuits is considerably different. An inspection of this circuit will show that this multivibrator will change state only if the applied trigger is of the proper polarity and will continue to change state only if the trigger polarity alternates. For example, assume that the right hand section is conducting and a negative trigger is applied at the point indicated. The right hand section will be turned off, and successive negative triggers will have no further effect upon the circuit. In order for negative input triggers to again effect the circuit, a "set-up" pulse of positive polarity must be applied to return the right hand section to conduction. These multivibrators

are connected in "chains" as indicated below as fig. 3.

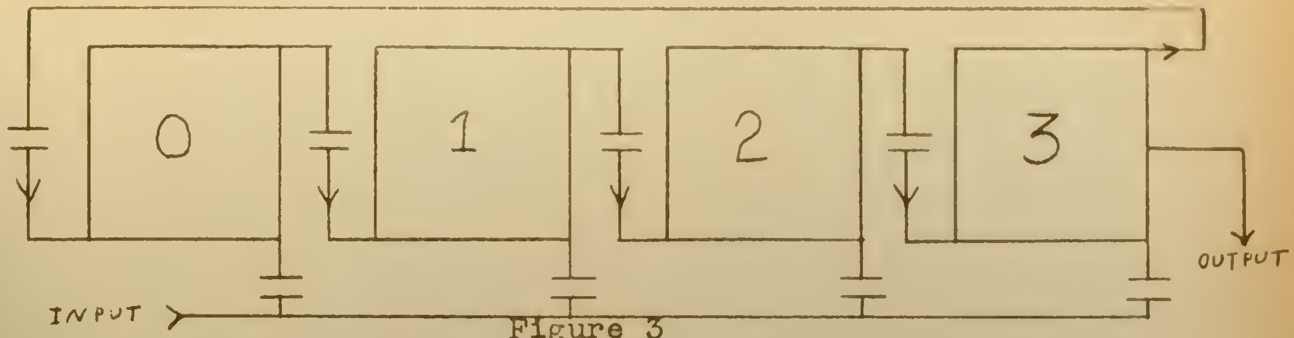


Figure 3

All multivibrators are connected in parallel to the trigger source and in series for "setting-up". When the first trigger is applied, multivibrator #0 changes state and the positive pulse output from #0 "sets-up" or forces #1 to change state. When this action is completed, #1 is the only multivibrator in the chain which will respond to the next negative trigger input. In this manner the "odd" state progresses down the chain, moving one multivibrator per input pulse. It can readily be seen that the number of multivibrators required is equal to the base to which it is desired to count, a decade counter requiring ten dual triodes. The number of input pulses which has occured can readily be determined by merely locating which multivibrator is in the "odd" state. In practice this is usually done by connecting a neon bulb between the normally high plate and the B+ supply. The neon bulb will then light on that particular multivibrator which has its normally high plate at a low potential. A typical decade ring would have the right hand section of the final tube connected back to the input of the first tube in order to return the decade to its zero condition after the tenth applied pulse. The left hand section of the tenth multivibrator would yield a negative output at one-tenth the frequency

of the applied trigger. This voltage could be used to trigger another decade, thereby again dividing the input by a factor of ten. To insure that the circuit will begin counting from its zero condition, a push-button switch could be installed to ground all left hand grids with the exception of multivibrator number 0 whose right hand grid would be grounded. This would force all multivibrators except number 0 to assume "normal" state and number 0 "odd" state, indicating zero count.

The main disadvantages of this type circuit are the large number of tubes required and its relatively low counting rate. The slow counting is caused by the fact that time must be allowed between triggers for "setting-up" the following state. The binary type circuit previously discussed briefly has neither of these disadvantages, although as far as the discussion here is concerned it has not yet been shown that a binary counter will divide by anything other than a number which can be represented as 2^n, where n is the number of multivibrators. For a long while this fact held back the development of binary counters, since the desired base of ten cannot be represented as 2^n and decades were impossible.

3. Mathmatical proof that binary counters may be constructed with any arbitrary integral counting base.

It has been shown [1] mathmatically that binary elements may be connected to count to any arbitrary integral base, and such a proof will be presented here followed by a specific example of applying the mathmatical theory to produce a decade counter composed of binary elements.

1. Bibliography reference number 10.

In order to carry forward the mathmatical proof, it will be assumed that any two bi-stable multivibrators may be connected by a feed-back circuit in such a manner that when the desired multivibrator changes state in a given direction, the "slave" multivibrator will be made to change state also; and furthermore that a change of state in the "slave" unit will have no effect on the controlling multivibrator. In other words, the feedback circuit is selective of the direction of change of the controlling multivibrator and is unilateral in the direction from the controlling unit to the "slave" unit. That a feed-back circuit of this type can be realized in practice will be demonstrated later.

Consider a cascaded chain of bi-stable multivibrators as indicated by the block diagram below as fig. 4:

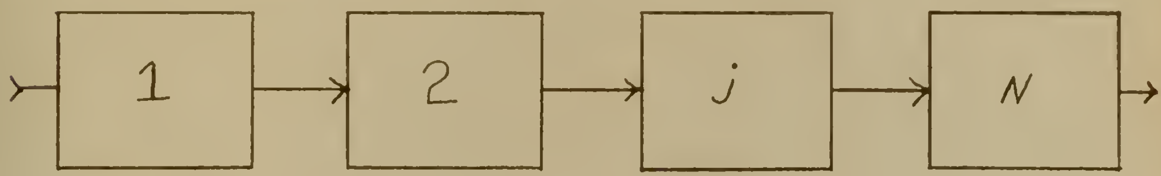


Figure 4

Each multivibrator in the chain changes state once for 2^{j-1} input pulses, where j is its position in the chain. For example: in the above block diagram reading from left to right, the number of input pulses required for one change of state is 2^0, 2^1, 2^{j-1}, 2^{n-1} respectively. In order for any multivibrator in the chain to change state twice, which amounts to exactly the same thing as producing one output pulse, the number of input pulses required is 2^1, 2^2, 2^j, 2^n respectively from left to right. The natural count or base of the

chain is defined as the total number of input pulses required to yield one output pulse and is 2^n.

Since it requires one change of state in each direction for any multivibrator to produce an output pulse to the next multivibrator in the chain, the total number of changes of state in a given direction during one natural count is 2^{n-j}, where j is again the position in the chain. In the block diagram, reading this time from right to left, the number of changes in a given direction is 2^0, 2^{n-j}, 2^{n-2}, 2^{n-1}.

Since the jth multivibrator changes state once for each 2^{j-1} pulses applied to the input, if this multivibrator is forced to change state by means of a feedback loop this will have exactly the same effect as 2^{j-1} input pulses. Furthermore, since the kth multivibrator changes state in a given direction 2^{n-k} times during one natural count, a feedback loop of the type described above when connected with k as the controlling multivibrator and j as the slave multivibrator will have the effect of subtracting 2^{n-k} times 2^{j-1} pulses from the number required to produce one output pulse. The natural count of the chain has now been reduced from 2^n to $2^n - 2^{n-k+j-1}$. Generalizing this reasoning to include an arbitrary number of feedback loops, the natural count C can be represented by:

$$C = 2^n - \sum_{k=2}^{n} 2^{n-k-1} \sum_{j=1}^{k-1} 2^j \delta_j^k$$

Where for simplicity the kth multivibrator is assumed to be further from the input than is the jth multivibrator. And:

$$\delta_j^k = 1 \quad \text{IF FEEDBACK EXISTS}$$
$$= 0 \quad \text{IF NO FEEDBACK EXISTS}$$

Since two chains of n and n-1 multivibrators without feedback will count to a base of 2^n and 2^{n-1} respectively, it is only necessary to prove that the formula developed for the natural count C will bridge this gap between 2^{n-1} and 2^n. Stated mathmatically it is necessary that:

$$\sum_{k=2}^{n} 2^{n-k-1} \sum_{j=1}^{k-1} 2^j \delta_j^k$$

take on all values between 1 and 2^{n-1}, since $2^n - 2^{n-1} = 2^{n-1}$. It is obvious that if k is taken equal to n the formula simplifies to:

$$\sum_{j=1}^{n-1} 2^{j-1} \delta_j^n$$

which is the binary representation for any number from 1 to $2^{n-1} - 1$. To illustrate let δ_j^n = 1 for all values of j, then:

$$\sum_{j=1}^{n-1} 2^{j-1} \delta_j^n = 1 + 2 + 2^2 + \cdots 2^{n-2} \equiv 2^{n-1} - 1$$

As δ_j^n is allowed to be zero for certain values of j which correspondes to opening various feedback loops, the summation will take on all values between 2^{n-1} and 1. If there is only one feedback loop present between the nth multivibrator and the first, then:

$$\sum_{j=1}^{n-1} 2^{j-1} \delta_j^n = 2^0 = 1$$

In order to design a chain of multivibrators which will count to a desired base, it is necessary to have n stages where 2^n is larger than the desired base and 2^{n-1} is smaller. Feedback loops can then be arranged to reduce the natural count of 2^n to any desired value larger than 2^{n-1}. As an example let it be desired to design a chain which will count to the base ten. Since 2^4 is 16 and 2^3 is 8 it will be necessary to employ 4 stages and a combination of feedback loops which will subtract 6 from the natural count of 16. Since:

$$C = 2^4 - \sum_{k=2}^{m} 2^{m-k-1} \sum_{j=1}^{k-1} 2^j \delta_j^k$$

If feedback is employed from the fourth stage to the third and from the fourth stage to the second:

$$C = 16 - \tfrac{1}{2}(8) - \tfrac{1}{2}(4) = 10$$

4. Practical feedback circuits for binary decades.

At this point the reader should have an understanding of the basic multivibrator circuit used for counting purposes, and it should be evident how to connect such multivibrators either in parallel for unitary counting or in series for binary counting. While it has been shown mathmatically that a binary type counter can be constructed to count with any desired base, no practical method for obtaining such feedback has been discussed. It is thought that this can best be illustrated by taking as an example a four tube scale of sixteen and by the use of feedback convering this circuit to the much more desirable scale of ten. First let us consider the waveforms present on some particular plate, say the right hand plate, for each tube of

the four in the chain, for sixteen consecutive input triggers as shown below in fig. 5.

