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## Elastic behavior of metal interfaces

Coyle, Arthur J.

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**ELASTIC BEHAVIOR  
OF METAL INTERFACES**

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Arthur J. Coyle  
and  
Herman A. Stromberg











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ELASTIC BEHAVIOR OF METAL INTERFACES

by

Arthur J. Coyle, Lieutenant, U. S. Navy  
Herman A. Stromberg, Jr., Lieutenant (j.g.), U. S. Navy  
U. S., U. S. Naval Academy 1949

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ELASTIC BEHAVIOR OF METAL  
INTERFACES

Arthur J. Coyle  
and  
Herman A. Stromberg





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BEHAVIOR OF METAL INTERFACES

by

W. Coyle, Lieutenant, U. S. Navy  
S. Berg, Jr., Lieutenant (j.g.), U. S. Navy  
U. S. Naval Academy, 1949

Submitted in Partial Fulfillment  
of the Requirements for the  
Degree of Naval Engineer  
from the  
Massachusetts Institute of Technology  
1954



ELASTIC BEHAVIOR OF METAL INTERFACES

by

Arthur J. Coyle, Lieutenant, U. S. Navy  
Herman A. Stromberg, Jr., Lieutenant (j.g.), U. S. Navy

Submitted to the Department of  
Naval Architecture and Marine Engineering

May 24, 1954

in partial fulfillment of the requirements for the  
degree of Naval Engineer

ABSTRACT

The object of this thesis is to investigate the reported existence of abnormal elastic effects in metal interfaces.

In general two separate methods of investigation were conceived and used. The first employed cylindrical specimens with truncated conical ends placed between flat surfaces. The second, devised to eliminate any possible elasticity contributed by the flat surfaces beyond the area of contact, employed two identical annular specimens placed end on end.

There was good correlation between the two methods since both produced excessive elastic angles of twist. Further, the following unusual results were observed. As the normal stress was increased in the interface, the amount of excess twist for a given torque increased with the degree of surface finish until a transition range was reached after which excess twist increased with the roughness of the surface.

It is concluded that these effects are caused by the asperities acting as cantilevers. This offers an explanation for the effect of normal stress and surface finish on the elastic angles of twist that is in agreement with the experimental data.

In view of the above, it is recommended that further studies be made using metals of different physical characteristics.

Thesis Supervisor: Brandon G. Bightaire  
Title: Associate Professor of  
Mechanical Engineering

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UNITED STATES DEPARTMENT OF JUSTICE

Washington, D. C. 20535

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Massachusetts Institute of Technology  
Cambridge 39, Massachusetts  
May 24, 1954

Professor L. F. Hamilton  
Secretary of the Faculty  
Massachusetts Institute of Technology  
Cambridge 39, Massachusetts

Dear Sir:

In accordance with the regulations of the faculty, we submit herewith a thesis entitled Elastic Behavior of Metal Interfaces in partial fulfillment of the requirements for the degree of Naval Engineer.

Respectfully,

---

Arthur J. Coyle  
Lieutenant, U. S. Navy

---

Herman A. Stromberg, Jr.  
Lieutenant (j.g.), U. S. Navy

Department of Chemistry  
University of Toronto  
128 St. George Street  
Toronto, Ontario

Professor J. V. Quirk  
Department of Chemistry  
University of Toronto  
128 St. George Street  
Toronto, Ontario

In accordance with the regulations of the Faculty,  
I have the honor to acknowledge the receipt of your  
application for admission to the M.Sc. program in  
Chemistry for the fall semester of 1964.

Yours sincerely,  
J. V. Quirk

Accepted for admission to the M.Sc. program in  
Chemistry for the fall semester of 1964.

### ACKNOWLEDGMENTS

The authors wish to express their gratitude to the personnel of the Lubrication Laboratory for their interest and help. Particularly to Professor B. G. Rigtsire for his encouragement, advice, and boundless patience; and to Professor I. M. Feng, Doctor S. Habinowicz, Mr. James Swartwout, and Mr. Jacques Bonneville, for their kind assistance.



MEMORANDUM

The report was prepared under the direction of the Director of the Bureau of Investigation and is a summary of the information received from the various sources mentioned in the report. It is intended to be a general statement of the facts and circumstances surrounding the case and is not intended to be a final report. The information contained herein is for the use of the Bureau and is not to be disseminated outside the Bureau without the approval of the Director.

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## INTRODUCTION

Although such information is available concerning the elastic behavior of bulk metals, data on the elastic behavior of metal interfaces is outstanding by its absence.

In a paper <sup>(1)</sup>, Tomlinson, Thorpe, and Gough reported observing deflection between two metal rings when subjected to a known torque far in excess of that predicted by theory and the excess was greater than could be accounted for by experimental error. Since this phenomenon was not directly connected with the objective of their work it was not investigated further. An element of uncertainty was associated with this report since the deflection was measured by means of the same arm with which the torque was applied. It was possible that the excessive deflection reported could have been caused by bending of the torque arm. Nonetheless, the report of this phenomenon was sufficiently enticing to invite further investigation.

Bowden and Tabor <sup>(2)</sup> show that the actual area of contact between two surfaces is different from the apparent. This difference is caused by the asperities to be found on all surfaces.

Herring and Galt <sup>(3)</sup> have shown that very small metal specimens (i.e., metal whiskers) can tolerate such larger strains without slip than the bulk metal.

Inasmuch as the volume of an asperity is small, it is considered possible that, like the metal whiskers, its elastic limit is high. A study of the literature did not reveal any investigation of this



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possibility other than the report by Youlinson, Thorpe, and Gough mentioned in the second paragraph. To us this seemed to present a challenge that was interesting, provocative, and stimulating.

The first part of the report is devoted to a general  
 description of the work done during the year. It  
 is followed by a detailed account of the various  
 experiments conducted and the results obtained.

The first experiment was designed to determine  
 the effect of temperature on the rate of  
 reaction. It was found that the rate of  
 reaction increased with increasing temperature.  
 This is in accordance with the theory that  
 the rate of reaction is proportional to the  
 number of molecules which possess sufficient  
 energy to overcome the energy barrier.  
 The second experiment was designed to determine  
 the effect of concentration on the rate of  
 reaction. It was found that the rate of  
 reaction increased with increasing concentration.  
 This is in accordance with the theory that  
 the rate of reaction is proportional to the  
 number of molecules which are in contact with  
 each other.

The third experiment was designed to determine  
 the effect of a catalyst on the rate of  
 reaction. It was found that the rate of  
 reaction increased with the addition of a  
 catalyst. This is in accordance with the  
 theory that a catalyst provides an alternative  
 reaction path with a lower energy barrier.  
 The fourth experiment was designed to determine  
 the effect of a solvent on the rate of  
 reaction. It was found that the rate of  
 reaction increased with the addition of a  
 solvent. This is in accordance with the  
 theory that a solvent provides a medium in  
 which the reactants can move more freely.

The results of these experiments are summarized  
 in the following table. It is seen that the  
 rate of reaction is affected by temperature,  
 concentration, a catalyst, and a solvent.

## PROCEDURE

Since two experimental arrangements were used in this work, it would be best to present these schemes separately.

The first scheme employed is illustrated by Figure XIX in which the solid steel specimens tested were machined from SAE 1030 cold rolled steel stock. The ends of most of the specimens were beveled to form truncated cones which reduced the contact surfaces and thereby increased the normal stress in the interfaces. The contact surfaces were lapped to the desired finish with emery polishing paper of varying degrees of roughness. The specimens were finally held in a vee-type block during the lapping process to make the test surfaces perpendicular to the specimen axis. The quality of the surface finish was checked microscopically using a magnification of 165x.

When the desired finish was obtained, the average diameter,  $d$ , and bevel height,  $l$ , (Figure XX) were measured by a microscope equipped with an optic vernier permitting accuracy to within .00007 of an inch. The ends of the specimens were made identical with respect to dimensions and surface finish.

The next step was to locate the torque lever exactly at the midpoint of the specimen to insure that one half the applied torque would act on each interface. This was accomplished by measuring the overall length and using a depth gage in positioning the lever which permitted accuracy to the nearest .001 inch.

The indicator arm was then fastened on the specimen with four





set screws so that the plane of the set screws and hence, the indicator, was perpendicular to the specimen axis. The set screws used were pointed to obtain a knife edge line from which the twist was transmitted. The screws were also barely tightened by hand to reduce the indentations to mere pin points.

With the positioning collar resting on the torque lever, the contact surfaces of the specimen and hardened plates were cleaned with solvent. The specimen was then placed on the lower plate in the Riehle testing machine. The upper plate, which was bolted to the movable head of the testing machine, was then lowered to a point slightly above the specimen. The vertical was then checked by inserting a feeler gage between the positioning collar and the upper plate. An axial load of one hundred pounds was then applied to anchor the specimen during the final adjustments to the apparatus.

Finally the frictionless pulleys were located so that the silk threads, which transmit force to the torque lever, were perpendicular to the lever in the vertical and horizontal planes.

For each run on a specimen a constant normal load was applied by the testing machine. Torque was next applied to the specimen by adding equal weights to the pans and the resulting twist read. After each reading of twist, the torque was removed to see if the indicator returned to the zero position. Increasing torques were applied until the indicator failed to return to zero which showed that slip or permanent deformation occurred in the interface ending the run. The normal load was increased before each of the succeeding

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is very faded.



runs on the same specimen until sufficient data was obtained.

Twist was read by measuring the motion of the indicator arm under a microscope equipped with an optic vernier. The indicator was double-ended to avoid any possibility of an unbalancing moment. The first indicator arm used was seven inches long and the magnification of the microscope was such that one scale unit of the optic vernier represented .001 inch of indicator movement.

After completion of all runs on a specimen, the apparatus was dismantled and the dimension  $l'$  was measured with the microscope which permitted accuracy to within .00007 of an inch.

A number of specimens were tested using this scheme; however, an uncertainty existed as to how much twist, if any, could be attributed to the hardened steel plate. Consequently scheme two was devised to eliminate this unknown factor.

The second scheme employed is illustrated in Figure XXI, in which the tubular steel specimens tested were machined from SAE 1080 cold rolled stock. The specimens had an outer diameter of 1/4 inch, an inner diameter of 8/32 inch, and a length of one inch. Two specimens with the same surface finish were mounted end on, and a rod with a diameter slightly less than the inner diameter of the annuli was threaded through the apparatus as shown in Figure XXI. The upper end of the rod was threaded into a circular disc which rested on top of the upper specimen, the lower end extended for some distance below the specimens and

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provided a means for attaching weights to furnish the normal load.

Two 7-1/2" double-ended indicator arms were used, one on either side of the interface. As before they were attached by pointed set screws and the distance between them measured microscopically after each run. The ends of the arms were bent as shown to facilitate reading the microscope. Deflections were obtained by reading the angle between the arms when no torque was applied and then reading the angle after the torque had been applied. A simple subtraction gave the deflection. As before, a check was made after each application of a torque to see that the arms returned to their initial setting.

Torques were applied in the same manner as in the first method.

An additional advantage obtained by use of this method was that it permitted the use of a control specimen. This consisted of using a single two inch specimen while holding all other conditions the same as they were for the runs involving two one inch specimens. The effect of the interface could thus be determined by comparing the runs on the control specimen with the runs on the other specimens.

The observed deflection was compared with the computed deflection as predicted by elastic theory. In all cases the observed was greater than the computed. This excess observed deflection which we shall hereafter refer to as residual deflection was plotted against the computed maximum tangential stress in the interface.

It is a common mistake to assume that the only

way to solve a problem is to try to find a

direct solution. In fact, it is often better to

try to understand the problem first and then to

try to solve it. This is the basic principle of

problem solving. It is often better to

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RESULTS

The results of this investigation are shown in  
Figures I - XVIII-A, inclusive.



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TABLE

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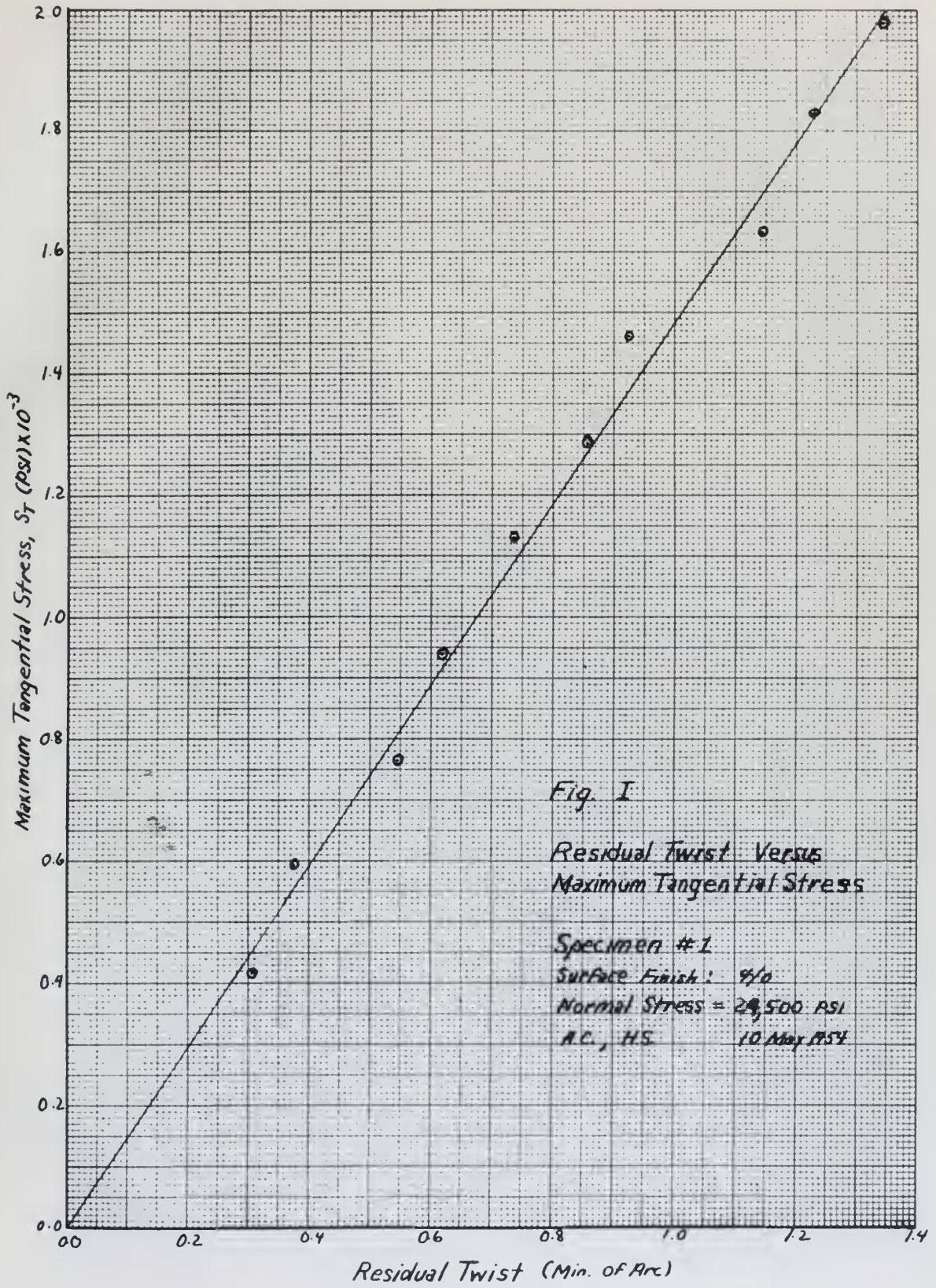
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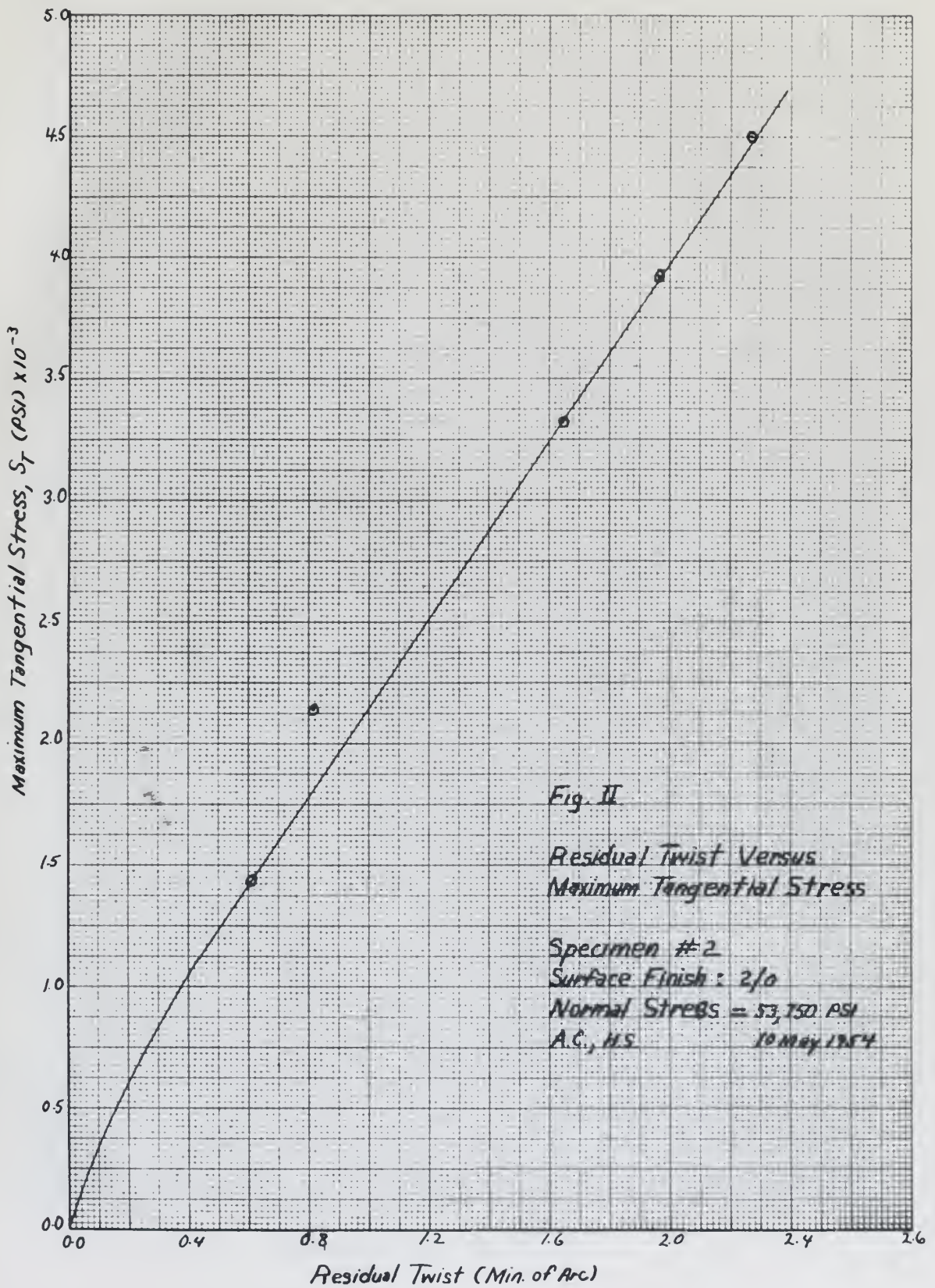
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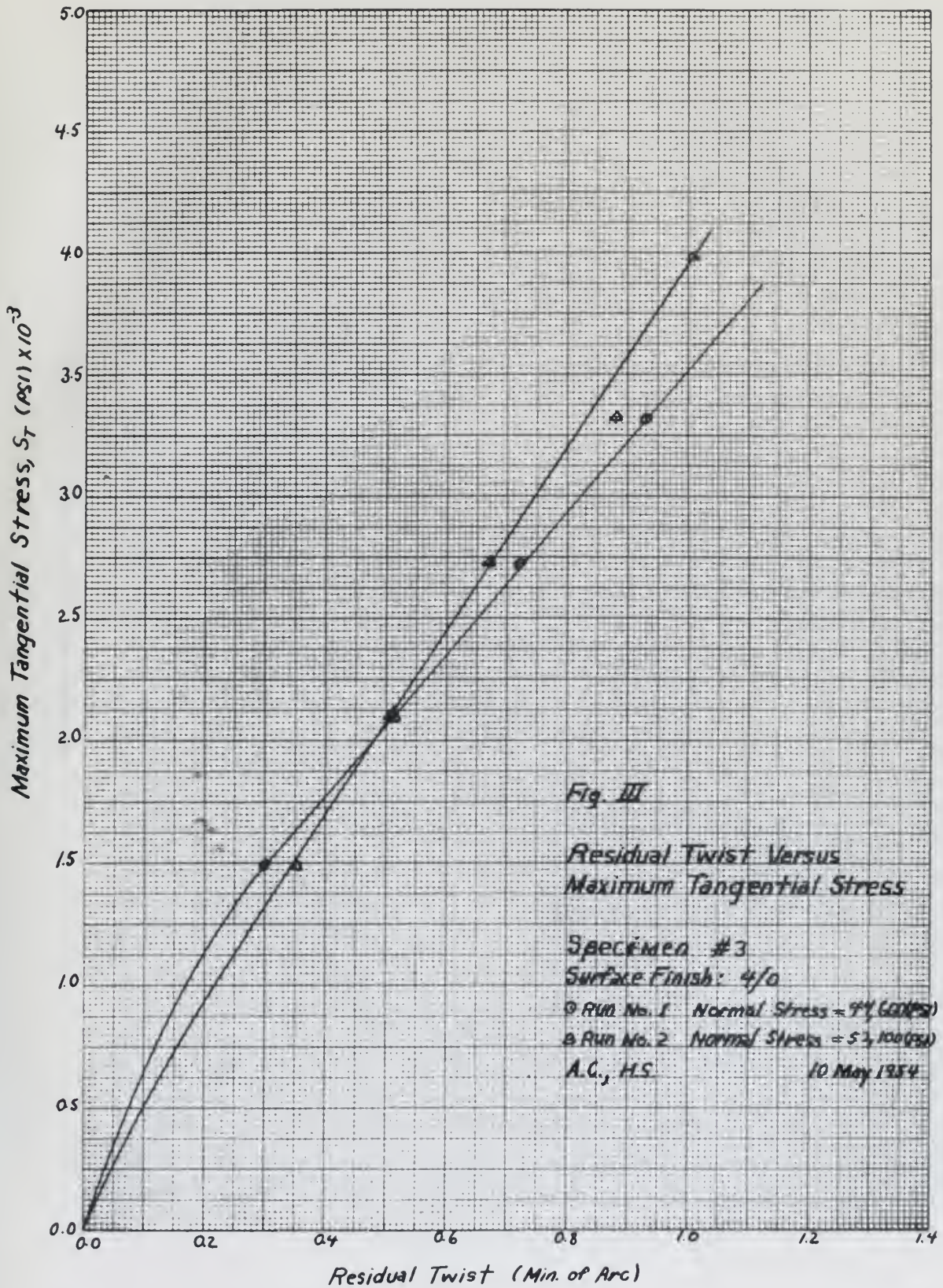


Fig. III

Residual Twist Versus  
Maximum Tangential Stress

Specimen #3

Surface Finish: 4/0

○ Run No. 1 Normal Stress = 97,600 (PSI)

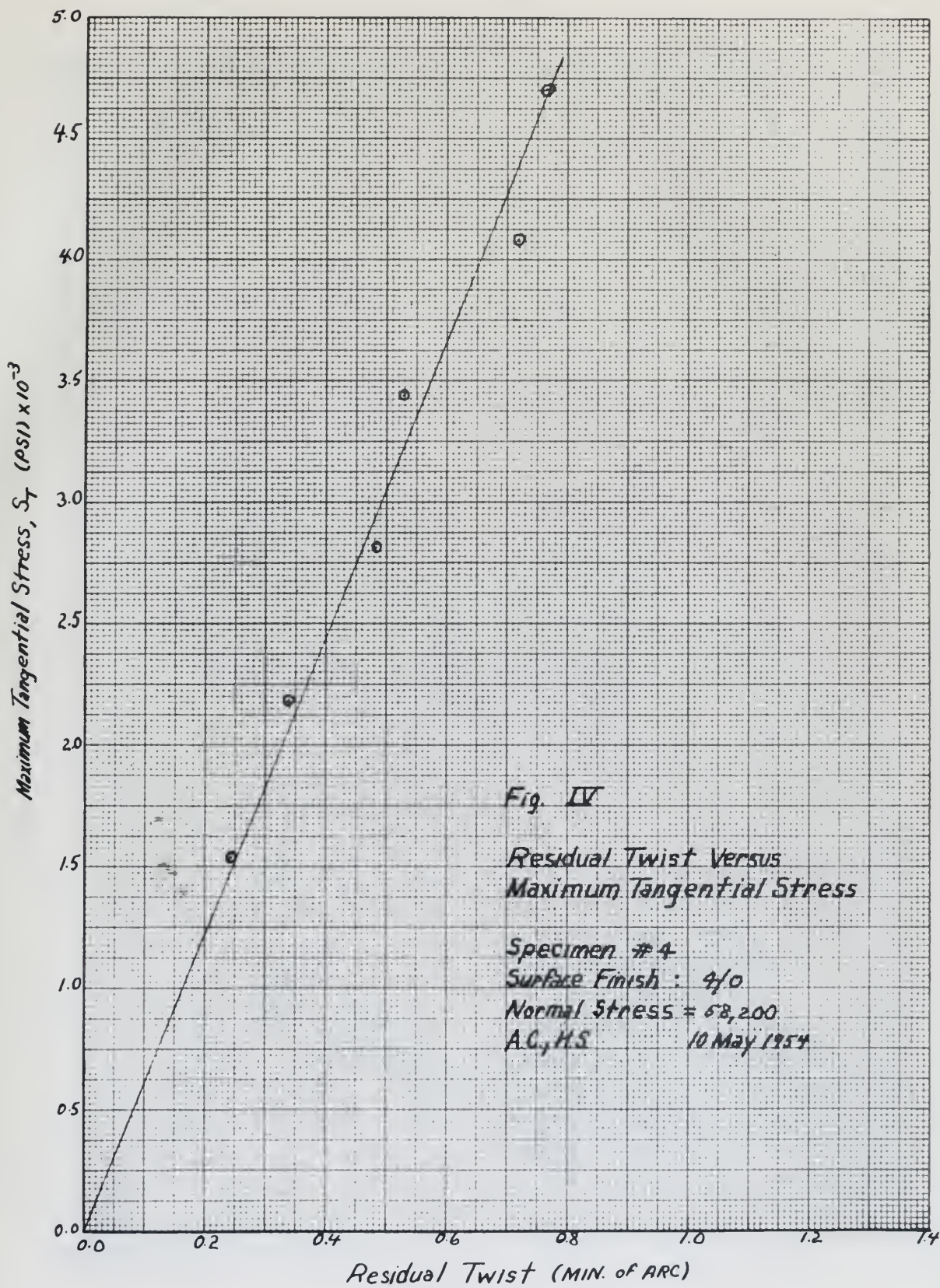
Δ Run No. 2 Normal Stress = 52,100 (PSI)

A.C., H.S.

