

THE EXPERIMENTAL DETERMINATION OF THE
BENDING AND TORSIONAL STIFFNESS OF A
BEAM WITH ROTATIONALLY CONSTANT
MOMENT OF INERTIA WITH VARYING
AMOUNTS OF PERMANENT TWIST

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AND
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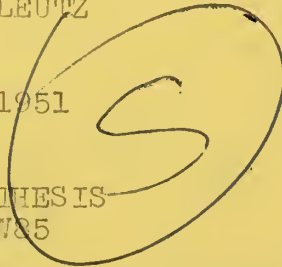
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THE EXPERIMENTAL DETERMINATION OF THE BENDING AND
TORSIONAL STIFFNESS OF A BEAM WITH ROTATIONALLY
CONSTANT MOMENT OF INERTIA WITH VARYING AMOUNTS OF
PERMANENT TWIST.

by

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1951

ABSTRACT

Title of Thesis: "The Experimental Determination of the Bending and Torsional Stiffness of a Beam with Rotationally Constant Moment of Inertia with Varying Amounts of Permanent Twist."

Authors: Lieutenant (J.G.) John Woolston, U. S. Navy.
Lieutenant (J.G.) Leon H. Leutz, U. S. Navy.

Submitted for the degree of Naval Engineer in the Department of Naval Architecture and Marine Engineering on May 18, 1951.

The object of this thesis was to investigate the variation of bending and torsional stiffness of a beam with permanent twist. The mild steel beam was cruciform in cross section with webs 0.102" thick and a total depth of 1.503" with .200" fillet radii at the center. The beam length was 50 inches. The effects noted on this beam must modify calculations for other twisted beams such as propeller blades, pump rotors, turbine blades, etc.

The torsional stiffness was calculated from the elastic angle of twist in the beam length under a constant torsional moment. The bending stiffness was calculated from bending deflections measured with the beam acted up on by constant bending moments. Bending stresses were in the elastic range.

The torsional stiffness increased with permanent twist approximately as the square of the helical angle of the outer beam fibers. The stiffness was doubled at a helical angle of 0.27 radians. This checked rather closely with the results of previous theoretical work. The overall results of the torsion tests conform to theory for cross sections approximating simple finned members.

EXPERIMENTAL

THE OBJECT OF THIS TEST WAS TO INVESTIGATE THE RELATION OF BENDING AND TORSIONAL STIFFNESS OF A BEAM WITH GEOMETRICAL CONSTANTS SUCH AS AREA, PERIMETER, AND MOMENT OF INERTIA.

EXPERIMENT (1) WAS CONDUCTED AT THE UNIVERSITY OF CALIFORNIA, BERKELEY, CALIF. BY THE AUTHOR, J. H. BAKER, IN 1921.

THE OBJECT OF THIS TEST WAS TO INVESTIGATE THE RELATION OF BENDING AND TORSIONAL STIFFNESS OF A BEAM WITH GEOMETRICAL CONSTANTS SUCH AS AREA, PERIMETER, AND MOMENT OF INERTIA.

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THE TORSIONAL STIFFNESS WAS CALCULATED FROM THE ELASTIC ANGLES OF TWIST IN THE BEAM UNDER A CONSTANT TORSIONAL MOMENT. THE BENDING STIFFNESS WAS CALCULATED FROM BENDING DEFLECTIONS MEASURED WITH THE BEAM UNDER A CONSTANT BENDING MOMENT. BENDING STIFFNESS WAS IN THE ORDER OF 1000 LB.-IN. PER RADIANT.

THE RESULTS OF THE TESTS WERE AS FOLLOWS: THE BENDING STIFFNESS WAS IN THE ORDER OF 1000 LB.-IN. PER RADIANT. THE TORSIONAL STIFFNESS WAS IN THE ORDER OF 1000 LB.-IN. PER RADIANT. THE RESULTS OF THE TESTS WERE AS FOLLOWS:

In the bending tests the ratio of deflection to the theoretical deflection, based on simple beam theory, increased approximately as the cube of the helical angle to a value of helical angle of about 0.15 radians. This indicates that the beam becomes less stiff as the helical angle increases. At higher angles of twist the curve droops, reaching a maximum deflection ratio of 1.32 at a helical angle of 0.23 radians. The last experimental point showed a deflection ratio of 1.20 at a helical angle of 0.314.

The results of the bending tests show quantitatively the effect of twist on bending stiffness of a member of a particular section. Because this effect is large and its cause unknown it is obvious that much more experimental and theoretical work must be done to establish theories for the many applications of twisted beams in practice.

In the meeting there the ratio of collection to the theoretical
collection based on energy taken theory. The record approximately as
the ratio of the actual work to a ratio of actual work of about 0.15
exists. This indicates that the actual power loss due to the actual
power loss. As a result of this the ratio of actual work to
a theoretical collection ratio is about 0.15 at a collection ratio of 0.15.
The last experimental work shows a collection ratio of 0.50 at a
ratio of 0.15.

The results of the existing ratio show practically the limit
of loss of energy, which is a measure of a practical design.
However, the ratio is large and the ratio shows that
more work is required and theoretical work ratio is about 0.15
exists for the ratio of actual work to theoretical.

Cambridge, Massachusetts
18 May, 1951

Professor J. S. Newell
Secretary of the Faculty
Massachusetts Institute of Technology
Cambridge, Massachusetts

Dear Sir:

In accordance with the requirements for the Degree of Naval Engineer, we submit herewith a thesis entitled "The Experimental Determination of the Bending and Torsional Stiffness of a Beam with Rotationally Constant Moment of Inertia with varying amounts of Permanent Twist."

Respectfully,

ACKNOWLEDGEMENT

The authors are deeply indebted to Professor J. P. DenHartog of the Massachusetts Institute of Technology whose inspiration and guidance made this thesis possible.

MEMORANDUM FOR THE RECORD

The report was duly filed in Volume 17, Serial 1000
at the Department of State in accordance with the
instructions of the Secretary of State.

Very truly yours,
[Signature]

[Signature]
[Title]
[Date]

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NOMENCLATURE

- L = Length of beam from load to load or 50".
- L' = Length of beam used in measurement of torsional stiffness in inches.
- α = Angle of permanent twist in the beam in degrees.
- ϕ = Angle of elastic twist of the beam under the action of torsional moment, T, where the moment was applied to the beam over a length L'; ϕ in degrees.
- β_0 = Helical angle of outer fiber of the beam = $\frac{\alpha \times r_0}{57.3 \times L}$ radians.
- r_0 = Radius of the outer fiber of the beam in inches or 0.751".
- J = Torsional stiffness of the beam, T/ θ
- J/J_s = Ratio of the torsional stiffness of the twisted beam to that of the straight beam.
- T = Torsional moment, inch pounds.
- θ = Angle of elastic twist per unit length as a result of the torsional moment; radians per unit length.
- Δ = Displacement of a point on the beam when loaded, measured from the unloaded position.
- δ = Displacement of a point on the loaded beam from the tangent at the center of the beam, corrected for lack of straightness in the unloaded beam.
- δ_0 = Theoretical displacement from horizontal tangent at center of beam, based on simple beam theory.
- δ/δ_0 = Ratio of displacement of beam to theoretical displacement.
 $\delta/\delta_0 = (EI)_0 / EI =$ ratio of original stiffness to stiffness at a given angle of permanent twist.

Beam Deflection

- 1. - Length of beam from fixed end to free end is L .
- 2. - Length of beam with displacement by vertical distance δ is $\sqrt{L^2 + \delta^2}$.
- 3. - Angle of deflection at free end is θ .
- 4. - Angle of elastic curve of the beam under the action of external moment M about the moment was applied to the beam over a distance x is ϕ .
- 5. - Deflection of beam from fixed end of the beam is δ .
- 6. - Slope of the beam at the free end is θ .
- 7. - External moment at the free end is M .
- 8. - Slope of the external moment at the fixed end is θ .
- 9. - External moment at the fixed end is M .
- 10. - Slope of elastic curve at the fixed end is θ .
- 11. - Deflection of the beam at the fixed end is δ .
- 12. - Deflection of the beam at the free end is δ .
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- 50. - Deflection of the beam at the free end is δ .

INTRODUCTION

Conventional beam theory states that if the EI product of a beam is constant, that is the stress-strain relationship is linear and the moment of inertia does not change, the beam will maintain the same bending stiffness, EI . Under these conditions the beam will always deflect the same amount under identical loadings.

The question then arises as to what happens to the bending stiffness when the beam has a longitudinal twist. If the modulus of elasticity is constant and the section has a rotationally constant moment of inertia, that is the I is the same about all axis through the center of gravity of the beam section, will the beam theory break down for a twisted beam? In the case of helical pump impellers and also in airplane propellers with their inherent pitch this question of twisted beams arises. The pump impeller designer will want to know the impeller stiffness for strength and for vibration characteristics. The propeller designer will pose the same questions concerning his design.

As far as is known no experimental or theoretical work has been done on the above question of bending stiffness. However, it is the belief of some engineers that the bending stiffness is not the same for a twisted beam as for a straight beam with the same EI . In one instance the designers of airplane propellers find it difficult to calculate the exact natural frequency of vibration of the blades and their results may be 15% in error from the actual value. This error may be due to using an incorrect value of the bending stiffness of the blade because

APPENDIX

2. The first part of the proof is to show that if ϵ is small enough

then the function ϕ is a solution of the problem.

It is clear that ϕ satisfies the boundary conditions.

It remains to show that ϕ satisfies the differential equation.

Let us first consider the case where $\epsilon < 1$.

The function ϕ is a solution of the problem if and only if

it satisfies the differential equation in the domain D .

Let us first consider the case where $\epsilon < 1$.

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they do not account for the twist.

The experimental determination of the variation of bending stiffness vs. angle of permanent twist is then begun without knowing the nature of the possible results or if there are any variations whatever. It is known, however, that in applying the angle of permanent twist to the beam that the outer fibers will be yielded in tension and the inner fibers will be yielded in compression. However, during the bending tests, since the beam is free to change its length longitudinally, the state of longitudinal stress will be well below the yield stress after the twisting moment is removed even though the beam has been yielded. The stress pattern of the beam will be quite complicated because of the bending stresses being superimposed upon the stresses that have been set up during the application of the permanent twist. It is felt that the latter stresses will have little effect upon the stiffness of the beam as long as the total stress is kept below the proportional limit. If there is a change in bending stiffness with changing angles of permanent twist it is most likely due to the interaction of the stresses caused by the geometry of the beam.

The other major topic to be examined here is the variation of the torsional stiffness of a beam as the angle of permanent twist is varied. This subject has been theoretically and experimentally studied and a bases for a comparison of results is at hand. Let it suffice to say that the torsional stiffness will increase with the angle of permanent twist and that for a rectangular beam this increase is primarily a function of the square of the height to thickness ratio of the beam cross section.

They are not known for the time.