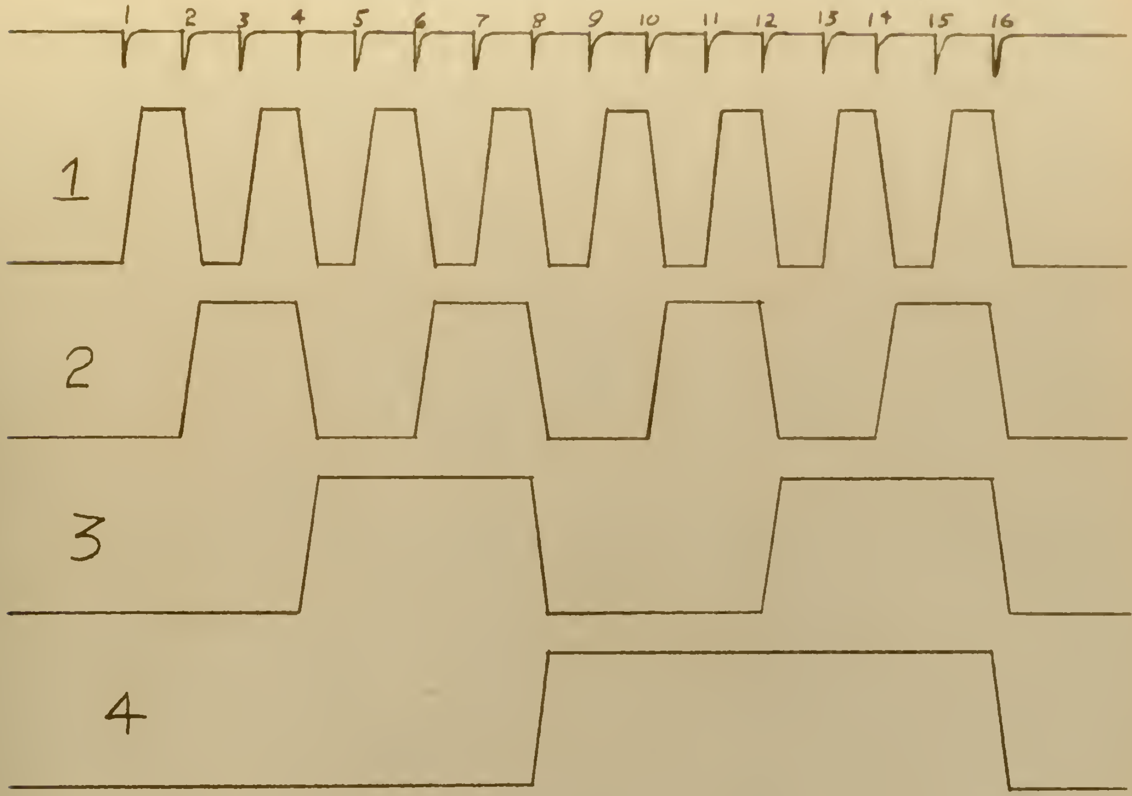


Figure 5

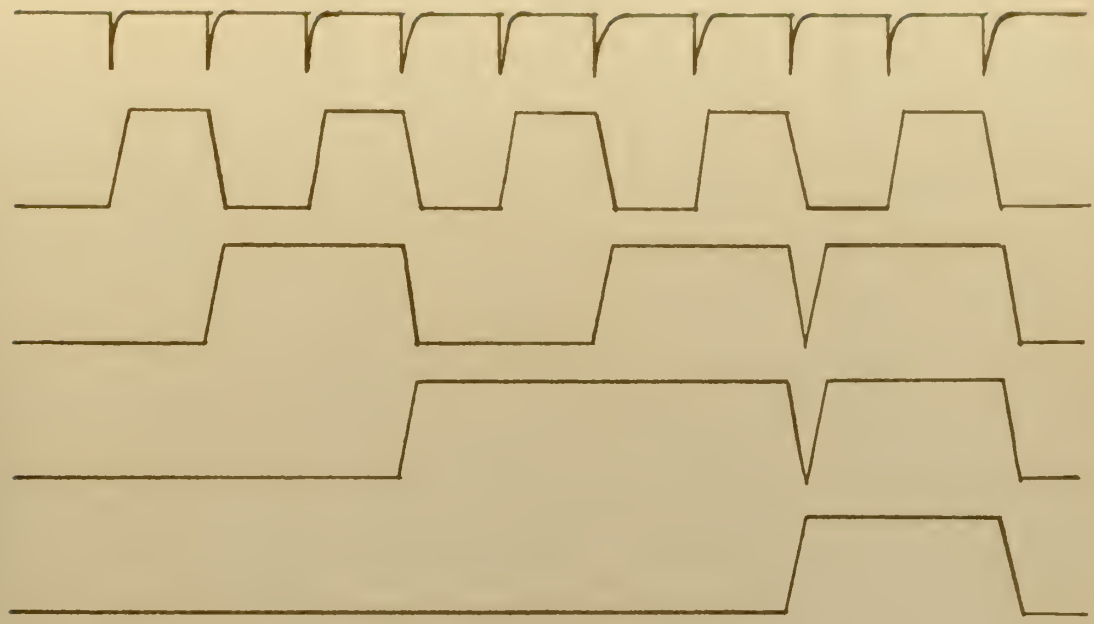
Looking at these waveforms it should be kept in mind that while each succeeding stage is triggered when the preceeding right hand plate moves in the negative direction, a pulse of either polarity might be taken from any of the multivibrators when it changes state provided the proper plate is chosen to furnish this pulse. It will be noticed that on the eighth applied trigger, multivibrator number four changes state in the positive direction while numbers two and three change in the negative direction. It will also be noticed that on the fourteenth applied trigger, number one changes negative, number two positive, and numbers three and four remain positive. If it were possible, therefore,

to use the pulse available from number four at the eighth applied trigger to force numbers two and three to remain positive, the circuit would then be in the same state as at the count of fourteen, and two more triggers would revert the entire circuit to its zero condition. This jump at the eighth trigger to that state which would normally require fourteen triggers is the same as subtracting six from the natural count and results in a decade counter. This method is called simultaneous feedback, and while it is perhaps the simplest circuit-wise it is seldom used due to the difficulty of obtaining display with the usual neon bulbs.

One other feedback arrangement is possible and is the more common. This system is known as "split feedback" and can be explained by means of the same waveforms used in the explanation of the simultaneous feedback system. In this system, on the fourth applied pulse, when number three changes state for the first time, its output is used to force number two to remain positive; thereby subtracting two from the natural count of sixteen. On the eighth applied pulse when number four changes state, its output is used to force number three to remain positive subtracting four more from the natural count. This results in a total of six subtracted and yields a decade as before. In fig. 6 are shown plate waveforms for one plate of each multivibrator; part (a) being simultaneous feedback and (b) split feedback.

The actual physical circuit details of this feedback have yet to be shown, and it must be remembered that the feedback loop must allow the high numbered multivibrator to control the lower numbered one, while the lower numbered one must have no effect on the operation of

/



(a) SIMULTANEOUS FEEDBACK

(b) SPLIT FEEDBACK

Figure 6

the higher numbered. To understand how this is possible, consider the circuit shown below as fig. 7:

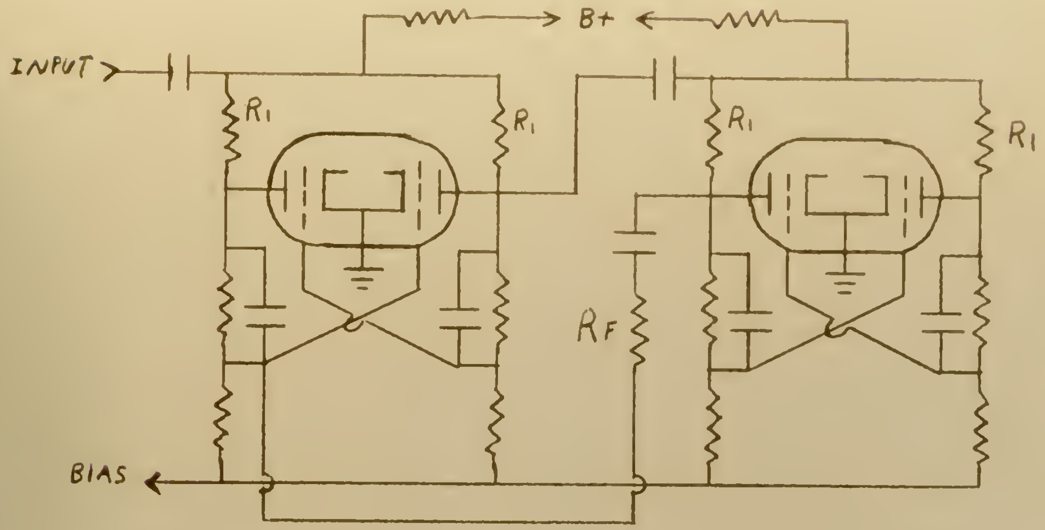


Figure 7

Consider this circuit to represent multivibrators number two and three of a binary type decade employing split feedback. On the third applied trigger a negative pulse is formed by the right hand plate of V_2, coupled to the right hand grid of V_3 cutting off that section of the tube. This in turn produces a negative pulse at the left hand plate of V_3 which is coupled by the feedback circuit to the right hand grid of V_2, returning this half of V_2 to the "off" condition as desired. That this action is unilateral can be seen by the fact that pulses of opposite polarity and of the same amplitude are generated at all times on the two plates of V_2. As long as R_f is considerably larger than R_1, then the pulse applied to V_3 from the right hand plate of V_2 will override that applied from the left hand plate, and as far as the operation of V_3 is concerned the feedback loop has no effect. As can be seen from

the waveforms of fig. 6 (b), on the eighth applied pulse the trigger delivered by the feedback circuit is a positive triangular pulse, and since the desired condition of V_2 is with the right hand section conducting, this feed-back is in the proper direction. The same reasoning will apply to the feedback between stages four and three.

5. A method of indicating the residual count.

Perhaps the best method of explaining count display is to take a specific example such as the case of a split feedback decade employing neon bulbs as indicators. This is perhaps the most popular circuit and can be understood by reference to the plate waveforms shown in fig. 6 (b) and the following explanation. Ten neon bulbs are arranged with one side connected in two groups of five, and the other side connected in five groups of two as shown below in fig. 8:

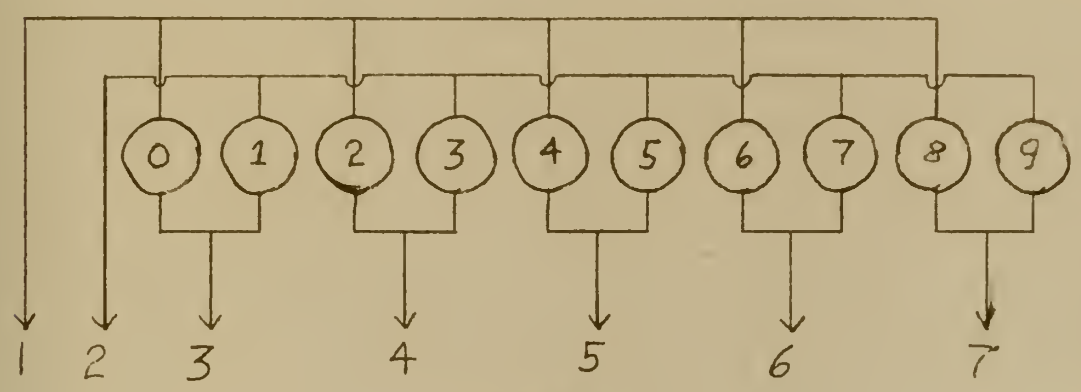


Figure 8

The connections marked one and two are connected to a tap on the opposite plate load resistors of multivibrator number one, and the state of this multivibrator will determine which of the two groups of five has its upper side at a high potential and which at a low potential. By connecting this to a tap on the plate load resistor the upper side

of the neon bulbs is permitted to go as high as the supply voltage but will not go as far in the negative direction as does the plate of the multivibrator. The usual tapping position is at the mid-point of the plate load resistor, provided that the difference in potential between the two plates is only slightly greater than that necessary to fire the bulb. The potential applied to the connections 3, 4, 5, 6 and 7 is determined in each case by two plates of multivibrators 2, 3 or 4 as shown schematically below in the case of lamps zero and one.

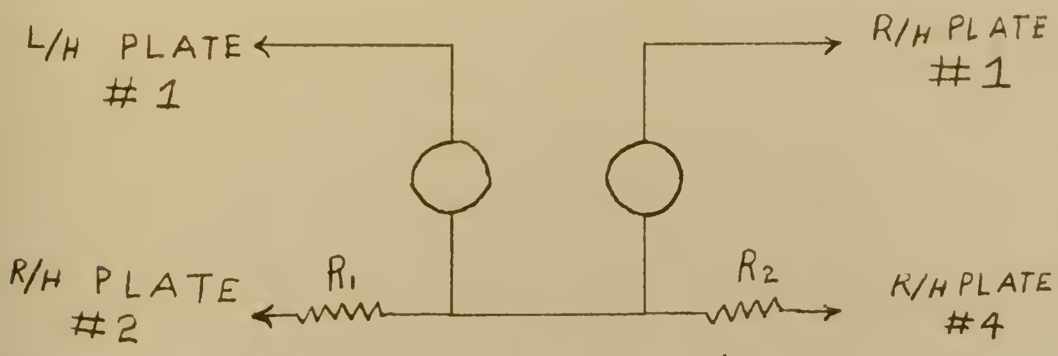


Figure 9

Due to the resistors R_1 and R_2 there are three possible potentials at the lower side of the neon bulbs corresponding to both plates high, both plates low, and one plate high while the other is low. Referring now to the waveforms of fig. 6 (b), it will be seen that only during counts zero and one are both right hand plates of multivibrators two and four low. Since the left hand plate of number one is high on the zero count and right hand side high on the count of one, the full possible potential is across lamp number zero and number one at the proper time. During no other count is it possible for the full

potential to appear across either of these lamps. Similar reasoning will apply to the other indicator lamps. To completely illustrate the method of indication and feedback, a schematic diagram of a complete binary decade employing split feedback and neon bulb indication is included in the appendix.

