1956

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# A NOMOGRAPHIC METHOD FOR PREDICTING THE BEHAVIOR OF A PETROLEUM RESERVOIR

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B.S., University of Washington, 1949

Submitted to the Graduate School of the University of Pittsburgh in partial fulfillment of the requirements for the degree of Master of Science

Pittsburgh, Pennsylvania

1956

# TABLE OF CONTENTS

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# I. INTRODUCTION

The Material Balance Method of calculating the behavior of petroleum reservoirs is one of the petroleum engineer's most important tools. Introduced in 1936, the Material Balance has undergone many improvements intended to simplify the mathematical techniques of handling the complex physical relationships involved. In spite of all advances in technique, the method remains unwieldy and laborious.

The basic concept of the Material Balance applies the Law of Conservation of Mass to the process of producing petroleum from an underground reservoir. The amount of hydrocarbons originally present in the reservoir must equal the amount produced plus the amount remaining in the ground at any given time during the production life of the reservoir. As normally used, the Material Balance is written volumetrically. It relates the volumes of oil and gas (produced and remaining) by means of their physical characteristics (gas volume, gas solubility and oil shrinkage) at the pressure prevailing in the reservoir.

The Material Balance has been most widely used for the solution of two general problems: the estimation of original oil in place and the prediction of future production performance of a reservoir. The reservoir discussed in this paper will be of the Solution Gas Drive Type, with fixed reservoir volume, and having no gas cap and no water encroachment into the reservoir.

## A. Estimation of the Original Oil in Place

The quantity of oil and gas originally present in a reservoir may be estimated by observing the drop in reservoir pressure during the production of a certain amount of fluids, and relating these data to the laboratory-measured behavior of the reservoir fluids caused by pressure changes. The volume of reservoir fluids (with known shrinkage and solubility characteristics) which will undergo a certain pressure change because of the removal of a given quantity of these fluids is a fixed quantity. Thus, the drop in reservoir pressure caused by a given amount of oil and gas being produced defines the volume of these fluids originally present.

## B. Prediction of the Future Production Performance of a Reservoir

For any given pressure assumed to prevail in the reservoir at some future time, the volumes of oil and gas produced (when related to their physical characteristics at that pressure) constitute a unique solution to the equation balancing the original volume of fluid with the volumes produced and remaining. An additional concept is involved however. The permeability of the porous reservoir medium to oil and to gas changes as production proceeds. Thus, as the saturation of liquid in the reservoir decreases as oil is produced, the relative amounts of oil and gas which flow to the well will change. This phenomenon cannot be directly related to the Material Balance itself, yet it determines the relative quantities of oil and gas which will be produced. The relationship between the relative permeability to oil and gas and the liquid saturation in the reservoir must be known or assumed, since it must be considered in arriving at the amounts of oil and gas to be produced. These amounts of oil and gas can then be used to predict a solution to the Material Balance at some assumed future point of pressure and production.

It will be the purpose of this thesis to develop a graphical means for predicting the performance of a depletion drive reservoir based on a modification of the Material Balance equation.

## II. REVIEW OF THE LITERATURE

The use of the Material Balance for estimating oil in place was developed by Schilthuis in 1936.[1] His equation (shown here for a solution gas drive reservoir without gas cap, water encroachment or water production)

$$N = \frac{n[u + (r_n - s_o)v]}{u - u_o}^{*} \qquad (1)$$

still remains the most used basic form of the Material Balance.

At about the same time Katz[2] proposed a tabular method for evaluating oil in place. This method was later shown by Pirson[3] to be equivalent to the Schilthuis method.

The use of the Material Balance Equation for prediction of future reservoir performance was proposed considerably later. Babson[4] developed a trial and error solution based on the Material Balance Equation and the Instantaneous Gas-Oil-Ratio Equation. His method was very cumbersome and required a great deal of computation. It is rarely used today. In the same year, 1944, Tarner[5] proposed a solution to the problem which was considerably simpler to use.

Muskat[6] developed a form of the Material Balance expressed as a differential equation and applied it to prediction of depletion drive reservoir performance.

The most widely used of the Material Balance prediction techniques is the method using a trial and error solution of Schilthuis' Equation[7, 8]. This method requires the simultaneous satisfaction of three equations:

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[1]References are listed in the Bibliography.
*Nomenclature shown in Appendix I.

1. The Material Balance Equation

$$N = \frac{n[u + (r_n - s_o)v]}{u - u_o} \tag{1}$$

2. The Instantaneous Gas-Oil-Ratio Equation

$$r_i = \frac{\mu_o}{\mu_g}\frac{\beta}{v}\frac{K_g}{K_o} + s \tag{2}$$

3. The Liquid Saturation Equation

$$S_L = S_W + (1 - S_W)\left(\frac{1 - n}{\beta_o}\right)\beta \tag{3}$$

The trial and error solution may be conducted in the following manner:

1. Assume a certain drop in reservoir pressure
2. Estimate the volume of oil which would be produced during this pressure drop
3. Using this estimated value of oil production, solve the Liquid Saturation Equation, (3), and obtain the corresponding value of $\frac{K_g}{K_o}$ from the known or assumed $\frac{K_g}{K_o}$ vs. $S_L$ relationship.
4. Solve Equation (2) for the value of Instantaneous Gas-Oil-Ratio which would obtain at the assumed pressure.
5. Using the assumed and estimated values of oil production, gas-oil-ratio and fluid characteristic solve the Material Balance Equation. If a balance is obtained, the assumed values were correct. If the equation does not balance, a new value of oil production or pressure is estimated and another trial is made.

This process is repeated until the equations are satisfied at sufficient pressure points to define the future performance of the reservoir.

The use of the Schilthuis Equation, while more flexible in the handling of such conditions as water encroachment and gas cap expansion, is extremely tedious. For the simplest reservoirs, twenty-five columns of calculations must be used, and five or six trials at each of seven or eight pressure points is not unusual. All this must be performed with a calculating machine. Despite this, it is still the most widely used mathematical prediction technique.

In January, 1955, a prediction method based on the Schilthuis method, but greatly simplified, was published by Tracy.[9] The Tracy method is the single biggest advance in the use of the Material Balance Equation. This method will be described for the case of a depletion drive reservoir without gas cap or water encroachment.

Tracy took the Schilthuis Equation

$$N = \frac{n[u + (r_n - s_o)v]}{u - u_o} \tag{1}$$

and substituted values for u, $u_o$ and $r_n$ as follows:

$$u = \beta + (s_o - s)\ v \tag{4}$$

$$u_o = \beta_o \tag{5}$$

$$r_n = \frac{G}{n} \tag{6}$$

This substitution resulted in:

$$N = \frac{n\left(\frac{\beta}{v} - s\right) + G}{\left(\frac{\beta}{v} - s\right) - \left(\frac{\beta_o}{v} - s_o\right)} \tag{7}$$

which may be expanded to:

$$N = \frac{n\left(\frac{\beta}{v} - s\right)}{\left(\frac{\beta}{v} - s\right) - \left(\frac{\beta_o}{v} - s_o\right)} + \frac{G}{\left(\frac{\beta}{v} - s\right) - \left(\frac{\beta_o}{v} - s_o\right)} \qquad (8)$$

Thus, n and G are multiplied by coefficients which are functions of pressure only. These coefficients are:

$$\phi_n = \frac{\left(\frac{\beta}{v} - s\right)}{\left(\frac{\beta}{v} - s\right) - \left(\frac{\beta_o}{v} - s_o\right)} \qquad (9)$$

$$\phi_g = \frac{1}{\left(\frac{\beta}{v} - s\right) - \left(\frac{\beta_o}{v} - s_o\right)} \qquad (10)$$

Equation (8) then reduces to:

$$N = n\,\phi_n + G\,\phi_g \qquad (11)$$

In this form, the equation may be used to estimate original oil in place.

Tracy's prediction method uses this equation in two forms with N taken as equal to one barrel of stock and oil.

1. As the Prediction Equation:

$$\Delta n = \frac{1 - (n_{i-1}\,\phi_n + G_{i-1}\,\phi_g)}{\phi_n + \frac{r_i + r_{i-1}}{2}\,\phi_g} \qquad (12)$$

2. As the Material Balance Equation:

$$n_i\,\phi_n + G_i\,\phi_g = 1 \qquad (13)$$

Equations (12) and (13) are used in conjunction with Gas-Oil-Ratio Equation:

$$r_i = \frac{\mu_o}{\mu_g}\ \frac{\beta}{V}\ \frac{K_g}{K_o} + s \qquad (2)$$

and Liquid Saturation Equation:

$$S_L = S_W + (1 - S_W)\left(\frac{1 - n_i}{\beta_o}\right)\beta \qquad (3)$$

Equations (2) and (3) being linked by knowledge of the $\frac{K_g}{K_o}$ vs. $S_L$ relationship.

The use of Tracy's Prediction Method calls for the following procedure:

1. Starting with a point in production history (with known values of $n_i$, $G_i$ $\phi$, $r_i$ for a given pressure), assume a new lower pressure point.

2. Estimate an instantaneous gas-oil ratio for this pressure

3. Using this pressure and gas-oil ratio in the Prediction Equation (12), compute a value for $\Delta n$.

4. Add this value of $\Delta n$ to $n_i - 1$ ($n_i$ from the previous pressure) to get the value of $n_i$ for the new assumed pressure.

5. Solve the Liquid Saturation Equation, (3), using this value of $n_i$ and obtain a value for $\frac{K_g}{K_o}$.

6. Using the $\frac{K_g}{K_o}$ value, solve the Gas-Oil Ratio Equation. This value of Instantaneous Gas-Oil Ratio should equal that estimated in step 2. If it does not, use the value of $r_i$ found in step 6 as the estimated value in step 2 and re-do steps 2 through 6. When the values of Instantaneous Gas-Oil Ratio in steps 2 and 6 are equal, substitute the values of $n_i$ and

$G_i$ into the Material Balance Equation (13) and solve. The sum of $n_i \phi_n + G_i \phi_g$ should be $1.00 \pm 0.02$.

The number of trials required is cut down to two or three in Tracy's method because Instantaneous Gas-Oil Ratio - a relatively insensitive factor - is estimated. This cannot be done in the Schilthuis Equation as the cumulative value of gas-oil ratio, rather than the instantaneous value, is used in the equation. The cumulative gas-oil ratio value cannot be estimated with any degree of accuracy.

Sturdivant,[10,11,12] in a series of three articles, expanded upon the use of the Tracy method developing a formula to be used in the event of gas reinjection into the reservoir. It was also shown[11] that accuracy to three significant figures in the calculation was sufficient.

## III. STATEMENT OF THE PROBLEM

This investigation was conducted with the object of developing a graphical method for solution of the Performance Prediction problem. It was felt that the Prediction Equation developed by Tracy, with its separation of the pressure factors into coefficients, would lend itself to such treatment. In the Schilthuis Equation, the complexity of form appeared to preclude any simple graphical treatment.

Of the various graphical problem solving methods available, nomography appeared as the best for trial and error solutions. The forms of the equations indicated that nomographs of reasonable simplicity could be constructed to solve them. For trial and error solutions, it was felt that the nomographs should be simple enough that the operator could see at a glance how much a given change in one factor would affect the other factors in the problem. Also, with simple, one-setting nomographs, the operator can readily enter the diagram with the dependent variable (or answer) and work backwards to find the value of independent variable (or normal input). This is of value on trial and error solutions. For these reasons, as well as for ease of construction, it was desirable that the nomographs be of the type in which a single line crosses all scales, obtaining the answer in a single setting.

There are a number of nomographs in the literature concerned with various aspects of the Material Balance problem[13,14,15,16]. All of them are quite complex, requiring entry with several individual pressure variables, and requiring several settings of a line before a solution is obtained. These nomographs are general in application and could be used on reservoirs having the same ranges of variables as the nomographs.