The theoretical consideration of the variation of bending stress

near an angle of curvature is of great importance in the design of

of the beams which are used in the design of bridges, etc. It is

found, however, that in such cases the angle of curvature is not

small and the stress varies with the position and the shape of the

will be varied in consequence. However, noting the bending stress, since

the beam is free to change its length longitudinally, the stress of longitudinal

stress will be well below the yield stress and the bending moment is

removed even though the beam has been twisted. The stress pattern of

the beam will be elastic throughout because of the bending stresses being

distributed over the cross-section and the beam is not being twisted.

and of the maximum stress. It is felt that the latter stresses will have

little effect upon the stiffness of the beam as long as the total stress is

well below the proportional limit. It is felt that a change in bending stress

near the support, which is maximum, will be small and will not

be a function of the stresses caused by the twisting of the beam.

The stress ratio to be considered here is the variation of

the maximum stress at a point on the edge of maximum twist is noted.

This subject was used theoretically and experimentally studied with a

beam for a comparison of results in its design. Let it be noted in any case

the maximum stresses will be noted with the angle of curvature being

and that for a rectangular beam the maximum is normally a function of the

square of the height in the case of the beam cross-section.

FIGURE I

THE ARRANGEMENT OF THE BEAM DURING BENDING TESTS

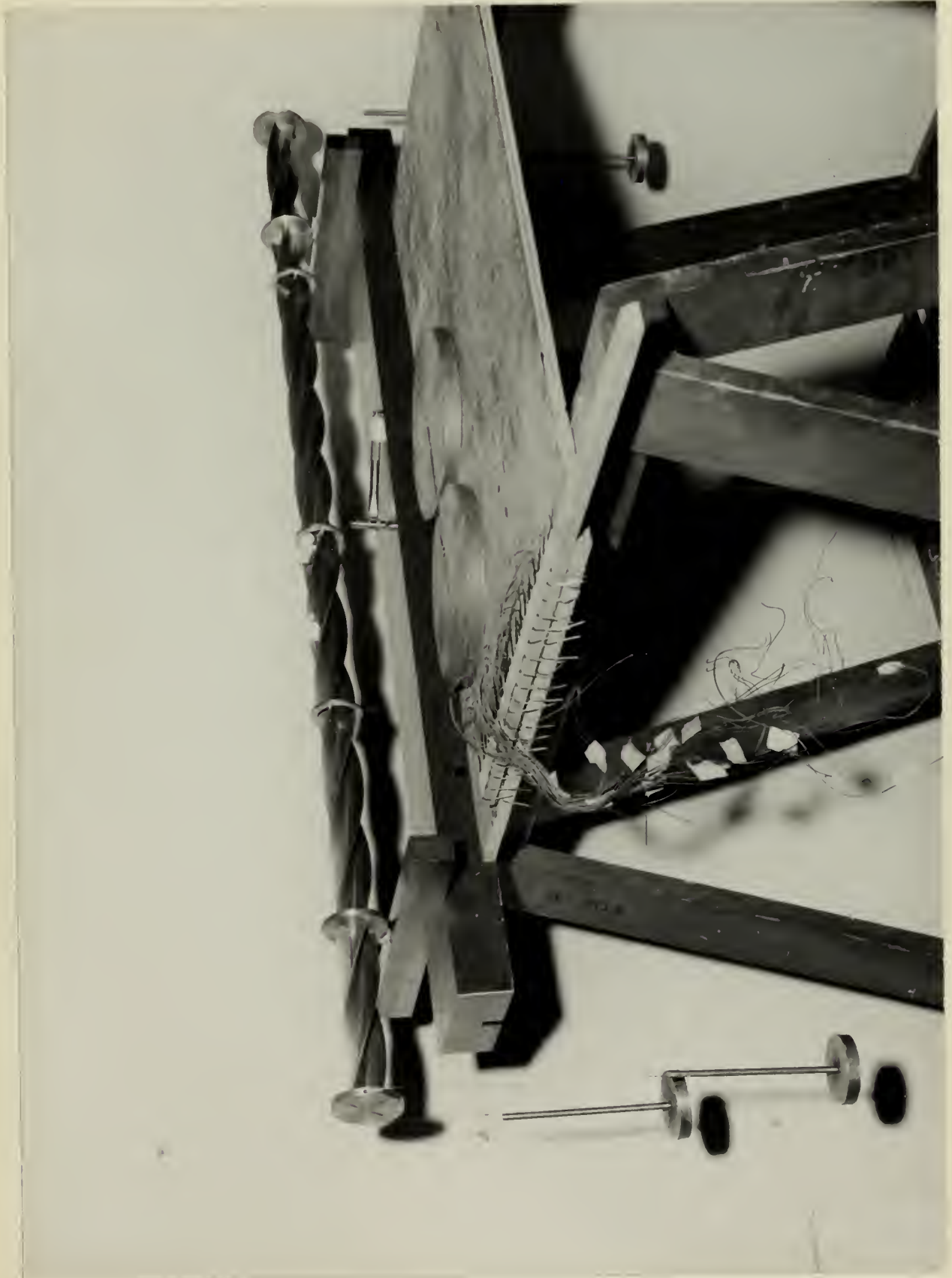


FIGURE II
CLOSE-UP OF THE BEAM AT $\beta_0 = .314$



PROCEDURE

The beam shown in Figure IV was designed with a rotationally constant moment of inertia in order that the conditions of the thesis could be met. The beam dimensions were chosen on the basis of predictable results to be obtained from the laboratory technique employed. The bending stiffness of the beam was to be obtained from the deflection of the beam loaded as shown in Figure III. This loading produces a constant bending moment on the beam between the supports. These deflections, to be measured with an inside micrometer (see Figure I), were to have an approximate maximum value of .100" at the center of the beam while keeping the stress in the beam well below the yield stress of the material, mild steel, or about 15,000 psi. The .100" maximum deflection figure was chosen since it was felt that an error of .001" would have to be accepted in the deflection measurements. This then would limit the error to 1% at the maximum deflection point. Furthermore, the loads to be used on the beam would have to be of a size that could be readily applied in the laboratory.

In order to obtain the variation of bending stiffness with the angle of permanent twist the beam was to be given additional twist prior to each run. Since there were no mechanical means of applying this twist available it would have to be applied manually. This condition further dictated the beam dimensions but it was found by using the membrane analogy that this condition of manual twisting of the beam did not necessitate a change in the beam dimensions derived from the above

PROCEDURE

The beam shown in Figure IV was designed with a rotationally constant moment of inertia in order that the conditions of the flexure could be varied. The beam dimensions were chosen on the basis of predictable results to be obtained from the laboratory technique employed. The bending stress area of the beam was to be obtained from the deflection of the beam loaded as shown in Figure III. This loading produces a constant bending moment on the beam between the supports. These deflections, to be measured with an inside micrometer (see Figure II), were to have an approximate maximum value of .100" at the center of the beam while keeping the stress in the beam well below the yield stress of the material, mild steel, or about 12,000 psi. The .100" maximum deflection figure was chosen since it was felt that an error of .001" would have to be accepted in the deflection measurements. This then would limit the error to 1% of the maximum deflection point. Furthermore, the loads to be used on the beam would have to be of a size that could be readily applied in the laboratory.

In order to obtain the variation of bending stresses with the angle of permanent twist the beam was to be given additional twist prior to each run. Since there were no mechanical means of applying this twist available it would have to be applied manually. This condition further dictated the beam dimensions but it was found by using the membrane analogy that this condition of manual twisting of the beam did not necessitate a change in the beam dimensions derived from the above

bending criteria. The final beam design is shown in Figure IV.

The beam and its fittings were manufactured at the Boston Naval Shipyard. It was planed from solid stock, heat treated and planed to its final dimensions. Because of the length of the beam and the play in the planer head it was found that the design tolerances could not be met. The beam micrometer readings are shown in Figure V. From these readings a mean value of flange thickness was taken as .1020" and mean beam depth of 1.5030". The moment of inertia of the section was calculated from these mean values and found to be .02925 in.⁴. The support rings, load rings and deflection rings were hand filed and fitted to the beam snugly with a hand fit. The bed plate was surface ground to a smooth finish.

The procedure used in the deflection tests is shown in Figure I where the supports are set up on parallels so that an inside micrometer might be used to measure the deflections. In the no load condition only the load rings (pulleys) were in place and deflections were read at each deflection ring between the supports. To apply the loads the weight supports (shown) were hung over the load rings and equal load weights, calibrated to .01#, were placed on the supports. Each separate weight (note two weights on table in Figure I) was $11.03 \pm .01\#$ and the weight supports weighed 1.39# at each end of the beam. The weight supports were designed so that no torsional moment would be applied to the beam when the load was applied. For each load condition 4 weights were placed on the beam, two at each end. The deflection rings were placed

ending criteria. The final beam design is shown in Figure IV.

The beam and its fittings were manufactured at the Boston Navy Yard.

It was fabricated from solid stock, heat treated and planed to its

final dimensions. Because of the length of the beam and the play in the

bracket head it was found that the best in tolerances could not be met.

The beam dimensional readings are shown in Figure V. From these

readings a mean value of flange thickness was taken as .1020" and

mean beam depth of .1030". The moment of inertia of the section was

calculated from these mean values and found to be .0195 in⁴. The

support rings, load rings and deflection rings were hand filed and fitted

to the beam snugly with a hand fit. The ball plate was surface ground

to a smooth finish.

The procedure used in the deflection tests is shown in Figure I

where the supports are set up on parallels so that an inside micrometer

might be used to measure the deflection. In the no load condition only

the load rings (bolleys) were in place and deflections were read at each

deflection ring between the supports. To apply the loads the weights

supports (shown) were hung over the load rings and equal load weights

calibrated to .014, were placed on the supports. Each support weight

(with two weights on table in Figure I) was 1.03 ± .014 and the weights

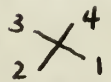
supports weighed 1.194 at each end of the beam. The weight supports

were designed so that no torsional moment would be applied to the beam

when the load was applied. For each load condition 4 weights were

placed on the beam, two at each end. The deflection rings were placed

to give a spread of readings. Deflection ring 3 was placed 3" from the support, or two beam diameters distance so that the effect of the support would not be felt. This is in accordance with Saint Venant's principle.

The beam stiffness in bending was checked in two rotational positions. The initial position of the beam was with flange 3 vertically up at the mid-span of the beam. No load and loaded beam deflections were taken with the beam in this position. When the beam was unloaded it was rolled through 45° with flanges 3 and 4 thus:  when looking at the beam from the left end in Figure III. At Run 15, when the largest value of β_0 was reached, the beam deflection was read with flange 3 at mid-span rotated through 360° with readings taken at each 45° interval. This beam rotation was accomplished to ascertain if the stiffness varied with the beam position on the supports. It seemed likely that if the beam stiffness varied with the angle of permanent twist that it might also vary with the position of the beam on the supports.

Strain gages, as shown in Figure VI, were placed on the beam to give possible aid in the analysis of results. These gages were all placed to indicate longitudinal strain near the outer fibers of the flanges in order that a longitudinal stress distribution might be had with the beam in a twisted condition. These gages were read only during the bending tests.