10 May 1954

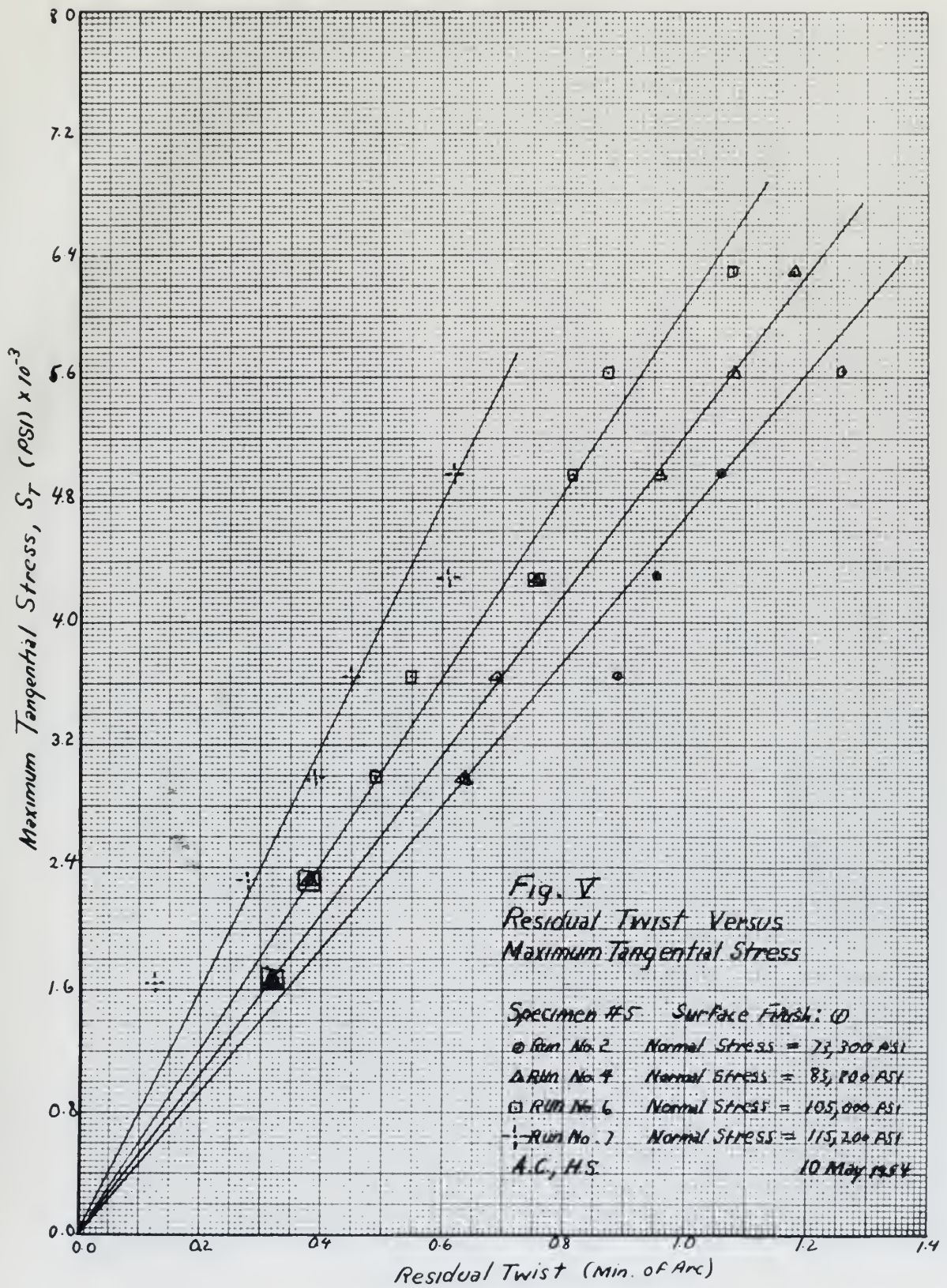






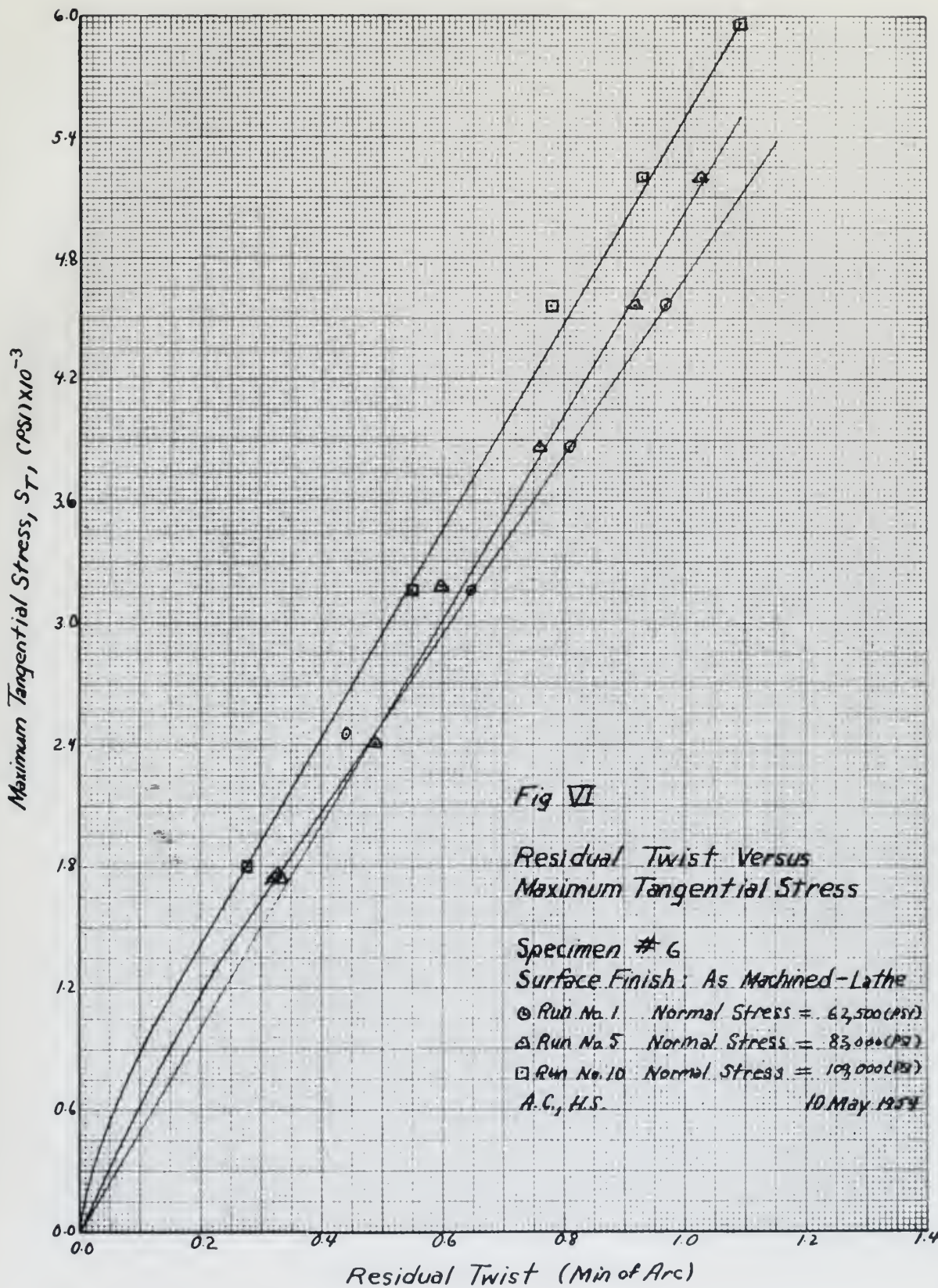






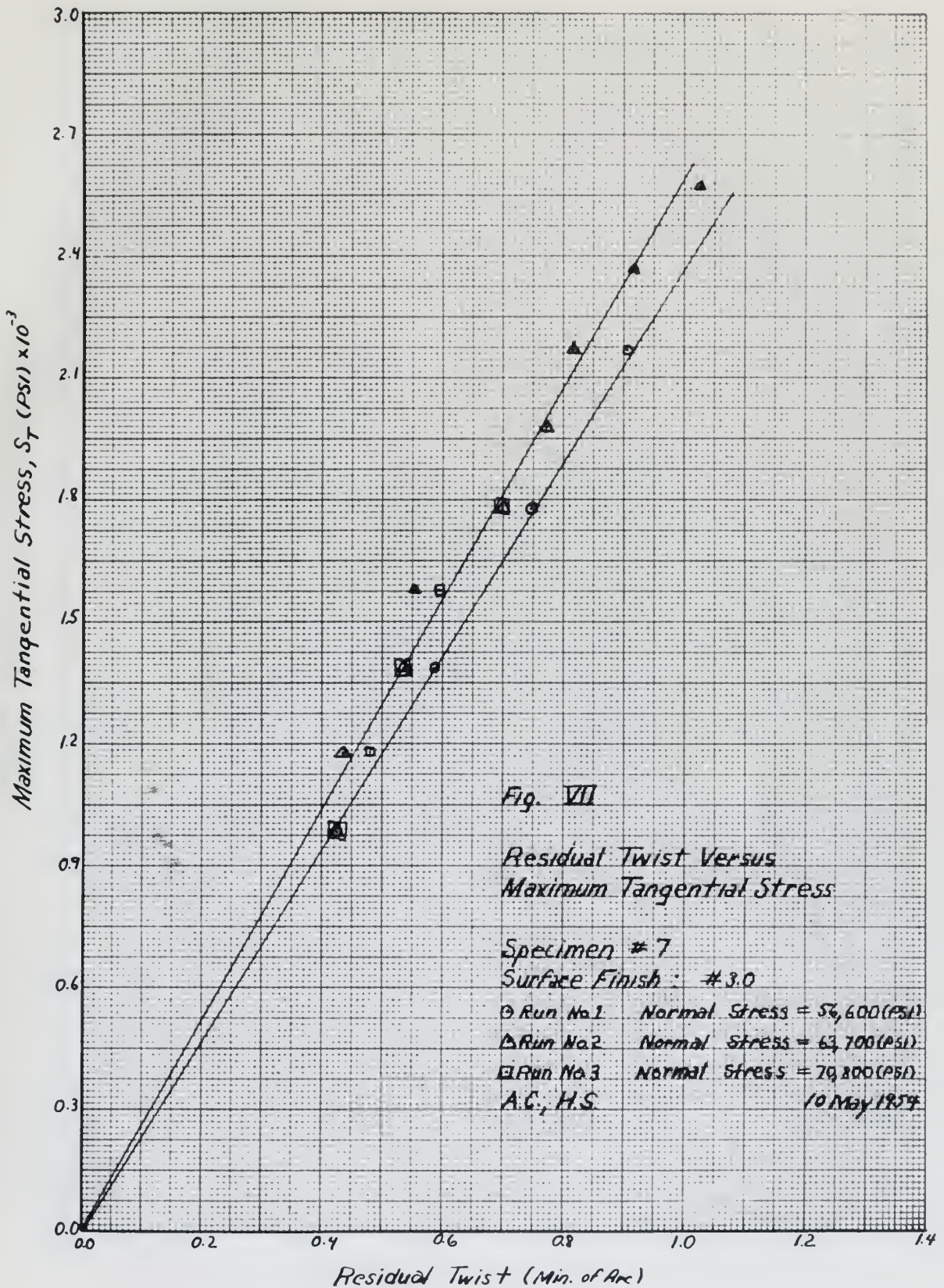




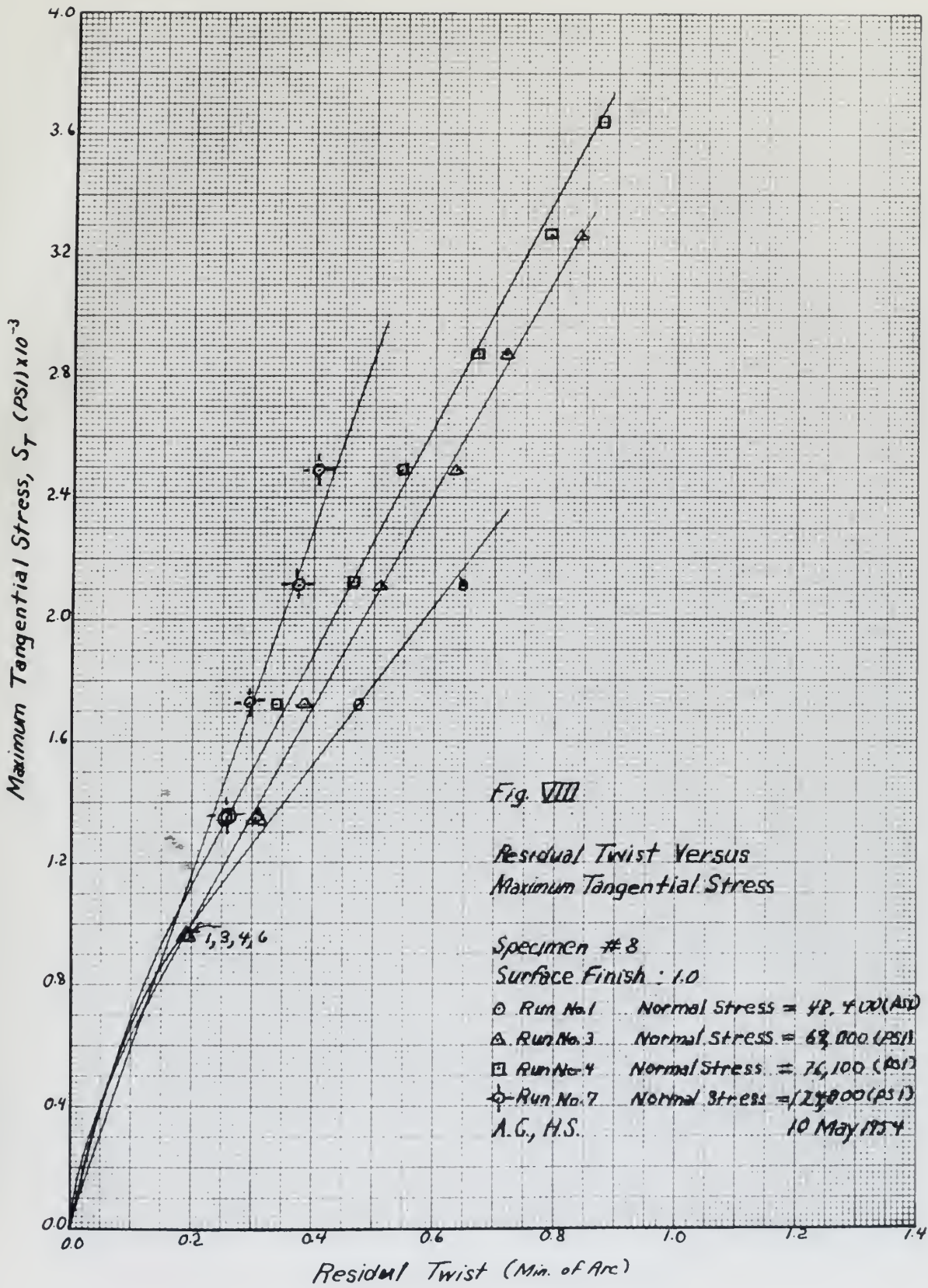
















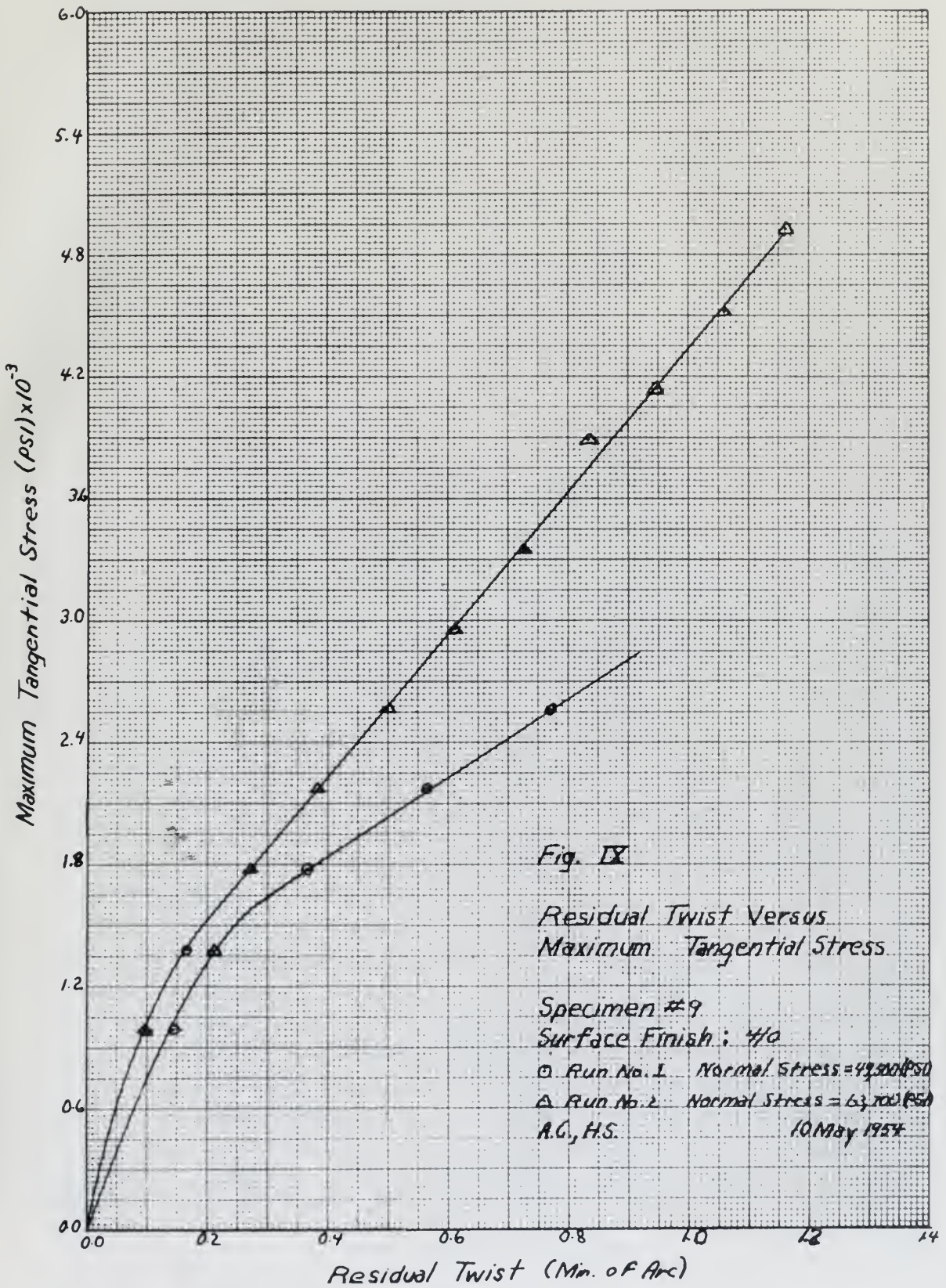


Fig. IX

Residual Twist Versus  
Maximum Tangential Stress

Specimen #9  
Surface Finish: 4/0

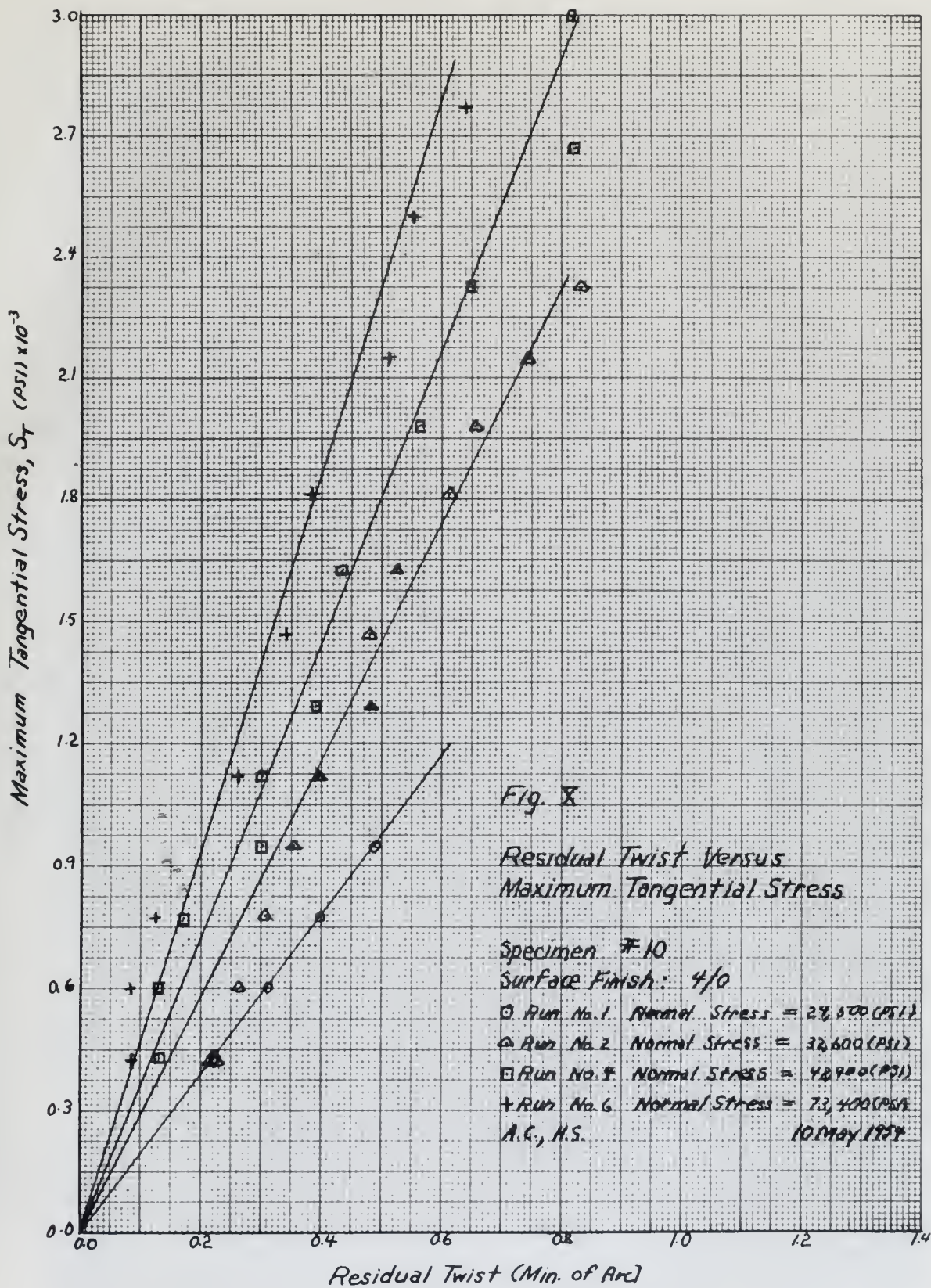
○ Run No. 1 Normal Stress = 49,500 PSI

△ Run No. 2 Normal Stress = 63,700 PSI

A.G., H.S. 10 May 1954

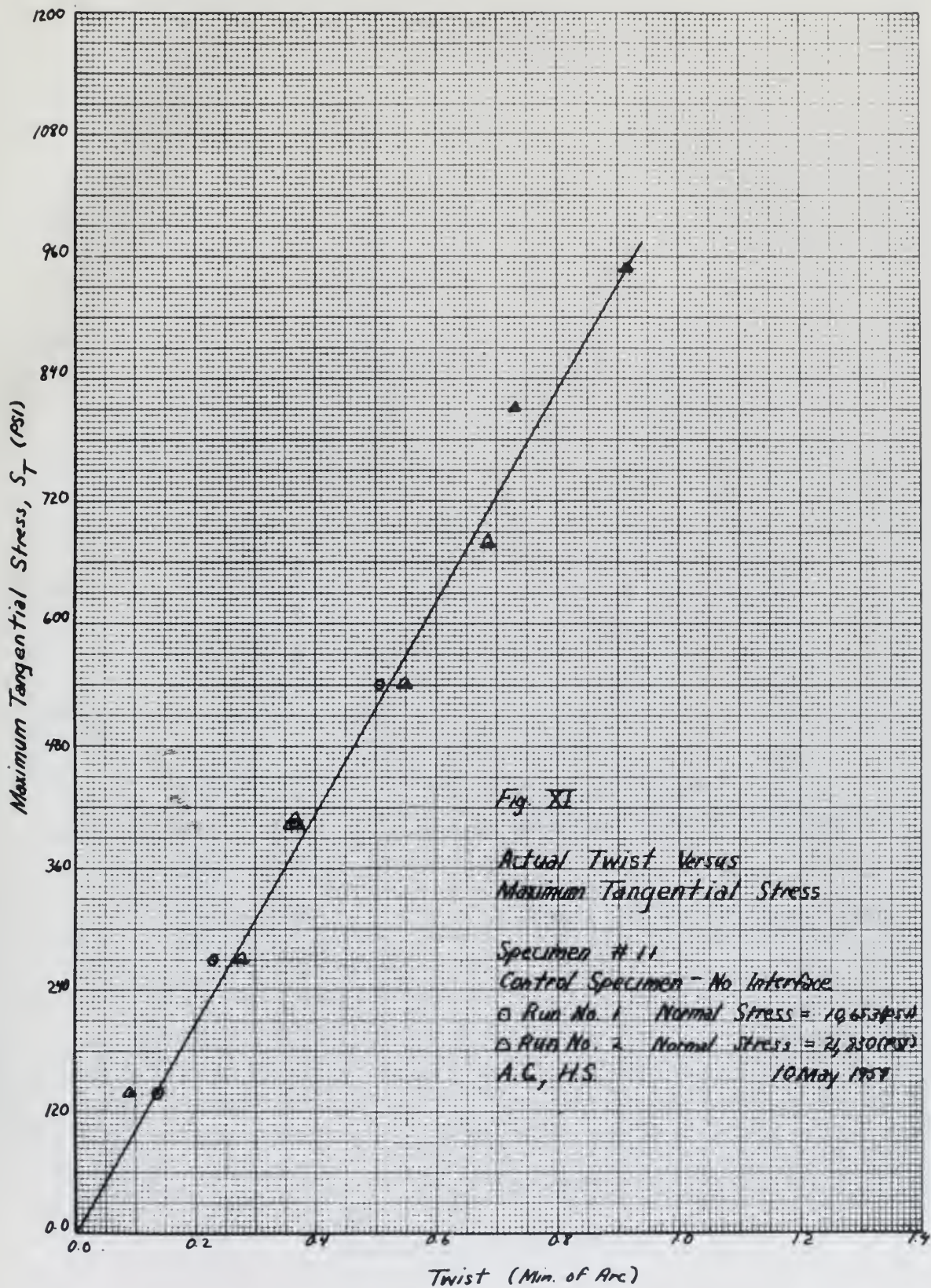
