6. Development and use of the "normalized flip-flop chart" for triode counter design and analysis.

In order to better tie down the subject of design and analysis of the scale-of-two circuits used in computers a series of performance curves termed the "normalized flip-flop chart" was conceived by Mr. Hal W. Boyd and set forth in Engineering Note E-525 of the MIT Digital Computer Laboratory in February 1953. The development and explanation of the chart to be given here is substantially as given by Mr. Boyd, with the example of the use of the chart for high speed counter work and the resulting circuit that worked out by the author during a ten week industrial experience tour at Sanders Associates Inc., Nashua, N. H.

Consider the triode bi-stable multivibrator shown below as fig. 10:

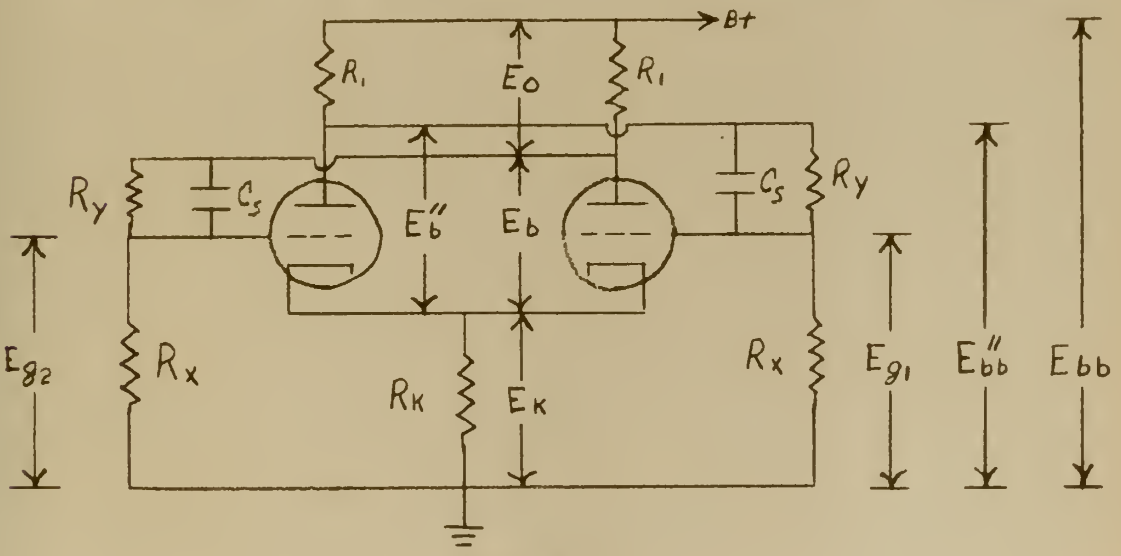


Figure 10

Assume that the right hand tube is conducting.

Let stability, s, be defined as the ratio of the grid swing available, E_s, to that required to cut off either tube, E_{co}, then:

$$s = E_s/E_{co} \tag{1}$$

and from fig. 10:

$$E_s = E_{g1} - E_{g2} \tag{2}$$

Defining the cut-off (u_o) as the magnitude of the ratio at cut-off of the plate-to-cathode voltage, E_b'', to the grid-to-cathode voltage, E_{co}, then:

$$E_{co} = E_b''/u_o \tag{3}$$

Assuming that the grid bias on the conducting tube is zero, then:

$$E_{g1} = E_k \tag{4}$$

substituting (4) into (2)

$$E_s = E_k - E_{g2} \tag{5}$$

From (1) and (5):

$$E_k - E_{g2} = sE_{co} \tag{6}$$

From figure 10:

$$E_b'' = E_{bb} - E_k \tag{7}$$

To simplify what is to follow and to make the chart more general, assume that $R_1 \ll R_y$ so that R_1 is negligible in comparison to R_y. Hence from the figures:

$$E_k = \frac{R_x}{R_x + R_y} E_{bb} \tag{8}$$

Inserting (3), (7), and (8) into (6):

$$\frac{R_x}{R_x + R_y} E_{bb} - E_{g2} = s\left[\frac{E_{bb}}{u_o} - \frac{R_x}{R_x + R_y}\frac{E_{bb}}{u_o}\right] \tag{9}$$

But from fig. 10:

$$E_{g2} = \frac{R_x}{R_x+R_y}\left(E_{bb} - E_o\right) \qquad (10)$$

Combining (9) and (10):

$$\frac{R_x}{R_x+R_y}E_{bb} - \frac{R_x}{R_x+R_y}\left(E_{bb}-E_o\right) = S\left[\frac{E_{bb}}{U_o} - \frac{R_x E_{bb}}{U_o(R_x+R_y)}\right] \qquad (11)$$

Simplifying:

$$E_o = (S/U_o)(R_y/R_x)E_{bb} \qquad (12)$$

From fig. 10:

$$E_b = E_{bb} - E_O - E_k \qquad (13)$$

Normalizing E_o, E_k, and E_b from equations (12), (8) and (13) with respect to E_{bb}, and letting primes denote normalized quantities:

$$E_o' = E_o/E_{bb} = (s/u_o)\ (R_y/R_x) \qquad (14)$$

$$E_k' = E_k/E_{bb} = R_x/R_x+R_y \qquad (15)$$

$$E_b' = E_b/E_{bb} = 1 - E_o' - E_k' \qquad (16)$$

From equations (1) and (3):

$$s/u_o = E_s/E_b'' \qquad (17)$$

Inserting (5) for E_s:

$$s/u_o = (E_k - E_{g2})/E_b'' \qquad (18)$$

Combining (10) and (7) in (18):

$$S/U_o = \frac{E_K - \frac{R_x}{R_x+R_y}\left(E_{bb}-E_o\right)}{E_{bb} - E_K} \qquad (19)$$

Normalizing (19) and substituting (15):

$$S/U_o = \frac{E_K' - E_K'\left(1 - E_o'\right)}{1 - E_K'} \qquad (20)$$

But, $1 - E_o' = E_b' + E_k'$, hence:

$$s/u_o = \frac{E_k' - E_k' E_b' - (E_k')^2}{1 - E_k'} \qquad (21)$$

To find the maximum s/u_o with respect to E_k' at a fixed E_b', let $\frac{\partial\, s/u_o}{\partial E_k'} = 0$, and solve for E_k'; hence:

$$\frac{\partial\, s/u_o}{\partial E_k'} = \frac{(1-E_k')(1-E_b'-2E_k') + [E_k' - E_k'E_b' - (E_k')^2]}{(1-E_k')^2} = 0 \qquad (22)$$

$$(E_k')^2 - 2E_k' + (1-E_b') = 0 \qquad (23)$$

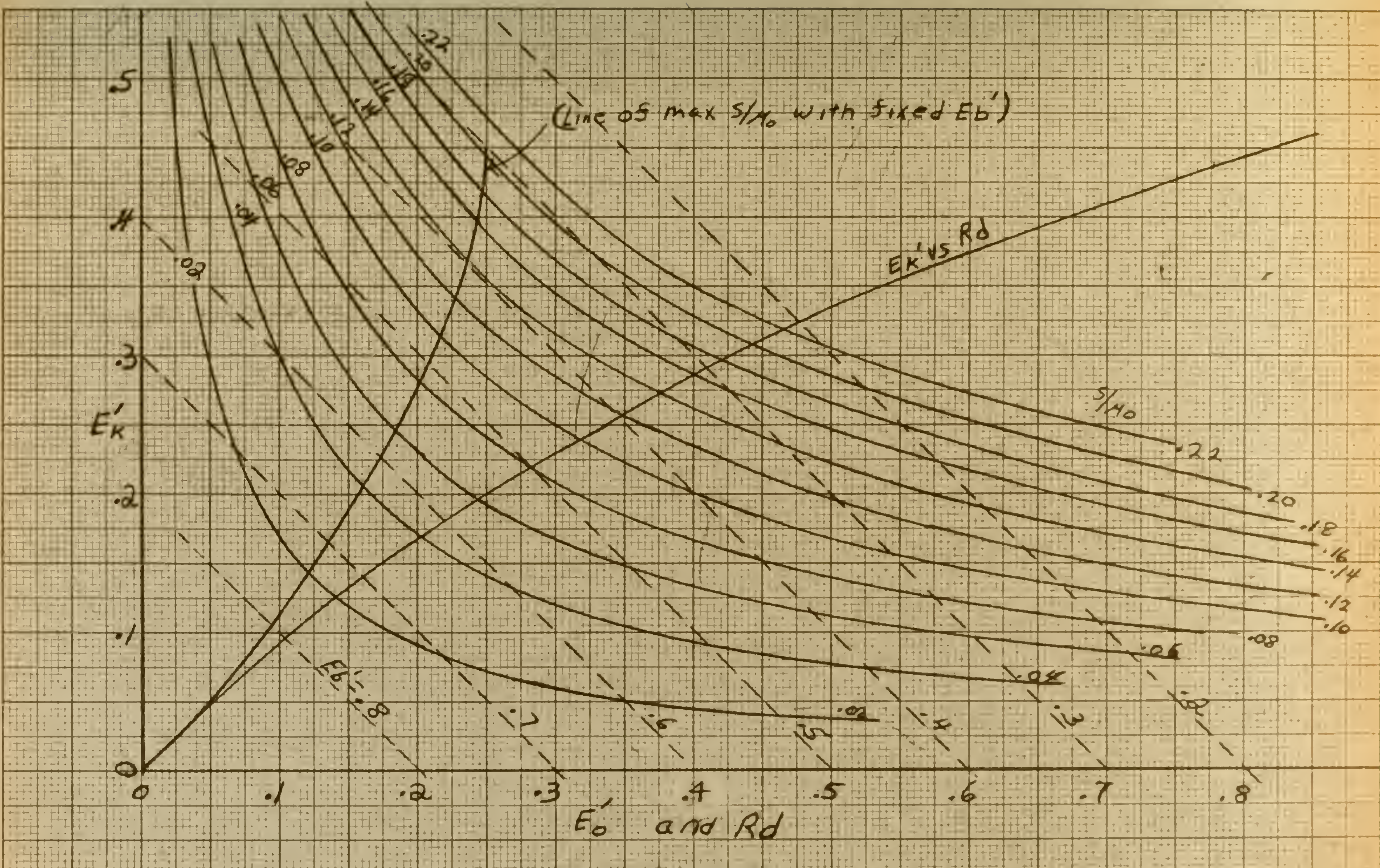
$E_k' = 1 \pm E_b'$; but since E_k' 1

$$E_k' = 1 - E_b' \qquad (24)$$

From equations (14), (15), and (16) plots can be made for constant values of s/u_o which relate E_k' to E_o' and E_b' to E_o'. Also, by letting $R_d = R_x/R_y$, a plot of R_d against E_k' can be made. From equation (24) a plot of E_k' for maximum s/u_o at fixed E_b's can be made versus E_o', thus completing the normalized flip-flop chart which is shown as fig. 11.

For fast operation it is desirable to switch large currents: i.e., to use high tube currents. In general, this may be interpreted on the normalized flip-flop chart to mean operating the flip-flop near the origin where the higher tube drops are located. The characteristics of the tube, its plate dissipation rating, over-all flip-flop reliability or stability, triggering characteristics, output voltage requirements, wattage ratings of components, and power supplies serve to define and limit the freedom of moving the operating point on the chart.