The beam was received in the straight condition from the Boston Naval Shipyard. The initial, straight beam deflection tests were made

to give a spread of energies. The electron ring was placed 1.7 cm from the support by two brass cylindrical supports and they had effect of the support would not be felt. This is in accordance with Yaman's principles.

The beam diameter in passing was checked in the cylindrical positions. The initial position of the beam was with range 3 vertically up at the mid-point of the beam. The beam and loaded beam detectors were taken with the beam in this position. When the beam was rotated



it was rotated through 45° with ranges 3 and 4 taken; looking at the beam from the left and in Figure III. At this point the largest value of A_{max} was recorded, the beam diameter was read with range 3 at mid-point rotated through 30° with readings taken at each 15° interval. This beam rotation was accomplished in seconds if the beam was varied with the beam position on the support. It seemed likely that if the beam diameter varied with the angle of curvature it was not it might also vary with the position of the beam on the support.

Beam ranges, as shown in Figure VI, were placed on the beam to give possible aid in the analysis of results. These ranges were all placed to indicate longitudinal strain near the outer fibers of the lenses in order that a longitudinal stress distribution might be had with the beam in a rotated condition. These ranges were read only during the bending tests.

The beam was received in the straight position from the position that it started. The initial straight beam diameter tests were made

and checked against the calculated values found by using standard beam theory. In order to determine the modulus of elasticity of the material a tensile test specimen was made by the shipyard and given the same heat treatment as the beam. A stress-strain curve was made from a tensile test and the modulus of elasticity was found to be 29.7×10^6 psi. The material had a proportional limit of 24,500 psi and an ultimate stress of 56,900 psi.

The torsional stiffness was found with the beam in the straight condition. This was done by holding the beam fixed at one end and applying a twisting moment at the other end. A twisting moment of 124.3 in. lbs. was used and the shear stresses set up in the beam were well within the elastic region of the beam material. The beam was held at one end by fastening a die stock to the load ring and holding the arms of the stock firmly to a stationary support. On the other end the load ring had been drilled and tapped (note holes in support ring at far end in Figure I) symmetrically so that an arm could be fitted to it. This arm was grooved 10" from the center of the beam thus giving the arm of the moment. From this groove the load supports were hung along with one lead weight, or a total of 12.43#. With this load applied the arm was made to be horizontal by setting the position of the die stock at the other end. Therefore, the full moment acted on the beam. The load was then removed and the angle through which the beam untwisted with the other end fixed was measured by using a protractor. This made possible the calculation of the torsional stiffness. This test was

and another against the vertical value found by using conventional beam theory. In order to determine the nature of the material a cantilever specimen was made by the elliptical and given the same load treatment as the beam. A stress-strain curve was made from a sample taken and the location of elastic limit was found to be 207 x 10⁶ psi. The material had a proportional limit of 24,000 psi and an ultimate stress of 50,000 psi.

The vertical deflection was found with the beam in the straight condition. This was done by holding the beam fixed at one end and applying a twisting moment at the other end. A rotating load was applied to the beam and the stress was measured at the beam ends. Well within the elastic region of the beam material. The beam was held at one end by turning a die stock so the load was applied to the other end of the beam. A stationary support. On the other end the beam was held and twisted and twisted (more stress is reported) at the end of the beam. It is specifically so that an equal load is applied to the beam was prepared. From the center of the beam the distance was of the moment. From this point the load supports were measured with one end fixed at a point in the air. With the load applied the end was made to be horizontal by setting the position of the die stock at the other end. Therefore, the full moment is on the beam. The load was then removed and the angle through which the beam twisted with the fixed end was measured by using a goniometer. This angle was the deflection of the beam at the fixed end. This was

also run with the beam on the bed plate.

The next phase was to apply a permanent twist to the beam. This was done by fastening the die stock to the load ring on one end of the beam and using the arm on the other end. The beam was placed freely on the bed plate and manually given a permanent twist. The beam was maintained straight by the bed plate in vertical plane but could possibly bend somewhat in the horizontal plane. But by carefully applying this torsional moment the bending of the beam could be minimized and it was found to be very small. Figure VIII shows the amounts of permanent twist put into the beam with each run.

With permanent twist in the beam the deflection and torsional tests were again made in the same manner as described above. The amount of permanent twist applied to the beam was to be small at the offset so that the initial trend of the stiffness curves could be accurately determined. After this trend had been found the angle of permanent twist between runs was increased as shown in Figure VIII. Permanent twist was applied to the beam up to the point where it became too stiff to twist manually. It was also necessary to check to see if the beam flanges warped from the twisting since if they did the moment of inertia of the section would be reduced. This was done by checking to see if the flanges were still at right angles and also by the tightness of the deflection rings on the beam.

Also you will find some of the best data.

The next phase was to supply a convenient table in the form of the

was done by fastening the lead to the end of the beam and

and using the wire on the other end. The beam was placed freely on the

rod and naturally gives a permanent deflection. The beam was indicated

straight by the two points in contact with the rod and the

what is the horizontal plane. But by carefully adjusting the horizontal

moment the bearing of the beam could be maintained and it was found to

be very small. Figure 11 shows the accounts of permanent deflection

into the beam with each run.

The permanent deflection in the beam the deflection and residual

deflection were again made in the same manner as described above. The

amount of permanent deflection applied to the beam was in the order of the

was so that the initial amount of the residual deflection could be accurately

determined. After this time had been found the weight of permanent

twice between time was increased as shown in Figure 11. The

twice was applied to the beam up to the point where it became too stiff

to twist manually. It was also necessary to check to see if the

deflection was from the twisting alone by twisting the beam at the

of the section would be removed. This was done by changing to see if

the deflection were still at right angles and also by the stiffness of the

deflection curve on the beam.

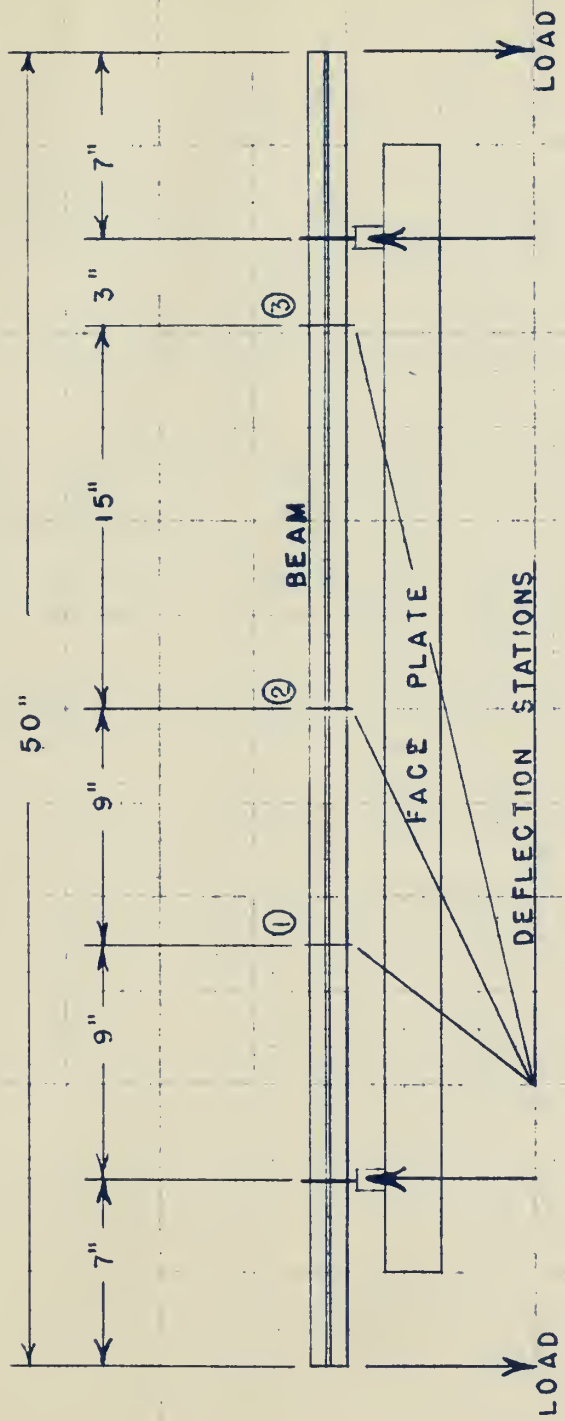
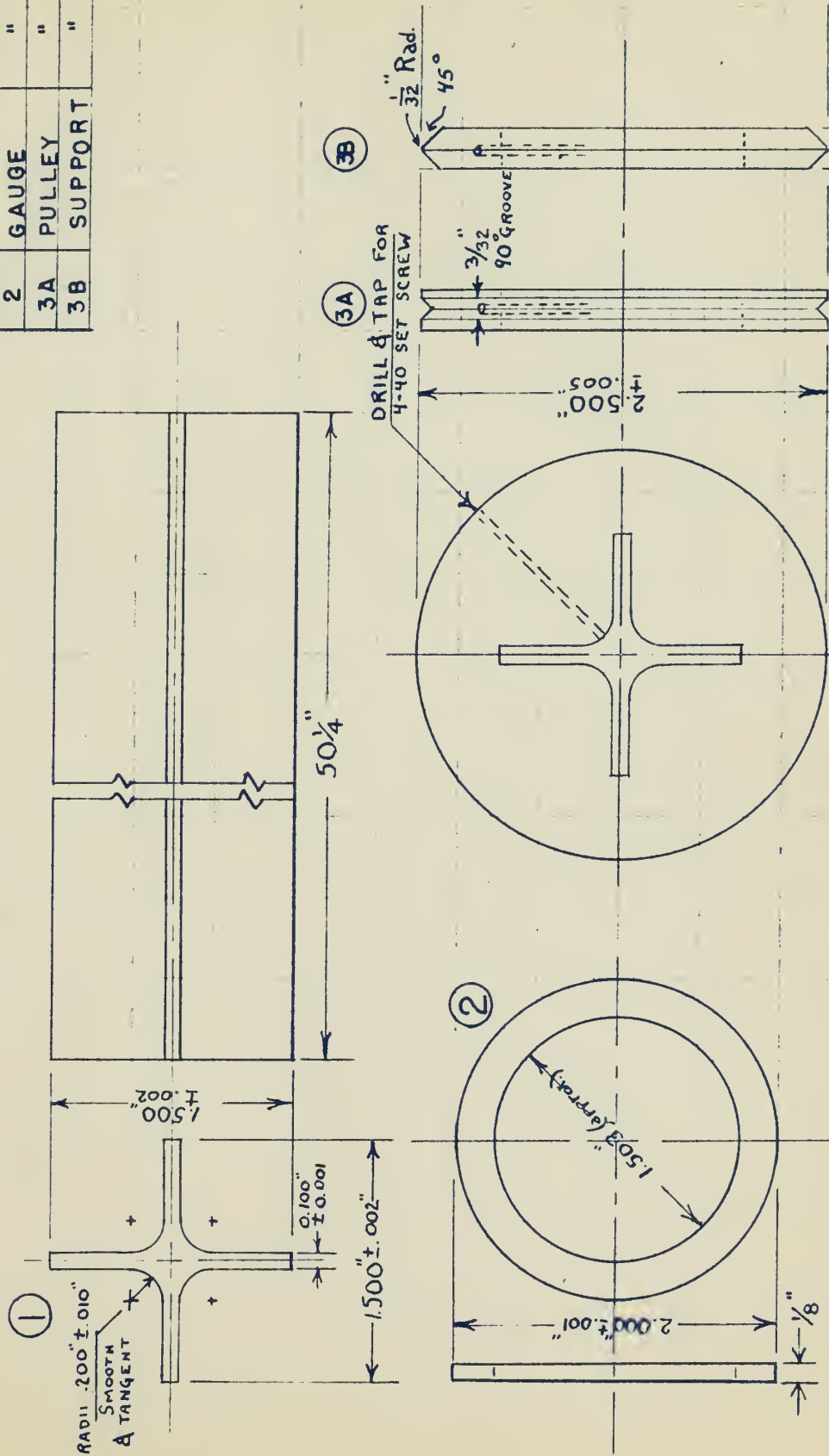


FIGURE III
 BEAM ARRANGEMENT
 IN BENDING TESTS

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FIGURE IV
 DESIGN DIMENSIONS OF
 BEAM AND FITTINGS.