By constructing nomographs for a specific reservoir, all of the fluid characteristics (which are functions of reservoir pressure) and combinations of them, can be treated as a single, or recurring, variable of pressure. This reduces the number of variables to the point where simple, single-setting nomographs can be constructed. Construction of the nomographs for a single reservoir allows selection of optimum scale lengths to specially handle the ranges of the variables for that reservoir. More generalized nomographs would require the ranges of the scales to be broad enough to fit all or several reservoirs. This would result in less accuracy for any particular reservoir.

## IV. METHOD OF INVESTIGATION

The four equations used in the Tracy method of prediction were separately investigated to find the forms most likely to yield the simplest nomographs. These equations, given previously as Equations (12), (13), (2) and (3) will be considered separately.

## A. The Prediction Equation

This equation was given[17] as:

$$\Delta n = \frac{1 - (n_{i-1}\,\phi_n + G_{i-1}\,\phi_g)}{\phi_n + r_a\,\phi_g} \qquad (14)$$

where

$$r_a = \frac{r_i + r_{i-1}}{2} \qquad (15)$$

It is readily rearranged into this form:

$$(n_{i-1} + \Delta n)\,\phi_n + (G_{i-1} + r_a\,\Delta n)\,\phi_g = 1 \qquad (16)$$

In this form, it is seen to be equivalent to the Material Balance Equation as:

$$n_{i-1} + \Delta n = n_i \qquad (17)$$

and

$$G_{i-1} + r_a\,\Delta n = G_i \qquad (18)$$

If these values are substituted into Equation (16), the result is the Material Balance Equation:

$$n_i\,\phi_n + G_i\,\phi_g = 1 \qquad (13)$$

Returning to the rearranged form of the Prediction Equation:

$$(n_{i-1} + \Delta n)\,\phi_n + (G_{i-1} + r_a\,\Delta n)\,\phi_g = 1 \qquad (16)$$

it is noted that there are four variables:

$(n_{i-1} + \Delta n)$, a function of oil production

Let this be $f_1$ (u)

$(G_{i-1} + r_a \Delta n)$, a function of the gas produced with the above amount of oil

Let this be $f_2$ (v)

$\phi_n$, a function of reservoir pressure

Let this be $f_3$ (W)

$\phi_g$, a different function of reservoir pressure

Let this be $f_4$ (W)

Thus, Equation (16) can be written as:

$$f_1 (u) \times f_3 (W) + f_2 (v) \cdot f_4 (W) = 1 \tag{17}$$

Note that the pressure, represented here by "W" is a recurrent variable.

Divide Equation (17) by $f_3$ (W), then:

$$f_1 (u) + f_2 (v) \cdot \frac{f_4 (W)}{f_3 (W)} = \frac{1}{f_3 (W)} \tag{18}$$

Now let

$$\frac{f_4 (W)}{f_3 (W)} = f_5 (W), \text{ a different function of pressure}$$

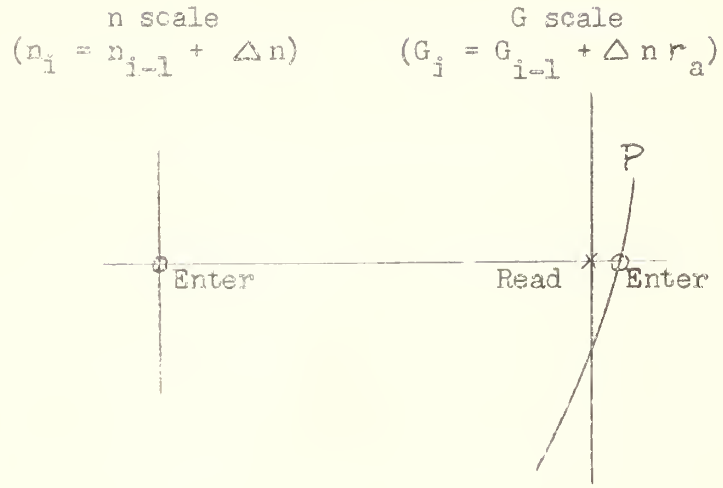
and

$$\frac{1}{f_3 (W)} = f_6 (W), \text{ another function of pressure.}$$

Equation (18) now becomes:

$$f_1 (u) + f_2 (v) \times f_5 (W) = f_6 (W) \tag{19}$$

This form is capable of representation by a simple nomograph having the following configuration:




The geometric proof for this nomograph is given in Appendix IV.

Now, evaluating the pressure functions $f_5$ (W) and $f_6$ (W) results in:

$$f_5\,(W) = \frac{f_4\,(W)}{f_3\,(W)} = \frac{\phi_g}{\phi_n} = \frac{1}{\left(\frac{\beta}{v} - s\right)} \tag{20}$$

$$f_6\,(W) = \frac{1}{\phi_n} = 1 - \frac{\left(\frac{\beta_o}{v} - s_o\right)}{\left(\frac{\beta}{v} - s\right)} \tag{21}$$

These values, when substituted into the Prediction Equation, give:

$$(n_{i-1} + \Delta n) + \frac{(G_{i-1} + r_a \Delta n)}{\left(\frac{\beta}{v} - s\right)} = 1 - \frac{\left(\frac{\beta_o}{v} - s_o\right)}{\left(\frac{\beta}{v} - s\right)} \tag{22}$$

Equation (22) is the form of Tracy's Prediction Equation which is solved by the Prediction Equation Nomograph.

## B. The Material Balance Equation

This equation is equivalent to the Prediction Equation and can use the same nomograph. One solution of the nomograph serves to satisfy both the Prediction Equation and the Material Balance Equation.

Placed in the form used to construct the nomograph, the Material Balance Equation becomes:

$$n_i + \frac{G_i}{\left(\frac{\beta}{v} - s\right)} = 1 - \frac{\left(\frac{\beta_o}{v} - s_o\right)}{\left(\frac{\beta}{v} - s\right)} \qquad (23)$$

Equation (23) is solved by the Prediction Equation nomograph.

[illegible]

This [illegible] Equations [illegible] the same [illegible]

the Prediction Equation [illegible]

[illegible]

Relation [illegible]

## C. The Liquid Saturation Equation

The Liquid Saturation Equation is used to obtain a value of $\frac{K_g}{K_o}$ from a known and extrapolated relationship between $S_L$ and $\frac{K_g}{K_o}$.

In the equation

$$S_L = S_W + (1 - S_W)\left(\frac{1 - n_i}{\beta_o}\right)\beta \tag{3}$$

The values of $S_W$ and $\beta_o$ remain constant.

Let $S_W$ be $K_1$ and let $\left(\frac{1 - S_W}{\beta_o}\right)$ be $K_2$.

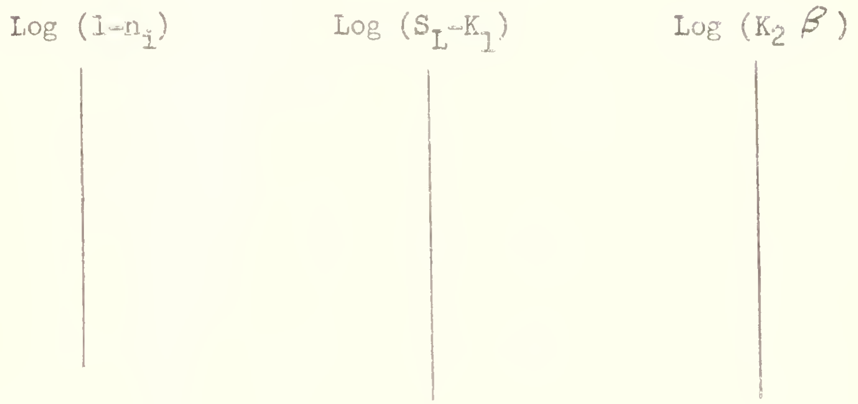
The equation can then be rearranged:

$$S_L - K_1 = (K_2)(\beta)(1 - n_i) \tag{24}$$

or

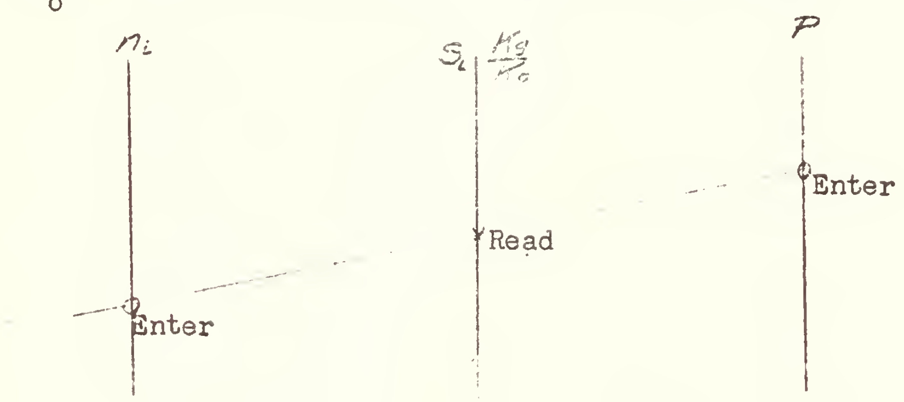
$$\text{Log}\,(S_L - K_1) = \text{Log}\,K_2\beta + \text{Log}\,(1 - n_i) \tag{25}$$

Equation (25) was used to construct a parallel line nomograph with logarithmic scales having the following form:*




*For proof of the geometry of this nomograph, see Appendix IV.

Log $(1-n_i)$ is a simple function of $n_i$ and the left scale was therefore graduated directly in units of $n_i$. Similarly, $\beta$, and therefore Log $K_2\beta$ is a function of pressure, so that the right scale was graduated in units of pressure. The center scale was graduated on one side in units of $S_L$ and by use of the known $\frac{K_g}{K_o}$ vs. $S_L$ relationship, the other side was graduated in units of $\frac{K_g}{K_o}$. The finished nomogram appeared as follows:



## D. The Gas-Oil Ratio Equation

In the instantaneous Gas-Oil Ratio Equation

$$r_i = \frac{\mu_o}{\mu_g} \frac{\beta}{v} \frac{K_g}{K_o} + s \qquad (2)$$

$\frac{\mu_o}{\mu_g}$, $\beta$, $v$, and $s$ are specific functions of the reservoir pressure. By combining all of these but $s$ into a single function of pressure, the equation was rearranged to:

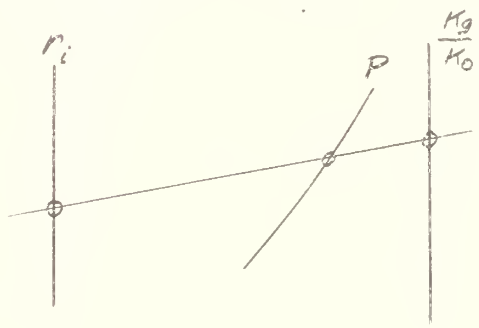
$$r_i - \frac{K_g}{K_o} \times (F) = s \qquad (26)$$

This form is identical to

$$f_1(u) - f_2(v) \cdot f_3(W) = f_4(W) \qquad (27)$$

where $f_3(W)$ and $f_4(W)$ are different functions of the recurrent variable, reservoir pressure.

A nomograph, similar in theory* to the Prediction Equation nomograph was constructed. The finished form appeared as follows:

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*See Appendix IV for geometric proof.

## E. Average Gas-Oil Ratio Equation

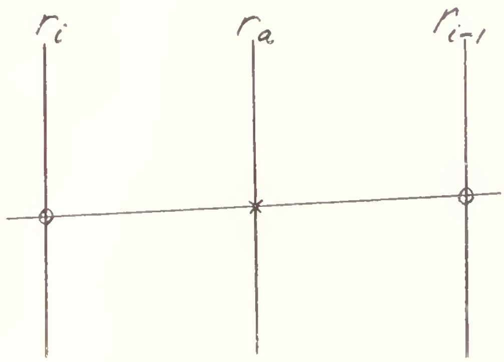
It was found desirable to construct a very simple nomograph to solve the equation

$$\frac{r_i + r_{i-1}}{2} = r_a \qquad (15)$$

The equation was rearranged to:

$$r_i + r_{i-1} = 2\ r_a$$

A nomograph with uniform parallel scales set equal distances apart was constructed.