For high flip-flop reliability it is desirable to have a sufficiently high stability factor. With respect to the normalized flip-flop chart,



Normalized Flip-Flop Chart

Figure 11

this means operating the flip-flop away from the origin of the chart. Again such things as tube characteristics, output requirements, etc. serve to define and limit the range over which the operating point can be moved.

For the fastest operation at the highest stability, the flip-flop should be operated on the "line of maximum s/u_o with fixed E_b". The tube characteristics, supply voltages, output swing, etc. determine whether or not operating on a point on this line is desirable. Operation below and to the right of the "line of maximum s/u_o with fixed E_b' " at either a fixed E_b' or s/u_o, yields a larger normalized output swing than operation above and to the left of the line. Two advantages are realized by operating in this region. These are: better triggering characteristics and, if the output swing is more than needed, isolation of the flip-flop from its load by tapping the plate resistor for the desired output swing.

The reason that triggering characteristics are better in this region is that the lower operating point corresponds to a lower normalized E_k which, in turn, determines a lower ratio of R_x to R_y (R_d). The combination of greater normalized output voltage swing and smaller divider arm ratio allows a greater difference in charge on the memory or speeding up capacitors. This allows better complementing characteristics and allows larger triggers before destroying the memory. A larger ratio of ac to dc gain in the divider arm also follows which allows easier triggering. In addition to the extended trigger amplitude range, the resolution time is decreased allowing faster operation.

The normalized flip-flop chart can be used for synthesizing as well as analyzing almost any type of triode flip-flop. To enter the chart one must know or determine at least two of the following:

1. u_o of the tube
2. s
3. E_o
4. E_k
5. E_b
6. R_d
7. E_{bb}

For design applications of the chart, some modifying considerations might be:

1. Resolution time
2. Qualitative reliability
3. Triggering characteristics
4. Tubes plate dissipation, I_b, or E_b
5. Wattage rating of components
6. Available power supplies

The more of the above that are known, providing there are no contradictions, the more rapid the solution.

In designing flip-flops by the normalized flip-flop chart method, synthesizing is the first step. After the synthesis of the flip-flop to meet its requirements, the nearest RMA size components are computed. Which side of the ideal is more desirable can be determined from the chart. The flip-flop with RMA components can then be resolved by the chart and an analysis made to determine whether further trials are needed.

If the plate resistor, R_1, cannot be considered negligible with respect to R_y as the derivation assumes, then a correction may be applied. For computations on the chart, substitute E_{bb}" for E_{bb} as the normalizing factor; hence, in converting from absolute to normalized and vice-versa, E_{bb}" is to be used. The only values thus affected are E_o, E_b, and E_k. E_o, instead of being the voltage across R_1, is now, however, the output voltage swing. The resistance at the plate of the flip-flop is now the parallel combination of R_1 and $R_x + R_y$.

If the "on" tube's bias is not zero, no corrections need be applied to the chart. In designing for a negative "on" tube bias, the values of the resistances R_1 and R_k are determined by E_o and E_k and by the I_b of the tube at the E_b and the negative bias desired.

As an example of the use of the chart to design one stage of a counter the following analysis is presented.

While any known quantities could be used for a start, in this example E_{bb}, s, and u_o will be decided upon first and all other quantities determined from the chart. First a 5963 was chosen as the tube to use, since it is especially designed for on-off service and its relatively high plate dissipation will allow the switching of high currents yielding high-speed response. Selection of the tube specifies u_o, and for the 5963 this quantity is approximately 12. The next quantity selected was E_{bb}, which was set as 210 volts because that voltage was conveniently available. The stability was selected next, and this was chosen equal to 2 on the basis that stability should be low enough to allow fast operation but high enough so that resistor tolerances and tube characteristics would not be too critical.

With s equal to 2, all resistors exactly as specified, and with a "normal" tube, there will be twice the voltage swing available at the grid as is necessary.

The following factors are now determined:

$E_{bb} = 210$

$u_o = 12$

$s = 2$

$s/u_o = 2/12 = .167$

Since the highest speed operation is obtained on the "line of maximum s/u_o with fixed E_b' ", the point of intersection of s/u_o and this line is located and the following quantities read from the chart:

$E_k' = .405$

$E_o' = .245$

$E_b' = .35$

$R_d = .685$

Then:

$E_k = .405 \times 210 = 85v$

$E_o = .245 \times 210 = 51.4v$

$E_b = .35 \times 210 = 73.5v$

From the plate characteristics of the 5963 with plate to cathode voltage equal to 73.5 volts and at a bias of zero, the plate current is determined to be 8.3ma. Therefore:

$R_k = 85/8.3 = 10200$ ohms

$R_1 = 210 - (85+73.5)/8.3 = 6200$ ohms

$R_d = R_x/R_y = .685$

Since $R_x + R_y$ is not critical, any convenient value may be chosen

for R_x or R_y so long as the ratio R_x/R_y is maintained at .685. Remembering that $R_x + R_y$ should be large compared to R_1, it was decided that R_y would be set at 82K. Therefore:

R_x = 82 X .685 = 56K (both RMA values)

Every component of the circuit has now been specified. This circuit was used as the basic stage of a binary decade constructed by the author and operated reliably as high as 1.6mc. The complete circuit is shown below as fig. 12.

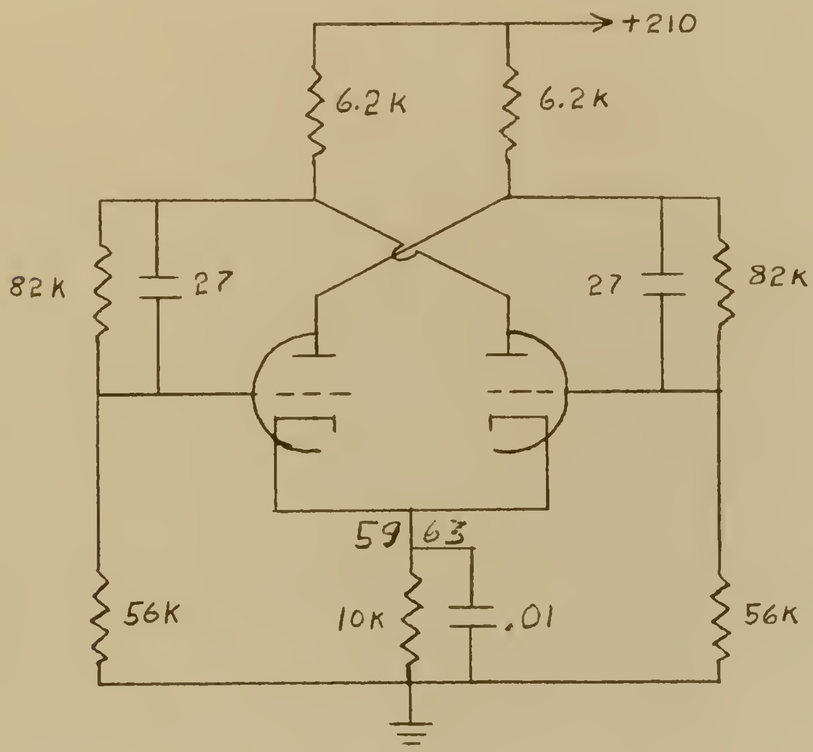


Figure 12

1. A brief sketch of the new tube and what it will do.

It seems that very often in the development of new type circuits there comes a point where a new type tube can be made which will permit circuits to be much simpler or much better. Such a tube development was undertaken in regard to counter circuits by the Philips Research Laboratories and resulted in the E1T Decade Counting Tube. This tube is basically a cathode-ray tube built into a normal octal receiving tube envelope which is capable of counting and displaying a count of ten at a maximum rate of 30kc. This one tube embodies within itself the functions of an entire decade which would normally require four dual triodes plus indicating circuits. While only a brief summary of the theory of operation of this tube can be presented here, the reader is strongly urged to read the article listed as reference 24 in the bibliography which contains a detailed description of the theory of operation of both the decade tube and associated pulse shaping circuitry.

In its simplest terms the E1T is composed of an electron gun which emits a beam towards the envelope of the tube, a deflecting device ensuring that the beam will come to rest in one of ten predetermined positions, and a fluorescent screen calibrated 0 to 9 to indicate the position of the beam. Either the tube structure itself or the circuitry associated with it must insure that the beam will return to its zero position after the tenth applied pulse, and that only one output pulse is obtained from the circuit for each ten applied

input pulses. The methods used to accomplish each of these functions will be briefly described.

Fig. 13 shows a cross-section of the decade counter tube and fig. 14 shows the schematic circuit diagram. The beam of electrons is focused into a ribbon shaped filament, rather than the conventional cylindrical shape, by electrodes p_1 and p_2 in order to provide a high beam current with relatively low beam voltage. This is desirable since the speed of response is in general proportional to I/VC, where V is the potential difference traversed by the electrons, I the current, and C the inter-electrode capacitance. This ribbon-shaped beam passes through the deflection electrodes D and D' which have been so positioned that at its extreme limits of deflection the beam almost touches the electrodes to increase the deflection sensitivity. At given values of deflection the beam passes through one of the ten vertical slots in the electrode g_4, with a feedback circuit ensuring that the only stable positions of the beam are when passing through one of these slots. Some of the electrons passing through these slots are collected by anode a_2 with the remainder passing through an aperture in this anode to strike the fluorescent screen painted on the envelope. The position of the beam in any of its ten stable positions is indicated by a lighted area on the envelope.

The major problem in the design of this tube is to insure 10 and only 10 stable beam positions and to insure that an input pulse will always move the beam from one stable position to the next without any possibility of "mis-fire" or overshoot.

In order to understand how the stable positions of the beam are

obtained, consider the simple case of a cathode-ray tube as shown in fig. 15 with anode "a" and one deflection plate D' fed from the same source via a common dropping resistor. In this case $V_a = V_{D'} = V_B - i_a R_a$, provided the deflection electrode current is negligible. This relationship can be represented as a straight line if i_a is plotted against $V_{D'}$ as shown at II in fig. 16 (a). If the other deflection electrode is connected to a separate voltage source, the anode current is independent of the value of this voltage and i_a versus V_D plots as a horizontal line shown at I in fig. 16 (a). Since both of these lines represent the same current (i_a), then the only possible state of equilibrium is at the intersection, and electrode D' will assume a potential corresponding to the abscissa of the point of intersection. Thus one point of stable equilibrium has been established, and if the straight horizontal line corresponding to the potential of V_D could be changed to some function which would intersect the other potential line at several points other stable states would be possible. This is exactly what was done in the case of the ElT by inserting a slotted plate between the deflection electrodes and the anode. As the beam is swept across this slotted plate a curve of the form shown in fig. 16 (a) results which has 19 points of intersection with the $V_{D'}$ potential line. Of these 19 points of intersection, the 10 points corresponding to the beam being centered in the slots represents stable conditions. In order to understand how the beam is shifted to the next position in line by each succeeding pulse, it should be remembered that the curve shown as I in fig. 16 (a) is for a constant voltage applied to the left deflection electrode. If a positive pulse is applied to the left deflection

electrode V_D will rise and the beam will tend to move toward the left, but since the anode current will decrease as less and less of the beam passes through the slot, the voltage on the right deflection electrode will increase pulling the beam back to its original position. This does not take into account the action of the interelectrode capacitance which is always present. If the rise time of the input pulse is short enough, then the anode and right deflection electrode voltage may be assumed constant during the pulse rise and the beam will actually be pulled to the left by an amount corresponding to the amplitude of the trigger. If this trigger amplitude is adjusted so that the beam will move approximately the distance between slots, and if the fall time of the trigger is long enough so that the voltage on the right hand electrode may change before the trigger has decayed to zero, then the beam will remain centered in this next slot in stable equilibrium. In the E1T tube the trigger voltage must be 13.6 volts ± 18% with rise time not slower than 20V/usec and fall time not faster than 2 V/usec. These conditions are met by using a pulse shaping amplifier prior to each counter tube.