P.C.	ITEM	MATERIAL	QUAN.
1	BEAM	COLD ROLLED MILD STEEL	1
2	GAUGE	"	3
3A	PULLEY	"	2
3B	SUPPORT	"	2



NOTE: INSIDE FINISHED FOR
 HAND PRESSED FIT OVER P.C. 1.

NOTE: P.C. 2 TO HAVE HAND FIT
 OVER P.C. 1 & NOT BE LOOSE.

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FIGURE V

MICROMETER READINGS OF BEAM DIMENSIONS.

STATION	FLANGE THICKNESS				BEAM DEPTH	
	Fl.#1	Fl.#2	Fl.#3	Fl.#4	Fls.1-3	Fls.2-4
0	.1023	.1017	.1023	.1030	1.5002	1.5033
1	.1003	.1008	.1015	.1002	1.5017	1.5035
2	.1006	.1010	.1016	.1010	1.5021	1.5041
3	.1014	.1018	.1028	.1025	1.5030	1.5047
4	.1015	.1024	.1033	.1021	1.5028	1.5040
5	.1018	.1012	.1033	.1004	1.5037	1.5037
6	.1023	.1016	.1030	.1013	1.5040	1.5038
7	.1028	.1019	.1040	.1028	1.5040	1.5037
8	.1020	.1016	.1038	.1025	1.5036	1.5032
9	.1010	.1002	.1015	.1006	1.5036	1.5028
10	.1011	.1003	.1015	.1016	1.5030	1.5035

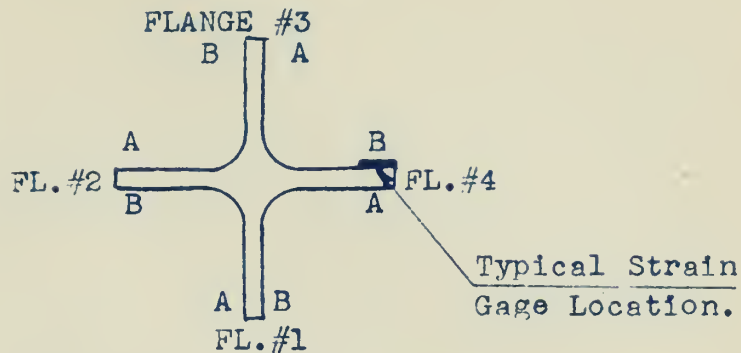
Stations are spaced each 5 inches along length of the beam. Station 0 is at left end of the beam as seen in Figure III.

All measurements are in inches.

Flange thicknesses were measured at the outer edges of the flange.

FIGURE VI

STRAIN GAGE DATA & LOCATION



BEAM CROSS SECTION LOOKING
AT BEAM IN FIGURE III FROM
THE LEFT END

STRAIN GAGE LOCATIONS

The strain gages are designated as to location by the flange number, the flange face letter (A or B) and by their distance in inches from the left end of the beam as shown in Fig. III. Then gage 3A 16 would be on flange 3, on the A face and 16 inches from the left end. All gages were oriented to give longitudinal strain and the center of the gage resistance wires were 0.1" in from the outer edge of the flange.

STRAIN GAGE DATA

Type: A-7
Res. in Ohms: 120
Gage Factor: 1.96
Lot Number: 501
Manufacturer: Baldwin Loco. Works.

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RESULTS

The results of the torsion tests are shown in figures VII and VIII.

It will be seen that the torsional stiffness increases with increased helical angle in approximately a parabolic manner and that the stiffness ratio J/J_s reaches 2.00 at a β_0 of .27.

The results of the bending tests are shown in figures IX and X.

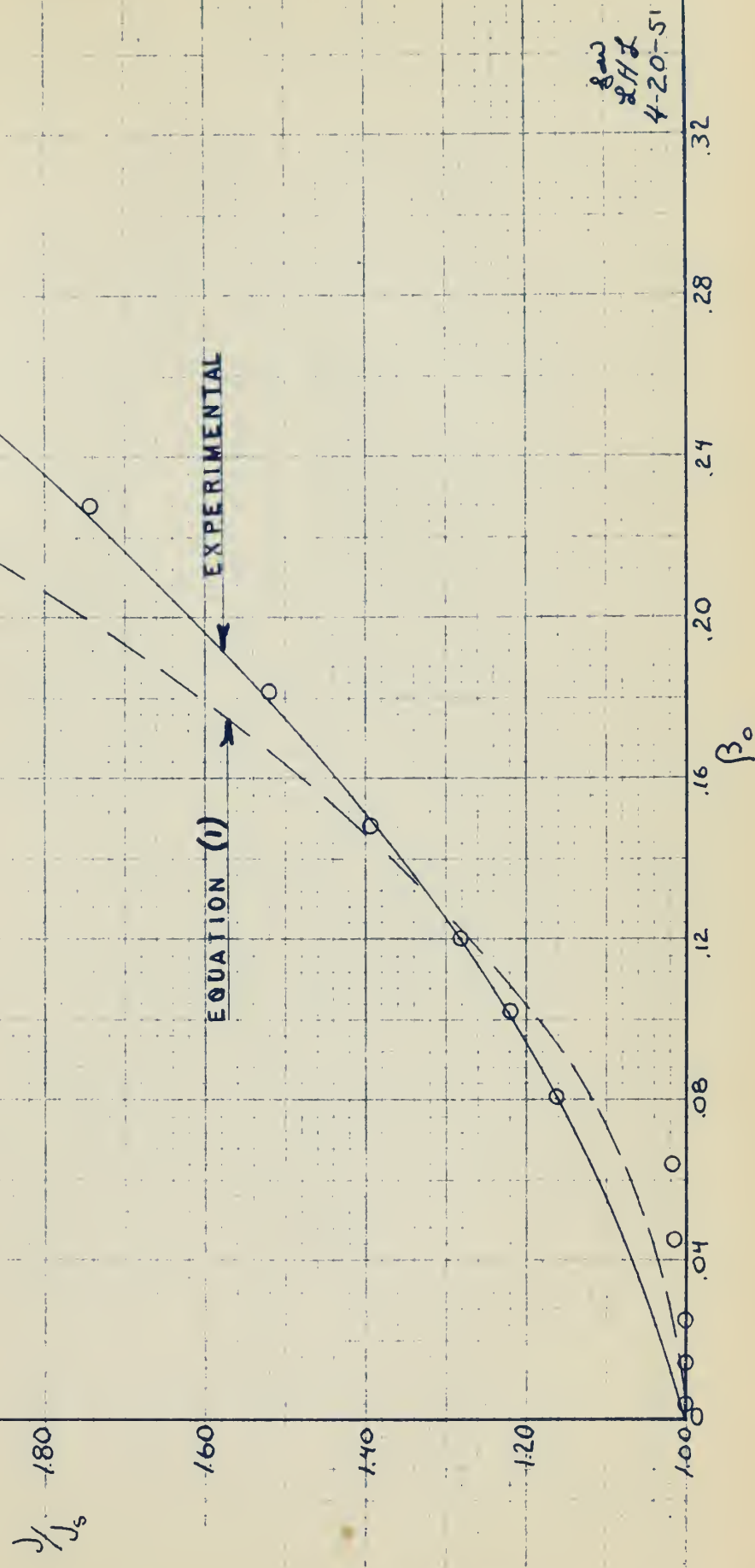
It will be seen that the displacement ratio δ/δ_0 , which is the reciprocal of the stiffness ratio $(EI)_0 / (EI)$, increases with helical angle exponentially to a β_0 of about .15. The exponent in this case is evidently slightly less than 3. Above $\beta_0 = .15$ the rate of increased δ/δ_0 decreases until a maximum value of $\delta/\delta_0 = 1.32$ at $\beta_0 = .23$ is reached. The trend of the results continues with this drooping characteristic to the last experimental point of $\delta/\delta_0 = 1.2$ at $\beta_0 = .314$.

RESULTS

The results of the various tests are shown in figures 11 and 12. It will be seen that the critical stresses increase with increased distance in approximately a parabolic manner and that the critical stress σ_c reaches 2.00 at a p_0 of 1.5.

The results of the bending tests are shown in figure 13 and 14. It will be seen that the displacement ratio δ/δ_0 which is the reciprocal of the stiffness ratio $(EI)_{eff}/(EI)_0$ increases with lateral angle exponentially to a δ_0 of about 1.5. The exponent in this case is evidently slightly less than 2. Above $p_0 = 1.5$ the rate of increase of δ/δ_0 decreases until a maximum value of $\delta/\delta_0 = 1.50$ at $p_0 = 1.5$ is reached. The trend of the results continues with the growing concentration in the last experimental point of $\delta/\delta_0 = 1.5$ at $p_0 = 1.5$.