Entry can be made on any two scales and a value read on the third scale.

## V. USE OF THE NOMOGRAPHS TO PREDICT THE PERFORMANCE OF A SPECIFIC RESERVOIR

To test the application of the nomographic prediction method, an actual reservoir was selected and nomographs constructed using the data available concerning this reservoir. The particular reservoir chosen was a mid-continent limestone reservoir, which is still in the early stages of production. For this reason, it will be called Reservoir "X." The basic data available on Reservoir "X" are listed in Appendix V. In addition, two sets of prediction curves (Pressure and Gas-Oil Ratio Vs. Cumulative Production) were available for comparison with the results obtained in this investigation by the nomographic method. One of these sets of prediction curves was obtained by trial and error use of the Schilthuis Equation. The other was obtained by using Tracy's method.

The insensitivity of instantaneous gas-oil ratio values over a range of incremental oil production values was mentioned in connection with the Tracy method. The relative positions of the n, G and P scales on the Prediction Nomograph show this feature graphically. The pressure scale is very close to the gas production scale. For this reason, it is best to enter the nomogram with an assumed value of oil production, rather than to assume a gas-oil ratio as in the tabulated Tracy method.

The following approach was found to give the easiest solution using the nomograms:

1. Set up a tabulation sheet as follows:

| | | | | |
|---|---|---|---|---|
| (0) | P | 1209 (A known point of Production History) | 1100 | 900 |
| (1) | $\Delta n$ | | 0.00773 | 0.01248 |
| (2) | $n_{i-1}$ | | 0.0319 | 0.03963 |
| (3) | $n_i$ | 0.0319 | 0.03963 | 0.05211 |
| (4) | $G_{i-1} + \Delta n\, r_a$ | 46.75 | 65.5 | 108.3 |
| (5) | $G_{i-1}$ | | 46.75 | 65.5 |
| (6) | $r_a \Delta n$ | | 18.75 | 42.8 |
| (7) | $r_a$ | | 2425 | 3430 |
| (8) | $r_{i-1}$ | | 2400 | 2450 |
| (9) | $r_i$ | 2400 | 2450 | 4420 |
| (10) | $\frac{K_g}{K_o}$ | | 0.0362 | 0.085 |
| (11) | $r_i$ | (From G.O.R. Equation) | 2450 | 4420 |
| (12) | $(n_i)\ (1.8 \times 10^7)$ | | 713.0 | 937.5 |
| (13) | +575,000 | | 575.0 | 575.0 |
| (14) | Cumulative Production (in $10^3$ bbls.) | | 1288.0 | 1512.5 |

2. Select a point of actual production history, knowing pressure, oil production from the bubble point to that pressure, $n_i$, expressed as a fraction of original oil in place, gas production from the bubble point to that pressure expressed as a function of original oil in place, and instantaneous gas-oil ratio, $r_i$.

3. Assume a new pressure, 100-200 psi lower than the historical point used. The $n_i$, $G_i$ and $r_i$ of the historical pressure point now become $n_{i-1}$, $G_{i-1}$, and $r_{i-1}$ in the above tabulation.

4. Estimate an increment of oil production, $\Delta n$. Enter this on

line one of the tabulation. Add to it the $n_{i-1}$. The sum is $n_i$.

5. Using this value of $n_i$ at the assumed pressure, enter the prediction nomograph on the n and P scales and read the value of $G_{i-1} + \Delta n\ r_a$ on the G scale. Enter this value on line 4 of the tabular sheet and subtract the $G_{i-1}$ from it. The difference is $\Delta n\ r_a$.

6. With a slide rule, divide the value of $\Delta n\ r_a$, (line 6) by $\Delta n$ (line 1) to get $r_a$ (line 7).

7. Enter the average gas-oil ratio nomograph with $r_{i-1}$ and $r_a$ and find $r_i$ (line 9).

8. Using $n_i$ (line 3) and Pressure, find $\frac{K_g}{K_o}$ from the liquid nomograph and enter this value on line 10.

9. Enter the gas-oil ratio nomograph with $\frac{K_g}{K_o}$ and Pressure and find the $r_i$ value based on the gas-oil ratio equation. Place on line 11.

10. Compare lines 9 and 11 which will be equal if the assumed value of $\Delta n$ was correct.

Note that as assumed values of $\Delta n$ are increased, resulting values of $r_i$ on line 9 (from prediction nomograph) decrease sharply, whereas values of $r_i$ on line 11 (from gas-oil ratio nomograph increase slowly. Thus, if the $r_i$ of line 11 is less than that of line 9, a <u>slight</u> increase of $\Delta n$ will bring them together. If the two values of $r_i$ are within a few hundred of each other, set the $\Delta n\ r_a$ value from line 6 on the slide rule "D" scale, place the value of $r_a$ (line 7) opposite it on the "C" scale, and read a nearly-correct value for $\Delta n$ under the index on the "D" scale.

If this new value of $\Delta n$ causes a <u>radical</u> change in $n_i$, it may be necessary to enter the nomograph for a new $G_{i-1} + \Delta n\ r_a$. However, from the scale positions, it can be seen that $G_{i-1} + \Delta n\ r_a$ is very insensitive to changes in $\Delta n$ at most pressures.

11. If the new value of $\Delta n$ caused more than a slight change in $n_i$, re-enter the liquid saturation and gas-oil ratio nomograms to obtain a corrected $r_i$ for line 11.

12. Lines 9 and 11 should now be within perhaps 50 or 60 of each other. Alter $\Delta n$ slightly to bring line 9 in exact agreement with line 10. Add the corrected $\Delta n$ to $n_{i-1}$. The result is $n_i$.

13. Multiply $n_i$ by the volume of original oil in place for line 12 and add line 13, the production from the original pressure to the bubble point. Line 14 is the predicted cumulative production at the assumed pressure.

14. Assume a new pressure about 200 psi lower and repeat the process.

Special care in the averaging of $r_i$ and $r_{i-1}$ must be taken when computing at about the peak of the Gas-Oil Ratio Vs. Production Curve, as any averaging process assumes local linearity of a curve. It is best to use smaller pressure increments in this region.

Using the method described above, about two hours were required to calculate the predicted performance using seven pressure steps.

An alternate method was developed which takes about the same time to use, but which may be a little more accurate:

1. Use the same tabulation sheet as shown on page 22 above. In addition, a large "scratch graph sheet" of squared paper (10 x 10 to the inch was used) and a sheet of scratch paper are necessary.

2. Same as step 2 above.

3. Same as step 3 above.

4. Estimate three values of $\Delta n$ quite close together. Jot on scratch sheet or carry mentally.

5. Enter the prediction nomogram with the middle value of $\Delta n$ and read a value of $G_{i-1} + \Delta n\ r_a$. (It will be seen that the change in $G_{i-1} + \Delta n\ r_a$ over the three values of $\Delta n$ is so small as to be unreadable.)

6. Subtract $G_{i-1}$ from $G_{i-1} + \Delta n\ r_a$. Divide $\Delta n\ r_a$ by each of the three values of $\Delta n$ from step 4 obtaining three values of $r_a$.

7. On the scratch graph sheet, plot the three values of $r_a$ vs. the respective $\Delta n$ values.

8. Add the three values of $\Delta n$ to $n_{i-1}$. Use the resulting 3 values of $n_i$ in the liquid saturation and gas-oil ratio nomographs to obtain three values of $r_i$.

9. Using the average gas-oil ratio nomograph and entering with $r_{i-1}$ and the three $r_i$ values of step 8, find three values of $r_a$.

10. Plot these values against $\Delta n$ values on the scratch graph sheet.

11. Where the lines of step 7 and step 10 cross, read the correct values of $\Delta n$ and $r_a$. Enter these values on the tabular sheet and check the computation on the nomographs, comparing tabulation lines nine and 11.

12. Make minor adjustments of $\Delta n$ to match line 9 to line 11.

13. Same as step 13 above.

14. Same as step 14 above.

Figure 1 is a graph comparing the results obtained by the nomographic method with those obtained by trial and error solution of the Schilthuis Equation and by Tracy's method. The results in general are in good agreement. The higher gas-oil ratio values shown on the Schilthuis method curve are thought to have resulted from use of different values of "v" than were used in the Tracy method and in the construction of the nomographs. The Schilthuis method curve was taken from an engineering consultant's report on Reservoir "X".

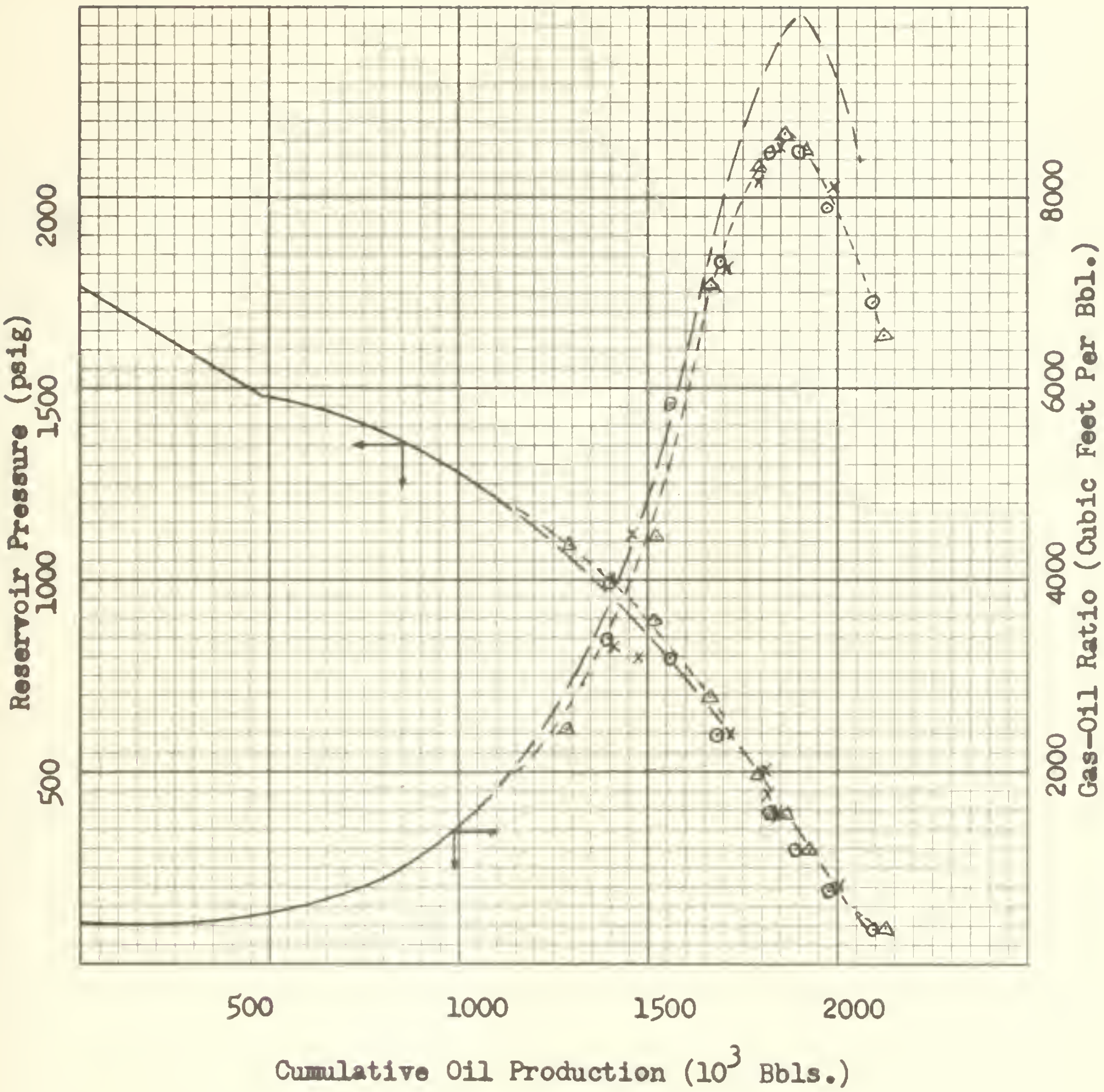



Figure 1

Comparison of Reservoir Performance Prediction Results

Legend

——— Production History

- - - - - Mean of Nomographic Predictions

— — — Prediction by Schilthuis Equation

⊙ Prediction by Nomographic Method First Described

△ Prediction by Alternate Nomographic Method

× Prediction by Tracy's Method

# VI. SUMMARY AND CONCLUSIONS

This investigation had as its object the development of a graphical technique for predicting production performance of a petroleum reservoir. A graphical technique involving the use of nomographs based on the Tracy method was worked out. Using this method nomographs were constructed to predict the performance of a specific actual reservoir called Reservoir "X." The performance of Reservoir "X" under depletion drive was predicted using these nomographs. The results, compared with the results obtained by other methods, show that the use of nomographs constructed for a specific reservoir gives a forecast of reasonable accuracy.