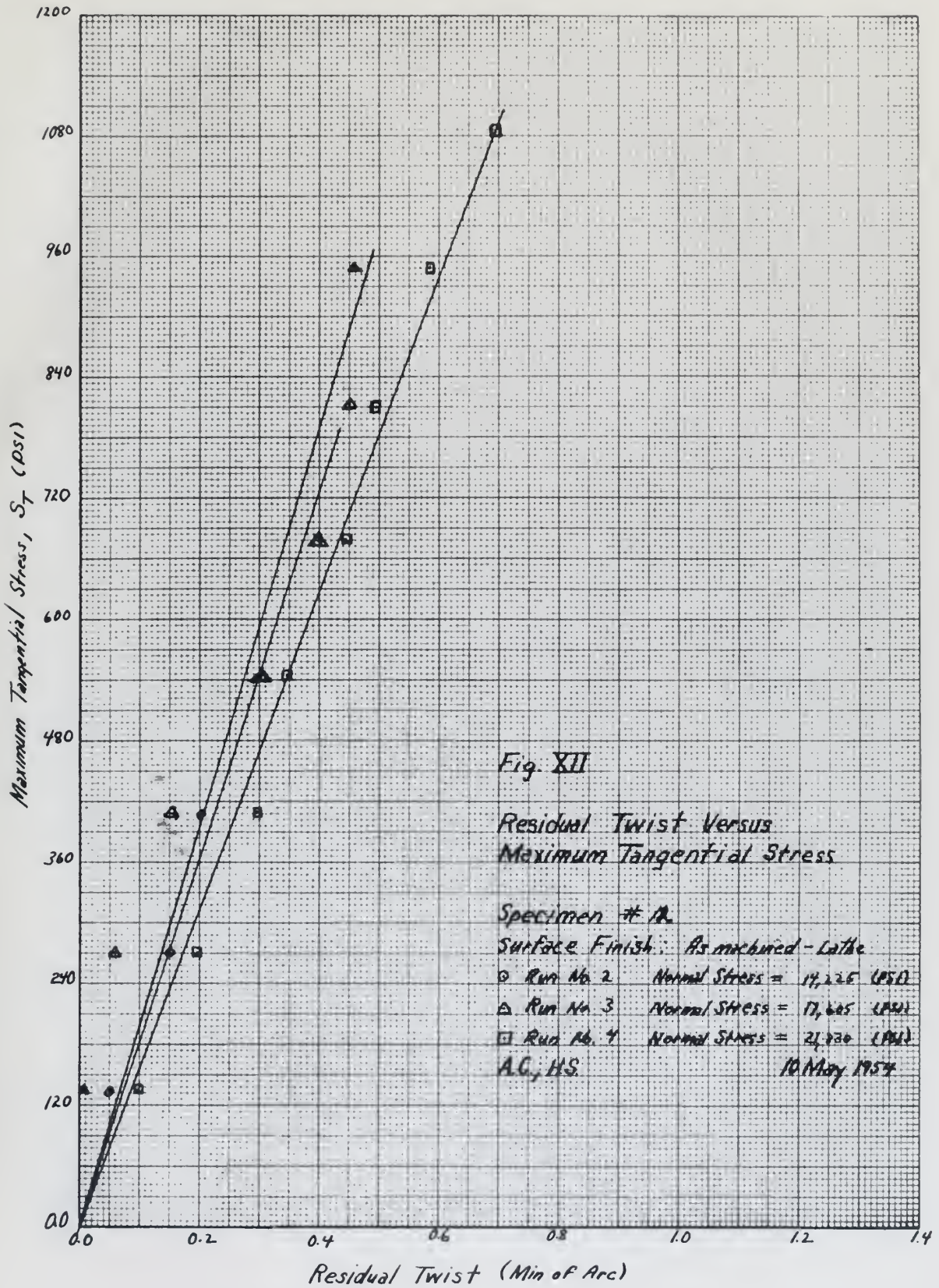


Fig. XII

Residual Twist Versus  
Maximum Tangential Stress

Specimen # 12

Surface Finish: As machined - Lathe

○ Run No. 2 Normal Stress = 14,225 (PSI)

△ Run No. 3 Normal Stress = 17,605 (PSI)

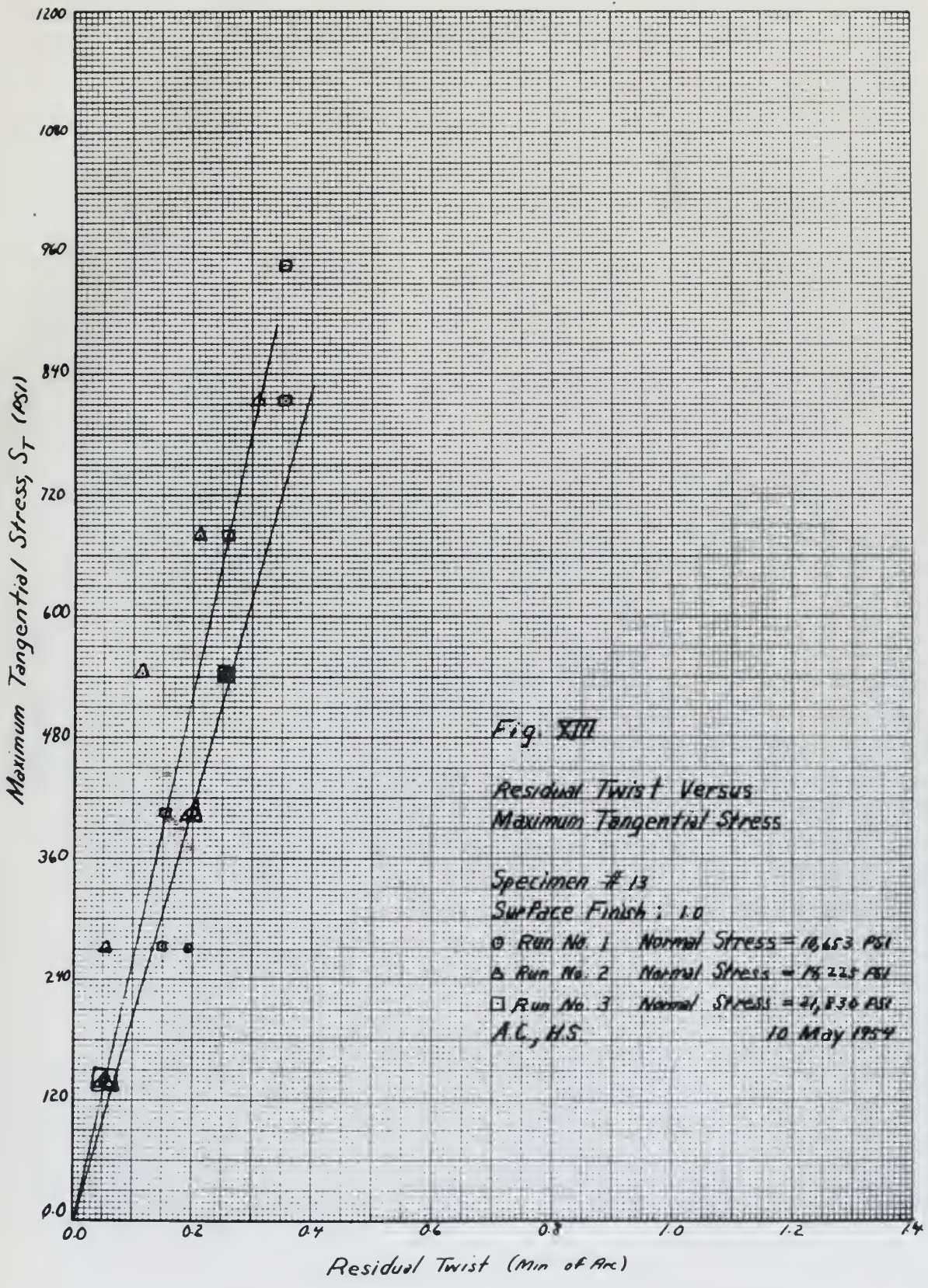
□ Run No. 4 Normal Stress = 21,930 (PSI)

A.C., H.S.

10 May 1954

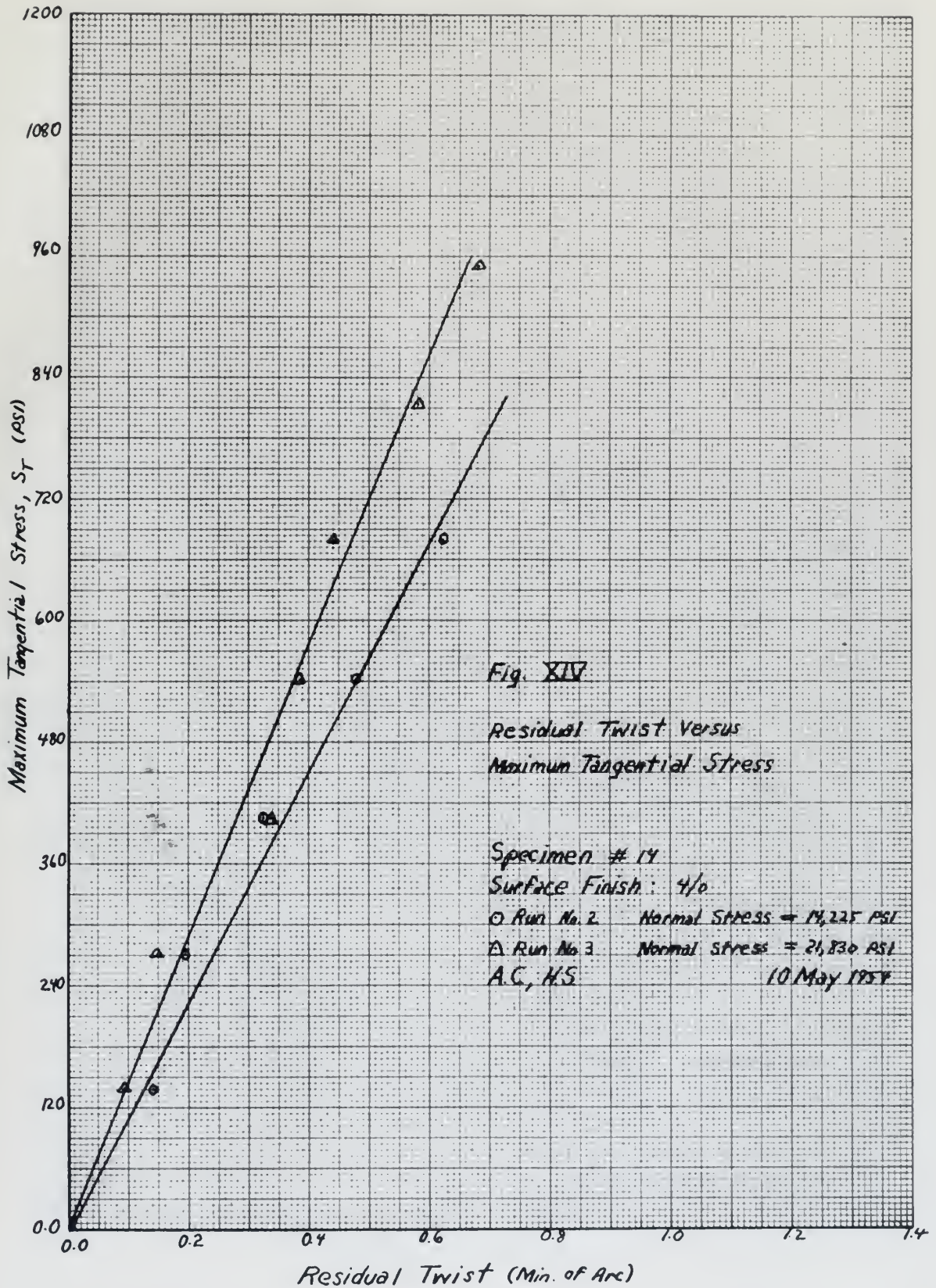
















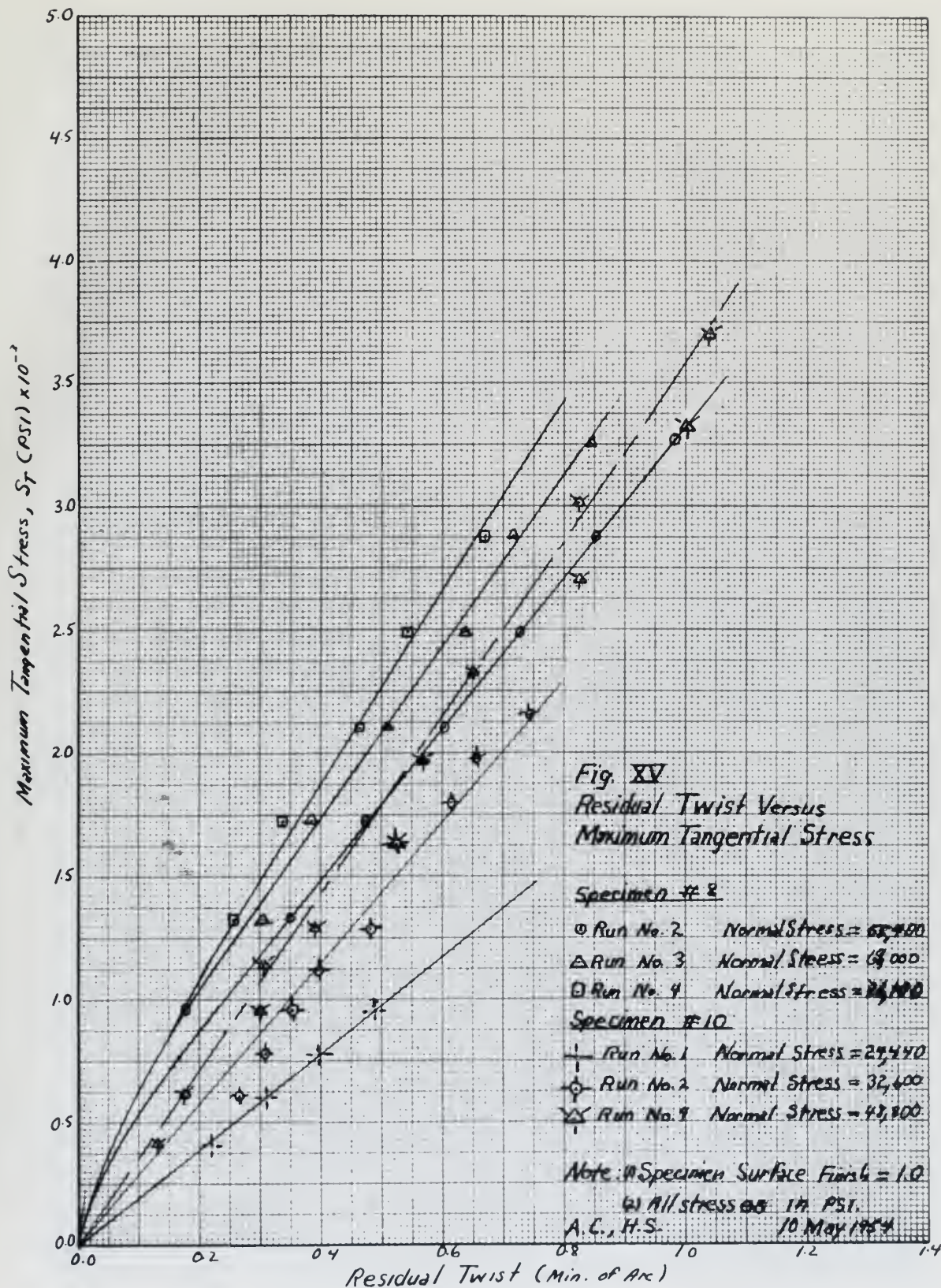






Fig. XVI

Residual Twist Versus Maximum Tangential Stress

Contact Surface Finish: All Specimens (4/a)