FIGURE VII
TORSIONAL STIFFNESS
VS.
HELICAL ANGLE



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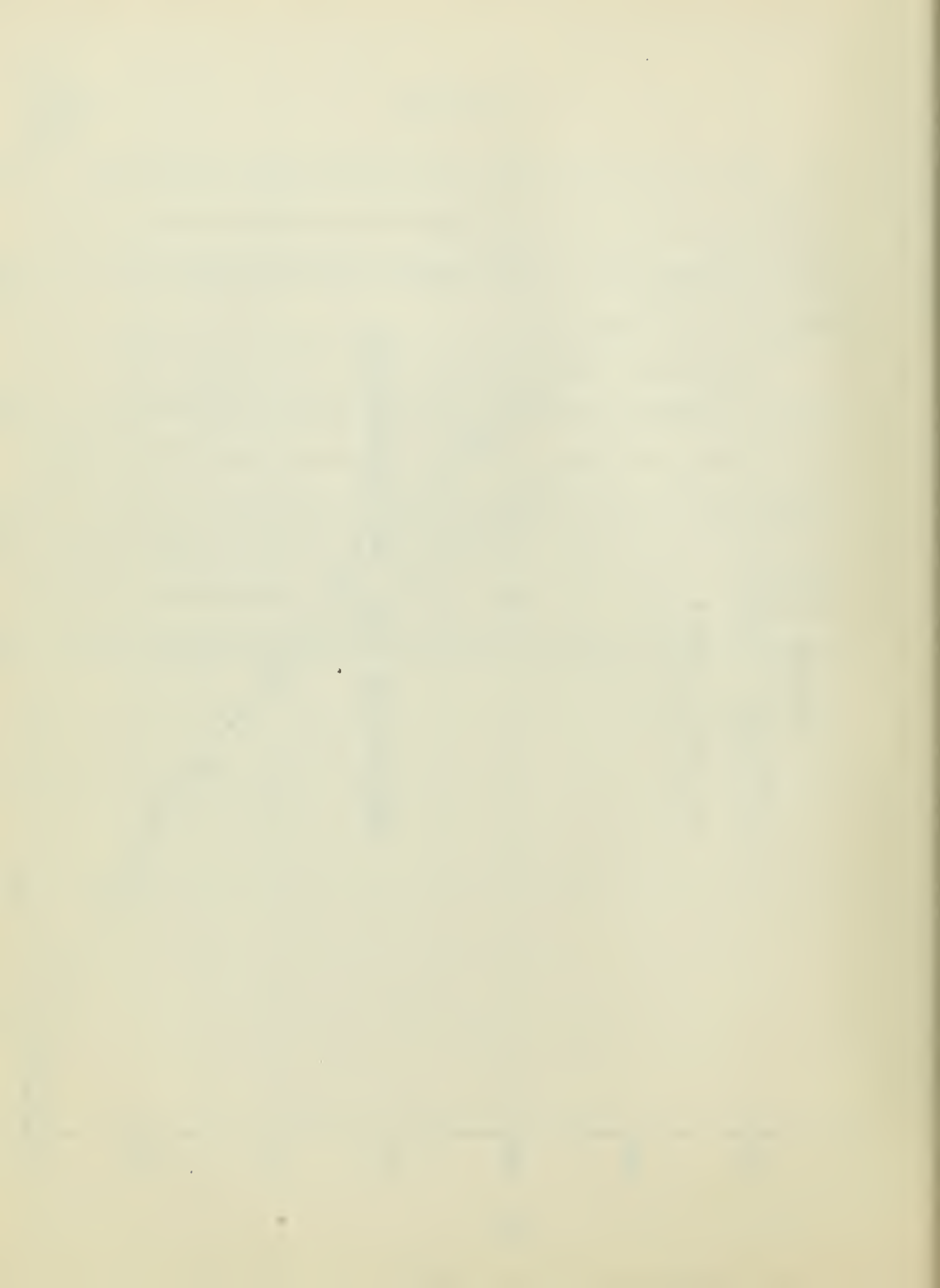


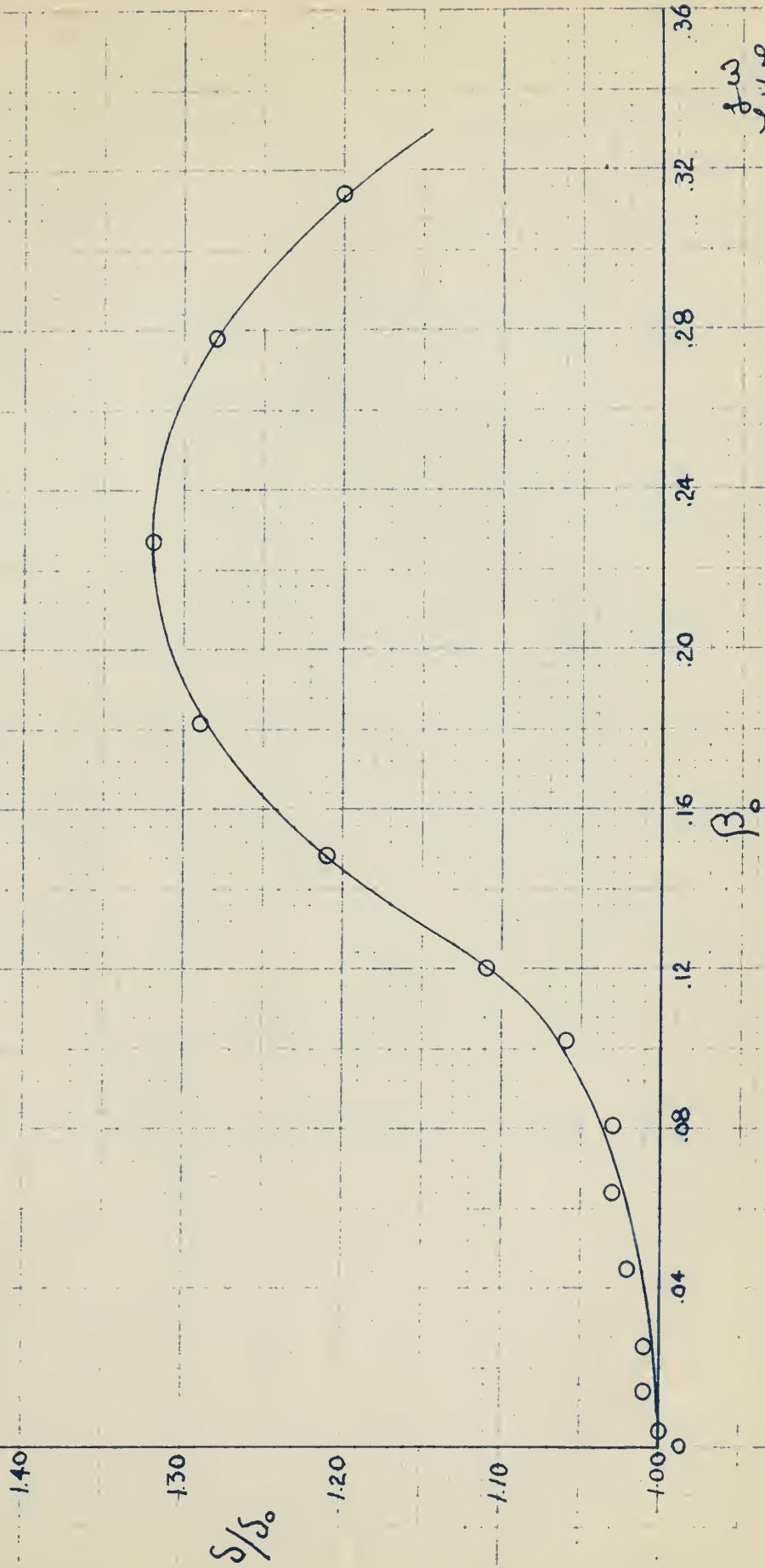
FIGURE VIII
TABLE OF TORSION
DATA & RESULTS

RUN	α°	β_0	ϕ°	L'	Θ	$\frac{J}{J_s} = \frac{.00426}{\Theta}$
1	0	0	11	45.1	.00426	1.000
2	16	.0042	11	45.1	.00426	1.000
3	55	.0144	11	45.1	.00426	1.000
4	98	.0257	11	45.1	.00426	1.000
5	170½	.0477	12	50	.00418	1.018
6	243	.0639	12	50	.00418	1.018
7	307½	.0807	10½	50	.00366	1.162
8	389	.1022	10	50	.00349	1.220
9	458	.1200	9½	50	.00332	1.282
10	567	.1488	8¾	50	.00305	1.395
11	692	.1814	8	50	.00279	1.525
12	866	.227	7	50	.00244	1.744
13	866	.227	7	50	.00244	1.744
14	1063	.278	5½	50	.00192	2.217
15	1199	.314	5½	50	.00192	2.217

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FIGURE IX

BENDING STIFFNESS ($\frac{s}{s_0}$) VS.
HELICAL ANGLE (β_0).



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FIGURE X

TABLE OF DISPLACEMENTS OF POINT 3 & SUPPORT
FROM POINT 2; CENTER OF BEAM.

Symbols as shown in Figure X1

X indicates beam rotated 45°.

RUN δ/δ_0	Load	δ_3	δ_5	δ_3/δ_{30}	δ_5/δ_{50}	RUN δ/δ_0	Load	δ_3	δ_5	δ_3/δ_{30}	δ_5/δ_{50}
Theory	Ld. 1	.021	.031			RUN 8	Ld. 1	.022	.032	1.05	1.03
	Ld. 2	.040	.059				Ld. 2	.043	.062	1.07	1.05
RUN 1	Ld. 1	.021	.030	1.00	.98	1.06 X	Ld. 1	.022	.032	1.05	1.03
	Ld. 2	.040	.059	1.00	1.00		Ld. 2	.043	.062	1.07	1.05
1.00 X	Ld. 1	.021	.031	1.00	1.00	RUN 9	Ld. 1	.023	.034	1.10	1.10
	Ld. 2	.040	.059	1.00	1.00		Ld. 2	.045	.066	1.12	1.12
RUN 2	Ld. 1	.021	.030	1.00	0.97	RUN 10	Ld. 1	.025	.037	1.19	1.19
	Ld. 2	.041	.060	1.02	1.02		Ld. 2	.050	.072	1.25	1.22
1.00 X	Ld. 1	.021	.030	1.00	0.97	RUN 11	Ld. 1	.027	.040	1.29	1.29
	Ld. 2	.040	.060	1.00	1.02		Ld. 2	.052	.076	1.30	1.29
RUN 3	Ld. 1	.022	.031	1.05	1.00	RUN 12&13	Ld. 1	.028	.040	1.33	1.29
	Ld. 2	.041	.059	1.02	1.00		Ld. 2	.053	.077	1.33	1.31
1.01 X	Ld. 1	.022	.031	1.05	1.00	1.32 X	Ld. 1	.028	.041	1.33	1.32
	Ld. 2	.040	.059	1.00	1.00		Ld. 2	.053	.078	1.33	1.32
RUN 4	Ld. 1	.021	.031	1.00	1.00	RUN 14	Ld. 1	.027	.040	1.29	1.29
	Ld. 2	.041	.059	1.02	1.00		Ld. 2	.051	.076	1.27	1.29
1.01 X	Ld. 1	.021	.031	1.00	1.00	1.28 X	Ld. 1	.027	.039	1.29	1.26
	Ld. 2	.042	.059	1.05	1.00		Ld. 2	.051	.075	1.27	1.27
RUN 5	Ld. 1	.021	.031	1.00	1.00	RUN 15	Ld. 1	.026	.037	1.24	1.19
	Ld. 2	.041	.059	1.02	1.00		Ld. 2	.048	.072	1.20	1.22
1.02 X	Ld. 1	.022	.031	1.05	1.00	1.20 X	Ld. 1	.025	.037	1.19	1.19
	Ld. 2	.042	.060	1.05	1.02		Ld. 2	.047	.071	1.18	1.20
RUN 6	Ld. 1	.022	.032	1.05	1.03	R 16 90°	Ld. 1	.025	.037	1.19	1.19
	Ld. 2	.043	.061	1.07	1.03	R 17 135°	Ld. 1	.025	.037	1.19	1.19
1.03 X	Ld. 1	.021	.031	1.00	1.00	R 18 180°	Ld. 1	.026	.037	1.24	1.19
	Ld. 2	.041	.060	1.02	1.02	R 19 225°	Ld. 1	.025	.037	1.19	1.19
RUN 7	Ld. 1	.022	.031	1.05	1.00	R 20 270°	Ld. 1	.026	.037	1.24	1.19
	Ld. 2	.043	.061	1.07	1.03	R 21 315°	Ld. 1	.025	.037	1.19	1.19
1.03 X	Ld. 1	.021	.031	1.02	1.00						
	Ld. 2	.041	.060	1.02	1.02						