The total time involved in nomograph construction and in solving the prediction problem with these nomographs proved to be about the same as that required to solve a reservoir problem by the Tracy method. However, reservoir performance predictions are often repeated, starting with a later point in production history, as the field develops. With this in mind, it is believed that the nomographic method would save considerable calculation. A repeated prediction could be run in about two hours using the same nomographs. A repeated calculation, even by the Tracy method, would take considerably longer.

As the production history of the field develops, it may be found that the basic field data relating relative permeability ratio and saturation were extrapolated in error. In this event, new and more correct values of $\frac{K_g}{K_o}$ may be marked on the center scale of the liquid saturation nomograph without affecting its accuracy for subsequent use.

The limited time available precluded investigating the application of this technique to a reservoir undergoing gas reinjection. It is almost

certain that a set of reasonably simple nomographs could be constructed to handle predictions under various conditions of gas reinjection.

The time available was also too short to allow the method to be applied to reservoirs of differing characteristics. In particular, it should be tried on a sandstone reservoir, on a reservoir with a high initial pressure, and a reservoir with a low initial pressure.

# APPENDIX I

## Nomenclature

$F$ = $\frac{\mu_o}{\mu_g} \cdot \frac{\beta}{v}$

$G$ = Cumulative gas production, standard cubic feet

$G_i$ = G at reservoir pressure being considered

$G_{i-1}$ = G at previously considered reservoir pressure

$\frac{K_g}{K_o}$ = Relative permeability ratio, a dimensionless number

$N$ = Total original oil in place, in stock tank barrels

$n$ = Cumulative oil production, expressed either in stock tank barrels (or as a fraction, when N is taken as equal to one barrel)

$n_i$ = The value of n at the reservoir pressure being considered

$n_{i-1}$ = The value of n at the last previously considered reservoir pressure

$r_a$ = Average gas-oil ratio, standard cubic feet per barrel of stock tank oil

$r_i$ = Instantaneous gas-oil ratio at reservoir pressure being considered standard cubic feet per barrel of stock tank oil

$r_{i-1}$ = Instantaneous gas-oil ratio at last previous reservoir pressure, standard cubic feet per barrel of stock tank oil

$r_n$ = Cumulative gas-oil ratio, or the ratio of total standard cubic feet of gas produced to total barrels of stock tank oil produced

$S_L$ = Total liquid saturation in the reservoir

$S_o$ = Oil saturation in the reservoir

$S_W$ = Saturation of connate water in the reservoir

$s$ = Solution gas-oil ratio, or the solubility of gas in crude oil at reservoir pressure and temperature, cubic feet per barrel

$s_o$ = Value of s at original reservoir pressure

$u$ = Volume in the reservoir occupied by one barrel of stock tank oil plus all the gas originally in solution in that oil, or the two-phase formation volume factor

$u_o$ = Value of u at original reservoir pressure $u_o = \beta_o$

$v$ = Barrels of free gas space occupied in the reservoir by one standard cubic foot of gas

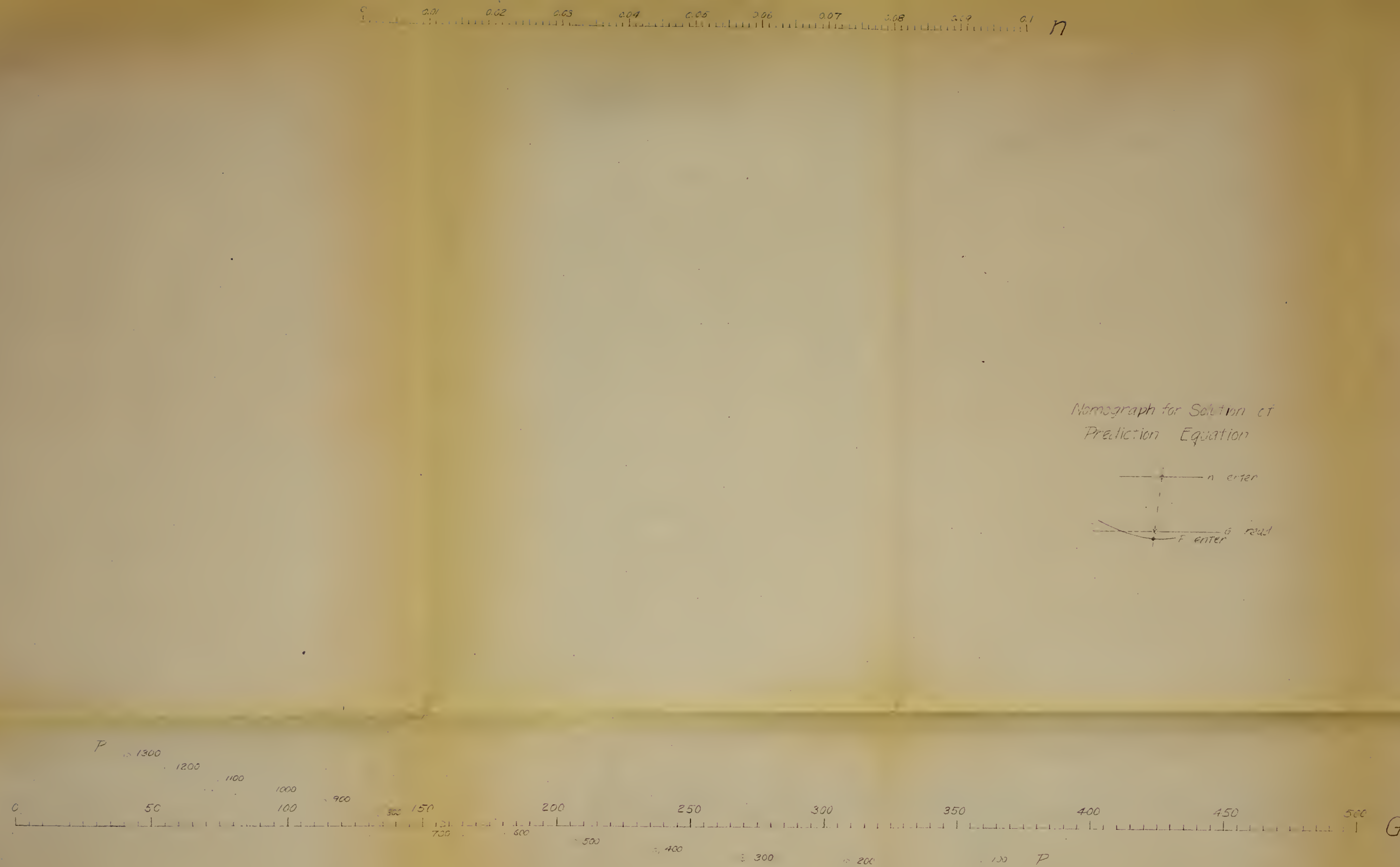
$\beta$ = Formation volume factor, or the volume occupied at reservoir conditions by one barrel of stock tank oil plus its dissolved gas

$\beta_o$ = Value of $\beta$ at original reservoir pressure

$\Delta n$ = Increment of oil production between two reservoir pressure steps

$\mu_o$ = Viscosity of reservoir oil at reservoir conditions, centipoises

$\mu_g$ = Viscosity of produced gas, at reservoir pressure and temperature centipoises