In order to reset the beam to its zero position and to obtain an output pulse after ten input pulses, a "reset" anode is added to the left of the main anode and is shown in fig. 13 as a_1. Every tenth pulse initially deflects the beam occupying position 9 still further to the left, so that it leaves the slotted electrode and terminates on the reset anode. The reset anode is connected to the +300V line via a resistor, so that its potential depends upon its current. The variations of this voltage are used for initiating the resetting operation

and for yielding an output pulse which is fed to the input of the following stage.

While this is admittedly only the briefest once-over of this counter tube, a modified circuit employing an ElT is included in the appendix. This circuit is an improved version of the original Philip's circuit that was developed at Airborne Instrument Laboratories employing American type tubes and has yielded consistent stable operation at rates of 30kc.

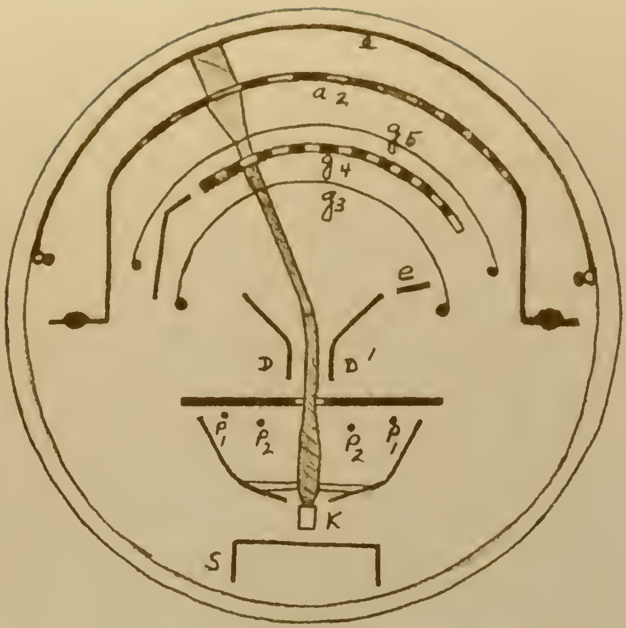


Figure 13

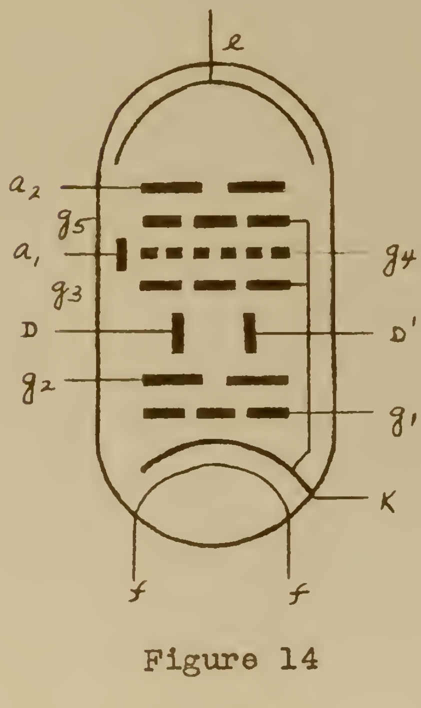


Figure 14

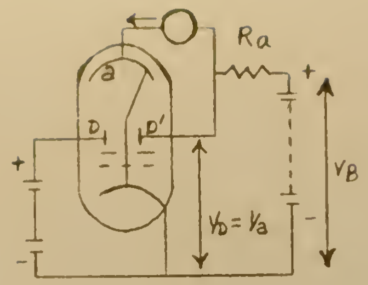


Figure 15

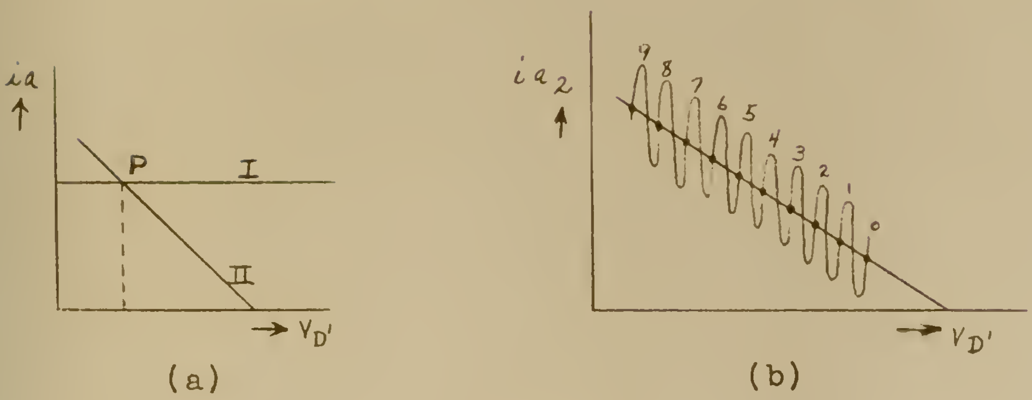


Figure 16

1. Triggering of multivibrator counters and its relationship to speed of response.

Although the triggering of multivibrator counter circuits may be discussed in a relatively small space, the subject should be given considerable attention when designing such circuits, and most particularly when designing high-speed circuits. Due to this increasing importance of proper trigger design as speed of response is increased, the discussion of this subject has been saved for this section of the paper. In order to study the available methods of triggering and compare them, consider the typical multivibrator counter stage as shown below as fig. 17.

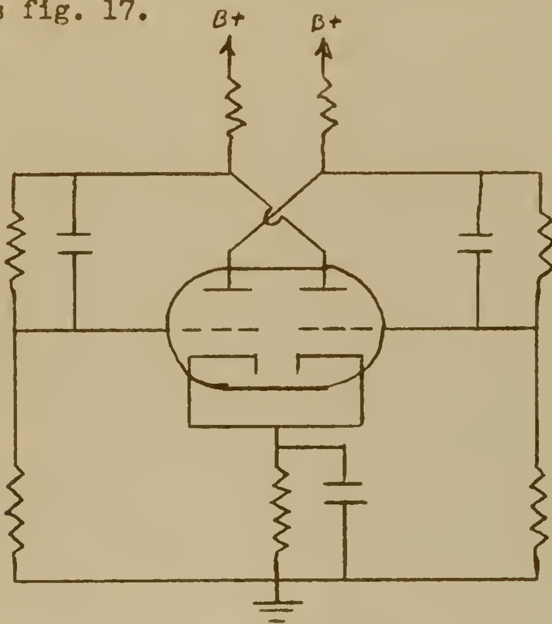


Figure 17

There are three possible points for applying the trigger corresponding to the three elements of the tube. All three elements have been used in practice, and it remains for us to discuss their relative advantages and differences. First let us see what changes to thebasic

circuit are necessary to apply a trigger at these points. Shown below as fig. 18 is the basic circuit slightly modified in three different ways in order that the trigger may be applied to plate, grid and cathode respectively.

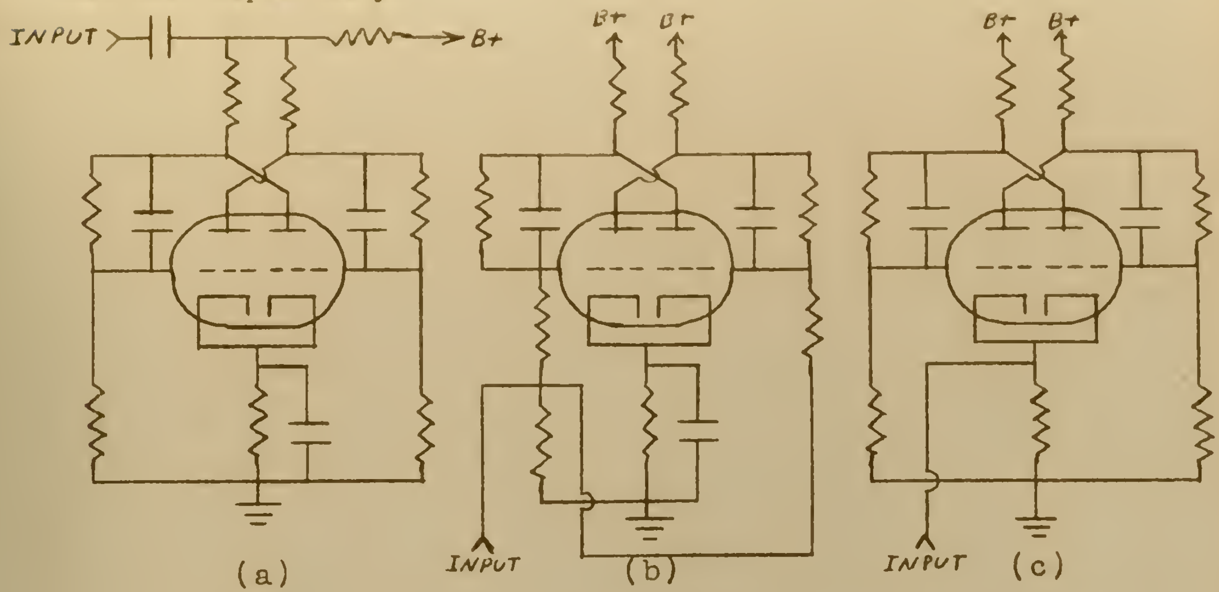


Figure 18

It should be remembered while considering the polarity of the trigger that it is desired to have the trigger start regeneration and not, of itself, change the state of the circuit. For example, in the circuit of fig. 18 (a) it should be apparent that a large enough positive trigger would force the grid of the "off" section above cut-off. The falling voltage on the plate of the "off" section would cancel the effect of the trigger on the other section and result in a change of state. This method of triggering is not reliable and is to be avoided in all cases, the design being such that the applied trigger will initiate regeneration and the resulting self-generated waveshapes cause the change of state.

The ideal triggering arrangement would be a fast acting switch

connecting the trigger source to the counter while it is moving in the desired direction, and disconnecting the trigger source as soon as regeneration occurs. If a square trigger is employed, the trailing edge will have a tendency to force the circuit in the wrong direction, and for high speed work where the duration of the trigger pulse must be short, this tendency can cause unreliable operation and incorrect counting.

Since the trigger will normally be applied to any element through a small condenser, some differentiation and overshoot can be expected to occur and this is another reason why a square pulse is not a good trigger. The ideal waveform appearing on any element would be a triangular wave with steep leading edge in the desired direction and tapering trailing edge.

If the trigger is to be applied by means of a condenser as shown in fig. 18, then there is no particular advantage or disadvantage to guide the choice of element. Any element chosen will normally yield a low impedance load to the trigger source, and if the resistors are properly chosen then capacative loading of the stage may be held to a minimum. Cathode triggering has the advantage that there is no capacitive loading of the circuit, but the un-bypassed cathode resistor results in a certain amount of degeneration and yields slower operation for that reason. The general tendency in practice with relatively low speed circuits (100kc or less) seems to be to use plate triggering and to by-pass the cathode so that balance between tubes is less important.

While any of the above methods will work fairly well with low speed circuits, it is necessary as speed is increased to have a triggering

arrangement and triggering waveform which more closely approaches the ideal. This can be accomplished very neatly by inserting a diode between the trigger source and the point of application of the trigger on the stage in question. This arrangement minimizes trigger source loading of the counter stage and when used with a square pulse triggering waveform results, after passing through the diode, in a waveform very closely approximating the ideal triangular shape. Capacitive loading can further be minimized by using crystal diodes rather than vacuum tubes and usually results in a somewhat less cumbersome layout. Diode triggering is normally applied to grid or plate with the cathode by-passed. As an illustration of this technique, one stage of a 1mc counter employing triodes and diode triggering is shown as fig. 19. This is one stage of a decade developed by the author while conducting experiments on these circuits at Sanders Associates, Inc. at Nashua, New Hampshire. This circuit was made to function at approximately 2 mc.