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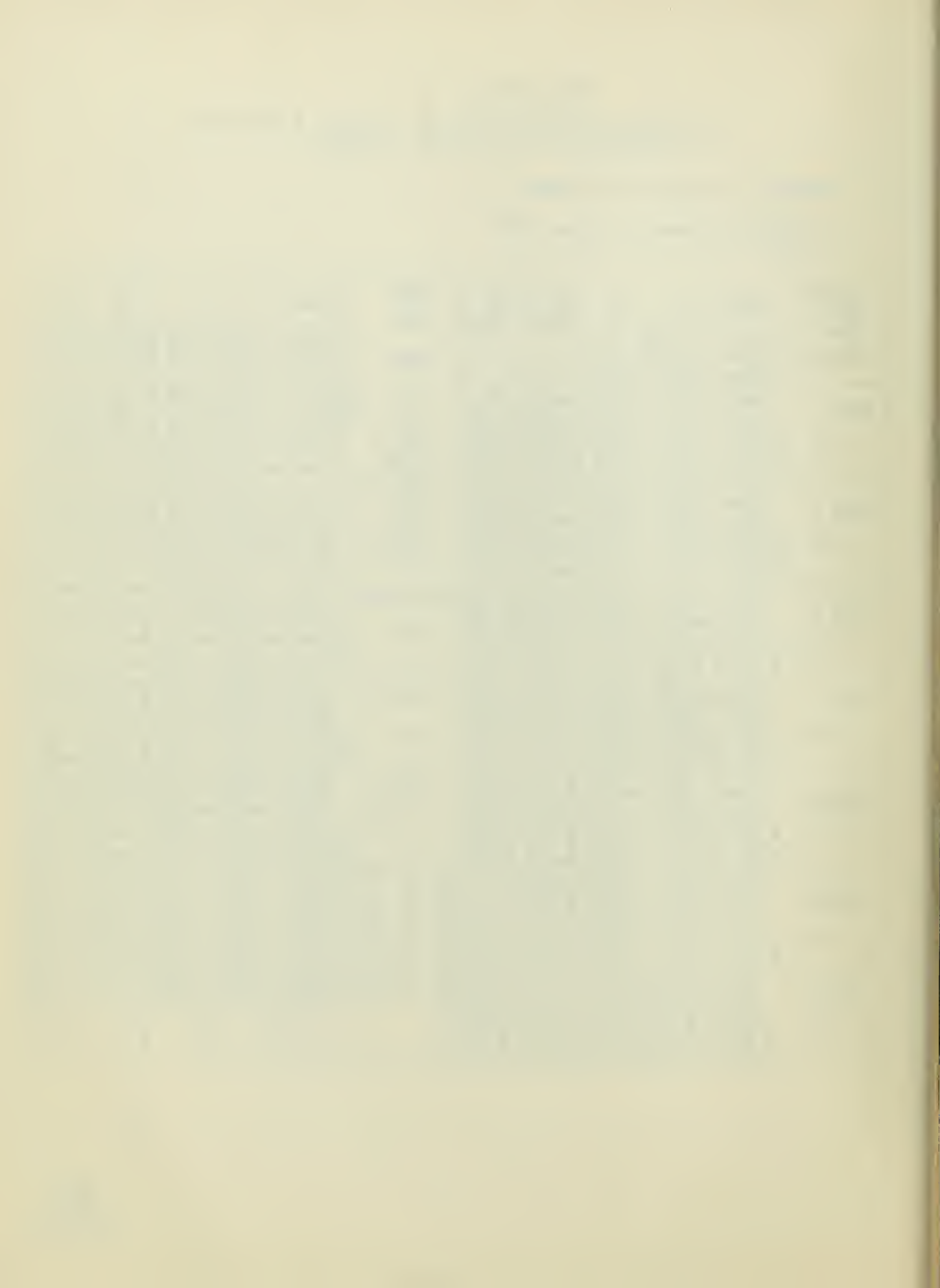
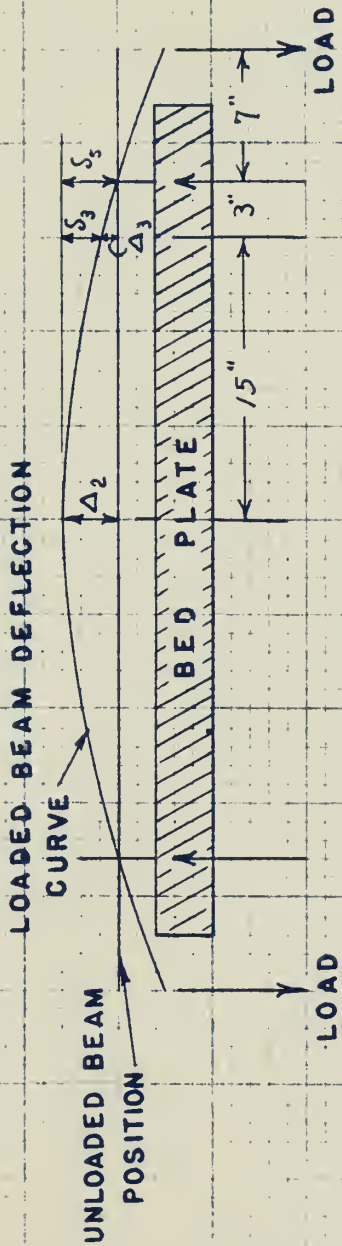


FIGURE XI
 BENDING DEFLECTION
 NOTATIONS



$$\delta_3 = \Delta_2 = \Delta_3$$

$$\delta_s = \Delta_2$$

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DISCUSSION OF RESULTS

The results of torsional experimentation are compared in Figure VII with the theoretical results of Chu in reference 1, which are based on the following equation:*

$$J/J_s = 1 + 2 \left[\frac{2}{15} (1 + \mu) \beta_0^2 \left(\frac{c}{h} \right)^2 \right] \quad (1)$$

Where μ = poisson's ratio, assumed .3

c = chord = 1.503"

h = thickness = .102"

J = torsional stiffness

$J_s = \frac{G}{3} c h^3$ from membrane analogy, in this case corrected for fillets.

It is readily observable that the results are comparable within limits set by the experimental limitations of the set up used in this thesis. Due to the lack of precision in measuring angles on the gauge rings the angles were measured from end to end of the entire beam. Consequently there is an indeterminent error due to the constraint of the support rings which may be noted in figures I and II. In order to bring the results more closely in line, rather complex changes would have to be made in the theory to account for fillets.

The results of the bending tests are; to the best of the author's knowledge, the first ever to be obtained, therefore there are no other

* Ref. 1, pg. 150.

DISCUSSION OF RESULTS

The results of theoretical calculations are compared in Figure

VII with the theoretical results of Chu in reference 1, which are based

on the following equation:

$$(1) \quad \left[\frac{2}{\pi} \left(\frac{2}{\pi} \right)^2 \left(1 + \mu \right) \left(\frac{2}{\pi} \right)^2 \right] \frac{2}{\pi} = 1 + \frac{2}{\pi}$$

where $\mu = \text{radius of circle, assumed } 1.$

$c = \text{radius} = 1.000$

$k = \text{thickness} = 1.005$

$l = \text{thickness of filter}$

From numerical analysis, it is

$$l = \frac{2}{3} c k^2$$

case corrected for filters.

It is readily observed that the results are comparable within

limits set by the experimental limitations of the set up used in this

study. Due to the lack of correlation in measuring angles on the gauge

rings the angles were measured from end to end of the entire beam.

Consequently there is an inherent error due to the constraint of

the support rings which may be noted in Figures I and II. In order to

bring the results more nearly in line, rather complex changes would

have to be made in the theory to account for filters.

The results of the heading tests are to the best of the author's

knowledge, the first ever to be obtained, therefore there are no other

results or equations available for comparison. At small angles of twist below $\beta_0 = .15$ the trend of δ/δ_0 is exponential at a rate slightly less than the cube of β_0 . Above this point the curve droops, reaches a maximum of $\delta/\delta_0 = 1.32$ at $\beta_0 = .23$ and continues to the last experimental point of $\delta/\delta_0 = 1.2$ at $\beta_0 = .314$. Calculations were made as shown in Appendix C. Point 1 was not used because the slight variation in beam dimensions accentuated the error for δ_1 to an unacceptable degree. The errors inherent in the system and due to the supports, as mentioned under the torsional results; and due to lack of straightness and the consequent error if there is a slight rotation of the beam in different load conditions. It is believed that the entire beam was elastic and that the E was nearly constant during these runs, as final no-load readings checked original no-load readings for every bending test.

It was noted that the beam did not warp from the application of the permanent twist nor did the deflection rings loosen appreciably.

Despite the limited scope of these results, they show a definite loss in bending stiffness in twisted members. They are the first quantitative results to be obtained to this problem and thus are important in themselves, and as proof that further research will be rewarding.

Strain gage readings have been included in the data section of the Appendix, however, no attempt has been made to analyse them. They do, however, indicate that no permanent set took place in the beam during the bending tests. This is readily seen by obtaining the

results an equation available for comparison. A small angle of
 twist below $\theta = 1.5$ the curve is exponential at a rate slightly
 less than the curve of $\theta = 1.5$. Above this point the curve deviates
 downward of $\theta = 1.5$ and continues to the last point.
 A point of $\theta = 1.5$ was found. Calculations were made as
 shown in Appendix C. Point 1 was used because the slight variation
 in beam dimensions accounted for the error in an acceptable
 degree. The stress intensity in the system due to the supports, as
 mentioned under the previous results, was not in fact of significance
 and the consequent error in the value of θ is small. The beam is
 different load conditions. It is believed that the entire beam was
 elastic and that the load was nearly constant during these tests as found
 by load readings checked against constant readings for every loading
 test. It was believed that the beam was nearly from the application of
 the permanent load and the deflection was nearly constant.
 It is believed that the beam was nearly from the application of
 load in loading stages in related members. The first question
 relative results to be obtained in this problem and that the important
 in this case, and as a result of the test results will be given below.
 The first question has been included in the last section of
 the Appendix because, as already has been noted in earlier sections,
 they are, however, included in the Appendix as they pertain to the
 beam during the loading test. This is usually seen by observing the

strain a 3A 25 for each run, for it is noted that for each run this strain is nearly constant at 220 micro inches per inch with Ld. 2 or beam.