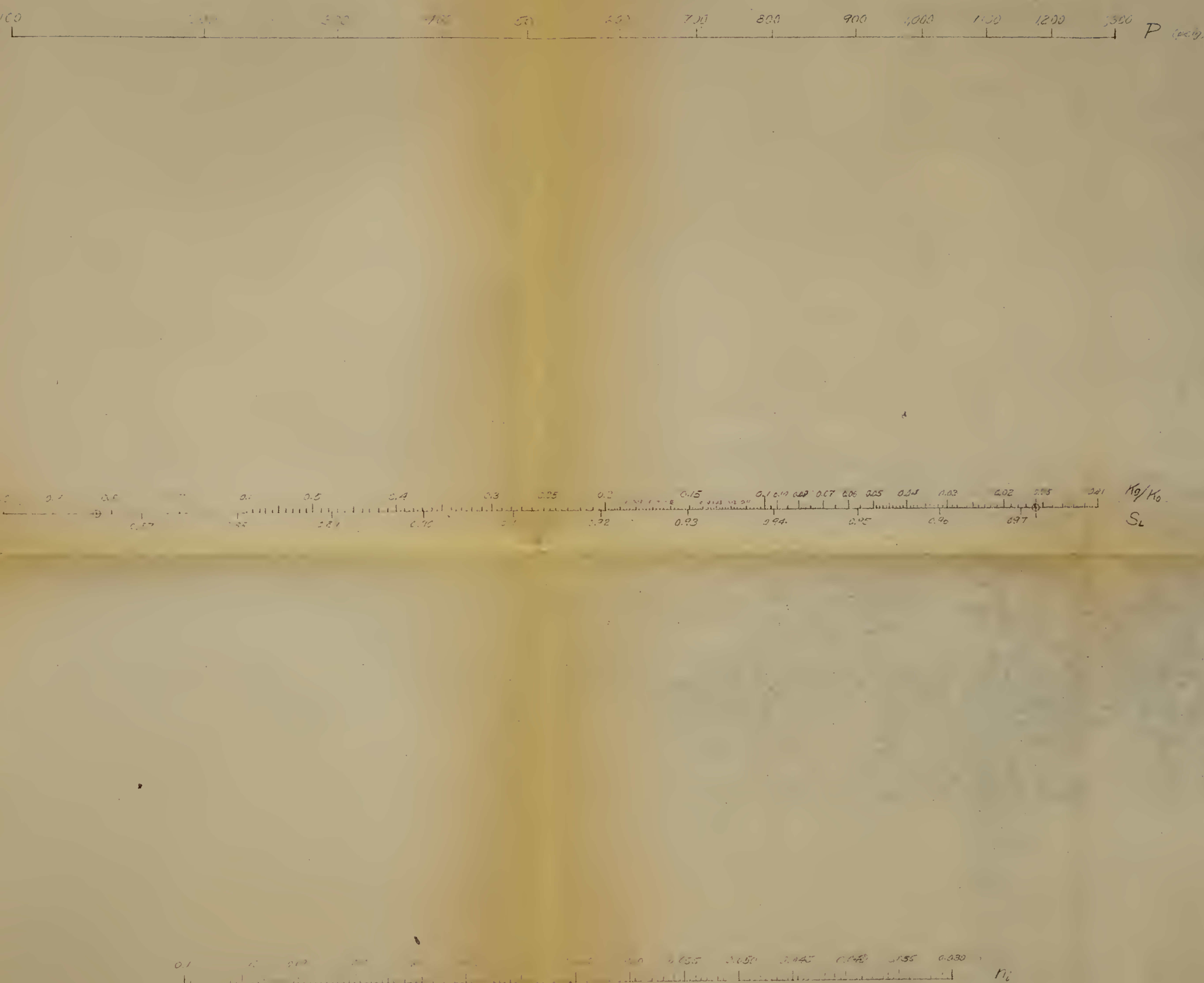


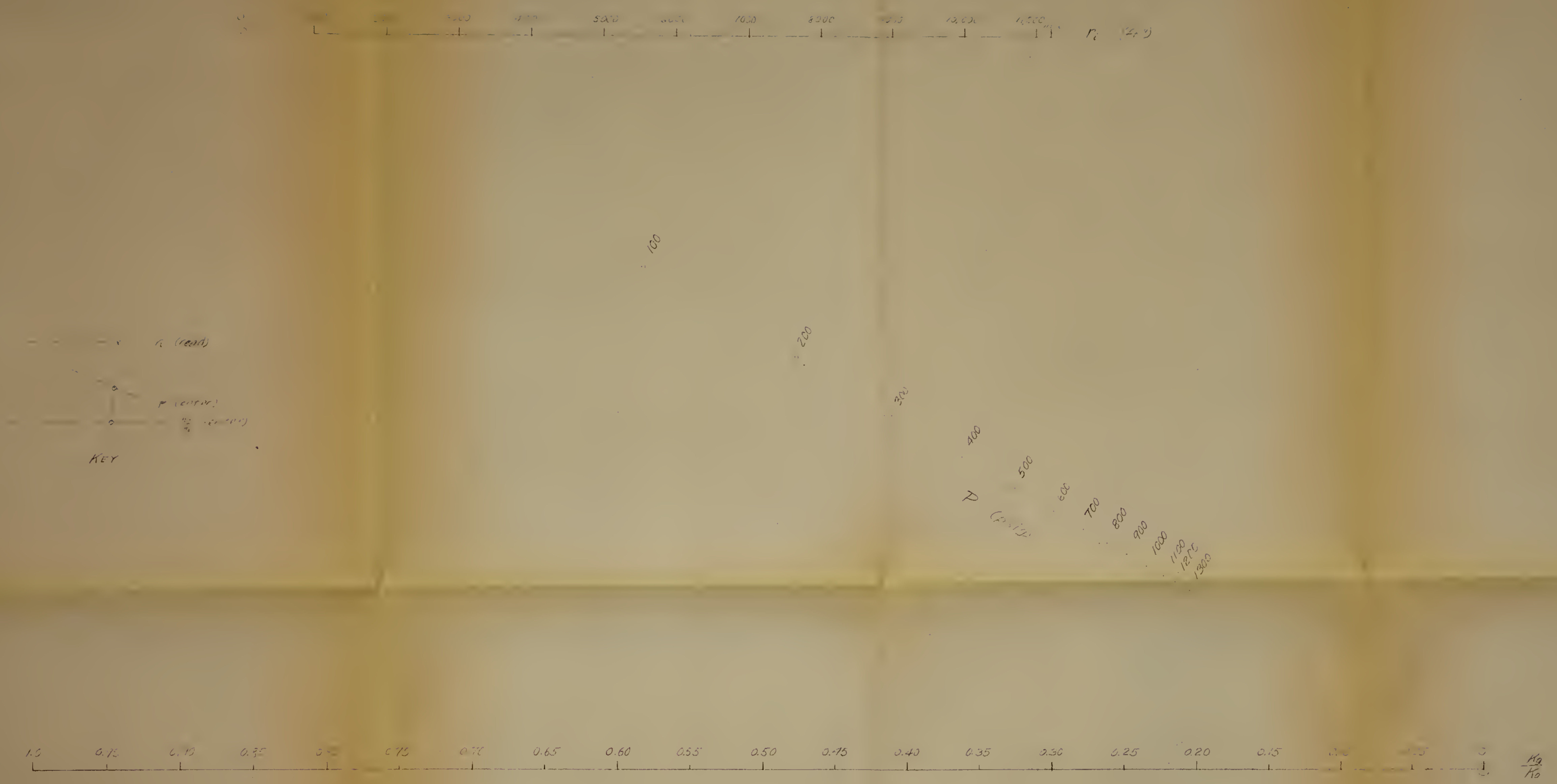

Nomograph for Solution of Prediction Equation

[illegible]

[illegible] OF

LIQUID SATURAT[illegible] EQUATION FOR $\frac{K_g}{K_o}$ - $S_L$



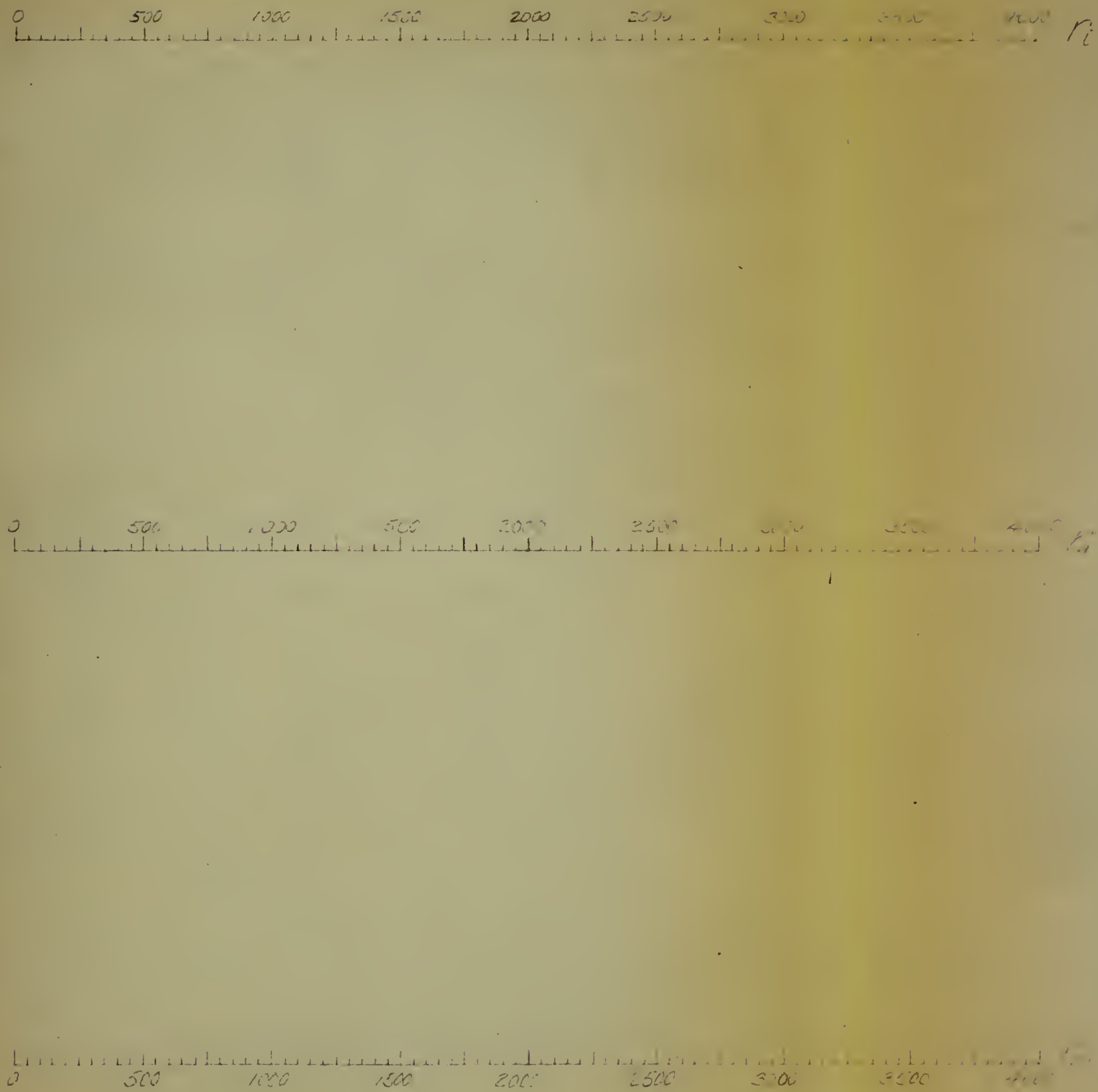



NOMOGRAPH FOR SOLUTION OF GAS-OIL RATIO EQUATION

$$r_i = \frac{\mu_o}{\mu_g} \frac{\beta}{V} \frac{K_g}{K_o} + S$$

0.2" = 0.001 unit $\frac{K_g}{K_o}$ 0.2" = 20 ft$^3$ of $r_i$

Nomograph for Solution of $\frac{r_o + r_i}{2} = r_a$



# APPENDIX III

## Considerations Affecting the Accuracy of Nomographs

1. Dimensions of the Diagram

The accuracy of a nomograph can be increased by increasing the dimensions of the diagram, within reasonable limits. This is due to the increase of relative accuracy and fineness of scale graduations.

2. Order of Accuracy

Nomographs, in general, have a degree of accuracy exceeding that of a slide rule of similar dimensions. In the nomograph, the ranges of the scales are tailored to the ranges of the variables involved. Short ranges are often expanded into long (and, therefore, more accurate) scales.

3. Decrease in Accuracy due to Complexity

The accuracy of a nomograph decreases sharply when more than one setting of a line must be made to obtain the answer. The more inter-related lines that must be drawn on a given nomograph to obtain a solution, the less accurate the answer will be.

4. Relative Placement of Scales

The accuracy of a nomograph is affected by the relative positions of the scales. If possible, the scale on which the answer is to be read should be between the other two scales.

# APPENDIX IV

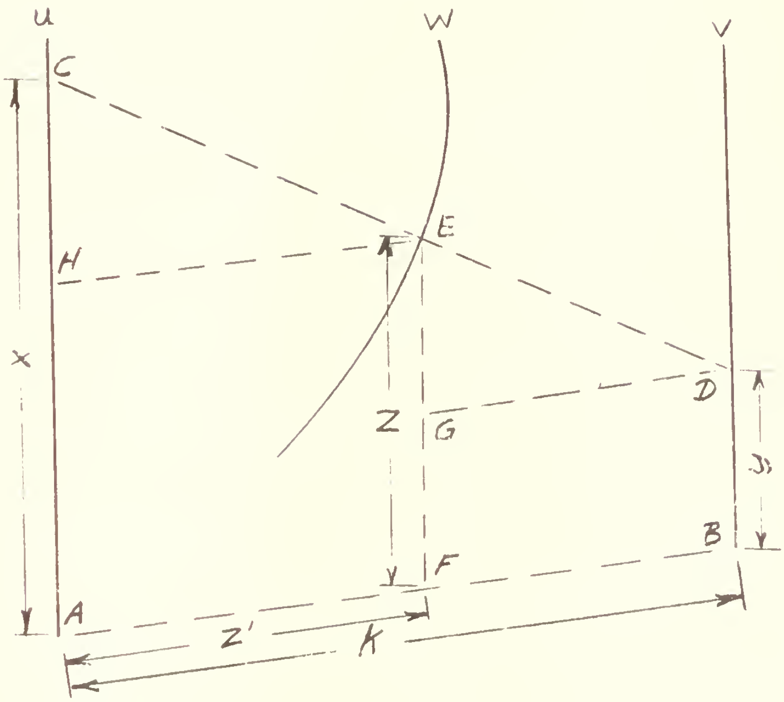
## Geometric Proof for Nomographic Forms Used

### A. Prediction Equation and Material Balance Equation

The equation form for the nomograph is:

$$f_1 (u) + f_2 (v) \times f_5 (W) = f_6 (W) \tag{19}$$

Take two parallel axes for the variables u and v, and a curved axis in general for the variable W:




The line AB, of length k, joins the zero points of the scales for u and v. Points x and y are any points on the u and v scale, $x = m_1 f_1 (u)$ and $y = m_2 f_2 (v)$, where $m_1$ and $m_2$ are the respective scale moduli.