○ Specimen # 1 Normal Stress = 24,470 PSI

Specimen # 3

△ Run # 1 Normal Stress = 44,600 PSI

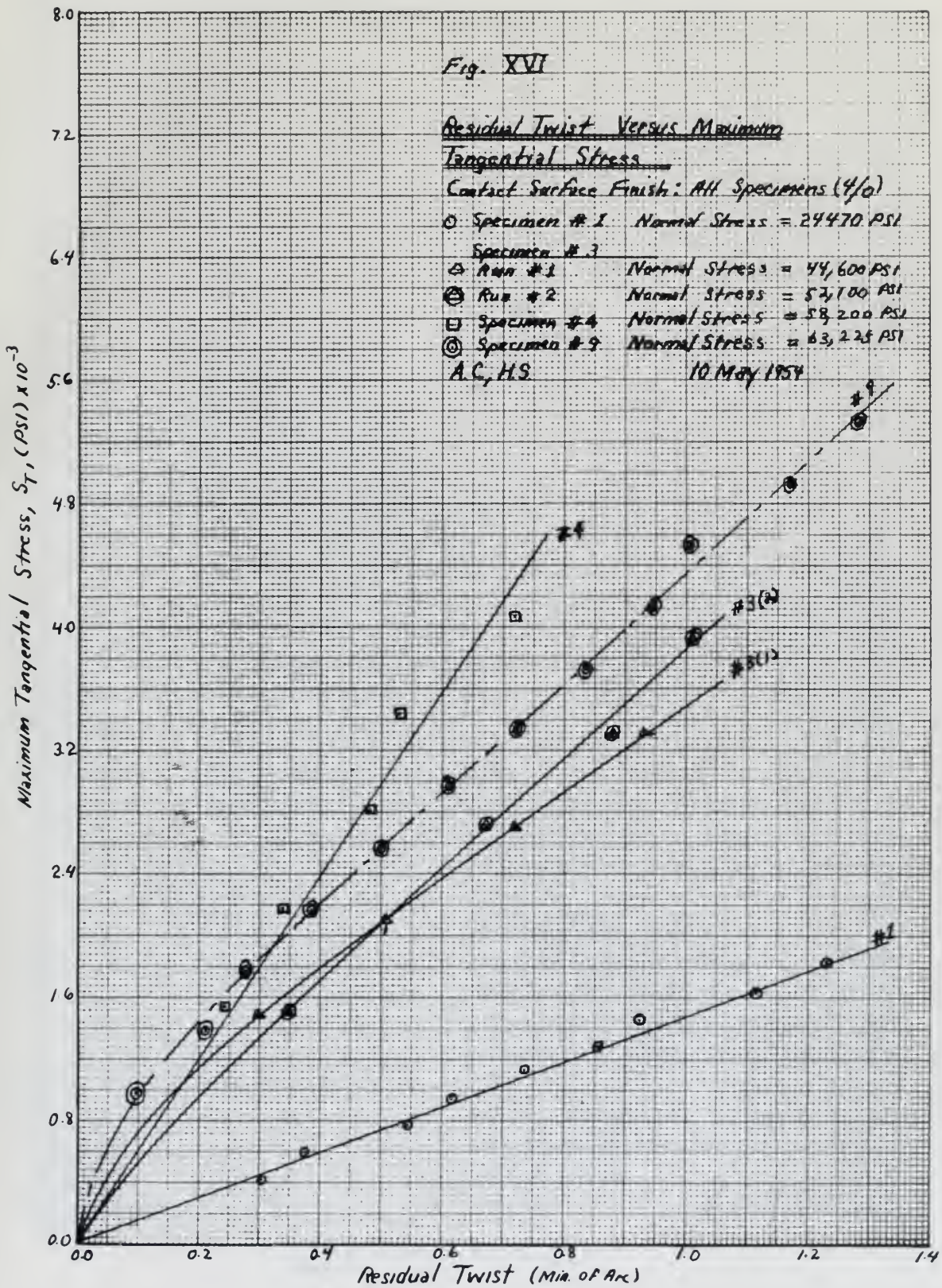
⊙ Run # 2 Normal Stress = 52,100 PSI

□ Specimen # 4 Normal Stress = 58,200 PSI

⊙ Specimen # 9 Normal Stress = 63,225 PSI

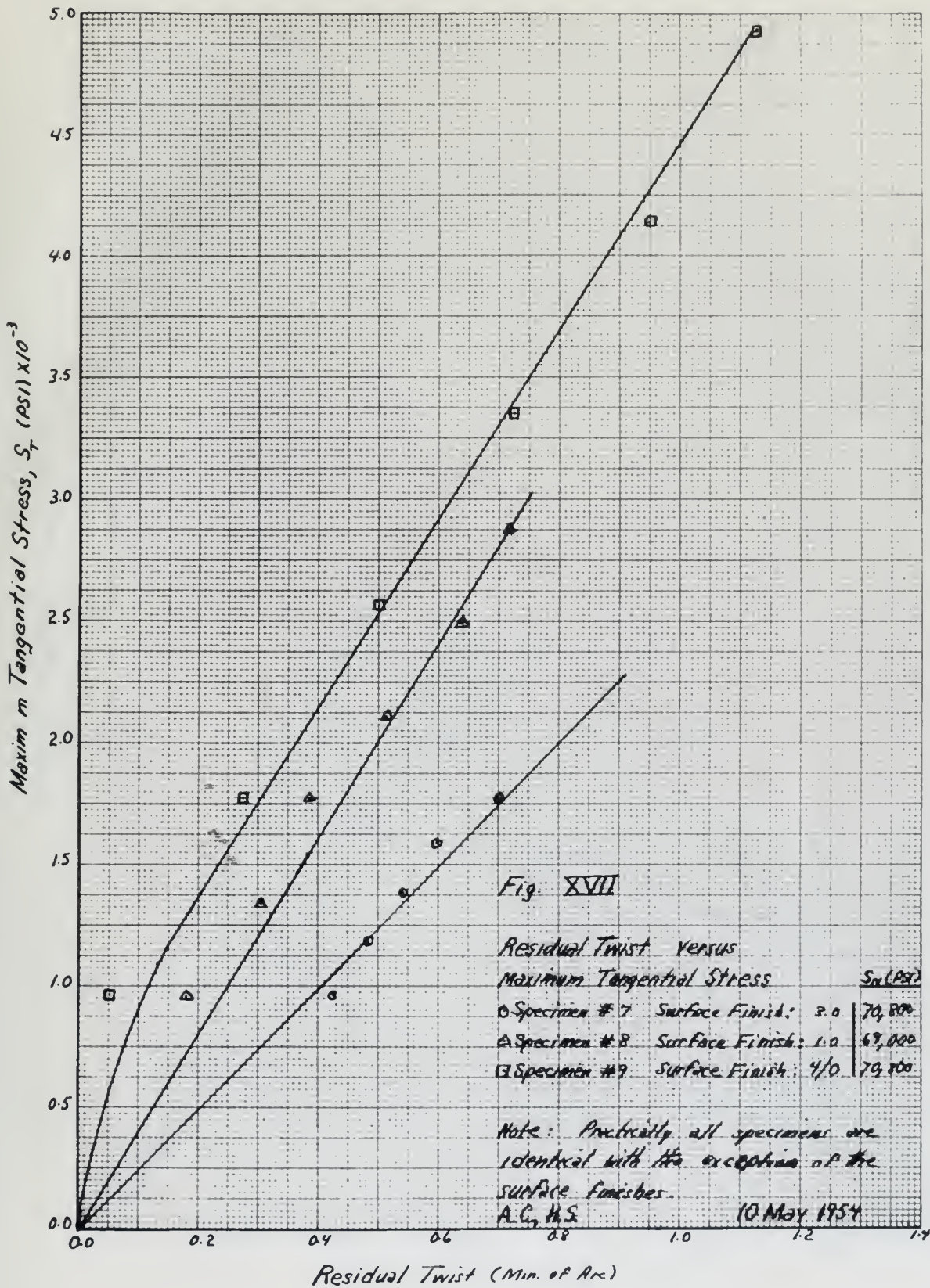
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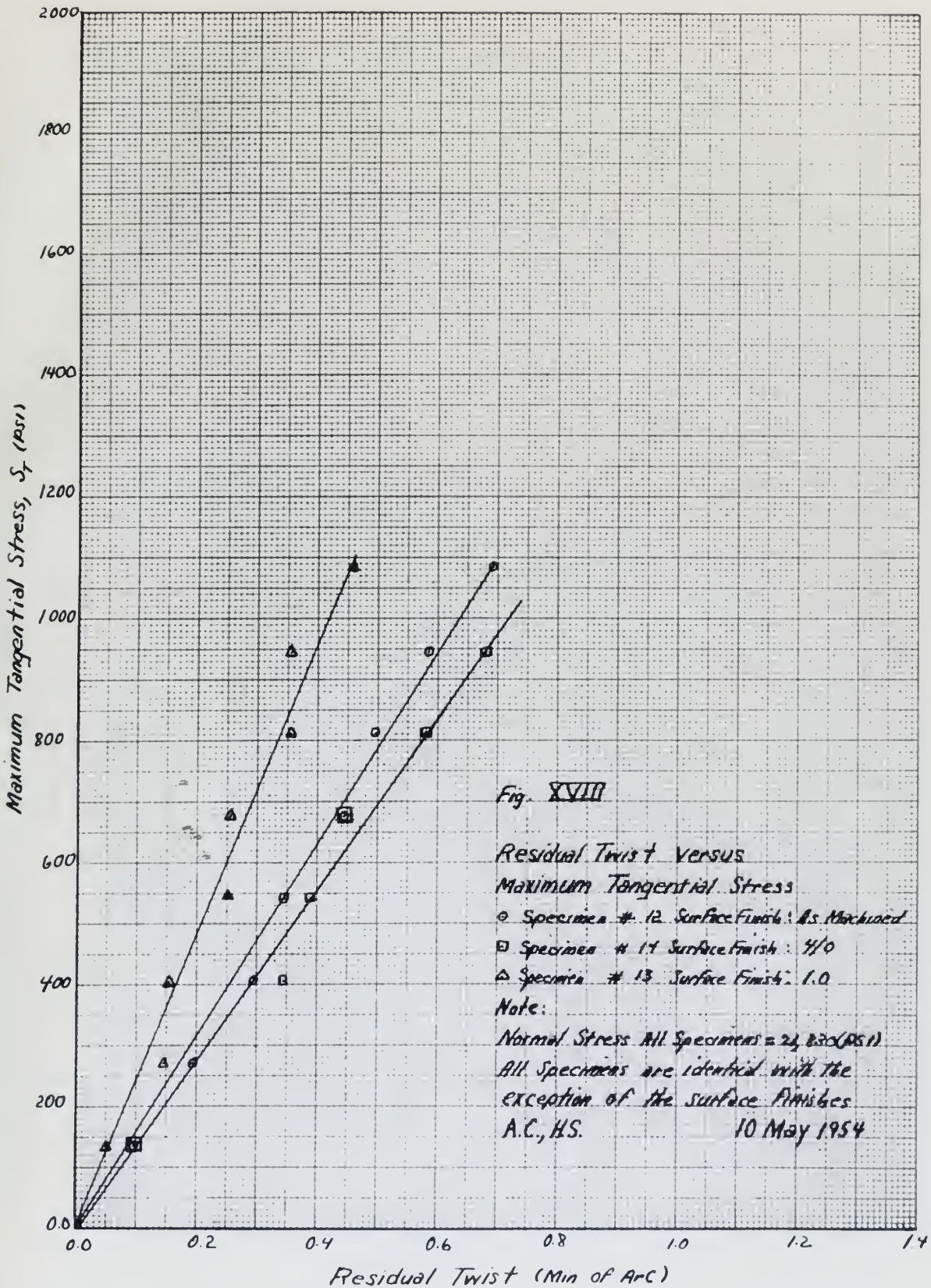






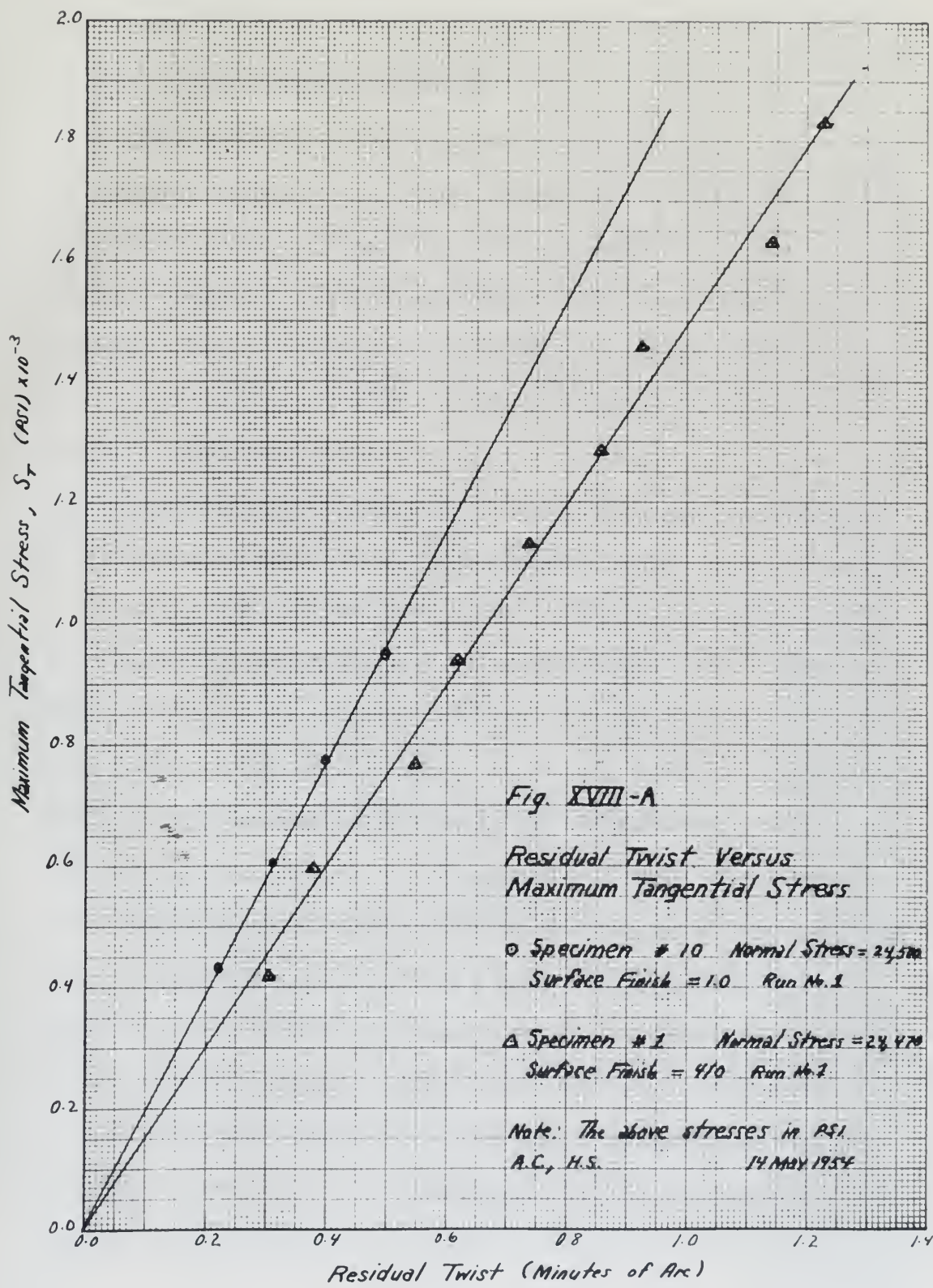














## DISCUSSION OF RESULTS

### Part 1. Test on Specimens 1 - 10

These tests were, on a qualitative basis, surprisingly consistent. Figures 1 to X show that on all specimens where more than one run was made, the residual twist for a given maximum tangential stress decreased as the normal stress increased. This is further substantiated by Figures XV and XVI. Figure XV shows the results for two specimens with the same surface finish (1.0), one specimen being beveled at the ends and the other unbeveled. Figure XVI compares four specimens having a 4/0 finish. These figures show the same effect of normal stress.

Figures XVII and XVIII-A show the effect when surface finish is varied while normal stress is maintained constant. Figure XVII compares three different finishes at approximately the same normal stress (70,000 psi). It is seen that, for a given value of maximum tangential stress, the residual twist varies with the roughness (i.e., the rougher the surface, the greater the residual twist). Figure XVIII-A, which compares two specimens with surface finishes of 4/0 and 1.0 at a low normal stress, (approximately 24,500 psi), shows that the residual twist increases with the surface smoothness.

If the above results are valid, there should be some value of normal stress at which the residual twist for a given maximum tangential stress for both 4/0 and 1.0 finish are the same. This



July 1, 1950

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second part of the report and the  
other dealing with the revision of the  
third part of the report.

is borne out by comparing the curve in Figure XV for specimen # 10, surface finish 1.0, Run 4, normal stress 48,800 psi with curves in Figure XVI for specimen #3, surface finish 4/0, Runs 1 and 2 with normal stresses of 44,600 and 52,100 psi respectively. This comparison shows that there is a transition range for normal stress in which the effect of surface finish on residual twist reverses (i.e., smooth finishes give more twist than the rough finishes below this range and less twist above it).

In evaluating our results some information upon the accuracy of the readings is in order. One scale unit of the optic vernier in the microscope eyepiece represents 0.490 minutes of twist for the first six specimens and 0.467 minutes for the succeeding specimens. The reason for the difference is that a 7-1/2 inch indicator arm was substituted for the original 7.0 inch indicator. Readings were estimated to the nearest tenth of a scale unit. This allowed an accuracy of plus or minus one tenth scale unit or 0.0490 and 0.046 minutes of twist respectively.

Part 2. Tests on Specimens 12 - 14.

Comparing Figures XII through XIV, it is noted that the results for these runs were not as consistent as those in Part 1. Figure XII does not show the residual twist decreasing with increasing normal stress as was observed in Part 1. Figures XIII and XIV show this trend only slightly.

Figure XVIII which compares the affects of surface finish for

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a normal stress of 21,830 psi shows that the 4/0 finish gives more residual twist than the 1.0 finish for any given maximum tangential stress. This is in accord with the results in Part 1, as shown in Figure XVIII-A.

As in Part 1, a brief explanation of the accuracy of the readings will be helpful in evaluating these results.

In this series of tests, as noted in the Procedure, two indicator arms were used. For one arm the accuracy of reading was plus or minus one tenth scale unit. For two arms, therefore, our accuracy was decreased to plus or minus two tenths of a scale unit. This means our readings of twist are accurate to only plus or minus 0.092 minutes of twist. Thus, in Figures XII through XIV the accuracy of the comparison is the sum of the accuracy of each curve or in other words the comparison can be accurate to only plus or minus 0.184 minutes of residual twist. This possible range of error is much greater than any of the observed differences shown in the curves, hence it is felt that this data cannot validly be used to determine the effect of normal stress on residual twist. Hindsight is admittedly better than foresight. It would have been better to have used a much wider range of normal loads. However, our apparatus would not permit this and time limitations prevented assembling new apparatus. Another solution would be the use of a more powerful microscope with a finer optic vernier scale.

Figure XVIII is not rendered invalid by the above limits on



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the accuracy of the readings since for the larger values of maximum tangential stress the difference in residual twist due to surface finish is greater than 0.184 minutes of residual twist and hence, these results are valid for qualitative analysis.

### Part 3. Authors' Theory on Residual Twist in Interfaces

The results of this work clearly show that metal interfaces do exhibit elastic behavior. The following explanation is offered.

It is well known that all metal surfaces regardless of the degree of surface finish are covered with small irregularities called asperities. These will vary with the degree of surface finish from jagged peaks to rolling downs. It is believed that these asperities act as cantilevers.

The calculations in Appendix G show that if the asperities are assumed to act as bulk metal, then to account for the residual twist would require an asperity height which is much too large. However, if the asperities are assumed to be cantilevers, an asperity height can be calculated that is of the same order of magnitude as that measured by a profilometer.

Further, the cantilever theory offers an explanation for the transition range mentioned earlier in Part 1.

Bowden and Tabor <sup>(4)</sup> state that the "real contact area" is directly proportional to the normal load and is independent of surface finish. It is known that the number of asperities will increase and the height will decrease with the degree of surface

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### Section 1

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finish. Therefore, for any normal load, a smoother surface finish will have more asperities in contact, but the asperity height will be less than that for the rougher surface.

The height of asperity also depends upon the normal load since the tips of the asperities will flow plastically to accommodate this load. There is a limit to this plastic flow in the asperities, apparently, for Bowden and Taber <sup>(4)</sup> show photographically (Plate II) that even when the normal load is great enough to cause the bulk metal to flow plastically, the asperities retain their identity.

Thus, the twist for a given maximum tangential stress is dependant on the number of asperities and the normal load, since this load affects the asperity height. For low normal stresses, apparently the number of cantilevers effects the effect of the shorter height in smooth surfaces as shown in Figures XVIII and XVIII-A. As the normal stress is increased and the limit of plastic flow is reached in the asperities, the height differential becomes the controlling factor affecting twist so that the asperities on the smoother surface display less twist than those on rougher surfaces.



These features, for my notes, are numbered 1 through 15. All my notes are arranged in order, but the subjects are not in any order. All the notes are in the same volume.

The weight of subjects is arranged from the most to the least. The list of the subjects are: (1) The history of the subject; (2) The nature of the subject; (3) The importance of the subject; (4) The scope of the subject; (5) The relationship of the subject to other subjects; (6) The role of the subject in the development of the subject; (7) The contribution of the subject to the development of the subject; (8) The impact of the subject on the development of the subject; (9) The future of the subject; (10) The present status of the subject; (11) The progress of the subject; (12) The current status of the subject; (13) The latest developments in the subject; (14) The most recent progress in the subject; (15) The most recent progress in the subject.

That, the first of these features, is arranged in the order of the subjects. The subjects are arranged in the order of their importance. The subjects are arranged in the order of their scope. The subjects are arranged in the order of their relationship to other subjects. The subjects are arranged in the order of their role in the development of the subject. The subjects are arranged in the order of their contribution to the development of the subject. The subjects are arranged in the order of their impact on the development of the subject. The subjects are arranged in the order of their future. The subjects are arranged in the order of their present status. The subjects are arranged in the order of their progress. The subjects are arranged in the order of their current status. The subjects are arranged in the order of their latest developments. The subjects are arranged in the order of their most recent progress.

The subjects are arranged in the order of their importance. The subjects are arranged in the order of their scope. The subjects are arranged in the order of their relationship to other subjects. The subjects are arranged in the order of their role in the development of the subject. The subjects are arranged in the order of their contribution to the development of the subject. The subjects are arranged in the order of their impact on the development of the subject. The subjects are arranged in the order of their future. The subjects are arranged in the order of their present status. The subjects are arranged in the order of their progress. The subjects are arranged in the order of their current status. The subjects are arranged in the order of their latest developments. The subjects are arranged in the order of their most recent progress.

## CONCLUSIONS AND RECOMMENDATIONS

### A. It is concluded that:

1. The asperities in steel interfaces contribute materially to the elastic twist.
2. The elastic twist due to an interface, for a given value of maximum tangential stress, decreases with increasing <sup>normal</sup> stress.
3. The effect of surface finish is most interestingly affected by normal stress in that a transition range exists for normal stress. Below this range the smoother finish gives the greater elastic twist, whereas above this range, the rougher finish gives the greater twist.
4. All the above effects are explained if the asperities are considered to behave as cantilevers.

### B. It is recommended that:

1. Further investigations be made of the transition range for degrees of surface finish other than 1.0 and 4/0.
2. Annular specimens be tested over a greater range of normal stresses.
3. Metals of different physical characteristics be studied.

EXPERIMENTAL RESULTS

1. It is concluded that the results in these experiments are consistent with the theory of the transition from the liquid to the solid state.
  2. The results show that the transition from the liquid to the solid state is a first order transition and is accompanied by a change in volume and a change in the refractive index.
  3. The effect of the transition on the optical properties of the material is studied in detail. It is shown that the transition is accompanied by a change in the refractive index and a change in the optical activity. The results are compared with the theory of the transition from the liquid to the solid state.
  4. All the above effects are explained if the transition is considered as a change in the order of the molecules.
- It is recommended that:
1. Further investigations be made of the transition from the liquid to the solid state.
  2. The transition from the liquid to the solid state be studied in detail.
  3. The transition from the liquid to the solid state be studied in detail.

APPENDIX A

Details of Procedure

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**A TABLE**

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**Appendix to [illegible]**

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DETAILS OF PROCEDURE

As in the section on Procedure, the discussion will be separated into two parts corresponding to the two experimental methods used.

In the first, practical considerations prevented location of the indicator arm exactly at the interface. Consequently the measured twist consisted of two parts: 1, that contributed by the interface; 2, that contributed by twist of the specimen between the interface and the angle indicator.

Since the angle indicator was attached to the specimen by pointed set screws, the distance between it and the interface could be determined with a high degree of accuracy. However, to determine the twist of the specimen required deriving an expression for computing the twist in that part of the specimen that was bevelled. The derivation is given below.

Refer to Figure XX. Consider a plane perpendicular to the axis of specimen intersecting the specimen at a distance "x" from the interface. The shear stress existing in this intersection is given by (4)

$$\text{Shear stress} = G r \frac{d\phi}{dx} = \sigma \quad (1)$$

$G$  = modulus of elasticity in shear (for our specimen  $G = 13 \times 10^6$  pounds per square inch).  $\frac{d\phi}{dx}$  = angle of twist per unit length.

Considering an element of area,  $dA$

GENERAL PRINCIPLES

As in the case of the other, the following will be  
applied to the case of the other, the following will be  
applied to the case of the other, the following will be

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As in the case of the other, the following will be  
applied to the case of the other, the following will be  
applied to the case of the other, the following will be

(1) of the other, the following will be

$$(1) \quad \frac{d}{dt} = \frac{d}{dt} + \frac{d}{dt}$$

As in the case of the other, the following will be  
applied to the case of the other, the following will be  
applied to the case of the other, the following will be

(2) of the other, the following will be

$$dA = 2\pi r dr \tag{2}$$

The moment of the shearing force about the axis of the specimen for this element of area is  $\sigma r dA$

The total moment about the axis of the specimen must equal the effective torque in the specimen. As explained in the Procedure, the effective torque equals one half the applied torque. Therefore,

$$T_E = 1/2 T_A = \int_0^r \sigma r dA \tag{3}$$

Substituting equations (1) and (2) into equation (3)

$$T_A = \int_0^r 4\pi G \frac{d\phi}{dx} r^3 dr \tag{4}$$

In any given plane perpendicular to the axis of the specimen  $\frac{d\phi}{dx}$  is independent of  $r$ . Therefore equation (4) can be integrated.

$$T_A = \pi G \frac{d\phi}{dx} r^4 \tag{5}$$

Rearranging gives

$$d\phi = \frac{T_A dx}{\pi G r^4} \tag{6}$$

It is evident in Figure XX that  $r$  is related to  $x$  by

$$r = r_0 + \frac{R - r_0}{l} x = r_0 + kx \tag{7}$$

$r_0$  = radius at  $x = 0$  (i.e.,  $r_0 = d/2$ ).