The first part of the report deals with the general situation of the country and the progress of the war. It then goes on to discuss the various aspects of the military and naval operations, and the state of the economy and the social conditions of the population. The author concludes by giving his own views on the future of the country and the world.

The second part of the report is a detailed account of the military operations in the East. It describes the various campaigns and battles, and the tactics used by the different sides. It also discusses the state of the army and the navy, and the progress of the war in the various theatres of operations.

The third part of the report deals with the naval operations. It describes the various sea battles and the state of the navy. It also discusses the progress of the war at sea, and the tactics used by the different sides.

The fourth part of the report deals with the economic and social conditions of the country. It discusses the state of the economy, the progress of the war, and the social conditions of the population. It also discusses the various aspects of the war, and the progress of the war.

The fifth part of the report deals with the future of the country and the world. It discusses the various aspects of the war, and the progress of the war. It also discusses the state of the economy and the social conditions of the population.

CONCLUSIONS AND RECOMMENDATIONS

It is concluded that the torsional results check those of Chu in reference 1, and that his equations may be used with confidence for cross sections that do not vary a great deal from simple finned forms.

The bending results show a definite loss of bending stiffness in twisted members and may be taken as the first results in a series of tests to establish workable theories for the many applications of twisted beams.

It is recommended that future tests be modified to maintain straightness and that the beam be annealed in each twisted position to assure constant E .

CONDITIONS AND RECOMMENDATIONS

It is concluded that the proposed results shall be of use in
reference 1, and that the questions may be used with confidence for
cross sections that do not vary a great deal from simple linear forms.
The bending results show a definite loss of bending stiffness
in certain members and may be taken as the first results in a series
of tests to establish suitable theories for the many applications of
twisted beams.
It is recommended that future tests be confined to maintain
straightness and that the beam be supported in each twisted position
to secure contact 1.

APPENDIX

SUMMARY

APPENDIX A

Application of the Membrane Analogy.

With the beam in the initial straight condition it would be well to calculate the torsional stiffness of the beam by using the membrane analogy. This analogy establishes certain relations between the deflection surface of a uniformly loaded membrane and the distribution of stress in a twisted bar. The portion of the analogy to be used here states that twice the volume included between the surface of the deflected membrane and the plane of its outline is equal to the torque of the twisted bar.

The problem of finding the volume under the membrane that would lie over the cross section of our beam is complicated by the fillets. This cross section is shown in Figure XII. It is assumed that the membrane takes a parabolic shape. Therefore, the area A is

$$A = b^3 G \theta / 6 \quad (2)$$

where b = width of cross section
 G = modulus of shear (11,500,000 psi)
 θ = angle of twist in radians per inch

The problem was resolved into finding the three volumes 1, 2 and 3, and because of the symmetry of these volumes the total volume could be found. Region 1 was readily solved since b is directly known, as is the length of this straight section. In region 2 the values of b_1 , b_2 , b_3 and b_4 were found by using trigonometry and thus their parabolic areas were found. The volume of this region was then found by using Simpson's rule utilizing five equally spaced stations. The volume in region 3 was found in the same manner. However, in region 3 stations b'_2 , b'_3 and b'_4

APPENDIX A

Application of the Membrane Analogy.

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The problem of finding the volume under the membrane that would

lie over the cross section of our beam is complicated by the ellipse.

This cross section is shown in Figure XII. It is assumed that the

membrane takes a parabolic shape. Therefore, the area A is

$$A = \frac{1}{2} C b^2 \quad (2)$$

where b = width of cross section
 C = modulus of shear (11,500,000 psi)
 θ = angle of twist in radians per inch

The problem was resolved into finding the three volumes V_1 , V_2 and V_3 , and because of the symmetry of these volumes the total volume could be found. Region 1 was readily solved since b is directly known as is the length of the straight section. In region 2 the values of b_1 , b_2 , and b_3 were found by using trigonometry and the fact that parabolic areas were found. The volume of this region was then found by using Simpson's rule utilizing five equally spaced stations. The volume in region 3 was found in the same manner. However, in region 3 stations b_1 , b_2 , and b_3

do not extend to the edge of the section and the areas at these stations are made up of a rectangle beneath a parabola. The parabolic area is found by using Equation (2). The length of the base of the rectangle is known since it is the same as the base length of the parabola. The height of the rectangle is obtained from the height of the parabola at the appropriate points on station b_4 . Therefore, this height would be the mid-point height for station b_3 and the quarter point height for stations b_4 and b_2 . The quarter point heights of b_4 will be three-quarters the mid-height because of parabolic shape of the section.

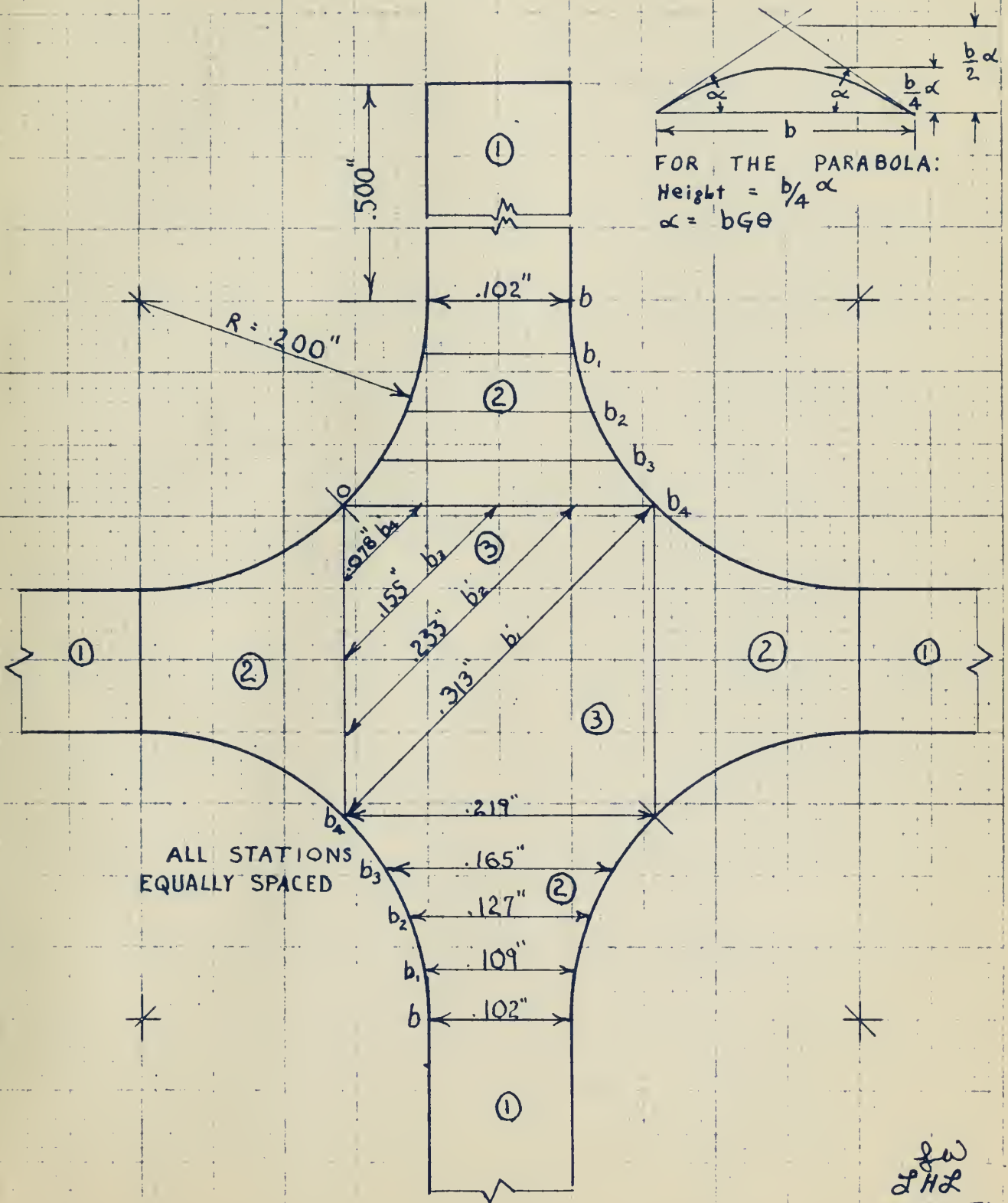
Since the torque of the bar is equal to twice the volume beneath the membrane, the torque in terms of Θ follow directly. The calculated results give $\Theta = .00379$ radians/inch. From Figure VIII it is seen that for the straight beam the experimental results are 11° twist in a length of 45.1". The value of the experimental twist was than .00426 radians/inch, or 11% greater than the calculated value. This difference in results can be accounted for by the membrane not having the exact parabolic shape that was assumed and by a possible error of 3% in measuring the angle of elastic twist. With these probable errors in mind the experimental and calculated angles of elastic twist are considered to be in good agreement.

do not extend to the edge of the section and the area at these stations
are made up of a rectangle between a parabola. The parabola area is
found by using Equation (7). The length of the base of the rectangle is
known since it is the same as the base length of the parabola. The
height of the rectangle is obtained from the height of the parabola at
the appropriate points on station x_i . Therefore, this height would be
the mid-point height for station x_i and the quarter point height for
stations $x_{i+1/2}$ and $x_{i+3/4}$. The quarter point heights of x_i will be twice
quarter the mid-height because of parabolic shape of the section.

Since the torque of the car is equal to twice the volume density
the membrane, the torque intensity of θ follows directly. The calculated
results give $\theta = 0.0017$ radians/inch. From Figure VIII it is seen that
for the straight beam the experimental results are 1.6° twist in a length
of 48.1". The value of the experimental twist was 0.00450 radians/
inch or 1.6° greater than the calculated value. This difference in
results can be recognized for by the membrane not having the exact
parabolic shape that was assumed and by a possible error of 3% in
measuring the angle of elastic twist. With these possible errors in
mind the experimental and calculated angles of elastic twist are con-
sidered to be in good agreement.

FIGURE XII

PORTION OF BEAM CROSS SECTION SHOWING STATIONS
USED IN CALCULATING MEMBRANE VOLUME USED
WITH MEMBRANE ANALOGY.



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APPENDIX B

DATA

A copy of all original data appears in Figures VIII, XIII and XIV.

ALPHABETIC

DATA

A copy of all original data appears in Figures VII, VIII and XIX.

FIGURE XIII

DEFLECTION READINGS IN INCHES AT VARIOUS
STATIONS, VALUES OF β_0 , & VALUES OF LOAD.

ML No load on beam.

Ld.1 Load of 23.47# at each end of beam

Ld.2 Load of 45.51# at each end of beam

S indicates deflection station as shown

on figure III.

Add 1" to all readings for actual deflection
above bed plate.