Take any index line CD intersecting the scale for W at E. Construct EF parallel to the u and v axes. Let EF = z and AF = z'. From similar triangles ECH and EDG:

$$\frac{x-z}{z'} = \frac{z-y}{k-z'}$$

$$(k-z')x = kz - zz' + zz' - yz'$$

$$(k-z')x + yz' = kz$$

$$x + \left(\frac{z'}{k-z'}\right)\left(y\right) = \left(\frac{k}{k-z'}\right)\left(z\right)$$

Substitute for x and y:

$$m_1\ f_1\ (u) + \left(\frac{z'}{k-z'}\right)\left[m_2\ f_2\ (v)\right] = \left(\frac{k}{k-z'}\right)\left(z\right)$$

$$f_1\ (u) + \left(\frac{m_2}{m_1}\ \ \frac{z'}{k-z'}\right)\left[f_2\ (v)\right] = \frac{k}{m_1(k-z')}\left(z\right)$$

This equation will become the original equation if $f_5$ (W) and $f_6$ (W) have the following values:

$$f_5\ (W) = \frac{m_2 z}{m_1(k-z')}$$

$$f_6\ (W) = \frac{k_z}{m_1(k-z')}$$

Solving for z and z':

$$z = \frac{m_2\ f_6\ (W)}{f_5\ (W) + \frac{m_2}{m_1}}$$

$$z' = \frac{k\ f_5\ (W)}{f_5(W) + \frac{m_2}{m_1}}$$

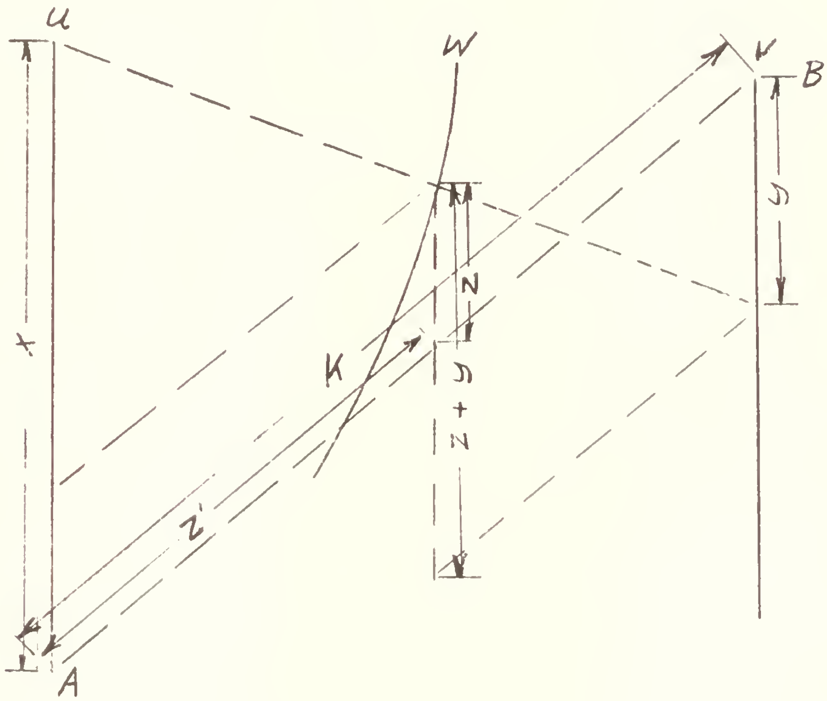
Points on the curve for the W-scale are obtained by taking different values of W and determining the corresponding values of z and z'. These points are plotted and a curve may be drawn through them.

B. Gas-Oil Ratio Equation

The form of the equation is:

$$f_1 (u) - f_2 (v) \cdot f_3 (W) = f_4 (W) \qquad (27)$$

This equation is similar in form to that discussed under "A Prediction Equation," and is subject to the same proof, except that the scale for $f_2$ (v) runs in the opposite direction from the $f_1$ (W) scale:




So that: $\frac{x-z}{z'} = \frac{z+y}{k-z'}$

After substitution for x and y, this reduces to:

$$f_1 (W) - \left(\frac{m_2}{m_1} \cdot \frac{z'}{k-z'}\right)\left[f_2 (v)\right] = \frac{k}{m_1 (k-z')}$$

and z and z' are as before:

$$z = \frac{m_2 f_4 (W)}{f_3 (W) + \frac{m_2}{m_1}}$$

$$z' = \frac{k f_3 (W)}{f_3 (W) + \frac{m_2}{m_1}}$$

Points on the W-scale are determined by taking different values of W and computing the corresponding values of z and z'.

Let ABC be a base line, perpendicular to the three scale axes, and let a and b be the distances between scales.

Draw any index line in general, FH, forming similar triangles DEF and DGH, where DE and GH are parallel to ABC.

From these similar triangles:

$$\frac{x - z}{a} = \frac{z - y}{b}$$

which rearranges to:

$$\frac{x}{a} + \frac{y}{b} = \frac{z}{ab/(a+b)}$$

Substituting particular functions for x, y, and z:

$$\frac{m_1\ f_1\ (u)}{a} + \frac{m_2\ f_2\ (v)}{b} = \frac{m_3\ f_3\ (W)}{ab/(a+b)}$$

To reduce this to the original form, let:

$$a = m_1$$

$$b = m_2$$

$$\frac{ab}{a + b} = m_3\ ,\ \text{or}\ \ m_3 = \frac{m_1\ m_2}{m_1 + m_2}$$

This also defines the scale moduli in terms of each other, showing that after $m_1$ and $m_2$ are selected, $m_3$ may be computed from them.

To find the position of the center scale in terms of distance between the outside scales, note that the distance from A to B in terms of AC is:

$$\frac{a}{a + b} \cdot (AC)$$

or

$$\frac{m_1}{m_1 + m_2} \cdot (AC)$$

### D. Average Gas-Oil Ratio Equation

The equation form is:

$$r_i + r_{i-1} = 2\ r_a$$

or

$$f_1\ (u) + f_2\ (v) = f_3\ (W)$$

The proof is identical to that for the liquid saturation equation nomograph.

# APPENDIX V

## Data Concerning Reservoir "X"

## A Mid-Continent Limestone Reservoir

### Basic Data

| | |
|---|---|
| Average Depth | 5300 ft. |
| Original Reservoir Pressure | 1773 psig |
| Reservoir Temperature | 128° F. |
| Average Connate Water Saturation | 33.4% |
| Original Formation Volume Factor, $\beta_o$ | 1.280 |
| Original Solution Gas-Oil Ratio | 500 $\frac{ft^3}{bbl.}$ |

### P.V.T. Data

| Pressure (psig) | S, (SCF/Bbl.STD) | $\beta$, F.V.F. | $\mu_o$ (C.P.) |
|---|---|---|---|
| 1500 | 500 | 1.280 | 1.28 |
| 1250 | 448 | 1.260 | 1.32 |
| 1000 | 392 | 1.240 | 1.41 |
| 750 | 336 | 1.218 | 1.54 |
| 500 | 278 | 1.193 | 1.72 |
| 250 | 208 | 1.164 | 2.0 |
| 100 | 145 | 1.136 | 2.37 |
| 0 | 0 | 1.07 | 3.4 |

## Volumetrically Weighted Average Bottom Hole Pressures

| Date | BHP (Psig) | Cumulative Oil Production (Bbls.) |
|---|---|---|
| Original | 1773 | -- |
| 10-3-52 | 1654 | 152,330 |
| 4-13-53 | 1558 | 353,995 |
| 7-6-53 | 1517 | 443,999 |
| 10-1-53 | 1473 | 543,783 |
| 1-4-54 | 1441 | 631,764 |
| 7-6-54 | 1342 | 866,063 |
| 1-3-55 | 1209 | 1,086,797 |

## Reservoir Fluid Characteristics:

| | |
|---|---|
| Gravity of stock tank oil | 41.5° API at 60° F. |
| Saturation pressure | 1487 PSIG at 128° F. |
| Solution gas-oil ratio | 484 Std. Cu.Ft./Resid. Bbl. |
| Formation volume factor | 1.28 Res. Bbl./Resid. Bbl. |
| Viscosity of reservoir oil | 1.28 Centipoise at 1487 PSI + 128°F. |

## Analysis of Reservoir Fluid Sample

| | Weight % | Mol % |
|---|---|---|
| Methane and Lighter | 3.39 | 26.35 |
| Ethane | 1.70 | 7.06 |
| Propane | 3.75 | 10.62 |
| Iso-butane | 0.82 | 1.77 |
| N-butane | 2.55 | 5.46 |
| Iso-pentane | 1.38 | 2.38 |
| N-pentane | 1.52 | 2.63 |
| Hexanes | 2.81 | 4.07 |
| Heavier | 82.08 | 39.66 |
| | 100.00 | 100.00 |

Gas Viscosity[18]

| Pressure (Psig) | |
|---|---|
| 1300 | 0.01477 |
| 1100 | 0.01401 |
| 900 | 0.01355 |
| 700 | 0.01280 |
| 500 | 0.01228 |
| 300 | 0.01193 |
| 100 | 0.01158 |

Reservoir Gas Analysis

| Analysis of Gas Sample | Mol % |
|---|---|
| Methane | 68.5 |
| Ethane | 8.0 |
| Propane | 7.6 |
| N-Butane | 2.5 |
| Iso-Butane | 1.2 |
| N-Pentane | 0.5 |
| Iso-Pentane | 0.8 |
| Cyclo-Pentane | 0.2 |
| Hexanes plus | 0.4 |
| Nitrogen | 9.9 |
| Oxygen | 0.1 |
| Helium | 0.1 |
| Hydrogen | 0.1 |
| $CO_2$ | 0.1 |
| $H_2S$ | 0.1 |
| | 100.00 |

Gas gravity at 60°F. + 14.7 psi = 0.819 (Air = 1.000)

Relative Permeability Data

| $S_L$ (%) | $\frac{K_g}{K_o}$ |
|---|---|
| 98 | 0.01 |
| 97 | 0.0181 |
| 96 | 0.033 |
| 95 | 0.057 |
| 90 | 0.373 |
| 85 | 1.15 |
| 80 | 3.20 |
| 75 | 8.0 |

Summary of Production:

| Year | Producing Wells at End of Year | Production: Oil (Bbls.) Year | Oil (Bbls.) Cumulative | Gas (MCF) Year | Gas (MCF) Cumulative |
|---|---|---|---|---|---|
| 1951 | 5 | 27,295 | 27,295 | 13,655 | 13,655 |
| 1952 | 26 | 185,930 | 213,225 | 93,105 | 106,760 |
| 1953 | 29 | 414,034 | 627,259 | 254,261 | 361,021 |
| 1954 | 31 | 456,238 | 1,083,497 | 753,605 | 1,114,626 |

Production prior to bubble point: 575,000 bbls. STO

Total Volume of Original Oil in Place:

$N = 18,000,000$ Bbls. STO (Calculated by Material Balance and verified by volumetric value)

On Basis of $N = 1$ Bbl. STO,

Oil Production, B.P. to 1209 psig

$n = 0.0319$ Bbl.

Gas Production, BP to 1209 psig

$G = 46.75$ SCF

Instantaneous Gas-Oil Ratio at 1209 psig

$r_i = 2400$ SCF/Bbl.