$R$  = radius at  $x = l$  (i.e.,  $R = D/2$ ).

Substituting equation (7) into equation (6)



The number of the starting force along the axis of the system

for this element is given by

The total moment about the axis of the system is given by

The effective radius of the system, as explained in the

preceding, the effective radius is not the physical

radius, but the

(3)

$$r_{eff} = \int_0^R r^2 \cdot 2\pi r \cdot dr$$

Substituting equation (1) into equation (3)

(4)

$$r_{eff} = \int_0^R \frac{1}{2} \pi r^2 \cdot 2\pi r \cdot dr$$

In the case of a uniform distribution of mass

the relationship of  $r_{eff}$  to the radius  $R$  can be determined

(5)

$$r_{eff} = \frac{1}{2} R$$

(6)

$$\frac{dr}{r} = \frac{1}{2} \frac{dr}{R}$$

It is noted in figure 11 that a radius of  $R$

(7)

$$r = \frac{1}{2} R$$

is the radius of the system,  $r = 0$  at the center.

is the radius of the system,  $r = R$  at the edge.

Substituting equation (7) into equation (5)

$$d\phi = \frac{T_A dx}{\pi G (r_0 + kx)^4} \quad (8)$$

Equation (8) can be integrated for the angle of twist over the length  $l$ .

$$\phi_l = \int_0^l d\phi = \int_0^l \frac{T_A dx}{\pi G (r_0 + kx)^4} = \frac{T_A}{3\pi Gk} \left[ \frac{1}{(r_0)^3} - \frac{1}{(r_0 + lk)^3} \right] \quad (9)$$

this can be simplified by substituting

$$k = \frac{R - r_0}{l}$$

$$r_0 = d/2$$

$$R = D/2$$

Performing this substitution gives

$$\phi_l = \frac{8 T_A}{3 \pi Gk} \left[ \left(\frac{1}{d}\right)^3 - \left(\frac{1}{D}\right)^3 \right] \quad (10)$$

this can be expressed more conveniently for our purposes by making the following substitutions

$$\psi_l = \text{twist in minutes} = \frac{(60)(180)}{\pi} \phi_l$$

$$Q_A = \text{applied torque in gram-inches} = \frac{1000}{2.205} T_A$$

$$G = 12 \times 10^6 \text{ lb/in}^2$$

This gives

$$\psi_l = \frac{(60)(180)}{\pi} \times \frac{8}{3\pi} \times \frac{2.205}{1000} Q_A (1/k) \left[ \left(\frac{1}{d}\right)^3 - \left(\frac{1}{D}\right)^3 \right]$$

$$\text{or, } \psi_l = 5.360 \times 10^{-7} (1/k) \left[ \left(\frac{1}{d}\right)^3 - \left(\frac{1}{D}\right)^3 \right] Q_A \quad (11)$$

$$\frac{1}{\sqrt{x^2 + 10x + 11}} = \dots$$

... to find the integral of the function ...

$$(10) \quad \frac{1}{\sqrt{x^2 + 10x + 11}} = \frac{1}{\sqrt{(x+5)^2 - 4}} = \dots$$

... of the function ...

$$\begin{aligned} \frac{1}{2} &= \frac{1}{2} \\ \frac{1}{2} &= \frac{1}{2} \\ \frac{1}{2} &= \frac{1}{2} \end{aligned}$$

... of the function ...

$$\left[ \frac{1}{2} - \frac{1}{2} \right] \frac{1}{\sqrt{x^2 + 10x + 11}} = \dots$$

... of the function ...

$$\frac{1}{2} = \frac{1}{2} \dots$$

... of the function ...

$$\left[ \frac{1}{2} - \frac{1}{2} \right] \frac{1}{\sqrt{x^2 + 10x + 11}} = \dots$$

$$(11) \quad \left[ \frac{1}{2} - \frac{1}{2} \right] \frac{1}{\sqrt{x^2 + 10x + 11}} = \dots$$

The twist in the specimen between  $x = l$  and  $x = l'$  (i.e., between the upper end of the bevel and the indicator arm) can be easily calculated (4).

$$\phi_{l'} = \frac{32 T_{\text{I}} l'}{\pi D^4} \quad (12)$$

This can be expressed in the same convenient form as equation (11)

$$\psi_{l'} = 3.205 \times 10^{-6} \frac{l'}{D^4} Q_{\text{I}} \quad (13)$$

Thus the total twist in the specimen between the interface and the indicator arm is

$$\psi_T = \psi_l + \psi_{l'} \quad (\text{Min. of Arc}) \quad (14)$$

The total twist as computed above is subtracted from the observed twist to obtain what we have called "residual twist" and designated  $\psi_R$ .

$$\psi_R = \psi_o - \psi_T \quad (15)$$

With the annular specimens, it was possible to use a control specimen as described in the procedure. This permitted us to obtain a value for twist per gram inch of torque per inch of specimen length under actual experimental conditions. For subsequent runs it was possible to compare the observed twist with what the twist would have been if there had been no interface. As before the difference was called "residual twist".



The total in the system between a = 2 and 2 = 2 (1-2) = 2  
 between the upper and the lower end of the indicator and the  
 middle indicator (1-2)

(10) 
$$2 = \frac{2}{2} = 2$$

This can be explained in two ways: (1) The total in the system (1-2)

(11) 
$$2 = \frac{2}{2} = 2$$

and the indicator can be

(12) 
$$2 = 2 + 2 = 4 \text{ (Min. of Arc)}$$

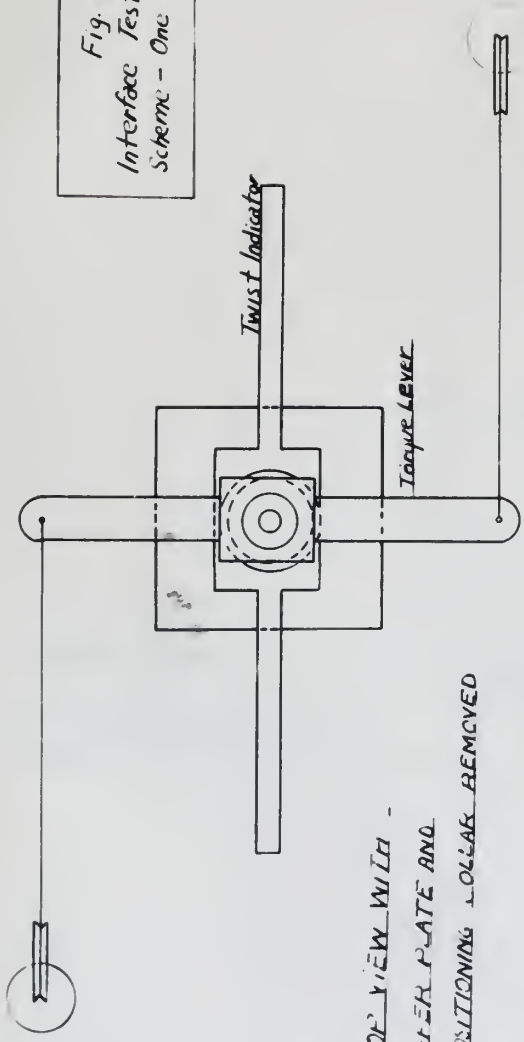
The total in the system is calculated from the  
 upper end to the lower end of the indicator (1-2)

$$2 = 2$$

(13) 
$$2 = 2 - 2 = 0$$

From the total indicator, it is possible to see a  
 number of indicators in the system. (1-2) (1-2) (1-2)  
 a value for the total of the indicator and the  
 indicator in the system. (1-2) (1-2) (1-2)  
 It is possible to explain the system with the  
 total in the system. (1-2) (1-2) (1-2)  
 This is the total indicator (1-2)

Fig. XIX  
 Interface Testing Apparatus  
 Scheme - One (NOT TO SCALE)



TOP VIEW WITH -  
 UPPER PLATE AND  
 POSITIONING COLLAR REMOVED

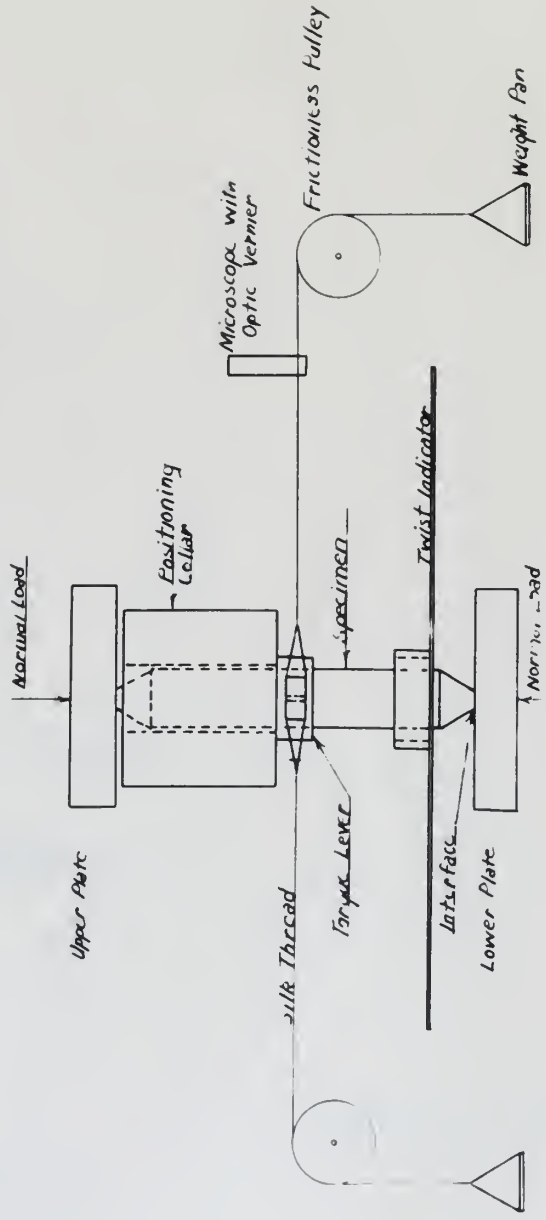




Fig. XX  
 Nomenclature Used In  
 Computations (From Fig XIX)

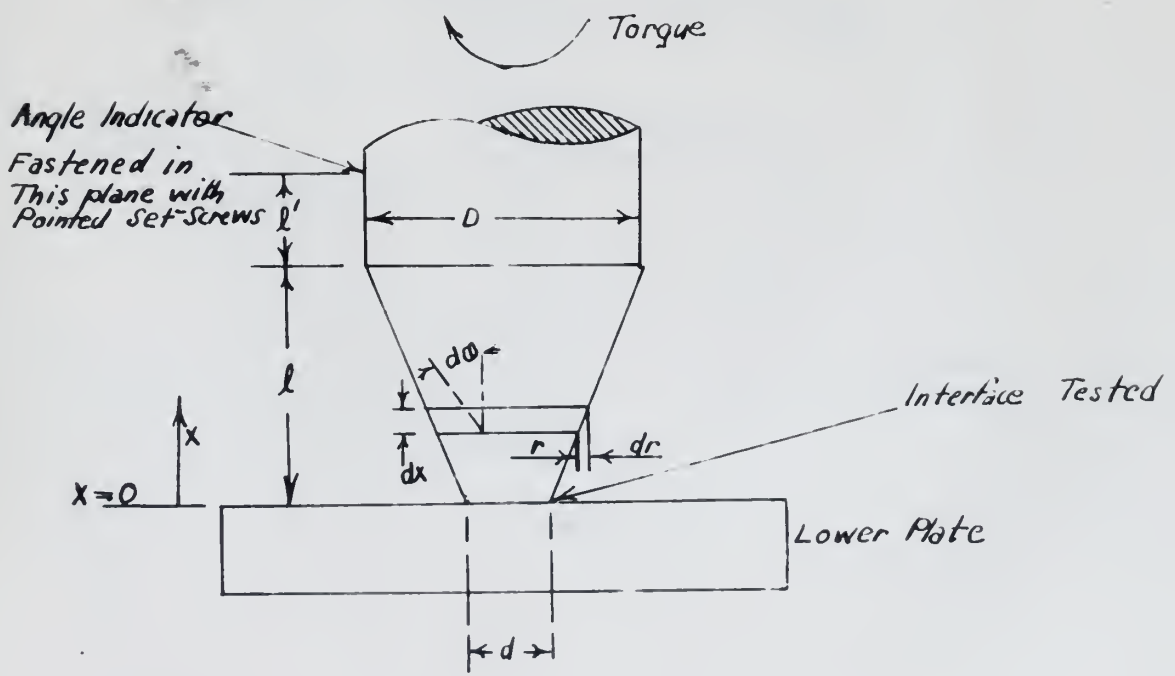
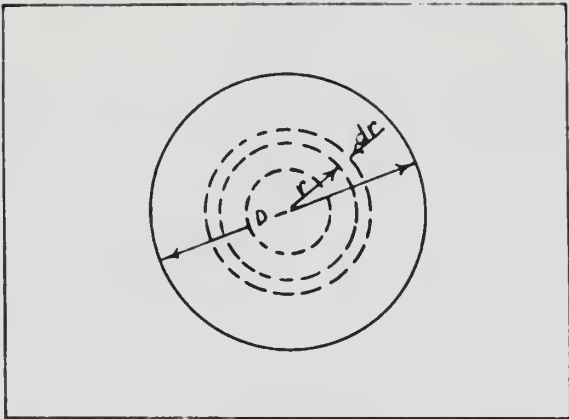
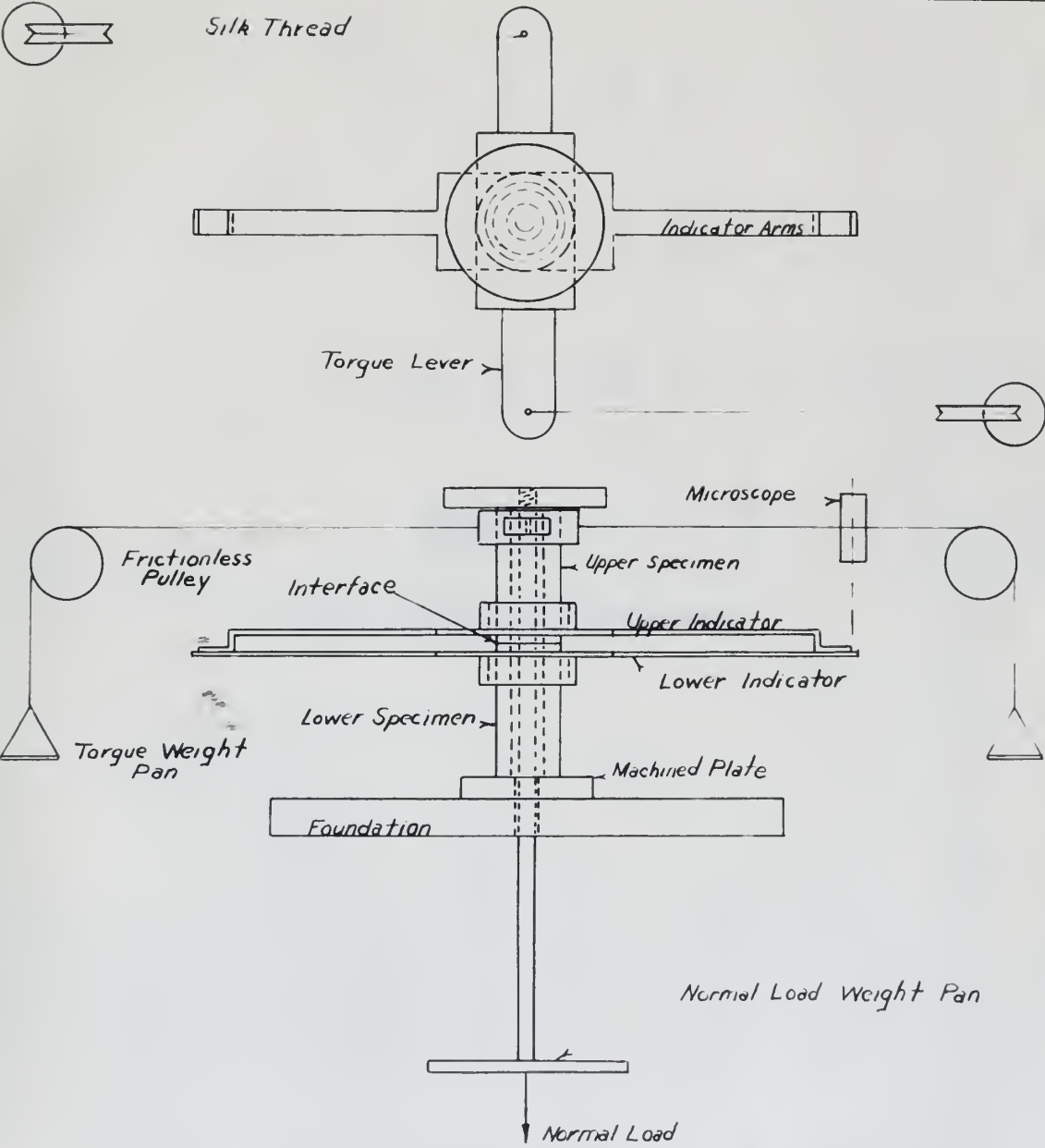






Fig XXI  
Interface Testing Apparatus  
Scherrie - Two (Not to Scale)





APPENDIX B

Summary of Data and Calculations



E. ALBERTA

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SPECIMEN # 1

$d = 0".125 = D$

$L = 0".000$

$L' = 0".096$

$F = 0.490$

$\psi_L = 0.000$

$\psi_L' = 3.205 \times 10^{-6} \frac{L'}{D^4} Q_A$

$\psi_L'' = 1.269 \times 10^{-3} Q_A$

$\psi_T = \psi_L + \psi_L'$

$\psi_T = 1.269 \times 10^{-3} Q_A$

$\psi_R = F \times \psi_L = 0.49 \psi_L$

$\psi_R = \psi_L - \psi_T$

$S_T = 5.610 \times 10^{-3} \frac{Q_A}{d^3}$

$S_T = 2.88 Q_A$

$g_N = \frac{P}{\pi/L}$

Surface Finish: 4/0

$Q_A$	$S_T$	$\psi_T$	$\psi_R$
145.86	418	0.185	0.305
205.86	593	0.261	0.376
265.86	765	0.337	0.545
325.86	938	0.413	0.617
385.86	1130	0.489	0.736
445.86	1285	0.565	0.856
505.86	1459	0.645	0.923
565.86	1630	0.718	1.143
625.86	1830	0.781	1.229
685.86	1978	0.858	1.347
745.86			

$P_N = 300$

$S_N = 24,500 \psi_R$

1.  $\frac{1}{x^2} = x^{-2}$

$$\frac{d}{dx} x^{-2} = -2x^{-3} = -\frac{2}{x^3}$$

$$\frac{d}{dx} x^3 = 3x^2$$

$$\frac{d}{dx} x^2 = 2x$$

$$\frac{d}{dx} x = 1$$

$$\frac{d}{dx} x^0 = 0$$

$$\frac{d}{dx} x^{-1} = -x^{-2} = -\frac{1}{x^2}$$

$$\frac{d}{dx} x^{-3} = -3x^{-4} = -\frac{3}{x^4}$$

$$\frac{d}{dx} x^{-4} = -4x^{-5} = -\frac{4}{x^5}$$

$$\frac{d}{dx} x^{-5} = -5x^{-6} = -\frac{5}{x^6}$$

$$\frac{d}{dx} x^{-6} = -6x^{-7} = -\frac{6}{x^7}$$

$$\frac{d}{dx} x^{-7} = -7x^{-8} = -\frac{7}{x^8}$$

$$\frac{d}{dx} x^{-8} = -8x^{-9} = -\frac{8}{x^9}$$

$$\frac{d}{dx} x^{-9} = -9x^{-10} = -\frac{9}{x^{10}}$$

$$\frac{d}{dx} x^{-10} = -10x^{-11} = -\frac{10}{x^{11}}$$

Exercise 1

Function	Derivative
$y = x^3$	$y' = 3x^2$
$y = x^2$	$y' = 2x$
$y = x$	$y' = 1$
$y = \frac{1}{x}$	$y' = -\frac{1}{x^2}$
$y = \frac{1}{x^2}$	$y' = -\frac{2}{x^3}$
$y = \frac{1}{x^3}$	$y' = -\frac{3}{x^4}$
$y = \frac{1}{x^4}$	$y' = -\frac{4}{x^5}$
$y = \frac{1}{x^5}$	$y' = -\frac{5}{x^6}$
$y = \frac{1}{x^6}$	$y' = -\frac{6}{x^7}$
$y = \frac{1}{x^7}$	$y' = -\frac{7}{x^8}$
$y = \frac{1}{x^8}$	$y' = -\frac{8}{x^9}$
$y = \frac{1}{x^9}$	$y' = -\frac{9}{x^{10}}$
$y = \frac{1}{x^{10}}$	$y' = -\frac{10}{x^{11}}$

SPECIMEN # 2

- d = 0".68045
- D = 0".1248
- Q = 0".08045
- L' = 0".000
- F = 0.490

$$\psi_L = \frac{5.360 \times 10^{-7}}{k} \left[ \left(\frac{L}{D}\right)^3 - \left(\frac{L}{D}\right)^2 \right] Q$$

$$\psi_{L'} = 0".000$$

$$\psi_T = \psi_L + \psi_{L'}$$

$$\psi_T = 1.229 \times 10^{-3} Q_A$$

$$\psi_0 = F \times \psi_S = 0.49 \psi_S$$

$$\psi_R = \psi_0 - \psi_T$$

$$S_T = 5.610 \times 10^{-3} \frac{Q_A}{D^3}$$

$$S_T = 9.83 Q_A$$

$$S_N = \frac{F_N}{\pi/4 d^2}$$

Surface Finish: 2/0

	$Q_A$	$S_T$	$\psi_T$	$\psi_0$	$\psi_R$
$F_N = 277.6$					
$S_N = 53,750$					
	145.86	1432	0.179	0.784	.605
	217.86	2140	0.267	1.078	.811
	337.86	3320	0.414	2.058	1.644
	398.34	3918	0.490	2.450	1.960
	458.82	4500	0.563	2.842	2.270
	519.30				



$$d_{i,j} = \frac{1}{2} \left( \frac{1}{\lambda_i} + \frac{1}{\lambda_j} \right)$$

$\lambda_i$	$\lambda_j$	$\lambda_k$	$\lambda_l$	$\lambda_m$
100.0	200.0	300.0	400.0	500.0
150.0	300.0	400.0	500.0	600.0
200.0	400.0	500.0	600.0	700.0
250.0	500.0	600.0	700.0	800.0
300.0	600.0	700.0	800.0	900.0
350.0	700.0	800.0	900.0	1000.0

$$d_{i,j} = \frac{1}{2} \left( \frac{1}{\lambda_i} + \frac{1}{\lambda_j} \right)$$