	THEORY		RUN 1		RUN 2		RUN 3		RUN 4		RUN 5		RUN 6		RUN 7		RUN 8	
	$\beta_0 = 0$		$\beta_0 = 0$		$\beta_0 = .0042$		$\beta_0 = .0144$		$\beta_0 = .0257$		$\beta_0 = .0447$		$\beta_0 = .0639$		$\beta_0 = .0807$		$\beta_0 = .1022$	
³ 2+ ₄	S2	S3	S2	S3	S2	S3	S2	S3	S2	S3	S2	S3	S2	S3	S2	S3	S2	S3
NL	.750	.750	.750	.750	.751	.751	.751	.752	.753	.753	.754	.753	.755	.753	.756	.752	.756	.753
Ld.1	.781	.760	.780	.759	.781	.760	.782	.761	.784	.763	.785	.763	.787	.763	.787	.761	.788	.763
Ld.2	.809	.769	.809	.769	.811	.770	.810	.770	.812	.771	.813	.771	.816	.771	.817	.770	.818	.772
NL	.750	.750	.750	.750	.751	.751	.752	.752	.753	.753	.754	.753	.755	.753	.756	.752	.756	.753
³ 2X ₁	BEAM ROTATED 45°																	
NL	.750	.750	.750	.750	.751	.751	.753	.752	.753	.753	.754	.754	.754	.753	.753	.750	.754	.751
Ld.1	.781	.760	.781	.760	.781	.760	.784	.761	.784	.763	.785	.763	.785	.763	.784	.760	.786	.761
Ld.2	.809	.769	.809	.769	.811	.771	.812	.771	.812	.770	.814	.772	.814	.772	.813	.769	.816	.770
NL	.750	.750	.750	.750	.751	.751	.753	.752	.753	.753	.754	.754	.754	.753	.753	.750	.754	.751

See PROCEDURE for method used in beam rotation.

	RUN 9		RUN 10		RUN 11		RUN 12&13		RUN 14		RUN 15		RUN 16 90°		RUN 18 180°		RUN 20 270°	
	$\beta_0 = .1200$		$\beta_0 = .1488$		$\beta_0 = .1814$		$\beta_0 = .227$		$\beta_0 = .278$		$\beta_0 = .314$		$\beta_0 = .314$		$\beta_0 = .314$		$\beta_0 = .314$	
³ 2+ ₄	S2	S3	S2	S3	S2	S3	S2	S3	S2	S3	S2	S3	S2	S3	S2	S3	S2	S3
NL	.760	.752	.761	.752	.765	.751	.763	.750	.760	.751	.755	.751	.750	.750	.745	.749	.750	.750
Ld.1	.794	.763	.798	.764	.805	.764	.803	.762	.800	.764	.792	.762	.787	.762	.782	.760	.787	.761
Ld.2	.826	.773	.833	.774	.841	.775	.840	.774	.836	.776	.827	.775	--	--	--	--	--	--
NL	.760	.752	.761	.752	.765	.751	.763	.750	.760	.751	.755	.751	.750	.750	.745	.749	.750	.750
	BEAM ROTATED						45°		45°		45°		R 17 135°		R 19 225°		R 21 315°	
NL	--	--	--	--	--	--	.755	.751	.754	.751	.753	.751	.747	.749	.747	.749	.753	.751
Ld.1	--	--	--	--	--	--	.796	.764	.793	.763	.790	.763	.784	.761	.784	.761	.790	.763
Ld.2	--	--	--	--	--	--	.833	.776	.829	.775	.824	.775	--	--	--	--	--	--
NL	--	--	--	--	--	--	.755	.751	.754	.751	.753	.751	.747	.749	.747	.749	.753	.751

J.H.Z.
4-16-51

FIGURE XIV

STRAIN GAGE READING IN MICRO INCHES PER INCH
FOR VARIOUS VALUES OF β_0 AND FOR VARIOUS LOADS.

N.L. = No load on the beam

Ld.1 = Load of 23.47# at each end of beam.

Ld.2 = Load of 45.51# at each end of beam

STRAIN GAGE	RUN 1 $\beta_0 = 0$		RUN 2 $\beta_0 = .0042$		RUN 3 $\beta_0 = .0144$		RUN 4 $\beta_0 = .0257$		RUN 5 $\beta_0 = .0447$		RUN 6 $\beta_0 = .0639$			
	N.L.	Ld.1	Ld.2	Ld.1	Ld.2	N.L.	Ld.1	Ld.2	N.L.	Ld.1	Ld.2	N.L.	Ld.1	Ld.2
3A 34	5719	5820	5920	5850	5950	5790	5890	6000	5950	6040	6150	6240	6330	6420
3A 25	6494	6595	6695	6545	6650	6440	6540	6660	6470	6570	6680	6680	6790	6905
3A 20½	5360	5470	5580	5480	5590	5400	5510	5630	5480	5590	5710	5700	5810	5920
3A 16	6800	6895	7000	6900	7000	6770	6870	6980	6910	7000	7110	7160	7270	7330
4B 34	6213	6220	6230	6250	6260	6250	6240	6250	6390	6370	6350	6740	6690	6650
4B 25	5917	5925	5930	5950	5960	5890	5840	5910	5990	6000	6020	6380	6400	6420
4B 16	5910	5910	5915	5930	5950	5990	6010	6050	6090	6120	6180	6430	6500	6570
4B 20½	5655	5665	5675	5720	5755				GAGE NOT GOOD					
2A 25	6810	6790	6785	6810	6790	6760	6740	6740	6840	6820	6800	7040	7020	7010
1B 25	6855	6735	6630	6820	6630	6900	6780	6680	6980	6860	6750	7210	7090	6980

STRAIN GAGE	RUN 7 $\beta_0 = .0807$		RUN 8 $\beta_0 = .1022$		RUN 9 $\beta_0 = .1200$		RUN 10 $\beta_0 = .1488$		RUN 11 $\beta_0 = .1814$		RUN 12 $\beta_0 = .227$			
	N.L.	Ld.1	Ld.2	Ld.1	Ld.2	N.L.	Ld.1	Ld.2	N.L.	Ld.1	Ld.2	N.L.	Ld.1	Ld.2
3A 34	7080	7130	7160	7800	7820	8380	8370	8370	0230	0210	0150			
3A 25	7390	7490	7600	7800	7920	8080	8210	8340	9930	0050	0150	1750	1850	1950
3A 20½	6670	6770	6880	7320	7400	7940	8030	8120	9410	9490	9560			
3A 16	8130	8170	8240	8600	8460	8130	8160	8170	9130	9100	9080	0300	0420	0590
4B 34	7530	7420	7320	8280	8150	9860	9700	9510	1160	1060	0930			
4B 25	7180	7180	7180	7900	7910	8390	8410	8420	0410	0430	0440			
4B 16	7220	7310	7400	8000	8100	8360	8490	8620	9940	0050	0160			
2A 25	8060	8040	8030	8860	8820	9330	9320	9300	0810	0790	0770			
1B 25	8040	7910	7780	8910	8760	9440	9320	9180	0820	0700	0560			

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APPENDIX C

SAMPLE CALCULATIONS

The following calculations were made for Run 10:

$$\alpha = 567^\circ \quad L' = 50'' \quad r_0 = 0.751'' \quad \phi = 8.75^\circ$$

$$\beta_0 = \frac{\alpha \times r_0}{57.3 \times L} = \frac{567 \times .751}{57.3 \times 50} = 0.1488 \text{ rad.}$$

Torsion Calculations:

$$\theta = \frac{\phi}{57.3 \times L'} = \frac{8.75}{57.3 \times 50} = 0.00305 \text{ rad/in.}$$

$$J/J_s = \frac{.00426}{\theta} = \frac{.00426}{.00305} = 1.395$$

Bending Calculations:

	S2	S3	Δ_2	Δ_3	δ_3	δ_s
NL	1.761	1.752	---	---	---	---
Ld.1	1.798	1.764	.037	.012	.025	.037
Ld.2	1.833	1.774	.072	.022	.050	.072
NL	1.761	1.752	---	---	---	---

Δ at Ld. 1 = reading at Ld. 1 - reading at NL

$$\delta_3 = \Delta_2 - \Delta_3$$

$$\delta_s = \Delta_2$$

	$\frac{\delta_3}{\delta_{30}}$	$\frac{\delta_s}{\delta_{s0}}$
Ld. 1	$\frac{.025}{.021} = 1.19$	$\frac{.037}{.031} = 1.19$
Ld. 2	$\frac{.050}{.040} = 1.25$	$\frac{.072}{.059} = 1.22$

$$\delta/\delta_0 = \frac{1.19 + 1.19 + 1.25 + 1.22}{4} = 1.21$$

EXAMPLE 2
SIMILAR CALCULATIONS

The following calculations were made for run 18:

$$\alpha = 267^\circ \quad \beta = 20^\circ \quad \gamma = 0.721 \quad \phi = 8.72^\circ$$

$$P_0 = \frac{\alpha \times \gamma}{27.3 \times 20} = \frac{122.7 \times 0.721}{27.3 \times 20} = 0.1488 \text{ rad.}$$

Correction Calculations:

$$\delta = \frac{\phi}{27.3 \times 20} = \frac{8.72}{27.3 \times 20} = 0.00302 \text{ rad/in.}$$

$$1/\rho^2 = \frac{0.0054}{0.00302} = 1.322$$

Bearing Calculations:

WT	WT	WT	Δ_1	Δ_2	Δ_3	δ_1	δ_2
1.781	1.781	1.781	---	---	---	---	---
1.788	1.788	1.788	---	---	---	---	---
1.783	1.783	1.783	---	---	---	---	---
1.781	1.781	1.781	---	---	---	---	---

$\Delta = \text{WT} - \text{reading at WT} = \text{reading at WT}$

$$\delta_2 = \Delta_2 - \Delta_3$$

$$\delta_1 = \Delta_1$$

$$\frac{\delta_1}{\delta_2} = \frac{0.721}{0.031} = 1.19$$

$$\frac{\delta_1}{\delta_2} = \frac{0.721}{0.051} = 1.19$$

$$1.19$$

$$0.051 / 0.020 = 1.52$$

$$0.020 / 0.040 = 1.52$$

$$1.52$$

$$\frac{\delta}{\rho} = \frac{1.19 + 1.19 + 1.19 + 1.52 + 1.52}{4} = 1.51$$

APPENDIX D

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APPENDIX B

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$$T = \frac{GJ}{L} \theta$$

$$J = \frac{\pi r^4}{2}$$

r	J	T	θ	$\frac{T}{J}$	$\frac{T}{\theta}$
0.5	0.098	1.0	0.01	102	102
1.0	0.393	4.0	0.04	100	100
1.5	0.883	9.0	0.09	100	100

$$T = \frac{GJ}{L} \theta$$

$$J = \frac{\pi r^4}{2}$$

$\frac{T}{J} = \frac{G}{L} \theta$

$$T = \frac{GJ}{L} \theta$$

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The experimental deter-
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and torsional stiffness of
a beam with rotationally
constant moment of inertia
with varying amounts of
permanent twist.
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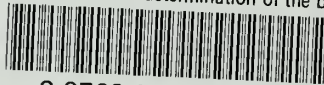
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W85 Woolston

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stiffness of a beam with rotation-
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