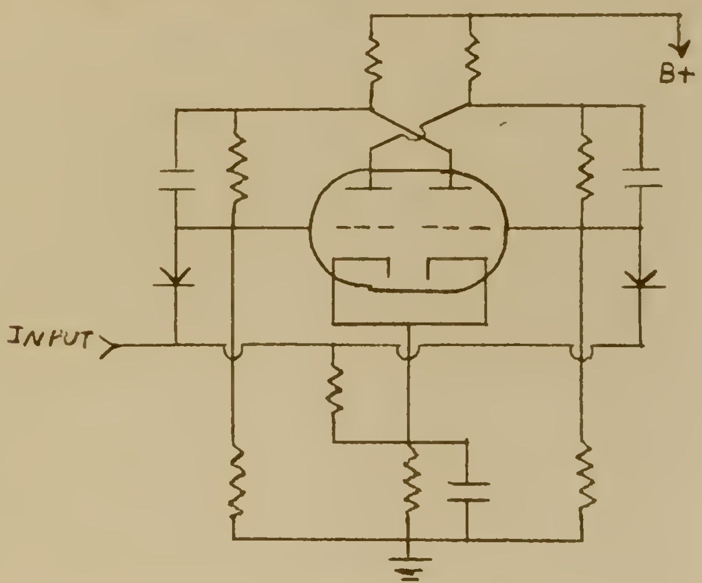


Figure 19

We have now arrived at that point in the discussion where the

emphasis is on high-speed operation, and for our purposes high-speed will be defined as operation above 200kc, as circuits operating below this frequency range pose no special problems. In order to get into this subject, let us continue our discussion of triggering to determine what limits the minimum trigger duration. Consider the triode multivibrator circuit shown below as fig. 20.

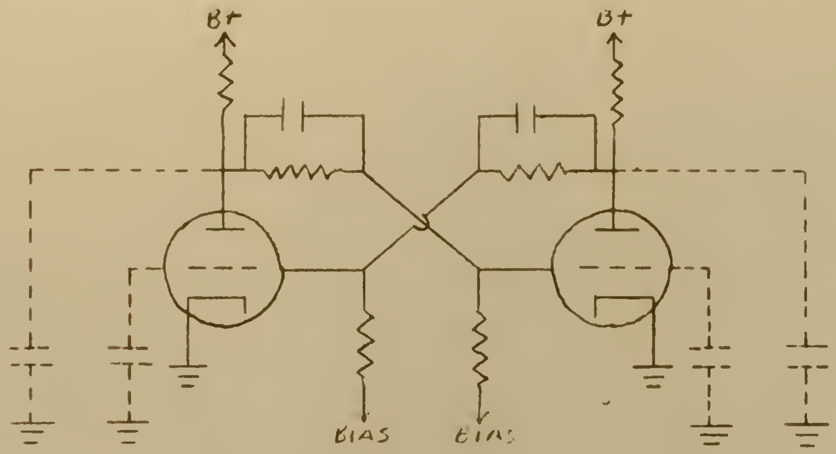


Figure 20

If the right hand section is conducting, then before regeneration can take place it is necessary that C_o of the conducting tube begin to charge and that it charge sufficiently prior to the removal of the trigger to lower the potential of the "off" plate enough to support regeneration. The limiting time constant of the circuit, and that which determines maximum counting rate, is the plate time constant of the tube which is being cut off.

It is apparent that if a trigger having an infiniately sharp leading edge can be impressed upon the plate, then that trigger will appear on the grid with the same leading edge slope. The tube can, therefore,

be turned off instantaneously, but its plate voltage cannot rise except as C_o charges. The duration of the trigger, therefore, must be of sufficient length to allow C_o to charge to a high enough voltage to cut on the opposite section. In this regard it can normally be assumed that the "speed-up" capacitor is large compared with C_i, and that there will be little reduction in magnitude of voltage rise between C_o and C_i. In the case of triode multivibrators, however, this is not the case, since C_i is not only a relatively large capacitor to start with but is greatly increased by the Miller effect as soon as the tube begins to conduct. With only this very basic discussion it can already be seen that high gm pentodes would seem to be the best solution to the problem of increasing speed, at least as far as high speed triggering is concerned.

2. Basic fast recurrence rate, short duration pulse circuit theory.

To determine those factors which limit the speed of response of the over-all circuit and to develop criteria to guide the choice of tubes for high speed work, let us first consider some basic fast recurrence rate, short duration pulse circuit theory. As an example take the circuit of a clipper amplifier shown below as fig. 21.

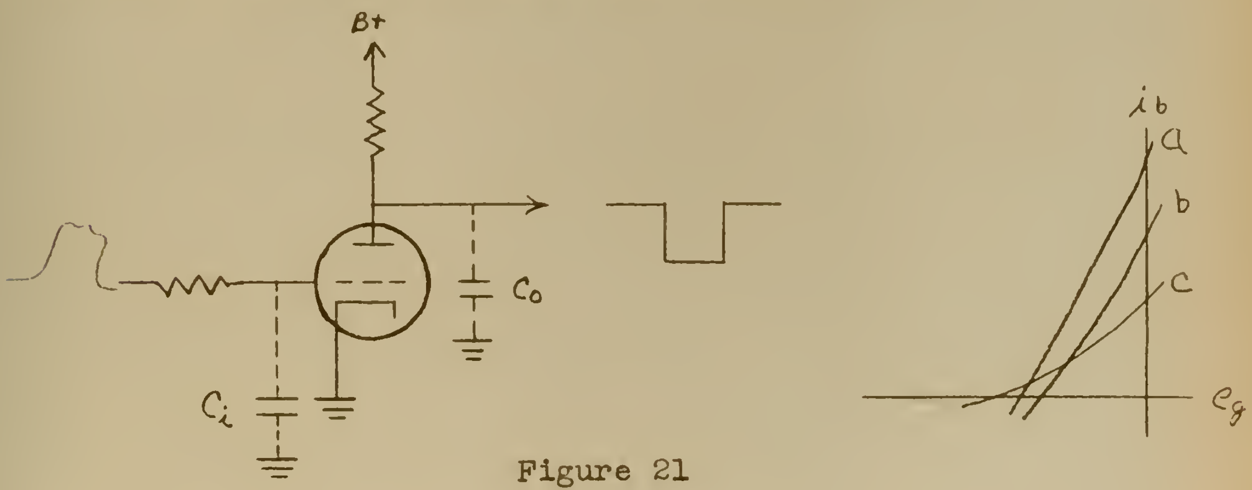


Figure 21

As a basic maxim for pulse circuit work, it should be borne in mind that the abruptness of change or maximum speed of a waveform at any point in a circuit is almost always limited by the capacity which is loading that point and the current available to charge or discharge that capacity. If optimum conditions are to be met, then loading capacities must be small and the peak currents through them must be large. It is obvious that for the three tube characteristics sketched "c" is inferior to either "a" or "b" because a larger grid swing and accordingly a longer time is required to cut the tube off. C_o should be kept as small as possible by choosing a tube with small output capacity, design of the circuit to have a small input capacity to the next stage, and by careful wiring to minimize stray capacitance. Once C_o has been made as small as practical, the limiting time of charge and discharge is determined by the current available on the charge and discharge cycle. The load resistance should be made as small as possible considering plate dissipation and desired output voltage so that a large charging current may flow from the high voltage supply on the trailing edge of the pulse, and the grid should be driven as far positive as practical so that a large discharge current may flow on the leading edge of the pulse.

Comparing the other two sketched tube characteristics it is evident that characteristic "a" is superior to "b", since for only a slightly larger grid swing a much larger current charge results. This very basically sets forth the general requirements of circuit and tube for fast pulse work. We will now proceed to examine these criteria in a more quantitative manner and as applied specifically to multivibrator

counting circuits.

3. Approximate quantitative analysis of the change of state in a multivibrator showing limiting factors to speed.

When first considering the problem of high speed multivibrator operation one might assume that it would be possible to break the desired output wave into its Fourier components and proceed with an analysis on the basis of the highest frequency which it is necessary to pass. This is not the case, however, due to the regenerative connections employed. As far as the author can determine no exact analysis of a multivibrator has been made to date. Several good approximate analyses have been presented, and the reader is referred particularly to the Ph. D. thesis of S. D. Snowdon, Cal. Inst. of Tech., 1945. This analysis concerns itself with the entire cycle of action and uses an analytical expression for tube characteristics. The analysis to be presented here is in substance the same as that derived at the MIT Radiation Laboratory and presented in Volume 19 of the Radiation Laboratory Series of reference books.

The analysis will be made on the basis of an astable circuit, but since only the transition period is to be analyzed the results will be essentially the same for any multivibrator circuit. The circuit is as shown below and its dynamic equivalent is as indicated.

The arrows indicate the direction of current flow, and for purposes of the analysis it will be assumed that V_1 is coming on and V_2 going off. There are two main simplifying assumptions necessary in order that the calculations will not be so cumbersome as to be useless. First is that the grid-plate capacitance may be replaced by a grid to ground capacitance

(Miller effect) which is in magnitude $(1 + A)C_{gp}$. The second is that the dynamic transfer characteristic of the tube may be represented as a straight line function of the form $i_p = gm\ E_c$. Therefore in the equivalent circuit $C_1' = C_{g_1} + C_{p_2} + (1+A_1)C_{gp_1}$.

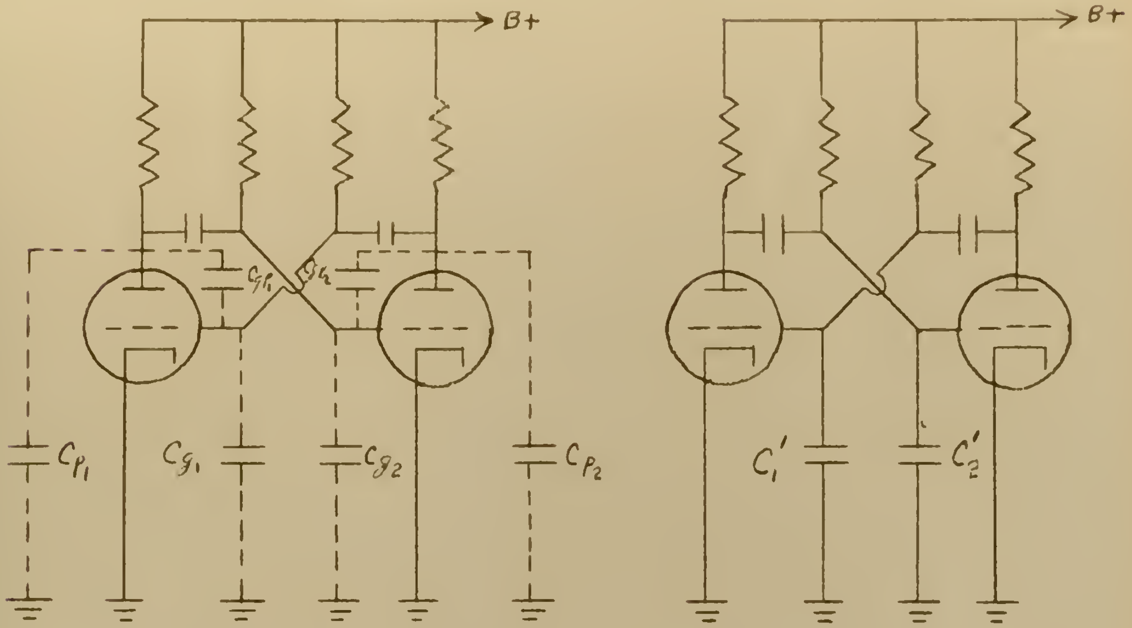


Figure 22

The method of attack will be to assume that the waveform at the grid of V_1 can be represented by a power series of time. This waveform may be translated to current through V_1 which is integrated in C_2' to yield the voltage waveform at the grid of V_2. This voltage waveform may in turn be translated into current through V_2 which is integrated in C_1' to yield another expression for the grid waveform of V_1. We then will have two power series representing the same voltage, and according to a well known mathmatical theorem, these two series may be equated term by term. In this manner all unknowns can be evaluated.