- 100.0
- 200.0
- 300.0
- 400.0
- 500.0
- 600.0
- 700.0
- 800.0
- 900.0
- 1000.0

$$d_{i,j} = \frac{1}{2} \left( \frac{1}{\lambda_i} + \frac{1}{\lambda_j} \right)$$

$$d_{i,j} = \frac{1}{2} \left( \frac{1}{\lambda_i} + \frac{1}{\lambda_j} \right)$$

$$d_{i,j} = \frac{1}{2} \left( \frac{1}{\lambda_i} + \frac{1}{\lambda_j} \right)$$

$$d_{i,j} = \frac{1}{2} \left( \frac{1}{\lambda_i} + \frac{1}{\lambda_j} \right)$$

$$d_{i,j} = \frac{1}{2} \left( \frac{1}{\lambda_i} + \frac{1}{\lambda_j} \right)$$

$$d_{i,j} = \frac{1}{2} \left( \frac{1}{\lambda_i} + \frac{1}{\lambda_j} \right)$$

$$d_{i,j} = \frac{1}{2} \left( \frac{1}{\lambda_i} + \frac{1}{\lambda_j} \right)$$

$$d_{i,j} = \frac{1}{2} \left( \frac{1}{\lambda_i} + \frac{1}{\lambda_j} \right)$$

$$d_{i,j} = \frac{1}{2} \left( \frac{1}{\lambda_i} + \frac{1}{\lambda_j} \right)$$

$d = 0".081$   
 $D = 0".125$   
 $Q = 0".00762$   
 $Q' = 0".1088$   
 $F = 0.490$

$$\psi_e = \frac{5.360 \times 10^{-7}}{k} \left[ \left(\frac{1}{Q}\right)^3 - \left(\frac{1}{D}\right)^3 \right] Q_A$$

$$= 0.267 \times 10^{-3} Q_A$$

$$\psi_i = 3.205 \times 10^{-6} \frac{Q'}{D^4} Q_A$$

$$= 1.430 \times 10^{-3} Q_A$$

$$\psi_T = \psi_e + \psi_i$$

$$= 1.697 \times 10^{-3} Q_A$$

$$\psi_0 = F \times \psi_s = 0.490 \psi_s$$

$$S_T = 5.610 \times 10^{-3} \frac{Q_A}{D^3}$$

$$S_T = 10.57 Q_A$$

$$S_N = \frac{F_H}{\pi/4 d^2}$$

Surface Finish: 4/0

$F_H = 300$	$S_N = 58,200$	$\psi_0$	$\psi_T$	$S_T$	$Q_A$	$\psi_R$
		.496	.245	1541	145.86	.242
		.687	.349	2175	205.86	.338
		.932	.451	2810	265.86	.421
		1.079	.552	3440	325.86	.527
		1.372	.654	4080	385.86	.718
		1.520	.756	4710	445.86	.764
		---			505.86	

	$\beta$	$\beta$	$\beta$	$\beta$	$\beta$
SKA.	371.	246.	242	241	241
EX.	141.	242.	473	242	242
DL.	221.	242.	242	242	242
PL.	241.	242.	242	242	242
MT.	241.	242.	242	242	242
MT.	241.	242.	242	242	242
MT.	241.	242.	242	242	242

one =  $\beta$   
 one =  $\beta$

$$\left[ \begin{matrix} \beta \\ \beta \\ \beta \end{matrix} \right] \frac{242 + 242 + 242}{3} = \beta$$

- $\beta = 242$
- $\beta = 242$
- $\beta = 242$
- $\beta = 242$
- $\beta = 242$

Answer: 242

$$\beta = 242$$

$$\beta = 242$$

$$\beta = 242$$

$$\beta = 242$$

$$\beta = 242$$

$$\beta = 242$$

$$\beta = 242$$

$$\beta = 242$$

$$\beta = 242$$

$$\beta = 242$$

SPECIMEN # 3

- d = 0".082
- D = 0".125
- Q = 0".1132
- Q' = 0".0756
- Y = 0.490

$$\psi_L = \frac{5.260 \times 10^{-7}}{k} \left[ \left(\frac{L}{d}\right)^3 - \left(\frac{L}{D}\right)^3 \right] Q_A$$

$$\psi_L = 3.680 \times 10^{-3} Q_A$$

$$\psi_{L'} = 3.205 \times 10^{-6} \frac{Q'}{D^4} Q_A$$

$$\psi_{L'} = 0.994 \times 10^{-3} Q_A$$

$$\psi_T = \psi_L + \psi_{L'}$$

$$\psi_T = 4.674 \times 10^{-3} Q_A$$

$$\psi_0 = F \times \psi_S = 0.49 \psi_S$$

$$\psi_R = \psi_0 - \psi_T$$

$$S_T = 5.610 \times 10^{-3} \frac{Q_A}{d^3}$$

$$S_T = 10.2 Q_A$$

$$S_H = \frac{F_N}{\pi/4 d^2}$$

Surface Finish = 4/0

RUN # 1		RUN # 2	
$\psi_0$	$\psi_R$	$\psi_0$	$\psi_R$
0.980	.299	1.029	.348
1.470	.508	1.470	.508
1.960	.719	1.911	.670
2.450	.930	2.890	.880
---	---	3.380	1.090
445.86			

$F_N = 300$   
 $S_N = 44,500$   
 $S_H = 52,100$



TABLE I

100	100	100	100	100	100	100	100	100	100
...	...	...	...	...	...	...	...	...	...

$$\left[ \frac{1}{1 + \frac{1}{2} + \frac{1}{3} + \dots} \right]$$

- 1. ...
- 2. ...
- 3. ...
- 4. ...
- 5. ...

$$\frac{1}{1 + \frac{1}{2} + \frac{1}{3} + \dots}$$

$$\frac{1}{1 + \frac{1}{2} + \frac{1}{3} + \dots}$$

$$\frac{1}{1 + \frac{1}{2} + \frac{1}{3} + \dots}$$

$$\frac{1}{1 + \frac{1}{2} + \frac{1}{3} + \dots}$$

$$\frac{1}{1 + \frac{1}{2} + \frac{1}{3} + \dots}$$

$$\frac{1}{1 + \frac{1}{2} + \frac{1}{3} + \dots}$$

R U N # 1                      R U N # 2                      R U N # 3

$Q_A$	$S_T$	$\psi_T$	$\psi_0$	$\psi_R$	$\psi_0$	$\psi_R$	$\psi_0$	$\psi_R$
150	1656	.464	.784	.320	.784	.320	.784	.320
210	2319	.649	1.029	.380	1.029	.380	1.029	.380
270	2981	.834	1.470	.636	1.470	.636	1.470	.636
330	3644	1.020	1.991	.891	1.911	.891	1.813	.793
390	4307	1.205	2.303	1.098	2.156	.951	2.009	.804
450	4969	1.390	--	--	2.450	1.060	2.401	1.010
510	5632	1.576			2.842	1.266	2.744	1.168
570	6294	1.761			3.185	1.424	3.081	1.326
630					--	--	--	--

$F_N = 300$                        $F_N = 350$                        $F_N = 373$   
 $S_N = 62,800$                    $S_N = 73,300$                    $S_N = 83,800$

$d = 0".0776$

$D = 0".125$

$l = 0".0562$

$l' = 0".0776$

$F = 0.490$

$$\psi_l = \frac{5.360 \times 10^{-7}}{k} \left[ \left(\frac{l}{d}\right)^3 - \left(\frac{l'}{D}\right)^3 \right] Q_A$$

$= 2.070 \times 10^{-3} Q_A$

$\psi_l' = 3.205 \times 10^{-6} \frac{Q_A}{D^4}$

$= 1.020 \times 10^{-3} Q_A$

$\psi_T = \psi_l + \psi_l' = 3.090 \times 10^{-3} Q_A$

$\psi_0 = F \times \psi_S = 0.490 \psi_S$

$S_T = 5.610 \times 10^{-3} \frac{Q_A}{d^2}$

$S_T = 11.042 Q_A$

$S_N = \frac{F_N}{\pi/4 d^2}$

Surface Finish:  $\phi$

L A M U B		S A M U B		L A M U B			
STC = 40	02E = 40	300 = 40	030 = 40	030 = 40	030 = 40		
000, 00 = 40	000, 00 = 40	000, 00 = 40	000, 00 = 40	000, 00 = 40	000, 00 = 40		
STC = 40	02E = 40	300 = 40	030 = 40	030 = 40	030 = 40		
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0200, 00 = b

0200, 00 = b

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0200, 00 = b

$$A \left[ \begin{pmatrix} 1 \\ 1 \end{pmatrix} - \begin{pmatrix} 1 \\ 1 \end{pmatrix} \right] \frac{1}{2} \times 0200, 00 = \frac{1}{2} \times 1000$$

0200, 00 = b

0200, 00 = b

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0200, 00 = b

0200, 00 = b

0200, 00 = b

q : relative position

RUN # 4

RUN # 5

RUN # 6

RUN # 7

 $F_N = 400$   
 $S_N = 83,800$ 
 $F_N = 450$   
 $S_N = 93,400$ 
 $F_N = 500$   
 $S_N = 105,000$ 
 $F_N = 550$   
 $S_N = 115,200$ 
 $\psi_T$   
.464

 $\psi_R$   
.320

 $\psi_R$   
.320

 $\psi_R$   
.320

 $S_T$   
1656

 $S_A$   
150

1.029

1.029

1.029

1.029

.649

210

.636

.538

.538

.636

.834

270

1.715

1.617

1.617

1.617

1.020

330

1.960

1.960

1.960

1.960

1.205

390

2.352

2.303

2.303

2.352

1.390

450

2.656

2.607

2.607

2.656

1.576

510

2.940

2.940

2.940

2.940

1.761

570

.931

1.225

1.225

1.225

.619

630

.588

.489

.489

.489

.874

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.124

.548

.548

.548

1.081

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.619

2.009

2.009

2.009

.874

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.874

2.450

2.450

2.450

1.081

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SPECIMEN # 6

- d = 0".0783
- D = 0".125
- d' = 0".0562
- d'' = 0".0783
- F = 0.490

$$\psi_L = \frac{5.36 \times 10^{-7}}{L} \left[ \left(\frac{L}{d}\right)^3 - \left(\frac{L}{d'}\right)^3 \right] Q_A$$

$$= 2.023 \times 10^{-3} Q_A$$

$$\psi_L' = 3.205 \times 10^{-3} \frac{d'}{L^2} Q_A$$

$$= 1.028 \times 10^{-3} Q_A$$

$$\psi_T = \psi_L + \psi_L' = 3.051 Q_A$$

$$\psi_0 = F \times \psi_S = 0.490 \psi_S$$

$$S_T = 5.610 \times 10^{-3} \frac{Q_A}{d^3}$$

$$S_T = 11.68 Q_A$$

$$S_H = \frac{F_N}{\pi/4 d^2}$$

	RUN # 1		RUN # 2		RUN # 3		RUN # 4	
	F <sub>N</sub> = 300	F <sub>N</sub> = 325	F <sub>N</sub> = 350	F <sub>N</sub> = 375	S <sub>N</sub> = 62,500	S <sub>N</sub> = 67,450	S <sub>N</sub> = 72,600	S <sub>N</sub> = 77,800
	ψ <sub>T</sub>	ψ <sub>T</sub>	ψ <sub>T</sub>	ψ <sub>T</sub>	ψ <sub>0</sub>	ψ <sub>0</sub>	ψ <sub>0</sub>	ψ <sub>0</sub>
Q <sub>A</sub>	150	210	270	330	390	450	510	570
S <sub>T</sub>	1752	2454	3156	3858	4560	5262	5964	6666
ψ <sub>0</sub>	.457	.640	.823	1.006	1.189	1.372	1.555	1.738
ψ <sub>T</sub>	.784	1.078	1.470	1.863	2.255	2.648	3.041	3.434
ψ <sub>R</sub>	.327	.438	.647	.954	1.310	1.967	2.107	2.287
ψ <sub>0</sub>	.833	1.127	1.421	1.813	2.107	2.450	2.842	3.234

Surface Finish: as machined

QUESTION 9: [Faint text]

TABLE 1: [Faint text]

- 37. [Faint text]
- 38. [Faint text]
- 39. [Faint text]
- 40. [Faint text]
- 41. [Faint text]
- 42. [Faint text]
- 43. [Faint text]
- 44. [Faint text]
- 45. [Faint text]
- 46. [Faint text]
- 47. [Faint text]
- 48. [Faint text]
- 49. [Faint text]
- 50. [Faint text]

TABLE 2: [Faint text]

- 37. [Faint text]
- 38. [Faint text]
- 39. [Faint text]
- 40. [Faint text]
- 41. [Faint text]
- 42. [Faint text]
- 43. [Faint text]
- 44. [Faint text]
- 45. [Faint text]
- 46. [Faint text]
- 47. [Faint text]
- 48. [Faint text]
- 49. [Faint text]
- 50. [Faint text]

$$p_{100} = 100.0$$

$$p_{100} = 100.0$$

$$p_{100} = 100.0$$

$$p_{100} = 100.0$$

$$p_{100} = 100.0$$

$$p_{100} = 100.0$$

TABLE 3: [Faint text]

SPECIMEN # 6 (continued)

RUN #5      RUN #6      RUN #7      RUN #8      RUN #9      RUN #10

$F_N = 400$        $F_N = 425$        $F_N = 440$        $F_N = 454$        $F_N = 460$        $F_N = 525$   
 $S_N = 83,000$        $S_N = 88,200$        $S_N = 91,300$        $S_N = 94,150$        $S_N = 95,500$        $S_N = 109,000$

$S_A$	$S_T$	$\psi_T$	$\psi_O$	$\psi_R$	$\psi_O$	$\psi_R$	$\psi_O$	$\psi_R$	$\psi_O$	$\psi_R$	$\psi_O$	$\psi_R$
150	1,752	.457	.784	.327	.656	.229	.637	.180	.784	.327	.784	.327
210	2,454	.640	1.127	.487	1.127	.487	1.078	.438	1.127	.487	1.024	.389
270	3,156	.823	1.421	.598	—	—	1.372	.549	1.421	.598	1.421	.588
330	3,858	1.006	1.764	.758	—	—	1.617	.611	—	—	1.764	.758
390	4,560	1.189	2.107	.918	—	—	—	—	—	—	2.009	.820
450	5,262	1.372	2.401	1.029	—	—	—	—	—	—	2.352	.980
510	5,964	1.555	2.744	1.189	—	—	—	—	—	—	—	—
570	6,666	1.738	3.087	1.349	—	—	—	—	—	—	—	—

630





- d = 0".0949
- D = 0".125
- f = 0".0410
- l = 0".3598
- F = 0.457

$$\psi_e = \frac{5.360 \times 10^{-7}}{k} \left[ \left(\frac{l}{d}\right)^3 - \left(\frac{l}{D}\right)^3 \right] c_A$$

$$= 0.980 \times 10^{-3} c_A$$

$$\psi_e' = 3.205 \times 10^{-3} \frac{D}{l} c_A$$

$$= 4.720 \times 10^{-3} c_A$$

$$\psi_T = \psi_e + \psi_e' = 5.700 \times 10^{-3} c_A$$

$$\psi_0 = F \times \psi_T = 0.457 \psi_T$$

$$S_T = 5.610 \times 10^{-3} \frac{c_A}{d^3}$$

$$S_T = 6.569 c_A$$

Surface Finish #3

RUN #1      RUN #2      RUN #3

F<sub>N</sub> = 400      F<sub>N</sub> = 450      F<sub>N</sub> = 500  
 S<sub>N</sub> = 56,600      S<sub>N</sub> = 63,700      S<sub>N</sub> = 70,800

Q <sub>A</sub>	S <sub>T</sub>	ψ <sub>T</sub>	ψ <sub>0</sub>	ψ <sub>R</sub>	ψ <sub>0</sub>	ψ <sub>R</sub>	ψ <sub>0</sub>	ψ <sub>R</sub>
150	985	.855	1.280	.425	1.280	.425	1.280	.425
180	1182	1.026	---	---	1.462	.436	1.508	.482
210	1320	1.197	1.782	.585	1.737	.540	1.737	.540
240	1577	1.368	---	---	1.919	.551	1.964	.596
270	1774	1.539	2.285	.746	2.239	.700	2.239	.700
300	1971	1.710	---	---	2.468	.758	---	---
330	2168	1.881	2.788	.907	2.696	.815	---	---
360	2365	2.052	---	---	2.970	.918	---	---
390	2562	2.223	---	---	3.245	1.022	---	---
420	---	---	---	---	---	---	---	---

Year	1946	1947	1948	1949	1950	1951	1952	1953	1954	1955	1956	1957	1958	1959	1960
1946	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000
1947	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000
1948	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000
1949	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000
1950	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000
1951	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000
1952	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000
1953	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000
1954	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000
1955	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000
1956	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000
1957	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000
1958	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000
1959	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000
1960	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000

$\frac{1}{2} = \frac{1}{2}$   
 $\frac{1}{3} = \frac{1}{3}$   
 $\frac{1}{4} = \frac{1}{4}$   
 $\frac{1}{5} = \frac{1}{5}$   
 $\frac{1}{6} = \frac{1}{6}$   
 $\frac{1}{7} = \frac{1}{7}$   
 $\frac{1}{8} = \frac{1}{8}$   
 $\frac{1}{9} = \frac{1}{9}$   
 $\frac{1}{10} = \frac{1}{10}$

Section 1000

$$\begin{aligned}
 & \frac{1}{2} = \frac{1}{2} \\
 & \frac{1}{3} = \frac{1}{3} \\
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 & \frac{1}{5} = \frac{1}{5} \\
 & \frac{1}{6} = \frac{1}{6} \\
 & \frac{1}{7} = \frac{1}{7} \\
 & \frac{1}{8} = \frac{1}{8} \\
 & \frac{1}{9} = \frac{1}{9} \\
 & \frac{1}{10} = \frac{1}{10}
 \end{aligned}$$

$$\frac{1}{2} = \frac{1}{2} \left[ \frac{1}{2} - \frac{1}{2} \right]$$

- 1 = 1
- 2 = 2
- 3 = 3
- 4 = 4
- 5 = 5
- 6 = 6
- 7 = 7
- 8 = 8
- 9 = 9
- 10 = 10

- d = 0".6958
- D = 0".325
- l = 0".03105
- l' = 0".3660
- P = 0.457

$$\psi_L = \frac{5.260 \times 10^{-7}}{k} \left[ \left(\frac{l}{d}\right)^3 - \left(\frac{l'}{D}\right)^3 \right] Q_A$$

$$= 0.715 \times 10^{-3} Q_A$$

$$\psi_L' = 3.205 \times 10^{-6} \frac{P'}{l^2} Q_A$$

$$= 4.800 \times 10^{-3} Q_A$$

$$\psi_T = \psi_L + \psi_L' = 5.515 \times 10^{-3} Q_A$$

$$\psi_0 = P \times \psi_S = 0.457 \psi_S$$

$$S_T = 5.610 \times 10^{-3} \frac{Q_A}{d^2}$$

$$S_T = 6.3098 Q_A$$

$$S_N = \frac{F_N}{\pi/4 d^2}$$

Surface finish: 1.0

RUN # 1                      RUN # 2                      RUN # 3

$F_N = 350$                        $F_N = 400$                        $F_N = 500$   
 $S_N = 48,400$                    $S_N = 55,400$                    $S_N = 69,000$

$Q_A$	$S_T$	$\psi_T$	$\psi_0$	$\psi_R$	$\psi_0$	$\psi_R$	$\psi_0$	$\psi_R$
150	958	.827	1.005	.178	1.005	.178	1.005	.178
210	1342	1.158	1.462	.304	1.508	.356	1.462	.304
270	1725	1.489	1.965	.476	1.965	.476	1.874	.385
330	2109	1.820	2.468	.648	2.422	.602	2.331	.511
390	2492	2.151	—	—	2.879	.722	2.788	.637
450	2875	2.482	—	—	3.336	.854	3.199	.717
510	3259	2.813	—	—	3.793	.980	3.656	.843
570	—	—	—	—	—	—	—	—

(continued on next page)



(logging from the basement)

100	100	100	100	100	100	100	100
200	200	200	200	200	200	200	200
300	300	300	300	300	300	300	300
400	400	400	400	400	400	400	400
500	500	500	500	500	500	500	500
600	600	600	600	600	600	600	600
700	700	700	700	700	700	700	700
800	800	800	800	800	800	800	800
900	900	900	900	900	900	900	900
1000	1000	1000	1000	1000	1000	1000	1000

11/15/2015

$$\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8}$$

$$\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{16}$$

$$\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{32}$$

$$\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{64}$$

$$\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{128}$$

$$\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{256}$$

$$\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{512}$$

$$\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{1024}$$

11/15/2015

- 1. 11/15/2015
- 2. 11/15/2015
- 3. 11/15/2015
- 4. 11/15/2015
- 5. 11/15/2015

SPKIPEN # 8 (continued)

	<u>RUN # 4</u>		<u>RUN # 5</u>		<u>RUN # 6</u>		<u>RUN # 7</u>		
	$F_H = 550$	$F_H = 600$	$F_H = 800$	$F_H = 900$	$F_H = 800$	$F_H = 900$	$F_H = 900$	$F_H = 900$	
	$S_H = 76,000$	$S_H = 83,100$	$S_H = 110,900$	$S_H = 124,800$	$S_H = 110,900$	$S_H = 124,800$	$S_H = 124,800$	$S_H = 124,800$	
$S_A$	$\psi_T$	$\psi_0$	$\psi_R$	$\psi_0$	$\psi_R$	$\psi_0$	$\psi_R$	$\psi_0$	$\psi_R$
150	958	.827	1.005	.178	1.005	.178	1.005	.178	1.005
210	1342	1.158	1.417	.259	1.417	.259	1.417	.259	1.417
270	1725	1.489	1.828	.339	1.874	.385	1.782	.293	1.737
330	2109	1.820	2.285	.465	2.285	.465	2.194	.374	—
390	2492	2.151	2.696	.595	2.696	.545	—	—	—
450	2875	2.482	3.153	.671	—	—	—	—	—
510	3259	2.813	3.610	.797	—	—	—	—	—
570	—	—	—	—	—	—	—	—	—



SPECIMEN # 9

- d = 0".0949
- D = 0".125
- Q = 0".0421
- Q' = 0".3629
- F = 0.457

$$\psi_1 = \frac{5.26 \times 10^{-7}}{k} \left[ \left(\frac{1}{d}\right)^3 - \left(\frac{1}{D}\right)^3 \right] Q_A$$

$$= 0.989 \times 10^{-3} Q_A$$

$$\psi_2 = 3.205 \times 10^{-6} \frac{Q}{D^2} Q_A$$

$$= 4.760 \times 10^{-3} Q_A$$

$$\psi_T = \psi_1 + \psi_2 = 5.749 \times 10^{-3} Q_A$$

$$\psi_0 = F \times \psi_s = 0.457 \psi_s$$

$$S_T = 5.610 \times 10^{-3} \frac{Q_A}{d^3}$$

$$S_T = 6.569 Q_A$$

Surface Finish: 4/0

	RUN # 1		RUN # 2		RUN # 3		
	F <sub>N</sub> = 350	F <sub>N</sub> = 450	F <sub>N</sub> = 450	F <sub>N</sub> = 500	S <sub>N</sub> = 49,500	S <sub>N</sub> = 63,700	S <sub>N</sub> = 70,800
Q <sub>A</sub>	ψ <sub>0</sub>	ψ <sub>R</sub>	ψ <sub>0</sub>	ψ <sub>R</sub>	ψ <sub>0</sub>	ψ <sub>R</sub>	ψ <sub>R</sub>
150	1.005	.143	.960	.098	.914	.052	
210	1.371	.164	1.418	.211			
270	1.919	.367	1.828	.276	1.828	.276	
330	2.468	.570	2.285	.387			
390	3.016	.770	2.742	.500	2.742	.500	
450	—		3.199	.612			
510	3.550	2.932	3.656	.724	3.656	.724	
570	3.745	3.277	4.113	.836			
630	4.134	3.622	4.570	.948	4.570	.948	
690	4.533	3.967	5.027	1.060			
750	4.927	4.312	5.484	1.172	5.438	1.126	
810	5.321	4.657	5.941	1.284			
870	5.715	5.002	6.398	1.396			
930							