# APPENDIX VI

Prediction of Performance of Reservoir "X" by Nomographic Method

| Line No. | P | (Historical Point) 1209 | 1000 | 800 | 600 | 400 | 300 | 200 | 100 |
|---|---|---|---|---|---|---|---|---|---|
| (1) | n | | 0.0135 | 0.010 | 0.00745 | 0.00733 | 0.00376 | 0.00462 | 0.00684 |
| (2) | $n_{i-1}$ | | 0.032 | 0.0455 | 0.0545 | 0.06195 | 0.06928 | 0.07304 | 0.07766 |
| (3) | $n_i$ | 0.0319 | 0.0455 | 0.0545 | 0.06195 | 0.06928 | 0.07304 | 0.07766 | 0.08450 |
| (4) | $G_{i-1} + r_a \ n$ | 46.75 | 85.90 | 131.9 | 181.1 | 239.1 | 270.9 | 308.7 | 358.0 |
| (5) | $G_{i-1}$ | | 46.75 | 85.9 | 131.9 | 181.1 | 239.1 | 270.9 | 308.7 |
| (6) | $r_a \ n$ | | 39.15 | 46.0 | 49.2 | 58.0 | 31.8 | 37.8 | 49.3 |
| (7) | $r_a$ | | 2900 | 4600 | 6600 | 7910 | 8450 | 8180 | 7210 |
| (8) | $r_{i-1}$ | | 2400 | 3400 | 5800 | 7380 | 8450 | 8450 | 7910 |
| (9) | $r_i$ | 2400 | 3400 | 5800 | 7380 | 8450 | 8450 | 7910 | 6510 |
| (10) | $\frac{K_g}{K_o}$ | | 0.057 | 0.124 | 0.194 | 0.305 | 0.384 | 0.494 | 0.692 |
| (11) | $r_i$ (from G.O.R.) | | 3400 | 5820 | 7380 | 8450 | 8450 | 7910 | 6520 |
| (12) | $(n_i)$ $(1.8 \times 10^7)$ | | 819.5 | 981 | 1,116 | 1,249 | 1,317 | 1,399 | 1,521 |
| (13) | +575,000 | | 575 | 575 | 575 | 575 | 575 | 575 | 575 |
| (14) | Cumulative Production (in $10^3$ bbls.) | | 1,394.5 | 1,556 | 1,691 | 1,824 | 1,892 | 1,974 | 2,096 |

Prediction of Performance of Reservoir "X" by the Nomographic Method
(Using Alternate Method)

| Line No. | P | 1209 (A known point of Production History) | 1100 | 900 | 700 | 500 | 300 | 400 | 300 | 100 |
|---|---|---|---|---|---|---|---|---|---|---|
| (1) | n | | 0.00773 | 0.01248 | 0.00845 | 0.00665 | 0.00741 | 0.00360 | 0.00372 | 0.01145 |
| (2) | $n_{i-1}$ | | 0.0319 | 0.03963 | 0.05211 | 0.06056 | 0.06721 | 0.06721 | 0.07081 | 0.07453 |
| (3) | $n_i$ | 0.0319 | 0.03963 | 0.05211 | 0.06056 | 0.06721 | 0.07462 | 0.07081 | 0.07453 | 0.08598 |
| (4) | $G_{i-1}$ + n $r_a$ | 46.75 | 65.5 | 108.3 | 156.8 | 208.5 | 271.2 | 239.2 | 271.2 | 358.0 |
| (5) | $G_{i-1}$ | | 46.75 | 65.5 | 108.3 | 156.8 | 208.5 | 208.5 | 239.2 | 271.2 |
| (6) | $r_a$ n | | 18.75 | 42.8 | 48.5 | 51.7 | 62.7 | 30.7 | 32.0 | 86.8 |
| (7) | $r_a$ | | 2425 | 3430 | 5745 | 7720 | 8465 | 8525 | 8615 | 7575 |
| (8) | $r_{i-1}$ | | 2400 | 2450 | 4420 | 7070 | 8370 | 8560 | 8680 | 8550 |
| (9) | $r_i$ | 2400 | 2450 | 4420 | 7070 | 8370 | 8560 | 8680 | 8550 | 6600 |
| (10) | $\frac{K_g}{K_o}$ | | 0.0362 | 0.085 | 0.1658 | 0.253 | 0.390 | 0.314 | 0.390 | 0.70 |
| (11) | $r_i$ (from G.O.R.) | | 2450 | 4420 | 7070 | 8370 | 8460 | 8680 | 8550 | 6590 |
| (12) | ($n_i$) ($1.8 \times 10^7$) | | 713.0 | 937.5 | 1090.5 | 1210.0 | 1342.0 | 1276.0 | 1341.0 | 1549.0 |
| (13) | +575,000 | | 575.0 | 575.0 | 575.0 | 575.0 | 575.0 | 575.0 | 575.0 | 575.0 |
| (14) | Cumulative Production (in $10^3$ bbls.) | | 1288.0 | 1512.5 | 1665.5 | 1785.0 | 1917.0 | 1851.0 | 1916.0 | 2124 |

$$z' = \frac{k\, f_5\,(W)}{f_5(W) + \frac{m_2}{m_1}}$$

$$z = \frac{m_2\, f_6\,(W)}{f_5(W) + \frac{m_2}{m_1}}$$

$k = 13"$

$m_1 = 100$ (scale = 10")

$m_2 = 0.04$ (scale = 20")

$$\frac{m_2}{m_1} = 0.0004$$

Scale Divisions:

n: 1" = 0.01 unit of n

$\frac{1}{50}$" = 0.0002 unit n

g: 1" = 25 units of g

$\frac{1}{50}$" = 0.5 unit of g

Calculations for Prediction Equation

| (1) P | (2) $f_5(W)$ $\frac{(\phi g)}{(\phi n)}$ | (3) $f_6(W)$ $(\frac{1}{\phi n})$ | (4) $k.f_5(W)$ | (5) $f_5(W)+\frac{m_2}{m_1}$ | (6) $m_2 f_6(W)$ | (7) $z''=\frac{(4)}{(5)}$ | (8) $z = \frac{(6)}{(5)}$ |
|---|---|---|---|---|---|---|---|
| 1300 | 0.004207 | 0.1396 | 0.054691 | 0.004607 | 0.005584 | 11.871 | 1.212 |
| 1200 | 0.005150 | 0.2623 | 0.06695 | 0.005550 | 0.010492 | 12.063 | 1.890 |
| 1100 | 0.006502 | 0.4644 | 0.084526 | 0.006902 | 0.018576 | 12.247 | 2.691 |
| 1000 | 0.008888 | 0.8064 | 0.115544 | 0.009238 | 0.032256 | 12.440 | 3.473 |
| 900 | 0.01306 | 1.471 | 0.16978 | 0.01346 | 0.05884 | 12.614 | 4.372 |
| 800 | 0.02435 | 3.278 | 0.31655 | 0.02475 | 0.13112 | 12.790 | 5.298 |
| 700 | 0.12139 | 19.135 | 1.57807 | 0.12179 | 0.76540 | 12.957 | 6.285 |
| 600 | (-)0.04390 | (-)7.918 | (-)0.57070 | 0.04350 | (-)0.31672 | 13.120 | 7.281 |
| 500 | (-)0.01985 | (-)4.078 | (-)0.25805 | 0.01945 | (-)0.16312 | 13.267 | 8.387 |
| 400 | (-)0.01392 | (-)3.262 | (-)0.18096 | 0.01352 | (-)0.13048 | 13.385 | 9.651 |
| 300 | (-)0.0110 | (-)2.911 | (-)0.1430 | 0.01060 | (-)0.11644 | 13.491 | 10.985 |
| 200 | (-)0.009848 | (-)2.971 | (-)0.128024 | 0.009448 | (-)0.11884 | 13.550 | 12.578 |
| 100 | (-)0.01010 | (-)3.538 | (-)0.13120 | 0.00970 | (-)0.14152 | 13.536 | 14.590 |

Calculation of $f_5(W) = \frac{1}{\frac{\beta}{v} - s}$ and $f_6(W) = 1 - \frac{\left(\frac{\beta_o}{v} - s_o\right)}{\left(\frac{\beta}{v} - s\right)}$

| P | $f_5$ (W) | $f_6$ (W) |
|---|---|---|
| 1300 | 0.004207 | 0.1396 |
| 1200 | 0.005150 | 0.2623 |
| 1100 | 0.006502 | 0.4644 |
| 1000 | 0.00888 | 0.8064 |
| 900 | 0.01306 | 1.471 |
| 800 | 0.02435 | 3.278 |
| 700 | 0.12139 | 19.135 |
| 600 | 0.04390 | (-)7.918 |
| 500 | 0.01985 | (-)4.078 |
| 400 | 0.01392 | (-)3.262 |
| 300 | 0.01100 | (-)2.911 |
| 200 | 0.009848 | (-)2.971 |
| 100 | 0.01010 | (-)3.538 |

Calculations for Nomograph to Sclve Gas-Oil Ratio Equation

Moduli

$\frac{K_g}{K_o}$ (or $f_2(v)$; $m_2$ = 20

$r_i$ (or $f_1(W)$; $m_1$ ] 0.001

Plotting Equations for

$\frac{K_g}{K_o}$ and $r_i$

$\frac{K_g}{K_o}$: Dist.=(20) $(\frac{K_g}{K_o})$

$r_i$ ] Dist.=(0.001)($r_i$)

Scale Lengths:

$\frac{K_g}{K_o}$ : 18" length to zero = 11.2"

k = 20"

| (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) |
|---|---|---|---|---|---|---|---|
| P (W) | F $f_3(W)$ | S $f_4(W)$ | K.F $Kf_3(W)$ | F + $\frac{m_2}{m_1}$ | $M_2S$ $m_2f_4(W)$ | $z'=\frac{(4)}{(5)}$ | $z=\frac{(6)}{(5)}$ |
| 1300 | 61935.057 | 458 | 1238701.140 | 81935.057 | 9160 | 15.118 | 0.111 |
| 1200 | 58189.950 | 437 | 1163799.00 | 78189.950 | 8740 | 14.884 | 0.111 |
| 1100 | 55522.293 | 414 | 1110445.860 | 75522.293 | 8280 | 14.703 | 0.109 |
| 1000 | 51724.011 | 393 | 1034480.220 | 71724.011 | 7860 | 14.423 | 0.109 |
| 900 | 47949.257 | 370 | 958985.140 | 67949.257 | 7400 | 14.113 | 0.108 |
| 800 | 44391.712 | 347 | 887834.240 | 64391.712 | 6940 | 13.788 | 0.107 |
| 700 | 40751.027 | 324 | 815020.54 | 60751.027 | 6480 | 13.415 | 0.106 |
| 600 | 36604.360 | 302 | 732087.20 | 56604.360 | 6040 | 12.933 | 0.106 |
| 500 | 31975.386 | 278 | 639507.72 | 51975.386 | 5560 | 12.304 | 0.106 |
| 400 | 26837.195 | 251 | 536743.90 | 46837.195 | 5020 | 11.459 | 0.107 |
| 300 | 21364.723 | 223 | 427294.46 | 41364.723 | 4460 | 10.329 | 0.107 |
| 200 | 15548.990 | 189 | 310979.80 | 35548.990 | 3780 | 8.747 | 0.106 |
| 100 | 9235.955 | 144 | 184719.10 | 29235.955 | 2880 | 6.318 | 0.098 |