Assume the voltage at the grid of V_1 to be:

$$v_1 = -E_c + bt + ct^2 + dt^3 + \cdots\cdots$$

Then if $i_p = gm\ E_c$; the current through V_1 is:

$$i_1 = bgm\ t + cgm\ t^2 + dgm\ t^3 + \cdots\cdot$$

Considering all of i_1 to pass through C'_2, then

$$\dot{v}_2 = -\left(\frac{bgm}{C'_2} t + \frac{cgm}{C'_2} t^2 + \frac{dgm}{C'_2} t^3 + \cdots\cdots\right)$$

$$v_2 = -\left(\frac{1}{2}\ \frac{bgm}{C'_2} t^2 + \frac{1}{3} \frac{cgm}{C'_2} t^3 + \frac{dgm}{C'_2} \frac{t^4}{4} + \cdots\cdot\right)$$

The current available for charging C'_1 is:

$$I_0 + \frac{1}{2}\ \frac{bgm^2}{C'_2}$$

Dividing by C'_1 and integrating:

$$v_1 = -E_c + \frac{I_0}{C_1} t + \frac{1}{6} \frac{bgm^2}{C'_1 C'_2} t^3 + \frac{1}{12} \frac{cgm^2}{C'_1 C'_2} t^4 + \frac{1}{20} \frac{dgm^2}{C'_1 C'_2} t^5 + \cdots\cdot$$

We now have two expressions in a power series representing the same quantity. If our original assumption that the voltage could be represented as a power series was corrent, then these two expressions must be equal term by term. Equating them and solving for the unknown constants:

$$b = \frac{I_0}{C_1} \qquad c = 0 \qquad d = \frac{1}{6} \frac{bgm^2}{C'_1 C'_2} = \frac{1}{6}\ \frac{I_0 gm^2}{C'_1 C'_2}$$

$$e = 0 \qquad f = \frac{1}{20} \frac{dgm^2}{C'_1 C'_2} = \frac{1}{120}\ \frac{I_0 gm^4}{C'^3_1 C'^2_2},\ etc \text{ ---}$$

$$v_1 = -E_c + \frac{I_0}{C}\left[t + \frac{1}{3!}\frac{g_m^2}{C_1' C_2'} t^3 + \frac{1}{5!}\left(\frac{g_m^2}{C_1' C_2'}\right)^2 t^5 + \cdots\right]$$

$$v_2 = -\frac{I_0 g_m}{C_1' C_2'}\left[\frac{1}{2!} t^2 + \frac{1}{4!}\frac{g_m^2}{C_1' C_2'} t^4 + \frac{1}{6!}\left(\frac{g_m^2}{C_1' C_2'}\right)^2 t^6 + \cdots\right]$$

These equations are admittedly the product of many assumptions, but for a typical triode circuit employing a type 6SN7 dual triode the equations yield a minimum rise time of approximately .3 microseconds which is in fairly good agreement with experimental minimums. It is considered, however, that the major contribution of these equations is that they very definitely indicate a figure of merit for tubes to be used in such circuits. It is very obvious that high gm tubes with low capacities must be used for fast rise times, but the reader should note that this figure of merit differs from that for straight amplifier tubes. In the multivibrator figure of merit the denominator is the product of the capacities of the tube, while for a normal amplifier this denominator would be the sum of the two. As a result of this reasoning it is thought by the author that the best modern day tube for use in fast acting multivibrators would be one such as the 6AH6 miniature pentode. This tube was used in experiments by the author and could be made to function in the vicinity of 10mc with the added circuitry to be described below.

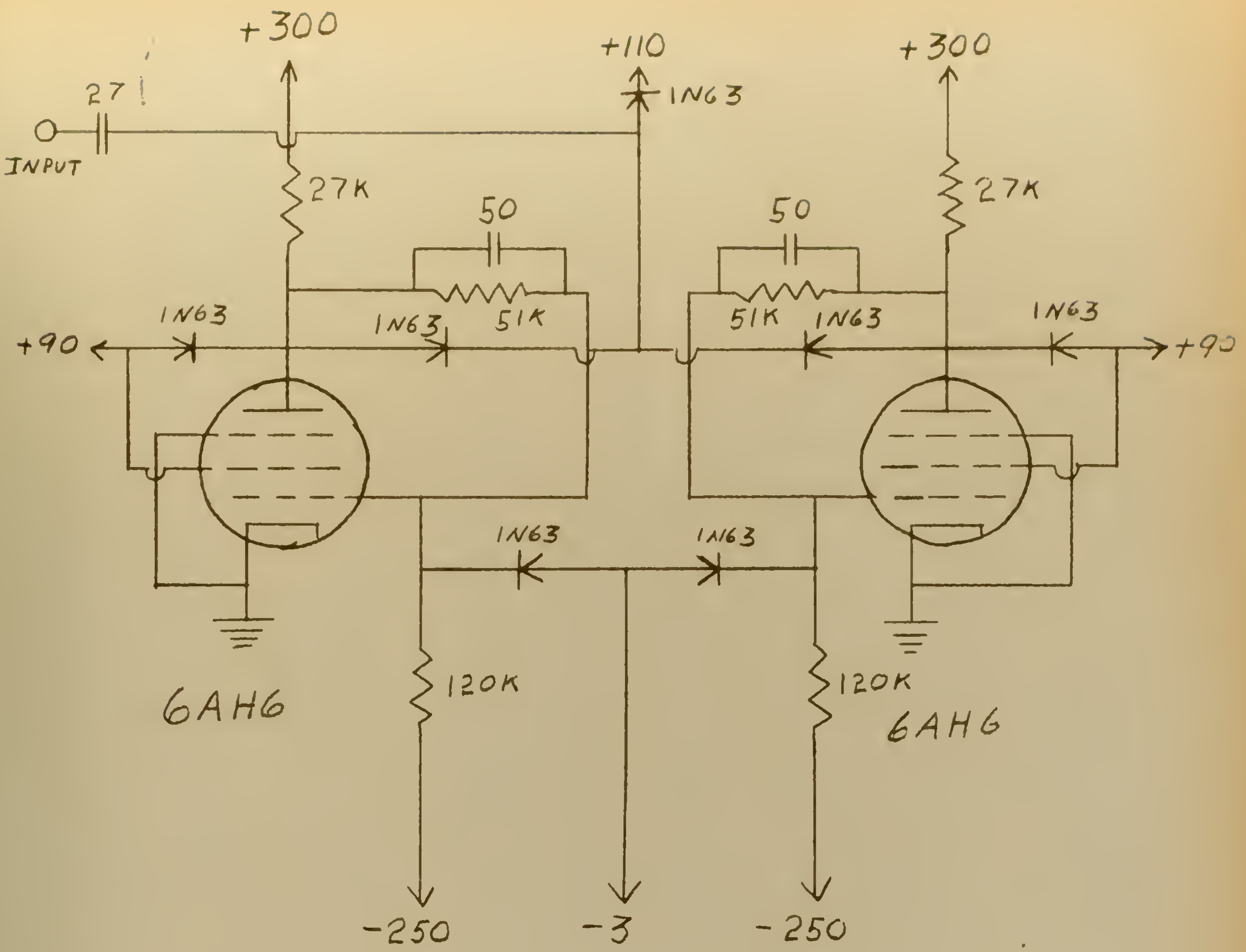
4. Use of clamping diodes to increase speed of counting.

Once the best possible tube has been chosen and the circuit designed for the fastest possible operation using the proceedures previously set forth, it will be found that the maximum counting rate will be slightly

greater than one megacycle. It would be very desirable if the counting rate could be increased to at least ten magacycles. Since no further improvement is possible in making the time constant shorter, it is rather obvious that increased speed can come only if some method is found to operate the circuit over only a portion of exponential. As far as the author can determine the first solution to this problem was obtained at the Radiation Laboratory. Although the method employed is not particularly complicated, it represents a very neat solution to the problem. In brief it consists of arranging diodes to limit the voltage travel of both grid and plate. By this means the change of state can be considered to be over and the circuit ready to accept another trigger in a fraction of the time normally required for a change of state. The circuit as developed by the author while working on this problem at Sanders Associates, Incorporated, Nashua, New Hampshire, during January and February of 1954 is shown below as an example of this technique.

One of the best features of this circuit is that it results in a relative freedom from change in tube characteristics. In the conventional circuit when designing for the highest possible speed the plate load resistor will be made small, and if tube characteristics change very much the circuit may not operate reliably. In this design, however, since the plate is allowed to move only over a small fraction of its swing, the actual voltage swing which would be present if no clamping were used is relatively unimportant.

The design of these circuits is relatively simple. First select a tube and design a normal multivibrator counter stage as has been described previously, using an intermediate value of plate load resistor.



TEN MEGACYCLE MULTIVIBRATOR

For high gm pentodes a load resistor in the range of ten to fifty thousand ohms will be satisfactory. Next a grid clamping level should be selected, and minus three volts is normally used for this purpose. This grid clamping will in itself limit the upper limit of plate travel. The new plate excursion can be obtained from the plate characteristics and then the plate clamping limits set. This should be done such that the midpoint of the plate clamping level corresponds to the plate voltage which exists due to the grid being at its clamping midpoint. A convenient plate excursion would be twenty volts arranged ten volts on either side of the plate midpoint.

There remains one last factor to overcome when designing for maximum speed, and that has to do with the delays encountered in the normal feedback circuits. At the present state of the art a single stage can be made to operate only slightly in excess of 10mc, and if it is desired to construct a 10mc decade then very little delay in the feedback circuit can be tolerated. Since no multivibrator can switch instantaneously, there will be some small time delay between the triggering of the first multivibrator in the chain and the actual switching of the last multivibrator in the chain. This delay would normally be of no consequence if it represented only a time differential between input and output, but in the case of the decade scaler employing simultaneous feedback it manifests itself as a delay between the eighth applied trigger and the action of the feedback circuit in resetting multivibrators two and three. With extremely high speeds this delay becomes as great as the time between triggers and results in the circuit

not being ready to accept the ninth trigger. This problem has been solved by Hewlett-Packard as follows: A gating circuit controlled by multivibrators two and three allow the eighth applied pulse to trigger both multivibrators one and four, with numbers two and three being reset by number four in the usual manner. In this way the delay throughout the entire circuit is only that of one stage rather than the sum of the delays in all four stages as is normally the case. Using all the techniques previously discussed, it is possible to design counter circuits which will operate reliably with inputs as high as 10mc or with randomly spaced input pulses separated by one-tenth micre-second or more.

6. Gating

The one remaining circuit normally associated with counters is the gating circuit. Although this circuit might be considered almost a separate equipment, the counter depends upon accurate gating for its usefulness. Without gating the entire counter could only perform the function of dividing the input by some fixed number, and the reading of its indicators would have no meaning. Gating circuits have been treated extensively in the literature, and only the uses of the gate as applied to counters will be discussed here. The function of the gating circuit as used in conjunction with a counter is that of an accurately controllable switch placed between the signal source to be counted and the actual counting circuits. While it is not necessary that this switch be able to operate at high recurrence rates, it is essential that the time between the opening and closing of the switch be accurately controlled. In order to get an idea as to the accuracy required,

consider the case of counter circuit capable of operating at a 10mc rate. Since the inherent error of the counter is plus or minus one count regardless of the frequency, then it will be necessary for the switch to be accurate to within one-tenth micresecond if it is not to indtroduce additional inaccuracies of its own. This is the greatest accuracy that will be required, since at any frequency below ten megacycles the inherent error of plus or minus one count of the scaling circuits will become an ever larger factor and the tolerance on the gate circuit accuracy may be accordingly relaxed. Whatever type gating circuit is eventually chosen to be used, it is usually driven by an accurate 100kc crystal oscillator followed by a string of frequency dividers. In this manner the gating tube can be controlled to yield accurate gate durations of .001, .01, .1, 1.0, and 10 seconds. By having available these accurate counting intervals the versatility of the instrument will be greatly increased over one which can count only over one fixed time duration. The time base generator which furnishes these time intervals is also usually made to reset the counting circuits for so called "automatic" counting. In this process let us assume that a 1 second duration is being employed by the counter to directly indicate the frequency of an oscillator under test. In the "automatic" counting made the instrument will count for one second, hold this count displayed for one second, and then repeat the process as long as desired. In this manner an observer could record the actual frequency of the oscillator under test every other second. It is rapid and precise measurements of this type which make the decade counter an indispensible laboratory tool.