Q.No.	Ans.	Q.No.	Ans.	Q.No.	Ans.	Q.No.	Ans.	Q.No.	Ans.	Q.No.	Ans.
1	10	11	11	21	11	31	11	41	11	51	11
2	10	12	11	22	11	32	11	42	11	52	11
3	10	13	11	23	11	33	11	43	11	53	11
4	10	14	11	24	11	34	11	44	11	54	11
5	10	15	11	25	11	35	11	45	11	55	11
6	10	16	11	26	11	36	11	46	11	56	11
7	10	17	11	27	11	37	11	47	11	57	11
8	10	18	11	28	11	38	11	48	11	58	11
9	10	19	11	29	11	39	11	49	11	59	11
10	10	20	11	30	11	40	11	50	11	60	11

- 1. 10
- 2. 10
- 3. 10
- 4. 10
- 5. 10

$$x^2 - 5x + 6 = 0$$

$$x^2 - 3x - 2x + 6 = 0$$

$$x(x - 3) - 2(x - 3) = 0$$

$$(x - 3)(x - 2) = 0$$

$$x - 3 = 0 \text{ or } x - 2 = 0$$

$$x = 3 \text{ or } x = 2$$

$$x = 2, 3$$

$$x = 2, 3$$

Ans: 2, 3

$d = 9.00 \text{ cm}$   
 $l = 0.000$   
 $l' = 0.3519$   
 $F = 0.457$   
 $\psi_l = 0.000$

$$\psi_l' = 3.205 \times 10^{-6} \frac{P}{l} Q_A$$

$$= 4.613 \times 10^{-3} Q_A$$

$$\psi_T = \psi_l + \psi_l' = 4.613 \times 10^{-3} Q_A$$

$$\psi_0 = F \times \psi_s = 0.457 \psi_s$$

$$S_T = 5.61 \times 10^{-3} \frac{Q_A}{d^3}$$

$$S_T = 2.8723 Q_A$$

$$S_H = \frac{F_H}{\pi/4 d^2}$$

Surface Finish: 1.0

	RUN # 1		RUN # 2		RUN # 3	
	$F_H = 300$	$S_N = 24,500$	$F_H = 400$	$S_N = 32,600$	$F_H = 500$	$S_N = 40,800$
	$\psi_0$	$\psi_R$	$\psi_0$	$\psi_R$	$\psi_0$	$\psi_R$
150	.914	.222	.914	.222	.868	.176
210	1.280	.311	1.234	.265	---	---
270	1.645	.399	1.554	.308	1.461	.216
330	2.011	.489	1.874	.352	---	---
390	---	---	2.194	.395	2.102	.403
450	---	---	2.559	.483	---	---
510	---	---	2.833	.480	2.788	.435
570	---	---	3.153	.524	---	---
630	---	---	3.519	.613	3.473	.567
690	---	---	3.893	.656	---	---
750	---	---	4.204	.744	4.204	.744
810	---	---	4.520	.833	---	---
870	---	---	---	---	---	---
930	---	---	---	---	---	---
1050	---	---	---	---	---	---
1170	---	---	---	---	---	---
1290	---	---	---	---	---	---
1470	---	---	---	---	---	---



SPLASH # 10 (Continued)

	<u>RUN # 4</u>	<u>RUN # 5</u>	<u>RUN # 6</u>			
	$F_N = 600$	$F_N = 700$	$F_N = 900$			
	$S_N = 45,900$	$S_N = 57,030$	$S_N = 73,400$			
$Q_A$	$\psi_T$	$\psi_0$	$\psi_R$			
	$\psi_T$	$\psi_0$	$\psi_R$			
150	431	.823	.131	.131	.777	.085
210	603	1.097	.128	1.417	1.051	.082
270	776	1.417	.171	2.102	1.371	.125
330	948	1.828	.306	2.742	1.782	.260
390	1120	2.102	.303	3.482	2.057	.258
450	1293	2.468	.392	---	---	---
510	1465	---	---	4.067	2.696	.343
570	1637	3.153	.524	---	---	---
630	1810	---	---	4.707	3.290	.384
690	1982	3.747	.564	---	---	---
750	2154	---	---	5.347	3.976	.516
810	2327	4.387	.650	---	---	---
870	2499	---	---	---	4.570	.557
930	2671	4.290	.828	---	---	---
990	2844	4.567	---	---	5.210	.643
1050	3016	4.844	.823	---	---	---
1170	3316	5.397	1.001	---	---	---
1290	3705	5.951	1.041	---	---	---
1470	---	---	---	---	---	---





SPECIMEN # 11

(Control Specimen For Runs on Annuli.)

$$D_1 = 0.171$$

$$D_0 = 0.251$$

$$Q = 0^{\circ}.5705$$

$$F = 0.457$$

$$\psi_0 = F \times \psi_s$$

$$\psi_0 = 0.457 \psi_s$$

$$S_T = 1.121 \times 10^{-2} \frac{I_0}{D_0^4 - D_1^4} Q_A$$

$$S_T = 0.903 Q_A$$

$$S_N = \frac{F_N}{\pi/4 (D_0^2 - D_1^2)}$$

RUN # 1    RUN # 2

$$F_N = 150 \quad F_N = 310$$

$$S_N = 10,563 \quad S_N = 21,830$$

$Q_A$	$S_T$	$\psi_0$	$\psi_0$
150	135	.137	.091
300	271	.229	.274
450	406	.366	.366
600	542	.503	.548
750	677	—	.686
900	813	—	.713
1050	948	—	.914
1200	—	—	—

From the plot of this data, Figure XI, it was found this specimen deflected

$$\frac{C \cdot Q_A}{1050} = 8.705 \times 10^{-4}$$

minutes per gram-inch of torque. Since the length between our indicator arms,  $Q$ , is  $0^{\circ}.5705$ , this gives an average deflection of

$$\frac{8.705 \times 10^{-4}}{0.5705} = 1.5258 \times 10^{-3}$$

minutes per gram-inch of torque per inch of length.

This number enables us to compare the deflections obtained in Runs on specimens, 12, 13, and 14, with the deflection for a solid specimen (i.e., no interface).

II. A. (continued)

(Please do not write beyond this line)

EXERCISES

$100 = 100 + 100$

$100 = 100 + 100$

$100 = 100$

$100 = 100$

$100 = 100$

$100 = 100$

$100 = 100$

$100 = 100$

$\frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \dots}}}$

$100 = 100$

$100 = 100$

$100 = 100$

100	100	100	100
100	100	100	100
100	100	100	100
100	100	100	100
100	100	100	100
100	100	100	100
100	100	100	100
100	100	100	100
100	100	100	100
100	100	100	100

... and ...

$100 = 100$

... and ...

$100 = 100$

... and ...

... and ...

... and ...

SPECIMEN # 12

$D_i = 0".171$   
 $D_o = 0".251$   
 $Q = 0".5702$   
 $F = 0.457$

$$\psi_c = 1.5258 \times 10^{-3} Q C_A$$

$$= 0.8700 \times 10^{-3} C_A$$

$$\psi_o = F \times \psi_c$$

$$= 0.457 \psi_c$$

$$S_T = 1.121 \times 10^{-2} \frac{F_o}{F_o^2 - D_i^2} C_A$$

$$S_N = \frac{F_N}{\pi/4 (D_o^2 - D_i^2)}$$

Surface finish:  
as machined 1380

	<u>RUN #1</u>	<u>RUN #2</u>	<u>RUN #3</u>	<u>RUN #4</u>
$F_N = 150$	$F_N = 202$	$F_N = 250$	$F_N = 310$	
$S_N = 10,563$	$S_N = 14,225$	$S_N = 17,605$	$S_N = 21,830$	
$C_A$	$\psi_o$	$\psi_R$	$\psi_o$	$\psi_R$
150	.229	.098	.137	.006
300	.457	.196	.320	.059
450	—	.594	.543	.156
600	.522	.823	.823	.301
750	.653	1.051	1.051	.398
900	.783	—	1.234	.451
1050	.914		1.571	.457
1200	1.044		—	1.737
1380				—





SPECIMEN # 13

$$D_1 = C'' .171$$

$$D_0 = C'' .251$$

$$l = C'' .5739$$

$$F = C .457$$

$$\psi_C = 1.5258 \times 10^{-3} Q_A$$

$$= C .8757 \times 10^{-3} Q_A$$

$$\psi_0 = F \times \psi_S$$

$$= C .457 \psi_S$$

$$S_T = 1.121 \times 10^{-2} \frac{B}{D_0 - D_1} \frac{D}{D_0 - D_1} Q_A$$

$$S_T = 0.903 Q_A$$

$$S_N = \frac{F_N}{\pi/4 (D_0^2 - D_1^2)}$$

Surface finish: 1.0

	<u>R U N # 1</u>		<u>R U N # 2</u>		<u>R U N # 3</u>	
	$F_N = 150$	$F_N = 202$	$F_N = 202$	$F_N = 350$	$F_N = 350$	$F_N = 350$
	$S_N = 10,653$	$S_N = 14,225$	$S_N = 14,225$	$S_N = 21,830$	$S_N = 21,830$	$S_N = 21,830$
$Q_A$	$\psi_0$	$\psi_R$	$\psi_0$	$\psi_R$	$\psi_0$	$\psi_R$
150	.183	.131	.183	.052	.183	.052
300	.457	.263	.320	.057	.411	.148
450	.594	.394	.594	.200	.548	.154
600	---	.525	.640	.115	.777	.252
750		.657	.868	.211	.914	.257
900		.788	---		1.143	.355
1050		.919			1.371	.352
1200					---	



$$D_1 = 0".171$$

$$D_0 = 0".251$$

$$l = 0".5749$$

$$F = 0.457$$

$$\psi_c = 1.5258 \times 10^{-3} \psi_A$$

$$= 0.8772 \times 10^{-3} \psi_A$$

$$\psi_0 = F \times \psi_s$$

$$= 0.457 \psi_s$$

$$S_T = 1.121 \times 10^{-2} \frac{D_0}{D_0 - D_1} \psi_A$$

$$S_T = 0.903 \psi_A$$

$$S_N = \frac{F_N}{\pi/4(D_0^2 - D_1^2)}$$

Surface finish: 4/0

	RUN # 1		RUN # 2		RUN # 3	
	$F_N = 150$	$S_N = 10,653$	$F_N = 202$	$S_N = 14,225$	$F_N = 350$	$S_N = 21,830$
$Q_A$	$\psi_0$	$\psi_R$	$\psi_0$	$\psi_R$	$\psi_0$	$\psi_R$
150	.132	.097	.274	.142	.229	.097
300	.263	.285	.457	.194	.411	.148
450	.395	.428	.731	.326	.731	.336
600	.526	—	1.005	.479	.914	.308
750	.658	—	1.280	.622	1.097	.439
900	.789	—	—	—	1.371	.582
1050	.921	—	—	—	1.600	.679
1200	—	—	—	—	—	—



Year	1790	1800	1810	1820	1830	1840	1850	1860	1870	1880	1890	1900
Population (Millions)	3.9	5.3	6.5	8.1	9.7	11.3	13.0	14.7	16.5	18.3	20.1	21.9
GDP (Billions)	15	25	35	50	70	100	140	190	250	320	400	480

Year	1790	1800	1810	1820	1830	1840	1850	1860	1870	1880	1890	1900
Population (Millions)	3.9	5.3	6.5	8.1	9.7	11.3	13.0	14.7	16.5	18.3	20.1	21.9
GDP (Billions)	15	25	35	50	70	100	140	190	250	320	400	480

Year	1790	1800	1810	1820	1830	1840	1850	1860	1870	1880	1890	1900
Population (Millions)	3.9	5.3	6.5	8.1	9.7	11.3	13.0	14.7	16.5	18.3	20.1	21.9
GDP (Billions)	15	25	35	50	70	100	140	190	250	320	400	480

Year	1790	1800	1810	1820	1830	1840	1850	1860	1870	1880	1890	1900
Population (Millions)	3.9	5.3	6.5	8.1	9.7	11.3	13.0	14.7	16.5	18.3	20.1	21.9
GDP (Billions)	15	25	35	50	70	100	140	190	250	320	400	480

APPENDIX C

Supplementary Discussion

11  
12  
13



## SUPPLEMENTARY DISCUSSION

In the Discussion of Results it was suggested that the residual deflection could be accounted for by assuming the asperities to act as cantilevers. The following calculations support this theory.

First, neglecting the cantilever effect and assuming the asperities behave as bulk metal leads to a calculated asperity height that is much too large. For example, consider a specimen of the type used in the first part of the work.

Equation (8) in Bowden and Tabor <sup>(4)</sup> gives

$$A_r = \frac{F_N}{p_m} \quad (1)$$

$A_r$  = real area of contact

$F_N$  = load normal to surface

$p_m$  = mean pressure over area of contact.

Moreover, the tips of the asperities will deform plastically when

$$p_m = cY$$

$c$  = a constant that depends upon size and shape of asperities. For the surface finishes of our specimens,  $c = 3$ .

$Y$  = elastic limit of deformed metal at tip of asperities. Here we will assume the elastic limit equals the proportional limit. Therefore,  $Y = 60,000$  psi for our specimens.

Substituting  $p_m = 3(60,000) = 180,000$  psi in equation (1)



EXPERIMENTAL RESULTS

In the discussion of results it was pointed out that the residual deflection could be accounted for by assuming the specimen to act as cantilevers. The following calculations support this theory.

First, regarding the cantilever effect and assuming the specimen behaves as half-wave loads to a calculated severity point that is well beyond the yield point, consider a specimen of the type used in the first part of the work.

Equation (8) in Gordon and Jones (4) gives

$$(1) \quad \frac{1}{r} = \frac{1}{r_0} + \frac{1}{r_1}$$

$r_1$  = total span of cantilever  
 $r_0$  = total span of specimen  
 $r_2$  = span between yield point of cantilever.

However, the sign of the exponent will depend practically

upon

$$r_2 > \text{or} < r_1$$

$r_2 > r_1$  = a condition that depends upon the end shape of specimen. For the section finished to the specimen,  $r_2 > r_1$ .  
 $r_2 < r_1$  = elastic limit of deflected metal at tip of specimen.  
Data will assume the elastic limit equals the proportional limit. Therefore,  $r_2 = 0.000001$  and for our specimen.

Substituting  $r_2 = 0.000001$  in equation (1)

$$A_r = \frac{F_N}{180,000} \quad (3)$$

From strength of materials, we have

$$A = \frac{F_N}{S_N} \quad (4)$$

A = apparent area of contact (i.e., area of bulk metal at surface)

$S_N$  = normal stress.

From equations (3) and (4)

$$\frac{A_r}{A} = \frac{S_N}{180,000} \quad (5)$$

Assume the total contact area,  $A_r$ , to be a circle of diameter  $d$ . This assumption results in the minimum calculated asperity height\*.

---

\* Consider, instead, that the contact area is an annulus with outside diameter,  $d_o$ , and inside diameter,  $d_i$ , and an area equal to the real area of contact

$$\therefore \frac{\pi}{4}(d_r)^2 = \frac{\pi}{4}(d_o^2 - d_i^2)$$

It is easily demonstrated from the formulae for twist in a shaft that if equal torques are to cause equal angles of twist,

$$\frac{h}{h_A} = \frac{(d_o^2 - d_i^2)}{(d_o^2 + d_i^2)} \quad \therefore h < h_A$$

where  $h$  is the height of a solid cylinder and  $h_A$  is the height of the corresponding annulus.

(1)

$$\frac{1}{100,000} = \frac{1}{100,000}$$

(2)

$$\frac{1}{100,000} = \frac{1}{100,000}$$

(3)

$$\frac{1}{100,000} = \frac{1}{100,000}$$

Consider, instead, that the contact area is an annulus with outside diameter  $d_o$  and inside diameter  $d_i$ , and so area  $A = \frac{\pi}{4}(d_o^2 - d_i^2)$ .

$$\frac{1}{100,000} = \frac{1}{100,000}$$

It is easily demonstrated from the formula for area in  $A$  that if equal forces are applied to equal areas of contact.

$$\frac{1}{100,000} = \frac{1}{100,000}$$

where  $H$  is the height of a solid cylinder and  $A$  is the height of the corresponding annulus.

Therefore,

$$\frac{A_r}{A} = \frac{d_r^2}{d^2} = \frac{S_N}{180,000} \quad (6)$$

$d$  = diameter of the specimen at the surface.

The area will increase from  $A_r$  to  $A$  in going from the tips of the asperities to the bulk metal at their roots. In the section on Procedure, an equation was derived for the twist in a cone. Applying this gives

$$\frac{\psi_R}{2} = \frac{5.36 \times 10^{-7}}{k'} \left[ \left( \frac{1}{d_r} \right)^3 - \left( \frac{1}{d} \right)^3 \right] Q_A \quad (7)$$

$\psi_R$  = the observed residual twist in minutes. It is divided by two since there are two surfaces in contact at an interface and we are considering the effect for one surface.

$$k' = \frac{d - d_r}{2h} \quad \text{where } h \text{ is asperity height.}$$

$Q_A$  = the applied torque in gram inches.

Substitute for  $k'$  in equation (7)

$$\psi_R = \frac{4h (5.36 \times 10^{-7})}{d - d_r} \left[ \left( \frac{1}{d_r} \right)^3 - \left( \frac{1}{d} \right)^3 \right] Q_A \quad (8)$$

Solving equation (8) for  $h$

$$h = \frac{\psi_R d^4}{21.44 \times 10^{-7}} \left[ \frac{1 - \frac{d_r}{d}}{\left( \frac{d}{d_r} \right)^3 - 1} \right] \frac{1}{Q_A} \quad (9)$$



$$(6) \quad \frac{1}{100,000} = \frac{1}{100,000} = \frac{1}{100,000}$$

The same will be true for the other side of the circuit. In fact, the equation for the other side is derived from the first in a similar manner. This gives

$$(7) \quad \frac{1}{100,000} = \frac{1}{100,000} = \frac{1}{100,000}$$

It is the above equation that is used in the derivation of the other side of the circuit. It is divided by the other side and the result is constant at all frequencies and we are considering the circuit for one side.

$$\frac{1}{100,000} = \frac{1}{100,000} = \frac{1}{100,000}$$

The applied voltage is given by

Substituting for  $V$  in equation (7)

$$(8) \quad \frac{1}{100,000} = \frac{1}{100,000} = \frac{1}{100,000}$$

Substituting equation (8) for  $V$

$$(9) \quad \frac{1}{100,000} = \frac{1}{100,000} = \frac{1}{100,000}$$

From equation (6) substitute  $\frac{d_r}{d} = \frac{S_N}{180,000}$

$$h = \frac{\psi_R d^4}{21.44 \times 10^{-7}} \left[ \frac{1 - \left(\frac{S_N}{180,000}\right)^{\frac{1}{2}}}{\left(\frac{180,000}{S_N}\right)^{3/2} - 1} \right] \frac{1}{Q_A} \quad (10)$$

Based on test data, the following asperity heights were computed using equation (10), to illustrate the orders of magnitude obtained when considering the asperities to behave as bulk metal.

Specimen 9, Run 2.

$$d = 0.0949", S_N = 63,700 \text{ psi}, Q_A = 630 \text{ gram inches}$$

$$\psi_R = 0.948 \text{ minutes, surface finish: } 4/0$$

$$h = 6.12 \times 10^{-3} \text{ inches.}$$

Specimen 1.

$$d = 0.125", S_N = 24,500 \text{ psi}, Q_A = 625.86, \psi_R = 1.229$$

minutes, surface finish: 4/0

$$h = 7.50 \times 10^{-3} \text{ inches}$$

Specimen 8, Run 1.

$$d = 0.0958, S_N = 69,000 \text{ psi}, Q_A = 330 \text{ gram inches}$$

$$\psi_R = 0.511 \text{ minutes, surface finish: } 1.0$$

$$h = 7.20 \times 10^{-3} \text{ inches.}$$

For purposes of comparison asperity heights were measured with a profilometer. For a 4/0 finish, before applying any normal

From equation (1) substitute

$$\frac{d^2}{100,000} = \frac{F}{b}$$

(11) 
$$b = \frac{1}{d^2} \left[ \frac{\left( \frac{100,000}{100,000} \right)^2 - 1}{1 - \frac{1}{100,000}} \right] \frac{\psi}{1.44 \times 10^{-2}}$$

Based on test data, the following specific weights were  
 computed using equation (10), to illustrate the order of  
 magnitude obtained when computing the specific weight to be used  
 as bulk weight.

Problem 1, Run 2.

$$d = 0.0040, \frac{d^2}{100,000} = 0.000016, \psi = 0.000016$$

$$\frac{\psi}{d^2} = 0.000016, \text{ specific weight: } \psi/d^2$$

$$b = 0.12 \times 10^{-2} \text{ inches.}$$

Problem 1, Run 1.

$$d = 0.100, \frac{d^2}{100,000} = 0.0001, \psi = 0.0001$$

$$\frac{\psi}{d^2} = 0.0001, \text{ specific weight: } \psi/d^2$$

$$b = 7.50 \times 10^{-2} \text{ inches}$$

Problem 2, Run 1.

$$d = 0.0025, \frac{d^2}{100,000} = 0.00000625, \psi = 0.00000625$$

$$\frac{\psi}{d^2} = 0.00000625, \text{ specific weight: } \psi/d^2$$

$$b = 7.50 \times 10^{-2} \text{ inches}$$

The purpose of computing specific weights were compared  
 also a procedure. For a  $\psi/d^2$  value, below specific weight

load.

$$h = 6.0 \times 10^{-5} \text{ inches}$$

When a normal stress of 21,830 psi was applied

$$h = 4.5 \times 10^{-5} \text{ inches.}$$

When the asperities are considered to act as cantilevers, the calculated height is found to be much closer to the measured. For the purposes of simplification consider an annular specimen such as was used in the second part of investigation.

Starting with equation (3) above,

$$A_r = \frac{F_N}{180,000}$$

The number of asperities in contact can be found by

$$N = \frac{A_r}{A_A} \quad (11)$$

$N$  = number of asperities in contact

$A_A$  = cross section area of average asperity.

Consider the average asperity to be at the mean diameter of the annulus. The torque action on this asperity is  $\frac{Q_A}{N}$  and the force deflecting the asperity is

$$\frac{\frac{Q_A}{N}}{\left(\frac{d_m}{2}\right)} \quad (12)$$

$d_m$  = mean diameter of the annulus.

The maximum deflection of a cantilever due to a load at its free end is



$$d = 2.0 \times 10^{-2} \text{ inches}$$

then a normal stress of 11,000 psi was applied

$$d = 4.8 \times 10^{-2} \text{ inches}$$

When the experiment was completed it was observed that the calculated deflection is found to be much closer to the measured deflection. For the purpose of simplification consider an average deflection such as was used in the second part of investigation. Identified with equation (8) above,

$$\frac{d^2}{100,000} = \dots$$

The number of repetitions in contact can be found by

$$(11) \quad \frac{d}{\lambda} = \dots$$

$\lambda$  = distance of repetition in contact

$d$  = three section area of average repetition.

Consider the average repetition to be at the mean diameter of the cylinder. The average section of this repetition is  $\frac{d}{2}$  and the force deflecting the repetition is

$$(12) \quad \frac{d}{\lambda} = \dots$$

$d$  = mean diameter of the cylinder.

The average deflection of a repetition was found to be  $\frac{d}{2}$  and the force deflecting the repetition is  $\frac{d}{2}$ .

$$\delta = \frac{P l^3}{3EI} \quad (13)$$

$\delta$  = maximum deflection of cantilever in inches.

$P$  = load at free end in pounds

$I$  = Moment of inertia of cross section about neutral axis.

$l$  = length of cantilever in inches

$E$  = modulus of elasticity in pounds per square inches.

Rearranging equation (13)

$$l^3 = \frac{3EI\delta}{P} \quad (14)$$

In this case the cantilever length,  $l$ , is equal to the asperity height,  $h$ , in inches. The deflection,  $\delta$ , is also equal to one-half the residual twist,  $\psi_R$ , (in radians) times the mean radius,  $\frac{d_m}{2}$ . One-half the residual twist was taken since there are two surfaces in contact that contributed to this displacement of

$$\delta = \frac{\psi_R \text{ (radians)}}{2} \times \frac{d_m}{2} \text{ (inches)} \quad (15)$$

Assuming that the asperities are of square cross section with an area of  $25 \times 10^{-8} \text{ cm}^2$  and substituting equations (3), (11), (12), and (15) into (14), gives

$$l^3 = h^3 = 2.67 \times 10^{-8} \frac{\psi_R F_N}{Q_A} (d_m)^2 \text{ (inches)}^3 \quad (16)$$

Where,  $d_m = 0.211$  inches for the specimens tested.