Computation of Scale Lengths for Liquid Saturation Equation

$$\text{Log } (S_L - K_1) = \text{Log } (K_2 \beta) + \text{Log } (1-n_i)$$

$$K_1 = S_W = 0.334 \qquad K_2 = \frac{(1-S_W)}{\beta_o} = \frac{0.666}{1.280} = 0.5203$$

| | Scale Label | Labeled Limits | Variable | Limits | Function | Range of Function | Scale Length | Actual Scale Modulus | Plotting Modulus | Scale Equation |
|---|---|---|---|---|---|---|---|---|---|---|
| | P | 1300 to 100 | | 1.264<br>1.135<br>0.129 | $\log K_2$ | 9.81803-10<br>9.77122-10<br>0.04681 | 14.043" | 300 | 300 | $x=300 \log K_2 \beta$ |
| 12" apart | | | | $1-n_i$ | | | | | | |
| | n | 0.03<br>0.10 | $n_i$ | 0.97<br>0.90 | $\log(1-n_i)$ | 9.98677-10<br>9.95424-10<br>0.03253 | 9.759" | 300 | 300 | $y=300 \log(1-n_i)$ |
| 6" from others | $S_L$ | | $S_L$ | | $\log(S_L-K_1)$ | | | 150 | 150 | |

| (1) | (2) | (3) | (4) | (5) | (6) |
|---|---|---|---|---|---|
| P(psig) | $\beta$ | $K_2 \beta$ | Log $K_2\beta$ | 300X$\log K_2\beta$ | (5)-* |
| 1300 | 1.264 | 0.6577 | 9.81803-10 | 2945.409 | 14.043 |
| 1200 | 1.256 | 0.6535 | 9.81525 | 2944.575 | 13.209 |
| 1100 | 1.248 | 0.6493 | 9.81245 | 2943.735 | 12.369 |
| 1000 | 1.240 | 0.6452 | 9.80969 | 2942.907 | 11.541 |
| 900 | 1.232 | 0.6410 | 9.80686 | 2942.058 | 10.692 |
| 800 | 1.222 | 0.6358 | 9.80332 | 2940.996 | 9.630 |
| 700 | 1.213 | 0.6311 | 9.80010 | 2940.030 | 8.664 |
| 600 | 1.204 | 0.6264 | 9.79685 | 2939.055 | 7.689 |
| 500 | 1.193 | 0.6207 | 9.79288 | 2937.864 | 6.498 |
| 400 | 1.183 | 0.6155 | 9.78923 | 2936.769 | 5.403 |
| 300 | 1.171 | 0.6093 | 9.78483 | 2935.449 | 4.083 |
| 200 | 1.156 | 0.6015 | 9.77924 | 2933.772 | 2.406 |
| 100 | 1.135 | 0.5905 | 9.77122-10 | 2931.366* | 0 |

| (1) | (2) | (3) | (4) | (5) |
|---|---|---|---|---|
| n | (1-n) | log(1-n) | 300X(3) | (4)-* |
| 0.03 | 0.970 | 9.98677-10 | 2996.031 | 9.759 |
| 0.035 | 0.965 | 9.98453 | 2995.359 | 9.087 |
| 0.040 | 0.960 | 9.98227 | 2994.681 | 8.409 |
| 0.045 | 0.955 | 9.98000 | 2994.000 | 7.728 |
| 0.050 | 0.950 | 9.97772 | 2993.316 | 7.044 |
| 0.055 | 0.945 | 9.97543 | 2992.629 | 6.357 |
| 0.060 | 0.940 | 9.97313 | 2991.939 | 5.667 |
| 0.065 | 0.935 | 9.97081 | 2991.243 | 4.971 |
| 0.070 | 0.930 | 9.968.48 | 2990.544 | 4.272 |
| 0.075 | 0.925 | 9.96614 | 2989.842 | 3.570 |
| 0.080 | 0.920 | 9.96379 | 2989.137 | 2.865 |
| 0.085 | 0.915 | 9.96142 | 2988.426 | 2.154 |
| 0.090 | 0.910 | 9.95904 | 2987.712 | 1.440 |
| 0.95 | 0.905 | 9.95665 | 2986.995 | 0.723 |
| 0.10 | 0.90 | 9.95424 | 2986.272* | 0 |

Limiting Values of $S_L$:

$P = 1300;\ n_i = 0.03$

$P = 100;\ n_i = 0.10$

Using

$$S_L = S_W + (1-S_W) \frac{(1-n_i)}{\beta_o} \beta$$

| (1) $\frac{1-S_W}{\beta_o}$ | (2) (1-n) | (3) (1)x(2) | (4) (3)+$S_W$=$S_L$ |
|---|---|---|---|
| 0.5203 | 1.226 | 0.6379 | 0.9719 |
| 0.5203 | 1.0215 | 0.5315 | 0.8655 |

Limit Values of

| $S_L-K_1$ | Log$S_L-K_1$ | Log x 150 | Actual Log $S_L-K_1$ | Actual Log x150 |
|---|---|---|---|---|
| 0.6379 | 9.80480-10 | 1470.720 | -0.19520 | -29.280 |
| 0.5315 | 9.72546-10 | 1458.819 | -0.27454 | -41.181 |
| | | 11.901<br>Scale Length | | -11.901<br>Diff. = Scale Length |

Liquid Saturation Equation Nomograph - Computation of $S_L$ Scale Values

Scale Modulus, $m_{S_L} = 150$; Scale Length = 11.901 inches

Scale Equation = $z = (150)(\log S_L - K_1)$, $K_1 = 0.334$

For Column (6) subtract $(150)(\text{Log } S_L - K_1)$ for T = 100, $n_i = 0.1$. This value is 1458.819 - 1500

| (1) $K_g/K_o$ | (2) $S_L$ | (3) $S_L - K_1$ | (4) $\text{Log}(S_L - K_1)$ | (5) 150x(4) | (6) (5) Zero Reading |
|---|---|---|---|---|---|
| 0.01 | 0.980 | 0.646 | 9.81023-10 | 1471.534 | 12.715 |
| 0.02 | 0.968 | 0.634 | 9.80209 | 1470.314 | 11.495 |
| 0.03 | 0.961 | 0.627 | 9.79727 | 1469.590 | 10.771 |
| 0.04 | 0.956 | 0.622 | 9.79379 | 1469.068 | 10.249 |
| 0.05 | 0.952 | 0.618 | 9.79099 | 1468.648 | 9.829 |
| 0.06 | 0.949 | 0.615 | 9.78888 | 1468.332 | 9.513 |
| 0.07 | 0.946 | 0.612 | 9.78675 | 1468.012 | 9.193 |
| 0.08 | 0.943 | 0.609 | 9.78462 | 1467.693 | 8.874 |
| 0.09 | 0.9405 | 0.6065 | 9.78283 | 1467.424 | 8.605 |
| 0.10 | 0.939 | 0.605 | 9.78176 | 1467.264 | 8.445 |
| 0.15 | 0.9302 | 0.5962 | 9.77539 | 1466.308 | 7.489 |
| 0.20 | 0.9205 | 0.5865 | 9.76827 | 1465.240 | 6.421 |
| 0.25 | 0.9135 | 0.5795 | 9.76305 | 1464.458 | 5.639 |
| 0.30 | 0.9075 | 0.5735 | 9.75853 | 1463.780 | 4.961 |
| 0.40 | 0.897 | 0.563 | 9.75051 | 1462.576 | 3.757 |
| 0.50 | 0.888 | 0.554 | 9.74351 | 1461.526 | 2.707 |
| 0.60 | 0.881 | 0.547 | 9.73799 | 1460.698 | 1.879 |
| 0.70 | 0.874 | 0.540 | 9.73239 | 1459.858 | 1.039 |
| 0.80 | 0.867 | 0.533 | 9.72673 | 1459.010 | 0.191 |
| 0.90 | 0.861 | 0.527 | 9.72181 | 1458.272 | -0.547 |
| 1.00 | 0.856 | 0.522 | 9.71767 | 1457.650 | -1.169 |
| | 0.97 | 0.636 | 9.8035-10 | 1470.525 | 11.706 |
| | 0.96 | 0.626 | 9.7966 | 1469.490 | 10.671 |
| | 0.95 | 0.616 | 9.7896 | 1468.440 | 9.621 |
| | 0.94 | 0.606 | 9.7825 | 1467.375 | 8.556 |
| | 0.93 | 0.596 | 9.7752 | 1466.280 | 7.461 |
| | 0.92 | 0.586 | 9.7679 | 1465.185 | 6.366 |
| | 0.91 | 0.576 | 9.7604 | 1464.060 | 5.241 |
| | 0.90 | 0.566 | 9.7528 | 1462.920 | 4.101 |
| | 0.89 | 0.556 | 9.7451 | 1461.765 | 2.446 |
| | 0.88 | 0.546 | 9.7372 | 1460.580 | 1.761 |
| | 0.87 | 0.536 | 9.7292 | 1459.380 | 0.561 |

Plotting values for $K_g/K_o$ side of scale (upper block, $K_g/K_o$ = 0.01 to 1.00)

Plotting values for $S_L$ side of scale (lower block, $S_L$ = 0.97 to 0.87)

# BIBLIOGRAPHY

## A. References Cited

1. Schilthuis, R. J., "Active Oil and Reservoir Energy," AIME Petroleum Development and Technology, 118 (1936), 33

2. Katz, D. L., "A Method of Estimating Oil and Gas Reserves," AIME Petroleum Development and Technology, 118, 10

3. Pirson, S. J., "On the Equivalence of Material Balance Equations for Calculating the Original Oil in Place," Oil Weekly, (April 3, 1944), 28

4. Babson, E. C., "Prediction of Reservoir Behavior from Laboratory Data," AIME Transactions, 155, (1944), 120

5. Tarner, J., "How Different Sized Gas Caps and Pressure Maintenance Programs Affect the Amount of Recoverable Oil," Oil Weekly, (June 12, 1944), 32

6. Muskat, M., "The Production Histories of Oil Producing Gas Drive Reservoirs," Journal of Applied Physics, 16 (March 1945) 147

7. Pirson, S. J., Elements of Oil Reservoir Engineering, New York, McGraw Hill, (1950), 345

8. Calhoun, J. C., Fundamentals of Reservoir Engineering, Norman, Oklahoma, University of Oklahoma Press, (1953)

9. Tracy, G. W., "A Simplified Form of the Material Balance Equation," Journal of Petroleum Technology, (January 1955), 53-56

10. Sturdivant, W. C., Jr., "Calculating Behavior of Closed Reservoirs - Part 1," Petroleum Engineer, 27, (1955), B-58

11. Sturdivant, W. C., Jr., "Calculating Behavior of Closed Reservoirs - Part 2," Petroleum Engineer, 27, (1955) B-105

12. Sturdivant, W. C., Jr., "Calculating Behavior of Closed Reservoirs - Part 3," Petroleum Engineer, 28, (1956), B-58

13. Anonymous, "Chart for Calculating Instantaneous Gas-Oil Ratio," Oil and Gas Journal, 47, (September 16, 1948), 139

14. Elliot, G. R. and Morris, W. C., "Oil Recovery Predictions," Oil and Gas Journal, 48, (June 16, 1949), 84

15. Anonymous, "Chart for Simplified Material Balance Equation," Oil and Gas Journal, 47, (December 16, 1948), 107

16. Anonymous, "Chart for Simplified Material Balance Equation," Oil and Gas Journal, 47, (December 30, 1948), 273

17. Tracy, G. W., "A Simplified Form of the Material Balance Equation," Journal of Petroleum Technology, (January 1955), 55

18. Carr, N. L., Kobayashi, R. and Burrow, D. B., "Viscosity of Hydrocarbon Gases under Pressure," Journal of Petroleum Technology, (October, 1954), 47-55

B. References not Cited

Bass, D. M. and Crawford, P. B., "Predicting Reservoir Performance," Petroleum Engineer, 27, (June 1955), B-59

Calhoun, J. C., "Variations of Material Balance," Oil and Gas Journal, 49 (December 21, 1950), 335

Calhoun, J. C., "Summary of Gas Drive Predictions," Oil and Gas Journal, 49, (November 2, 1950), 97

Davis, Dale S., Nomography and Empirical Equations, New York, Reinhold Publishing Corp., (1955)

Johnson, L. H., Nomography and Empirical Equations, New York, John Wiley and Sons, Inc., (1952)

Kullmann, C. A., Nomographic Charts, New York, McGraw-Hill, (1951)

Osoba, J. S., "Relative Permeability," Oil and Gas Journal, 52, (July 27, 1953), 326

Patton, E. C., "Evaluation of Pressure Maintenance by Internal Gas Injection in Volumetrically Controlled Reservoirs," AIME Transactions, 170, (1947), 112-155

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