BIBLIOGRAPHY

1. Bagley, A. S. A 10mc Scaler for Nuclear Counting and Frequency Measurement. Hewlett-Packard Journal. 2:1-4, October 1950.

2. Bagley, A. S. The High-Speed Frequency Counter. Hewlett-Packard Journal. 2:1-4, January 1951.

3. Brown, Cyril H. Plug-in Scaler for Industrial Counting. Electronics. 21:90-91, July 1948.

4. Buckley, Elery F. and Manley, John C. Neon Diode Ring Counter. Electronics. 23:84-87, January 1950.

5. Chance, Britton et al. Waveforms New York, McGraw Hill. 1949 (Massachusetts Institute of Technology. Radiation Laboratory Series. No. 19).

6. Deeter, E. L. and Dau, W. K. Interval Timer. Electronics. 20:87- . July 1947.

7. Easton, Allan and Odessey, Paul H. Counter Circuits for Television. Electronics. 21:120-126, May 1948.

8. Eccles, W. H. and Jordan, F. W. A Trigger Relay Utilizing Three-electrode Thermionic Vacuum Tubes. Electronics. 83:298- . September 1919.

9. Gerlough, D. L. and Daiser, H. R. Multiple-Output Predetermined Counter. Electronics. 24:122- . February 1951.

10. Gossick, B. R. Predetermined Electronic Counter. Proceedings of the IRE. 37:813- . July 1949.

11. Grosdoff, I. E. Electronic Counters. RCA Review. 7:438-447, September 1946.

12. Hertz, Hellmuth. A Magnetic Scaling Circuit. Journal of Applied Physics. 22:107- . January 1951.

13. Howland, Bradford. Capacitor Counting Circuit. Electronics. 21:182- . June 1948.

14. Howland, Bradford. Scale of N Counting Circuits. Electronics. 20:138- . July 1947.

15. Lifshutz, Harold and Lawson, J. L. A Triode Vacuum Tube Scale-of-Two Circuit. Review of Scientific Instruments. 9:83- . 1938.

16. Lifshutz, Harold. New Vacuum Tube Scaling Circuit of Arbitrary Integral or Fractional Scaling Ratio. Physical Review. 57:243-244. 1940.

17. Phelps, Byron E. Dual-Triode Trigger Circuits. Electronics. 18:110- . July 1945.

18. Potter, John T. A Four Tube Counter Decade. Electronics. 17:110- . June 1944.

19. Ratz, A. G. Positive Gate Counter Circuit. Electronics. 23:194- . November 1950.

20. Regener, V. H. Decade Counting Circuits. Review of Scientific Instruments. 17:185-189, May 1946.

21. Sharpless, T. K. High-Speed N-Scale Counters. Electronics. 21:122-125, March 1948.

22. Shea, Gregory. Electronic True Decade Counters. Electronic Industries. 5:82-84, September 1946.

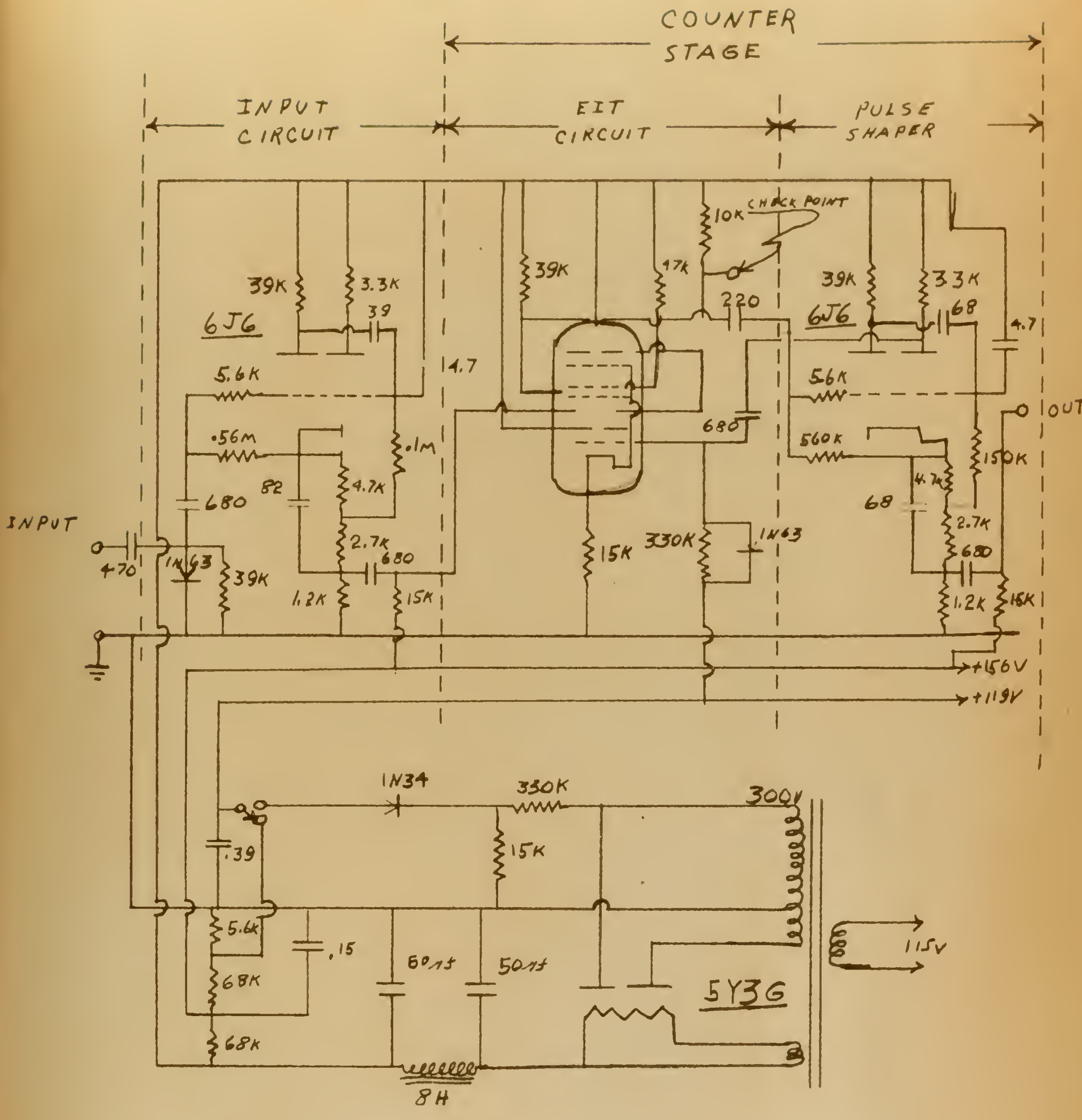
23. Vin Zeluff. New Count Detector. Electronics. 22:122, April 1949.

24. Van Overbeek, A. J. W. M.; Jonker, J. L. H.; Rodenhuis, K. A Decade Counter Tube for High Counting Rates. Philips Technical Review. 14:313-326, May 1953.

25. Weissman, Richard. Stable Ten-Light Decade Scaler. Electronics. 21:90-91, May 1949.

26. Wells, F. H. Fast Pulse Circuit Techniques for Scintillation Counters. Necleaonics. 10:28-33, April 1952.

27. Wild, J. J. Predetermined Counters. Electronics. 20:120-123, March 1947.



COUNTER STAGE
INPUT CIRCUIT
EIT CIRCUIT
PULSE SHAPER
INPUT
OUT
6J6
6J6
5Y3G
CHECK POINT
+156V
+119V
300V
115V
8H
1N34
1N63

Figure 24

Appendix II

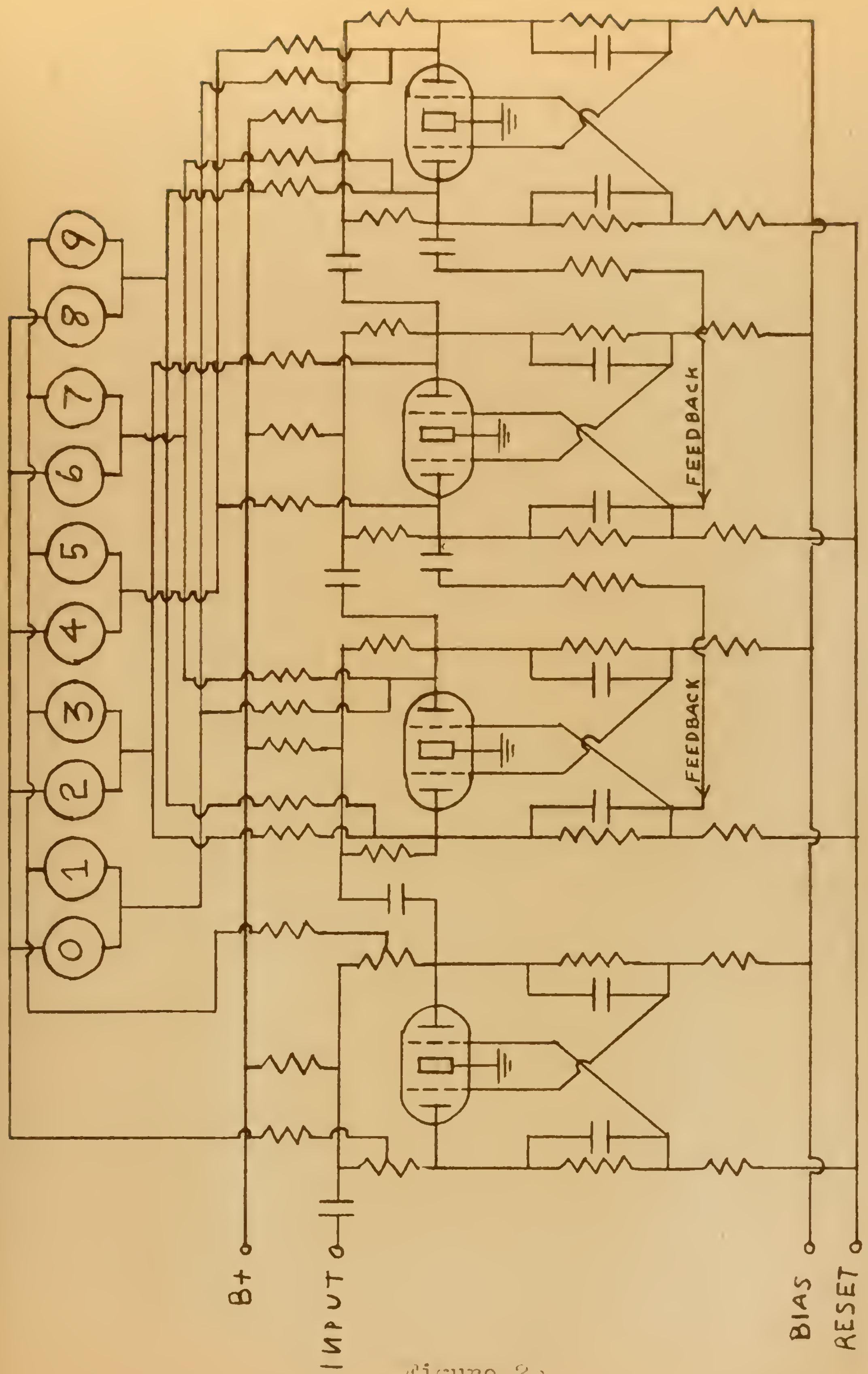


Figure 25