Taking data for Specimen # 14, Run 3, as an example,

$$\frac{c}{v} = \beta$$

$\beta$  = velocity of propagation of radiation in vacuum.

$p$  = path of ray and its length

$l$  = length of particle of cross section about which axis.

$l$  = length of particle in vacuum

$w$  = number of rotations in vacuum and space frame.

Substituting equation (18)

$$l^2 = \frac{2 \pi l \beta}{v} \quad (19)$$

In this case the radiation length  $l$ , is equal to the quantity  $h$ , in vacuum. The distance  $\beta$ , is also equal to one-half the radiation length  $l$ , in vacuum. Thus the wave length  $\frac{h}{v}$ . One-half the radiation length since there are two surfaces in contact that contribute to this displacement of

$$l = \frac{\psi \text{ (inches)}}{2} \times \frac{2}{2} \text{ (inches)} \quad (20)$$

Assuming that the separation between the two surfaces with an area of  $2 \pi \times 10^{-8}$  cm<sup>2</sup> and an initial separation of  $10^{-8}$  cm, (19), (20), and (21) gives

$$l = 0.07 \times 10^{-8} \times \frac{\psi \text{ (inches)}}{2} \times \frac{2}{2} \text{ (inches)} \quad (22)$$

where  $l = 0.07$  inches for the conditions stated. Taking care for equation (19), then  $l$ , as an example,



$$F_N = 310\# \quad \psi_R = 0.582 \text{ minutes}$$

$$Q_R = 900 \text{ gm-inches, surface finish} = 4/0$$

Substituting the above data into equation (16) gives the following estimate of asperity height.

$$h = 6.02 \times 10^{-4} \text{ inches.}$$

In contrast, the asperity height as measured by profilometer and as calculated by equation (10) are  $4.5 \times 10^{-5}$  and  $7.5 \times 10^{-3}$  inches respectively.

As an additional point of interest, the maximum bending stress can be calculated using the data for specimen 14, Run 3, above.

$$\sigma_B = \frac{M c}{I} = \frac{p h c}{I} \quad (17)$$

$\sigma_B$  = maximum bending stress in a cantilever due to load p.

c = maximum distance from neutral axis =  $2.5 \times 10^{-4}$  cm for the assumed asperity.

Other quantities, as defined in equation (13).

Substituting equations (8), (11), and (12) into equation (17) and solving,

$$\sigma_B = 1.97 \times 10^5 \text{ psi.}$$

This is further support to the theory that asperities can withstand much larger stress than the bulk metal and still retain elastic properties.



$$\psi = 0.005 \text{ cm}^{-1} \quad \psi = 0.005 \text{ cm}^{-1}$$

Following values of  $\psi$  are given in the following table (16) from the

$$a = 0.005 \times 10^{-2} \text{ cm}^{-1}$$

In column 1, the energy levels are calculated by equation (10) and  $4.8 \times 10^{-2}$  and  $7.8 \times 10^{-2}$  are the values of  $\psi$  calculated.

As an additional point of interest, the values of  $\psi$  are also calculated using the data for equation (10), and  $4.8 \times 10^{-2}$  and  $7.8 \times 10^{-2}$  are the values of  $\psi$  calculated.

$$(17) \quad \psi = \frac{1}{2} \left( \frac{1}{2} + \frac{1}{2} \right)$$

$\psi = 0.005 \text{ cm}^{-1}$  is the value of  $\psi$  calculated in a similar manner as in equation (10). The values of  $\psi$  are  $4.8 \times 10^{-2}$  and  $7.8 \times 10^{-2}$  are the values of  $\psi$  calculated.

Other quantities, as defined in equation (17), are calculated using equation (17), and the values of  $\psi$  are  $4.8 \times 10^{-2}$  and  $7.8 \times 10^{-2}$  are the values of  $\psi$  calculated.

$$\psi = 1.07 \times 10^{-2} \text{ cm}^{-1}$$

This is further support to the theory that the values of  $\psi$  are  $4.8 \times 10^{-2}$  and  $7.8 \times 10^{-2}$  are the values of  $\psi$  calculated.

APPENDIX D  
Original Data

11  
12  
13

The first part of the document discusses the importance of maintaining accurate records and the role of the auditor in this process.

It is noted that the auditor's primary responsibility is to provide an independent opinion on the financial statements.

The document also highlights the need for transparency and accountability in the auditing process.

Furthermore, it emphasizes the importance of communication between the auditor and the management of the entity.

In conclusion, the document stresses the significance of the auditor's role in ensuring the integrity of financial reporting.

The document concludes with a statement of the auditor's commitment to the highest standards of professional conduct.

Finally, it reiterates the auditor's duty to the public and the confidence placed in them by the stakeholders.

The document ends with a formal declaration of the auditor's independence and a statement of their professional opinion.

SPECIMEN # 1  
Surface Finish: 4/0  
RUN # 1  
SN = 24,500

SPECIMEN # 2  
Surface Finish: 2/0  
RUN # 1  
SN = 53,750

SPECIMEN # 3  
Surface Finish: 4/0  
RUN # 1  
SN = 44,600

SPECIMEN # 4  
Surface Finish: 4/0  
RUN # 1  
SN = 58,200

ψ S

ψ S

ψ S

ψ S

ψ S

QA	ψ S	ψ S	ψ S	ψ S
145.86	1.0	1.6	2.0	1.0
205.86	1.3	---	3.0	1.4
217.86	---	2.2	---	---
265.86	1.8	---	4.0	1.9
325.86	2.1	---	5.0	---
337.86	---	4.2	---	2.2
385.86	2.5	---	---	2.8
398.34	---	5.0	---	---
445.86	2.9	---	---	3.1
458.82	---	5.8	---	---
505.86	3.2	---	---	---
519.30	---	---	---	---
565.86	3.8	---	---	---
625.86	4.1	---	---	---
685.86	4.5	---	---	---
745.86	---	---	---	---





PRECIPITATION # 5

Surface Finish:  $\phi$

	<u>RUN # 1</u>	<u>RUN # 2</u>	<u>RUN # 3</u>	<u>RUN # 4</u>	<u>RUN # 5</u>	<u>RUN # 6</u>	<u>RUN # 7</u>
	$S_M = 62,800$	$S_M = 73,300$	$S_M = 78,100$	$S_M = 83,800$	$S_M = 93,400$	$S_M = 105,000$	$S_M = 115,200$
$\psi_A$	$\psi_B$	$\psi_C$	$\psi_D$	$\psi_E$	$\psi_F$	$\psi_G$	$\psi_H$
150	1.6	1.6	1.6	1.6	1.6	1.6	1.2
210	2.1	2.1	2.1	2.1	2.1	2.1	1.9
270	3.0	3.0	3.0	3.0	2.8	2.8	2.5
330	3.9	3.9	3.7	3.5	3.3	3.2	3.0
390	4.7	4.4	4.1	4.0	3.9	4.0	3.7
450	--	5.0	4.9	4.8	4.7	4.5	4.1
510		5.8	5.6	5.4	5.3	5.0	--
570		6.5	6.3	6.0	6.0	--	--
630		--	--	--	--	--	--



SPECIMEN # 6

Surface Finish: As Machined

	<u>RUN # 1</u> $S_H = 62,500$	<u>RUN # 2</u> $S_H = 67,450$	<u>RUN # 3</u> $S_H = 72,600$	<u>RUN # 4</u> $S_H = 77,800$	<u>RUN # 5</u> $S_H = 83,000$
$Q_A$	$\psi_s$	$\psi_s$	$\psi_s$	$\psi_s$	$\psi_s$
150	1.6	1.6	1.6	1.7	1.6
210	2.2	2.2	2.2	2.3	2.3
270	3.0	3.0	3.0	2.9	2.9
330	4.0	3.7	3.7	3.7	3.6
390	—	4.4	4.3	4.4	4.3
450	—	—	5.0	5.0	4.9
510	—	—	—	5.2	5.6
570	—	—	—	—	6.3
630	—	—	—	—	—





SPECIMEN # 6 (continued)

Surface Finish: As Machined

	RUN # 6 S <sub>H</sub> = 85,200	RUN # 7 S <sub>H</sub> = 91,300	RUN # 8 S <sub>H</sub> = 94,150	RUN # 9 S <sub>H</sub> = 95,500	RUN # 10 S <sub>H</sub> = 108,000
Q <sub>A</sub>	ψ <sub>S</sub>	ψ <sub>S</sub>	ψ <sub>S</sub>	ψ <sub>S</sub>	ψ <sub>S</sub>
150	1.4	1.3	1.6	1.6	1.5
210	2.3	2.2	2.3	2.1	2.8
270	—	2.8	2.9	2.9	4.0
330	—	3.3	—	3.6	4.7
390	—	—	—	4.1	—
450	—	—	—	4.8	—
510	—	—	—	—	—
570	—	—	—	—	—
630	—	—	—	—	—

(Worksheet) 3. A unit circle

Position: (x, y) = (cos(t), sin(t))

$$\frac{d(\cos t)}{dt} = -\sin t$$

$$\frac{d(\sin t)}{dt} = \cos t$$

$$\frac{d(\cos^2 t)}{dt} = -2\cos t \sin t$$

$$\frac{d(\sin^2 t)}{dt} = 2\sin t \cos t$$

$$\frac{d(\cos^2 t + \sin^2 t)}{dt} = 0$$

✓

✓

✓

✓

✓

✓

6.1

6.2

6.3

6.4

6.5

6.6

6.7

6.8

6.9

6.10

6.11

6.12

6.13

6.14

6.15

6.16

6.17

6.18

6.19

6.20

6.21

6.22

6.23

6.24

6.25

6.26

6.27

6.28

6.29

6.30

6.31

6.32

6.33

6.34

6.35

6.36

6.37

6.38

6.39

6.40

6.41

6.42

6.43

6.44

6.45

6.46

6.47

6.48

SPECIMEN # 7

Surface Finish: # 3

RUN # 1  
S<sub>N</sub> = 56,000

ψ S

RUN # 2  
S<sub>N</sub> = 63,700

ψ S

RUN # 3  
S<sub>N</sub> = 70,800

ψ S

RUN # 1  
S<sub>N</sub> = 48,400

ψ S

RUN # 2  
S<sub>N</sub> = 55,400

ψ S

SPECIMEN # 8

Surface Finish: 1.0

Q<sub>A</sub>

150

2.8

2.8

2.8

2.2

2.2

180

—

3.2

3.3

—

—

210

3.9

3.8

3.8

3.2

3.3

240

—

4.2

4.3

—

—

270

5.0

4.9

4.9

4.3

4.3

300

—

5.4

—

—

—

330

6.1

5.9

—

5.4

5.3

360

—

6.5

—

—

—

390

—

7.1

—

—

6.3

420

—

—

—

—

—

450

—

—

—

—

7.3

510

—

—

—

—

8.3

570

—

—

—

—

—





PICUPIN / S

Surface Finish: 1.0

	<u>RUN # 2</u> S <sub>N</sub> = 69,000	<u>RUN # 4</u> S <sub>N</sub> = 76,100	<u>RUN # 5</u> S <sub>N</sub> = 83,100	<u>RUN # 6</u> S <sub>N</sub> = 110,900	<u>RUN # 7</u> S <sub>N</sub> = 124,800
Q <sub>A</sub>	ψ <sub>S</sub>	ψ <sub>S</sub>	ψ <sub>S</sub>	ψ <sub>S</sub>	ψ <sub>S</sub>
150	2.2	2.2	2.2	2.2	2.2
180	—	—	—	—	—
210	3.2	3.1	3.1	3.1	3.0
240	—	—	—	—	—
270	4.1	4.0	4.1	3.9	3.8
300	—	—	—	—	—
330	5.1	5.1	5.0	4.8	—
360	—	—	—	—	—
390	6.1	5.9	5.9	—	—
420	—	—	—	—	—
450	7.0	6.9	—	—	—
510	8.0	7.9	—	—	—
570	—	—	—	—	—

RESULTS

TABLE I

	$\frac{P_{1,1}}{P_{1,1} + P_{1,2}}$ deg, out = $\theta^1$	$\frac{P_{2,1}}{P_{2,1} + P_{2,2}}$ deg, in = $\theta^2$	$\frac{P_{3,1}}{P_{3,1} + P_{3,2}}$ deg, in = $\theta^3$	$\frac{P_{4,1}}{P_{4,1} + P_{4,2}}$ deg, in = $\theta^4$	$\frac{P_{5,1}}{P_{5,1} + P_{5,2}}$ deg, in = $\theta^5$
1	0.1	0.1	0.1	0.1	0.1
2	0.2	0.2	0.2	0.2	0.2
3	0.3	0.3	0.3	0.3	0.3
4	0.4	0.4	0.4	0.4	0.4
5	0.5	0.5	0.5	0.5	0.5
6	0.6	0.6	0.6	0.6	0.6
7	0.7	0.7	0.7	0.7	0.7
8	0.8	0.8	0.8	0.8	0.8
9	0.9	0.9	0.9	0.9	0.9
10	1.0	1.0	1.0	1.0	1.0

PART 19

Surface Finish: 4/C

RUN # 1  
S<sub>N</sub> = 49,500

4/8

RUN # 2  
S<sub>N</sub> = 63,700

4/8

RUN # 3  
S<sub>N</sub> = 70,800

4/8

150  
210  
270  
330  
390  
450  
510  
570  
630  
690  
750  
810  
870  
930  
990  
1050  
1110  
1170  
1230  
1290  
1470

2.2  
3.0  
4.2  
5.4  
6.6  
—  
2.1  
3.1  
4.0  
5.0  
6.0  
7.0  
8.0  
9.0  
10.0  
11.0  
12.0  
13.0  
—  
2.0  
—  
4.0  
—  
6.0  
—  
8.0  
—  
10.0  
—  
11.9  
—



BY LINEAR METHOD

$$\frac{1.0 \times 1.0}{\sin^2 \theta = \lambda^2}$$

2

0.2 | 0.4 | 0.6 | 0.8 | 1.0

$$\frac{1.0 \times 1.0}{\cos^2 \theta = \lambda^2}$$

2

0.2 | 0.4 | 0.6 | 0.8 | 1.0

$$\frac{1.0 \times 1.0}{\sin^2 \theta = \lambda^2}$$

2

0.2 | 0.4 | 0.6 | 0.8 | 1.0

0.2 0.4 0.6 0.8 1.0

0.2 0.4 0.6 0.8 1.0

SPIN DOWN # 10

Surface Finish: 1.0

	<u>RUN # 1</u>	<u>RUN # 2</u>	<u>RUN # 3</u>	<u>RUN # 4</u>	<u>RUN # 5</u>	<u>RUN # 6</u>
	$S_H = 24,500$	$S_H = 32,600$	$S_H = 40,800$	$S_H = 48,900$	$S_H = 57,030$	$S_H = 73,400$
$\psi_s$	$\psi_s$	$\psi_s$	$\psi_s$	$\psi_s$	$\psi_s$	$\psi_s$
150	2.0	2.0	1.9	1.8	1.8	1.7
210	2.8	2.7	—	2.4	—	2.3
270	3.6	3.4	3.2	3.1	3.1	3.0
330	4.4	4.1	—	4.0	—	3.9
390	—	4.8	4.6	4.6	4.6	4.5
450	—	5.6	—	5.4	—	—
510	—	6.2	6.1	—	6.0	5.9
570	—	6.9	—	6.9	—	—
630	—	7.7	7.6	—	7.5	7.2
690	—	8.4	—	8.2	—	—
750	—	9.2	9.2	—	8.9	8.7
810	—	10.0	—	9.6	—	—
870	—	—	—	—	10.3	10.0
930	—	—	—	11.2	—	—
990	—	—	—	—	11.7	11.4
1050	—	—	—	12.4	—	—
1110	—	—	—	—	—	—
1170	—	—	—	14.0	—	—
1290	—	—	—	15.3	—	—
1470	—	—	—	—	—	—

MLA 8th Edition

0-1 Values Worksheet

0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.0000	0.0044	0.0088	0.0133	0.0177	0.0221	0.0265	0.0309	0.0353
0.0397	0.0441	0.0485	0.0529	0.0573	0.0617	0.0661	0.0705	0.0749
0.0793	0.0837	0.0881	0.0925	0.0969	0.1013	0.1057	0.1101	0.1145
0.1189	0.1233	0.1277	0.1321	0.1365	0.1409	0.1453	0.1497	0.1541
0.1585	0.1629	0.1673	0.1717	0.1761	0.1805	0.1849	0.1893	0.1937
0.1981	0.2025	0.2069	0.2113	0.2157	0.2201	0.2245	0.2289	0.2333
0.2377	0.2421	0.2465	0.2509	0.2553	0.2597	0.2641	0.2685	0.2729
0.2773	0.2817	0.2861	0.2905	0.2949	0.2993	0.3037	0.3081	0.3125
0.3169	0.3213	0.3257	0.3301	0.3345	0.3389	0.3433	0.3477	0.3521
0.3565	0.3609	0.3653	0.3697	0.3741	0.3785	0.3829	0.3873	0.3917
0.3961	0.4005	0.4049	0.4093	0.4137	0.4181	0.4225	0.4269	0.4313
0.4357	0.4401	0.4445	0.4489	0.4533	0.4577	0.4621	0.4665	0.4709
0.4753	0.4797	0.4841	0.4885	0.4929	0.4973	0.5017	0.5061	0.5105
0.5149	0.5193	0.5237	0.5281	0.5325	0.5369	0.5413	0.5457	0.5501
0.5545	0.5589	0.5633	0.5677	0.5721	0.5765	0.5809	0.5853	0.5897
0.5941	0.5985	0.6029	0.6073	0.6117	0.6161	0.6205	0.6249	0.6293
0.6337	0.6381	0.6425	0.6469	0.6513	0.6557	0.6601	0.6645	0.6689
0.6733	0.6777	0.6821	0.6865	0.6909	0.6953	0.6997	0.7041	0.7085
0.7129	0.7173	0.7217	0.7261	0.7305	0.7349	0.7393	0.7437	0.7481
0.7525	0.7569	0.7613	0.7657	0.7701	0.7745	0.7789	0.7833	0.7877
0.7921	0.7965	0.8009	0.8053	0.8097	0.8141	0.8185	0.8229	0.8273
0.8317	0.8361	0.8405	0.8449	0.8493	0.8537	0.8581	0.8625	0.8669
0.8713	0.8757	0.8801	0.8845	0.8889	0.8933	0.8977	0.9021	0.9065
0.9109	0.9153	0.9197	0.9241	0.9285	0.9329	0.9373	0.9417	0.9461
0.9505	0.9549	0.9593	0.9637	0.9681	0.9725	0.9769	0.9813	0.9857
0.9901	0.9945	0.9989	1.0000					

SPECIMEN # 11

SPECIMEN # 12

Control Specimen for Runs  
on Arnulf

Surface Finish: As Machined

	<u>RUN # 1</u> $S_H = 10,653$	<u>RUN # 2</u> $S_H = 21,830$	<u>RUN # 1</u> $S_H = 10,653$	<u>RUN # 2</u> $S_H = 14,225$	<u>RUN # 2</u> $S_H = 17,605$	<u>RUN # 4</u> $S_H = 21,830$
$\psi_s$	$\psi_s$	$\psi_s$	$\psi_s$	$\psi_s$	$\psi_s$	$\psi_s$
150	0.3	0.2	0.5	0.4	0.3	0.5
300	0.5	0.6	1.0	0.9	0.7	1.0
450	0.8	0.8	--	1.3	1.2	1.5
600	1.1	1.2		1.8	1.8	1.9
750	--	1.5		2.2	2.2	2.4
900		1.6		--	2.7	2.8
1050		2.0			3.0	3.3
1200		--			--	3.8
1380						4.0
1500						4.5
1620						--





SPECIMEN # 13

Surface Finish: 1.0

SPECIMEN # 14

Surface Finish: 4/0

$Q_A$	SPECIMEN # 13			SPECIMEN # 14		
	RUN #1 $S_H = 10,653$	RUN #2 $S_H = 14,225$	RUN #3 $S_H = 21,830$	RUN #1 $S_H = 10,653$	RUN #2 $S_H = 14,225$	RUN #3 $S_H = 21,830$
	$\psi_s$	$\psi_s$	$\psi_s$	$\psi_s$	$\psi_s$	$\psi_s$
150	0.4	0.4	0.4	0.5	0.6	0.5
300	1.0	0.7	0.9	1.2	1.0	0.9
450	1.3	1.3	1.2	1.8	1.6	1.6
600	--	1.4	1.7	--	2.2	2.0
750		1.9	2.0		2.8	2.4
900		--	2.5		--	3.0
1050			3.0			3.5
1200			--			--



DEFINITION OF SYMBOLS

- A Area (square inches).
- d Diameter of solid specimens at interface (inches).
- D Maximum diameter of solid specimens (inches).
- $D_1$  Inner diameter of annular specimens (inches).
- $D_o$  Outer diameter of annular specimens (inches).
- E Young's modulus of elasticity (pounds per square inch).
- F Conversion factor to convert angle of twist in scale units,  $\frac{1}{8}$  , to minutes,  $\psi_o$  .
- $F_N$  Force normal to interface (pounds).
- G Modulus of rigidity (pounds per square inch).
- k  $\frac{R - r_o}{l}$
- $l$  Length along solid specimens from  $r=r_o$  to  $r = R$  (inches).
- $l$  Length along annular specimens between indicator arms, (inches).
- $l'$  Length along solid specimens between base of bevel and indicator arm.
- $T_A$  Applied torque (gram inches).
- r Radius (inches).
- $r_o$  Radius of solid specimens at interface (inches).
- R Maximum radius of solid specimens (inches).
- $S_N$  Normal stress at interface due to  $F_N$ .  $S_N$  equals  $F_N/A$  divided by the area of the interface, (psi).
- $S_T$  Maximum tangential stress in interface due to applied torque, (psi).



DEFINITION OF TERMS

1	Area (square inches).
2	Diameter of solid specimen at interface (inches).
3	Original diameter of solid specimen (inches).
4	Initial diameter of smaller specimen (inches).
5	Outer diameter of smaller specimen (inches).
6	Young's modulus of elasticity (pounds per square inch).
7	Government factor to convert units of stress in units of stress $\psi$ to pounds $\psi$ .
8	Force normal to interface (pounds).
9	Radius of cylinder (pounds per square inch).
10	$\frac{L}{r}$
11	Length along which specimens are fixed to $r = R$ (inches).
12	Length along which specimens are fixed to interface.
13	(inches).
14	Length along which specimens are fixed to base of lower and indicator etc.
15	Applied torque (pound inches).
16	Radius (inches).
17	Radius of solid specimen at interface (inches).
18	Volume weight of solid specimen (inches).
19	Applied stress at interface due to $\psi$ . $\psi$ equals $\frac{1}{2} \psi$ .
20	Radius of the ends of the interface (inches).
21	Volume weight of specimen at interface due to applied torque, (psi).

- $T_A$  Torque applied to specimen (pound inches).
- $T_E$  Effective torque at interface (pound inches) =  $\frac{T_A}{2}$
- $\sigma$  Shear stress (psi).
- $\frac{d\theta}{dx}$  Angle of twist (radians) per unit length (inches).
- $\phi_l$  Total theoretical angle of twist in length  $l$ . (radians).
- $\phi_{l'}$  Total theoretical angle of twist in length  $l'$  (radians).
- $\psi_c$  Calculated corresponding angle of twist in minutes for annular specimens if there had been no interface between arms.
- $\psi_l$  Total theoretical angle of twist in length  $l$ . (minutes).
- $\psi_{l'}$  Total theoretical angle of twist in length  $l'$ . (minutes).
- $\psi_o$  Observed angle of twist (minutes).
- $\psi_R$  Residual angle of twist or residual twist (minutes).
- $\psi_s$  Observed angle of twist in scale units of the optic vernier.
- $\psi_T$  Total theoretical angle of twist in length  $l + l'$ . (minutes).

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